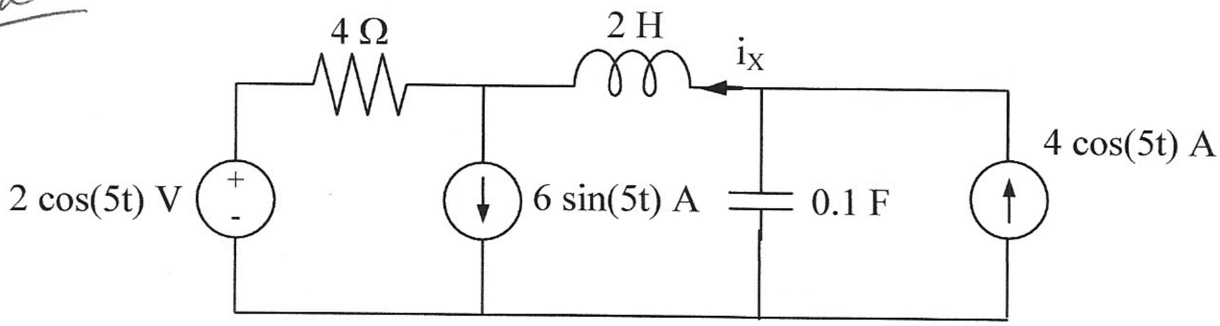
Solutions

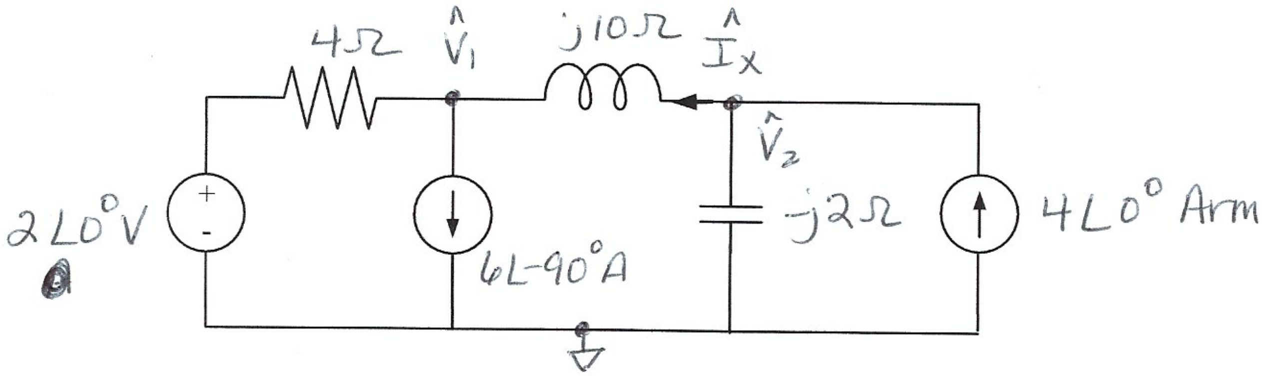
Nodal

$\omega = 5\,rad/s$



$2\cos 5t\,V \rightarrow 2\angle 0^\circ\,V$ $\quad$ $4\cos(5t)A \rightarrow 4\angle 0^\circ A$ $\quad$ $2H \rightarrow j10\Omega$

$6\sin(5t)A \rightarrow 6\angle -90^\circ A$ $\quad$ $4\Omega \rightarrow 4\Omega$ $\quad$ $0.1F \rightarrow -j2\Omega$



Know:

$$\hat{I}_X = \frac{\hat{V}_2 - \hat{V}_1}{j10\Omega}$$

(N1) $$\frac{\hat{V}_1 - 2\angle 0^\circ}{4} + 6\angle -90^\circ + \frac{\hat{V}_1 - \hat{V}_2}{j10} = 0$$

(N2) $$\frac{\hat{V}_2 - \hat{V}_1}{j10} + \frac{\hat{V}_2}{-j2} + (-4\angle 0^\circ) = 0$$

simplify:

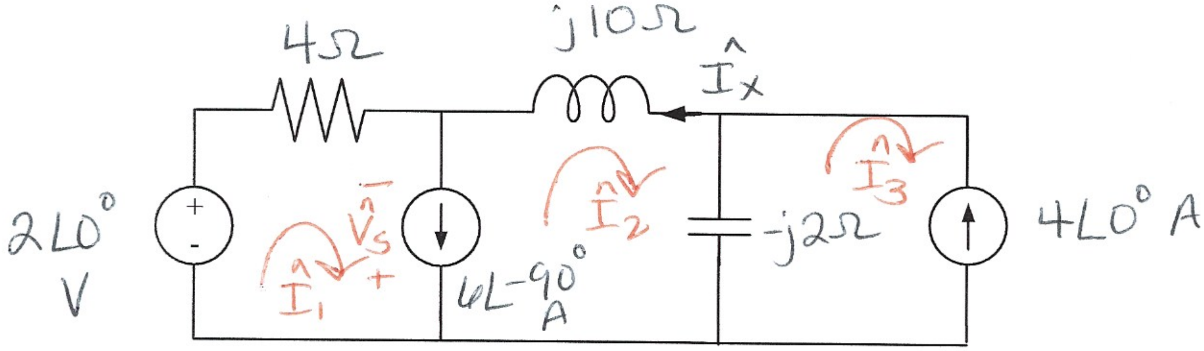
(N1) $\hat{V}_1(.27\angle -21.80^\circ) + \hat{V}_2(0.1\angle 90^\circ) = 6\angle 90^\circ$

