CPE 212 - Fundamentals of Software Engineering

Heaps

Outline

- Heap definition
- Heap examples
- Heap implementations
- Heap Efficiency

Heap ADT

- Heap
  - A complete binary tree, each of whose elements satisfies the heap ordering property
    - Min-heap : the value of each node is greater than or equal to the value of its parent, with the min-value element at the root
    - Max-heap : the value of each node is less than or equal to the value of its parent, with the max-value element at the root
  - Shape property & order property
  - A heap is not a sorted structure and can be regarded as partially ordered.

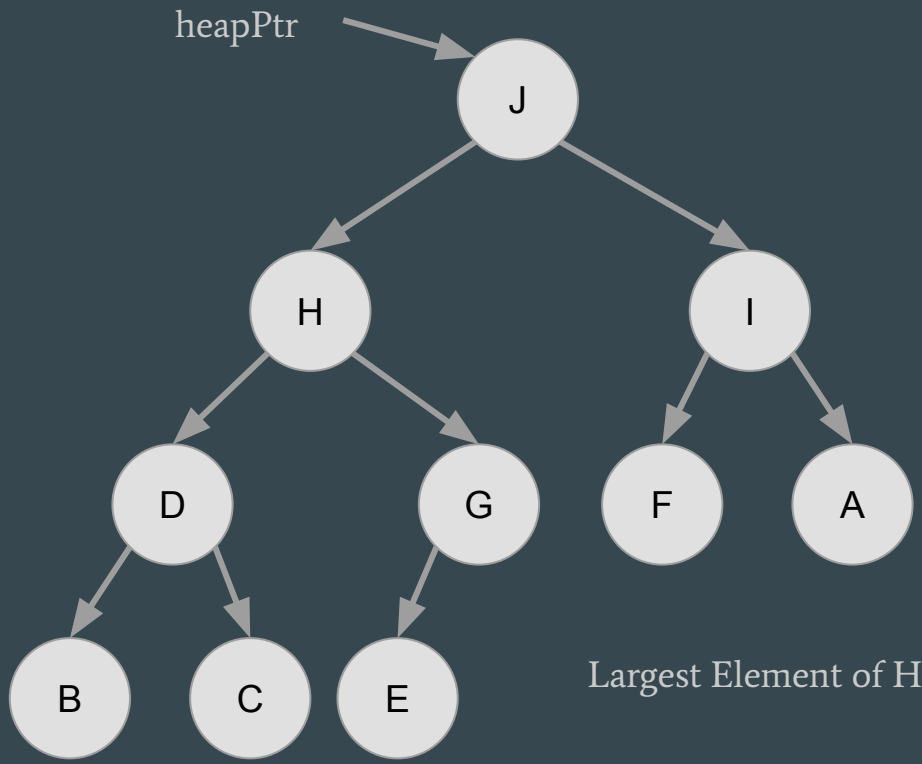
Complete Binary Tree

- Complete Binary Tree
  - A complete binary tree of height *h* is full down to height *h* - 1.
  - Example:
    - Height = 5
    - Full from height = 1 to height = 4
  - When a node at height 4 has children all nodes at the same height and to it's left have two children each
  - When a node at height 4 has one child it's a left child
  - All nodes at *h* - 2 and above have two children each

Heap ADT

- ReheapDown(root,bottom)
  - Restores the order property of the heaps to the tree between root and bottom
  - Precondition
    - The order property of heaps may be violated only by the root node of the tree
  - Postcondition
    - The order property applies to all elements of the heap

