$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

$\vec{u} \times \vec{v} = \vec{0}$

$\vec{u}$ and $\vec{v}$ are parallel.

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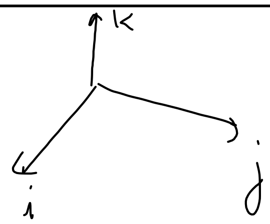

$i \times j = k$

$j \times k = i \; ; \quad i \times i = \vec{0}$

$k \times i = j \qquad j \times j = \vec{0}$

$k \times k = \vec{0}$

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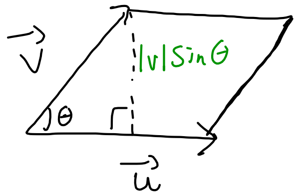
Area of a parallelogram



Area = base · height

$= |\vec{u}||\vec{v}| \sin\theta$

$= |\vec{u} \times \vec{v}|$

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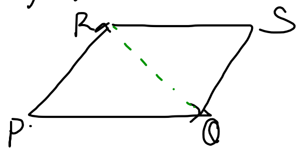
Ex) Find the area a $\triangle PQR$ with vertices $P(1,0,0)$, $Q(1,1,1)$ & $R(2,-1,3)$



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$\vec{PQ} = \langle 0, 1, 1 \rangle$

$\vec{PR} = \langle 1, -1, 3 \rangle$

Area of // PQRS

$= |\vec{PQ} \times \vec{PR}|$

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$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix}$

$= i \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} - j \begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$

$= 4i + j - k$

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Area of ‖ PQRS

$$= |\vec{PQ} \times \vec{PR}|$$

$$= \sqrt{16+1+1}$$

$$= \sqrt{18}$$

Area of $\Delta PQR = \boxed{\dfrac{1\sqrt{18}}{2}}$

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Triple scalar of
Box product:

Volume of the parallelopiped is given by the absolute value of

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$$|(\vec{u} \times \vec{v}) \cdot \vec{w}|$$

$$(\vec{u} \times \vec{v}) \cdot \vec{w}$$

$$= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

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Given

$$\vec{u} = i + 2j - k$$

$$\vec{v} = -2i + 3k$$

$$\vec{w} = 7j - 4k$$

Find the volume of the box.

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$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & 3 \\ 7 & -4 \end{vmatrix} - 2\begin{vmatrix} -2 & 3 \\ 0 & -4 \end{vmatrix}$$

$$+(-1)\begin{vmatrix} -2 & 0 \\ 0 & 7 \end{vmatrix}$$

$$= -21 - 16 + 14$$

$$= -23 \text{ units}$$

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$$V = |-23|$$

$$= 23 \text{ units.}$$

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{r}$ represents the length of lever arm

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and $\vec{F}$ represents the force.

$$\tau = |\vec{r}||\vec{F}| \sin\theta \, \hat{n}$$

units Nm

The magnitude of the torque generated

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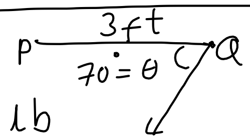
by the force F is given by

$$|\vec{r} \times \vec{F}|$$

$$= |r||F| \sin\theta$$

ex



F = 20 lb

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Find the magnitude of the torque generated by force F

$$= |\vec{PQ}||F| \sin\theta$$

$$= |3||20| \sin 70^\circ$$

$$\approx 56.4 \text{ ft lb}$$

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