

Lecture 5:

Performance Optimization Part 1: Work Distribution and Scheduling

**Parallel Computer Architecture and Programming
CMU 15-418/15-618, Fall 2025**

Programming for high performance

- Optimizing the performance of parallel programs is an **iterative process** of refining choices for decomposition, assignment, and orchestration...
- **Key goals** (that are **at odds with each other**)
 - **Balance workload** onto available execution resources
 - **Reduce communication** (to avoid stalls)
 - **Reduce extra work** (overhead) performed to increase parallelism, manage assignment, reduce communication, etc.
- We are going to talk about a rich space of techniques

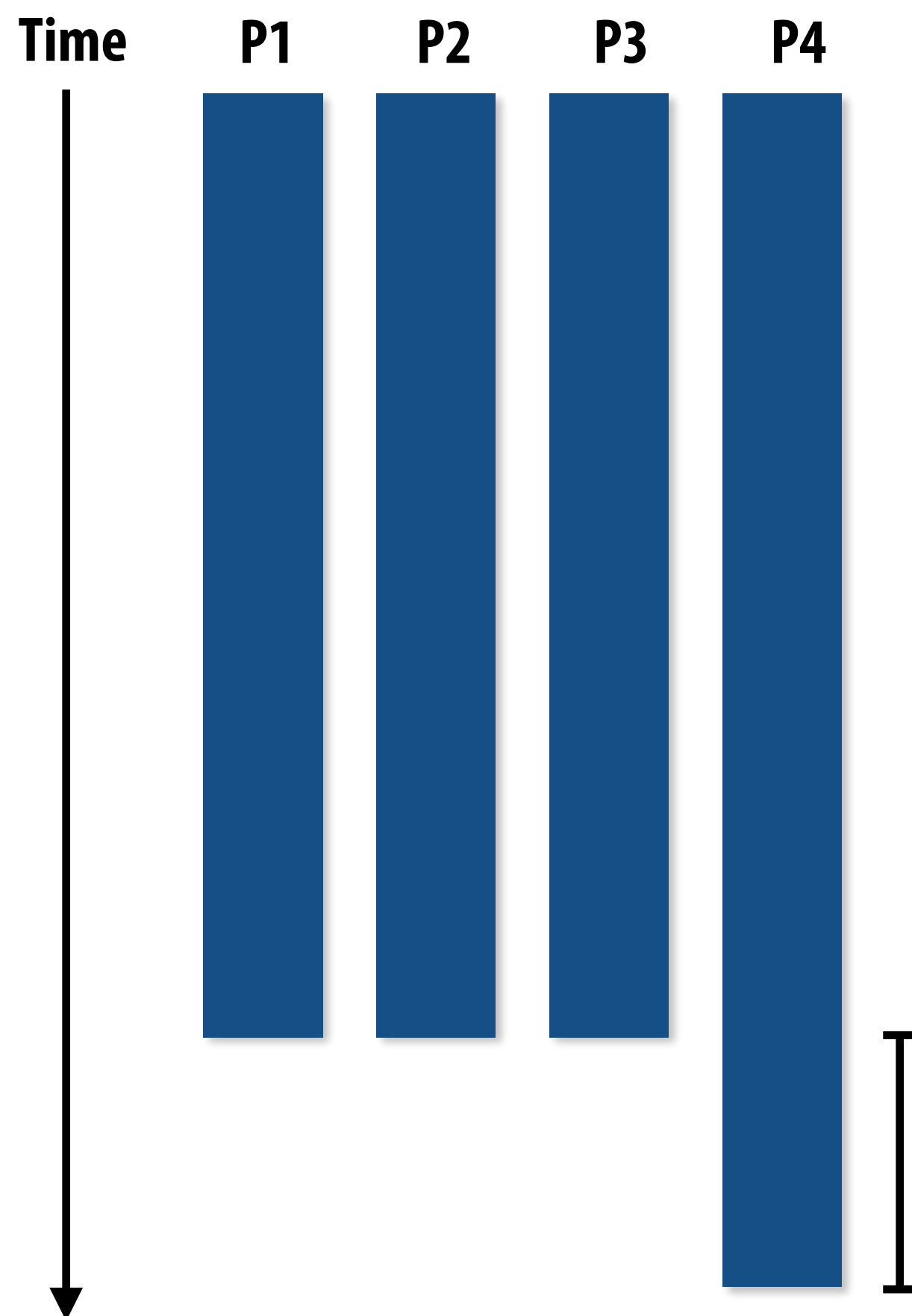
TIP #1: Always **implement the simplest solution first, then **measure performance** to determine if you need to do better.**

“My solution scales” = your code scales as much as you need it to.

(if you anticipate only running low-core count machines, it may be unnecessary to implement a complex approach that creates and hundreds or thousands of pieces of independent work)

Balancing the workload

**Ideally: all processors are computing all the time during program execution
(they are computing simultaneously, and they finish their portion of the work at the same time)**



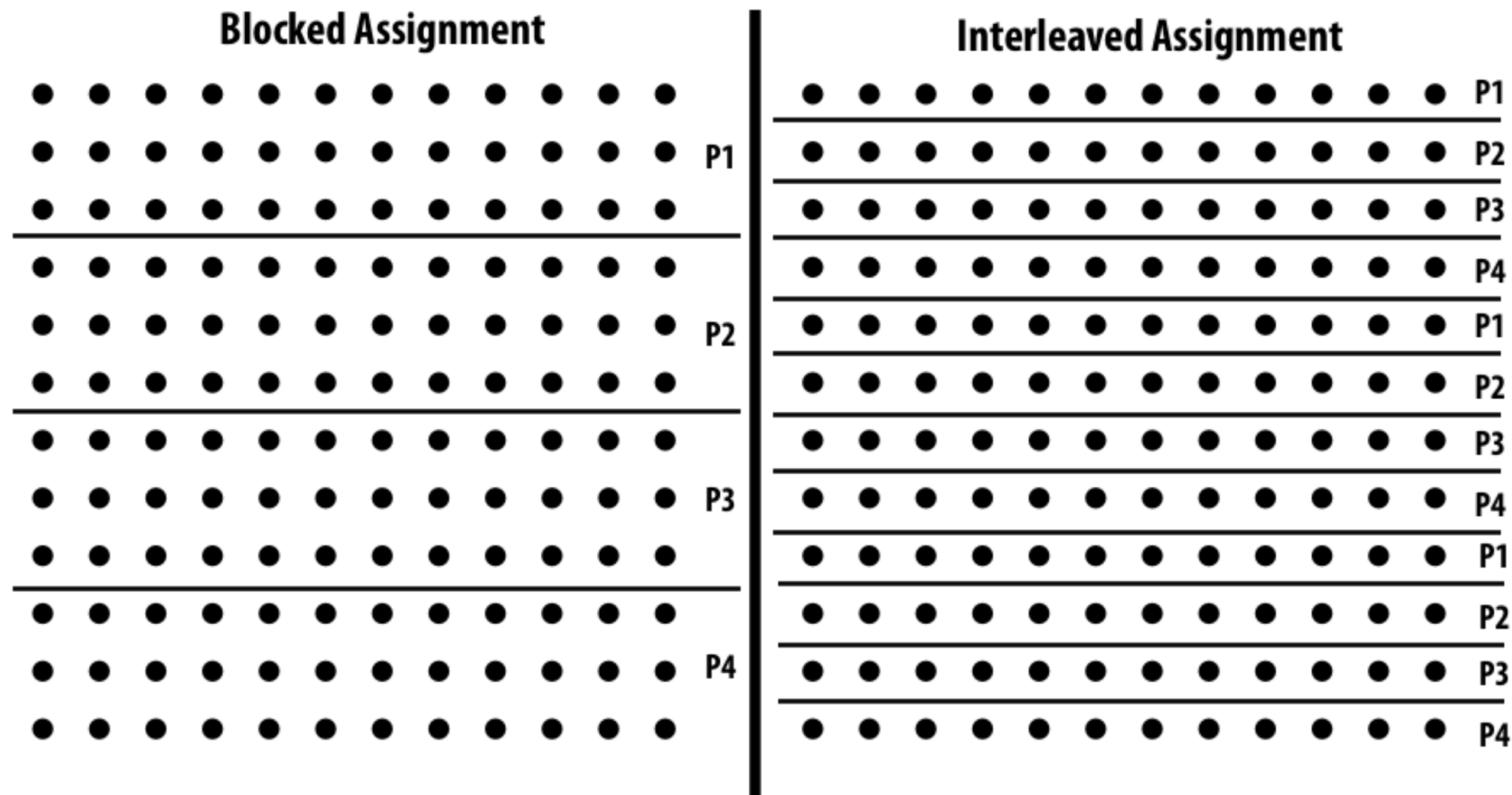
**Recall Amdahl's Law:
Only small amount of load imbalance can
significantly bound maximum speedup**

**P4 does 20% more work → P4 takes 20% longer to complete
→ 20% of parallel program's
runtime is serial execution**

**(work in serialized section here is about 5% of the work of the whole program:
 $S=.05$ in Amdahl's law equation)**

Static assignment

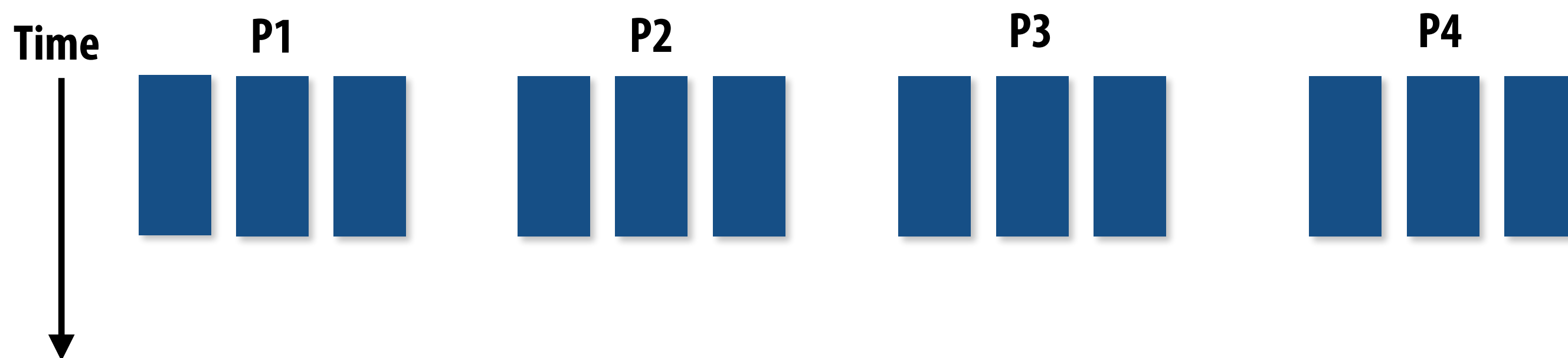
- Assignment of work to threads is **pre-determined**
 - Not necessarily determined at compile-time (assignment algorithm may depend on runtime parameters such as input data size, number of threads, etc.)
- Recall solver example: assign equal number of grid cells (work) to each thread (worker)
 - We discussed two static assignments of work to workers (**blocked** and **interleaved**)



- **Good properties of static assignment:** simple, essentially **zero runtime overhead** (in this example: extra work to implement assignment is a little bit of indexing math)

When is static assignment applicable?

- When the cost (execution time) of work and the amount of work is **predictable** (so the programmer can work out a good assignment in advance)
- Simplest example: it is known up front that all work has the same cost



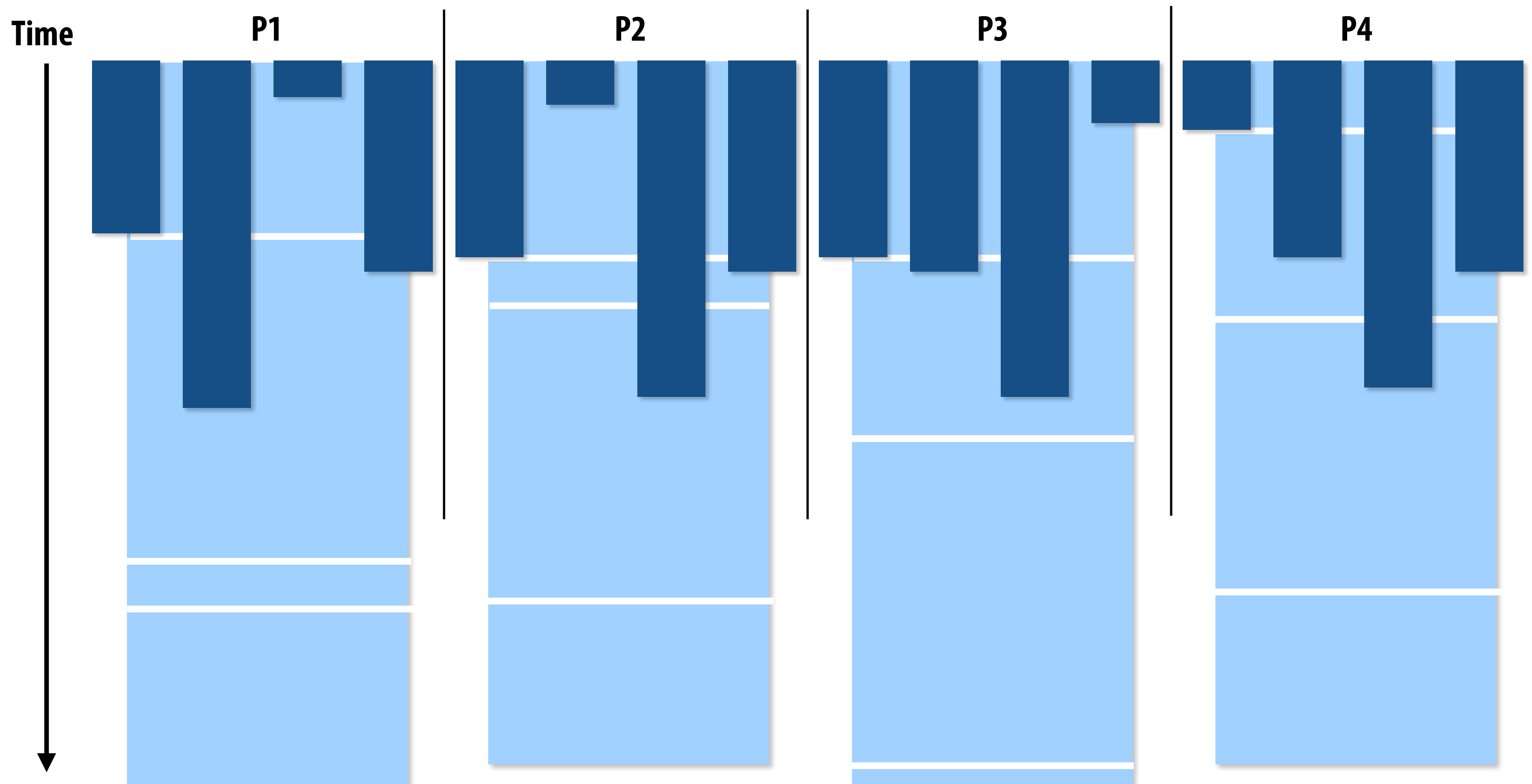
In the example above:

There are 12 tasks, and it is known that each have the same cost.

Assignment solution: statically assign three tasks to each of the four processors.

When is static assignment applicable?

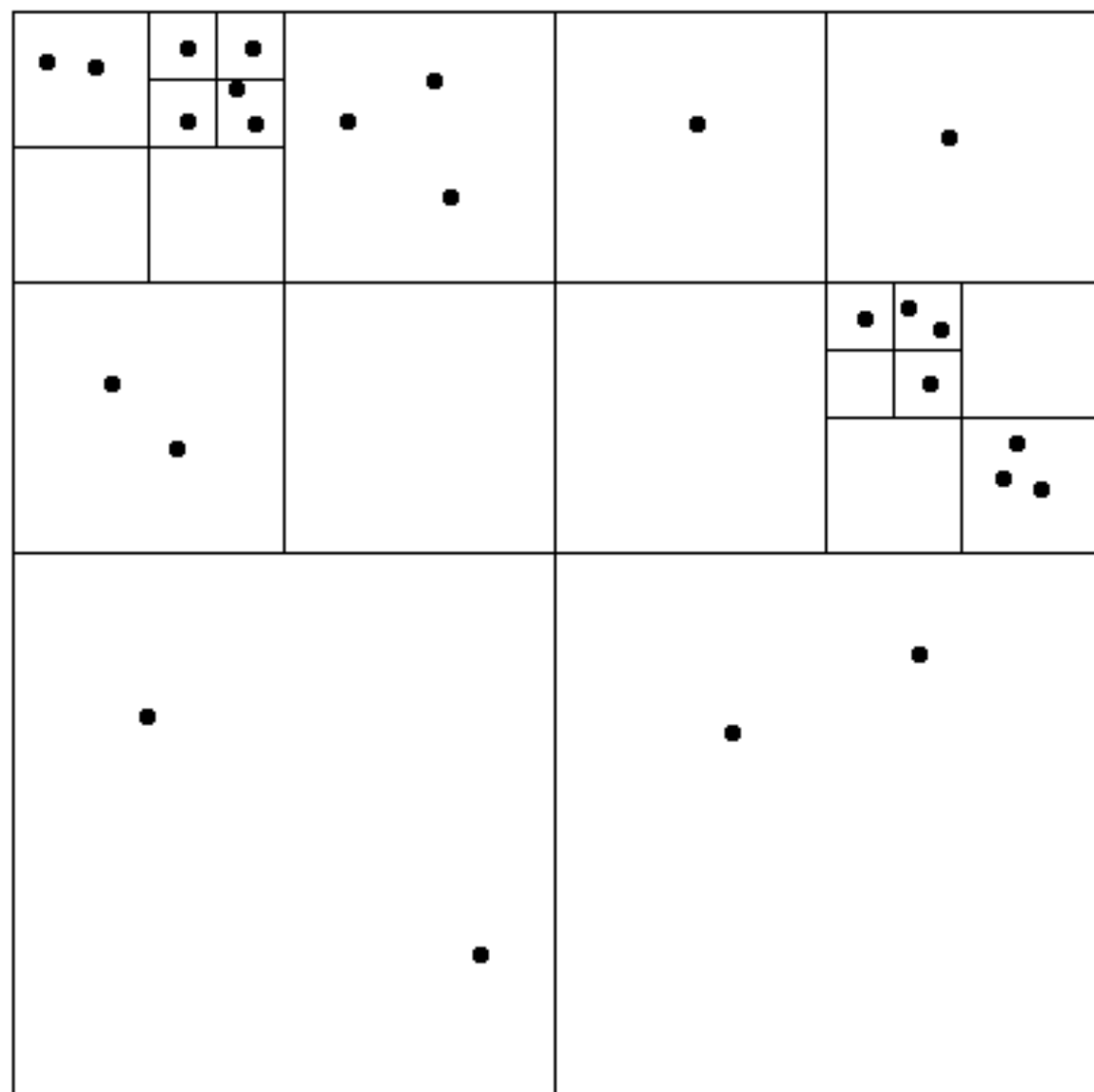
- When work is **predictable**, but not all jobs have same cost (see example below)
- When **statistics about execution time are known** (e.g., same cost on average)



Jobs have unequal, but known cost: assign to processors to ensure overall good load balance

“Semi-static” assignment

- Cost of work is **predictable for near-term future**
 - Idea: recent past good predictor of near future
- Application **periodically profiles itself and re-adjusts assignment**
 - Assignment is “static” for the interval between re-adjustments



