Chapter 1

Theoretical framework

1.1 Introduction

The behaviour of fundamental particles and forces are described by the Standard Model (SM) of particle physics. For a long time validation of the SM relied upon the discovery of a scalar Higgs boson, which was observed in 2012 CMS and ATLAS collaborations. This final piece of the picture has made the SM a remarkably robust theory with no predictions deviating from experimental observations. Indeed, the fine structure constant, α, which characterizes the coupling strength of electromagnetic interactions is one of the most accurately predicted physical values. The value of α^{-1} is measured to be:

$$\alpha^{-1} = 137.035999074(44), \tag{1.1}$$

with a relative uncertainty of 0.32 parts per billion [1], while the current best theoretical prediction, which accounts for up to tenth order Feynman diagrams, is:

$$\alpha^{-1} = 137.035999073(35), \tag{1.2}$$

to an accuracy of 0.25 parts per billion [2]. The agreement of these values makes the theory of Quantum Electrodynamics, which describes interactions of photons and charged particles in the SM, one of the most accurate theories yet constructed.

Despite its countless successes, there are many experimental and theoretical arguments indicating that the SM is an incomplete picture of particle physics. Many theoretical problems arise from the idea of naturalness, that is that...

Experimentally, there are observed phenomena which are left unexplained by the SM. Neutrinos are treated as massless in the SM, but they are seen to oscillate in flavour space indicating that they must, in fact, have mass. Flat rotation curves of galaxies and gravitational lensing indicate the existence of Dark Matter, which is entriely unaccounted for by the SM.

Another problem is that the SM cannot reconcile the matter-antimatter asymmetry observable in the Universe today. The hypothesized process which caused this asymmetry is known as baryogenesis. Whatever this process may be, it must satisfy:

- at least one baryon number (B) violating process,
- Charge and Charge-Parity (CP) violation,
- interactions out of thermal equilibrium.

These are the Sakharov conditions [3], and outline the minimum requirements of baryogenesis. The first of these criteria is an obvious one: at the time of the Big Bang $B = 0$, whereas today $B \gg 0$; hence B must not be conserved in some process. If a process conserves charge then

$$\Gamma(X \to Y + B) = \Gamma(\bar{X} \to \bar{Y} + \bar{B}), \tag{1.3}$$

so B will be conserved over time. However, this condition is insufficient. Consider a process $X \to q_L q_L$ which has a CP-conjugate process $\bar{X} \to \bar{q}_R \bar{q}_R$; then

$$\Gamma(X \to q_L q_L) + \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \bar{q}_R \bar{q}_R) + \Gamma(\bar{X} \to \bar{q}_L \bar{q}_L) \tag{1.4}$$

would still result in B conservation even if C is violated. Thus, the process must be CP violating. The final criteria ensures that baryogenesis occurs at a higher rate than anti-baryogenesis.

The flavour sector is the only source of CPV in the SM, but comes up short by around 10 orders of magnitude when explaining the matter dominated nature of the Universe [4,5]. The following chapter will elucidate as to how the flavour sector is the only source of CPV in the SM.

1.2 The Standard Model

The current formulation of the SM of particle physics was concocted in the 1970s, when the Higgs mechanism was incorporated into Glashow's electroweak theory by Salam and Weinberg. The theory prescribes a treatment as to how fundamental particles interact via three of the four fundamental forces, namely: the strong, weak and electromagnetic forces.

Mathematically, the SM is a locally gauge invariant quantum field theory. It inhabits a space-time with a global Poincaré symmetry that obeys a local $SU(3) \times SU(2) \times U(1)$ symmetry. Each generator in this group corresponds to a gauge-boson; so the strong force ($SU(3)$ group) has eight gluons that mediate the interaction, and the electroweak force ($SU(2) \times U(1)$ group) has $3+1$ gauge bosons, which are the weak gauge bosons (Z, $W^{\pm}$) and the photon (γ). These are all vector fields. Fermions are described by spinor fields, ψ, which obey the Dirac equation:

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\psi = 0. \tag{1.5}$$

The fermions of the SM constitute six leptons (electron, electron neutrino, muon, muon neutrino, tau and tau neutrino) and six quarks (up, down, charm, strange, top and bottom), which are organized into pairs forming three generations. For each fermion there is a corresponding antiparticle with the same mass and opposite charge — charge being the conserved quantity resulting from the global gauge symmetry (by Noether's theorem). There is also a single scalar field in the SM, that of the Higgs boson.

