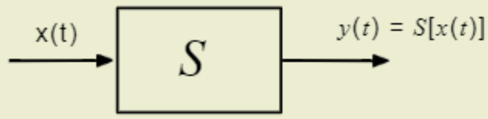
Unit Step: $S(s) = X(s)H(s)$

and $X(s) = \frac{1}{s}$

Chapter 2:

$x(t)$	$\Rightarrow$	$y(t) = S[x(t)]$
Input		Output



A system S is linear if for inputs $x(t)$ and $v(t)$, and constants α and β, superposition holds, i.e.,

$$\begin{aligned} S[\alpha x(t) + \beta v(t)] &= S[\alpha x(t)] + S[\beta v(t)] \\ &= \alpha S[x(t)] + \beta S[v(t)] \end{aligned}$$

Biased averager

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau + B, \quad \text{linear if } B = 0$$

Non–linear systems

$$\begin{aligned} &(i) \quad y(t) = |x(t)| \\ &(iii) \quad v(t) = x^2(t) \end{aligned}$$

RLC

$$\begin{aligned} &\text{resistor} \quad v(t) = Ri(t), \quad \text{linear} \\ &\text{capacitor} \quad v_c(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v_c(0), \quad \text{linear if } v_c(0) = 0 \\ &\text{inductor} \quad i_L(t) = \frac{1}{L}\int_0^t v(\tau)d\tau + i_L(0), \quad \text{linear if } i_L(0) = 0 \end{aligned}$$

System S is time-invariant if

$$\begin{aligned} x(t) \quad &\Rightarrow \quad y(t) = S[x(t)] \\ x(t \mp \tau) \quad &\Rightarrow \quad y(t \mp \tau) = S[x(t \pm \tau)] \end{aligned}$$

Time–varying system

$x(t)$, $y(t)$ input and output of system defined by

$y(t) = f(t)x(t)$, TV if $f(t)$ not constant

Amplitude modulation (AM) communication system

$$y(t) = m(t)\cos(\Omega_0 t), \quad \text{LTV}$$

Frequency modulation (FM) communication system

$$z(t) = \cos\left(\Omega_c t + \int_{-\infty}^{t} m(\tau)d\tau\right), \quad m(t) \;\; \text{message}$$

FM system non–linear

scale message $\gamma m(t)$ then output is

$$\cos\left(\Omega_c t + \gamma\int_{-\infty}^{t} m(\tau)d\tau\right) \neq \gamma z(t)$$

FM system time–varying

delay message $m(t-\lambda)$ then output is

$$\cos\left(\Omega_c t + \int_{-\infty}^{t} m(\tau-\lambda)d\tau\right) \neq z(t-\lambda)$$

System represented by linear, constant coefficient differential equation: System S, with input x (t) and output y (t), represented by

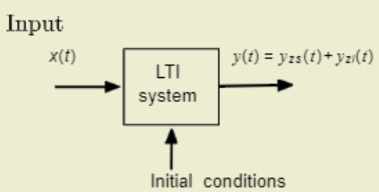
$$a_0 y(t) + a_1\frac{dy(t)}{dt} + \cdots + a_N\frac{d^N y(t)}{dt^N} = b_0 x(t) + b_1\frac{dx(t)}{dt} + \cdots + b_M\frac{d^M x(t)}{dt^M} \qquad t \geq 0$$

is linear time-invariant (LTI) if

- IC are zero
- input x (t) is causal (i.e., zero for $t<0$)

i.e., the system is not initially energized

If IC$\neq 0$, $x(t)$ causal consider superposition



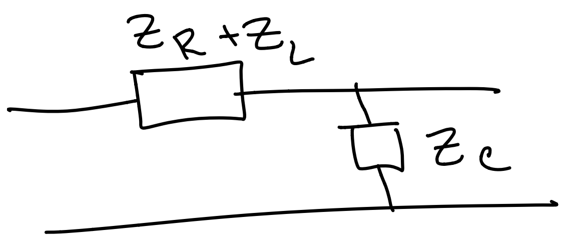
LTI system with x (t) and IC as inputs

Impedence → combines like resistors.

