Department of Electrical and Computer Engineering
The University of Alabama in Huntsville
Spring 2021

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #5

Due: Wednesday, April 21 at 9:35 am
Please bring hardcopy to the class and upload softcopy to Canvas

Student name:
Nolan Anderson

1 20	2 20	3 16	4 18	5 20	6 6	Σ

1. (20 points) A discrete time IIR system with input x[n] and output y[n] is represented by the equation:

$$y[n] = 0.2 \cdot y[n-2] + x[n] \qquad n \geq 0$$

a) find the impulse response *h(n)* of the system, by assuming that initial conditions are zerc (y[n]=h[n]=0, n<0) and x[n]=δ[n].

$h[0] = 0.2 \cdot h[-2] + 1 = 1$ | $h[1] = 0.2 \cdot h[-1] + 0 = 0$

$h[2] = 0.2 \cdot h[0] + 0 = .2$ | $h[3] = 0.2 \cdot h[1] + 0 = 0$

$h[4] = 0.2 \cdot h[2] + 0 = 0.2^2$

$$h[n] = \begin{cases} 0.2^{n/2} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

b) find the impulse response alternatively by using recursive relation between x[n] and y[n].

$h[n] = 0.2 * h[n-2] + \delta[n]$

$h[n-2] = 0.2 * h[n-4] + \delta[n-2]$

eventually you'll just use

$h[n] = \delta[n] + 0.2 \times \delta[n-2] + 0.2 * \delta[n-4] + \ldots$

c) plot h[n] using MATLAB function filter.

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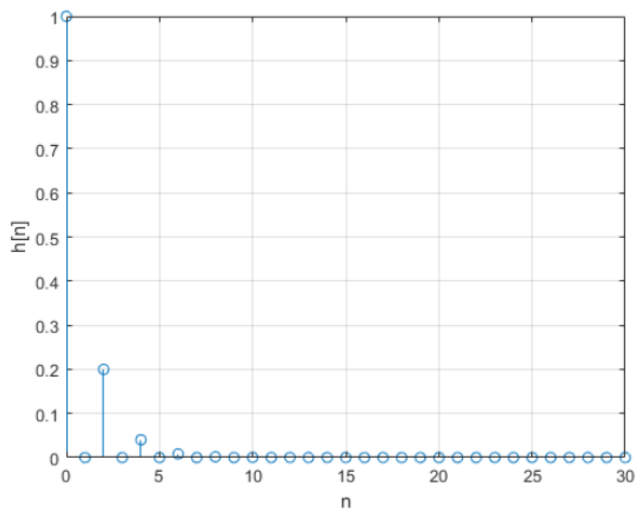
Variables

```
i = [1 0 -0.2];
j = 1;
x = [1 zeros(1,30)];
h = filter(j,i,x);
n = 0:30;
```

Plot

```
stem(n,h); axis([0 30 0 1]);
grid;
ylabel('h[n]');
xlabel('n')
```



2. (20 points) An FIR filter is represented as:

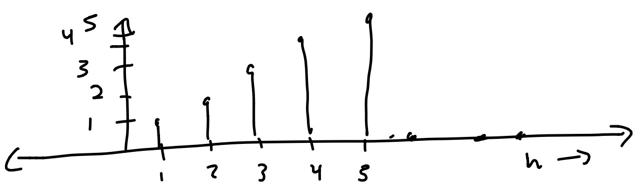
$$y[n]=\sum_{k=0}^{5} k \cdot x[n-k]$$

a) find and plot the impulse response of this filter.
b) is this a causal and stable filter? Explain.
c) find and plot the unit-step response s[n] for this filter.
d) what is the maximum value of the output if the maximum input is 5?
e) plot h[n] and s[n] using MATLAB function filter.

a) $y[n] = 0 * \delta[n] + 1 * \delta[n-1] + 2 * \delta[n-2]$
$+ 3 * \delta[n-3] + 4 * \delta[n-4] + 5 * \delta[n-5]$

$\frac{y(z)}{x(z)} = z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 5z^{-5}$

$= h[n] = \delta(n-1) + 2 \times \delta(n-2) + 3 \times \delta(n-3) + 4\delta(n-4) + 5\delta(n-5)$

