CPE 381 Equation Sheet

Chapter 0:

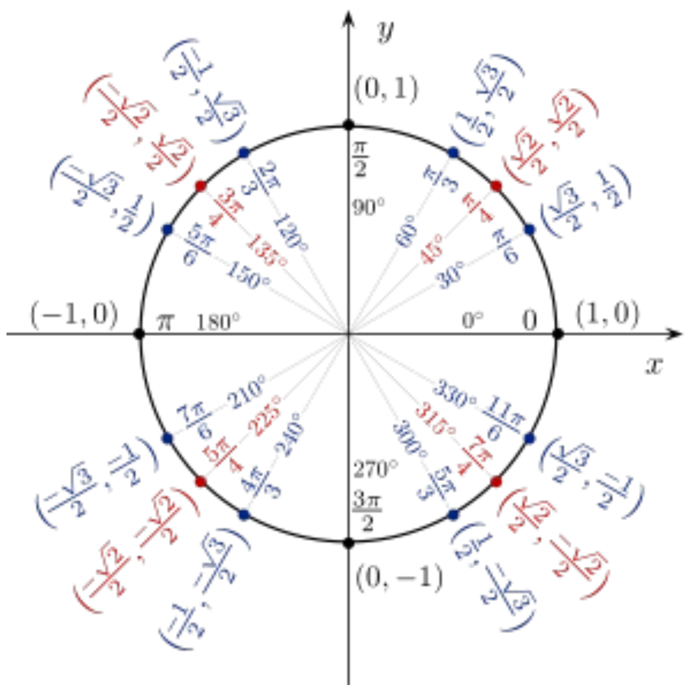
n	2^n	Significance
0	1	
1	2	
2	4	
4	16	
8	256	
10	1024	
16	65536	

Range of numbers:

0 to $2^n - 1$ for unsigned

-2^{n-1} to $2^{n-1}-1$

Unit Circle:



Arithmetic Series:

$a_n = a_1 + (n-1)d$

$$\sum_{i=1}^{n} a_i = n \cdot \frac{a_1 + a_n}{2}$$

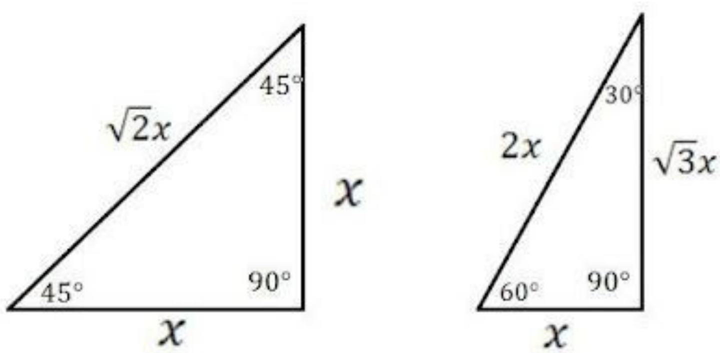
Geometric Series:

$$\sum_{k=1}^{n} a_k r^k = a \cdot \frac{1-r^n}{1-r}$$

for inf.

$$= \frac{a_1}{1-r}$$

Triangles to Know:



Integrals to Know:

$$\int x^n \, dx = \frac{1}{n+1} \cdot x^{n+1}$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \sin(ax)\,dx = \frac{-1}{a}\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

Euler's Identity:

$$e^{j\theta} = cos\theta + jsin\theta$$

Other forms:

$$e^{-j\theta} = \cos(-\theta) + jsin(-\theta)$$

$$e^{-j\theta} = cos\theta - jsin\theta$$

$$e^{j\theta} + e^{-j\theta} = 2\,cos\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Sampling continuous time signal x(t) into discrete time signal sequence x[n]:

$$x[n] = x(nT_s) = x(t) \vee t = nT_s$$

Derivative and Forward Difference:

$$D[x(t)] = \frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}$$

$$\Delta[x(nT_s)] = x((n+1)T_s) - x(nT_s)$$

Integral and Summation:

$$I(t)=\int_{t_0}^{t} x(\tau)\,d\tau \quad x(t)=\frac{dI(t)}{dt}$$

$$I(t)\approx\sum_{n} x(nT_s)\,p(n)$$ p(n) pulses of width T_s

DE for RC circuit with constant voltage:

$$v_i(t)=v_c(t)+\frac{d\,v_c(t)}{dt}\quad t\geq 0$$

Approximate Integral for a Trapezoid & DE:

$$v_c(t)=\int_{0}^{t}\left[v_i(\tau)-v_c(\tau)\right]d\tau+v_c(0)$$

$$v_c(nT)=\frac{T}{2+T}\left[v_i(nT)+v_i((n-1)T)\right]+\frac{2-T}{2+T}v_c((n-1)T),\, v_c(0)=0,\, n\geq 1$$

Converting between Polar and Rectangular:

$z=x+jy=|z|e^{j\angle(z)}$

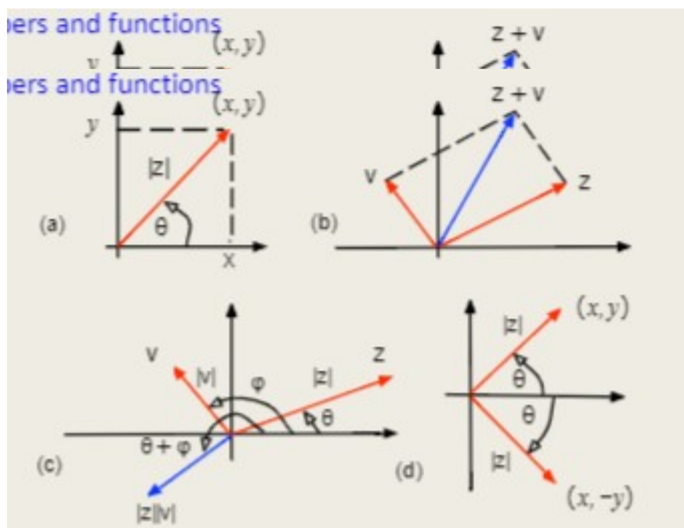
$v=u+jw=|v|e^{j\angle(v)}$

$z+v=(x+u)+j(y+w)$

$zv=|z||v|e^{j(\angle(z)+\angle(v))}$

$z^{¿}=x-jy=|z|e^{-j\angle z}$

- Be able to convert between the two in any quadrant



Euler's Identity:

$e^{j\theta}=\cos(\theta)+j\sin(\theta)$

$\cos(\theta)=\Re\left[e^{j\theta}\right]=\frac{e^{j\theta}+e^{-j\theta}}{2}$

$\sin(\theta)=Im\left[e^{j\theta}\right]=\frac{e^{j\theta}-e^{-j\theta}}{2j}$

Trig. Identities:

$$\sin(-\theta)=\frac{e^{-j\theta}-e^{j\theta}}{2j}=-\sin(\theta)$$

$$\cos(\pi+\theta)=e^{j\pi}\frac{e^{j\theta}+e^{-j\theta}}{2}=-\cos(\theta)$$

$$cos^2(\theta)=\left[\frac{e^{j\theta}+e^{-j\theta}}{2}\right]^2=\frac{1}{2}+\frac{1}{2}\cos(2\theta)$$

$$\sin(\theta)\cos(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j}\cdot\frac{e^{j\theta}+e^{-j\theta}}{2}=\frac{1}{2}\sin(2\theta)$$

Sinusoids and phasors

$$x(t) = A\cos(\Omega_0 t + \psi) \qquad -\infty < t < \infty$$

A amplitude, $\Omega_0 = 2\pi f_0$ frequency (rad/sec), ψ phase (rad)

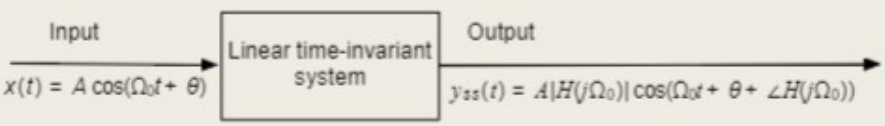
$$\boxed{\text{Phasor}: X = Ae^{j\psi},\ x(t) = \text{Re}[Xe^{j\Omega_0 t}]}$$

Eigenfunction property of LTI systems

Input: $x(t) = \text{Re}[Xe^{j\Omega_0 t}]$, input phasor $X = Ae^{j\theta}$

Output: $y(t) = \text{Re}[Ye^{j\Omega_0 t}]$, output phasor $Y = XH(j\Omega_0)$

Steady-state response



Frequency response of system

$$H(j\Omega_0) = |H(j\Omega_0)|e^{\angle H(j\Omega_0)}$$

Chapter 1:

$$\boxed{\begin{array}{l} x(.) : \mathcal{R} \to \mathcal{R} \;\; (\mathcal{C}) \\ \quad\quad t \to \;\; x(t) \end{array}}$$

Example: complex signal $y(t) = (1+j)e^{j\pi t/2}$, $0 \le t \le 10$, 0 otherwise

$$y(t) = \begin{cases} \sqrt{2}\left[\cos(\pi t/2 + \pi/4) + j\sin(\pi t/2 + \pi/4)\right], & 0 \le t \le 10, \\ 0, & \text{otherwise} \end{cases}$$

If $\quad x(t) = \sqrt{2}\cos(\pi t/2 + \pi/4), -\infty < t < \infty$

$p(t) = 1, \;\; 0 \le t \le 10, \;\; 0$ otherwise

then

$y(t) = [x(t) + jx(t-1)]p(t)$

Given signals x (t), y (t), constants α and τ , and function w(t):

