$\boxed{8.1}$ Integration by parts:

$$\frac{d}{dx} f(x)g(x)$$
$$= f'(x)g(x)$$
$$+ f(x)g'(x)$$

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$$\int \frac{d}{dx} f(x)g(x)\,dx$$
$$= \int f'(x)g(x)\,dx$$
$$+ \int f(x)g'(x)\,dx$$
$$f(x)g(x) = \int f'(x)g(x)\,dx$$

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$$+ \int f(x)g'(x)\,dx$$

$$\int f(x)g'(x)\,dx$$
$$= f(x)g(x) - \int f'(x)g(x)\,dx$$

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Let $u = f(x)$

$v = g(x)$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

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$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

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Ex $\int x \sin x\,dx$

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$= -x\cos x - \int -\cos x\,dx$

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$$= -x\cos x + \int \cos x\,dx$$
$$= -x\cos x + \sin x + C$$

Ex $\int x^2 \sin x\,dx$

$u = x^2 \qquad dv = \sin x\,dx$

$du = 2x\,dx \qquad v = -\cos x$

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$$= -x^2\cos x + \int 2x\cos x\,dx$$

$u = 2x \qquad dv = \cos x\,dx$

$du = 2dx \qquad v = \sin x$

$$= -x^2\cos x + \left[2x\sin x - \int 2\sin x\,dx\right] + C$$

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$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

Ex $\int_1^e \ln x\,dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

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$$= x\ln x\Big|_1^e - \int_1^e \cancel{x}\cdot\frac{1}{\cancel{x}}\,dx$$
$$= (e\ln e - 1\cdot \underbrace{\ln 1}_{0}) - x\Big|_1^e$$
$$= e - [e - 1]$$
$$= e - e + 1 = \boxed{1}$$

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Ex $\int x^3 \ln x\,dx$

$u = \ln x \qquad dv = x^3\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^4}{4}$

$$= \frac{x^4}{4}\ln x - \int \frac{x^{\cancel{4}}}{4}\cdot\frac{1}{\cancel{x}}\,dx$$

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$$= \frac{x^4\ln x}{4} - \frac{1}{4}\int x^3\,dx$$
$$= \frac{x^4\ln x}{4} - \frac{x^4}{16} + C$$

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(Ex) $\int_0^1 \tan^{-1}x \, dx$

$u = \tan^{-1}x \qquad dv = dx$

$du = \dfrac{1}{1+x^2}dx \qquad v = x$

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$= x\tan^{-1}x \Big|_0^1$

$- \int_0^1 \dfrac{x}{1+x^2} dx$

$= \Big[(1)\underbrace{\tan^{-1}(1)}_{\frac{\pi}{4}} - 0\underbrace{\tan^{-1}(0)}_{0}\Big]$

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$-\frac{1}{2}\int \frac{du}{u}$

$u = 1+x^2$

$du = 2x\,dx$

$= \left(\frac{\pi}{4} - 0\right) - \frac{1}{2}\ln|u|$

$= \frac{\pi}{4} - \frac{1}{2}\ln|1+x^2| \Big|_0^1$

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$= \frac{\pi}{4} - \frac{1}{2}\Big[\ln|1+1^2| - \underbrace{\ln|1+0|}_{=0}\Big]$

$\boxed{= \frac{\pi}{4} - \frac{1}{2}\ln(2)}$

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(Ex) $\underbrace{\int e^x \cos x \, dx}_{I}$

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x dx \qquad v = \sin x$

$= e^x \sin x - \int e^x \sin x \, dx$

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$u = e^x \qquad dv = \sin x\,dx$

$du = e^x dv \qquad v = -\cos x$

$I = e^x \sin x - \Big[-e^x \cos x + \int e^x \cos x \, dx\Big]$

$I = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$I = e^x \sin x + e^x \cos x - I$

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$$2I = e^x \sin x + e^x \cos x + C$$

$$I = \int e^x \cos x\, dx$$

$$I = \frac{e^x \sin x + e^x \cos x + C}{2}$$

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$$\int e^{\sqrt{9x+15}}\, dx$$

$$a = \sqrt{9x+15}$$

$$da = \left(\frac{1}{2}(9x+15)^{-1/2} \cdot 9\right) dx$$

$$da = \frac{9}{2\sqrt{9x+15}}\, dx$$

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$$da = \frac{9}{2a}\, dx$$

$$\frac{2}{9}\int e^{a} \cdot a\, da$$

$u = a \qquad dv = e^a\, da$

$du = da \qquad v = e^a$

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$$= \frac{2}{9}\left[a e^a - \int e^a\, da\right]$$

$$= \frac{2}{9}\left[a e^a - e^a\right] + C$$

$$= \frac{2}{9}\left[\sqrt{9x+15}\, e^{\sqrt{9x+15}} - e^{\sqrt{9x+15}}\right] + C$$

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