$\boxed{7.3}$ Hyperbolic functions:

Basic hyperbolic functions:

(1) $y = \sinh x$

$$= \frac{e^{x} - e^{-x}}{2}$$

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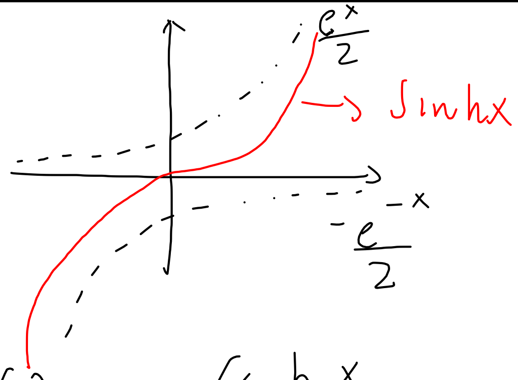

(2) $y = \cosh x$

$$= \frac{e^{x} + e^{-x}}{2}$$

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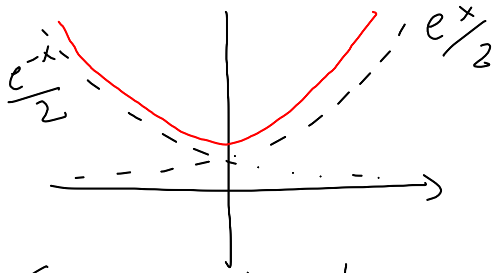

(3) $y = \tanh x$

$$= \frac{\sinh x}{\cosh x}$$

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$$y = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \tanh x$$

(4) $y = \operatorname{sech} x = \frac{1}{\cosh x}$

$$y = \frac{2}{e^{x} + e^{-x}}$$

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(5) $y = \operatorname{csch} x = \frac{1}{\sinh x}$

$$= \frac{2}{e^{x} - e^{-x}}$$

(6) $y = \coth x = \frac{\cosh x}{\sinh x}$

$$= \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

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Evaluate:

$$\cosh 5 = \frac{e^{5} + e^{-5}}{2}$$

$$\tanh 2 = \frac{e^{2} - e^{-2}}{e^{2} + e^{-2}}$$

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Identities for hyperbolic functions:

1) $\cosh^2 x - \sinh^2 x = 1$

2) $\sinh 2x = 2\sinh x \cosh x$

3) $\cosh 2x = \cosh^2 x + \sinh^2 x$

4) $\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

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5) $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

6) $\tanh^2 x = 1 - \operatorname{sech}^2 x$

7) $\coth^2 x = 1 + \operatorname{coch}^2 x$

Verify

$\cosh^2 x - \sinh^2 x = 1$

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$$LHS = \cosh^2 x - \sinh^2 x$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x} + 2e^x \cdot e^{-x} + e^{-2x}}{4} - \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4}$$

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$$= \frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}}{4}$$

$$= \frac{4}{4} = \boxed{1}$$

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Derivatives of hyperbolic functions:

Let u be a differentiable function of x.

1) $\dfrac{d}{dx}(\sinh u) = \cosh u \dfrac{du}{dx}$

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$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \frac{1}{2}\left[e^x + e^{-x}\right]$$

$$= \cosh x$$

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(2) $\frac{d}{dx} \cosh u = \sinh u \cdot \frac{du}{dx}$

(3) $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$

(4) $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$

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(5) $\frac{d}{dx}(\operatorname{sech} u) = - \operatorname{sech} u \tanh u \cdot \frac{du}{dx}$

(6) $\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \frac{du}{dx}$

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Integral formulas for hyperbolic functions:

(1) $\int \sinh x \, dx = \cosh x + C$

(2) $\int \cosh x \, dx = \sinh x + C$

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(3) $\int \operatorname{sech}^2 x \, dx = \tanh x + C$

(4) $\int \operatorname{csch}^2 x \, dx = -\coth x + C$

(5) $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$

(6) $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$

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(1) find $\frac{dy}{dx}$

(a) $y = 6 \sinh \frac{x}{3}$

(b) $y = \cosh \sqrt{1+x^2}$

(c) $y = \ln(\cosh x)$

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(d) $y = (1-x) \tanh x$

(a) $y' = 6 \cosh\left(\frac{x}{3}\right) \cdot \frac{1}{3}$

$y' = 2\cosh\left(\frac{x}{3}\right)$

(b) $y' = \sinh\sqrt{1+x^2} \cdot \frac{1}{\cancel{2}}(1+x^2)^{-1/2} \cancel{2}x$

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$$y' = \frac{x \sinh\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$$

(c) $y' = \frac{1}{\cosh x} \cdot \sinh x$

$= \tanh x$

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(d) $y = (1-x)\tanh x$

$y' = -\tanh x + (1-x)\operatorname{sech}^2 x$

Integrate

(a) $\int \coth 5x \, dx$

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(b) $\int \tanh 2x \, dx$

(c) $\int_0^1 \sinh^2 x \, dx$

(d) $\int_0^{\ln 2} 4e^x \sinh x \, dx$

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(a) $\int \coth 5x \, dx$

$= \int \frac{\cosh 5x}{\sinh 5x} \, dx$

Let $u = \sinh 5x$

$du = 5\cosh 5x \, dx$

$= \frac{1}{5} \int \frac{1}{u} \, du$

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$= \frac{1}{5} \ln|u| + C$

$$\boxed{= \frac{1}{5} \ln|\sinh 5x| + C}$$

(b) $\int \tanh 2x \, dx$

$= \int \frac{\sinh 2x}{\cosh 2x} \, dx$

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$u = \cosh 2x$

$du = 2\sinh 2x \, dx$

$\frac{1}{2} \int \frac{1}{u} \, du$

$= \frac{1}{2} \ln|u| = \frac{1}{2} \ln|\cosh 2x| + C$

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ⓒ $\int_0^1 \sinh^2 x \, dx$

$= \int_0^1 \left(\frac{\cosh 2x - 1}{2} \right) dx$

$= \frac{1}{2} \int_0^1 (\cosh 2x - 1) \, dx$

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$= \frac{1}{2} \left[\frac{\sinh 2x}{2} - x \right]_0^1$

$= \frac{1}{2} \left[\frac{\sinh 2}{2} - 1 - \cancel{\frac{\sinh 0}{2}}^{\,0} + 0 \right]$

$= \frac{1}{2} \left[\frac{\sinh 2}{2} - 1 \right]$

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