Observation Matrix

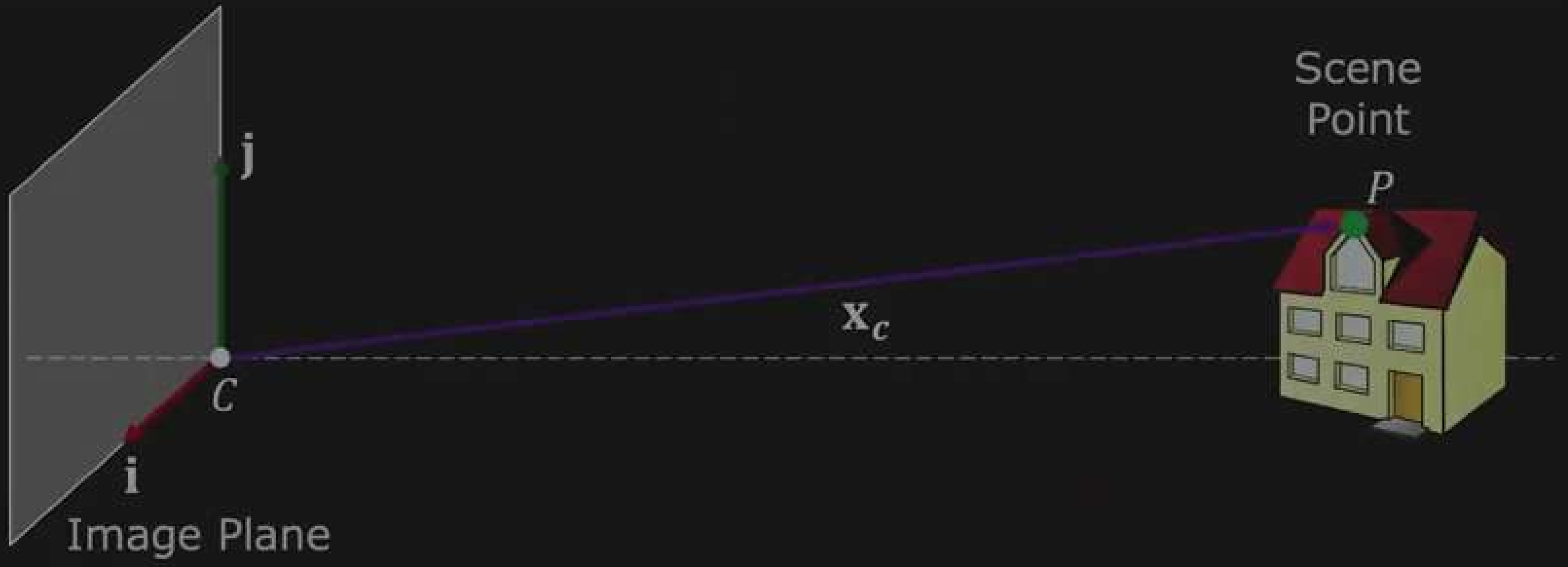
Shree K. Nayar

Columbia University

Topic: Structure from Motion, Module: Reconstruction II

First Principles of Computer Vision

From 3D to 2D: Orthographic Projection



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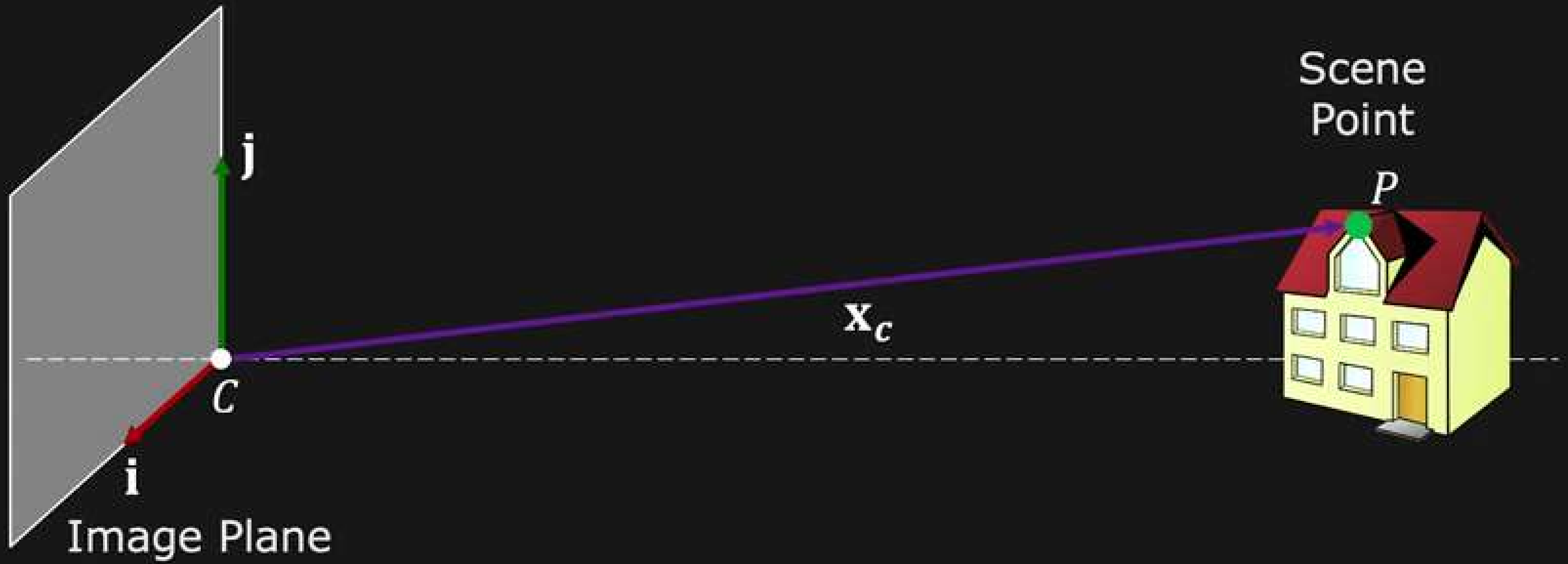

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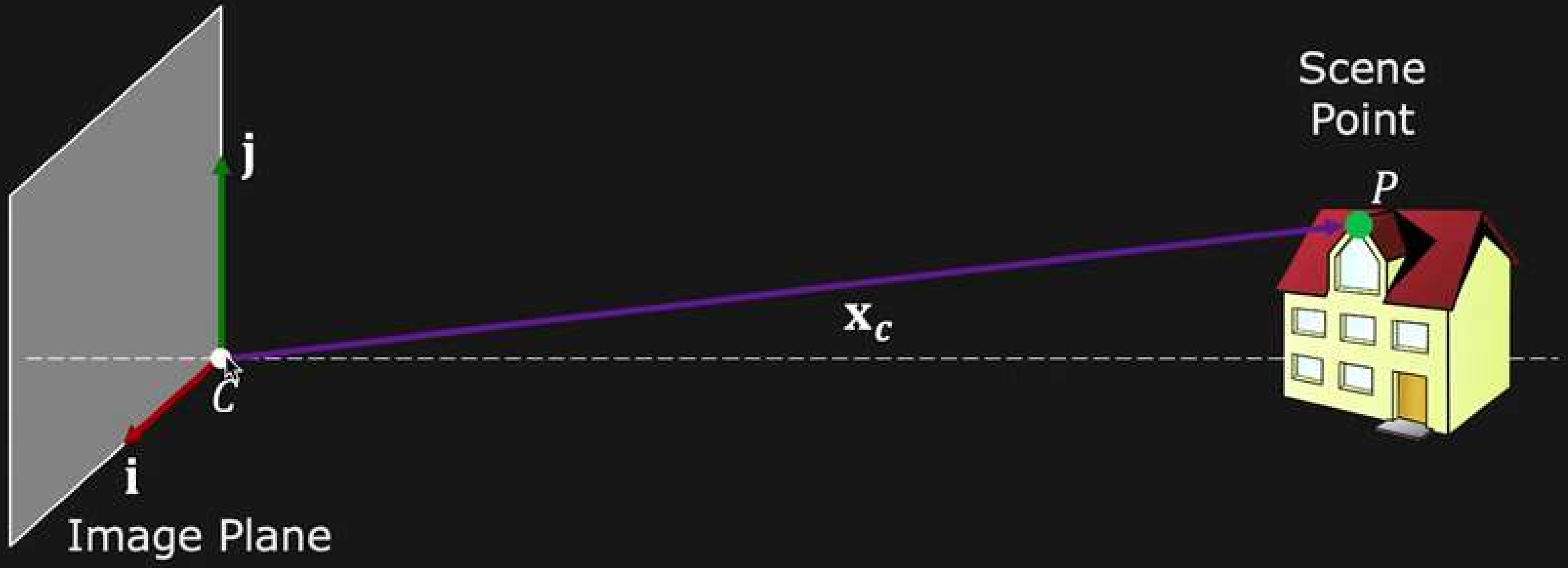

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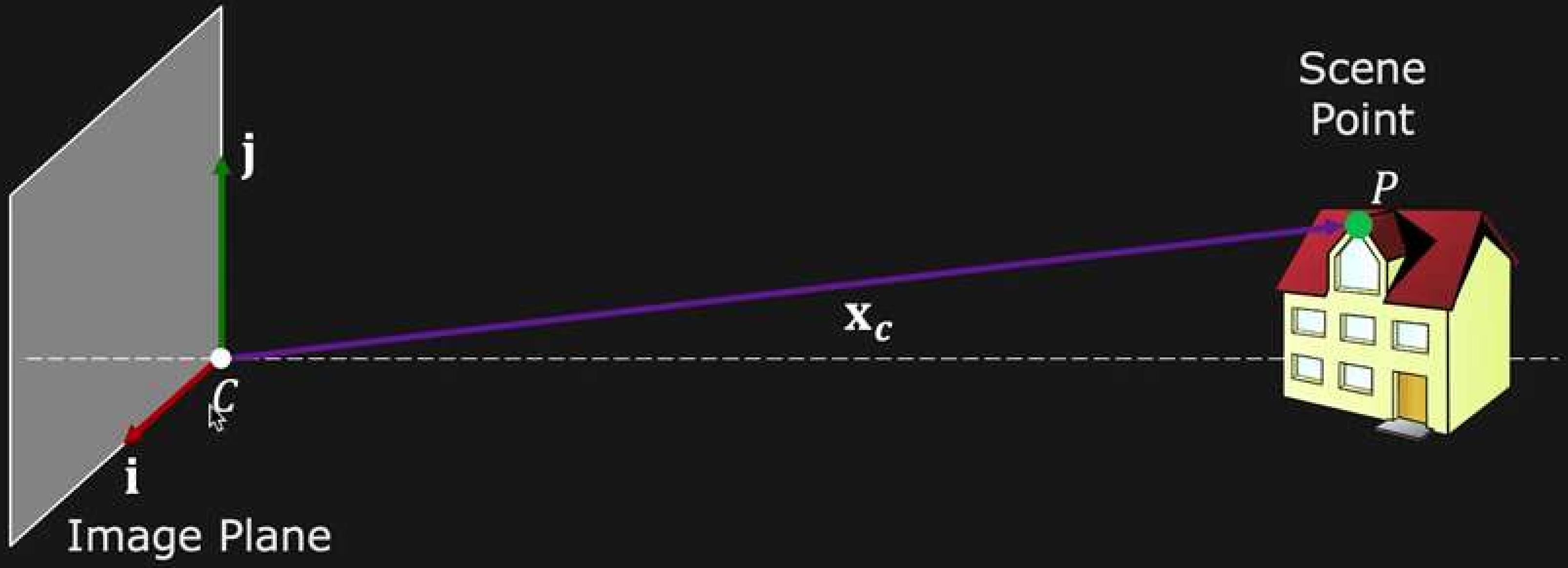

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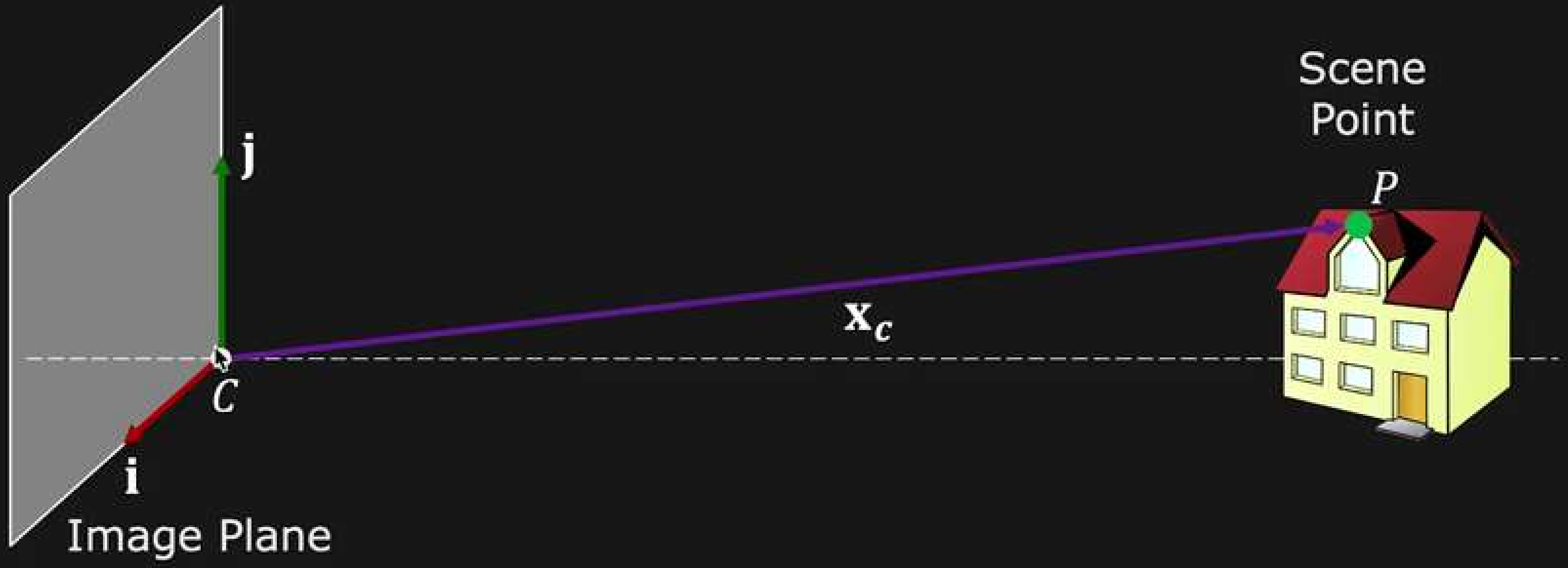

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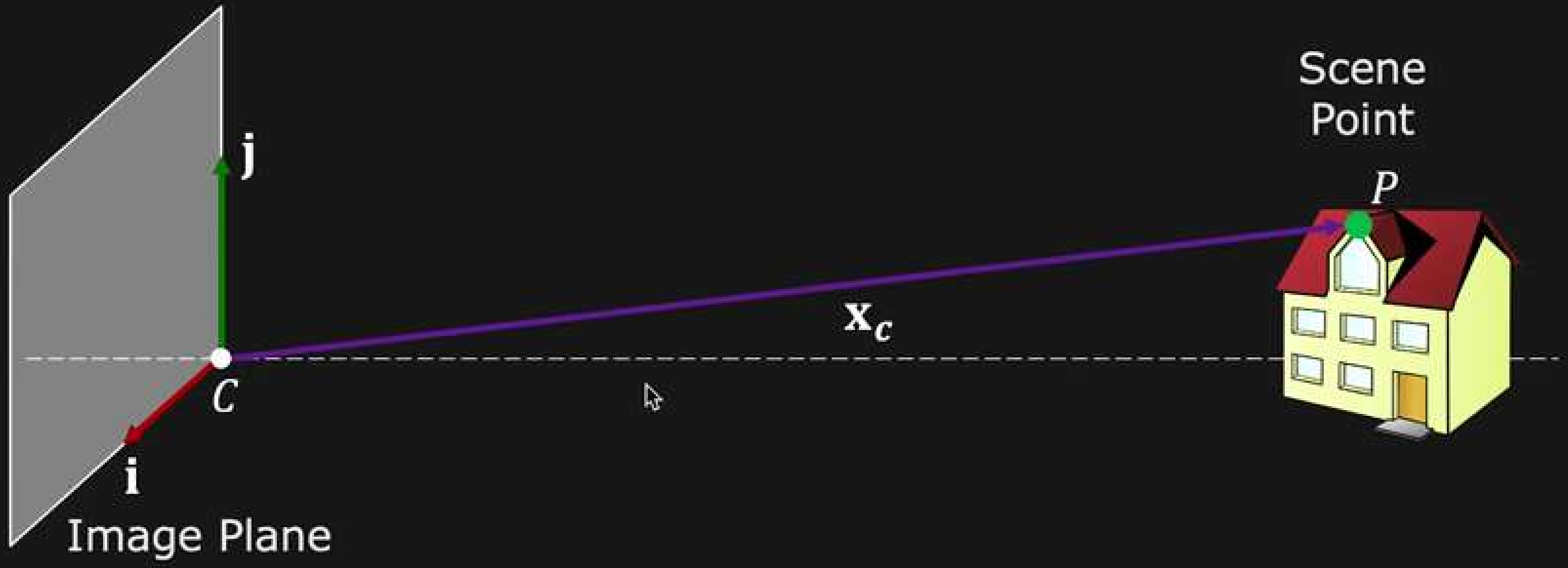

