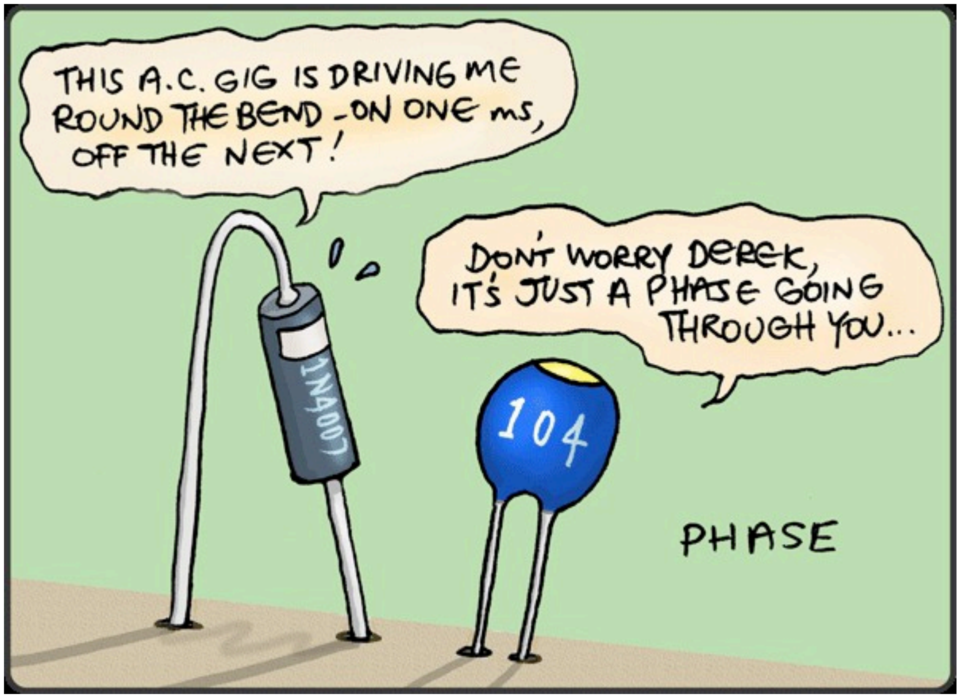

Lab 115 - 04

RC circuits

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PHYSICS/UAH

Please refrain from printing this document and have it available in an electronic form. We could save a few trees like that…

Ver 3.01

Author's note: The content herein, although it covers some introductory concepts, it goes above and beyond the material discussed in the lectures. Therefore, an adequate preparation for the lab must encompass the studying of this manuscript as well as the related lecture material. Online lecture screencasts can be found at https://sites.google.com/uah.edu/chronis (*Google UAH account required*).

Please make sure you stream (YouTube) and study the links highlighted with red color throughout this manual.(Google UAH account required).

1. Background

Think of a garbage bag. Usually we stuff as much as possible. When it looks full, we keep on pressing down, until we can still shove in this tiny piece of trash…Now **think of a metal plate**, which we are trying to "push in" as many electrons as possible. At some point, when the potential (remember ch. 24) between our charging device (say a charged rod touching our metal plate or a battery) is not high enough, then now enough electrons are transferred from our charging device to the metal plate. How can we keep transferring electrons? By *squishing* more and more electrons on the plate, exactly as the garbage bag. We do this by increasing the potential across the capacitor's plates. The capacitance is defined as

$$C = \frac{Q}{V}$$

but be extra cautious! Capacitance is **only a geometrical property** and as such is defined and determined by the dimensions of the capacitor (watch screencast ~min 16:00).

2. Behavior of an inductor in different voltage inputs

2.1 Constant voltage input

If voltage source E is constant with time, then after some time t, the capacitor will acquire its maximum charge (maximum voltage) and eventually, after this point **no current will run through** the circuit. The capacitor's voltage will be equal to the source voltage E. We **will observe** this behavior in our first experiment in this lab 115.

2.2 Square wave voltage input

Imagine if we wanted to make a circuit where we would like to observe how fast a capacitor charges or discharges. Hooking up the capacitor, charging it, unhook it to discharge it and repeat; the entire process is cumbersome. This is why we use a **function generator (FG) constant square wave.** With this configuration the capacitor will **charge while the source provides a constant +V** and **discharge while the source provides a constant -V**.

