Tomasi-Kanade Factorization

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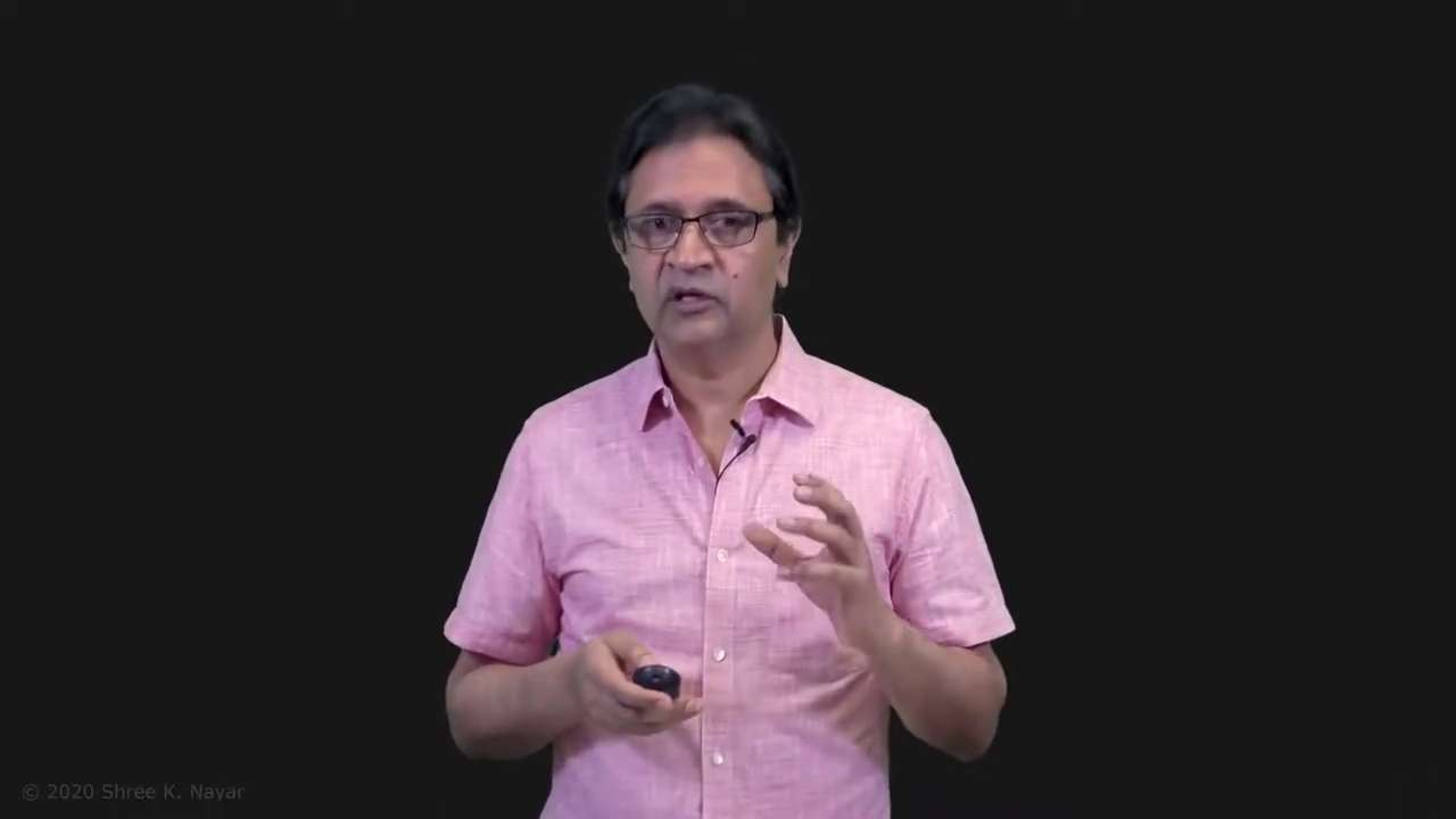
Topic: Structure from Motion, Module: Reconstruction II

First Principles of Computer Vision



Rank of Observation Matrix

$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

$$Rank(MS) \leq Rank(M) \qquad Rank(MS) \leq Rank(S)$$

$$\Rightarrow \quad Rank(MS) \leq \min(3,2F) \qquad Rank(MS) \leq \min(3,N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \leq \min(3,N,2F)$$

$$\boxed{\text{Rank Theorem:} \quad Rank(W) \leq 3}$$

[Tomasi 1992]

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We can "factorize" W into M and S!