(N2) $\hat{V}_1(0.1\angle 90^\circ) + \hat{V}_2(0.4\angle 90^\circ) = 4\angle 0^\circ$

$\hat{V}_1 = 21.48\angle 121.32^\circ\,V$ $\quad$ $\hat{I}_X = 3.57\angle -156.92^\circ A$

$\hat{V}_2 = 14.85\angle -79.17^\circ V$ $\quad$ $i_X(t) = 3.57\cos(5t - 156.92^\circ)A$

Mesh



Know: $\hat{I}_x = -\hat{I}_2$

$\hat{I}_3 = -4\angle 0°$ A

$\hat{I}_1 - \hat{I}_2 = 6\angle -90°$ A

m1: $2\angle 0° - 4\hat{I}_1 + \hat{V}_s = 0$

m2: $-\hat{V}_s - j10\hat{I}_2 - (-j2)(\hat{I}_2 - \hat{I}_3) = 0$

add m1 & m2: $2\angle 0° - 4\hat{I}_1 - j10\hat{I}_2 - (-j2)(\hat{I}_2 - \hat{I}_3) = 0$

simplify: $\hat{I}_1(-4) + \hat{I}_2(-j8) = \hat{I}_3(j2) - 2\angle 0$

$\hat{I}_1(-4) + \hat{I}_2(-j8) = 8.25\angle -104.04°$

known $\longrightarrow$ $\hat{I}_1(1) + \hat{I}_2(-1) = 6\angle -90°$

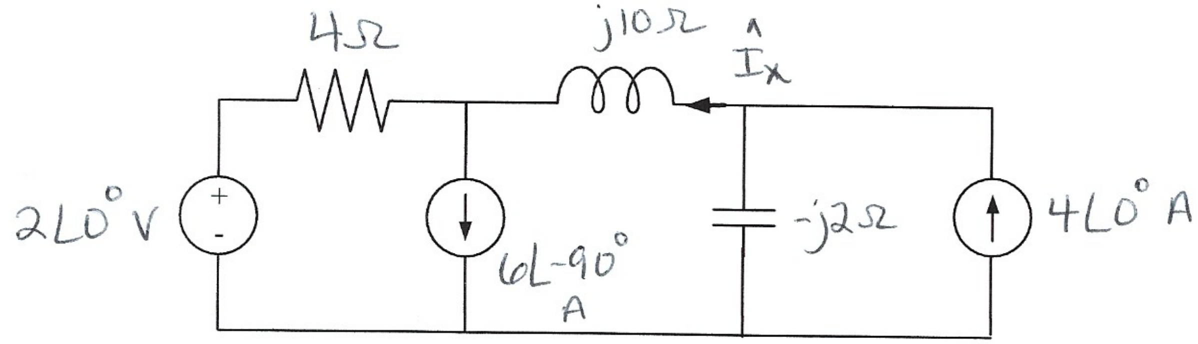
solve: $\hat{I}_1 = 5.66\angle -54.34°$ A

$\hat{I}_2 = 3.59\angle 22.99°$ A

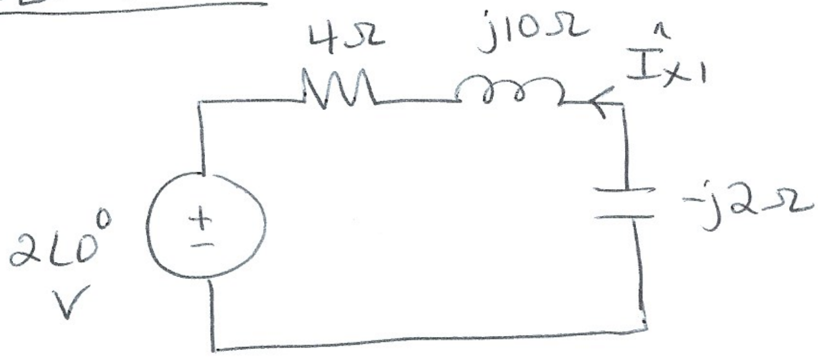
$\hat{I}_x = -\hat{I}_2 = 3.59\angle -157.01°$ A

