Quantum Computing
Homework 1
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Exercise chapter 1 Introduction

1. How many bits are necessary to represent the alphabet using a binary code if we only allow uppercase characters? How about if we allow both uppercase and lowercase characters
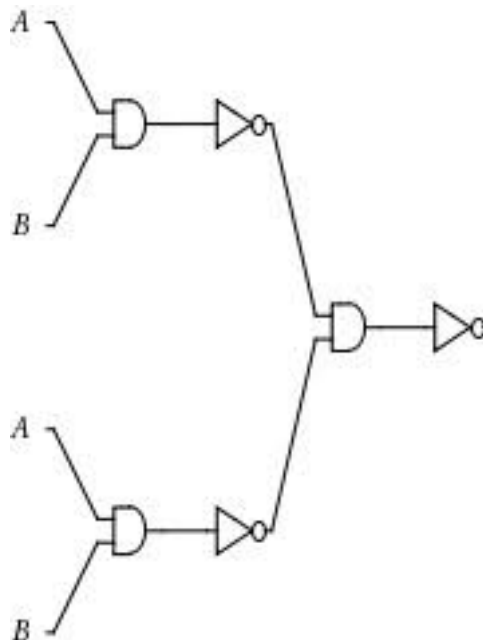
   Solution

   Uppercase:
   $$2^x = 29 \Rightarrow x = 5$$

   Lowercase:
   $$2^x = 58 \Rightarrow x = 6$$

2. Describe how can create an OR gate using NOT gates and AND gates

3. A kilobyte is 1024 bytes.How many message can it store?

   $2^1024 = 179,769,313,486,231,590,772,930,...,304,835,356,329,624,224,137,216 \; digits = 39$

4. What is the entropy associated with the tossing of a fair coin?

   $$H(x) = -\sum_{i=1}^{n} P_i \log_2 P_i$$

   We known that information is
   $$I = -\log_2 P_i$$

   Then we can write entrophy equation as
   $$H(x) = -\sum_{i=1}^{n} P_i I_i$$

   for tossing of a fair coin
   $$H(x) = -\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{2}\log_2 \frac{1}{2} = \frac{1}{2} * \frac{1}{2} = 1$$

5. Suppose that X consist of characters A,B,C,D that occur in a signal with respective probabilities 0,1,0,4,0,25 and 0,25. What is the Shannon entrophy?

$$H(x) = -\sum_{i=1}^{n} P_i \log_2 P_i$$

$$H(x) = -0,1\log_2 0,1 - 0,4\log_2 0,4 - 0,25\log_2 0,25 - 0,25\log_2 0,25 = 1,86096$$

6. A room full of people has incomes distributed in the follow way:

   - $n(25,5) = 3$
   - $n(30) = 5$
   - $n(42) = 7$
   - $n(50) = 3$
   - $n(63) = 1$
   - $n(75) = 2$
   - $n(90) = 1$

What is the most probable income?

$$n(42) = 7$$

What is the average income?

$$E(x) = 25,5(\frac{3}{22}) + 30(\frac{5}{22}) + 42(\frac{7}{22}) + 50(\frac{3}{22}) + 63(\frac{1}{22}) + 75(\frac{2}{22}) + 90(\frac{1}{22}) = 44,25$$

What is the variance of this distribution?

$$<j^2> = 25,5^2(\frac{3}{22}) + 30^2(\frac{5}{22}) + 42^2(\frac{7}{22}) + 50^2(\frac{3}{22}) + 63^2(\frac{1}{22}) + 75^2(\frac{2}{22}) + 90^2(\frac{1}{22}) = 2255,35$$

$$< (\Delta j)^2 > = <j^2> - <j>^2 = 297,29$$