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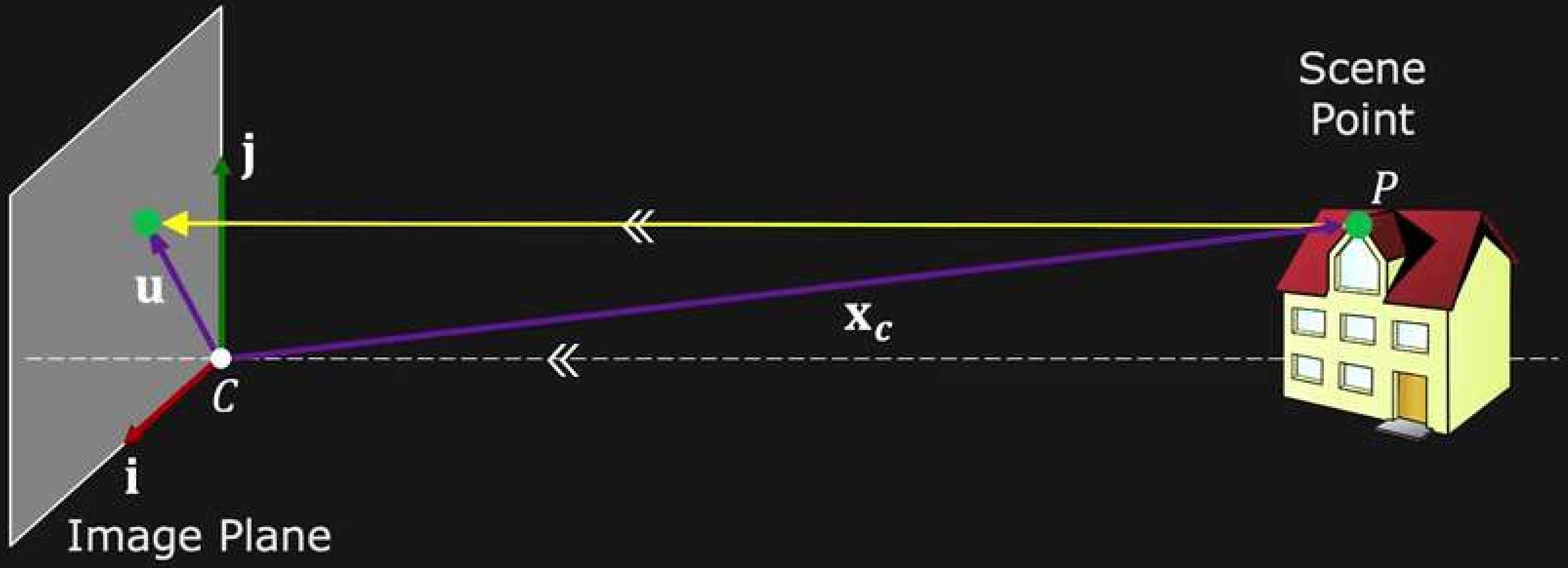

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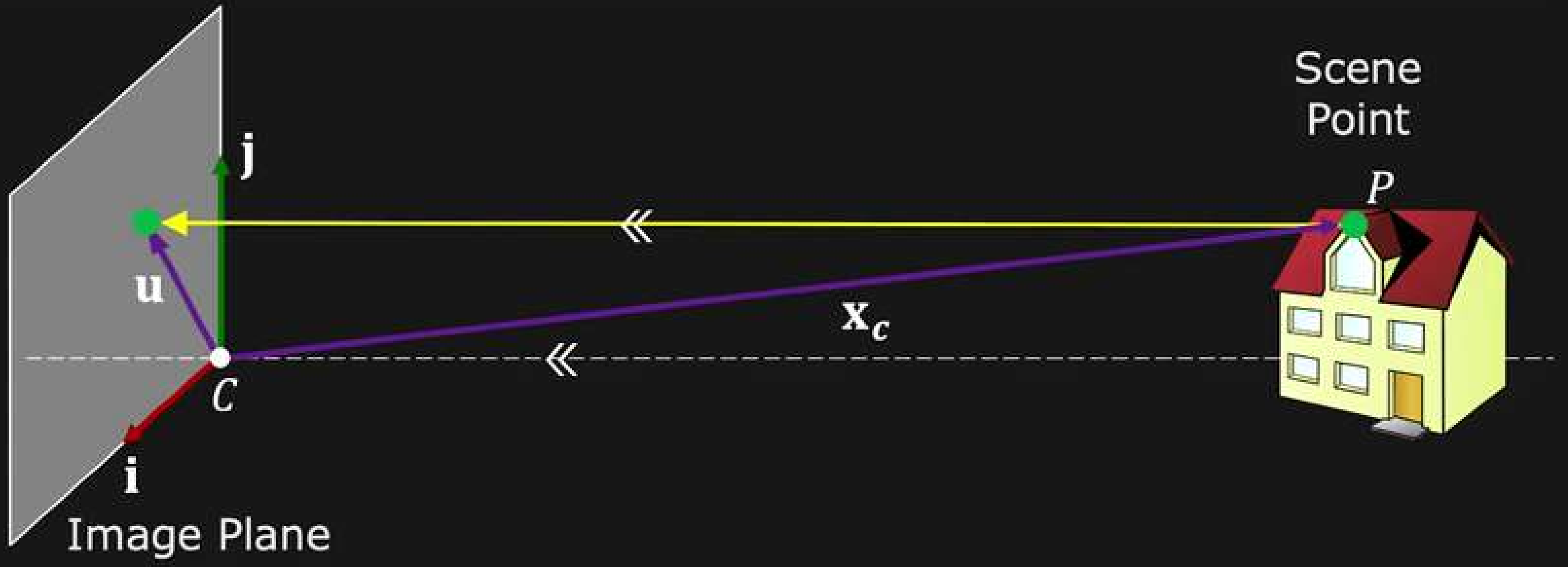

$$u = \mathbf{i} \cdot \mathbf{x}_c = \mathbf{i}^T \mathbf{x}_c$$

$$v = \mathbf{j} \cdot \mathbf{x}_c = \mathbf{j}^T \mathbf{x}_c$$

From 3D to 2D: Orthographic Projection

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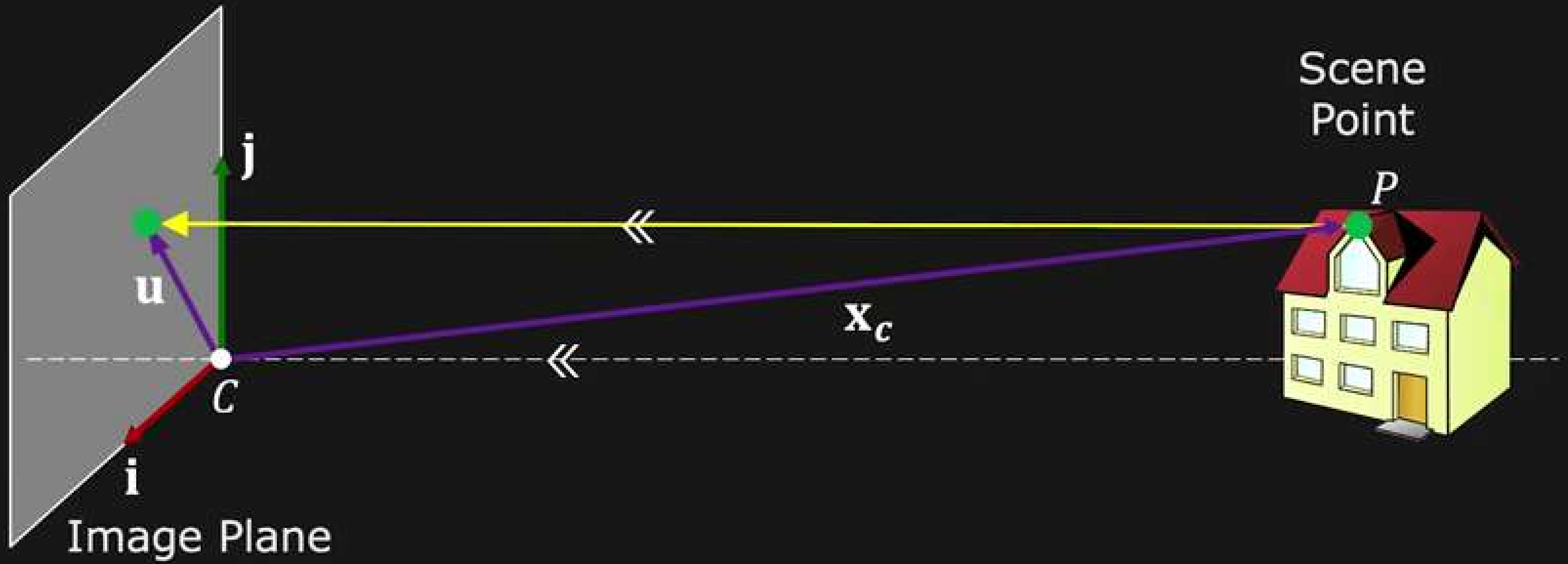
Perspective cameras exhibit orthographic projection when distance of scene from camera is large compared to dept variation within scene (magnification is nearly constant).

From 3D to 2D: Orthographic Projection



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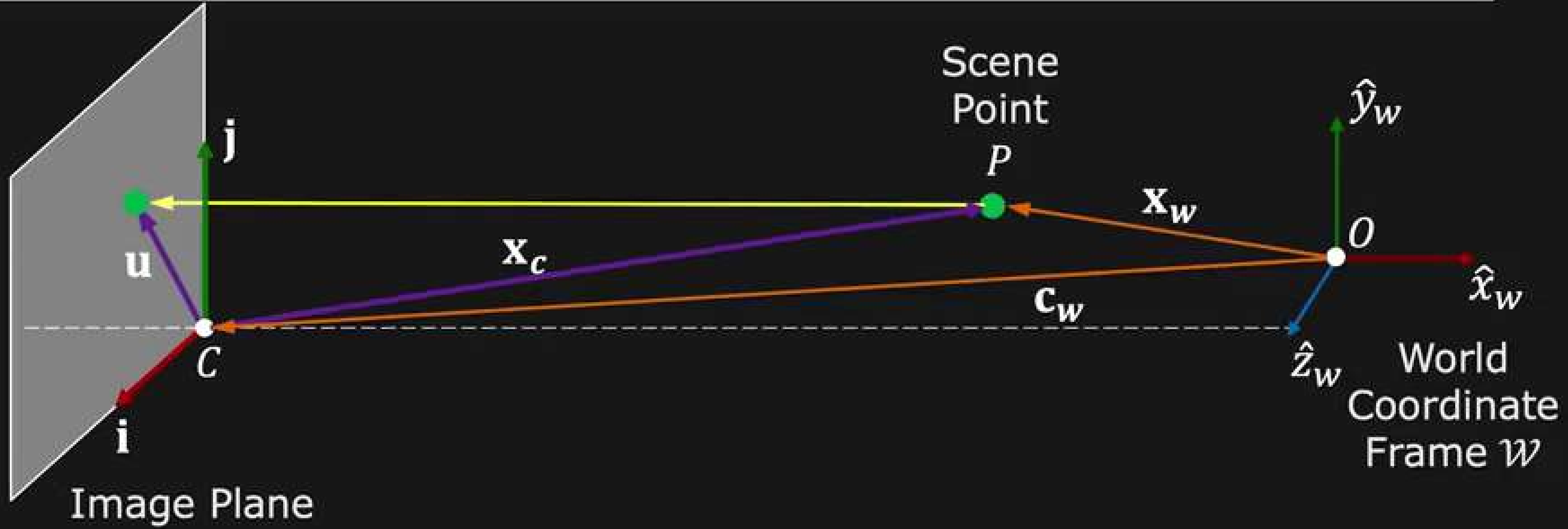

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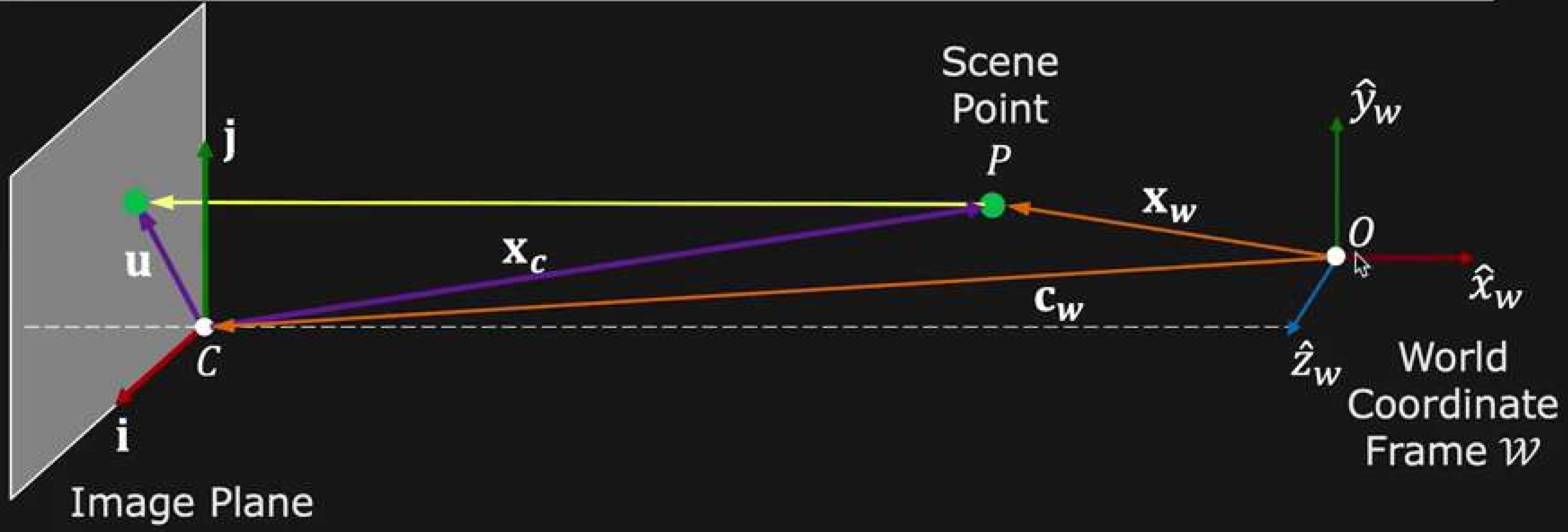

