

# Circuits Cheat Sheet

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## Exam 1

### • Amplifiers

• Gain

$$A_v = \frac{v_o}{v_i} \left( \frac{V}{V} \right) A_i = \frac{i_o}{i_i} \left( \frac{A}{A} \right) A_p = \frac{P_o}{P_i} = \frac{v_o i_o}{v_i i_i} \left( \frac{\omega}{\omega} \right)$$

in dB

$$A_v \text{ or } A_i = 20 \log |A_{v(i)}|$$

$$A_p = 10 \log |A_p|$$

### • Power Balance

✓ d.s. in amp.

$$P_{dc} + P_i = P_L + P_{loss}$$

$$\frac{P_L}{P_{dc}} \times 100 = \% \text{ efficiency.}$$

### • Standard Amp.

(Amp. Models concept lec)

$$\frac{v_o}{v_i} = A_{vo} \left( \frac{R_L}{R_L + R_o} \right) \left( \frac{R_i}{R_s + R_i} \right)$$

### • Ideal Op-Amps

$$v_o = A(v_+ - v_-)$$

① Draws zero input current:  $i_- = i_+ = 0 A$

②  $v_o$  is not a func. of  $i_o$ :  $R_o = 0$

③ Open loop gain is inf:  $v_+ = v_-$

④ Inf. Common Mode Rejection:  $v_o = 0$

⑤ Inf. Bandwidth:  $A$  is not a func. of freq.

### • Non-Ideal Op-Amps. (Finite A)

inverting:  $\frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + \frac{1}{A} \left( 1 + \frac{R_2}{R_1} \right)}$

non-inverting:  $\frac{v_o}{v_i} = \frac{\left( 1 + R_2/R_1 \right)}{1 + \frac{1}{A} \left( 1 + \frac{R_2}{R_1} \right)}$

This is the  
one where you  
make tables

\* Know how to Draw all Op-Amp Configs.

\* Know how to Solve w/ Pots or dividers.

- Inverting Op-Amps

$$\frac{V_o}{V_i} = \frac{-R_2}{R_1} \rightarrow G = \frac{-R_2}{R_1}$$

Closed Loop Gain:

$$R_i = \frac{V_i}{I_i} = R_{\text{in}} R_1 \quad R_o = 0$$

$$I_i = \frac{V_i}{R_1}$$

$$I_2 = \frac{0 - V_o}{R_2}$$

\* Know how to Design an Op-Amp → see ex. lec.

- Weighted Summer

$$V_o = V_{o1} + V_{o2} + V_{o3}$$

$$V_o = -\frac{R_f}{R_1} V_1 - \frac{R_f}{R_2} V_2 - \frac{R_f}{R_3} V_3 \dots$$

$$i_f = \frac{V_{\text{max}}}{R_f} \quad (?)$$

- Non-Inverting Op-Amps (Rev. V.S.P.)

$$R_i = \frac{V_i}{I_i} = \infty \quad R_o = 0$$

$$G = 1 + \frac{R_2}{R_1}$$

- Voltage Follower:

$$R_1 = \infty \quad R_2 = 0 \quad V_o = V_i$$

- Difference Amplifier:

$$V_o = V_{o1} + V_{o2}$$

$$V_{o1} = \left( -\frac{R_2}{R_1} \right) V_{I1}$$

$$V_{I1} = V_{ICM} - \frac{1}{2} V_{ID}$$

$$V_{I2} = V_{ECM} + \frac{1}{2} V_{ID}$$

$$V_o = A_{cm} V_{ECM} + A_d V_{ID}$$

$$V_{o2} = \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{R_u}{R_3 + R_u} \right) V_{EC2}$$

$V_{EC}$  = common mode

$V_{ID}$  = differential

- Common Mode Rejection Ratio:

$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| \text{ (dB)}$$

\* if  $A_{cm}$  is zero (ideal) then  $CMRR = \infty$ .

$$A_{cm} = -\frac{R_2}{R_1} + \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{R_u}{R_3 + R_u} \right) \text{ ***}$$

$$A_d = \frac{1}{2} \left( \frac{R_2}{R_1} + \left( 1 + \frac{R_2}{R_1} \right) \left( \frac{R_u}{R_3 + R_u} \right) \right)$$

\* When  $A_{cm} = 0$ :

$$A_d = \frac{R_2}{R_1}$$

When true  
 $A_d = \frac{R_2}{R_1}$

$$R_{ID} = 2R_1$$

$$R_2 = R_u$$

\* Know how to solve when  $R_s$  have a ~~tolerance~~ % tolerance.

