Real-time Filtering

CPE 381 Fundamentals of Signals and Systems for Computer Engineers

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Matlab environment

- De-facto standard for scientific computing
  - http://www.mathworks.com/
- Mathematical computation, analysis, visualization, and algorithm development
- Toolboxes
  - Signal Processing, Image Processing, Data Acquisition
- Vector/Array oriented computation
  - Custom procedures
- Simulink
- Real-time interfaces & hardware

Signal processing environment

- Function generators
  - Convenient preparation of constant arrays
    - DA conversion
- Signal processing tool
  - `sptool`
- Filter Design and Analysis tool
  - `fdatool`
  - Generate C-header files with your filter coefficients
    - Targets/Generate C header
  - Export filters to Simulink modules
- Wavelet toolbox
  - `wavemenu`

Real-time Processing

- What is real-time?
  - Processing your samples fast enough to influence the system you control
- Hard real time and soft real time systems
  - Always evaluate performance
- Matlab processes the whole array
- Real-time systems receive data sample-by-sample from signal generators, such as AD converter
- Buffering, processing, output
  - Latency handling – signal processing

Digital filters

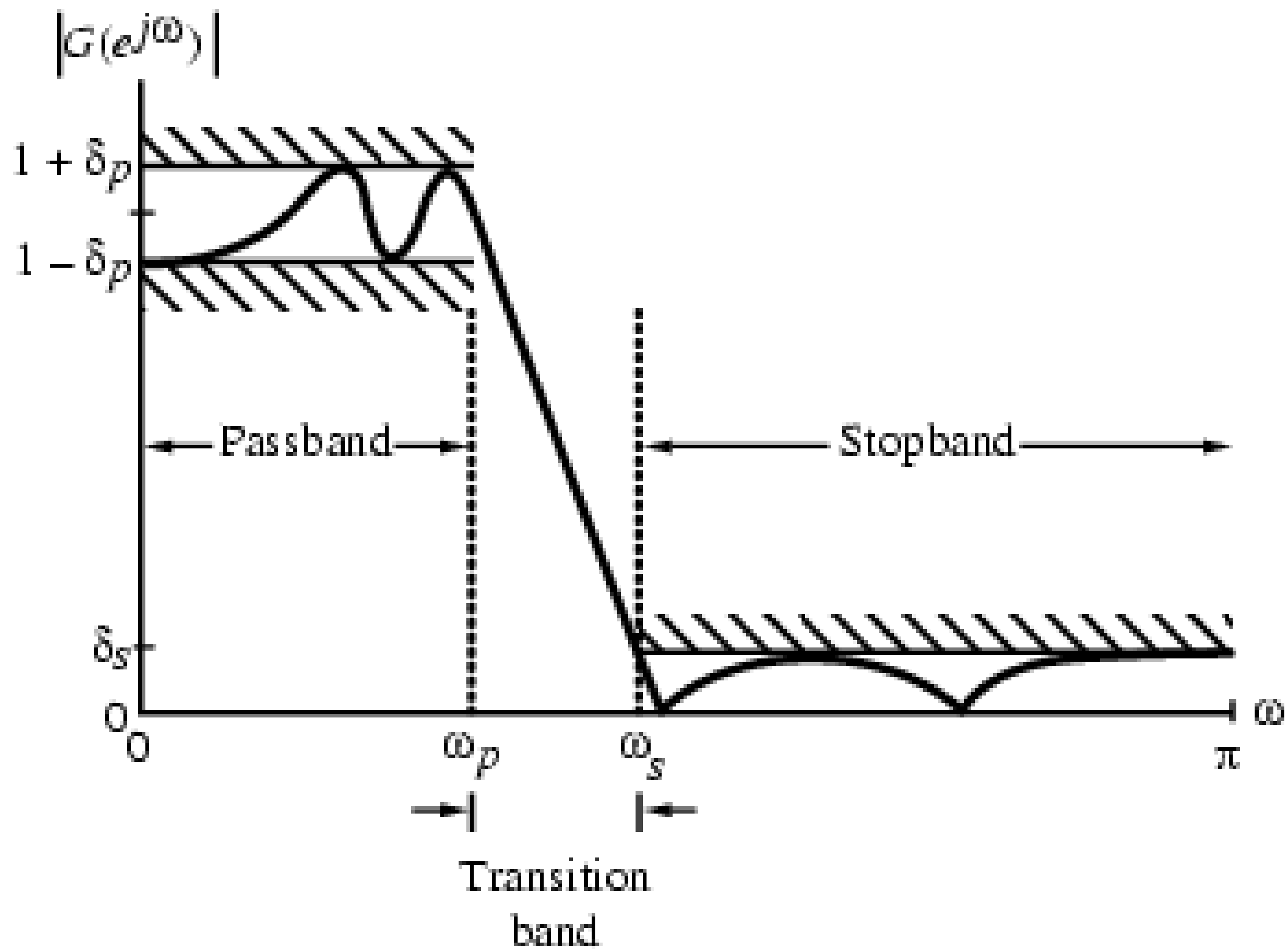
- Processing previous inputs (X[i]) and outputs (Y[i]) to evaluate the value of the current output sample
  - nb input samples and coefficients
  - na previous output samples and coefficients
  - current output is the sum of products of previous samples and coefficients
- Types of filters
  - IIR (Infinite Impulse Response) → general form
  - FIR (Finite Impulse Response) → only input samples X[i]
- Matlab function – filter
  - run Matlab "filtering" demo

```
FILTER One-dimensional digital filter.
   Y = FILTER(B,A,X) filters the data in vector X with the
   filter described by vectors A and B to create the filtered
   data Y.  The filter is a "Direct Form II Transposed"
   implementation of the standard difference equation:

   a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
                         - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
```

