Linear Camera Model

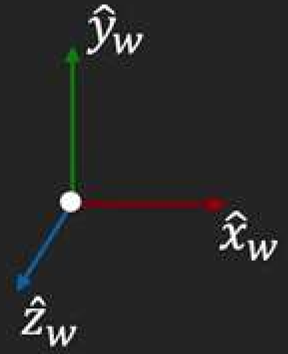
Shree K. Nayar

Columbia University

Topic: Camera Calibration, Module: Reconstruction II

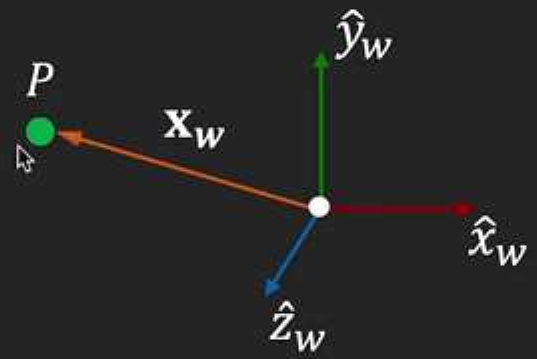
First Principles of Computer Vision

Forward Imaging Model: 3D to 2D



World Coordinate Frame $\mathcal{W}$

Forward Imaging Model: 3D to 2D



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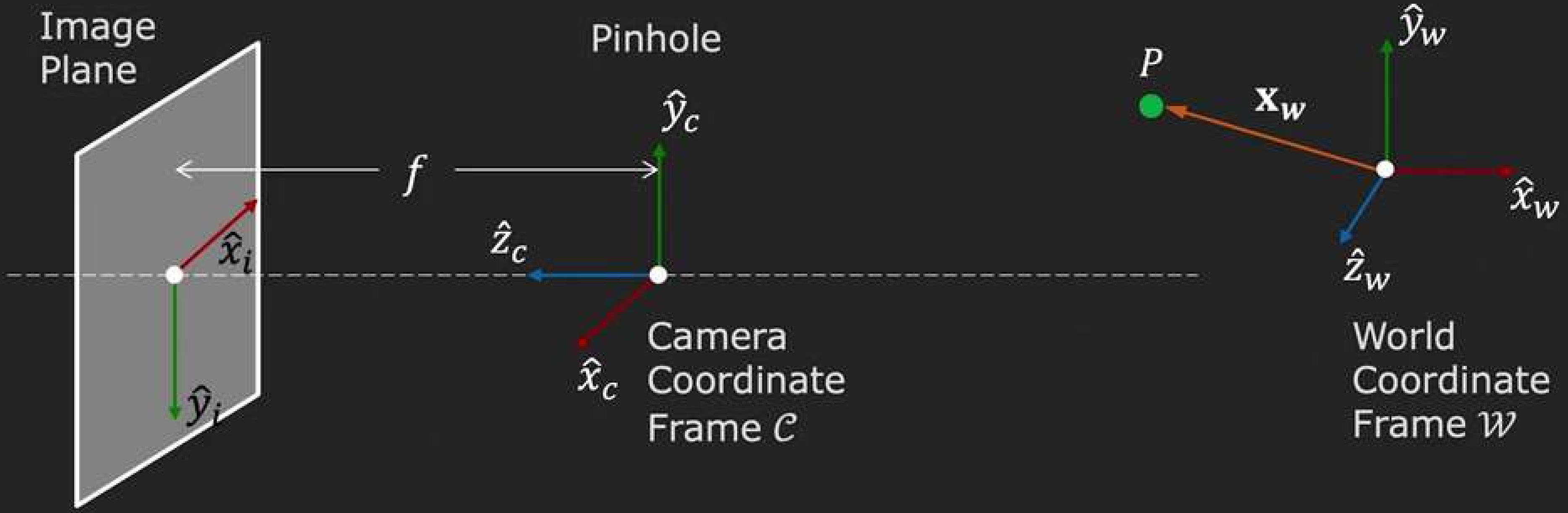
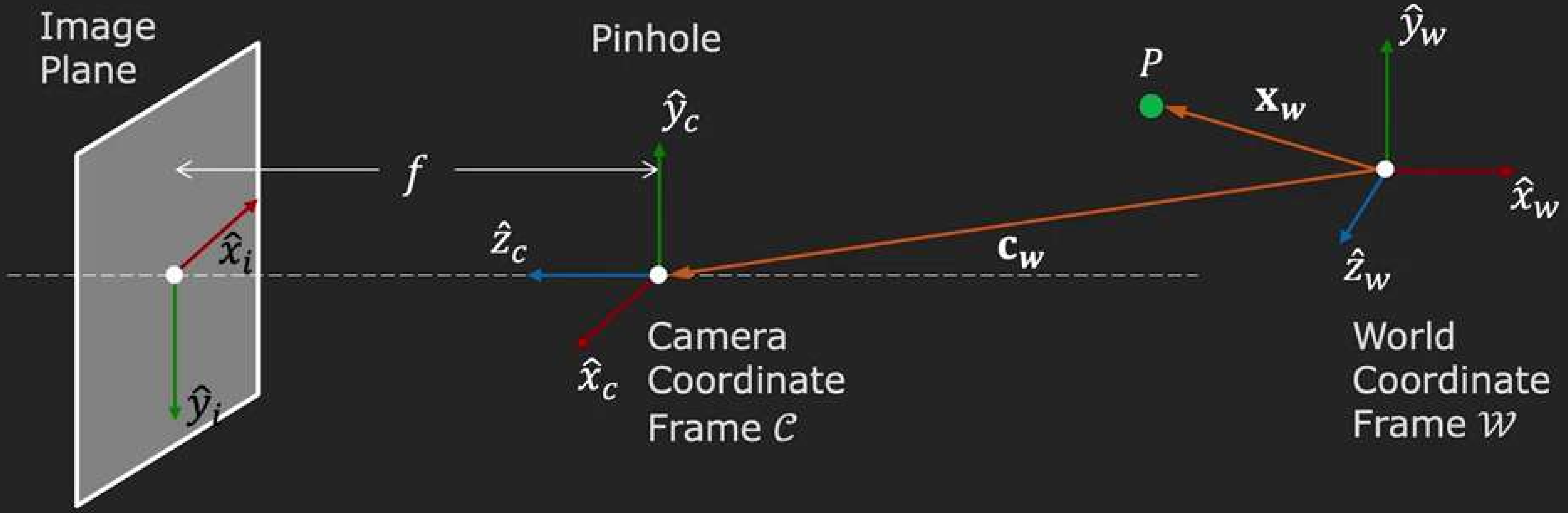