Max Heap Example

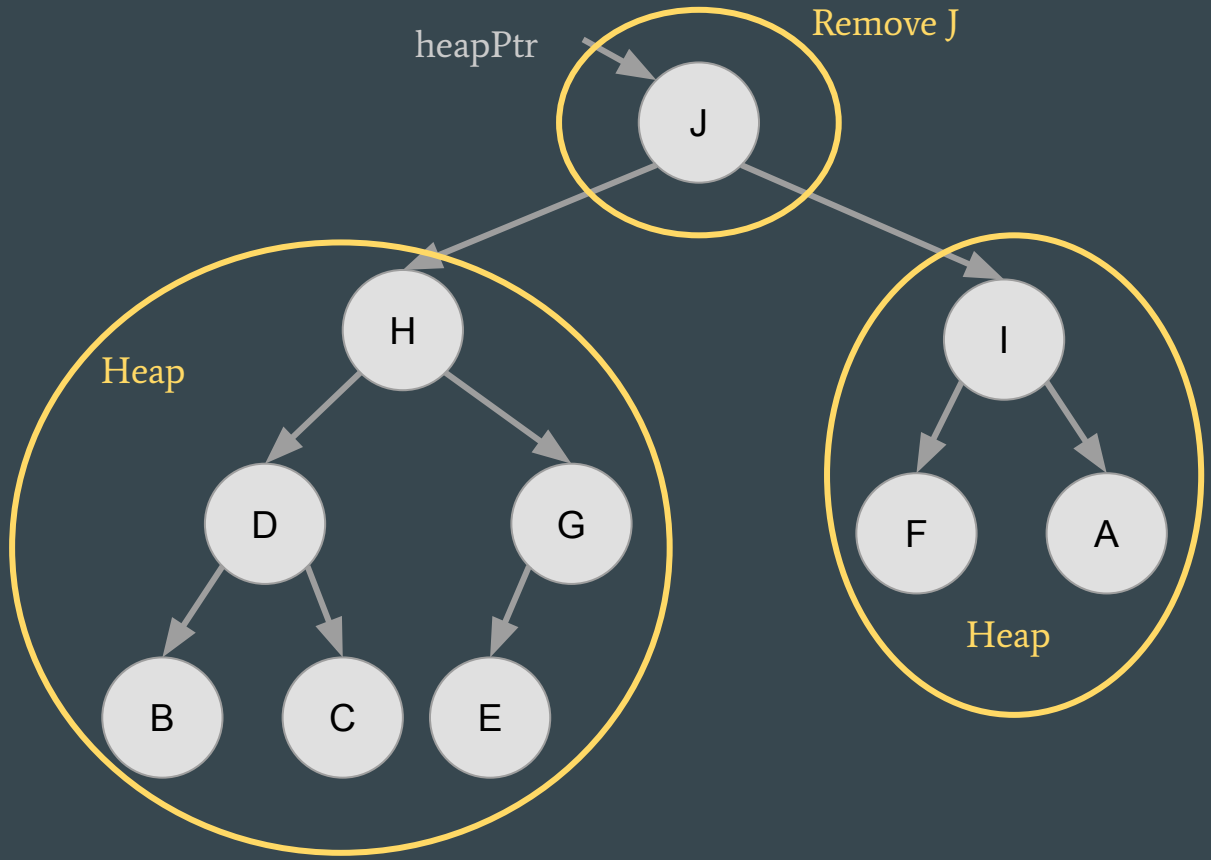


Largest Element of Heap resides in the root node

myHeap.elements

[0]	J
[1]	H
[2]	I
[3]	D
[4]	G
[5]	F
[6]	A
[7]	B
