Lab 1: Calculations

EE316-08 Spring 2021

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$

- Calculate…
  - Branch Voltages
  - Branch Currents
  - Node Voltages
  - Loop Currents

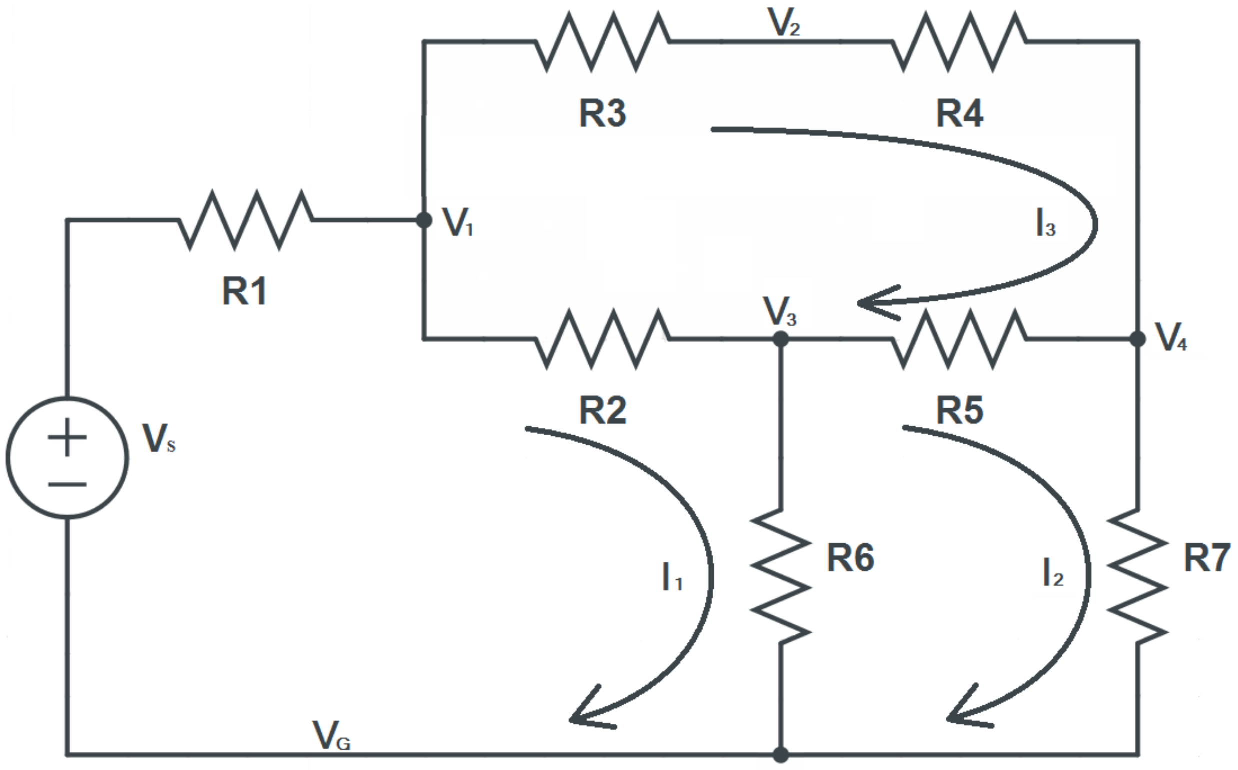
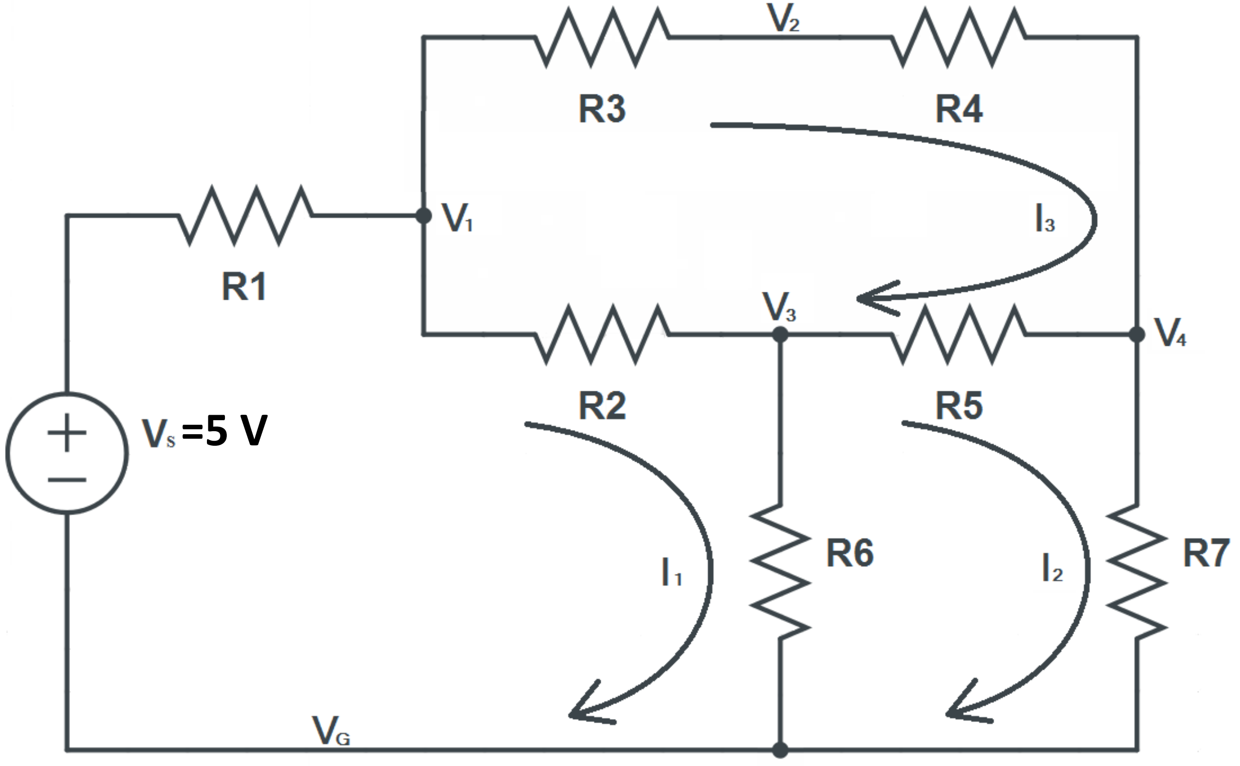


