Quiz #3 Wednesday, Thev. Eq. Ckt

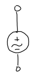
$v(t) = V_m \cos(\omega t + \theta)$ V

phase shift (deg)

magnitude

angular freq ($\frac{rad}{s}$)

$\omega = 2\pi f$



⇐ ckt symbol for sinusoidal source.

to convert a time domain source to the frequency domain

1. sinusoidal and in cosine form
2. Amplitude must be positive
3. phase shift must be between -180° to $+180^\circ$

1. quarter period shift $\quad \sin\omega t = \cos(\omega t - 90^\circ)$
2. half period shift $\quad -\cos(\omega t) = \cos(\omega t \pm 180^\circ)$; $\; -\sin(\omega t) = \sin(\omega t \pm 180^\circ)$
3. full period shift $\quad \cos\omega t = \cos(\omega t \pm 360)$

Mar 2-2:45 PM

$v(t) = 10\cos(2t + 20^\circ)$ V $\quad$ no changes $\quad \hat{V} = 10\angle 20^\circ$ V

$v(t) = 4\sin(8t - 120^\circ)$ V

$= 4\cos(8t - 120^\circ - 90^\circ)$ V

$= 4\cos(8t - 210^\circ)$ V

$= 4\cos(8t - 210^\circ + 360^\circ)$

$\boxed{v(t) = 4\cos(8t + 150^\circ) \text{ V}}$ $\quad \hat{V} = 4\angle 150^\circ$ V

$v(t) = -12\cos(3t + 20^\circ)$ V

$= 12\cos(3t + 20^\circ - 180^\circ)$

$\boxed{v(t) = 12\cos(3t - 160^\circ) \text{ V}}$ $\quad \hat{V} = 12\angle -160^\circ$ V

Mar 2-2:56 PM

Constructing a complex source: $\quad \cos x + j\sin x = e^{jx}$

$v(t) = V_m\cos(\omega t + \theta)$

Step
1) Add a complex part to $v(t)$
2) convert to complex exponential notation
3) Suppress the time domain part
4) change variable to frequency domain

$v(t) = V_m\cos(\omega t + \theta) + j V_m \sin(\omega t + \theta)$

$v^*(t) = V_m e^{j(\omega t + \theta)}$

$= V_m \cancel{e^{j\omega t}} e^{j\theta}$

$v^*(t) \longrightarrow V_m e^{j\theta}$

phasors $\mathbf{V}, \mathbf{I}, \quad \hat{V}, \hat{I}$

$\hat{V} = V_m \angle\theta$

$\hat{V} = 7\angle 25^\circ$ V $\quad (\omega = 2\text{ rad/s})$

$v^*(t) = 7e^{j2t} e^{j25^\circ}$

$= 7e^{j(2t + 25^\circ)}$

$= 7\cos(2t + 25^\circ) + j\,7\sin(2t + 25^\circ)$

$v(t) = 7\cos(2t + 25^\circ)$ V

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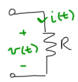


$v(t) = V_m\cos(\omega t + \theta) + j V_m \sin(\omega t + \theta)$

$v^*(t) = V_m e^{j(\omega t + \theta)}$

$i(t) = I_m\cos(\omega t + \phi) + j I_m \sin(\omega t + \phi)$

$i^*(t) = I_m e^{j(\omega t + \phi)}$

$v(t) = R\,i(t)$

$v^*(t) = R\,i^*(t)$

$\Rightarrow V_m e^{j(\omega t + \theta)} = R \cdot I_m e^{j(\omega t + \phi)}$

$V_m \cancel{e^{j\omega t}} e^{j\theta} = R \cdot I_m \cancel{e^{j\omega t}} e^{j\phi}$

$\Rightarrow V_m e^{j\theta} = R \cdot I_m e^{j\phi}$

$V_m\angle\theta = R \cdot I_m \angle\phi$

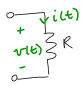
$\hat{V} = R\hat{I}$

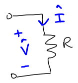
$V_m = R I_m$

$e^{j\theta} = e^{j\phi}$

$\theta = \phi$

voltage and current for a resistor are "in phase"





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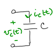


$v_c(t) = V_m e^{j(\omega t + \theta)}$

$i_c(t) = I_m e^{j(\omega t + \phi)}$

$i_c(t) = C\dfrac{dv_c(t)}{dt}$

$I_m e^{j(\omega t + \phi)} = C\dfrac{d}{dt}\left(V_m e^{j(\omega t + \theta)}\right)$

$I_m e^{j(\omega t + \phi)} = C\left(j\omega \cdot V_m e^{j(\omega t + \theta)}\right)$

