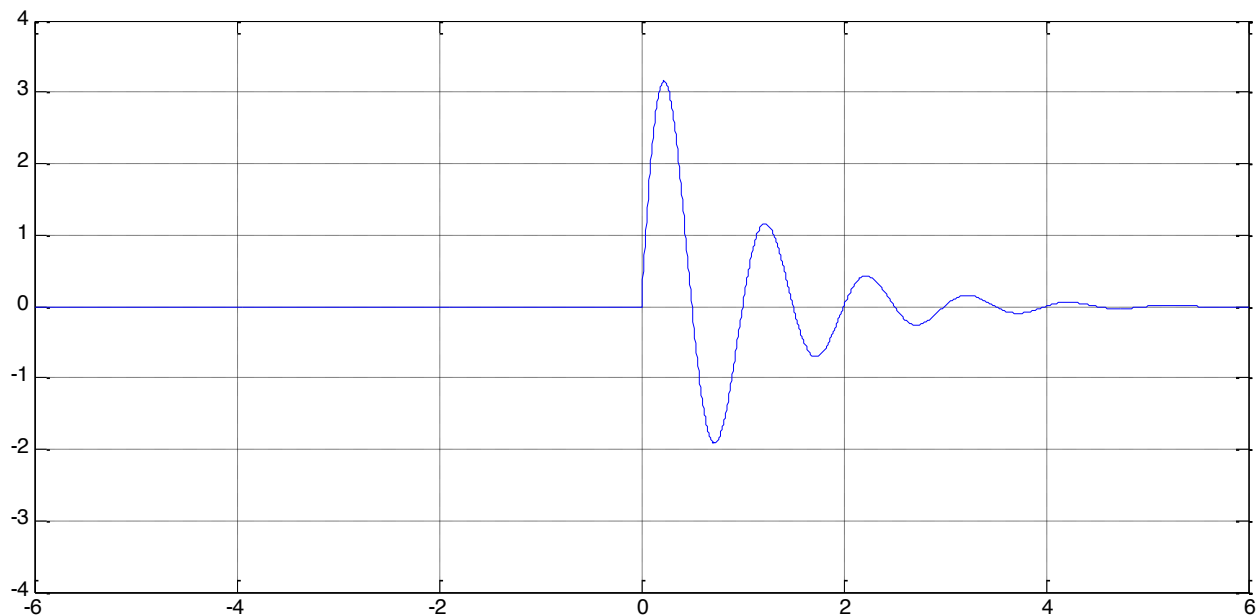


1. A microcontroller system has a 16 bit AD converter, 64,000 bytes of available RAM, and samples stereo audio signal with sampling frequency of 10 KHz. How many seconds of audio signal can be stored in memory?

$$T = 64000 \text{ bytes} / (2 \text{ channels} * 2 \text{ bytes/sample} * 10000 \text{ samples/s}) = 1.6 \text{ s}$$

2. Plot function $x(t) = e^{-t} \cdot 4 \sin(2\pi t) \cdot u(t)$



(2 points) What is the value of $x(t)$ for $t = 0$?

$x(t) =$ undefined

3. Consider the periodic signal $x(t) = \cos(12\pi \cdot t) + 3 \cdot \cos(16\pi \cdot t)$, $-\infty < t < \infty$.

What is the period T_0 of $x(t)$? $T_0 = 0.5$ second

$$x(t) = x_1(t) + x_2(t)$$

$$T_1 = 2\pi / (2\pi \cdot 6) = 1/6 \text{ s}$$

$$T_2 = 2\pi / (2\pi \cdot 8) = 1/8 \text{ s}$$

$$T_0 = N \cdot T_1 = M \cdot T_2 \rightarrow 8N = 6M \rightarrow N/M = 3/4 \rightarrow T_0 = 4 \cdot (1/8) = 0.5 \text{ s}$$

What is the average power of $x(t)$?

