Verifying Heap-Manipulating Programs with Unknown Procedure Calls

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Abstract. Verification of programs with invocations to unknown procedures is a practical problem, because in many scenarios not all codes of programs to be verified are available. Those unknown calls also pose a challenge for their verification. This paper addresses this problem with an attempt to verify the full functional correctness of such programs using pointer-based data structures. Provided with a Hoare-style specification $\{\Phi_{pr}\}$ prog $\{\Phi_{po}\}$ where program prog contains calls to some unknown procedure unknown, we infer a specification $mspec_u$ for unknown from the calling contexts, such that the problem of verifying prog can be safely reduced to the problem of proving that the procedure unknown (once its code is available) meets the derived specification $mspec_u$. The expected specification $mspec_u$ for the unknown procedure unknown is automatically calculated using an abduction-based shape analysis specifically designed for a combined abstract domain. We have also done some experiments to validate the viability of our approach.

Key words: Program verification, full-functional correctness, unknown procedures, abduction, combined domain

1 Introduction

While automated verification of heap-manipulating programs remains a big challenge [9, 19], significant advances have been seen recently since the emergence of separation logic [8, 16]. For instance, SpaceInvader [20, 3] can verify the pointer safety of a large portion of the Linux kernel and many device drivers using shared mutable data structures; THOR [12] employs additional numerical analysis to help gain better precision for data structure properties such as list length; Hip/Sleek [4, 13, 14] can verify more sophisticated properties involving both shape and numerical information, such as sortedness, height-balanced and red-black properties. These are all successful examples of verification/analysis of heap-manipulating programs, esp. those processing pointer-based shared mutable data structures.

However, a recent prevalent trend of component-based software engineering [10] poses great challenge for quality assurance and verification of programs. This methodology involves the integration of software components from both native development and third-parties, and thus the source code of some components/procedures might be unknown for verification.This problem is quite practical and has multiple forms in various scenarios.For example, some programs may have calls to third-party library procedures whose code is not accessible (e.g. in binary form). Some components may be invoked by remote procedure calls only with a native interface such as COM/DCOM [17]. Still, some components could be used for dynamic upgrading of running systems whose cost of being stopped/restarted is too expensive to bear [18]. Other scenarios include function pointers (e.g. in C), interface method invocation (e.g. in OO) and mobile code, which all contain procedures not available for static verification.

To verify such programs, existing approaches generally do not provide elegant solutions. For example, black-box testing [2] regards the unknown procedures as black-boxes to test their functionality, which cannot formally prove the absence of program bugs, therefore may not be enough for safety-critical systems. Likewise, specification mining [1] discovers possible specifications for the (unknown part of the) program by observing its execution and traces, which is also dynamically performed and bears the same problem. For static verifiers/analysers, SpaceInvader [3] simply assumes the program and the unknown procedure have disjoint memory footprints so that the unknown call can be safely ignored due to the hypothetical frame rule [15], whereas this assumption does not hold in many cases. Some methods [5, 7] try to take into account all possible implementations for the unknown procedure; however there can be too many such candidates in general, and hence the verification might be infeasible for large-scaled programs. Finally, some verifiers will just stop at the first unknown procedure call and provide an incomplete verification [14], which is obviously undesirable.

Approach and contributions. We propose a different approach in this paper to the verification of programs with unknown procedure calls. Given a specification $\mathsf{S} = \{\Phi_{pr}\}\ \mathsf{prog}\ \{\Phi_{po}\}$ for the program prog containing calls to an unknown procedure $\mathsf{unknown}$, our solution is to proceed with the verification for the known fragments of prog, and at the same time infer a specification S_u that is expected for the unknown procedure $\mathsf{unknown}$ based on the calling context(s). The problem of verifying the program prog against the specification S can now be safely reduced to the problem of verifying the procedure $\mathsf{unknown}$ against the inferred specification S_u, provided that the verification of the known fragments does not cause any problems. The inferred specification is subject to a later verification when an implementation or a specification for the unknown procedure becomes available. This is essentially an improvement of our previous work [11] by extending the program properties to be verified from simple pointer safety to full functional correctness of linked data structures. Such properties include structural numerical ones like size and height, relational numerical ones like sortedness, and multi-set ones like symbolic content. Our paper makes the following technical contributions:

– We propose a novel framework in a combined abstract domain (involving both shape and pure properties) for the verification of full functional correctness of programs with unknown calls.
– Our approach is essentially *top-down*, as it can be used to infer the specification for callee procedures based on the specification for the caller procedure. Hence it may benefit the general software development process as a complement for current *bottom-up* approaches [3, 14].
– We have invented an abduction mechanism which can be applied in this combined domain. It not only can infer shape-based anti-frames for an entailment, but also can discover corresponding pure information (numerical and/or multi-set) as well. We also defined a partial order as a guidance for the quality of abduction results.
– We have conducted some initial experimental studies to test the viability and performance of our approach. Preliminary results show that our approach can derive expressive specifications which fully capture the behaviours of the unknown code in many cases.

Outline. Section 2 employs a motivating example to informally illustrate our approach. Section 3 presents the programming language and the abstract domain for our analysis. Section 4 introduces our abductive reasoning. Section 5 depicts our verification algorithms. Experimental results are shown in Section 6, followed by some concluding remarks.

2 The Approach

We first introduce our specification mechanism, followed by an illustrative example for the verification.

2.1 Separation Logic and User-defined Predicates

Separation logic [8, 16] extends Hoare logic to support reasoning about shared mutable data structures. It adds two more connectives to classical logic: separation conjunction $*$ and spatial implication $-\!\!*$. The formula $\mathtt{p_1} * \mathtt{p_2}$ asserts that two heaps described by $\mathtt{p_1}$ and $\mathtt{p_2}$ are domain-disjoint, while $\mathtt{p_1} -\!\!* \mathtt{p_2}$ asserts that if the current heap is extended with a disjoint heap described by $\mathtt{p_1}$, then $\mathtt{p_2}$ holds in the extended heap. In this paper we only use separation conjunction.

Our abstract domain is founded on a hybrid logic of both separation logic and classical first-order logic to specify both separation and pure properties. Over this domain we allow user-defined inductive predicates. For example, with a data structure definition for a node in a list `data node { int val; node next; }`, we can define a predicate for a list with the content stored in its nodes as

$$\mathtt{root}{::}\mathtt{ll}\langle\mathtt{n}\rangle \equiv (\mathtt{root}{=}\mathtt{null}\wedge\mathtt{n}{=}0)\vee(\exists\mathtt{v},\mathtt{q},\mathtt{m}\cdot\mathtt{root}{::}\mathtt{node}\langle\mathtt{v},\mathtt{q}\rangle{*}\mathtt{q}{::}\mathtt{ll}\langle\mathtt{m}\rangle\wedge\mathtt{n}{=}\mathtt{m}{+}1)$$

The parameter `root` for the predicate `ll` is the root pointer referring to the list. Its length is denoted by `n`. A uniform notation $\mathtt{p}{::}\mathtt{c}\langle\mathtt{v}^*\rangle$ is used for either a singleton heap or a predicate. If $\mathtt{c}$ is a data node, the notation represents a singleton heap, $\mathtt{p}{\mapsto}\mathtt{c}[\mathtt{v}^*]$, e.g. the $\mathtt{root}{::}\mathtt{node}\langle\mathtt{v},\mathtt{q}\rangle$ above. If $\mathtt{c}$ is a predicate name, then the data structure pointed to by $\mathtt{p}$ has the shape $\mathtt{c}$ with parameters $\mathtt{v}^*$, e.g., the $\mathtt{q}{::}\mathtt{ll}\langle\mathtt{m}\rangle$ above.

We can also upgrade this predicate to one with symbolic content using multi-set as follows:

$$\mathtt{llB}\langle\mathtt{S}\rangle \equiv (\mathtt{root}{=}\mathtt{null} \wedge \mathtt{S}{=}\emptyset) \vee (\mathtt{root}{::}\mathtt{node}\langle\mathtt{v},\mathtt{q}\rangle * \mathtt{q}{::}\mathtt{llB}\langle\mathtt{S_1}\rangle \wedge \mathtt{S}{=}\mathtt{S_1}{\sqcup}\{\mathtt{v}\})$$

where we use the following shortened notation: (i) default $\mathtt{root}$ parameter in LHS may be omitted, (ii) unbound variables, such as $\mathtt{q}$ and $\mathtt{S_1}$, are implicitly existentially quantified. Meanwhile, later we may still use underscore _ to denote an implicitly quantified variable, for example $\mathtt{x}{::}\mathtt{ll}\langle\_\rangle$ meaning "we don't care about the length of the list".

If users want to verify a sorting algorithm, they can incorporate sortedness property into the above predicate as follows:

$$\begin{aligned}&\mathtt{sllB}\langle\mathtt{S}\rangle \equiv (\mathtt{root}{=}\mathtt{null} \wedge \mathtt{S}{=}\emptyset) \vee \\ &\quad(\mathtt{root}{::}\mathtt{node}\langle\mathtt{v},\mathtt{q}\rangle * \mathtt{q}{::}\mathtt{sllB}\langle\mathtt{S_1}\rangle \wedge \mathtt{S}{=}\{\mathtt{v}\}{\sqcup}\mathtt{S_1} \wedge (\forall \mathtt{u}{\in}\mathtt{S_1} \cdot \mathtt{v}{\leq}\mathtt{u}))\end{aligned}$$

Such user-supplied predicates can be used to specify method specifications. Meanwhile, we also allow user-defined lemmas to express coercion relations between predicates [13]. For example, we can express with lemma that a list with content $\mathtt{S}$ can be viewed as another list of length $|\mathtt{S}|$ as

$$\mathtt{root}{::}\mathtt{ll}\langle|\mathtt{S}|\rangle \longleftarrow \mathtt{root}{::}\mathtt{llB}\langle\mathtt{S}\rangle$$

This lemma mechanism is also supported by our verification. Sometimes it can benefit us significantly (e.g. in the abductive reasoning mentioned later).

2.2 Illustrative Example

In this section, we illustrate informally, via an example, how our approach verifies a program by inferring the specification for the unknown procedure it invokes.

Example 1 (Motivating example). Our goal is to verify the procedure `sort` against the given specification shown in Figure 1. According to the specification, the procedure takes in a non-empty linked list `x` and returns a sorted list referenced as `res`. The (symbolic) content of these two lists are identical (`S`). Note that `sort` calls an unknown procedure `unknown` at line `4`. As we do not have available knowledge about it, the discovery of its specifications is essential for both the verification and our understanding of the program (such that we may find out what sorting algorithm this procedure implements).

We conduct a forward analysis on the program body starting with the pre-condition $\mathtt{x}{::}\mathtt{llB}\langle\mathtt{S}\rangle$ (line `0`). The results of our analysis (e.g. the abstract states) are marked as comments in the code. The analysis carries on until it reaches the unknown procedure call at line `4`.

```
0 node sort(node x) requires $x::llB\langle S\rangle$ ensures $res::sllB\langle S\rangle$
1 {  // res is the value returned by the procedure
1a    // Forward analysis begins with current state $\sigma : x::llB\langle S\rangle$
2   if (x == null) return null;
2a    // $\sigma : x::llB\langle S\rangle \wedge x{=}null \wedge res{=}null$
2b    // Check whether current state meets the postcondition: $\sigma \vdash res::sllB\langle S\rangle$
2b    //  which succeeds; the verification on this branch terminates
3   else {
3a    // $\sigma : x::llB\langle S\rangle \wedge x{\neq}null$
3b    // Unknown call is now encountered (line 4); extract its precondition from $\sigma$:
3c    // $\Phi^{u}_{pr} := \mathsf{Local}(\sigma, \{x\}) := x::llB\langle S\rangle \wedge x{\neq}null$
3d    // Also distinguish the frame part not touched by unknown call:
3e    // $R_0 := \mathsf{Frame}(\sigma, \{x\}) := emp \wedge x{\neq}null$
4   node y = unknown(x);
4a    // Immediately after the unknown call we know nothing about its effect, so
4b    // we begin to discover its post-effect starting from emp (saved in $\sigma'$):
4c    // $\sigma'_0 : emp \wedge x{=}a \wedge y{=}res_u$      $\sigma := R_0 * \sigma'_0$      $\sigma' := \sigma'_0$
4d    // Next instruction (y.next) requires y be a node
4e    // But the entailment checking $\sigma \vdash y::node\langle v,p\rangle$ fails
4f    // This requirement might be part of the unknown call's post-effect; we use
4g    // abduction to find it and add it to current state and unknown call's post:
4h    // $\sigma * [\sigma'_1] \rhd y::node\langle v,p\rangle$ (s.t. $\sigma * \sigma'_1 \vdash y::node\langle v,p\rangle * true$)
4i    // $\sigma'_1 : y::node\langle v,p\rangle$              $\sigma := \sigma * \sigma'_1$      $\sigma' := \sigma' * \sigma'_1$
5   node z = y.next;
5a    // Current state $\sigma : y::node\langle v,z\rangle$
5b    // Next instruction invokes this procedure recursively and requires its pre, but
5c    //  $\sigma \vdash z::llB\langle S_1\rangle$ fails possibly due to lack of knowledge about unknown call
5d    // Again we use abduction to find the missing part of unknown call's post-effect
5e    // $\sigma * [\sigma'_2] \rhd z::llB\langle S_1\rangle$ (s.t. $\sigma * \sigma'_2 \vdash z::llB\langle S_1\rangle * true$)
5f    // $\sigma'_2 : z::llB\langle S_1\rangle$               $\sigma := \sigma * \sigma'_2$      $\sigma' := \sigma' * \sigma'_2$
6   node w = sort(z);
6a    // Current state $\sigma : y::node\langle v,z\rangle * w::sllB\langle S_1\rangle$ (w already refers to a sorted list)
7   y.next = w;
7a    // Current state $\sigma : y::node\langle v,w\rangle * w::sllB\langle S_1\rangle$
8   return y;
8a    // $\sigma : y::node\langle v,w\rangle * w::sllB\langle S_1\rangle \wedge res{=}y$; it should imply sort's postcondition
8b    // But $\sigma \vdash res::sllB\langle S\rangle$ still fails, suggesting more post-effect of unknown call
8c    // A final abduction is conducted to find it: $\sigma * [\sigma'_3] \rhd res::sllB\langle S\rangle$
8d    // $\sigma'_3 : S{=}\{v\}{\sqcup}S_1 \wedge \forall u{\in}S_1{\cdot}v{\leq}u$   $\sigma := \sigma * \sigma'_3$      $\sigma' := \sigma' * \sigma'_3$
8e    // All abduction results will be combined at last to form unknown call's post
9   } }
9a // $\Phi^{u}_{pr} : a::llB\langle S\rangle \wedge a{\neq}null$ (a is the unknown procedure's formal parameter)
9b // $\Phi^{u}_{po} : res_u::node\langle v,b\rangle * b::llB\langle S_1\rangle \wedge S{=}\{v\}{\sqcup}S_1 \wedge \forall u{\in}S_1{\cdot}v{\leq}u$
```

Fig. 1. Verification of sort which invokes an unknown procedure unknown.

