CPE 323
Intro to Embedded Computer Systems
Number Representation

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Admin

→ Watch for Q1a

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Numeral Systems

- Decimal (base 10): 456_{10} = $4\cdot10^2+5\cdot10^1+6\cdot10^0$

  $\{0,1,...9\}$, base = 10

- Binary (base 2): $\overset{3}{0}\overset{2}{1}\overset{1}{1}\overset{0}{0}_{2}$ = $0\cdot2^3+1\cdot2^2+1\cdot2^1+0\cdot2^0=6_{10}$

  $\{0,1\}$

- Octal (base 8): $\overset{2}{1}\overset{1}{2}\overset{0}{5}_8$ = $1\cdot8^2+2\cdot8^1+5\cdot8^0=85_{10}$

  $\{0,1,...,7\}$

- Hexadecimal (base 16): $10A_{16}$ = $1\cdot16^2+0\cdot16^1+10\cdot16^0$

  $=256+10=266_{10}$

  $\{0,1,...9,\underset{10}{A},\underset{11}{B},\underset{12}{C},\underset{13}{D},\underset{14}{E},\underset{15}{F}\}$

Decimal to Binary Conversion

- $A = 27_{10}$

$27 / 2 = 13 \quad 1$ (LSB)

$13 / 2 = 6 \quad 1$

$6 / 2 = 3 \quad 0$

$3 / 2 = 1 \quad 1$

$1 / 2 = 0 \quad 1$ (MSB)

$27_{10} = 11011_2 = 33_8 = 1B_{16}$

Representing Integers, Unsigned, Binary Format

- E.g., 1 byte or 8 bits, unsigned

n-bit

$[A_{n-1} A_{n-2} \ldots A_0]$

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	0	0	1	0	1	0	1	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- Convert to decimal: $1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^1 = 42_{10}$
- Convert to octal: 052_8
- Convert to hex: $2A_{16}$
- Range : $[0 \div 2^8 - 1]$
  255

Representing Integers, Signed, Binary Format

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

$[A_{n-1} \ldots A_0] = -A_{n-1} 2^{n-1} + A_{n-2} 2^{n-2} + A_{n-3} 2^{n-3} + \ldots + A_0 \cdot 2^0$

A:

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$100\ldots0 = -128$

$2^6 + 2^5 + \ldots + 2^1 + 2^0 = 2^7 - 1 = 127$

- Convert to decimal: -4_{10}
- Convert to octal:
- Convert to hex:
- Range → $[-128 \div 127]$

$-A$: 0000_0011
+ 1

0000_0100 $= 4_{10}$

Representing Integers, Signed

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- Convert to decimal:
- Convert to octal:
- Convert to hex:
- Range

Properties of 2's complement

00011000

- A = 11101000b
- Find –A:

-24_{10}

0111
00010111
+ 1
00011000b

$= 18_{16} = 1\cdot 16^1 + 8\cdot 16^0$
$= 24$

$V_{ADD} = \overline{A_{n-1}}\cdot\overline{B_{n-1}}\cdot R_{n-1} + A_{n-1}\cdot B_{n-1}\cdot\overline{R_{n-1}}$

- Assume 4-bit machine
- A = 1010b
- B = 0011b
- Find A + B

$\boxed{0}$010
1010
+ 0011
1101

$\boxed{1}$
1010
1000
0010

2's complement

$C_4 C_3 C_2 C_1$
0 0 1 0
1010
0011
1101

$V = C_4 \oplus C_3$
$= 0 \oplus 0 = 0$

Arithmetic Operations

$A - B = A + (\overline{B} + 1)$

- Addition
- Subtraction

$-B = \overline{B} + 1$

- Multiplication
- Flags
  - Carry (C) →
  - Overflow (V) →
  - Negative (N) $N = R_{n-1}$
  - Zero (Z) → set if the result of the operation is equal to zero

Arithmetic Operation Examples

$n = 4$

$$
\begin{array}{rrr}
 & (0110 & \\
 & A: 0111 & 7 \\
+ & B: 0110 & 6 \\
\hline
A+B: & 1101 & \\
\end{array}
$$

$C = \cancel{1}\ 0 \quad (C = C_4 = 0)$

$V = C_4 \oplus C_3 = 0 \oplus 1 = 1$

$Z = 0$

$N = 1$

Fraction Numbers

- Fixed-point, unsigned

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

00000.000 → 0

0000.001 → 0.125

Fraction Numbers

- Floating-point (IEEE 754 standard)

Type	Sign	Exponent	Exponent bias	Significand	Total
Half (IEEE 754-2008)	1	5	15	10	16
Single	1	8	127	23	32
Double	1	11	1023	52	64
Quad	1	15	16383	112	128

- Single-precision, normalized: $(-1)^{S} * 2^{E-127} 1.F$

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	S	E7	E6	E5	E4	E3	E2	E1	E0	F22	F21																					F0

Floating-point

Sign (s)	Exponent (e)	Fraction (f)	Value
0	00 ... 00	000....0	+0
0	00 ... 00	00 ... 01 11 ... 11	Positive denormalized real $0.f \times 2^{(-b+1)}$
0	00 ... 01 11 ... 10	xx ... xx	Positive normalized real $1.f \times 2^{(e-b)}$
0	11 ... 11	00 ... 00	+Infinity
0	11 ... 11	00 ... 01 01 ... 11	SNaN
0	11 ... 11	10 ... 00 11 ... 11	QNaN
1	00 ... 00	000....0	-0
1	00 ... 00	00 ... 01 11 ... 11	Negative denormalized real $-0.f \times 2^{(-b+1)}$
1	00 ... 01 11 ... 10	xx ... xx	Negative normalized real $-1.f \times 2^{(e-b)}$
1	11 ... 11	00 ... 00	-Infinity
1	11 ... 11	00 ... 01 01 ... 11	SNaN
1	11 ... 11	10 ... 00 11 ... 11	QNaN

88_{10}

$88/8 = 11 \;\; \llcorner 0$

$11/8 = 1 \;\; \llcorner 3$

$1/8 = 0 \;\; \llcorner 1$

$$88_{10} = 130_8 = 1\cdot 8^2 + 3\cdot 8 + 0\cdot 8^0$$
$$= 64 + 24 + 0 = 88$$

negative $s = 1$

-128.25

$(-1)^{s} \cdot 2^{E-127} \cdot 1.F$

$128.25 \rightarrow 10000000.01_2$

$= 1.\underbrace{000000001}_{\text{Fraction}} \times 2^{7}$

$\boxed{E=0}$

$\boxed{E=255}$

$0.25 \times 2 = 0.5$ | 0

$0.5 \times 2 = 1.0$ | 1

$0.25_{10} = .01_2$

$2^{-1}\ 2^{-2}$

S	E	
1	10000110	00000000100... 0
1	8	23

$E - 127 = 7$

$E = 134 = 10000110_2$

Binary Coded Decimal Numbers (BCD)

7 … 4	3 … 0
0100	0011
digit ×10	unit digits

43_{10}

$\{0, \ldots, 9\}$: 0 → 0000, 9 → 1001

~~1101~~	0011

packed BCD

$43_{10} / 16 = 2$ B

$2 / 16 = 0$ 2

$43_{10} = 2B_{16}$

0010	1011