[Tomasi 1992]

Singular Value Decomposition (SVD)

For any matrix A there exists a factorization:

$$A_{M \times N} = U_{M \times M} \cdot \Sigma_{M \times N} \cdot V^{T}{}_{N \times N}$$

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MATLAB: `[U,S,V] = svd(A)`

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$$\Sigma_{M\times N} = \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_4 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_N \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

$\sigma_1, \dots, \sigma_N$: Singular Values

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$\sigma_1, \dots, \sigma_N$: Singular Values

If $Rank(A) = r$ then A has r non-zero singular values.

Enforcing Rank Constraint

Using SVD: $$W = U \, \Sigma \, V^T$$

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Using SVD: $W = \; U \, \Sigma \, V^T$

$$
= \begin{bmatrix} \\ \\ & & U & & \\ \\ \\ \end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & \sigma_4 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & \sigma_N \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}
\begin{bmatrix} \\ \\ & & V^T & & \\ \\ \\ \end{bmatrix}
$$

$2F \times 2F$ $\quad$ $2F \times N$ $\quad$ $N \times N$

Enforcing Rank Constraint

Using SVD: $W = U \Sigma V^T$

$$= \begin{bmatrix} \\ \\ & U & \\ \\ \\ \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_4 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_N \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix} \begin{bmatrix} \\ \\ & V^T & \\ \\ \\ \end{bmatrix}$$

$2F \times 2F$ $\quad$ $2F \times N$ $\quad$ $N \times N$

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Using SVD: $W = U\,\Sigma\,V^T$

$$
= \underbrace{\begin{pmatrix} & & \\ & U & \\ & & \end{pmatrix}}_{2F\times 2F}
\underbrace{\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{pmatrix}}_{2F\times N}
\underbrace{\begin{pmatrix} & & \\ & V^T & \\ & & \end{pmatrix}}_{N\times N}
$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Enforcing Rank Constraint

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$$= \begin{pmatrix} & & \\ & U & \\ & & \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix} \begin{pmatrix} & & \\ & V^T & \\ & & \end{pmatrix}$$

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$$
= \begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}
\begin{bmatrix} & & \\ & V^T & \\ & & \end{bmatrix}
$$

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Enforcing Rank Constraint

Using SVD: $W = U \, \Sigma \, V^T$

$$= \underbrace{\begin{bmatrix} \underset{3}{U_1} & \underset{2F-3}{U_2} \end{bmatrix}}_{2F\times 2F} \underbrace{\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix}}_{2F\times N} \underbrace{\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}}_{N\times N} \begin{array}{l} 3 \\ N-3 \end{array}$$

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$$
= \underset{2F\times 2F}{\begin{pmatrix} \underset{3}{U_1} & \underset{2F-3}{U_2} \end{pmatrix}}
\underset{2F\times N}{\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{pmatrix}}
\underset{N\times N}{\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}}
\begin{matrix} 3 \\ N-3 \end{matrix}
$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Enforcing Rank Constraint

Using SVD: $$W = U\,\Sigma\,V^T$$

$$= \underset{2F\times 2F}{\begin{array}{cc} \left(\begin{array}{c|c} \quad U_1 \quad & \quad U_2 \quad \end{array}\right) \\ \;\;3 \qquad\quad 2F-3 \end{array}} \; \underset{2F\times N}{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}} \; \underset{N\times N}{\begin{pmatrix} V_1^T \\ \hline V_2^T \end{pmatrix}} \begin{array}{l} 3 \\ N-3 \end{array}$$