$$\int_0^x \cos^2(x) dx = \int_0^x \frac{1}{2} (1 + \cos(2x)) = \frac{1}{2} \int_0^x dx + \frac{1}{4} \int_0^x \cos(y) dy = \left(\frac{x}{2} + \frac{1}{4} \sin(2x) \right) \Big|_0^x$$

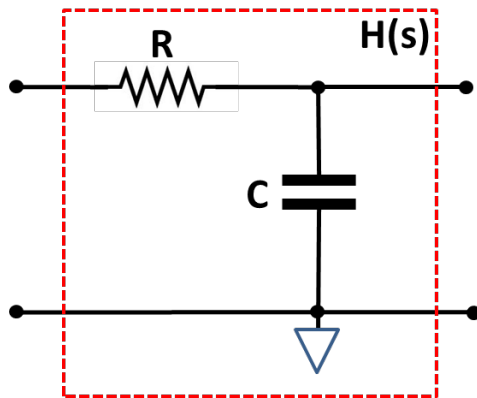
$$\int_0^t \cos^2(x) dx = \left(\frac{x}{2} + \frac{1}{4} \sin(2x) \right) \Big|_0^t = \frac{t}{2} + \frac{1}{4} \sin(2t) \Rightarrow \text{for } t = T \int_0^T \cos^2(x) dx = \frac{1}{2}$$

$$P_{x1} = \frac{1}{T_1} \int_0^{T_1} x_1^2(t) dt = \frac{1}{0.5} \cdot \left(\frac{x}{2} + \frac{1}{4} \sin(2x) \right) \Big|_0^{T_1} = \frac{1}{T_1} \left(\frac{T_1}{2} + \frac{1}{4} \sin\left(12\pi \cdot \frac{1}{6}\right) \right) = 1 \cdot \frac{1}{2} = 0.5$$

$$P_{x2} = \frac{1}{T_2} \int_0^{T_2} x_2^2(t) dt = \frac{1}{T_2} \int_0^{T_2} (3 \cos(16\pi t))^2 dt = 9 \cdot \frac{1}{T_2} \int_0^{T_2} \cos^2(16\pi t) dt = 9 \cdot \frac{1}{2} = 4.5$$

$$P = P_{x1} + P_{x2} = 0.5 + 4.5 = 5$$

4. (4 points)



$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

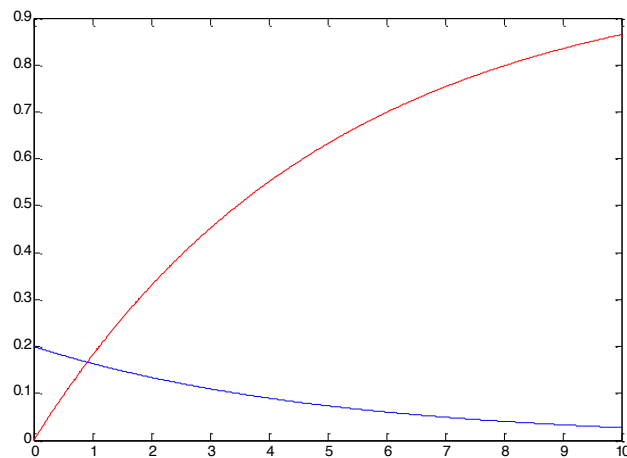
$$h(t) = \frac{1}{RC} \cdot e^{-\frac{t}{RC}}$$

b) (6 points) Step response:

$$\frac{1}{RC} = \frac{1}{5M\Omega \cdot 1\mu F} = 0.2$$

$$S(s) = \frac{1}{s} H(s) = \frac{1}{s} \frac{0.2}{s + 0.2} = \frac{A}{s} + \frac{B}{s + 0.2} = \frac{1}{s} - \frac{1}{s + 0.2}$$

$$s(t) = (1 - e^{-0.2t}) \cdot u(t)$$



5. A system with input $x(t)$ and output $y(t)$ is defined by the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = x(t)$$

Find the impulse response $h(t)$ and the unit-step response $s(t)$.

