EXAM 3, PH111 4/5/~~18~~ 19 **STRONG/CHRONIS**

NAME KEY '19

Please PRINT your name as it appears in my class roster.

Exam Instructions:

1. Circle one answer for each problem.
2. Enter your answer using your clicker.
3. Since the problems have imbedded partial credit, you should not leave any problems blank either on your paper or on your clicker!
4. Turn in exam and all scratch paper used during exam.

1. Masses of 3 particles are given in kg and the *xy* coordinates in meters: $m_1 = 2$ @ (0,20); $m_2 = 5$ @ (−20, −12); and $m_3 = 3$ @ (10, 15). Find the *xy* coordinates of the center of mass.

- ***A.*** (0, 20)
- m_1, m_3 2 ***B.*** (6, 17)
- (−) 4 ***C.*** (13, 2.5)
- (−) 4 ***D.*** (−7, 14.5)
- ***E.*** (−7, 2.5)

$$x_{cm} = \frac{2(0) + 5(-20) + 3(10)}{2+5+3} = -7$$

$$y_{cm} = \frac{2(20) + 5(-12) + 3(15)}{2+5+3} = 2.5$$

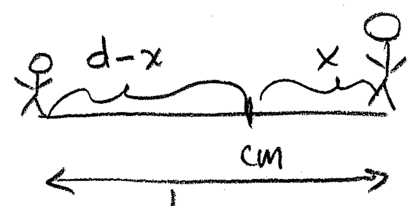
2. Two boys with masses of 45 kg and 75 kg stand on a horizontal frictionless surface holding the ends of a light 18-m long rod. The boys pull themselves together along the rod. How far will the 75-kg boy have moved when the two boys meet?

- M=m 2 ***A.*** 5.0 m
- ***B.*** 6.8 m
- ***C.*** 9.0 m
- for 45 3 ***D.*** 11 m
- ***E.*** 18 m

$$Mx = m(d-x)$$

$$\Rightarrow (M+m)x = md$$

$$x = \frac{md}{(M+m)} = \frac{45(18\text{ m})}{45+75} = 6.75\text{ m}$$

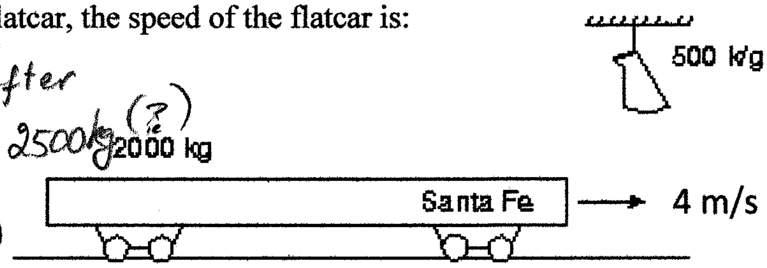


3. A 1.0-kg ball moving at 2.0 m/s perpendicular toward a wall rebounds back from the wall at 1.5 m/s, still perpendicular to the wall. The change in linear momentum is, including direction:

- ***A.*** zero
- 2 ***B.*** 0.5 N·s, directed away from the wall
- 3 ***C.*** 0.5 N·s, directed toward the wall
- ***D.*** 3.5 N·s, directed away from the wall
- 4 sign ***E.*** 3.5 N·s, directed toward the wall

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
$$= 1\text{ kg}\left(1.5\tfrac{m}{s}\text{ away} - 2.0\tfrac{m}{s}\text{ toward}\right)$$
$$= 1\text{ kg}\left(1.5\tfrac{m}{s}\text{ away} + 2.0\tfrac{m}{s}\text{ away}\right)$$
$$= 3.5\text{ kg}\cdot\text{m/s away}$$

4. A 500kg sack of coal is dropped on a 2000kg railroad flatcar which was initially moving at 4m/s as shown. After the sack rests on the flatcar, the speed of the flatcar is:



A. 2.4 m/s
B. 3.2 m/s
K.E. 3 **C.** 3.6 m/s
D. 4.0 m/s
upside down 2 **E.** 5 m/s

$\vec{p}_{before} = \vec{p}_{after}$

$2000kg(4m/s) = 2500kg(?)$

$? = v_f = \frac{2000}{2500}(4)$

$= 3.2 m/s$

5. A sled of mass m is coasting at a constant velocity, v_i, on the ice covered surface of a lake. Three birds, with a combined mass of 1.5m, gently land at the same time on the sled with no horizontal velocity. The sled and the birds continue sliding along together in the original direction of motion. In terms of m and v_i, what is the final kinetic energy of the system?

