Student name:

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1 37	2 5	3 12	4 20	5 6	6 5	7 15	Total

1. An embedded microcontroller program is used to measure temperature with thermistor with the following characteristics:

temp

$$R(t) = -2 * t + 200\ [K\Omega]$$

ADC 0 , 2.5
Circ. 0.75 1.5

where *t* is temperature in degrees Fahrenheit [°F]. Thermistor is used in voltage divider configuration with fixed resistor of 120KΩ connected to power supply and thermistor connected to the ground. Microcontroller uses a 10 bit AD converter with 2.5V positive reference and ground as negative reference. The system has 1,200 bytes of RAM available to store samples from the temperature sensor. Signal conditioning circuit uses power supply of 3V. Maximum expected frequency of the signal is 3Hz.

Nyquist Thm.: $\Omega_s \geq 2\Omega_{max} = \boxed{6\text{ Hz}}$

Q1. (2 points) What is the minimum sampling frequency of the signal?

$R_f(80) = -2(80) + 200 = 40\ k\Omega$

$V_{out} = \frac{40}{40+120} k\Omega \cdot 3 = \boxed{0.75\ V}$

Q2. (2 points) What is the minimum voltage of the signal if the expected range of temperatures is 40-80 °F?

$R_f(40) = -2(40) + 200 = 120\ k\Omega$

$V_{out} = \frac{120\ k\Omega}{120 + 120} \cdot 3V = \boxed{1.5\ V}$

Q3. (2 points) What is the maximum voltage of the signal if the expected range of temperatures is 40-80 °F?

$\boxed{100\ s}$

Q4. (2 points) How many seconds of the signals you can buffer on the microcontroller without optimization?

Equations

$BW = Fs \cdot Ss$

$T_o = \frac{MEM}{BW}$

Samp Size = 10 bits = 1.25 B

↳ Round up to 2B/sample

$BW = 6\text{ Hz} \cdot 2\text{ B/sample} = 12\text{ Bps}$

$T_o = \frac{1200\text{ B}}{12\text{ Bps}} = \boxed{100\ s}$

$BW = 6 \cdot 1.25 = 7.5\ Bps \quad T_0 = \frac{1200B}{7.5Bps} = \boxed{160s}$

Q5. (2 points) How many seconds of the signals you can buffer on the microcontroller with optimization? → $S_s = 1.25\ B$

$\Delta = \frac{V_+ - V_-}{2^n - 1} = \frac{2.5-0}{1023} = \boxed{0.0024\ V}$ keep all digits.

Q6. (5 points) What is the quantization step of the AD converter in volts [V]?

$\Delta T^\circ = \frac{80-40\ [F^\circ]}{1023} = \boxed{0.0391\ F^\circ}$

Q7. (5 points) What is the maximum temperature error caused by the AD conversion in [°F] assuming that signal conditioning circuit is optimized to span the range of the input signal (40-80 °F)?

output of circuit

$R_f(70) = -2(70)+200 = 60 \qquad N_{ADC} = \frac{V_{in} - V_-}{\Delta}$

$V_{in} = \frac{60}{60+120} \cdot 3 = 1V \qquad N_{ADC} = \frac{1-0}{0.0024} = \boxed{409}$

Q8. (5 points) What is the output of the AD converter when temperature is 70°F? keep all digits

Q9. (10 points) The embedded microcontroller runs at clock speed of 1MHz and spends 20,000 cycles per sample. Total data acquisition time is 1 ms and sampling frequency is 10 Hz. In addition, every second the controller is running spectral processing that takes 0.7 seconds. What is the ratio of average processing time (including data acquisition time) and sampling interval Ts?

$T_s = 0.1s$

avg. for all 10 samples per second

$\underbrace{700\,ms}_{t_{proc.}} + \underbrace{\left(\frac{20000}{1\,MHz}\right)}_{s/samp}^{\times 1000 \text{ for ms}} \cdot \underbrace{10}_{\#samp/s} + (\underbrace{1ms}_{t_{aq}} \cdot \underbrace{10}_{\#samp/s}) = \underbrace{910\,ms}_{\text{all samp proc. in a s}}$

$\frac{910}{10} = 91\,ms$ avg. for 1 sample

$\frac{91\,ms}{0.1\,s} = \boxed{0.91}$ (unitless)

Q10. (2 points) Can system run in real-time?

$\boxed{\text{Yes, processing time only takes 91\% of Ts.}}$

2. A low-pass filter is implemented using R=1$K\Omega$ and C = 1 μF (series of resistor and capacitor, and capacitor is parallel to output).

$$\frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(2000 \cdot 1K\Omega \cdot 1\mu F)^2}} = \boxed{0.447}$$

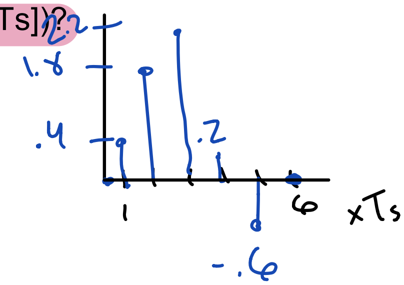
Q11. (5 points) What is the magnitude of the transfer function at frequency 2000 rad/s?