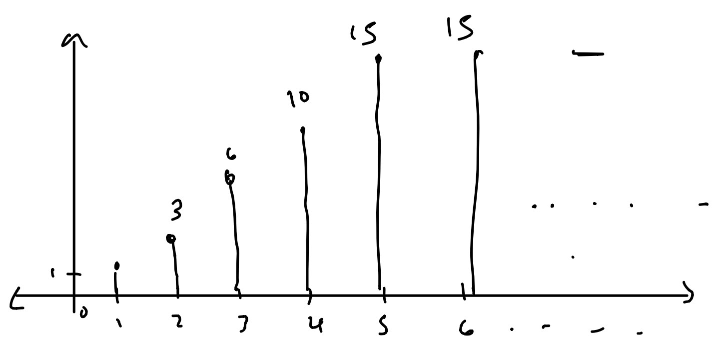


b) This filter depends on previous values for its inputs, therefore it can be casual. Also, $h[n]=0$ for $n<0$

c) $s[n] = \sum_{k=1}^{5} k \cdot u[n-k]$

$= u[n-1] + 2u[n-2] + 3u[n-3] + 4u[n-4] + 5u[n-5]$

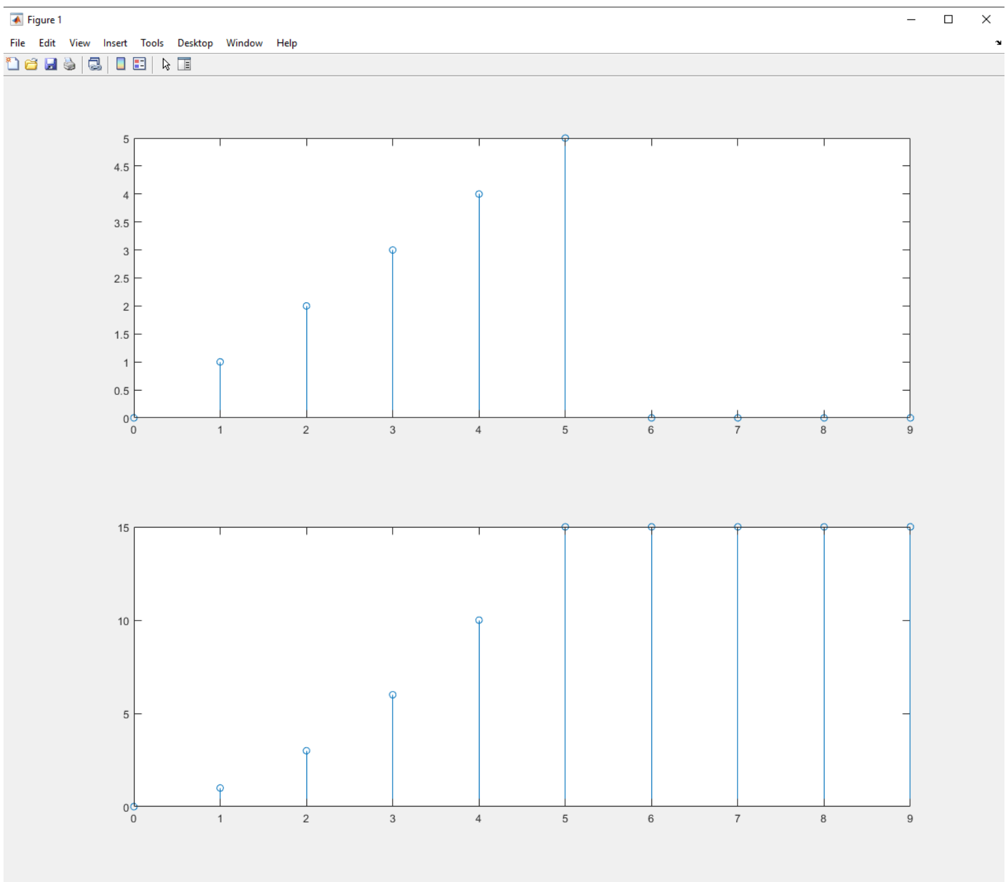
$s[1] = 1$
$s[2] = 3$
$s[3] = 6$
$s[4] = 10$
$s[5] = 15$
...
...
...



d) if $x[n] < 5$

$$y[n] < 5 \cdot \sum_{k=0}^{5} k\,[x[k]] = \boxed{75}$$

e) plots



3. (16 points) Let x[n] = {0, 1, 1, 1, 0} and h[n] = {1.5, 1, 0.6}.

Compute and plot the convolution y[n] = x[n] * h[n].

