Big O Notation

- **Big O Notation is a mathematical Approximation for the “Slowest” an algorithm or operation will take to complete.**
- **This is not a measurement of SPEED but EFFICIENCY.   This Counts the total number of operations needed to complete the algorithm in question.**
- **BigO ignores all terms of an equation except the Largest Term**

$f(N)$=N 〖+ N〗 ^5+log(N)+ 〖2500N 〗 ^3

The order of the above equation is N^5

Big-O

- **O(1)** - Constant time
  - It requires the same amount of time regardless of input size
    - array: access any element
    - fixed-size stack: push and pop
- **O(N)** - Linear Time
  - If the execution time is directly proportional to the input size. Time grows linearly as the input size increases
    - array: linear search, find min
    - queue: contains method
- **O(log n)** - Logarithmic Time
  - If the execution time is proportional to the logarithm of the input size.
    - binary search
- **O(n^2)** - Quadratic Time
  - If the execution time is proportional to the square of the input size.
    - bubble sort, selection sort, insertion sort

Linear Search

- Consider a sequential list of elements
- If we wish to insert elements as quickly as possible, then ***InsertItem*** is **O(1)**
  - New element placed in next array slot
  - Element added at first position in linked list
- To find a particular value, one may have to scan the entire sequence of values **O(N)**
- Average N/2 comparisons for successful search

Self-Organizing / Self-Adjusting Lists

- Put frequently accessed items near front
  - Strategy #1
    - Move each item accessed to the front
  - Strategy #2
    - Upon access to an element, swap it with the element that precedes it
- Worst case performance still **O(N)**
- Average performance on successful searches improves

Binary Search

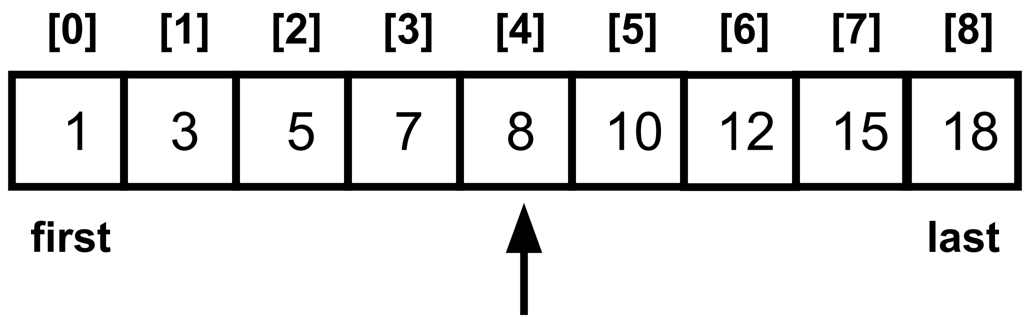
- Assume elements are stored in a sorted array
  $A[0] < A[1] < A[2] < \ldots A[n-1]$
- Use divide and conquer to determine if a value is in the list
- $O(\log_2 n)$ search

Binary Search Example - 1

List	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	1	3	5	7	8	10	12	15	18

Is 15 in the array?

Binary Search Example - 2



midpoint = (first+last)/2 = 4

List[4] == 15 No

List[4] < 15 Yes

first = midpoint + 1 = 5

Binary Search Example - 3

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
1	3	5	7	8	10	12	15	18

Item can't be here **first** **last**

midpoint = (first+last)/2 = 6 (integer division)

List[6] == 15 No

List[6] < 15 Yes

first = midpoint + 1 = 7

Binary Search Example - 4

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
1	3	5	7	8	10	12	15	18

first last

Item can't be here

midpoint = (first+last)/2 = 7 (integer division)

