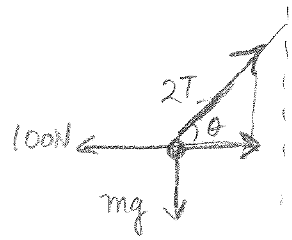
SAMPLE FINAL KEY

1. A 160-N child sits in a light swing and is pulled back and held with a horizontal force of 100 N. The tension in each of the two supporting ropes is:

A) 60 N
B) 94 N
C) 120 N
D) 190 N
E) 260 N



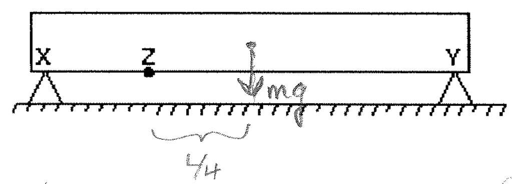
$2T\cos\theta = 100\,N$

$2T\sin\theta = mg = 160N$ then $T = \dfrac{160\,N}{2\sin 57.99°} = \underline{94.3\,N}$

divide equations:

$\tan\theta = \dfrac{160}{100} \Rightarrow \theta = 57.99°$

2. A uniform plank XY of weight 240 N is supported by two equal 120-N forces at X and Y, as shown. The support at X is then moved to Z (half-way to the plank center). The supporting force at Y then becomes:



A) 240 N
B) 160 N
C) 80 N
D) 60 N
E) 40 N

$\Sigma\tau = 0$

Choose z as "hinge position"

$\Sigma\tau_z = 0 = -\frac{L}{4}(mg)\sin 90° + \frac{3L}{4}(F_y)\sin 90°$

L's cancel, $\sin 90° \Rightarrow 1$

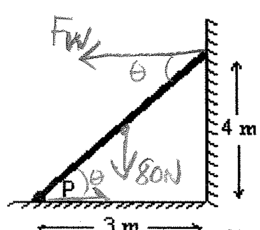
$\frac{1}{4}mg = \frac{3}{4}F_y$

$F_y = \frac{mg}{3} = \frac{240N}{3} = \underline{80N}$

3. An 80-N uniform plank leans against a frictionless wall as shown. The torque (about point P) applied to the plank by the wall is:

means Fw is only horizontal!

only asking for torque due to Fw



A) 40 N·m
B) 60 N·m
C) 120 N·m
D) 160 N·m
E) 240 N·m

$\sin(90-\theta)$ $\sin\theta$ also

$\Sigma\tau_p = 0 = -\frac{L}{2}(80)\left(\frac{3}{5}\right) + LF_W\left(\frac{4}{5}\right)$

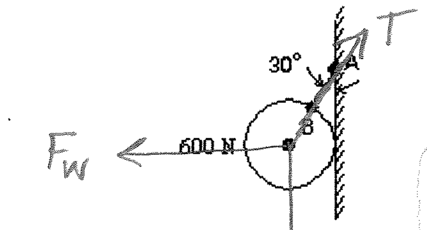
So $F_W = \dfrac{\frac{1}{2}(80)(3/5)}{(4/5)} = 30\,N$

and τ_p due to $F_W = LF_W\left(\frac{4}{5}\right)$

$= (5m)(30N)\left(\frac{4}{5}\right)$

$= \underline{120N\cdot m}$

4. The 600-N ball shown is suspended on a string AB and rests against the frictionless vertical wall. The string makes an angle of 30° with the wall. The ball presses against the wall with a force of:



$\tan 30° = \frac{F_W}{mg}$

$F_W = mg \tan 30°$

$= 600N \tan 30°$

$= 346.4\ N$

A) 120 N
B) 300 N
C) 350 N
D) 600 N
E) 690 N

x: $T \sin 30° = F_W$

y: $T \cos 30° = mg$

divide equations to get rid of T

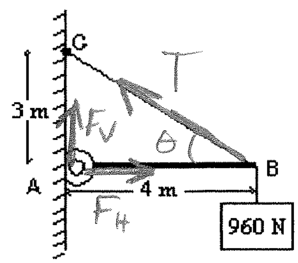
5. A 960-N block is suspended as shown. The beam AB is weightless and is hinged to the wall at A. The tension in the cable BC is:

x: $F_H - T\cos\theta = 0$

y: $F_V + T\sin\theta - mg = 0$

$\Rightarrow F_H = T\cos\theta$

$F_V = mg - T\sin\theta$



A) 720 N
B) 1200 N
C) 1280 N
D) 1600 N
E) none of these

and $\sum \tau_{hinge} = 0 = +L(T)\sin\theta - L(mg)\sin 90°$

L's cancel, $\sin 90° = 1$, $\sin\theta = 3/5$

$T = \frac{mg}{\sin\theta} = \frac{960N}{(3/5)} = 1600N$

6. A 4.0 m steel beam with a cross sectional area of 1.0×10^{-2} m^2 and a Young's modulus of 2.0×10^{11} N/m^2 is wedged horizontally between two vertical walls. In order to wedge the beam, it is compressed by 0.020 mm. How much force must be applied to compress the beam?

A) 0 N
B) 10,000 N
C) 1 x 10^9 N
D) 1 x 10^{11} N
E) cannot be determined

$\frac{F}{A} = \mathcal{E}\frac{\Delta L}{L} = \mathcal{E}\frac{(L_f - L_i)}{L_i}$

$F = \mathcal{E}A(L_f - L_i)/L_i$

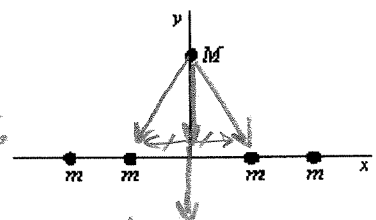
$= \frac{(2\times10^{11} N/m^2)(1\times10^{-2} m^2)(0.02\times10^{-3} m)}{4m}$

$= 10\,000\ N$

7. In the formula $F = Gm_1m_2/r^2$, the quantity G:
   A) depends on the local value of g
   B) is used only when the Earth is one of the two masses
   C) is greatest at the surface of the Earth
   D) is a universal constant of nature   always $6.67 \times 10^{-11}\ N \cdot m^2/kg^2$
   E) is related to the Sun in the same way that g is related to the Earth

8. Four particles, each with mass m are arranged symmetrically about the origin on the x axis. A fifth particle, with mass M, is on the y axis. The direction of the gravitational force on M is:

for each pair of forces on M, the horizontal components cancel, only the vertical components add.



   A) ↑
   B) ↓
   C) ←
   D) →
   E) none of these directions

9. The mass of a hypothetical planet is 1/100 that of Earth and its radius is 1/4 that of Earth. If a person weighs 600 N on Earth, what would he weigh on this planet?
   A) 24 N
   B) 48 N
   C) 96 N
   D) 192 N
   E) 600 N

$F = \frac{GM_1M_2}{r^2}$ $F_1 = 600\,N = \frac{GM_E m}{R_E^2}$

$F_2 = \frac{G\left(\frac{1}{100}M_E\right)m}{\left(\frac{1}{4}R_E\right)^2} = 0.16\,\frac{GM_E m}{R_E^2} = 0.16\,(600N) = 96N$

10. The escape velocity at the surface of Planet X is approximately 8 km/s. What is the escape velocity for a planet whose radius is 4 times and whose mass is 100 times that of Planet X?
    A) 1.6 km/s
    B) 8 km/s
    C) 40 km/s
    D) 200 km/s
    E) none of the above

$V_{escape_x} = \sqrt{\frac{2GM_x}{R_x}} = 8\ km/s$

$V_{escape_{new}} = \sqrt{\frac{2G(100M_x)}{4R_x}} = 5\sqrt{\frac{2GM_x}{R_x}} = 5(8\ km/s)$

$= 40\ km/s$

11. Neglecting air resistance, a 1.0 kg projectile has an escape velocity of about 11 km/s at the surface of Earth. The corresponding escape velocity for a 2.0 kg projectile is:

A) 3.5 km/s
B) 5.5 km/s
C) 7.1 km/s
D) 10 km/s
E) 11 km/s

Same, doesn't depend on the projectile's mass!

