6.5

Work done by a variable force along a line:

Def$^{n}$: The work done by a variable force $F(x)$ in moving

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an object along the x-axis from $x=a$ to $x=b$ is

$$W = \int_a^b F(x)\,dx$$

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Ex) When a particle is located at a distance $x$ m from the origin, a force of $x^2+2x$ pounds acts on it. How much work is

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done in moving it from $x=1$ to $x=3$ m.

$$W = \int_1^3 F(x)\,dx$$

$$= \int_1^3 (x^2+2x)\,dx$$

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$$= \frac{x^3}{3} + \frac{2x^2}{2}\Big|_1^3$$

$$= \left(\frac{3^3}{3} + 3^2\right) - \left(\frac{1^3}{3} + 1^2\right)$$

$$= \frac{50}{3}\ \text{N m} = \boxed{\frac{50}{3}\ \text{J}}$$

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Hooke's law for springs:

Hooke's law states that the force required to maintain a spring stretched or compressed

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x units from its natural length is proportion to x.

$F = Kx$

Where K is a positive constant called spring constant.

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(Ex) Find the force required to stretch a 20 cm long spring, with a spring constant of 100 N/m, to a length of 21 cm.

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$K = 100\ N/m$

$x = 21 - 20 = 1\ cm$

$= 0.01\ m$

$F = Kx$

$F = (100)(0.01)$

$F = 1\ N$

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(Ex) A spring has a natural length of 2 ft. A body of 10 lb hanging on the spring stretches it to the total length of 2.4 ft.

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(a) Find the spring constant.

(b) How far beyond its natural length would a body of 50 lb stretch the spring?

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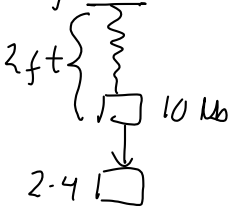
(c) How much work is required to stretch the spring from its natural length to a length of 3 ft?

(a)

$F = Kx$

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$\Rightarrow 10 = K(0.4)$

$\Rightarrow \boxed{K = 25 \text{ lb/ft}}$

(b) $F = kx$

$50 = 25(x)$

$\boxed{x = 2 \text{ ft}}$

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(c) $W = \int_0^1 F(x)\,dx$

$= \int_0^1 25x\,dx$

$= 25\frac{x^2}{2}\Big|_0^1$

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$= \frac{25}{2}$ lb ft

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