Nolan Anderson CPE 381 Homework 4

1. (20 points) Consider the following filters with the given poles and zeros and DC constant.

$H_1(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -0.5 \pm j2\pi$; $zeros\ z_{1,2} = \pm j2\pi$;

$H_2(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -1 \pm j2\pi$; $zeros\ z_1 = 1,\ z_{2,3} = -1 \pm j2\pi$;

$H_1(s)$: $K = 1$; $poles\ p_1 = -1,\ p_{2,3} = -1 \pm j2\pi$; $zeros\ z_1 = 1$

Use MATLAB to plot the magnitude response of these filters and indicated the type of filters they represent.

$$H1: \frac{(s+2j\pi)(s-2j\pi)}{(s+1)(\cancel{s}+\cancel{0.5}+2j\pi)(s+0.5-2j\pi)} \rightarrow \frac{s^2+4\pi^2}{s^3+2s^2+(2.25+4\pi^2)s+(4\pi^2+0.25)}$$

$$= (s+1)[s^2+0.5s-\cancel{2j\pi s}+0.5s+0.25-\cancel{j\pi}\cancel{+2j\pi s}+\cancel{j\pi}+4\pi^2]$$

$$= (s+1)[s^2+s+(4\pi^2+0.25)]$$

$$= (s^3+s^2+(4\pi^2+0.25)s+s^2+s+(4\pi^2+0.25)$$

$$= s^3+2s^2+(2.25+4\pi^2)s+(4\pi^2+0.25) \rightarrow \frac{0 \quad 1 \quad 0 \quad 4\pi^2}{1 \quad 2 \quad (2.25+4\pi^2) \quad (4\pi^2+0.25)}$$

$$H2: 1 \cdot \left[\frac{(s-1)\cancel{(s+1+2j\pi)}\cancel{(s+1-2j\pi)}}{(s+1)\cancel{(s+1+2j\pi)}\cancel{(s+1-2j\pi)}}\right]$$

$$\boxed{H_2 = \frac{s-1}{s+1}}$$

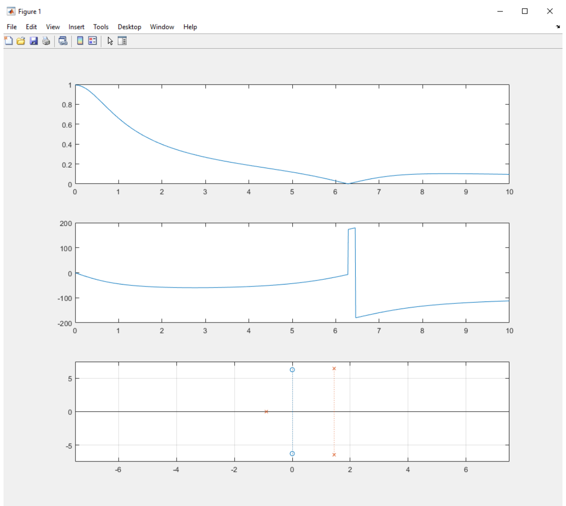
$$H_3(s) = \frac{(s-1)}{(s+1)(s+1+2j\pi)(s+1-2j\pi)}$$

$$(s+1)[s^2+\cancel{s}-\cancel{2j\pi s}+\cancel{s}+1-\cancel{2j\pi}+\cancel{2j\pi s}+\cancel{2j\pi}+4\pi^2]$$