$i_x(t) = 3.59\cos(5t - 157.01°)$ A

Superposition



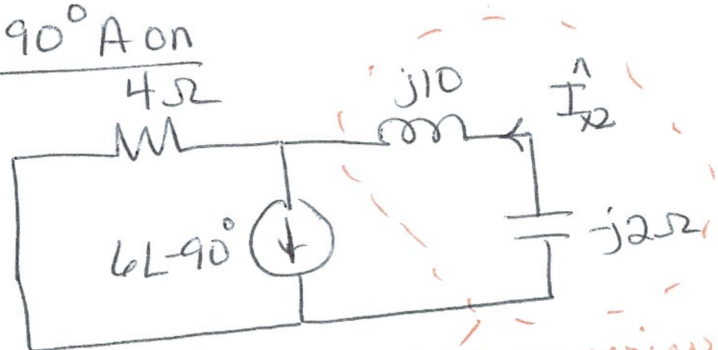
2∠0° V on



$$\hat{I}_{x1} = \frac{-(2\angle 0)}{4+j10-j2}$$

$$= 0.224 \angle 116.56^\circ \text{ A}$$

6∠-90° A on



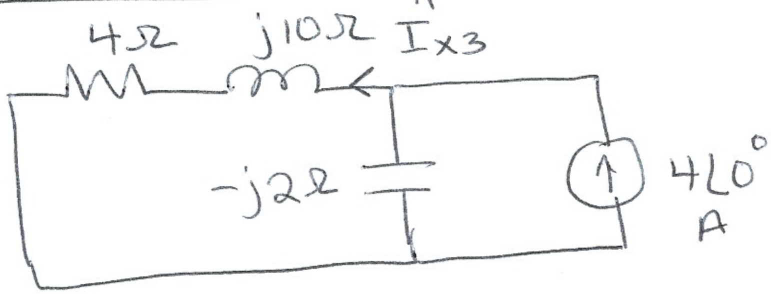
j8Ω in series

by current division

$$\hat{I}_{x2} = 6\angle -90 \left(\frac{4 \| j8}{j8} \right)$$

$$\hat{I}_{x2} = 2.68 \angle -153.43^\circ \text{ A}$$

4∠0° A on



by current division

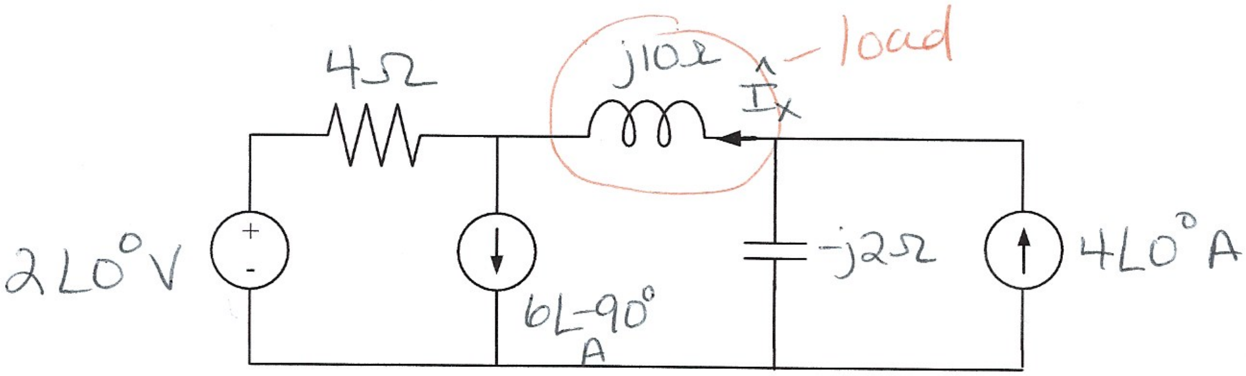
$$\hat{I}_{x3} = 4\angle 0 \left(\frac{(4+j10) \| -j2}{4+j10} \right)$$

