$\boxed{6.1}$ Volumes using cross-sectional area :

Def^n: The volume of a solid of integrable cross sectional area $A(x)$ from $x=a$ to

Jan 22-11:16 AM

$x=b$ is the integral of A from a to b

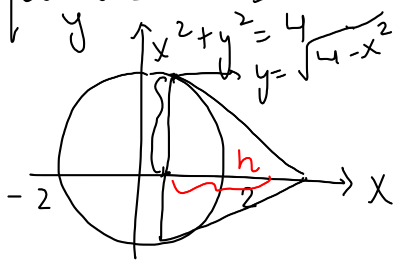
$$\boxed{V = \int_a^b A(x)\,dx}$$

Jan 22-11:38 AM

Ex Find the volume of a solid whose base is bounded by the circle $x^2+y^2=4$, the cross section perpendicular to the x-axis

Jan 22-11:38 AM

are equilateral Δ's.



$$V = \int_{-2}^{2} A(x)\,dx$$

Jan 22-11:40 AM

$$A(x) = \frac{1}{2} b h$$

$$b = 2\sqrt{4-x^2}$$

$$h^2 = \left(2\sqrt{4-x^2}\right)^2 - \left(\sqrt{4-x^2}\right)^2$$

$$h^2 = 4(4-x^2) - (4-x^2)$$

$$h^2 = 16 - 4x^2 - 4 + x^2$$

$$= 12 - 3x^2 = 3(4-x^2)$$

Jan 22-11:41 AM

$$h = \sqrt{3}\sqrt{4-x^2}$$

$$A(x) = \frac{1}{\cancel{2}} \cdot \cancel{2}\sqrt{4-x^2} \cdot \sqrt{3}\sqrt{4-x^2}$$

$$= \sqrt{3}(4-x^2)$$

$$\Rightarrow V = \int_{-2}^{2} \sqrt{3}(4-x^2)\,dx$$

Jan 22-11:45 AM

$$V = \sqrt{3}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$V = \sqrt{3}\left[\left(4(2) - \frac{2^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$

$$\boxed{V = 18.475}$$

Jan 22-11:47 AM

Solids of Revolution:

The solid generated by rotating a plane region about an axis in its plane is called a solid of revolution.

Jan 22-11:51 AM

(1) Disk method:

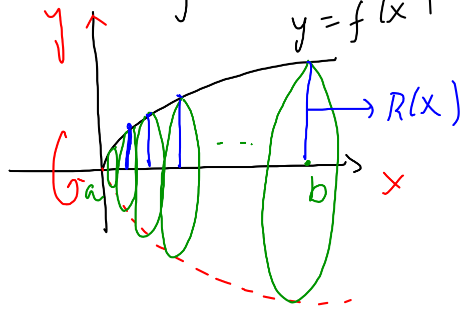
Volume by disks for rotation about x-axis:

$$V = \int_a^b A(x)\,dx$$

Where $A = \pi R(x)^2$

Jan 22-11:52 AM

Where $R(x)$ is the radius of the circle.

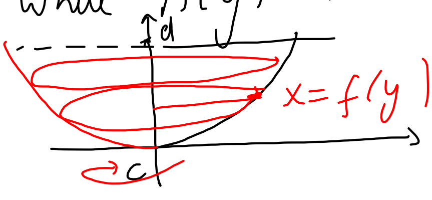


Jan 22-11:53 AM

Disk method: about y-axis

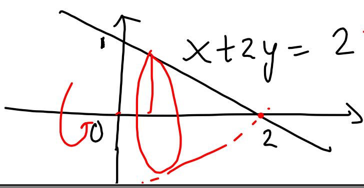
$$V = \int_c^d A(y)\,dy$$

Where $A(y) = \pi R(y)^2$



Jan 22-11:56 AM

Ex Find the volume of the solid generated by revolving the shaded region about the x-axis.



$\Rightarrow 2y = 2 - x$

$y = 1 - \frac{x}{2}$

Jan 22-11:58 AM

$$V = \int_0^2 A(x)\,dx$$

$$= \int_0^2 \pi R^2\,dx$$

$$= \int_0^2 \pi\left(1 - \frac{x}{2}\right)^2 dx$$

Jan 22-12:00 PM

$$V = \pi \int_0^2 \left(1 - x + \frac{x^2}{4}\right) dx$$

$$V = \pi\left[x - \frac{x^2}{2} + \frac{x^3}{12}\right]_0^2$$

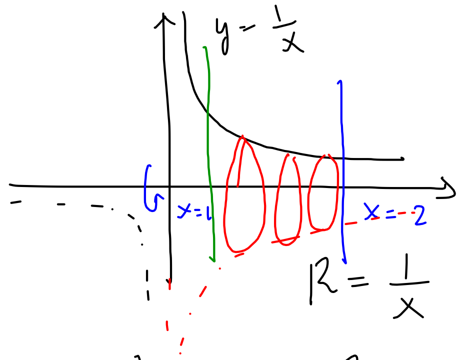
$$V = \pi\left[2 - \frac{2^2}{2} + \frac{2^3}{12}\right]$$

$$= \boxed{2\pi/3}$$

Jan 22-12:01 PM

Ex) Find the volume of the solid enclosed under the region $y = \frac{1}{x}$, $x = 1$ and $x = 2$ by revolving about x-axis.

Jan 22-12:03 PM



$$V = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx$$

Jan 22-12:04 PM

$$V = \pi \int_1^2 x^{-2}\,dx$$

$$= \pi\left(-\frac{1}{x}\right)\Big|_1^2$$

$$= \pi\left(-\frac{1}{2}\right) + \pi \cdot \frac{1}{1}$$

$$V = \boxed{\frac{\pi}{2}}$$

Jan 22-12:06 PM

Washer method:

About x-axis:

$$V = \int_a^b A(x)\,dx$$

$$= \int_a^b \left[\pi R(x)^2 - \pi r(x)^2\right] dx$$

Jan 22-12:08 PM

$$V = \int_a^b \left(\pi R_{out}^2 - \pi r_{in}^2\right) dx$$

About y-axis :

$$V = \int_c^d A(y)\, dy$$

Jan 22-12:10 PM

$$V = \int_c^d \left(\pi R_{out}^2 - \pi r_{in}^2\right) dy$$

Jan 22-12:11 PM