MA 201-05
Test 1

Name Key

No books, notes, scratch paper or calculators allowed. Show your work to get full credit.

1. (12 points) Identify the quadric surfaces below by name:

a) $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{9}=1$

ellipsoid

b) $\frac{x^2}{4}+\frac{y^2}{1}-\frac{z^2}{2}=1$

hyperboloid of one sheet

b) $\frac{x^2}{1}+\frac{y^2}{4}=z^2$

elliptical cone

d) $x^2+y^2=z$

paraboloid

2. (10 points) Write the equation of a sphere with radius 4 and center (-2, 3, 1).

$(x+2)^2+(y-3)^2+(z-1)^2=16$

3. (12 points) Given the points P(5,-2,4), Q(2,6,1) and R(0,-5,12) answer the following questions:

$\vec{PQ}=\langle -3,8,-3\rangle$ $\vec{PR}=\langle -5,-3,8\rangle$

a) Find a normal vector to the plane containing P, Q and R.

$\vec{n}=\vec{PQ}\times\vec{PR}$

$=\begin{vmatrix} i & j & k \\ -3 & 8 & -3 \\ -5 & -3 & 8 \end{vmatrix}=i(64-9)-j(-24-15)+k(9+40)$

$=55i+39j+49k$

b) Find an equation of the plane containing P, Q, and R.

P(5,-2,4)

$55(x-5)+39(y+2)+49(z-4)=0$

4. (12 points) Find the arc length of the curve

$\boldsymbol{r}(t) = 12t\boldsymbol{i} + 8t^{3/2}\boldsymbol{j} + 3t^2\boldsymbol{k}$ for $0 \le t \le 1$.

$L = \int_0^1 |v(t)|\,dt$

$v(t) = 12i + 8\cdot\frac{3}{2}t^{1/2}j + 6tk$

$= 12i + 12t^{1/2}j + 6tk$

$|v(t)| = \sqrt{144 + 144t + 36t^2} \Rightarrow L = \int_0^1 \sqrt{144+144t+36t^2}\,dt$

$= \int_0^1 \sqrt{(6t+12)^2}\,dt$

$= \int_0^1 (6t+12)\,dt = \frac{6t^2}{2} + 12t\Big|_0^1 = 3 + 12$

$= \boxed{15}$

5. (12 points) Solve the initial value problem for $\boldsymbol{r}(t)$ if

$\boldsymbol{r}'(t) = (4t)\boldsymbol{i} + (4t)\boldsymbol{j} + (3t^2)\boldsymbol{k}$ and $\boldsymbol{r}(0) = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$.

$r(t) = \frac{4t^2}{2}i + \frac{4t^2}{2}j + \frac{3t^3}{3}k + C_1 + C_2 + C_3$

$r(t) = 2t^2 i + 2t^2 j + t^3 k + C_1 i + C_2 j + C_3 k$

$r(0) = i + 2j + 3k = 0i + 0j + 0k + C_1 i + C_2 j + C_3 k$

$\Rightarrow C_1 = 1,\ C_2 = 2,\ C_3 = 3$

$\boxed{\vec{r}(t) = (2t^2+1)i + (2t^2+2)j + (t^3+3)k}$

6. (10 points) The position vector of a moving body at time t is

$\vec{r}(t) = \frac{1}{3}t^3\,\mathbf{i} + \frac{1}{\sqrt{2}}t^2\mathbf{j} + t\,\mathbf{k}$. Find the unit tangent vector to the curve

$\vec{v} = \frac{1}{3}3t^2 i + \frac{1}{\sqrt{2}}2t\,j + k = t^2 i + \sqrt{2}t\,j + k$

$|\vec{v}| = \sqrt{t^4 + 2t^2 + 1} = \sqrt{(t^2+1)^2} = (t^2+1)$

$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{t^2}{t^2+1}i + \frac{\sqrt{2}t}{t^2+1}j + \frac{k}{t^2+1}$

7. (12 points) Evaluate the integral.