Particle simulation:

Redistribute particles as they move over course of simulation
(if motion is slow, redistribution need not occur often)

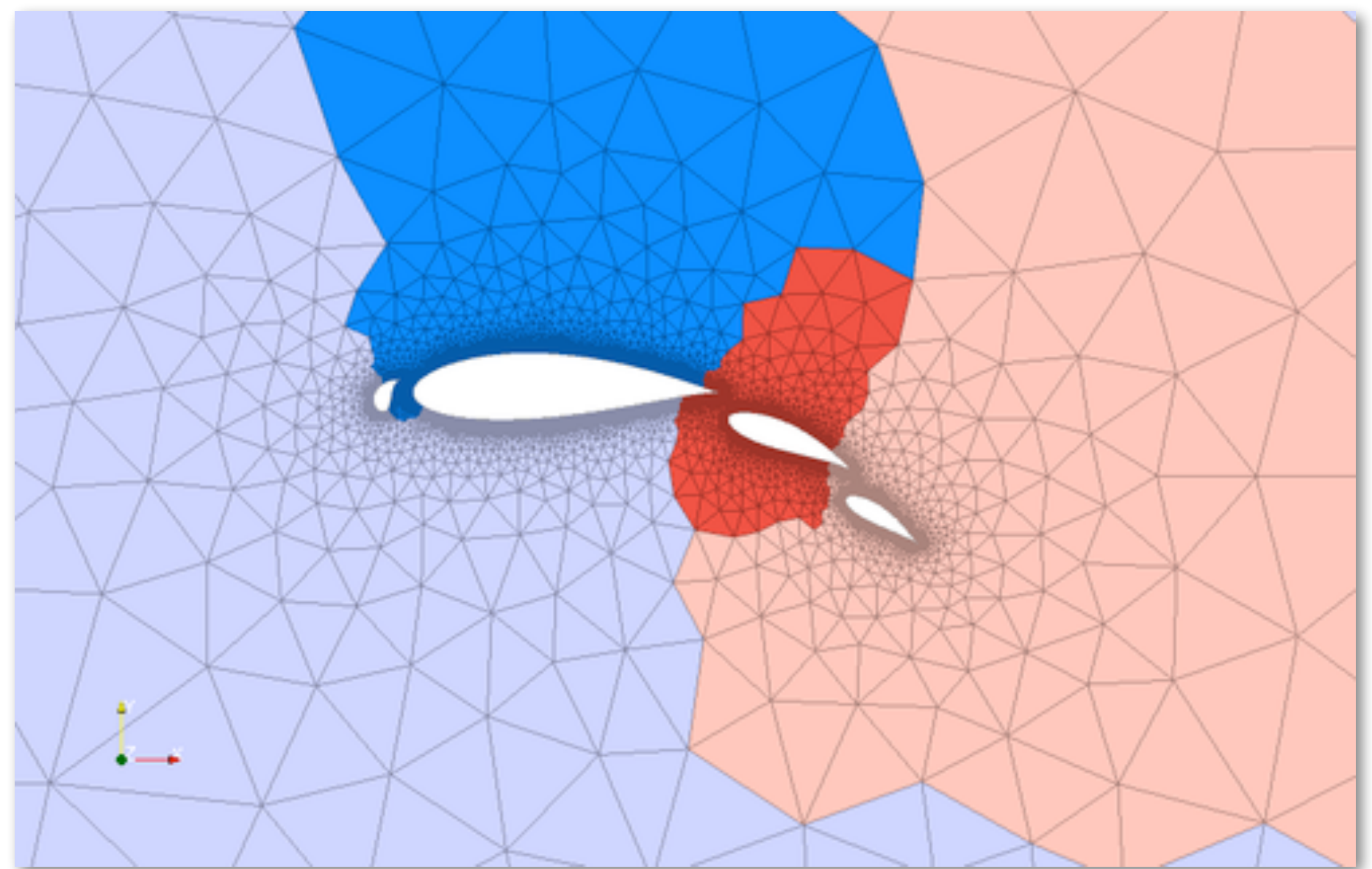


Image credit: <http://typhon.sourceforge.net/spip/spip.php?article22>

Adaptive mesh:

Mesh is changed as object moves or flow over object changes, but changes occur slowly (color indicates assignment of parts of mesh to processors)

Dynamic assignment

Program determines assignment dynamically at runtime to ensure a well distributed load. (The execution time of tasks, or the total number of tasks, is **unpredictable**.)

Sequential program
(independent loop iterations)

```
int N = 1024;
int* x = new int[N];
bool* prime = new bool[N];

// initialize elements of x here

for (int i=0; i<N; i++)
{
    // unknown execution time
    is_prime[i] = test_primalty(x[i]);
}
```

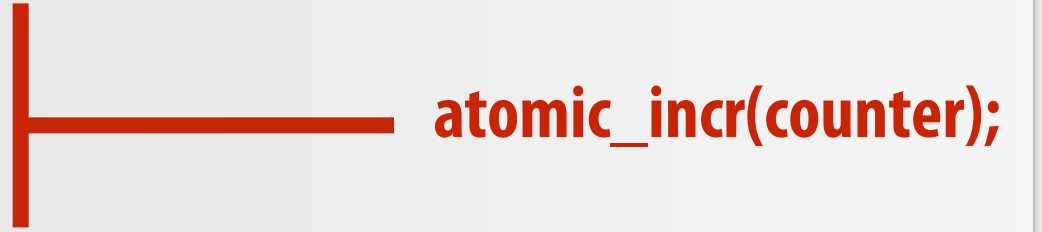
Parallel program
(SPMD execution by multiple threads,
shared address space model)

```
int N = 1024;
// assume allocations are only executed by 1 thread
int* x = new int[N];
bool* is_prime = new bool[N];

// initialize elements of x here

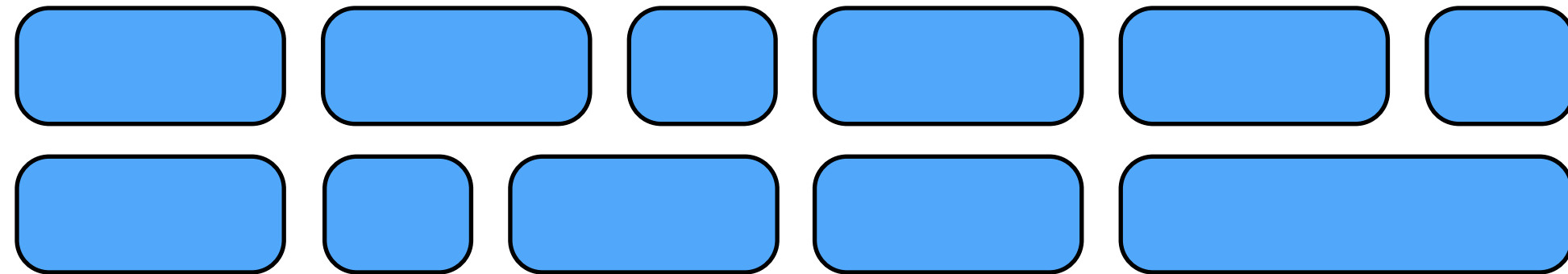
LOCK counter_lock;
int counter = 0;    // shared variable

while (1) {
    int i;
    lock(counter_lock);
    i = counter++;
    unlock(counter_lock);
    if (i >= N)
        break;
    is_prime[i] = test_primalty(x[i]);
}
```

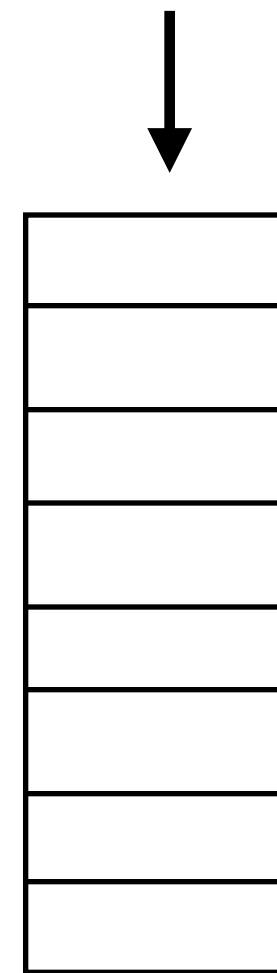


Dynamic assignment using a work queue

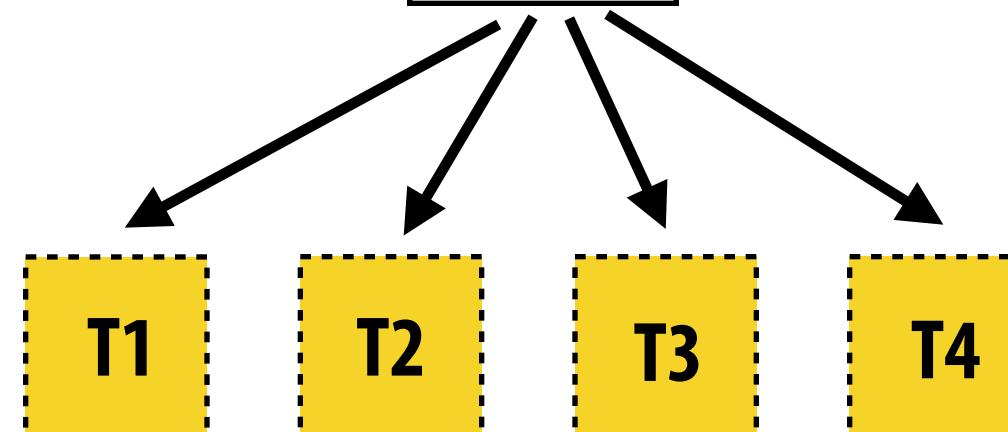
Sub-problems
(a.k.a. “tasks”, “work”)



Shared work queue: a list of work to do
(for now, let’s assume each piece of work is independent)



Worker threads:
Pull data from shared work queue
Push new work to queue as it is created



What constitutes a piece of work?

What is a potential problem with this implementation?

```
const int N = 1024;
// assume allocations are only executed by 1 thread
float* x = new float[N];
bool* prime = new bool[N];

// initialize elements of x here

LOCK counter_lock;
int counter = 0;

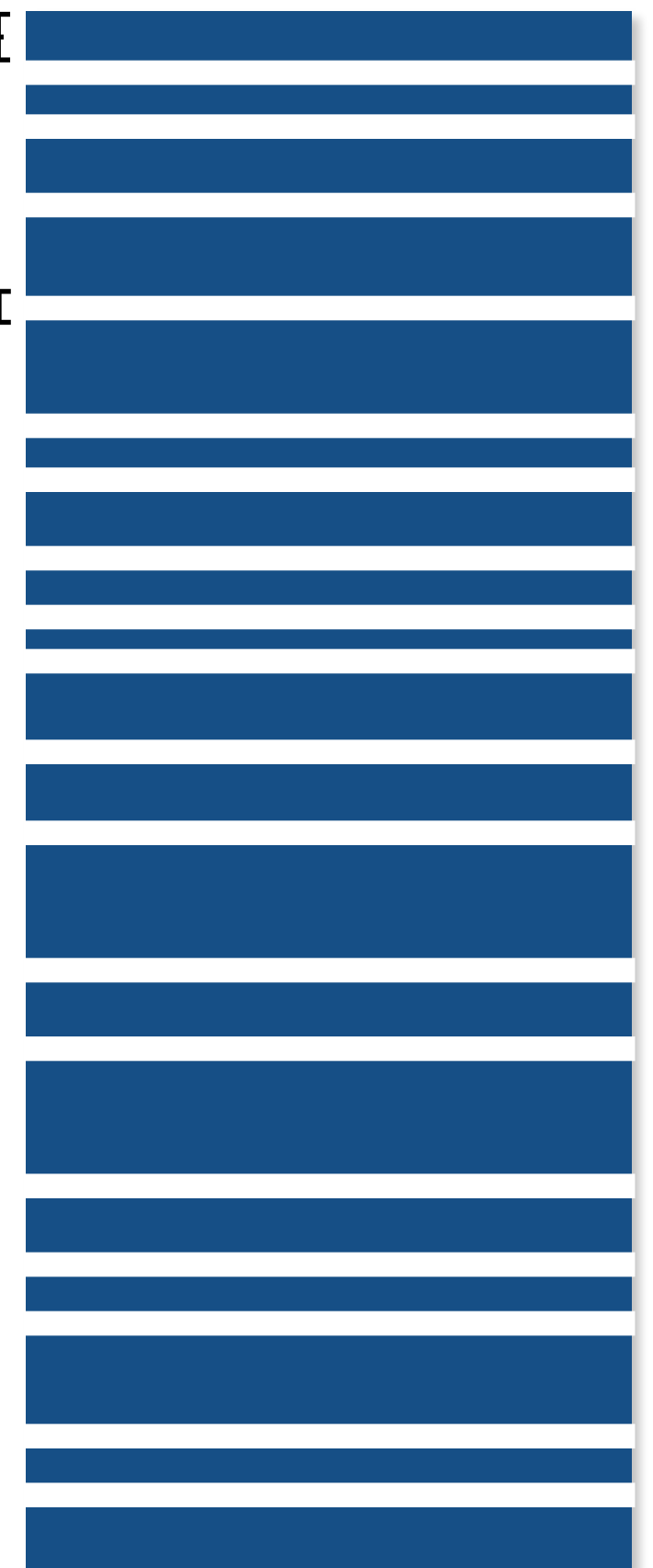
while (1) {
    int i;
    lock(counter_lock);
    i = counter++;
    unlock(counter_lock);
    if (i >= N)
        break;
    is_prime[i] = test_primalty(x[i]);
}
```

Time in task 0 ————— I

Time in critical section ————— I

This is overhead that
does not exist in serial
program

And.. it's serial execution
(recall Amdahl's Law)



Fine granularity partitioning: 1 “task” = 1 element

Likely **good workload balance** (many small tasks)

Potential for **high synchronization cost**
(serialization at critical section)

So... IS IT a problem?

Increasing task granularity

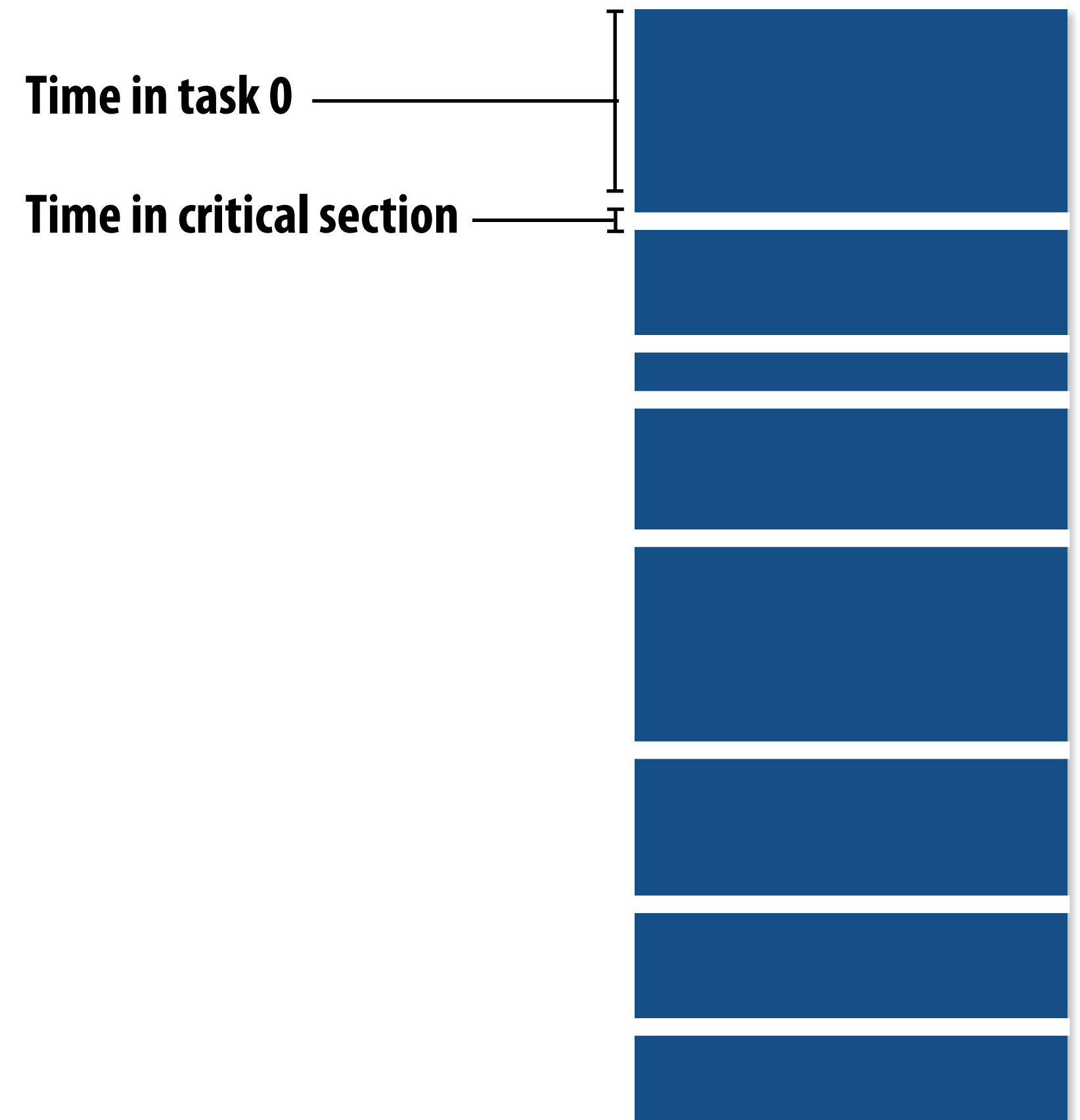
```
const int N = 1024;
const int GRANULARITY = 10;
// assume allocations are only executed by 1 thread

float* x = new float[N];
bool* prime = new bool[N];

// initialize elements of x here

LOCK counter_lock;
int counter = 0;

while (1) {
    int i;
    lock(counter_lock);
    i = counter;
    counter += GRANULARITY;
    unlock(counter_lock);
    if (i >= N)
        break;
    int end = min(i + GRANULARITY, N);
    for (int j=i; j<end; j++)
        is_prime[j] = test_primalty(x[j]);
}
```



Coarse granularity partitioning: 1 “task” = 10 elements

Decreased synchronization cost

(Critical section entered 10 times less)

Larger tasks increase work but very large tasks can lead to load balancing issues

Choosing task size

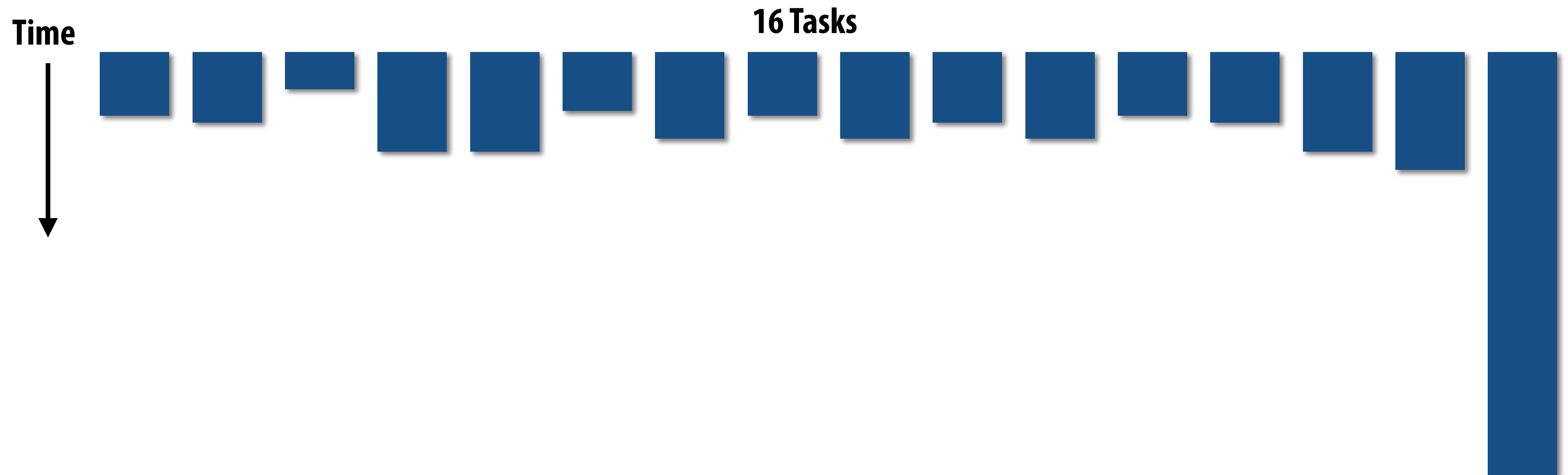
- **Useful to have many more tasks* than processors**
(many small tasks enables good **workload balance** via dynamic assignment)
 - Motivates **small granularity** tasks
- **But want as few tasks as possible to **minimize overhead** of managing the assignment**
 - Motivates **large granularity** tasks
- **Ideal granularity depends on many factors**
(Common theme in this course: must know your workload, and your machine)

* I had to pick a term for a piece of work, a sub-problem, etc.

