If $\vec{v} \neq \vec{0}$

(1) $\dfrac{\vec{v}}{|\vec{v}|}$ is a unit vector called the direction of $\vec{v}$

(2) The equation

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$\vec{v} = |\vec{v}| \dfrac{\vec{v}}{|\vec{v}|}$

expresses $\vec{v}$ as its length times its direction.

Ex If $\vec{v} = 3i - 5j$ is a velocity vector,

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express $\vec{v}$ as a product of its speed times its direction of motion.

$\vec{v} = 3i - 5j = \langle 3, -5 \rangle$

$|\vec{v}| = \sqrt{9 + 25} = \sqrt{34}$

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unit vector, $\dfrac{\vec{v}}{|\vec{v}|}$

$= \dfrac{3i - 5j}{\sqrt{34}}$

$\vec{v} = |\vec{v}| \dfrac{\vec{v}}{|\vec{v}|} \rightarrow$ direction

$\swarrow$

speed

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$\vec{v} = \sqrt{34}\left(\dfrac{3}{\sqrt{34}} i - \dfrac{5}{\sqrt{34}} j\right)$

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$\boxed{11.3}$ The Dot product

Def^n:

The dot product denoted by $\vec{u} \cdot \vec{v}$ ($\vec{u}$ dot $\vec{v}$) of

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Vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is the scalar quantity given by

$$\boxed{\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3}$$

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Ex) Given

$\vec{u} = 5i - 4j$

$\vec{v} = 2i - 2j + 3K$

find $\vec{u} \cdot \vec{v}$

$= (5)(2) + (-4)(-2) + (0)(3)$

$= 10 + 8 = \boxed{18}$

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Def^n:

The angle θ between two nonzero vectors $\vec{u}$ and $\vec{v}$ is given

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by $\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}\right)$

$0 \le \theta \le \pi$

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta}$$

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Two nonzero vectors $\vec{u}$ and $\vec{v}$ are orthogonal (perpendicular) if and only if

$\vec{u} \cdot \vec{v} = 0$

$\boxed{\text{i.e. } \theta = \frac{\pi}{2}}$

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Ex) $\vec{i} = \langle 1, 0 \rangle$

$\vec{j} = \langle 0, 1 \rangle$

$\vec{i} \cdot \vec{j} = 0 + 0 = 0$

Ex) $\vec{u} = \langle -1, 0, 5 \rangle$

$\vec{v} = \langle 0, 1, 2 \rangle$

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$\vec{u} \cdot \vec{v} = 0 + 0 + 10$

$= 10 \neq 0$

Not orthogonal

$\theta = 0 \Rightarrow \vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos 0$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cdot 1$

$\theta = \pi$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos \pi$

$= -|\vec{u}||\vec{v}|$

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Dot product properties

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and c is a scalar, then

(1) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

(2) $(c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v})$

$= c(\vec{u} \cdot \vec{v})$

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(3) $\vec{u} \cdot (\vec{v} + \vec{w})$

$= \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

(4) $\vec{u} \cdot \vec{u} = |\vec{u}|^2$

(5) $\vec{0} \cdot \vec{u} = 0$

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The vector projection of $\vec{u}$ onto $\vec{v}$ is the vector

$$\text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|v|^2} \right) \vec{v}$$

The scalar component of $\vec{u}$ in the direction

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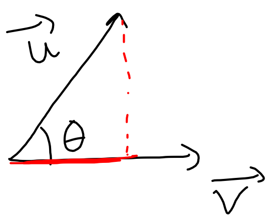

of $\vec{v}$ is the scalar

$$|u| \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

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(Ex) Determine the projection of

$\vec{u} = \langle 2, 1, -1 \rangle$

onto $\vec{v} = \langle 1, 0, -2 \rangle$

$$\text{Proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

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$$= \frac{2+0+2}{1+0+4}\left(\langle 1, 0, -2\rangle\right)$$

$$= \frac{4}{5}\langle 1, 0, -2\rangle$$

$$= \frac{4}{5} i - \frac{8}{5} K$$

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