$u = t \quad dv = e^t$

$du = dt \quad v = e^t$

$$\int_0^1 (te^t)\mathbf{i} + (3t^2)\mathbf{j} + (\sin t)\mathbf{k} \;\; dt$$

3

$= \left[te^t \Big|_0^1 - \int_0^1 e^t dt \right] i + \left[\frac{3t^3}{3} \Big|_0^1 \right] j$

3

$+ \left[(-\cos t) \Big|_0^1 \right] k$

3

$= \left[e - 0 - e^t \Big|_0^1 \right] i + j + (-\cos 1 + 1) k$

3

$\cancel{e - 0 - e + 1}$

$\boxed{= i + j + (1 - \cos 1) k}$

8. (20 points) Given $r(t) = (2cost)\boldsymbol{i} + (3sint)\boldsymbol{j} + (4t)\boldsymbol{k}$ describes the path of the particle at time **t**, find the **velocity, acceleration vectors** and **the speed and the direction** at $t = \pi/2$.

$\vec{v}(t) = (-2\sin t) i + (3\cos t) j + 4k$ 4

$\vec{a}(t) = (-2\cos t) i - (3\sin t) j + 0k$ 4

$\vec{v}\left(\frac{\pi}{2}\right) = -2i + 3(0) j + 4k = \boxed{-2i + 4k}$ 3

$\vec{a}\left(\frac{\pi}{2}\right) = \boxed{-3j}$ 3

speed $= |v(t)| = \sqrt{4\sin^2 t + 9\cos^2 t + 16}$ 3

speed at $t = \frac{\pi}{2} = \sqrt{4 + 16} = \sqrt{20}$

direction at $t = \frac{\pi}{2} = \dfrac{v\left(\frac{\pi}{2}\right)}{\left|v\left(\frac{\pi}{2}\right)\right|} = \boxed{\dfrac{-2i + 4k}{\sqrt{20}}}$ 3

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Test 2

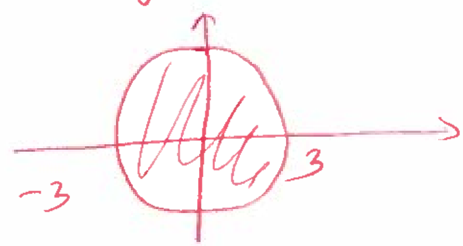
FALL 2019

NAME: KEY

1. Find and sketch the domain of the function. (10 pts)

$$f(x,y) = \sqrt{9 - x^2 - y^2}$$

$9 - x^2 - y^2 \geq 0 \Rightarrow 9 \geq x^2 + y^2$ ie $x^2 + y^2 \leq 9$.



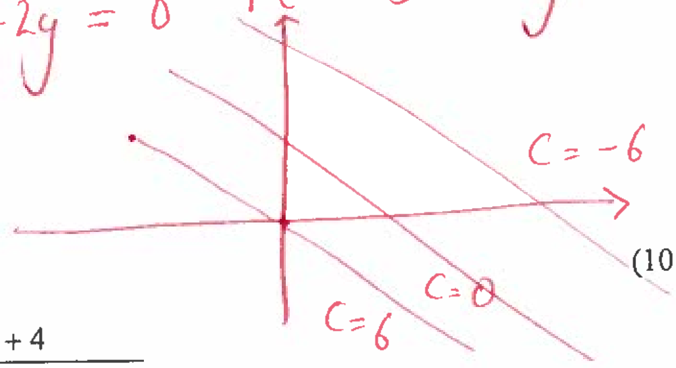
All the pts. (x,y) on & inside the circle $x^2+y^2=9$