$Z_R = R$

$Z_C = \frac{1}{Cs}$

$Z_L = Ls$



$$Y(s) = \frac{Z_C}{Z_{RL} + Z_C}\cdot X(s)$$

RL circuit: $R = 1$, $L = 1$ and voltage source $v(t) = Bu(t)$

$$v(t) = i(t) + \frac{di(t)}{dt}, \qquad t > 0, \quad i(0) = I_0$$

$$\text{solution} \quad i(t) = [I_0 e^{-t} + B(1 - e^{-t})]u(t)$$

IC$\neq$ 0: (i) $I_0 = 1$ and $B = 1$

$$\text{complete response: } i_1(t) = [e^{-t} + (1 - e^{-t})]u(t) = u(t)$$

$$\text{zero-state response: } i_{1zs}(t) = (1 - e^{-t})u(t)$$

$$\text{zero-input response: } i_{1zi}(t) = e^{-t}u(t)$$

(ii) $I_0 = 1$ and $B = 2$ (double input)

$$\text{complete response: } i_2(t) = (2 - e^{-t})u(t) \neq 2i_1(t)$$

zero-state response: $i_{2zs}(t) = 2(1 - e^{-t})u(t)$, doubled

zero-input response: $i_{2zi}(t) = e^{-t}u(t)$, same

Averager

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau, \quad \text{(L)}$$

shifted input $x(t - \lambda)$, then output is

$$\frac{1}{T}\int_{t-T}^{t} x(\tau - \lambda)d\tau = \frac{1}{T}\int_{t-T-\lambda}^{t-\lambda} x(\sigma)d\sigma = y(t - \lambda), \quad \text{(TI)}$$

$$\begin{aligned}
\delta(t) \quad &\rightarrow \quad h(t) \quad \text{(definition)} \\
\delta(t - \tau) \quad &\rightarrow \quad h(t - \tau) \quad \text{(TI)} \\
x(\tau)h(t - \tau) \quad &\rightarrow \quad x(\tau)h(t - \tau) \quad \text{(L)} \\
x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau \quad &\rightarrow \quad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \quad \text{(L)}
\end{aligned}$$

$$\boxed{\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \\
&= \int_{-\infty}^{\infty} x(t - \tau)h(\tau)d\tau \\
&= [x * h](t) = [h * x](t)
\end{aligned}}$$

Example: for averager

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau, \quad x(t) \text{ input}, \ y(t) \text{ output}$$

impulse response $h(t) = \frac{1}{T}\int_{t-T}^{t} \delta(\tau)d\tau$

$$= \begin{cases} 1/T & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

ramp response $\rho(t) = \frac{1}{T}\int_{t-T}^{t} \sigma u(\sigma)d\sigma$

$$= \begin{cases} 0 & t < 0 \\ t^2/(2T) & 0 \le t < T \\ t - T/2 & t \ge T \end{cases}$$

Note that

$$\frac{d^2\rho(t)}{dt^2} = h(t)$$

Impulse response $h(t)$, unit-step response $s(t)$, and ramp response $\rho(t)$ are related by

$$h(t) = \begin{cases} ds(t)/dt \\ d^2\rho(t)/dt^2 \end{cases}$$

Cascade

$y(t) = [[x * h_1] * h_2](t) = [x * [h_1 * h_2]](t) = [x * [h_2 * h_1]](t),$ (commute)

Parallel

$$y(t) = [x * h_1](t) + [x * h_2](t) = [x * (h_1 + h_2)](t)$$

Negative feedback

$y(t) = [h_1 * e](t)$

error signal $e(t) = x(t) - [y * h_2](t)$

Closed loop impulse response $h(t) = [h_1 - h * h_1 * h_2](t),$ (implicit)

$$s(t) = \frac{1}{T}\int_{t-T}^{t} u(\tau)d\tau = \begin{cases} 0 & t < 0 \\ t/T & 0 \le t < T \\ 1 & t \ge T \end{cases}$$

$$y(t) = \frac{1}{\Delta}[s(t) - s(t-\Delta)] \quad \text{approximate impulse response of averager}$$

System S is causal if

- $x(t) = 0$, IC$= 0$, output $y(t) = 0$,
- output $y(t)$ does not depend on future inputs

S is LTI and causal, $h(t) = 0, t < 0$, input is causal, $x(t) = 0$, $t < 0$, output

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau = \int_0^t h(\tau)x(t-\tau)d\tau$$

Graphical procedure

- Choose time t_0 to compute $y(t_0)$,
- Plot as functions of τ, $x(\tau)$ and the reflected and delayed $h(t_0 - \tau)$,
- Obtain $x(\tau)h(t_0 - \tau)$ and integrate it from 0 to t_0 to obtain $y(t_0)$.
- Increase t_0, move from $-\infty$ to ∞

Equal results obtained if $x(t - \tau)$ and $h(\tau)$ used

LTI S is BIBO stable if

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty, \quad \text{(absolutely integrable)}$$

Chapter 3:

LTI system with h(t) as impulse response:

$$\begin{aligned}
\text{input} \quad x(t) &= e^{s_0 t}, \quad s_0 = \sigma_0 + j\Omega_0, \quad -\infty < t < \infty \\
\text{convolution} \quad y(t) &= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \\
&= e^{s_0 t} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-\tau s_0} d\tau}_{H(s_0)} = x(t) H(s_0)
\end{aligned}$$

The two-sided Laplace transform of $f(t)$ is

$$\boxed{\begin{array}{c} F(s) = \mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st} dt \qquad s \in \text{ROC} \\ s = \sigma + j\Omega, \ \ \text{damping } \sigma, \ \text{frequency } \Omega \end{array}}$$

The inverse Laplace transform is

$$\boxed{f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} ds \qquad \sigma \in \text{ROC}}$$

Rational function $F(s) = \mathcal{L}[f(t)] = N(s)/D(s)$

- zeros: values of s such that $F(s) = 0$
- poles: values of s such that $F(s) \to \infty$

ROC: where $F(s)$ is defined (integral converges) where $\{\sigma_i\} = \{Re(p_i)\}$

- Causal $f(t)$, $f(t) = 0$ for $t < 0$,

$$R_c = \{(\sigma, \Omega) : \sigma > \max\{\sigma_i\}, -\infty < \Omega < \infty\},$$

right of poles

- Anti-causal $f(t)$, $f(t) = 0$ for $t > 0$,

$$R_{ac} = \{(\sigma, \Omega) : \sigma < \min\{\sigma_i\}, -\infty < \Omega < \infty\},$$

left of poles

- Non-causal $f(t)$ defined for $-\infty < t < \infty$,

$R_c \cap R_{ac}$, poles in middle

$\delta(t)$ and $u(t)$

$$\mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-st}dt = \int_{-\infty}^{\infty} \delta(t)e^{-s0}dt = 1, \; ROC \text{ whole s-plane}$$

$$\begin{aligned} U(s) &= \mathcal{L}[u(t)] = \int_{-\infty}^{\infty} u(t)e^{-st}dt = \int_{0}^{\infty} e^{-st}dt = \int_{0}^{\infty} e^{-\sigma t}e^{-j\Omega t}dt \\ &= \frac{1}{s}, \quad ROC = \{(\sigma, \Omega) : \sigma > 0, -\infty < \Omega < \infty\} \end{aligned}$$

Pulse $p(t) = u(t) - u(t-1)$

$$\begin{aligned} P(s) &= \mathcal{L}[u(t) - u(t-1)] = \int_{0}^{1} e^{-st}dt = \frac{-e^{-st}}{s}\Big|_{t=0}^{1} \\ &= \frac{1}{s}[1 - e^{-s}] \quad ROC = \text{whole s–plane} \end{aligned}$$

For function $f(t), -\infty < t < \infty$, its one-sided Laplace transform is

$$\boxed{F(s) = \mathcal{L}[f(t)u(t)] = \int_{0-}^{\infty} f(t)e^{-st}dt, \quad \text{ROC}}$$

$$P(s) = \mathcal{L}[p_{ac}(-t)u(t)]_{(-s)} + \mathcal{L}[p_c(t)u(t)] \qquad \mathcal{R}_c \bigcap \mathcal{R}_{ac}$$

Causal functions and constants

Linearity	$\alpha f(t), \beta g(t)$	$\alpha F(s), \beta G(s)$
	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
Time shifting	$f(t-\alpha)u(t-\alpha)$	$e^{-\alpha s}F(s)$
Frequency shifting	$e^{\alpha t}f(t)$	$F(s-\alpha)$
Multiplication by t	$t\, f(t)$	$-\dfrac{dF(s)}{ds}$
Derivative	$\dfrac{df(t)}{dt}$	$sF(s) - f(0-)$
Second derivative	$\dfrac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0-) - f$
Integral	$\int_{0-}^{t} f(t')dt'$	$\dfrac{F(s)}{s}$
Expansion/contraction	$f(\alpha t), \; \alpha \neq 0$	$\dfrac{1}{\lvert\alpha\rvert}F\left(\dfrac{s}{\alpha}\right)$
Initial value	$f(0-) = \lim_{s\to\infty} sF(s)$	

$$\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] = \frac{e^{j\theta}}{s - j\Omega_0} \qquad \text{ROC:} \;\; \sigma > 0.$$

Laplace transform of $x(t) = \cos(\Omega_0 t + \theta)u(t)$

$$\begin{aligned} X(s) &= 0.5\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] + 0.5\mathcal{L}[e^{-j(\Omega_0 t+\theta)}u(t)] \\ &= \frac{s\cos(\theta) - \Omega_0 \sin(\theta)}{s^2 + \Omega_0^2}, \quad ROC: \sigma > 0 \end{aligned}$$