List[7] == 15 Yes

Done

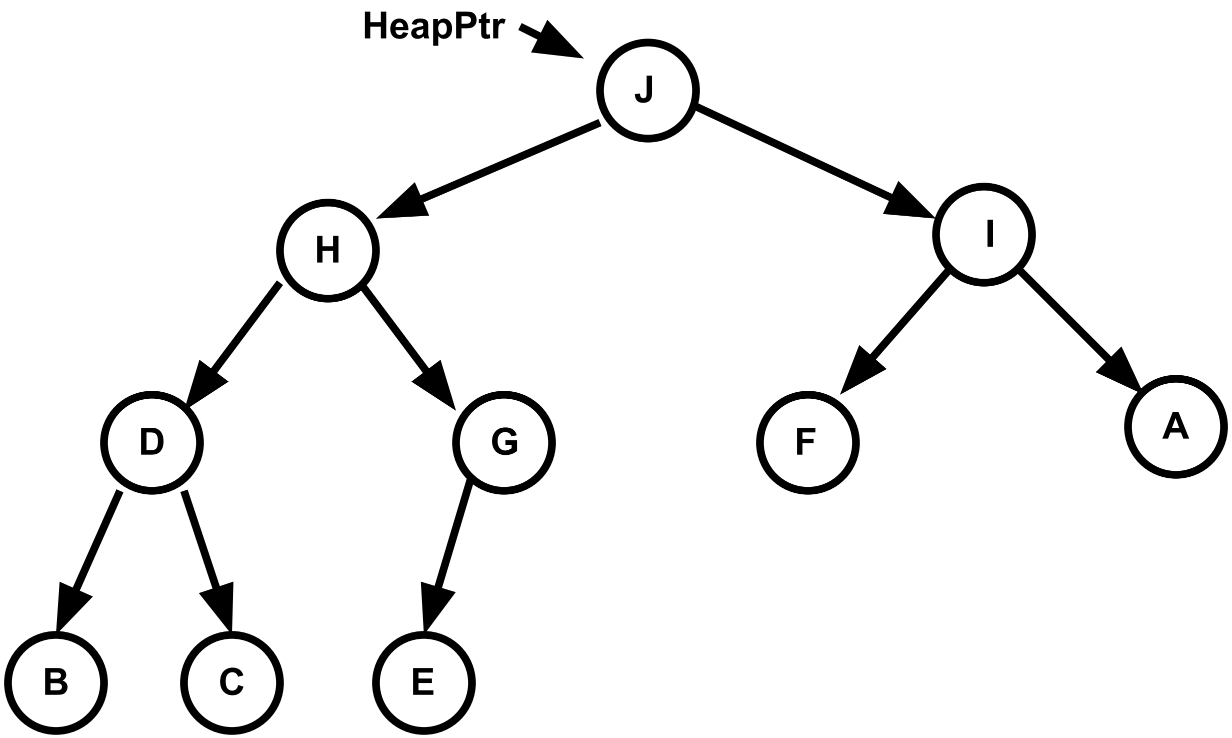
Binary Search Algorithm

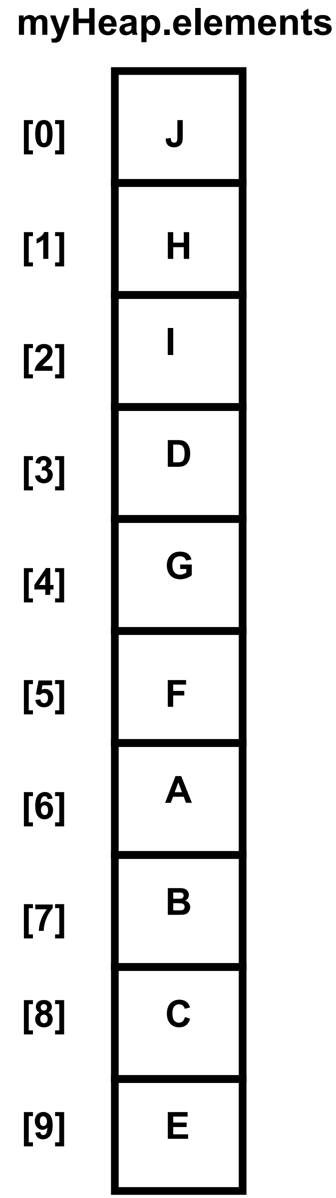
```
void SortedType::RetrieveItem(ItemType& item, bool& found)
{
  int midPoint;
  int first = 0;
  int last = length - 1;

  bool moreToSearch = first <= last;
  found = false;
   while (moreToSearch && !found)
  {
    midPoint = (first + last) / 2;
    switch (item.ComparedTo(info[midPoint]))
    {
      case LESS    : last = midPoint - 1;
                     moreToSearch = first <= last;
                     break;
      case GREATER : first = midPoint + 1;
                     moreToSearch = first <= last;
                     break;
      case EQUAL   : found = true;
                     item = info[midPoint];
                     break;
    }
  }
}
```

Heap ADT

- Heap
  - A complete binary tree, each of whose elements contains a value that is greater than or equal to the value of each of its children (aka maximum heap)
  - Shape property & order property
- Complete Binary Tree
  - A binary tree that is either full or full through the next-to-last level, with the leaves on the last level as far to the left as possible