Forward Imaging Model: 3D to 2D



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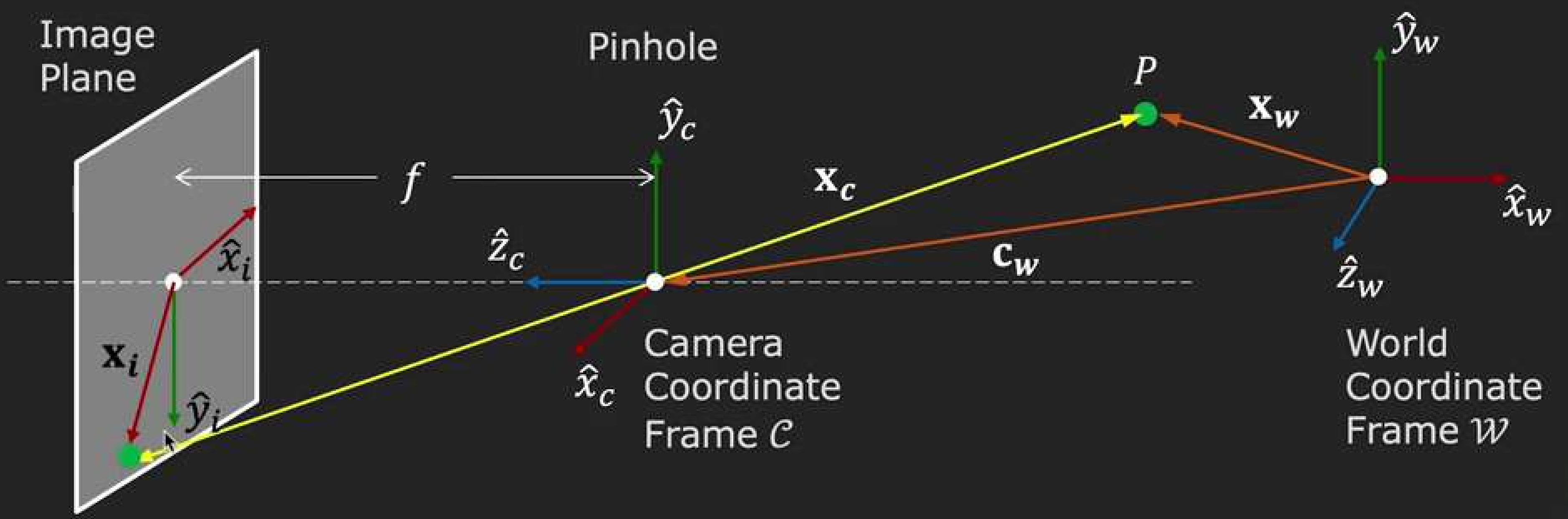

