CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #4 Solution

1.

Magnitude response from poles and zeros—MATLAB
Consider the following filters with the given poles and zeros and dc constant:

$H_1(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zeros $z_1 = 1, z_{2,3} = 1 \pm j\pi$

$H_2(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zeros $z_{1,3} = \pm j\pi$

$H_3(s)$: $K = 1$ poles $p_1 = -1, p_{2,3} = -1 \pm j\pi$; zero $z_1 = 1$

Use MATLAB to plot the magnitude responses of these filters and indicate the type of filters they are.

See section 5.7.3 in the textbook:

$$H(s) = \frac{\prod_i (s - z_i)}{\prod_k (s - p_k)}$$

$$H_1(s) = \frac{(s-1)(s-1-j\pi)(s-1+j\pi)}{(s+1)(s+1-j\pi)(s+1+j\pi)} = \frac{(s-1)\left((s-1)^2+\pi^2\right)}{(s+1)\left((s+1)^2+\pi^2\right)} = \frac{s^3 - 3s^2 + (3+\pi^2)s - (1+\pi^2)}{s^3 + 3s^2 + (3+\pi^2)s + (1+\pi^2)}$$

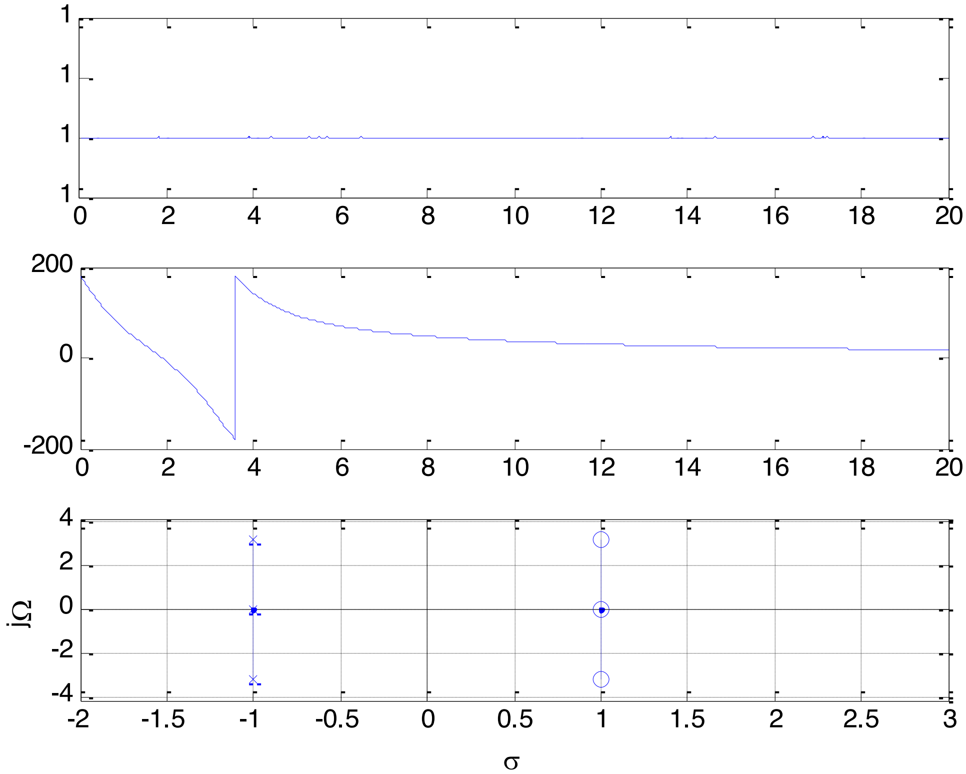
$$H_2(s) = \frac{s^2 + \pi^2}{(s+1)\left((s+1)^2+\pi^2\right)}$$