Figure 1.3: Solve for I_1, I_2, and I_3



Use Mesh Analysis

Loop I_1: $R_1I_1 + R_2(I_1 - I_3) + R_6(I_1 - I_2) - 5 = 0$

$R_1I_1 + R_2I_1 - R_2I_3 + R_6I_1 - R_6I_2 = 5$

$(R_1 + R_2 + R_6)I_1 - R_6I_2 - R_2I_3 = 5$ (1)

Loop I_2: $R_5(I_2 - I_3) + R_7I_2 + R_6(I_2 - I_1) = 0$

$R_5I_2 - R_5I_3 + R_7I_2 + R_6I_2 - R_6I_1 = 0$

$-R_6I_1 + (R_5 + R_6 + R_7)I_2 - R_5I_3 = 0$ (2)

Loop I_3: $R_3I_3 + R_4I_3 + R_5(I_3 - I_2) + R_2(I_3 - I_1) = 0$

$R_3I_3 + R_4I_3 + R_5I_3 - R_5I_2 + R_2I_3 - R_2I_1 = 0$

$-R_2I_1 - R_5I_2 + (R_2 + R_3 + R_4 + R_5)I_3 = 0$ (3)

Figure 1.3: Solve for I_1, I_2, and I_3 (cont.)

Recall,

$$(R_1 + R_2 + R_6)I_1 - R_6 I_2 - R_2 I_3 = 5 \qquad (1)$$

$$- R_6 I_1 + (R_5 + R_6 + R_7)I_2 - R_5 I_3 = 0 \qquad (2)$$

$$- R_2 I_1 - R_5 I_2 + (R_2 + R_3 + R_4 + R_5)I_3 = 0 \qquad (3)$$

Now, we have 3 equations and 3 unknown variables (I_1, I_2, and I_3). We can solve for I_1, I_2, and I_3 by using Matrices.

$$\underbrace{\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\mathbf{X}} = \underbrace{\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{B}}$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\boxed{\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \end{bmatrix}}$$

$$\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) \end{bmatrix} \boxed{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}} = \begin{bmatrix} \boxed{5} \\ 0 \\ 0 \end{bmatrix}$$

$$(R_1 + R_2 + R_6)I_1 - R_6 I_2 - R_2 I_3 = 5 \qquad (1)$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \\ \boxed{-R_6 \quad (R_5 + R_6 + R_7) \quad -R_5} \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) \end{bmatrix} \boxed{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}} = \begin{bmatrix} 5 \\ \boxed{0} \\ 0 \end{bmatrix}$$

$$-R_6 I_1 + (R_5 + R_6 + R_7) I_2 - R_5 I_3 = 0 \qquad (2)$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$-R_2\mathbf{I_1} - R_5\mathbf{I_2} + (R_2 + R_3 + R_4 + R_5)\mathbf{I_3} = 0 \qquad (3)$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\begin{bmatrix} (\mathrm{R_1} + \mathrm{R_2} + \mathrm{R_6}) & -\mathrm{R_6} & -\mathrm{R_2} \\ -\mathrm{R_6} & (\mathrm{R_5} + \mathrm{R_6} + \mathrm{R_7}) & -\mathrm{R_5} \\ -\mathrm{R_2} & -\mathrm{R_5} & (\mathrm{R_2} + \mathrm{R_3} + \mathrm{R_4} + \mathrm{R_5}) \end{bmatrix} . \begin{bmatrix} \mathrm{I_1} \\ \mathrm{I_2} \\ \mathrm{I_3} \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

*** Reduced row echelon form ***

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \mathrm{I_1} \\ \mathrm{I_2} \\ \mathrm{I_3} \end{bmatrix} = \begin{bmatrix} Answer\ for\ \mathrm{I_1} \\ Answer\ for\ \mathrm{I_2} \\ Answer\ for\ \mathrm{I_3} \end{bmatrix}$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\begin{bmatrix} (R_1 + R_2 + R_6) & -R_6 & -R_2 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

*** augmented matrix form ***

$$\left[\begin{array}{ccc|c} (R_1 + R_2 + R_6) & -R_6 & -R_2 & 5 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 & 0 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \textit{Answer for } I_1 \\ 0 & 1 & 0 & \textit{Answer for } I_2 \\ 0 & 0 & 1 & \textit{Answer for } I_3 \end{array}\right]$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\left[\begin{array}{ccc|c} (R_1 + R_2 + R_6) & -R_6 & -R_2 & 5 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 & 0 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) & 0 \end{array}\right] \sim$$

Plug in R_1- R_7 under the first assumption (the exact values)

$$\left[\begin{array}{ccc|c} (100 + 100 + 2{,}200) & -2{,}200 & -100 & 5 \\ -2{,}200 & (1{,}000 + 2{,}200 + 2{,}200) & -1{,}000 & 0 \\ -100 & -1{,}000 & (100 + 100 + 1{,}000 + 1{,}000) & 0 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 2{,}400 & -2{,}200 & -100 & 5 \\ -2{,}200 & 5{,}400 & -1{,}000 & 0 \\ -100 & -1{,}000 & 2{,}200 & 0 \end{array}\right] \underset{\sim}{\textbf{Divided by 100}} \left[\begin{array}{ccc|c} 24 & -22 & -1 & 0.05 \\ -22 & 54 & -10 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right] \sim$$

Figure 1.3: Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ (cont.)