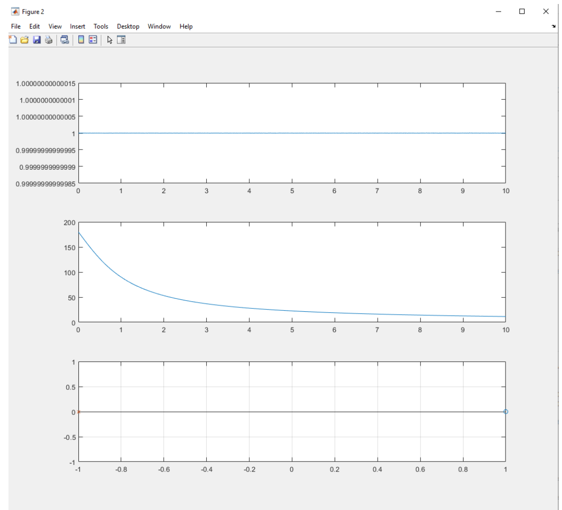
$$(s+1)[s^2+2s+(4\pi^2+1)]$$

$$= s^3+3s^2+(3+4\pi^2)s+(4\pi^2+1)$$

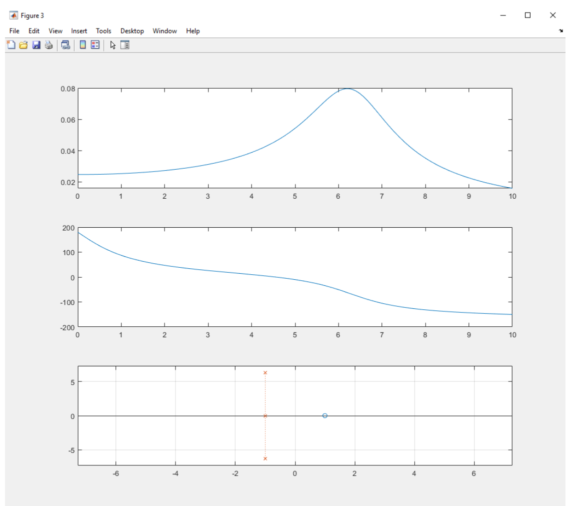
$$= \frac{s-1}{s^3+3s^2+(3+4\pi^2)s+(4\pi^2+1)} \rightarrow \frac{[0 \quad 0 \quad 1 \quad -1]}{[1 \quad 3 \quad (3+4\pi^2) \quad (4\pi^2+1)]}$$



H1 -> low pass notched



H2 -> High pass



H3 -> low pass

```matlab
%% Header
% Nolan Anderson
% CPE 381 - 01 Homework 4 # 1
clear all; clf
m = 10;
%% H1
n1 = [0 1 0 4*pi^2];
d1 = [1 -2 2.25+4*pi^2 4*pi^2+0.25];
figure(1)
freq(n1,d1,m)

%% H2
% n2 = [1 -3 3+4*pi^2 -(4*pi^2-1)];
% d2 = [1 3 3+4*pi^2 4*pi^2-1];
n2 = [0 0 1 -1];
d2 = [0 0 1 1];
figure(2)
freq(n2,d2,m)

%% H3
n3 = [0 0 1 -1];
d3 = [1 3 4*pi^2+3 4*pi^2+1];
figure(3)
freq(n3,d3,m)

%% Frequency Function
function[w,Hm,Ha]=freq(b,a,max)
    w = 0:0.01:max;
    H = freqs(b,a,w);
    Hm = abs(H);
    Ha = angle(H)*180/pi;
    subplot(311)
    plot(w,Hm)
    subplot(312)
    plot(w,Ha)
    subplot(313)
    splane(b,a)
end

%% Splane function
function splane(num,den)
    z=roots(num);
    p=roots(den);
    A1=[min(imag(z)) min(imag(p))];A1=min(A1)-1;
    B1=[max(imag(z)) max(imag(p))];B1=max(B1)+1;
    N=20;
    D=(abs(A1)+abs(B1))/N;
    im=A1:D:B1;
    Nq=length(im);
    re=zeros(1,Nq);
    A=[min(real(z)) min(real(p))];A=min(A)-1;
    B=[max(real(z)) max(real(p))];B=max(B)+1;
    stem(real(z),imag(z),'o:')
    hold on
    stem(real(p),imag(p),'x:')
    hold on
    grid
    axis([min(im) max(im) min(im) max(im)]);
    hold off
end
```

2. (20 points) An ideal low pass filter *H(s)* with zero phase and magnitude response:

$$|H(j\Omega)| = \begin{cases} 1 & -\pi \le \Omega \le \pi \\ 0 & otherwise \end{cases}$$

a) Find the impulse response *h(t)* of the low-pass filter. Plot it and indicate whether this filter is causal system or not.

b) What is the effect of shifting the central frequency of the ideal filter for 5π ?

