

Quantum Computing
Homework 1
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Exercise chapter 1 Introduction

1. How many bits are necessary to represent the alphabet using a binary code if we only allow uppercase characters? How about if we allow both uppercase and lowercase characters

Solution

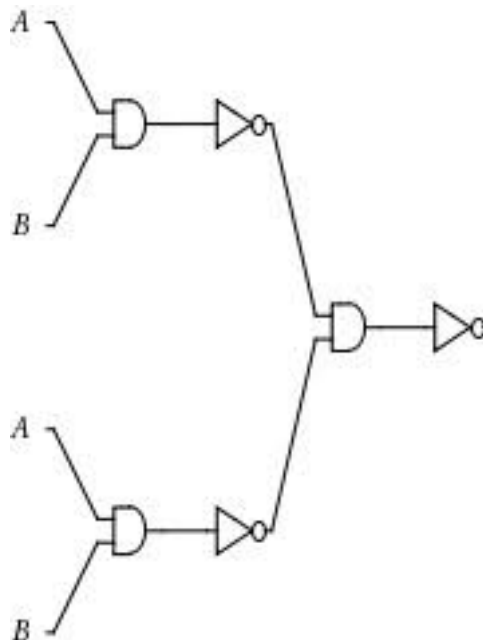
Uppercase:

$$2^x = 29 \Rightarrow x = 5$$

Lowercase:

$$2^x = 58 \Rightarrow x = 6$$

2. Describe how can create an OR gate using NOT gates and AND gates



3. A kilobyte is 1024 bytes. How many messages can it store?

$$2^{1024} = 179, 769, 313, 486, 231, 590, 772, 930, \dots, 304, 835, 356, 329, 624, 224, 137, 216 \text{ digits} = 39$$

4. What is the entropy associated with the tossing of a fair coin?

$$H(x) = - \sum_{i=1}^n P_i \log_2 P_i$$

We know that information is

$$I = - \log_2 P_i$$

Then we can write entropy equation as

$$H(x) = - \sum_{i=1}^n P_i I_i$$

for tossing of a fair coin

$$H(x) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = \frac{1}{2} * \frac{1}{2} = 1$$

5. Suppose that X consist of characters A, B, C, D that occur in a signal with respective probabilities 0,1,0,4,0,25 and 0,25. What is the Shannon entropy?

$$H(x) = - \sum_{i=1}^n P_i \log_2 P_i$$

$$H(x) = -0,1 \log_2 0,1 - 0,4 \log_2 0,4 - 0,25 \log_2 0,25 - 0,25 \log_2 0,25 = 1,86096$$

6. A room full of people has incomes distributed in the follow way:

- $n(25,5) = 3$
- $n(30) = 5$
- $n(42) = 7$
- $n(50) = 3$
- $n(63) = 1$
- $n(75) = 2$
- $n(90) = 1$

What is the most probable income?

$$n(42) = 7$$

What is the average income?

$$E(x) = 25,5\left(\frac{3}{22}\right) + 30\left(\frac{5}{22}\right) + 42\left(\frac{7}{22}\right) + 50\left(\frac{3}{22}\right) + 63\left(\frac{1}{22}\right) + 75\left(\frac{2}{22}\right) + 90\left(\frac{1}{22}\right) = 44,25$$

What is the variance of this distribution?

$$\langle j^2 \rangle = 25,5^2\left(\frac{3}{22}\right) + 30^2\left(\frac{5}{22}\right) + 42^2\left(\frac{7}{22}\right) + 50^2\left(\frac{3}{22}\right) + 63^2\left(\frac{1}{22}\right) + 75^2\left(\frac{2}{22}\right) + 90^2\left(\frac{1}{22}\right) = 2255,35$$

$$\langle (\Delta j)^2 \rangle = \langle j^2 \rangle - \langle j \rangle^2 = 297,29$$