$$\left[\begin{array}{ccc|c} 24 & -22 & -1 & 0.05 \\ -22 & 54 & -10 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right]\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 24 & -22 & -1 & 0.05 \\ 2.2 & -5.4 & 1 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right]\begin{array}{c} R_1 \\ \frac{-R_2}{10} \\ R_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 26.2 & -27.4 & 0 & 0.05 \\ 48.4 & -118.8 & 22 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right]\begin{array}{c} R_1 + R_2 \\ 22R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 26.2 & -27.4 & 0 & 0.05 \\ 49.4 & -108.8 & 0 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right]\begin{array}{c} R_1 \\ R_2 - R_3 \\ R_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & \frac{1}{524} \\ 49.4 & -108.8 & 0 & 0 \\ -1 & -10 & 22 & 0 \end{array}\right]\begin{array}{c} \frac{R_1}{26.2} \\ R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 49.4 & -108.8 & 0 & 0 \\ -49.4 & -494 & 1086.8 & 0 \end{array}\right]\begin{array}{c} R_1 \\ R_2 \\ 49.4R_3 \end{array} \sim$$

Figure 1.3: Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ (cont.)

$$\left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 49.4 & -108.8 & 0 & 0 \\ 0 & -602.8 & 1086.8 & 0 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ 49.4\mathrm{R}_3 + R_2 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 49.4 & -108.8 & 0 & 0 \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ \frac{\mathrm{R}_3}{1086.8} \end{array} \sim \left[\begin{array}{ccc|c} 49.4 & -\frac{(49.4)137}{131} & 0 & {}^{49.4}/_{524} \\ 49.4 & -108.8 & 0 & 0 \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} 49.4R_1 \\ R_2 \\ R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 0 & {}^{7485}/_{131} & 0 & {}^{49.4}/_{524} \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 \\ R_1 - R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 0 & 1 & 0 & 0.00165 \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 \\ \frac{R_2}{\left({}^{7485}/_{131}\right)} \\ R_3 \end{array} \sim$$

Figure 1.3: Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ (cont.)

$$\left[\begin{array}{ccc|c} 1 & -\frac{137}{131} & 0 & {}^{1}/_{524} \\ 0 & \frac{137}{131} & 0 & 0.00165\left(\frac{137}{131}\right) \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.003634 \\ 0 & 1 & 0 & 0.00165 \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 + R_2 \\ R_2 \\ R_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.003634 \\ 0 & \frac{137}{247} & 0 & 0.00165\left(\frac{137}{247}\right) \\ 0 & -\frac{137}{247} & 1 & 0 \end{array}\right] \begin{array}{c} R_1 \\ \left(\frac{137}{247}\right)R_2 \\ R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.003634 \\ 0 & 1 & 0 & 0.00165 \\ 0 & 0 & 1 & 0.0009152 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_2+R_3 \end{array}$$

Figure 1.3: Solve for $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ (cont.)

$$\left[\begin{array}{ccc|c} (R_1 + R_2 + R_6) & -R_6 & -R_2 & 5 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 & 0 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) & 0 \end{array}\right]$$

Assumption 1: All resistors are exact value.

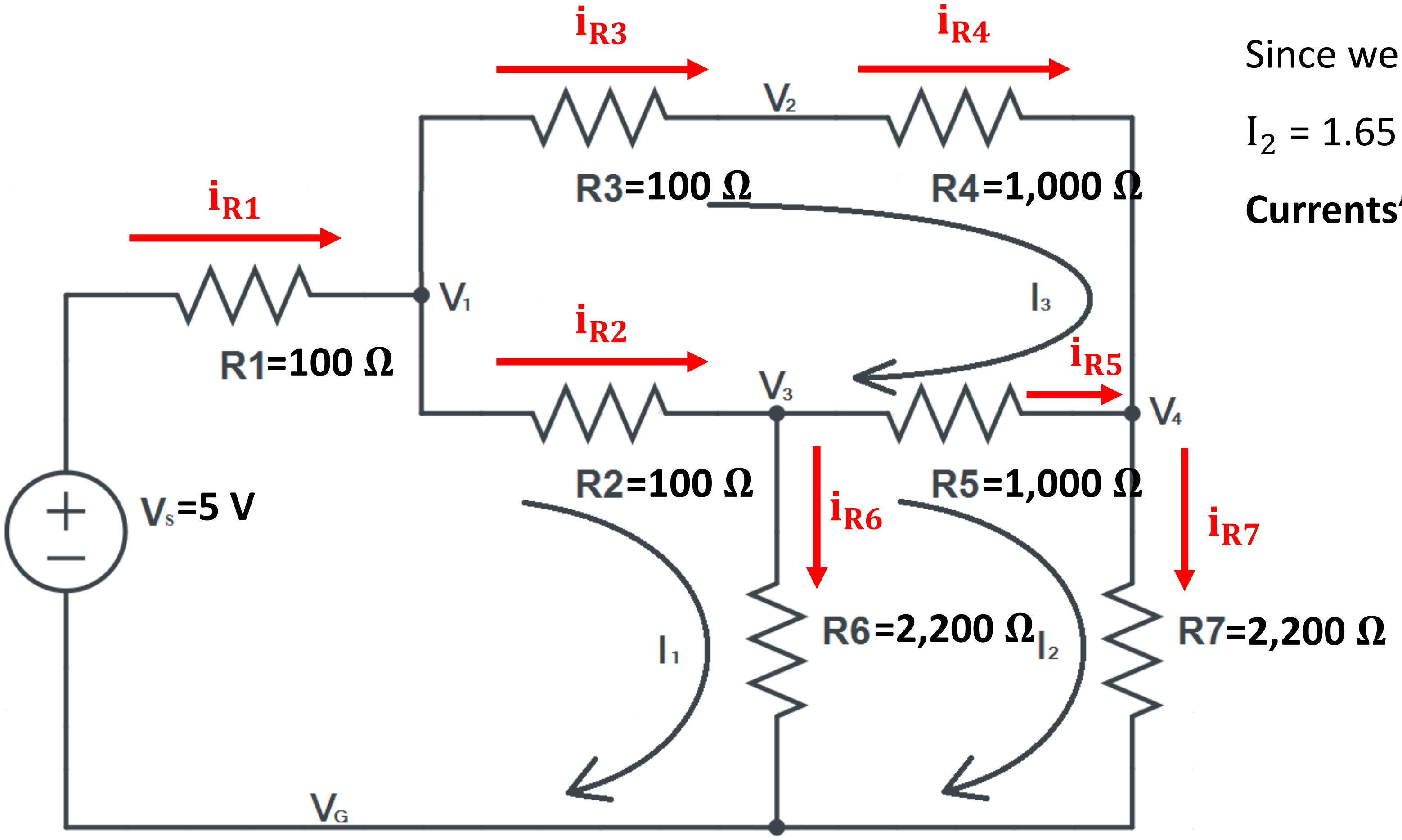
$$\left[\begin{array}{ccc|c} 2{,}400 & -2{,}200 & -100 & 5 \\ -2{,}200 & 5{,}400 & -1{,}000 & 0 \\ -100 & -1{,}000 & 2{,}200 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \textit{Answer for } I_1 \\ 0 & 1 & 0 & \textit{Answer for } I_2 \\ 0 & 0 & 1 & \textit{Answer for } I_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.003634 \\ 0 & 1 & 0 & 0.00165 \\ 0 & 0 & 1 & 0.0009152 \end{array}\right]$$

Therefore, I_1= 0.003634 A, I_2 = 0.00165 A, and I_3 = 0.0009152 A.