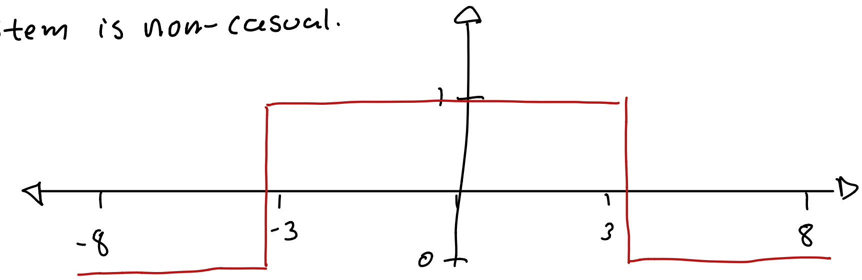
$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{\pi j t} - e^{-\pi j t}}{jt}\right]$$

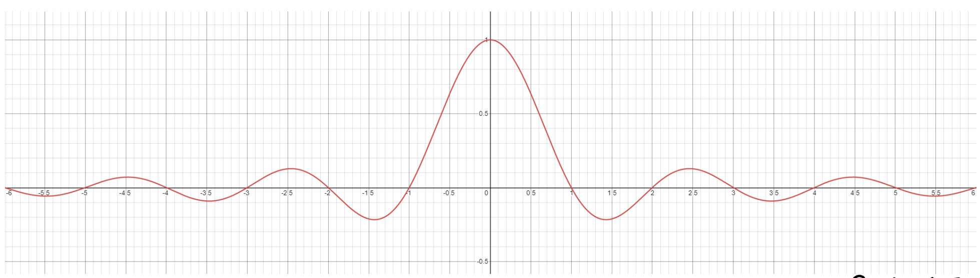
$$= \frac{e^{\pi j t} - e^{-\pi j T}}{2\pi j t} = \frac{\sin(\pi t)}{\pi t}$$

a) $h(t) = \dfrac{\sin(\pi t)}{\pi t}$

Since $h(t) \neq 0$ for all $t < 0$, we can assume that the system is non-casual.



$h(j\Omega)$

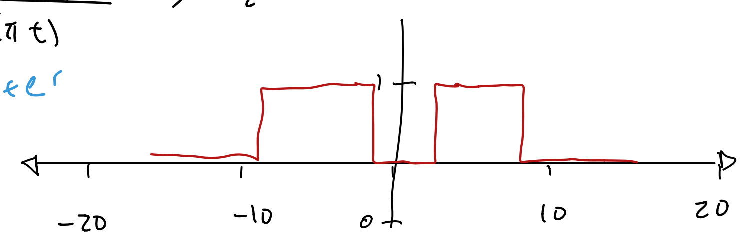


$h(t)$

b) $h_2(t)$ shifted by 5π $\quad H(5j\Omega) = \begin{cases} 1 & \Omega - 5\pi <= \Omega \le \Omega + 5\pi \\ 0 & otherwise \end{cases}$

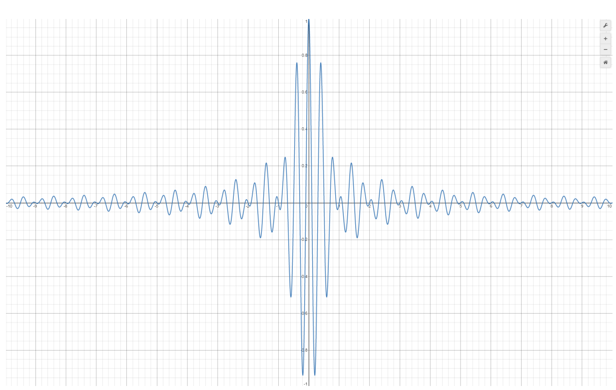
$$\frac{\sin(\pi t + 5\pi)}{(\pi t)} \rightarrow 2 \times h(t) \times \cos(5\pi t) \quad h_2(t) = \frac{2\sin(\pi t)}{\pi t} \times \cos(5\pi t)$$