(4)

- Instrumentation Amplifier

$$v_o = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) v_{id}$$

w/  $R_2$  mismatch:

$$6 \quad v_o = \frac{R_4}{R_3} \left( 1 + \frac{R_{21} + R_{22}}{R_1} \right) v_{id}$$

$$v_{id} = v_{i2} - v_{i1}$$

$$i_1 = \frac{v_{i1} - v_{o1}}{R_2}$$

\* Knows how to  
solve w/ a  
pot.

$$i_2 = \frac{v_{i2} - v_{o2}}{R_2}$$

$$i_x = \frac{v_{id}}{2R_1}$$

- IDEAL DIODES

OFF: Reverse Bias Region:  $v < 0, i = 0$

↳ looks like an open circuit

ON: Forward Bias Region:  $v > 0, i > 0$

↳ looks like a short circuit

→ Assume Diode state & solve to check.

\* Know how to read both types of circs.

(5)

- Forward Bias Diodes

$$V_T = \frac{kT}{q}$$

$$V_T = 25 \text{ mV} \quad (\text{room temp.})$$

$$k = 8.62 \times 10^{-5} \text{ eV/K}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$i \approx I_s \exp\left(\frac{v}{nV_T}\right)$$

$$\frac{I_2}{I_1} = \exp\left(\frac{V_2 - V_1}{V_T}\right)$$

$$V_2 - V_1 = V_T \ln\left(\frac{I_2}{I_1}\right)$$

- Modeling Diodes:

iterate:  $I_D = \frac{V_{DD} - V_D}{R}$

constant drop: use ↑ w/  $V_D = 0.7$  unless told otherwise

ideal: solve w/  $V_D = 0$ .

- Zener Diodes

$$V_Z = V_{Z0} + I_Z r_Z \quad \underline{\text{or}} \quad V_{Z0} = V_Z - I_Z r_Z$$

$$\Delta V_0 = \frac{r_Z}{R+r_Z} (\Delta V_S)$$

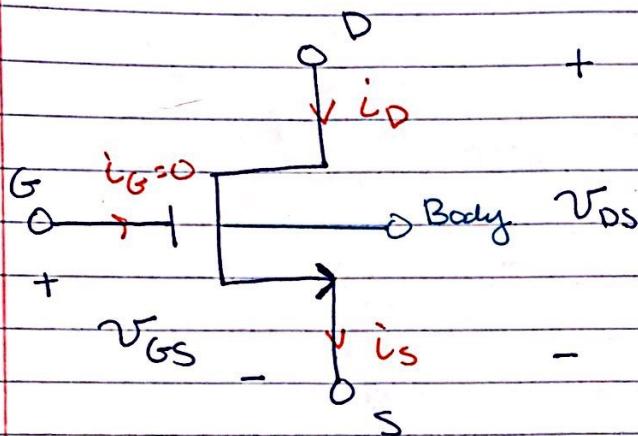
# Circuits Eq. Sheet

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## Exam 2

①

- MOSFETS: NMOS \* see MOSFET Eq. Summary



Symmetrical Device:

$$i_D = i_S$$

$$\text{so: } i_G = 0$$

$$\mu_n C_{ox}, V_{tn} = V_t$$

Triode:

- ①  $V_{GS} > V_t$ ,  $V_{DS}$  is very small

$$i_D = k'n \left( \frac{w}{l} \right) \frac{1}{2} (V_{GS} - V_t) V_{DS}$$

$$V_{DS} = \frac{k'n \left( \frac{w}{l} \right) (V_{GS} - V_t)}{2}$$

- ②  $V_{GS} > V_t$ ,  $V_{DS}$  is small

$$V_{DS} < (V_{GS} - V_t)$$

$$i_D = k'n \left( \frac{w}{l} \right) \left[ (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

Saturation Region:

$$V_{GS} > V_t$$

$$V_{DS} > V_{GS} - V_t$$

Edge of Saturation:

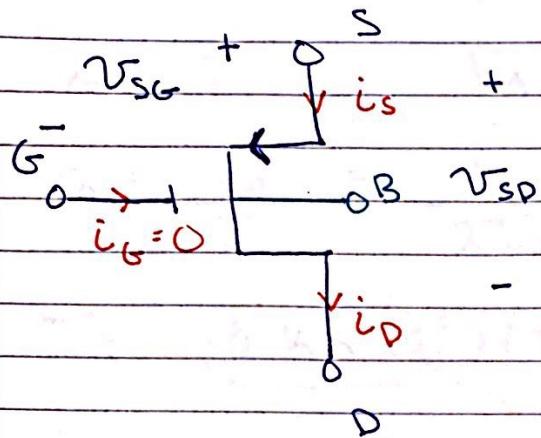
$$V_{DS} = V_{GS} - V_t$$

$$i_D = \frac{1}{2} k'n \left( \frac{w}{l} \right) (V_{GS} - V_t)^2$$

for  $I_{DQ}$ :  $I_{DQ} = \frac{1}{2} k'n \left( \frac{w}{l} \right) (V_{GSQ} - V_t)^2$

(2)

- MOSFETs: PMOS  $\rightarrow$  also in MOSFET Eq. Summary



Symmetrical Device:

$$i_D = i_S$$

$$\text{so: } i_D = 0$$

$$M_p C_{ox}, |V_{tp}| < 0 \text{ so } |V_{tp}|$$

### Triode:

$$\textcircled{1} \quad V_{SG} > |V_{tp}|, V_{SD} \text{ is very small}$$

$$i_D = k'_p \left(\frac{W}{L}\right) (V_{SG} - |V_{tp}|) V_{SD}$$

$$r_{ds} = \frac{1}{k'_p \left(\frac{W}{L}\right) (V_{SG} - |V_{tp}|)}$$

$$\textcircled{2} \quad V_{SG} > V_t, V_{SD} \text{ is small}$$

$$V_{SD} \ll V_{SG} - |V_{tp}|$$

$$i_D = k'_p \left(\frac{W}{L}\right) \left[ (V_{SG} - |V_{tp}|) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$$

### Saturation:

$$V_{SG} > |V_{tp}|$$

### Edge of saturation:

$$V_{SD} \geq V_{SG} - |V_{tp}| \rightarrow V_{SD} = V_{SG} - |V_{tp}|$$

$$i_D = \frac{1}{2} k'_p \left(\frac{W}{L}\right) (V_{SG} - |V_{tp}|)^2$$

(3)

- MOSFETs w/ Finite Output Resistance:

$$\lambda \sim \frac{1}{L} \quad V_A = \frac{1}{\lambda}$$

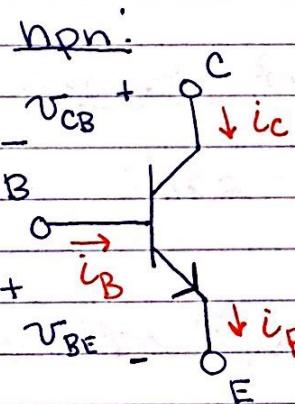
$$r_o = \frac{V_A}{I_D} = \frac{1}{\lambda I_D}$$

\* Assume  $\lambda = 0$  when no  $V_A$ ,  $\lambda$ , or  $r_o$  values are given.

$$I_D = \frac{1}{2} k' n \left( \frac{W}{L} \right) (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

Original  $I_D$  eq.

- BJT Operation + DC Biasing:



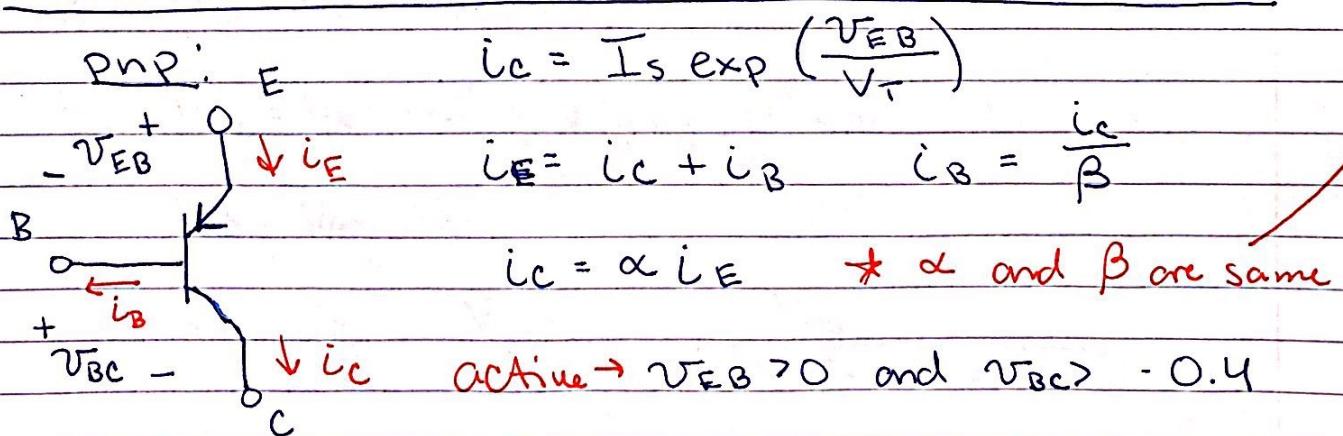
$$i_C = I_S \exp \left( \frac{V_{BE}}{V_T} \right)$$

$$i_B = \frac{i_C}{\beta} \quad i_E = i_B + i_C$$

$$i_C = \alpha i_E$$

$$\beta = \frac{\alpha}{\alpha - 1} \quad \alpha = \frac{\beta}{\beta + 1}$$

→ active:  $V_{BE} > 0$  and  $V_{CB} > -0.4V$



$$i_C = I_S \exp \left( \frac{V_{EB}}{V_T} \right)$$

$$i_E = i_C + i_B \quad i_B = \frac{i_C}{\beta}$$

$$i_C = \alpha i_E \quad * \alpha \text{ and } \beta \text{ are same}$$

active →  $V_{EB} > 0$  and  $V_{BC} > -0.4V$

\* See Note on Diode Eq. →

- Forward Bias Eq.: (Came back from diodes)

→ These equations were used in the diode forward bias lectures and the BJT forward bias lecture.

\* Know how & when to use them.

General Forms:

$$\frac{I_2}{I_1} = \exp \left( \frac{V_2 - V_1}{V_T} \right)$$

$$V_2 - V_1 = V_T \ln \left( \frac{I_2}{I_1} \right)$$

Example for BJT to find  $V_{BE2}$ :

$$V_{BE2} - V_{BE1} = V_T \ln \left( \frac{I_{C2}}{I_{C1}} \right)$$

- Transition Equations

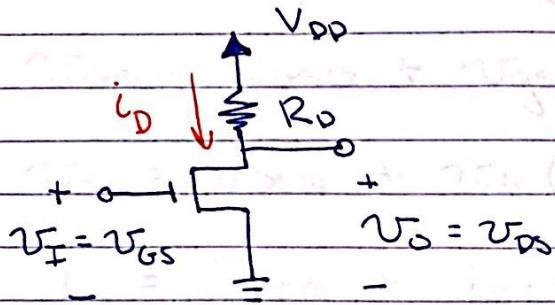
A:  $V_{GS} = V_t \quad V_{DS} = V_{DD}$

B:  $V_{GS} = V_t + \sqrt{1 + 2R_D k' \ln(\frac{\omega}{\tau}) V_{DD}} - 1$   
 $R_D k' \ln(\frac{\omega}{\tau})$

$$V_{DS} = V_{GS} - V_t$$

C:  $V_{GS} = V_{DD} \quad V_{DS} = \frac{V_{DD}}{(1 + R_D k' \ln(\frac{\omega}{\tau})(V_{DD} - V_t))}$

- MOSFETs & Amplifiers ★ Know how we are using prev. MOSFET Eq.



Always true: (Ohm's Law)

$$i_D = \frac{V_{DD} - V_{DS}}{R_D}$$

combine w/ MOSFET eqs.

