Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #5 Solution

1. (15 points) A discrete time IIR system with input x[n] and output y[n] is represented by the equation:

$$y[n] = 0.2 \cdot y[n-2] + x[n] \qquad n \geq 0$$

a) find the impulse response *h(n)* of the system, by assuming that initial conditions are zero (y[n]=h[n]=0, n<0) and x[n]=δ[n].

h[0] = 0.2*h[-2] + 1 = 1
h[1] = 0.2*h[-1] + 0 = 0
h[2] = 0.2*h[0] + 0 = 0.2
h[3] = 0.2*h[1] + 0 = 0
h[4] = 0.2*h[2] + 0 = 0.2^2

or

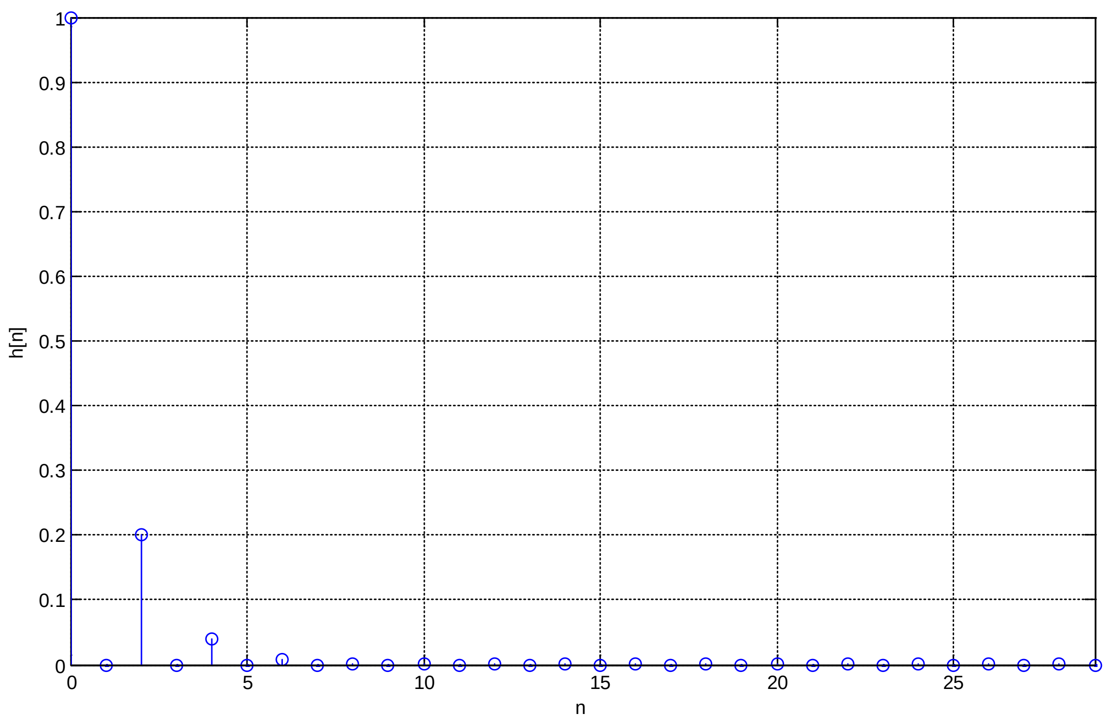
$$h[n] = \begin{cases} 0.2^{n/2} & \text{for } n \geq 0 \text{ and even} \\ 0 & \text{otherwise} \end{cases}$$

b) find the impulse response alternatively by using recursive relation between x[n] and y[n].

h[n] = 0.2*h[n-2] + δ[n]
h[n-2] = 0.2*h[n-4] + δ[n-2]
…
h[n] = δ[n] + 0.2*δ[n-2] + 0.2*δ[n-4] + … (the same result as above)

c) plot h[n] using MATLAB function filter.

```
clear all;clf
a=[1 0 -0.2];
b=1;
x=[1 zeros(1,29)];
h=filter(b,a,x);
n=0:29;
figure(1)
stem(n,h); axis([0 29 0 1]);
grid;ylabel('h[n]'); xlabel('n')
```



2. An FIR filter is represented as:

$$y[n]=\sum_{k=0}^{5} k \cdot x[n-k]$$

a) find and plot the impulse response of this filter.

y[n] = 0*δ[n] + 1*δ[n-1] +2*δ[n-2] +3*δ[n-3] +4*δ[n-4] +5*δ[n-5]

b) is this a causal and stable filter? Explain.

The filter is causal since the output depends only on previous values of the input and h[n]=0 for n < 0.

c) find and plot the unit-step response s[n] for this filter.

for x[n] = u[n]

$$s[n]=\sum_{k=1}^{5} k \cdot u[n-k] = u[n-1]+2u[n-2]+3u[n-3]+4u[n-4]+5u[n-5]$$

d) what is the maximum value of the output if the maximum input is 5?

if |x[n]| <5 → |y[n]| < 5 * $\sum_{k=0}^{5} k \cdot |x[k]|$ = 75 the bound is 75.

