Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #3

Due: Monday, March 8 at 9:35 am
Please upload PDF files of the assignment to Canvas

Student name:

Nolan Anderson

1 20	2 20	3 20	4 20	5 20	Total

---

1. (20 points) What is the inverse Laplace transform of the function

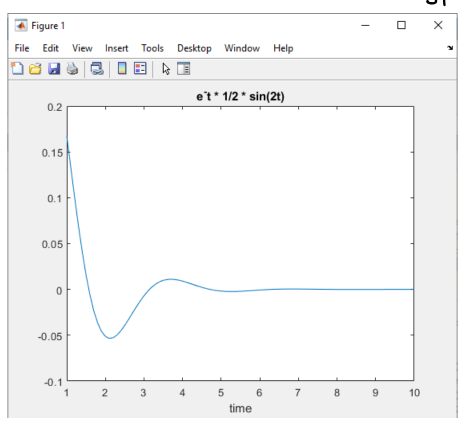
$$X(s) = \frac{1}{(s+1)^2 + 4}$$

$\frac{1}{s^n} = \frac{t^{n-1}}{(n-1)!} \rightarrow \frac{t^{2-1}}{(2-1)!} = \frac{t}{1!}$

$e^{-at}u(t) = \frac{1}{s+a} \rightarrow e^{-4t}u(t)$

Plot the function in time domain.

$te^{-t}u(t) = \frac{1}{(s+1)^2} \rightarrow \boxed{e^{-t}\frac{1}{2}\sin(2t)}$



2. (20 points) A system with input x(t) and output y(t) is defined by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Find the impulse response *h(t)* and the unit-step response *s(t)*. $h(t)$; $s(t)$

$\left.\begin{matrix} Y(s) = \mathcal{L}[y(t)] \\ X(s) = \mathcal{L}[x(t)] \end{matrix}\right\}$ $Y(s)[s^2+3s+2] = X(t)$; $Y(s) = H(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

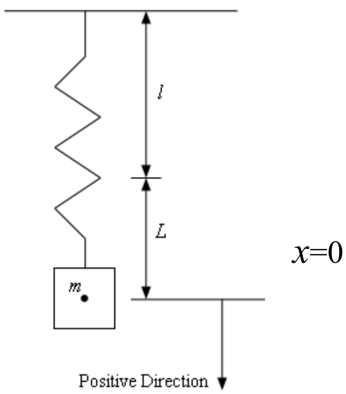
$A = H(s)(s+1)\Big|_{-1} = \frac{1}{-1+2} = 1$ $B = H(s)(s+2)\Big|_{-2}$ $\frac{1}{-2+1} = -1 \longrightarrow \boxed{h(t) = [e^{-t} - e^{-2t}]\cdot u(t)}$

$S(s) = \frac{H(s)}{s} = \frac{1}{s\cdot(s+1)(s+2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$;

$$\boxed{S(t) = 0.5\cdot u(t) - e^{-t}\cdot u(t) + 0.5e^{-2t}\cdot u(t)}$$

3. (20 points) A 1 kg weight is hung on the end of a vertically suspended spring, thereby stretching the spring L= 20 cm. The weight is raised 10 cm above its equilibrium position and released from rest at time t=0. Find the displacement x of the weight from its equilibrium position at time t=2.5s. Use g=10m/s^2.

All forces, velocities, and displacements in the upward direction will be negative, according to the Figure below.



$F = kL$, $k = \frac{F}{L} = \frac{mg}{L} = \frac{1[kg]\ 10[m/s^2]}{0.2[m]}$

$k = 50\ kg/s^2$

$m\ddot{x} + kx = 0$

$-0.10m \quad x(0) = 0$

$(s^2 + 50)\, X(s) = -0.10$

$X(s) = -0.10/\, s^2 + 50$

$x(t) = \mathcal{L}(x(s)) = -0.01 \sin(7.07t)$

$x(2.5) = -0.01 \sin(7.07 \times 2.5)$

$\boxed{x(2.5) = -0.043m}$

4. (20 points) An unstable system can be stabilized by using negative feedback with gain K in the feedback loop. For the given unstable system with pole in the right-hand s-plane:

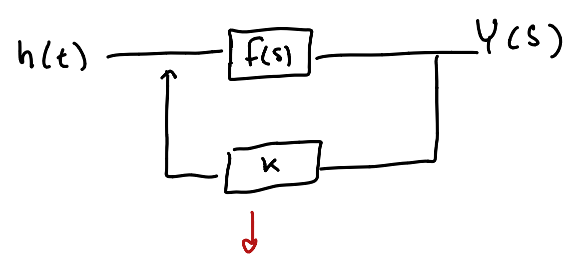
$\mathcal{L}\left\{\frac{1}{-4s+3}\right\} = \boxed{-\frac{1}{4}e^{3/4t}} \to h(t)$

$$F(s) = \frac{1}{2 \cdot s - 3}$$

what should be the value of the gain K (K is integer greater than zero) with $F(s)$ in the forward loop that will make the system BIBO stable.

Draw block diagram of the system, write overall transfer function of the system $H(s)$, and impulse response $h(t)$.

$F(s) = \frac{1}{2s-3} \to \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2}e^{3/2t}$



block diagram

$Y(s) = \frac{F(s)}{1 + k F(s)} = \frac{\frac{1}{2s-3}}{1 + k\left(\frac{1}{2s-3}\right)}$

Let k : $3 < k < \infty$

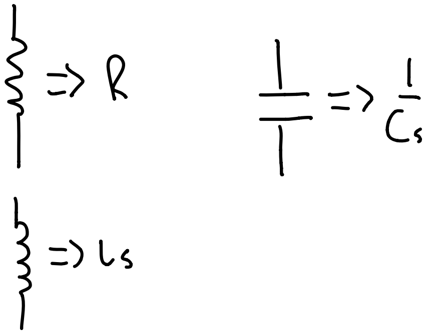
$k = 4 \cdot \frac{1}{2s-3} = \frac{4}{2s-3}$

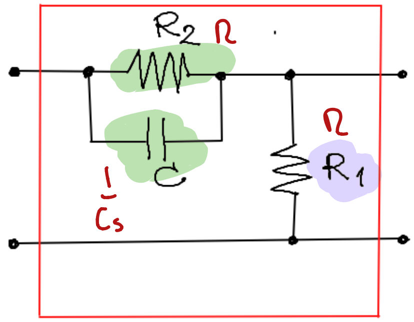
$\mathcal{L}^{-1}\left\{\frac{4}{2s-3}\right\} = \boxed{2e^{3/2t}} - h(t)$

$$H(s) = \frac{Y(s)}{Y(s) + X(s)}$$

$R_1 \to$ numerator
R_2 ; $C \to$ denom.

5. a) (4 points) What is the transfer function of the following circuit:





$$\frac{R}{R + R \parallel \frac{1}{C_1}} = \frac{R}{R + \frac{R}{RCs+1}}$$

$$= \frac{RCs+1}{RCs+2} = \frac{s + 1/RC}{s + 2/RC}$$

$$\boxed{H(s) = \frac{s + 1/RC}{s + 2/RC}}$$

b) (6 points) What is the transfer function of the following circuit?
Hints:

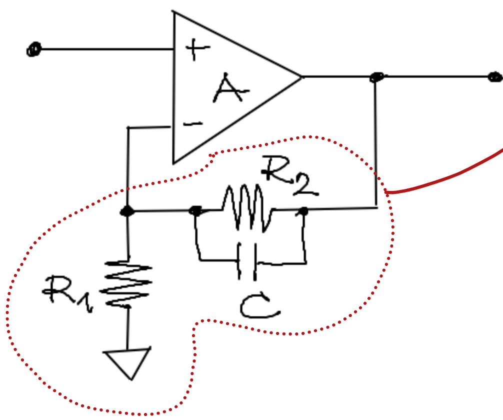
- you can use solutions of problems #4 and #5a
- to simplify the result you can assume that A → ∞

