Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

# Homework #2

Due: Monday, March 1 at 9:35 am
Please submit your work as PDF on Canvas

**Student name:**

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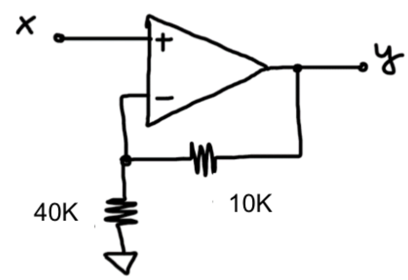
| 1 10 | 2 20 | 3 10 | 4 15 | 5 20 | 6 10 | 7 15 | Total |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

1. (10 points) What is the transfer function of the following circuit:

$X = \frac{40}{40+10} y = y = \frac{5}{4}x$

$\frac{y}{x} = \frac{5}{4}$




2. (20 points) Simulate the effect of multipath in wireless communication. Generate dumped sine wave $x(t)$ with amplitude A=2 and frequency $f$=200Hz sampled at $F_s$= 11,025Hz with time constant 1 second (i.e. $e^{-t}$). Assume that the signal is transmitted over three paths, so that the received signal is

$$y(t) = x(t) + 0.4x(t\text{-}0.1) + 0.2x(t\text{-}0.4)$$

Determine the number of samples corresponding to delay using sampling frequency Fs.
Plot the function x(t) and output y(t) and use *sound* function in Matlab to listen to original and received signals.

**Effect of multipath**

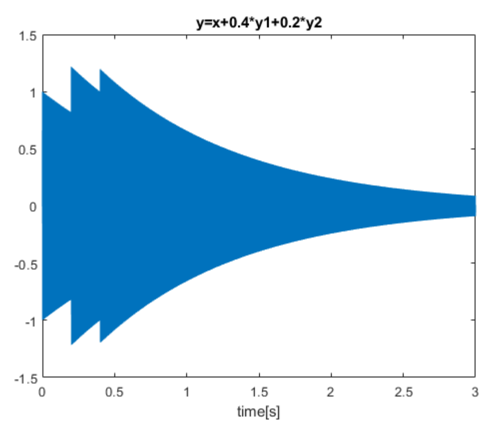
```
A = 1;
f = 400;
Fs = 11025;         % Sampling frequency, 11,025Hz
Ts = 1/Fs;           % Sampling interval
t = 0:Ts:3;         % Time
x = exp(-t).*sin(2*pi*f.*t);
N = length(t);
```

**Signal delays**

```
%This is the (t-0.2) portion.
delay = 0.2*Fs;
delay2 = 0.4*Fs;
intdelay = round(0.2*Fs);       % integer delay, based on sampling frequency.
intdelay2 = round(0.4*Fs);      % second integer delay.
y1 = [zeros(1, intdelay) x(1:N-intdelay)];      % Delayed signal
y2 = [zeros(1, intdelay2) x(1:N-intdelay2)];    % Delayed signal
```

**Signal and plot**

```
y = x + 0.4*y1 + 0.2*y2;
% plot the function
plot(t,y),title('y=x+0.4*y1+0.2*y2'),xlabel('time[s]')
% and listen the result
sound(y,Fs);
```



3. (10 points)

Find impulse response of capacitor and its unit step response. How step response depends on the capacitance of the capacitor?

$$V_C(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$i(t) = \delta(t) \Rightarrow V_c(t) = h(t) = \frac{1}{C}\int_0^t \delta(\tau)d\tau = \frac{1}{C}u(t)$$

$$V_c(t) = \int_{-\infty}^{\infty} h(t-\tau)i(\tau)d\tau = \int_{\infty}^{\infty} \frac{1}{c}(u-\tau)u(\tau)d\tau$$

$$V_c(t) = \frac{1}{C}\int_0^t d\tau = \frac{1}{c}r(t)$$

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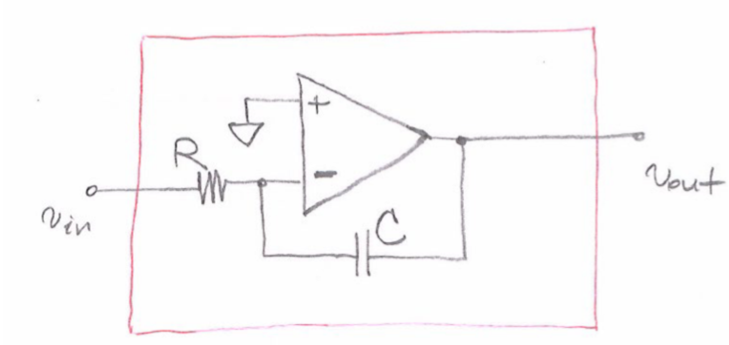
4. (15 points) Find transfer function of the following circuit. What is the output voltage at t=0.4s, for R=10KΩ and C=1 nF?