2. Sketch the level curves of the function $f(x,y) = 6 - 3x - 2y$ for the values $c = -6, 0, 6$. (12 pts)

$c=-6$, $6-3x-2y=-6$ ie $3x+2y=12$

$c=0$, $6-3x-2y=0$ ie $6=3x+2y$

$c=6$, $6-3x-2y=6$ ie $3x+2y=0$



3. Evaluate the limit. (10 pts)

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$$

$$\lim_{(x,y)\to(2,-4)} \frac{y+4}{xy(x-1)+4x(x-1)}$$

$$\frac{y+4}{(x-1)(xy+4x)} = \frac{(y+4)}{(x-1)x(y+4)}$$

$$\lim_{(x,y)\to(2,-4)} = \frac{1}{x(x-1)} = \frac{1}{2(2-1)} = \boxed{\frac{1}{2}}$$

4. Given $f(x,y) = x^3e^y + x^2y^2 - 3y + 2x$, find each of the following: (15 pts)

3 pts each

a) f_x $= 3x^2e^y + 2xy^2 + 2$

b) f_y $= x^3e^y + 2x^2y - 3$

c) f_{xx} $= 6xe^y + 2y^2$

d) f_{yy} $= x^3e^y + 2x^2$

e) f_{xy} $= 3x^2e^y + 4xy$

5. Let $w = 4x^2 + 5xy - 2e^{3y}$; $x = 3u + \sin 5v$; $y = 7u^2v$. Find $\frac{\partial w}{\partial u}$. (10 pts)

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= (8x+5y)3 + (5x - 6e^{3y})14uv$$

$$= (24u + 8\sin 5v + 35u^2v)3$$

$$+ (15u + 5\sin 5v - 6e^{21u^2v})14uv$$

6. Find the directional derivative of f at the point (1,6,2) in the direction of $\vec{v} = 3\vec{\imath} + 4\vec{\jmath} + 12\vec{k}$. (10 pts)

$$f(x,y,z) = z^3 - x^2y$$

$$|v| = \sqrt{9+16+144} = \sqrt{169} = 13$$

$$\vec{u} = \frac{v}{|v|} = \frac{3}{13}i + \frac{4}{13}j + \frac{12}{13}k$$

$$\nabla f = f_x i + f_y j + f_z k$$

$$f_x = -2xy, \quad f_y = -x^2, \quad f_z = 3z^2$$

$$\nabla f|_{(1,6,2)} = \langle f_x, f_y, f_z\rangle = \langle -12, -1, 12\rangle$$

$$D_u f(1,6,2) = \nabla f|_{(1,6,2)} \cdot \vec{u}$$

$$= \langle -12, -1, 12\rangle \cdot \langle \tfrac{3}{13}, \tfrac{4}{13}, \tfrac{12}{13}\rangle$$

$$= -\frac{36}{13} - \frac{4}{13} + \frac{144}{13} = \boxed{\frac{104}{13}} = 8$$

7. Find $\frac{dy}{dx}$. Use implicit differentiation formula. (10 pts)

$$y^2 - x^2 - sin(xy) = 0$$

$F_x = -2x - (\cos xy) \cdot y$

$F_y = 2y - (\cos xy)\, x$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

$$= -\frac{\{-2x - y\cos(xy)\}}{\{2y - x(\cos xy)\}}$$

8. A particle moves with position function $\mathbf{r}(t) = (\cos t)\boldsymbol{i} + (sint)\boldsymbol{j} + t\boldsymbol{k}$. Find the tangential and normal components of acceleration (i.e. a_T and a_N repectively). (12 pts)

