1. Write the formula + plot the roots of $z^7 + 1 = 0$

$$z^n + a = 0 \Rightarrow z_k^n = -a = |a| e^{j(2k+1)\pi}$$

$$z_k = |a|^{(1/n)} e^{j((2k+1)\pi)/n} \quad \text{for } k = 0, 1, \dots n-1$$

$$z^7 = -1 \Rightarrow a = -1 \text{ and } |a| = 1^{(1/n)} = 1 \qquad n = 7$$

$$z_0 = e^{j(\cancel{2(0)}+1)\pi/7} = e^{j\pi/7}$$

$$z_1 = e^{j(2(1)+1)\pi/7} = e^{j3\pi/7}$$

$$z_2 = e^{j(2(2)+1)\pi/7} = e^{j5\pi/7}$$

$$z_3 = e^{j(2(3)+1)\pi/7} = e^{j\pi} = -1$$

$$z_4 = e^{j(2(4)+1)\pi/7} = e^{j9\pi/7} = \cancel{e^{2\pi}} e^{j-5\pi/7}$$

$$z_5 = e^{j(2(5)+1)\pi/7} = e^{j11\pi/7} = \cancel{e^{2\pi}} e^{j-3\pi/7}$$

$$z_6 = e^{j(2(6)+1)\pi/7} = e^{j13\pi/7} = \cancel{e^{2\pi}} e^{j-\pi/7}$$

Roots

$z_0 = e^{j\pi/7}$

$z_1 = e^{j3\pi/7}$

$z_2 = e^{j5\pi/7}$

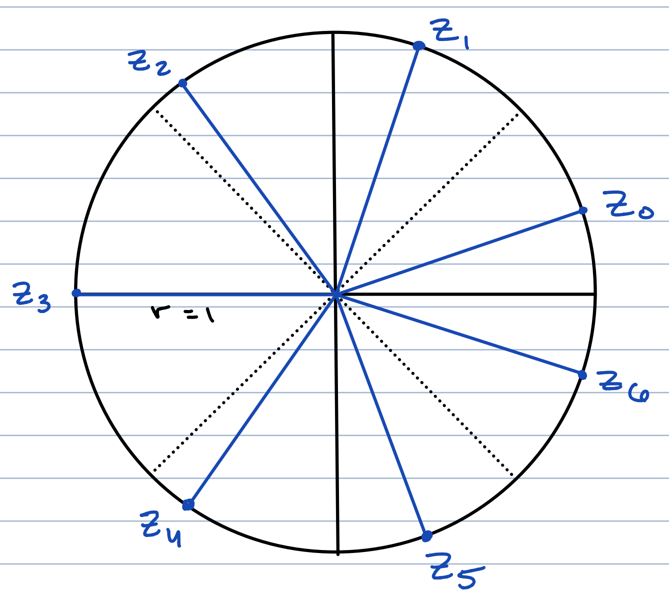
$z_3 = e^{j\pi} = -1$

$z_4 = e^{-j5\pi/7}$

$z_5 = e^{-j3\pi/7}$

$z_6 = e^{-j\pi/7}$

Plot Polar Coordinates

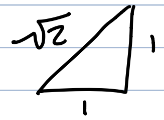


* know how to have MATLAB do this

roots([1 0 0 0 0 0 0 1])

2. Represent the following complex numbers in an alternate form.

a. $1 + j$



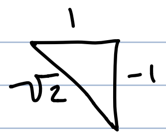
$\rightarrow$ first quadrant

$z = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4} = 45°$

$\Rightarrow \boxed{z = \sqrt{2}\ e^{j\angle\frac{\pi}{4}} = \sqrt{2}e^{j\angle 45°}}$

b. $1 - j$



$\rightarrow$ fourth quadrant.

$z = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\tan^{-1}\left(\frac{-1}{1}\right) = \frac{7\pi}{4} = 315°$

$\Rightarrow \boxed{z = \sqrt{2}e^{j\angle\frac{7\pi}{4}} = \sqrt{2}\,e^{j\angle 315°}} = \sqrt{2}\,e^{j\angle -\frac{\pi}{4}}$

c. $5e^{j\angle 210°}$ $\rightarrow$ 210° is 3rd quad.

$5e^{j\angle 210} = 5\cos(210) + 5j\sin(210)$

$= -\frac{5\sqrt{3}}{2} - \frac{5}{2}j = \boxed{-4.33 - 2.5j}$ $\boxed{\{Re, Im\} = \{-4.33, -2.5\}}$

d. $5e^{-j\angle 210°}$ $\rightarrow$ flips y $\rightarrow$ 2nd quad.