The SM Lagrangian can be expressed as a sum of components:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Strong}} + \mathcal{L}_{\text{V}} + \mathcal{L}_{\ell} + \mathcal{L}_{q} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yuk}}. \tag{1.6}$$

The first three terms describe: interactions of the strong force between colour carrying particles, weak vector boson self-interactions, and the electroweak behavior of leptons. The remaining terms describe the electroweak behavior of quarks, the Higgs interaction and Yukawa couplings, respectively. These latter terms are of fundamental importance as to how the flavour changing currents and CPV occur in the SM, and will be be discussed in detail.

The structure of CP and flavour violation emerges as a direct consequence of the Higgs mechanism breaking the local electroweak symmetry. The Lagrangian of the scalar Higgs field is:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \left(D_{\mu}\Phi\right)^{\dagger}\left(D^{\mu}\Phi\right) - V(\Phi) && (1.7) \\ &= \left(D_{\mu}\Phi\right)^{\dagger}\left(D^{\mu}\Phi\right) - \mu^2\left(\Phi^{\dagger}\Phi\right) + \lambda\left(\Phi^{\dagger}\Phi\right), && (1.8) \end{aligned}$$

where μ and λ are constants, D_{μ} is the covariant derivative, and Φ is the Higgs doublet, defined by:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{1.9}$$

Taking $\mu^2 < 0$ and $\lambda > 0$ moves the minimum of the potential $V(\Phi)$ away from zero to a distance v:

$$v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{1.10}$$

At this point the Higgs field gets a vacuum expectation value (VEV) of $\langle\phi\rangle = \frac{1}{\sqrt{2}}v$. The direction of the VEV from the origin is arbitrary, but the choice of:

$$\langle 0|\phi_1|0\rangle = \langle 0|\phi_2|0\rangle = \langle 0|\phi_4|0\rangle = 0 \qquad\qquad \langle 0|\phi_3|0\rangle = v, \tag{1.11}$$

is convenient, and changes Eq. 1.9 to:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\eta_1 + i\eta_2 \\ v + i\eta_4\end{pmatrix}. \tag{1.12}$$

Here, η_1, η_2 and η_4, are Goldstone bosons which, by choosing an appropriate gauge, become the longitudinal components of the weak bosons. This choice of gauge simplifies Φ to:

$$\Phi = \begin{pmatrix}0 \\ v + H\end{pmatrix}, \tag{1.13}$$

where H is the physical Higgs boson. Inserting Eq. 1.13 into Eq. 1.8 gives:

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\left(\partial_\mu H\right)\left(\partial^\mu H\right) + \mu^2 H^2 + \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{m_Z^2}{2} Z_\mu Z^\mu\right)\cdot\left(1 + \frac{H}{v}\right)^2 \tag{1.14}$$

where g and g' are coupling constants and other terms are three- and four-point interactions of the Higgs with itself and weak gauge bosons. Thus, the $U(1)$ local gauge symmetry is broken, weak gauge boson acquire a mass while photons remain massless; as is consistent with observations.

All fermions (except neutrinos) also get a mass after the spontaneous symmetry breaking (SSB) of the $U(1)$ symmetry. The Dirac mass term for a chiral field should be of the form:

$$\mathcal{L}_{\text{mass}} = -m_\psi\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right), \tag{1.15}$$

but the left- and right-handed fields have different $U(1)$ charges and so transform differently under local gauge transformations and so cannot be added to $\mathcal{L}_{\text{SM}}$. However, masses can be generated through the Yukawa couplings ($\mathcal{L}_{\text{Yuk}}$ in Eq. 1.6), which describe integrations between all fermionic fields and the Higgs doublet, and can be written:

$$\mathcal{L}_{\text{Yuk}} = \sum_{\substack{\ell= \\ e,\mu,\tau}}\left(\mathcal{L}^{\ell}_{\text{Yuk}}\right) + \mathcal{L}^{q}_{\text{Yuk}}, \tag{1.16}$$

where ℓ and q denote the lepton and quark sectors respectively. First considering the lepton term, after SSB:

$$\mathcal{L}^{\ell}_{\text{Yuk}} = -g_\ell \left(\bar{\chi}_L \Phi \ell_R + \bar{\ell}_R \Phi^\dagger \chi_L \right) \tag{1.17}$$

$$= -\frac{g_\ell v}{\sqrt{2}} \left(\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L \right) \cdot \left(1 + \frac{H}{v} \right) \tag{1.18}$$

where g_ℓ is a coupling constant, and

$$\chi_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}. \tag{1.19}$$

Thus the leptons get a mass of $m_\ell = \frac{1}{\sqrt{2}} g_\ell v$, and interact with the Higgs field.

The story for $\mathcal{L}^{q}_{\text{Yuk}}$ is a bit more involved. Before SSB:

$$\mathcal{L}^{q}_{\text{Yuk}} = -y^u_{ij} \bar{Q}^i_L \Phi u^j_R - y^d_{ij} \bar{Q}^i_L \tilde{\Phi} d^j_R + \text{h.c.}, \tag{1.20}$$

where there is an implicit sum over all generations i and j, $y^{u,d}$ is a 3×3 matrix characterizing the Yukawa couplings between quark generations,

$$\tilde{\Phi}_i = \varepsilon_{ij} \Phi_j, \qquad \text{and} \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}. \tag{1.21}$$

After SSB, and the Higgs acquires a VEV, Eq. 1.20 becomes:

$$\mathcal{L}^{q}_{\text{Yuk}} = -\frac{v}{\sqrt{2}} \left(y^u_{ij} \bar{u}^i_L u_{R,j} + y^d_{ij} \bar{d}^i_L d_{R,j}, + \text{h.c.} \right) \cdot \left(1 + \frac{H}{v} \right), \tag{1.22}$$

where the mass of the quarks is $m_q = \frac{v}{\sqrt{2}} y^q_{ij}$. However, it is more convenient to change a basis in which the matrix m^q is diagonal such that $m^{\text{diag}}_{ij} = V_{Lik} m_{kl} (V^\dagger_R)_{lj}$. This is exactly equivalent to transforming the chiral quark fields for up- and down-type quarks accordingly:

$$q^\alpha_L = \left(V^q_L \right)_{\alpha i} q_L \qquad\qquad q^\alpha_R = \left(V^q_R \right)_{\alpha i} q_R, \tag{1.23}$$

where the index of the original basis is identified with i and the mass basis uses α.

The rotations of the basis of the chiral quark fields leave much of $\mathcal{L}_{\text{SM}}$ unchanged since $V^\dagger_{qL} V_{qL} = V^\dagger_{qR} V_{qR} = \mathbb{1}$. However, this is not the case in the charged current (CC) part of $\mathcal{L}_q$; which transforms as:

$$
\begin{aligned}
\mathcal{L}_q^{\mathrm{CC}} &= \frac{g}{2} i\gamma^\mu \left[\bar{u}_L d_L W_\mu^+ + \bar{d}_L u_L W_\mu^-\right] && (1.24)\\
&= \frac{g}{2} i\gamma^\mu \left[\bar{u}_L \left(V_{uL} V_{dL}^\dagger\right) d_L W_\mu^+ + \bar{d}_L \left(V_{dL} V_{uL}^\dagger\right) u_L W_\mu^-\right] && (1.25)\\
&= \frac{g}{2} i\gamma^\mu \left[V \bar{u}_L d_L W_\mu^+ + V^\dagger \bar{d}_L u_L W_\mu^-\right]. && (1.26)
\end{aligned}
$$

The matrix V is defined is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix and parameterizes the couplings between up- and down-type quarks in charged weak currents.