$a_T = \frac{d}{dt}|v(t)|$

$v(t) = (-\sin t)\,i + (\cos t)\,j + k$

$|v(t)| = \sqrt{\sin^2 t + \cos^2 t + 1}$

$= \sqrt{1+1} = \sqrt{2}$

$a_T = \frac{d}{dt}\sqrt{2} = \boxed{0}$

$\vec{a} = (-\cos t)\,i - (\sin t)\,j + 0\vec{k}$

$|\vec{a}| = \sqrt{\cos^2 t + \sin^2 t} = \sqrt{1} = 1$

$a_N = \sqrt{|a|^2 - a_T^{\,2}} = \sqrt{1^2 - 0} = \sqrt{1} = \boxed{1}$

9. Find the equations for the tangent plane and the normal line to the given surface at the specified point. (11 pts)

$$x^2 - xy - y^2 - z = 0, \quad P_0(1, 1, -1)$$

$f_x = 2x - y \qquad f_y = -x - 2y, \quad f_z = -1$

$f_x|_{(1,1,-1)} = 2-1 = 1, \quad f_y|_{(1,1,-1)} = -1-2 = -3, \quad f_z = -1$

eqn of the tangent plane then $P_0(1,1,-1)$ is

$f_x(1,1,-1)(x-1) + f_y(y-1) + f_z(z+1) = 0$

$\Rightarrow 1(x-1) - 3(y-1) - 1(z+1) = 0 \Rightarrow \boxed{x - 3y - z = -1}$

$\boxed{\text{Normal line: } x = 1+t, \quad y = 1-3t, \quad z = -1-t}$

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TEST 3

NAME : KEY

Show all work to receive full credit.

1. (10 points) Evaluate $\int_1^2 \int_1^4 \frac{1}{xy}\,dy\,dx$

$$\int_1^2 \frac{1}{x}\ln|y|\Big|_1^4 dx$$

$$= \int_1^2 \left(\frac{1}{x}\ln|4| - \frac{1}{x}\underbrace{\ln|1|}_{=0}\right) dx$$

$$= \ln|x|\ln(4)\Big|_1^2 = \boxed{\ln(2)\ln(4)}$$

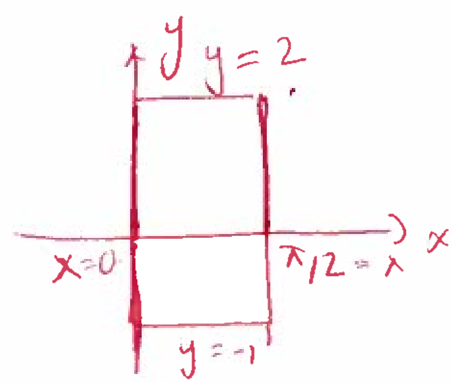
2. (15 points) For the integral below, **sketch** the region of integration, and **evaluate** the double integral.

$$\int_{-1}^{2}\int_{0}^{\pi/2} y\sin x\,dx\,dy$$

$$\int_{-1}^2 -y\cos x\Big|_0^{\pi/2} dy$$

$$= \int_{-1}^2 \left(-y\underbrace{\cos\frac{\pi}{2}}_{=0} + y\underbrace{\cos 0}_{=1}\right) dy$$

$$= \frac{y^2}{2}\Big|_{-1}^2 = \frac{4}{2} - \frac{1}{2} = \boxed{\frac{3}{2}}$$



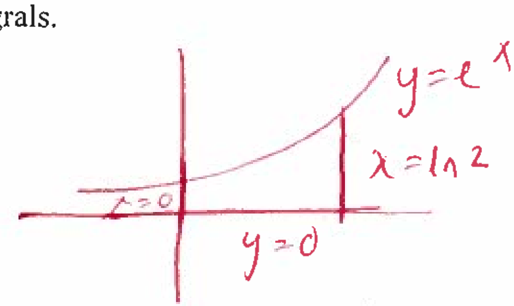
3. (15 points) Find the area of the region bounded by
$y = e^x, y = 0, x = 0$ *and* $x = ln2$, using double integrals.

$$\int_0^{\ln 2}\int_0^{e^x} dy\,dx$$

$$= \int_0^{\ln 2} y\Big|_0^{e^x} dx$$

$$= \int_0^{\ln 2} e^x\,dx = e^x\Big|_0^{\ln 2} = e^{\ln 2} - e^0$$

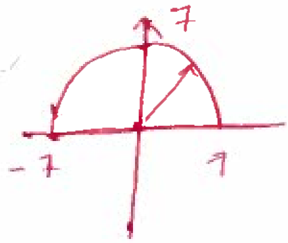
$$= 2 - 1 = \boxed{1}$$



4. (10 points) Change the cartesian integral into equivalent polar integral and evaluate it.