Figure 1.3: Assume all resistors are the exact value



Since we know that **'Loop Currents'** are $I_1 = 3.634$ mA, $I_2 = 1.65$ mA, and $I_3 = 0.9152$ mA, we can find **'Branch Currents'**.

$i_{R1} = I_1 = 3.634$ A

$i_{R2} = I_1 - I_3 = 3.634 - 0.9152 = 2.719$ mA

$i_{R3} = I_3 = 0.9152$ mA

$i_{R4} = I_3 = 0.9152$ mA

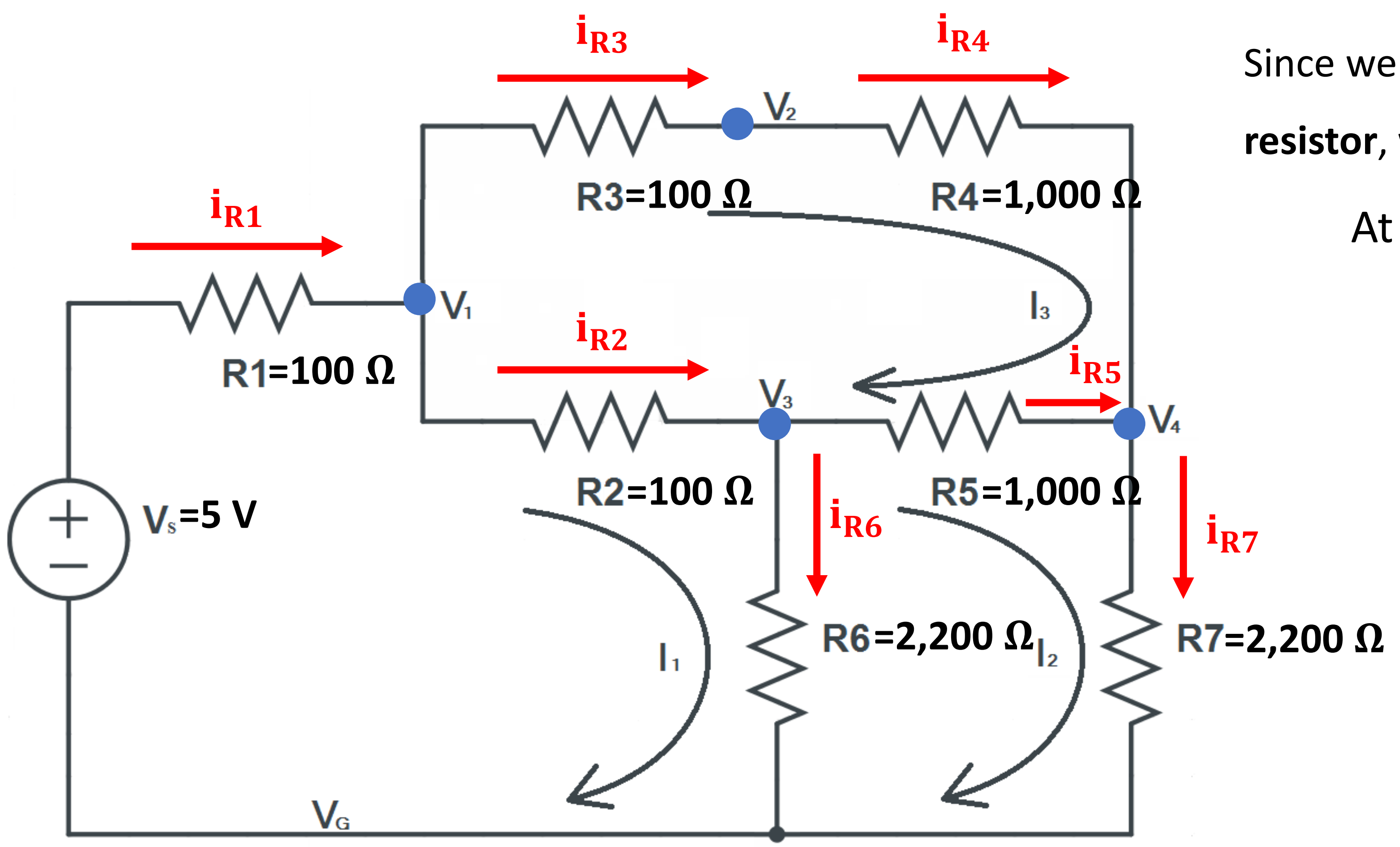
$i_{R5} = I_2 - I_3 = 1.65 - 0.9152 = 0.735$ mA

$i_{R6} = I_1 - I_2 = 3.634 - 1.65 = 1.984$ mA

$i_{R7} = I_2 = 1.65$ mA

Note: I use **RED ARROW** as a reference direction of the current.

Figure 1.3: Assume all resistors are the exact value



Note: I use **RED ARROW** as a reference direction of the current.