A. $\frac{5}{2}mv_i^2$
Kbefore 2 **B.** $\frac{1}{2}mv_i^2$
$(\frac{1}{2})$ 3 **C.** $\frac{2}{5}mv_i^2$
D. $\frac{1}{5}mv_i^2$
Δp? 3 **E.** $\frac{5}{4}mv_i^2$ only

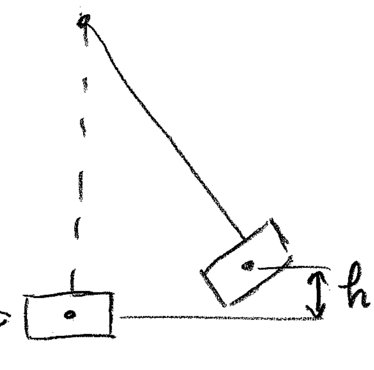
$\Delta\vec{p} = 0 \Rightarrow m_i v_i = M_f v_f$

$m v_i = (1.5+1) m v_f \Rightarrow v_f = \frac{v_i}{2.5}$

$K_f = \frac{1}{2} M_f v_f^2 = \frac{1}{2}(2.5 m_i)\left(\frac{v_i}{2.5}\right)^2$

$= \frac{1}{2}\left(\frac{1}{2.5}\right) v_i^2 \Rightarrow \frac{1}{5} m_i v_i^2$

6. A bullet of mass m is fired horizontally into a block of mass M suspended by a rope from the ceiling. The combined mass $(m + M)$ rises to a height h above the lowest position of the swing. What is the original velocity v of the bullet before the collision?



A. $2gh$
V only 2 **B.** $\sqrt{2gh}$
4K 3 **C.** $gh\sqrt{m+M}$
D. $\left(\frac{m+M}{m}\right)\sqrt{2gh}$
4 **E.** $\left(\frac{m}{m+M}\right)\sqrt{2gh}$

Work backwards:

$\frac{1}{2}(\cancel{m+M})V^2 = (\cancel{m+M})gh$

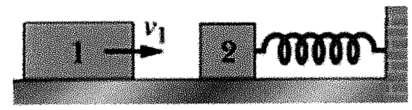
$V = \sqrt{2gh}$ velocity together

$\Delta\vec{p} = 0$:

$mv = (M+m)V$

$v = \left(\frac{M+m}{m}\right)V$

7. Block 2 is at rest on a frictionless, flat table and is just barely touching an un-stretched spring which is secured to a wall as shown. Block 1 is moving and hits Block 2. The two blocks stick together and move as one at a speed of 3 m/s. If the spring constant k is 200 N/m and the total mass of the blocks is 2.5 kg, what is the compression of the spring in cm?



$\sqrt{}$ 2 **A.** 3.4 cm
$\frac{1}{2}$ 3 **B.** 16 cm
$\sqrt{3}$ 3 **C.** 18 cm
D. 32 cm
E. 64 cm

$\cancel{\frac{1}{2}} kx^2 = \cancel{\frac{1}{2}}(m_1+m_2)V^2$

$x = \sqrt{\frac{(m_1+m_2)}{k}}\,V$

$= \sqrt{\frac{2.5kg}{220 N/m}}(3 m/s) = 0.32 m$

8. During a bat'leth "training exercise", a Klingon of $M = 90$ kg is kicked by an officer into the door of a shuttle bay. The door malfunctions and the Klingon is flung into space at $v_i = 3.5$ m/s. After travelling 20 m from the ship, the Klingon realizes he can throw his bat'leth away from the ship thereby propelling himself backwards to the ship, but he has only 60 seconds left to reach his ship. At what speed must he throw his bat'leth of $m = 10$kg away from the ship so that he may reach his ship in time?

(-) 4 *A.* 34 m/s
B. 38 m/s
C. 52 m/s
K 3 *D.* 62 m/s
E. 97 m/s

$$(M+m)v_i = m v_f - M V_f$$

$$v_f = \frac{(M+m)v_i + M V_f}{m} = \frac{(90+10)(3.5\,m/s) + 90\left(\frac{20\,m}{60\,s}\right)}{10\,kg}$$

$$= 38\ m/s$$

9. A proton of atomic mass 1u with a speed of 750 m/s collides elastically with another proton at rest. The original proton is scattered $+60^\circ$ from its initial direction with a final speed of 375 m/s. The protons scatter from the collision at a total of 90° to each other. Therefore, the scattered angle for the second proton is -30°. What is the final speed of the target proton after the collision? *(Hint: Since the masses of the protons are equal, they cancel. You know all angles and two of the three velocities for conservation of momentum in the $\hat{x}$ direction, therefore, only one equation is needed!)*