[8]	C
[9]	E

Max Heap Example

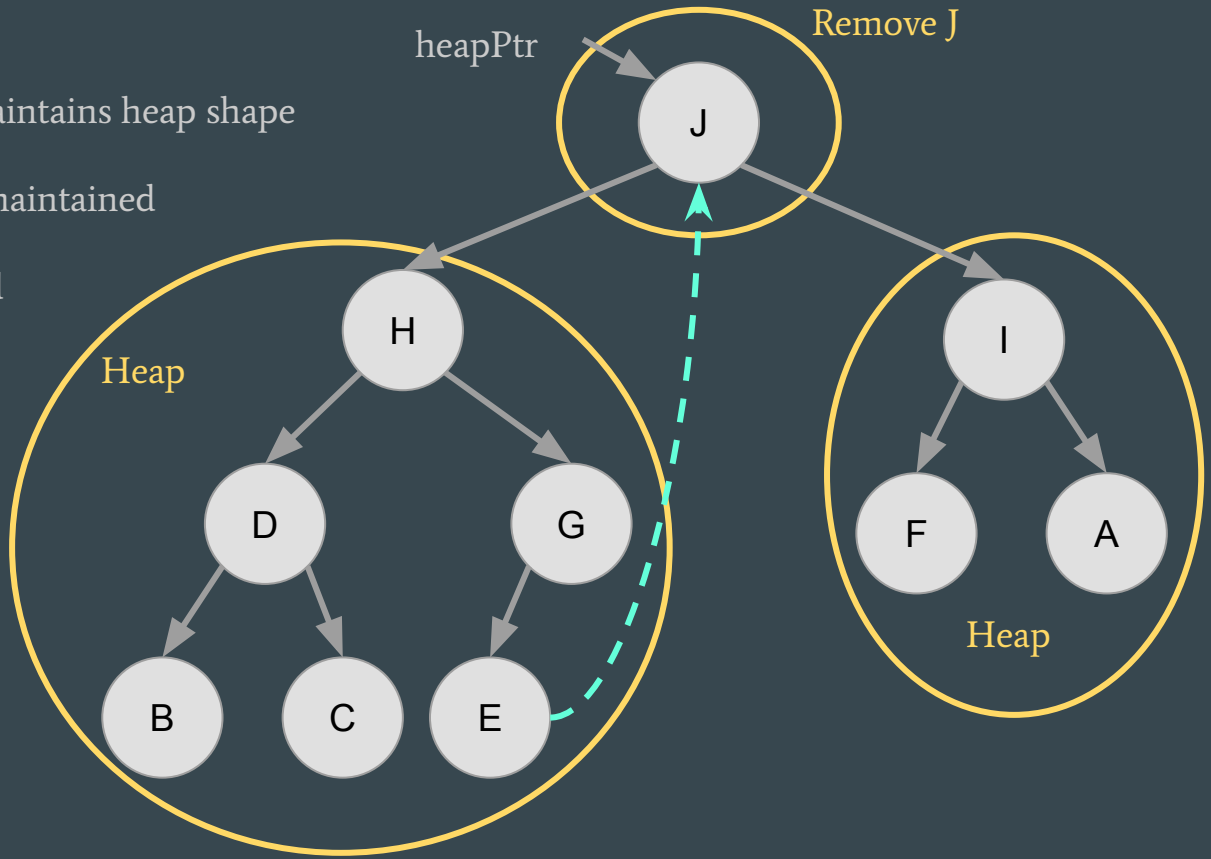


Max Heap Example

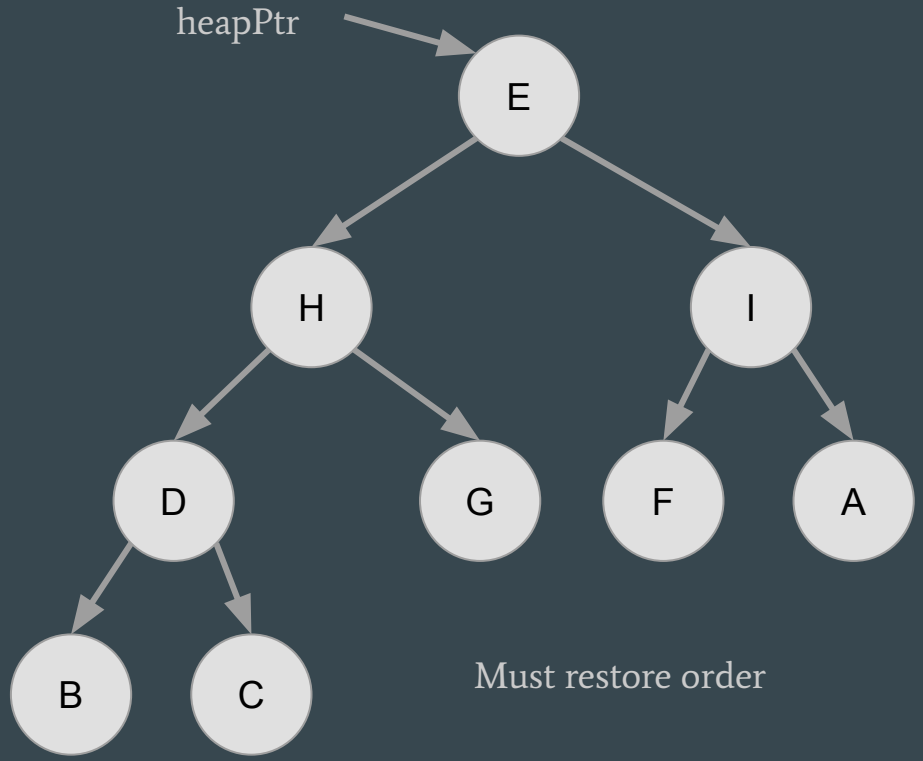
Promoting E to root maintains heap shape