Since we know **'Branch Currents'** and **all values of resistor**, we can find **'Node Voltage'.**

At R_1: $5 - V_1 = (\mathbf{i_{R1}})(R1)$

$V_1 = 5 - (\mathbf{i_{R1}})(R1)$

$V_1 = 5 - (0.003634)(100)$

$\mathbf{V_1 = 4.64\ V}$

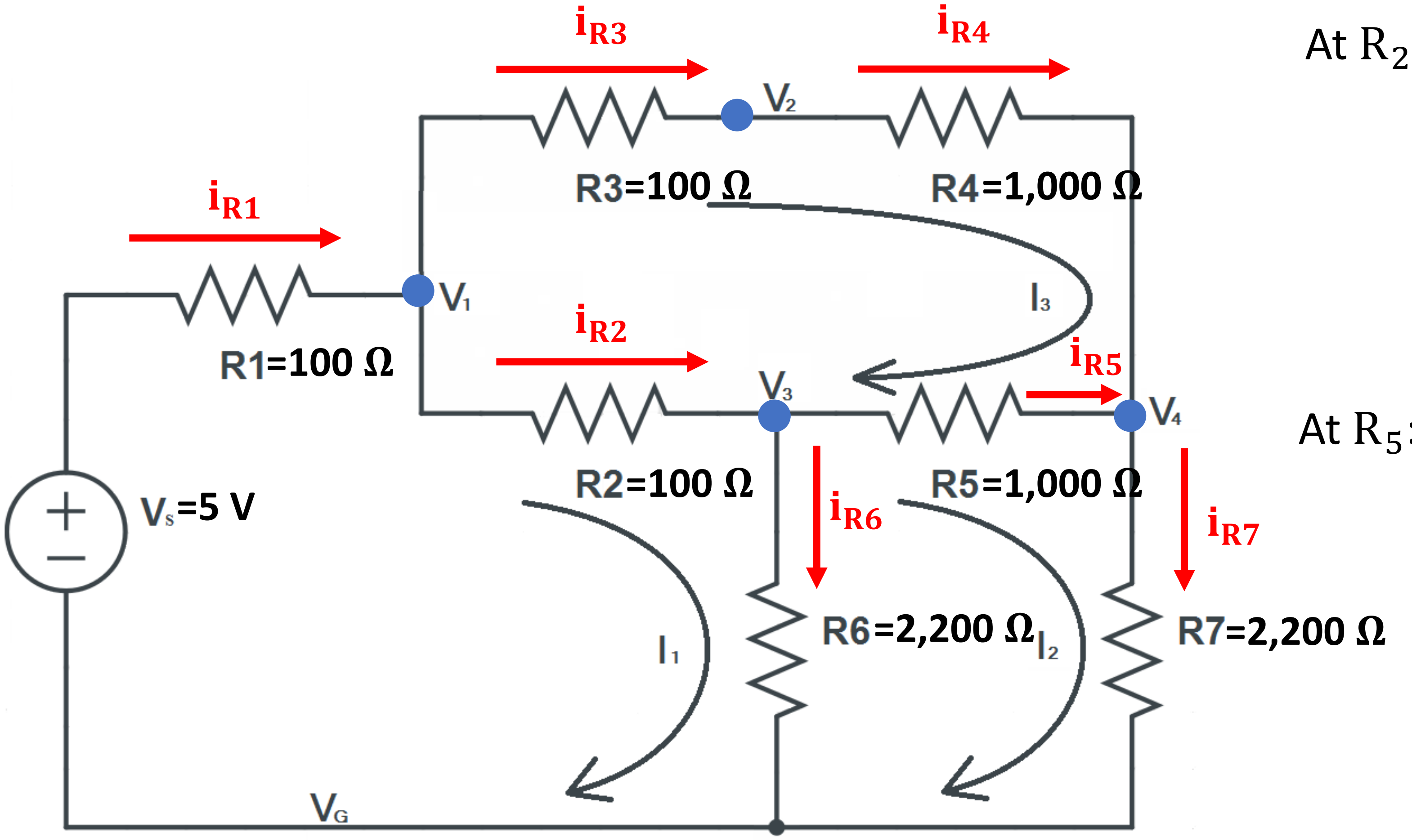
At R_3: $V_1 - V_2 = (\mathbf{i_{R3}})(R3)$

