7.1

Logarithm defined as an integral:

Def^n: The natural logarithm is the function

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given by

$$\ln x = \int_1^x \frac{1}{t}\,dt,$$

$x > 0$

$y = f(x) = \ln x$

$D: (0, \infty)$

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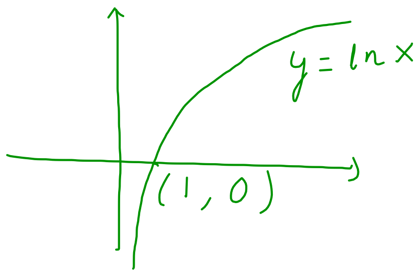

$$\ln 2 = \int_1^2 \frac{1}{t}\,dt$$

$$= \ln t \Big|_1^2$$

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$= \ln 2 - \ln 1$

$= \ln 2$

$$\ln e = \int_1^e \frac{1}{t}\,dt$$

$$= \ln t \Big|_1^e$$

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$= \ln e$

$= 1$

Derivative of $y = \ln x$

$y = \ln u$, where u is a differentiable function of x,

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$$\boxed{\frac{dy}{dx} = y' = \frac{1}{u} \cdot \frac{du}{dx}}$$

$y = \ln x$

$\Rightarrow y' = \frac{1}{x}$

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(ex)

$$y = \ln(x^2+1)$$

$$y' = \frac{1}{x^2+1}\cdot 2x$$

(ex) $y = \ln\sqrt{x^2+1}$

$$y' = \frac{1}{\sqrt{x^2+1}}\cdot\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot\cancel{2}x$$

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$$y' = \frac{x}{x^2+1}$$

or, $y = \ln(x^2+1)^{1/2}$

$$y = \frac{1}{2}\ln(x^2+1)$$

$$y' = \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+1}\cdot\cancel{2}x = \frac{x}{x^2+1}$$

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Properties:

① $\ln(bx)$

$= \ln b + \ln x$

② $\ln\left(\frac{b}{x}\right)$

$= \ln b - \ln x$

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③ $\ln\frac{1}{x} = -\ln x$

④ $\ln x^r = r\ln x$

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Integrals:

If u is a differentiable fcn that is never zero, then

$$\int\frac{1}{u}\,du = \ln|u| + C$$

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① $\int \tan x\,dx$

$$\boxed{= \ln|\sec x| + C}$$

$$\int\frac{\sin x}{\cos x}\,dx$$

$u = \cos x \Rightarrow du = -\sin x\,dx$

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$$-\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$\boxed{\int \tan x\,dx = \ln|\sec x| + C}$$

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(2) $\int \sec x\,dx$

$$\boxed{= \ln|\sec x + \tan x| + C}$$

(3) $\int \cot x\,dx$

$$= \ln|\sin x| + C$$

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(4) $\int \csc x\,dx$

$$= -\ln|\csc x + \cot x| + C$$

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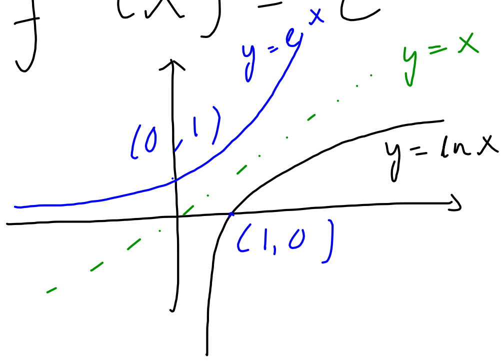
Inverse of $y = \ln x$

$$f^{-1}(x) = e^x$$



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$y = e^x$

D: $(-\infty, \infty)$

R: $(0, \infty)$

$y = e^x$

$\frac{dy}{dx} = e^x$

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$y = e^u$, u is a differentiable fcn of x, then

$y' = e^u \cdot \frac{du}{dx}$

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Inverse equations for e^x and $\ln x$:

$e^{\ln x} = x \quad , x > 0$

$\ln(e^x) = x$, for all x.

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$\int e^x \, dx$

$= e^x + C$

$\int e^{kx} \, dx$

$= \frac{e^{kx}}{k} + C$

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Ex $\int e^{-\frac{1}{2}x} \, dx$

$= -2e^{-\frac{x}{2}} + C$

Ex $\int e^{5x} \, dx$

$= \frac{e^{5x}}{5} + C$

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Laws of exponents:

1) $e^{x_1} \cdot e^{x_2} = e^{x_1 + x_2}$

2) $e^{-x} = \frac{1}{e^x}$

3) $\frac{e^{x_1}}{e^{x_2}} = e^{x_1 - x_2}$

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4) $(e^{x_1})^{x_2}$

$= e^{x_1 x_2}$

$y = f(x) = a^x$,

$a > 0$

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$y = 2^x$

$y = 2^{-x} = \left(\frac{1}{2}\right)^x$

Ex $y = 2^x$

$y' = \cancel{x 2^{x-1}} + C$

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① $y = a^x$

$y' = a^x \ln a$

② $\int a^x \, dx$

$= \frac{a^x}{\ln a} + c$

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Ex) $y = 3^x$

$y' = 3^x \ln 3$

Ex) $y = 2^{-x} = \left(\frac{1}{2}\right)^x$

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$y' = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^x (-\ln 2)$

$= 2^{-x}(-\ln 2)$

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Ex) $\int_0^1 2^{-\theta} \, d\theta$

$= -\frac{2^{-\theta}}{\ln 2} \Big|_0^1$

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$= -\frac{1}{\ln 2}\left[2^{-1} - 2^{-0}\right]$

$= -\frac{1}{\ln 2}\left[\frac{1}{2} - 1\right]$

$= \boxed{\frac{1}{2\ln 2}}$

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Change of base theorem:

$\log_a x = \frac{\ln x}{\ln a}$

ex) $\log_5 16 = \frac{\ln 16}{\ln 5}$

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Ex) $\int \frac{\log_{10} x}{x} \, dx$

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