CPE 323
Intro to Embedded Computer Systems
Number Representation

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Admin

→ Watch for Q1a

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Numeral Systems

- Decimal (base 10): $456_{10} = 4 \cdot 10^2 + 5 \cdot 10^1 + 6 \cdot 10^0$

  $\{0, 1, \ldots 9\}$, base = 10

- Binary (base 2): $0110_2 = 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 6_{10}$

  $\{0, 1\}$

- Octal (base 8): $125_8 = 1 \cdot 8^2 + 2 \cdot 8^1 + 5 \cdot 8^0 = 85_{10}$

  $\{0, 1, \ldots, 7\}$

- Hexadecimal (base 16): $10A_{16} = 1 \cdot 16^2 + 0 \cdot 16^1 + 10 \cdot 16^0$

  $= 256 + 10 = 266_{10}$

  $\{0, 1, \ldots 9, \underset{10}{A}, \underset{11}{B}, \underset{12}{C}, \underset{13}{D}, \underset{14}{E}, \underset{15}{F}\}$

Decimal to Binary Conversion

- A = 27_{10}

$27 / 2 = 13 \quad 1$

$13 / 2 = 6 \quad 1$

$6 / 2 = 3 \quad 0$

$3 / 2 = 1 \quad 1$

$1 / 2 = 0 \quad 1$

LSB

MSB

$27_{10} = 11011_2 = 33_8 = 1B_{16}$

Representing Integers, Unsigned, Binary Format

- E.g., 1 byte or 8 bits, unsigned [n-bit $A_{n-1}\, A_{n-2} \ldots A_0$]

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	0	0	1	0	1	0	1	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- Convert to decimal: $1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^1 = 42_{10}$
- Convert to octal: 052_8
- Convert to hex: $2A_{16}$
- Range : $[0 \div 2^8 - 1]$ 255

Representing Integers, Signed, Binary Format

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

$[A_{n-1} \dots A_0] = -A_{n-1} \cdot 2^{n-1} + A_{n-2} \cdot 2^{n-2} + A_{n-3} \cdot 2^{n-3} + \dots + A_0 \cdot 2^0$

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

A:

$100\dots0 = -128$

$2^6 + 2^5 + \dots + 2^1 + 2^0 = 2^7 - 1 = 127$

- Convert to decimal: -4_{10}
- Convert to octal:
- Convert to hex:
- Range $\longrightarrow [-128 \div 127]$

$$
\begin{array}{r}
-A: \ 0000\_0011 \\
+ \qquad\qquad 1 \\
\hline
0000\_0100 \\
= 4_{10}
\end{array}
$$

Representing Integers, Signed

- E.g., 1 byte or 8 bits, signed in 2's complement
- Bit 7 is sign bit (0 for positive integers, 1 for negative integers)

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- Convert to decimal:
- Convert to octal:
- Convert to hex:
- Range

Properties of 2's complement

00011000

- A = 11101000b $\boxed{-24_{10}}$
- Find –A:

$$
\begin{array}{r}
0111 \\
00010111 \\
+ \quad 1 \\
\hline
00011000b
\end{array}
$$

$= 18_{16} = 1 \cdot 16^1 + 8 \cdot 16^0$

$= 24$

$$V_{ADD} = \overline{A_{n-1}} \cdot \overline{B_{n-1}} \cdot R_{n-1} + A_{n-1} \cdot B_{n-1} \cdot \overline{R_{n-1}}$$

- Assume 4-bit machine
- A = 1010b
- B = 0011b
- Find A + B

$$
\begin{array}{r}
\boxed{0}010 \\
1010 \\
+ 0011 \\
\hline
1101
\end{array}
$$

$$
\begin{array}{r}
\boxed{1} \\
1010 \\
1000 \\
\hline
0010
\end{array}
$$

2's complement

$$
\begin{array}{r}
C_4 C_3 C_2 C_1 \\
0\,0\,1\,0 \\
1010 \\
0011 \\
\hline
1101
\end{array}
$$