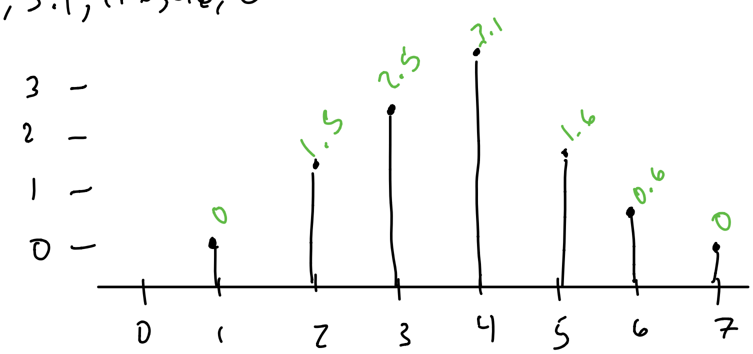
$$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$$

$$y(n) = \{0, 1, 1, 1, 0\} * \{1.5, 1, 0.6)$$

		0	1	1	1	0	Sum
0.6	1	1.5					0
	0.6	1	1.5				1.5
		0.6	1	1.5			2.5
			0.6	1	1.5		3.1
				0.6	1	1.5	1.6
					0.6	1	0.6
							0

1 2 3 4 5 6 7

0, 1.5, 2.5, 3.1, 1.6, 0.6, 0



$h(0)\, x(0) = 0$

$h(1)\; x(1) = 1.5$

$h(2)\; x(2) + h(1)\, h(1) =$

4. (18 points)
   a) (6 points) Explain the difference between hard and soft real-time systems.
   b) (7 points) Maximum frequency of the input is 600Hz. The microcontroller processes each sample in 1200 clock cyles with clock frequency F_c = 1MHz. Can this system run in real-time?
   c) (5 points) What is the minimum frequency of the clock that allows real-time operation with 2x oversampling of the input?

a) hard → generate total system failure on missed deadline

soft → If a deadline is missed, it wont stop the system. It will just slow the system down until it gets resolved.

Realtime systems are classified by how they react to a missed deadline. This is where hard and soft come in to Play.

b) 600Hz real time?

1200 cc

$F_c = 1MHz$

$T_s = 1/F_s = 1/1200 = 833\mu s$

T_p = Cycles x T_{cycle} = $1200 \times 1\mu s = 1.2ms$

$T_p > T_s$, So no it cannot run in real time. This is because it takes longer to process than it does to take a sample

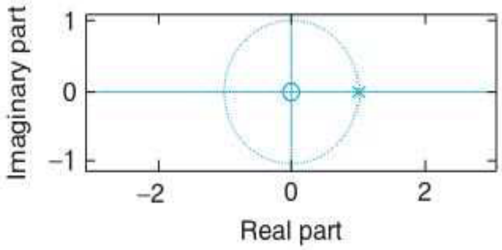
c) Nyquist? Processing time must be less than sample, "Solution" to part B's issue.

$T_p = 1200 \times T_{cycle} = 1200 \times 1/\text{clock} = 1/F_s$

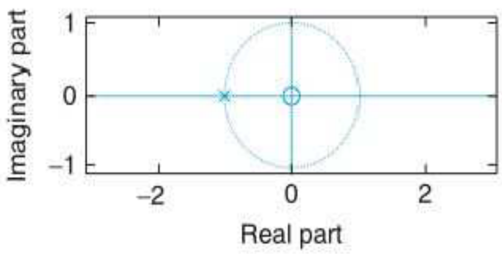
$F_{clock} > 1200 \times F_s$

$\boxed{F_{clock} > 1.44 MHz}$

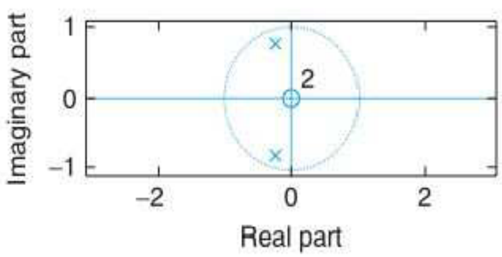
5. (20 points) Describe the effect of pole location on the inverse Z-transform for the following cases.



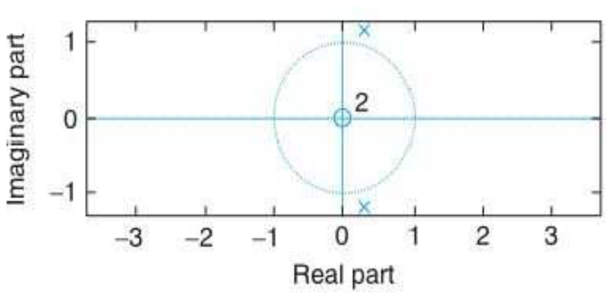
Constant function of u[n]



Cosine of frequency π
constant amplitude



decaying modulated exponential



growing modulated signal

6. (6 points) If X(z) is the Z-transform of a causal signal x[n], then

Initial value is x[0] = $\lim_{z \to \infty} X(z)$

Final value is $\lim_{n\to\infty} x[n] =$ $\lim_{z \to 1} (z-1) X(z)$