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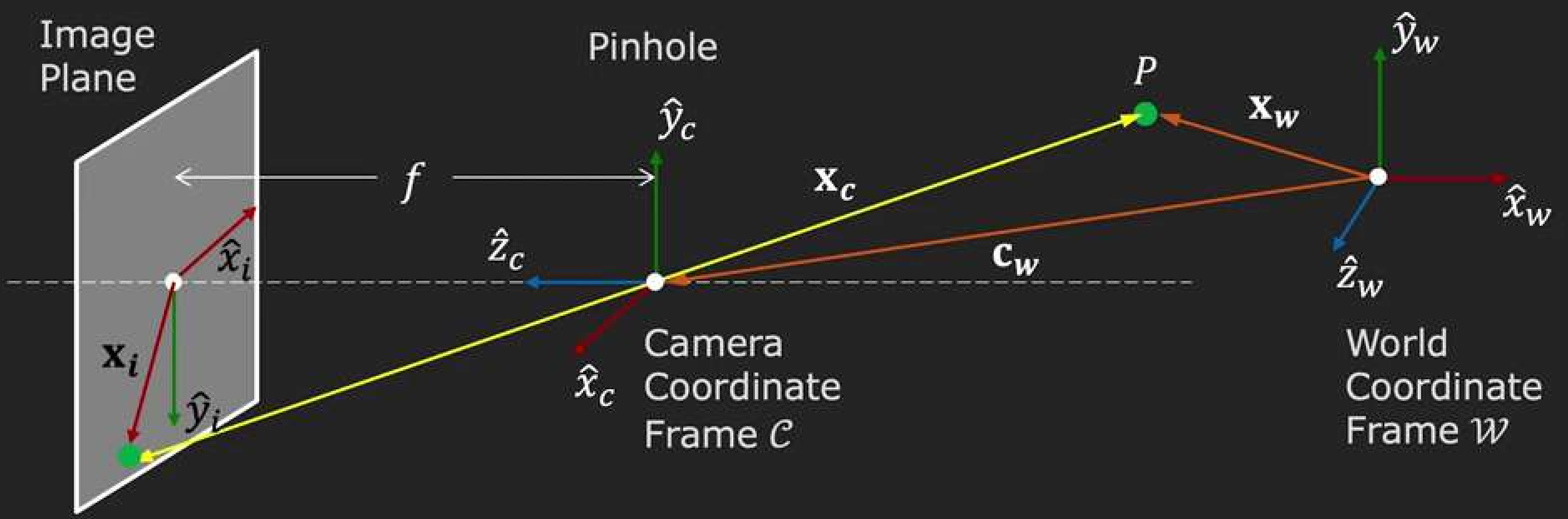
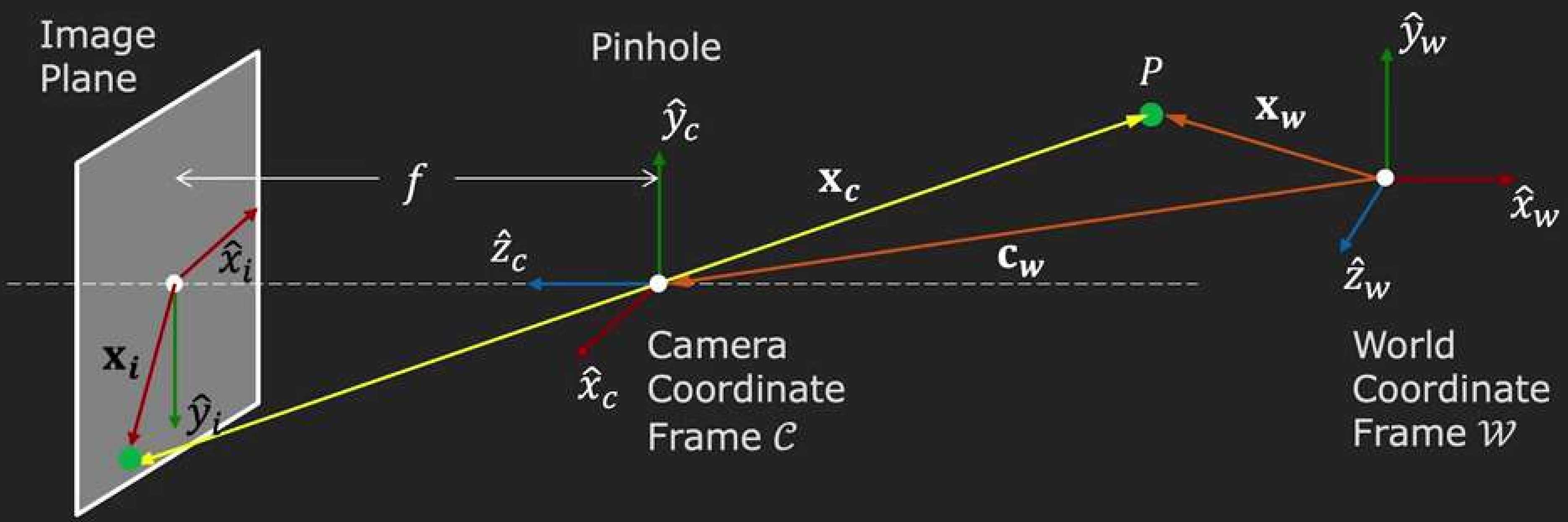


World
Coordinates

$$\mathbf{x}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Forward Imaging Model: 3D to 2D



Camera Coordinates

$$\mathbf{x}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

World Coordinates

$$\mathbf{x}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Coordinate Transformation