$$H(s) = \frac{s + 1/RC}{s + 2/RC}$$

$$F(s) = A,\quad G(s) = \frac{s + 1/RC}{s + 2/RC}$$

$$H(s) = \frac{A}{1 + A(G(s))}$$

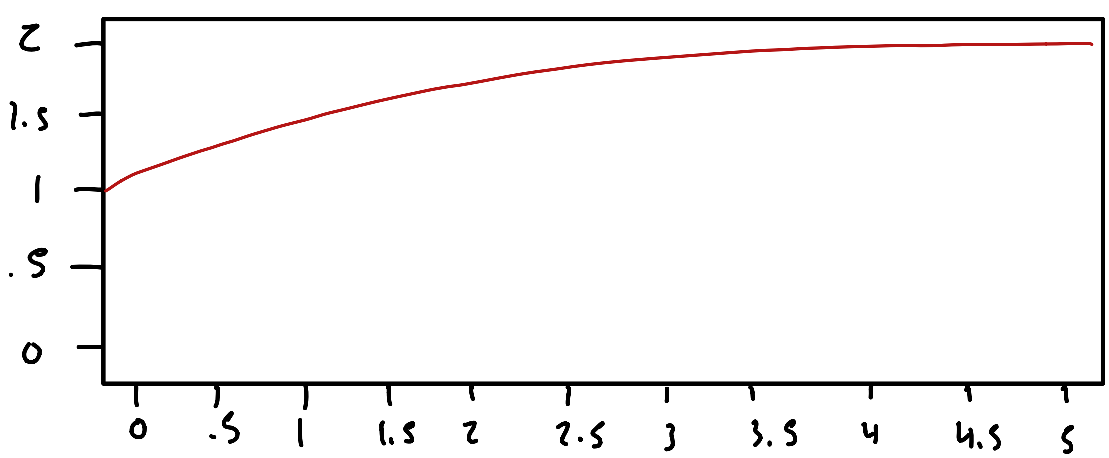
$$A \to \infty \quad H(s) = \frac{s + 2/RC}{s + 1/RC}$$

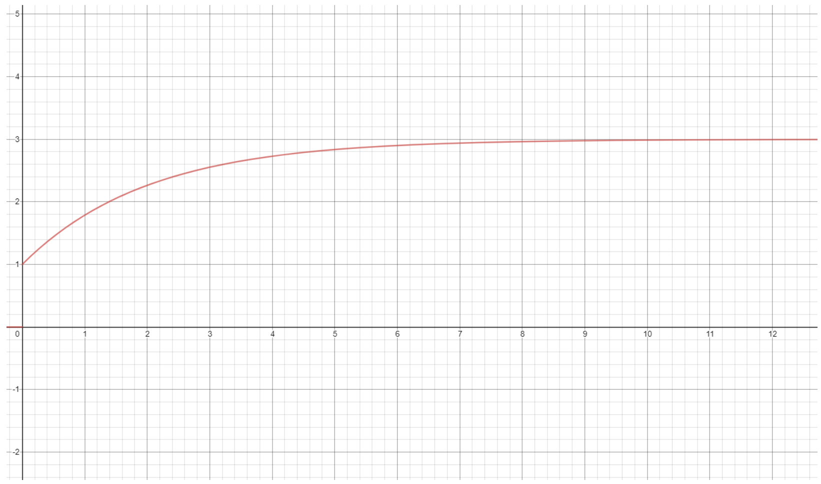


c) (10 points) Find and plot the unit-step response *s*(t) of the system.
Assume R_1=1Ω, R_2=2Ω, C=1F.

$$S(s) = \frac{1}{s} \cdot \frac{s + 2/RC}{s + 1/RC} = \frac{A}{s} + \frac{B}{s + 1/RC} = \frac{2}{s} - \frac{1}{s + 1/RC}$$

$$s(t) = (2 - e^{-t/RC}) \cdot u(t)$$




5
4
3
2
1
0
-1
-2
1
2
3
4
5
6
7
8
9
10
11
12