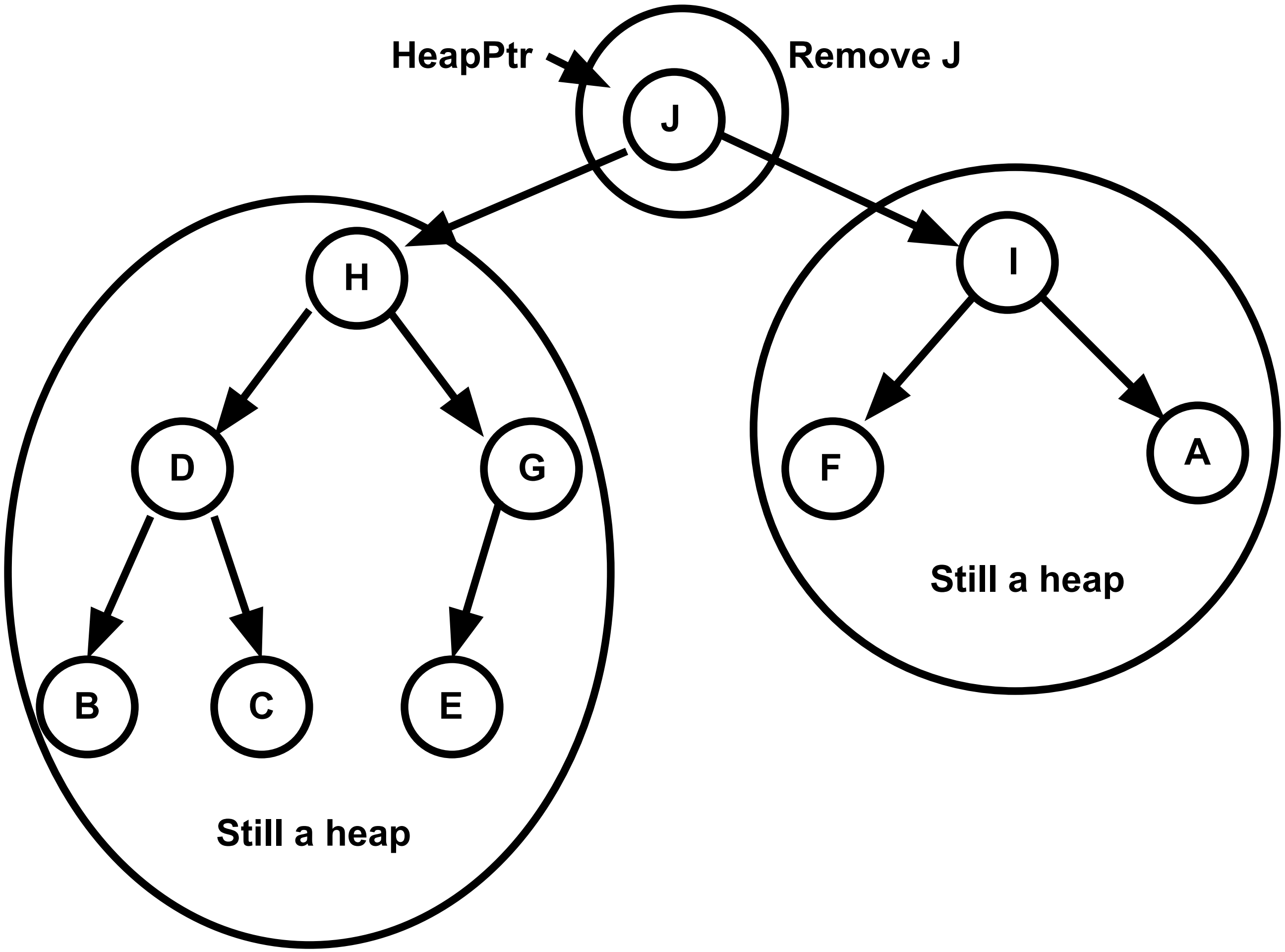
Heap Example





Largest Element of Heap resides in the root node

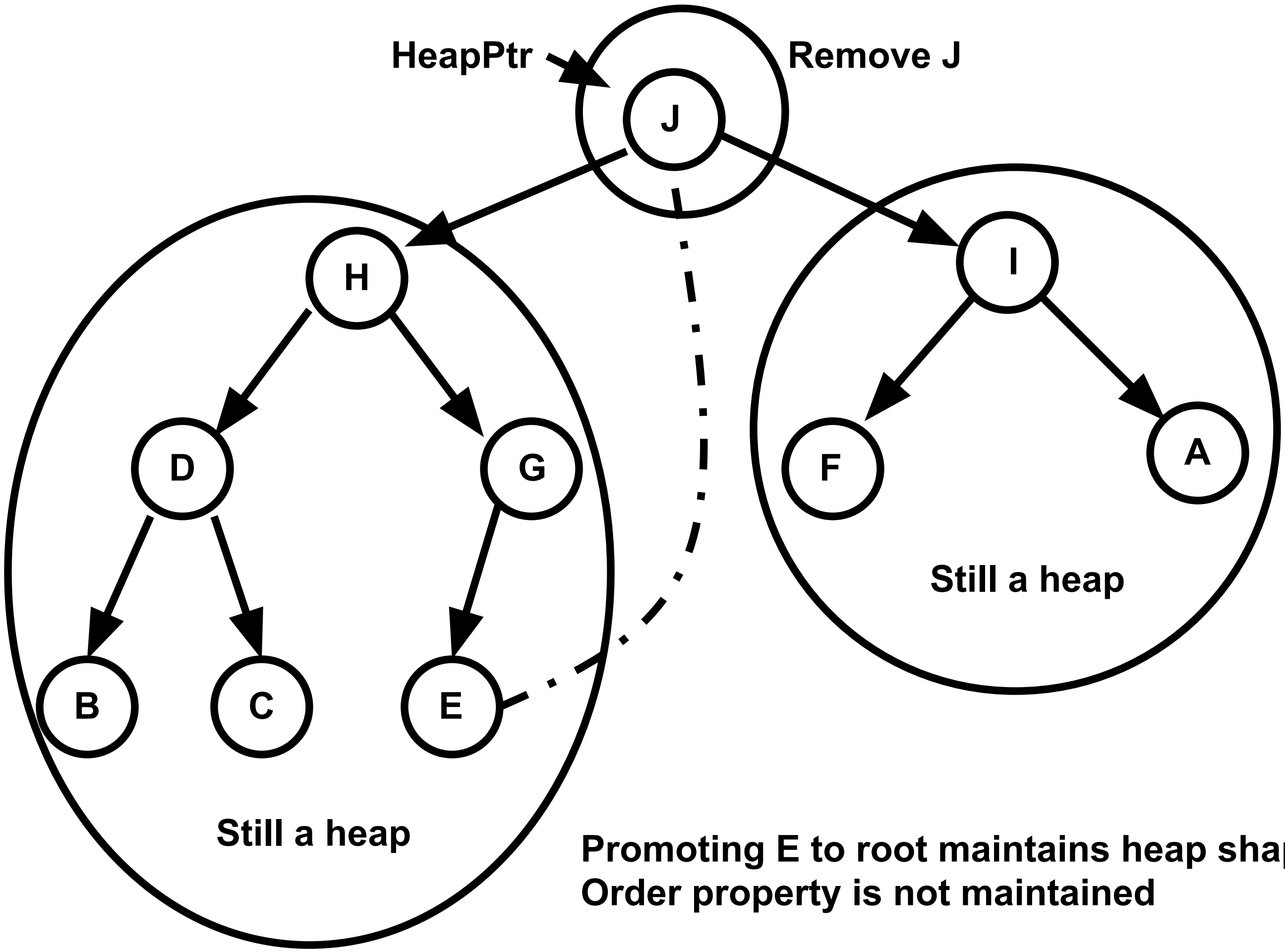
Heap Example



myHeap.elements

[0]	J
[1]	H
[2]	I
[3]	D
[4]	G
[5]	F
[6]	A
[7]	B
[8]	C
[9]	E

Heap Example



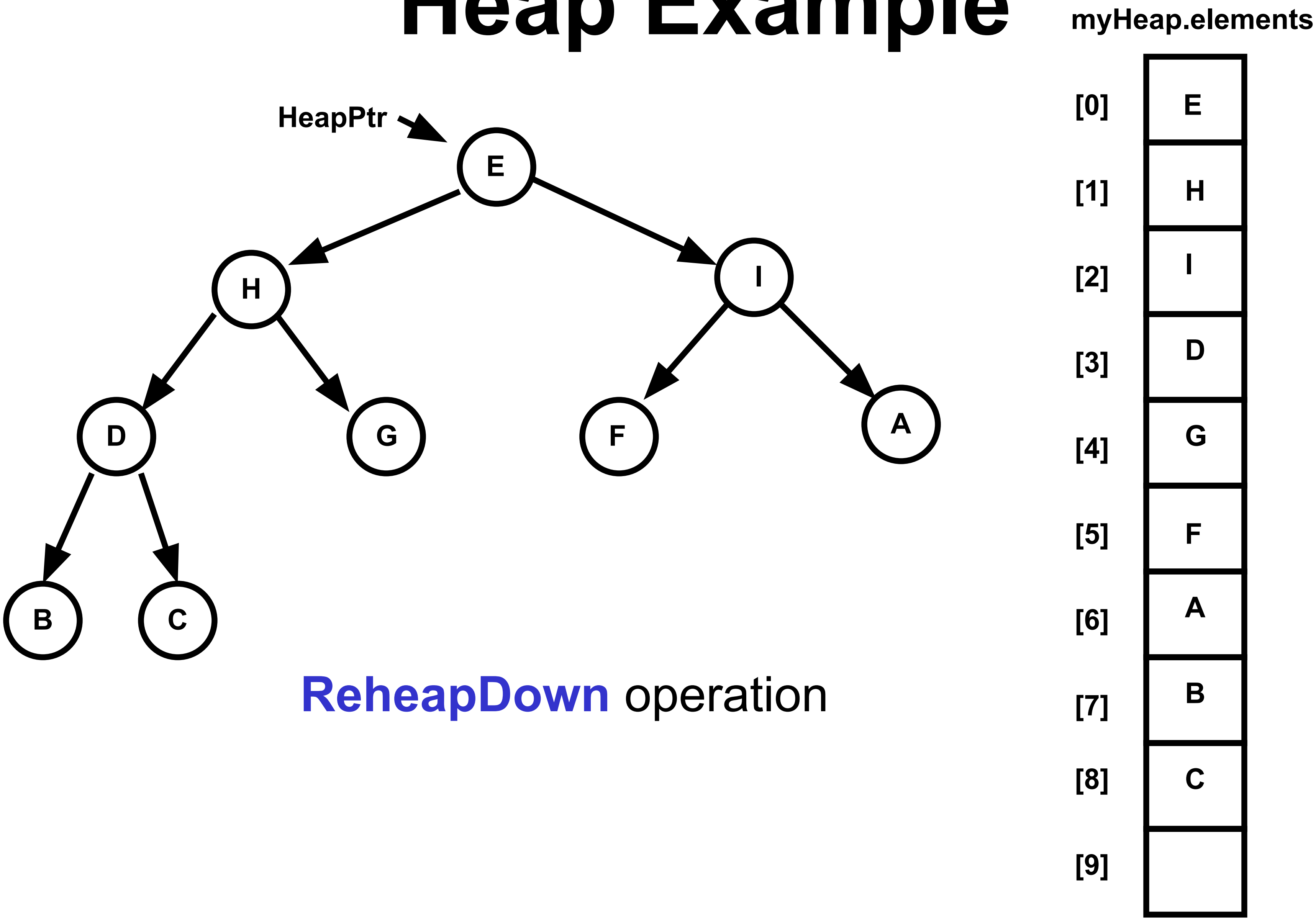
Promoting E to root maintains heap shape
Order property is not maintained