3. Let x[n] = {0, 1, 2, -1, 0} and h[n] = {0.4, 1, 0.6}. Compute and plot the convolution y[n] = x[n] * h[n].

	0	1	2	-1	0
0.4	0	0.4	0.8	-0.4	0
1	0	1	2	-1	0
0.6	0	0.6	1.2	-0.6	0

$y[3 \cdot T_s] = 2.2$

Q12. (12 points) What is the value of the fourth sample of the output (y[3*Ts])?



4. Consider a second order differential equation,

$$\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Assume the above equation represents a system with input x(t) and output y(t). Find the impulse response h(t) and the unit step response s(t) of the system, assuming that the initial conditions are $y(0^-) = 1,\ \frac{dy(t)}{dt}|_{t=0} = 0,\ x(t) = u(t)$.

0.698

Q13. (17 points) What is the value of the step response s(t) at time t=1.5 s?

1/2

Q14. (3 points) What is the steady state value of s(t)?

$-.6$

$$\frac{d^2y(t)}{dt}+3\frac{dy(t)}{dt}+2y(t)=x(t)$$

Assume the above equation represents a system with input x(t) and output y(t). Find the impulse response h(t) and the unit step response s(t) of the system, assuming that the initial conditions are $y(0^-)=1,\ \frac{dy(t)}{dt}\big|_{t=0}=0,\ x(t)=u(t)$.

impulse response $X(s)=1$ ↳ $\mathcal{L}[u(t)]=\frac{1}{s}=X(s)$

step response $X(s)=\frac{1}{s}$

$$X(s)=\mathcal{L}\left[\ddot{y}(t)+3\dot{y}(t)+2y(t)\right]$$

$$X(s)=s^2Y(s)-s\cancelto{1}{y(0^-)}-\cancelto{0}{\dot{y}(0)}+3sY(s)-3\cancelto{1}{y(0)}+2Y(s)$$

$$X(s)=s^2Y(s)+3sY(s)+2Y(s)$$

$$\frac{1}{s}=Y(s)\left[s^2+3s+2\right]-s-3$$

$$Y(s)\left[s^2+3s+2\right]=\frac{1}{s}+s+3$$

$$Y(s)=\frac{\frac{1}{s}+s+3}{s^2+3s+2}\left[\frac{s}{s}\right]=\frac{s^2+3s+1}{s(s+1)(s+2)}$$

Partial Frac. Decomp.

$$Y(s)=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$

↳ $s+1=0 \Rightarrow s=-1$

$$A=Y(s)(s)\Big|_{s=0}=\frac{s^2+3s+1}{(s+1)(s+2)}\Big|_{s=0}=\frac{1}{2}$$

$$B=Y(s)(s+1)\Big|_{s=-1}=\frac{s^2+3s+1}{s(s+2)}\Big|_{s=-1}=\frac{1-3+1}{-1(1)}=1$$

$$C=Y(s)(s+2)\Big|_{s=-2}=\frac{s^2+3s+1}{s(s+1)}=\frac{4-6+1}{-2(-1)}=-1/2$$

$$\mathcal{L}\left\{ S(s) = \frac{1/2}{s} + \frac{1}{s+1} - \frac{1/2}{s+2} \right]$$

$$S(s) = \left[\frac{1}{2} + e^{-t} - \frac{1}{2}e^{-2t}\right] \cdot u(t)$$

$$S(1.5) = \boxed{0.698}$$ → quiz didn't use init. conds. (0.301)

$$\text{Steady State} = \boxed{\frac{1}{2}}$$

Steady State

$\lim_{t \to \infty}$ = finite value

Transient State

$\lim_{t \to \infty}$ = not finite

5. A and B are vectors with coefficients of a 4-point averaging filter in C program. $\frac{B}{A}$

1 → denominator is 1

Q15. (3 points) What is the value of A[0]?

$$\frac{.25\,b_0 \overset{(.25)}{+b_1} \overset{(.25)}{+b_2} \overset{(.25)}{+b_3}}{1}$$

$$\frac{1}{4} = 0.25$$

Q16. (3 points) What is the value of B[0]?

6. Signal is sampled at Fs=200 Hz and discrete Fourier transform is performed by using N=1024 point window.

$$\Delta f = \frac{Fs}{NFFT} = \frac{200}{1024} = 0.1953$$

Q17. (5 points) What is the frequency resolution of the result - Δf?

7. Load **`fintest.mat`** from `Canvas/exams`. Signal x is sampled at Fs=200 Hz. Analyze signal using Matlab function **`fft`** and NFFT=1024 point window and a 1024 point **`hanning`** window. Plot the spectrum and publish the file

3 (or 4)

Q18. (6 points) How many discrete frequency components do you have in spectrum of the real signal x?

5.859 (on graph)

Q19. (9 points) What is the frequency of the component with maximum magnitude in spectrum of x?