For $\theta = 0, -\pi/2$

$$\mathcal{L}[\cos(\Omega_0 t)u(t)] = \frac{s}{s^2 + \Omega_0^2},$$
$$\mathcal{L}[\sin(\Omega_0 t)u(t)] = \frac{\Omega_0}{s^2 + \Omega_0^2}, \quad ROC: \sigma > 0$$

One-sided Laplace Transforms

(1)	$\delta(t)$	1, whole s-plane
(2)	$u(t)$	$\frac{1}{s}$, $Re[s] > 0$
(3)	$r(t)$	$\frac{1}{s^2}$ $Re[s] > 0$
(4)	$e^{-at}u(t),\ a > 0$	$\frac{1}{s+a}$ $Re[s] > -a$
(5)	$\cos(\Omega_0 t)u(t)$	$\frac{s}{s^2+\Omega_0^2}$, $Re[s] > 0$
(6)	$\sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$, $Re[s] > 0$
(7)	$e^{-at}\cos(\Omega_0 t)u(t),\ a > 0$	$\frac{s+a}{(s+a)^2+\Omega_0^2}$, $Re[s] > -a$
(8)	$e^{-at}\sin(\Omega_0 t)u(t),\ a > 0$	$\frac{\Omega_0}{(s+a)^2+\Omega_0^2}$, $Re[s] > -a$
(9)	$2Ae^{-at}\cos(\Omega_0 t + \theta)u(t),\ a > 0$	$\frac{A\angle\theta}{s+a-j\Omega_0} + \frac{A\angle-\theta}{s+a+j\Omega_0}$, $Re[s] > -a$
(10)	$\frac{1}{(N-1)!}\ t^{N-1}u(t)$	$\frac{1}{s^N}$ N an integer, $Re[s] > 0$

Simple real poles

$$X(s) = \frac{N(s)}{(s+p_1)(s+p_2)}, \{-p_i, i = 1, 2\} \text{ real poles}$$

partial fraction expansion and inverse

$$X(s) = \frac{A_1}{s+p_1} + \frac{A_2}{s+p_2} \Rightarrow x(t) = [A_1 e^{-p_1 t} + A_2 e^{-p_2 t}]u(t)$$

$$A_k = X(s)(s+p_k)|_{s=-p_k} \quad k = 1, 2$$

Simple complex conjugate poles

$$X(s) = \frac{N(s)}{(s+\alpha)^2 + \Omega_0^2} = \frac{N(s)}{(s+\alpha-j\Omega_0)(s+\alpha+j\Omega_0)}, \quad \text{poles: } \{-\alpha \pm j\Omega_0\}$$

partial fraction expansion and inverse

$$X(s) = \frac{A}{s+\alpha-j\Omega_0} + \frac{A^*}{s+\alpha+j\Omega_0} \Rightarrow x(t) = 2|A|e^{-\alpha t}\cos(\Omega_0 t + \theta)u(t)$$

$$A = X(s)(s+\alpha-j\Omega_0)|_{s=-\alpha+j\Omega_0} = |A|e^{j\theta}$$

Double real poles

$$X(s) = \frac{N(s)}{(s+\alpha)^2} \quad \text{proper rational,} \qquad \text{poles } s_{1,2} = -\alpha$$

partial fraction expansion and inverse

$$X(s) = \frac{a + b(s+\alpha)}{(s+\alpha)^2} = \frac{a}{(s+\alpha)^2} + \frac{b}{s+\alpha}$$

$$x(t) = [ate^{-\alpha t} + be^{-\alpha t}]u(t)$$

$$a = X(s)(s+\alpha)^2|_{s=-\alpha}$$

b found by computing $X(s_0)$ for $s_0 \neq -\alpha$

$$y(t) = \mathcal{L}^{-1}\left[Y(s) = \frac{B(s)}{A(s)}X(s) + \frac{1}{A(s)}I(s)\right]$$

$$Y(s) = H(s)X(s) + H_1(s)I(s), \quad H(s) = \frac{B(s)}{A(s)}, \quad H_1(s) = \frac{1}{A(s)}$$

$$y(t) = y_{zs}(t) + y_{zi}(t)$$

$$y_{zs}(t) = \mathcal{L}^{-1}[H(s)X(s)] \quad \text{system's zero-state response}$$

$$y_{zi}(t) = \mathcal{L}^{-1}[H_1(s)I(s)] \quad \text{system's zero-input response}$$

$$\text{LTI, BIBO system} \quad y(t) = \underbrace{y_t(t)}_{transient} + \underbrace{y_{ss}(t)}_{steady-state}$$