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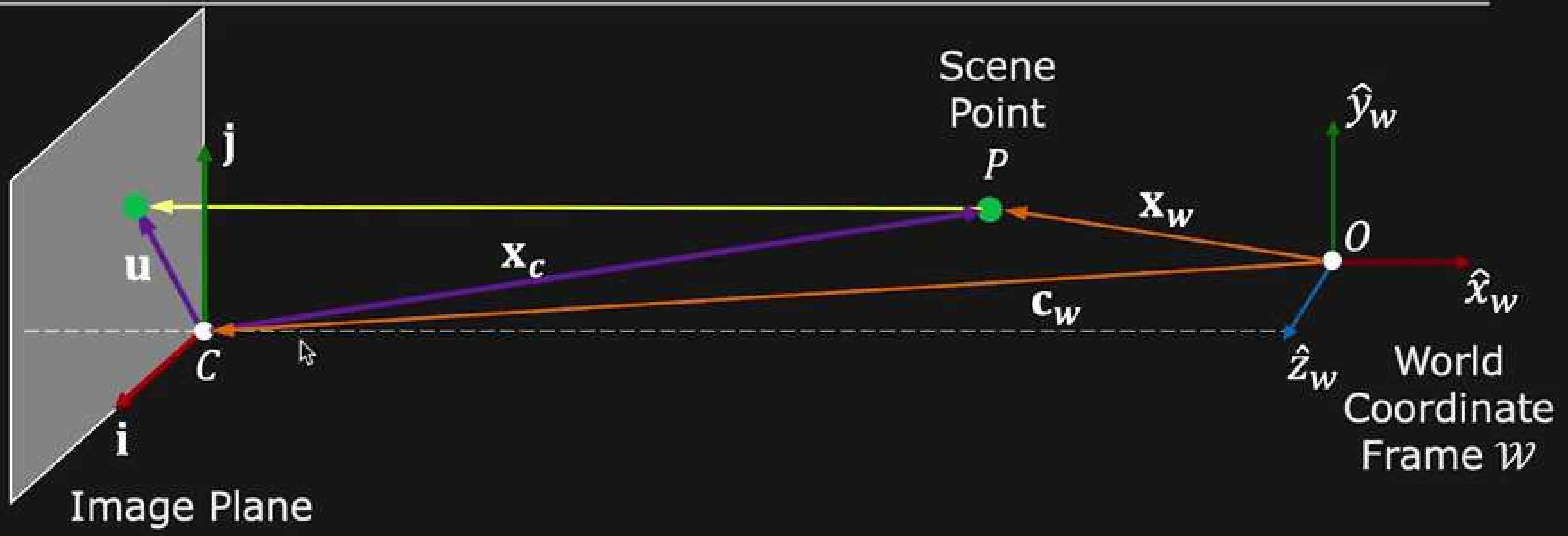

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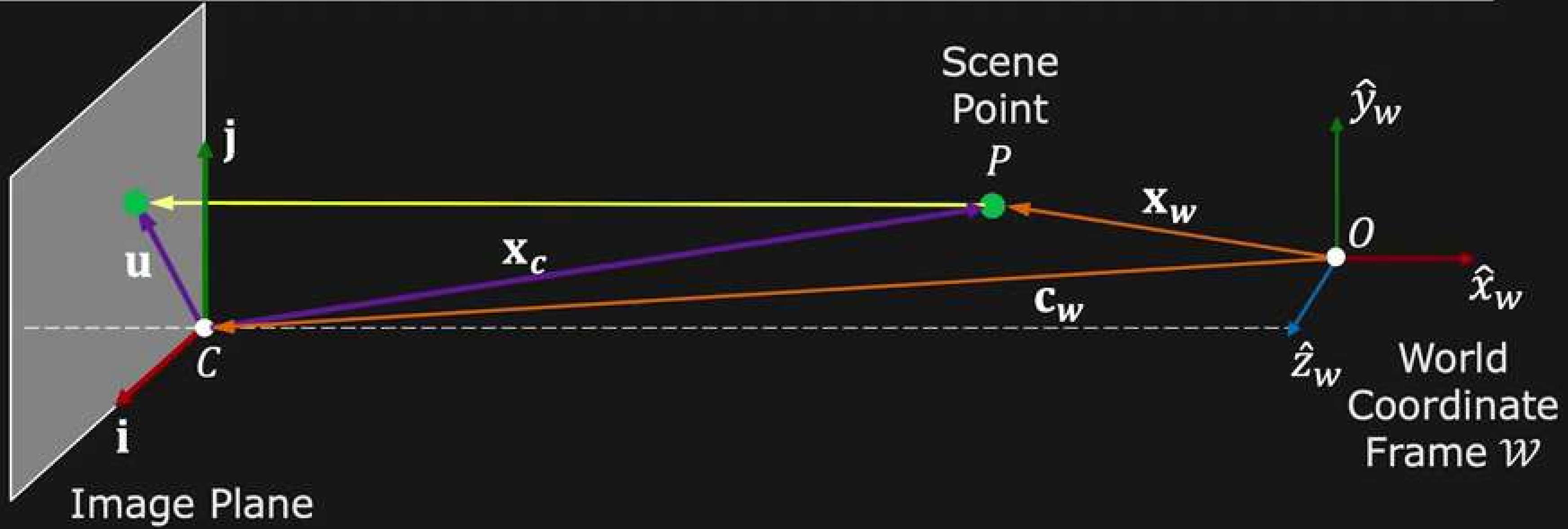

$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w)$$

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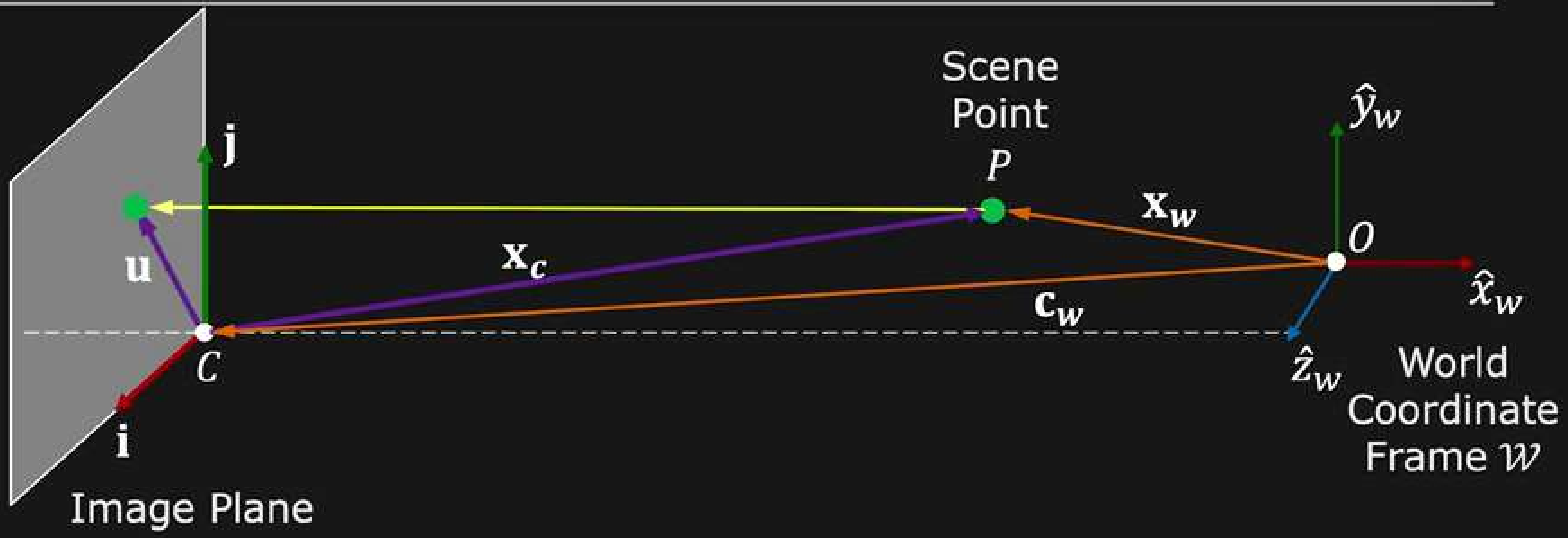

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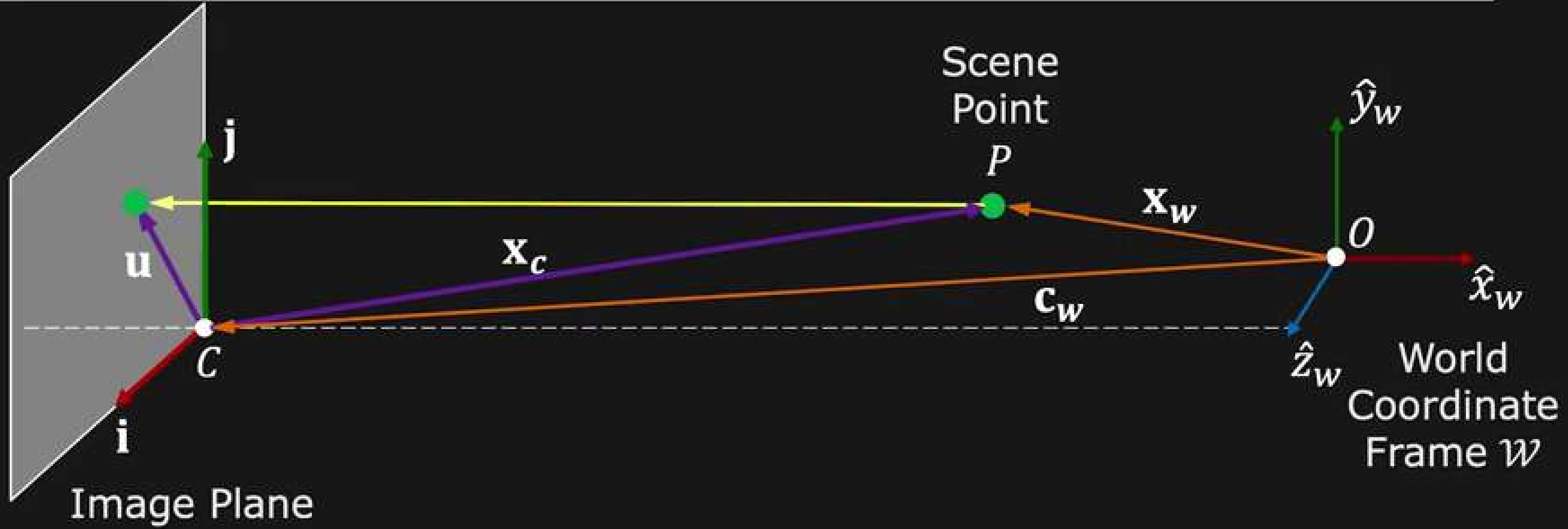

$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{i}^T (P - C)$$

$$v = \mathbf{j}^T \mathbf{x}_c = \mathbf{j}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{j}^T (P - C)$$

$$\boxed{\begin{aligned} u &= \mathbf{i}^T (P - C) \\ v &= \mathbf{j}^T (P - C) \end{aligned}}$$

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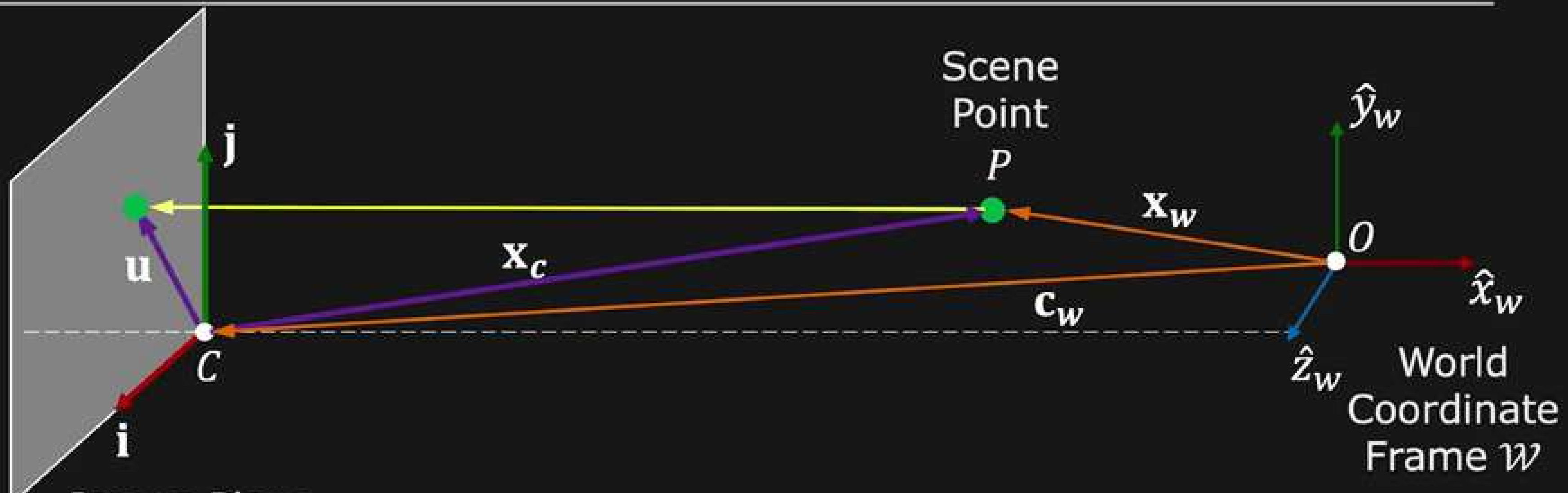

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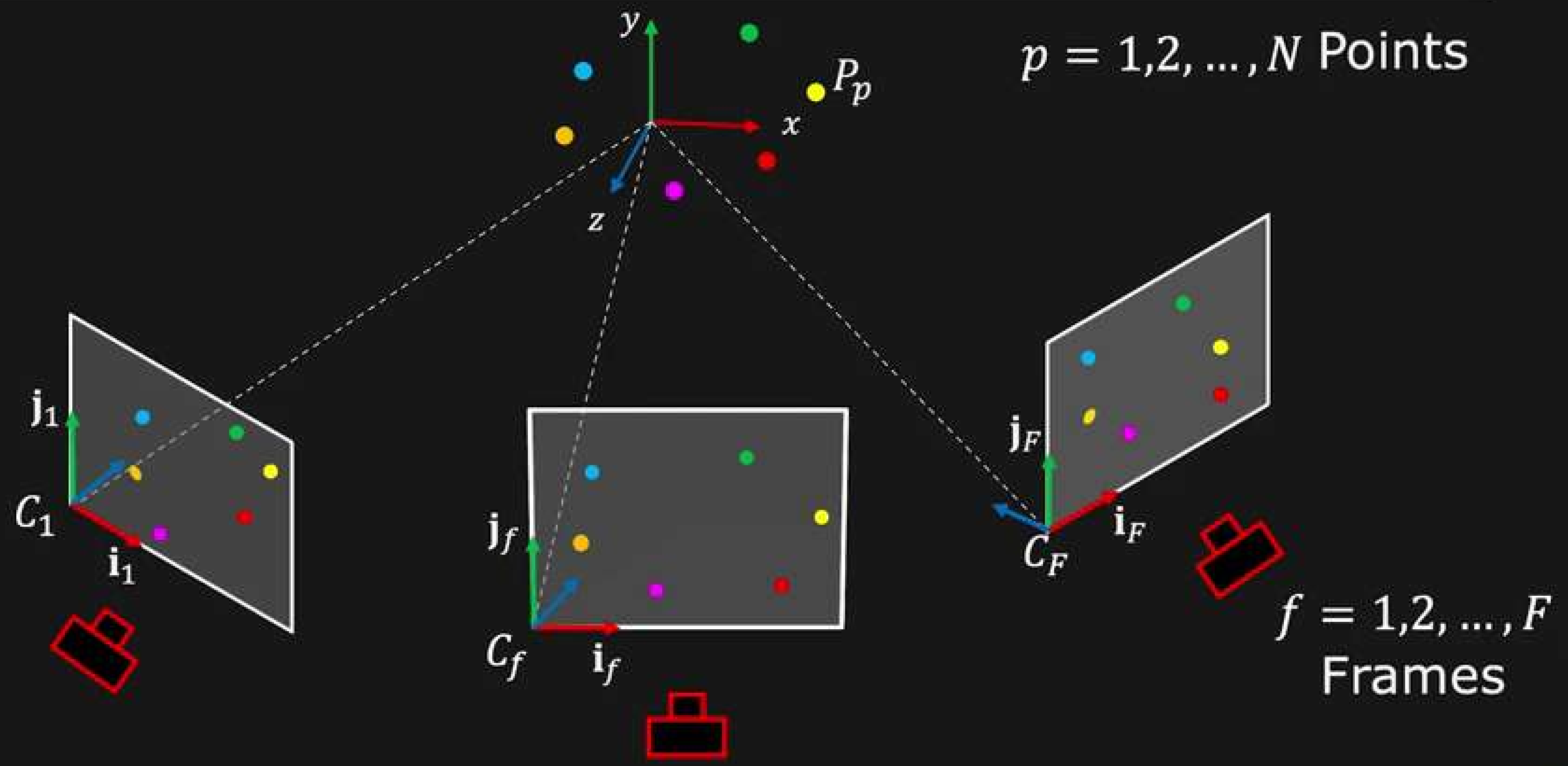
Orthographic SFM