Matlab filter Design and Analysis

- Matlab tools
  - sptool for preliminary signal processing and analysis
    - Example 3D accelerometer with LP and HP filtering
      - Signal viewer
      - Filter design and testing
      - Spectral analysis
  - fdatool for filter design and export
    - Tools → generate C header file
- Other tools …
- Specialized filter controllers
  - http://www.quickfiltertech.com/index.php

Filter Specification



Filter Order Estimation

- Filter order (Fred Harris' guide):

$$N \cong \frac{A}{20(\omega_s - \omega_p)/2\pi}$$

  - A – attenuation
  - Pass and stop band
  - Add 10%

Example

- Generate a signal that consists of two sinewaves with frequency of 5Hz and 40Hz and sampling frequency of 200Hz.
- Design FIR and IIR low pass filter with cut-off frequency of 7 Hz and attentuaion of 40 dB at 40 Hz.

Filtering

- Init


```c
// input & output samples
#define FILT_LEN 12

int NB=FILT_LEN;                    // filter length


/*** Filter initialization ***/
void filt_init_var(int *x, int *y) {
  register int ii;

  for (ii=0; ii<FILT_LEN; ii++)
    x[ii] = y[ii] = 0;
}
```

Filtering

- FIR filter (floating point)

```
void xiir_filter(int * x, int * y, int sample)
{
        /* fixed point filter procedure
                xin - input signal
                yout - filtered input signal
        */
        long templ;
        register int i;

        /* the latest sample is at index 0, all other are shifted */
        for (i=NB-1;i>0;i--) {
                x[i]=x[i-1];
                y[i]=y[i-1];
        }
        x[0]=sample;

// FIR filter
        templ=0;
        for (i=0;i<NB;i++) {
                templ += x[i]*B[i];
                }

        y[0]=(int)templ;
}
```

ffilt.c

```
/******
   file: ffilt.c
   description: Code snippets for floating point FIR filter implementation
   author: Emil Jovanov
   date: November 3, 2011. */

// input & output samples
#define FILT_LEN 12
// FIR filter coefficients
const double B[]={0.005308, 0.022428, 0.055974, 0.102684, 0.149662, 0.179419, …
      0.179419, 0.149662, 0.102684, 0.055974, 0.022428, 0.005308};
int NB=FILT_LEN;                        // filter length

/*** FIR filter - fixed point ***/
void xfir_filter(int * x, int * y, int sample) {
        /* fixed point filter procedure, xin - input signal ,  yout - filtered input signal */
             double tempf;
             int ii;

             /* the latest sample is at index 0, all other are shifted */
             for (ii=NB-1;ii>0;ii--) {
                  x[ii]=x[ii-1];
                  y[ii] = y[ii-1];
                  }

             x[0]=sample;

             /*** convolution sum */
             tempf=0.0;
             for (ii=0; i<NB;ii++) {
                 tempf += x[ii]*B[ii];
                 }
             y[0]=(int) tempf;
}
```