Forward Imaging Model: 3D to 2D

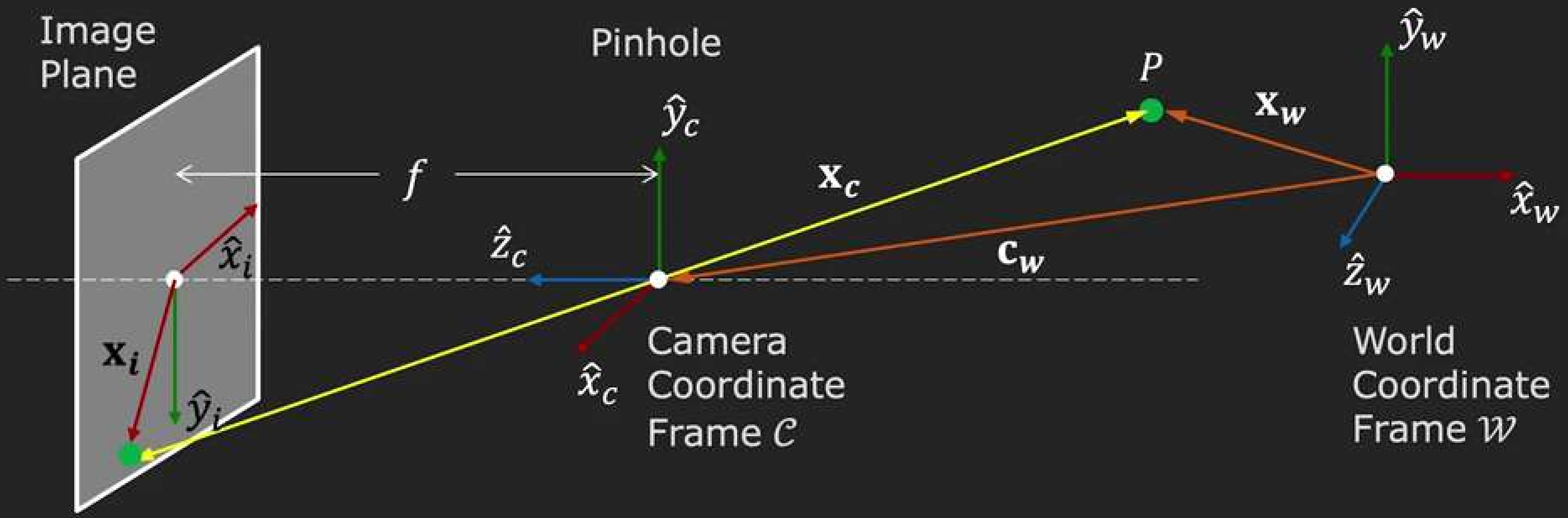


Image Coordinates		Camera Coordinates		World Coordinates
$\mathbf{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$	⟵	$\mathbf{x}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$	⟵	$\mathbf{x}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$
	Perspective Projection		Coordinate Transformation	

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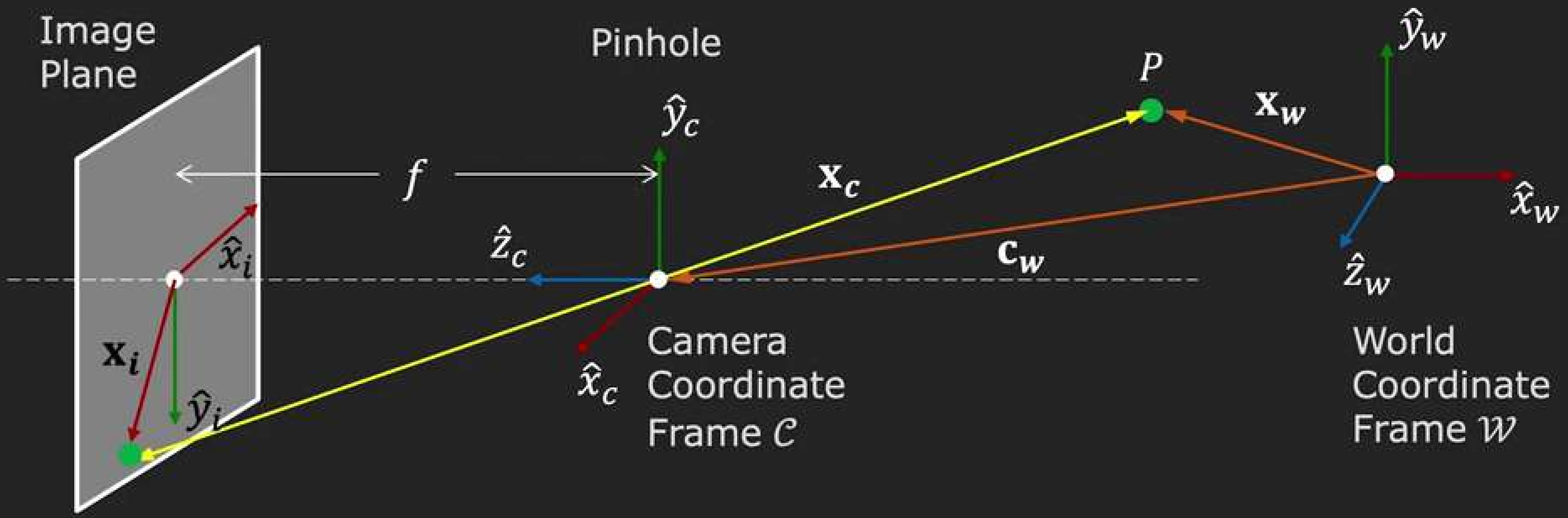