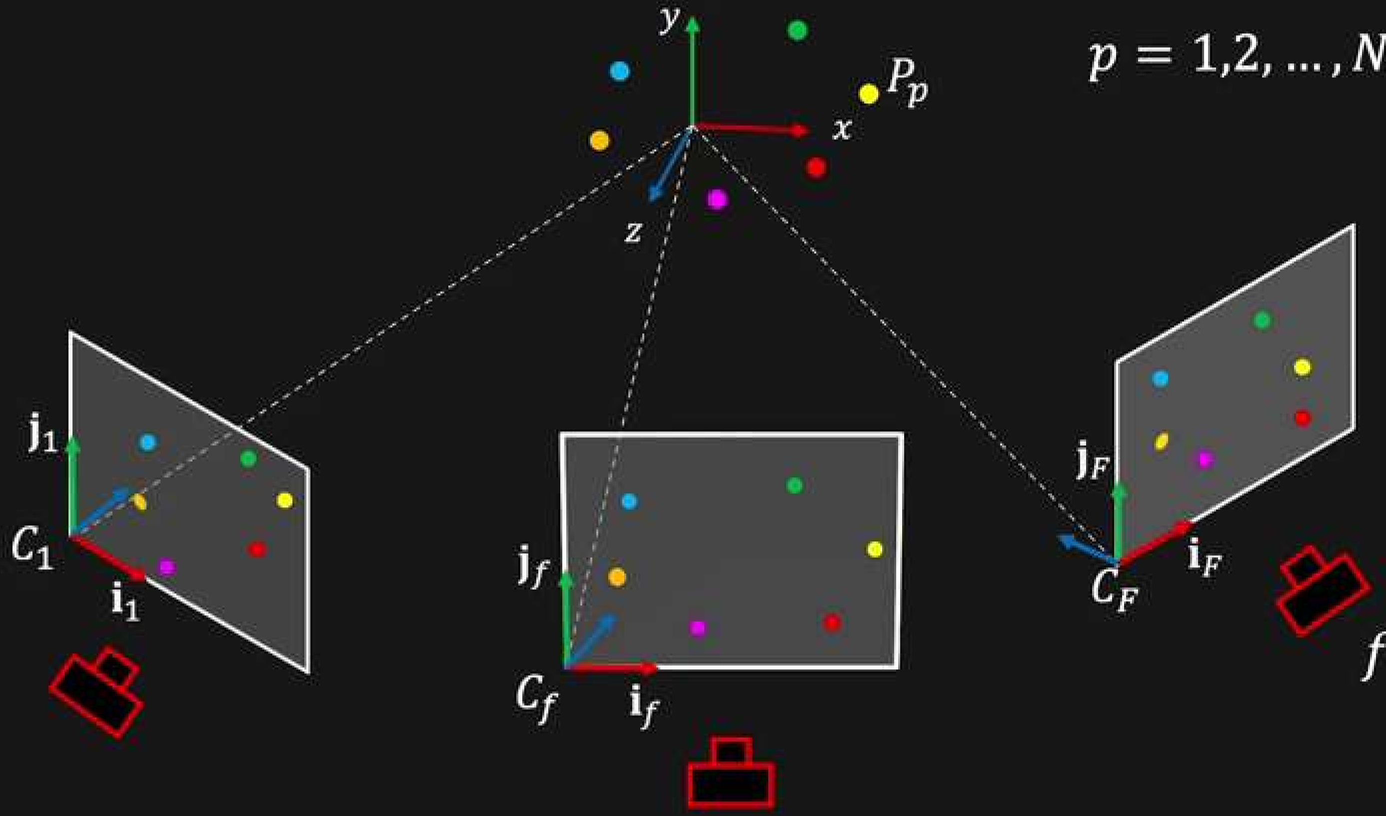


Given corresponding image points (2D) $(u_{f,p}, v_{f,p})$

Find scene points $\{P_p\}$.

Camera Positions $\{C_f\}$, camera orientations $\{(\mathbf{i}_f, \mathbf{j}_f)\}$ are unkno

Orthographic SFM



$p = 1,2,\ldots,N$ Points

$f = 1,2,\ldots,F$ Frames

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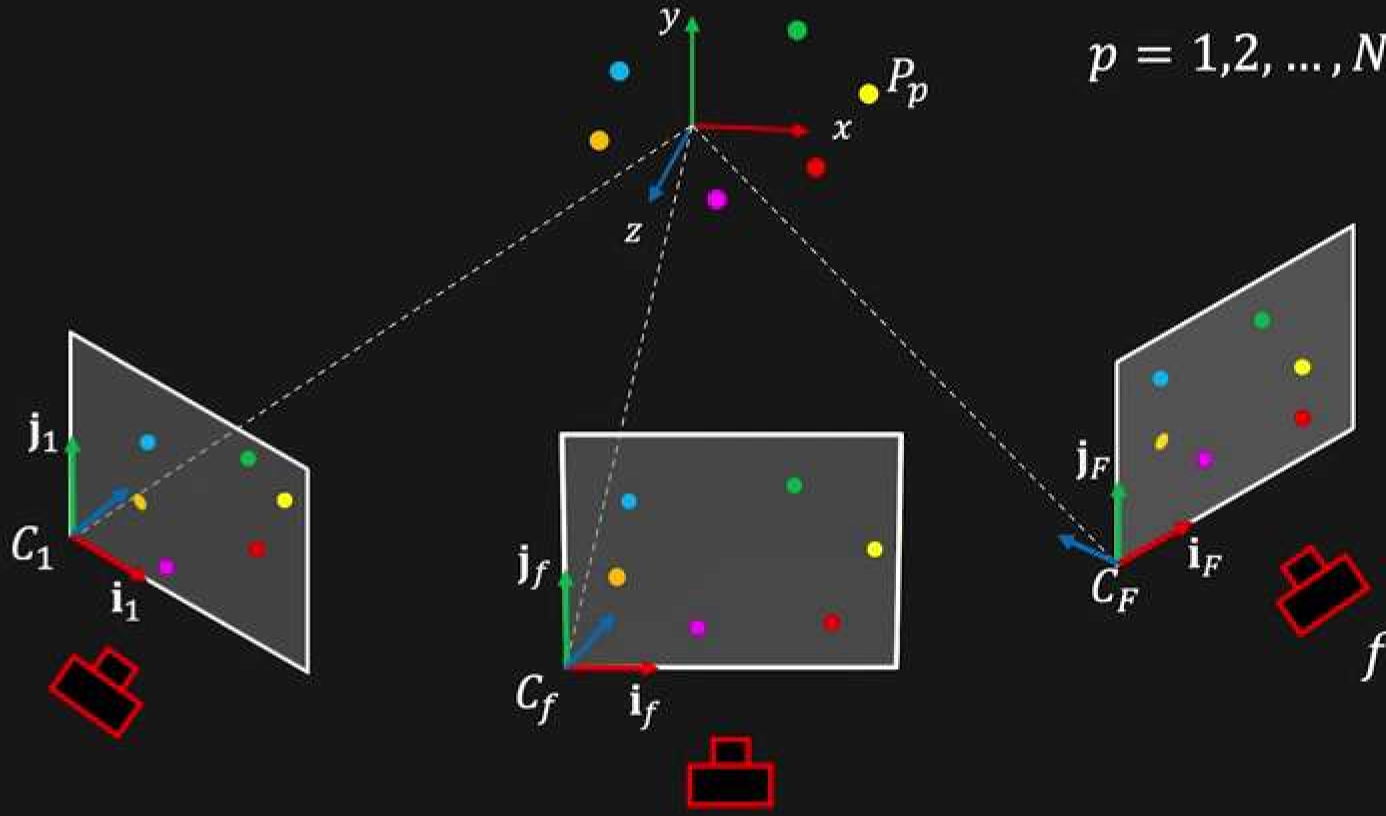

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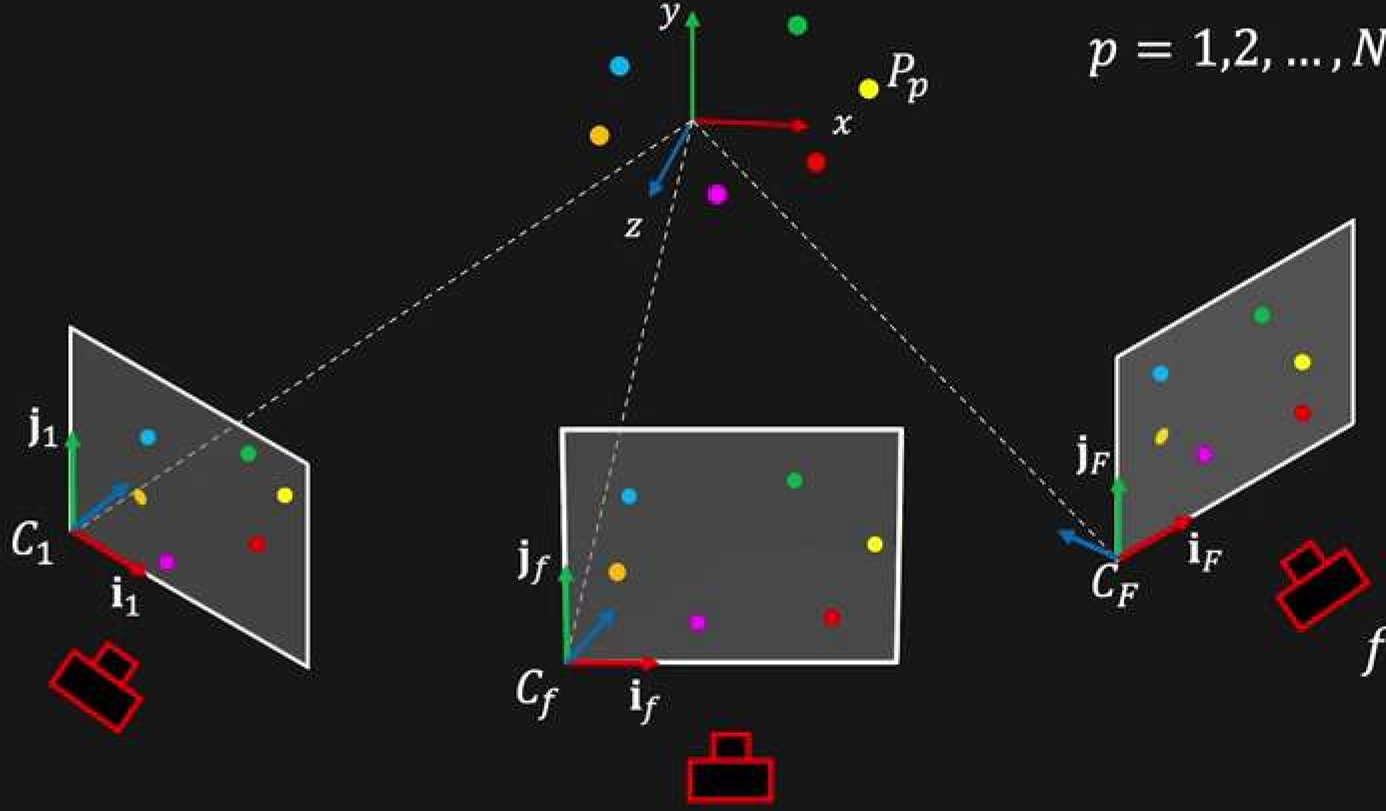

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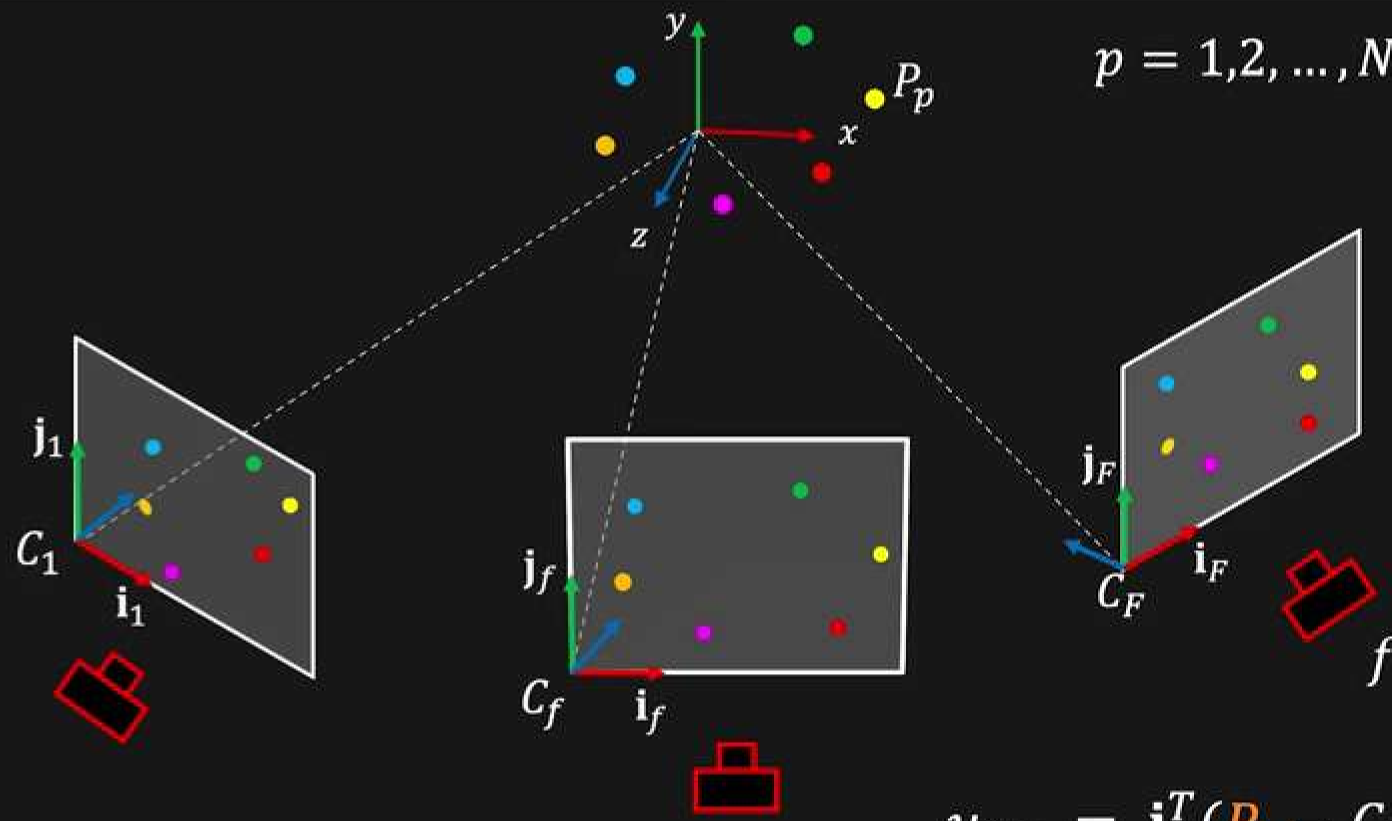