$$\hat{I}_{x3} = 0.89 \angle -153.43^\circ \text{ A}$$

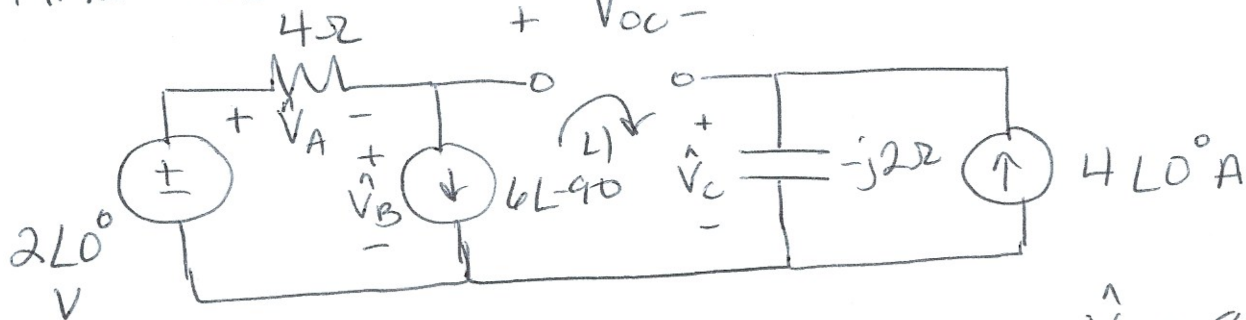
$$\hat{I}_x = \hat{I}_{x1} + \hat{I}_{x2} + \hat{I}_{x3} = 3.58 \angle -157.01^\circ \text{ A}$$

$$\boxed{i_x(t) = 3.58 \cos(5t - 157.01^\circ) \text{ A}}$$

Solutions: thevenin



Find $\hat{V}_{oc}$



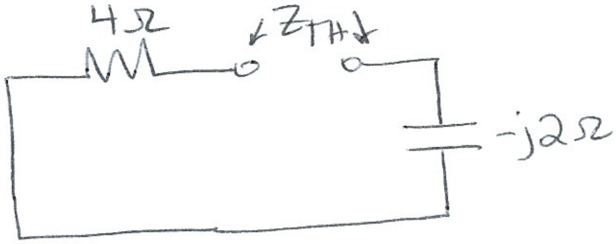
$$\hat{V}_C = 8\angle -90^\circ \text{ V}$$

$$\hat{V}_A = 24\angle -90^\circ \text{ V}$$

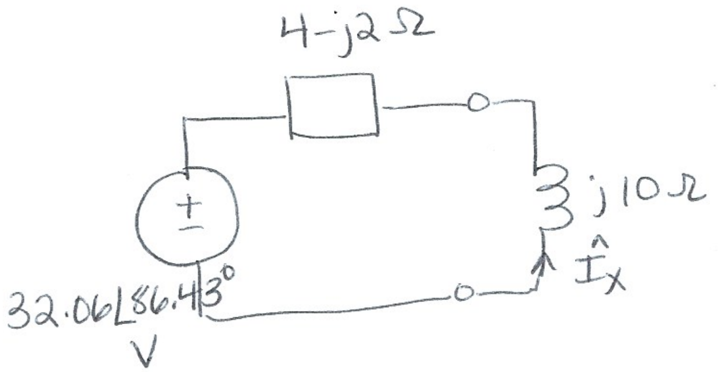
$$\hat{V}_B = 2\angle 0^\circ - \hat{V}_A = 24.08\angle 85.24^\circ \text{ V}$$

by KVL @ L1: $\hat{V}_B - \hat{V}_{oc} - \hat{V}_C = 0$ $\quad \hat{V}_{oc} = 32.06\angle 86.43^\circ$ V

Find Z_{TH}



$$Z_{TH} = 4 - j2\ \Omega$$

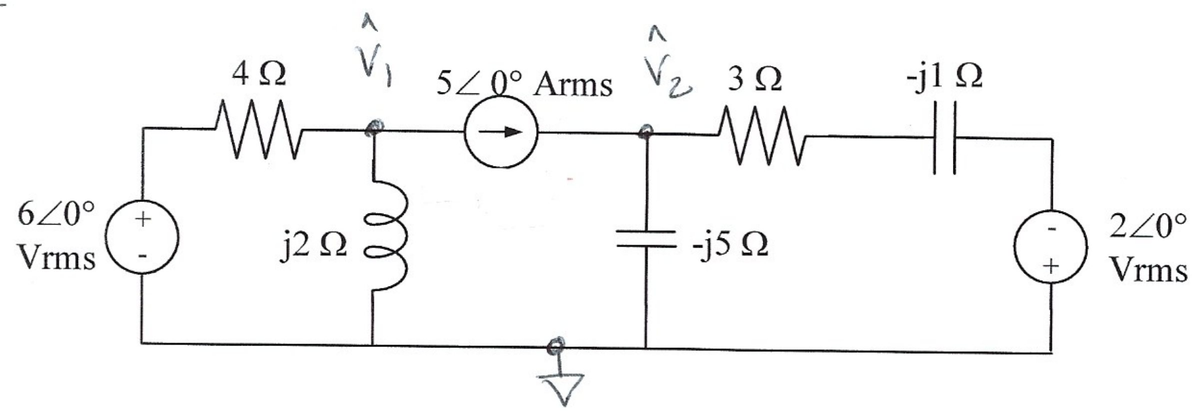


$$\hat{I}_x = \frac{-(32.06\angle 86.43^\circ)}{4-j2+j10}$$

$$\hat{I}_x = 3.58\angle -157.01^\circ \text{ A}$$

$$i_x(t) = 3.58 \cos(5t - 157.01^\circ \text{ A})$$

AC power
nodal



(N1) $$\frac{\hat{V}_1 - 6\angle 0^\circ}{4} + \frac{\hat{V}_1}{j2} + 5\angle 0^\circ = 0$$ $\hat{V}_1 = 6.26\angle -116.57^\circ$ Vrms