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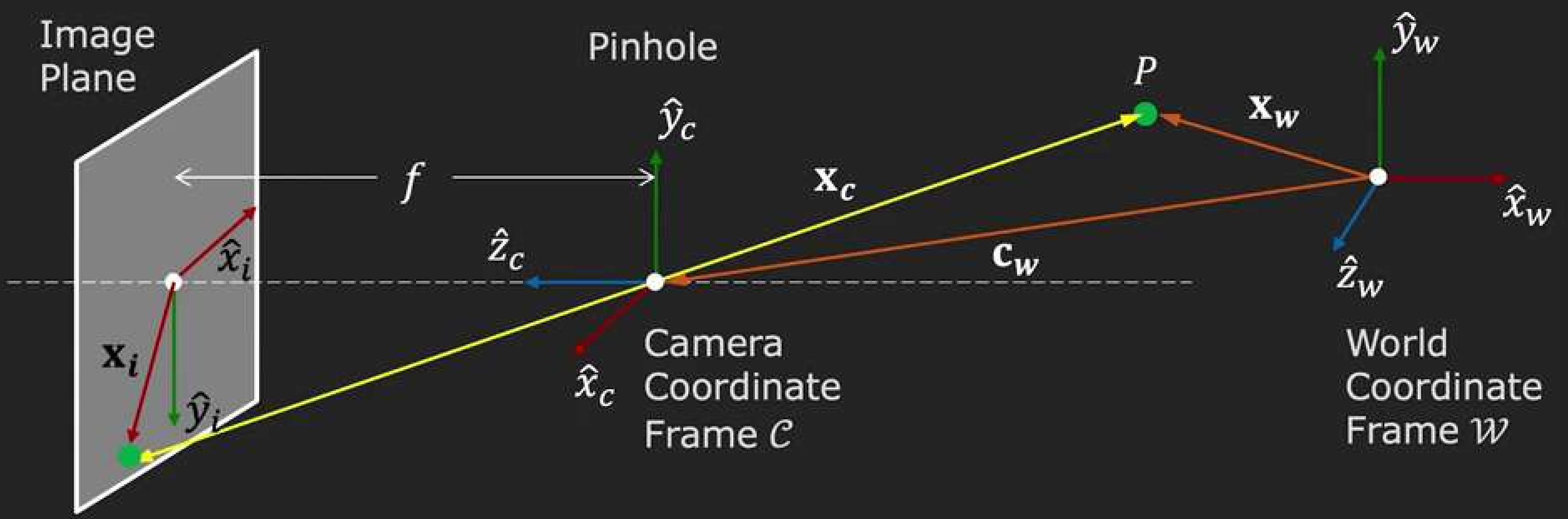
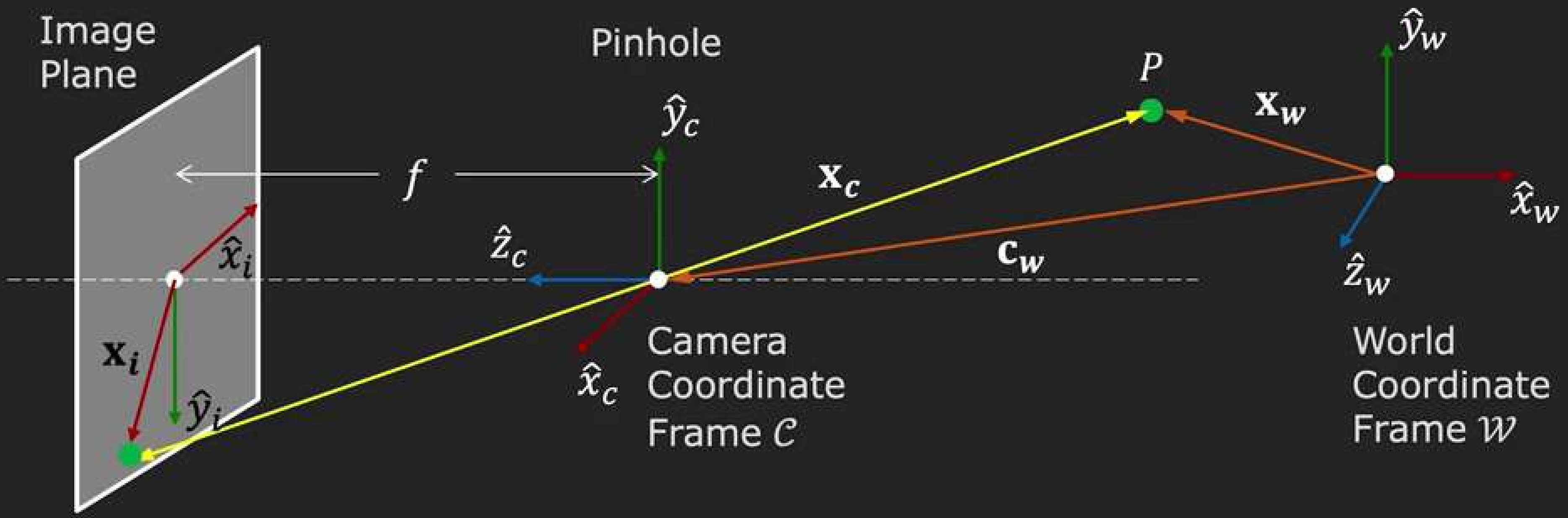
	Perspective Projection		Coordinate Transformation	

Perspective Projection



We know that: $\dfrac{x_i}{f} = \dfrac{x_c}{z_c}$ and $\dfrac{y_i}{f} = \dfrac{y_c}{z_c}$

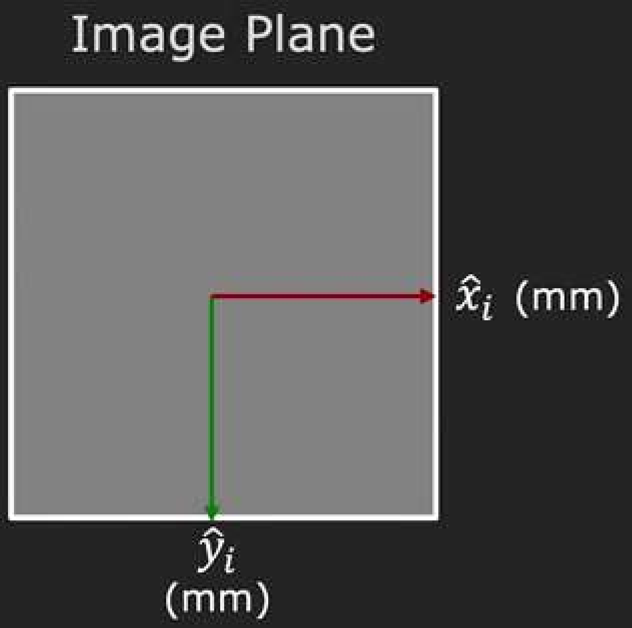
Perspective Projection



We know that: $\dfrac{x_i}{f} = \dfrac{x_c}{z_c}$ and $\dfrac{y_i}{f} = \dfrac{y_c}{z_c}$