Smarter task scheduling

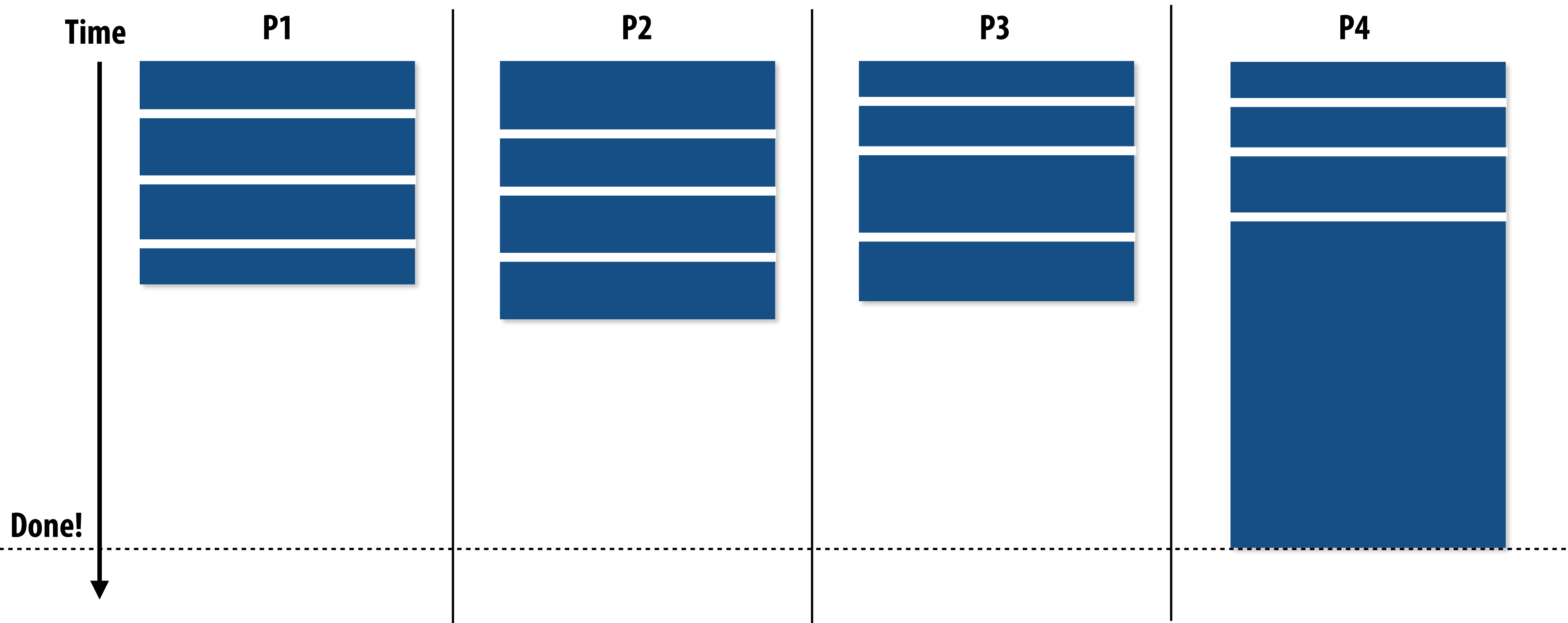
Consider dynamic scheduling via a shared work queue

What happens if the system assigns these tasks to workers in left-to-right order?



Smarter task scheduling

What happens if scheduler runs the long task last? Potential for load imbalance!



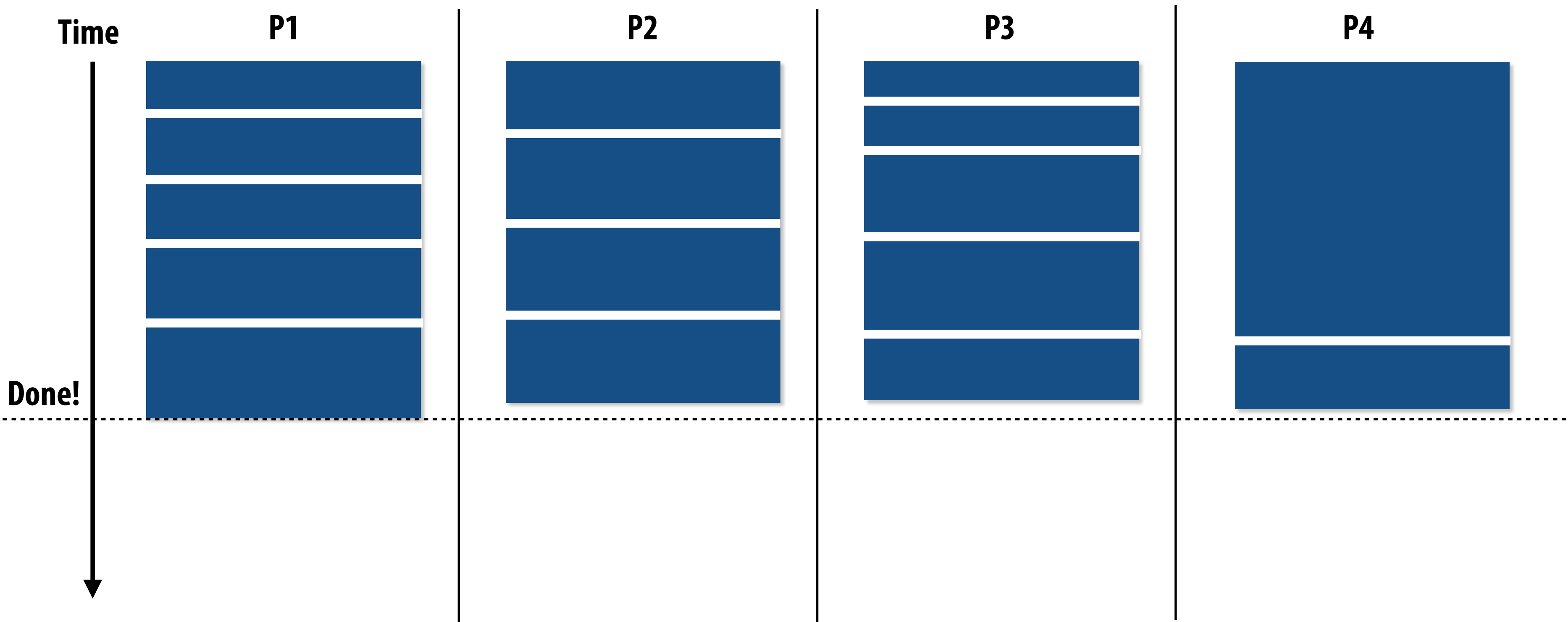
One possible solution to imbalance problem:

Divide work into a larger number of smaller tasks

- Hopefully “long pole” gets shorter relative to overall execution time
- May increase synchronization overhead
- May not be possible (perhaps long task is fundamentally sequential)

Smarter task scheduling

Schedule long task first to reduce “slop” at end of computation



Another solution: smarter scheduling

Schedule long tasks first

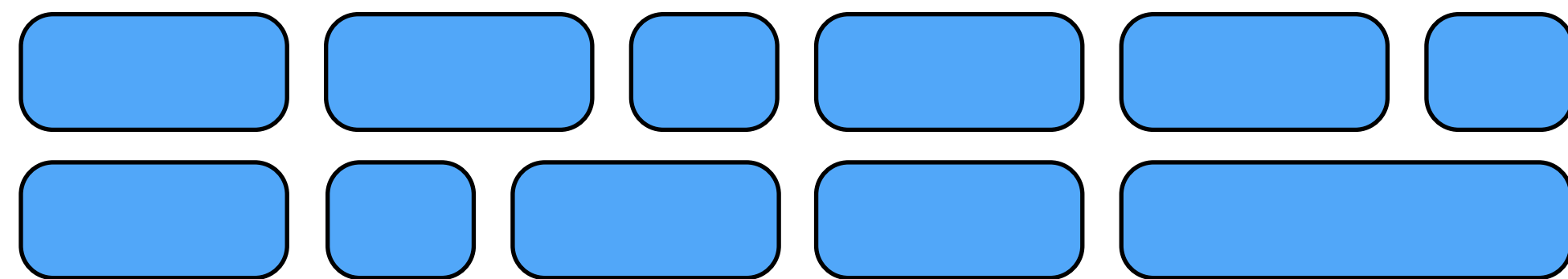
- Thread performing long task performs fewer overall tasks, but approximately the same amount of work as the other threads.
- Requires some knowledge of workload (some predictability of cost)

Decreasing synchronization overhead using a distributed set of queues

(avoid need for all workers to synchronize on single work queue)

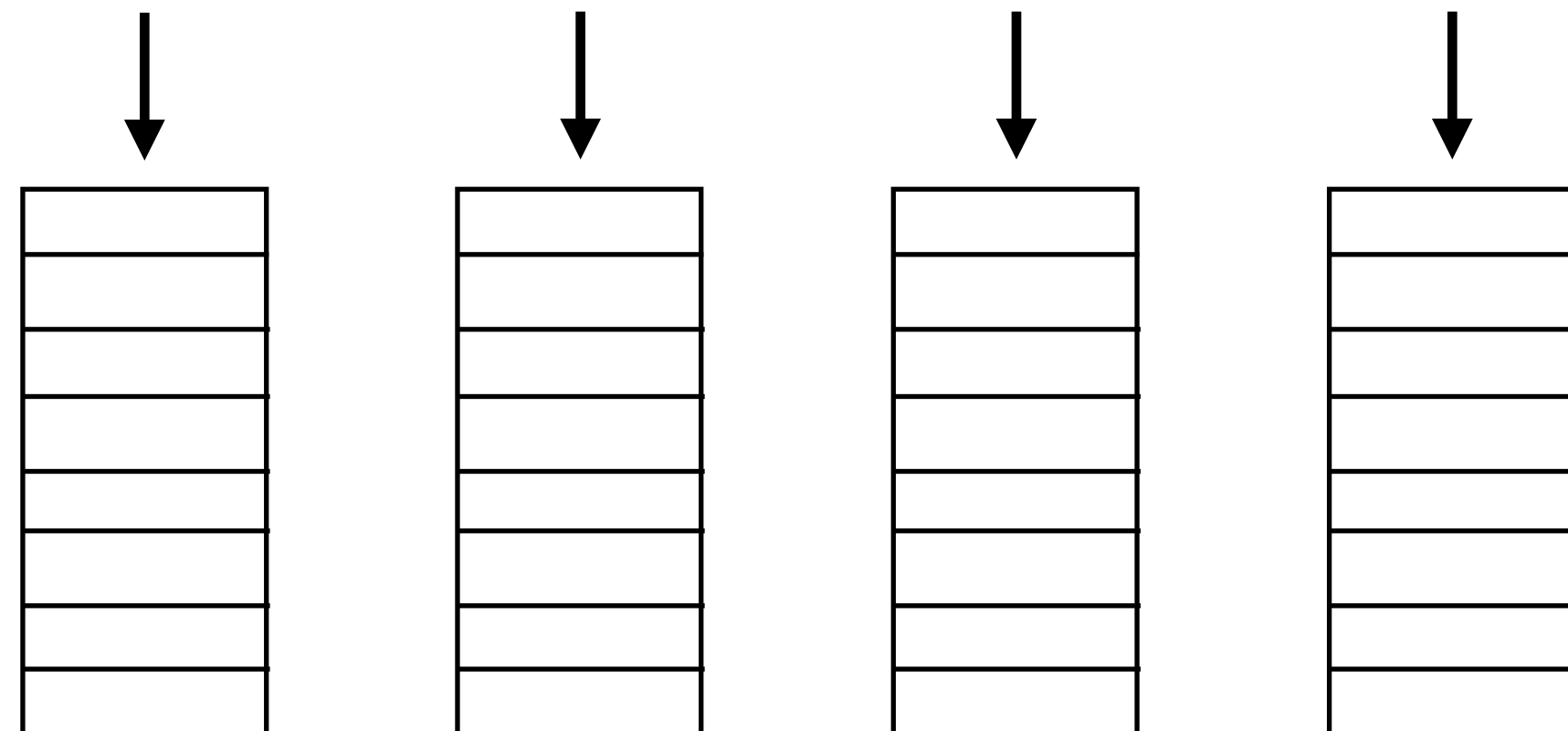
Subproblems

(a.k.a. “tasks”, “work to do”)



Set of work queues

(In general, one per worker thread)



Worker threads:

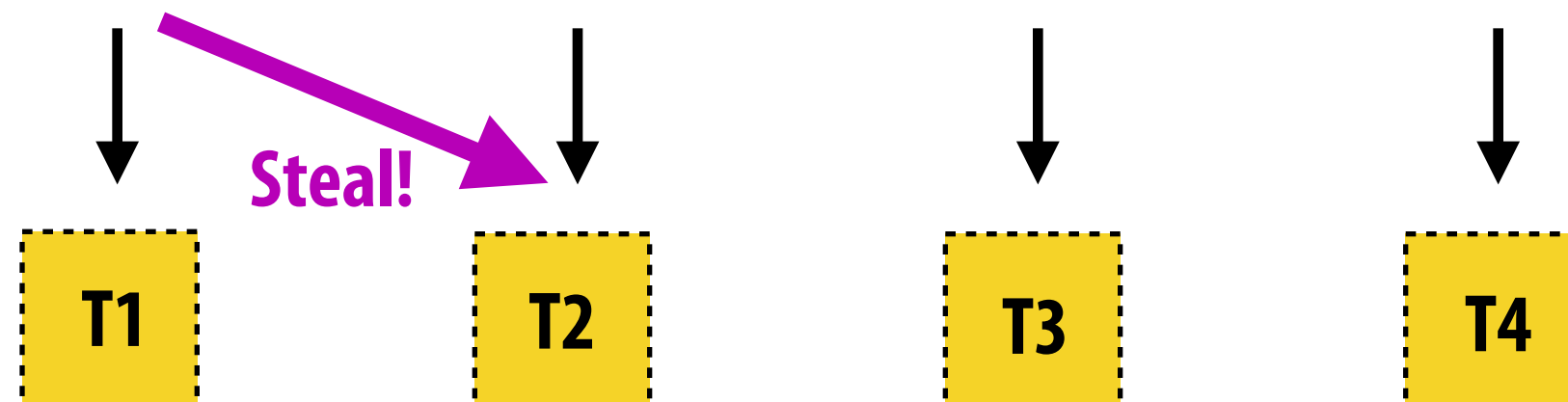
Pull data from OWN work queue

Push new work to OWN work queue

When local work queue is empty...

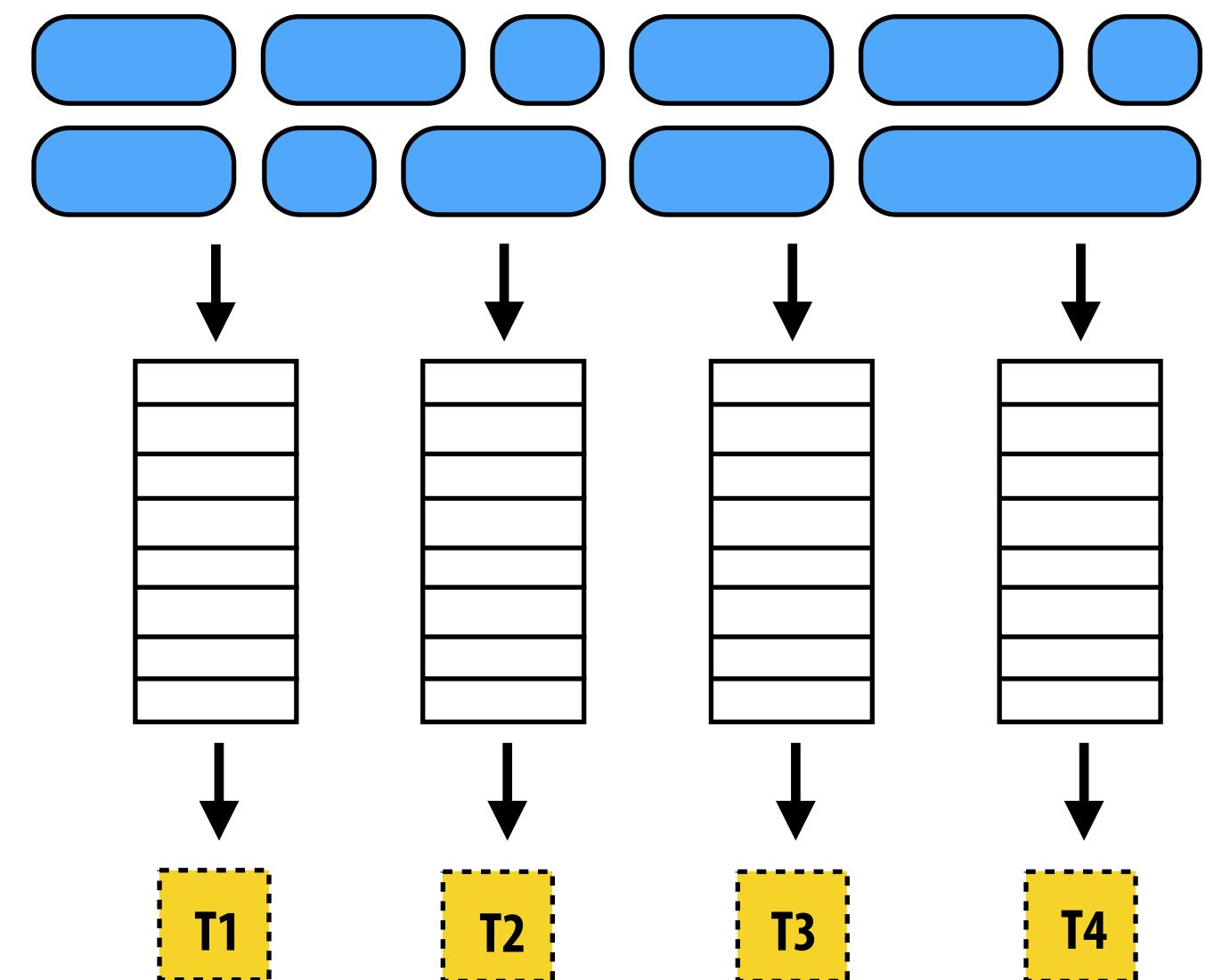
STEAL work from another work queue

Steal!



