AC Steady State Power

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$p(t) = v(t) \cdot i(t)$$

$$p(t) = V_m \cos(\omega t + \theta) \cdot I_m \cos(\omega t + \phi)$$

$$= V_m I_m \left[\cos(\omega t + \theta) \cdot \cos(\omega t + \phi) \right]$$

$$\boxed{\text{Aside:} \quad \cos x \cdot \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]}$$

$$p(t) = \frac{V_m I_m}{2} \left[\cos(\omega t + \theta - (\omega t + \phi)) + \cos(\omega t + \theta + (\omega t + \phi)) \right]$$

$$p(t) = \frac{V_m I_m}{2} \left[\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi) \right]$$

$P \equiv$ average power $[W]$

$$P = \frac{1}{T} \int_0^T p(t)\,dt$$

$$P = \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \left(\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi) \right) dt$$

$$P = \frac{1}{T} \left(\frac{V_m I_m}{2} \right) \int_0^T \left[\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi) \right] dt$$

$$P = \frac{1}{T} \left(\frac{V_m I_m}{2} \right) \left[\int_0^T \cos(\theta - \phi)\, dt + \cancel{\int_0^T \cos(2\omega t + \theta + \phi)\, dt}^{\,0} \right]$$

$$P = \frac{1}{T} \left(\frac{V_m I_m}{2} \right) \int_0^T \cos(\theta - \phi)\, dt$$

$$P = \frac{1}{T} \left(\frac{V_m I_m}{2} \right) \left[\cos(\theta - \phi)\, t \Big|_0^T \right]$$

$$P = \frac{1}{\cancel{T}} \left(\frac{V_m I_m}{2} \right) \left[\cos(\theta - \phi) (\cancel{T} - 0) \right]$$

$$\boxed{P = \frac{V_m I_m}{2} \cos(\theta - \phi) \quad [\text{Watts}]}$$

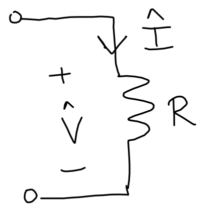
$\hat{I} = 10 \angle 30^\circ$ Arms

$\hat{I} = 2 \angle -19^\circ$ A

$$\frac{V_m}{\sqrt{2}} = V_{rms} \qquad \frac{I_m}{\sqrt{2}} = I_{rms}$$

$$\boxed{P = V_{rms} \cdot I_{rms} \cos(\theta - \phi) \quad [\text{Watts}]}$$

Resistor



Voltage + current are in phase

$\theta = \phi$

$V_{rms} = R I_{rms}$

$\hat{V} = V_{rms} \angle \theta$

$\hat{I} = I_{rms} \angle \phi$

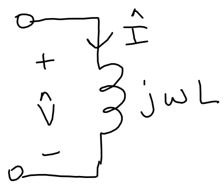
$$P = V_{rms} I_{rms} \cos(\theta - \phi)$$

$$P = V_{rms} I_{rms} \overset{1}{\cos(0)}$$

$$\boxed{\begin{aligned} P &= V_{rms} I_{rms} \\ P &= (I_{rms})^2 R \\ P &= \frac{(V_{rms})^2}{R} \end{aligned}}$$

Resistors as impedances absorb average power.

Inductor



$\hat{V} = V_{rms} \angle \theta$

$\hat{I} = I_{rms} \angle \phi$

$P = V_{rms} I_{rms} \cos(\theta - \phi)$

$P = V_{rms} I_{rms} \cos(\cancel{90})$ 0

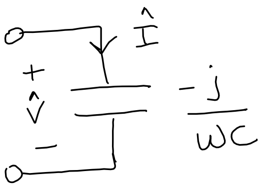
$P = 0$

inductors absorb <u>Zero</u> average power.

Voltage and current are $+90°$ out of phase

$\theta - \phi = 90°$

Capacitors

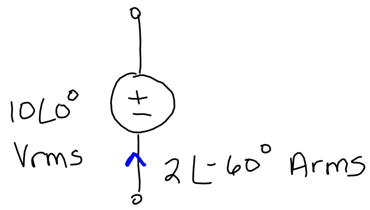


voltage and current are $-90°$ out of phase: $\theta - \phi = -90°$

$P = V_{rms} I_{rms} \cos(\cancel{-90})$ 0

$P = 0$

capacitors absorb <u>ZERO</u> average power.



$$P = V_{rms} I_{rms} \cos(\theta - \phi)$$

$$= (10)(2)\cos(0 - (-60))$$

$$= 20 \cos(60)$$

$$= 10 \text{ W}, \text{ Del}$$

Complex Power: $\hat{S} = P + jQ$

average power (P)

reactive power (Q)

Units = $P \Rightarrow$ watts [W]

$Q \Rightarrow$ volt-amperes, reactive [VAR]

$\hat{S} \Rightarrow$ volt-amperes [VA]

$$\hat{S} = \hat{V} \cdot \hat{I}^*$$

$\hat{V} = V_{rms} \angle \theta$

$\hat{I} = I_{rms} \angle \phi$

$x = 2 + j7$

$x^* = 2 - j7$

$y = 10 \angle 30^\circ$

$y^* = 10 \angle -30^\circ$

$$\hat{S} = (V_{rms} \angle \theta)(I_{rms} \angle -\phi)$$

$$\hat{S} = (V_{rms} I_{rms}) \angle (\theta - \phi)$$

$$\hat{S} = V_{rms} I_{rms} \cos(\theta - \phi) + j V_{rms} I_{rms} \sin(\theta - \phi)$$

$$= P + jQ$$

$P = V_{rms} I_{rms} \cos(\theta - \phi)$ [W]

