- Decrease and Conquer:

Three variations

-> Decrease by a constant

$$f(n) = \begin{cases} f(n-1) \cdot a & \text{if } n > 0, \\ 1 & \text{if } n = 0, \end{cases}$$

-> Decrease by a constant Factor:

ex. decreces by a Factor of two.

$$a^n = \begin{cases} (a^{n/2})^2 & \text{if } n \text{ is even and positive,} \\ (a^{(n-1)/2})^2 \cdot a & \text{if } n \text{ is odd,} \\ 1 & \text{if } n = 0. \end{cases}$$

-> Variable Size decrease:

ex: Euclids algorithm

- Straight insertion sort:

Recursive ->

Bottom-up / Iterative approach

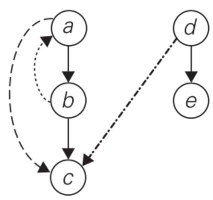
ALGORITHM *InsertionSort*($A[0..n-1]$)

```
//Sorts a given array by insertion sort
//Input: An array A[0..n − 1] of n orderable elements
//Output: Array A[0..n − 1] sorted in nondecreasing order
for i ← 1 to n − 1 do
    v ← A[i]
    j ← i − 1
    while j ≥ 0 and A[j] > v do
        A[j + 1] ← A[j]
        j ← j − 1
    A[j + 1] ← v
```

-> more efficient!

$$\underbrace{A[0] \le \cdots \le A[j]}_{\text{smaller than or equal to } A[i]} < \underbrace{A[j+1] \le \cdots \le A[i-1]}_{\text{greater than } A[i]} \mid A[i] \cdots A[n-1]$$

•Direct graph:



Tree edges:

Back edges:

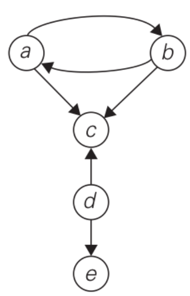
Forward edges:

Cross edges

(b)

Dag:

(Directed acyclic graph)



Directed cycle:

(a)

(a b a)

* If a digraph has no directed cycles, the topological sorting problem for it has a solution.
* If the DFS finds a back edge, then the topological sort for the vertices is impossible.

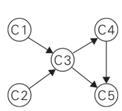
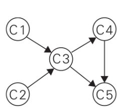


FIGURE 4.6 Digraph representing the prerequisite structure of five courses.

→ DFS for topological sort.

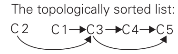


(a)

$C5_1$
$C4_2$
$C3_3$
$C1_4$ $C2_5$

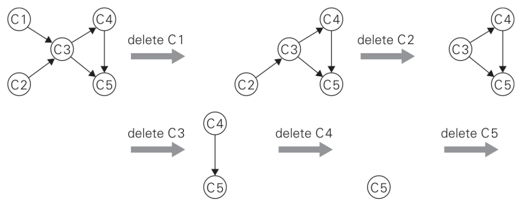
(b)

The popping-off order:
C5, C4, C3, C1, C2
The topologically sorted list:



(c)

FIGURE 4.7 (a) Digraph for which the topological sorting problem needs to be solved. (b) DFS traversal stack with the subscript numbers indicating the popping-off order. (c) Solution to the problem.



The solution obtained is C1, C2, C3, C4, C5

- Source:

- The order in which the vertices are removed is the solution to the topological sorting algorithm.

Binary Search:

ALGORITHM *BinarySearch*$(A[0..n-1], K)$

```
//Implements nonrecursive binary search
//Input: An array A[0..n − 1] sorted in ascending order and
//        a search key K
//Output: An index of the array's element that is equal to K
//        or −1 if there is no such element
l ← 0;   r ← n − 1
while l ≤ r do
    m ← ⌊(l + r)/2⌋
    if K = A[m] return m
    else if K < A[m] r ← m − 1
    else l ← m + 1
return −1
```

- efficiency: count the number of times the search key is compared with an element of the array.

• Fake Coin problem

• Russian peasant multiplication

n & m

instance size by the value of n. Now, if n is even, an instance of half the size has to deal with $n/2$, and we have an obvious formula relating the solution to the problem's larger instance to the solution to the smaller one:

$$n \cdot m = \frac{n}{2} \cdot 2m.$$

If n is odd, we need only a slight adjustment of this formula:

$$n \cdot m = \frac{n-1}{2} \cdot 2m + m.$$

- Josephus problem