A. 750 m/s
B. 700 m/s
C. 650 m/s
$(\cos 30^\circ)$ 4 *D.* 560 m/s
3 *E.* 190 m/s
only + $v_{1f}\cos 60^\circ$

$$\hat{x} \Rightarrow \cancel{m} v_{1i} = \cancel{m} v_{1f}\cos 60^\circ + \cancel{m}\, v_{2f}^{?} \cos 30^\circ$$

$$v_{2f} = \frac{v_{1i} - v_{1f}\cos 60^\circ}{\cos 30^\circ}$$

$$= \frac{750 - 375\cos 60^\circ}{\cos 30^\circ} = 649.5\ m/s$$

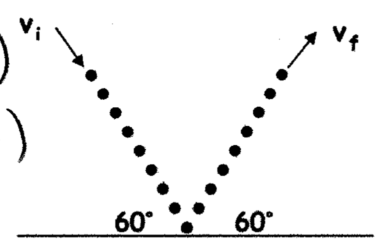
10. A stream of gas consists of n molecules. Each molecule has a mass m and a speed v. The stream is reflected **elastically** from a rigid surface as shown. The magnitude of the change in the total momentum of the stream is:

3 *A.* $2mnv$
2 *B.* mnv
2 *C.* $mnv\cos 60°$
3 *D.* $mnv\sin 60°$
E. $2mnv\sin 60°$

$$P_f = mnv(\cos 60^\circ \hat{x} + \sin 60^\circ \hat{y})$$

$$P_i = mnv(\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y})$$

$$P_f - P_i = ?$$



11. A flywheel rotating at 18.0 rev/s is brought to rest in 3.00 s. The magnitude of the average angular acceleration in rad/s² of the wheel during this process is:

A. 37.7 rad/s²
B. 0 rad/s²
*3s 3 *C.* 339 rad/s²
(2π) 3 *D.* 6.00 rad/s²
2 *E.* 12.6 rad/s²
$\frac{18(2\pi)}{(3s)^2}$

$$\cancel{\omega_f}^{\,0} = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega_0}{t} = \frac{\frac{18\,rev}{s}\left(\frac{2\pi\,rad}{1\,rev}\right)}{3\,s} = 37.7\ \frac{rad}{s^2}$$

12. A wheel starts from rest and has an angular acceleration of 5.0 rad/s^2. The time it takes to make 23 revolutions is:

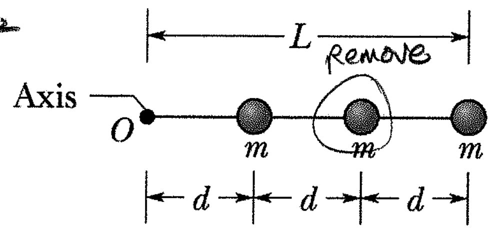
$$\Delta\theta = \cancel{\omega_i} t + \tfrac{1}{2}\alpha t^2$$

- **A.** 7.6 s
- $\sqrt{2}$ 4 **B.** 5.4 s
- 2π 3 **C.** 3.0 s
- **D.** 23 s
- $\sqrt{\ }$ 4 **E.** 58 s

$$t = \sqrt{\frac{2\Delta\theta}{\alpha}} = \sqrt{\frac{2\left(23\text{ rev} \times \frac{2\pi}{\text{rev}}\right)}{5.0\text{ rad/s}^2}} = \sqrt{58}$$

$$= 7.6\text{ s}$$

13. The figure shows three particles of mass m fastened to a rod of length $L = 3d$. The rod has negligible mass. The assembly can rotate around a perpendicular axis through point O at the left end as shown. If we remove the middle particle from the system, by what percentage does the rotational inertia of the assembly around the rotation axis decrease?