$$H_3(s) = \frac{s-1}{(s+1)\left((s+1)^2+\pi^2\right)}$$

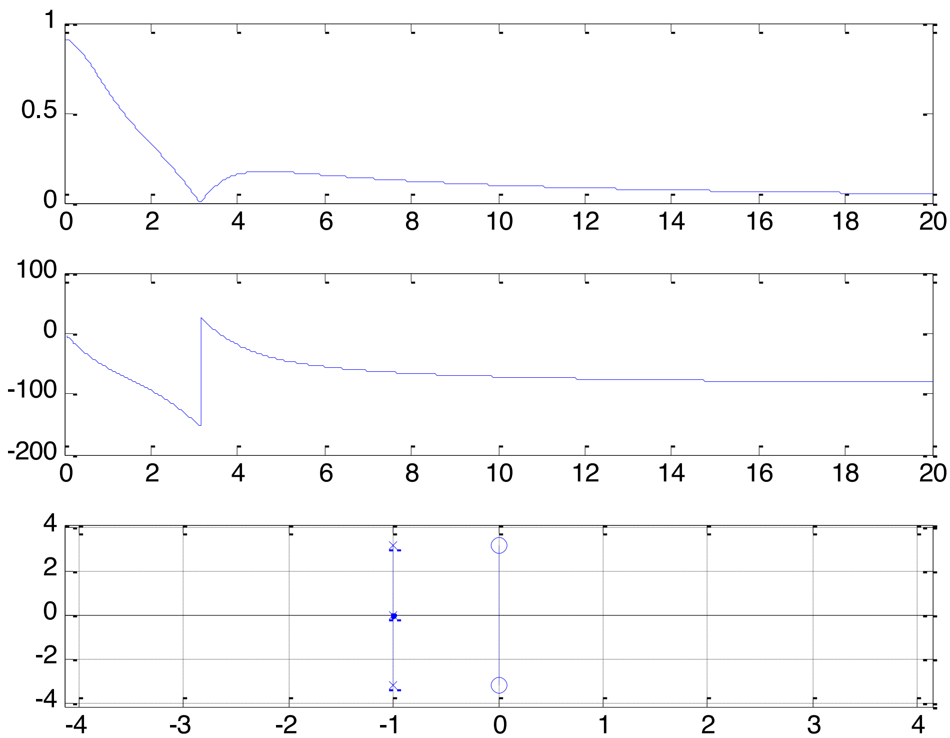
```
% CPE381 HW4_3
clear all; clf
n=[1 -3 3+pi^2 -(1+pi^2)];
d=[1 3 3+pi^2 (1+pi^2)];
figure(1)
wmax=20;
freqresp_s(n,d,wmax)
n1=[0 1 0 pi^2]; d1=d;
figure(2)
freqresp_s(n1,d1,wmax)
n2=[0 0 1 -1]; d2=d;
figure(3)
freqresp_s(n2,d2,wmax)

function [w,Hm,Ha]=freqresp_s(b,a,wmax)
w=0:0.01:wmax;
H=freqs(b,a,w);
Hm=abs(H);
Ha=angle(H)*180/pi;
figure
subplot(311)
plot(w,Hm)
subplot(312)
plot(w,Ha)
subplot(313)
splane(b,a)
```

