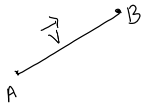
11.2 Vectors



$\vec{v} = \overrightarrow{AB}$ directed line segment

with initial pt. A and terminal pt. B

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Two vectors $\vec{u}$ and $\vec{v}$ are equal if and only if their length (magnitude) and have the

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same direction.

The length of the vector $\vec{u}$ is denoted by $|\vec{u}|$.

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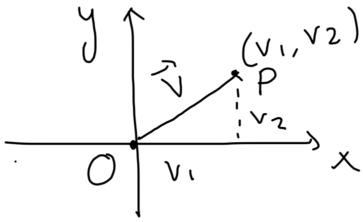
Vectors in two-dimensional



$\vec{v} = \overrightarrow{OP} = \langle v_1, v_2 \rangle$

v_1 and v_2 are called the components of $\vec{v}$.

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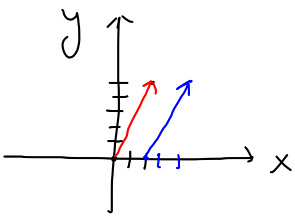
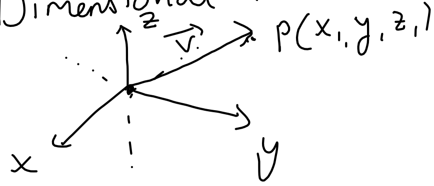
ex) $\vec{v} = \langle 2, 5 \rangle$



3 Dimensional:



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$\vec{v} = \langle x_1, y_1, z_1 \rangle$

x_1, y_1, z_1 are components of $\vec{v}$.

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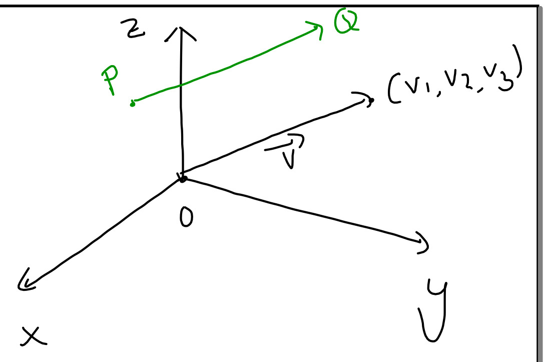

$P(x_1, y_1, z_1)$
$Q(x_2, y_2, z_2)$

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$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

ex

$P(2, 5, 6)$
$Q(-5, 2, 0)$
$\overrightarrow{PQ} = \langle -7, -3, -6 \rangle$

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$\boxed{\text{Def}^n:}$

The magnitude (length) of $\vec{v} = \overrightarrow{PQ}$ is the nonnegative number given by

$|\vec{v}| = |\overrightarrow{PQ}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

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$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$\boxed{\text{Vector operations}}$

Def^n: Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ be any vectors and k is

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any scalar.

Addition:

$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

Scalar multiplication:

$k\vec{u} = \langle ku_1, ku_2, ku_3 \rangle$

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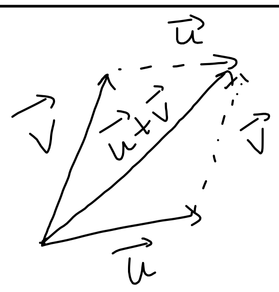

$\vec{u}$ 

$k\vec{u}$ if $k > 0$ $k\vec{u}$ is in the direction of $\vec{u}$.

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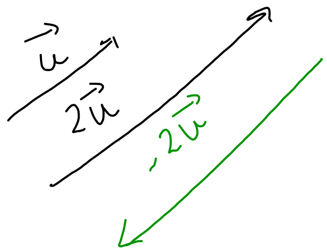
If $K < 0$, then $K\vec{u}$ is in the opposite direction of $\vec{u}$.



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Ex) Find the length of

(a) $\vec{v} = \langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \rangle$

(b) $A(1, 3)$ and $B(2, -1)$

$|\vec{AB}| = ?$

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(a) $|\vec{v}| = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{-2}{\sqrt{5}}\right)^2}$

$= \sqrt{\frac{1}{5} + \frac{4}{5}} = 1$

(b)

$|\vec{AB}| = \sqrt{(2-1)^2 + (-1-3)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$

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Ex) Given

$\vec{u} = \langle 4, 0, 3 \rangle$

$\vec{v} = \langle -2, 1, 5 \rangle$

(1) $\vec{u} + \vec{v}$

(2) $3\vec{v}$

(3) $\vec{u} - \vec{v}$

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(1) $\vec{u} + \vec{v} = \langle 2, 1, 8 \rangle$

(2) $3\vec{v} = \langle -6, 3, 15 \rangle$

(3) $\vec{u} - \vec{v}$

$= \vec{u} + (-\vec{v})$

$= \langle 6, -1, -2 \rangle$

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Properties of vector operations:

Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors and a and b are scalars.

(1) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

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(2) $(\vec{u}+\vec{v})+\vec{w}$
$= \vec{u}+(\vec{v}+\vec{w})$

(3) $\vec{u}+\vec{0}=\vec{u}$

(4) $\vec{u}+(-\vec{u})=\vec{0}$

(5) $0\vec{u}=\vec{0}$

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(6) $1\vec{u}=\vec{u}$

(7) $a(b\vec{u})$
$=(ab)\vec{u}$

(8) $a(\vec{u}+\vec{v})$
$=a\vec{u}+a\vec{v}$

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(9) $(a+b)\vec{u}$
$=a\vec{u}+b\vec{u}$

Unit Vectors:

A vector $\vec{v}$ of length 1 is called a unit vector.

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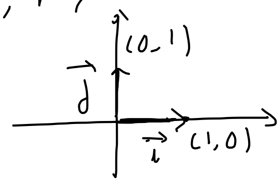
The standard unit vectors in 2-D are:

$\vec{i}=\langle 1,0\rangle$

$\vec{j}=\langle 0,1\rangle$



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$\vec{i}=\langle 1,0,0\rangle$

$\vec{j}=\langle 0,1,0\rangle$

$\vec{k}=\langle 0,0,1\rangle$

$\vec{v}=\langle v_1, v_2, v_3\rangle$

$\vec{v}=v_1 i+v_2 j+v_3 k$

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(ex) $\vec{v}=\langle -1,4\rangle$
$\vec{v}=-i+4j$

(ex) $\vec{u}=-5i-5j+10k$
$\vec{u}=\langle -5,-5,10\rangle$

(ex) $\vec{v}=2i+15k$
$\vec{v}=\langle 2,0,15\rangle$

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Let $\vec{u}$ be a unit vector and define $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|}$

ex) $\vec{v} = \langle 2, 3 \rangle$

$|\vec{v}| = \sqrt{4+9} = \sqrt{13}$

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$\vec{u} = \dfrac{\langle 2, 3 \rangle}{|\vec{v}|}$

$= \dfrac{\langle 2, 3 \rangle}{\sqrt{13}}$

$= \left\langle \dfrac{2}{\sqrt{13}}, \dfrac{3}{\sqrt{13}} \right\rangle$

unit vector

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