$$V_+ = V_- = 0, \quad V_{in} - R \cdot i(t) = V_- = 0$$

$$i(t) = \frac{V_{in}(t)}{R}; \quad V_{out} = V_- - \frac{1}{c}\int_0^t i(\tau)d\tau + V_c^0 = -\frac{1}{c}\int_0^t \frac{V_{in}(t)}{R}d\tau.$$

$$\Rightarrow -\frac{1}{RC}\int_0^t v_{in}(\tau)d\tau \qquad \frac{V_{out}}{V_{in}} = -\frac{t}{RC} \qquad V_{out} = \frac{0.4}{10E3 \times 1.0E-9} \times V_{in}$$

$$I = \frac{V_{in}}{R} \qquad V_{out} = -\frac{1}{Cs}I = -\frac{1}{RCs}V_{in} \qquad \boxed{V_{out} = 40{,}000 \cdot V_{in}}$$

$$\frac{4(s+4)}{(s+4)^2+64} \qquad \frac{4(s+2)}{(s+2)^2+64} \qquad \begin{matrix}(s+2)(s+2) \\ s^2+4s+4\end{matrix} \qquad \begin{matrix}(s+4)(s+4)+64 \\ s^2+8s+16\end{matrix}$$

5. (20 points)

Use Matlab symbolic computation to find the Laplace transform of a real exponential

$$x(t) = 4e^{-2t}\cos(8t)\,u(t)$$

Plot the signal and the poles and zeros of their Laplace transform.

Repeat the analysis and plot the results for $x(t) = 4e^{-4t}\cos(8t)\,u(t)$

Discuss the changes in the $s$ plane and describe their effect on function in time domain.

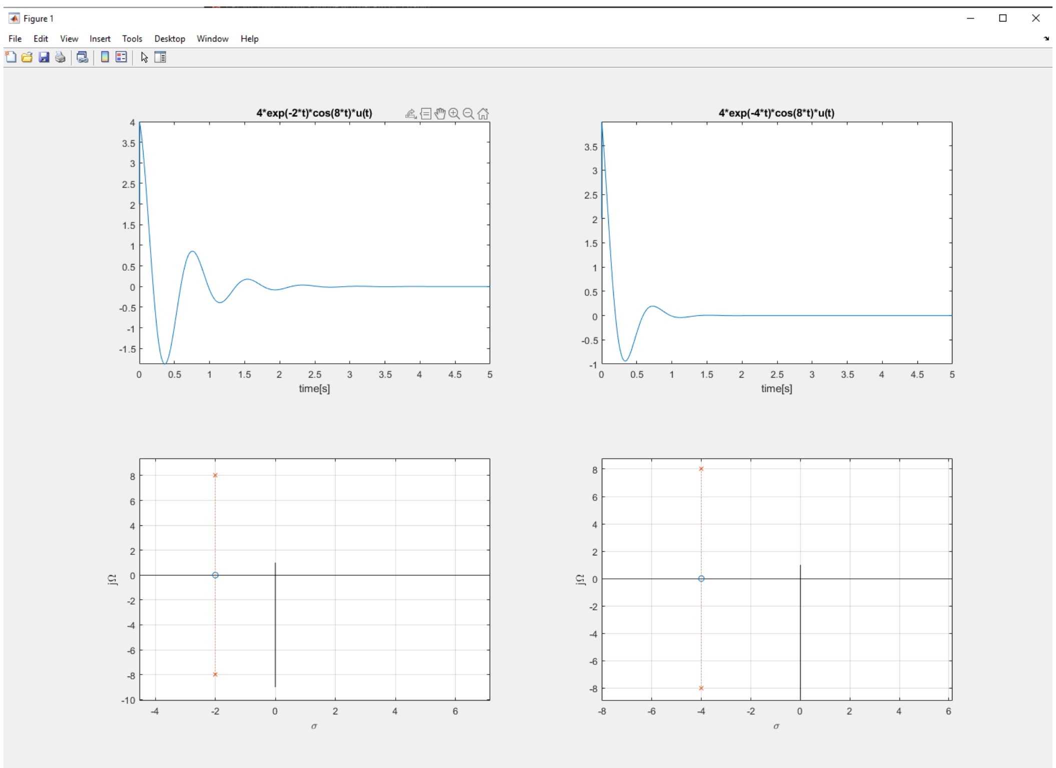
The time domain will be more dampened as the zeroes and poles are shifted to the left.

```
syms t x1 x2 s
x1=4*exp(-2*t)*cos(8*t)*heaviside(t);
x2=4*exp(-4*t)*cos(8*t)*heaviside(t);

figure
X1=laplace(x1)
X12 = (4*(s + 2))/((s + 2)^2 + 64);
subplot(221)
fplot(x1, [0,5]),title('4*exp(-2*t)*cos(8*t)*u(t)'),xlabel('time[s]')


X2=laplace(x2)
X13 = (4*(s + 4))/((s + 4)^2 + 64);
subplot(222)
fplot(x2, [0,5]),title('4*exp(-4*t)*cos(8*t)*u(t)'),xlabel('time[s]')

subplot(223)
splane([4 8],[1 4 68])
subplot(224)
splane([4 16],[1 8 80])
```