Order property is not maintained

Order must be adjusted

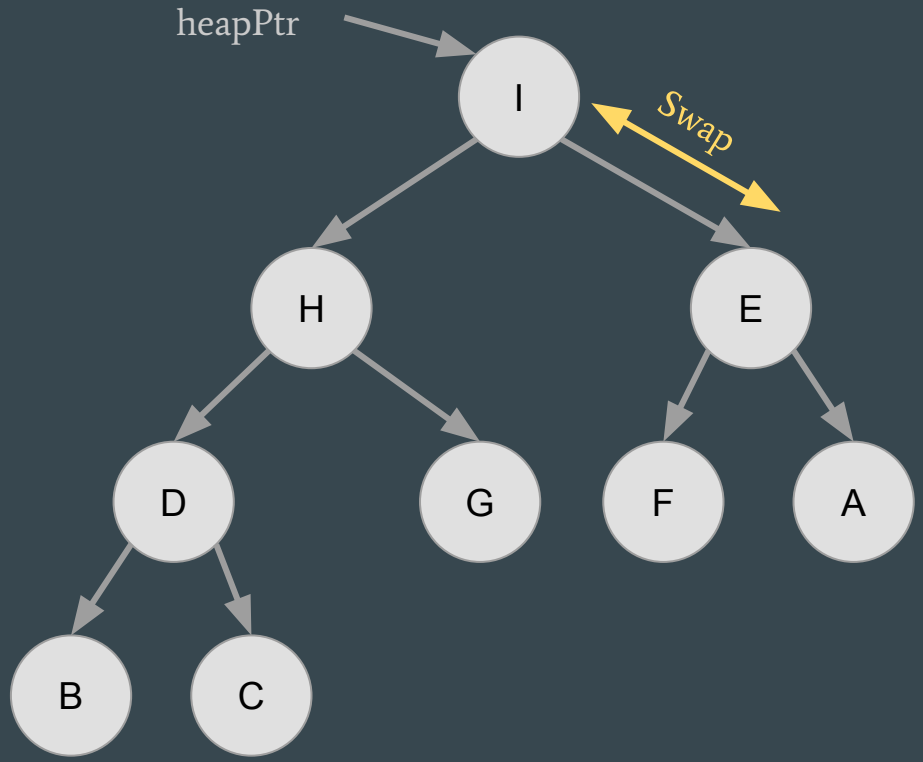


Max Heap Example



Must restore order

Max Heap Example



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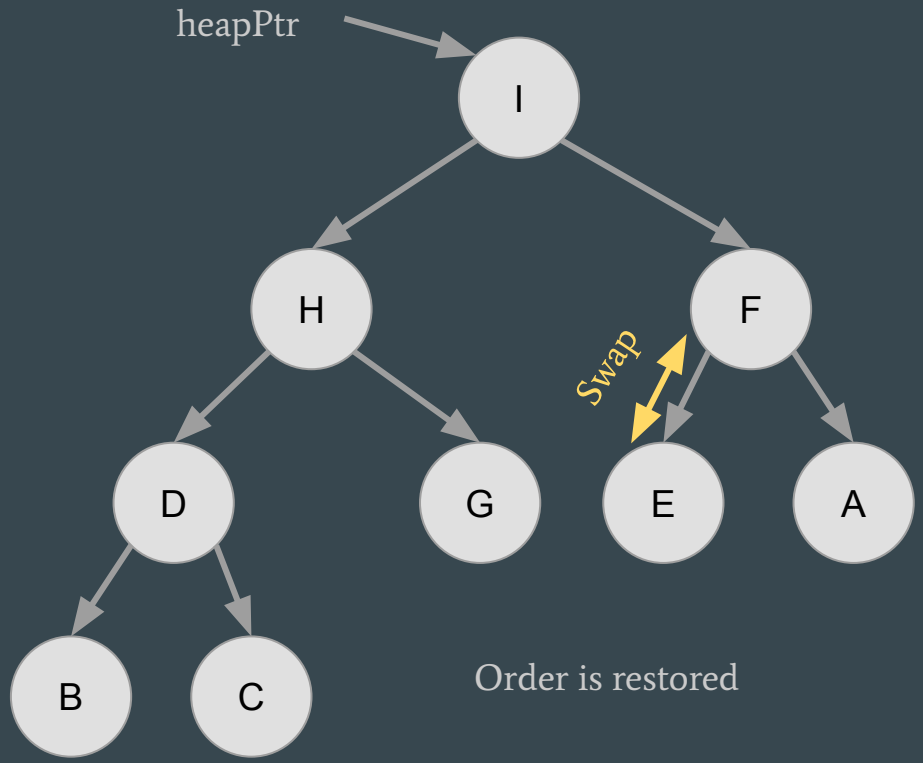