Therefore: $x_i = f\dfrac{x_c}{z_c}$ and $y_i = f\dfrac{y_c}{z_c}$

Image Plane to Image Sensor Mapping



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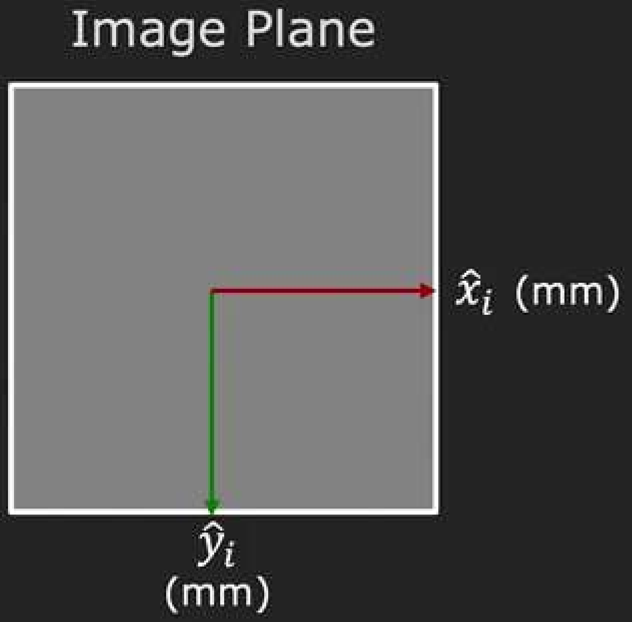

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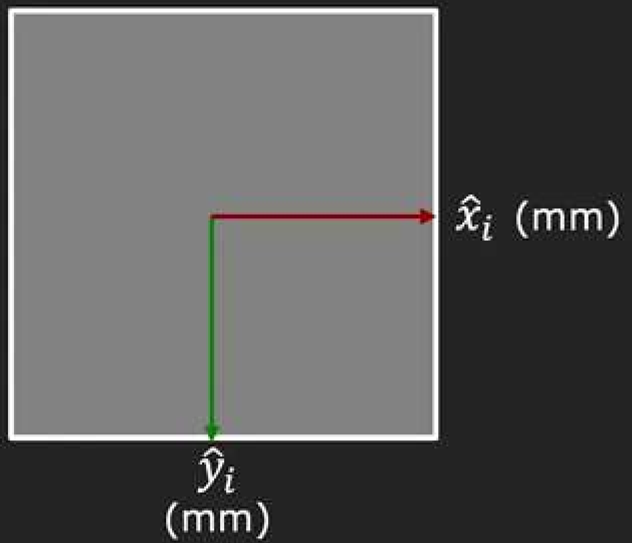
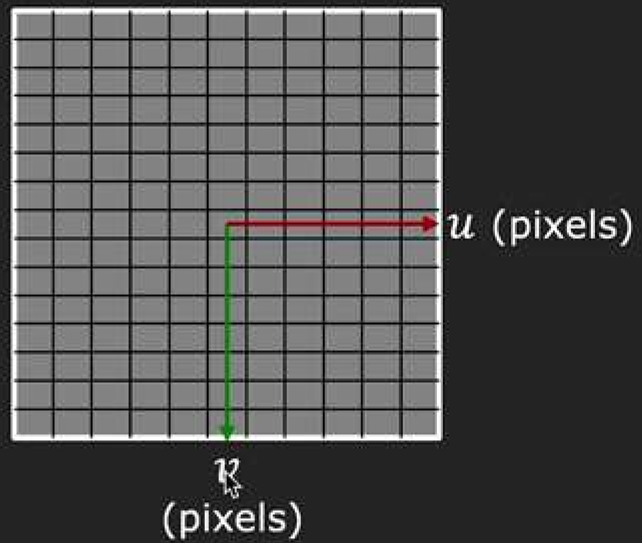


Image Sensor



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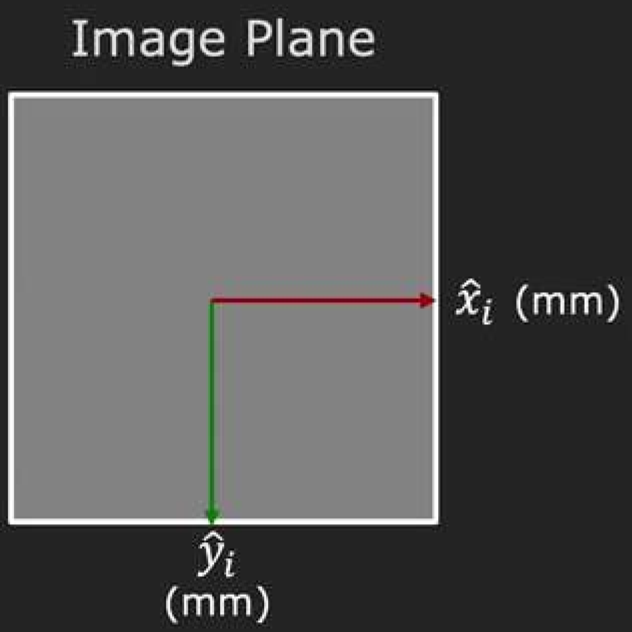