$$\int_{-7}^{7}\int_{0}^{\sqrt{49-x^2}} dy\,dx$$

$x^2+y^2=49$



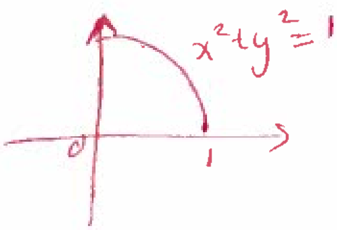
$0 \le y \le \sqrt{49-x^2}$, $-7 \le x \le 7$

$0 \le r \le 7$, $\pi \le \theta \le 0$

$$\int_0^{\pi}\int_0^{7} r\,dr\,d\theta = \int_0^{\pi} \frac{r^2}{2}\Big|_0^7 d\theta = \int_0^{\pi} \frac{49}{2}\,d\theta = \frac{49}{2}\theta\Big|_0^{\pi}$$

$$\boxed{= \frac{49\pi}{2}}$$

5. (10 points) Change the integral $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}(x^2+y^2+z^2)^2\,dzdydx$ to an integral in spherical coordinates. **Do not evaluate.**



$0 \le z \le \sqrt{1-x^2-y^2}$

$0 \le y \le \sqrt{1-x^2}$

$0 \le x \le 1$

$z^2 = 1-x^2-y^2$

$x^2+y^2+z^2=1$

$\rho^2 = 1$

$\Rightarrow 0 \le \rho \le 1$

$0 \le \theta \le \frac{\pi}{2}$

$z = \rho\cos\phi$

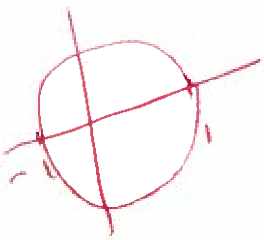
$0 = \rho\cos\phi \Rightarrow \cos\phi = 0$

$0 \le \phi \le \frac{\pi}{2}$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} (\rho^2)^2 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

6. (10 points) Change the Cartesian integral to an equivalent integral in cylindrical coordinates. **Do not evaluate the integral**:

$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{3/2}dz\ dy\ dx.$



$$\int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2}(r^2)^{3/2}\cdot r\,dz\,dr\,d\theta$$

$$r^4\,dz\,dr\,d\theta$$

7. (20 points) Let D be the region bounded below by $z=0$, above by the sphere $x^2+y^2+z^2=4$, and on the sides by the cylinder $x^2+y^2=1$.

a) Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration $dzdrd\theta$.

$z^2=4-x^2-y^2$

$z^2=4-r^2$

$0\le\theta\le 2\pi$

$0\le r\le 1$

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

b) Evaluate the integral.

$$\int_0^{2\pi}\int_0^1 rz\Big|_0^{\sqrt{4-r^2}}\,dr\,d\theta$$

$u=4-r^2$

$du=-2r\,dr$

$$=\int_0^{2\pi}\int_0^1 r\sqrt{4-r^2}\,dr\,d\theta=\int_0^{2\pi}-\frac{1}{2}\sqrt{u}\,du\,d\theta=\int_0^{2\pi}-\frac{1}{3}(4-r^2)^{3/2}\Big|_0^1 d\theta$$

$$=\left(-\frac{1}{3}3\sqrt{3}+\frac{1}{3}8\right)\theta\Big|_0^{2\pi}$$

$$=\boxed{\left(\frac{8-3\sqrt{3}}{3}\right)2\pi}$$

8. (10 points) Find coordinates (x,y,z) corresponding to spherical coordinates $(\rho,\theta,\varphi)=(8,\frac{\pi}{3},\frac{2\pi}{3})$.

$x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$

$(x,y,z)=(2\sqrt{3},6,-4)$

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TEST 4 FALL 2019

Name KEY

Show your work to receive full credit.