$p = 1,2,\ldots,N$ Points

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Image of point P_p in camera frame f:

$$u_{f,p} = \mathbf{i}_f^T(P_p - C_f)$$

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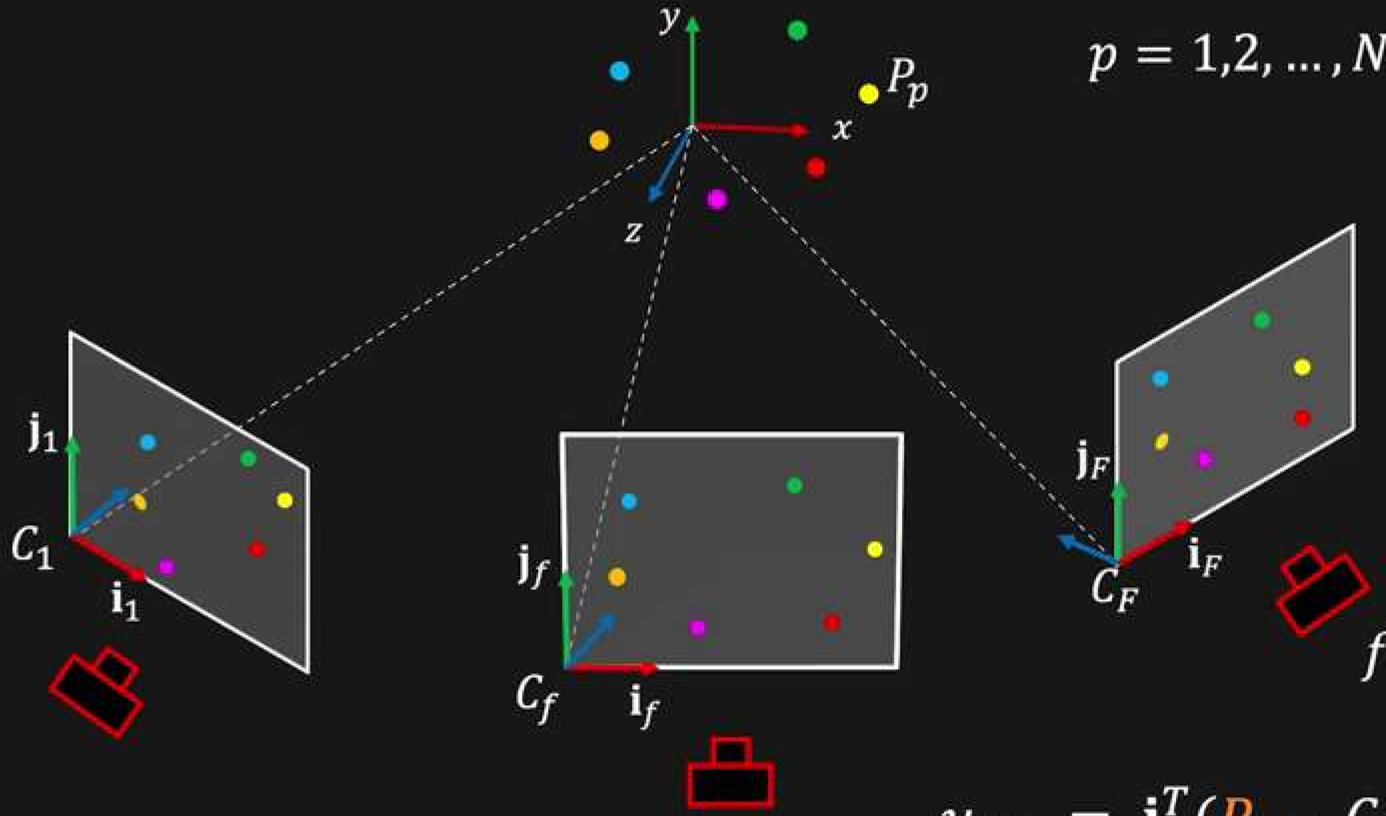
Orthographic SFM



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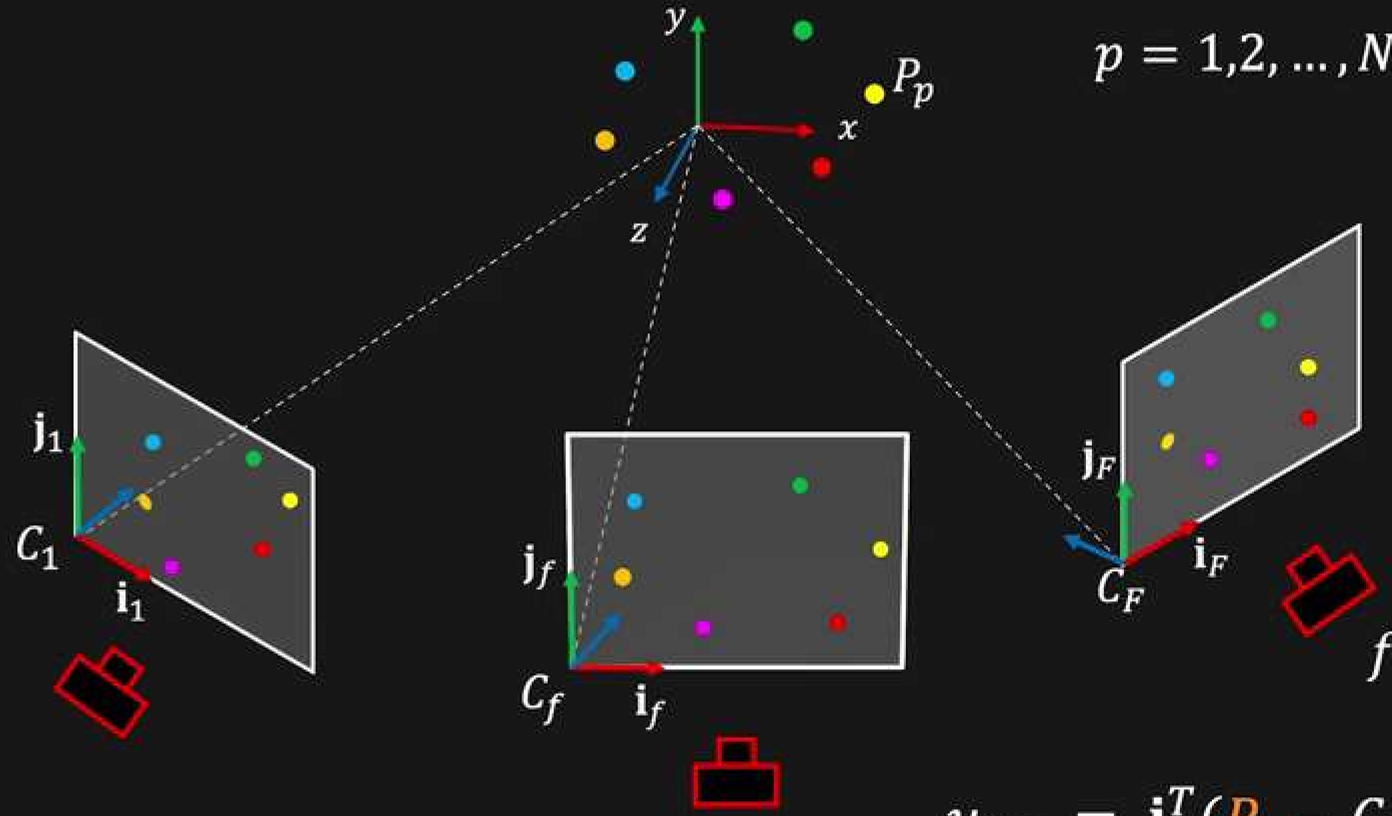

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$$u_{f,p} = \mathbf{i}_f^T (P_p - C_f)$$

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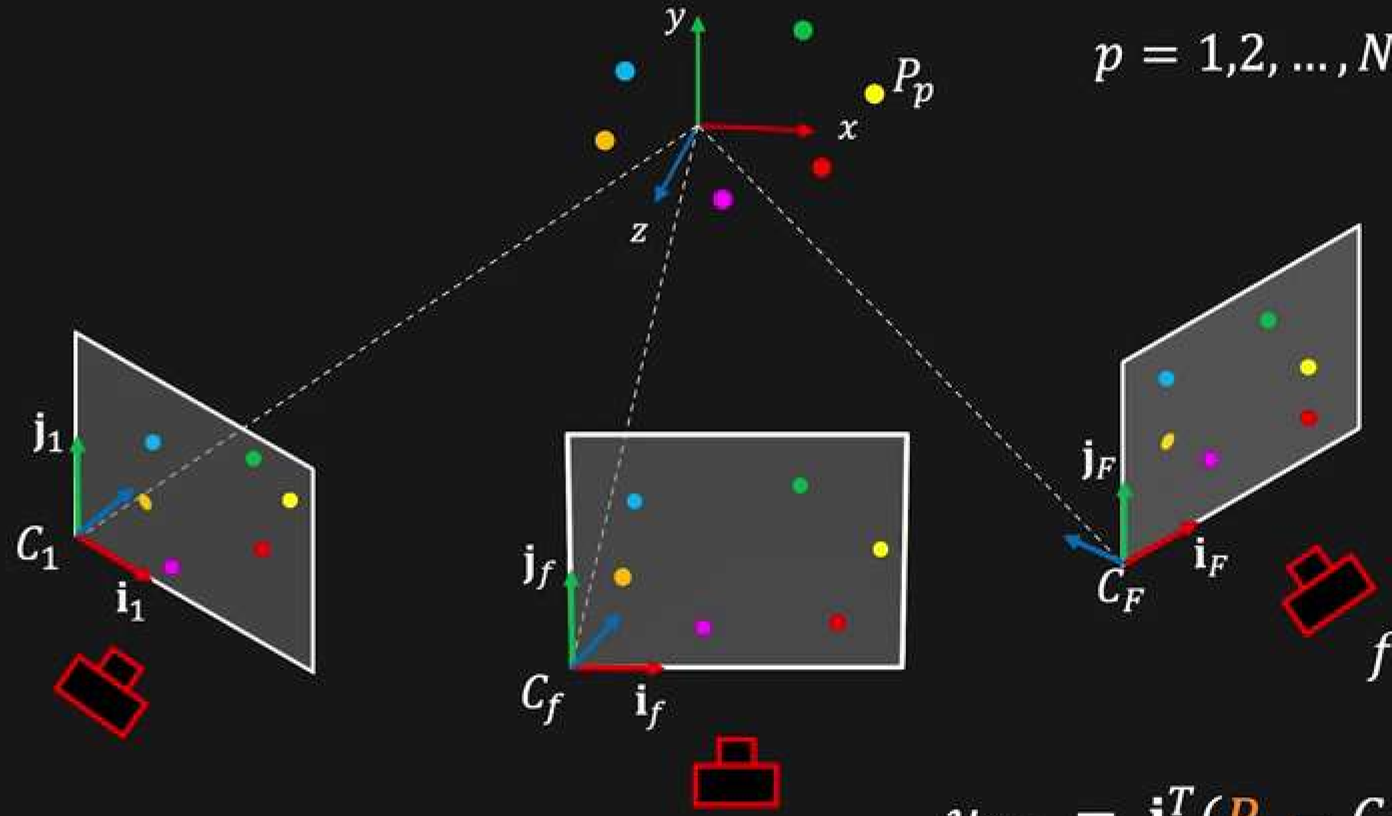
Orthographic SFM



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Known — Unknown

Orthographic SFM

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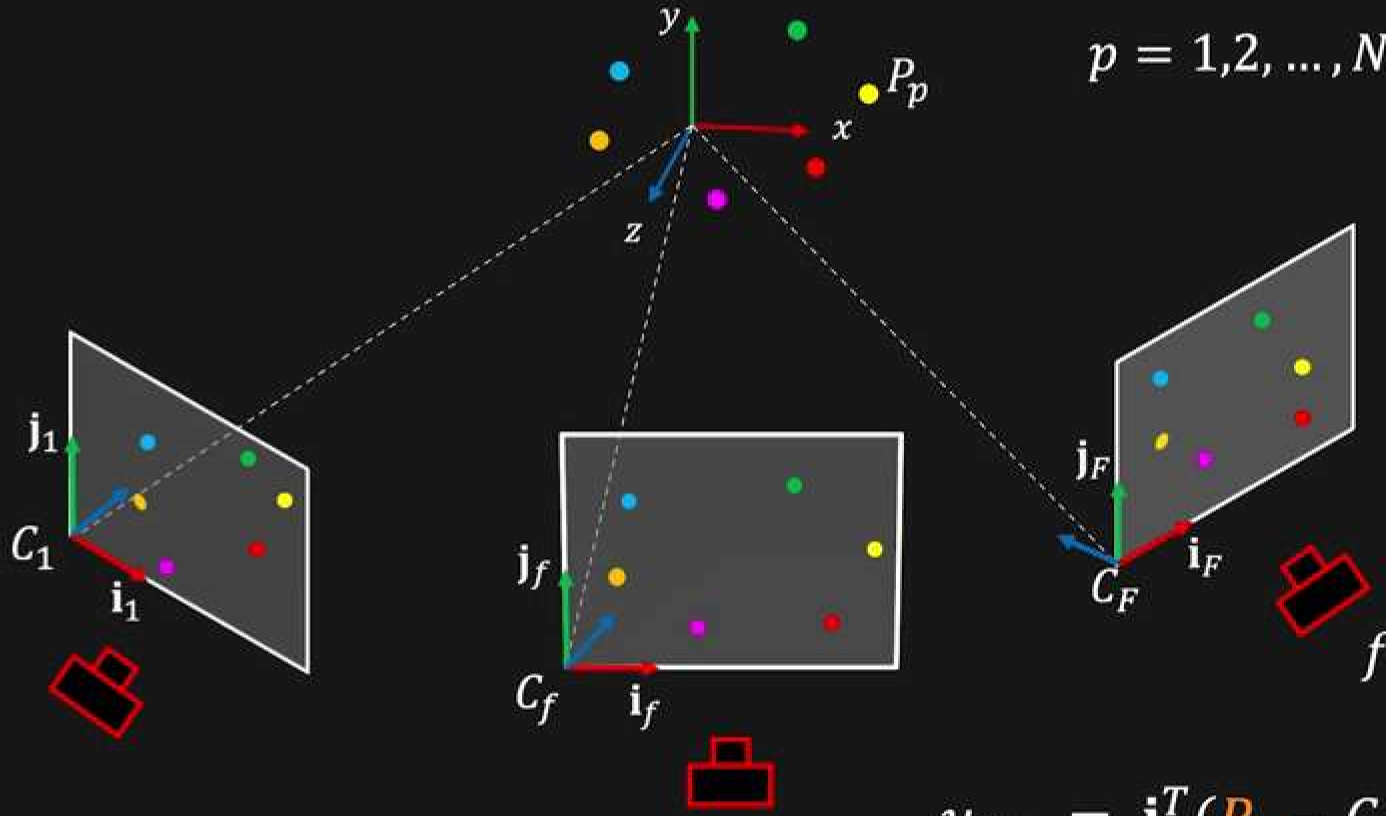

$f = 1,2,\ldots,F$ Frames

Image of point P_p in camera frame f:

$$\underbrace{u_{f,p}}_{\text{Known}} = \underbrace{\mathbf{i}_f^T(P_p - C_f)}_{\text{Unknown}}$$

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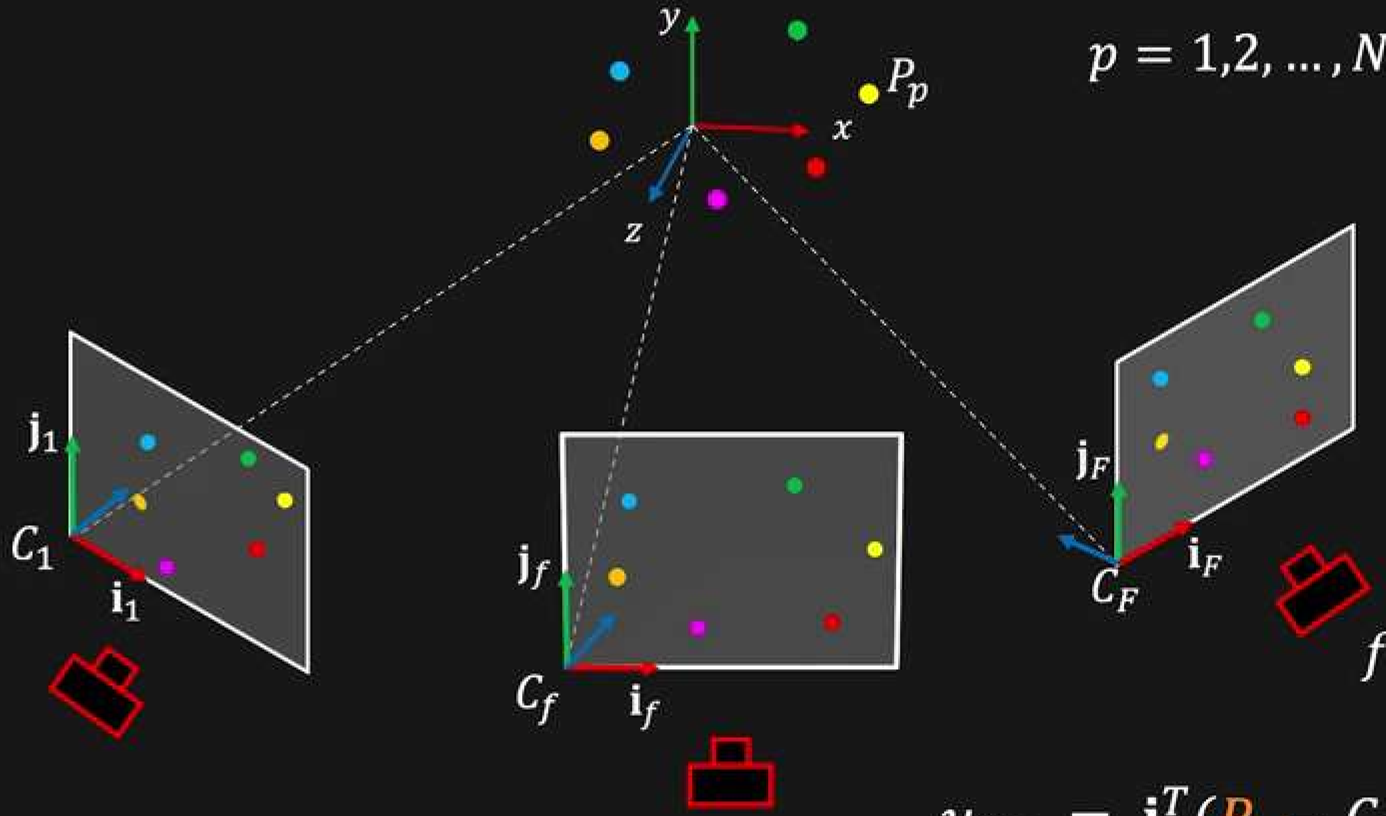