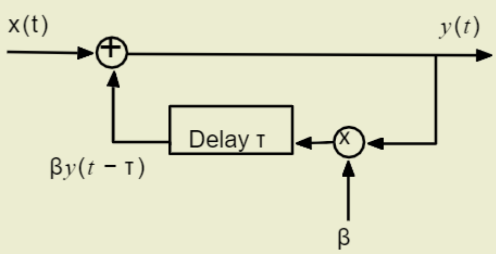
1. Steady state is due to simple real or complex conjugate pairs poles of $Y(s)$ in $j\Omega$-axis
2. Transient is due to poles of $Y(s)$ in the left-hand s-plane
3. Multiple poles in the $j\Omega$-axis and poles in the right-hand s-plane give unbounded responses

$$y(t) = [x * h](t) \quad \text{convolution} \quad \Rightarrow \quad Y(s) = X(s)H(s)$$

$$H(s) = \mathcal{L}[h(t)] = \frac{Y(s)}{X(s)} \quad \text{trasnfer function of system}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

Example: Positive feedback created by closeness of a microphone to a set of speakers



- Impulse response $x(t) = \delta(t)$, IC$= 0$, $y(t) = h(t)$

$$y(t) = x(t) + y(t-1) \quad \Rightarrow \quad h(t) = \delta(t) + \beta h(t-1)$$
$$H(s) = 1 + H(s)e^{-s}$$
$$H(s) = \frac{1}{1 - \beta e^{-s\tau}} = \frac{1}{1 - e^{-s}} = \sum_{k=0}^{\infty} e^{-sk} = 1 + e^{-s} + e^{-2s} + e^{-3s} + \cdots$$
$$h(t) = \delta(t) + \delta(t-1) + \delta(t-2) + \cdots = \sum_{k=0}^{\infty} \delta(t-k)$$

****positive feedback system – not BIBO stable? – see final slide of chap. 3**

Extra Laplace:

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	$\mathbf{e}^{a\,t}$	$\frac{1}{s-a}$
3.	$t^n, \quad n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
4.	$t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
6.	$t^{n-\frac{1}{2}}, \quad n = 1, 2, 3, \ldots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$
8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$
10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
12.	$\sin(at) + at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at) - at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$

14. $\cos(at) + at\sin(at)$	$\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
15. $\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$
16. $\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$
18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
20. $\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$
22. $\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n\mathbf{e}^{at}, \quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$
24. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$
26. $\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$

27. $u_c(t)\, f(t-c)$	$\mathbf{e}^{-cs}F(s)$
28. $u_c(t)\, g(t)$	$\mathbf{e}^{-cs}\mathcal{L}\{g(t+c)\}$
29. $\mathbf{e}^{ct} f(t)$	$F(s-c)$
30. $t^n f(t), \quad n = 1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t} f(t)$	$\int_s^\infty F(u)\, du$
32. $\int_0^t f(v)\, dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)\, g(\tau)\, d\tau$	$F(s)\,G(s)$
34. $f(t+T) = f(t)$	$\dfrac{\int_0^T \mathbf{e}^{-st} f(t)\, dt}{1-\mathbf{e}^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$
36. $f''(t)$	$s^2F(s) - sf(0) - f'(0)$
37. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0)\cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$

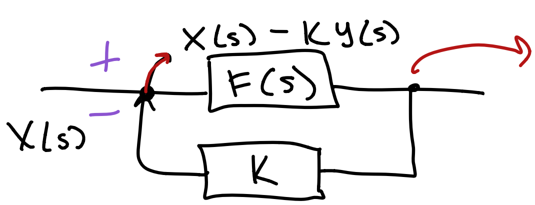
Spring Problem

$$m\frac{d^2x}{dt^2} + kx + c\frac{dx}{dt} = 0$$

$$k = \frac{mg}{L}$$

L → disp. from eq.

Negative Feedback



$$F(s)\left(X(s) - K\cdot Y(s)\right) = Y(s)$$

$$F(s)X(s) - F(s)\cdot K\cdot Y(s) = Y(s)$$

$$F(s)X(s) = (1 + F(s)K)\,Y(s)$$

$$Y(s) = \frac{F(s)}{1+KF(s)}\cdot X(s)$$

$$H(s) = \frac{F(s)}{1+KF(s)}$$

modify K to be stable

$K > 0$ ✓

as $F(s)\to\infty$

$$\frac{F(s)}{1+KF(s)} = \frac{1}{\cancel{\frac{1}{F(s)}}^{\,0} + K} = \frac{1}{K}$$