Our goal here will be to quantify **how fast** this happens. We call this variable **the time constant τ of the circuit**. To mathematically derive **τ** we need some basis and this will be the **Kirchhoff rules** and of

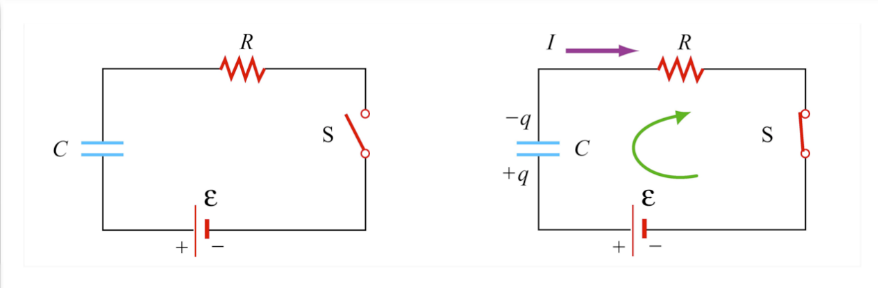

Figure 1: A simple RC circuit charging configuration.

course what we define as capacitance. First we are going to solve the "charging" scenario (see fig.1). Going around a closed loop in an RC circuit we get:

$$0 = \varepsilon - i(t)R - \frac{Q}{C} \Leftrightarrow 0 = \varepsilon - \frac{dQ}{dt}R - \frac{Q}{C} \Leftrightarrow \frac{dQ}{dt} = \frac{1}{R}\left(\varepsilon - \frac{Q}{C}\right)$$

This can be re-written as

$$\frac{dQ}{\left(\varepsilon - \frac{Q}{C}\right)} = \frac{1}{R}dt \Leftrightarrow \frac{dQ}{Q - C\varepsilon} = \frac{1}{RC}dt.$$

This is an equation that can be integrated by parts, with a final solution

$$Q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right).$$

The voltage across the capacitor is

$V_c = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$. When $t \to \infty$ then $V_c = \varepsilon$. The current thought the circuit is

$$i = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}.$$

The exponential term RC is the so-called time constant τ (in seconds i.e. Ω·F=sec).

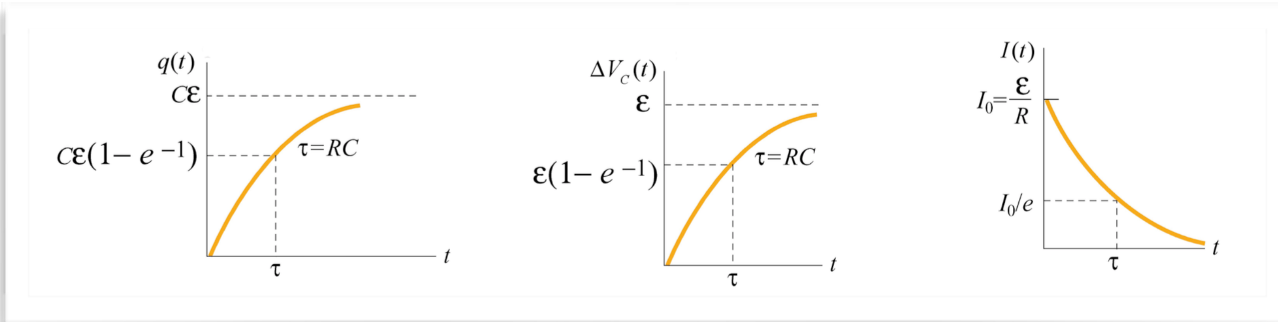

Figure 2: Capacitor charge (left) Capacitor voltage (middle) and current in circuit (right) as a function of time.

After time=τ has elapsed the capacitor is (1-e^{-1})~0.63 or ~63% charged. After **time=5 τ has elapsed the capacitor is ~99.99% charged**. The exact same solution approach is used for the discharging scenario, with the only difference being that in equation (1) we will use $\varepsilon = -\varepsilon$ (the negative voltage of the square-wave). The respective Vc solution will be

$$V_c = \varepsilon e^{-\frac{t}{RC}}$$

which is simply the **mirror image** of figure 2 (middle). During the discharging process, after time=1τ has elapsed the capacitor is (e^{-1})~0.37 which means about 37% of the initial charge remains (i.e. after 1τ, the voltage across the capacitor will be 63% of its maximum). After **time=5 τ has elapsed the capacitor is ~99.99% discharged**.