6. (10 points)

Describe the basic properties of the one sided Laplace transform.

| | | |
|---|---|---|
| Causal functions and constants | | |
| Linearity | $\alpha f(t),\ \beta g(t)$ | $\alpha F(s),\ \beta G(s)$ |
| | $\alpha f(t) + \beta g(t)$ | $\alpha F(s) + \beta G(s)$ |
| Time shifting | $f(t-\alpha)u(t-\alpha)$ | $e^{-\alpha s}F(s)$ |
| Frequency shifting | $e^{\alpha t}f(t)$ | $F(s-\alpha)$ |
| Multiplication by $t$ | $t\,f(t)$ | $-\dfrac{dF(s)}{ds}$ |
| Derivative | $\dfrac{df(t)}{dt}$ | $sF(s) - f(0-)$ |
| Second derivative | $\dfrac{d^2f(t)}{dt^2}$ | $s^2F(s) - sf(0-) - f^{(1)}(0)$ |
| Integral | $\int_{0-}^{t} f(t')dt'$ | $\dfrac{F(s)}{s}$ |
| Expansion/contraction | $f(\alpha t),\ \alpha \neq 0$ | $\dfrac{1}{\lvert\alpha\rvert}F\left(\dfrac{s}{\alpha}\right)$ |
| Initial value | $f(0-) = \lim_{s\to\infty} sF(s)$ | |

For any function $f(t)$, $-\infty < t < \infty$

Laplace: $F(s) = \mathcal{L}[f(t)\,u(t)]$

$$= \int_{0-}^{\infty} f(t)e^{-st}\,dt.$$

Considers casual signals and functions.

7. (15 points)

Find and use the Laplace transform of $e^{j(\Omega_0 t+\Theta)}u(t)$ to obtain the Laplace transform of

$$x(t) = cos(\Omega_0 t + \Theta) \cdot u(t)$$

Consider the special cases for Θ =0, Θ = -π/2, and Θ = π/4.

$$0 \quad \mathcal{L}[e^{j(\Omega_0 t+\Theta)} u(t)] = \int_0^\infty e^{j(\Omega_0 t+\theta)} e^{-st} dt = e^{-st} \int_0 e^{-(s-j\Omega_0)t} dt$$

$$= \frac{-e^{j\theta}}{s-j\Omega_0} e^{-\alpha t - j(\Omega - \Omega_0)t} \Big|_0^\infty = \frac{e^{j\theta}}{s-j\Omega_0}$$

$$x(t) = cos(\Omega_0 t + \Theta) \cdot u(t) = \frac{e^{j(\Omega_0 t+\theta)} + e^{-j(\Omega_0 t+\theta)}}{2}$$

$$\mathcal{L}[\cos(\Omega_0 t + \theta \cdot u(t)] = 0.5\, \mathcal{L}[e^{j(\Omega_0 + \theta)} u(t)] + 0.5\, \mathcal{L}[e^{-j(\Omega_0 t + \theta} u(t)]$$

$$= 0.5 \frac{e^{j\theta}(s+j\Omega_0) + e^{-j\theta}(s-j\Omega_0)}{s^2 + \Omega_0^2}$$

$$= \frac{s\cos(\theta) - \Omega_0 \sin(\theta)}{s^2 + \Omega_0^2}$$

$$\Theta = 0 \rightarrow \frac{s\cos(0) - \Omega_0 \sin(0)}{s^2 + \Omega_0^2} = \boxed{\frac{s}{s^2 + \Omega_0^2}}$$

$$\Theta = -\frac{\pi}{2} \rightarrow \frac{s\cos(-\pi/2) - \Omega_0 \sin(-\pi/2)}{s^2 + \Omega_0^2} = \boxed{\frac{\Omega_0}{s^2 + \Omega_0^2}}$$

$$\theta = \frac{\pi}{4} \rightarrow \frac{s\cos(\pi/4) - \Omega_0 \sin(\pi/4)}{s^2 + \Omega_0^2} = \frac{\frac{\sqrt{2}}{2}s - \frac{\sqrt{2}}{2}\Omega_0}{s^2 + \Omega_0^2} = \boxed{\frac{\frac{\sqrt{2}}{2}(s - \Omega_0)}{s^2 + \Omega_0^2}}$$