As afore-shown, the current state before line 4 is $\texttt{x}::\texttt{llB}\langle\texttt{S}\rangle \wedge \texttt{x}{\neq}\texttt{null}$ (σ at line 3a). Then we want to discover the precondition for the unknown call from it. To do that, we split σ into two disjoint parts: the local part $\Phi^{\mathtt{u}}_{pr}$ (line 3c) that is depended on, and possibly mutated by, the unknown procedure; and the frame part $\mathtt{R_0}$ (line 3e) that is not accessed by the unknown procedure. Intuitively, the local part of a state w.r.t. a set of variables X is the part of the heap reachable from variables in X (together with pure information); while the frame part denotes the unreachable heap part (together with pure information). For example, for a program state $\texttt{x}::\texttt{node}\langle\texttt{a},\texttt{w}\rangle * \texttt{y}::\texttt{node}\langle\texttt{b},\texttt{z}\rangle * \texttt{z}::\texttt{node}\langle\texttt{c},\texttt{null}\rangle \wedge \texttt{w}{=}\texttt{z}$, its local part w.r.t. $\{\texttt{x}\}$ is $\texttt{x}::\texttt{node}\langle\texttt{a},\texttt{w}\rangle * \texttt{z}::\texttt{node}\langle\texttt{c},\texttt{null}\rangle \wedge \texttt{w}{=}\texttt{z}$, and its frame part w.r.t. $\{\texttt{x}\}$ is $\texttt{y}::\texttt{node}\langle\texttt{b},\texttt{z}\rangle \wedge \texttt{w}{=}\texttt{z}$. We will have their formal definitions in Section 5. Thus we take $\Phi^{\mathtt{u}}_{pr}$ (line 3c) as a crude precondition for the unknown procedure, since it denotes the part of program state that is accessible, and hence potentially usable, by the unknown call. The frame part $\mathtt{R_0}$ is not touched by the unknown call and will remain in the post-state, as shown in line 4c.

At line 4c, the abstract state after the unknown call (σ) consists of two parts: one is the aforesaid frame $\mathtt{R_0}$ not accessed by the call, and the other is the procedure's postcondition which is unfortunately not available. Our next step is to discover the postcondition by examining the code fragment after the unknown call (lines 4a to 8e). For this task, a traditional approach is a backward reasoning from the caller's postcondition towards the unknown call's postcondition. However, this is proven infeasible for separation logic based shape domain by previous works [3], and hence we employ another approach with a forward reasoning from the unknown call towards the caller's postcondition, using *abduction* to discover the unknown call's postcondition.

Initially, we assume the unknown procedure having an empty heap σ'_0 as its postcondition[1], and gradually discover the missing parts of the postcondition during the symbolic execution of the code fragment after the unknown call. To do that, our analysis keeps track of a pair (σ, σ') at each program point, where σ refers to the current heap state, and σ' denotes the expected postcondition discovered so far for the unknown procedure. The notations σ'_i are used to represent parts of the discovered postcondition.

At line 5, y.next is dereferenced, whose value is then assigned to z. Such dereference causes a problem, as we have an empty heap beforehand (σ in line 4c). However, this is not necessarily due to a program error; it might be attributed to the fact that the unknown call's postcondition is still unknown. Therefore, our analysis performs an abduction (line 4h) to infer the missing part σ'_1 for σ such that $\sigma * \sigma'_1$ implies that y points to a node. As shown in line 4i, σ'_1 is inferred to be $\texttt{y}::\texttt{node}\langle\texttt{v},\texttt{p}\rangle$, which is accumulated into σ' as part of the expected postcondition of the unknown procedure. (We will explain the details for abduction in Section 4.) Now the heap state combined with the inferred σ'_1 meets the requirement of the dereference, and thus the forward analysis continues.

[1] Note that we introduce fresh logical variables a and $\mathtt{res_u}$ to record the value of x and y when unknown returns.

At line 6, the procedure `sort` is called recursively. Here the current heap state still does not satisfy the precondition of `sort` (as shown in line 5c). Blaming the lack of knowledge about the unknown call's postcondition, we conduct another abduction (line 5e) to infer the missing part σ_2' for σ such that $\sigma * \sigma_2'$ entails the precondition of `sort` w.r.t. some substitution $[\mathtt{z}/\mathtt{x}]$. Updated with the abduction result $\mathtt{z}{::}\mathtt{llB}\langle\mathtt{S}_1\rangle$, the program state now meets the precondition of `sort`, which is later transformed to $\mathtt{w}{::}\mathtt{sllB}\langle\mathtt{S}_1\rangle$ as the effect of sorting over `z`.

After that, line 7 links `y` and the sorted list `w` together. Then `y` is returned as the procedure's result at last. The corresponding state σ at line 8a is expected to establish the postcondition of `sort` for the overall verification to succeed. However, it does not (as shown in line 8b). Again this might be because part of the unknown call's postcondition is still missing. Therefore, we perform a final abduction (line 8c) to infer the missing σ_3' as follows:

$$(\mathtt{y}{::}\mathtt{node}\langle\mathtt{v},\mathtt{w}\rangle * \mathtt{w}{::}\mathtt{sllB}\langle\mathtt{S}_1\rangle \wedge \mathtt{res}{=}\mathtt{y}) * [\sigma_3'] \rhd \mathtt{res}{::}\mathtt{sllB}\langle\mathtt{S}\rangle$$

such that $\sigma * \sigma_3'$ implies the postcondition. In this case, our abductor returns σ_3' as a sophisticated pure constraint $\mathtt{S}{=}\{\mathtt{v}\}{\sqcup}\mathtt{S}_1 \wedge \forall \mathtt{u}{\in}\mathtt{S}_1{\cdot}\mathtt{v}{\le}\mathtt{u}$ as the result which is then added into σ', as shown in line 8d.

Finally, we generate the expected pre/post-specification for the unknown procedure (lines 9a and 9b). The precondition is obtained from the local pre-state of the unknown call, $\Phi^{\mathtt{u}}_{pr}$ at line 3c, by replacing all variables that are aliases of `a` with the formal parameter `a`. The postcondition is obtained from the accumulated abduction result, σ', after performing a similar substitution (which also involves formal parameter $\mathtt{res_u}$). Our discovered specification for the unknown procedure `node unknown(node a)` is:

$\Phi^{\mathtt{u}}_{pr} : \mathtt{a}{::}\mathtt{llB}\langle\mathtt{S}\rangle \wedge \mathtt{a}{\neq}\mathtt{null}$
$\Phi^{\mathtt{u}}_{po} : \exists \mathtt{b} \cdot \mathtt{res_u}{::}\mathtt{node}\langle\mathtt{v},\mathtt{b}\rangle * \mathtt{b}{::}\mathtt{llB}\langle\mathtt{S}_1\rangle \wedge \mathtt{S}{=}\{\mathtt{v}\}{\sqcup}\mathtt{S}_1 \wedge \forall \mathtt{u}{\in}\mathtt{S}_1{\cdot}\mathtt{v}{\le}\mathtt{u}$

This derived specification has two implications. The first is that the entire program is verified on the condition that `unknown` meets such specification. The second is a hint of the behaviours of both the caller (`sort`) and the callee (`unknown`), that is, `unknown` should take as input a list and returns another list with identical content as the input, whose smallest element is exactly at its head. After calling it, `sort` only needs to sort the rest of the list to accomplish the whole task of sorting. From this we may guess that `sort` could be a selection-sort algorithm if `unknown` always selects the smallest element and puts it on the list head, or it could be bubble-sort algorithm if `unknown` is a bubbling procedure to exchange two adjacent elements, where the first is larger than the second, from the list tail. Therefore this enhances our understanding of the whole program, and we can verify it as soon as we have the code of `unknown`.

3 Language and Abstract Domain

To simplify presentation, we focus on a strongly-typed C-like imperative language in Figure 2. A program *Prog* consists of two parts: type declarations

$$
\begin{array}{lll}
\mathit{Prog} & ::= \mathit{tdecl}\ \mathit{meth}\ \mathit{munk} & \mathit{tdecl} ::= \mathit{datat} \mid \mathit{spred} \mid \mathit{lemma} \\
\mathit{datat} & ::= \texttt{data}\ c\ \{\ \mathit{field}\ \} \quad \mathit{field} ::= t\ x & t ::= c \mid \tau \\
\mathit{meth} & ::= t\ mn\ ((\boldsymbol{t}\ \boldsymbol{x}); (\boldsymbol{t}\ \boldsymbol{y}))\ \mathit{mspec}\ \{e\} & \tau ::= \texttt{int} \mid \texttt{bool} \mid \texttt{void} \\
\mathit{munk} & ::= t\ mn\ ((\boldsymbol{t}\ \boldsymbol{x}); (\boldsymbol{t}\ \boldsymbol{y}))\ \mathit{mspec}\ \{v\} & \\
e & ::= d \mid d[x] \mid x{=}e \mid e_1; e_2 \mid t\ x;\ e \mid \texttt{if}\ (x)\ e_1\ \texttt{else}\ e_2 \mid \texttt{while}\ x\ \{e\}\ \texttt{inv}\ \Delta & \\
u & ::= \mathit{unk}(\boldsymbol{x}; \boldsymbol{y}) \mid \mathit{unk}(\boldsymbol{x_0}; \boldsymbol{y_0}); e_1; \mathit{unk}_1(\boldsymbol{x_1}; \boldsymbol{y_1}); e_2; ...; e_{n-1}; \mathit{unk}_n(\boldsymbol{x_n}; \boldsymbol{y_n}) \mid & \\
 & \quad \texttt{if}\ (x)\ v\ \texttt{else}\ e \mid \texttt{if}\ (x)\ e\ \texttt{else}\ v \mid \texttt{if}\ (x)\ v_1\ \texttt{else}\ v_2 \mid \texttt{while}\ x\ \{v\}\ \texttt{inv}\ \Delta & \\
v & ::= e_1; u; e_2 & \\
d & ::= \texttt{null} \mid k^{\tau} \mid x \mid \texttt{skip} \mid \texttt{new}\ c(\boldsymbol{x}) \mid mn(\boldsymbol{x}; \boldsymbol{y}) & \\
d[x] & ::= x.f \mid x.f{:=}z \mid \texttt{free}(x) &
\end{array}
$$

Fig. 2. A core (C-like) imperative language.

and method declarations. The type declarations *tdecl* can define either data type *datat* (e.g. `node`) or predicate *spred* (e.g. `llB`) or lemma *lemma*. The method declarations include *meth* and *munk*, of which the second contains invocations to unknown procedures while the first does not. The definitions for *spred*, *lemma* and *mspec* are given later in Figure 3.

Note that the language is expression-oriented, so the body of a method is an expression composed of standard instructions and constructors of an imperative language. e is the (recursively defined) program constructor and d and $d[x]$ are atom instructions. Note also that the language allows both call-by-value and call-by-reference method parameters (which are separated with a semicolon ; where the ones before ; are call-by-value and the ones after are call-by-reference).

To address the unknown calls, we employ *unknown constructors* u and v to denote expressions that involve invocations to the unknown procedures ($\mathit{unk}(\boldsymbol{x}, \boldsymbol{y})$). An *unknown block* v is defined as a sequence of normal expressions sandwiching an *unknown expression* u, which can be a single unknown call, or a sequence of unknown calls, or an if-conditional statement/while loop containing an unknown block. Our aim is to discover the specifications for the unknown procedures in u and v to verify the whole program.

Our specification language (in Figure 3) allows (user-defined) shape predicates to specify both separation and pure properties. The shape predicates *spred* and lemmas *lemma* are constructed with disjunctive constraints Φ. We require that the predicates be well-formed [14].

A conjunctive abstract program state σ is composed of a heap (shape) part κ and a pure part π, where π consists of γ, ϕ and φ as aliasing, numerical and bag information, respectively. We use SH to denote a set of such conjunctive states. During our verification, the abstract program state at each program point will be a disjunction of σ's, denoted by Δ (and the set of such disjunctions $\mathcal{P}_{\mathsf{SH}}$). An abstract state Δ can be normalised to the Φ form [14].