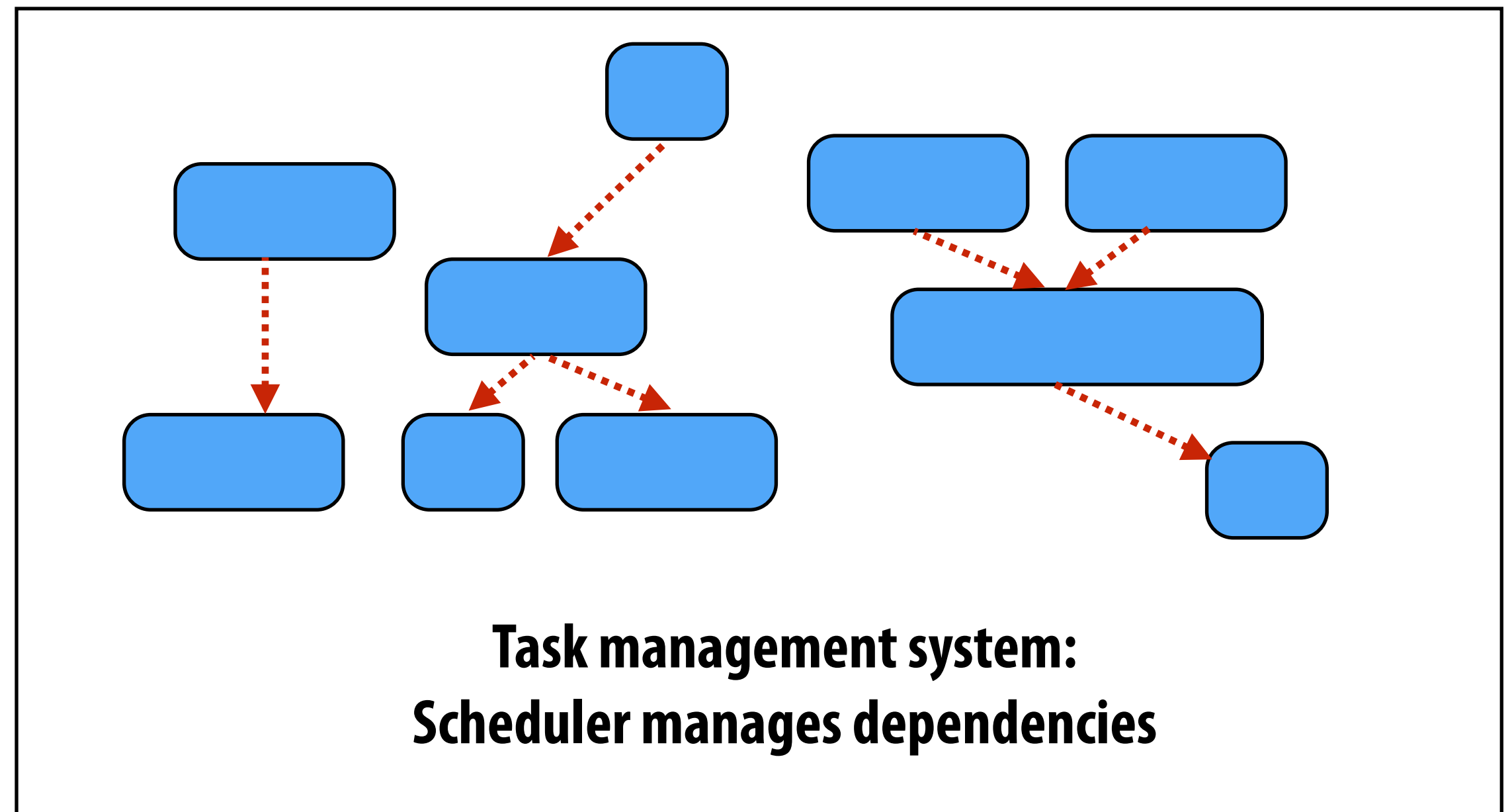
Distributed work queues

- **Costly synchronization/communication occurs during stealing**
 - But not every time a thread takes on new work
 - **Stealing occurs only when necessary** to ensure good load balance
- **Leads to increased locality**
 - Common case: threads work on tasks they create (producer-consumer locality)
- **Implementation challenges**
 - Who to steal from?
 - How much to steal?
 - How to detect program termination?
 - Ensuring local queue access is fast (while preserving mutual exclusion)

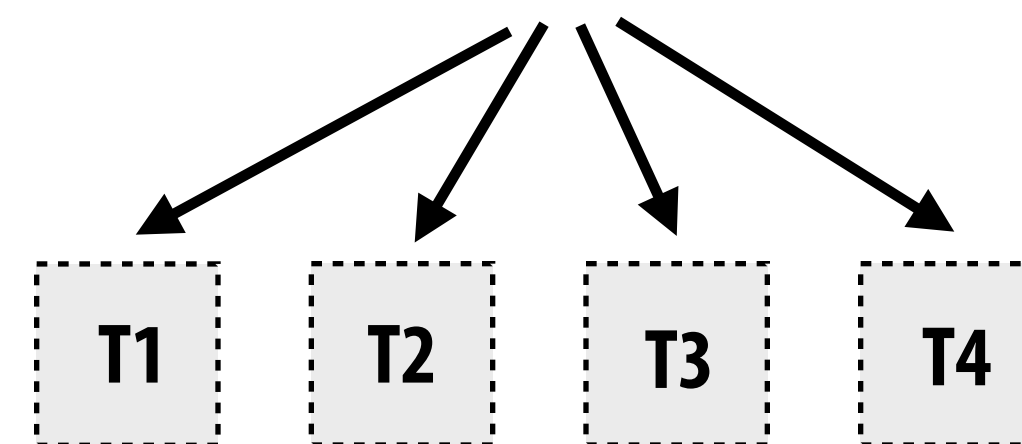


Work in task queues need not be independent

.....→ = application-specified dependency



A task is not removed from queue and assigned to worker thread until all task dependencies are satisfied



Workers can submit new tasks (with optional explicit dependencies) to task system

```
foo_handle = enqueue_task(foo);           // enqueue task foo (independent of all prior tasks)
bar_handle = enqueue_task(bar, foo_handle); // enqueue task bar, cannot run until foo is complete
```

Summary

- **Challenge: achieving good workload balance**
 - Want all processors working all the time (otherwise, resources are idle!)
 - But want low-cost solution for achieving this balance
 - Minimize computational overhead (e.g., scheduling/assignment logic)
 - Minimize synchronization costs
- **Static assignment vs. dynamic assignment**
 - Really, it is not an either/or decision, there's a continuum of choices
 - Use up-front knowledge about workload as much as possible to reduce load imbalance and task management/synchronization costs (in the limit, if the system knows everything, use fully static assignment)
- **Issues discussed today span decomposition, assignment, and orchestration**

Quizz

- **bring balance to the force**

Scheduling fork-join parallelism

Common parallel programming patterns

Data parallelism:

Perform same sequence of operations on many data elements

// ISPC foreach

```
foreach (i=0 ... N) {  
    B[i] = foo(A[i]);  
}
```

// ISPC bulk task launch

```
launch[numTasks] myFooTask(A, B);
```

// CUDA bulk launch

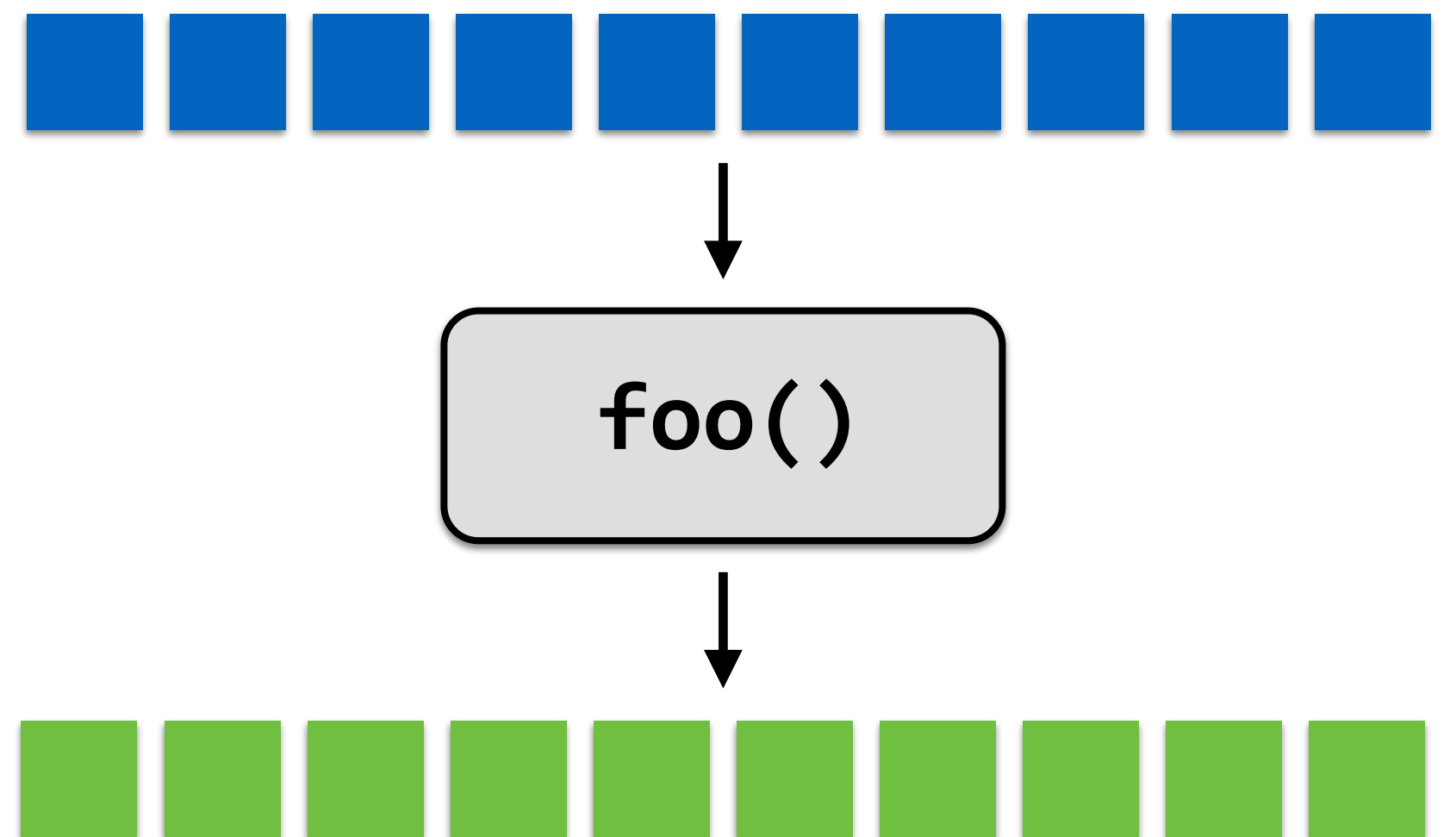
```
foo<<<numBlocks, threadsPerBlock>>>(A, B);
```

// using higher-order function 'map'

```
map(foo, A, B);
```

// openMP parallel for

```
#pragma omp parallel for  
for (int i=0; i<N; i++) {  
    B[i] = foo(A[i]);  
}
```



Common parallel programming patterns

Explicit management of parallelism with threads:

Create one thread per execution unit (or per amount of desired concurrency)

- Example below: C code with pthreads

```
struct thread_args {  
    float* A;  
    float* B;  
};  
  
int thread_id[MAX_THREADS];  
  
thread_args args;  
args.A = A;  
args.B = B;  
  
for (int i=0; i<num_cores; i++) {  
    pthread_create(&thread_id[i], NULL, myFunctionFoo, &args);  
}  
  
for (int i=0; i<num_cores; i++) {  
    pthread_join(&thread_id[i]);  
}
```

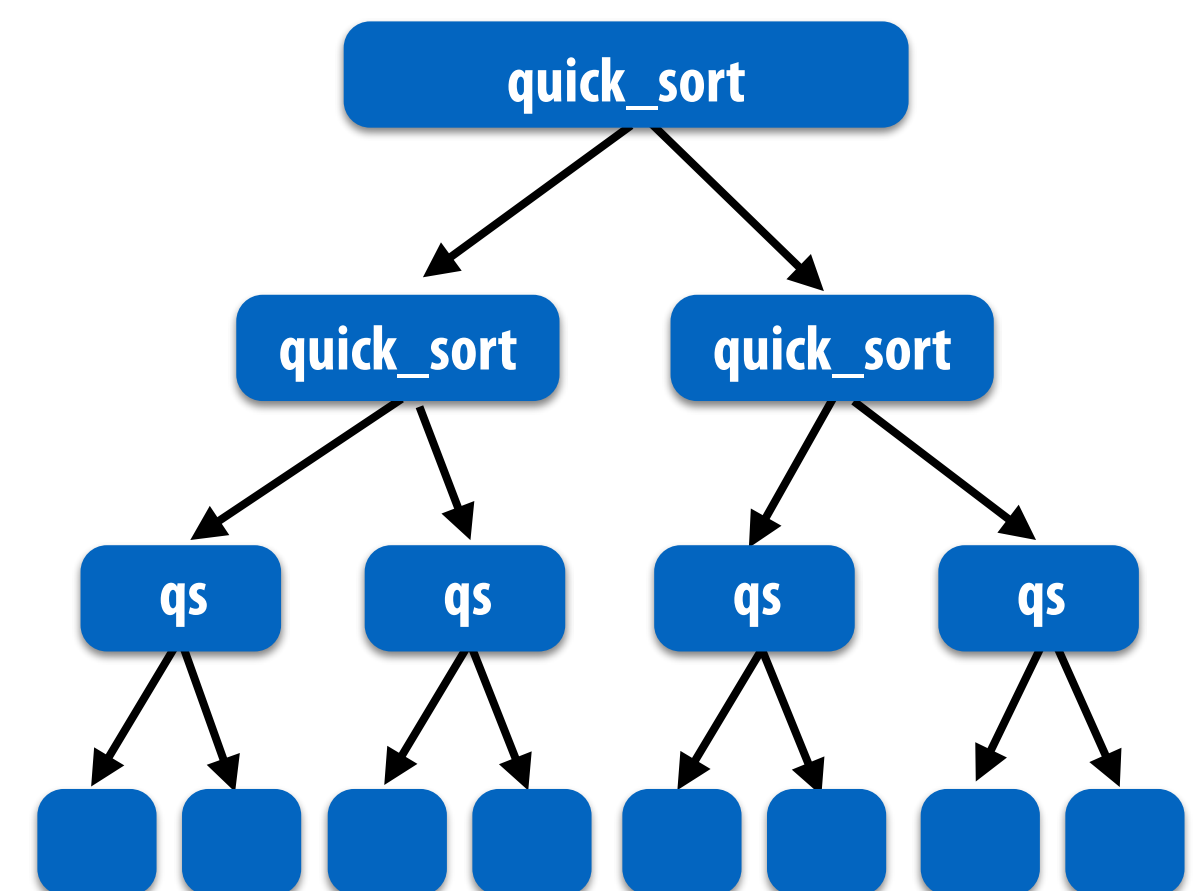
Consider divide-and-conquer algorithms

Quick sort:

```
// sort elements from 'begin' up to (but not including) 'end'
void quick_sort(int* begin, int* end) {
    if (begin >= end-1)
        return;
    else {
        // choose partition key and partition elements
        // by key, return position of key as 'middle'
        int* middle = partition(begin, end);
        quick_sort(begin, middle);
        quick_sort(middle+1, last);
    }
}
```

independent work!

Dependencies



Fork-join pattern

- Natural way to express independent work inherent in divide-and-conquer algorithms
 - This lecture's code examples will be in Cilk Plus
 - C++ language extension
 - Originally developed at MIT, now adapted as open standard (in GCC, Intel ICC)
- `cilk_spawn` foo(args); ← **“fork” (create new logical thread of control)**

Semantics: invoke foo, but unlike standard function call, caller may continue executing asynchronously with execution of foo.

