

ALGORITHM Comput $V(n)$ // input : $n \geq 1$

$V \leftarrow 0$

For $i \leftarrow 1$ to n do

$V \leftarrow V + i \times i$

①

return v

a) What does this algorithm compute?

$$V = V + i^2$$

$$V = 0 + 1^2 = 1$$

$$V = 1 + 2^2 = 5$$

$$V = 5 + 3^2 = 14$$

The sum of the squares from $1 \rightarrow n$

If n is 50

$n=3$

b) What is its basic operation?

While there is addition, its basic operation is $i \times i$, or n^2 . So multiplication is the basic operation.

c) How many times is the operation computed?

n times. It only goes n times because there is only one operation inside one loop from $1 \rightarrow n$

d) Give an alternate solution that can execute in $O(1)$

$$G(n) = G(n-1) + n^2$$

$$G(1) = G(0) + 1^2 = 1$$

$$G(2) = G(1) + 2^2 = 5$$

$$\frac{n(n-1)}{2}$$

2)

	Work	Answer
a) $\log_2 n$	$\log_2 4n - \log_2 n =$	2 times
b) n^2	$(4n^2)/n^2 =$	4^2 times
c) $\sqrt{4n}$	$\sqrt{16n} / \sqrt{4n} =$	2 times

3)

a) Alg (n)

// computes 2^n recursively w/ : $2^n = 2^{n-1} + 2^{n-1}$

IF $n=0$, return 1

else return Alg(n-1) + Alg(n-1)

→ put both b and c here.

b/c) Alg(n) = 2Alg(n-1) + 1 → b

$$= 2[2Alg(n-2) + 1] + 1 = 2^2 Alg(n-2) + 2 + 1$$

$$= 2^2 [2Alg(n-3) + 1] + 2 + 1 = 2^3 Alg(n-3) + 2^2 + 2 + 1$$

$$= 2^i Alg(n-i) + 2^{i-1} + 2^{i-2} + \dots + 1$$

$$= 2^n Alg(0) + 2^{n-1} + 2^{n-2} + \dots + 1$$

$$= 2^{n-1} + 2^{n-2} + \dots + 1$$

$$\boxed{= 2^{n-1} + 1} \rightarrow c$$

d)

For $i \rightarrow n$ do ? This might be better since it is only performing one operation per iteration.
 $n = 2 \cdot i$