$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

$\boxed{\text{Work}}$

The work done by a constant force

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$\vec{F}$ acting through a displacement $\vec{D} = \overrightarrow{PQ}$ is given by

$W = \vec{F} \cdot \vec{D}$

$= |\vec{F}||\vec{D}| \cos \theta$

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Ex A father pulls a child in a sleigh with 150 N force at an angle of 30° to the ground. How much work is done over a 2 km $= 2000\,m$

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distance ?

$W = \vec{F} \cdot \vec{D}$

$= |\vec{F}| |\vec{D}| \cos \theta$

$= (150)(2000\,m) \cos 30°$

$= (150)(\overset{1000}{20\cancel{00}}) \dfrac{\sqrt{3}}{\cancel{2}}$

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$= 150{,}000\sqrt{3}$ Nm

Joules

$\boxed{11.4}$ The Cross Product

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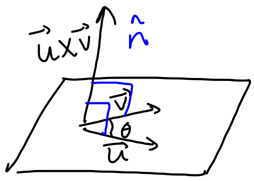

Defⁿ: The cross product of $\vec{u}$ and $\vec{v}$ is given by

$\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}| \sin \theta)\, \hat{n}$

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Where $\hat{n}$ is the normal (unit) vector.

The cross product $\vec{u} \times \vec{v}$ is a vector quantity.

Note: Two nonzero vectors $\vec{u}$ and $\vec{v}$ are

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parallel if and only if $\vec{u} \times \vec{v} = \vec{0}$

Properties of cross-product:

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and c and d any scalars,

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1) $(c\vec{u}) \times (d\vec{v}) = (cd)(\vec{u} \times \vec{v})$

2) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

3) $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$

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4) $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$

5) $\vec{0} \times \vec{u} = \vec{0}$

6) $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$

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Calculating the cross product as a determinant:

If $\vec{u} = u_1 i + u_2 j + u_3 k$ and $\vec{v} = v_1 i + v_2 j + v_3 k$ then
$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

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$$\vec{u} \times \vec{v} = i\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= (u_2 v_3 - u_3 v_2) i - (u_1 v_3 - u_3 v_1) j$$

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$+ (u_1 v_2 - u_2 v_1) k$

Ex Given

$\vec{u} = 2i + j - k$

$\vec{v} = -3i + 4j + k$

1) $\vec{u} \times \vec{v}$

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2) $\vec{v} \times \vec{u}$

1) $\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ -3 & 4 & 1 \end{vmatrix}$

$= i \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} - j \begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix}$

$+ k \begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix}$

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$= i(1+4) - j(2-3)$

$+ k(8+3)$

$= 5i + j + 11k$

$= \langle 5, 1, 11 \rangle$

2) $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$

$= -5i - j - 11k$

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