

CPE 323 Intro to Embedded Computer Systems Number Representation

Aleksandar Milenkovic milenka@uah.edu







> Watch for 21a





Numeral Systems

- Decimal (base 10): $456_{10} = 4.10^2 + 5.10^3 + 6.10^2$
- Binary (base 2): $0110_2^2 = 0.2^3 + 1.2^2 + 1.2^1 + 0.2^0 = 6_{16}$
- Octal (base 8): $125_8 = 1 \times 8^2 + 2 \cdot 8^3 + 5 \cdot 8^2 = 85_{10}$





Decimal to Binary Conversion

•
$$A = 27_{10}$$

$$27/2 = 13 11$$

$$13/2 = 6 11$$

$$61 2 = 3 10$$

$$31 2 = 1 11$$

$$1/2 = 0$$

$$27_{10} = 11011_2 = 33_8 = 18_{16}$$





Representing Integers, Unsigned, Binary Format

• E.g., 1 byte or 8 bits, unsigned
$$\begin{bmatrix} A_{n-1} + A_{n-2} & A_{n-1} \end{bmatrix}$$

Bit position	7 (MSB)	6	5	4	3	2	1 (0 (LSB)
Value	0	0 ر	1	0	1	0	1	0
Weights	27	2 ⁶	2 ⁵	24	23	22	21	20

• Convert to decimal:
$$1.2^{5} + 1.2^{3} \rightarrow 1.2^{1} = 42.00$$

- Convert to octal: 052x
- Convert to hex:
- Range : $[0 \div 2^8 1]$





Representing Integers, Signed, Binary Format

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

LAU-1	· Ao	= - A	n-12"	+ An-	2.2 7	An-32	. +	10.2				
Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)				
Value	1	1	1	1	1	1	0	0				
Weights	2 ⁷	2 ⁶	2 ⁵	24	2 ³	2 ²	21	20				
Conve	Convert to decimal: -410 $1000 = -128$ $26+2^{4}++2^{6}=2^{7}-1=127$											
Convert to octal: $-A: 0000-0011$												
C 0 10 10	Convert to bow											

- Convert to decimal: -4 10
- Convert to octal:
- Convert to hex:
- Range

$$\rightarrow 1-128 \div 127$$





Representing Integers, Signed

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	27	2 ⁶	2 ⁵	24	23	22	21	20

- Convert to decimal:
- Convert to octal:
- Convert to hex:
- Range

Properties of 2's complement

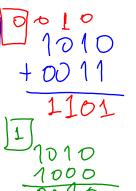
•
$$A = 11101000b$$
 -24_{10}

• Find –A:

$$\begin{array}{r}
00010111 \\
+ 00010000 \\
= 18_{16} = 116^{1} + 8.16^{0} \\
= 24
\end{array}$$

+ An-1 Bn-1 · Rn-

Assume 4-bit machine



$$V = C_4 \oplus C_3$$

$$\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc = \bigcirc$$





Arithmetic Operations

$$A - B = A + (\overline{B} + 1)$$

- Negative (N)
$$V = -$$

$$N = R_{n-1}$$





Arithmetic Operation Examples

$$n=4$$
 $+3:0110$
 $N=4$
 $N=4$

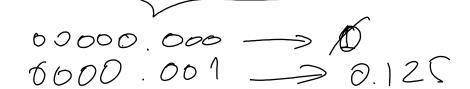




Fraction Numbers

Fixed-point, unsigned

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	24	23	22	21	20	2-1	2-2	2-3







Fraction Numbers

Floating-point (IEEE 754 standard)

Туре	Sign	Exponent	Exponent bias	Significand	Total
Half (IEEE 754- 2008)	1	5	15	10	16
Single	1	8	127	23	32
Double	1	11	1023	52	64
Quad	1	15	16383	112	128

• Single-precision, normalized: (-1)^{S*}2^{E-127}1.F

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
S	E7	E6	E5	E4	E3	E2	E1	EO	F22	F21																					F0





Floating-point

Sign (s)	Exponent (e)	Fraction (f)	Value
0	00 00	0000	+0
0	00 00	00 01 11 11	Positive denormalized real 0.f \times 2 ^(-b+1)
0	00 01 11 10	xx xx	Positive normalized real 1.f \times 2 ^(e-b)
0	11 11	00 00	+Infinity
0	11 11	00 01 01 11	SNaN
0	11 11	10 00 11 11	QNaN
1	00 00	0000	-0
1	00 00	00 01 11 11	Negative denormalized real $-0.f \times 2^{(-b+1)}$
1	00 01 11 10	xx xx	Negative normalized real $-1.f \times 2^{(e-b)}$
1	11 11	00 00	-Infinity
1	11 11	00 01 01 11	SNaN
1	11 11	10 00 11 11	QNaN



LaCASA

$$88_{10}$$

$$88/8 = 11 0
$$11/8 = 1 13$$

$$1/8 = 0 11$$$$

$$88_{10} = 130_8 = 18^2 + 3.8 + 0.8^\circ$$

= 64 + 24 + 0 = 88

THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

8/26/2020



Binory Coded Decimal Numbers (BCD) LaCASA

\bigcirc		
0100 0011	packed BCD	40
D = 3	4310/16 -2	B
x post out dig 15	$\frac{2}{16} = 0$	2
43)0 0000 1001	' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '	 2 B.
	43,0=	2016
1/01/0011	0010 10	11

8/26/2020