## Department of Electrical and Computer Engineering The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

## Homework #3

Due: Monday, February 24 at 9:35 am
Please bring hardcopy to the class and upload softcopy to Canvas

| Student name: | 1<br>15 | 2<br>20 | 3<br>20 | 4<br>10 | 5<br>15 | 6<br>20 | Total |
|---------------|---------|---------|---------|---------|---------|---------|-------|
|               |         |         |         |         |         |         |       |

**1.** (15 points) Write differential equation describing displacement x of suspended weight m on spring with elastic constant k.

**2.** (20 points) A system with input x(t) and output y(t) is defined by the following differential equation:

$$\frac{d^{2}y(t)}{dt^{2}} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Find the impulse response h(t) and the unit-step response s(t).

3. (20 points) Consider a second order differential equation,

$$\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

with initial conditions y(0) = 1 and  $\frac{dy(t)}{dt}|_{t=0} = 0$  and  $\mathbf{x}(t) = \mathbf{u}(t)$ .

- Find the complete response y(t)
- Find the steady state response and the transient response.

4. (10 points) Suppose that the transfer function of the LTI system is

$$H(s) = \frac{s}{s^2 + s + 1}$$

Find the unit-step response s(t), and then use it to find the response to

$$x_1(t) = u(t) - u(t-1)$$

$$x_2(t) = \delta(t) - \delta(t-1)$$

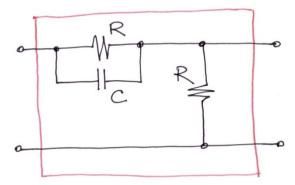
**5.** (15 points)

An unstable system can be stabilized by using negative feedback with a gain K in the feedback loop. For instance, consider an unstable system with transfer function

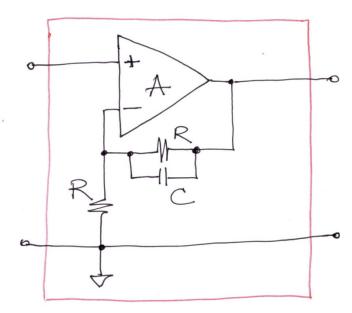
$$H(s) = \frac{2}{s-1}$$

which has a pole in the right-hand s-plane, making the impulse response of the system h(t) grow as t increases. Use negative feedback with a gain K>0 in the feedback loop, and put H(s) in the forward loop. Draw a block diagram of the system. Obtain the transfer function G(s) of the feedback system and determine the value of K that makes the overall system BIBO stable (i.e., its poles in the open left-hand s-plane).

**6.** a) (5 points) What is the transfer function of the following circuit:



- b) (5 points) What is the transfer function of the following circuit? Hints:
  - you can use solutions of problem #5 and #6a
  - to simplify the result you can assume that A  $\rightarrow \infty$



c) (10 points) Find and plot the unit-step response s(t) of the system?