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Using SVD: $W = U \, \Sigma \, V^T$

$$
= \underset{2F\times 2F}{\begin{bmatrix} \underset{3}{U_1} & \underset{2F-3}{U_2} \end{bmatrix}}
\underset{2F\times N}{\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \cdots & 0 \\
0 & \sigma_2 & 0 & 0 & \cdots & 0 \\
0 & 0 & \sigma_3 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots
\end{bmatrix}}
\underset{N\times N}{\begin{array}{c} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \end{array}
\begin{array}{l} 3 \\ N-3 \end{array}}
$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Enforcing Rank Constraint

Using SVD: $W = U\,\Sigma\,V^T$

$$= \underset{2F\times 2F}{\begin{array}{cc} \left[\begin{array}{cc} U_1 & U_2 \end{array}\right] \\ \begin{array}{cc} 3 & 2F-3 \end{array} \end{array}} \underset{2F\times N}{\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix}} \underset{N\times N}{\begin{array}{cc} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} & \begin{array}{c} 3 \\ N-3 \end{array} \end{array}}$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

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Using SVD: $W = U \, \Sigma \, V^T$

$$= \underset{2F\times 2F}{\begin{pmatrix} \underset{3}{U_1} & \underset{2F-3}{U_2} \end{pmatrix}} \underset{2F\times N}{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \sigma_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \end{pmatrix}} \underset{N\times N}{\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}} \begin{array}{l} 3 \\ N-3 \end{array}$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

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Using SVD: $W = U \Sigma V^T$

$$= \underbrace{\begin{pmatrix} \underbrace{U_1}_{3} & \underbrace{U_2}_{2F-3} \end{pmatrix}}_{2F\times 2F} \underbrace{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}}_{2F\times N} \underbrace{\begin{array}{c} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \begin{array}{l} 3 \\ N-3 \end{array} \end{array}}_{N\times N}$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Submatrices U_2 and V_2^T do not contribute to W.

Enforcing Rank Constraint

Using SVD: $W = U \, \Sigma \, V^T$

$$
= \underbrace{\begin{bmatrix} \underbrace{U_1}_{3} & \underbrace{U_2}_{2F-3} \end{bmatrix}}_{2F\times 2F}
\underbrace{\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}}_{2F\times N}
\underbrace{\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}}_{N\times N}
\begin{array}{l} \}\, 3 \\ \}\, N-3 \end{array}
$$

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$$
= \underbrace{\begin{pmatrix} \underbrace{U_1}_{3} & \underbrace{U_2}_{2F-3} \end{pmatrix}}_{2F \times 2F}
\underbrace{\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{pmatrix}}_{2F \times N}
\underbrace{\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}}_{N \times N}
\begin{matrix} 3 \\ N-3 \end{matrix}
$$

$$
\boxed{W = U_1 \;\; \Sigma_1 \;\; V_1^T}
$$

$$
(2F \times 3)(3 \times 3)(3 \times P)
$$

Enforcing Rank Constraint

Using SVD: $W = U \, \Sigma \, V^T$

$$= \begin{array}{c} \begin{bmatrix} U_1 & U_2 \end{bmatrix} \\ \begin{matrix} 3 & \quad 2F-3 \end{matrix} \\ 2F\times 2F \end{array} \begin{array}{c} \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix} \\ 2F\times N \end{array} \begin{array}{c} \begin{array}{cc} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} & \begin{matrix} 3 \\ N-3 \end{matrix} \end{array} \\ N\times N \end{array}$$

$$\boxed{W = U_1 \;\; \Sigma_1 \;\; V_1^T}$$

$(2F\times 3)\,(3\times 3)\,(3\times P)$

Factorization (Finding M, S)

$$W = U_1 \, (\Sigma_1)^{1/2} \;\; (\Sigma_1)^{1/2} V_1^T$$

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$$W = U_1 \, (\Sigma_1)^{1/2} \; (\Sigma_1)^{1/2} V_1^T$$

Factorization (Finding M, S)

$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

Factorization (Finding M, S)

$$W = \underbrace{U_1 \, (\Sigma_1)^{1/2}}_{(2F\times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

$$= M? \qquad = S?$$

Factorization (Finding M, S)

$$W = \underbrace{U_1 \, (\Sigma_1)^{1/2}}_{(2F \times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3 \times N)}$$

$= M?$ $\qquad$ $= S?$

Factorization (Finding M, S)

$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

$$= M? \qquad = S?$$

Not so fast. Decomposition not unique!

