Quantum Computing
Homework 2

Raymundo Santana Carrillo

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Exercise chapter 2 Qbits and quantum states

1. A quantum system is in the state

$$\frac{(1-i)}{\sqrt{3}}\left|0\right\rangle + \frac{(1)}{\sqrt{3}}\left|1\right\rangle$$

if a measurment is made, what is the probability the system is in state or in state ?
Solution.

$$|\frac{(1-i)}{\sqrt{3}}|^2 = \frac{(1+i)}{\sqrt{3}}\frac{(1-i)}{\sqrt{3}} = \frac{(1-i+i-i^2)}{3} = \frac{2}{3}$$

$$|\frac{(1)}{\sqrt{3}}|^2 = \frac{1}{3}$$

$$\frac{2}{3} + \frac{1}{3} = 1$$

2. Two quantum states are given by

$$\left|a\right\rangle = \begin{pmatrix} -4i \\ 2 \end{pmatrix}, \left|b\right\rangle = \begin{pmatrix} 1 \\ -1+i \end{pmatrix}$$

a) Find $\left|a+b\right\rangle$

b) Calculate $3\left|a\right\rangle - 2\left|b\right\rangle$

c) Normalize $\left|a\right\rangle, \left|b\right\rangle$

$$\left|a+b\right\rangle = \begin{pmatrix} -4i \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1+i \end{pmatrix} = \begin{pmatrix} 1-4i \\ 1+i \end{pmatrix}$$

$$3\left|a\right\rangle - 2\left|b\right\rangle = \frac{-2-12i}{8-2i}$$

$$|a| = \sqrt{\langle a|a\rangle} = \begin{pmatrix} 4i & 2 \end{pmatrix}\begin{pmatrix} -4i \\ 2 \end{pmatrix} = -16i^2 + 4 = \sqrt{20}$$

$$|b| = \sqrt{\langle b|b\rangle} = \begin{pmatrix} 1 & -1-i \end{pmatrix}\begin{pmatrix} 1 \\ -1+i \end{pmatrix} = 1 + (-1-i)(-1-i) = \sqrt{3}$$

$$\left|a\right\rangle = \frac{\left|a\right\rangle}{|a|} = \frac{\begin{pmatrix} -4i \\ 2 \end{pmatrix}}{\sqrt{20}}$$

$$\left|b\right\rangle = \frac{\left|b\right\rangle}{|b|} = \frac{\begin{pmatrix} 1 \\ -1+i \end{pmatrix}}{\sqrt{3}}$$

3. Another basis for $\mathbb{C}^2$ is

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

   Invert this relation to express $|0\rangle, |1\rangle$ in terms of $|+\rangle, |-\rangle$.

4. A quantum system is in the state

$$|\psi\rangle = \frac{3i\,|0\rangle + 4\,|1\rangle}{\sqrt{5}}$$

   a) Is the state normalized?

   b) Express the state in the $|+\rangle, |-\rangle$ basis.

5. Use the Gram-Schmidt process to find an orthonormal basis for a subspace of the four-dimensional space $\mathbb{R}^4$ spanned by

$$|u_1\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, |u_2\rangle = \begin{pmatrix} 1 \\ 2 \\ 4 \\ 5 \end{pmatrix}, |u_3\rangle = \begin{pmatrix} 1 \\ -3 \\ -4 \\ -2 \end{pmatrix}$$

6. Photon horizontal and vertical polarization states are written as $|h\rangle$ and $|v\rangle$, respectively. Suppose

$$|\psi_1\rangle = \frac{1}{2}|h\rangle + \frac{\sqrt{3}}{2}|v\rangle$$

$$|\psi_2\rangle = \frac{1}{2}|h\rangle - \frac{\sqrt{3}}{2}|v\rangle$$

$$|\psi_3\rangle = |h\rangle$$