Fixed point filter implementation

- Microcontrollers emulate floating point operations
  - Running fixed point operations much faster
  - The precision may not be sufficient for some applications
  - Example ffilt.c on our web-site
- Representing floating point numbers using fixed point values (arithmetic operations)
- Assume:
  - max(coefficient value) = MAX_INT
  - scale all coefficients to MAX_INT
- Optimize individual terms

Filter implementation - example

- Optimized processing
- Example - low pass IIR filter, coefficients:

```
b(1) -> 0.009236
b(2) -> 0.018472        a(2) -> -1.710329
b(3) -> 0.009236        a(3) -> 0.747274
```

  - filter coefficient: 0.009236
  - fixed point coef. value: 65536*()=605.290496 (0x25D)
  - binary value 0b 0000 0010 0101 1101 010010
  - Loosing 6-bits of precision!!!
  - `Old value: 0b0000001001011101`
  - `New value:     0b1001011101010010` (dec. 38738)
- Processing
  - temporary result: unsigned long templ;
  - templ += (38738 * x[0]) >> 6;

Filter implementation – example (2)

- Matlab sequence of the previous example:

```
>> c=0.009236;    % coefficient value
>> sfact=floor(log2(1/c)) % scale factor
sfact = 6
>> cint=round(c2*65535*2^sfact) % int value
cint = 38738
```

fix_filt.c

```c
/*******
  file: fix_filt.c
  description: Fixed point FIR and IIR filtering routines,
               all procedures manually optimized for maximum precision.
  author: Emil Jovanov
  date: November 3, 2001. */

// input & output samples
#define FILT_LEN 12
int NB=FILT_LEN;                                   // filter length

/*** FIR filter - fixed point ***/
void xfir_filter(int * x, int * y, int sample) {
    /* fixed point filter procedure, xin - input signal ,  yout - filtered input signal */
        long templ;
        register int ii;

        /* the latest sample is at index 0, all other are shifted */
        for (ii=NB-1;ii>0;ii--) {
                x[ii]=x[ii-1];
                y[ii] = y[ii-1];
                }

        x[0]=sample;

        /*** B coefficients */
        templ=0;
        templ += (44530 * x[0]) >> 7;       /* b(1) -> 0.005308 */
        templ += (47034 * x[1]) >> 5;       /* b(2) -> 0.022428 */
        templ += (58693 * x[2]) >> 4;       /* b(3) -> 0.055974 */
        templ += (53836 * x[3]) >> 3;       /* b(4) -> 0.102684 */
        templ += (39233 * x[4]) >> 2;       /* b(5) -> 0.149662 */
        templ += (47034 * x[5]) >> 2;       /* b(6) -> 0.179419 */
        templ += (47034 * x[6]) >> 2;       /* b(7) -> 0.179419 */
        templ += (39233 * x[7]) >> 2;       /* b(8) -> 0.149662 */
        templ += (53836 * x[8]) >> 3;       /* b(9) -> 0.102684 */
        templ += (58693 * x[9]) >> 4;       /* b(10) -> 0.055974 */
        templ += (47034 * x[10]) >> 5;      /* b(11) -> 0.022428 */
        templ += (44530 * x[11]) >> 7;      /* b(12) -> 0.005308 */

        y[0]=templ >> 16;
}
```