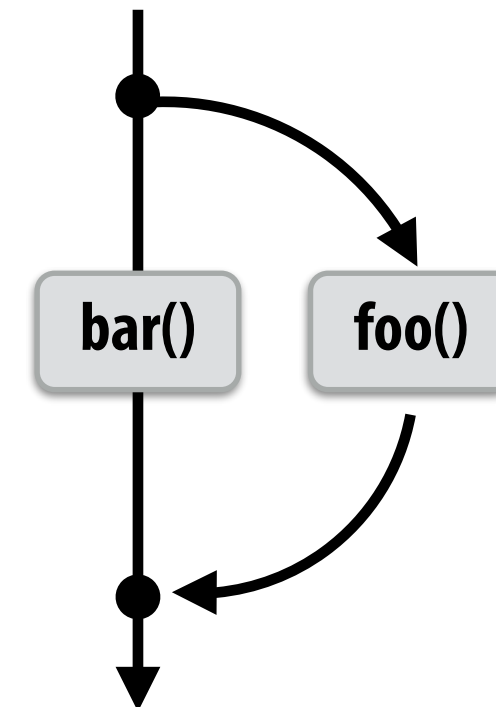
`cilk_sync;` ← **“join”**

Semantics: returns when all calls spawned by current function have completed.
(“sync up” with the spawned calls)

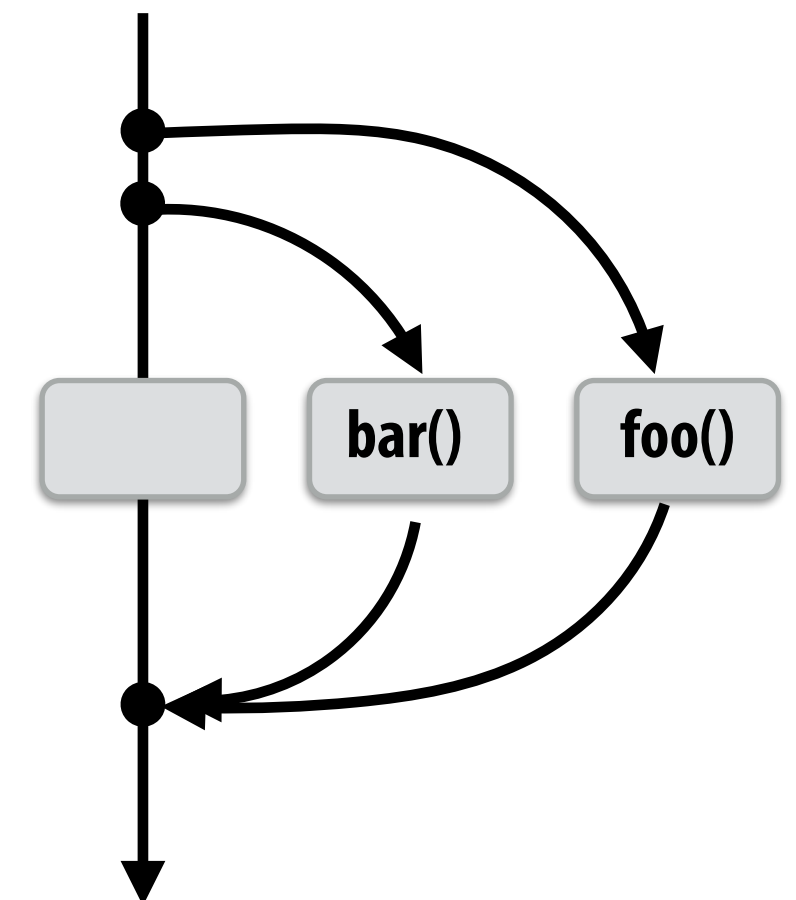
Note: there is an implicit `cilk_sync` at the end of every function that contains a `cilk_spawn` (implication: when a Cilk function returns, all work associated with that function is complete)

Basic Cilk Plus examples

```
// foo() and bar() may run in parallel
cilk_spawn foo();
bar();
cilk_sync;
```

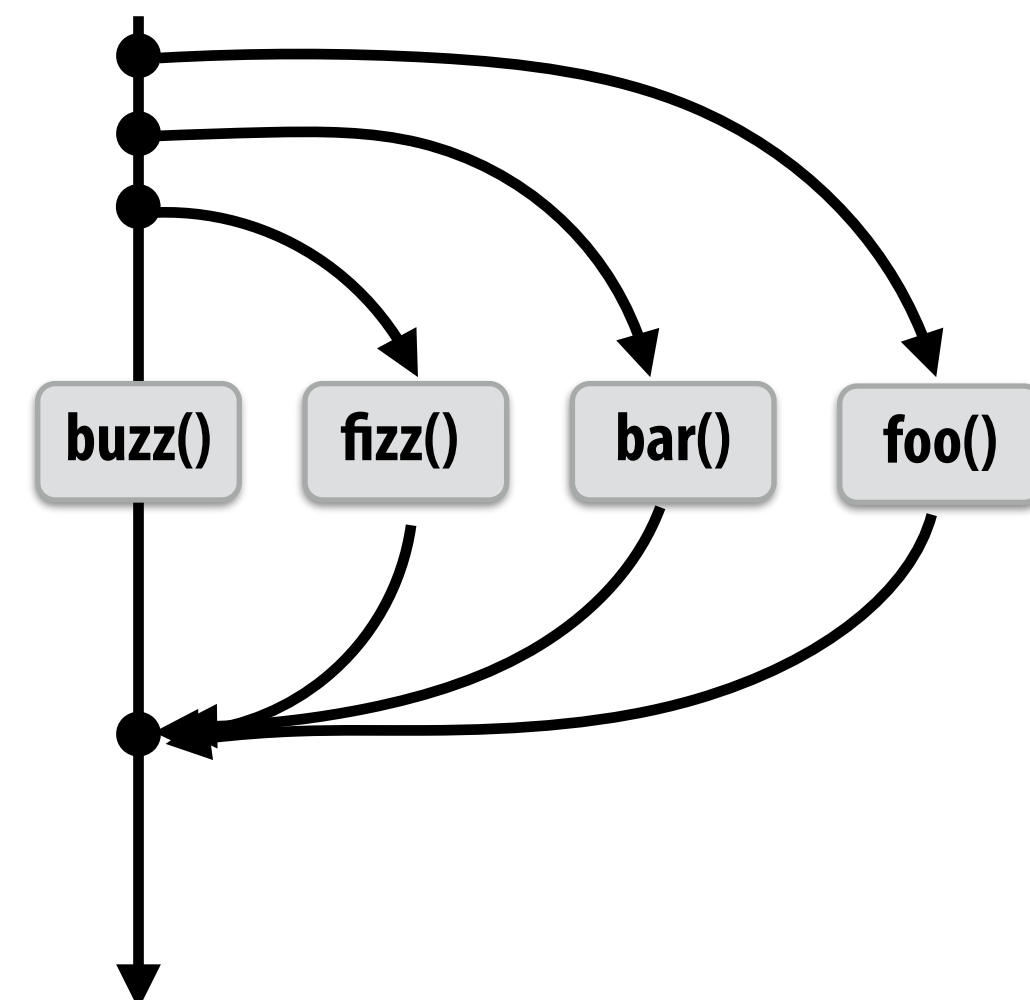


```
// foo() and bar() may run in parallel
cilk_spawn foo();
cilk_spawn bar();
cilk_sync;
```



Same amount of independent work first example, but potentially higher runtime overhead (due to two spawns vs. one)

```
// foo, bar, fizz, buzz, may run in parallel
cilk_spawn foo();
cilk_spawn bar();
cilk_spawn fizz();
buzz();
cilk_sync;
```



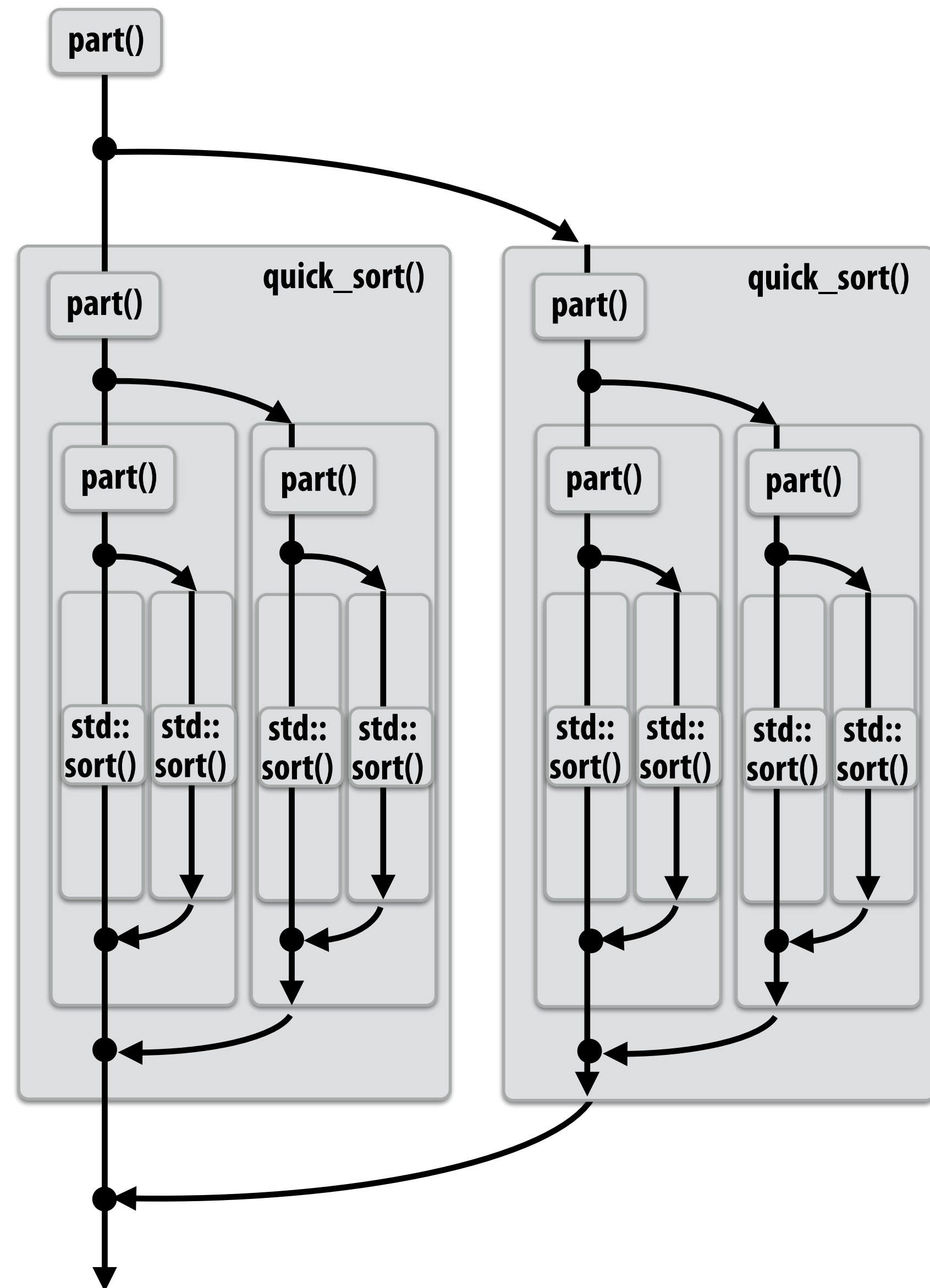
Abstraction vs. implementation

- Notice that the **cilk_spawn abstraction** does not specify how or when spawned calls are scheduled to execute
 - Only that they **may be run concurrently with caller** (and with all other calls spawned by the caller)
- But **cilk_sync** does serve as a **constraint on scheduling**
 - All spawned calls must complete before **cilk_sync** returns

Parallel quicksort in Cilk Plus

```
void quick_sort(int* begin, int* end) {  
    if (begin >= end - PARALLEL_CUTOFF)  
        std::sort(begin, end);  
    else {  
        int* middle = partition(begin, end);  
        cilk_spawn quick_sort(begin, middle);  
        quick_sort(middle+1, last);  
    }  
}
```

Sort sequentially if problem size is sufficiently small (overhead of spawn trumps benefits of potential parallelization)



Writing fork-join programs

- Main idea: **expose independent work** (potential parallelism) to the system using `cilk_spawn`
- Recall parallel programming rules of thumb
 - Want **at least as much work** as **parallel execution capability** (e.g., program should probably spawn at least as much work as there are cores)
 - Want **more independent work** than execution capability to allow for good workload balance of all the work onto the cores
 - “**parallel slack**” = ratio of independent work to machine’s parallel execution capability (in practice: ~8 is a good ratio)
 - But **not too much independent work** so that **granularity of work is too small** (too much slack incurs overhead of managing fine-grained work)

Scheduling fork-join programs

■ Consider very simple scheduler:

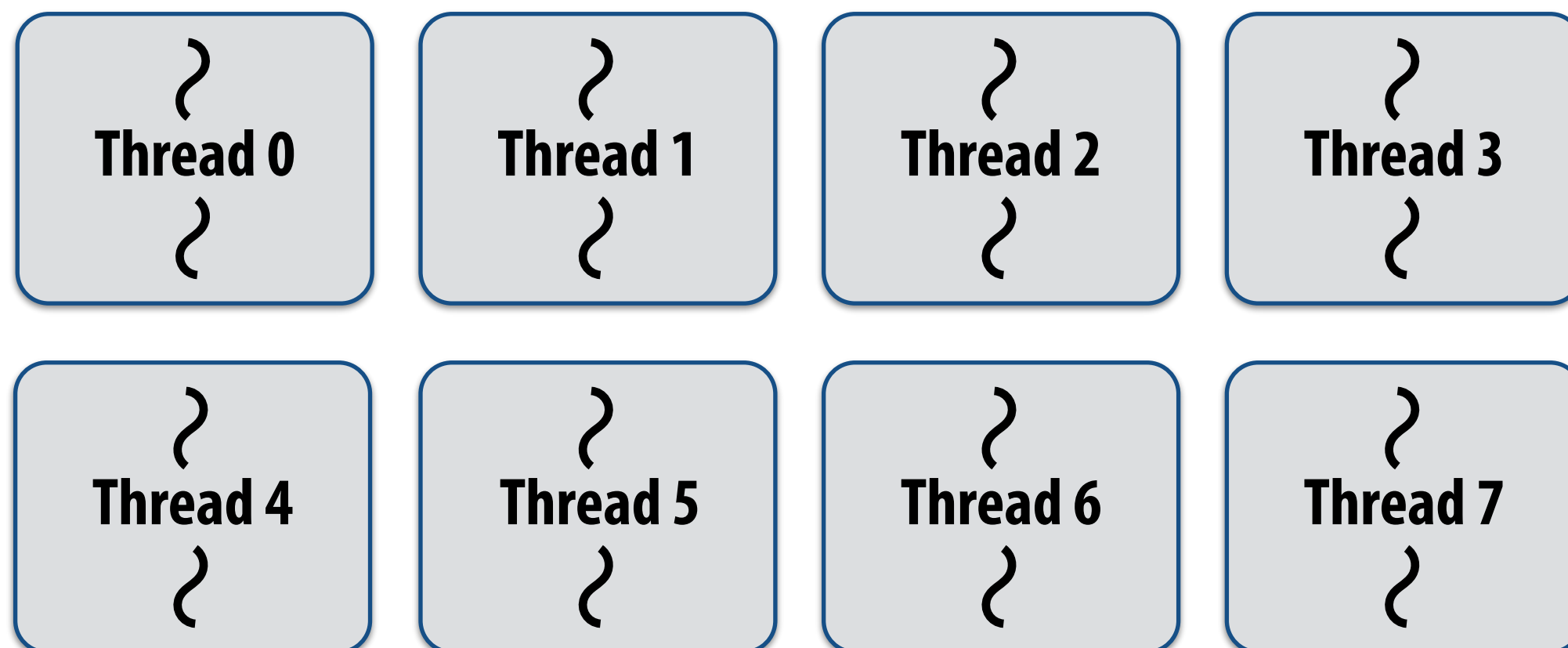
- **Launch pthread** for each `cilk_spawn` using `pthread_create`
- Translate `cilk_sync` into appropriate `pthread_join` calls

■ Potential performance problems?

- Heavyweight spawn operation
- Many more concurrently running threads than cores
 - Context switching overhead
 - Larger working set than necessary, less cache locality

Pool of worker threads

- The Cilk Plus runtime maintains pool of worker threads
 - Think: **all threads created at application launch***
 - Exactly **as many worker threads as execution contexts** in the machine



Example: Eight thread worker pool for my quad-core laptop with Hyper-Threading

```
while (work_exists()) {  
    work = get_new_work();  
    work.run();  
}
```

* It's perfectly fine to think about it this way, but in reality, runtimes tend to be lazy and initialize worker threads on the first Cilk spawn. (This is a common implementation strategy, ISPC does the same with worker threads that run ISPC tasks.)

Consider execution of the following code

Specifically, consider execution from the point `foo()` is spawned

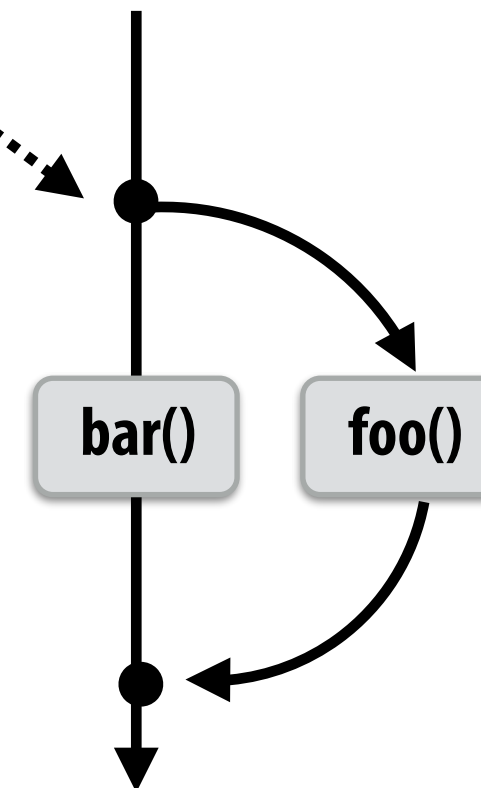
```
cilk_spawn foo();
```

```
bar();
```

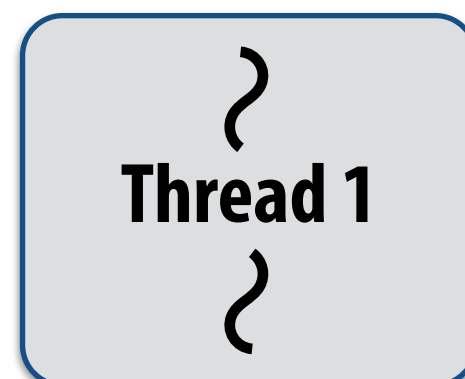
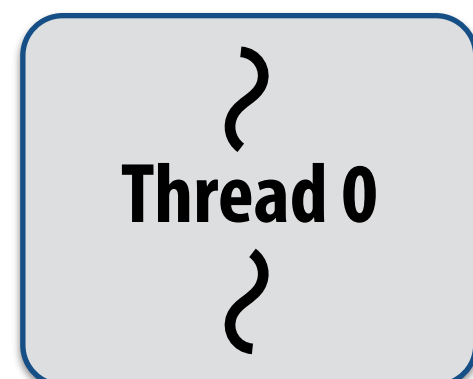
```
cilk_sync;
```

spawned child

continuation (rest of calling function)



What threads should `foo()` and `bar()` be executed by?



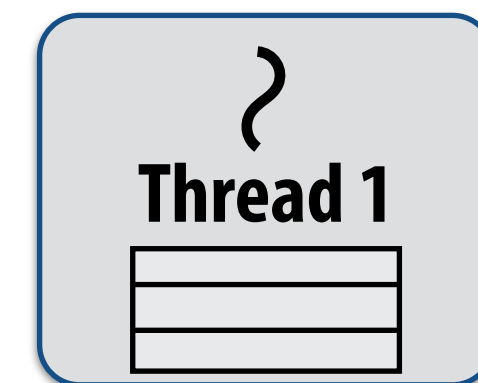
First, consider a serial implementation

Run child first... via a regular function call

- Thread runs `foo()`, then returns from `foo()`, then runs `bar()`
- Continuation is implicit in the thread's stack



Executing `foo()`...

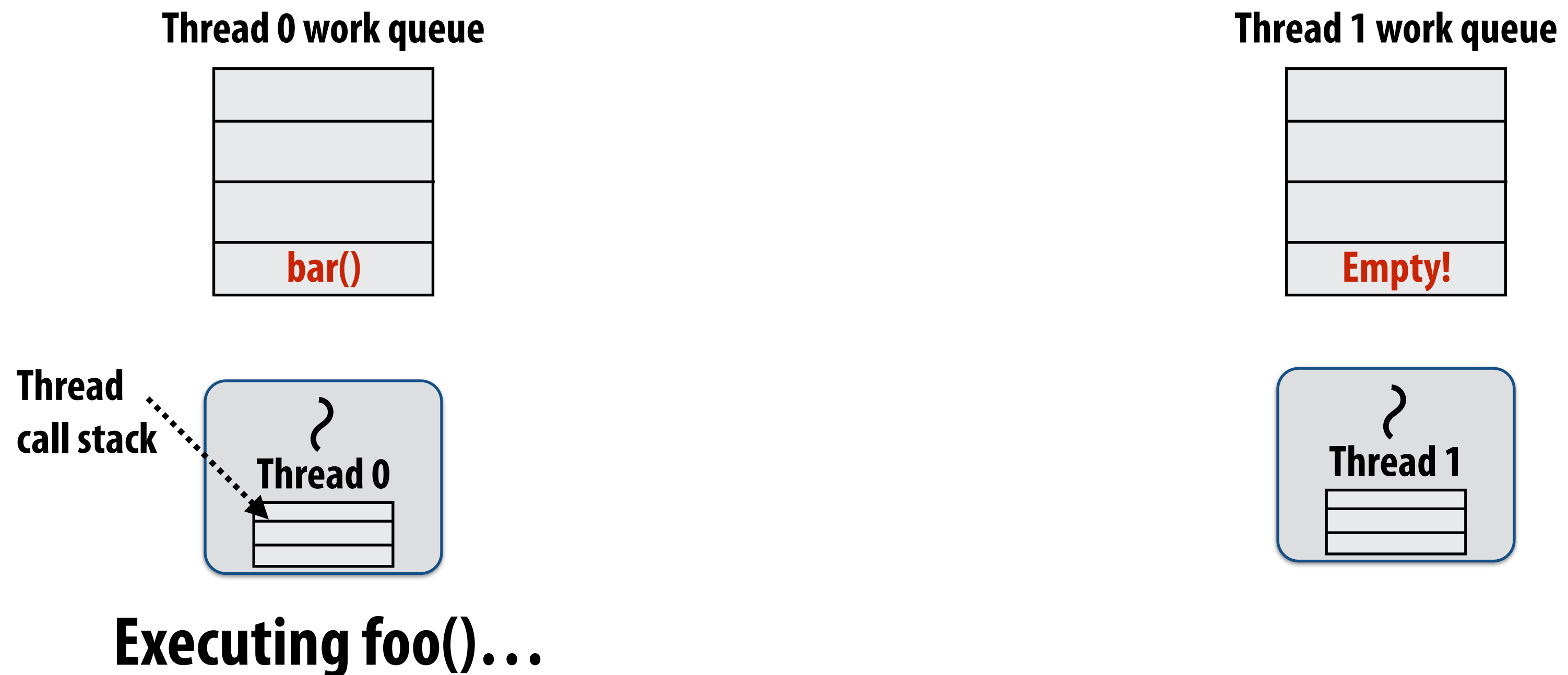


What if, while executing `foo()`, thread 1 goes idle...