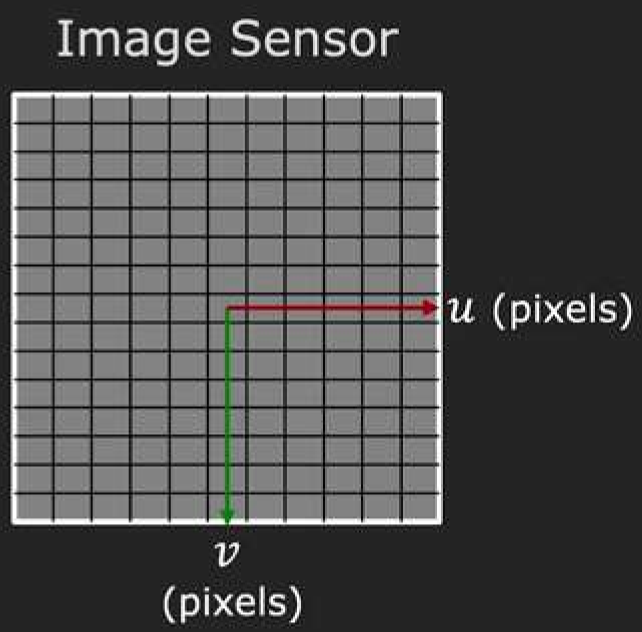

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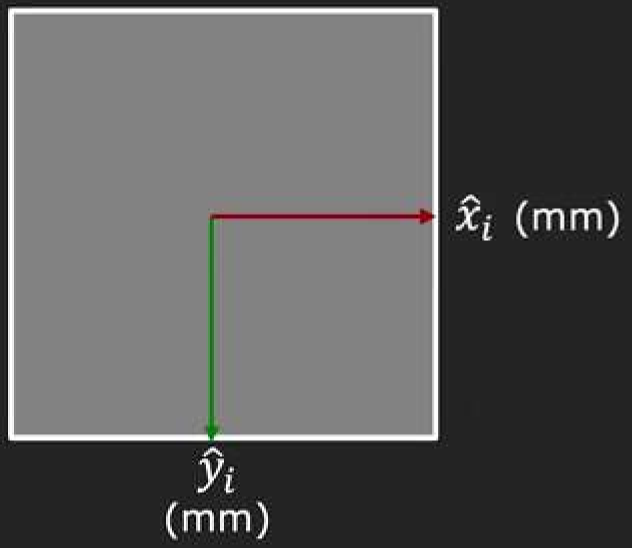

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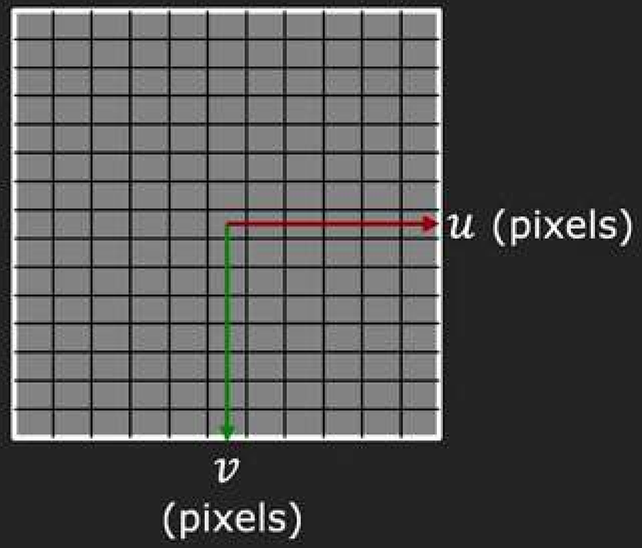

Pixels may be rectangular.

If m_x and m_y are the pixel densities (pixels/mm) in x and y directions, respectively, then pixel coordinates are:

$$u = m_x x_i = m_x f \frac{x_c}{z_c} \qquad v = m_y y_i = m_y f \frac{y_c}{z_c}$$

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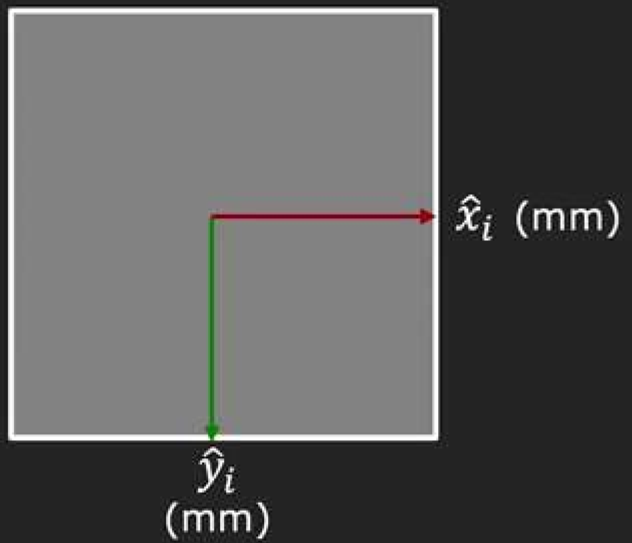