For any 3x3 non-singular matrix Q:

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Scene Structure

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Summary: Orthographic SFM

1. Detect and track feature points.

[Tomasi 1992]

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Results

Input Image Sequence

Results

Input Image Sequence

Estimated 3D Points

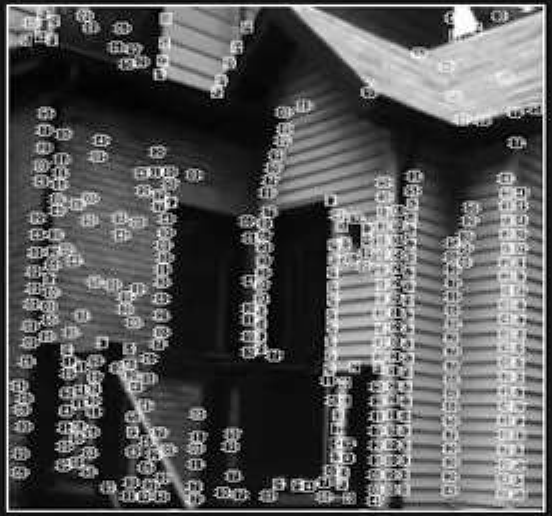
Results

Input Image Sequence

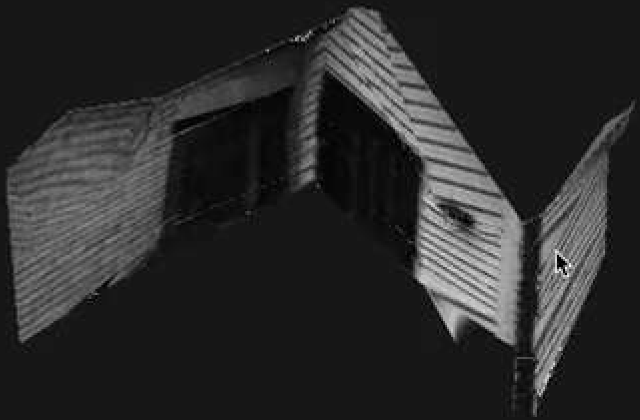
Tracked Features

Results

Input Image Sequence



Tracked Features



3D Reconstruction

3D Reconstruction

Results

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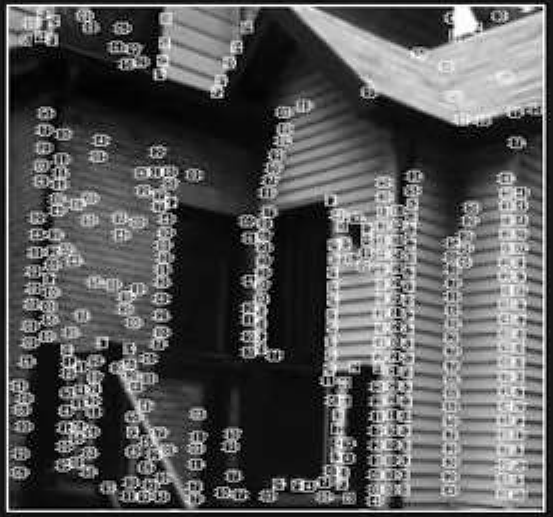
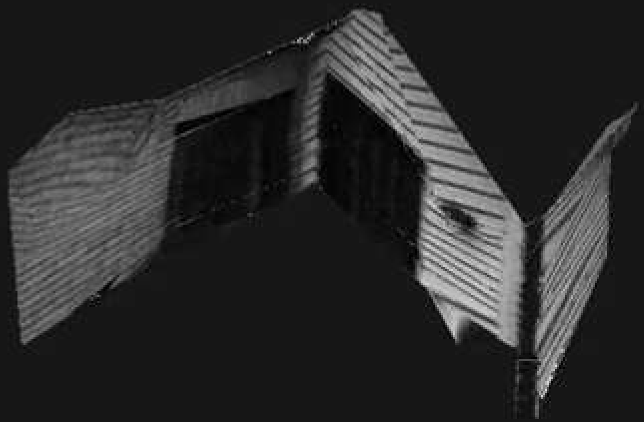


