2. (20 points) An ideal low pass filter *H(s)* with zero phase and magnitude response:

$$|H(j\Omega)| = \begin{cases} 1 & -\pi \le \Omega \le \pi \\ 0 & otherwise \end{cases}$$

a) Find the impulse response *h(t)* of the low-pass filter. Plot it and indicate whether this filter is causal system or not.

b) What is the effect of shifting the central frequency of the ideal filter for 5π ?

a.) Pulse in freq. domain = sinc func. in time domain

inverse Fourier Transform:

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\Omega)e^{j\Omega t}\, d\Omega$$

$$h(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} 1 \cdot e^{j\Omega t}\, d\Omega = \frac{1}{2\pi}\left[\frac{e^{j\Omega t}}{jt}\right]_{-\pi}^{\pi}$$

$$h(t) = \frac{1}{2\pi}\left[\frac{e^{\pi j t}}{jt} - \frac{e^{-\pi j t}}{jt}\right]$$

$$= \frac{e^{\pi j t} - e^{-\pi j t}}{2\pi j t} \rightarrow$$

related Eq.

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

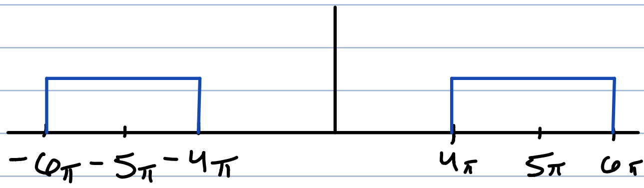
$$h(t) = \frac{\sin(\pi t)}{\pi t}$$

[sinc. function]

Frequency Domain Plot:

$\boxed{\text{Not causal because it is defined as something (other than 0) when } t < 0.}$

b.) Graph when shifted by 5π



→ Band pass now

→ modulated by shifting over by 5π

↳ from class + Office hours, to modulate you can multiply by $\cos(xt)$

⇒ $h(t)$ when shifted by 5π you multiply by $\cos(5\pi t)$

$$\boxed{h(t) = \frac{\sin(\pi t)}{\pi t} \cdot \cos(5\pi t)}$$

<u>Effect</u>: If you look at the magnitude plot when shift by $5\pi t$, it now represents a band pass filter.

3. (20 points)

A 12-bit AD converter is used to digitize signal with negative reference V_{R-} = 0.5V and positive reference V_{R+} = 2.5V.

a) (3 points) What is the quantization step?
b) (3 points) What is the output of the AD converter for V_{in} = 2.3 V ?
c) (2 points) What is the output of the AD converter for V_{in} = 0.4 V ?
d) (2 points) What is the output of the AD converter for V_{in} = 2.9 V ?

a.) $\Delta = \frac{\text{range}}{\text{\# of steps}} = \frac{V_{r+} - V_{r-}}{2^n - 1}$

$\Delta = \frac{2.5 - 0.5}{2^{12} - 1} = \boxed{0.0004884 = 4.884 \times 10^{-4}}$

b.) output if $V_{in} = 2.3V$

$N_{adc} = \frac{V_{in} - V_{r-}}{\Delta} = \frac{2.3 - 0.5}{4.884 \times 10^{-4}} = \boxed{3686}$

c.) output if $V_{in} = 0.4V$

↳ lower than negative reference so it reports lowest possible value

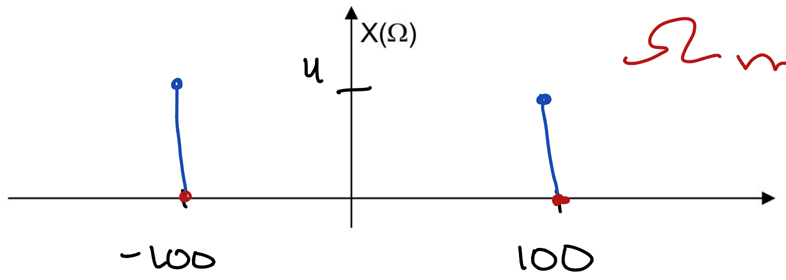
$\boxed{N_{adc} = 0}$

d.) output if $V_{in} = 2.9$

↳ higher than positive reference, so it reports the highest possible value.

$\boxed{N_{adc} = 2^{12} - 1 = 4095}$

4. (40 points) Represent spectrum of the signal $x(t) = 8cos(100t)$.



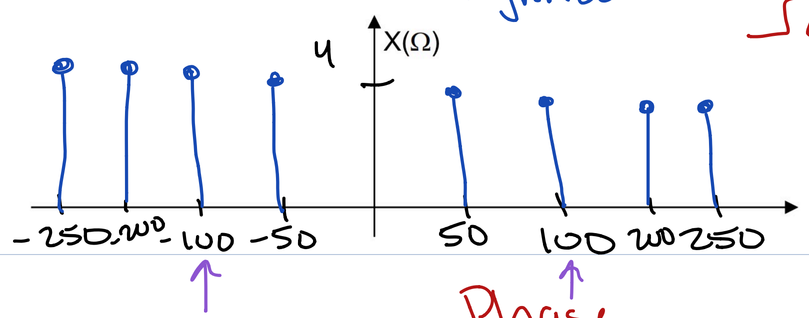
$\Omega_{max} = 100$

magnitude
phase

Represent magnitude and phase spectrum of the same signal sampled at Fs = 150 rad/s.
Describe the effect.

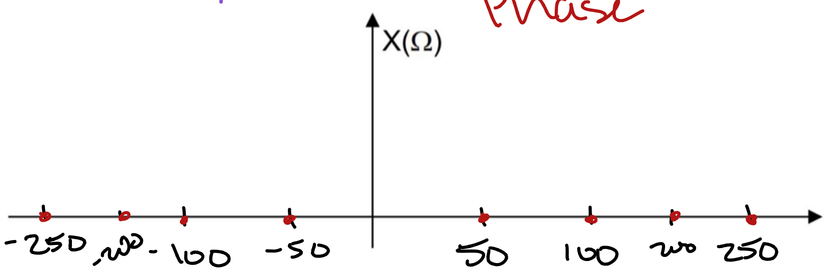
Magnitude

$\Omega_s = 150$



30

Phase



$150 \geq 200$

When you sample a signal at a sampling period of Fs, then the signal repeats at every multiple of Fs. So, this one repeats every 150 rad/s, with the multiple behaving like the zero point.

☆ Note: for this problem there is aliasing because it does not satisfy the Nyquist Sampling rate condition:

$$\Omega_s \geq 2\Omega_{max} \rightarrow 150 \not\geq 200$$