8.4 Integration of rational functions by partial fractions

(ex) $\displaystyle\int \frac{5x+7}{x^2-2x-3}\,dx$

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$$\frac{5x+7}{x^2-2x-3} = \frac{5x+7}{(x+1)(x-3)}$$

$$= \frac{A}{x+1} + \frac{B}{x-3}$$

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Proper rational functions:

$\dfrac{P(x)}{Q(x)}$ When $P(x)$ and $Q(x)$ are polynomial functions

When degree $P(x) <$ degree $Q(x)$

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Case 1: denominator $Q(x)$ is a product of distinct linear factors.

$$\frac{P(x)}{Q(x)} = \frac{A}{a_1 x + b_1} + \frac{B}{a_2 x + b_2} + \ldots$$

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Case 2: $Q(x)$ is a product of linear factors, some of which are repeated.

$$\frac{P(x)}{Q(x)} = \frac{A}{a_1 x + b_1} + \frac{B}{(a_1 x + b_1)^2} + \ldots$$

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Case 3: $Q(x)$ contains irreducible quadratic factor, none of which is repeated.

$$\frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2 + bx + c}$$

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Case 4: $Q(x)$

Contains irreducible quadratic factors, some of which is repeated

$$\frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2} + \cdots$$

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Improper rational functions:

$$\frac{P(x)}{Q(x)} \quad \text{degree } P(x) \geq \text{degree } Q(x)$$

By using long division method.

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Write out the partial fraction decomposition for the following:

① $\dfrac{2}{x^2+3x-4} = \dfrac{2}{(x+4)(x-1)}$

$$= \frac{A}{x+4} + \frac{B}{x-1}$$

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② $\dfrac{x^2}{(x-1)(x^2+x+1)}$

$$= \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$$

③ $\dfrac{x^2}{(x-1)^2(x^2+x+1)^2}$

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$$= \frac{A}{x-1} + \frac{B}{(x-1)^2}$$
$$+ \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{(x^2+x+1)^2}$$

Ex) $\displaystyle\int \frac{5x+7}{x^2-2x-3}\, dx$

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$$\frac{5x+7}{x^2-2x-3} = \frac{5x+7}{(x-3)(x+1)}$$

$$\frac{5x+7}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$5x+7 = A(x+1) + B(x-3)$$

$$\left(\frac{5x+7}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1} \right)(x-3)(x+1)$$

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$$5x + 7 = Ax + A$$
$$+ Bx - 3B$$
$$5x + 7 = (A + B)x$$
$$+ (A - 3B)$$
$$\left. \begin{array}{l} A + B = 5 \\ A - 3B = 7 \end{array} \right\} \times 3$$

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$$3A + 3B = 15$$
$$A - 3B = 7$$
$$+ \overline{\hspace{5cm}}$$
$$4A = 22 \Rightarrow A = \frac{11}{2}$$

$$\frac{11}{2} + B = 5$$
$$\Rightarrow B = 5 - \frac{11}{2} = -\frac{1}{2}$$

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$$\frac{5x + 7}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$
$$= \frac{11}{2}\left(\frac{1}{x-3}\right)$$
$$+ \left(-\frac{1}{2}\right) \cdot \frac{1}{x+1}$$
$$\int \frac{5x+7}{(x-3)(x+1)} \, dx$$
$$= \frac{11}{2} \int \frac{1}{x-3} \, dx - \frac{1}{2} \int \frac{dx}{x+1}$$

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$$= \frac{11}{2} \ln|x - 3|$$
$$- \frac{1}{2} \ln|x + 1| + C$$

$$\int \frac{1}{(x+5)^2(x-1)} \, dx$$
$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}$$

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$$1 = A(x+5)(x-1)$$
$$+ B(x-1)$$
$$+ C(x+5)^2$$

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