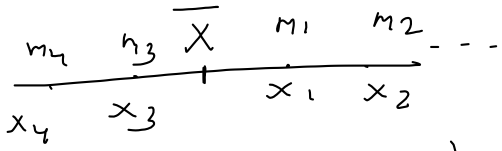
6.6 Moments and Center of mass

1-Dimensional



$\bar{X}$ (Center of mass)

$$= \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}$$

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$$\bar{X} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

$$\boxed{\bar{X} = \frac{M}{m} \quad \frac{\text{(Moments)}}{\text{Total mass}}}$$

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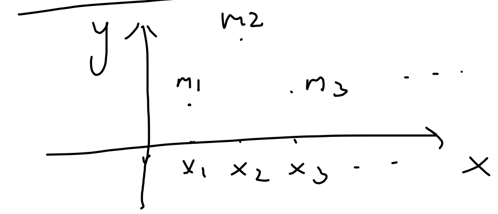
In 2-dimensional:



$$\bar{X} = \frac{M_y}{M}$$

$$\bar{y} = \frac{M_x}{M}$$

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Where $M_y = \sum_{i=1}^{n} x_i m_i$

moments across the y-axis

and $M_x = \sum_{i=1}^{n} y_i m_i$

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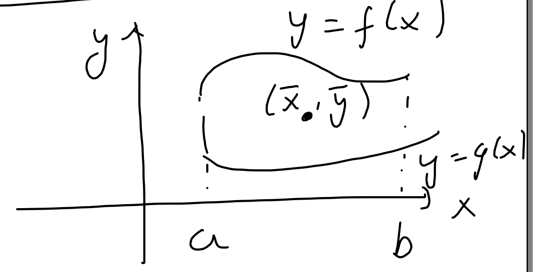
Plates bounded by two curves:



Let's suppose that the plate is the region

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bounded by two curves $y=f(x)$ and $y=g(x)$ where $f(x) \geq g(x)$ and $a \leq x \leq b$.

The center of mass of the region is

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given by below:

Mass of the plane,

$$M = \text{density} \cdot \text{area}$$

$$= \delta \int_a^b [f(x) - g(x)]\, dx$$

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$$M_x = \delta \int_a^b \frac{1}{2}\left[f^2(x) - g^2(x)\right] dx$$

$$M_y = \delta \int_a^b x\,[f(x) - g(x)]\, dx$$

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$$\bar{x} = \frac{M_y}{M}$$

$$\bar{y} = \frac{M_x}{M}$$

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Ex) Find the center of mass for the region bounded by $y = 25 - x^2$ and x-axis. (Assuming density δ is constant).

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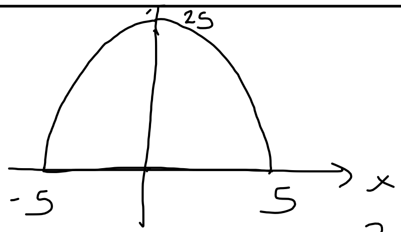

$$f(x) = 25 - x^2$$

$$g(x) = 0$$

$$M = \delta \int_{-5}^{5} [f(x) - g(x)]\, dx$$

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$$M = \delta \int_{-5}^{5} (25 - x^2)\, dx$$

$$= \delta \left[25x - \frac{x^3}{3}\right]_{-5}^{5}$$

$$= \delta \left[25(5) - \frac{5^3}{3} - 25(-5) + \frac{(-5)^3}{3}\right]$$

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$$= \delta \, \frac{4}{3}(5)^3$$

$$M_x = \delta \int_{-5}^{5} \frac{1}{2}\left[f^2(x) - \cancel{g^2(x)}^{\,0}\right]dx$$

$$= \frac{\delta}{2}\int_{-5}^{5} (25-x^2)^2 \, dx$$

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$$= \frac{\delta}{2}\int_{-5}^{5} (625 - 50x^2 + x^4)\,dx$$

$$= \frac{\delta}{2}\left[625x - \frac{50x^3}{3} + \frac{x^5}{5}\right]_{-5}^{5}$$

$$= \delta \, 625\left(\frac{8}{3}\right)$$

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$$M_y = \delta \int_{-5}^{5} x(25-x^2)\,dx$$

$$= \delta \int_{-5}^{5} (25x - x^3)\,dx$$

$$= \delta \left[\frac{25x^2}{2} - \frac{x^4}{4}\right]_{-5}^{5}$$

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$$= \delta \cdot 0$$

$$= 0$$

$$\bar{x} = \frac{\cancel{\delta} M_y}{\cancel{\delta} M} = 0$$

$$\bar{y} = \frac{\cancel{\delta} M_x}{\cancel{\delta} M} = 10$$

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$$(\bar{x}, \bar{y}) = (0, 10)$$

Ex $y = x^3$

and $y = \sqrt{x}$

δ is constant.

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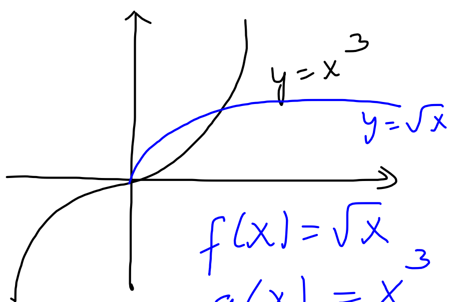

$f(x) = \sqrt{x}$

$g(x) = x^3$

$x^3 = \sqrt{x}$

$x^6 = x$

$x^6 - x = 0 \Rightarrow x(x^5 - 1) = 0$

$\Rightarrow x = 0$ or $x^5 = 1$

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$x = 0$, $x = 1$

$$M = \delta \int_0^1 [f(x) - g(x)]dx$$

$$= \delta \int_0^1 (\sqrt{x} - x^3) dx$$

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$$= \delta \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{x^4}{4} \right]_0^1$$

$$= \delta \left[\frac{2}{3} - \frac{1}{4} \right]$$

$$M = \boxed{\frac{5\delta}{12}}$$

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$$M_x = \delta \int_0^1 \frac{1}{2} \left[(\sqrt{x})^2 - (x^3)^2 \right] dx$$

$$= \frac{\delta}{2} \int_0^1 [x - x^6] dx$$

$$= \frac{\delta}{2} \left[\frac{x^2}{2} - \frac{x^7}{7} \right]_0^1$$

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$$M_x = \frac{5\delta}{28}$$

$$M_y = \delta \int_0^1 x [\sqrt{x} - x^3] dx$$

$$= \delta \int_0^1 (x^{\frac{3}{2}} - x^4) dx$$

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$$M_y = \delta \left[\frac{2}{5} x^{\frac{5}{2}} - \frac{x^5}{5} \right]_0^1$$

$$= \boxed{\frac{\delta}{5}}$$

$$\bar{x} = \frac{M_y}{M} = \frac{\frac{\delta}{5}}{\frac{5\delta}{12}} = \frac{12}{25}$$

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$$\bar{y} = \frac{M_x}{M} = \frac{\frac{5\delta}{28}}{\frac{5\delta}{12}}$$

$$= \frac{12}{28} = \frac{3}{7}$$

$$(\bar{x}, \bar{y}) = \left(\frac{12}{25}, \frac{3}{7} \right)$$

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