- Cutoff:  $V_{GS} < V_t$   $\rightarrow O = \frac{V_{DD} - V_{DS}}{R_D}$   
 $i_D = 0$   
 $V_{DS} = V_{DD}$   $\Rightarrow V_{DS} = V_{DD}$

- Saturation:  $(V_{DS} \geq V_{GS} - V_t)$

$$V_{DS} = V_{DD} - \frac{1}{2} R_D k' n \left( \frac{w}{l} \right) (V_{GS} - V_t)^2$$

$R_D i_D$

- Triode:  $(V_{DS} < V_{GS} - V_t)$

$$V_{DS} = \frac{V_{DD}}{1 + R_D k' n \left( \frac{w}{l} \right) (V_{GS} - V_t)}$$

$V_{GS}^{\max}$

$$V_{DS} = \frac{V_{DD}}{1 + R_D k' n \left( \frac{w}{l} \right) (V_{DD} - V_t)}$$

- Edge of Saturation:

$$V_{GS} = V_t + \sqrt{1 + 2(R_D k' n \left( \frac{w}{l} \right) V_{DD})} - 1$$

$R_D k' n \left( \frac{w}{l} \right)$

(6)

### MOS Amplifier Voltage Gain + the Q-Point:

$$V_{GS} = V_{GSQ} + v_{gs}(t)$$

$$V_{DS} = V_{DSQ} + v_{ds}(t)$$

$$i_D = \underbrace{I_{DQ}}_{\substack{\text{DC-bias} \\ \text{point} \\ (\text{Q-point})}} + \underbrace{i_d(t)}_{\substack{\text{Small} \\ \text{Signal}}}$$

Finding Max  $v_i$ :

$$V_{DSQ} - |Av|v_i = V_{GSQ} + v_i - V_t$$

→ solve for  $v_i$

comes from:

$$V_{DS\min} = V_{GS\max} - V_t$$

$$v_{ds} = Av v_{gs}$$

↑ small signal volt. gain

$$Av = -R_D k'n \left(\frac{w}{l}\right) (V_{GSQ} - V_t)$$

\* can pick resistors, transistors, and Q-Point to get desired gain when designing.

### Small Signal Eq.:

$$i_d = k'n \left(\frac{w}{l}\right) [v_{gs}(V_{GSQ} - V_t)]$$

$$g_m = \frac{i_d}{v_{gs}} = k'n \left(\frac{w}{l}\right) (V_{GSQ} - V_t)$$

units:  
[A/V]

$$Av = -g_m R_D$$

$$\text{reminder: } r_o = \frac{1}{2I_{DQ}} = \frac{V_A}{I_{DQ}}$$

\* coupling capacitors act as open circuits for DC and as short circuits for AC.

- MOS Amps: \* Combined w/ the Small sig. eqs.

$$R_{in} = \frac{V_i}{I_i}$$

$$V_i = \frac{R_{in}}{R_{in} + R_{sig}} \cdot V_{sig}$$

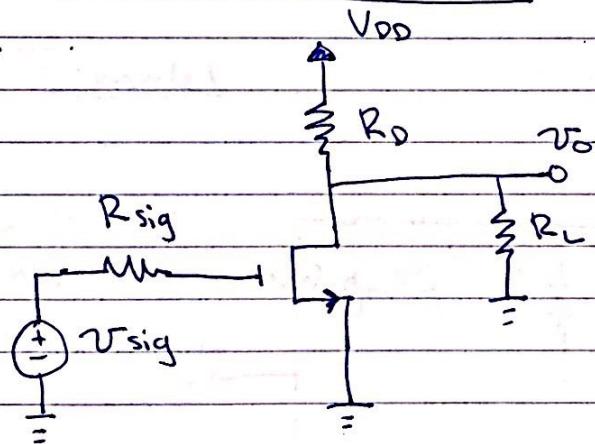
$$A_{v0} = \frac{V_0}{V_i} \mid R_o \rightarrow \infty$$

$$A_v = \frac{V_0}{V_i}$$

$$G_v = \frac{V_0}{V_{sig}}$$

- Common Source Amp: ("Source Grounded")

