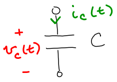
Quiz 3 Wed (3/4) Thevenin Eq ckt

Capacitors & Inductors

Capacitors



- are passive elements
- units => Farads [F]

$i_c(t) = C\frac{dv_c}{dt}$ $\quad v_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt$

if $v_c(t) \equiv$ constant

$\frac{dv_c}{dt} = 0 \Rightarrow i_c = 0$

at DC, capacitors behave like open ckts.

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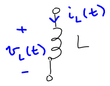
Inductors



- passive elements
- units Henrys [H]

$v_L(t) = L\frac{di_L}{dt}$ $\quad i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt$

if $i_L(t) \equiv$ constant.

$v_L(t) = 0 \Rightarrow$ short ckt

at DC, inductors behave like short ckt

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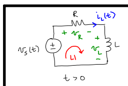

initial condition $i_L(0) = I_L$

by KVL at L1:

$v_s - v_R - v_L = 0$

$v_s - (Ri_L) - (L\frac{di_L}{dt}) = 0$

$L\frac{di_L}{dt} + Ri_L = v_s$

$\frac{di_L}{dt} + \frac{R}{L}i_L = \frac{v_s}{L}$

$i_L(t) = \underset{\text{forced response}}{\text{particular}} + \boxed{\underset{\text{natural response}}{\text{homogeneous}}}$

$\frac{di_L}{dt} + \frac{R}{L}i_L = 0$

$\frac{di_L}{dt} = -\frac{R}{L}i_L$

$\int_{i_L(0)}^{i_L(t)} \frac{1}{i_L}\,di_L = -\frac{R}{L}\int_0^t dt$

$\ln(i_L)\Big|_{i_L(0)}^{i_L(t)} = -\frac{R}{L}\left(t\Big|_0^t\right)$

$\ln(i_L(t)) - \ln(i_L(0)) = -\frac{R}{L}t$

$\left(\ln\left(\frac{i_L(t)}{i_L(0)}\right) = -\frac{R}{L}t\right)\exp$ $\quad e^{-\frac{R}{L}t}$

$\frac{i_L(t)}{i_L(0)} = \exp(-\frac{R}{L}t)$

$i_L(t) = i_L(0)\exp(-\frac{R}{L}t)$

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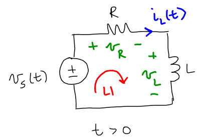

$i_L(t) = $ particular + homogeneous

$i_L(t) = \underset{i_{L_p}}{\text{particular}} + \underbrace{i_L(0)\exp(-\frac{R}{L}t)}_{i_{L_h}}$

$\frac{di_L}{dt} + \frac{R}{L}i_L = \frac{v_s}{L}$

$v_s(t) = V_m\cos(\omega t)$

$i_{L_p}(t) = A\cos\omega t + B\sin\omega t$

$-A\omega\sin\omega t + B\omega\cos(\omega t) + \frac{R}{L}(A\cos\omega t + B\sin\omega t) = \frac{V_m\cos(\omega t)}{L}$

$\sin\omega t\underbrace{\left[-A\omega + \frac{R}{L}B\right]}_{=0} + \cos\omega t\left[B\omega + \frac{R}{L}A\right] = \frac{V_m\cos(\omega t)}{L}$

$-A\omega + \frac{R}{L}B = 0$

$B\omega + \frac{R}{L}A = \frac{V_m}{L}$

$i_L(t) = \frac{V_0\omega L}{R^2 + \omega^2L^2}\sin(\omega t) + \frac{V_0R}{R^2+\omega^2L^2}\cos\omega t + i_L(0)\exp(-\frac{R}{L}t)$

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Complex numbers

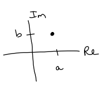
$i = \sqrt{-1}$

$j = \sqrt{-1}$

cartesian $C = a + jb$

a = real part

b = imaginary part



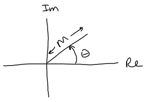
↓ angle

$\angle$

polar $C = m\angle\theta$

$m \equiv$ magnitude of C

$\theta \equiv$ angle of C



magnitude $\angle$angle $\quad$ magnitude . angle

cartesian

a+jb => polar

$m = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$X_1 = 3 + j4$

$m = \sqrt{3^2+4^2}$

$= \sqrt{25}$

$= 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$= 53.13°$

$X_2 = -3 + j7$

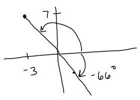
$m = \sqrt{-3^2 + 7^2}$

$= \sqrt{58}$

$= 7.61$

$\theta = \tan^{-1}\left(\frac{7}{-3}\right)$

$= -66.8°$



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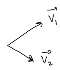
polar to cartesian



$10\angle45°$ => a+jb
cartesian

$a = 10\cos(45°) = 7.07$

$b = 10\sin(45°) = 7.07$

addition => $(a+jb) + (c+jd) = (a+c) + j(b+d)$

$m_1\angle\theta_1 + m_2\angle\theta_2$ => we don't add polar numbers

subtraction => $(a+jb) - (c+jd) = (a-c) + j(b-d)$

multiplication => $(m_1\angle\theta_1)(m_2\angle\theta_2) = (m_1m_2)\angle(\theta_1+\theta_2)$

$(a+jb)(c+jd) = ac + jad + jbc + jj\,bd$

$(ac - bd) + j(bc + ad)$

division => $\frac{m_1\angle\theta_1}{m_2\angle\theta_2} = \left(\frac{m_1}{m_2}\right)\angle(\theta_1-\theta_2)$

$c+jd \Rightarrow c-jd$ $\quad \frac{a+jb}{c+jd}\cdot\frac{c-jd}{c-jd} = \frac{(ac+bd)+j(bc-ad)}{c^2+d^2}$

$X \Rightarrow X^*$

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Euler's Identity: $\cos\theta + j\sin\theta = e^{j\theta}$

$\exp(j\theta)$

$v(t) = V_m \cos(\omega t + \theta)$ volts

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