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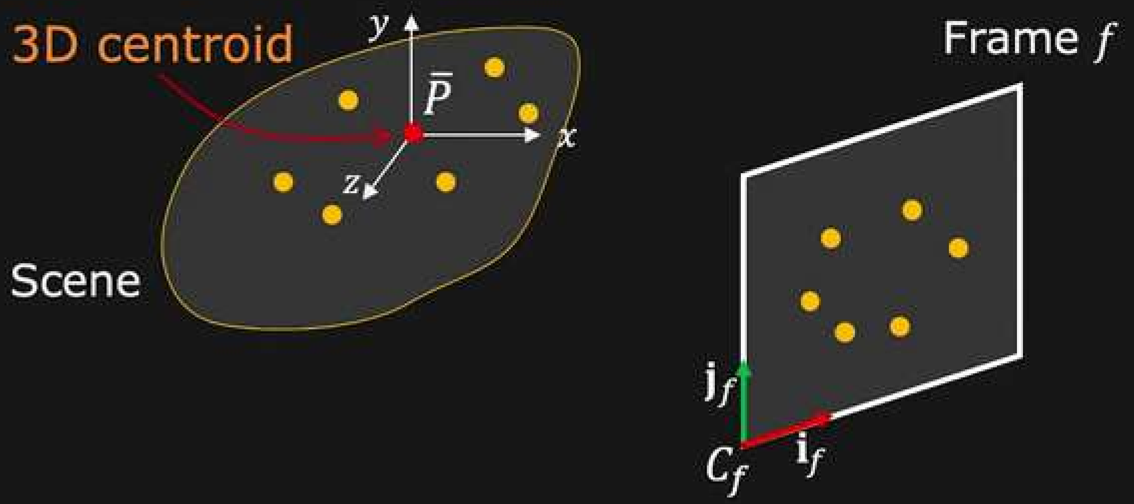
Known Unknown

We can remove C_f from equations to simply SFM problem.

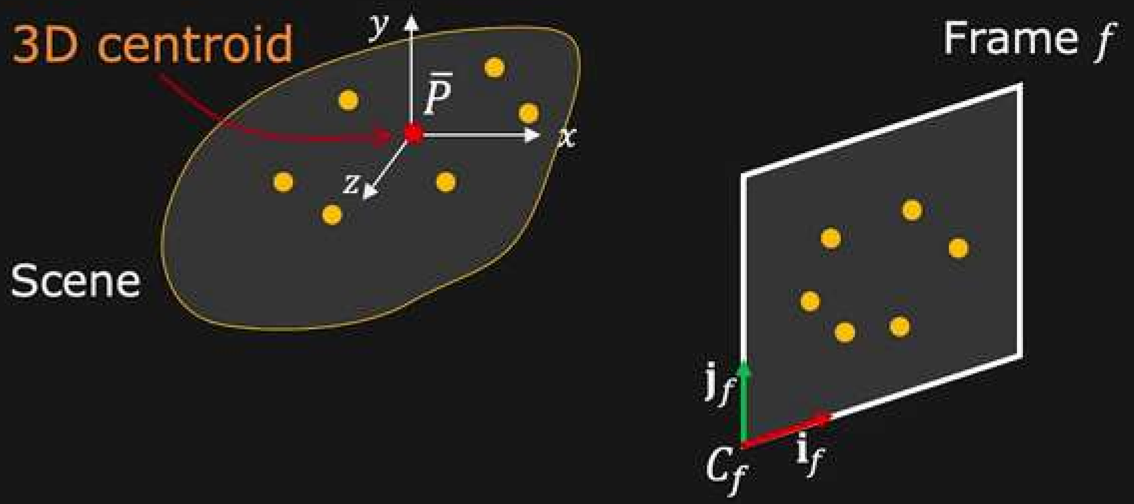
Centering Trick



Assume origin of world at centroid of scene points:

$$\frac{1}{N}\sum_{p=1}^{N} P_p = \bar{P} = \mathbf{0}$$

Centering Trick



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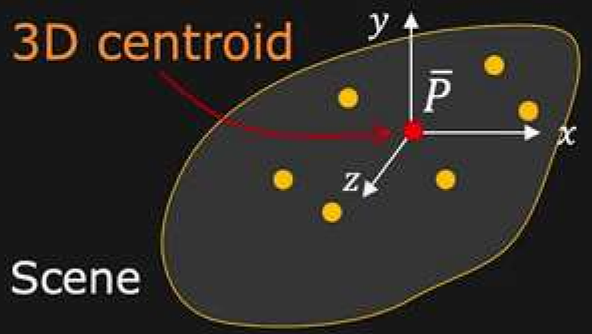

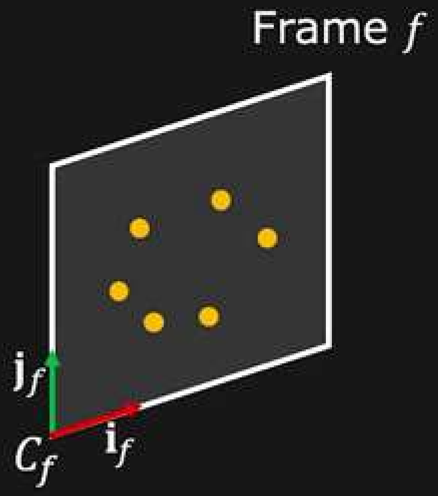

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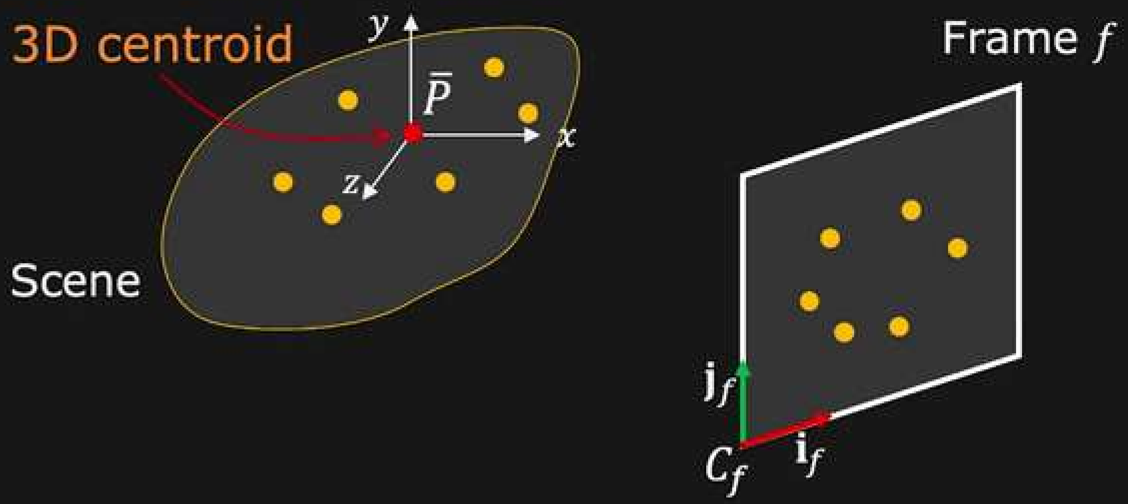

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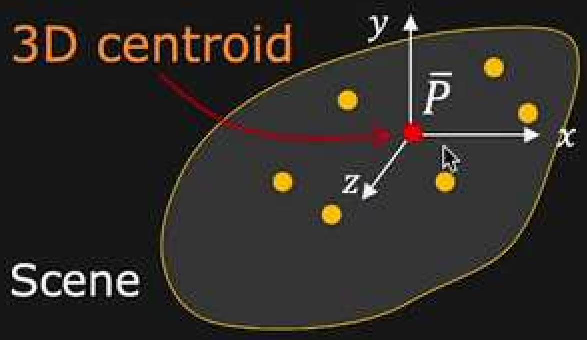
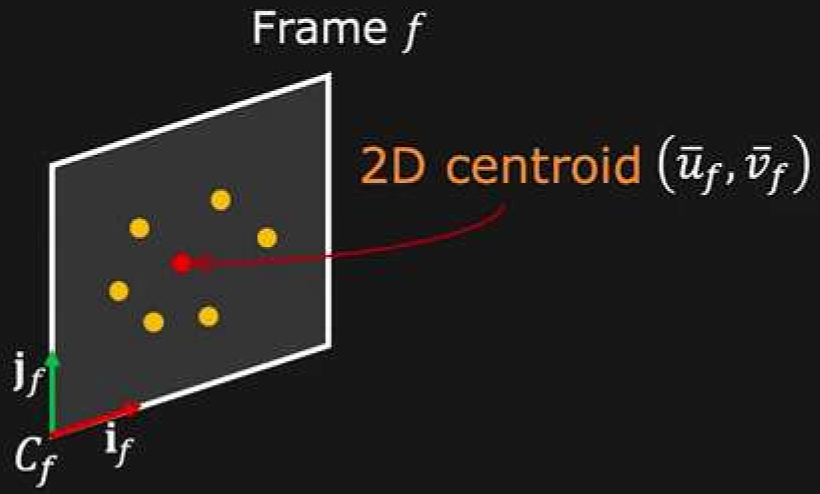
We will compute scene points w.r.t their centroid!

Centering Trick

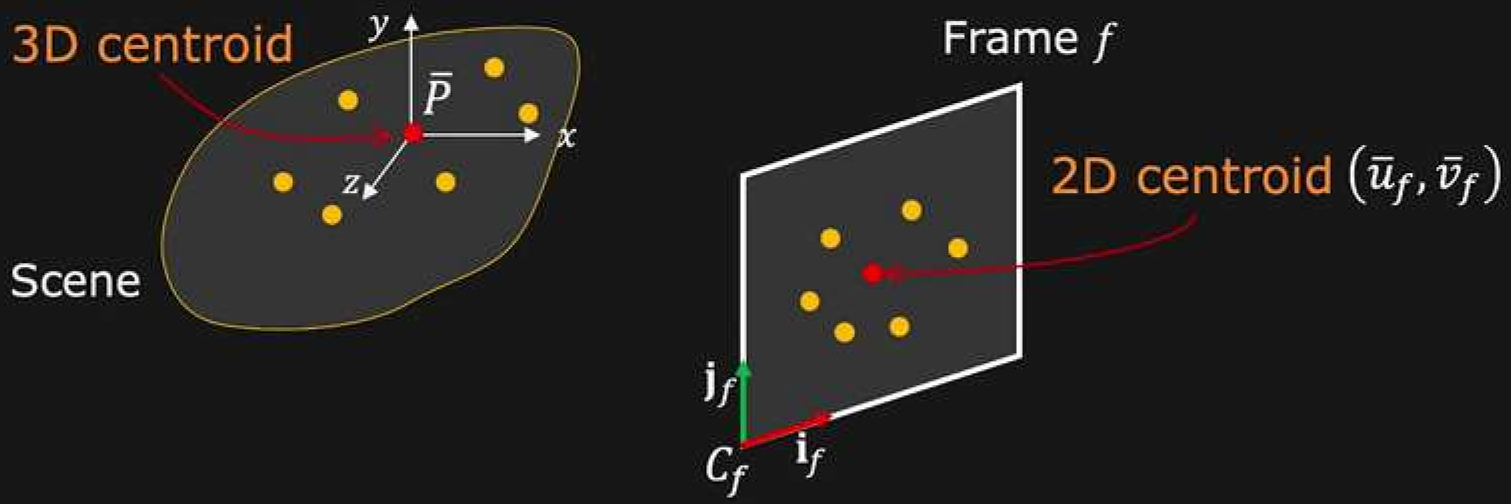




Centroid $(\bar{u}_f, \bar{v}_f)$ of the image points in frame f:

$$\bar{u}_f = \frac{1}{N}\sum_{p=1}^{N} u_{f,p}$$

Centering Trick



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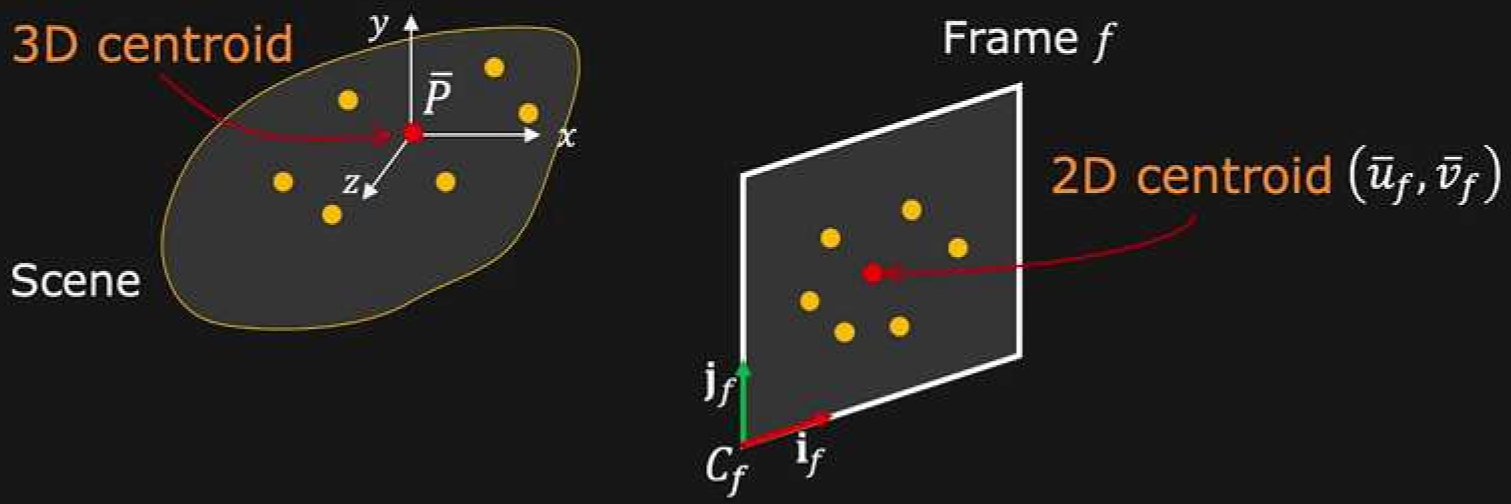

