

$$\int \frac{1}{(x+5)^2(x-1)} dx$$

$$= \frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{(x-1)}$$

$$\Rightarrow 1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

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$$(x+5)^2(x-1)$$

$$1 = A(x^2 + 4x - 5) + B(x-1) + C(x^2 + 10x + 25)$$

$$A + C = 0 \Rightarrow A = -C$$

$$4A + B + 10C = 0$$

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$$-5A - B + 25C = 1$$

$$-4C + B + 10C = 0$$

$$\Rightarrow B + 6C = 0$$

$$\Rightarrow B = -6C$$

$$\Rightarrow C = 1/36$$

$$A = -\frac{1}{36}, B = -\frac{1}{6}$$

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$$\int \frac{1}{(x+5)^2(x-1)} dx$$

$$= \int \frac{A}{x+5} dx + \int \frac{B}{(x+5)^2} dx + \int \frac{C}{x-1} dx$$

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$$= -\frac{1}{36} \int \frac{1}{x+5} dx$$

$$- \frac{1}{6} \int \frac{1}{(x+5)^2} dx \xrightarrow{u=x+5, du=dx} \int \frac{1}{u^2} du = -\frac{1}{u}$$

$$+ \frac{1}{36} \int \frac{1}{x-1} dx$$

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$$= -\frac{1}{36} \ln|x+5|$$

$$- \frac{1}{6} \left(-\frac{1}{x+5} \right)$$

$$+ \frac{1}{36} \ln|x-1| + C$$

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$$= -\frac{1}{36} \ln|x+5| + \frac{1}{6(x+5)} + \frac{1}{36} \ln|x-1| + C$$

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$$\int \frac{x^3 + x + 1}{x^2 - 1} dx$$

improper rational function

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$$\begin{array}{r} x \\ x^2 - 1 \overline{) x^3 + x + 1} \\ \underline{-x^3 - x} \\ 0 + 2x + 1 \end{array}$$

$$\frac{x^3 + x + 1}{x^2 - 1} = x + \frac{2x + 1}{x^2 - 1}$$

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$$= \int x + \frac{2x + 1}{(x + 1)(x - 1)} dx$$

$$\frac{2x + 1}{(x + 1)(x - 1)} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$\Rightarrow 2x + 1 = A(x - 1) + B(x + 1)$$

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$$\begin{array}{r} A + B = 2 \\ -A + B = 1 \end{array}$$

$$\hline 2B = 3$$

$$\Rightarrow B = \frac{3}{2}$$

$$\Rightarrow A + \frac{3}{2} = 2$$

$$\Rightarrow A = 2 - \frac{3}{2} = \frac{1}{2}$$

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$$\int \frac{2x + 1}{(x + 1)(x - 1)} dx$$

$$= \frac{1}{2} \int \frac{1}{x + 1} dx$$

$$+ \frac{3}{2} \int \frac{1}{x - 1} dx$$

$$= \frac{1}{2} \ln|x + 1| + \frac{3}{2} \ln|x - 1| + C$$

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$$\int x dx + \int \frac{2x+1}{(x+1)(x-1)} dx$$

$$= \frac{x^2}{2} + \frac{1}{2} \ln|x+1| + \frac{3}{2} \ln|x-1| + C$$

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