Heap ADT

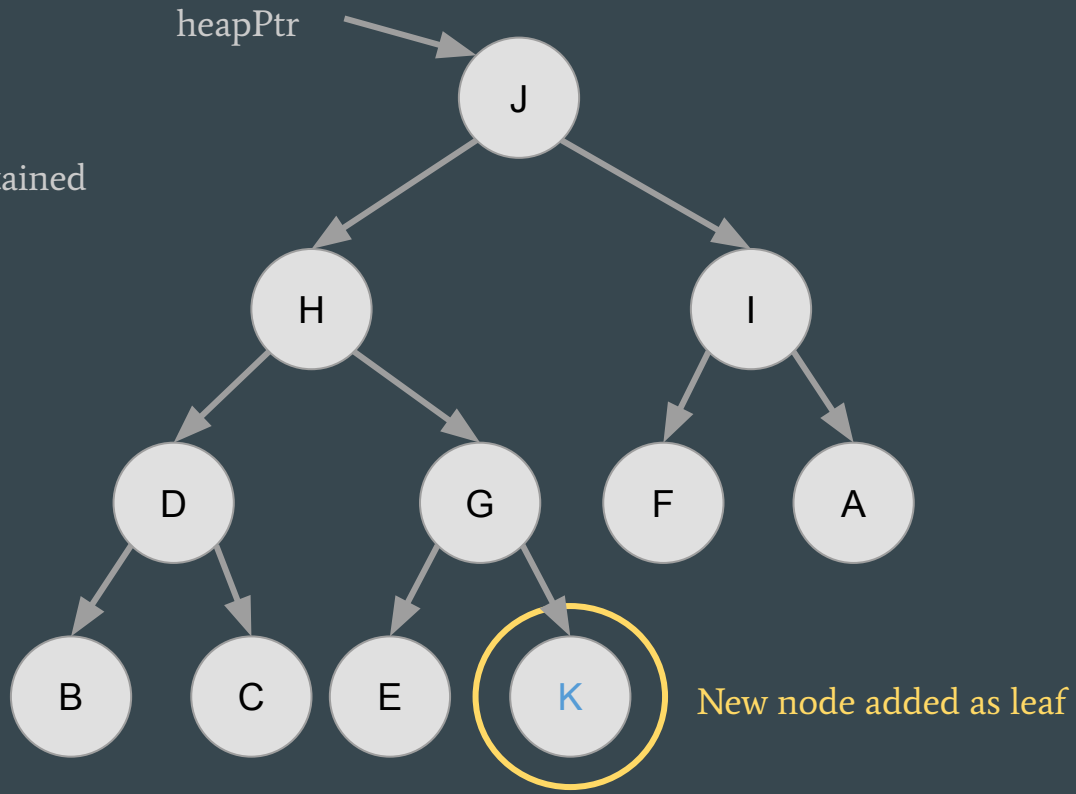
- ReheapUp(root,bottom)
  - Restores the order property of the heap between root and bottom
  - Precondition
    - The order property is satisfied from the root of the heap through the next-to-last node; the last (bottom) node may violate the order property
  - Postcondition
    - The order property applies to all elements of the heap