The memory model of our specification formulae is adapted from the model given for "early versions" of separation logic [16], except that we have extensions to handle user-defined shape predicates and related pure properties. We assume

$$\begin{array}{lll}
\mathit{mspec} & ::= \mathit{requires}\ \Phi_{pr}\ \mathit{ensures}\ \Phi_{po} & \mathit{spred} ::= \texttt{root}{::}c\langle\boldsymbol{v}\rangle \equiv \Phi \\
\mathit{lemma} & ::= \texttt{root}{::}c\langle\boldsymbol{v}\rangle \wedge \pi \bowtie \Phi & \bowtie ::= \longleftarrow \mid \longrightarrow \mid \longleftrightarrow \\
\Delta & ::= \Phi \mid \Delta_1 \vee \Delta_2 \mid \Delta \wedge \pi \mid \Delta_1 * \Delta_2 \mid \exists v \cdot \Delta & \\
\Phi & ::= \bigvee \boldsymbol{\sigma} & \sigma ::= \exists \boldsymbol{v} \cdot \kappa \wedge \pi \\
\kappa & ::= \texttt{emp} \mid v{::}c\langle\boldsymbol{v}\rangle \mid \kappa_1 * \kappa_2 & \pi ::= \gamma \wedge \phi \\
\gamma & ::= v_1{=}v_2 \mid v{=}\texttt{null} \mid v_1{\neq}v_2 \mid v{\neq}\texttt{null} \mid \texttt{true} \mid \gamma_1 \wedge \gamma_2 & \\
\phi & ::= \varphi \mid b \mid a \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \neg\phi \mid \exists v \cdot \phi \mid \forall v \cdot \phi & \\
b & ::= \texttt{true} \mid \texttt{false} \mid v \mid b_1 = b_2 & a ::= s_1{=}s_2 \mid s_1 {\leq} s_2 \\
s & ::= k^{\texttt{int}} \mid v \mid k^{\texttt{int}} \times s \mid s_1 + s_2 \mid -s \mid \mathit{max}(s_1, s_2) \mid \mathit{min}(s_1, s_2) \mid |\texttt{B}| & \\
\varphi & ::= v \in \texttt{B} \mid \texttt{B}_1 {=} \texttt{B}_2 \mid \texttt{B}_1 \sqsubset \texttt{B}_2 \mid \texttt{B}_1 \sqsubseteq \texttt{B}_2 \mid \forall v \in \texttt{B} \cdot \phi \mid \exists v \in \texttt{B} \cdot \phi & \\
\texttt{B} & ::= \texttt{B}_1 \sqcup \texttt{B}_2 \mid \texttt{B}_1 \sqcap \texttt{B}_2 \mid \texttt{B}_1 - \texttt{B}_2 \mid \emptyset \mid \{v\} &
\end{array}$$

Fig. 3. The specification language.

sets Loc of memory locations, Val of primitive values (with $0 \in \mathsf{Val}$ denoting `null`), Var of variables (program and logical variables), and ObjVal of object values stored in the heap, with $c[f_1 {\mapsto} \nu_1, .., f_n {\mapsto} \nu_n]$ denoting an object value of data type c where $\nu_1, .., \nu_n$ are current values of the corresponding fields $f_1, .., f_n$. Let $s, h \models \Delta$ denote the model relation, i.e. the stack s and heap h satisfy Δ, with h, s from the following concrete domains:

$$h \in \ \mathit{Heaps} \ =_{df} \mathsf{Loc} \rightharpoonup_{fin} \mathsf{ObjVal} \qquad s \in \ \mathit{Stacks} \ =_{df} \mathsf{Var} \rightarrow \mathsf{Val} \cup \mathsf{Loc}$$

Note that each heap h is a finite partial mapping while each stack s is a total mapping, as in the classical separation logic [8, 16]. The detailed model definitions can be found in Nguyen et al. [14].

In the verification we use three kinds of variables in the Var set: program variables (PVar), logical variables as unknown procedure's formal parameters (SVar), and logical variables to record intermediate states (LVar). For the first two groups we use variables without subscription (such as `x` and `a`), and denote a program variable's initial value as unprimed, and its current (and hence final) value as primed [4, 14]. For the third group, we use subscript ones like $\texttt{x}_1$. For instance, for a code segment $\texttt{x} := \texttt{x} + 1;\ \texttt{x} := \texttt{x} - 2$ starting with state $\{\texttt{x}{>}1\}$, we have the following reasoning procedure:

$$\{\texttt{x}'{=}\texttt{x} \wedge \texttt{x}{>}1\}\ \texttt{x:=x+1}\ \{\texttt{x}{>}1 \wedge \texttt{x}'{=}\texttt{x}{+}1\}\ \texttt{x:=x-2}\ \{\texttt{x}{>}1 \wedge \texttt{x}'{=}\texttt{x}_1{-}2 \wedge \texttt{x}_1{=}\texttt{x}{+}1\}$$

where the final value of `x` is recorded in variable $\texttt{x}'$ and $\texttt{x}_1$ keeps an intermediate state of `x`.

4 Abduction

As shown in Section 2, when analysing the code after an unknown call, it is possible that the current state cannot meet the required precondition for the next instruction due to the lack of information about the unknown procedure.

Therefore we need to infer the unknown procedure's specification with *abduction* (or abductive reasoning) [3, 6]. It works as follows: for a failed entailment checking $\sigma_1 \vdash \sigma_2 * \texttt{true}$, it attempts to compute an anti-frame σ', such that $\sigma_1 * \sigma' \vdash \sigma_2 * \texttt{true}$ succeeds. For instance, the entailment checking $\texttt{emp} \vdash \texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$ fails as the antecedent contains an empty heap. Then $\texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$ will be found to strengthen the antecedent and validate the entailment $\texttt{emp} * \texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle \vdash \texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$.

An abduction $\sigma_1 * [\sigma'] \rhd \sigma_2$ can also be written as $\sigma_1 * [\sigma'] \rhd \sigma_2 * \sigma_3$, where σ_1 and σ_2 are inputs, σ' is the abduction result (the anti-frame), and σ_3 is the frame part resulted from the entailment checking $\sigma_1 * \sigma' \vdash \sigma_2$.

$$\frac{\sigma \nvdash \sigma_1 * \texttt{true} \quad \sigma_1 \vdash \sigma * \sigma' \quad \sigma * \sigma' \vdash \sigma_1 * \sigma_2}{\sigma * [\sigma'] \rhd \sigma_1 * \sigma_2}$$

$$\frac{\begin{array}{c}\sigma \nvdash \sigma_1 * \texttt{true} \quad \sigma_1 \nvdash \sigma * \texttt{true} \quad \sigma_0 \in \mathsf{unroll}(\sigma) \quad \mathsf{data\_no}(\sigma_0) \leq \mathsf{data\_no}(\sigma_1) \\ \sigma_0 \vdash \sigma_1 * \sigma' \text{ or } \sigma_0 * [\sigma_0'] \rhd \sigma_1 * \sigma' \quad \sigma''{=}\mathit{XPure}_1(\sigma') \quad \sigma \wedge \sigma'' \vdash \sigma_1 * \sigma_2\end{array}}{\sigma \wedge [\sigma''] \rhd \sigma_1 * \sigma_2}$$

$$\frac{\sigma \nvdash \sigma_1 * \texttt{true} \quad \sigma_1 \nvdash \sigma * \texttt{true} \quad \sigma_1 * [\sigma_1'] \rhd \sigma * \sigma' \quad \sigma''{=}\mathit{XPure}_1(\sigma') \quad \sigma \wedge \sigma'' \vdash \sigma_1 * \sigma_2}{\sigma \wedge [\sigma''] \rhd \sigma_1 * \sigma_2}$$

$$\frac{\sigma \nvdash \sigma_1 * \texttt{true} \quad \sigma_1 \nvdash \sigma * \texttt{true} \quad \sigma * \sigma_1 \nvdash \texttt{false}}{\sigma * [\sigma_1] \rhd \sigma_1 * \sigma_2}$$

Fig. 4. Abduction rules.

Our abduction rules given in Figure 4 deal with four different cases. The first rule triggers when the LHS (σ) does not imply the RHS (σ_1) but the RHS implies the LHS with some formula (σ') as the frame. This rule is quite general and applies in many cases, such as the state immediately after an unknown call where we start with emp as the heap state. For the example above $\texttt{emp} \nvdash \texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$, the RHS can entail the LHS with frame $\texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$. The abduction then checks whether σ plus the frame information σ' entails σ_1 with some frame formula σ_2 (emp in this example), and returns the result $\texttt{x}{::}\texttt{llB}\langle\texttt{S}\rangle$.

In the case described by the second rule, neither side implies the other, e.g. for $\texttt{x}{::}\texttt{sllB}\langle\texttt{S}\rangle$ as LHS (σ) and $\exists \texttt{p}, \texttt{u}, \texttt{v} \cdot \texttt{x}{::}\texttt{node}\langle\texttt{u}, \texttt{p}\rangle * \texttt{p}{::}\texttt{node}\langle\texttt{v}, \texttt{null}\rangle$ as RHS (σ_1). As the shape predicates in the antecedent σ are formed by disjunctions according to their definitions (like sllB), its certain disjunctive branches may imply σ_1. As the rule suggests, to accomplish abduction $\sigma * [\sigma''] \rhd \sigma_1 * \sigma_2$, we first unfold σ ($\sigma_0 \in \mathsf{unroll}(\sigma)$) and try entailment or further abduction with the results (σ_0) against σ_1. If it succeeds with a frame σ', then we first obtain a pure approximation of σ' with *XPure* [14], and confirm the abduction by ensuring $\sigma \wedge \sigma'' \vdash \sigma_1 * \sigma_2$, for some σ_2. For the example above, the abduction returns $|\texttt{S}|{=}2$ as the anti-frame σ' and discovers the nontrivial frame $\texttt{S}{=}\{\texttt{u}, \texttt{v}\} \wedge \texttt{u}{\leq}\texttt{v}$ (σ_2). Note the function $\mathsf{data\_no}$ returns the number of data nodes in a state, e.g. it returns one for $\texttt{x::node}\langle\texttt{v},\texttt{p}\rangle * \texttt{p::llB}\langle\texttt{T}\rangle$. This syntactic check is important for the termination of the abduction. The unroll unfolds all shape predicates once in σ, normalises the result to a disjunctive form ($\bigvee_{i=1}^{n}\sigma_i$), and returns the result as a set of formulae ($\{\sigma_1, ..., \sigma_n\}$).An instance is that it expands $\texttt{x::node}\langle\texttt{v},\texttt{p}\rangle * \texttt{p::llB}\langle\texttt{T}\rangle$ to be $\{\texttt{x::node}\langle\texttt{v},\texttt{p}\rangle \wedge \texttt{p=null} \wedge \texttt{T}=\emptyset, \exists \texttt{u},\texttt{q},\texttt{T}_1,\texttt{k}\cdot \texttt{x::node}\langle\texttt{v},\texttt{p}\rangle*\texttt{p::node}\langle\texttt{u},\texttt{q}\rangle*\texttt{q::llB}\langle\texttt{T}_1\rangle\wedge\texttt{T}=\texttt{T}_1\cup\{\texttt{k}\}\}$.The *XPure* is a strengthened version of that in [14], as it also keeps the pure part of σ' in the result.

In the third rule, neither side entails the other, and the second rule does not apply, for example $\exists \texttt{p},\texttt{u},\texttt{v}\cdot \texttt{x::node}\langle\texttt{u},\texttt{p}\rangle * \texttt{p::node}\langle\texttt{v},\texttt{null}\rangle$ as LHS (σ) and $\exists \texttt{S}\cdot \texttt{x::sllB}\langle\texttt{S}\rangle$ as RHS (σ_1). In this case the antecedent cannot be unfolded as they are already data nodes. As the rule suggests, it reverses two sides of the entailment and applies the second rule to uncover the constraints σ_1' and σ'. Then it checks that the LHS (σ), with σ' added, does imply the RHS (σ_1) before it returns σ'. For the example above, the abduction returns $\texttt{u}\leq\texttt{v}$ which is essential for the two nodes to form a sorted list (σ_1).

When an abduction is conducted, the first three rules should be attempted first; if they do not succeed in finding a solution, the last rule is invoked to simply add the consequent to the antecedent, provided that they are consistent. It is effective for situations like $\texttt{x::node}\langle\_,\_\rangle \nvdash \texttt{y::node}\langle\_,\_\rangle$, where we should add $\texttt{y::node}\langle\_,\_\rangle$ to the LHS directly (as the other three rules do not apply here).

One observation on abduction is that there can be many solutions of the anti-frame σ' for the entailment $\sigma_1 * \sigma' \vdash \sigma_2 * \texttt{true}$ to succeed. For instance, $\texttt{false}$ is always a solution but should be avoided where possible. For all possible solutions to an abduction, we can compare their "quality" with a partial order $\preceq$ over SH defined by the entailment relationship ($\vdash$):

$$\sigma_1 \preceq \sigma_2 =_{df} \sigma_2 \vdash \sigma_1 * \texttt{true}$$

and the smaller (weaker) one in two abduction solutions is regarded as better. We prefer to find solutions that are (potentially locally) minimal with respect to $\preceq$ and consistent. However, such solutions are generally not easy to compute and could incur excess cost (with additional disjunction in the analysis). Therefore, our abductive inference is designed more from a practical perspective to discover anti-frames that should be suitable as specifications for unknown procedures, and the partial order $\preceq$ is more a guidance of the decision choices of our abduction implementation, rather than a guarantee to find the theoretically best solution.

5 Verification

This section presents our algorithms to verify programs with unknown calls.