Inefficient: thread 1 could be performing `bar()` at this time!

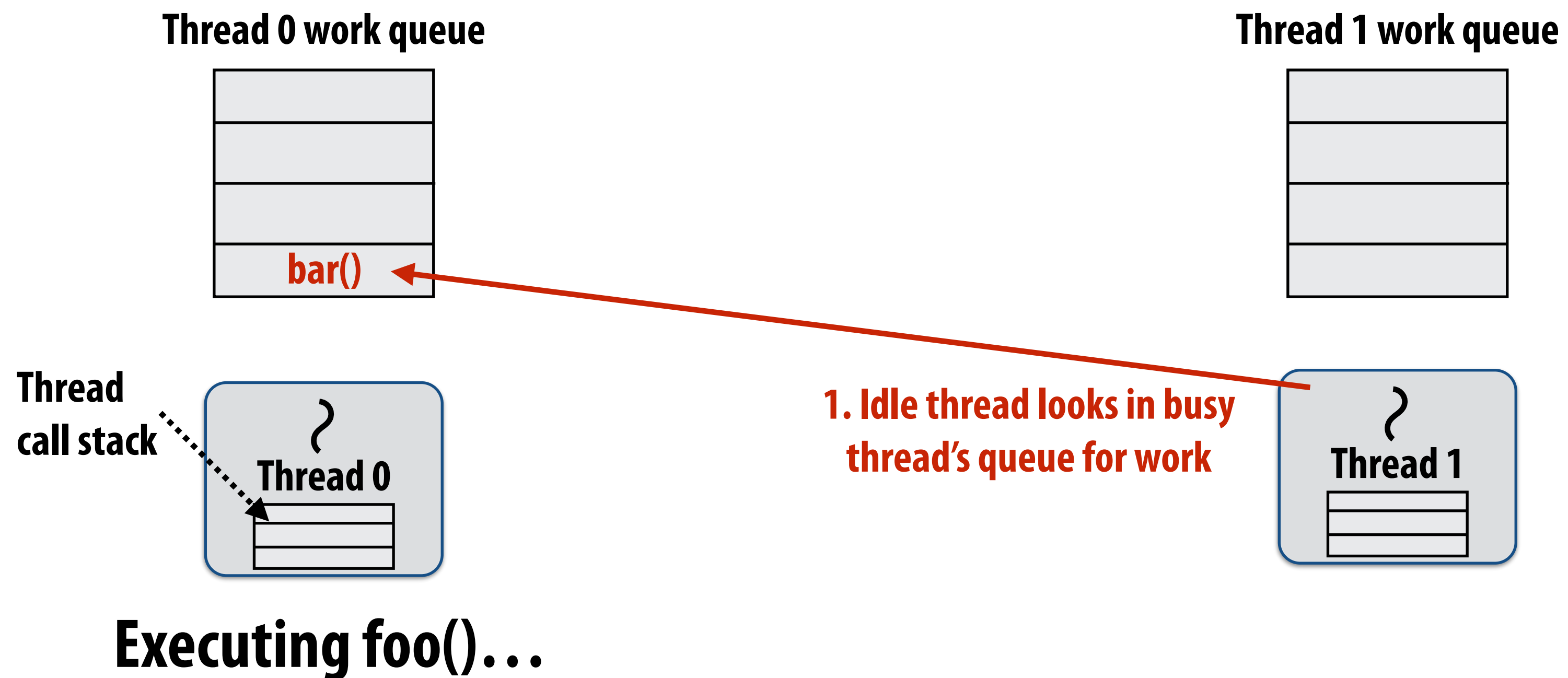
Per-thread work queues store “work to do”

Upon reaching `cilk_spawn foo()`, thread places continuation in its work queue, and begins executing `foo()`.



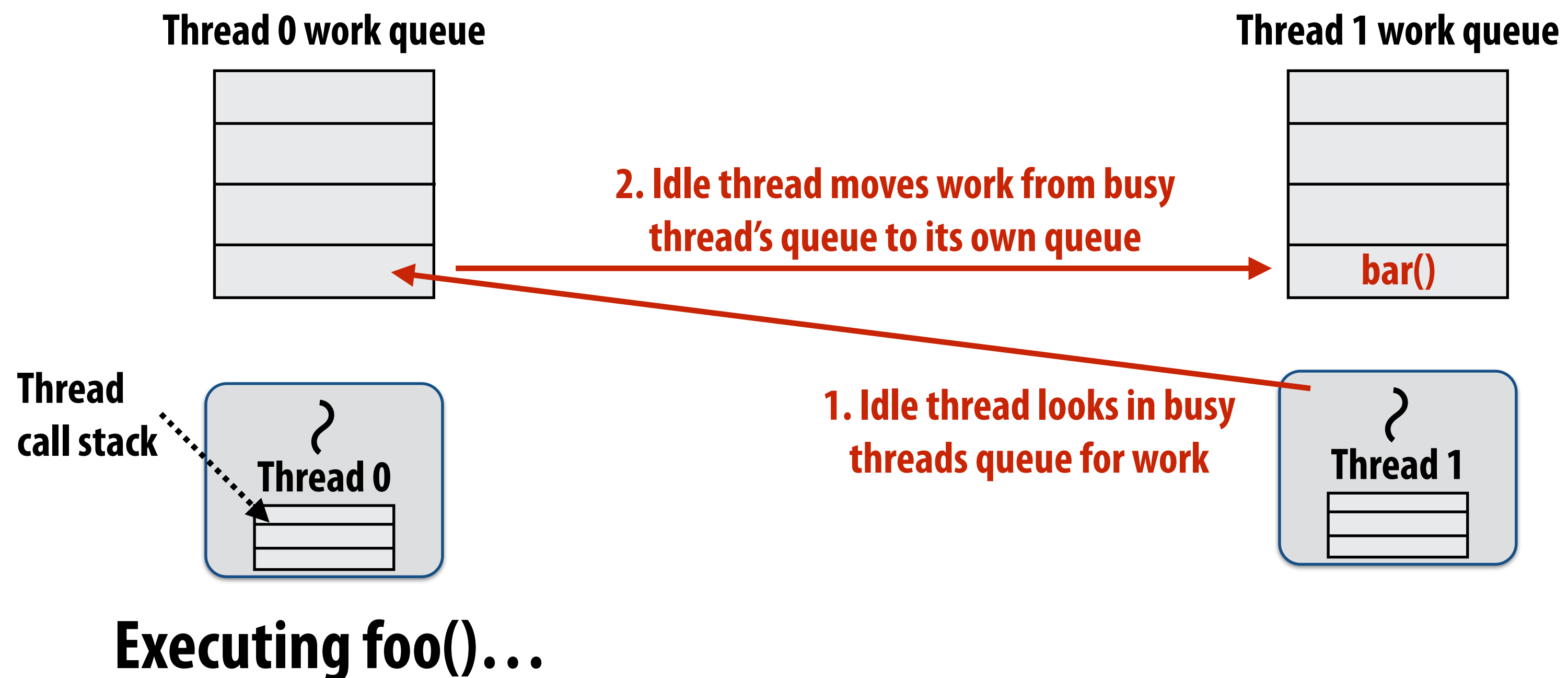
Idle threads “steal” work from busy threads

If thread 1 goes idle (a.k.a. there is no work in its own queue), then it looks in thread 0’s queue for work to do.



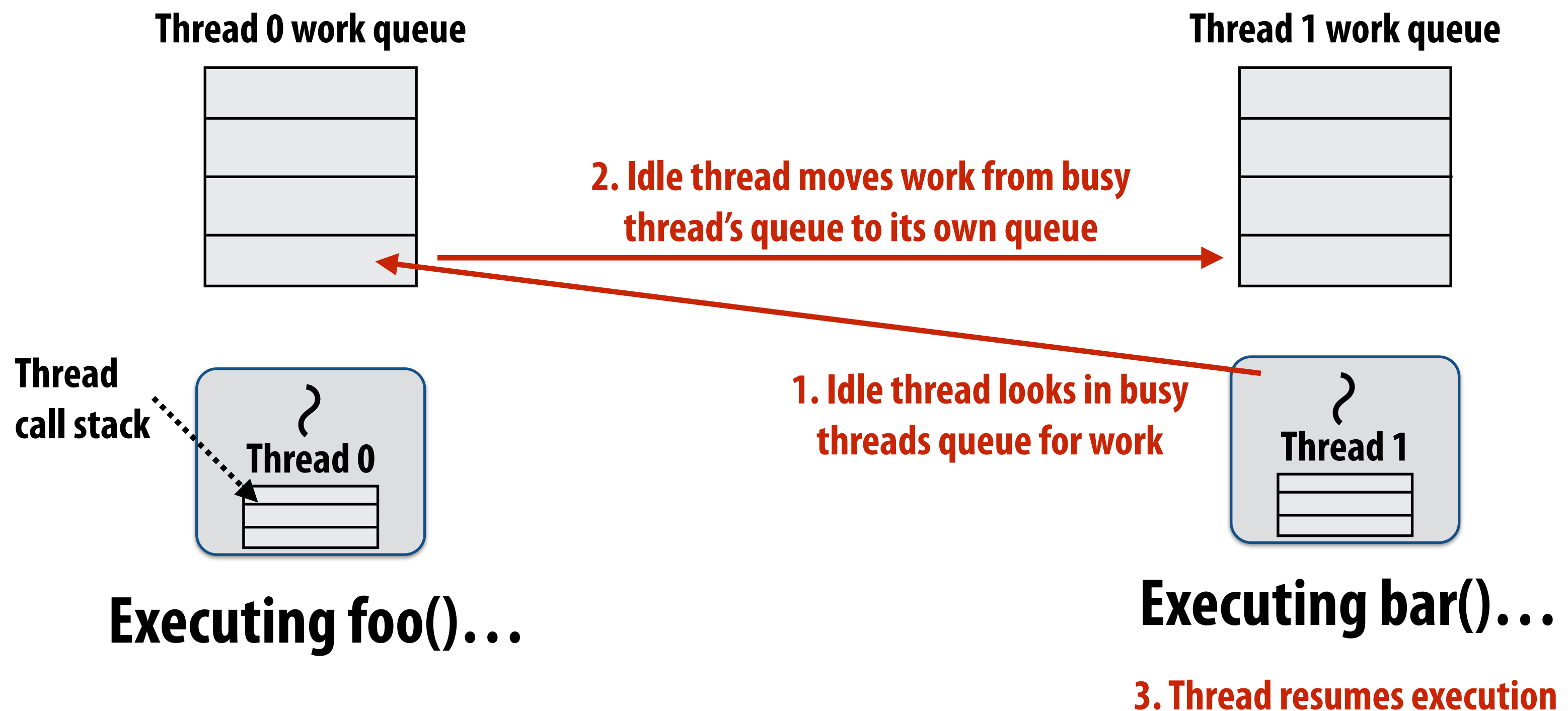
Idle threads “steal” work from busy threads

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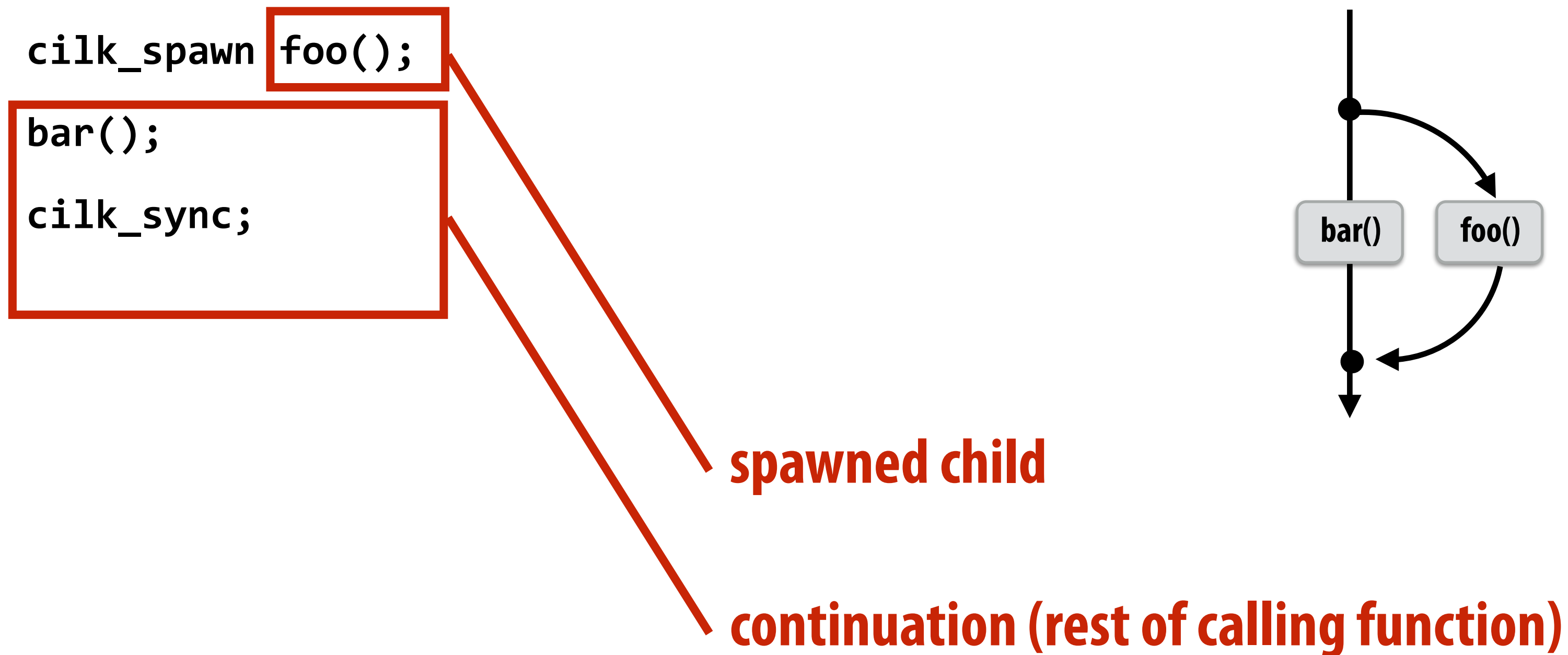


Idle threads “steal” work from busy threads

If thread 1 goes idle (a.k.a. there is no work in its own queue), then it looks in thread 0’s queue for work to do.



At spawn, should thread run child or



Run **continuation first**: record child for later execution

- Child is made available for stealing by other threads ("**child stealing**")

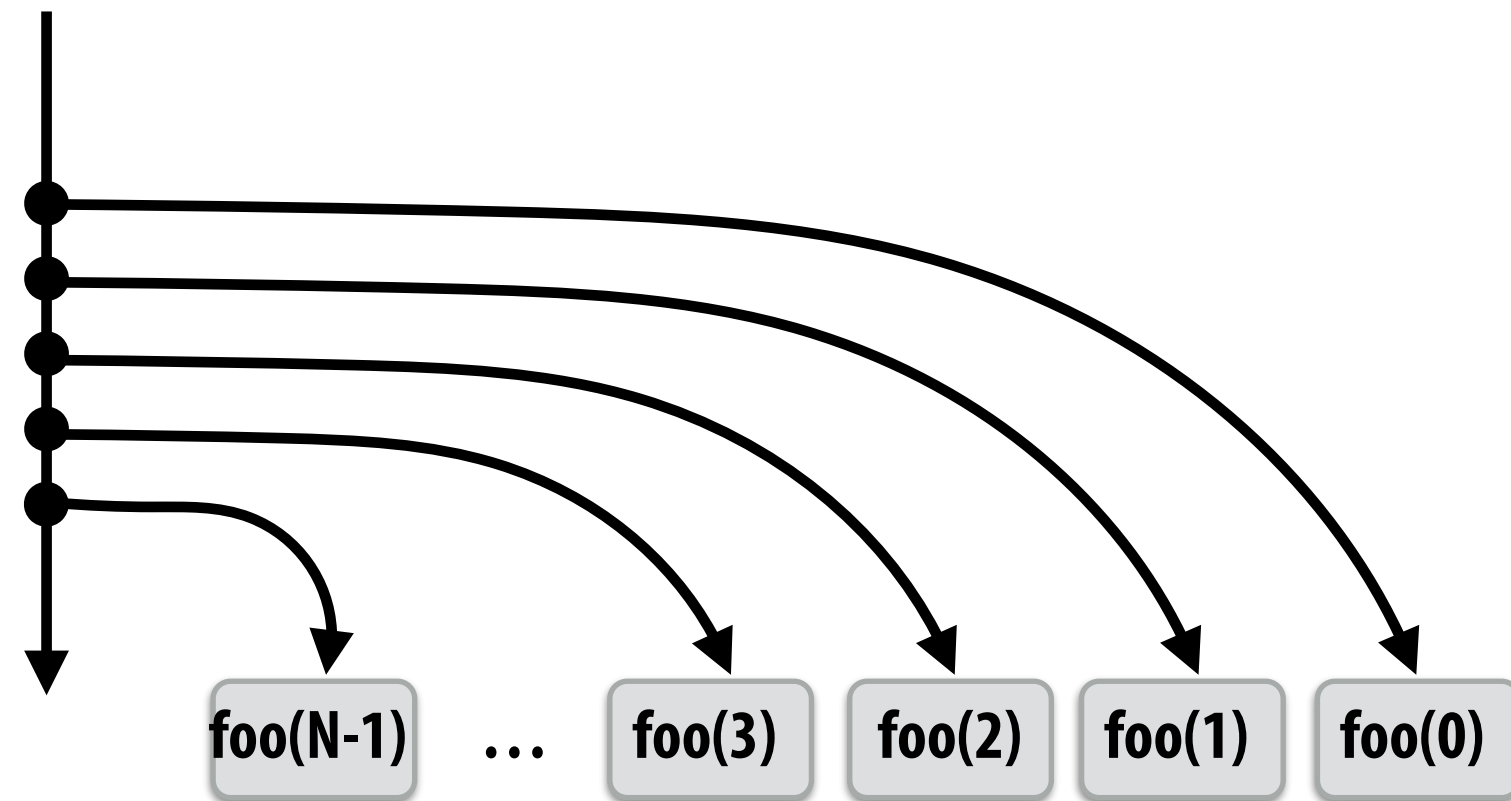
Run **child first**: record continuation for later execution

- Continuation is made available for stealing by other threads ("**continuation stealing**")

Which implementation do we choose?