Now we will use a two channel **scope** to monitor the important **voltage changes** and estimate, based on these observations, the time constant τ of the RC circuit [see Q3 and fig. 3]. To review basic information about the scope see this video. It is **very important to understand** that (see figure 3) CH2 returns the electric potential from one side of the capacitor **with respect to the ground** (i.e. V_C) and CH1 estimates the electric potential across the entire RC circuit (V or as indicated in the equations

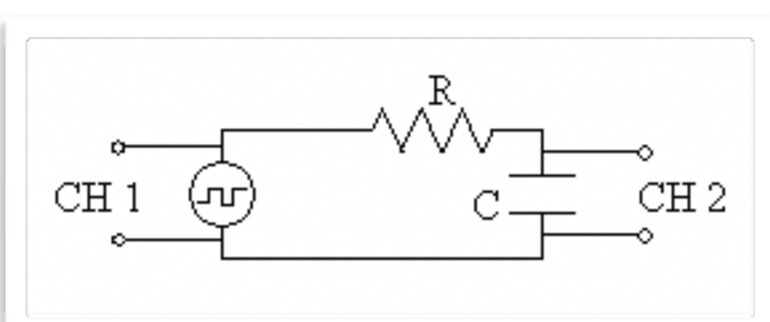

Figure 3: Oscilloscope connectivity in a simple RC circuit.

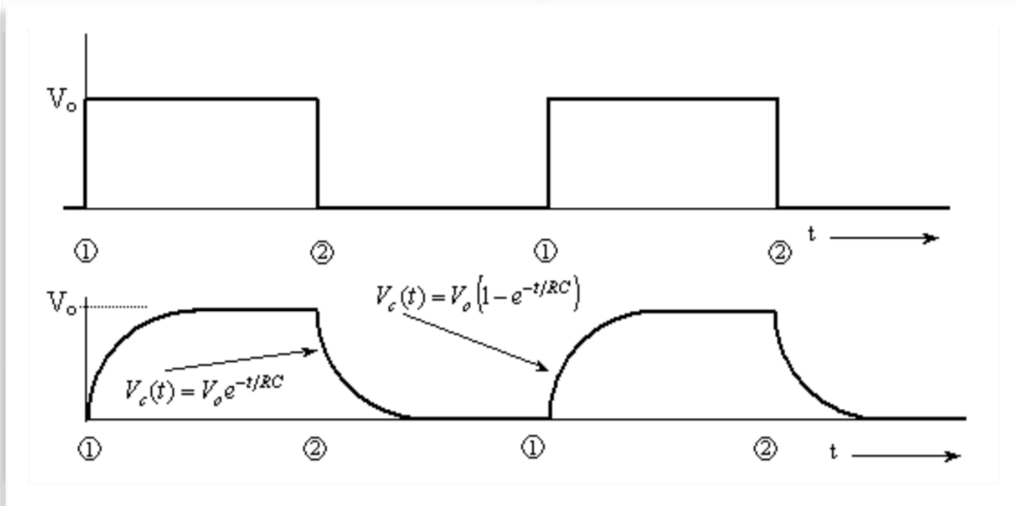

Figure 4: Source voltage (square wave) at CH1 a the shark fin indicating the capacitor charging-discharging in CH2.

above the calligraphic E). This is in contrast to the DM that can measure the voltage across any element of the circuit, but not its temporal variations.

The second step is to setup our scope and FG in order to estimate the constant τ. See Q1 for video instructions. If you use the other scope channel to probe the voltage across our entire circuit you should see the square wave input and if you overlay the channel probing the capacitor you should also see the so-called "shark-fin" which simply relates to the charging-discharging phases of the capacitor (i.e. the exponential equations we derived above). In short your scope should display something like this:

As figure 4 illustrates, the capacitor's voltage "lags" with respect to the source voltage is expected since the capacitor **does not acquire the maximum voltage instantaneously**. We are going to revisit this behavior in the AC section in this lab. To review the entire lab you can watch this online instructional video (Google UAH account required).

REPORT TASKS/QUESTIONS

[Q$_1$-Lab] Connect the R=5 Ω from the box to the 100 µF capacitor on the PASCO board **in series**. Provide 1 V DC. What is **the voltage** across the capacitor, the **voltage** across the resistor and the **current** after 10-20 seconds have elapsed? *[see screencast ~minute 1:35-3:20]*

[Q$_2$-Lab] The LED we used in Lab 2 displays different colors, depending on the polarity of the voltage applied. **Find a way** to use this property and **demonstrate** that when a capacitor discharges, it does so with the **opposite polarity** that was charged. Watch the demo here.

