Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

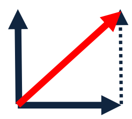
Homework #1 Solution

1. (20 points)

Represent the following complex numbers in alternative form (polar ←→ {Re,Im} z=x +*j*y)

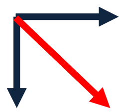
a) 1 + *j*

$\sqrt{2}e^{j\pi/4}$



b) 1 – *j*

$\sqrt{2}e^{-j\pi/4}$



c) 5 e $^{j210°}$

$5e^{j\,210} = 5e^{j(180+30)} = 5e^{j180}\,e^{j30}$

$e^{j180°} = \cos(180°) + j\sin(180°) = -1 + j\,0 = -1$

➔ $-\,5e^{j30} = -5\cos(30) - j5\sin(30) = -4.33 - j\,2.5$

d) 5 e $^{-j210°}$

$5e^{-j\,210} = 5e^{-j(180+30)} = 5e^{j180}\,e^{-j30} = -\,5e^{-j30} = -5\cos(-30) - j5\sin(-30) = -4.33 + j\,2.5$

e) z z* =

$z\,z^* = (\mathrm{Re} + j\,\mathrm{Im})(\mathrm{Re} - j\,\mathrm{Im}) = \mathrm{Re}^2 + \mathrm{Im}^2 = |z|^2$

f) if $w = e^z$ and z=1+*j* , find log(*w*)

$\log(e^z) = z = 1 + j$

2. (10 points)

Find and plot the roots of

$$Z^7 + 1 = 0$$

$z^7 = -1$ →

$$z_k = e^{j(2k+1)\pi/7}, k = 0,1,\ldots 6$$

3. (20 points)

Use Euler's identity to find trigonometric identities in terms of sin(α), sin(β), cos(α), and cos(β):

a) cos(α + β)

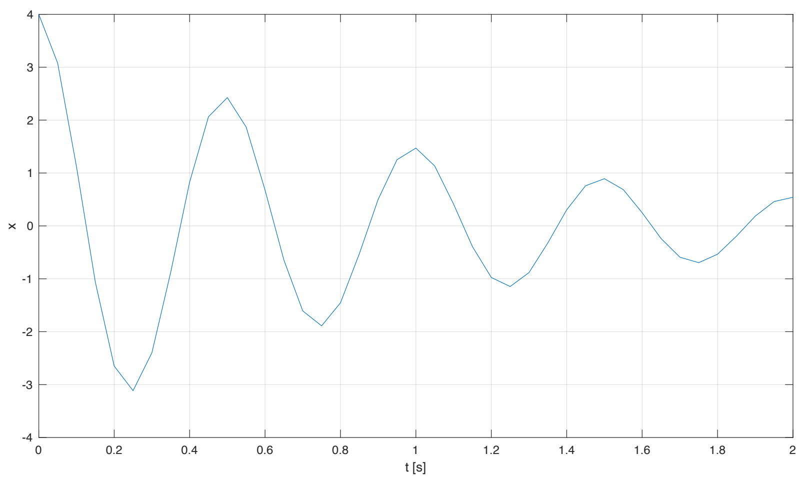
$$\cos(\alpha + \beta) = \frac{e^{j(\alpha + \beta)} + e^{-j(\alpha+\beta)}}{2} = \frac{\big(\cos(\alpha) + jsin(\alpha)\big)\big(\cos(\beta) + \mathrm{jsin}(\beta)\big) + \big(\cos(\alpha) - jsin(\alpha)\big)\big(\cos(\beta) - \mathrm{jsin}(\beta)\big)}{2}$$
$$= \cos(\alpha)\cos(\beta) - sin\,(\alpha)\sin\,(\beta)$$

