Definite Integral
(Indefinite Integral)

(1) $\int x(x^2+4)^{-\frac{1}{3}}\,dx$

(2) $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$

Jan 18-11:18 AM

(3) $\int \frac{(\ln x)^5}{x}\,dx$

(4) $\int \cos\theta\, e^{\sin\theta}\,d\theta = e^{\sin\theta} + C$

(5) $\int \sqrt{e^x}\,dx = 2e^{\frac{x}{2}} + C$

(6) $\int_0^7 \sqrt{4+3x}\,dx$

Jan 18-11:28 AM

(1) $u = x^2+4$

$\frac{du}{dx} = 2x$

or $du = 2x\,dx$

$\int x(x^2+4)^{-\frac{1}{3}}\,dx$

$= \frac{1}{2}\int u^{-\frac{1}{3}}\,du$

Jan 18-11:32 AM

$= \frac{1}{2}\cdot\frac{3}{2}u^{2/3}$

$= \frac{3}{4}u^{\frac{2}{3}} + C$

$= \frac{3}{4}(x^2+4)^{\frac{2}{3}} + C$

Jan 18-11:34 AM

(3) $\int \frac{(\ln x)^5}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^5\,du = \frac{u^6}{6} + C$

Jan 18-11:37 AM

$= \frac{(\ln x)^6}{6} + C$

$\int_1^e \frac{(\ln x)^5}{x}\,dx$

$= \left.\frac{(\ln x)^6}{6}\right|_1^e$

Jan 18-11:39 AM

$$= \frac{(\ln e)^6}{6} - \frac{(\ln 1)^6}{6}$$

$$= \frac{1}{6} - \frac{0}{6}$$

$$= \boxed{\frac{1}{6}}$$

Jan 18-11:40 AM

(4) $\int \cos\theta\, e^{\sin\theta}\, d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\int e^u\, du$

$= e^u + C = e^{\sin\theta} + C$

Jan 18-11:42 AM

(7) $\int_0^1 \frac{1+e^{3x}}{e^{3x}+3x}\, dx$

(8) $\int x\sqrt{4+x}\, dx$

(9) $\int_0^{\pi} \cos^2\left(\frac{\theta}{4}\right)\sin\left(\frac{\theta}{4}\right) d\theta$

Jan 18-11:44 AM

(6) $u = 4+3x$

$du = 3\,dx$

$\frac{1}{3}\int \sqrt{u}\, du$

$= \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}}$

$= \frac{2}{9}(4+3x)^{\frac{3}{2}}\Big|_0^7$

Jan 18-11:51 AM

$= \frac{2}{9}(4+3\cdot 7)^{3/2} - \frac{2}{9}(4+0)^{3/2}$

$= \frac{2}{9}\left[25^{\frac{3}{2}} - 4^{3/2}\right]$

$= \frac{2}{9}[125-8]$

Jan 18-11:52 AM

$= \frac{234}{9} = 26$

(8) $\int x\sqrt{4+x}\, dx$

$u = 4+x \Rightarrow x = u-4$

$du = dx$

$\int (u-4)\sqrt{u}\, du$

Jan 18-11:53 AM

$$= \int (u\sqrt{u} - 4\sqrt{u})\,du$$

$$= \int u^{3/2} - 4u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} - 4\cdot\frac{2}{3}u^{\frac{3}{2}}$$

$$= \frac{2}{5}(4+x)^{\frac{5}{2}} - \frac{8}{3}(4+x)^{3/2} + C$$

Jan 18-12:01 PM

$$\int \frac{e^{3x}+1}{e^{3x}+3x}\,dx$$

$u = e^{3x} + 3x$

$du = (3e^{3x} + 3)\,dx$

$= 3(e^{3x}+1)\,dx$

Jan 18-12:06 PM

$$\frac{1}{3}\int \frac{du}{u}$$

$$= \frac{1}{3}\ln|u|$$

$$= \frac{1}{3}\ln|e^{3x}+3x|\Big|_0^1$$

Jan 18-12:07 PM

$$= \frac{1}{3}\ln|e^3+3| - \frac{1}{3}\ln|e^0+0|$$

$\ln 1 = 0$

$$= \boxed{\frac{1}{3}\ln|e^3+3|}$$

Jan 18-12:08 PM