1. (15 points) Show that the vector field is conservative. Find a potential function for the vector field. $\boldsymbol{F}(x,y,z) = \underbrace{(y-z)}_{M}\boldsymbol{i} + \underbrace{(x+2y-z)}_{N}\boldsymbol{j} - \underbrace{(x+y)}_{P}\boldsymbol{k}$

$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z} \Rightarrow -1 = -1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow 1 = 1$

$\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \Rightarrow -1 = -1$

$\Rightarrow$ F is conservative.

$M = y - z$, $M = \frac{\partial f}{\partial x}$

$N = x + 2y - z$, $N = \frac{\partial f}{\partial y}$

$P = -x - y$, $P = \frac{\partial f}{\partial z}$

$\Rightarrow y - z = \frac{\partial f}{\partial x} \Rightarrow f(x,y,z) = xy - xz + g(y,z)$

$f_y = x + \frac{\partial g}{\partial y} = x + 2y - z \Rightarrow g(y,z) = y^2 - z\cdot y + h(z)$

$f(x,y,z) = xy - xz + y^2 - zy + h(z)$

$f_z = -x - y + \frac{\partial h}{\partial z} = -x - y \Rightarrow \frac{dh}{dz} = 0 \Rightarrow h(z) = C$

$\Rightarrow \boxed{f(x,y,z) = xy - xz + y^2 - zy + C}$

2. (15 points) Find the work done by the force field $\boldsymbol{F}(x,y,z) = 10z\,\boldsymbol{i} + 7x\,\boldsymbol{j} + 10y\,\boldsymbol{k}$ over the curve C: $\boldsymbol{r}(t) = t\,\boldsymbol{i} + t\,\boldsymbol{j} + t\,\boldsymbol{k}, 0 \le t \le 1$

$z = t,\ x = t,\ y = t$

$F = 10t\,i + 7t\,j + 10t\,k$

$r'(t) = i + j + k$

$F \cdot r'(t) = 10t + 7t + 10t = 27t$

$$\int_C F \cdot dr = \int_C F \cdot r'(t)\,dt = \int_0^1 27t\,dt = 27\frac{t^2}{2}\Big|_0^1 = \boxed{\frac{27}{2}}$$

3. (15 points) Evaluate the line integral $\int_C x^2z + y^2z \;\; ds$ along the curve C parametrized by $\boldsymbol{r}(t) = \sin t\,\boldsymbol{i} + \cos t\,\boldsymbol{j} + t\sqrt{3}\,\boldsymbol{k}, 0 \le t \le 2\pi$.

$x = \sin t,\ y = \cos t,\ z = t\sqrt{3}$

$v(t) = r'(t) = \cos t\,i - \sin t\,j + \sqrt{3}\,k$

$|v(t)| = \sqrt{\cos^2 t + \sin^2 t + 3} = \sqrt{1+3} = \sqrt{4} = 2$

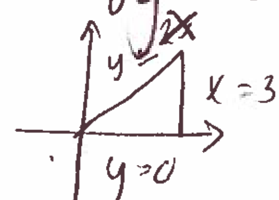
$$\int_0^{2\pi} [\sin^2 t \cdot t\sqrt{3} + \cos^2 t \cdot t\sqrt{3}]\,|v(t)|\,dt$$

$$= \int_0^{2\pi} t\sqrt{3}\,[\underbrace{\sin^2 t + \cos^2 t}_{1}]\,2\,dt = 2\sqrt{3}\frac{t^2}{2}\Big|_0^{2\pi} = \boxed{4\sqrt{3}\pi^2}$$

4. (15 points) Use Green's theorem to find the outward Flux for the field **F** and the curve C.
$\mathbf{F} = \underbrace{(y^2 - x^2)}_{M}\boldsymbol{i} + \underbrace{(x^2 + y^2)}_{N}\boldsymbol{j}$; C is the triangle bounded by $y = 0, x = 3$ and $y = x$.

$$\frac{\partial M}{\partial x} = -2x \qquad \frac{\partial N}{\partial y} = 2y$$

$$\int_0^3 \int_0^x (-2x + 2y)\, dy\, dx.$$



$$\text{Flux} = \int_0^3 -2xy + y^2 \Big|_0^x dx = \int_0^3 -2x^2 + x^2\, dx$$
$$= -\frac{x^3}{3}\Big|_0^3$$
$$= \boxed{-9}$$

5. (20 points) Find the surface area of the portion of the plane
$y + 2z = 2$ inside the cylinder $x^2 + y^2 = 1$.

a. Find the parametrization of the surface.