1.3 The CKM matrix and Unitarity Triangle

The CKM matrix is defined as:

$$
V = \left(V_{uL} V_{dL}^\dagger\right) = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{1.27}
$$

where each $|V_{ij}|$ parameterizes the probability of an up-type quark, i, transistioning to a down-type quark j. In the SM, it is assumed that the total charged current couplings of up- to down-type quarks is the same as down- to up-type. This means that the CKM matrix is unitary, $V^\dagger V = \mathbb{1}$, and therefore it contains four physical parameters: three angles (θ_{12}, θ_{13} and θ_{23}) and one complex phase (δ). In fact, the observation of CPV in kaon mixing led to the prediction of a third generation before its discovery because a 3×3 matrix is the smallest necessary for a phase to enter a unitary matrix.

There are many ways of representing the CKM matrix, one way is as a product of three rotation matrices, one of which contains the complex phase, this is the *standard* parameterization. The *Wolfenstein* parameterization is obtained by defining

$$
\sin\theta_{12} = \lambda, \qquad \sin\theta_{23} = A\lambda^2, \qquad \text{and} \qquad e^{-i\delta}\sin\theta_{13} = A\lambda^3(\rho - i\eta), \tag{1.28}
$$

which results in

$$
V \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{1.29}
$$

Since $A, \lambda \neq 0$, it is clear that V is not diagonal, and therefore flavour changing currents are allowed in the SM. However, the diagonal elements are close to unity and the CKM matrix

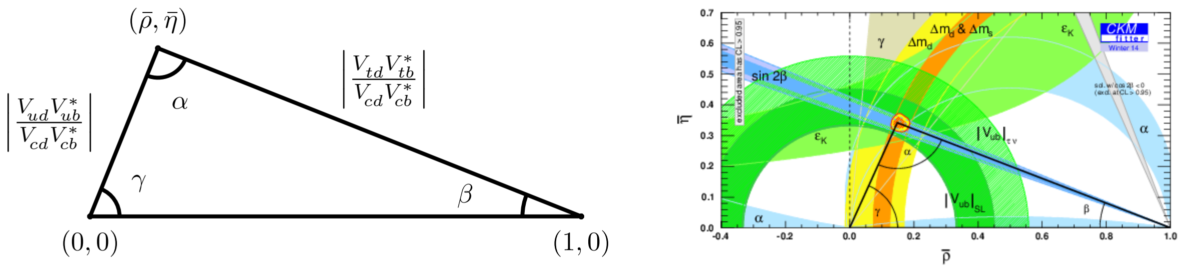

Figure 1.1: Schematic diagram of the Unitarity triangle given in Eq. 1.31 on the complex plane, where the base has been normalized to unit length. Alongside is shown the same triangle with coloured bands indicating various constraints on side lengths, angles and position of the apex.

exhibits a strong hierarchichal structure, for which there is no explaination in the SM.

Unitarity can be expressed as $V_{\alpha\beta}V^*_{\beta\gamma} = \delta_{\alpha\gamma}$ which gives six equations, when $\delta_{\alpha\gamma} = 0$, of the form:

$$\sum_{\alpha=1}^{3} V^*_{\alpha\beta}V_{\gamma\alpha} = 0 \qquad \sum_{\alpha=1}^{3} V^*_{\beta\alpha}V_{\alpha\gamma} = 0, \qquad \beta \neq \gamma. \tag{1.30}$$

These equations map closed triangles on the complex plane. Two of these triangles have all sides of similar length ($\mathcal{O}(\lambda^3)$). One of these is referred to as *the* Unitarity Triangle (UT) and is described by

$$1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0. \tag{1.31}$$

The UT has a base of unit length and an apex at

$$\bar\rho + i\bar\eta = (1 - \tfrac{1}{2}\lambda^2)(\rho + i\eta) = \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} \tag{1.32}$$

and the angles are

$$\alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right), \qquad \beta = \pi - \arg\left(\frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}\right), \qquad \text{and} \qquad \gamma = \arg\left(\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right). \tag{1.33}$$

This triangle is depicted in Fig. 1.1.