5.1 Main Verification Algorithm

Our main verification algorithm is given in Figure 5. It verifies an unknown block v (the third parameter) against given specifications mspec_v (the second parameter). The first parameter includes the specifications of already available procedures which might be invoked as well as the unknown ones in the program to be verified. Upon successful verification, this algorithm returns specifications that should be met by the unknown procedures in v. If the verification fails, it suggests that the current program cannot meet one or more given specifications due to a potential program bug. The specifications for unknown procedures will be expressed in terms of special variables $\boldsymbol{a}, \boldsymbol{b}$, etc. as in the earlier example.

```
Algorithm Verify($\mathcal{T}, mspec_v, v$)
1  Denote $v$ as $\{e_1; u; e_2\}$ ;  $mspec_u := \emptyset$
2  $(\boldsymbol{x_0}, \boldsymbol{y_0}) :=$ prog_var($v$) ;  $(\boldsymbol{x}, \boldsymbol{y}) :=$ prog_var($u$)
3  foreach ($requires\ \Phi_{pr}\ ensures\ \Phi_{po}$) $\in mspec_v$ do
4     $\mathsf{S}_0 := [\![e_1]\!]_{\mathcal{T}}\{\Phi_{pr} \wedge \boldsymbol{y_0'}{=}\boldsymbol{y_0}\}$
5     if false $\in \mathsf{S}_0$ then return fail endif
6     foreach $\sigma \in \mathsf{S}_0$ do
7        $\Phi^u_{pr} :=$ Local($\sigma, \{\boldsymbol{x}, \boldsymbol{y}\}$)
8        $\boldsymbol{z} :=$ fv($\Phi^u_{pr}$) $\setminus \{\boldsymbol{x}, \boldsymbol{y}\}$
9        $\mathsf{S} := [\![e_2]\!]^{\mathsf{A}}_{\mathcal{T}}\{([\boldsymbol{b}/\boldsymbol{y}]\ \mathsf{Frame}(\sigma, \{\boldsymbol{x}, \boldsymbol{y}\}) \wedge \boldsymbol{x}{=}\boldsymbol{a} \wedge$
              $\boldsymbol{y}{=}\boldsymbol{b} \wedge \boldsymbol{z}{=}\boldsymbol{c}, \mathsf{emp} \wedge \boldsymbol{x}{=}\boldsymbol{a} \wedge \boldsymbol{y}{=}\boldsymbol{b} \wedge \boldsymbol{z}{=}\boldsymbol{c})\}$
10       $\mathsf{S}' := \{\ (\sigma, \sigma')\ |\ (\sigma, \sigma'){\in}\mathsf{S} \wedge \sigma \vdash \Phi_{po} * \mathtt{true}\ \} \ \cup$
                 $\{\ (\sigma * \sigma'', \sigma' * \sigma'')\ |\ (\sigma, \sigma'){\in}\mathsf{S} \wedge$
                   $\sigma \nvdash \Phi_{po}{*}\mathtt{true} \wedge \sigma{*}[\sigma''] \rhd \Phi_{po}{*}\mathtt{true}\ \}$
11       if $\exists(\sigma, \sigma'){\in}\mathsf{S}'$ . fv($\sigma'$) $\nsubseteq$ ReachVar($\sigma, \{\boldsymbol{a}, \boldsymbol{b}\}$)
            then return (fail, $\sigma'$) endif
12       foreach $(\sigma, \sigma') \in \mathsf{S}'$ do
13          $\Phi^u_{pr} := [\boldsymbol{a}/\boldsymbol{x}, \boldsymbol{b}/\boldsymbol{y}, \boldsymbol{c}/\boldsymbol{z}]\ \Phi^u_{pr}$
14          $\Phi^u_{po} :=$ sub_alias($\sigma', \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$)
15          $g := (\mathsf{fv}(\Phi^u_{pr}) \cap \mathsf{fv}(\Phi^u_{po})) \cup \{\boldsymbol{a}, \boldsymbol{b}\}$
16          $mspec_u := mspec_u \cup \{(requires\ \exists(\mathsf{fv}(\Phi^u_{pr}) \backslash g) \cdot \Phi^u_{pr}$
                                    $ensures\ \Phi^u_{po})\}$
17       end foreach
18    end foreach
19 end foreach
20 $\mathcal{T}_u :=$ CaseAnalysis($\mathcal{T}, mspec_u, u$)
21 return $\mathcal{T} \uplus \mathcal{T}_u$
end Algorithm
```

Fig. 5. The main verification algorithm.

The algorithm initialises in the first two lines. It distinguishes the body of the unknown block v (as an unknown expression u in between two normal expressions e_1 and e_2), sets up the set to store discovered specifications (line 1), and finds the program variables that are potentially accessed by v and u, respectively (prog_var in line 2). Note that $\boldsymbol{x_0}$ and $\boldsymbol{x}$ are the variables read by v and u, and

$\boldsymbol{y_0}$ and $\boldsymbol{y}$ are those mutated. For example, if v contains an assignment $\texttt{y = x}$ then $\texttt{x}$ will be in $\boldsymbol{x_0}$ and $\texttt{y}$ in $\boldsymbol{y_0}$.

After the initialisation, for each specification (*requires* Φ_{pr} *ensures* Φ_{po}) to verify against (line 3), the algorithm works in three steps. The first step is to compute the preconditions of u (lines 4–7). It first conducts a symbolic execution from Φ_{pr} over e_1 (the program segment before u) to obtain its post-states, from which the preconditions for u will be extracted (line 4). The symbolic execution is essentially a forward analysis whose details are presented later. If the post-states include `false`, then it means the given Φ_{pr} cannot guarantee e_1's memory safety, and thus fail is returned (line 5). Otherwise, each post-state of e_1 is processed by function Local as a candidate precondition for u (line 7). Intuitively, it extracts the part of each σ reachable from the variables that may be accessed by u, namely, $\boldsymbol{x}$ and $\boldsymbol{y}$. The function Local is defined as follows:

$$\mathsf{Local}(\exists \boldsymbol{z} \cdot \kappa \wedge \pi, \{\boldsymbol{x}\}) =_{df} \exists \texttt{fv}(\kappa{\wedge}\pi){\cup}\{\boldsymbol{z}\} \setminus \texttt{ReachVar}(\kappa \wedge \pi, \{\boldsymbol{x}\}) \cdot \\ \texttt{ReachHeap}(\kappa \wedge \pi, \{\boldsymbol{x}\}) \wedge \pi$$

where $\mathsf{fv}(\sigma)$ stands for all free (program and logical) variables occurring in σ, and $\texttt{ReachVar}(\kappa \wedge \pi, \{\boldsymbol{x}\})$ is the minimal set of variables reachable from $\{\boldsymbol{x}\}$:

$$\{\boldsymbol{x}\} \cup \{z_2 \mid \exists z_1, \pi_1 \cdot z_1{\in}\texttt{ReachVar}(\kappa{\wedge}\pi, \{\boldsymbol{x}\}) \wedge \pi{=}(z_1{=}z_2 \wedge \pi_1)\} \cup \{z_2 \mid \\ \exists z_1, \kappa_1 \cdot z_1{\in}\texttt{ReachVar}(\kappa{\wedge}\pi, v) \wedge \kappa{=}(z_1{::}c\langle .., z_2, ..\rangle * \kappa_1)\} \subseteq \texttt{ReachVar}(\kappa{\wedge}\pi, \{\boldsymbol{x}\})$$

That is, it is composed of aliases of $\boldsymbol{x}$ as well as variables reachable from $\boldsymbol{x}$. And the formula $\texttt{ReachHeap}(\kappa{\wedge}\pi, \{\boldsymbol{x}\})$ denotes the part of κ reachable from $\{\boldsymbol{x}\}$ and is formally defined as the $*$-conjunction of the following set of formulae:

$$\{\kappa_1 \mid \exists z_1, z_2, \kappa_2 \cdot z_1{\in}\texttt{ReachVar}(\kappa{\wedge}\pi, \{\boldsymbol{x}\}) \wedge \kappa{=}\kappa_1{*}\kappa_2 \wedge \kappa_1{=}z_1{::}c\langle .., z_2, ..\rangle\}$$

The second step is to discover the postconditions for u (lines 9–11). This is mainly completed with another symbolic execution with abduction over e_2 (line 9), whose details are also introduced later. Here we denote u's post-state as `emp`, since its knowledge is not available yet. Therefore, the initial state for the symbolic execution of e_2 is simply the frame part of state not touched by u. The function Frame is formally defined as

$$\mathsf{Frame}(\exists \boldsymbol{z} \cdot \kappa \wedge \pi, \{\boldsymbol{x}\}) =_{df} \exists \boldsymbol{z} \cdot \texttt{UnreachHeap}(\kappa \wedge \pi, \{\boldsymbol{x}\}) \wedge \pi$$

where $\texttt{UnreachHeap}(\exists \boldsymbol{z} \cdot \kappa \wedge \pi, \{\boldsymbol{x}\})$ is the formula consisting of all $*$-conjuncts from κ which are not in $\texttt{ReachHeap}(\exists \boldsymbol{z} \cdot \kappa \wedge \pi, \{\boldsymbol{x}\})$.

The conjunctions $\boldsymbol{x}{=}\boldsymbol{a} \wedge \boldsymbol{y}{=}\boldsymbol{b} \wedge \boldsymbol{z}{=}\boldsymbol{c}$ in line 9 are to keep track of variable snapshot accessed by u using the special variables $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$. Then the symbolic execution returns a set S of pairs (σ, σ') where σ is a possible post-state of e_2 and σ' records the discovered effect of u. However, maybe u still has some effect that is only exposed in the expected postcondition Φ_{po} for the whole program; therefore we need to check whether or not σ can establish Φ_{po}. If not, another abduction $\sigma{*}[\sigma''] \triangleright \Phi_{po}$ is invoked to discover further effect σ'' which is then added into σ'.

There can still be some complication here. Note that the effect discovered during e_2's symbolic execution may not be attributed all over to u; it is also possible that there is a bug in the program, or the given specification is not sufficient. As a consequence of that, the result σ' returned by our abduction may contain more information than what can be expected from u, in which case we cannot simply regard the whole σ' as the postcondition of u.For example, consider the code fragment `unk(x); z=y.next` with the precondition $\texttt{x}{::}\texttt{node}\langle\_, \texttt{null}\rangle$. Before the assignment (and dereference of `y.next`) we use abduction to get $\texttt{y}{::}\texttt{node}\langle\_, \_\rangle$. However, noting the fact $\texttt{y}{\notin}\texttt{ReachVar}(\sigma, \{\texttt{x}\})$ where $\sigma = \texttt{emp}{*}\texttt{y}{::}\texttt{node}\langle\_, \_\rangle$ is the state immediately after the unknown call with the abduction result, we know that from the unknown call's parameters (`x`), `y` is not reachable, and hence the unknown call will never establish a state to satisfy $\texttt{y}{::}\texttt{node}\langle\_, \_\rangle$. In that case we are assured that the program being verified cannot meet its given specification.

To detect such a situation, we introduce the check in line 11. It tests whether the whole abduction result is reachable from variables accessed by u. If not, then the unreachable part cannot be expected from u, which indicates a possible bug in the program or some inconsistency between the program and its specification. In such cases, the algorithm returns an additional formula that can be used by a further analysis to either identify the bug or strengthen the specification. Recall the example presented in the previous paragraph: since $\texttt{y}{::}\texttt{node}\langle\_, \_\rangle$ cannot be established by the unknown call, if we add it to the precondition of the code fragment (to form a new precondition $\texttt{x}{::}\texttt{node}\langle\_, \texttt{null}\rangle * \texttt{y}{::}\texttt{node}\langle\_, \_\rangle$), then the verification with the new specification can move on and will potentially succeed.

The third step (lines 12–17) is to form the derived specifications for u in terms of variables $\boldsymbol{a}, \boldsymbol{b}$ and g. Here g denotes logical variables not explicitly accessed by u, but occurring in both pre- and postconditions (ghost variables). The formula $\mathsf{sub\_alias}(\sigma', \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\})$ is obtained from σ' by replacing all variables with their aliases in $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$. It is defined as

$$\begin{aligned}\mathsf{sub\_alias}(\sigma', \{\boldsymbol{x}\}) =_{df} \; & (\{[x/x_0] \mid x{\in}\{\boldsymbol{x}\} \wedge x_0{\in}\mathsf{aliases}(\sigma', x)\} \; \sigma') \wedge \\ & \bigwedge \{x = x_0 \mid x, x_0{\in}\{\boldsymbol{x}\} \wedge x_0{\in}\mathsf{aliases}(\sigma', x)\}\end{aligned}$$

where a set of substitutions before a formula σ' denotes the result of applying each of those substitutions to σ' (where the ordering is not important), and $\mathsf{aliases}(\sigma', x)$ returns all the aliases of x in σ'.

Finally, at line 20, the obtained specifications mspec_u for u are passed to the case analysis algorithm (given in Figure 6) to derive the specifications of unknown procedures invoked in u.

5.2 Case Analysis Algorithm

In order to discover specifications for unknown procedures invoked in u, the algorithm in Figure 6 conducts a case analysis according to the structure of u. In the first case (line 2), u is simply a single unknown call. In this situation, the algorithm returns all the pre-/postcondition pairs from mspec_u as the unknown procedure's specifications.

```
Algorithm CaseAnalysis($\mathcal{T}$, $mspec_u$, $u$)
1  switch $u$
2    case $unk(\boldsymbol{x};\boldsymbol{y})$
3      return $\{\,(unk(\boldsymbol{x};\boldsymbol{y}), mspec_u)\}$
4    case if $(x)$ $v_1$ else $v_2$
5      $mspec_T := \{(requires\ \Phi_{pr}{\wedge}x\ ensures\ \Phi_{po}) \mid$
                 $(requires\ \Phi_{pr}\ ensures\ \Phi_{po}) \in mspec_u\}$
6      $mspec_F := \{(requires\ \Phi_{pr}{\wedge}\neg x\ ensures\ \Phi_{po}) \mid$
                 $(requires\ \Phi_{pr}\ ensures\ \Phi_{po}) \in mspec_u\}$
7      $R_1 :=$ Verify($\mathcal{T}$, $mspec_T$, $v_1$)
8      $R_2 :=$ Verify($\mathcal{T}$, $mspec_F$, $v_2$)
9      return $R_1 \uplus R_2$
10   case if $(x)$ $v$ else $e$
11     $mspec_T := \{(requires\ \Phi_{pr}{\wedge}x\ ensures\ \Phi_{po}) \mid$
                 $(requires\ \Phi_{pr}\ ensures\ \Phi_{po}) \in mspec_u\}$
12     $R :=$ Verify($\mathcal{T}$, $mspec_T$, $v$)
13     if $\exists(requires\ \Phi_{pr}\ ensures\ \Phi_{po}) \in mspec_u, \sigma \in [\![e]\!]_{\mathcal{T}}\{\Phi_{pr} \wedge \neg x\} \cdot$
             $\sigma{=}\mathtt{false} \vee \sigma \nvdash \Phi_{po}{*}\mathtt{true}$ then return fail
14     else return $R$ endif
15   case if $(x)$ $e$ else $v$   (Similar to the previous case)
16   case while $x$ $\{v\}$ inv $\Delta$
17     return Verify($\mathcal{T}$, $requires\ \Delta{\wedge}x\ ensures\ \Delta$, $v$)
18   case $unk_0(\boldsymbol{x_0};\boldsymbol{y_0})$ $\{\ ;e_i; unk_i(\boldsymbol{x_i};\boldsymbol{y_i})\}_{i=1}^{n}$
19     return $\{\,(unk_i(\boldsymbol{x_i};\boldsymbol{y_i}),$ SeqUnkCalls($\mathcal{T}$, $mspec_u$, $u$)$)\}_{i=0}^{n}$
end Algorithm
```

Fig. 6. The case analysis algorithm.

