Quantum Computing
Homework 1

Raymundo Santana Carrillo

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Exercise chapter 1 Introduction

1. How many bits are necessary to represent the alphabet using a binary code if we only allow uppercase characters? How about if we allow both uppercase and lowercase characters

   Solution

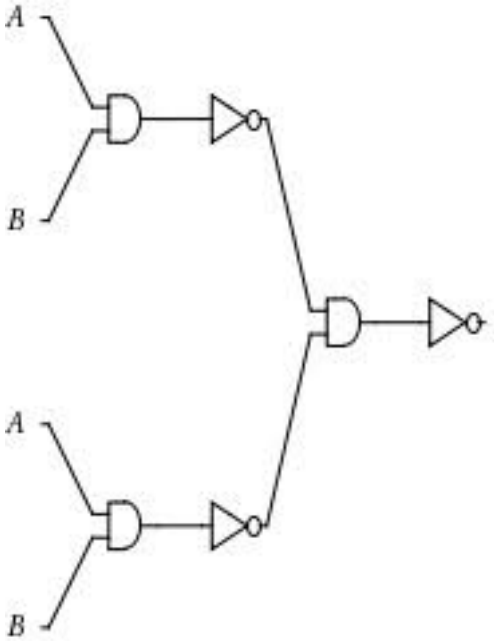
   Uppercase:

   $$2^x = 29 \Rightarrow x = 5$$

   Lowercase:

   $$2^x = 58 \Rightarrow x = 6$$

2. Describe how can create an OR gate using NOT gates and AND gates



3. A kilobyte is 1024 bytes.How many message can it store?

   $2^{1024} = 179,769,313,486,231,590,772,930,...,304,835,356,329,624,224,137,216\ digits = 39$

4. What is the entropy associated with the tossing of a fair coin?

   $$H(x) = -\sum_{i=1}^{n} P_i \log_2 P_i$$

   We known that information is

   $$I = -\log_2 P_i$$

   Then we can write entrophy equation as

   $$H(x) = -\sum_{i=1}^{n} P_i I_i$$

   for tossing of a fair coin

   $$H(x) = -\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{2}\log_2 \frac{1}{2} = \frac{1}{2} * \frac{1}{2} = 1$$

5. Suppose that X consist of characters A,B,C,D that occur in a signal with respective probabilities 0,1,0,4,0,25 and 0,25. What is the Shannon entrophy?

$$H(x) = -\sum_{i=1}^{n} P_i \log_2 P_i$$

$$H(x) = -0{,}1 \log_2 0{,}1 - 0{,}4 \log_2 0{,}4 - 0{,}25 \log_2 0{,}25 - 0{,}25 \log_2 0{,}25 = 1{,}86096$$

6. A room full of people has incomes distributed in the follow way:

   - $n(25{,}5) = 3$
   - $n(30) = 5$
   - $n(42) = 7$
   - $n(50) = 3$
   - $n(63) = 1$
   - $n(75) = 2$
   - $n(90) = 1$

   What is the most probable income?

   $$n(42) = 7$$

   What is the average income?

   $$E(x) = 25{,}5(\frac{3}{22}) + 30(\frac{5}{22}) + 42(\frac{7}{22}) + 50(\frac{3}{22}) + 63(\frac{1}{22}) + 75(\frac{2}{22}) + 90(\frac{1}{22}) = 44{,}25$$

   What is the variance of this distribution?

   $$< j^2 >= 25{,}5^2(\frac{3}{22}) + 30^2(\frac{5}{22}) + 42^2(\frac{7}{22}) + 50^2(\frac{3}{22}) + 63^2(\frac{1}{22}) + 75^2(\frac{2}{22}) + 90^2(\frac{1}{22}) = 2255{,}35$$

   $$< (\Delta j)^2 >=< j^2 > - < j >^2 = 297{,}29$$