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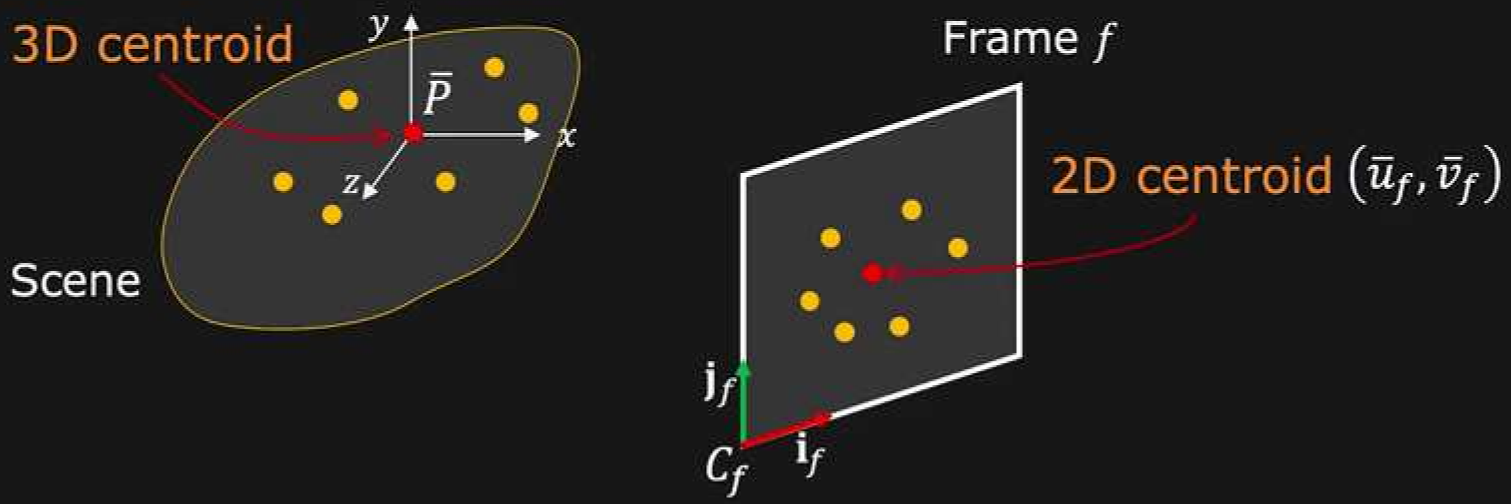

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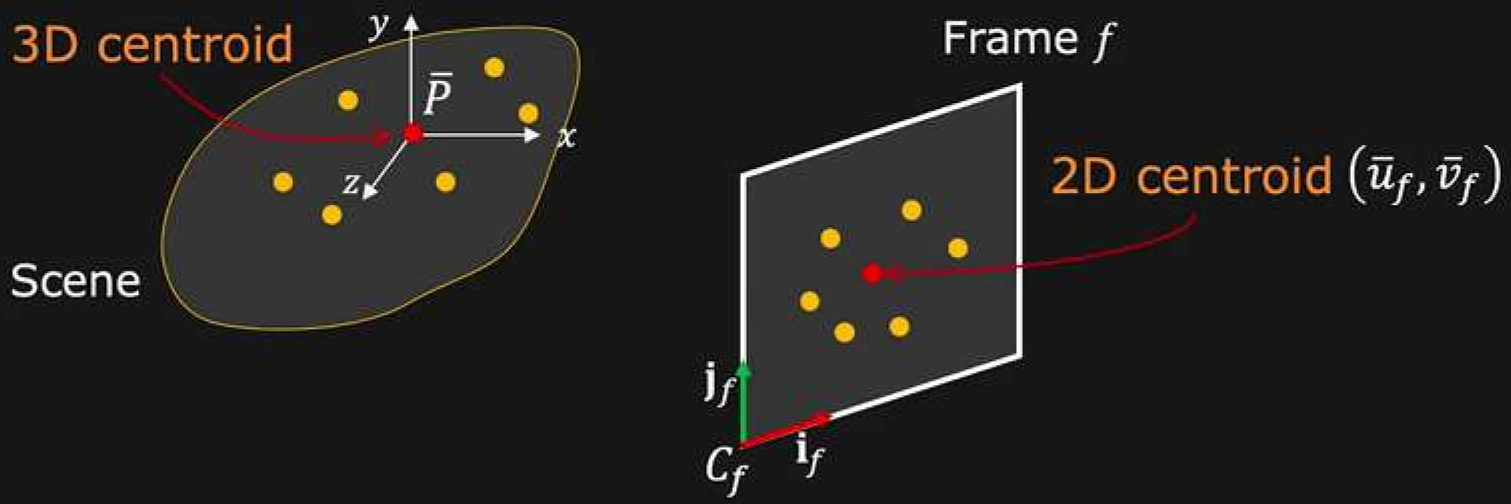

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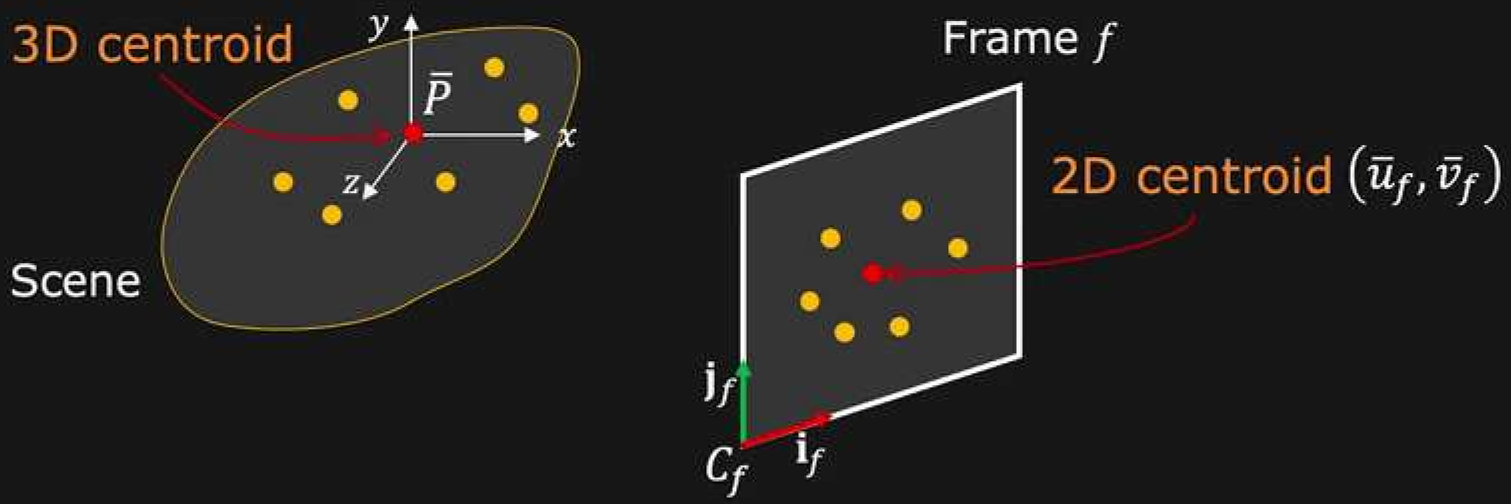

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$$\boxed{\bar{u}_f = -\mathbf{i}_f^T C_f}$$

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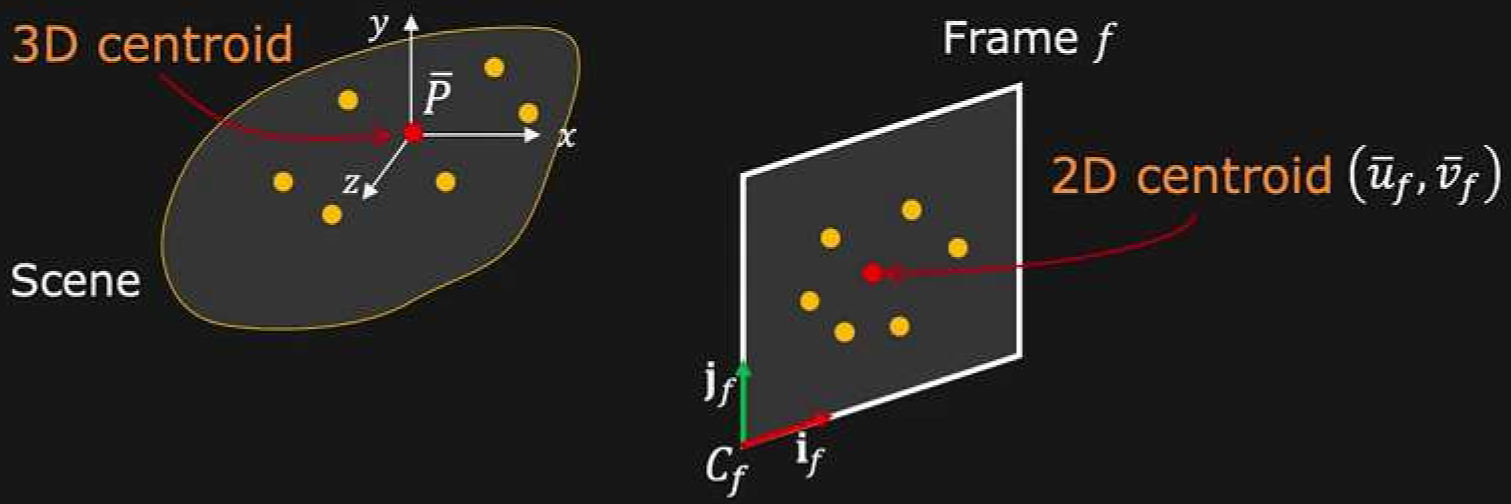

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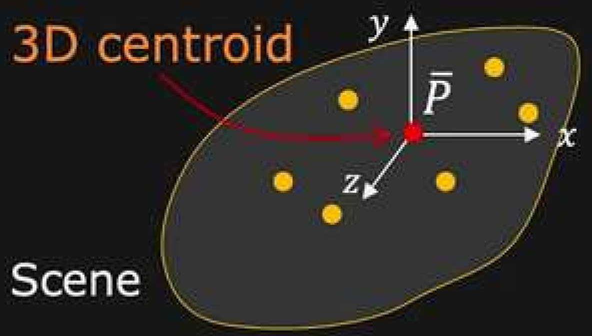

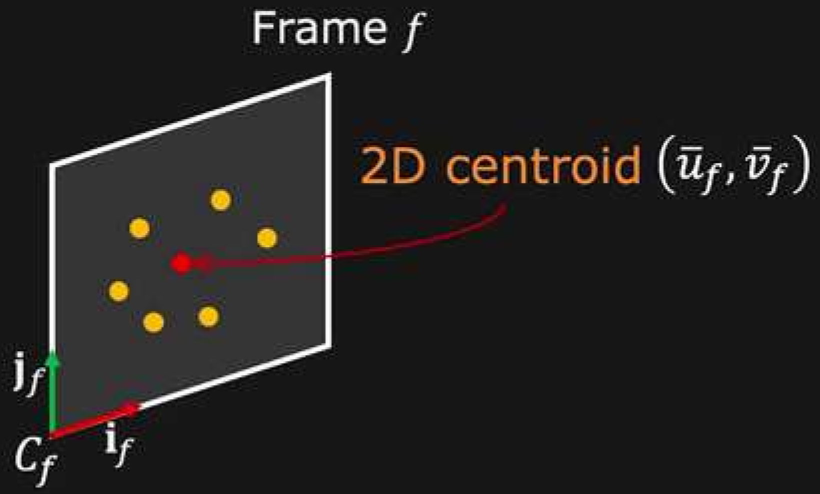

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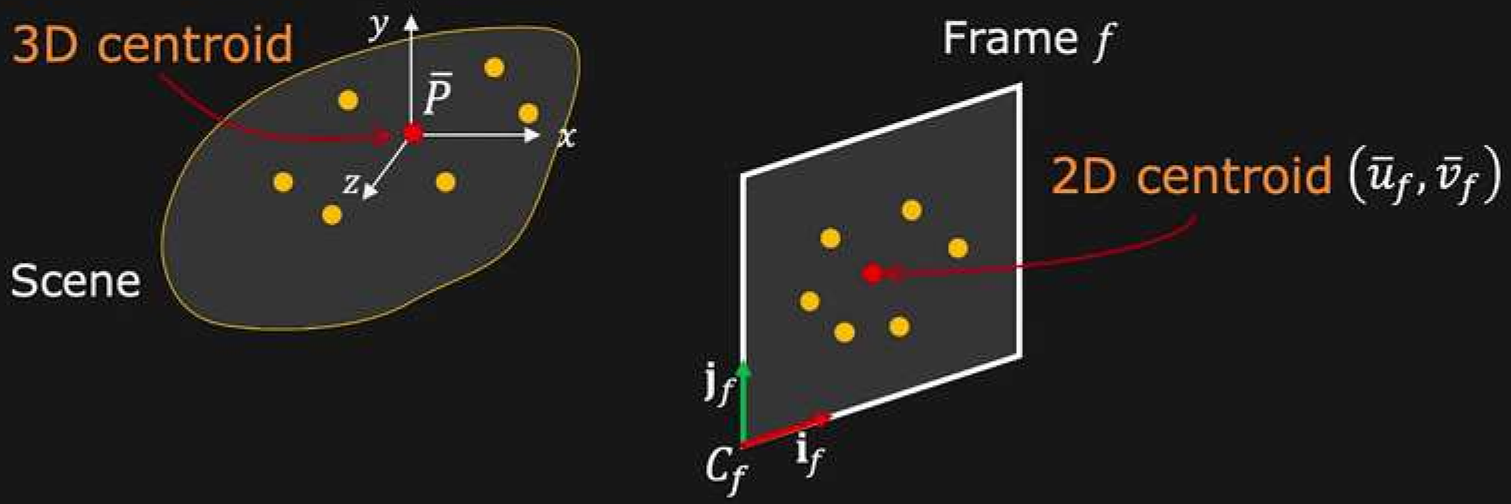

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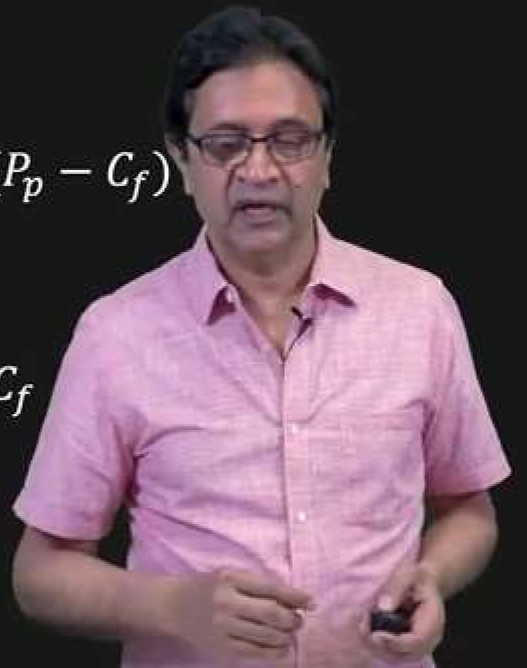
$$\bar{v}_f = \frac{1}{N}\sum_{p=1}^{N} v_{f,p} = \frac{1}{N}\sum_{p=1}^{N} \mathbf{j}_f^T (P_p - C_f)$$

$$\bar{u}_f = \frac{1}{N}\cancel{\mathbf{i}_f^T \sum_{p=1}^{N}} P_p - \frac{1}{N}\sum_{p=1}^{N} \mathbf{i}_f^T C_f$$