Consider thread executing the following

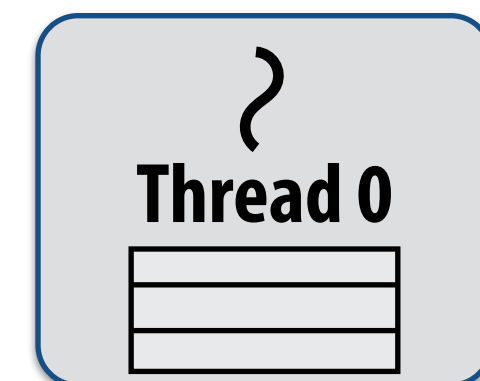
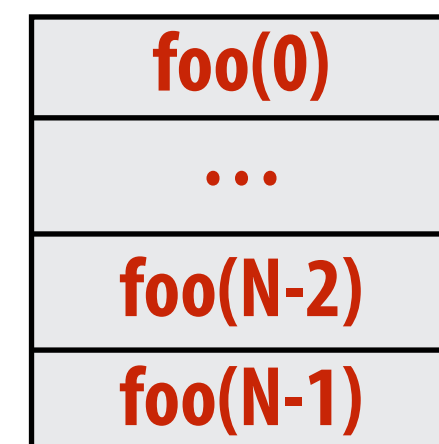
```
for (int i=0; i<N; i++) {  
    cilk_spawn foo(i);  
}  
cilk_sync;
```



■ Run **continuation first** (“**child stealing**”)

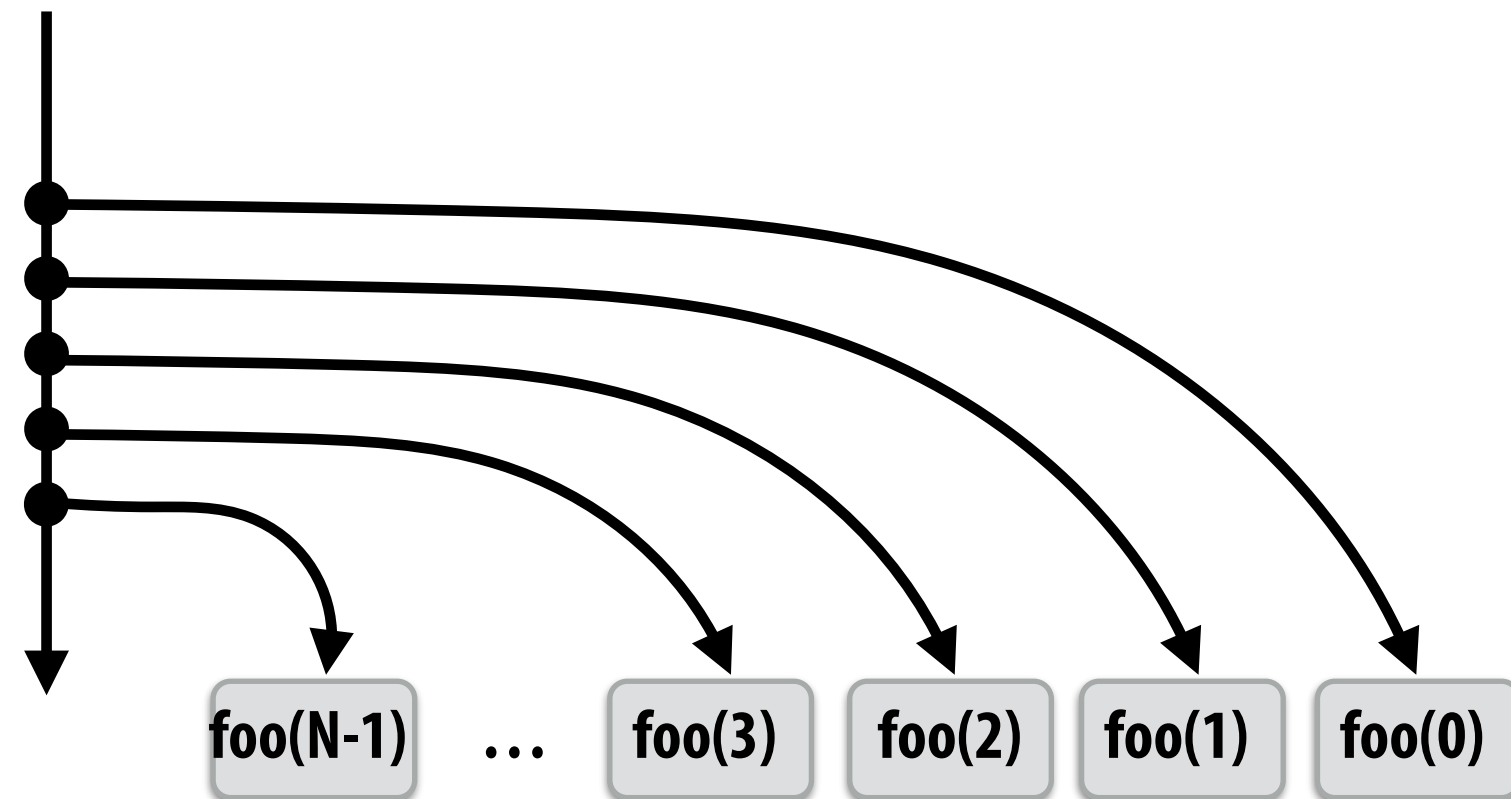
- Caller thread spawns work for all iterations before executing any of it
- Think: **breadth-first traversal of call graph**. $O(N)$ space for spawned work (maximum space)
- If no stealing, **execution order is very different than that of program with `cilk_spawn` removed**

Thread 0 work queue



Consider thread executing the following

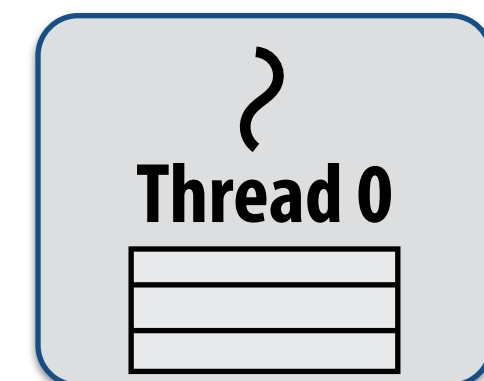
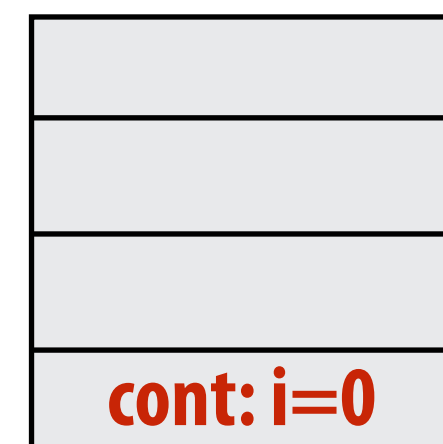
```
for (int i=0; i<N; i++) {  
    cilk_spawn foo(i);  
}  
cilk_sync;
```



■ Run **child first** (“**continuation stealing**”)

- Caller thread only creates one item to steal (continuation that represents all remaining iterations)
- If no stealing occurs, thread continually pops continuation from work queue, enqueues new continuation (with updated value of `i`)
- **Order of execution is the same as for program with spawn removed.**
- **Think: depth-first traversal of call graph**

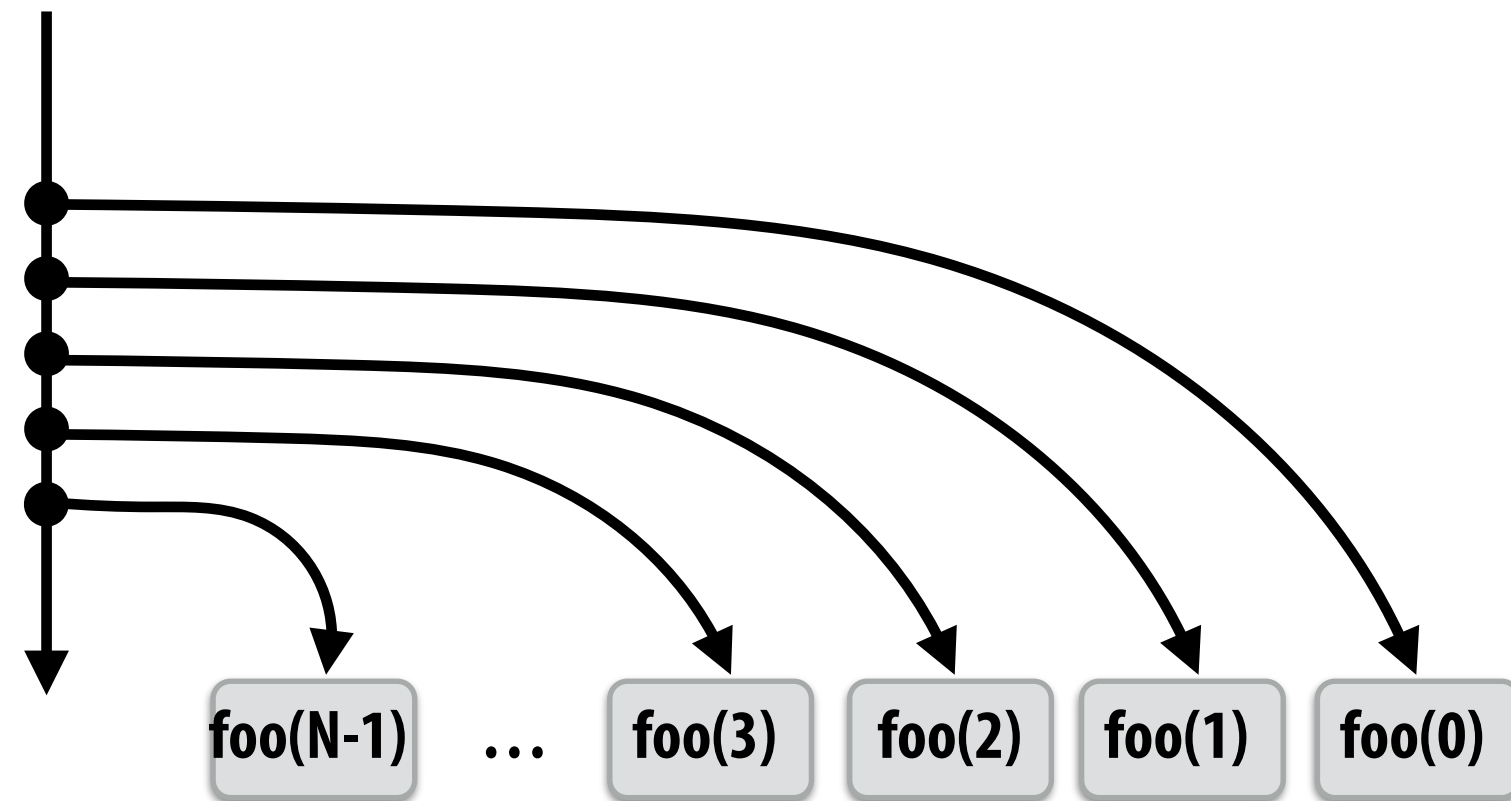
Thread 0 work queue



Executing `foo(0)`...

Consider thread executing the following

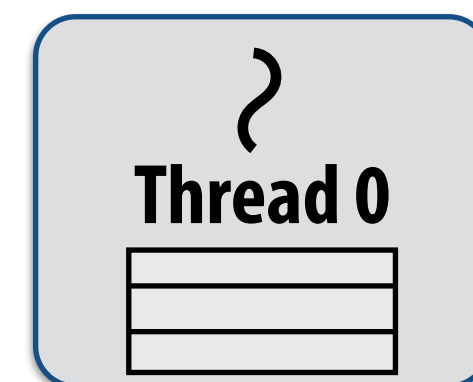
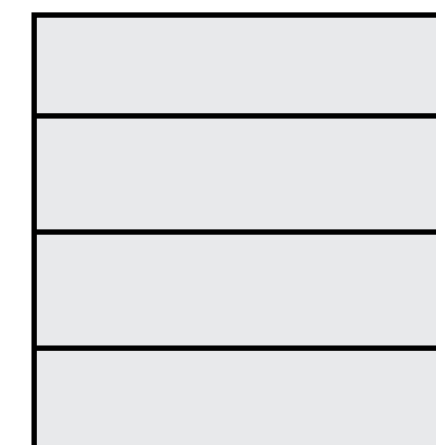
```
for (int i=0; i<N; i++) {  
    cilk_spawn foo(i);  
}  
cilk_sync;
```



■ Run **child first** (“**continuation stealing**”)

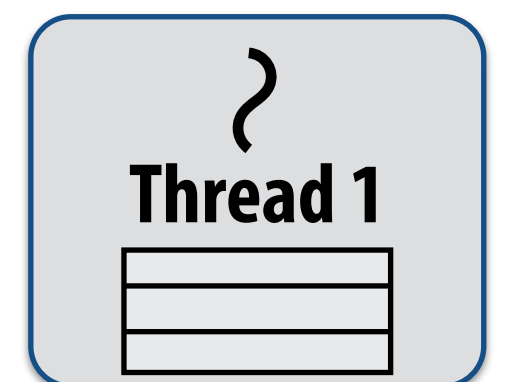
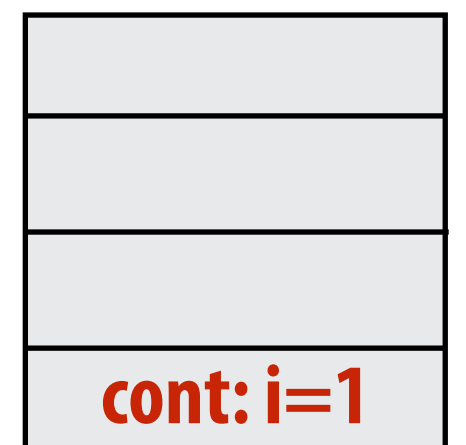
- If continuation is stolen, stealing thread spawns and executes next iteration
- Enqueues continuation with `i` advanced by 1
- Can prove that **work queue storage** for system with `T` threads is **no more than `T` times that of stack storage for single threaded execution**

Thread 0 work queue



Executing `foo(0)...`

Thread 1 work queue



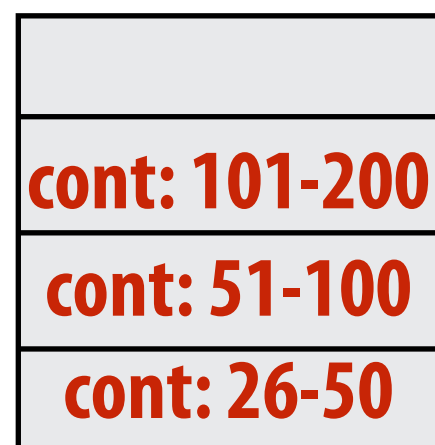
Executing `foo(1)...`

Scheduling quicksort: assume 200 elements

```
void quick_sort(int* begin, int* end) {  
    if (begin >= end - PARALLEL_CUTOFF)  
        std::sort(begin, end);  
    else {  
        int* middle = partition(begin, end);  
        cilk_spawn quick_sort(begin, middle);  
        quick_sort(middle+1, last);  
    }  
}
```

**What work in the queue
should other threads steal?**

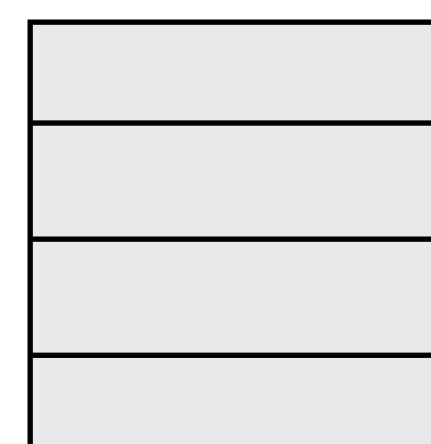
Thread 0 work queue



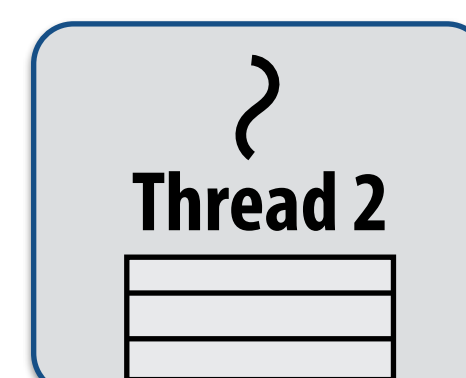
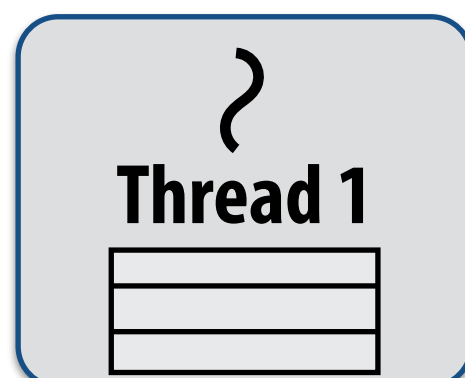
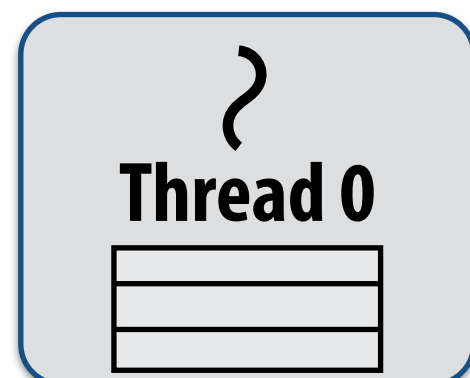
Thread 1 work queue



Thread 2 work queue



...