Order must be adjusted

myHeap.elements

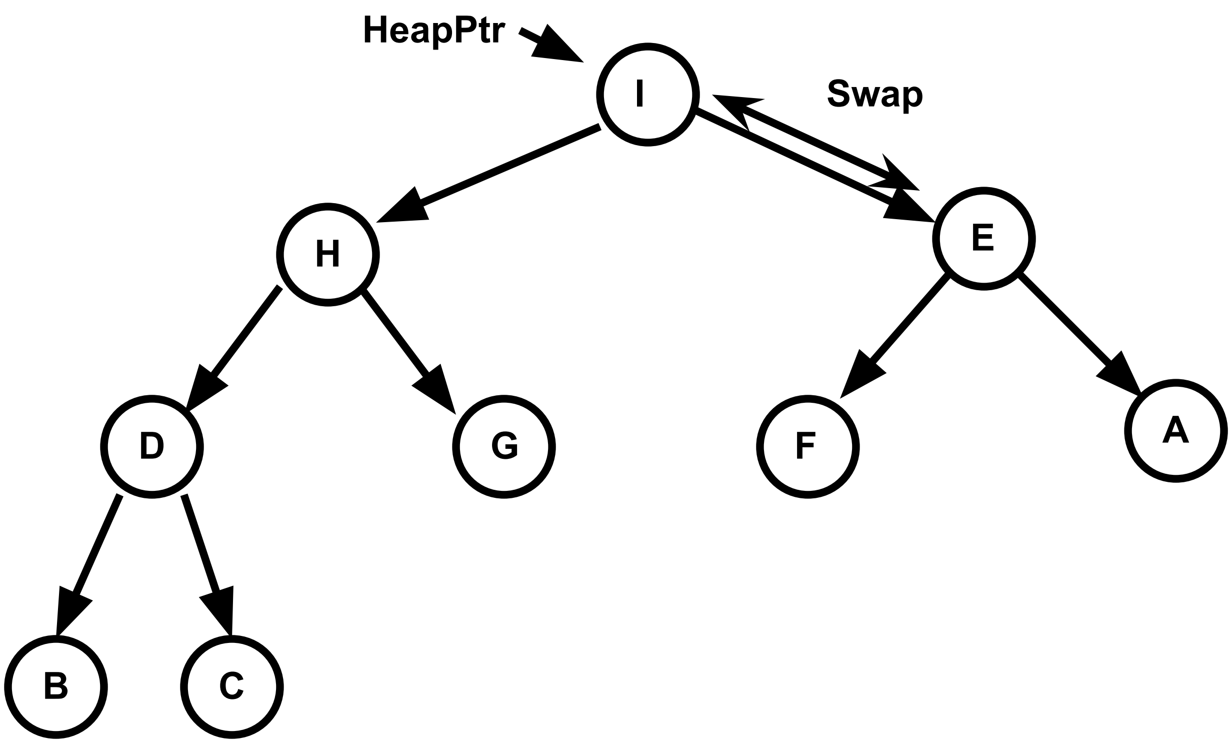
[0]	E
[1]	H
[2]	I
[3]	D
[4]	G
[5]	F
[6]	A
[7]	B
[8]	C
[9]	