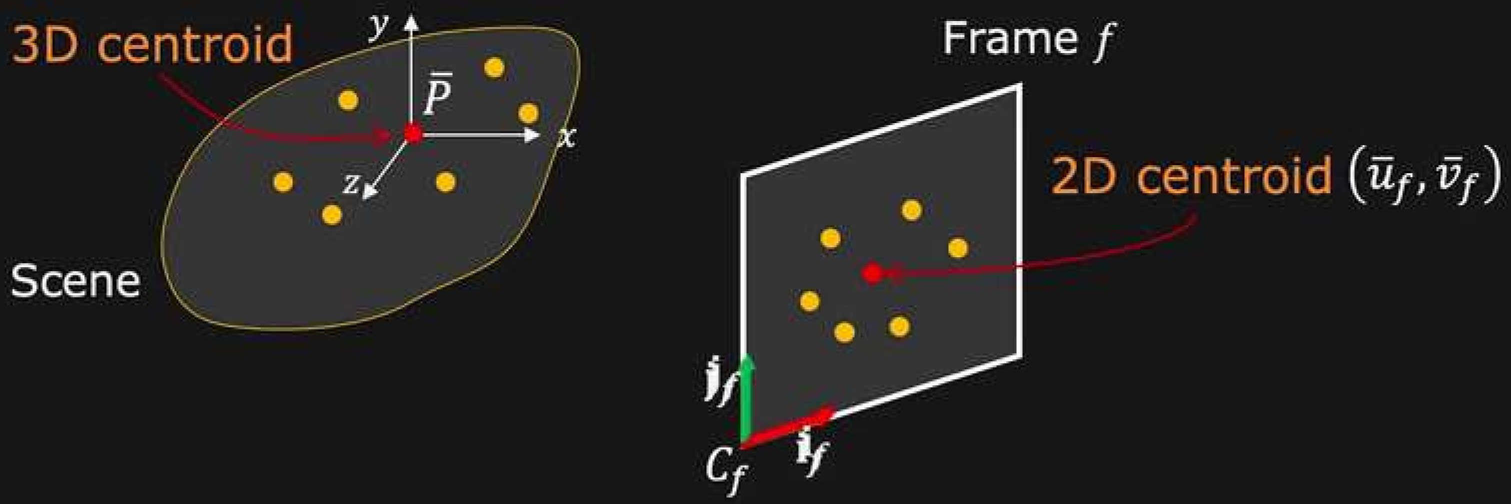
$$\bar{v}_f = \frac{1}{N}\cancel{\mathbf{j}_f^T \sum_{p=1}^{N}} P_p - \frac{1}{N}\sum_{p=1}^{N} \mathbf{j}_f^T C_f$$

$$\boxed{\bar{u}_f = -\mathbf{i}_f^T C_f}$$

$$\boxed{\bar{v}_f = -\mathbf{j}_f^T C_f}$$

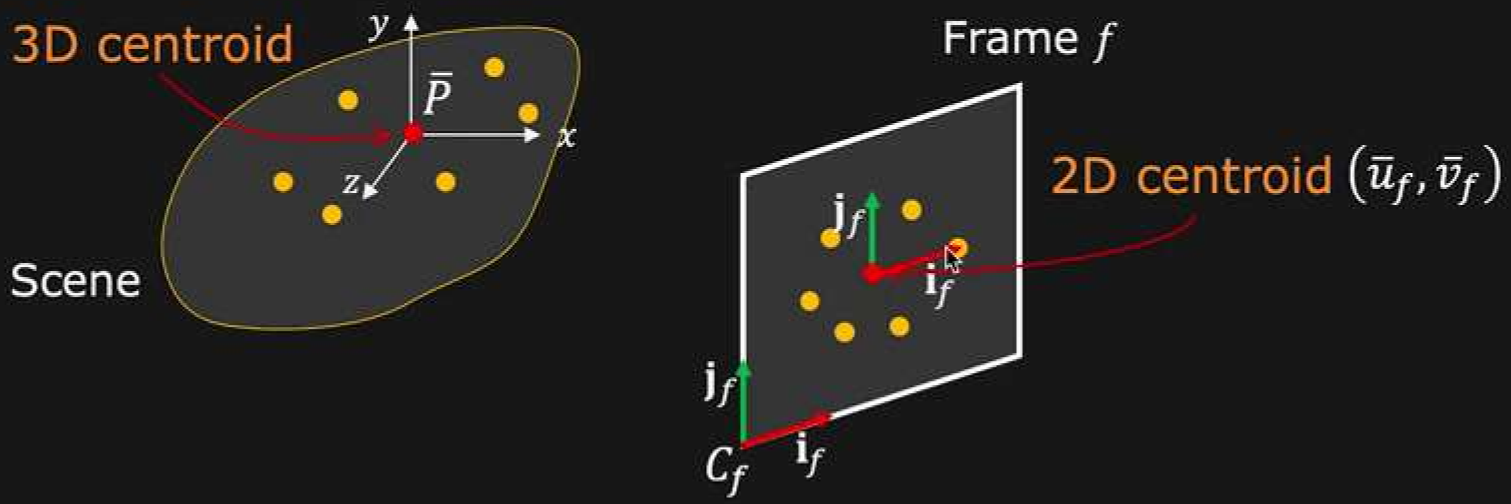


Centering Trick



Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

Centering Trick

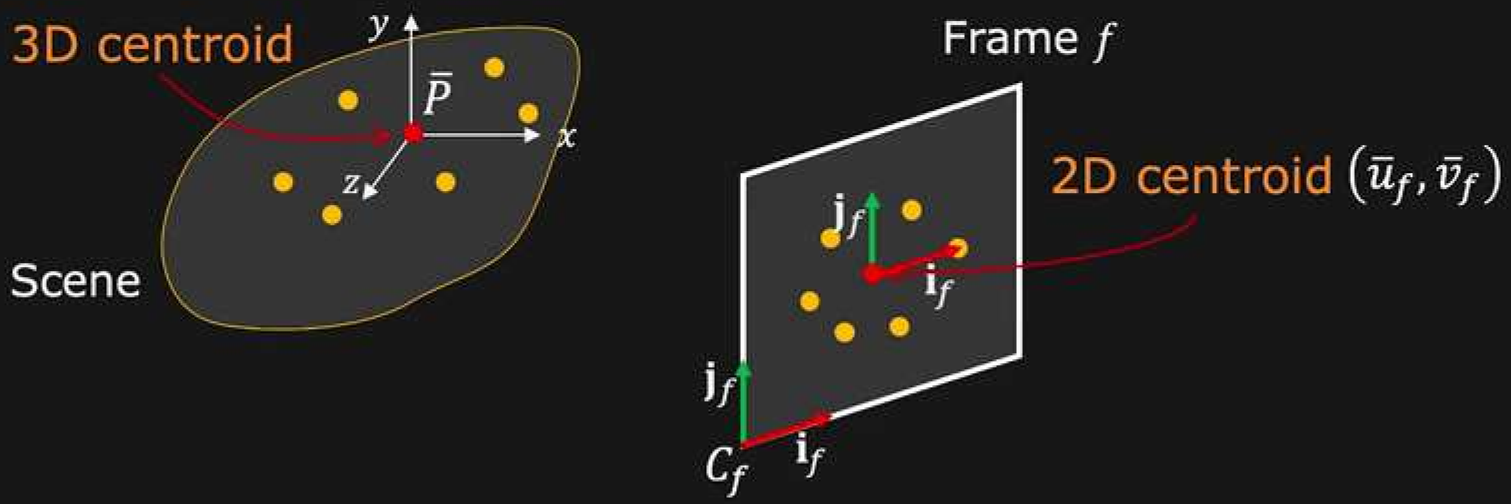


Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

Image points w.r.t. $(\bar{u}_f, \bar{v}_f)$:

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

Centering Trick



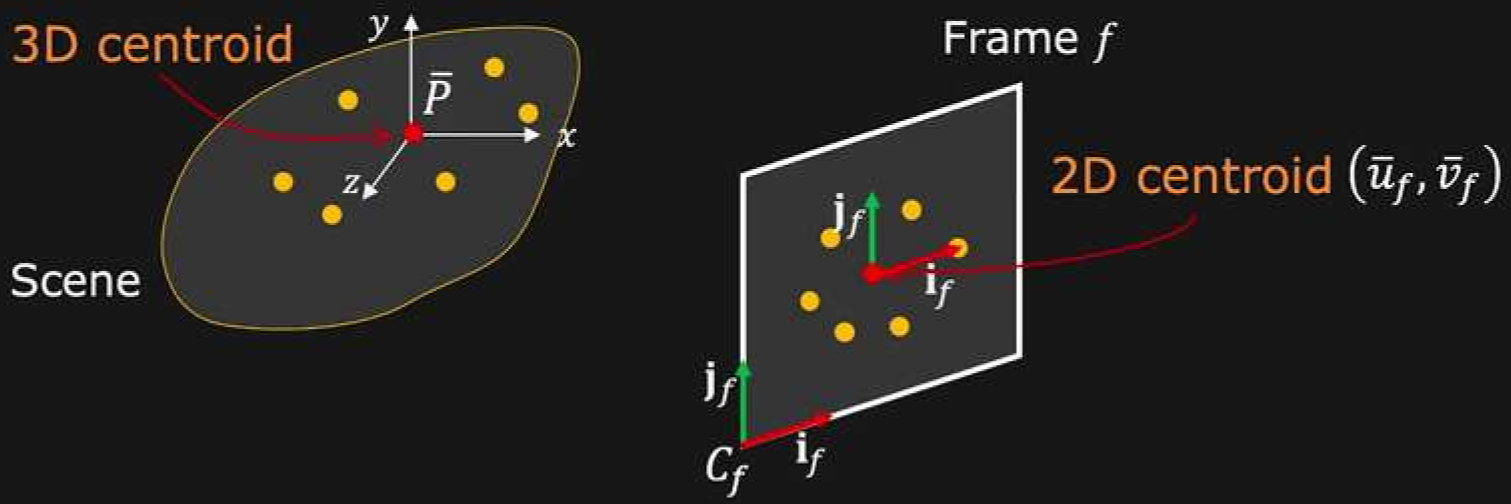
Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

Image points w.r.t. $(\bar{u}_f, \bar{v}_f)$:

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

$$= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f$$

Centering Trick



Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

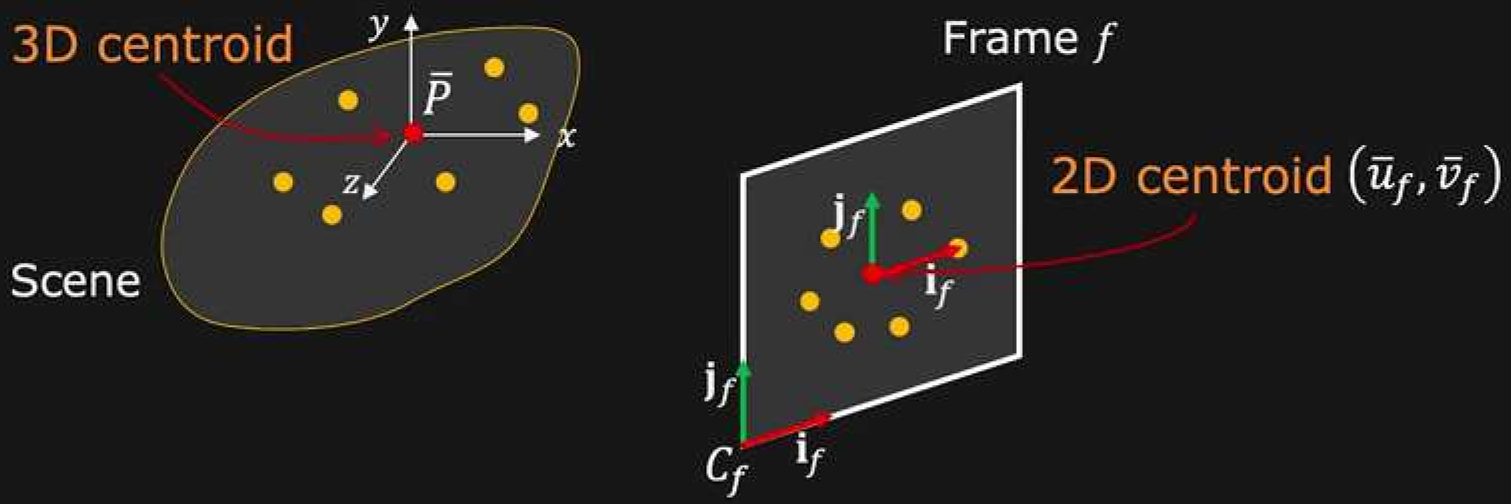
Image points w.r.t. $(\bar{u}_f, \bar{v}_f)$:

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

$$= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f$$

$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p}$$

Centering Trick



Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

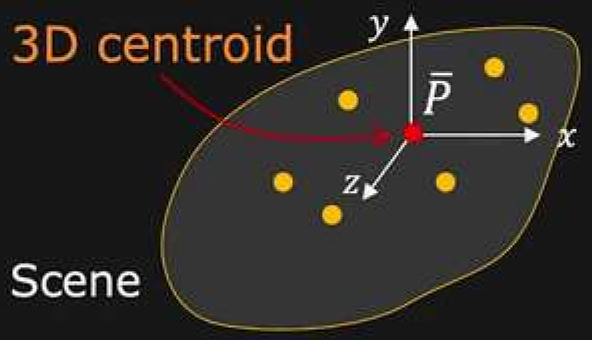
Image points w.r.t. $(\bar{u}_f, \bar{v}_f)$:

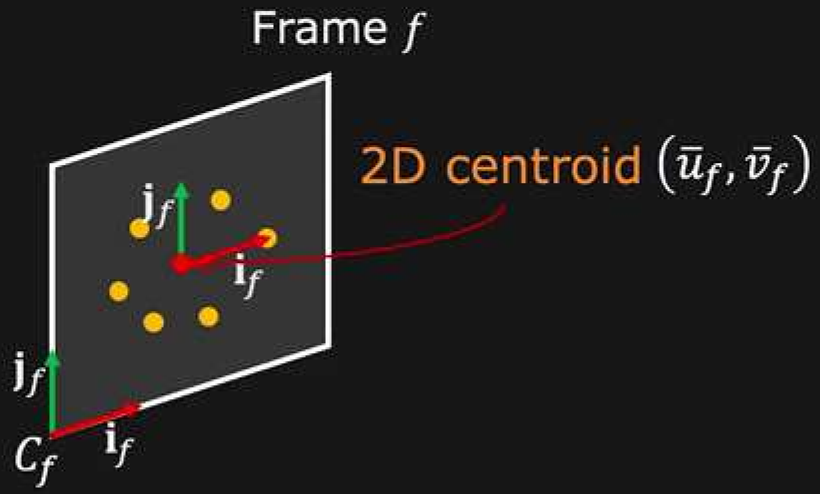
$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f \qquad \tilde{v}_{f,p} = v_{f,p} - \bar{v}_f$$

$$= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f \qquad = \mathbf{j}_f^T (P_p - C_f) - \mathbf{j}_f^T C_f$$

$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p} \qquad \boxed{\tilde{v}_{f,p} = \mathbf{j}_f^T P_p}$$

Centering Trick





Shift camera origin to the centroid $(\bar{u}_f, \bar{v}_f)$.

Image points w.r.t. $(\bar{u}_f, \bar{v}_f)$:

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

$$= \mathbf{i}_f^T(P_p - C_f) - \mathbf{i}_f^T C_f$$

$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p}$$

$$\tilde{v}_{f,p} = v_{f,p} - \bar{v}_f$$

$$= \mathbf{j}_f^T(P_p - C_f) - \mathbf{j}_f^T C_f$$

$$\boxed{\tilde{v}_{f,p} = \mathbf{j}_f^T P_p}$$

Camera locations C_f now removed from equations.

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p \qquad \tilde{v}_{f,p} = \mathbf{j}_f^T P_p \quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$
\begin{array}{rc}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hdashline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$W_{2F\times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{cc} & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\ \begin{array}{c} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} & \begin{bmatrix} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\ \tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\ \hdashline \tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\ \tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N} \end{bmatrix} \end{array} = \begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} \begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\ \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \end{array}$$

$W_{2F\times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{r c}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hdashline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}$$

$W_{2F\times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{r} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix} \end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \end{array}$$

$W_{2F \times N}$
Centroid-Subtracted Feature Points (Known)

$M_{2F \times 3}$
Camera Motion (Unknown)

$S_{3 \times N}$
Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$

$$\Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hdashline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix} \end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\ \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \end{array}$$

$W_{2F\times N}$
Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$
Camera Motion (Unknown)

$S_{3\times N}$
Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p \qquad \tilde{v}_{f,p} = \mathbf{j}_f^T P_p \quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{l} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\ \tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\ \tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\ \tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N} \end{bmatrix} \end{array}
=
\begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\ \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \end{array}$$

$W_{2F\times N}$
Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$
Camera Motion (Unknown)

$S_{3\times N}$
Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p \qquad \tilde{v}_{f,p} = \mathbf{j}_f^T P_p \quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c c}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{c} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}$$

$W_{2F \times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F \times 3}$

Camera Motion (Unknown)

$S_{3 \times N}$

Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p \qquad \tilde{v}_{f,p} = \mathbf{j}_f^T P_p \quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$
\begin{array}{rc}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hdashline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$W_{2F\times N}$
Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$
Camera Motion (Unknown)

$S_{3\times N}$
Scene Structure (Unknown)

Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p \qquad \tilde{v}_{f,p} = \mathbf{j}_f^T P_p \quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{l} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\hdashline
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix} \end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \hdashline \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c} \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\ \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix} \end{array}$$

$W_{2F \times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F \times 3}$

Camera Motion (Unknown)

$S_{3 \times N}$

Scene Structure (Unknown)

Observation Matrix W

$$
\begin{array}{l}
\\
\text{Image 1} \\
\text{Image 2} \\
\\
\text{Image F} \\
\text{Image 1} \\
\text{Image 2} \\
\\
\text{Image F}
\end{array}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$W_{2F \times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F \times 3}$

Camera Motion (Unknown)

$S_{3 \times N}$

Scene Structure (Unknown)

Can we find M and S from W?

Observation Matrix W

$$
\begin{array}{r c}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$W_{2F\times N}$

Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)

Can we find M and S from W?