$5e^{-j\angle 210} = 5\cos(210) - 5j\sin(210)$

$= \frac{-5\sqrt{3}}{2} + \frac{5}{2}j = \boxed{-4.33 + 2.5j}$ $\boxed{\{Re, Im\} = \{-4.33, 2.5\}}$

e. $z \cdot z^*$ $\rightarrow$ multiplying conjugates

for $z = x + jy$ and $z^* = x - jy$

$(x + jy)(x - jy)$

$x^2 + y^2 = \boxed{|z|^2}$ or $\rightarrow$ $||z|^2|e^{j\angle|z|^2}$

3. Use Euler's Identity to find trig. identities

a. $\sin(\alpha+\beta) \rightarrow \theta = \alpha+\beta$

Euler's Identity:
$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$= e^{j(\alpha+\beta)} = e^{j\alpha} \cdot e^{j\beta}$$

$$= \left(\cos(\alpha) + j\sin(\alpha)\right) \cdot \left(\cos(\beta) + j\sin(\beta)\right)$$

$$= \cos(\alpha)\cos(\beta) + j\cos(\alpha)\sin(\beta) + j\sin(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$= \underbrace{\left[\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)\right]}_{\cos(\alpha+\beta)} + j\underbrace{\left[\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)\right]}_{\sin(\alpha+\beta)}$$

a. $\sin(\alpha+\beta)$ is the imaginary part $\nearrow$

$$\boxed{\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha) + \sin(\beta)}$$

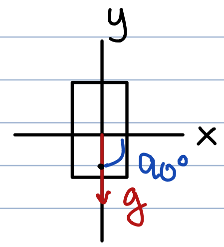
b. $\cos(\alpha+\beta)$ is the real part

$$\boxed{\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)}$$

6. $a_{swing} = \pm 2g \qquad \text{Power} = +3V$

$$s = \frac{V_{cc}}{a_{swing}} = \frac{3}{4} = 0.75 \left[\frac{V}{g}\right]$$

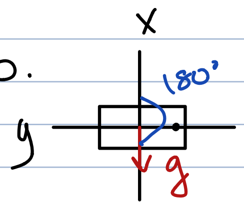
a.



$\theta = 90^\circ \qquad a_o = \frac{V_{cc}}{2} = \frac{3}{2} = 1.5\ V$

$$X = 1.5 + 0.75\cos(90) = \boxed{1.5\ V}$$
$$Y = 1.5 - 0.75\sin(90) = \boxed{0.75\ V}$$

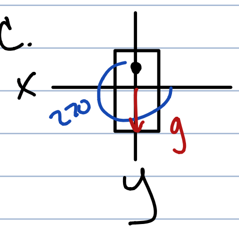
b.



$\theta = 180^\circ \qquad a_o = \frac{V_{cc}}{2} = \frac{3}{2} = 1.5V$

$$X = 1.5 + 0.75\cos(180) = \boxed{0.75V}$$
$$Y = 1.5 - 0.75\sin(180) = \boxed{1.5V}$$

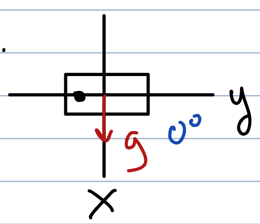
c.



$\theta = 270^\circ \qquad a_o = \frac{V_{cc}}{2} = \frac{3}{2} = 1.5$

$$X = 1.5 + 0.75\cos(270) = \boxed{1.5V}$$
$$Y = 1.5 - 0.75\sin(270) = \boxed{2.25V}$$

d.



$\theta = 0^\circ \qquad a_o = \frac{V_{cc}}{2} = \frac{3}{2} = 1.5$

$$X = 1.5 + 0.75\cos(0) = \boxed{2.25V}$$
$$Y = 1.5 - 0.75\sin(0) = \boxed{1.5V}$$

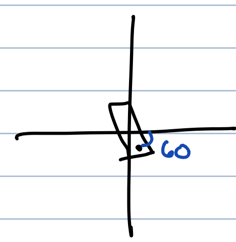
e. $X = 1.875V \rightarrow 1.5 + 0.75\cos(\alpha) = 1.875$

$Y = 0.8505V$ $\qquad \cos(\alpha) = 0.5$

$\rightarrow 1.5 - 0.75\sin(\alpha) = 0.8505$

$a_x = a_o + g\cos(\theta)$ $\qquad \sin(\alpha) = 0.866$

$$\tan^{-1}\left(\frac{0.866}{0.5}\right) \approx \boxed{60^\circ}$$



f. $X = 2.1495V \rightarrow 1.5 + 0.75\cos(\alpha) = 2.1495 \Rightarrow \cos(\alpha) = 0.866$

$Y = 1.875V \rightarrow 1.5 - 0.75\sin(\alpha) = 1.875 \Rightarrow \sin(\alpha) = -0.5$

$$\tan^{-1}\left(\frac{-0.5}{0.866}\right) \approx \boxed{30}$$