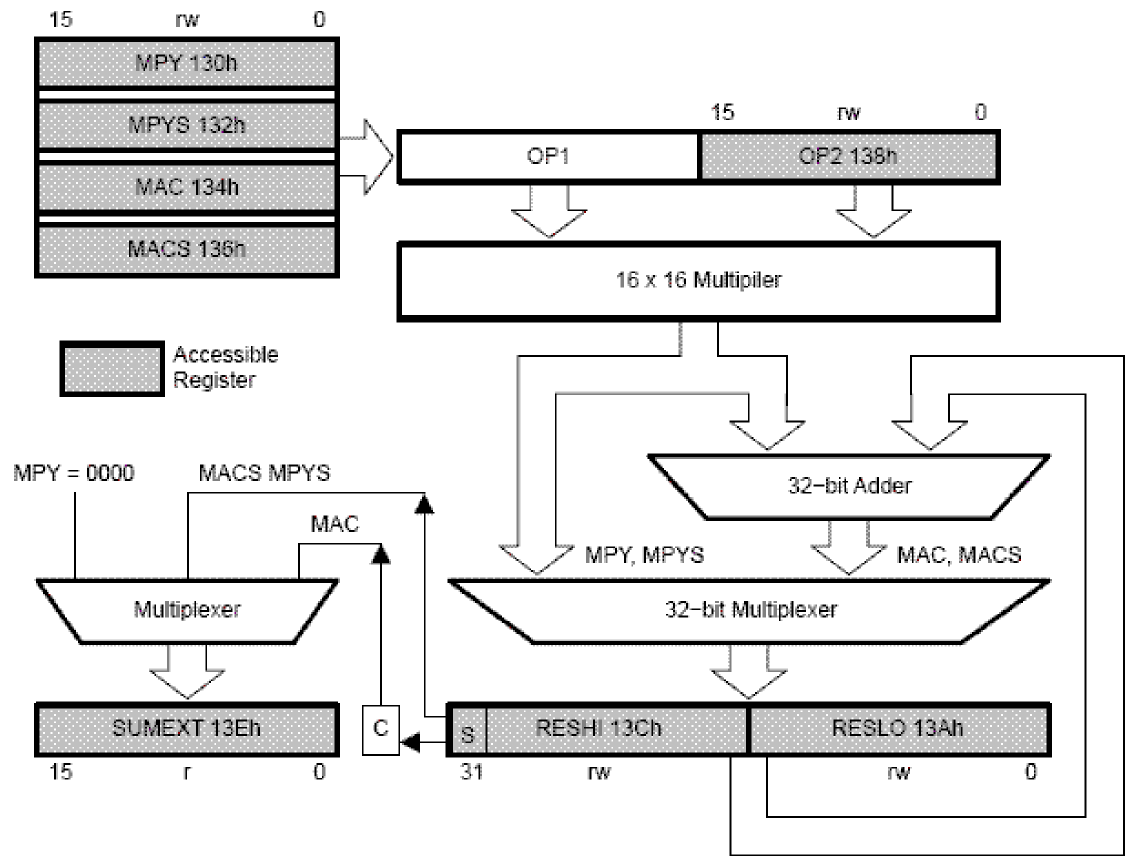
```c
*** Filter initialization ***/
void filt_init_var(int *x, int *y) {
  register int ii;

  for (ii=0; ii<FILT_LEN; ii++)
    x[ii] = y[ii] = 0;
}

/*** IIR filter - fixed point ***/
void xiir_filter(int * x, int * y, int sample) {
        /* fixed point filter procedure
                xin - input signal
                yout - filtered input signal
        */
        long templ;
        register int ii;

        /* the latest sample is at index 0, all other are shifted */
        for (ii=NB-1;ii>0;ii--) {
                x[ii]=x[ii-1];
                y[ii]=y[ii-1];
        }
        x[0]=sample;

        /*** B coefficients */
        templ=0;
        templ += (38740 * x[0]) >> 6;       /* b(1) -> 0.009236 */
        templ += (38740 * x[1]) >> 5;       /* b(2) -> 0.018472 */
        templ += (38740 * x[2]) >> 6;       /* b(3) -> 0.009236 */
        /*** A coefficients */
        templ += (56044 * y[1]) << 1;       /* a(2) -> -1.710329 */
        templ -= (48973 * y[2]);            /* a(3) -> 0.747274 */

        y[0]=templ >> 16;
}
```

Filter Coefficients

- Group scaling of coefficients
  - Scale factor 1/max_coeff_value:
    - 1/1.710329 = 0.58 (1/2)
  - Coeff_shift = 0.5*65536
- Individual scaling of coefficients
  - Scale factor 1/coeff_value:
    - 1/0.005308 = 188
  - Coeff_shift = the largest power of two → 128 (2^7)
    - Coefficient value: round(128*65536*0.005308) = 44,527
- Temporary result:
  - templ += (sample * coeff) >> coeff_shift

