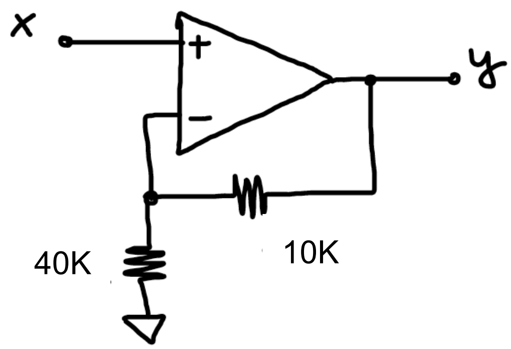
Department of Electrical and Computer Engineering
The University of Alabama in Huntsville

CPE 381: Fundamentals of Signals and Systems for Computer Engineers

Homework #2 Solution

1. (10 points) What is the transfer function of the following circuits



$$x = \frac{40K}{40K + 10K} y \rightarrow y = \frac{5}{4} x$$

Transfer function is therefore:

$$\frac{y}{x} = \frac{5}{4}$$

2. (20 points) Simulate the effect of multipath in wireless communication. Generate dumped sine wave $x(t)$ with amplitude A=1 and frequency f=400Hz sampled at F_s= 11,025Hz with time constant 1 second (i.e. e^{-t}). Assume that the signal is transmitted over three paths, so that the received signal is

$$y(t) = x(t) + 0.4x(t\text{-}0.2) + 0.2x(t\text{-}0.4)$$

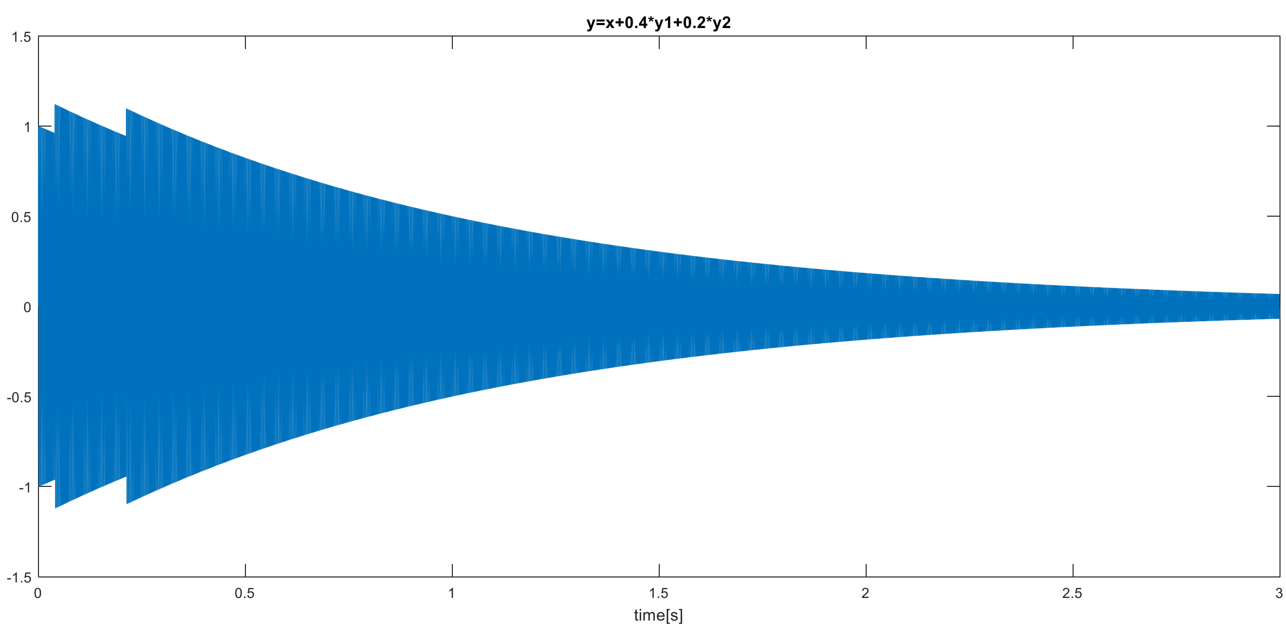
Determine the number of samples corresponding to delay using sampling frequency Fs from the file. Plot the function x(t) and output y(t) and use *sound* function in Matlab to listen to original and received signals.

```
%% Multipath
Fs=11025;  % sampling frequency
Ts=1/Fs;    % sampling interval
t=0:Ts:3;   % time
x=exp(-t).*sin(2*pi*400.*t);    % original signal
N=length(t);    % length of vector

td1=0.2*Fs;   % time delay
i1=round(td1)   % integer delay
i2=round(0.4*Fs)   % integer delay
y1=[zeros(1,i1) x(1:N-i1)]; % delayed signal
y2=[zeros(1,i2) x(1:N-i2)]; % second delayed signal
y=x+0.4*y1+0.2*y2;  % received signal

% plot the function
plot(t,y),title('y=x+0.4*y1+0.2*y2'),xlabel('time[s]')

% and listen the result
sound(y,Fs);
```



3. (10 points)
Find impulse response of capacitor and its unit step response.