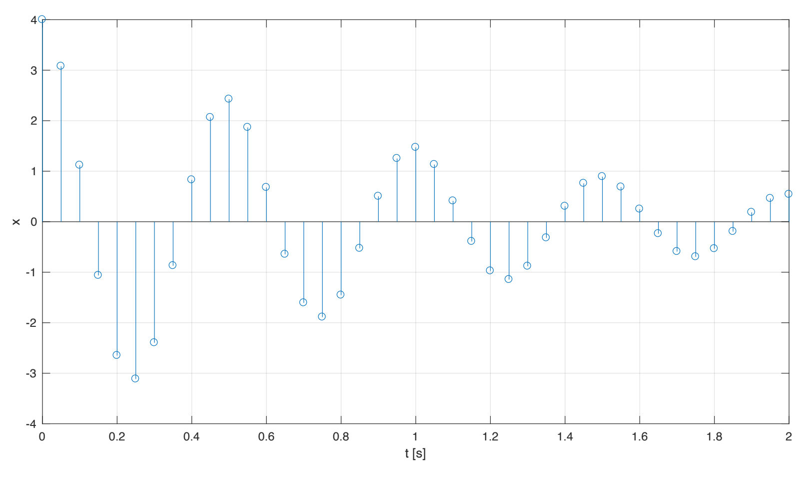
b) sin(α + β)

$$\sin(\alpha + \beta) = \frac{e^{j(\alpha + \beta)} - e^{-j(\alpha+\beta)}}{2j} = \cos(\alpha)\sin(\beta) + sin\,(\alpha)\cos\,(\beta)$$

4. (10 points)

```
% HW#1
Fs=20;  % sampling frequency
Ts=1/Fs; % sampling interval
f=2; % 2 Hz
t=0:Ts:2; % time [s]
A=4;    % Amplitude
xenv=exp(-t);
x=A*xenv.*cos(2*pi*f*t);
figure
plot(t,x),xlabel('t [s]'),ylabel('x'),grid
figure
stem(t,x),xlabel('t [s]'),ylabel('x'),grid
```





3. (30 points)

Accelerometer (± 2g) with analog output and power supply of +3V is used in smartphone to determine orientation of the smartphone according to the figure below.

Sensitivity 1*g* → *s = 3V / 4 g* = 0. 75 [V / g]

Acceleration output for sensitivity *s*, acceleration *a, and DC offset (0g)* A_0:

A = A_0 + *s*a*

A_0 (0 g) = 1.5V

A_1 (+1 g) = 1.5V + 0. 75[V/g]*1[g] = 2.25V

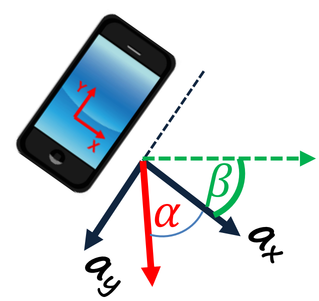
A_{-1} (-1 g) = 1.5V + 0. 75[V/g]*(-1[g]) = 0.75V

What are the values of X and Y components [in Volts] for the following positions

a)	b)	c)	d)
X = 1.5V (0 g)	X = 0.75V (-1g)	X = 1.5V (0 g)	X = 2.25V (1g)
Y = 0.75V (-1g)	Y = 1.5V (0 g)	Y = 2.25V (1g)	Y = 1.5V (0 g)

What is the angle of the smartphone if:

e) a_x = 1.875V, a_y = 0.8505V



acceleration is:

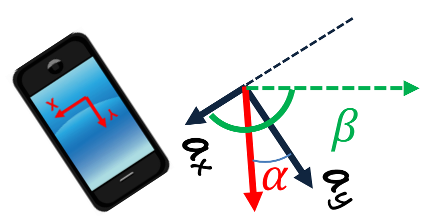
$$a_x = \frac{A-A_0}{s} = \frac{1.875V-1.5V}{0.75\frac{V}{g}} = 0.5\,g, a_y = \frac{A-A_0}{s} = \frac{0.8505V-1.5V}{0.75\frac{V}{g}} = -0.866g,$$

and angle is

$$\alpha = \operatorname{atan}(-0.866/0.5) = -60°, \beta = -30°$$

f) a_x = 2.1495V, a_y = 1.875V

$$a_x = 0.866\,g, a_y = 0.5g,$$



$$\alpha = \operatorname{atan}\left(\frac{0.5}{0.87}\right) = 30°, \quad \beta = -150° (between\ horizontal\ and\ X\ axis)$$