12. Two planets are orbiting a star in a distant galaxy. The first has a semimajor axis of 150×10^6 km, an eccentricity of 0.20, and a period of 1.0 Earth years. The second has a semimajor axis of 250×10^6 km, an eccentricity of 0.30, and a period of:

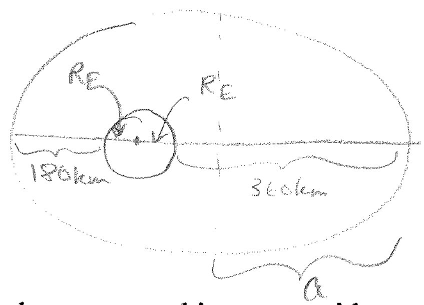
A) 0.46 Earth yr
B) 0.57 Earth yr
C) 1.4 Earth yr
D) 1.8 Earth yr
E) 2.2 Earth yr

$$T_1 = \sqrt{\frac{4\pi^2 a_1^3}{GM_{star}}} \qquad \frac{T_1}{T_2} = \frac{a_1^{3/2}}{a_2^{3/2}}$$

$$T_2 = \sqrt{\frac{4\pi^2 a_2^3}{GM_{star}}} \qquad \text{OR} \quad T_2 = T_1\left(\frac{a_2}{a_1}\right)^{3/2} = 1\,yr\left(\frac{250\times10^6}{150\times10^6}\right)^{3/2} = 2.15\ yr$$

13. A satellite, moving in an elliptical orbit, is 360 km above Earth's surface at its farthest point and 180 km above at its closest point. Calculate the semimajor axis of the orbit. (The radius of the Earth is 6.37×10^6 m.)

A) 6910 km
B) 6640 km
C) 3455 km
D) 540 km
E) 270 km



$$a = \frac{360km + 180km + 2R_E}{2} = \frac{360 + 180 + 2(6370)}{2}\ km = 6640\ km$$

14. A planet in another solar system orbits a star with a mass of 4.0×10^{30} kg. At one point in its orbit it is 250×10^6 km from the star's center and is moving at 35 km/s. Take the universal gravitational constant to be 6.67×10^{-11} N· m^2/kg^2 and calculate the semimajor axis of the planet's orbit. The result is:

A) 79×10^6 km
B) 160×10^6 km
C) 290×10^6 km
D) 320×10^6 km
E) 590×10^6 km

$$E \Rightarrow -\frac{GMm}{2a} = \frac{1}{2}mv^2 - \frac{GMm}{r} \Leftarrow K+U$$

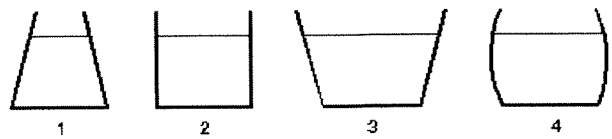
m's cancel, solve for a:

$$a = \frac{-GM/2}{\frac{1}{2}v^2 - \frac{GM}{r}} = \frac{-GMr}{rv^2 - 2GM}$$

$$= \frac{-(6.67\times10^{-11} N\cdot m^2/kg^2)(250\times10^9 m)(4\times10^{30} kg)}{(250\times10^9 m)(35\,000 m/s)^2 - 2(6.67\times10^{-11} \frac{N.m^2}{kg^2})(4\times10^{30} kg)}$$

$$= 2.93\times10^{11} m = 293\times10^9 m = 293\times10^6 km$$

15. The vessels shown below all contain water to the same height. Rank them according to the pressure exerted by the water on the vessel bottoms, least to greatest.



A) 1, 2, 3, 4
B) 3, 4, 2, 1
C) 4, 3, 2, 1
D) 2, 3, 4, 1
E) All pressures are the same

same density of fluid

$P = P_0 + \rho g h$

same height

Same outside pressure

16. Calculate the hydrostatic difference in blood pressure between the brain and the foot in a person of height 1.83 m. The density of blood is 1.06×10^3 kg/m^3.
A) 19 Pa
B) 120 Pa
C) 1.9 kPa
D) 19 kPa
E) 120 kPa

P_0 @ brain is same as P_0 @ foot, only $\rho g h$ changes

$P \Rightarrow \rho g h$

$= (1.06 \times 10^3 \frac{kg}{m^3})(9.8 \frac{m}{s^2})(1.83 m)$

$= 19010\ Pa = 19\ kPa$

17. A rock, which weighs 1400 N in air, has an apparent weight of 900 N when submerged in fresh water (998 kg/m^3). The volume of the rock is:
A) 0.14 m^3
B) 0.60 m^3
C) 0.90 m^3
D) 5.1×10^{-2} m^3
E) 9.2×10^{-2} m^3