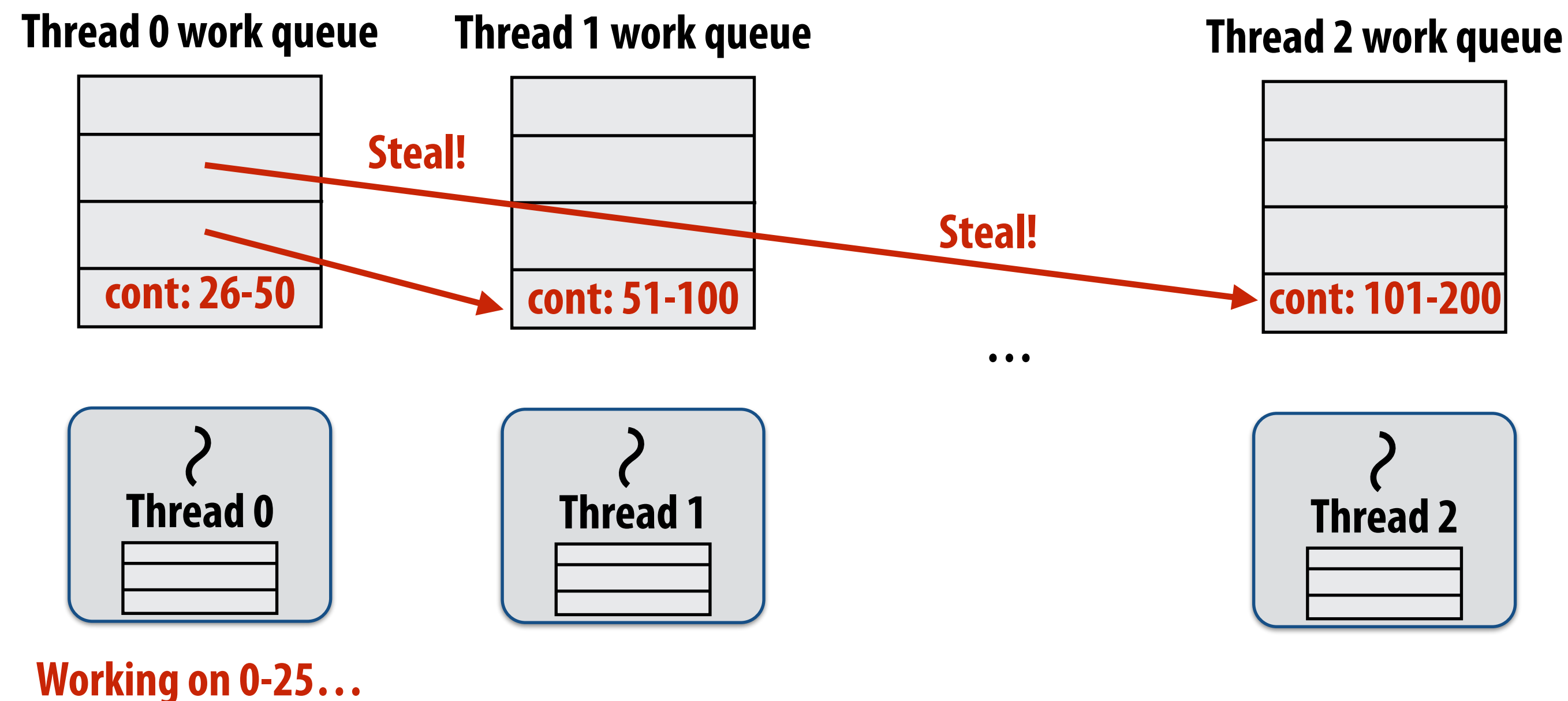


Working on 0-25...

Implementing work stealing: dequeue per worker

Work queue implemented as a **dequeue** (double ended queue)

- Local thread pushes/pops from the “tail” (bottom)
- Remote threads **steal from “head”** (top)
- Efficient lock-free dequeue implementations exist

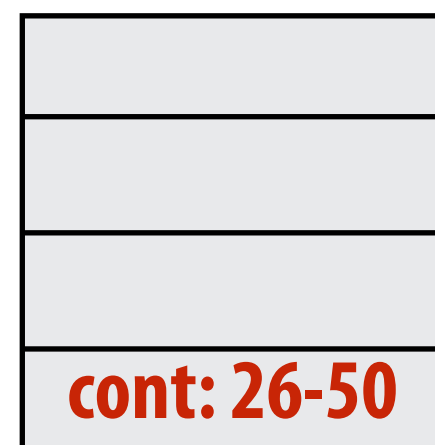


Implementing work stealing: dequeue per worker

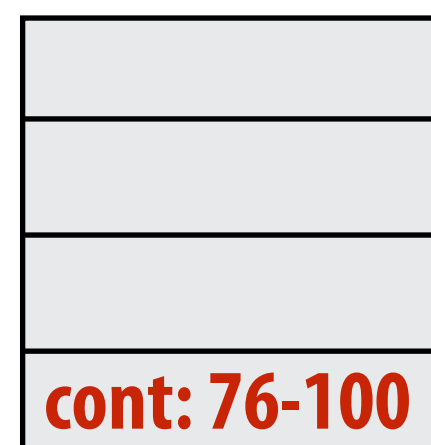
Work queue implemented as a **dequeue** (double ended queue)

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- Efficient lock-free dequeue implementations exist

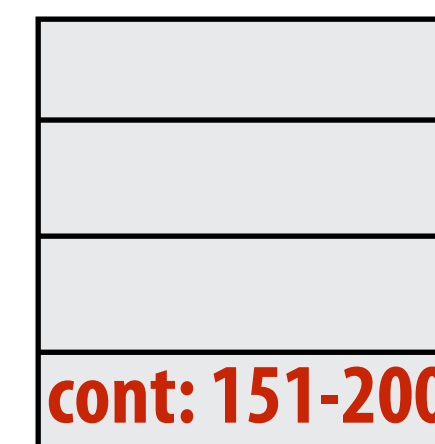
Thread 0 work queue



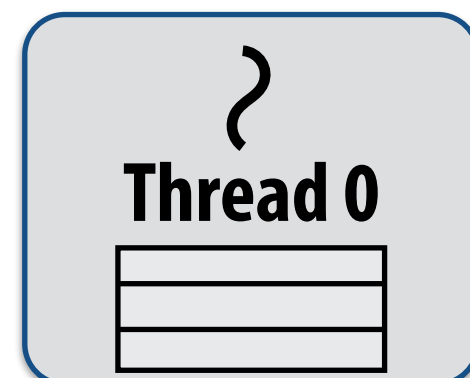
Thread 1 work queue



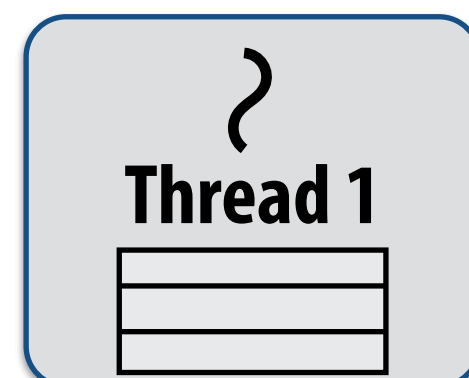
Thread 2 work queue



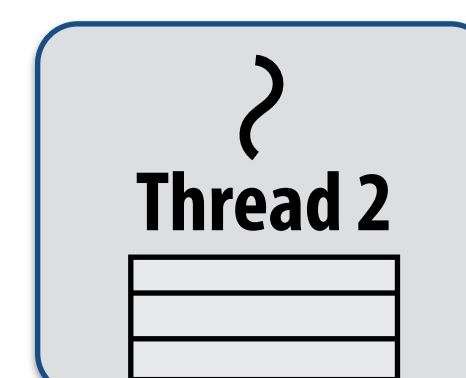
...



Working on 0-25...



Working on 51-75...

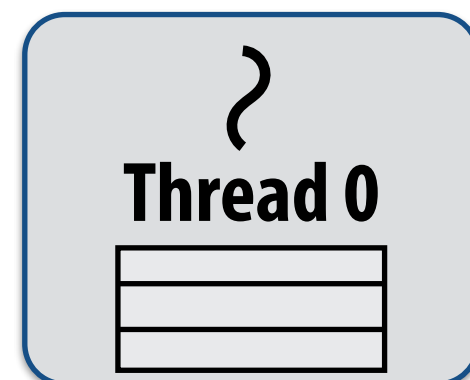
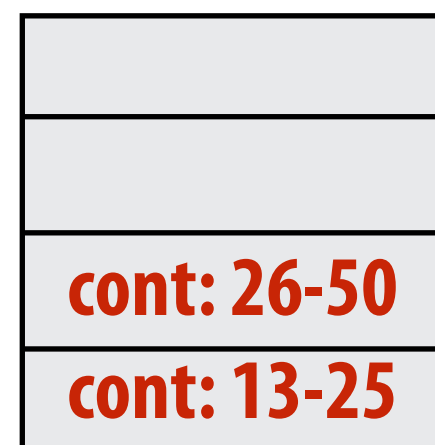


Working on 101-150...

Implementing work stealing: **random choice of victim**

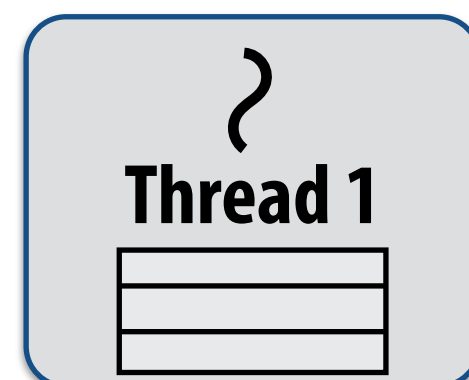
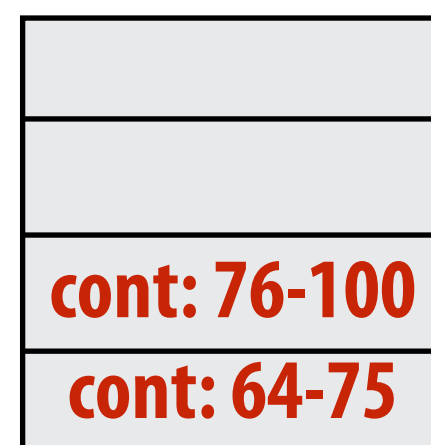
- Idle threads randomly choose a thread to attempt to steal from
- Stealing from top of dequeue...
 - Reduces contention with local thread: local thread is not accessing same part of dequeue that stealing threads do!
 - Steals work at beginning of call tree: this is a “larger” piece of work, so the cost of performing a steal is amortized over longer future computation
 - Maximizes locality: (in conjunction with run-child-first policy) local thread works on local part of call tree

Thread 0 work queue



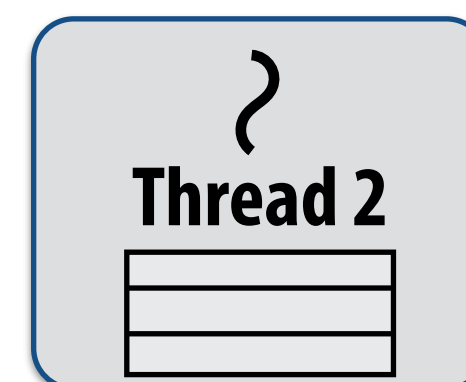
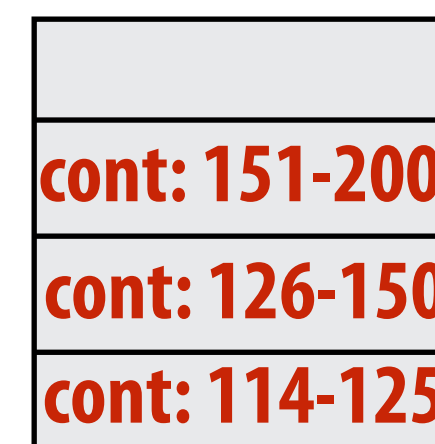
Working on 0-12...

Thread 1 work queue



Working on 51-63...

Thread 2 work queue

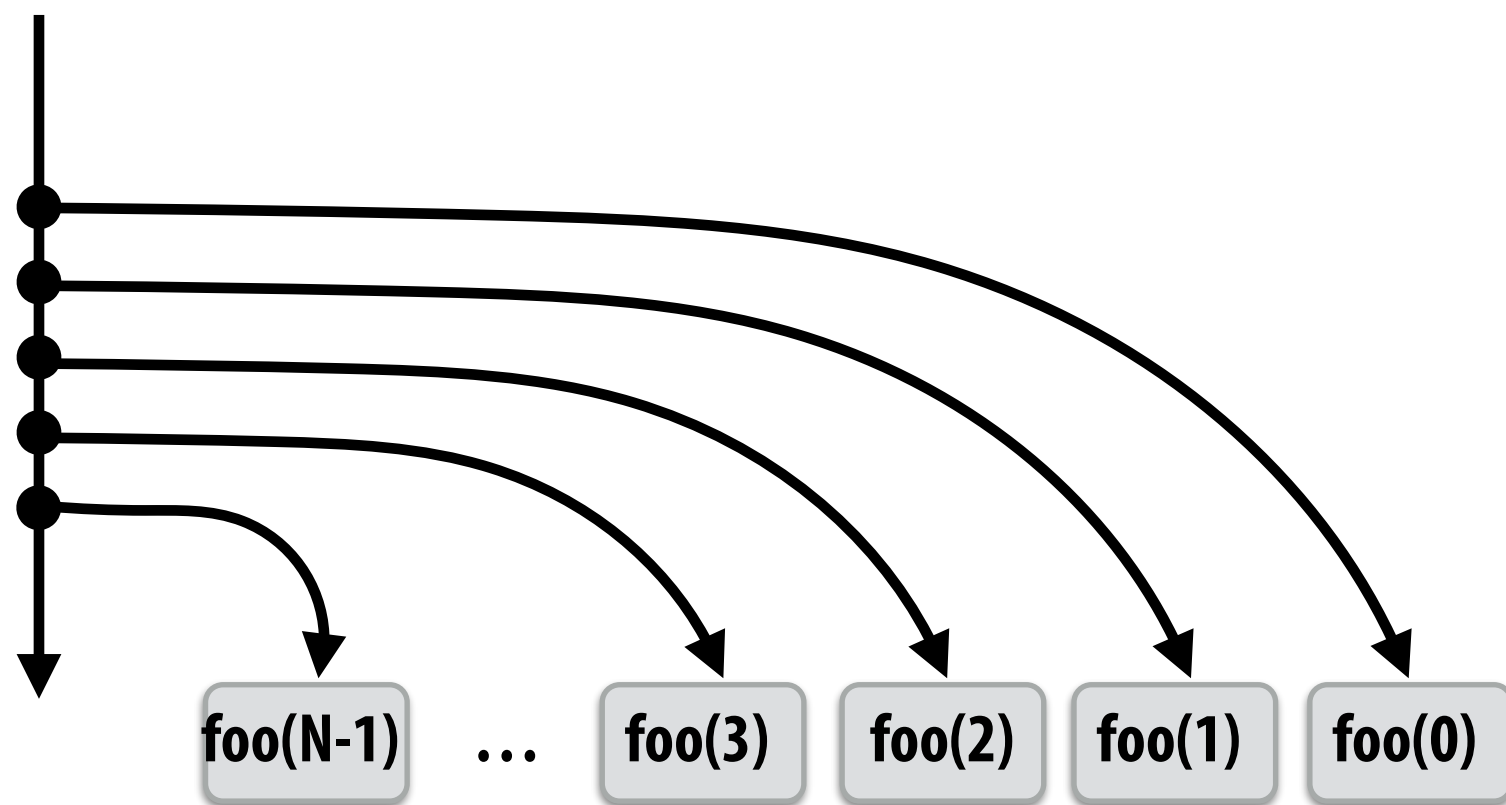


Working on 101-113...

...

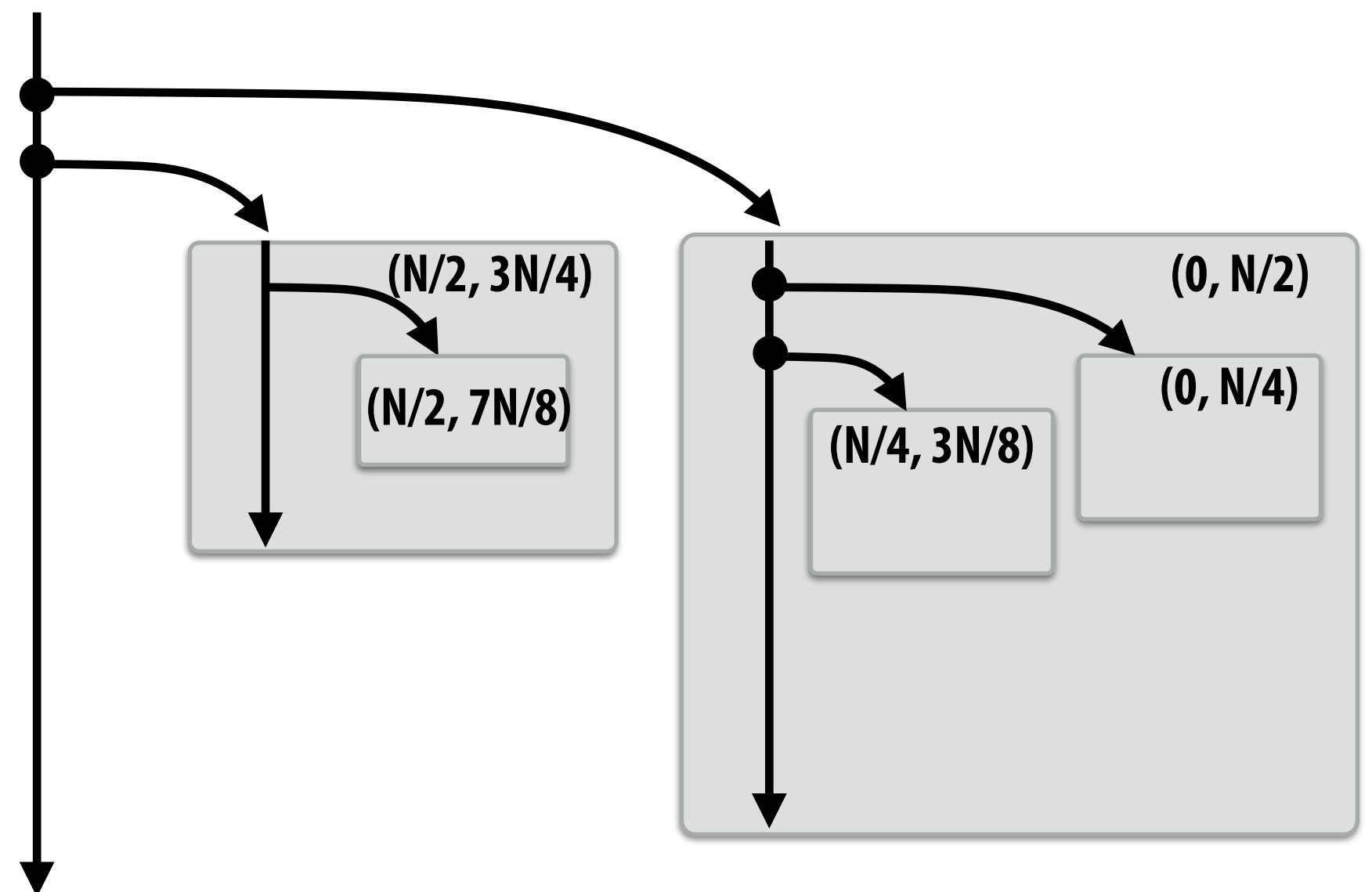
Child-first work stealing scheduler anticipates divide-and-conquer parallelism

```
for (int i=0; i<N; i++) {  
    cilk_spawn foo(i);  
}  
cilk_sync;
```



Code at right generates work in parallel, unlike the code at left (so it more quickly fills machine)

```
void recursive_for(int start, int end) {  
  
    while (start <= end - GRANULARITY) {  
        int mid = (end - start) / 2;  
        cilk_spawn recursive_for(start, mid);  
        start = mid;  
    }  
  
    for (int i=start; i<end; i++)  
        foo(i);  
}  
  
recursive_for(0, N);
```



Summary

- **Fork-join parallelism: a natural way to express divide-and-conquer algorithms**
 - Discussed Cilk Plus, but OpenMP also has fork/join primitives
- **Cilk Plus runtime implements spawn/sync abstraction with a locality-aware work stealing scheduler**
 - Always run spawned child (continuation stealing)