Heap Example

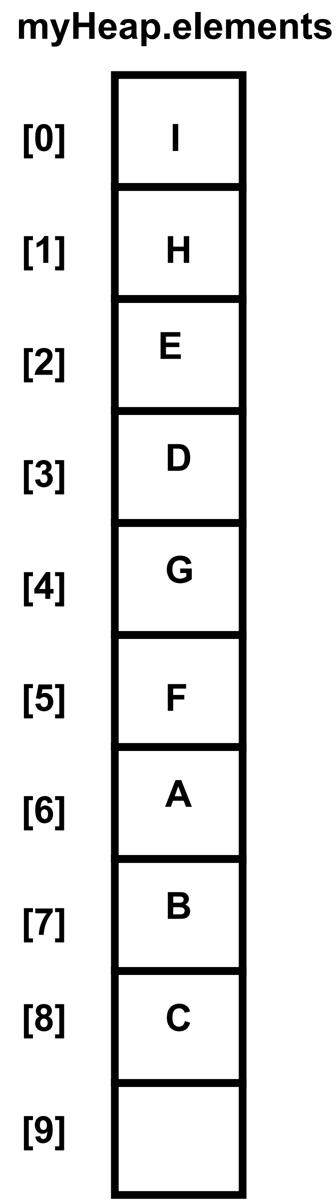


ReheapDown operation

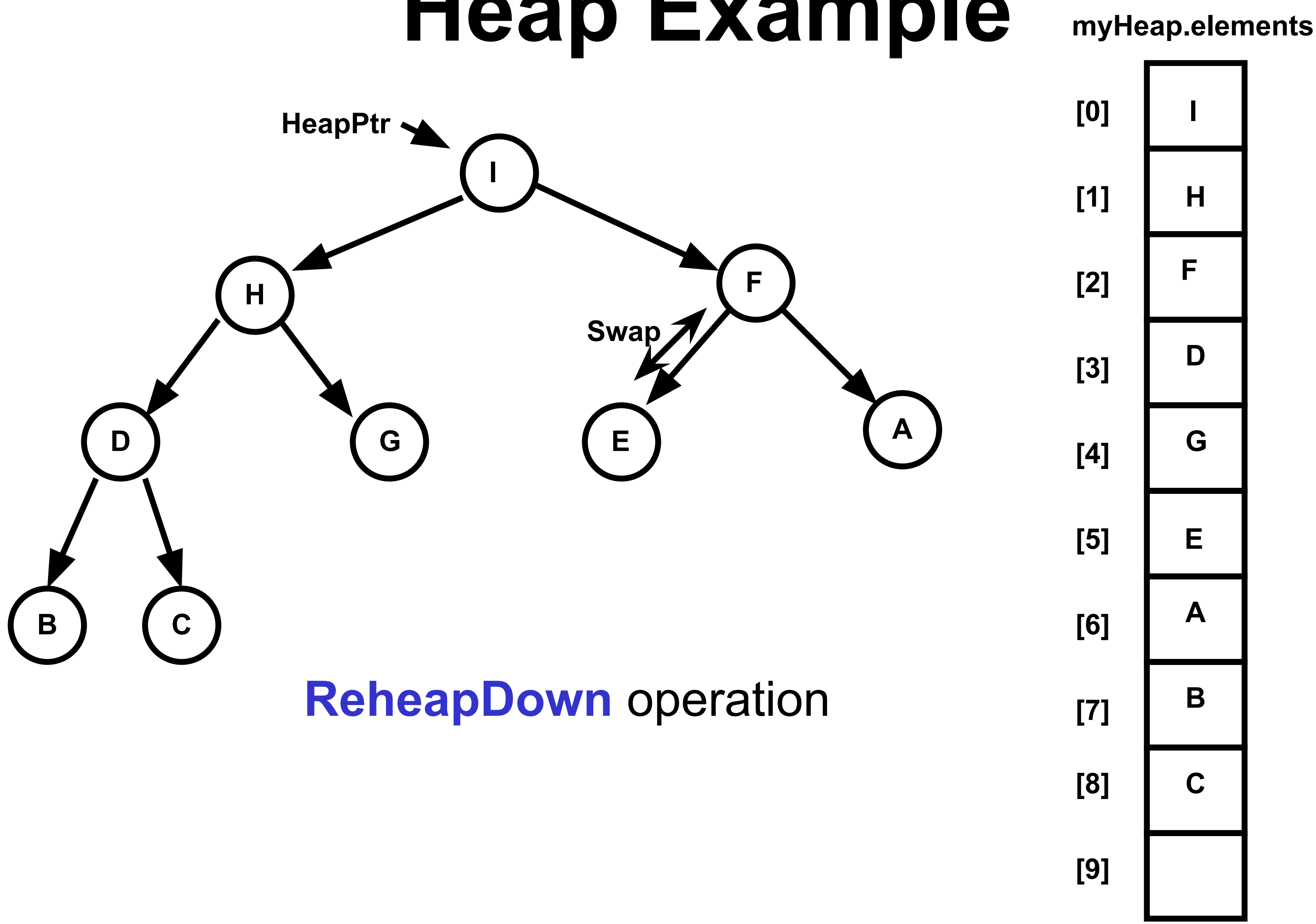
Heap Example



ReheapDown operation



Heap Example

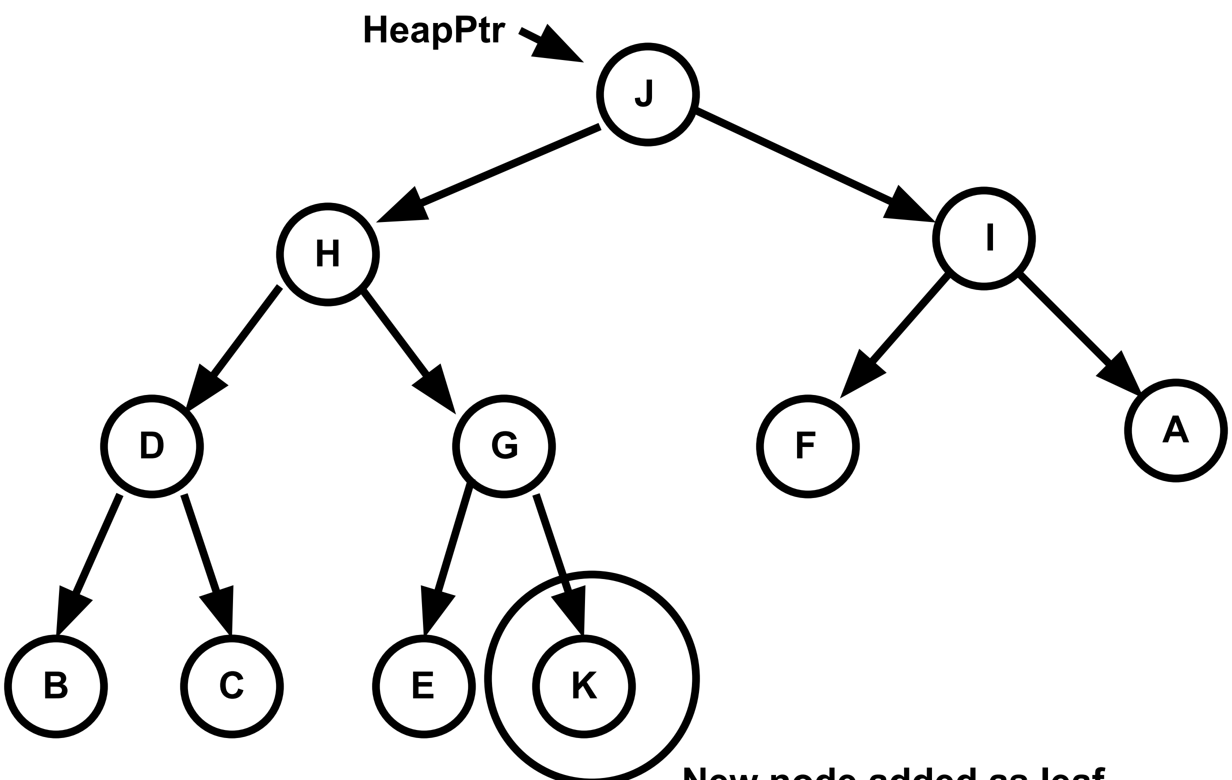


ReheapDown operation

Heap ADT

- ReheapDown(root,bottom)
  - Restores the order property of the heaps to the tree between root and bottom
  - Precondition
    - The order property of heaps may be violated only by the root node of the tree
  - Postcondition
    - The order property applies to all elements of the heap

