7.2 Seperable differential equations:

Def^n: A first order differential equation $\boxed{y' = f(x, y)}$

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is called a seperable equation if the function $f(x, y)$ can be factored into the product of two functions of x and y

$$f(x, y) = p(x)\,h(y)$$

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Where $p(x)$ and $h(y)$ are continuous functions.

$$y' = f(x, y)$$

$$\Rightarrow \frac{dy}{dx} = p(x)\,h(y)$$

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$$\Rightarrow \frac{dy}{h(y)} = p(x)\,dx$$

Let $g(y) = \dfrac{1}{h(y)}$, $h(y) \neq 0$

$$\Rightarrow g(y)\,dy = p(x)\,dx$$

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$$\Rightarrow \int g(y)\,dy = \int p(x)\,dx$$

$$\Rightarrow \boxed{G(y) = P(x) + C}$$

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Ex) Solve the differential equation

$$(x^2 + 4)\,y' = 2xy$$

$$(x^2 + 4)\frac{dy}{dx} = 2xy$$

$$\Rightarrow (x^2 + 4)\,dy = 2xy\,dx$$

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$$\Rightarrow \frac{dy}{y} = \frac{2x}{x^2+4}\,dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{x^2+4}\,dx$$

$$\ln|y| = \int \frac{du}{u} \qquad \begin{array}{|l} u = x^2+4 \\ du = 2x\,dx \end{array}$$

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$$\Rightarrow \ln|y| = \ln|u| + C$$

$$\Rightarrow \underbrace{\ln|y| = \ln|x^2+4| + C}_{\text{implicit solution}}$$

$$\Rightarrow e^{\ln|y|} = e^{\ln|x^2+4|}\,C$$

$$\Rightarrow \boxed{|y| = C e^{\ln|x^2+4|}}$$

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$$\Rightarrow |y| = C\,|x^2+4|$$

Ex Solve the diff. equation

$$(x^2-1)y^3\,dx + x^2\,dy = 0$$

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$$x^2\,dy = -(x^2-1)y^3\,dx$$

$$\frac{dy}{y^3} = -\frac{(x^2-1)}{x^2}\,dx$$

$$\int \frac{dy}{y^3} = -\int \frac{x^2-1}{x^2}\,dx$$

$$\Rightarrow \int y^{-3}\,dy = -\int\left(\frac{x^2}{x^2} - \frac{1}{x^2}\right)dx$$

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$$\Rightarrow \int y^{-3}\,dy = -\int (1 - x^{-2})\,dx$$

$$\Rightarrow \boxed{-\frac{1}{2y^2} = -x - \frac{1}{x} + C}$$

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Ex Solve

$$(1+e^x)\,dy = e^x\,dx$$

$$\int dy = \int \frac{e^x}{1+e^x}\,dx \qquad \begin{array}{l} u = 1+e^x \\ du = e^x\,dx \end{array}$$

$$\Rightarrow y = \int \frac{du}{u}$$

$$\Rightarrow y = \ln|u| + C \Rightarrow \boxed{y = \ln|1+e^x| + C}$$

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Ex) Solve the initial value problem

$$y' = \frac{x(e^{x^2}+2)}{6y^2}, \quad y(0)=1.$$

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$$\frac{dy}{dx} = \frac{x(e^{x^2}+2)}{6y^2}$$

$$\Rightarrow \int 6y^2\,dy = \int x(e^{x^2}+2)\,dx$$

$$\Rightarrow \int 6y^2\,dy = \int xe^{x^2}dx + \int 2x\,dx$$

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$$\Rightarrow 6\frac{y^3}{3} = \frac{1}{2}\int e^u\,du + \frac{2x^2}{2} + C$$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x\,dx \end{aligned}}$$

$$\Rightarrow 2y^3 = \frac{1}{2}e^{x^2} + x^2 + C$$

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$y(0) = 1$

$x = 0,\ y = 1$

$$\Rightarrow 2(1)^3 = \frac{1}{2}e^0 + 0 + C$$

$$\Rightarrow 2 = \frac{1}{2}(1) + C$$

$$\Rightarrow \boxed{C = \frac{3}{2}}$$

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$$\boxed{2y^3 = \frac{1}{2}e^{x^2} + x^2 + \frac{3}{2}}$$

$$y = \left(\frac{1}{4}e^{x^2} + \frac{x^2}{2} + \frac{3}{4}\right)^{1/3}$$

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$\boxed{\text{D.E}}$ Solve

$$\frac{dy}{dt} = Ky$$

K is a constant

Initial condition: $y = y_0$ when $t = 0$.

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$$y = \underbrace{\pm y_0}_{\sim a \sim} e^{kt}$$

$$y = Ae^{kt}$$

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