$V_2 = V_1 - (\mathbf{i_{R3}})(R3)$

$V_2 = 4.64 - (0.0009152)(100)$

$\mathbf{V_2 = 4.55\ V}$

Figure 1.3: Assume all resistors are the exact value



At R_2: $V_1 - V_3 = (i_{R2})(R2)$

$V_3 = V_1 - (i_{R2})(R2)$

$V_3 = 4.64 - (0.002719)(100)$

$\mathbf{V_3 = 4.37\ V}$

At R_5: $V_3 - V_4 = (i_{R5})(R5)$

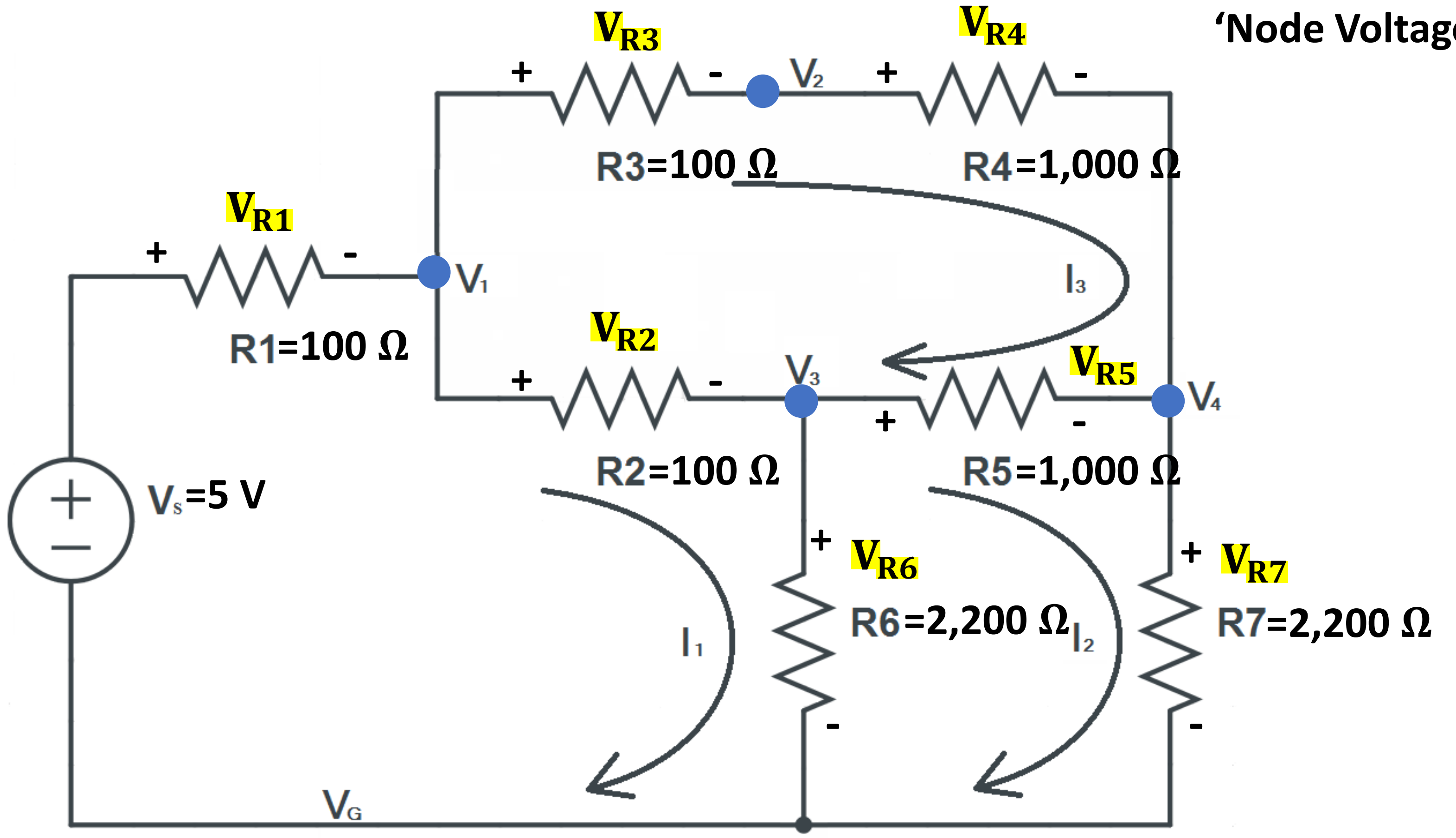
$V_4 = V_3 - (i_{R5})(R5)$

$V_4 = 4.37 - (0.000735)(1{,}000)$

$\mathbf{V_4 = 3.64\ V}$

Note: I use **RED ARROW** as a reference direction of the current.

Figure 1.3: Assume all resistors are the exact value



Since we know **'Branch Currents', all values of resistor**, and **'Node Voltage',** we can find **'Branch Voltage'**.

At R_1: V_{R1} = ($\mathbf{i_{R1}}$)(R1) **or** $V_{R1} = 5 - V_1$

V_{R1} = 0.36 V

At R_2: V_{R2} = ($\mathbf{i_{R2}}$)(R2) **or** $V_{R2} = V_1 - V_3$

V_{R2} = 0.27 V

At R_3: V_{R3} = ($\mathbf{i_{R3}}$)(R3) **or** $V_{R3} = V_1 - V_2$

V_{R3} = 0.09 V

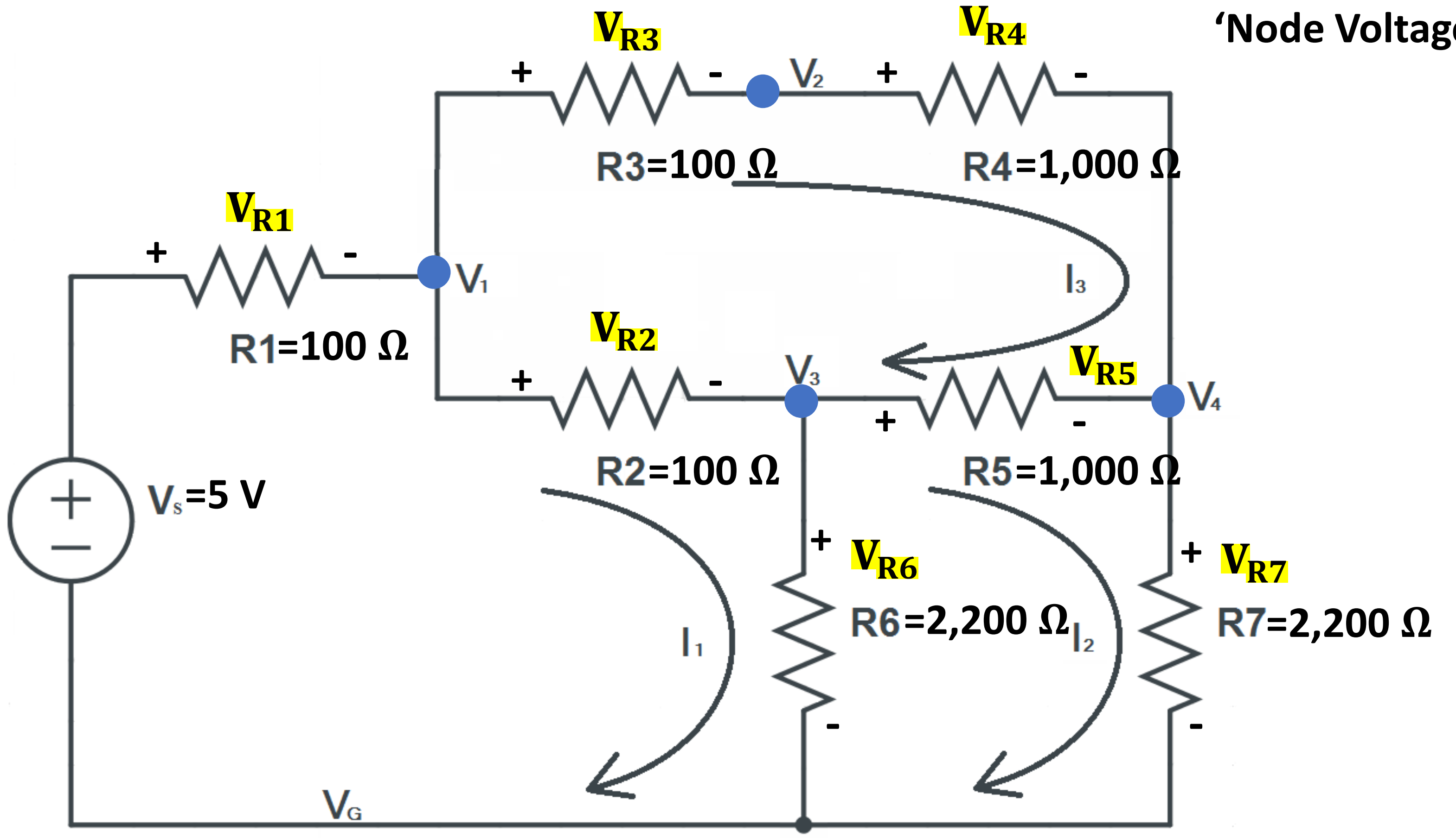
At R_4: V_{R4} = ($\mathbf{i_{R4}}$)(R4) **or** $V_{R4} = V_2 - V_4$