Heap Example

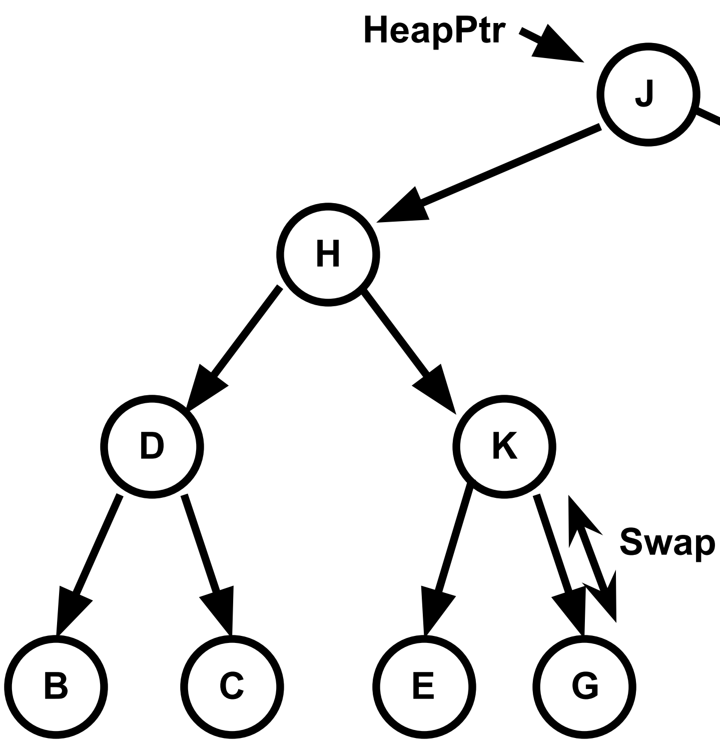


New node added as leaf

Shape property maintained
Order property is not maintained

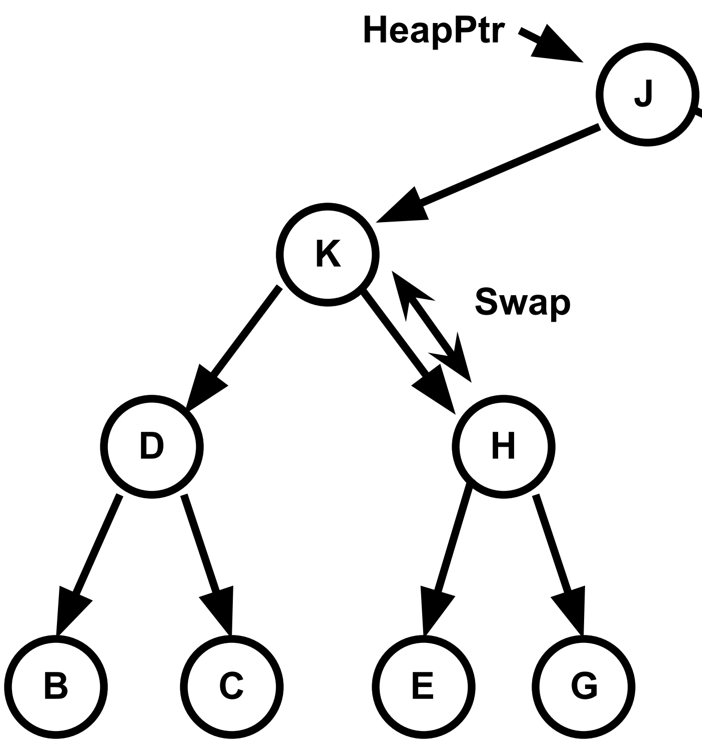
Order must be adjusted

Heap Example



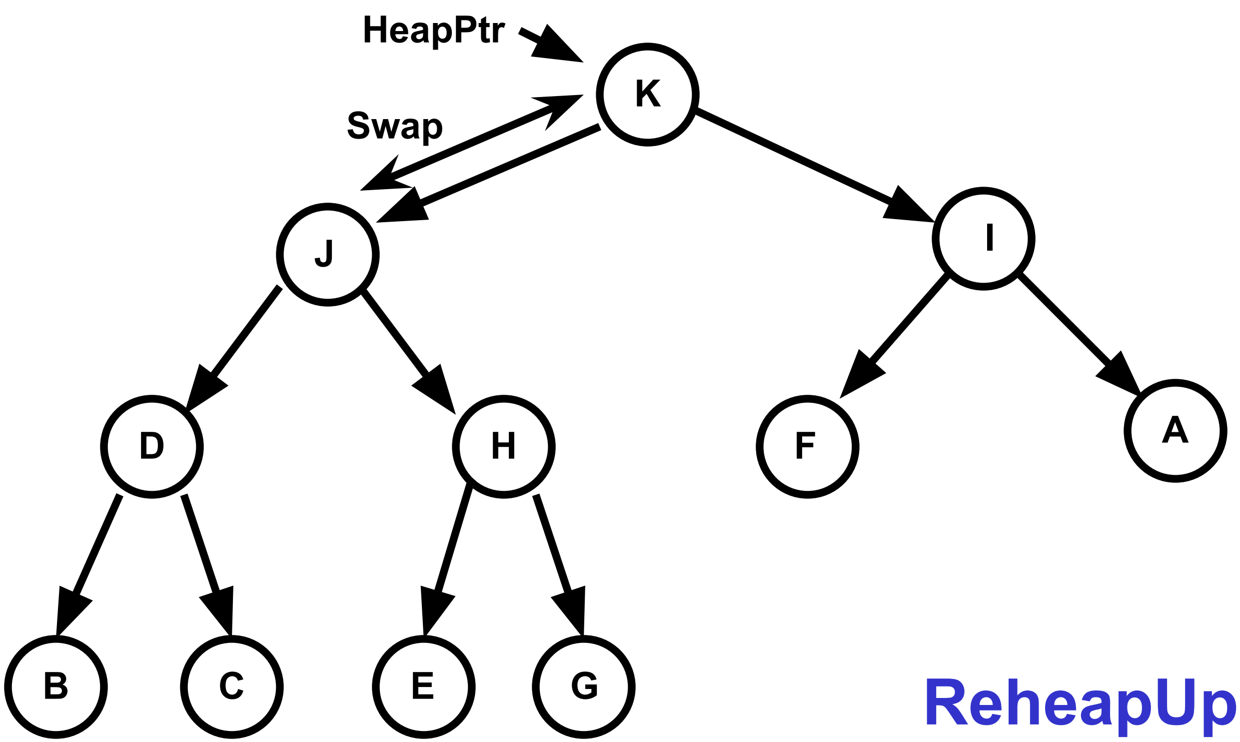
ReheapUp operation

Heap Example



ReheapUp operation

Heap Example



ReheapUp operation

Heap ADT

- ReheapUp(root,bottom)
  - Restores the order property of the heap between root and bottom
  - Precondition
    - The order property is satisfied from the root of the heap through the next-to-last node; the last (bottom) node may violate the order property
  - Postcondition
    - The order property applies to all elements of the heap

Heap Implementation - 1

```
typedef int ItemType;

template<class ItemType>
// Assumes ItemType is either a built-in simple type or a class
// with overloaded relational operators.
struct HeapType
{
  void ReheapDown(int root, int bottom);
  void ReheapUp(int root, int bottom);
  ItemType* elements;   // Array to be allocated dynamically
  int numElements;
};
```

Heap Implementation - 2

```
template<class ItemType>
void HeapType<ItemType>::ReheapUp(int root, int bottom)
// Post: Heap property is restored.
{
  int parent;

  if (bottom > root)
  {
    parent = (bottom-1) / 2;
    if (elements[parent] < elements[bottom])
    {
      Swap(elements[parent], elements[bottom]);
      ReheapUp(root, parent);
    }
  }
}
```

Heap Implementation - 3

```
template<class ItemType>
void HeapType<ItemType>::ReheapDown(int root, int bottom)
// Post: Heap property is restored.
{
  int maxChild;
  int rightChild;
  int leftChild;

  leftChild = root*2+1;
  rightChild = root*2+2;
  if (leftChild <= bottom)
  {
    if (leftChild == bottom)
      maxChild = leftChild;
    else
    {
      if (elements[leftChild] <= elements[rightChild])
        maxChild = rightChild;
      else
        maxChild = leftChild;
    }
    if (elements[root] < elements[maxChild])
    {
      Swap(elements[root], elements[maxChild]);
      ReheapDown(maxChild, bottom);
    }
  }
}
```

Heap Efficiency

- Insert Item
  - O(log n)
- Remove maximum
  - O(log n)
- Access maximum
  - O(1)

