

Quiz #4 Solution

Q1. Impulse response of the system with transfer function

$$H(s) = \frac{1}{s + 1.3}$$

is

$$h(t) = e^{-1.3t}$$

The system is **stable** since

$$\int_0^{\infty} e^{-1.3t} dt = \frac{e^{-\infty} - e^{-0}}{-1.3} = 0.769 < \infty$$

Q2. Integral

$$\int_0^{\infty} e^{-1.3t} dt = \frac{e^{-\infty} - e^{-0}}{-1.3} = \mathbf{0.769} < \infty$$

Q2. A low-pass filter is implemented using $R=1\text{K}\Omega$ and $C = 1 \mu\text{F}$ (series of resistor and capacitor, and capacitor is parallel to output).

What is the magnitude of the transfer function at frequency 1000 rad/s?

Transfer function of the filter is

$$H(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

and

$$H(j\Omega) = \frac{1}{jRC\Omega + 1}$$

Magnitude of the transfer function is:

$$|H(j\Omega)| = \left| \frac{1}{jRC\Omega + 1} \right| = \left| \frac{1}{j10^3 10^{-6} 10^3 + 1} \right| = \left| \frac{1}{j+1} \right| = 0.707$$

Q3. Fourier transform of the modulated signal

$$x(t)=3+8*\cos(10t)*\cos(100t)$$

How many frequency components do you have in Fourier transform $X(\Omega)$?

Spectrum has **5** frequency components.

Signal $\cos(10t)$ has two frequency components: $\{ X(-10), X(10) \}$

The Fourier transform of the modulated signal is

$$0.5 [X((\Omega - \Omega_0)) + X((\Omega + \Omega_0))]$$

Signal $\cos(10t)*\cos(100t)$ has four frequency components: $\{ X(-110), X(-90), X(90), X(110) \}$

and signal $x(t)$ has 5 components: $\{ X(-110), X(-90), X(0), X(90), X(110) \}$

with values: $\{ 2, 2, 3, 2, 2 \}$

Q4. $X(90) = 2$ (see Q3)

Q5. $X(100) = 0$ (see Q3)