Tracked Features



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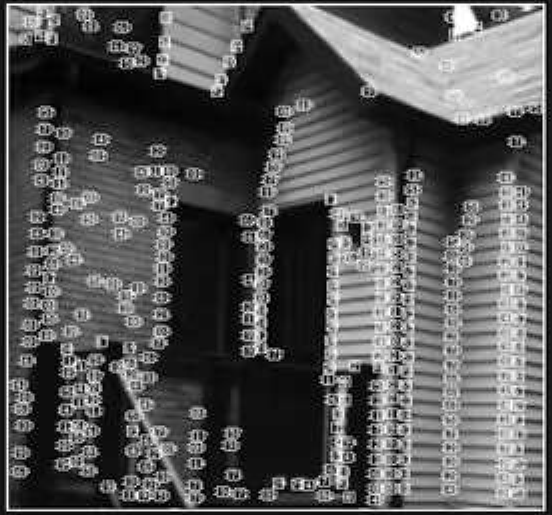

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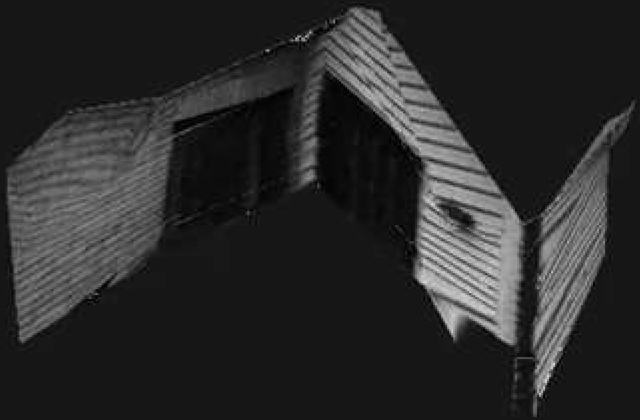

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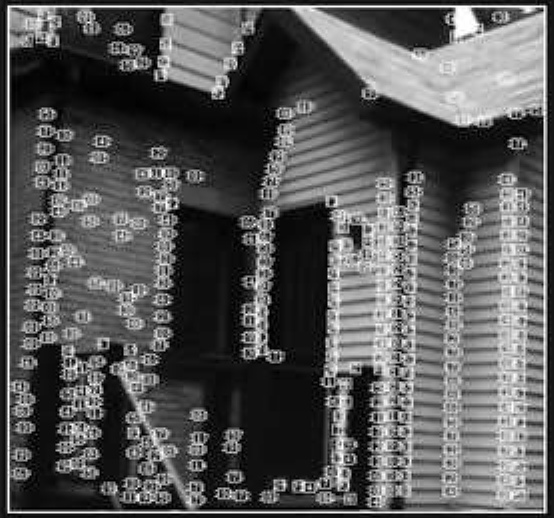

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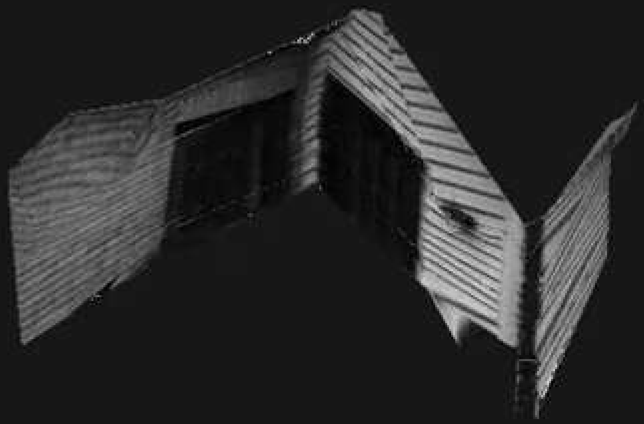

3D Reconstruction

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Structure From Motion: Result

[Pollefeys 2002]