$I_m e^{j(\omega t + \phi)} = j\omega C\left(V_m e^{j(\omega t + \theta)}\right)$

$I_m \cancel{e^{j\omega t}} e^{j\phi} = (j\omega C) V_m \cancel{e^{j\omega t}} e^{j\theta}$

$I_m e^{j\phi} = (j\omega C) V_m e^{j\theta}$

$\dfrac{1}{j\omega C} \equiv$ impedance of the capacitor, C

$V_m e^{j\theta} = \dfrac{1}{j\omega C} I_m e^{j\phi}$

$\dfrac{1}{j} = -j$

$\Rightarrow V_m\angle\theta = \dfrac{1}{j\omega C} I_m\angle\phi$

$\hat{V} = \dfrac{1}{j\omega C}\hat{I}$

$V_m\angle\theta = \underbrace{-j\left(\tfrac{1}{\omega C}\right)}_{\frac{1}{\omega C}\angle -90^\circ} I_m\angle\phi$

$0 - j\left(\frac{1}{\omega C}\right)$

mag $= \sqrt{0^2 + \left(-\frac{1}{\omega C}\right)^2} = \dfrac{1}{\omega C}$

angle $= \tan^{-1}\left(\dfrac{-\frac{1}{\omega C}}{0}\right) = -90^\circ$

$V_m\angle\theta = \left(\frac{1}{\omega C}\angle -90\right)(I_m\angle\phi)$

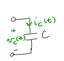
$V_m\angle\theta = \left(\frac{1}{\omega C}\cdot I_m\right)\angle(\phi - 90)$

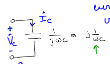
$V_m = \dfrac{1}{\omega C} I_m$

$\theta = \phi - 90^\circ$

$\theta - \phi = -90^\circ$

voltage leads current by -90° or current leads the voltage by $+90^\circ$





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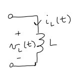


$v_L(t) = V_m e^{j(\omega t + \theta)}$

$i_L(t) = I_m e^{j(\omega t + \phi)}$

$v_L(t) = L\dfrac{di_L}{dt}$

$V_m e^{j(\omega t + \theta)} = L\dfrac{d}{dt}\left(I_m e^{j(\omega t + \phi)}\right)$

$V_m e^{j(\omega t + \theta)} = L\left(j\omega\, I_m e^{j(\omega t + \phi)}\right)$

$V_m \cancel{e^{j\omega t}} e^{j\theta} = (j\omega L) I_m \cancel{e^{j\omega t}} e^{j\phi}$

$V_m e^{j\theta} = (j\omega L) I_m e^{j\phi}$

$\Rightarrow V_m\angle\theta = \underbrace{j\omega L}\, I_m\angle\phi$

$\hat{V} = j\omega L\,\hat{I}$

$V_m\angle\theta = (\omega L\angle 90^\circ) I_m\angle\phi$

$V_m\angle\theta = \omega L \cdot I_m \angle(\phi + 90)$

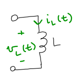
$j\omega L$ = impedance of the inductor L

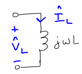
$V_m = \omega L \cdot I_m$

$\theta = \phi + 90^\circ$

$\theta - \phi = 90^\circ$

voltage leads the current by 90°





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Ohm's Law for Impedance (Z)

Resistor $\hat{V} = R\hat{I}$ $\Rightarrow$ $\hat{V} = Z_R \hat{I}$

$Z_R = R + j0$

$= R \angle 0$

Capacitor $\hat{V} = \frac{-j}{\omega C}\hat{I}$ $\Rightarrow$ $\hat{V} = Z_C \hat{I}$

$Z_C = \frac{-j}{\omega C}$

$= \frac{1}{\omega C} \angle -90^\circ$

Inductor $\hat{V} = j\omega L \hat{I}$ $\hat{V} = Z_L \hat{I}$

$Z_L = j\omega L$

$= \omega L \angle 90^\circ$

all impedances have units of OHMS.

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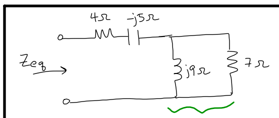
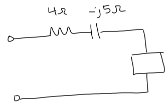
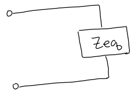


$$7 \| j9 = \frac{7(j9)}{7+j9} = \frac{j63}{7+j9}$$

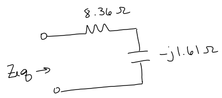


$Z_{eq_1} = \frac{j63}{7+j9}\ \Omega$



$Z_{eq} = (4 - j5) + \frac{j63}{7+j9}$

$= 8.36 - j1.61\ \Omega$



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