Heap Example

Shape is maintained

Order property is not maintained

Order must be adjusted



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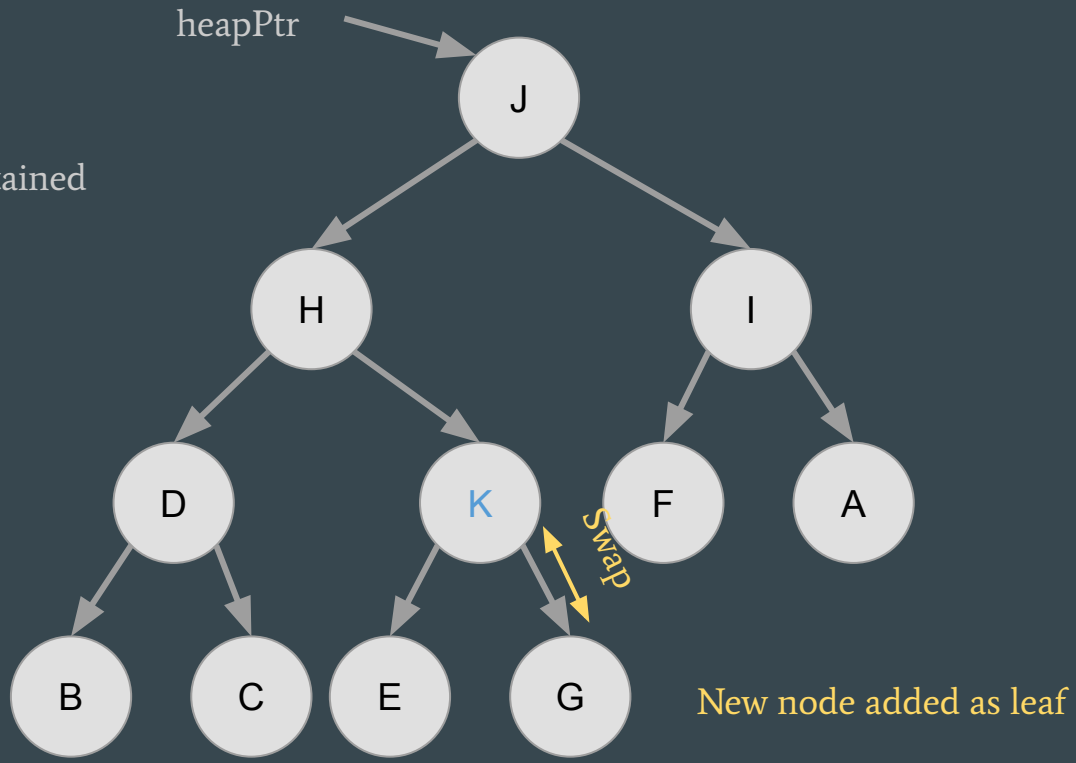

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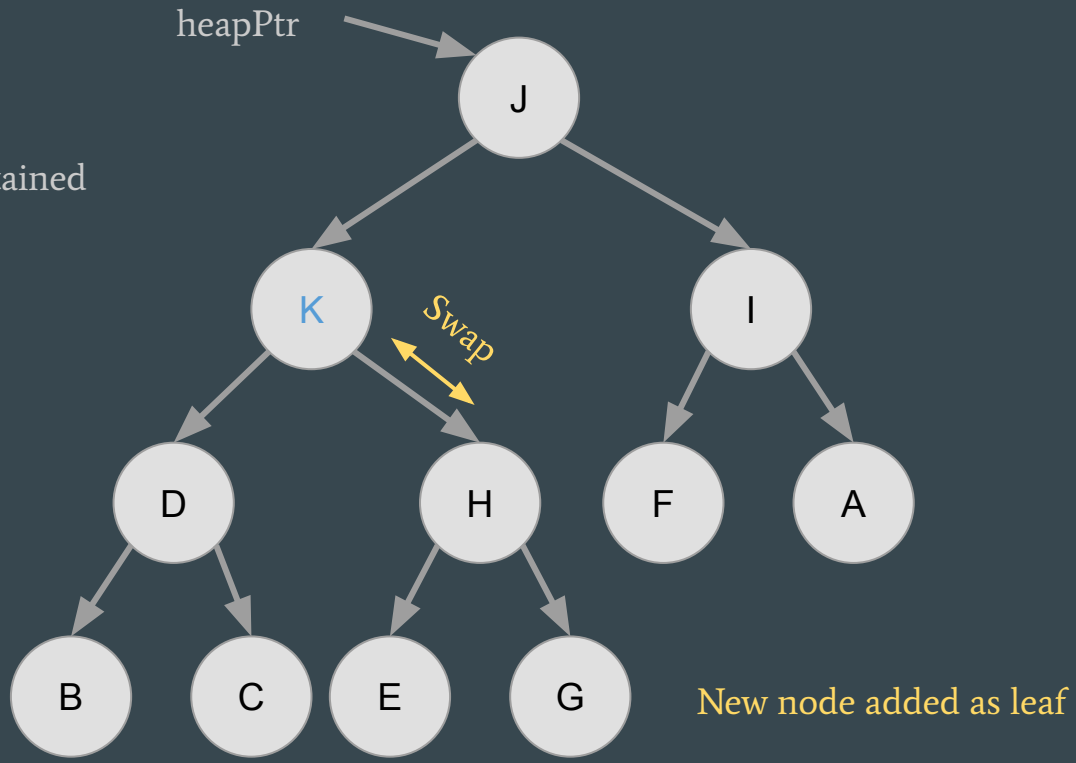