MSP430 Hardware multiplier

- The hardware multiplier supports:
  - Unsigned multiply
    - MPY
  - Signed multiply
    - MPYS
  - Unsigned multiply accumulate
    - MAC
  - Signed multiply accumulate
    - MACS
  - 16×16 bits, 16×8 bits, 8×16 bits, 8×8 bits



Fiter implemenation using DMA

- Direct use of the hardware multiplier
- DMA used to access coefficients and data
- Precision/Performance

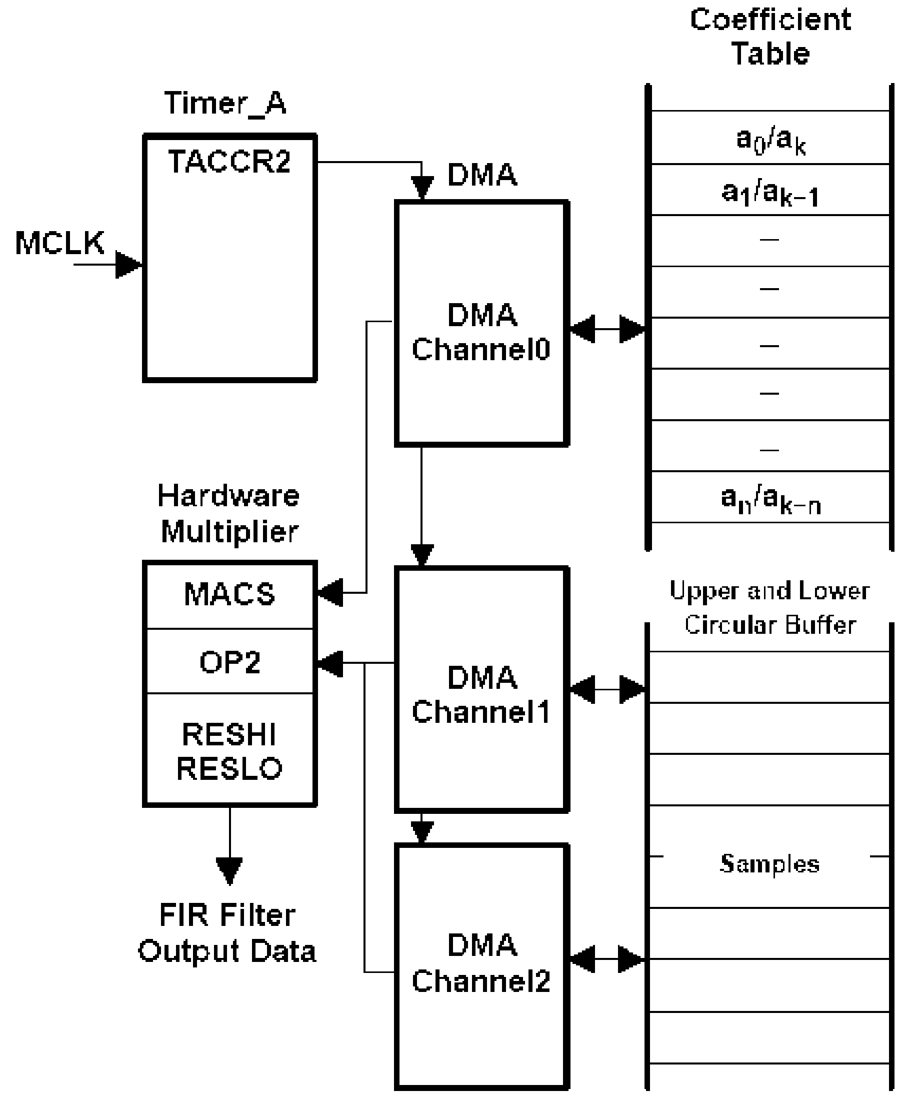


Figure 1. FIR Implementation Using the MSP430F169

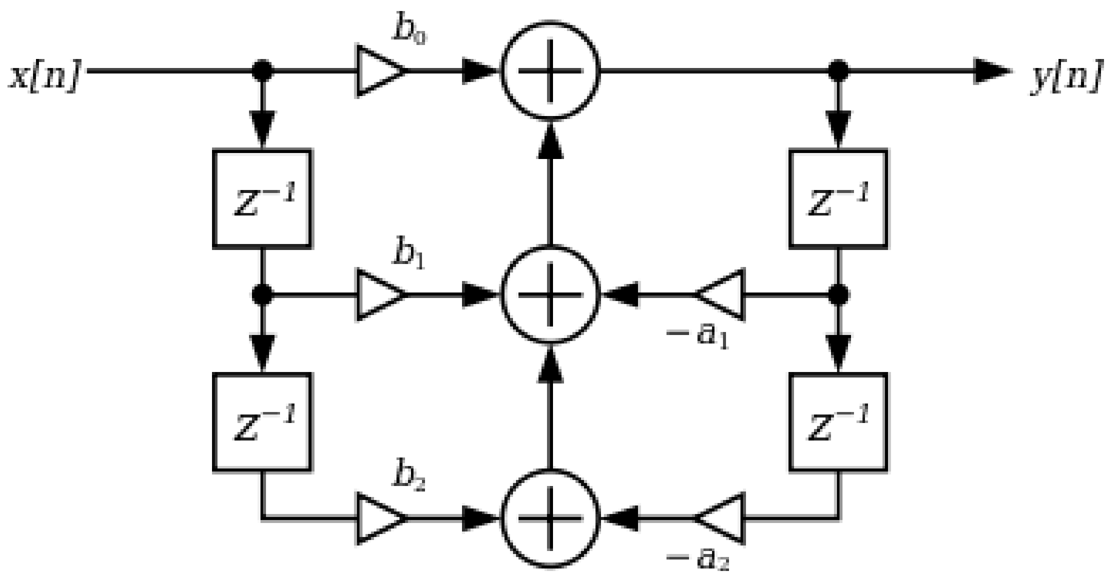
http://focus.ti.com/general/docs/litabsmultiplefilelist.tsp?literatureNumber=slaa228

