

Name \_\_\_\_\_

**Solve the problem.**

- 1) Find parametric equations for the normal line to the surface  $z = 5x^2 - 2y^2$  at the point  $(2, 1, 18)$ .
- 2) Find the equation for the tangent plane to the surface  $z = -9x^2 - 3y^2$  at the point  $(2, 1, -39)$ .
- 3) Write parametric equations for the tangent line to the curve of intersection of the surfaces  $x + y + z = 9$  and  $x - y + 2z = 11$  at the point  $(1, 2, 6)$ .

**Find the absolute maxima and minima of the function on the given domain.**

- 4)  $f(x, y) = x^2 + 8x + y^2 + 14y + 2$  on the rectangular region  $-1 \leq x \leq 1, -2 \leq y \leq 2$

**Find all the local maxima, local minima, and saddle points of the function.**

- 5)  $f(x, y) = (x^2 - 25)^2 + (y^2 - 16)^2$

**Find the volume under the surface  $z = f(x, y)$  and above the rectangle with the given boundaries.**

- 6)  $z = 8x + 4y + 7; 0 \leq x \leq 1, 1 \leq y \leq 3$

**Reverse the order of integration and then evaluate the integral.**

- 7)  $\int_0^{392} \int_{\sqrt{y/8}}^7 \frac{\sin x^2}{x} dx dy$

- 8)  $\int_1^4 \int_{\sqrt{y}}^2 \sin\left(\frac{x^3}{3} - x\right) dx dy$

**Find the area of the region specified in polar coordinates.**

- 9) one petal of the rose curve  $r = 3 \cos 3\theta$

**Find the volume of the indicated region.**

- 10) the region bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $x + z = 7$
- 11) the region bounded by the coordinate planes, the parabolic cylinder  $z = 36 - x^2$ , and the plane  $y = 3$

- 12) the region enclosed by the paraboloids

$$z = x^2 + y^2 - 2 \text{ and } z = 198 - x^2 - y^2$$

- 13)

$$\int_0^6 \int_0^y x dx dy$$

**The position vector of a particle is  $\mathbf{r}(t)$ . Find the requested vector.**

- 14) The velocity and acceleration at  $t = \frac{\pi}{8}$  for  $\mathbf{r}(t) = (7$

$$\sin 4t)\mathbf{i} - (9 \cos 4t)\mathbf{j} + (2 \csc 4t)\mathbf{k}$$

**For the smooth curve  $\mathbf{r}(t)$ , find the parametric equations for the line that is tangent to  $\mathbf{r}$  at the given parameter value  $t = t_0$ .**

- 15)  $\mathbf{r}(t) = (7 \sin t)\mathbf{i} - (3 \cos 4t)\mathbf{j} + e^{-10t}\mathbf{k}; t_0 = 0$

**Find  $\mathbf{T}$ ,  $\mathbf{N}$ , and  $\mathbf{B}$  for the given space curve.**

- 16)  $\mathbf{r}(t) = (\ln(\cos t) + 2)\mathbf{i} + 3\mathbf{j} + (8 + t)\mathbf{k}, -\pi/2 < t < \pi/2$

**Find  $f_x$ ,  $f_y$ , and  $f_z$ .**

- 17)  $f(x, y, z) = \sin(xy) \cos(yz^2)$

**Solve the problem.**

- 18) Evaluate  $\frac{dw}{dt}$  at  $t = 6$  for the function  $w = e^y - \ln x$ ;

$$x = t^2, y = \ln t.$$

**Find the derivative of the function at  $P_0$  in the direction of  $\mathbf{u}$ .**

- 19)  $f(x, y) = \ln(6x + 2y), P_0(-8, 3), \mathbf{u} = 6\mathbf{i} + 8\mathbf{j}$

**Solve the problem.**

- 20) Write an equation for the tangent line to the curve  $x^2 - 9xy + y^2 = 11$  at the point  $(-1, 1)$ .

**Find the area of the region specified by the integral(s).**

- 21)  $\int_0^9 \int_{9-x}^{e^x} dy dx$

**Use a spherical coordinate integral to find the volume of the given solid.**

- 22) the solid between the spheres  $\rho = 5 \cos \varphi$  and  $\rho = 8 \cos \varphi$

## Answer Key

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- 1)  $x = 20t + 2$ ,  $y = -4t + 1$ ,  $z = -t + 18$
- 2)  $-36x - 6y - z = -39$
- 3)  $x = 3t + 1$ ,  $y = -t + 2$ ,  $z = -2t + 6$
- 4) Absolute maximum: 43 at (1, 2); absolute minimum: -29 at (-1, -2)
- 5)  $f(0, 0) = 881$ , local maximum;  $f(0, 4) = 625$ , saddle point;  $f(0, -4) = 625$ , saddle point;  
 $f(5, 0) = 881$ , saddle point;  $f(5, 4) = 0$ , local minimum;  $f(5, -4) = 0$ , local minimum;  
 $f(-5, 0) = 256$ , saddle point;  $f(-5, 4) = 0$ , local minimum;  $f(-5, -4) = 0$ , local minimum
- 6) 38
- 7)  $4(1 - \cos 49)$
- 8)
- 9)  $\frac{3}{4}\pi$
- 10)  $28\pi$
- 11) 432
- 12)  $10,000\pi$
- 13)
- 14)  $\mathbf{a}\left(\frac{\pi}{8}\right) = -112\mathbf{i} + 32\mathbf{k}$
- 15)  $x = 7t$ ,  $y = -3$ ,  $z = 1 - 10t$
- 16)  $\mathbf{T} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{k}$ ;  $\mathbf{N} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{k}$ ;  $\mathbf{B} = -\mathbf{j}$
- 17)  $f_x = y \cos(xy) \cos(yz^2)$ ;  $f_y = x \cos(xy) \cos(yz^2) - z^2 \sin(xy) \sin(yz^2)$ ;  $f_z = -2yz \sin(xy) \sin(yz^2)$
- 18)  $\frac{2}{3}$
- 19)  $-\frac{13}{105}$
- 20)  $x - y + 2 = 0$
- 21)  $e^9 - \frac{83}{2}$
- 22)  $\frac{129}{2}\pi$