Band pass-filter



$\rightarrow h_2(j\Omega)$

)



$\rightarrow h_2(t)$

3. (20 points)

A 12-bit AD converter is used to digitize signal with negative reference V_{R-} = 0.5V and positive reference V_{R+} = 2.5V.

a) (3 points) What is the quantization step?
b) (3 points) What is the output of the AD converter for V_{in} = 2.3 V ?
c) (2 points) What is the output of the AD converter for V_{in} = 0.4 V ?
d) (2 points) What is the output of the AD converter for V_{in} = 2.9 V ?

a) $\Delta = (V_{r+} - V_{r-})/(2^{12}-1) \rightarrow \left(\frac{\cdot \quad \cdot}{2^{12} \quad -1}\right) = 0.49mV$

b) $V_{in} = 2.3v$

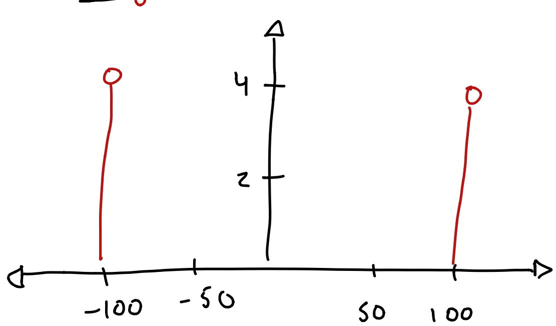
$(V_{in} - V_{r-})/\Delta \rightarrow \frac{(2.3-0.5)}{0.49mV} = 3673.5$

c) 0 because the output cannot be negative. (-204 otherwise)

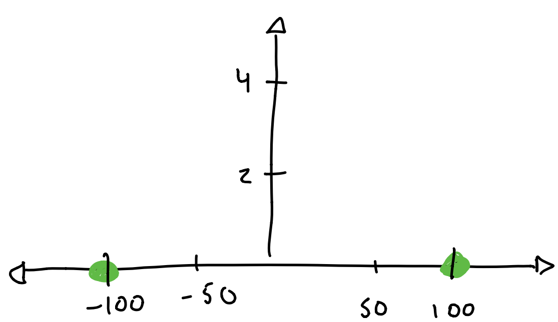
d) $\frac{2.9-0.5}{0.49mV} = \boxed{4095}$. Max value is $2^{12}-1$, 4897 is larger than that. 2.5-0.5

4. (40 points) Represent spectrum of the signal $x(t) = 8cos(100t)$.