Filter Implementation: Direct Form 1

The most straightforward implementation is the Direct Form 1, which has the following different equation:

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) - a_1 y(n-1) - a_2 y(n-2)$$

Here the b_0, b_1 and b_2 coefficients determine zeros, and a_1, a_2 determine the position of the poles. Flow graph of biquad filter:



Filter Implementation: Direct Form 2

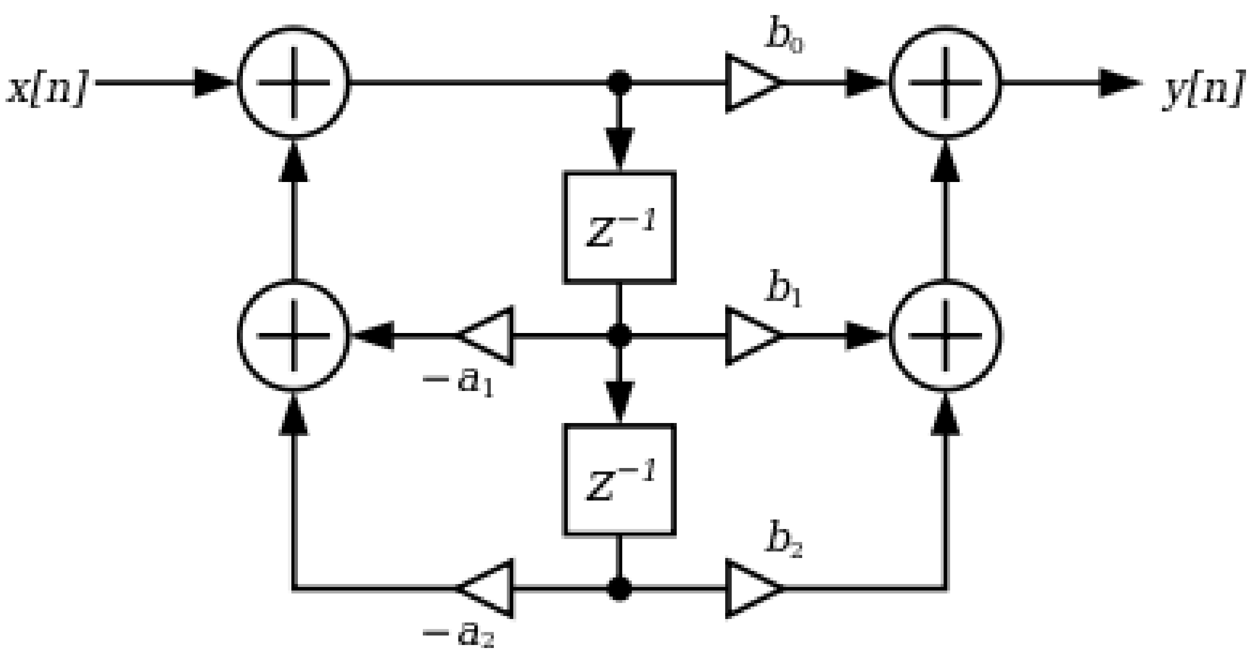
The Direct Form 1 implementation requires four delay registers. An equivalent circuit in the Direct Form 2 implementation requires two delay registers.