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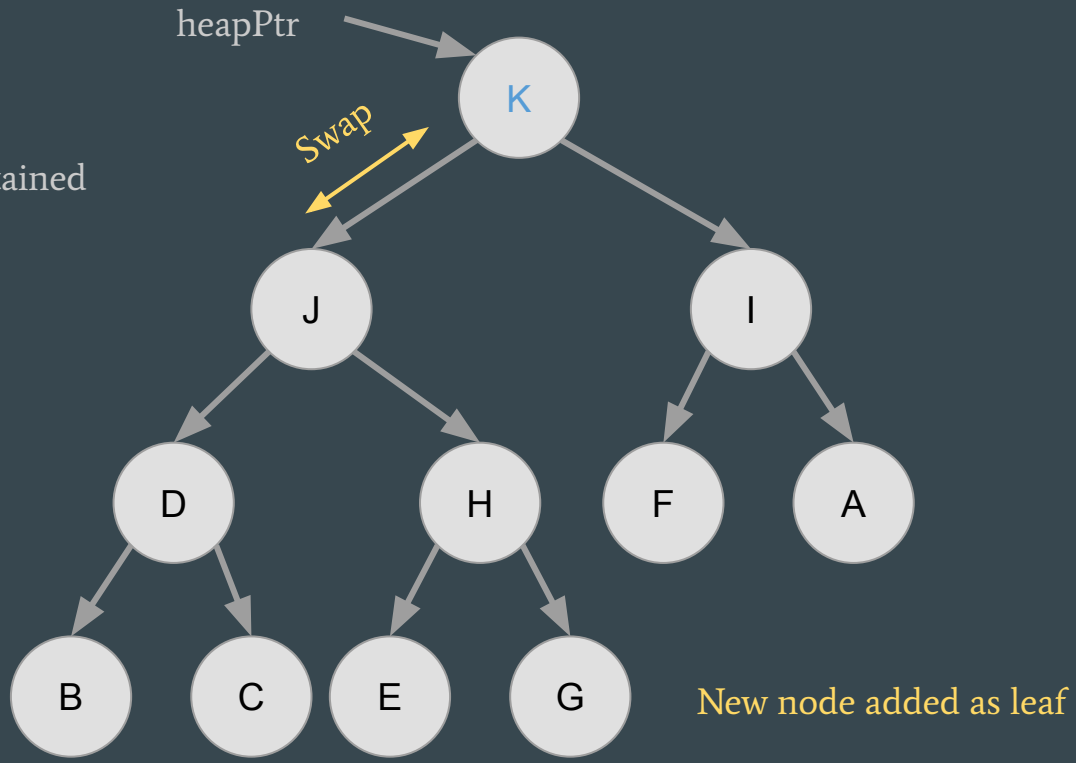

Heap Efficiency

- Inserting
  - O(log n)
- Remove Maximum
  - O(log n)
- Access Maximum
  - O(1)

Examples