[Q3-Lab] We have seen that the *shark fin* is a good indication for a fully charged cap (see Fig. 4). Connect your 100 μF cap in series with a 5 Ω resistor. On the FG **use square wave 3 V @ 100 Hz**. It is **VERY important** to change your FG square wave to have an **"offset" of +3.00 V** (you do that by pressing the Menu on your FG). Use the scope and **estimate the time constant** τ(=RC) and **compare** this to the theoretical value. Make sure that the scope's channels are set to **DC coupling**. Now that you estimated τ **re-estimate the capacitance**. How close to the nominal 100 μF is your estimate? To review this experiment see this *screencast [~minute 4:00-5:50]*

Switch back to "offset Voltage" to zero BEFORE you proceed and scope's channels are set to AC coupling.

[Q4-Lab] How would the **voltage waveform across the cap** look like if we were to **increase the resistance** from 5 Ω to e.g. 50 Ω, 100 Ω or higher? **Explain** what is the **physical process** leading to this outcome. *[see this screencast ~minute 6:00-6:30]*

2.3 Sine Wave (AC) Voltage input

In the AC regime **a few important things change** when it comes to the RC circuits. What makes this RC special is of course the presence of the capacitor. We are going to use the FG's sinusoidal waveform at various frequencies at a given maximum voltage (please make sure you **NEVER exceed** the indicated value).

A **major difference in an AC circuit** that includes a capacitor is that now the **capacitor has a resistance**, which, we are going to call **reactive capacitance** (X_C). Let's see what this is: The charge (q) on the capacitor will be

$$q = CV_c = CV_0 \sin(\omega t)$$

since the voltage input is AC (i.e. sinusoidal). We also know that the current is the change of current with time i.e.

$$i = \frac{dq}{dt} = \omega C V_o \cos(\omega t).$$

The maximum current in the circuit will be

$$i_0 = \omega C V_0 \Leftrightarrow \frac{V_0}{i_0} = \frac{1}{\omega C} \Leftrightarrow \frac{V_0}{i_0} = \frac{1}{2\pi f C}.$$

We call the right hand side of this last equation **reactive capacitance** X_C. Here C stands for the capacitance (in Farad) and f is the FG's frequency (in Hz). The units for are Ohms. These derivations show that in an AC circuit, the **higher (lower) the frequency the lesser (greater) the resistance of the capacitor** given that

$$X_C = \frac{1}{2\pi f C}.$$

Unlike the **reactive capacitance's** dependency **on frequency**, always keep in perspective that as we derived from the Gauss' law**, the capacitance is ONLY a geometric property** (e.g. for parallel plates $C = \frac{\varepsilon_0 A}{L}$).

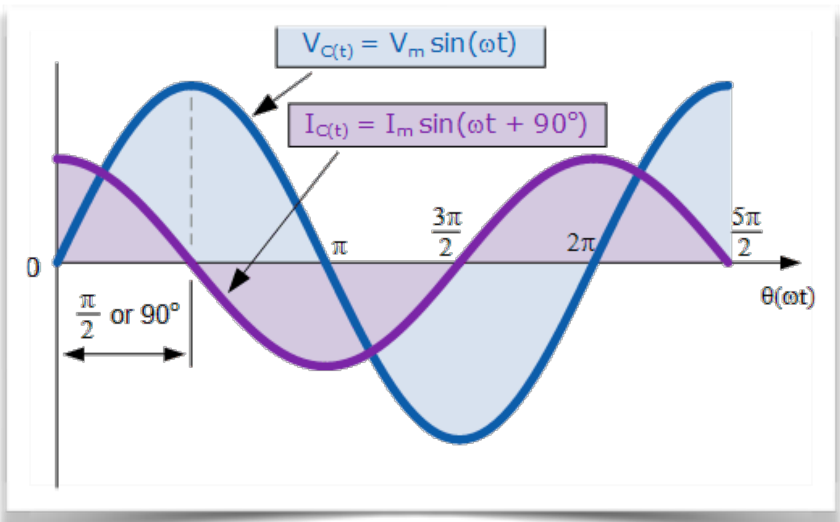

Figure 5: The phase angle relationship between current and voltage across the capacitor in an AC circuit.