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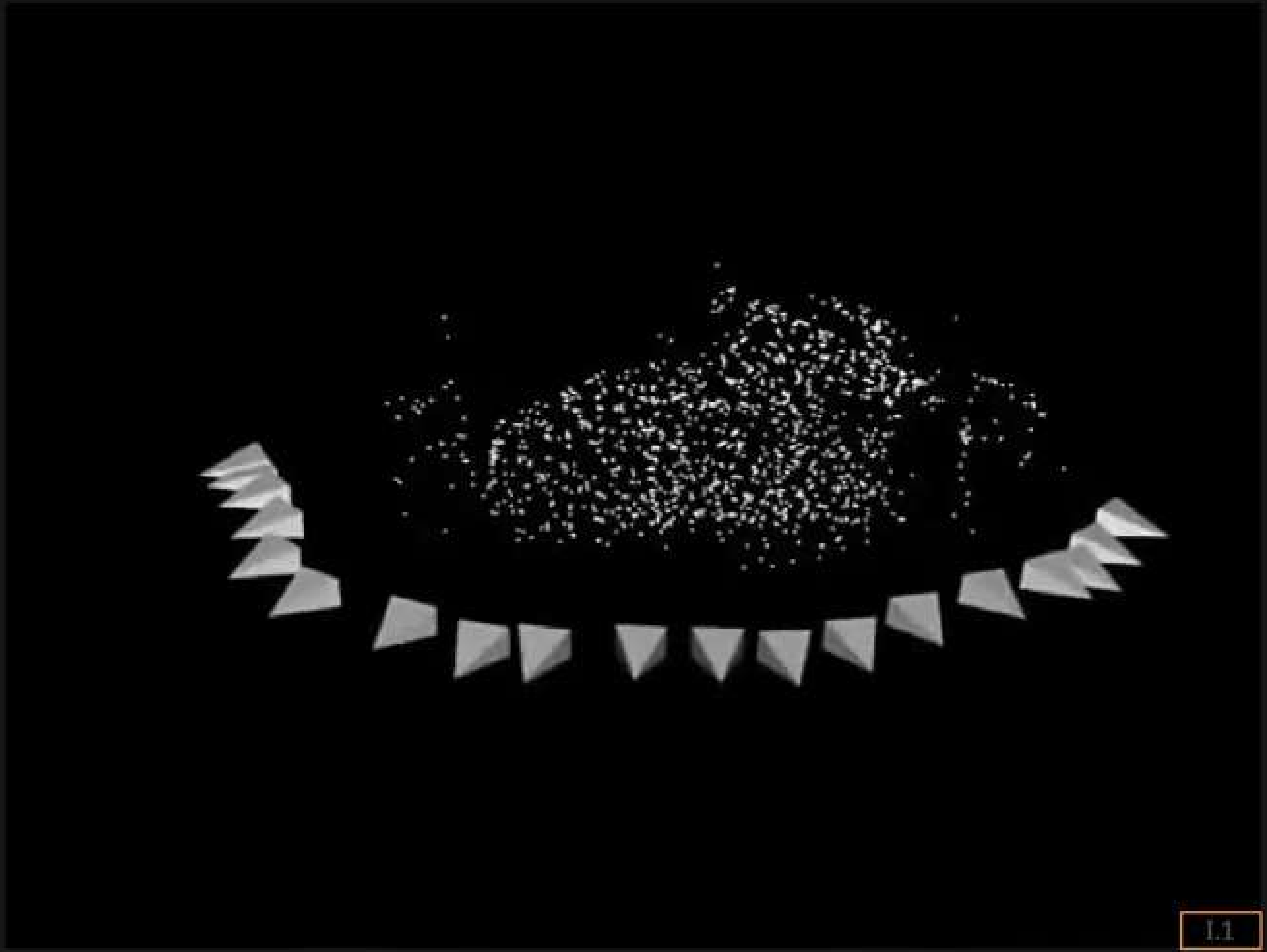