(N2) $$\frac{\hat{V}_2 + 2\angle 0^\circ}{3-j1} + \frac{\hat{V}_2}{-j5} + (-5\angle 0^\circ) = 0$$ $\hat{V}_2 = 10.38\angle -47.60^\circ$ Vrms

$\hat{S} = \hat{V} \cdot \hat{I}^*$

$6\angle 0^\circ$ Vrms: $\hat{S} = (6\angle 0^\circ)\left(\frac{\hat{V}_1 - 6\angle 0^\circ}{4}\right)^* = 15.65\angle 147.53^\circ$ VA Abs

$$\boxed{\hat{S} = 15.65\angle -32.46^\circ \text{ VA Del}}$$

$5\angle 0^\circ$ Arms: $\hat{S} = (\hat{V}_2 - \hat{V}_1)(5\angle 0)^* = \boxed{50.07\angle -11.91^\circ \text{ VA, Del}}$

$2\angle 0^\circ$ Vrms $\hat{S} = (2\angle 0^\circ)\left(\frac{\hat{V}_2 + 2\angle 0^\circ}{3-j1}\right)^*$

$$\boxed{\hat{S} = 7.48\angle 21.99^\circ \text{ VA, del}}$$

$$\sum \hat{S}_{del} = 70.94\angle -12.98^\circ \text{ VA}$$

AC power nodal continued.

impedances

4Ω: voltage across 4Ω is $\hat{V}_1 - 6\angle 0 = 10.43\angle -147.53^\circ$ Vrms

$P = \frac{(10.43)^2}{4} = 27.20\text{ W} \Rightarrow \boxed{\hat{S} = 27.20\angle 0^\circ \text{ VA Abs}}$

$Q = 0$

3Ω: current through 3Ω is $\frac{\hat{V}_2 + 2\angle 0^\circ}{(3-j1)} = 3.74\angle -21.99^\circ$ Arms

$P = (3.74)^2(3) = 41.92\text{ W}$, $\boxed{\hat{S} = 41.92\angle 0^\circ \text{ VA, Abs}}$

$Q = 0$

$j2\Omega$: $P = 0$ $\boxed{\hat{S} = 19.59\angle 90^\circ \text{ VA, Abs}}$

$Q = \frac{|\hat{V}_1|^2}{2} = 19.59\text{ VAR}$

$-j5\Omega$: $P = 0$ $\boxed{\hat{S} = 21.55\angle -90^\circ \text{ VA, Abs}}$

$Q = \frac{|\hat{V}_2|^2}{-5} = -21.55\text{ VAR}$

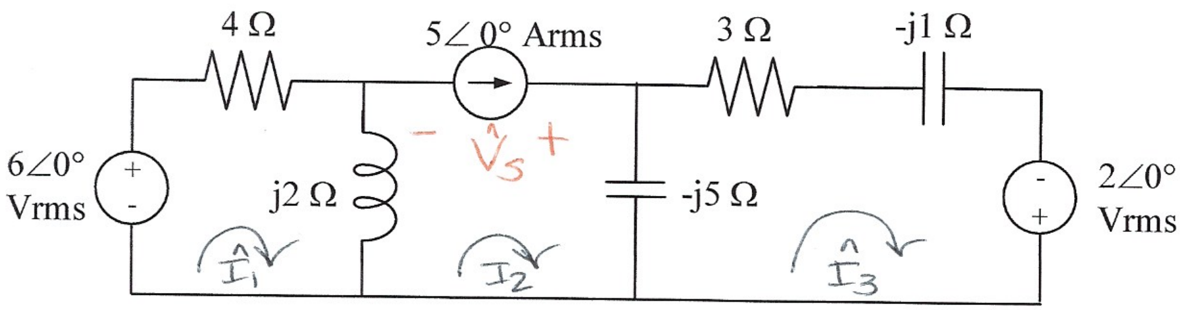
$-j1\Omega$: current is $3.74\angle -21.99^\circ$ Arms

$P = 0$ $Q = (3.74)^2(-1) = -13.97\text{ VAR}$

$\boxed{\hat{S} = 13.97\angle -90^\circ \text{ VA, Abs}}$

$\Sigma \hat{S}_{abs} = 70.93\angle -12.98^\circ\text{ VA}$ checks w/ $\Sigma \hat{S}_{del}$:)