Another **major difference** in the AC is that **because of the presence of the capacitor** and the fact that it requires **some time for the capacitor to charge**, the voltage and the current across the capacitor **exhibit a phase difference**; You can think of the following situation to better understand it; When the switch is closed in the presence of an AC source a high current will start to flow into the capacitor since it is initially uncharged. Since the rate of change of the potential difference across the plates is now at its maximum value, the flow of current into the capacitor will also be at its maximum rate. As the sinusoidal supply voltage reaches its its maximum (~90°) the waveform it begins to slow down therefore the current decreases to zero as there is no rate of voltage change. **This is the physical process** that introduces the 90° difference between the voltage and the current across the capacitor (hence the expression, **the current "leads the voltage, see Fig. 5**). The 90° phase difference between V_C and I_C is also explained if one compares the two respective algebraic expressions side-by-side i.e.

$V_c = V_0 \sin(\omega t)$ and

$$i = \frac{dq}{dt} = \omega C V_o \cos(\omega t).$$

You can see that the *sin* and *cos* functions are **by definition 90° out-of-phase**. Understand that we write e.g. i or I_C, these refer to the same current since the current that runs through the capacitor(*) is the current that runs through the circuit. *[I just used a phrase which must sound weird "current that runs through a capacitor"...I will explicitly address this in a subsequent section*].

2.3.1 RC in series

If we now have a **resistor in series** then since the capacitor and the resistor must be run by the **same current** (Ohm's law) and since the current and V_C have 90° phase difference then, we should further expect that **V_R and V_C must also be 90° out of phase**. This creates some complications in terms of

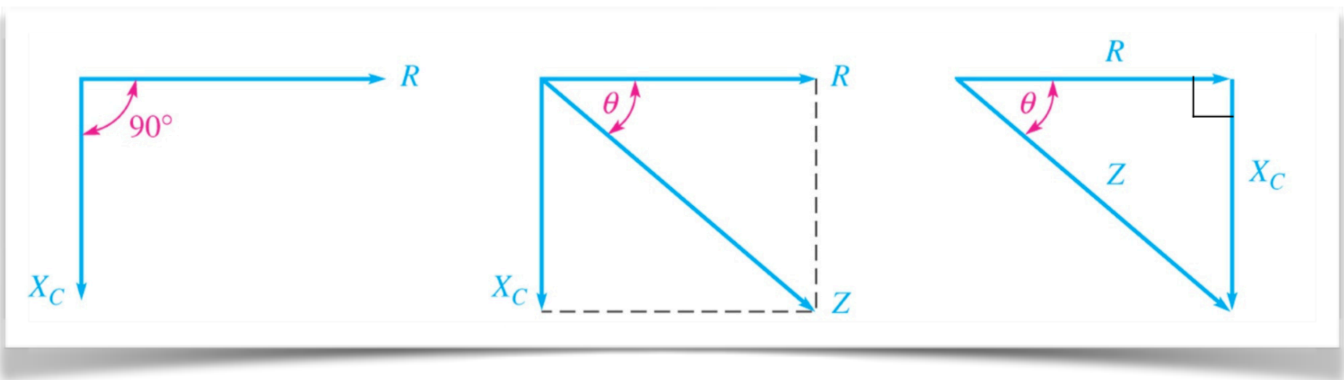

Figure 6: The phase angle relationship between R, X_C and Z.

calculating the "total equivalent resistance" as we have been used to in DC resistive circuits. To address this "phase difference" we treat these voltages as "vectors" and their summation, instead of V_R+V_C, $V = \sqrt{V_R^2 + V_C^2}$ (i.e. the Pythagorean theorem), where V is the voltage source. Note that this last equation **is usually used in rms terms**. Given that the resistor and capacitor are **run by the same current**, their total "equivalent" resistance will also be:

$Z = \sqrt{R^2 + X_C^2}$.

We call Z the **impedance** of the circuit (also measured in Ω, see fig.6). The **phase angle θ is going to be** $\theta = tan^{-1}\frac{V_C}{V_R} = tan^{-1}\frac{X_C}{R}$.

2.3.2 RC in parallel

Let's now think of a capacitor in **parallel** with a **resistor.** We know that components in parallel **share the same potential**

$V_R(t) = V_C(t) = V(t)$.

In light of the 2.3.1 it is intuitive that instead of

$$V = \sqrt{V_R^2 + V_C^2}$$

we should now involve **current** hence

$$I = \sqrt{I_R^2 + I_C^2}$$

since (i.e. in phase) while the impedance Z will be

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2}}.$$

In light of the aforementioned it would make sense that the equivalent of fig.6 (for RC in parallel) would be identical but instead of voltages we would have currents.