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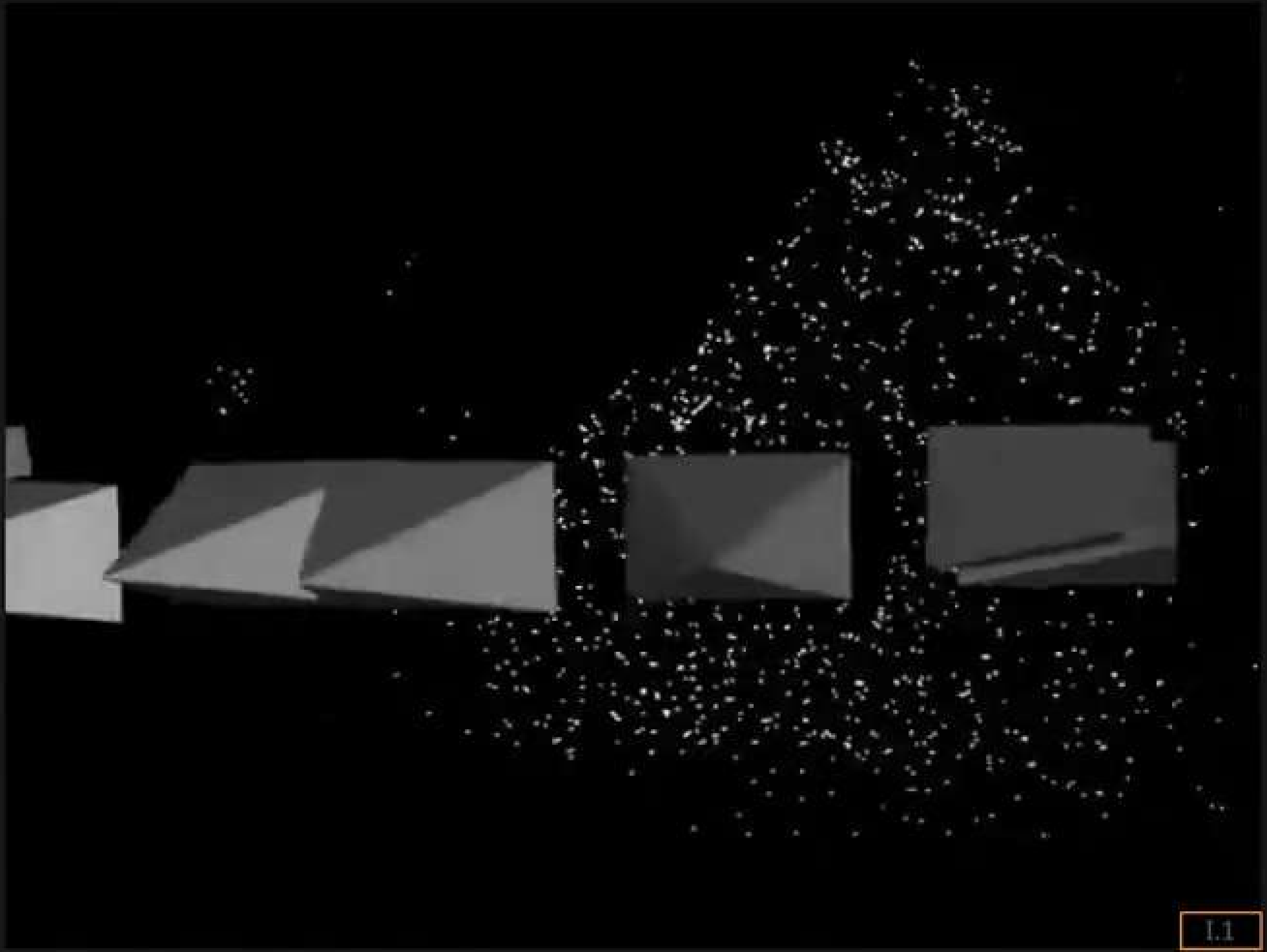

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References: Papers

[Tomasi 1992] Tomasi, C. and Kanade, T. "Shape and Motion from Image Streams under Orthography: A Factorization Method" IJCV, 1992.

[Pollefeys 2002] M. Pollefeys and L. Van Gool. Visual modeling: from images to images, The Journal of Visualization and Computer Animation, 13: 199-209, 2002.