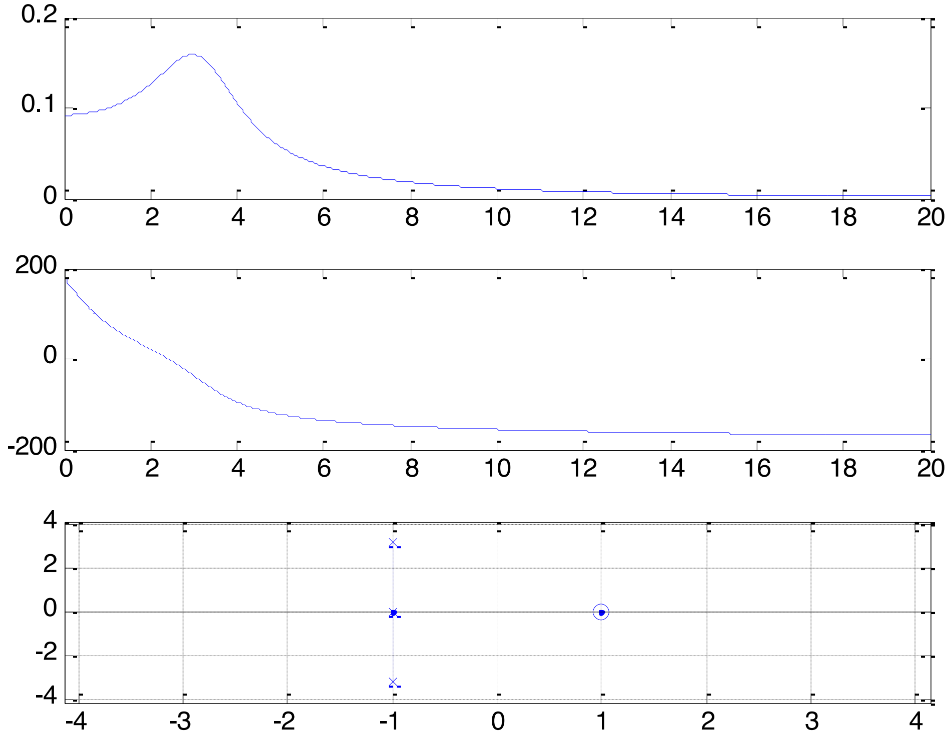
```
function splane(num,den)
% function splane
% input: coefficients of numerator (num) and denominator (den) in
% decreasing order
% output: pole/zero plot
% use: splane(num,den)
%
z=roots(num);
p=roots(den);
A1=[min(imag(z)) min(imag(p))];A1=min(A1)-1;
B1=[max(imag(z)) max(imag(p))];B1=max(B1)+1;
N=20;
D=(abs(A1)+abs(B1))/N;
im=A1:D:B1;
Nq=length(im);
re=zeros(1,Nq);
A=[min(real(z)) min(real(p))];A=min(A)-1;
B=[max(real(z)) max(real(p))];B=max(B)+1;
stem(real(z),imag(z),'o:')
hold on
stem(real(p),imag(p),'x:')
hold on
%plot(re,im,'k');xlabel('\sigma');ylabel('j\Omega')
grid
% axis([A -A min(im) max(im)])
axis([min(im) max(im) min(im) max(im)]);
hold off
```



H1 is an all-pass filter.



H2 is a notch filter; it behaves like low-pass filter at low frequencies.



H3 is a low-pass filter.

2. An ideal low pass filter *H(s)* with zero phase and magnitude response:

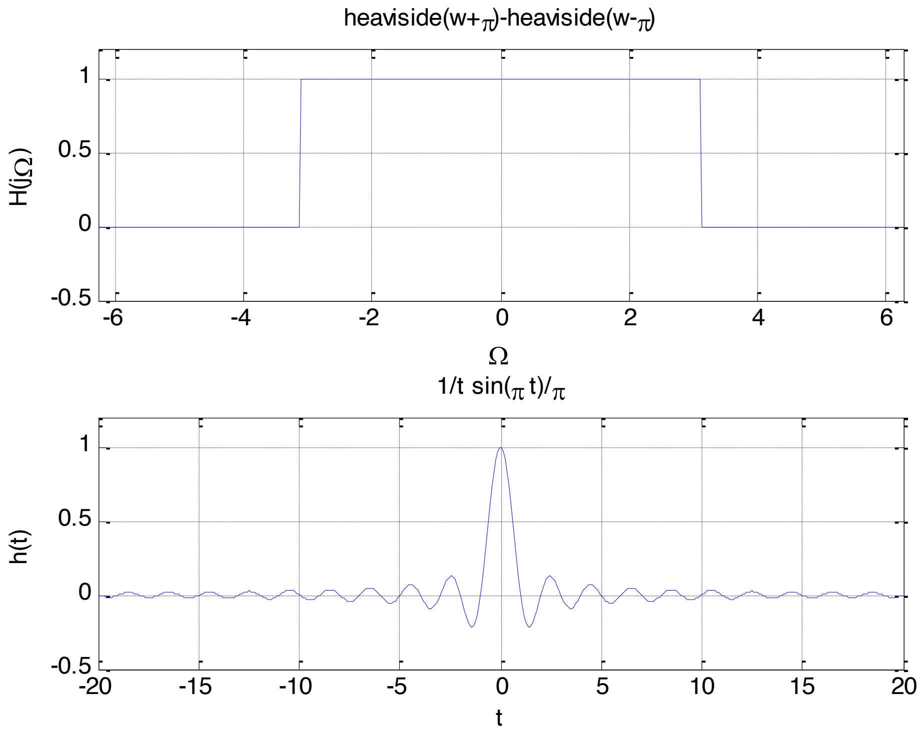
$$|H(j\Omega)| = \begin{cases} 1 & -\pi \le \Omega \le \pi \\ 0 & otherwise \end{cases}$$