$Q = V_{rms} I_{rms} \sin(\theta - \phi)$ [VAR]

$$\hat{S} = \underbrace{V_{rms} I_{rms}}_{[VA]} \angle (\theta - \phi) \ [VA] \Rightarrow \hat{S} = |\hat{S}| \angle (\theta - \phi)$$

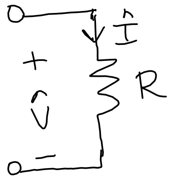
apparent power ($|\hat{S}|$)

power angle ($\theta - \phi$)

$\hat{V} = V_{rms} \angle \theta$

$\hat{I} = I_{rms} \angle \phi$

$Z = R + jX$



$\theta = \phi$

$$Q = V_{rms} I_{rms} \sin(\theta - \phi)$$

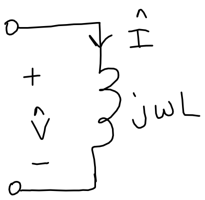
$$= V_{rms} I_{rms} \cancelto{0}{\sin(0)}$$

$$Q = 0$$

$$\hat{S} = P + jQ$$

$$\hat{S} = P$$

$$= (V_{rms} I_{rms}) \angle 0^\circ$$



$\theta - \phi = 90^\circ$

$$Q = V_{rms} I_{rms} \cancelto{1}{\sin(90^\circ)}$$

$$Q = V_{rms} I_{rms}$$

$$V_{rms} = \omega L I_{rms}$$

$$Q = (I_{rms})^2 \omega L$$

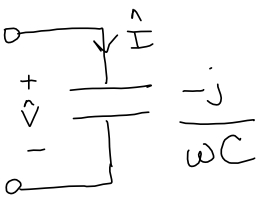
$$Q = \frac{(V_{rms})^2}{\omega L}$$

$$\hat{S} = P + jQ$$

$$= 0 + jQ$$

$$= jQ$$

$$\hat{S} = \underbrace{Q}_{V_{rms} I_{rms}} \angle 90^\circ$$



$(\theta - \phi) = -90^\circ$

$$V_{rms} = \frac{I_{rms}}{\omega C}$$

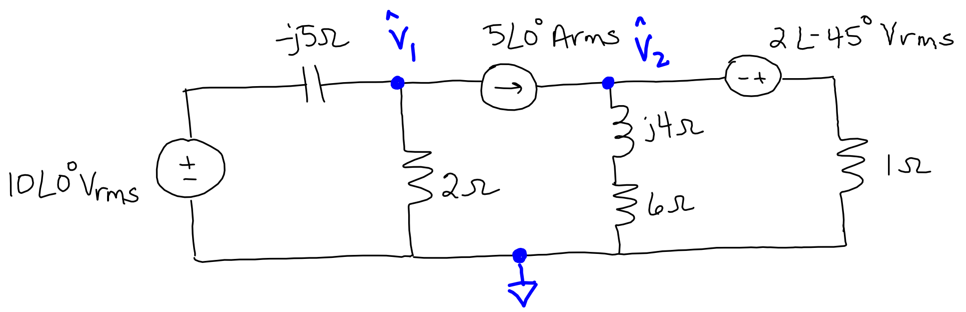
$$Q = V_{rms} I_{rms} \cancelto{-1}{\sin(-90^\circ)}$$

$$Q = -V_{rms} I_{rms}$$

$$Q = \frac{-(I_{rms})^2}{\omega C}$$

$$= -(V_{rms})^2 \cdot \omega C$$

$$\hat{S} = P + jQ \Rightarrow \hat{S} = V_{rms} I_{rms} \angle -90^\circ$$

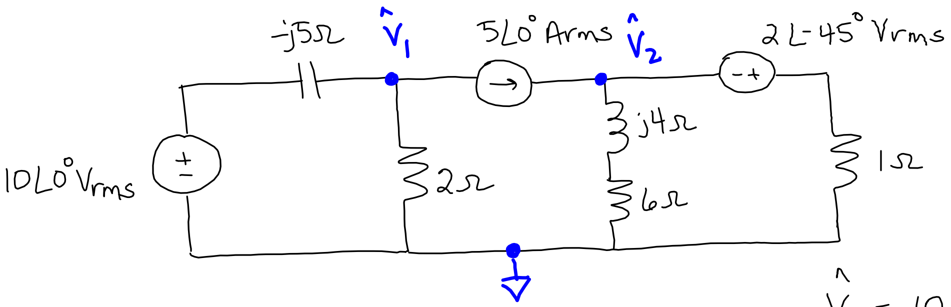


(N1) $$\frac{\hat{V}_1 - 10\angle 0}{-j5} + \frac{\hat{V}_1}{2} + 5\angle 0 = 0$$

$\hat{V}_1 = 10.0\angle 136.4°$ Vrms

$\hat{V}_2 = 3.45\angle 25.47°$ Vrms

(N2) $$\frac{\hat{V}_2 + 2\angle -45°}{1} + \frac{\hat{V}_2}{(6+j4)} + (-5\angle 0) = 0$$



$\hat{V}_1 = 10.0\angle 136.4^\circ$ Vrms

$\hat{V}_2 = 3.45\angle 25.47^\circ$ Vrms

Impedances

$-j5\Omega$: $P = 0$

$j4\Omega$: $P = 0$

2Ω : $P = \dfrac{(V_{rms})^2}{2} = \dfrac{(10.0)^2}{2} = 50W$, Abs

6Ω : $\dfrac{\hat{V}_2}{6+j4} = \dfrac{3.45\angle 25.47}{7.2\angle 33.69} = 0.48\angle -8.22^\circ$ Arms

$P = (.48)^2(6) = 1.38$ W, Abs

1Ω : $\dfrac{\hat{V}_2 + 2\angle -45}{1} = 4.53\angle 0.88^\circ$ Arms

$P = (4.53)^2(1) = 20.52$ W, Abs