In the second case (line 4), u is an `if`-conditional and both branches contain an unknown block. The algorithm uses the main algorithm to verify the two branches separately with preconditions $\Phi_{pr}{\wedge}x$ and $\Phi_{pr}{\wedge}\neg x$ respectively, where Φ_{pr} is one of the preconditions of the whole `if`. The results obtained from the two branches are then combined using the $\uplus$ operator:

$$R_1 \uplus R_2 =_{df} \{(\mathsf{f}, \mathsf{Refine}(mspec^1_{\mathsf{f}} \cup mspec^2_{\mathsf{f}})) \mid (\mathsf{f}, mspec^1_{\mathsf{f}}){\in}R_1 \wedge (\mathsf{f}, mspec^2_{\mathsf{f}}){\in}R_2\}$$

where Refine is used to eliminate any specification $(requires\ \Phi_{pr}\ ensures\ \Phi_{po})$ from a set if there exists a "stronger" one $(requires\ \Phi'_{pr}\ ensures\ \Phi'_{po})$ such that $\Phi'_{pr}{\preceq}\Phi_{pr}$ and $\Phi_{po}{\preceq}\Phi'_{po}$. It is defined as

$$\begin{array}{l}
\mathsf{Refine}(\emptyset) =_{df} \emptyset \\
\mathsf{Refine}(\{(requires\ \Phi_{pr}\ ensures\ \Phi_{po})\} \cup \mathsf{Spec}) =_{df} \\
\quad \textbf{if } \exists(requires\ \Phi'_{pr}\ ensures\ \Phi'_{po}){\in}\mathsf{Spec} \cdot \Phi'_{pr}{\preceq}\Phi_{pr} \wedge \Phi_{po}{\preceq}\Phi'_{po} \\
\quad \textbf{then } \mathsf{Refine}(\mathsf{Spec}) \textbf{ else } \{(requires\ \Phi_{pr}\ ensures\ \Phi_{po})\} \cup \mathsf{Refine}(\mathsf{Spec})
\end{array}$$

and $\uplus$ is to refine the union of two specification sets.

The third and fourth cases (lines 10 and 15) are for `if`-conditionals which contain only one unknown block in one of the two branches. This is handled in a similar way as in the second case. The only difference is, for the branch without unknown blocks, we need to verify it with the underlying semantics (line 13).

The fifth case is the `while` loop. As we assume its invariant is already given for the verification, we simply verify its body with the main algorithm, regarding the invariant as both pre- and postconditions (line 17).

In the last case (line 21), where u consists of multiple unknown procedure calls in sequence, another algorithm SeqUnkCalls is invoked to deal with it.

5.3 Verifying Sequential Unknown Calls

We provide a solution to the most complicated case (unknown procedure calls in sequence) under a strong assumption, namely, we can find a common specification to capture all these unknown procedures' behaviours. First we illustrate the brief idea using two sequential unknown procedure calls as an example, followed by the general algorithm.

Suppose we have

$$\{\Phi_{pr}\}\ \{unk_0(\boldsymbol{x_0};\boldsymbol{y_0}); e; unk_1(\boldsymbol{x_1};\boldsymbol{y_1})\}\ \{\Phi_{po}\}$$

where e is the only known code fragment within the block. The algorithm works in three steps. In the first step, it extracts the precondition for the first procedure, say Φ^u_{pr}, from the given precondition Φ_{pr} by extracting the part of heap that may be accessed by the call via $\boldsymbol{x_0}$ and $\boldsymbol{y_0}$, which is similar to the first step of the main algorithm Verify. Aiming at a general specification for both unknown calls, it then assumes that the second procedure has a similar precondition Φ^u_{pr}. In the second step, it symbolically executes the code fragment e with the help of the abductor, to discover a crude postcondition, say Φ^0_{po}, expected from the first unknown call. This is similar to the second step of the main algorithm Verify, except that the postcondition for e is now assumed to be Φ^u_{pr}. In the third step, the algorithm takes Φ^0_{po} (with appropriate variable substitutions) as the postcondition of the second unknown call, and checks whether or not the derived post (Φ^0_{po}) satisfies Φ_{po}. If not, it invokes another abduction to strengthen Φ^0_{po} to obtain the final postcondition Φ^u_{po} for the unknown procedures. Note that this strengthening does not affect soundness: the strengthened Φ^u_{po} can still be used as a general postcondition for both unknown procedures.

Figure 7 presents the algorithm to infer specifications for n ($n \geq 2$) unknown calls in sequence. As aforementioned, given a block of (n+1) unknown procedure calls with n pieces of known code blocks sandwiched among them ($unk_0(\boldsymbol{x_0};\boldsymbol{y_0})\ \{; e_i; unk_i(\boldsymbol{x_i};\boldsymbol{y_i})\}^n_{i=1}$ in line 1), and the specification (*requires* Φ_{pr} *ensures* Φ_{po}) (line 3) for such a block, our approach generally works in three steps: first, to compute a precondition for the unknown calls; second, to verify each code fragment e_i ($i = 1, ..., n$) with abduction to collect expected behaviour of the unknown calls (as part of their postcondition); third, to guarantee that the collected postcondition satisfies Φ_{po}. If not, then another abduction is conducted to strengthen the gained postcondition to ensure this.

```
Algorithm SeqUnkCalls($\mathcal{T}$, $mspec_u$, $u$)
1  Denote $u$ as $unk_0(\boldsymbol{x_0};\boldsymbol{y_0})$ $\{;e_i;unk_i(\boldsymbol{x_i};\boldsymbol{y_i})\}_{i=1}^{n}$
2  $R := \emptyset$
3  foreach $(requries\ \Phi_{pr}\ ensures\ \Phi_{po}) \in mspec_u$ do
4    $\Phi^u_{pr} := \mathsf{Local}(\Phi_{pr}, \{\boldsymbol{x_0}, \boldsymbol{y_0}\})$
5    $\boldsymbol{z_0} := \mathsf{fv}(\Phi^u_{pr}) \setminus \{\boldsymbol{x_0}, \boldsymbol{y_0}\}$
6    $\Phi^u_{pr} := [\boldsymbol{a}/\boldsymbol{x_0}, \boldsymbol{b}/\boldsymbol{y_0}, \boldsymbol{c}/\boldsymbol{z_0}]\Phi^u_{pr}$
7    $\mathsf{S}'_0 := \{(\Phi_{pr} \wedge \boldsymbol{y_0}'{=}\boldsymbol{y_0}, \mathtt{emp} \wedge \boldsymbol{a}{=}\boldsymbol{x_0} \wedge \boldsymbol{b}{=}\boldsymbol{y_0} \wedge \boldsymbol{c}{=}\boldsymbol{z_0})\}$
8    for $i := 1$ to $n$ do
9      $\mathsf{S}_i := [\![e_i]\!]^{\mathsf{A}}_{\mathcal{T}}\{\ (\Phi^{i-1}_{po} * [\boldsymbol{b}/\boldsymbol{y}_{i-1}]\ \mathsf{Frame}(\sigma_{i-1}, \{\boldsymbol{x}_{i-1}, \boldsymbol{y}_{i-1}\}), \Phi^{i-1}_{po})\ |$
           $(\sigma_{i-1}, \sigma'_{i-1}){\in}\mathsf{S}'_{i-1} \wedge \Phi^{i-1}_{po} = ([\boldsymbol{x}_{i-1}/\boldsymbol{a}, \boldsymbol{y}_{i-1}/\boldsymbol{b}, \boldsymbol{z}_{i-1}/\boldsymbol{c}]\ \mathsf{sub\_alias}($
             $\sigma'_{i-1}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\})) \wedge \boldsymbol{a}{=}\boldsymbol{x}_{i-1} \wedge \boldsymbol{b}{=}\boldsymbol{y}_{i-1} \wedge \boldsymbol{c}{=}\boldsymbol{z}_{i-1}\}$ where $\boldsymbol{z}_{i-1}$ is fresh
10     $\mathsf{S}'_i := \{(\sigma, \sigma')\ |\ (\sigma, \sigma'){\in}\mathsf{S}_i \wedge \rho\sigma \vdash \exists \boldsymbol{c} \cdot \Phi^u_{pr}{*}\mathtt{true}\} \cup \{(\sigma * \sigma'', \sigma' * \sigma'')\ |$
                 $(\sigma, \sigma'){\in}\mathsf{S}_i \wedge \rho\sigma \nvdash \exists \boldsymbol{c} \cdot \Phi^u_{pr}{*}\mathtt{true} \wedge \rho\sigma{*}[\sigma''] \rhd \exists \boldsymbol{c} \cdot \Phi^u_{pr}\}$
                                                   where $\rho{=}[\boldsymbol{a}/\boldsymbol{x_i}, \boldsymbol{b}/\boldsymbol{y_i}]$
11     if $\exists(\sigma, \sigma'){\in}\mathsf{S}'_i{\cdot}\mathsf{fv}(\sigma') \nsubseteq \mathsf{ReachVar}(\sigma, \{\boldsymbol{a}, \boldsymbol{b}\})$ then return $(\mathsf{fail}, \sigma')$ endif
12   end for
13   $\mathsf{S}_{n+1} := \{\ (\Phi^n_{po}{*}[\boldsymbol{b}/\boldsymbol{y_n}]\mathsf{Frame}(\sigma_n, \{\boldsymbol{x_n}, \boldsymbol{y_n}\}), \Phi^n_{po})\ |\ (\sigma_n, \sigma'_n){\in}\mathsf{S}'_n \wedge$
             $\Phi^n_{po} = ([\boldsymbol{x_n}/\boldsymbol{a}, \boldsymbol{y_n}/\boldsymbol{b}, \boldsymbol{z_n}/\boldsymbol{c}]\ \mathsf{sub\_alias}(\sigma'_n, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\})) \wedge$
               $\boldsymbol{a}{=}\boldsymbol{x_n} \wedge \boldsymbol{b}{=}\boldsymbol{y_n} \wedge \boldsymbol{c}{=}\boldsymbol{z_n}\}$ where $\boldsymbol{z_n}$ is fresh
14   $\mathsf{S}'_{n+1} := \{(\sigma, \sigma')\ |\ (\sigma, \sigma'){\in}\mathsf{S}_{n+1} \wedge \sigma \vdash \Phi_{po}{*}\mathtt{true}\} \cup \{(\sigma * \sigma'', \sigma' * \sigma'')\ |$
             $(\sigma, \sigma'){\in}\mathsf{S}_{n+1} \wedge \sigma \nvdash \Phi_{po}{*}\mathtt{true} \wedge \sigma{*}[\sigma''] \rhd \Phi_{po}\}$
15   if $\exists(\sigma, \sigma'){\in}\mathsf{S}'_{n+1} \cdot \mathsf{fv}(\sigma') \nsubseteq \mathsf{ReachVar}(\sigma, \{\boldsymbol{a}, \boldsymbol{b}\})$ then return $(\mathsf{fail}, \sigma')$ endif
16   foreach $(\sigma, \sigma') \in \mathsf{S}'_{n+1}$ do
17     $\Phi^u_{po} := \mathsf{sub\_alias}(\sigma', \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\})$
18     $g := \mathsf{fv}(\Phi^u_{pr}) \cap \mathsf{fv}(\Phi^u_{po}) \setminus \{\boldsymbol{a}, \boldsymbol{b}\}$
19     $R := \mathsf{Refine}(R \cup \{(requires\ \exists \mathsf{fv}(\Phi^u_{pr}) \setminus (g \cup \{\boldsymbol{a}, \boldsymbol{b}\}) \cdot \Phi^u_{pr}\ ensures\ \Phi^u_{po})\})$
20   end foreach
21 end foreach
22 return $R$
end Algorithm
```

Fig. 7. Algorithm for sequential unknown calls.

The first step is completed by lines 4 to 6. The local part of Φ_{pr} is extracted w.r.t. the first unknown call's parameters $\boldsymbol{x_0}$ and $\boldsymbol{y_0}$. Other free variables are distinguished as $\boldsymbol{z_0}$, which may be ghost variables. Finally the precondition is found in terms of special logical variables $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$.

The second step is performed over each $e_i; unk_i(\boldsymbol{x_i}; \boldsymbol{y_i})$. Its main idea is to take the postcondition generated for the last unknown call (Φ^{i-1}_{po}), plus the frame part during the entailment check against Φ^{i-1}_{pr}, as the post-state of $unk_{i-1}(\boldsymbol{x_{i-1}}; \boldsymbol{y_{i-1}})$, and try to verify e_i beginning with such a state, using abduction when necessary (line 9). After the verification we get S_i containing abstract states before $unk_i(\boldsymbol{x_i}; \boldsymbol{y_i})$, and we want those states to satisfy its precondition Φ^u_{pr} subject to substitution. Note that during the verification of e_i and the last satisfaction checking we may use abduction to strengthen the program state, whose results reflect the expected behaviour of $unk_{i-1}(\boldsymbol{x_{i-1}};\ \boldsymbol{y_{i-1}})$ and are accumulated as its expected postcondition. Hence we achieve a sufficiently strong postcondition for each unknown call.

The third step is similar to the first algorithm: it checks whether the final abstract state entails the postcondition of the whole block, and strengthens the final abstract state with abduction if it cannot. Then the ghost variables are recognised and processed analogously to the first algorithm. Finally the strongest specifications discovered for those unknown procedures are returned.

Note that our current solution tries to find a common specification (*requires* Φ_{pr} *ensures* Φ_{po}) suitable for all the unknown procedures. Generally we may allow the unknown procedures to have different specifications. In theory, this can be achieved by a more in-depth analysis which examines the known code fragments in between those unknown calls. That is, by analysing the code fragment e_i we would hopefully obtain a postcondition for the $(i{-}1)$-th procedure and a precondition for the i-th. In the case of two unknown calls $unk_0(\boldsymbol{x_0};\boldsymbol{y_0}); e_1; unk_1(\boldsymbol{x_1};\boldsymbol{y_1})$, the precondition for unk_0 and the postcondition for unk_1 can be obtained as usual (by analysing the code before unk_0 and after unk_1 respectively). To derive the postcondition Φ^0_{po} for unk_0 and the precondition Φ^1_{pr} for unk_1, we initialise Φ^0_{po} to be `emp` to start a forward analysis over e_1 with abduction, to accumulate (via abduction) the expected behaviour of unk_0 (for e_1 to be verified) as Φ^0_{po}, and extract a formula (which is relevant to the footprint of unk_1) from the abstract state at the end of e_1 as Φ^1_{pr}. However, our initial experiments show that, unless the fragment e_1 is sufficiently complex to expose enough information expected from unk_0, the derived Φ^0_{po} and Φ^1_{pr} can be rather weak. As a consequence, the derived specification for unk_0 can be too weak (with a weak postcondition) and the one for unk_1 can be too strong (with a weak precondition). It remains an open problem how we might tune the derived results to obtain more reasonable specifications. We conjecture that certain heuristics might help and we will explore this further in our future work.

