Department of Electrical and Computer Engineering
The University of Alabama in Huntsville
Spring 2021

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Due: Monday February 8 at 9:35 am
Please bring hardcopy to the class and upload softcopy to Canvas

Student name:
Nolan Anderson

1 10	2 15	3 20	4 10	4 15	5 30	Total

Homework #1

1. (10 points) Write the formula and plot the roots of

$z^7 + 1 = 0$

$z_k^7 = -1 = e^{j(2k+1)\pi}$, $k = 0, 1, \ldots 6$

$z_k = |a|^{(1/n)} e^{j((2k+1)\pi + 2a)/n}$ (n will always be 1 in this case.) $\rightarrow$ $z_k = e^{j((2k+1)\pi + 2a)/n}$

$z_0 = e^{j\pi/7}$

$z_1 = e^{j(3\pi + 2a)/7} = e^{j3\pi/7}$

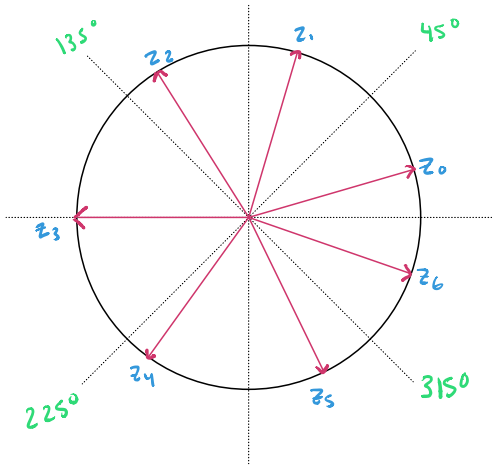
$z_2 = e^{j(5\pi)/7}$

$z_3 = e^{j(7\pi)/7} = e^{j\pi} = -1$

$z_4 = e^{j9\pi/7}$

$z_5 = e^{j11\pi/7}$

$z_6 = e^{j13\pi/7}$



2. (15 points) Represent the following complex numbers in alternative form (polar ←→ {Re,Im} z=x +jy)
   a) $1 + j$
   b) $1 - j$
   c) $5\,e^{j210°}$
   d) $5\,e^{-j210°}$
   e) $z\,z^*$

a)
- $|z| = \sqrt{1+1} = \sqrt{2}$
- $\angle z = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$ or $\pi/4$
- $z = \sqrt{2}\,e^{j\pi/4}$

b)
- $|z| = \sqrt{1+1} = \sqrt{2}$
- $\angle = \tan(-1) = -\pi/4$
- $z = \sqrt{2}\,e^{j\pi/4}$

c)
- $5e^{j210^\circ} = 5e^{j(180)} + 5e^{j(30)}$
- $5e^{j180} = \cos(180) + j\sin(180) = -1$
- $-5e^{j(30)} = -5\cdot\cos(30) + 5\cdot j\sin(30) = -4.33 - j2.5$

d) $5e^{j-210} = 5e^{-j(180+30)} = -1\left(-5\cos(-30) + j5\sin(-30)\right) = -4.33 + j2.5$

e) $zz^* = (Re + j\,Im)(Re - j\,Im) = Re^2 + Im^2 = |z|^2$

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3. (20 points) Use Euler's identity to find trigonometric identities in terms of sin(α), sin(β), cos(α), and cos(β):
   a) $\sin(\alpha + \beta)$
   b) $\cos(\alpha + \beta)$
   Demonstrate all the steps in formula evaluation.

a) $\sin(\alpha+\beta) = \frac{e^{j(\alpha+\beta)} + e^{-j(\alpha+\beta)}}{2} = \frac{(\cos(\alpha)+j\sin(\beta))(\cos(\beta)+j\sin(\beta)) + (\cos(\alpha)-j\sin(\alpha))(\cos(\beta)-j\sin(\beta))}{2}$

$[\cos\alpha\cos\beta + j\cos\alpha\sin\beta + j\sin\alpha\cos\beta - \sin\alpha\sin\beta - \cos\alpha\cos\beta + \cos\alpha\sin\beta + j\sin\alpha\cos\beta + \sin\alpha\sin\beta] \times \frac{1}{2j} = \frac{2j\cos\alpha\sin\beta + 2j\sin\alpha\cos\beta}{2j}$

$= \cos\alpha\sin\beta - \sin\alpha\sin\beta$

b) $\cos(\alpha+\beta) = \frac{e^{j(\alpha+\beta)} - e^{-j(\alpha+\beta)}}{2} = \frac{(\cos(\alpha)+j\sin(\beta))(\cos(\beta)+j\sin(\beta)) - (\cos(\alpha)-j\sin(\alpha))(\cos(\beta)-j\sin(\beta))}{2}$

$\cos\alpha\cos\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta - \cos\alpha\sin\beta - \sin\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}{2} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

4. (10 points) Write a script in Matlab to plot function

$$y(t) = Ae^{-t}\sin(2\pi ft), t \geq 0, \qquad y(t) = 0 \; for \; t < 0$$

for *f* = 2Hz, *A*= 2, sampling frequency (F_s) of 20 Hz, and $-4 \leq t \leq 4$.

Plot the signal using blue line and envelope (positive and negative) of the signal using dotted red line.