Before $I_i = md^2 + m(2d)^2 + m(3d)^2$

After $I_f = md^2 + m(3d)^2$

- **A.** 0%
- 1-()4 **B.** 7.1%
- **C.** 29%
- **D.** 36%
- 2 **E.** 64%

last particle

$$\frac{I_f}{I_i} = \frac{1+9}{1+4+9} = \frac{10}{14} = 0.714$$

$$\therefore\ 1 - I_f/I_i = 1 - 0.714 = 28.6\%$$

14. A pulley with a radius of 3.0 cm has a rotational inertia of 4.0×10^{-3} $\text{kg}\cdot\text{m}^2$ and is suspended from a ceiling. A rope is wrapped around the pulley and connected to a 2.0 kg hanging block. Assume the pulley has no friction and the rope does not slip (only unwinds as the block falls). When the block falls and reaches a speed of 2.0 m/s, what is the total kinetic energy of the pulley and block system? *(Note: Trick!* $r_{pulley}\omega = v_{block}$ *)*

$\frac{1}{2}mv^2$

$(\ )^2$

$*2$

pulley

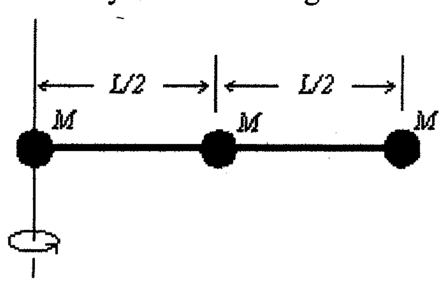
- 2 **A.** 4.0 J
- 4 **B.** 4.3 J
- 4 **C.** 6.5 J
- 3 **D.** 8.9 J
- **E.** 13 J

$$\tfrac{1}{2}I\left(\frac{v^2}{r^2}\right) + \tfrac{1}{2}mv^2 = \tfrac{1}{2}v^2\left[I/r^2 + m\right]$$

$$\Rightarrow \tfrac{1}{2}(2\,\text{m/s})^2\left[\frac{4\times10^{-3}}{0.03^2} + 2.0\right] = 12.9\text{ J}$$

15. Three identical particles, each of mass M, are fastened to a rod of total length L and total mass m as shown. What is the rotational kinetic energy about the left end if the system is rotating at ω?

- 2 **A.** $\frac{1}{2}(\frac{1}{3}mL^2)\omega^2$
- **B.** $\frac{1}{2}(ML^2)\omega^2$
- 3 **C.** $\frac{1}{2}(\frac{1}{4}ML^2+\frac{1}{3}mL^2)\omega^2$
- 3 **D.** $\frac{1}{2}(ML^2+\frac{1}{3}mL^2)\omega^2$
- **E.** $\frac{1}{2}(\frac{5}{4}ML^2+\frac{1}{3}mL^2)\omega^2$



Rod about end

$\frac{1}{3}mL^2$

M @ L/2:

$M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{4}$

M @ L $\Rightarrow ML^2$

$K = \frac{1}{2}(I_{total})\,\omega^2$

16. The mass and radius of a **solid sphere** rotating about its central axis are respectively 11.0 kg and 0.40 m. When a 2.00 kg point mass is added to its surface, i.e. 0.40 m from the axis, the rotational inertia becomes:

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(11.0kg)(0.4m)^2 = 0.704\ kg\cdot m^2$$

m @ surface $\Rightarrow$ $I_m = mR^2$

$$= (2.0kg)(0.4m)^2$$

$$= 0.32\ kg\cdot m^2$$

$$0.32 + 0.704 = 1.024 \approx \boxed{1.02\ kg\cdot m^2}$$

only mass 3 *A.* 0.32 kg m^2
sphere only 3*B.* 0.70 kg m^2
C. 0.54 kg m^2
(*D.*) 1.02 kg m^2
4 *E.* 1.50 kg m^2
spherical shell

17. A **spherical shell** of radius r = 0.3 m and v_{cm} = 8.5 m/s rolls without sliding along level ground. The shell continues rolling up an incline. How high does the center of mass rise before the shell stops?

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\left(\frac{v_{cm}}{r}\right)^2 = mgh$$

$$\frac{1}{2}\cancel{m}v_{cm}^2 + \frac{1}{2}\left(\frac{2}{3}\cancel{m}r^2\right)\left(\frac{v_{cm}}{r}\right)^2 = \cancel{m}gh$$

$$\left(\frac{1}{2}+\frac{1}{3}\right)v_{cm}^2 = gh$$

$$h = \frac{\left(\frac{1}{2}+\frac{1}{3}\right)v_{cm}^2}{g} = \frac{\left(\frac{1}{2}+\frac{1}{3}\right)(8.5)^2}{9.8\,m/s^2} = 6.14\,m$$

only $\frac{1}{2}I\omega^2$ 3 *A.* 2.5 m
sphere 4 *B.* 5.2 m
(*C.*) 6.1 m
only $\frac{1}{2}mv^2$ 2 *D.* 3.7 m
$()^2$ 4 *E.* 0.72 cm

18. A 0.5 kg particle is moving in a straight line with position vector $\vec{r} = (-2.0t^2 - t)\hat{\imath} + 5\hat{\jmath}$, where $\vec{r}$ is in meters, t is in seconds, and $\vec{r}$ points from the origin to the particle. Find the angular momentum, $\vec{l}$, of the particle about the origin at t = 2 s.