Magnitude

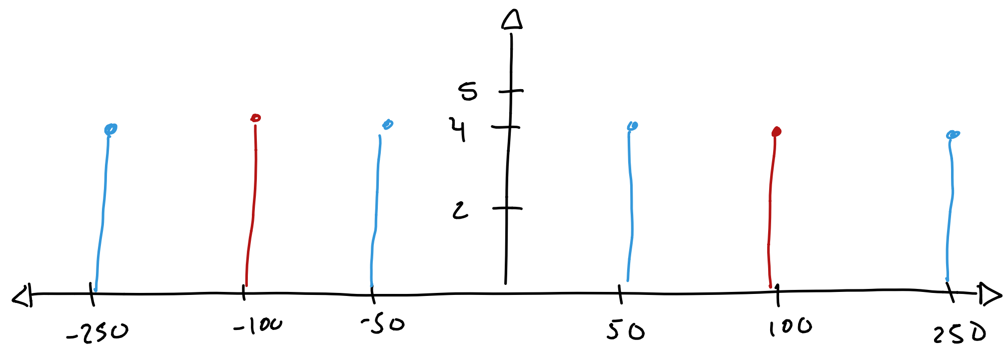


Phase

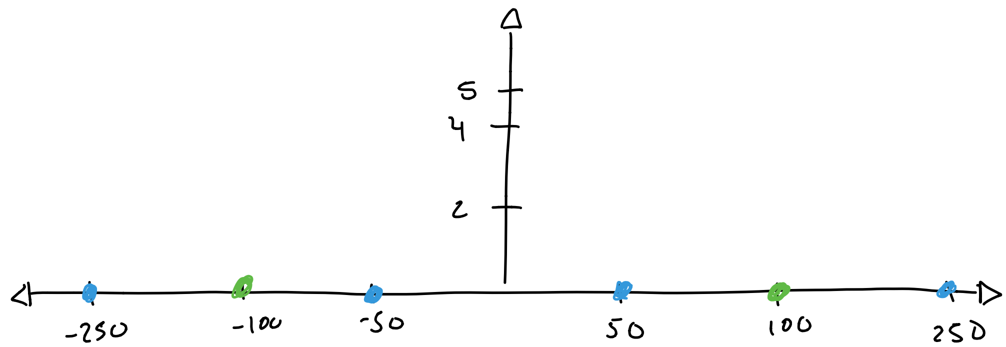


Represent magnitude and phase spectrum of the same signal sampled at Fs = 150 rad/s. Describe the effect.

Magnitude



Phase



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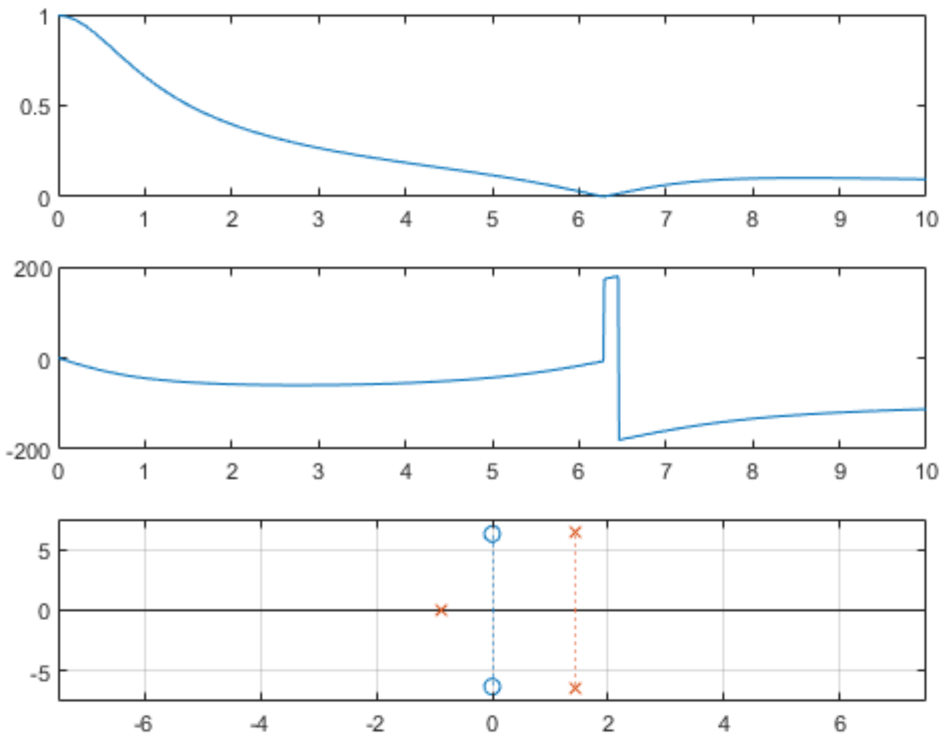
Header