2.4 Practical calculations for RC in AC domain

Let's see a numerical example and its solution to gain more insight on how to solve problems like this.

RC series circuit, AC input is 10 V (this is peak voltage) at a frequency 1000 Hz, resistor is 10 Ω. ***Estimate*** *the capacitance based on the current reading of FG (i_{RMS} of ~0.69 A).*

Solution: We know that the impedance Z of the circuit as

$$Z = \sqrt{R^2 + X_C^2}$$

where R=10 Ω

$$X_C = \frac{1}{2\pi f C}$$

where C is unknown.

Given that $\boldsymbol{i_{RMS}}$ =0.69 A then

$$Z_{rms} = \frac{V_{rms}}{i_{rms}} \sim 10.13\ \Omega.$$

Note that V_{rms} is $\frac{10}{\sqrt{2}}$ (the source voltage rms).

Solving for $X_C = \sqrt{Z^2 - R^2} \sim 1.6\ \Omega$

$$C = \frac{1}{2\pi f X_C} \sim 99.5\ \mu\text{F}.$$

From here the rest is straightforward in that,

$$V_{C_{rms}} = i_{rms} \cdot X_C$$

$$V_{R_{rms}} = i_{rms} \cdot R.$$

The phase angle $\theta = tan^{-1}\frac{V_C}{V_R}.\ \sim 9°$.

This last equation stands for just C and R (since the i_{RMS} cancel out from numerator and denominator). ***Caution***: When you measure this angle with the scope then the actual angle will be (90°-θ°) since the voltage across the circuit is leading the voltage across the capacitor i.e. θ in fig.6 is measured counter-clockwise. We will solve this problem again making actual measurements.

2.5 Displacement current

[*Note: This is a more advanced topic and will not be included in any way, shape or form in a PH-115 exam*]

I owe you an explanation to a question that was raised earlier in the text and interestingly enough, is rarely covered in mainstream Physics textbooks. **Is there a current flowing through a capacitor**? If you ask a Physicist and an Electrical Engineer this question, you will witness a **fierce debate**! Let's start from the basics; as we already discussed in fig.2 that when the switch is closed, the current in the circuit **will be at its maximum** and then taper off (to ~zero) as the capacitor becomes fully charged. Isn't though a capacitor essentially an open switch? Certainly, when we have an open switch, there is no current flowing through. Then why current does flow through when we have a capacitor? As it turns out, **James Clerk Maxwell raised a very similar question**; What we know as current is simply the flux of charges through an area. But in Maxwell's equations there is a key-term that is called the **"displacement current"**

$$(I_d = \varepsilon_0 \frac{d\Phi_E}{dt})$$

This are type of current is computed from the changing flux of electric field (Φ_E). The existence of the **displacement current is not just a "correction"** Maxwell introduced to pre-existing laws (the so-called Ampere law) but it is essential to the very existence of electromagnetic radiation. In a nutshell, it states that "a changing electric field creates current" which, as you will find out later, further creates a magnetic field.

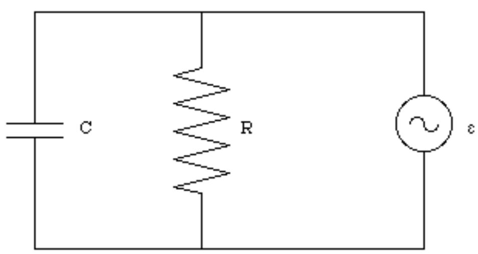

Figure 7: Example of induction current

Let's see how we can implement this with an example:

A parallel-plate capacitor has plates of area A separated by a distance d. A thin straight wire of length d lies along the axis of the capacitor and connects the two plates. This wire has a resistance R. The exterior terminals of the plates are connected to a source of alternating voltage $V = V_0 \cdot \sin(\omega \cdot t)$ *a) What is the **displacement current** through the capacitor? b) What is the current in the thin wire? c) What is the total current through the circuit? (fig.7)*

The displacement current defined in Maxwell's equation is

$$I_d = \varepsilon_0 \frac{d\Phi}{dt}.$$

But how does this link to our capacitor? Remember that

$$Q = CV$$

$$I = \frac{dQ}{dt}$$

$$I = C\frac{dV}{dt}.$$

Now the electric field across the capacitor is

$$E(t) = \frac{V_0 \sin(\omega t)}{d}$$

(i.e. E=V/d for parallel plate).