Giver



so  $V_i = V_{sig}$

$$R_{in} = \infty \quad I_i = 0$$

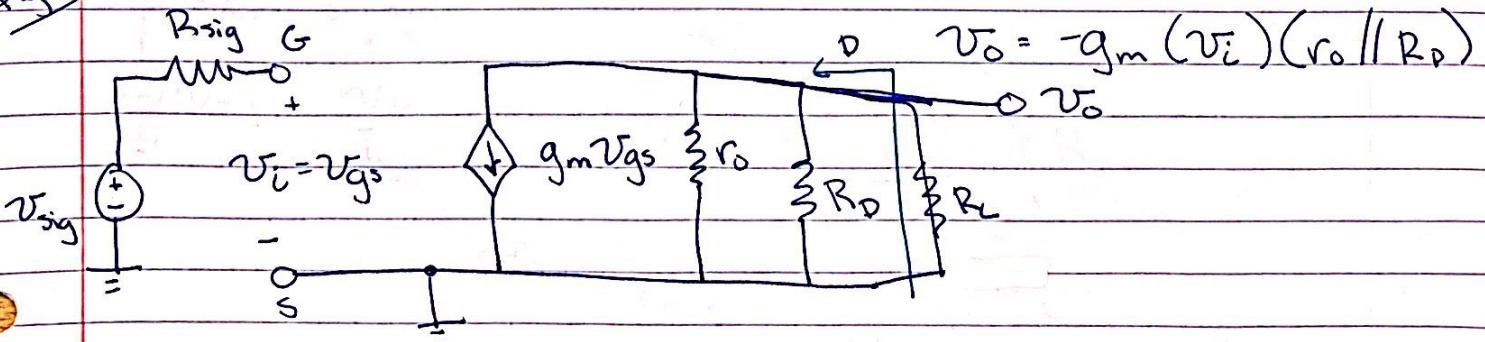
$$R_o = R_D \parallel r_o$$

$$A_{v0} = -g_m (r_o \parallel R_D)$$

$$A_v = -g_m (r_o \parallel R_D \parallel R_L)$$

$$G_v = -g_m (r_o \parallel R_D \parallel R_L)$$

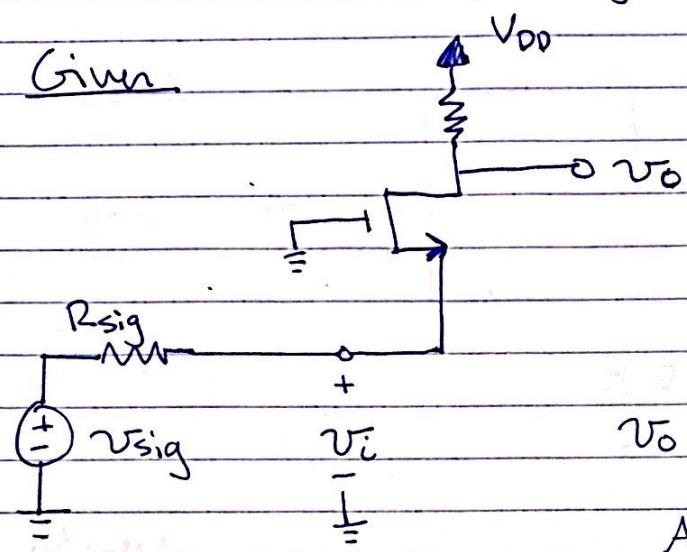
hybrid- $\pi$



$$V_0 = -g_m (V_i) (r_o \parallel R_D)$$

(8)

- Common Gate Amp: (gate grounded,  $v_i$  is @ source)