Completely under

$1400 N - 900 N = 500 N = F_b$

$F_b = \rho_w V_w g = \rho_w V_{Rock}\, g$

$V_{Rock} = \frac{F_b}{\rho_w g} = \frac{500 N}{(998 kg/m^3)(9.8 m/s^2)} = 0.051\ m^3$

18. A 210-gram object apparently loses 30 g when suspended in a liquid of density 2.0 g/cm^3. The density of the object is:
A) 7.0 g/cm^3
B) 3.5 g/cm^3
C) 1.4 g/cm^3
D) 14 g/cm^3
E) none of these

$\rho_f V_f g = F_b \Rightarrow \rho_f V_{object}\, g$ for submerged

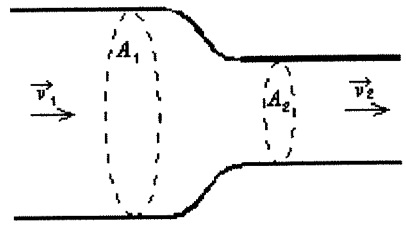
but $V_{object} = \frac{M_{object}}{\rho_{object}}$

also $F_b = m_{lost}\, g$

So: $F_b = \rho_f \left(\frac{m_{object}}{\rho_{object}}\right) g$

$\Rightarrow \rho_{object} = \frac{\rho_f (m_{object}) g}{F_b} = \frac{2\ g/cm^3 (210 g)\ g}{(30 g)\ g} = 14\ g/cm^3$

19. An incompressible liquid flows along the pipe as shown. The ratio of the speeds v_2/v_1 is:



A) A_1/A_2
B) A_2/A_1
C) $\sqrt{A_1 / A_2}$
D) $\sqrt{A_2 / A_1}$
E) v_1/v_2

$$R = \underbrace{A_1 v_1 = A_2 v_2}$$

$$\frac{v_2}{v_1} = \frac{A_1}{A_2}$$

v faster when A is smaller

20. A constriction in a pipe reduces its diameter from 4.0 cm to 2.0 cm. Where the pipe is wide the water velocity is 8.0 m/s. Where it is narrow the water velocity is:
A) 2.0 m/s
B) 4.0 m/s
C) 8.0 m/s
D) 16 m/s
E) 32 m/s

$$\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

conversions would have cancelled, so don't bother!

$$v_2 = v_1 \left(\frac{r_1}{r_2}\right)^2 = 8\,\text{m/s} \left(\frac{4\,\text{cm}}{2\,\text{cm}}\right)^2 = 32\,\text{m/s}$$

21. A water line enters a house 2.0 m below ground. A smaller diameter pipe carries water to a faucet 5.0 m above ground, on the second floor. Water flows at 2.0 m/s in the main line and at 7.0 m/s on the second floor. Take the density of water to be 1.0×10^3 kg/m^3. If the pressure in the main line is 2.0×10^5 Pa, then the pressure on the second floor is:
A) 5.3×10^4 Pa
B) 1.1×10^5 Pa
C) 1.5×10^5 Pa
D) 2.5×10^5 Pa
E) 3.4×10^5 Pa

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_2 = P_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

$$= (2\times10^5\,\text{Pa}) + \tfrac{1}{2}\left(1000\,\tfrac{\text{kg}}{\text{m}^3}\right)(2^2 - 7^2) + (1000)(9.8)(-2 - 5\,\text{m})$$

$$= 108900\,\text{Pa} = 1.1\times10^5\,\text{Pa}$$

22. An object attached to one end of a spring makes 20 vibrations in 10s. Its period is:
A) 2 Hz
B) 10 s
C) 0.5 Hz
D) 2 s
E) 0.50 s

$$T = \frac{\#\,\text{sec}}{\text{cycle}} = \frac{1}{f} = \frac{10\,\text{s}}{20\,\text{cycles}}$$

$$\boxed{T = 0.5\,\text{s}}$$

23. An object attached to one end of a spring makes 20 vibrations in 10 seconds. Its frequency is:
A) 2 Hz
B) 10 s
C) 0.05 Hz
D) 2 s
E) 0.50 s

$f = \frac{\#\text{ cycles}}{\text{sec}} = \frac{20\text{ cycles}}{10\text{ sec}} = 2\text{ Hz}$