$x = r\cos\theta,\ y = r\sin\theta,\ z = 1 - \frac{y}{2} = 1 - \frac{r\sin\theta}{2}$

$\vec{r}(r,\theta) = (r\cos\theta)i + (r\sin\theta)j + (1 - \frac{r\sin\theta}{2})k, \quad 0 \le \theta \le 2\pi,\ 0 \le r \le 1$

b. Find the partial derivatives.

$\vec{r_r} = \cos\theta\, i + \sin\theta\, j - \frac{\sin\theta}{2} k$

$\vec{r_\theta} = -r\sin\theta\, i + r\cos\theta\, j - \frac{r}{2}\cos\theta\, k$

c. Find the cross product of your partial derivatives

$$\vec{r_r} \times \vec{r_\theta} = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & -\frac{\sin\theta}{2} \\ -r\sin\theta & r\cos\theta & -\frac{r}{2}\cos\theta \end{vmatrix} = \frac{r}{2} j + r k$$

$$|\vec{r_r} \times \vec{r_\theta}| = \sqrt{\frac{r^2}{4} + r^2} = \frac{\sqrt{5}\, r}{2}$$

d. Using your work above, calculate the surface area of S.

$$S.A = \int_0^{2\pi}\int_0^1 \frac{\sqrt{5}}{2} r \cdot dr\, d\theta = \int_0^{2\pi} \frac{\sqrt{5}}{2}\frac{r^2}{2}\Big|_0^1 d\theta$$
$$= \frac{\sqrt{5}}{4}\theta\Big|_0^{2\pi} = \boxed{\frac{\sqrt{5}\cdot\pi}{2}}$$

6. (10 points) Find a parametrization of the surface S where S is the portion of the sphere $x^2 + y^2 + z^2 = 3$ between the planes $z = \frac{\sqrt{3}}{2}$ *and* $z = -\frac{\sqrt{3}}{2}$

$x = \sqrt{3}\sin\phi\cos\theta$

$y = \sqrt{3}\sin\phi\sin\theta$

$z = \sqrt{3}\cos\phi$

$0 \le \theta \le 2\pi$

$\frac{\pi}{3} \le \phi \le \frac{2\pi}{3}$

$-\frac{\sqrt{3}}{2} = \sqrt{3}\cos\phi \Rightarrow \cos\phi = -\frac{1}{2} \Rightarrow \phi = 2\pi/3$

$\frac{\sqrt{3}}{2} = \sqrt{3}\cos\phi \Rightarrow \cos\phi = \frac{1}{2} \Rightarrow \phi = \pi/3$

$\vec{r}(\phi, \theta) = (\sqrt{3}\sin\phi\cos\theta)i + (\sqrt{3}\sin\phi\sin\theta)j + (\sqrt{3}\cos\phi)K.$

7. (10 points) Use Green's theorem to evaluate the integral

$\oint_C (3y\,dx + 2x\,dy)$; C: the boundary of $0 \le x \le \pi, 0 \le y \le sinx.$

$M = 3y \qquad N = 2x$

$\frac{\partial M}{\partial y} = 3 \qquad \frac{\partial N}{\partial x} = 2.$

$$\int_0^{\pi}\int_0^{\sin x} (2-3)\,dy\,dx$$

$$= \int_0^{\pi} -y\Big|_0^{\sin x} dx$$

$$= \int_0^{\pi} -\sin x\,dx = \cos x\Big|_0^{\pi}$$

$$= \cos\pi - \cos 0$$

$$= (-1) - 1 = \boxed{-2}$$