AC power mesh



$\hat{I}_2 = 5\angle 0^\circ$ Arms

m1 : $6\angle 0^\circ - 4\hat{I}_1 - j2(\hat{I}_1 - \hat{I}_2) = 0$

m2 : don't need now

m3 : $-(3-j1)\hat{I}_3 + 2\angle 0 - (-j5)(\hat{I}_3 - \hat{I}_2) = 0$

solve

$\hat{I}_1 = 2.61\angle 32.47^\circ$ Arms

$\hat{I}_3 = 3.74\angle -21.99^\circ$ Arms

m2:

$-j2(\hat{I}_2 - \hat{I}_1) + \hat{V}_s - (-j5)(\hat{I}_2 - \hat{I}_3) = 0$

$\hat{V}_s = 10.02\angle -11.89^\circ$ Vrms

Complex Power

$6\angle 0^\circ$ Vrms : $\hat{S} = (6\angle 0^\circ)(\hat{I}_1)^* = 15.66\angle -32.47^\circ$ VA Del

$5\angle 0^\circ$ Arms : $\hat{S} = \hat{V}_s \cdot (5\angle 0)^* = 50.10\angle -11.89^\circ$ VA Del

$2\angle 0^\circ$ Vrms $\hat{S} = 2\angle 0^\circ \cdot \hat{I}_3^* = 7.48\angle 21.99^\circ$ VA, Del

$$\boxed{\Sigma \hat{S}_{Del} = 70.98\angle -12.97^\circ \text{ VA}}$$

4Ω : $P = I_1^2 \cdot 4 = 27.25$ W ; $Q = 0$ $\hat{S} = 27.25\angle 0^\circ$ VA, Abs

3Ω : $P = I_3^2 \cdot 3 = 41.96$ W , $Q = 0$ $\hat{S} = 41.96\angle 0^\circ$ VA, Abs

$j2\Omega$: $P = 0$; $Q = |\hat{I}_1 - \hat{I}_2|^2(2) = (3.13)^2(2) = 19.58$ VAR