Example Find (i) impulse response of capacitor and (ii) its unit step response. $C = 1$ F.

C: $v_c(0) = 0$,

$$v_c(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$$

Impulse response:

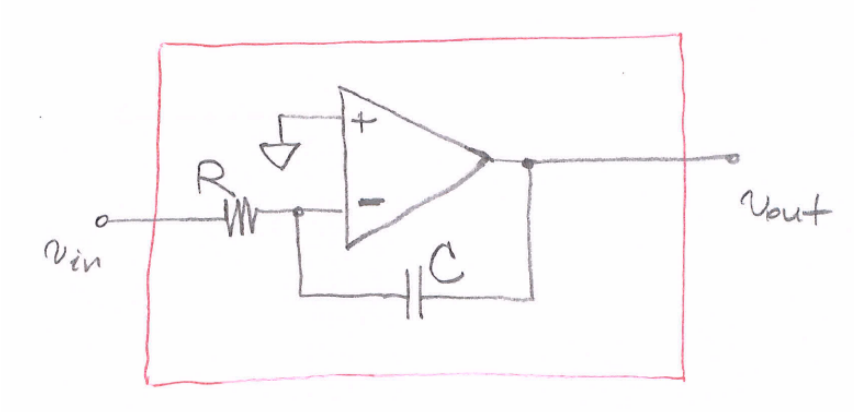
$$i(t) = \delta(t) \quad \Rightarrow \quad v_c(t) = h(t) = \frac{1}{C}\int_0^t \delta(\tau)d\tau = \frac{1}{C}u(t)$$

$C = 1$F, unit-step response

$$v_c(t) = \int_{-\infty}^{\infty} h(t-\tau)i(\tau)d\tau = \int_{-\infty}^{\infty} \frac{1}{C}u(t-\tau)u(\tau)d\tau$$

$$v_c(t) = \frac{1}{C}\int_0^t d\tau = \frac{1}{C}r(t)$$

4. (15 points)
Find transfer function of the following circuit



Standard solution in time domain:

$$V_+ = V_- = 0,\ V_{in} - R \cdot i(t) = V_- = 0$$

$$i(t) = \frac{V_{in(t)}}{R}$$

$$V_{out} = V_- - \frac{1}{C}\int_0^t i(\tau)d\tau + V_C^0 = -\frac{1}{C}\int_0^t \frac{V_{in}(t)}{R} d\tau = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$$

For unit-step function V_{in} is constant and

$$\frac{V_{out}}{V_{in}} = -\frac{t}{RC}$$

Solution using Laplace transform:

$$I = \frac{v_{in}}{R},\quad v_{out} = -\frac{1}{Cs}I = -\frac{1}{RCs}v_{in}$$

5. (20 points)

Use Matlab symbolic computation to find the Laplace transform of a real exponential

$$x(t) = 5e^{-2t}\cos(8t)u(t)$$

Plot the signal and the poles and zeros of their Laplace transform.

Repeat the analysis and plot the results for $x(t) = 5e^{-4t}\cos(8t)u(t)$

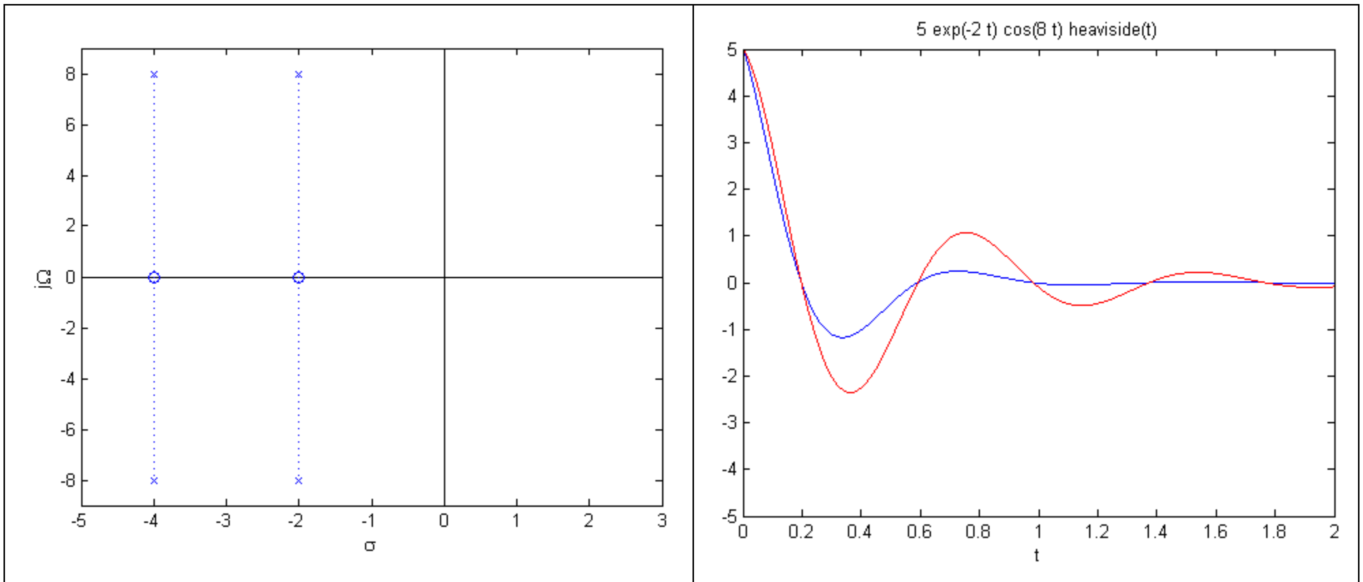
Discuss the changes in the s plane and describe their effect on function in time domain.

```
%CPE381: HW2_5

syms t x1 x2
x1=5*exp(-2*t)*cos(8*t)*heaviside(t);
x2=5*exp(-4*t)*cos(8*t)*heaviside(t);

X1=laplace(x1)
% X1 = 5*(s+2)/(s^2+4*s+68)
% plot
splane([5 10],[1 4 68])

X2=laplace(x2)
% X2 = 5*(s+4)/(s^2+8*s+80)
figure
% plot
splane([5 20],[1 8 80])
```