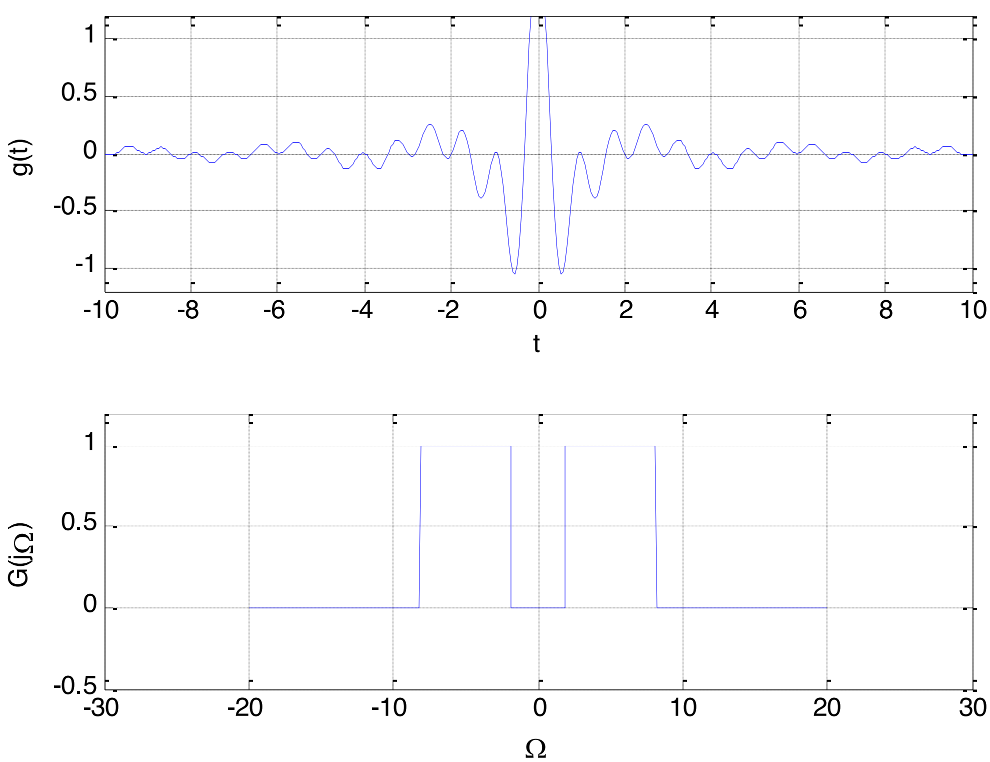
a) The impulse response is h(t) = sin(πt)/ πt , which is non-causal since h(t) $\neq$ 0 for t<0. (textbook 5.7.2.)

b) What is the effect of shifting the central frequency of the ideal filter for 5π ?