Searching

- We have seen two techniques for determining if a particular element is stored in a data structure
  - Brute-force linear search $O(N)$
  - Binary search $O(\log_2 N)$

Hashing

- ***Hashing***
  - A technique for ordering and accessing list elements which can permit O(1) searching
  - Key value is used to place an item into the list at a particular location
  - Key also enables access to that location later
  - Trades memory space for access speed

Hash Function

- A function used to manipulate the key of an element in a list to identify its location in the list
- Hash functions have two purposes
  - Used to access a particular item
  - Determines where to place the item
- Collision
  - The condition resulting when two or more keys produce the same hash location

Hashing Example - 1

- Suppose one wishes to store employee records in an array
- Currently there are 100 employees
- Each employee has a unique employee ID number in the range 00000-99999
- Employee ID numbers are not necessarily contiguous

Hashing Example - 2

- **Option #1**
  - Declare an array of 100,000 records
  - Use the unique employee ID number directly as an array index
- Analysis
  - Wasteful, only 100 records needed but memory is reserved for 100,000
  - Provides O(1) access

Hashing Example - 3

- **Option #2**
  - Declare an array just large enough to hold 100 records ==> valid indices 0-99
  - Use a Hashing Function to transform the employee ID number to a particular array index in the range 0-99

  - Consider the following hash function

    ID % 100

Hashing Example - 4

Examples:

–Employee #53330

53330 % 100 = 30

–Employee #81235

81235 % 100 = 35

	…
[30]	Employee 53330
[31]	Empty
[32]	Empty
[33]	Empty
[34]	Empty
[35]	Employee 53335
[36]	Empty
[37]	Empty
	…

Hashing Example - 5

```
int ItemType::Hash() const
// NOTE: Hash function return value is a member of ItemType
// Post: Returns an integer between 0 and MAX_ITEMS -1.
{
  return (idNum % MAX_ITEMS);  // idNum is a member of ItemType
}


template<class ItemType>
void ListType<ItemType>::InsertItem(ItemType item)
// Post: item is stored in the array at position item.Hash().
{
  int location;

  location = item.Hash();
  info[location] = item;
  length++;
}


template<class ItemType>
void ListType<ItemType>::RetrieveItem(ItemType& item)
// Post: Returns the element in the array at position
//       item.Hash().
{
  int location;

  location = item.Hash();
  item = info[location];
}
```

Hashing Example - 6

Examples:

–Employee #53330

53330 % 100 = 30

–Employee #81235

81235 % 100 = 35

–Employee #10935

10935 % 100 = 35 Collision!!

Bin 35 is occupied but the next bin is empty so use it.

	...
[30]	Employee 53330
[31]	Empty
[32]	Empty
[33]	Empty
[34]	Empty
[35]	Employee 53335
[36]	Employee 10935
[37]	Empty
	...

Hashing Example - 7

Examples:

–Employee #53330

53330 % 100 = 30

–Employee #81235

81235 % 100 = 35

–Employee #10935

10935 % 100 = 35   Collision!!

–Employee #35

35 % 100 = 35   Another Collision!!

What happens if an employee ID hashes to the last bin in the array but it is occupied?

	...
[30]	Employee 53330
[31]	Empty
[32]	Empty
[33]	Empty
[34]	Empty
[35]	Employee 53335
[36]	Employee 10935
[37]	Employee 35
	...

Hashing with Collision Resolution

```
int ItemType::Hash() const
// Post: Returns an integer between 0 and MAX_ITEMS -1.
{
  return (idNum % MAX_ITEMS);
}

template<class ItemType>
void ListType<ItemType>::InsertItem(ItemType item)
// Post: item is stored in the array at position item.Hash()
//       or the next free spot.
{
  int location;

  location = item.Hash();
  while (info[location] != emptyItem)
    location = (location + 1) % MAX_ITEMS;
  info[location] = item;
  length++;
}

template<class ItemType>
void ListType<ItemType>::RetrieveItem(ItemType& item, bool& found)
{
  int location;
  int startLoc;
  bool moreToSearch = true;

  startLoc = item.Hash();
  location = startLoc;
  do
  {
    if (info[location] == item || info[location] == emptyItem)
      moreToSearch = false;
    else
      location = (location + 1) % MAX_ITEMS;
  } while (location != startLoc && moreToSearch);
  found = (info[location] == item);
  if (found)
    item = info[location];
}
```

Deletions

- How does one delete an item from the array if linear probing has been used to resolve collisions?

Collision Resolution - 1

- ***Linear Probing***
  - A technique which resolves a hash collision be sequentially searching a hash table for an empty location starting at the location returned by the hash function
- ***Clustering***
  - The tendency of elements to become unevenly distributed in the hash table, creating clusters about hash locations

Collision Resolution - 2

- ***Rehashing***
  - Resolving a collision by computing a new hash location from a hash function that manipulates the original location rather than the element's key

Linear Probing

Rehash function = (hash value + 1) % 100

Could also use (hash value + constant) % 100

Note: constant and array_size should be relatively prime
[successive rehashes will eventually cover every array index]

Collision Resolution - 3

- ***Quadratic Probing***
  - Resolving has collisions by using the rehashing formula