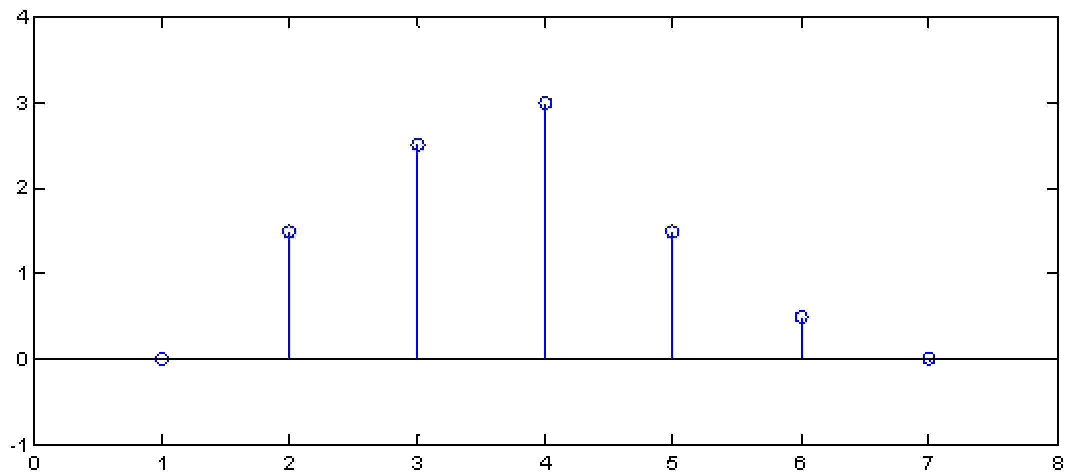
3. (10 points) Let x[n] = {0, 1, 1, 1, 0} and h[n] = {1.5, 1, 0.5}.
Compute and plot the convolution y[n] = x[n] * h[n].

Convolution for causal LTI system:

$$y(t) = \int_{0}^{t} x(\tau)h(t-\tau)d\tau$$

		0	1	1	1	0	
0.5	1	1.5					→ **0**
	0.5	1	1.5				→ **1.5**
		0.5	1	1.5			→ **2.5**

Result: { 0 1.5 2.5 3.0 1.5 0.5 0 }



4. (18 points)

a) (6 points) Explain the difference between hard and soft real-time systems.

A system is said to be real-time if the total correctness of an operation depends not only upon its logical correctness, but also upon the processing time for each system state. A system state is created by sampling the system.
Real-time systems perform their operation fast enough to influence the system they control. They are classified according to the consequences of missing a deadline.

<u>**Hard real-time systems**</u> may generate a total system failure if the deadline is missed.

<u>**Soft real-time systems**</u> may tolerate missing a processing deadline for a limited period of time that will degrade only the system's quality of service (such as latency)

b) (7 points) Maximum frequency of the input is 600Hz. The microcontroller processes each sample in 1200 clock cyles with clock frequency F_c = 1MHz. Can this system run in real-time?

If the maximum frequency of the signal is 600Hz, the sampling frequency must be at least 1,200Hz (Nyquist criterion). Therefore, sampling time is

$$T_s = 1 / F_s = 1/1200 = 833\ \mu s$$

Processing time is

$$T_p = 1{,}200 \text{ cycles} * T_{cycle} = 1{,}200 * 1\ \mu s = 1.2 \text{ ms}$$

Since Tp > Ts, the system CAN NOT work in real time.

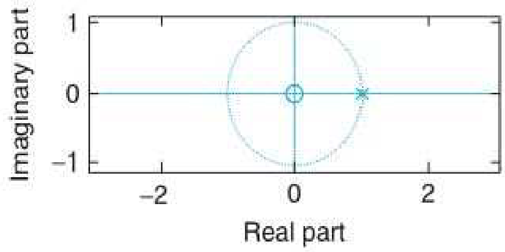
c) (5 points) What is the minimum frequency of the clock that allows real-time operation with 2x oversampling of the input?

Processing time must be less than sampling time

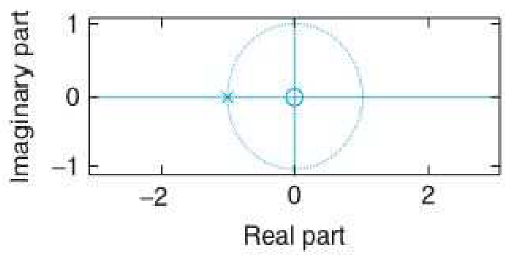
$$T_p = 1{,}200 \text{ cycles} * T_{cycle} = 1200 * 1/F_{clock} < T_s = 1/F_s \rightarrow$$

$$F_{clock} > 1{,}200 * F_s, \qquad \text{Fclock} > 1.44 \text{ MHz}$$

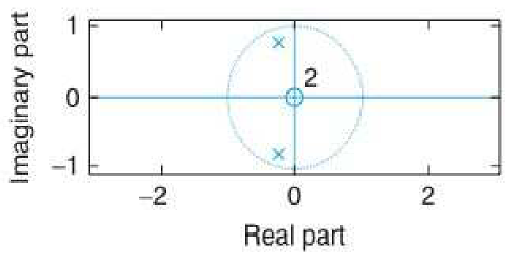
5. (12 points) Describe the effect of pole location on the inverse Z-transform for the following cases.



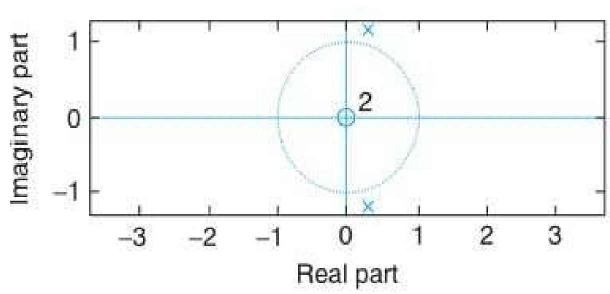
u[n], constant



cosine of frequency π, constant amplitude



a decaying modulated exponential



a growing modulated exponential

6. (4 points) If X(z) is the Z-transform of a causal signal x[n], then

Initial value is x[0] = $\lim_{Z\to\infty} X(Z)$

Final value is $\lim_{n\to\infty} x[n] = \lim_{Z\to 1}(Z-1)X(Z)$