The bandpass filter. It can be implemented using ideal low-pass filter by shifting the central of the ideal low-pass filter

g(t) = 2*h(t)*cos(5*π*t)
and G(jΩ) = H(j(Ω - 5π)) + H(j(Ω + 5π))


heaviside(w+π)-heaviside(w-π)
H(jΩ)
Ω
1/t sin(π t)/π
h(t)
t


g(t)
t
G(jΩ)
Ω

3. A 12-bit AD converter is used to digitize signal with negative reference V_{R-} = 0.5V and positive reference V_{R+} = 2.5V.

a) (3 points) What is the quantization step?
b) (3 points) What is the output of the AD converter for V_{in} = 2.2 V ?
c) (2 points) What is the output of the AD converter for V_{in} = 0.4 V ?
d) (2 points) What is the output of the AD converter for V_{in} = 3 V ?

a) The quantization step is

$$\Delta = (V_{r+} - V_{r-})/(2^{12}-1) = (2.5-0.5)/4095 \approx 0.49 \text{ mV}$$

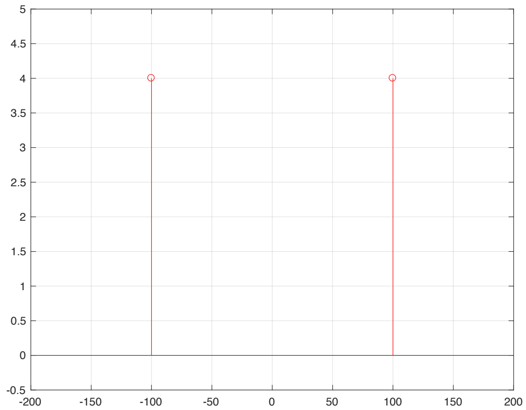
b) The output of the AD converter is

$$ADout = (V_{in} - V_{r-})/\Delta = (2.2 - 0.5)/\Delta = 3480$$

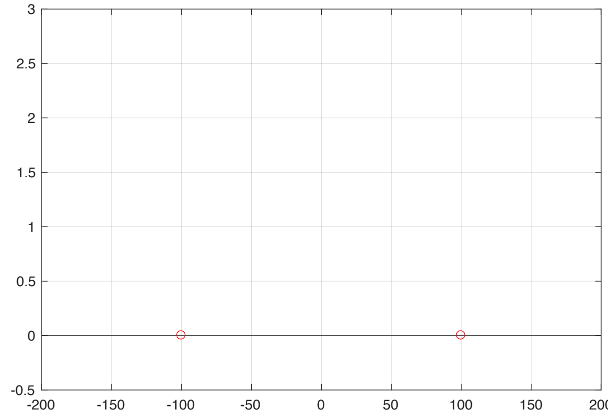
c) The output of the AD converter is 0

d) The output of the AD converter is 4095 (all ones)

4.

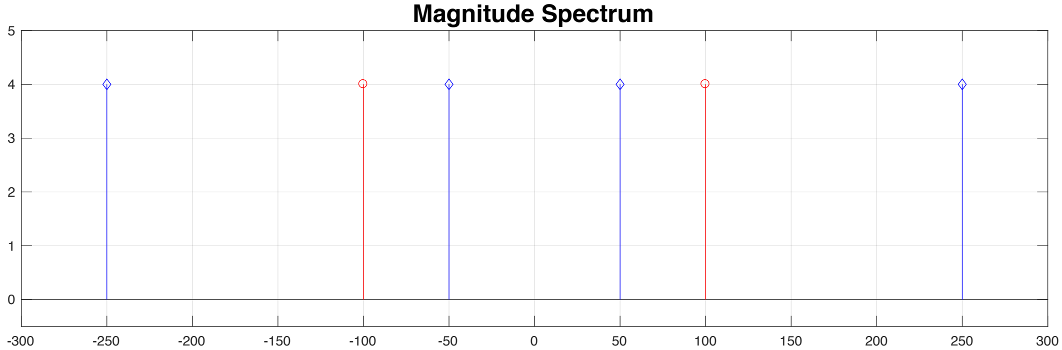


Magnitude



Phase

Magnitude Spectrum



Phase Spectrum