Since by definition

$\Phi = EA$ (i.e. electric flux)

then the displacement current across the capacity will be

$I_d = \frac{AV_0\omega}{d} cos(\omega t)$. This is the **displacement** current.

Now the current across the resistor is simply the Ohm's law i.e.

$$I_R = \frac{V_R}{R} = \frac{V_0 \sin(\omega t)}{R}$$

The **total current across** the RC circuit is the addition of the two ($I_d + I_R$)

3. Practical considerations of Capacitors

The RC circuits with DC or AC input have many **traditional** and **modern applications.** These range e.g. from digital filters, microphones and accelerometers to smartphone touchscreens. In this lab we are going to experiment with a couple of the aforementioned applications.

3.1 The low pass filter

First we are going to build a "digital filter", a device that is extensively used to remove (or allow) different sound frequencies. The difference will be that **instead of sound we all be filtering voltages** using a resistor and a capacitor in series (there is another process that converts sound to voltages) and we are going to call this a "**low-pass**" (also known as *integrator, see fig.8*)

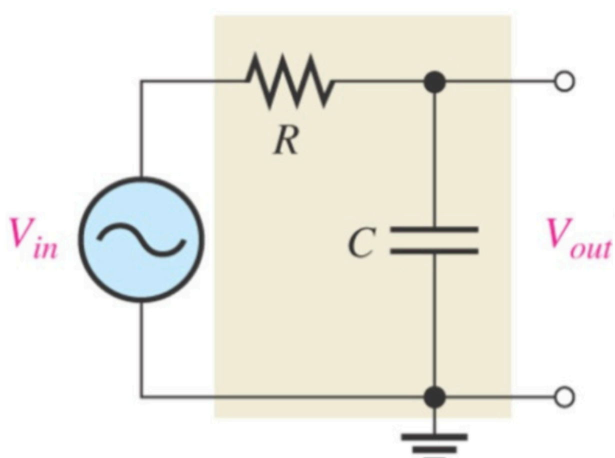

Figure 8: a simple RC low-pass filter

REPORT TASKS/QUESTIONS

Switch back to "offset Voltage" to zero BEFORE you proceed and scope's channels are set to AC coupling. Also - change the waveform to sine wave (i.e. from square wave)

[Q5-Home] *(This question has 4 parts)* You have a RC circuit connected to AC sine wave of 10 Volts peak. Your capacitor is 100 µF and your resistor is 10 Ω **in series**. For frequencies ranging from f=20 Hz to f=100 Hz **at 20 Hz intervals** and then discrete values of 200, 500, 1000 and 2000 Hz, **compute** and **plot frequency** (x-axis, logarithmic) *vs.* Q_{5a}) i_{RMS} and Q_{5b}) V_C. Use the solved problem in 2.4 as your guide Q_{5c}) **What is the impact of frequency** on i_{RMS} and V_C? Q_{5d}) **Discuss** the physical interpretation of these results. [*Hint: For each frequency calculate X_C, then the impedance (Z), then i_{RMS} and so forth]*

[Q6-Lab] *(This question has 3 parts)* This activity shows you how to estimate the capacitance solely based on the scope. R=10 Ω C=100 µF in series (AC sine wave Vp=10 V and **f=100 Hz),** hook the scope for **CH_1** to probe the **entire circuit** and **CH_2** to probe the **100 µF cap.** Q_{7a}) **Use** your DM to measure V, V_C and V_R Is the Ohm's law **corroborated**? Q_{7b}) Using the previous info calculate the **phase difference** between V *vs.* V_C Q_{7c}) **Explicitly discuss** what would happen to the **phase angle** between V *vs.* V_C and V_R *vs.* V_C as you **increase the frequency**. *[see this screencast ~minute 14:30-24:00].*

3.2 Touchscreen

Touchscreens are also a direct result of how capacitance works; If somehow we consider our finger as one "plate" of the capacitor and the touchscreen the other, then by touching the screen we effectively increase the capacitance (i.e. reducing d since $C = \dfrac{\epsilon_0 A}{d}$).

