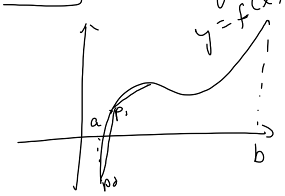
$\boxed{6.3}$ Arc length



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Def^n:

If f' is continuous on $[a,b]$, then the length (arc length) of the curve $y = f(x)$ from the point $A = (a, f(a))$

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to the point $B = (b, f(b))$ is the value of the integral

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

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$$\boxed{L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}$$

Or

Dealing with discontinuities in dy/dx

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$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy,$$

$x = g(y)$

$c \leq y \leq d$

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Ex) Given

$$y = f(x) = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - 1$$

Find the length of the curve from $x = 0$ to $x = 1$.

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$$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{4\sqrt{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{2} x^{\frac{1}{2}}$$

$$= 2\sqrt{2}\sqrt{x}$$

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$$\left(\frac{dy}{dx}\right)^2 = \left(2\sqrt{2}\sqrt{x}\right)^2$$

$$= 8x$$

$$L = \int_0^1 \sqrt{1 + 8x}\, dx$$

$$= \int \sqrt{u} \cdot \frac{1}{8} du$$

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Let $u = 1 + 8x$

$$\frac{1}{8} du = \frac{\cancel{8}}{\cancel{8}} dx \Rightarrow$$

$$L = \frac{1}{8}\int \sqrt{u}\, du$$

$$= \frac{1}{8} \cdot \frac{2}{3} u^{\frac{3}{2}}$$

$$= \frac{1}{12} u^{3/2}$$

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$$L = \frac{1}{12}(1+8x)^{3/2}\Big|_0^1$$

$$L = \frac{1}{12}(1+8)^{3/2} - \frac{1}{12}(1+8(0))^{3/2}$$

$$= \frac{1}{12} 9^{3/2} - \frac{1}{12} = \frac{26}{12}$$

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$$= \boxed{\frac{13}{6}}$$

Ex $y = \ln(\sec x)$

$$0 \le x \le \frac{\pi}{4}$$

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$$L = \int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = \ln(\sec x)$$

$$dy = \frac{1}{\cancel{\sec x}} \cdot \cancel{\sec x} \tan x \cdot dx$$

$$= \tan x \; dx$$

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$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x \, dx$$

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$$= \ln|\sec x + \tan x| \Big|_0^{\pi/4}$$

$$= \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right|$$

$$- \ln|\sec 0 + \tan 0|$$

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$$= \ln|\sqrt{2} + 1|$$

$$- \underbrace{\ln|1 + 0|}_{= 0}$$

$$\boxed{= \ln|\sqrt{2} + 1|}$$

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