5.4 Abstract Semantics

As shown in the algorithms, we use two kinds of abstract semantics to analyse the program: an underlying semantics and another semantics based on both the first one and abduction to improve the postcondition of unknown calls.

The type of our underlying semantics is defined as

$$[\![e]\!]\ :\ \mathsf{AllSpec} \to \mathcal{P}_{\mathsf{SH}} \to \mathcal{P}_{\mathsf{SH}}$$

where $\mathsf{AllSpec}$ contains procedure specifications (extracted from the program *Prog*). For some expression e, given its precondition, the semantics will calculate the postcondition.

The foundation of the semantics is the basic transition functions from a conjunctive abstract state (σ) to a conjunctive or disjunctive abstract state (σ or Δ) below:

$$\begin{array}{lll}
\mathsf{unfold}(x) & : \mathsf{SH} \to \mathcal{P}_{\mathsf{SH}[x]} & \text{Unfolding} \\
\mathsf{exec}(d[x]) & : \mathsf{AllSpec} \to \mathsf{SH}[x] \to \mathsf{SH} & \text{Heap-sensitive execution} \\
\mathsf{exec}(d) & : \mathsf{AllSpec} \to \mathsf{SH} \to \mathsf{SH} & \text{Heap-insensitive execution}
\end{array}$$

where $\mathsf{SH}[x]$ denotes the set of conjunctive abstract states in which each element has x exposed as the head of a data node ($x{::}c\langle v^*\rangle$), and $\mathcal{P}_{\mathsf{SH}[x]}$ contains all the (disjunctive) abstract states, each of which is composed by such conjunctive states. Here $\mathsf{unfold}(x)$ rearranges the symbolic heap so that the cell referred to by x is exposed for access by heap sensitive commands $d[x]$ via the second transition function $\mathsf{exec}(d[x])$. The third function defined for other (heap insensitive) commands d does not require such exposure of x.

The unfolding function is defined by the following two rules:

$$\frac{\mathit{isdatat}(c) \quad \sigma \vdash x{::}c\langle v^*\rangle * \sigma'}{\mathsf{unfold}(x)\sigma \rightsquigarrow \sigma} \qquad \frac{\mathit{isspred}(c) \quad \sigma \vdash x{::}c\langle u^*\rangle * \sigma' \quad \mathtt{root}{::}c\langle v^*\rangle \equiv \Phi}{\mathsf{unfold}(x)\sigma \rightsquigarrow \sigma' * [x/\mathtt{root}, u^*/v^*]\Phi}$$

where the test $\mathit{isdatat}(c)$ returns `true` only if c is a data node and $\mathit{isspred}(c)$ returns `true` only if c is a shape predicate.

The symbolic execution of heap-sensitive commands $d[x]$ (i.e. $x.f_i$, $x.f_i := w$, or $\mathtt{free}(x)$) assumes that the rearrangement $\mathsf{unfold}(x)$ has been done prior to the execution:

$$\frac{\mathit{isdatat}(c) \qquad \sigma \vdash x{::}c\langle v_1, .., v_n\rangle * \sigma'}{\mathsf{exec}(x.f_i)(\mathcal{T})\sigma \rightsquigarrow \sigma' * x{::}c\langle v_1, .., v_n\rangle \wedge \mathtt{res}{=}v_i}$$

$$\frac{\mathit{isdatat}(c) \qquad \sigma \vdash x{::}c\langle v_1, .., v_n\rangle * \sigma'}{\mathsf{exec}(x.f_i := w)(\mathcal{T})\sigma \rightsquigarrow \sigma' * x{::}c\langle v_1, .., v_{i-1}, w, v_{i+1}, .., v_n\rangle}$$

$$\frac{\mathit{isdatat}(c) \qquad \sigma \vdash x{::}c\langle u^*\rangle * \sigma'}{\mathsf{exec}(\mathtt{free}(x))(\mathcal{T})\sigma \rightsquigarrow \sigma'}$$

The symbolic execution rules for heap-insensitive commands are as follows:

$$\mathsf{exec}(k)(\mathcal{T})\sigma \rightsquigarrow \sigma \wedge \mathtt{res}{=}k \qquad\qquad \mathsf{exec}(x)(\mathcal{T})\sigma \rightsquigarrow \sigma \wedge \mathtt{res}{=}x$$

$$\frac{\mathit{isdatat}(c)}{\mathsf{exec}(\mathtt{new}\ c(v^*))(\mathcal{T})\sigma \rightsquigarrow \sigma * \mathtt{res}{::}c\langle v^*\rangle}$$

$$\frac{\begin{array}{c} t\ mn\ ((t_i\ u_i)_{i=1}^m; (t_i\ v_i)_{i=1}^n)\ \mathit{requires}\ \Phi_{pr}\ \mathit{ensures}\ \Phi_{po} \in \mathcal{T} \\ \rho = [x'_i/u'_i]_{i=1}^m \circ [y'_i/v'_i]_{i=1}^n \quad \sigma \vdash \rho\Phi_{pr} * \sigma' \quad \rho_o = [y_i/v_i]_{i=1}^n \circ \rho \\ \rho_l = [r_i/y'_i]_{i=1}^n \qquad \rho_{ol} = [r_i/y_i]_{i=1}^n \qquad \mathit{fresh\ logical}\ r_i \end{array}}{\mathsf{exec}(mn(x_1, .., x_m; y_1, .., y_n))(\mathcal{T})\sigma \rightsquigarrow (\rho_l\sigma') * (\rho_{ol} \circ \rho_o\Phi_{po})}$$

Note that the first three rules deal with constant (k), variable (x) and data node creation ($\mathtt{new}\ c(v^*)$), respectively, while the last rule handles method invocation. In the last rule, the call site is ensured to meet the precondition of mn, as signified by $\sigma \vdash \rho\Phi_{pr} * \sigma'$. In this case, the execution succeeds and the post-state of the method call involves mn's postcondition as signified by $\rho_{ol} \circ \rho_o\Phi_{po}$.

A lifting function † is defined to lift unfold's domain to $\mathcal{P}_{\mathsf{SH}}$:

$$\mathsf{unfold}^{\dagger}(x) \bigvee \sigma_i =_{df} \bigvee(\mathsf{unfold}(x)\sigma_i)$$

and this function is overloaded for exec to lift both its domain and range to $\mathcal{P}_{\mathsf{SH}}$:

$$\mathsf{exec}^{\dagger}(d)(\mathcal{T}) \bigvee \sigma_i =_{df} \bigvee(\mathsf{exec}(d)(\mathcal{T})\sigma_i)$$

Based on the transition functions above, we can define the abstract semantics for a program expression e as follows:

$$\begin{array}{lll}
[\![d[x]]\!]_{\mathcal{T}}\Delta & =_{df} & \mathsf{exec}^{\dagger}(d[x])(\mathcal{T}) \circ \mathsf{unfold}^{\dagger}(x)\Delta \\
[\![d]\!]_{\mathcal{T}}\Delta & =_{df} & \mathsf{exec}^{\dagger}(d)(\mathcal{T})\Delta \\
[\![e_1;e_2]\!]_{\mathcal{T}}\Delta & =_{df} & [\![e_2]\!]_{\mathcal{T}} \circ [\![e_1]\!]_{\mathcal{T}}\Delta \\
[\![x := e]\!]_{\mathcal{T}}\Delta & =_{df} & [x'/x, r'/\mathtt{res}]([\![e]\!]_{\mathcal{T}}\Delta) \wedge x{=}r' \quad \textit{fresh logical } x', r' \\
[\![\mathtt{if}\ (v)\ e_1\ \mathtt{else}\ e_2]\!]_{\mathcal{T}}\Delta & =_{df} & ([\![e_1]\!]_{\mathcal{T}}(v{\wedge}\Delta)) \vee ([\![e_2]\!]_{\mathcal{T}}(\neg v{\wedge}\Delta))
\end{array}$$

$$\frac{\Delta \vdash \Delta_i * \mathtt{R}}{[\![\mathtt{while}\ x\ \{e\}\ \mathtt{inv}\ \Delta_i]\!]_{\mathcal{T}}\Delta =_{df} \mathtt{R} * \Delta_i \wedge \neg x}$$

Next we define the abstract semantics with abduction used in our analysis, whose type is

$$[\![e]\!]^{\mathsf{A}} \ : \ \mathsf{AllSpec} \to \mathcal{P}(\mathsf{SH} \times \mathsf{SH}) \to \mathcal{P}(\mathsf{SH} \times \mathsf{SH})$$

It takes a piece of program and a specification table, to map a (disjunctive) set of pair of symbolic heaps to another such set (where the first in the pair is the current state and the second is the accumulated postcondition for unknown call).

This semantics also consists of the basic transition functions which compose the atomic instructions' semantics and then the program constructors' semantics. Here the basic transition functions are lifted as

```
$\mathsf{Unfold}(x)(\sigma, \sigma') =_{df}$
        **let** $\Delta{=}\mathsf{unfold}(x)\sigma$ and $\mathsf{S}{=}\{(\sigma_1, \sigma')|\sigma_1 \in \Delta\}$
        **in if** $(\mathtt{false} \notin \Delta)$ **then** $\mathsf{S}$
           **else if** $(\Delta \vdash x{=}a$ for some $a{\in}\mathsf{SVar})$ and
                 $(\sigma' \nvdash a{::}c\langle \boldsymbol{y}\rangle * \mathtt{true}$ for fresh $\{\boldsymbol{y}\}{\subseteq}\mathsf{LVar})$
           **then** $\mathsf{S} \cup \{(\Delta * x{::}c\langle \boldsymbol{y}\rangle, \sigma' * x{::}c\langle \boldsymbol{y}\rangle)\}$
           **else**  $\mathsf{S} \cup \{(\mathtt{false}, \sigma')\}$
$\mathsf{Exec}(ds)(\sigma, \sigma') =_{df}$ **let** $\sigma_1{=}\mathsf{exec}(d)\sigma$ **in** $\{(\sigma_1, \sigma')|\sigma_1 \in \Delta\}$
        where $ds$ is either $d[x]$ or $d$, except procedure call
```

In the definition of Exec we need special treatment for instructions that may alter variable values, say procedure call. As can be seen in the rule below, when a call-by-reference variable y is assigned to a new value after the call, the original value is still preserved with a substitution $\rho = [y_0/y]$ where y_0 is fresh. Doing this allows us to keep the connection among the history values of a variable and its latest value, which may be essential as a link from the unknown procedure's postcondition to its caller's postcondition.

$$\frac{\begin{array}{c} t\ mn\ ((t_i\ u_i)_{i=1}^{m}; (t_i\ v_i)_{i=1}^{n})\ requires\ \Phi_{pr}\ ensures\ \Phi_{po} \in \mathcal{T} \\ \rho = [x'_i/u'_i]_{i=1}^{m} \circ [y'_i/v'_i]_{i=1}^{n} \qquad \sigma \vdash \rho\Phi_{pr} * \sigma_1 \text{ and } \sigma'_1{=}\texttt{emp}, \text{ or } \sigma * [\sigma'_1] \rhd \rho\Phi_{pr} * \sigma_1 \\ \rho_o = [y_i/v_i]_{i=1}^{n} \circ \rho \qquad \rho_l = [r_i/y'_i]_{i=1}^{n} \qquad \rho_{ol} = [r_i/y_i]_{i=1}^{n} \quad fresh\ logical\ r_i \end{array}}{\mathsf{Exec}(mn(x_1,..,x_m;y_1,..,y_n))(\mathcal{T})(\sigma,\sigma') \rightsquigarrow ((\rho_l\sigma_1) * (\rho_{ol} \circ \rho_o\Phi_{po}), \rho_{ol} \circ \rho_o(\sigma' * \sigma'_1))}$$

A similar lifting function $\dagger$ is defined to lift Unfold's and Exec's domains:

$$\begin{array}{ll} \mathsf{Unfold}^{\dagger}(x) \bigvee(\sigma_i,\sigma'_i) & =_{df} \bigvee(\mathsf{Unfold}(x)(\sigma_i,\sigma'_i)) \\ \mathsf{Exec}^{\dagger}(ds)(\mathcal{T}) \bigvee(\sigma_i,\sigma'_i) & =_{df} \bigvee(\mathsf{Exec}(ds)(\mathcal{T})(\sigma_i,\sigma'_i)) \end{array}$$

Based on the above transition functions, the abstract semantics with abduction is as follows:

$$\begin{array}{ll} [\![d[x]]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') & =_{df}\ \mathsf{Exec}^{\dagger}(d[x])(\mathcal{T}) \circ \mathsf{Unfold}^{\dagger}(x)(\Delta,\Delta') \\ [\![d]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') & =_{df}\ \mathsf{Exec}^{\dagger}(d)(\mathcal{T})(\Delta,\Delta') \\ [\![e_1;e_2]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') & =_{df}\ [\![e_2]\!]^{\mathsf{A}}_{\mathcal{T}} \circ [\![e_1]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') \\ [\![\texttt{if}\ (v)\ e_1\ \texttt{else}\ e_2]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') & =_{df}\ ([\![e_1]\!]^{\mathsf{A}}_{\mathcal{T}}(v{\wedge}\Delta,\Delta')) \vee ([\![e_2]\!]^{\mathsf{A}}_{\mathcal{T}}(\neg v{\wedge}\Delta,\Delta')) \end{array}$$

$$\frac{\rho = [x_1/x, r_1/\texttt{res}] \quad fresh\ logical\ x_1, r_1}{[\![x := e]\!]^{\mathsf{A}}_{\mathcal{T}}(\Delta,\Delta') =_{df} ((\rho[\![e]\!]^{\mathsf{A}}_{\mathcal{T}}\Delta) \wedge x{=}r_1, \rho\Delta')}$$

$$\frac{\Delta \vdash \Delta_i * \texttt{R} \text{ and } \Delta'_1{=}\texttt{emp}, \text{ or } \Delta * [\Delta'_1] \rhd \Delta_i * \texttt{R}}{[\![\texttt{while}\ x\ \{e\}\ \texttt{inv}\ \Delta_i]\!]_{\mathcal{T}}(\Delta,\Delta') =_{df} (\texttt{R} * \Delta_i \wedge \neg x, \Delta' * \Delta'_1)}$$

which is used in both the first and the third verification algorithms.