$\hat{S} = 19.58\angle 90^\circ$ VA, Abs

Mesh AC power cont

$-j5\Omega$: $P = 0$

$$Q = |\hat{I}_2 - \hat{I}_3|^2(-5)$$

$$= (2.08)^2(-5)$$

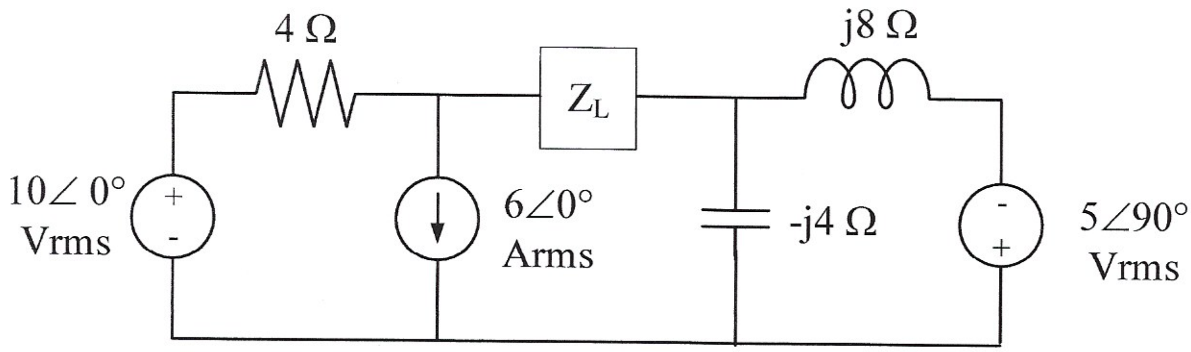
$$= -21.54 \text{ VAR}$$

$$\hat{S} = 21.54\angle -90^\circ \text{ VA, Abs}$$

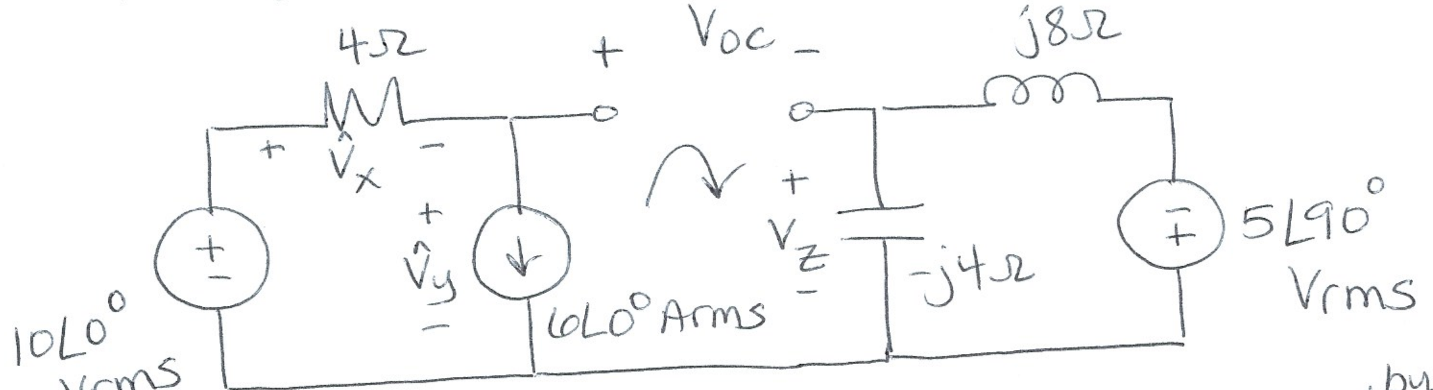
$-j1\Omega$: $P = 0$

$$Q = I_3^2(-1) = -13.99 \text{ VAR}$$

$$\hat{S} = 13.99\angle -90^\circ \text{ VA Abs}$$



Find $\hat{V}_{oc}$



by KVL

$\hat{V}_x = 4(6\angle 0^\circ) = 24\angle 0^\circ$ Vrms $\quad \hat{V}_y = (10\angle 0) - \hat{V}_x$

$= 14\angle 180^\circ$ Vrms

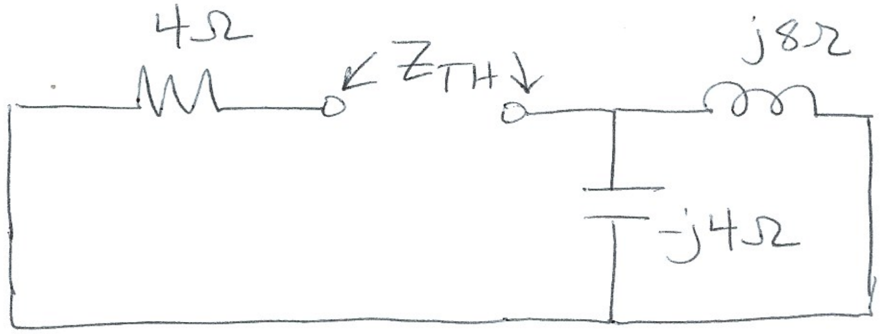
$\hat{V}_Z = -(5\angle 90)\left(\dfrac{-j4}{-j4+j8}\right) = 5\angle 90^\circ$ Vrms

by volt. division

by KVL: $\hat{V}_y - \hat{V}_{oc} - \hat{V}_Z = 0$

$\hat{V}_{oc} = 14.87\angle -160.35^\circ$ Vrms max power cont

Find Z_{TH} & Z_L



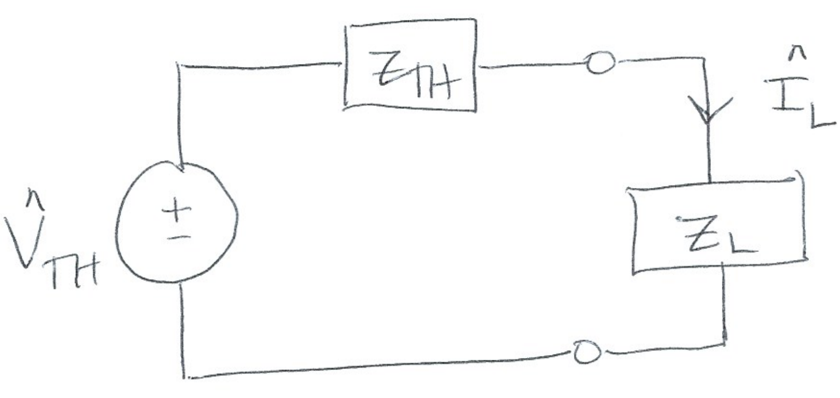
$$Z_{TH} = 4 + (j8 \,||\, -j4)$$

$$= 8.94 \angle -63.43^\circ \, \Omega$$

$$= 4 - j8 \, \Omega$$

$$Z_L = Z_{TH}^{*} = 4 + j8 \, \Omega$$

$$= 8.94 \angle 63.43^\circ \, \Omega$$



$$\hat{I}_L = \frac{\hat{V}_{TH}}{Z_{TH} + Z_L}$$

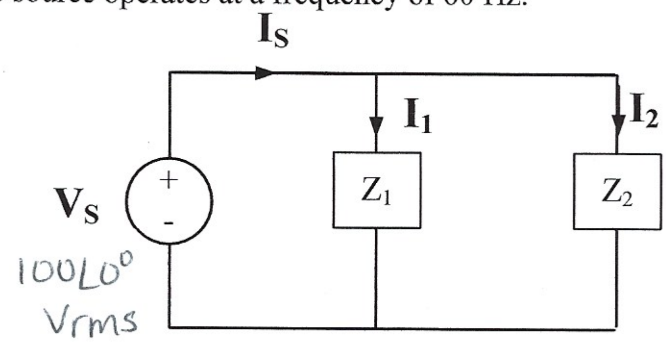
$$\hat{I}_L = 1.86 \angle -160.35^\circ \text{ Arms}$$

$$P_L = (I_L)^2 \cdot \mathrm{Re}[Z_L]$$

$$P_L = (1.86)^2 (4) = 13.82 \text{ W Abs}$$

A voltage source provides 100 Vrms to 2 loads. The source operates at a frequency of 60 Hz. The details of the loads are given below.