$\vec{l} = \vec{r} \times m\vec{v}$ $\quad \vec{v} = \frac{d\vec{r}}{dt}$ $\quad \vec{v} = -(4t+1)\hat{\imath}$

$$\vec{l} = m\begin{vmatrix} i & j & k \\ -2t^2-t & 5 & 0 \\ -4t-1 & 0 & 0 \end{vmatrix}$$

$$= (0.5kg)\{\hat{k}((-1)(\underbrace{-4t-1}_{m/s})(\underbrace{5}_{m}))\}$$

$$= 0.5\,kg\,(20t+5)\,m^2/s \quad @\ t=2s \Rightarrow 22.5\hat{k}\ \frac{kg\cdot m^2}{s}$$

A. $-10\hat{\imath} + 5\hat{\jmath}$ m
$\vec{v}$ 2 *B.* $-3.5\hat{\jmath}$ m/s
mass 3 *C.* $11.3\hat{k}$ kg·m^2/s
(*D.*) $22.5\hat{k}$ kg·m^2/s
sign 4 *E.* $-22.5\hat{k}$ kg·m^2/s

19. An ice skater with rotational inertia I_0 is spinning with angular speed ω_0. She pulls her arms in until her rotational inertia is one third of the initial inertia. Her angular speed becomes:

$$I_o\omega_o = I_f\omega_f$$

$$\omega_f = \frac{I_o}{I_f}\omega_o = \frac{I_o}{\frac{1}{3}I_o}\omega_o = 3\omega_o$$

AK 3 *A.* $\sqrt{3}\,\omega_0$
B. $\omega_0/\sqrt{3}$
(*C.*) $3\omega_0$
upside down 3 *D.* $\omega_0/3$
E. ω_0

20. A playground merry-go-round has a radius of 2.7 m and a rotational inertia of 600 kg m^2. When the merry-go-round is at rest, a 200kg child runs at 2.0 m/s along a line tangent to the rim and jumps on. The angular velocity of the merry-go-round and the child is then:

Item *A.* 5.4 rad/s
3*B.* 1.8 rad/s
( ) 4*C.* 0.95 rad/s
Item 3 *D.* 0.74 rad/s
E. 0.53 rad/s

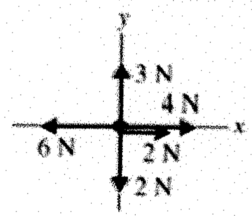
$l_i = rmv = 2.7m(200kg)(2 m/s)\sin 90° = 1080 \frac{kg\cdot m^2}{s}$

$L_f = [600\, kg\cdot m^2 + (200kg)(2.7m)^2]\,\omega_f$

Solve for ω_f:

$$\omega_f = \frac{(2.7)(200)(2)\sin 90}{(600 + (200)(2.7)^2)} = 0.525\ rad/s$$

21. (2 points Bonus) This diagram shows the forces acting on a box of chocolates as it slides across a frictionless counter from an overhead view. Is linear momentum of the box conserved in both the *x* and *y* directions?



***A.* I don't care to answer this question**
B. Yes in both directions.
C. Only in the ***x*** direction
D. Only in the ***y*** direction

22. (2 points Bonus) The table shows data for 3 shapes all with uniform mass distribution, all with the same radii, all spinning about the center of mass. Which of the 3 has the largest rotational inertia, *I*?

***A.* I don't care to answer this question**
B. Hoop
C. Solid Sphere
D. Disk

Hoop	Solid Sphere	Disk
M	M	3M
MR^2	$\frac{2}{5}MR^2$	$3(\frac{1}{2}MR^2)$

23. (2 points Bonus) A rhinoceros beetle rides the rim of a small disk that rotates like a merry-go-round. If the beetle crawls toward the center of the disk does the angular speed increase, decrease or remain the same?

***A.* I don't care to answer this question**
B. increase
C. decrease
D. remains the same

$I_i \omega_i = I_f \omega_f$

$\frac{I_i \omega_i}{I_f} = \omega_f$

$I_f < I_i$

$\therefore \omega_f > \omega_i$