- Signal addition/subtraction: x(t) + y (t), x (t) − y (t)
- Constant multiplication: αx (t)
- Time shifting
  - x (t − τ) is x (t) delayed by τ
  - x (t + τ) is x (t) advanced by τ
- Time scaling x(αt)
  - $\alpha = -1$, x (−t) reversed in time or reflected
  - $\alpha > 1$, x (αt) is x (t) compressed
  - $\alpha < 1$, x (αt) is x (t) expanded
- Time windowing x(t)w(t), w(t) window
- Integration

$$y(t) = \int_{t_0}^{t} x(\tau)d\tau + y(t_0)$$

Example

$$x(t) = \begin{cases} t & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

delayed by 1: $x(t-1) = \begin{cases} t-1 & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$

advanced by 1: $x(t+1) = \begin{cases} t+1 & -2 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$

reflected: $x(-t) = \begin{cases} -t & -1 < t < 1 \\ 0 & \text{otherwise} \end{cases}$

reflected and delayed by 1: $x(-t+1) = \begin{cases} -t+1 & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$

reflected and advanced by 1: $x(-t-1) = \begin{cases} -t-1 & -2 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$

compressed by 2: $x(2t) = \begin{cases} 2t & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

expanded by 2: $x(t/2) = \begin{cases} t/2 & -2 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$

x(t) even: x (t) = x (−t)
odd : x (t) = −x (−t)

sine → odd
cos → even

$x(t)$ is periodic if
(i) $x(t)$ defined in $-\infty < t < \infty$. and
(ii) there is $T_0 > 0$, the fundamental period of $x(t)$, such that $x(t + kT_0) = x(t)$, integer k

Energy of $x(t)$: $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt,$

Power of $x(t)$: $P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$T = \frac{2\pi}{b}$$

$\sin(bt)$
or $\cos(bt)$

- x (t) is finite-energy, or square integrable, if $E_x < \infty$
- x (t) is finite-power if $P_x < \infty$

x (t) period of fundamental period T_0 is

$$\boxed{P_x = \frac{1}{2T_0}\int_{t_0}^{t_0+T_0} x^2(t)dt}$$

- Complex exponential

$$\boxed{\begin{aligned} x(t) &= Ae^{at} = |A|e^{j\theta}e^{(r+j\Omega_0)t} \\ &= |A|e^{rt}[\cos(\Omega_0 t + \theta) + j\sin(\Omega_0 t + \theta)] \quad -\infty < t < \infty \end{aligned}}$$

Sinusoid

$$\boxed{A\cos(\Omega 0t + \theta) = A\sin(\Omega 0t + \theta + \pi/2) \qquad -\infty < t < \infty}$$

Modulation systems

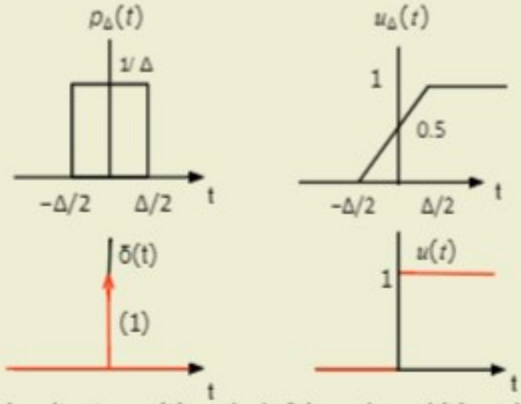
$$A(t)\cos(\Omega(t)t + \theta(t))$$

- Amplitude modulation or AM: A(t) changes according to the message, frequency and phase constant,
- Frequency modulation or FM: Ω(t) changes according to the message, amplitude and phase constant,
- Phase modulation or PM: θ(t) changes according to the message, amplitude and frequency constant

$$y = A\sin(B(x + C)) + D$$

- amplitude is **A**
- period is **2π/B**
- phase shift is **C** (positive is to the **left**)
- vertical shift is **D**

Unit-impulse signal



Unit-impulse δ(t) and unit−step u(t) as Δ → 0 in pulse pΔ(t) and its integral uΔ(t).

Unit-impulse

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \text{undefined} & t = 0 \end{cases}$$

$$\int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Unit–step signal

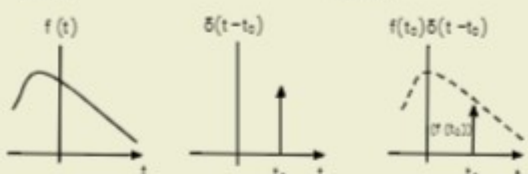
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t \leq 0 \end{cases}$$

Ramp signal

$$r(t) = tu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Sifting property of $\delta(t)$

$$\int_{-\infty}^{\infty} f(t)\delta(t-\tau)dt = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)dt = f(\tau)\int_{-\infty}^{\infty} \delta(t-\tau)dt = f(\tau), \quad \text{for any } \tau$$



Generic representation

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$