V_{R4} = 0.91 V

At R_5: V_{R5} = ($\mathbf{i_{R5}}$)(R5) **or** $V_{R5} = V_3 - V_4$

V_{R3} = 0.73 V

Figure 1.3: Assume all resistors are the exact value



Since we know **'Branch Currents', all values of resistor**, and **'Node Voltage',** we can find **'Branch Voltage'**.

At R_6: V_{R6} = ($\mathbf{i_{R6}}$)(R6) **or** $V_{R6} = V_3 - V_G$

$V_{R6} = V_3$

$\mathbf{V_{R1}}$ **= 4.37 V**

At R_7: V_{R7} = ($\mathbf{i_{R7}}$)(R7) **or** $V_{R7} = V_4 - V_G$

$V_{R7} = V_4$

$\mathbf{V_{R2}}$ **= 3.64 V**

Next, let the fun part begin…

Re-calculate I_1, I_2, and I_3 again under two assumptions:

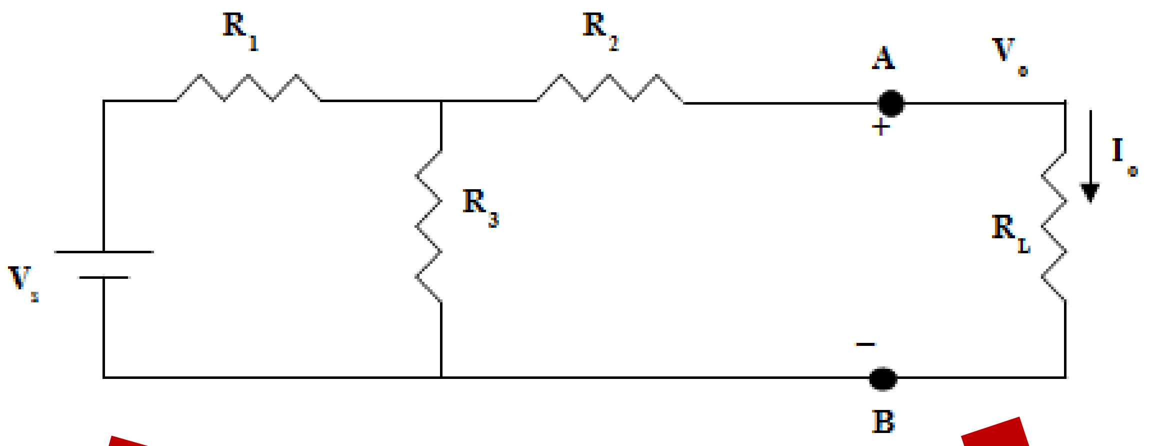
1. All resistors are <u>10% above</u> the exact value.

2. All resistors are <u>10% below</u> the exact value.

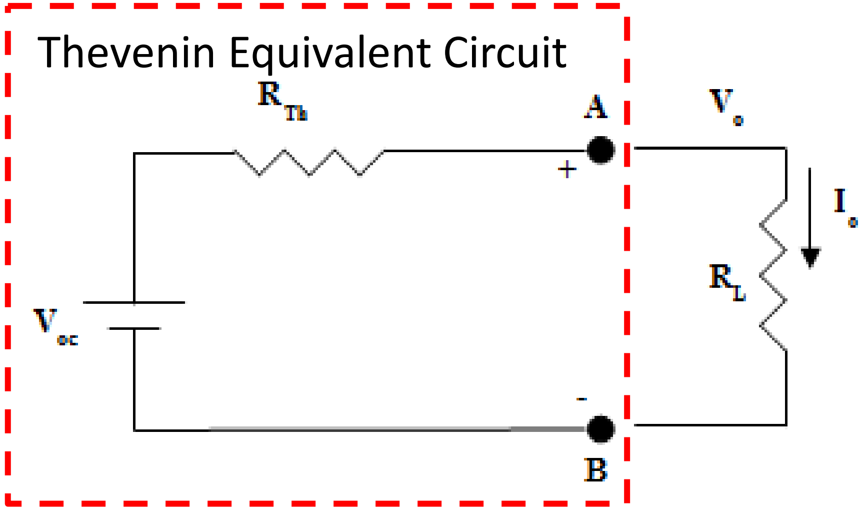
Recall,

$$\left[\begin{array}{ccc|c} (R_1 + R_2 + R_6) & -R_6 & -R_2 & 5 \\ -R_6 & (R_5 + R_6 + R_7) & -R_5 & 0 \\ -R_2 & -R_5 & (R_2 + R_3 + R_4 + R_5) & 0 \end{array}\right]$$