24. A certain spring elongates 9 mm when it is suspended vertically and a block of mass M is hung on it. The natural frequency of this mass-spring system is:
A) 0.014
B) 5.3 Hz
C) 31.8 Hz
D) 181.7 Hz
E) need to know M

$\omega = \sqrt{\frac{k}{m}}$ and $F = -kx$

$-Mg = -kx$

$k = \frac{Mg}{x}$

$\omega = \sqrt{\frac{Mg/x}{M}} = \sqrt{\frac{g}{x}}$

$f = \frac{1}{2\pi}\sqrt{\frac{g}{x}} = \frac{1}{2\pi}\sqrt{\frac{9.8}{9\times10^{-3}}}$

$f = 5.25\text{ Hz}$

25. A 0.200-kg mass attached to a spring whose spring constant is 500 N/m executes simple harmonic motion with amplitude 0.100 m. Its maximum speed is:
A) 25 m/s
B) 5 m/s
C) 1 m/s
D) 15.8 m/s
E) 0.2 m/s

$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{500\text{ N/m}}{0.2\text{ kg}}} = 50\text{ rad/s}$

$v_{max} = \omega x_{max} = (50\text{ rad/s})(0.1\text{ m}) = \underline{5\text{ m/s}}$

26. A particle is in simple harmonic motion along the x axis. The amplitude of the motion is x_m. At one point in its motion its kinetic energy is $K = 5$J and its potential energy (measured with $U = 0$ at $x = 0$) is $U = 3$J. When it is at $x = x_m$, the kinetic and potential energies are:
A) $K = 5$J and $U = 3$J
B) $K = 5$J and $U = -3$J
C) $K = 8$J and $U = 0$
D) $K = 0$ and $U = 8$J
E) $K = 0$ and $U = -8$J

@ x_m, $K \to 0$ and $U \Rightarrow U_{max} = E$

but $E = K + U$ at any time

$= 5 + 3 = 8J$

27. A 0.25-kg block oscillates on the end of the spring with a spring constant of 200 N/m. If the system has an energy of 6.0 J, then the amplitude of the oscillation is:
A) 0.06 m
B) 0.17 m
C) 0.24 m
D) 4.9 m
E) 6.9 m

$E = U_{max} = \frac{1}{2}kx_{max}^2$

$x_{max} = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(6J)}{(200\text{ N/m})}} = 0.2449\text{ m}$

28. A 0.25-kg block oscillates on the end of the spring with a spring constant of 200 N/m. If the oscillation is started by elongating the spring 0.15 m and giving the block a speed of 3.0 m/s, then the maximum speed of the block is:

A) 0.13 m/s
B) 0.18 m/s
C) 3.7 m/s
D) 5.2 m/s
E) 13 m/s

$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$= \frac{1}{2}(0.25)(3)^2 + \frac{1}{2}(200)(0.15)^2 = 3.375\ J$

@ v_{max} $U=0$, $K_{max} = \frac{1}{2}m v_{max}^2 = E$

So $v_{max} = \sqrt{2E/m} = \sqrt{\frac{2(3.375)}{0.25}} = 5.2\ m/s$

29. A simple pendulum has length L and period T. As it passes through its equilibrium position, the string is suddenly clamped at its mid-point. The period then becomes:

A) $2T$
B) T
C) $T/2$
D) $T/4$
E) $T/\sqrt{2}$

$T_1 = 2\pi\sqrt{L/g}$

$T_2 = 2\pi\sqrt{\frac{L/2}{g}} = \frac{2\pi}{\sqrt{2}}\sqrt{L/g} = \frac{1}{\sqrt{2}} T_1$

30. The rotational inertia of a uniform thin rod about its end is $ML^2/3$, where M is the mass and L is the length. Such a rod is hung vertically from one end and set into small amplitude oscillation. If $L = 1.0$ m this rod will have the same period as a simple pendulum of length:

A) 33 cm
B) 50 cm
C) 67 cm
D) 100 cm
E) 150 cm

$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg(L/2)}}$

h is distance from CM to pivot

$T_{\text{meter stick}} = 2\pi\sqrt{\frac{2L}{3g}}$

$T_{\text{simple pend.}} = 2\pi\sqrt{\frac{L'}{g}}$ if $L' = \frac{2}{3}L$

$= \frac{2}{3}(1m)$

$= 67 cm$

then same period.