5.5 Soundness and Termination

Informally, in the presence of unknown procedure calls, the soundness of the verification signifies that, a program is successfully verified against its specifications, if all the unknown procedures that it invokes conform to the specifications discovered by the verification algorithm. Therefore, the correctness of the program depends on a (possible) further verification for the unknown procedures. It can be defined as follows:

Definition 1 (Soundness). *Suppose for specification table $\mathcal{T}$, program to be verified $v = \{e_1; u; e_2\}$ and its specifications $mspec_v$, our verification succeeds and returns $\mathcal{T}_u$ as the specification table for unknown procedures invoked in v. Then we say our verification is sound, if the following holds:*

$$\forall \sigma \in [\![e_1;u;e_2]\!]_{\mathcal{T} \uplus \mathcal{T}_U}\{[\boldsymbol{x_0}/\boldsymbol{a}, \boldsymbol{y_0}/\boldsymbol{b}]\Phi_{pr}\} \cdot \sigma \vdash [\boldsymbol{x_0}/\boldsymbol{a}, \boldsymbol{y_0}/\boldsymbol{b}, \boldsymbol{y'_0}/\boldsymbol{b'}]\Phi_{po} * \texttt{true}$$

which means that, with respect to the underlying semantics, if all the unknown procedures can be verified to satisfy their specifications in $\mathcal{T}_u$, then the whole program v should meet all the specifications in $mspec_v$.

The soundness of our verification algorithm is guaranteed with several aspects: the soundness of the entailment checking, the soundness of our abduction, and the soundness of our abstract semantics. The proof for entailment checking is by structural induction over abstract domain [14]. For abduction, as its rules show, the abduction result σ' is always checked together with the antecedent σ such that they can entail the consequence, and hence its soundness follows that of entailment checking's. Finally, the soundness of abstract semantics is proven by induction over program constructors. Therefore we have

Theorem 1 (Soundness). *Our analysis is sound due to the soundness of entailment checking, abduction and abstract semantics.*

The termination of our verification algorithms is based on the termination of abduction, abstract semantics and the verification algorithms. For the abduction, we ensure in the second rule that the number of possible unfoldings is limited, and hence the recursion of abduction will eventually converge. For the abstract semantics, as we do not need to calculate fixed-points of certain program segments (such as loops and recursive method calls) based on the user-supplied invariants and specifications, its termination is straightforward to achieve. Finally, our verification algorithms only perform instructions to the scale of the input (program and specifications). As the input is finite, we can conclude the following:

Theorem 2 (Termination). *Our verification will terminate in finite steps for finite input of programs and specifications.*

6 Experimental Results

We have implemented the verification algorithms and the abstract semantics with Objective Caml and evaluated them over some heap-manipulating programs. The results are in Tables 1, 2 and 3. In each table, the first and second columns denote the programs used for evaluation and their time consumption, respectively. During the experiments, we manually hide some instructions in the original programs as calls to unknown procedures, whose specifications we try to discover during the verification process. Accordingly, the third column in the first two tables contain both the specifications of the programs to be verified (upper line), and the derived specifications for the unknown procedure (upper line). For the third table, as we used the same specification $\texttt{x::llB}\langle\texttt{S}\rangle \ast\!\!\!\to \texttt{res::sllB}\langle\texttt{T}\rangle\wedge\texttt{T=S}$ to verify all the sorting algorithms, the third column (from the second line on) states the discovered specification for the unknown call only. Some of the programs with the same name have different versions, say, the ones processing (sorted) singly-linked lists (`ll` and `sll`) are different from their counterparts for doubly-linked lists (`dll`).

It can be seen that all programs are successfully verified, with some obligations on the unknown calls discovered. We note down two observations on the experimental results. The first is that the discovered specifications for the unknown procedures are usually more general than what we expect. Bear in mind

Prog.	**Time**	**Main spec.** ($\Phi_{pr} \mathrel{*\!\!\rightarrow} \Phi_{po}$) and **Derived unknown spec.** ($\Phi^u_{pr} \mathrel{*\!\!\rightarrow} \Phi^u_{po}$)
List processing programs		
create	0.405	$\mathtt{emp} \wedge \mathtt{n}\geq 0 \mathrel{*\!\!\rightarrow} \mathtt{res::llB}\langle \mathtt{S}\rangle \wedge \mathtt{n}=\lvert\mathtt{S}\rvert \wedge \forall \mathtt{v}\in\mathtt{S}\cdot 1\leq\mathtt{v}\leq\mathtt{n}$
		$\mathtt{emp} \wedge \mathtt{a}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{res::node}\langle \mathtt{c},\mathtt{b}\rangle \wedge 1\leq\mathtt{c}\leq\mathtt{n}$
	1.895	$\mathtt{emp} \wedge \mathtt{n}\geq 0 \mathrel{*\!\!\rightarrow} \mathtt{res::dllB}\langle \mathtt{rp},\mathtt{S}\rangle \wedge \mathtt{n}=\lvert\mathtt{S}\rvert \wedge \forall \mathtt{v}\in\mathtt{S}\cdot 1\leq\mathtt{v}\leq\mathtt{n}$
		$\mathtt{emp} \wedge \mathtt{a}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{res::node2}\langle \mathtt{c},\mathtt{d},\mathtt{b}\rangle \wedge 1\leq\mathtt{c}\leq\mathtt{n}$
	1.020	$\mathtt{emp} \wedge \mathtt{n}\geq 0 \mathrel{*\!\!\rightarrow} \mathtt{res::sllB2}\langle \mathtt{S}\rangle \wedge \mathtt{n}=\lvert\mathtt{S}\rvert \wedge \forall \mathtt{v}\in\mathtt{S}\cdot 1\leq\mathtt{v}\leq\mathtt{n}$
		$\mathtt{emp} \wedge \mathtt{a}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{res::node}\langle \mathtt{c},\mathtt{b}\rangle \wedge \mathtt{a}-1\leq\mathtt{c}\leq\mathtt{a}$
sort_ insert	0.667	$\mathtt{x::ll}\langle \mathtt{n}\rangle \wedge \mathtt{n}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::ll}\langle \mathtt{m}\rangle \wedge \mathtt{m}=\mathtt{n}+1$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{c}\rangle * \mathtt{c::ll}\langle \mathtt{d}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::node}\langle \mathtt{b},\mathtt{e}\rangle * \mathtt{e::ll}\langle \mathtt{d}+1\rangle$
	0.842	$\mathtt{x::dll}\langle \mathtt{p},\mathtt{n}\rangle \wedge \mathtt{n}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::dll}\langle \mathtt{q},\mathtt{m}\rangle \wedge \mathtt{n}\geq 1 \wedge \mathtt{m}=\mathtt{n}+1 \wedge \mathtt{p}=\mathtt{q}$
		$\mathtt{a::node2}\langle \mathtt{b},\mathtt{g},\mathtt{c}\rangle * \mathtt{c::dll}\langle \mathtt{a},\mathtt{d}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::node2}\langle \mathtt{b},\mathtt{g},\mathtt{e}\rangle * \mathtt{e::dll}\langle \mathtt{a},\mathtt{d}+1\rangle$
	0.764	$\mathtt{x::sll}\langle \mathtt{n},\mathtt{xs},\mathtt{xl}\rangle \wedge \mathtt{v}\geq\mathtt{xs} \mathrel{*\!\!\rightarrow} \mathtt{x::sll}\langle \mathtt{n}+1,\mathtt{mn},\mathtt{mx}\rangle \wedge \mathtt{mn}=\mathtt{xs} \wedge \mathtt{mx}=\mathsf{max}(\mathtt{xl},\mathtt{v})$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{c}\rangle * \mathtt{c::sll}\langle \mathtt{d},\mathtt{g},\mathtt{h}\rangle \wedge \mathtt{b}\leq\mathtt{f}\leq\mathtt{g} \mathrel{*\!\!\rightarrow} \mathtt{a::node}\langle \mathtt{b},\mathtt{e}\rangle * \mathtt{e::sll}\langle \mathtt{d}+1,\mathtt{f},\mathtt{h}\rangle$
tail_ insert	0.498	$\mathtt{x::ll}\langle \mathtt{n}\rangle \wedge \mathtt{n}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::ll}\langle \mathtt{m}\rangle \wedge \mathtt{m}=\mathtt{n}+1$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{null}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::ll}\langle 2\rangle$
	0.627	$\mathtt{x::sll}\langle \mathtt{n},\mathtt{xs},\mathtt{xl}\rangle \wedge \mathtt{v}\geq\mathtt{xl} \mathrel{*\!\!\rightarrow} \mathtt{x::sll}\langle \mathtt{m},\mathtt{mn},\mathtt{mx}\rangle \wedge \mathtt{v}=\mathtt{mx} \wedge \mathtt{mn}=\mathtt{xs} \wedge \mathtt{m}=\mathtt{n}+1$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{null}\rangle \wedge \mathtt{b}\leq\mathtt{c} \mathrel{*\!\!\rightarrow} \mathtt{a::sll}\langle 2,\mathtt{b},\mathtt{c}\rangle$
rand_ insert	0.514	$\mathtt{x::ll}\langle \mathtt{n}\rangle \wedge \mathtt{n}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::ll}\langle \mathtt{m}\rangle \wedge \mathtt{m}=\mathtt{n}+1$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{c}\rangle * \mathtt{c::ll}\langle \mathtt{d}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::node}\langle \mathtt{b},\mathtt{e}\rangle * \mathtt{e::ll}\langle \mathtt{d}+1\rangle$
	0.697	$\mathtt{x::dll}\langle \mathtt{p},\mathtt{n}\rangle \wedge \mathtt{n}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::dll}\langle \mathtt{q},\mathtt{m}\rangle \wedge \mathtt{m}=\mathtt{n}+1 \wedge \mathtt{p}=\mathtt{q}$
		$\mathtt{a::node2}\langle \mathtt{b},\mathtt{g},\mathtt{c}\rangle * \mathtt{c::dll}\langle \mathtt{a},\mathtt{d}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::node2}\langle \mathtt{b},\mathtt{g},\mathtt{e}\rangle * \mathtt{e::dll}\langle \mathtt{a},\mathtt{d}+1\rangle$
delete	0.646	$\mathtt{x::llB}\langle \mathtt{S}\rangle \wedge \lvert\mathtt{S}\rvert\geq 2 \mathrel{*\!\!\rightarrow} \mathtt{x::llB}\langle \mathtt{T}\rangle \wedge \exists \mathtt{a}\cdot\mathtt{S}=\mathtt{T}\sqcup\{\mathtt{a}\}$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{c}\rangle * \mathtt{c::node}\langle \mathtt{d},\mathtt{e}\rangle * \mathtt{e::llB}\langle \mathtt{E}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::node}\langle \mathtt{b},\mathtt{e}\rangle * \mathtt{e::llB}\langle \mathtt{E}\rangle$
	0.916	$\mathtt{x::sllB}\langle \mathtt{S}\rangle \wedge \lvert\mathtt{S}\rvert\geq 2 \mathrel{*\!\!\rightarrow} \mathtt{x::sllB}\langle \mathtt{T}\rangle \wedge \exists \mathtt{a}\cdot\mathtt{S}=\mathtt{T}\sqcup\{\mathtt{a}\}$
		$\mathtt{a::node}\langle \mathtt{b},\mathtt{c}\rangle * \mathtt{c::node}\langle \mathtt{d},\mathtt{e}\rangle * \mathtt{e::sllB}\langle \mathtt{E}\rangle \wedge \forall \mathtt{f}\in\mathtt{E}\cdot\mathtt{b}\leq\mathtt{d}\leq\mathtt{f} \mathrel{*\!\!\rightarrow}$ $\mathtt{a::node}\langle \mathtt{b},\mathtt{e}\rangle * \mathtt{e::sllB}\langle \mathtt{E}\rangle \wedge \forall \mathtt{f}\in\mathtt{E}\cdot\mathtt{b}\leq\mathtt{f}$
	1.430	$\mathtt{x::dllB}\langle \mathtt{p},\mathtt{S}\rangle \wedge \lvert\mathtt{S}\rvert\geq 2 \mathrel{*\!\!\rightarrow} \mathtt{x::dllB}\langle \mathtt{q},\mathtt{T}\rangle \wedge \exists \mathtt{a}\cdot\mathtt{S}=\mathtt{T}\sqcup\{\mathtt{a}\} \wedge \mathtt{p}=\mathtt{q}$
		$\mathtt{a::node2}\langle \mathtt{b},\mathtt{f},\mathtt{c}\rangle * \mathtt{c::node}\langle \mathtt{d},\mathtt{a},\mathtt{e}\rangle * \mathtt{e::dllB}\langle \mathtt{c},\mathtt{E}\rangle \mathrel{*\!\!\rightarrow}$ $\mathtt{a::node}\langle \mathtt{b},\mathtt{f},\mathtt{e}\rangle * \mathtt{e::dllB}\langle \mathtt{a},\mathtt{E}\rangle$
travrs	0.272	$\mathtt{x::ll}\langle \mathtt{m}\rangle \wedge \mathtt{n}\geq 0\wedge\mathtt{m}\geq\mathtt{n} \mathrel{*\!\!\rightarrow} \mathtt{x::ls}\langle \mathtt{p},\mathtt{k}\rangle * \mathtt{res::ll}\langle \mathtt{r}\rangle \wedge \mathtt{p}=\mathtt{res} \wedge \mathtt{k}=\mathtt{n} \wedge \mathtt{m}=\mathtt{n}+\mathtt{r}$
		$\mathtt{a::ll}\langle \mathtt{b}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::ls}\langle \mathtt{c}\rangle * \mathtt{res::ll}\langle \mathtt{d}\rangle \wedge \mathtt{b}=\mathtt{c}+\mathtt{d} \wedge \mathtt{c}\leq\mathtt{n}$
	2.322	$\mathtt{x::sllB}\langle \mathtt{S}\rangle \wedge \mathtt{n}\geq 0\wedge\lvert\mathtt{S}\rvert\geq\mathtt{n} \mathrel{*\!\!\rightarrow} \mathtt{x::slsB}\langle \mathtt{p},\mathtt{T}\rangle * \mathtt{res::sllB}\langle \mathtt{S_2}\rangle \wedge \mathtt{p}=\mathtt{res} \wedge$ $\lvert\mathtt{T}\rvert=\mathtt{n} \wedge \mathtt{S}=\mathtt{T}\sqcup\mathtt{S_2} \wedge \forall \mathtt{u}\in\mathtt{T},\mathtt{v}\in\mathtt{S_2}\cdot\mathtt{u}\leq\mathtt{v}$
		$\mathtt{a::sllB}\langle \mathtt{A}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::slsB}\langle \mathtt{A_1}\rangle * \mathtt{res::sllB}\langle \mathtt{R}\rangle \wedge$ $\mathtt{A}=\mathtt{A_1}\sqcup\mathtt{R} \wedge \lvert\mathtt{A_1}\rvert\leq\mathtt{n} \wedge \forall \mathtt{b}\in\mathtt{A_1},\mathtt{c}\in\mathtt{R}\cdot\mathtt{b}\leq\mathtt{c}$
append	0.523	$\mathtt{x::ll}\langle \mathtt{xn}\rangle * \mathtt{y::ll}\langle \mathtt{yn}\rangle \wedge \mathtt{xn}\geq 1 \mathrel{*\!\!\rightarrow} \mathtt{x::ll}\langle \mathtt{m}\rangle \wedge \mathtt{m}=\mathtt{xn}+\mathtt{yn}$
		$\mathtt{a::ll}\langle \mathtt{b}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::ls}\langle \mathtt{c}\rangle * \mathtt{res::ll}\langle \mathtt{d}\rangle \wedge \mathtt{b}=\mathtt{c}+\mathtt{d}$
	0.861	$\mathtt{x::sll}\langle \mathtt{xn},\mathtt{xs},\mathtt{xl}\rangle * \mathtt{y::sll}\langle \mathtt{yn},\mathtt{ys},\mathtt{yl}\rangle \wedge \mathtt{xl}\leq\mathtt{ys} \mathrel{*\!\!\rightarrow}$ $\mathtt{x::sll}\langle \mathtt{m},\mathtt{rs},\mathtt{rl}\rangle \wedge \mathtt{yl}=\mathtt{rl} \wedge \mathtt{m}\geq 1+\mathtt{yn} \wedge \mathtt{m}=\mathtt{xn}+\mathtt{yn}$
		$\mathtt{a::sllB}\langle \mathtt{A}\rangle \mathrel{*\!\!\rightarrow} \mathtt{a::slsB}\langle \mathtt{A_1}\rangle * \mathtt{res::sllB}\langle \mathtt{R}\rangle \wedge \mathtt{A}=\mathtt{A_1}\sqcup\mathtt{R} \wedge \forall \mathtt{b}\in\mathtt{A_1},\mathtt{c}\in\mathtt{R}\cdot\mathtt{b}\leq\mathtt{c}$