Figure 1.5: Norton and Thevenin Equivalent Circuits



Calculate V_{oc} , I_{sc} , and R_{Th}



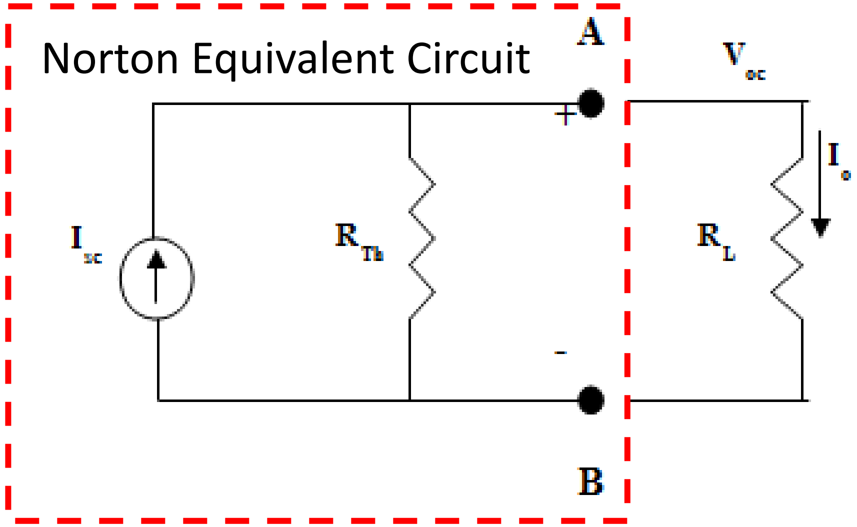
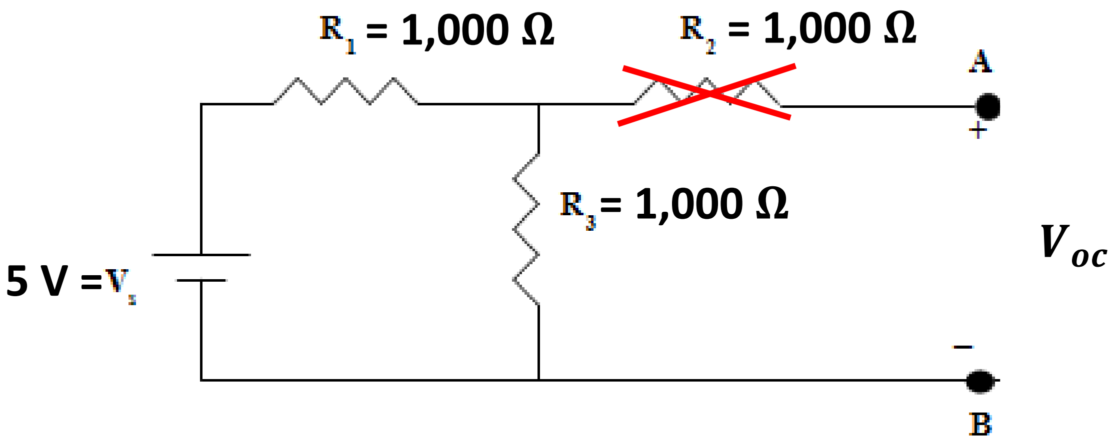


Figure 1.5: Norton and Thevenin Equivalent Circuits

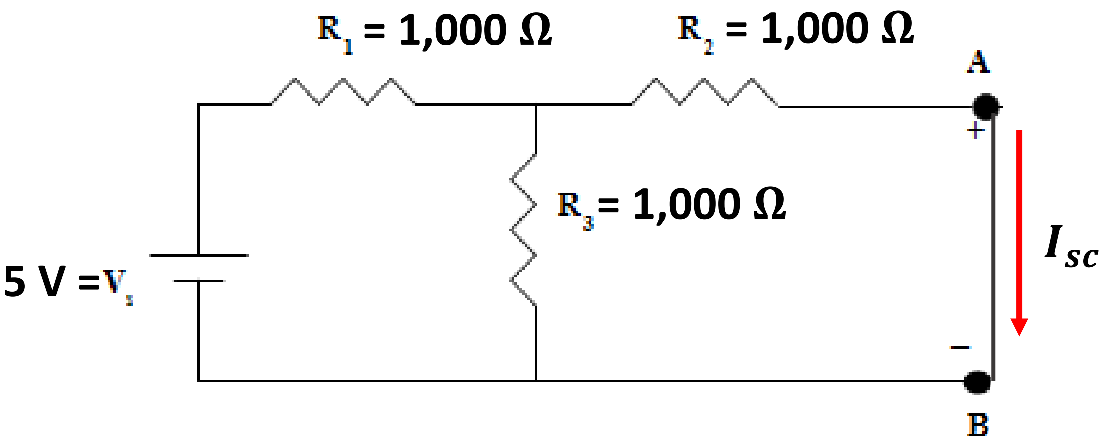


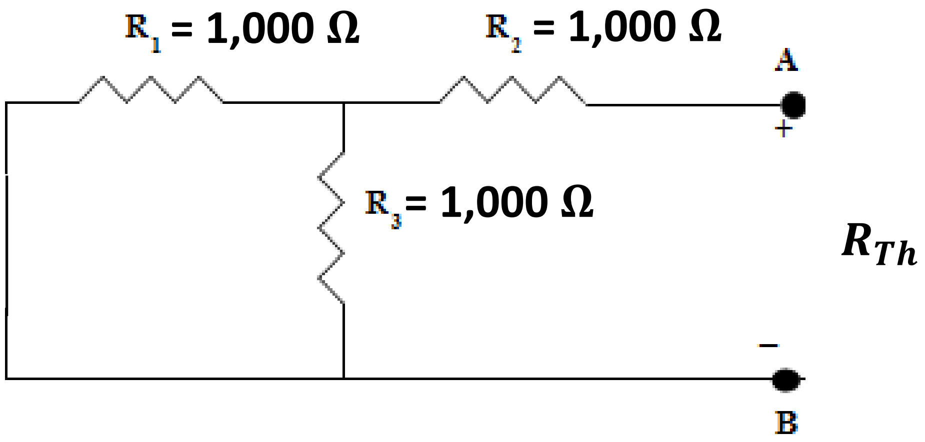
$\mathbf{V_{oc}}$ is a voltage drop across $\mathbf{R_3}$.

$$\mathbf{V_{oc}} = \frac{\mathbf{V_s R_3}}{\mathbf{R_1 + R_3}}$$

$$\mathbf{V_{oc}} = \mathbf{I_{sc} R_{Th}}$$

$$\mathbf{R_{Th}} = \mathbf{R_2} + \frac{\mathbf{R_1 R_3}}{\mathbf{R_1 + R_3}}$$





Note: The maximum power transfer to the load is when $\mathbf{R_L} = \mathbf{R_{Th}}$. Therefore, $P_{max} = \frac{V_{oc}^2}{4\mathbf{R_{Th}}}$ (Watt)