If $Y(s) = \mathcal{L}[y(t)]$ and $X(s) = \mathcal{L}[x(t)]$, then

$$Y(s) [s^2 + 3s + 2] = X(s)$$

To find impulse response, we let $x(t) = \delta(t)$, and $X(s) = 1$, then $Y(s)$ is equal to $H(s)$:

$$Y(s) = H(s) = \frac{1}{s^2 + 3s + 2} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

We find

$$A = H(s)(s+1) \Big|_{s=-1} = \frac{1}{-1+2} = 1$$

and

$$B = H(s)(s+2) \Big|_{s=-2} = \frac{1}{-2+1} = -1$$

therefore:

$$h(t) = [e^{-t} - e^{-2t}] \cdot u(t)$$

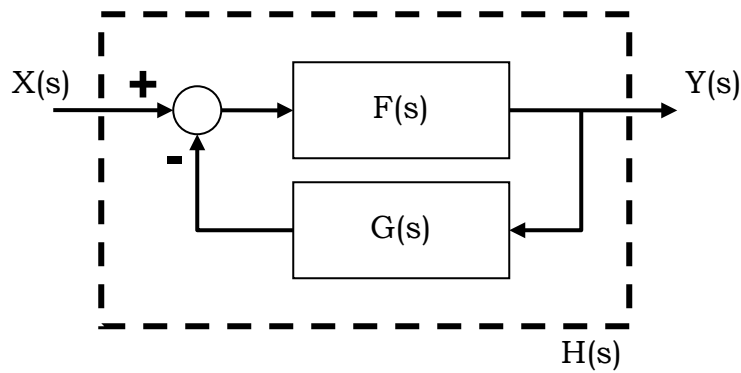
Similarly, unit step response is:

$$S(s) = \frac{H(s)}{s} = \frac{1}{s \cdot (s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

and $A=0.5$, $B= -1$, $C=0.5$, therefore:

$$s(t) = 0.5 \cdot u(t) - e^{-t} \cdot u(t) + 0.5e^{-2t} \cdot u(t)$$

6. (5 points) What is the transfer function $H(s)$ of the system represented below?



$$Y(s) = F(s) * (X(s) - G(s) * Y(s)) = F(s) * X(s) - F(s) * G(s) * Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

b) (10 points) Transfer function of the feedback block is

$$G(s) = \frac{Z_{R||C}}{Ls + Z_{R||C}}$$

$$Z_{R||C} = \frac{R \frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{R}{RCs + 1}$$

$$G(s) = \frac{R}{R + Ls(RCs + 1)} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Transfer function of the system is:

$$H(s) = \frac{F(s)}{1 + F(s) \cdot G(s)}$$

$$F(s) = A \quad \text{and} \quad G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$H(s) = \frac{A}{1 + A \left(\frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \right)} \quad \text{for } A \rightarrow \infty \quad H(s) = LCs^2 + \frac{L}{R}s + 1$$

7. Determine the fundamental frequency ω_0 of

$$x(t) = 2 + 8 \cdot \sin\left(\frac{2\pi}{6} t\right)$$

and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\omega_0 = 2\pi/6$$

$$\begin{aligned} x(t) &= 2 + 8 \cdot \frac{1}{2} \left(e^{j\frac{2\pi}{6}t} - e^{-j\frac{2\pi}{6}t} \right) = a_{-1}e^{-j\omega_0 t} + a_0e^{j0t} + a_1e^{j\omega_0 t} \\ &= a_{-1}e^{-j\frac{2\pi}{6}t} + a_0 + a_1e^{j\frac{2\pi}{6}t} \\ &= -4e^{-j\frac{2\pi}{6}t} + 2 + 4e^{j\frac{2\pi}{6}t} \end{aligned}$$

and $a_{-1} = -4$, $a_0 = 2$, $a_1 = 4$.

8. Represent magnitude and phase line spectra of raised sine signal:

$$x(t) = 2 + 8 \cdot \sin(100t) + 6 \cdot \cos(200t)$$