Z_1: $P_1 = 20$ kW with $pf_1=0.5$ lagging
Z_2: $\mathbf{S}_2=10\angle 63°$ kVA



Load 1 : $P_1 = V_S I_1 pf_1$

$20\times10^3 = 100(I_1)(0.5)$ $\qquad I_1 = 400$ Arms

$\angle Z_1 = \cos^{-1}(pf_1) = 60°$

$\boxed{\hat{I}_1 = 400\angle{-60°} \text{ Arms} \qquad Z_1 = \frac{\hat{V}_S}{\hat{I}_1} = 0.25\angle 60°\ \Omega}$

Load 2: $\hat{S}_2 = \hat{V}_S \cdot \hat{I}_2^*$

$\hat{I}_2 = \left(\frac{\hat{S}_2}{\hat{V}_S}\right)^*$

$\hat{I}_2 = \left(\frac{(10\times10^3)\angle 63°}{100\angle 0°}\right)$

$\boxed{\hat{I}_2 = 100\angle{-63°} \text{ Arms} \qquad Z_2 = \frac{\hat{V}_S}{\hat{I}_2} = 1\angle 63°\ \Omega}$

$\hat{I}_S = \hat{I}_1 + \hat{I}_2$

$\boxed{\hat{I}_S = 499.89\angle{-60.6°} \text{ Arms}}$

check power

$P_S = P_1 + P_2$

$P_S = (20 + Re[\hat{S}_2])$ kW

$P_S = (20 + 4.54)$ kW

$= (20 + 4.54)$ kW

$= 24.54$ kW

$\boxed{P_S = 24.54 \text{ kW}}$

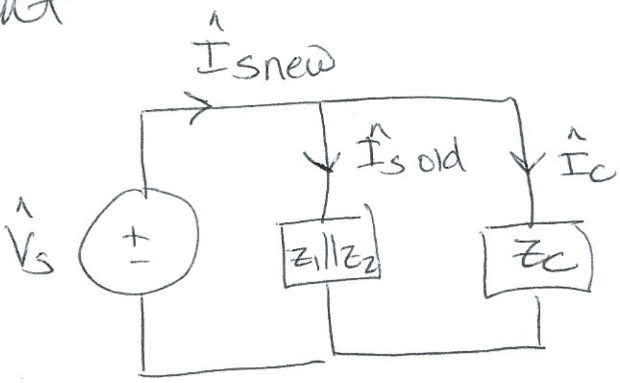
$P_S = V_S I_S \cos(\theta - \phi)$

$= (100)(499.89)\cos(0 - (-60.6))$

$\boxed{P_S = 24.54 \text{ kW}}$

Compensating Load cont

d)



$pf_{CL} = 0.95$ lag

$P_s = 24.54\,kW$

$\hat{I}_{sold} = 499.89 \angle -60.6^\circ$ Arms

$\hat{V}_s = 100 \angle 0^\circ$ Vrms

$\angle Z_{CL} = \cos^{-1}(.95)$
$= 18.19^\circ$

$P_s = V_s (I_{snew})\, pf_{CL}$

$24.54 \times 10^3 = 100 (I_{snew})(0.95)$

$I_{snew} = 258.31$ Arms

$\hat{I}_{snew} = 258.31 \angle -18.19^\circ$ Arms

$f = 60\,Hz$
$\downarrow$
$\omega \cong 377 \frac{rad}{s}$

by KCL $\hat{I}_c = \hat{I}_{snew} - \hat{I}_{sold}$

$\boxed{\hat{I}_c = 354.87 \angle 90^\circ \text{ Arms}}$

$\boxed{Z_c = \frac{\hat{V}_s}{\hat{I}_c} = 0.282 \angle -90^\circ \Omega}$

$\frac{1}{\omega C} = 0.282$

$\boxed{C = 9.41\,mF}$

e) Design #2

$I_{snew} = 0.15\, I_{sold} = 0.15(499.89) = 74.98$ Arms

$P_s = V_s I_{snew}\, pf_{CL}$

$24.54 \times 10^3 = (100)(74.98)\, pf_{CL}$

$\boxed{pf_{CL} = 3.273}$

Not possible because maximum pf is 1.