Precise determination of the CKM matrix elements is important, for they are each fundamental parameters of the SM. They also contain all the information about flavour violation and CPV that is allowed within the framework of the SM. This information is reprtesented by the UT, a current fit of measurement of angles and side lengths is shown in Fig. 1.1. If additional CP-violating phases were to exist beyond the SM, their effect would be seen in global fits of the

UT.

Each CKM matrix element is ususally measured in a particular way. For example, the elements V_{td} and V_{ts} are measured using $B-\bar{B}$ oscillations since tree level determination using t quarks is difficult to do with precision. However, the nature of NP is unknown and could effect different processes in different ways. A good example of this is V_{ub}, which is the CKM element known to the lowest degree of accuracy.

A determination of V_{ub} can be made using inclusive and exculsive measurements of $B \rightarrow X_u \ell \bar{\nu}_\ell$ decays. Inclusive measurements are made difficult from large $B \rightarrow X_c \ell \bar{\nu}_\ell$ backgrounds, while exclusive semi-leptonic modes suffer from uncertainties introduced by form factors. A combination of these results leads to a value of $|V_{ub}|_{\mathrm{SL}} = (4.13 \pm 0.49) \times 10^{-3}$ [1]. But, V_{ub} can also be obtained using the tree level decay of $B^+ \rightarrow \tau^+ \nu_\tau$: $|V_{ub}|_{\tau\nu} = (4.22 \pm 0.42) \times 10^{-3}$ [1]. The branching fraction which is used to calculate this latter value [6] is somewhat higher than the SM prediction and particularly sensitive to NP models which include a charged Higgs. Figure 1.1 shows the length of the side $|V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$ calculated by both values of V_{ub}. The decay $B^+ \rightarrow D_s^+ \phi$ has a very similar topology to $B^+ \rightarrow \tau^+ \nu_\tau$ in that it proceeds via the annihilation of the B^+ meson constituent quarks and the resulting W^+ decays into quarks that form the final state. This decay is discussed in Sec **??**.

The other side is sensitive to the value of V_{tb} and V_{td}. The value of V_{tb} is determined from decays of t quarks using the ratio $\mathcal{B}(t \rightarrow Wb)/\mathcal{B}(t \rightarrow Wq)$, where $q = d, b, s$. Oscillation frequencies of B^0 and B_s^0 mesons (Δm_d and Δm_s respectively) are used to measure the V_{td} and V_{ts}.

1.3.1 FCNCs

Since the SM is such a good description of physics at energy scales that have been probed to date, it is resonable to assume that this diverges at some cut off energy scale, Λ. This scale can be set based on the solution of the hierarchy problem, which indicates that Λ should be less than a few TeV. A bound can also be set on Λ by considering processes which are absent from SM processes at tree level, such as flavour changing neutral currnents (FCNCs). This results in a value of Λ which far exceeds the few TeV from solving the hierarchy problem. The conflict between these two determinations is named the *Flavour Problem.*

The most pessimistic solution to the flavour problem is *Minimal Flavour Violation* (MFV) which simply assumes that beyond SM physics follows a Yukawa coupling like structure in the flavour sector, this would lead to no discernable new physics in the flavour sector.

Bibliography

[1] Particle Data Group, J. Beringer *et al.*, *Review of particle physics*, Phys. Rev. **D86** (2012) 010001.

[2] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Tenth-Order QED Contribution to the Electron g-2 and an Improved Value of the Fine Structure Constant*, Phys. Rev. Lett. **109** (2012) 111807, `arXiv:1205.5368`.

[3] A. D. Sakharov, *SPECIAL ISSUE: Violation of CP in variance, C asymmetry, and baryon asymmetry of the universe*, Soviet Physics Uspekhi **34** (1991) 392.

[4] J. M. Cline, *Baryogenesis*, `arXiv:hep-ph/0609145`.

[5] P. Huet and E. Sather, *Electroweak baryogenesis and standard model CP violation*, Phys. Rev. **D51** (1995) 379, `arXiv:hep-ph/9404302`.

[6] Heavy Flavor Averaging Group, Y. Amhis *et al.*, *Averages of B-Hadron, C-Hadron, and tau-lepton properties as of early 2012*, `arXiv:1207.1158`.