The Direct Form 2 implementation is called the canonical form, because it uses the minimal amount of delays, adders and multipliers, yielding in the same transfer function as the Direct Form 1 implementation. The difference equations for DF2 are:

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2),$$

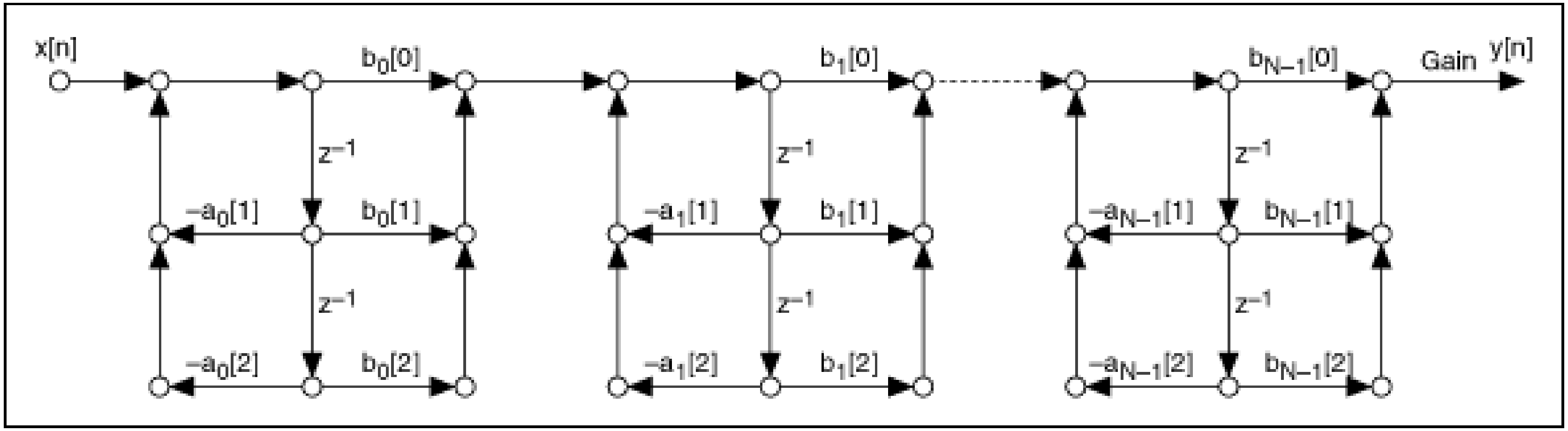
where

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2).$$



Filter Implementation: Second Order Sections

$$H(z) = Gain \bullet \prod_{n=0}^{N-1} \frac{b_n[0] + b_n[1]z^{-1} + b_n[2]z^{-2}}{1 + a_n[1]z^{-1} + a_n[2]z^{-2}}$$




```
function [sos,g] = tf2sos(B,A)
[z,p,g]=tf2zp(B(:)',A(:)');  % Direct form to (zeros,poles,gain)
sos=zp2sos(z,p,g); % (z,p,g) to series second-order sections
```

Conclusion

- Real-time processing is critical for many embedded systems
- Check the performance of your programs
- Support the worst case
- Optimize power after satisfying processing requirements