$V = C_4 \oplus C_3$

$= 0 \oplus 0 = 0$

Arithmetic Operations

$A - B = A + (\overline{B} + 1)$

$-B = \overline{B} + 1$

- Addition
- Subtraction
- Multiplication
- Flags
  - Carry (C) $\longrightarrow$
  - Overflow (V) $\longrightarrow$
  - Negative (N) $N = R_{n-1}$
  - Zero (Z) $\longrightarrow$ set if the result of the operation is equal to zero

Arithmetic Operation Examples

$n = 4$

$$
\begin{array}{rll}
 & (0110 & \\
 & A: 0111 & 7 \\
+ & B: 0110 & 6 \\
\hline
A+B: & 1101 &
\end{array}
$$

$C = \cancel{1}\ \emptyset \quad (C = C_4 = \emptyset)$

$V = C_4 \oplus C_3 = 0 \oplus 1 = 1$

$Z = \emptyset$

$N = 1$

Fraction Numbers

- Fixed-point, unsigned

Bit position	7 (MSB)	6	5	4	3	2	1	0 (LSB)
Value	1	1	1	1	1	1	0	0
Weights	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

00000.000 → 0

0000.001 → 0.125

Fraction Numbers

- Floating-point (IEEE 754 standard)

Type	Sign	Exponent	Exponent bias	Significand	Total
Half (IEEE 754-2008)	1	5	15	10	16
Single	1	8	127	23	32
Double	1	11	1023	52	64
Quad	1	15	16383	112	128

- Single-precision, normalized: $(-1)^{S} * 2^{E-127} 1.F$

	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	S	E7	E6	E5	E4	E3	E2	E1	E0	F22	F21																					F0

Floating-point

Sign (s)	Exponent (e)	Fraction (f)	Value
0	00 … 00	000….0	+0
0	00 … 00	00 … 01 11 … 11	Positive denormalized real 0.f $\times$ $2^{(-b+1)}$
0	00 … 01 11 … 10	xx … xx	Positive normalized real 1.f $\times$ $2^{(e-b)}$
0	11 … 11	00 … 00	+Infinity
0	11 … 11	00 … 01 01 … 11	SNaN
0	11 … 11	10 … 00 11 … 11	QNaN
1	00 … 00	000….0	-0
1	00 … 00	00 … 01 11 … 11	Negative denormalized real -0.f $\times$ $2^{(-b+1)}$
1	00 … 01 11 … 10	xx … xx	Negative normalized real -1.f $\times$ $2^{(e-b)}$
1	11 … 11	00 … 00	-Infinity
1	11 … 11	00 … 01 01 … 11	SNaN
1	11 … 11	10 … 00 11 … 11	QNaN

88_{10}

$88 / 8 = 11 \;\; \llcorner 0$

$11 / 8 = 1 \;\; \llcorner 3$

$1 / 8 = 0 \;\; \llcorner 1$

$$88_{10} = 130_8 = 1 \cdot 8^2 + 3 \cdot 8 + 0 \cdot 8^0$$
$$= 64 + 24 + 0 = 88$$

-128.25 → negative $s = 1$

$$(-1)^S \cdot 2^{E-127} \cdot 1.F$$

$128.25 \rightarrow 1000\,0000.01_2$

$= \underbrace{1.0000\,00001}_{\text{Fraction}} \times 2^7$

$\boxed{E=0}$

$\boxed{E=255}$

$0.25 \times 2 = 0.5 \quad | \; 0$

$0.5 \times 2 = 1 \quad | \; 1$

$0.25_{10} = .01_2$

$2^{-1} \; 2^{-2}$

S	E	
1	1000 0110	0000 0000100…
1	8	23

$E - 127 = 7$

$E = 134 = 10000110_2$

Binary Coded Decimal Numbers (BCD)

7	4	3	0
0100		0011	

digit ×10 | unit digits

43_{10}

{0, ..., 9}

0000 ... 1001

1101	0011

packed BCD

$43_{10} / 16 = 2$ B

$2 / 16 = 0$ 2

$43_{10} = 2B_{16}$

0010	1011