[Q7-Lab] *(This question has 4 parts)* For our experiment consider the following connectivity (see fig.9). From the FG at ~140 kHz (aka "floor" the FG as high as it goes) of sine @10 V go to ~100 **kΩ** resistor

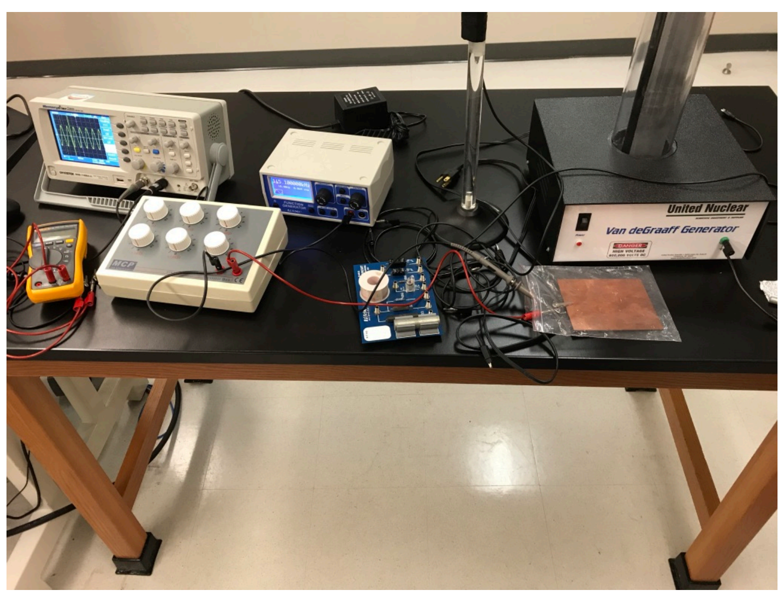

Figure 9: A simple touchscreen

to the aluminum foil inside the ziplock bag (see fig.9). Note that image shows a copper plate instead of the aluminum foil. Connect the foil to the scope's channels (either one is fine).

Make sure that the scope displays your waveform. Now approach your finger and touch the ziplock bag**. Explain** the physical processes and in particular Q_{8a}) why does the V_C amplitude change? Q_{8b}) what is the role of the ziplock bag? Q_{8c}) why there is no ground in the connectivity? Q_{8d}) why do we use such a high frequency and resistance?

1.1.1 ADDITIONAL CONCEPTUAL INSIGHT

Have you ever wondered how your smartphone is able to **estimate acceleration**? Inside its complex circuitboard is has a mechanical part that **entails a capacitor and spring**. Acceleration causes **one of the plates of the capacitor to move** opposite to the acceleration direction therefore creating a differential voltage. Another important capability of RC circuits is the **modulation of the DC signal**. For instance, what do you think would happen if you were to **increase the resistance** to which the capacitor is connected (see last part of Q4). How would this affect the V_C?

Other typical examples of capacitors entail electrical energy storage/use upon request (e.g. flashing lights), keyboards, the old fashioned radio antennas and much much more.

Finally another important consideration that goes beyond capacitors; any simple wire can behave as a capacitor. Imagine a wire that is connected to a simple DC input. When we turn on the power we are essentially pushing the existing gore electrons of the wire to move though. This does NOT happen instantaneously (i.e. nothing in Nature happens instantaneously, except of course some quantum properties…) but it takes time until the current reaches its maximum value. Parasitic effects are most prominent **at high frequencies**. For example, a metal foil 1 kΩ resistor at 100 MHz would, in fact, behave as a 1.001 kΩ resistor, when all parasitic effects are considered. This is an example of a good frequency response for a resistor. For comparison, a wire wound resistor is only usable up to 50 kHz, because of both inductive and capacitive parasitic effects. We cannot demonstrate this behavior in our lab since our FG cannot reach high enough frequencies where the parasitic effect would become obvious.

Before we wrap up this lab **take a moment to consider the bigger picture** i.e. far more important than any of the above derivations, which, in fact have been hundreds thus far; Which physical law(s) have we employed? Amazingly enough we find out that it is only the **Coulomb law** (plus a bit of Newton to derive the ohm's law): this is how we defined the electric field and electric potential, the fact that it is a conservative force, hence the Kirchhoff rule. We further used the Gauss' theorem to derive capacitance, which never forget that it is a **geometric property** (*aka it is not the charge and the voltage that determines capacitance, despite that they belong in the same equation!*). All the rest are mathematical derivations but in essence, the stepping stone is the **electric force** *(see this screencast's few last minutes)*.

4. Lab Checkpoints

- Capacitor charging discharging parameters, RC constant in DC, capacitance properties
- Capacitor in AC, reactive capacitance, frequency effects on voltage across capacitor
- Standard RC calculations in DC (based on τ) and AC (based on the Ohm's law, rms values)

5. On-line study material

https://sites.google.com/uah.edu/chronis/home/ph-112/past-lectures (*Google UAH account required*)