Nolan Anderson CPE 381 - 01 Homework 4 # 1

```
clear all; clf
m = 10;
```

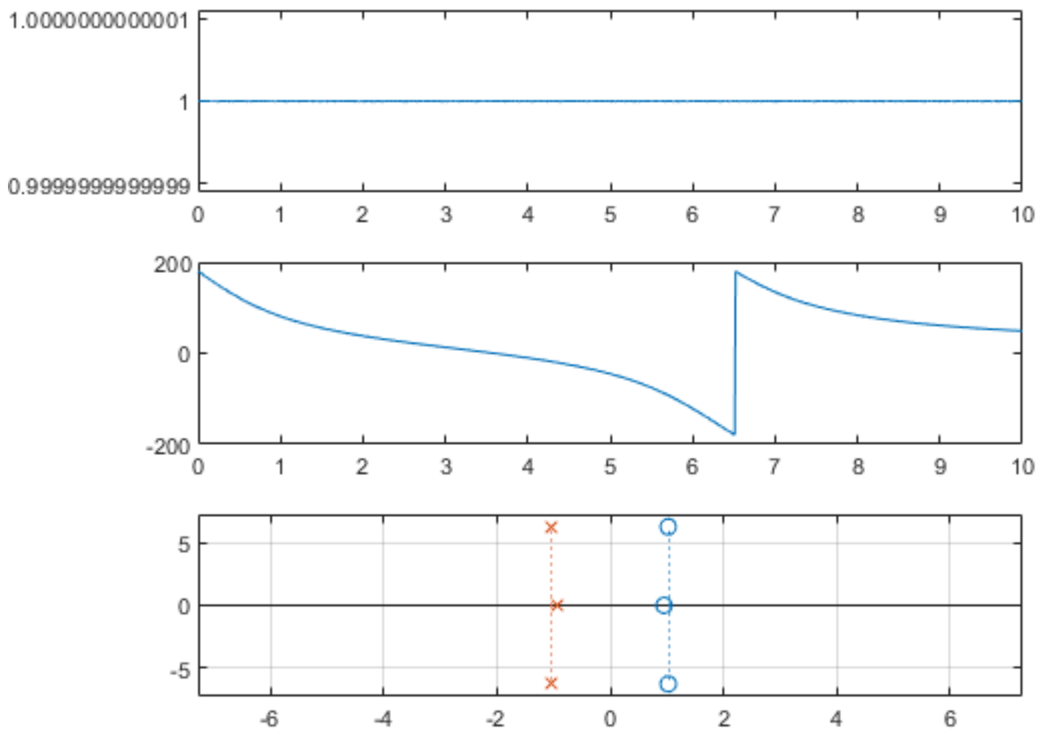
H1

```
n1 = [0 1 0 4*pi^2];
d1 = [1 -2 2.25+4*pi^2 4*pi^2+0.25];
figure(1)
freq(n1,d1,m);
```



H2

```
n2 = [1 -3 3+4*pi^2 -(4*pi^2-1)];
d2 = [1 3 3+4*pi^2 4*pi^2-1];
figure(2)
freq(n2,d2,m);
```

