8.3 Trigonometric substitution:

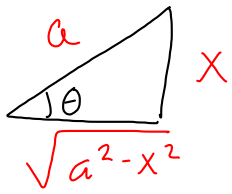
(1) $\sqrt{a^2 - x^2}$ $\quad \sin\theta = \frac{x}{a}$

Let $x = a\sin\theta$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Feb 22-11:20 AM

$\cos^2\theta = 1 - \sin^2\theta$



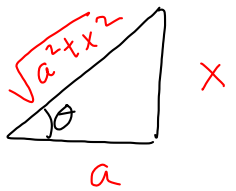
(2) $\sqrt{x^2 + a^2}$ $\quad \tan\theta = \frac{x}{a}$

Let $x = a\tan\theta$

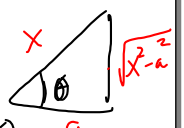
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Feb 22-11:29 AM

$\tan^2\theta + 1 = \sec^2\theta$



(3) $\sqrt{x^2 - a^2}$



Let $x = a\sec\theta$ $\quad \sec\theta = \frac{x}{a}$

$0 \le \theta \le \pi/2$

Feb 22-11:32 AM

$\sec^2\theta - 1 = \tan^2\theta$

(Ex) $\int \frac{dx}{\sqrt{9 + x^2}}$

$= \int \frac{dx}{\sqrt{3^2 + x^2}}$

Feb 22-11:34 AM

Let $x = a\tan\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\int \frac{3\sec^2\theta\, d\theta}{\sqrt{9 + 9\tan^2\theta}}$

Feb 22-11:38 AM

$= \int \frac{3\sec^2\theta\, d\theta}{\sqrt{9(1 + \tan^2\theta)}}$

$= \int \frac{3\sec^2\theta\, d\theta}{\sqrt{9\sec^2\theta}}$

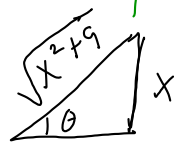
$= \int \frac{3\sec^2\theta}{3\sec\theta}\, d\theta$

Feb 22-11:39 AM

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$\tan\theta = \frac{x}{3}$



$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

Feb 22-11:42 AM

(Ex) $\int x^3\sqrt{9-x^2}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$= \int (3\sin\theta)^3 \sqrt{9-9\sin^2\theta}\; 3\cos\theta \, d\theta$

Feb 22-11:47 AM

$= \int 3^3 \sin^3\theta \sqrt{9(1-\sin^2\theta)}\; 3\cos\theta \, d\theta$

$= \int 3^4 \sin^3\theta \; 3\cos\theta \cos\theta \, d\theta$

$= 3^5 \int \sin^3\theta \cos^2\theta \, d\theta$

Feb 22-11:48 AM

$= 3^5 \int \sin^2\theta \sin\theta \cos^2\theta \, d\theta$

$= 3^5 \int (1-\cos^2\theta)\sin\theta \cos^2\theta \, d\theta$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$= -3^5 \int (1-u^2)u^2 \, du$

Feb 22-11:52 AM

$= -3^5 \int (u^2 - u^4)\, du$

$= -3^5 \left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C$

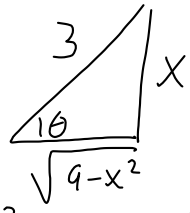
$= -3^5 \left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right] + C$

Feb 22-11:54 AM

$x = 3\sin\theta$

$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$



$$= -3^5 \left[\frac{\left(\frac{\sqrt{9-x^2}}{3}\right)^3}{3} - \frac{\left(\frac{\sqrt{9-x^2}}{3}\right)^5}{5}\right] + C$$

Feb 22-11:55 AM

Ex $\int \frac{dx}{\sqrt{25x^2-4}}$,

$x > \frac{2}{5}$

Feb 22-11:58 AM