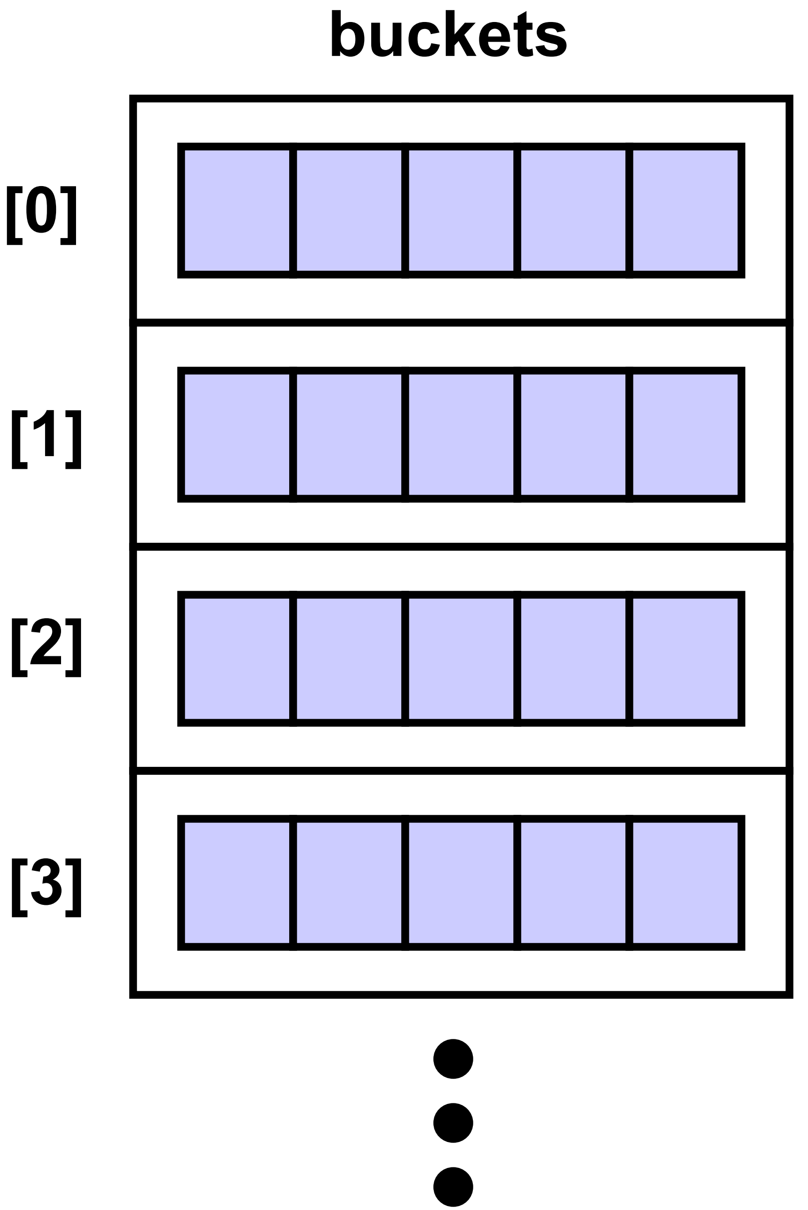
    (HashValue + J^2) % ArraySize

    where J is the number of times that the rehash function has been applied

- ***Buckets***
  - A collection of elements associated with a particular hash location
- ***Chains***
  - A linked list of elements that share the same hash location

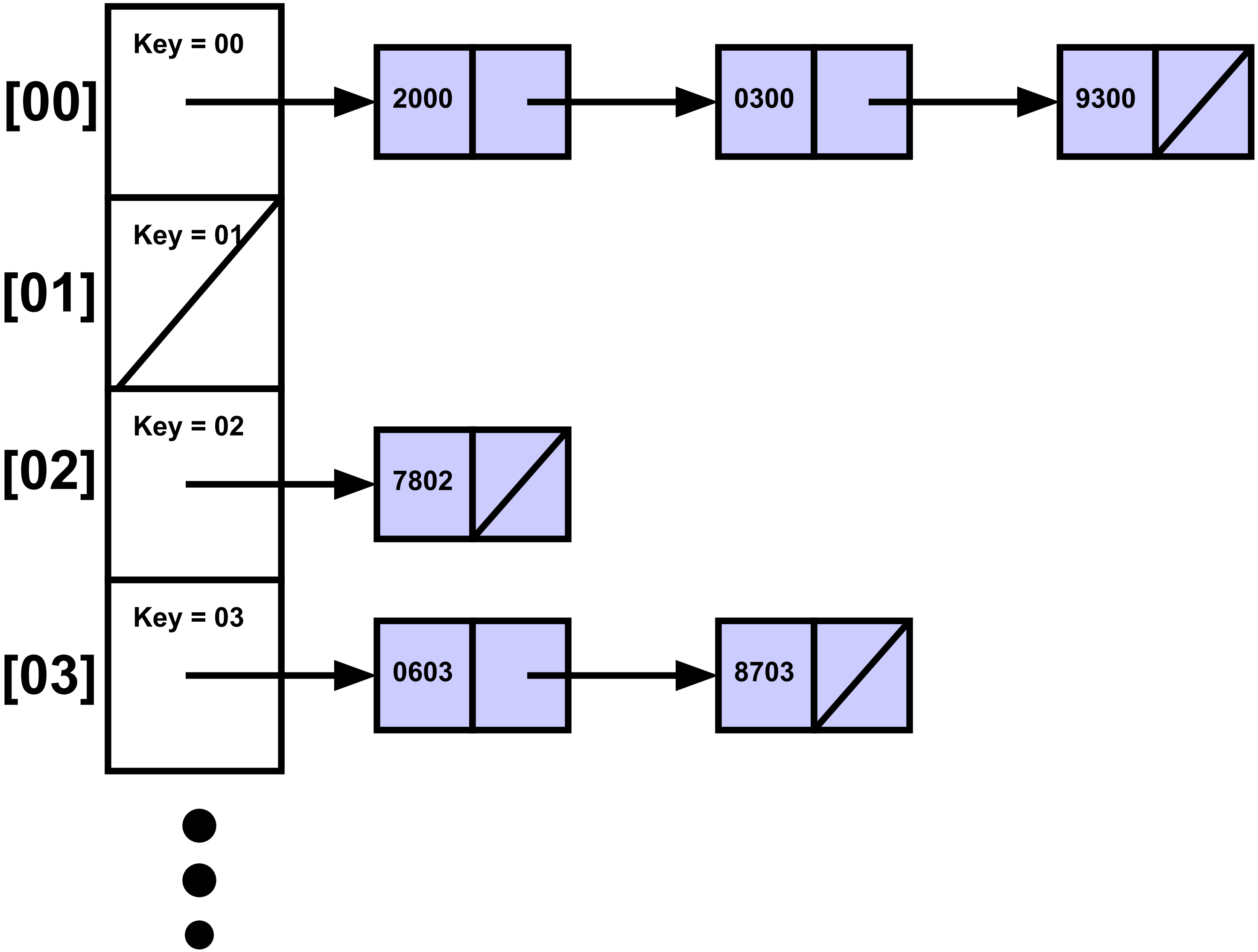
Buckets

- Each hash location can hold an array of element values
- A collision causes the new item to be stored in one of the other local array bins
- If the local array is already full, something else must be done to resolve the collision


buckets
[0]
[1]
[2]
[3]

Chains

- Hash value provides an index into an array of pointers
- At a given location, the pointer is a link to a linked list which stores all items which hashed to the base location
- Eliminates collision resolution
- Simplifies deletions


[00]
Key = 00
2000
0300
9300
[01]
Key = 01
[02]
Key = 02
7802
[03]
Key = 03
0603
8703

Designing Hash Functions

- Goals
  - Uniformly distribute items across array
  - Minimize collisions
- Need to know something about the distribution of the keys
- Division Method is most common
  - Key % TableSize

Folding

- A hash method that breaks the key into several pieces and concatenates or XORs some of the pieces to form the hash value

What if your key is a string??

- Characters are represented internally by integers
- Can convert one or more characters of the string to a numeric value using the equivalent ASCII codes
- Can then use Division Method

Load Factor

- For n items stored in an array of size N, the ***Load Factor*** is $\dfrac{n}{N}$
- When the load factor approaches 1, the probability of a collision also approaches 1

Goodrich, Tamassia, and Mount, ***Data Structurs and Algorithms in C++***

Big-O Video

https://www.youtube.com/watch?v=fHNmRkzxHWs