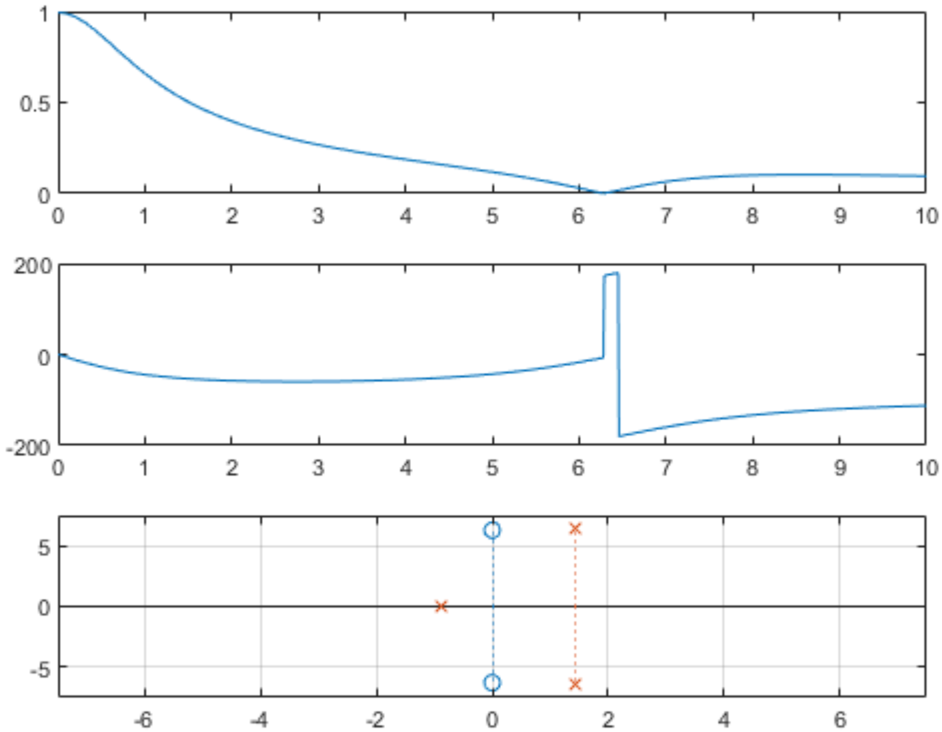


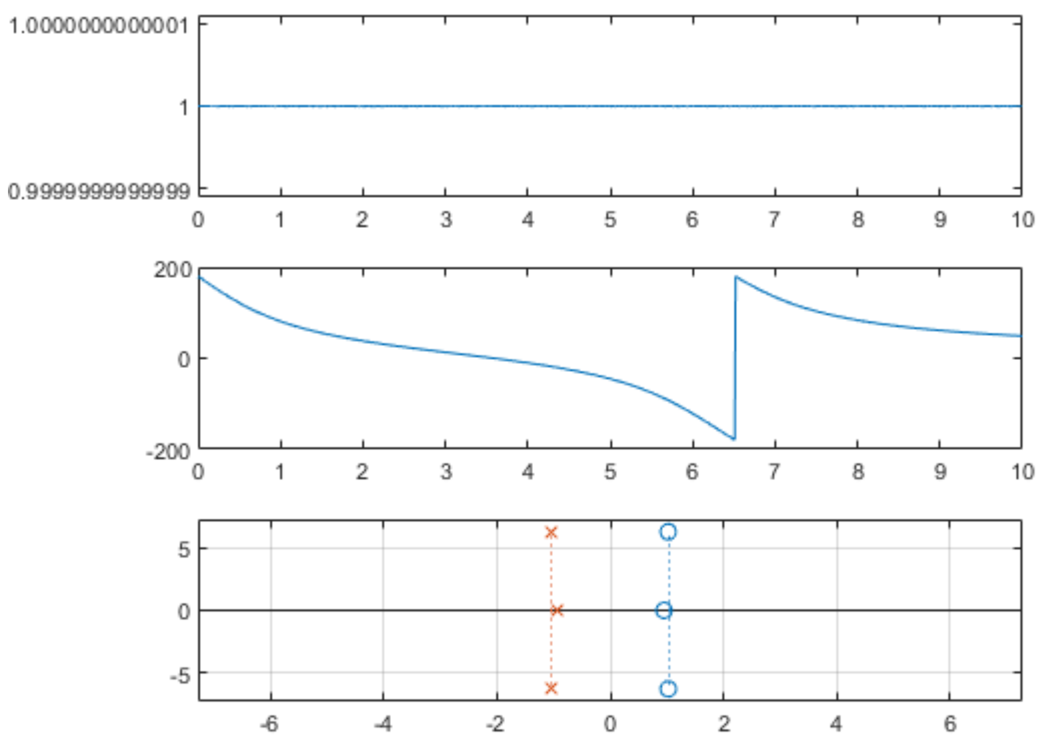
H3

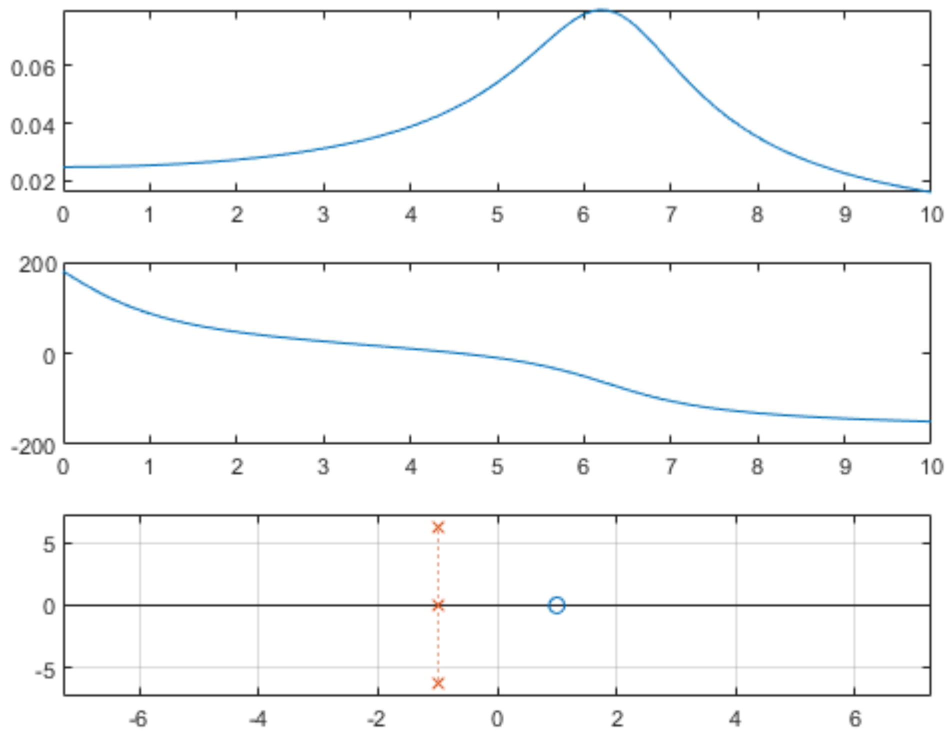
```
n3 = [0 0 1 -1];
d3 = [1 3 4*pi^2+3 4*pi^2+1];
figure(3)
freq(n3,d3,m);
```

Frequency Function

```
function[w,Hm,Ha]=freq(b,a,max)
    w = 0:0.01:max;
    H = freqs(b,a,w);
    Hm = abs(H);
    Ha = angle(H)*180/pi;
    subplot(311);
    plot(w,Hm);
    subplot(312);
    plot(w,Ha);
    subplot(313);
    splane(b,a);
end
```


1
0.5
0
0 1 2 3 4 5 6 7 8 9 10
200
0
-200
0 1 2 3 4 5 6 7 8 9 10
5
0
-5
-6 -4 -2 0 2 4 6


1.0000000000001
1
0.9999999999999
0 1 2 3 4 5 6 7 8 9 10
200
0
-200
0 1 2 3 4 5 6 7 8 9 10
5
0
-5
-6 -4 -2 0 2 4 6



Splane function

```
function splane(num,den)
    z=roots(num);
    p=roots(den);
    A1=[min(imag(z)) min(imag(p))];A1=min(A1)-1;
    B1=[max(imag(z)) max(imag(p))];B1=max(B1)+1;
    N=20;
    D=(abs(A1)+abs(B1))/N;
    im=A1:D:B1;
    Nq=length(im);
    re=zeros(1,Nq);
    A=[min(real(z)) min(real(p))];A=min(A)-1;
    B=[max(real(z)) max(real(p))];B=max(B)+1;
    stem(real(z),imag(z),'o:');
    hold on
    stem(real(p),imag(p),'x:');
    hold on
    grid
    axis([min(im) max(im) min(im) max(im)]);
    hold off
end
```

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