MA 172_03
TEST 2 Spring 2019

Name: KEY

Show all work to receive credit.

1. It took 1800J of work to stretch a spring from its natural length of 2m to a length of 5m. Find the spring's force constant. (10 pts)

3 pts

$F(x) = kx$

$$W = \int_0^3 F(x)\,dx = \int_0^3 kx\,dx$$ 3pts

$$\Rightarrow 1800 = k\frac{x^2}{2}\Big|_0^3 \Rightarrow 1800 = k\cdot\frac{9}{2}$$ 4pts

$$\Rightarrow \boxed{k = 400\ \text{Nm}}$$

2. Solve the differential equation $y^2\frac{dy}{dx} = 3x^2y^3 - 6x^2$. (10 pts)

$$y^2\frac{dy}{dx} = 3x^2(y^3 - 2)$$ 2 pts

$$\int \frac{y^2\,dy}{y^3-2} = \int 3x^2\,dx$$ 2pts

$u = y^3 - 2$

2pts $du = 3y^2\,dy$

$$\frac{1}{3}\int \frac{1}{u}\,du = 3\cdot\frac{x^3}{3} + C$$ 1pt

$$\Rightarrow \frac{1}{3}\ln|u| = x^3 + C$$ 2pt

$$\Rightarrow \boxed{\frac{1}{3}\ln|y^3 - 2| = x^3 + C}$$ 1pt

3. Verify the identity $tanh^2 x = 1 - sech^2 x$. Show your work. (10 pts)

$$\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = 1 - \left(\frac{2}{e^x + e^{-x}}\right)^2$$

$$= 1 - \frac{4}{(e^x + e^{-x})^2} = \frac{(e^x + e^{-x})^2 - 4}{(e^x + e^{-x})^2}$$

$$= \frac{e^{2x} + 2 + e^{-2x} - 4}{(e^x + e^{-x})^2}$$

$$= \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$

4. Evaluate the following integrals. (36 pts)

a. $\int \cosh\frac{x}{5}\, dx$

$u = \frac{x}{5}$ $\quad du = \frac{1}{5} dx$

$5\int \cosh u\, du$

$= 5 \sinh u + C$

$= 5 \sinh \frac{x}{5} + C$

b. $\int_0^9 \frac{2\log_{10}(x+1)}{x+1}\, dx$

$= \int 2 \frac{\ln(x+1)}{\ln 10} \cdot \frac{1}{x+1}\, dx$ $\quad u = \ln(x+1)$, $du = \frac{1}{x+1} dx$

$= \frac{2}{\ln 10} \int \frac{\ln(x+1)}{x+1}\, dx$

$= \frac{2}{\ln 10}\int u\, du = \frac{2}{\ln 10} \cdot \frac{u^2}{2} = \frac{u^2}{\ln 10} = \frac{1}{\ln 10}\left[\ln(x+1)\right]^2 \Big|_0^9$

$= \frac{1}{\ln 10}\left[(\ln 10)^2 - \ln(1)^2\right]$

$= \boxed{\ln 10}$

c. $\int 4e^{-\theta}\sinh\theta\, d\theta$

$= \int 4e^{-\theta}\left(\frac{e^{\theta} - e^{-\theta}}{2}\right) d\theta$

$= 2\int \left(e^{0} - e^{-2\theta}\right) d\theta$

$= 2\left[\theta + \frac{e^{-2\theta}}{2}\right] + C$

d. $\int_{-1}^{0} \frac{3}{3x-2}\, dx$

$u = 3x - 2$

$du = 3dx$

$\int \frac{1}{u}\, du = \ln|u| \Big|_{-1}^{0}$

$= \ln|3x-2| \Big|_{-1}^{0}$

$= \ln|-2| - \ln|-5|$

$= \ln 2 - \ln 5$

$= \ln \frac{2}{5}$

e. $\int x2^{(x^2)}\, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int 2^u\, du$

$= \frac{1}{2}\frac{2^u}{\ln 2} + C$

$= \frac{1}{2\ln 2} 2^{x^2} + C$

f. $\int_1^2 4^{-\theta}\, d\theta$

$-\frac{4^{-\theta}}{\ln 4}\Big|_1^2$

$= -\frac{4^{-2}}{\ln 4} + \frac{4^{-1}}{\ln 4}$

5. Find the derivative. (24 pts)

a) $y = \tanh\sqrt{7x}$ $\qquad (7x)^{1/2}$

$$y' = \operatorname{sech}^2\sqrt{7x}\cdot\frac{1}{2}(7x)^{-1/2}\cdot 7$$
$$= \frac{7}{2}\,\frac{\operatorname{sech}^2\sqrt{7x}}{\sqrt{7x}}$$

b) $y = \sqrt{x}\sinh\sqrt{x}$

$$y' = \frac{1}{2}x^{-1/2}\sinh\sqrt{x} + \sqrt{x}\cosh\sqrt{x}\cdot\frac{1}{2}(x)^{-1/2}$$

c) $f(x) = \ln(\operatorname{sech} x)$

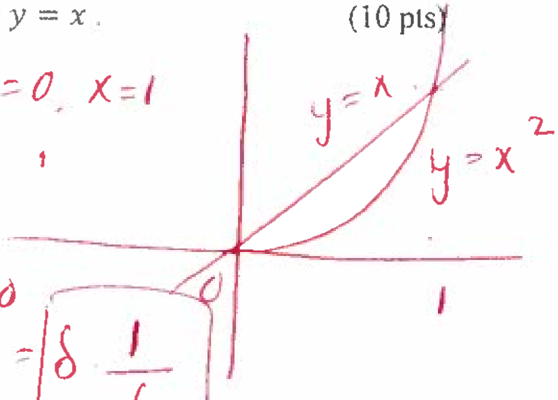
$$f'(x) = \frac{1}{\operatorname{sech} x}(-\operatorname{sech} x\tanh x)$$
$$= -\tanh x$$

d) $f(x) = \coth(1-3x)$

$$f'(x) = -(-3)\operatorname{csch}^2(1-3x)$$
$$= 3\operatorname{csch}^2(1-3x)$$

6. Find the center of mass of a thin plate bounded below by *the parabola* $y = x^2$ *and above by the line* $y = x$. (10 pts)

$$x = x^2 \Rightarrow x^2 - x = 0 \qquad x = 0,\ x = 1$$
$$x(x-1) = 0$$




$$M = \int_0^1 \delta\cdot[x - x^2]\,dx = \delta\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= \delta\left[\frac{1}{2} - \frac{1}{3}\right] = \boxed{\delta\,\frac{1}{6}}$$

$$M_x = \frac{\delta}{2}\int_0^1 [x^2 - x^4]\,dx = \frac{\delta}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{\delta}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$$
$$= \frac{\delta}{2}\left[\frac{2}{15}\right] = \boxed{\frac{\delta}{15}}$$

$$M_y = \delta\int_0^1 x[x - x^2]\,dx = \delta\int_0^1 (x^2 - x^3)\,dx$$
$$= \delta\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \delta\left[\frac{1}{3} - \frac{1}{4}\right]$$
$$= \delta\,\frac{1}{12} = \boxed{\frac{\delta}{12}}$$

$$= \frac{M_y}{M} = \boxed{\frac{1}{2}}, \quad \bar{y} = \frac{M_x}{M} = \boxed{\frac{2}{5}}$$