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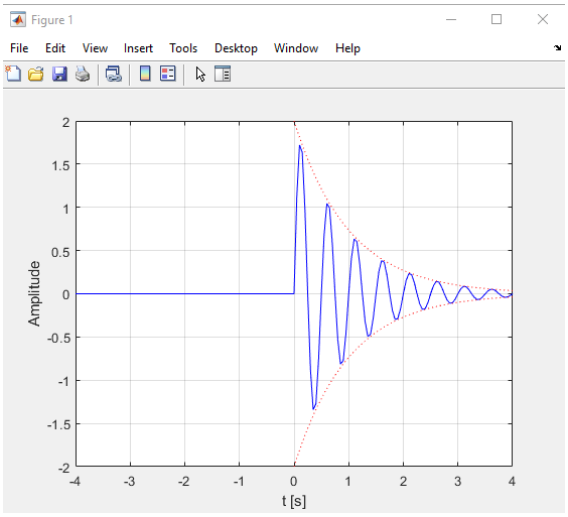
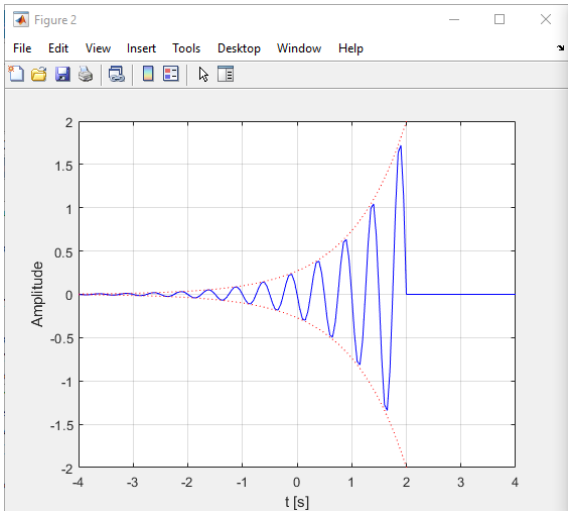
HW1.m

```
%% Initial Declaration
Fs = 20;          % Sampling Frequency, 20hz.
Ts = 1/Fs;        % Sampling interval.
f = 2;            % 2 Hz
t = 0:Ts:4;       % Time
t2 = -4:Ts:0;     % t <= 0 time
A = 2;            % Amplitude
t3 = 0:Ts:6;
t4 = -2:Ts:0;
ee3 = exp(-t3);
%% Number 4
env = A*ee;                    % Envelope
env2 = -A*ee;                  % Envelope
ee = exp(-t);
y = A*ee.*sin(2*pi*f*t);       % Function
y2 = zeros(size(t2));          % t < = 0
figure
plot(t, y,'b', t2, y2, 'b', t, env, 'r:', t, env2, 'r:'), xlabel('t [s]'),ylabel('Amplitude'),grid
%% Number 5
env3 = A*ee3;                  % Envelope
env4 = -A*ee3;                 % Envelope
y3 = A*ee3.*sin(2*pi*f*t3);    % Function with new time
y4 = zeros(size(t4));          % t < = 0
tow = 2-t3;                    % Function shift
tow2 = 2-t4;                   % Function shift
figure
plot(tow, y3,'b', tow2, y4, 'b', tow, env3, 'r:', tow, env4, 'r:'), xlabel('t [s]'),ylabel('Amplitude'),grid
```

Code for #4 and #5 ↑



5. (15 points) Write a script in Matlab and plot the function $y(2-\tau)$ where $y(t)$ is function from problem #4. Use Matlab arrays to manipulate samples from function in problem #4.



Code is shown above, below #4.

6. (30 points)

Accelerometer with analog output, sensitivity ±2g, and power supply of +3V is used in smartphone to determine orientation of the smartphone according to the figure below.

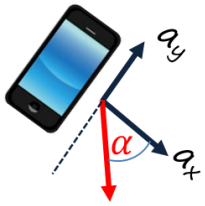
Sensitivity: $1g \rightarrow S = {}^{3V}/_{4g}$

$S = 0.75\,V/g$

$0g = 1.5V$

$1g = 1.5V + .75 \cdot 1 = 2.25V$

$-1g = 1.5 + .75 \times -1 = .75V$



What are the values of X and Y components [in Volts] for the following positions

a) $0g$ X = 1.5V, $-1g$ Y = .75V

b) $-1g$ X = .75V, $0g$ Y = 1.5V

c) $0g$ X = 1.5V, $1g$ Y = 2.25V

d) $1g$ X = 2.25V, $0g$ Y = 1.5V

What is the angle of the smartphone if:

e) X = 1.875 V, Y = 0.8505 V → α =

f) X = 2.1495 V, Y = 1.875V → α =

Please draw a phone as a part of the solution to avoid confusion.

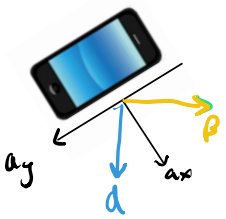
e) $X = 1.875V$, $Y = 0.850SV$ $\alpha = ?$

$a_x = \frac{A - A_0}{S} = \frac{1.875 - 1.5}{0.75} = 0.5g$

$a_y = \frac{A - A_0}{S} = \frac{0.8505 - 1.5}{0.75} = -0.866g$

$\alpha = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-.866}{0.5}\right) = -60^\circ$

$\beta = \tan^{-1}\left(\frac{a_x}{a_y}\right) = \tan^{-1}\left(\frac{0.5}{-.866}\right) = -30^\circ$



f) $X = 2.1495V$, $Y = 1.875V$

$a_x = \frac{A - A_0}{S} = \frac{2.1495 - 1.5}{0.75} = .866g$

$a_y = \frac{A - A_0}{S} = \frac{1.875 - 1.5}{0.75} = 0.5g$

$\alpha = \tan^{-1}\left(\frac{.5}{0.866}\right) = 30^\circ$

$\beta = -150^\circ$