Zeros and poles of X1 and X2	Output y1 and y2

Discuss the changes in the s plane and describe their effect on function in time domain

Zeros and poles shifted to the left (larger absolute values of σ); consequently, signal in time domain is more attenuated (damped).

6. (10 points)
Describe the basic properties of the one sided Laplace transform.

Causal functions and constants:	$\alpha f(t)$	$\Leftrightarrow$	$\alpha F(s)$
Linearity:	$\alpha f(t) + \beta g(t)$	$\Leftrightarrow$	$\alpha F(s) + \beta G(s)$
Time shifting:	$f(t - \alpha)$	$\Leftrightarrow$	$e^{-\alpha s} F(s)$
Frequency shifting:	$e^{\alpha t} f(t)$	$\Leftrightarrow$	$F(s - \alpha)$
Multiplication by t:	$tf(t)$	$\Leftrightarrow$	$-\frac{dF(s)}{ds}$
Derivative:	$\frac{df(t)}{dt}$	$\Leftrightarrow$	$sF(s) - f(0-)$
Second derivative:	$\frac{d^2 f(t)}{dt^2}$	$\Leftrightarrow$	$s^2 F(s) - sf(0 -) - f^{(1)}(0)$
Integral:	$\int_{0-}^{t} f(t')dt$	$\Leftrightarrow$	$\frac{F(s)}{s}$
Expansion/Contraction:	$f(\alpha t) \alpha \neq 0$	$\Leftrightarrow$	$\frac{1}{\lvert a \rvert} F(\frac{s}{\alpha})$

7. (15 points) Example 3.3. (page 192)

The Laplace transform of the complex causal signal $e^{j(\Omega_0 t+\theta)}u(t)$ is found to be

$$\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] = \int_0^\infty e^{j(\Omega_0 t+\theta)} e^{-st} dt = e^{j\theta} \int_0^\infty e^{-(s-j\Omega_0)t} dt$$

$$= \frac{-e^{j\theta}}{s-j\Omega_0} e^{-\sigma t - j(\Omega-\Omega_0)t} \Big|_{t=0}^{\infty} = \frac{e^{j\theta}}{s-j\Omega_0} \qquad \text{ROC: } \sigma > 0$$

According to Euler's identity

$$\cos(\Omega_0 t+\theta) = \frac{e^{j(\Omega_0 t+\theta)} + e^{-j(\Omega_0 t+\theta)}}{2}$$

by the linearity of the integral and using the above result, we get that

$$\begin{aligned}\mathcal{L}[\cos(\Omega_0 t+\theta)u(t)] &= 0.5\mathcal{L}[e^{j(\Omega_0 t+\theta)}u(t)] + 0.5\mathcal{L}[e^{-j(\Omega_0 t+\theta)}u(t)] \\ &= 0.5\frac{e^{j\theta}(s+j\Omega_0) + e^{-j\theta}(s-j\Omega_0)}{s^2+\Omega_0^2} \\ &= \frac{s\cos(\theta) - \Omega_0\sin(\theta)}{s^2+\Omega_0^2}\end{aligned}$$

and a region of convergence $\{(\sigma,\Omega) : \sigma > 0, -\infty < \Omega < \infty\}$.

Now if we let $\theta = 0, -\pi/2$ in the above equation we have the following Laplace transforms:

$$\mathcal{L}[\cos(\Omega_0 t)u(t)] = \frac{s}{s^2+\Omega_0^2}$$

$$\mathcal{L}[\sin(\Omega_0 t)u(t)] = \frac{\Omega_0}{s^2+\Omega_0^2}$$

as $\cos(\Omega_0 t - \pi/2) = \sin(\Omega_0 t)$. The ROC of the above Laplace transforms is $\{(\sigma,\Omega) : \sigma > 0, -\infty < \Omega < \infty\}$, or the open right-hand s-plane (i.e., not including the $j\Omega$ axis). See Figure 3.6 for the pole-zero plots and the corresponding signals for $\theta = 0$, $\theta = \pi/4$, and $\Omega_0 = 2$. ■