Given

$$R_{in} = 1/g_m$$

$$R_o = R_D$$

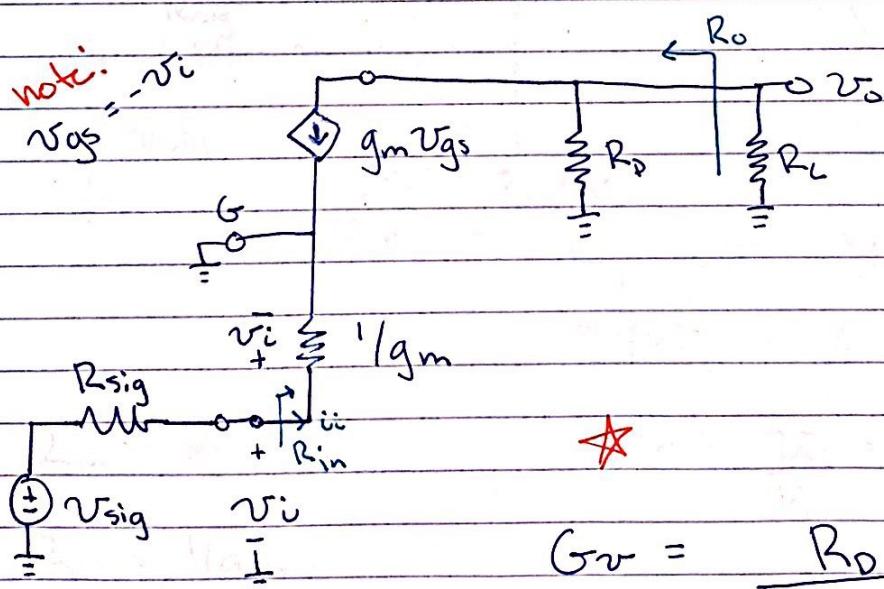
$$v_{gs} = -v_i$$

$$v_o = -g_m (R_D) (-v_i) = g_m R_D v_i$$

$$A v_o = g_m R_D$$

T-model

$$A v = g_m (R_D \parallel R_L)$$



$$G_v = \frac{R_D \parallel R_L}{R_{sig} + 1/g_m}$$

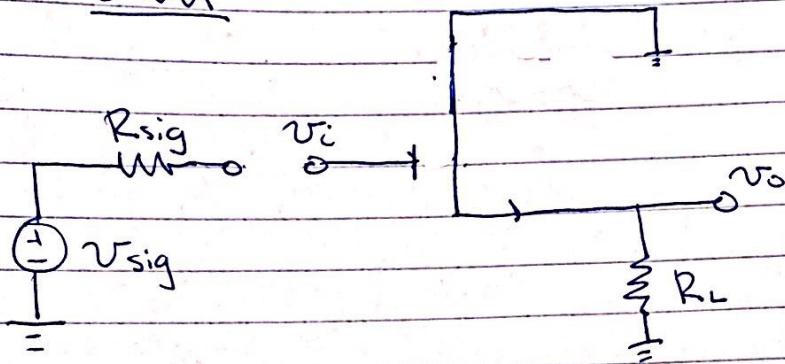
$$G_v = \frac{R_D \parallel R_L}{R_{sig} + R_{in}}$$

or

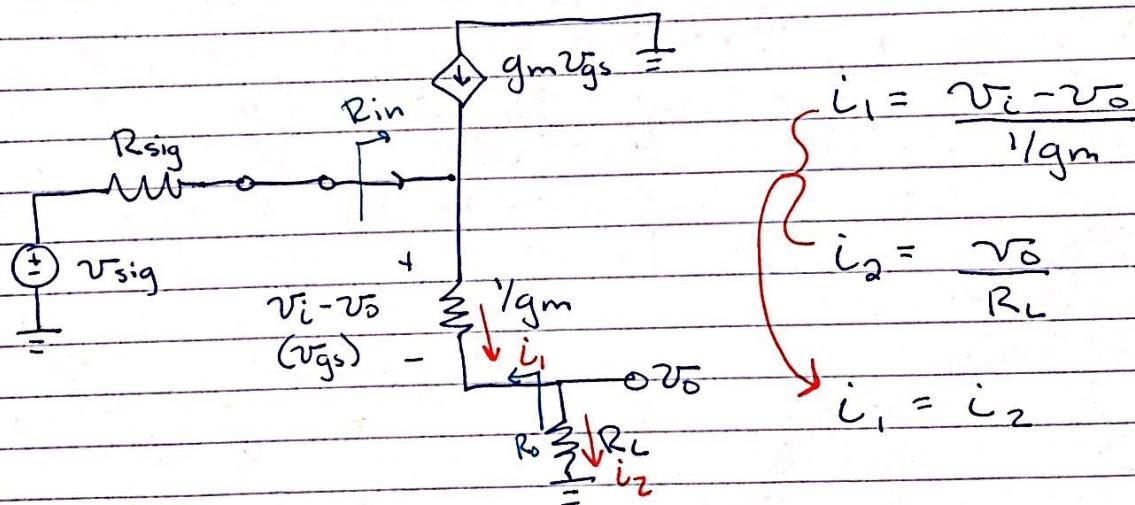
(9)

Common Drain Amp: (voltage follower,  $V_i$  at gate)

Given



Model



$$R_{in} = \infty$$

$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_L + 1/g_m}$$

$$R_o = 1/g_m$$

$$G_v = \frac{V_o}{V_{sig}} = \frac{R_L}{R_L + 1/g_m}$$

$$A_v \Big|_{R_L \rightarrow \infty} = \frac{V_o}{V_i} = 1$$

if  $R_L$  is  $\gg 1/g_m$  then  $G_v \approx 1$