Table 1. Experimental results (lists).

that we have replaced some instructions from those programs with unknown calls. We have compared the inferred specifications for those unknown calls with the original instructions. The results show that the specifications derived by

Prog.	**Time**	**Main spec.** ($\Phi_{pr} \mathrel{*\!\!\rightarrow} \Phi_{po}$) and **Derived unknown spec.** ($\Phi^u_{pr} \mathrel{*\!\!\rightarrow} \Phi^u_{po}$)
Binary tree, binary search tree, AVL tree and red-black tree processing programs		
`travrs`	0.417	$x{::}bt\langle S,h\rangle \mathrel{*\!\!\rightarrow} x{::}bt\langle T,k\rangle \wedge S{=}T \wedge h{=}k$
		$a{::}bt\langle A,b\rangle \wedge a{\neq}null \mathrel{*\!\!\rightarrow} a{::}node2\langle c,d,e\rangle * d{::}bt\langle D,f\rangle * e{::}bt\langle E,g\rangle \wedge A{=}\{c\}{\sqcup}D{\sqcup}E \wedge b{=}max(f,g){+}1 \wedge (res{=}d \vee res{=}e)$
`count`	0.705	$x{::}bt\langle S,h\rangle \mathrel{*\!\!\rightarrow} x{::}bt\langle T,k\rangle \wedge res{=}\lvert S\rvert \wedge S{=}T \wedge h{=}k$
		$a{::}bt\langle A,b\rangle \wedge a{\neq}null \mathrel{*\!\!\rightarrow} a{::}node2\langle c,d,e\rangle * d{::}bt\langle D,f\rangle * e{::}bt\langle E,g\rangle \wedge A{=}\{c\}{\sqcup}D{\sqcup}E \wedge b{=}max(f,g){+}1 \wedge (res{=}d \vee res{=}e)$
`height`	0.821	$x{::}bt\langle S,h\rangle \mathrel{*\!\!\rightarrow} x{::}bt\langle T,k\rangle \wedge res{=}h{=}k \wedge S{=}T$
		$a{::}bt\langle A,b\rangle \wedge a{\neq}null \mathrel{*\!\!\rightarrow} a{::}node2\langle c,d,e\rangle * d{::}bt\langle D,f\rangle * e{::}bt\langle E,g\rangle \wedge A{=}\{c\}{\sqcup}D{\sqcup}E \wedge b{=}max(f,g){+}1 \wedge (res{=}d \vee res{=}e)$
`insert`	1.354	$x{::}bt\langle S,h\rangle \wedge \lvert S\rvert{\geq}1 \wedge h{\geq}1 \mathrel{*\!\!\rightarrow} x{::}bt\langle T,k\rangle \wedge T{=}S{\sqcup}\{v\} \wedge h{\leq}k{\leq}h{+}1$
		$a{::}node2\langle b,null,null\rangle \mathrel{*\!\!\rightarrow} a{::}bt\langle A,2\rangle \wedge A{=}\{b,c\}$
`delete`	1.019	$x{::}bt\langle S,h\rangle \wedge \lvert S\rvert{\geq}2 \wedge h{\geq}2 \mathrel{*\!\!\rightarrow} x{::}bt\langle T,k\rangle \wedge \exists a \cdot S{=}T{\sqcup}\{a\} \wedge h{-}1{\leq}k{\leq}h$
		$a{::}node2\langle b,c,null\rangle{*}c{::}node2\langle d,null,null\rangle \mathrel{*\!\!\rightarrow} a{::}node2\langle b,null,null\rangle$, & $a{::}node2\langle b,null,c\rangle{*}c{::}node2\langle d,null,null\rangle \mathrel{*\!\!\rightarrow} a{::}node2\langle b,null,null\rangle$
`search`	1.851	$x{::}bst\langle sm,lg\rangle \mathrel{*\!\!\rightarrow} x{::}bst\langle mn,mx\rangle \wedge sm{=}mn \wedge lg{=}mx \wedge 0{\leq}res{\leq}1$
		$a{::}bst\langle b,c\rangle{\wedge}a{\neq}null \mathrel{*\!\!\rightarrow} a{::}node2\langle d,e,f\rangle{*}e{::}bst\langle b,g\rangle{*}f{::}h\langle c\rangle{\wedge}g{\leq}d{\leq}h$
`bst_ insert`	1.822	$x{::}bst\langle sm,lg\rangle \mathrel{*\!\!\rightarrow} x{::}bst\langle mn,mx\rangle \wedge (v{<}sm{\wedge}v{=}mn{\wedge}lg{=}mx \vee lg{<}v{\wedge}v{=}mx{\wedge}sm{=}mn \vee sm{=}mn{\wedge}lg{=}mx)$
		$a{::}node2\langle b,null,c\rangle * c{::}bst\langle d,e\rangle \wedge f{<}b{<}d \mathrel{*\!\!\rightarrow} a{::}bst\langle f,e\rangle$, or $a{::}node2\langle b,c,null\rangle * c{::}bst\langle d,e\rangle \wedge e{<}b{<}f \mathrel{*\!\!\rightarrow} a{::}bst\langle d,f\rangle$
`avl_ins`	5.202	$x{::}avl\langle S,h\rangle \mathrel{*\!\!\rightarrow} res{::}avl\langle T,k\rangle \wedge T{=}S{\sqcup}\{v\} \wedge h{\leq}k{\leq}h{+}1$
		$a{::}avl\langle A,b\rangle \mathrel{*\!\!\rightarrow} a{::}avl\langle A,b\rangle \wedge res{=}b$
`rbt_ins`	9.093	$x{::}rbt\langle S,cl,bh\rangle \mathrel{*\!\!\rightarrow} res{::}rbt\langle T,cl_1,bh_1\rangle \wedge T{=}S{\sqcup}\{v\}$
		$a{::}rbt\langle A,b,c\rangle \mathrel{*\!\!\rightarrow} a{::}rbt\langle A,b,c\rangle \wedge res{=}b$
`sdl2nbt`	5.238	$x{::}sdlB\langle p,q,S\rangle \wedge \lvert S\rvert{\geq}1 \wedge p{=}null \wedge q{=}tail \mathrel{*\!\!\rightarrow} res{::}nbt\langle T\rangle \wedge T{=}S$
		$a{::}sdlB\langle null,c,A\rangle \wedge a{=}b{=}d \mathrel{*\!\!\rightarrow} a{::}sdlB\langle null,b,A_1\rangle * b{::}sdlB\langle e,c,B\rangle \wedge d{=}c \wedge A{=}A_1{\sqcup}B \wedge (\forall f{\in}A_1, g{\in}B{\cdot}f{\leq}g) \wedge 0{\leq}\lvert B\rvert{-}\lvert A_1\rvert{\leq}1$

Table 2. Experimental results (trees).

our algorithm not only fully capture the behaviours of those instructions, but also suggest other possible implementations. A case in point is list's `travrs`. Its "unknown call" was originally an assignment `x = x.next` which traverses the list towards its end by one node. Armed with list segment predicates and corresponding lemmas, we are able to infer that the unknown call may actually traverse the list for arbitrary number of nodes, provided it does not go beyond the list's tail or where the user has specified as input. Compared with this, we did not provide tree's `travrs` with appropriate predicates or lemmas, and hence we only have the unknown call unrolls the tree once to expose its root. To conclude, our verification always tries its best to find a sound (w.r.t. the program being verified) and general (w.r.t. the unknown call) specification for unknown procedure calls.

The second observation is that the precision of unknown calls' discovered specifications depends on its caller's given specification. As can be seen we have

Prog.	**Time**	**Main spec.** ($\Phi_{pr} \ast\!\!\!\rightarrow \Phi_{po}$) or **Derived unknown spec.** ($\Phi^u_{pr} \ast\!\!\!\rightarrow \Phi^u_{po}$)
Sorting (main)		$\mathtt{x{::}llB}\langle\mathtt{S}\rangle \ast\!\!\!\rightarrow \mathtt{res{::}sllB}\langle\mathtt{T}\rangle \wedge \mathtt{T{=}S}$
`merge`	4.099	$\mathtt{a{::}sllB}\langle\mathtt{A}\rangle * \mathtt{b{::}sllB}\langle\mathtt{B}\rangle \ast\!\!\!\rightarrow \mathtt{res{::}sllB}\langle\mathtt{R}\rangle \wedge \mathtt{R{=}A}\sqcup\mathtt{B}$
`flatten`	2.680	$\mathtt{a{::}bstB}\langle\mathtt{A}\rangle \ast\!\!\!\rightarrow \mathtt{res{::}sllB}\langle\mathtt{R}\rangle \wedge \mathtt{R{=}A}$
`insert`	1.667	$\mathtt{a{::}sllB}\langle\mathtt{A}\rangle * \mathtt{b{::}node}\langle\mathtt{c},\mathtt{d}\rangle \ast\!\!\!\rightarrow \mathtt{res{::}sllB}\langle\mathtt{R}\rangle \wedge \mathtt{R{=}A}\sqcup\{\mathtt{c}\}$
`quick`	2.064	$\mathtt{a{::}lbd}\langle\mathtt{A}\rangle \ast\!\!\!\rightarrow \mathtt{a{::}lbd}\langle\mathtt{A_1}\rangle * \mathtt{res{::}lbd}\langle\mathtt{R}\rangle \wedge \mathtt{A{=}A_1}\sqcup\mathtt{R} \wedge \forall\mathtt{c}{\in}\mathtt{A_1},\mathtt{d}{\in}\mathtt{R}\cdot\mathtt{c}{\leq}\mathtt{b}{\leq}\mathtt{d}$
`unknown`	1.824	$\mathtt{a{::}llB}\langle\mathtt{A}\rangle{\wedge}\mathtt{a}{\neq}\mathtt{null} \ast\!\!\!\rightarrow \mathtt{res{::}node}\langle\mathtt{c},\mathtt{b}\rangle{*}\mathtt{b{::}llB}\langle\mathtt{B}\rangle{\wedge}\mathtt{A}{=}\{\mathtt{c}\}{\sqcup}\mathtt{B}{\wedge}\forall\mathtt{d}{\in}\mathtt{B}{\cdot}\mathtt{c}{\leq}\mathtt{d}$

Table 3. Experimental results (sorting).

verified several list-processing programs where each one has various specifications. Within these programs we want to point out that the ones with specifications of both normal lists and sorted lists share the same code (but just with two different specifications). Such examples include `create`, `sort_insert`, `delete`, and so on. For `create` which creates a list containing numbers from 1 to `n` in descending order, we can see once incorporated with `llB` as specification predicates, the unknown call is expected to return a node whose value `c` is within 1 to `n`. Comparatively, when verified for sortedness, `c` is inferred to be between `a`−`1` and `a`, as for sortedness to hold. For `delete`'s sorted version, we also have the extra information that the list with one node removed is still a sorted list (with the multi-set value constraints), whose result is stronger than the normal list version.

7 Conclusion

It is a practical and challenging problem to verify the full functional correctness of heap-manipulating imperative programs with unknown procedure calls. Our proposed solution infers expected specifications for unknown procedures from their calling contexts. The program is verified correct on condition that the invoked unknown procedures meet the inferred specifications. We employ a forward program analysis over a combined domain and invent a novel abduction for it to synthesise the specifications of the unknown procedure. As a proof of concept, we have also implemented a prototype system to test the viability of the proposed approach. Our main future work is to explore more general solution for unknown calls in sequence to achieve more reasonable specifications for them.

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