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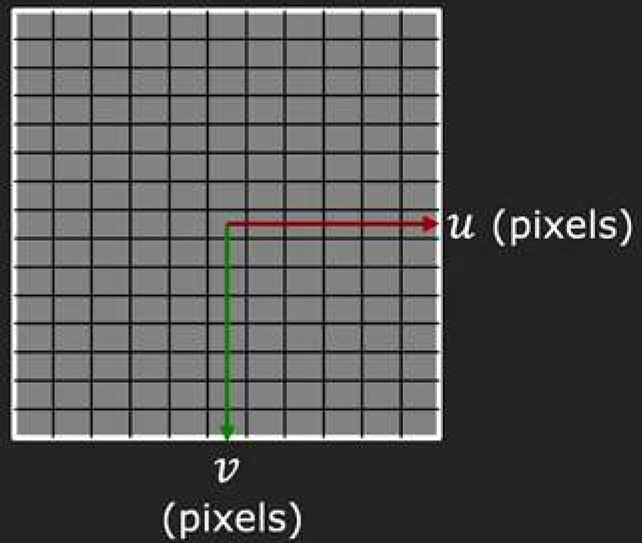

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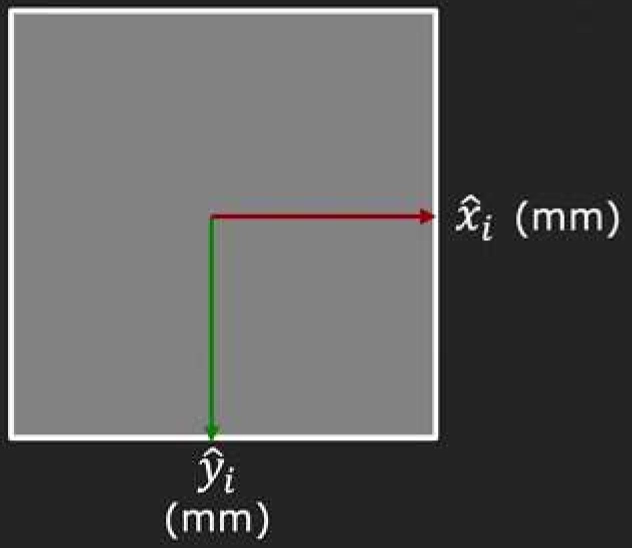

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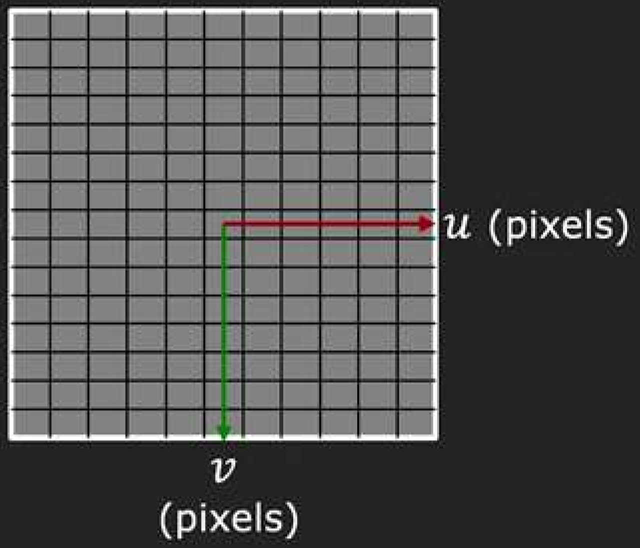

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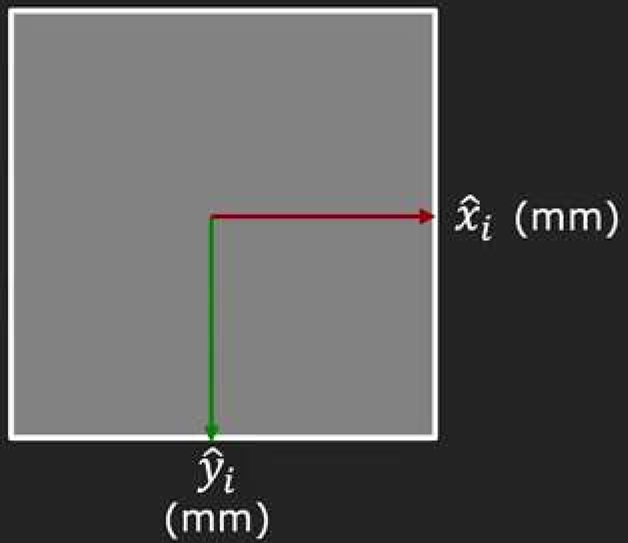

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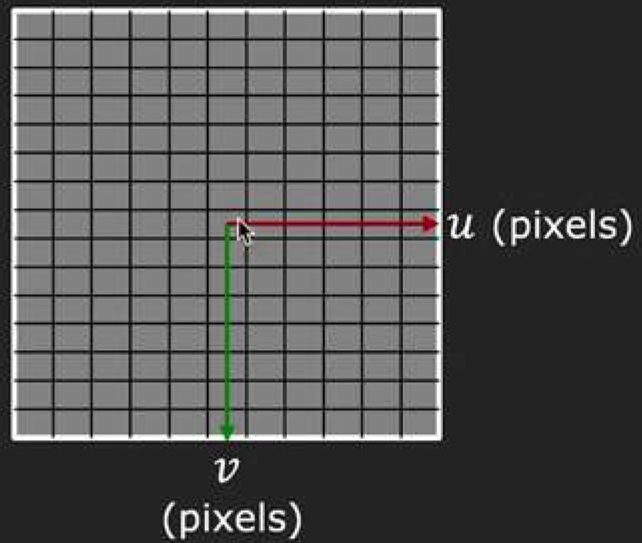

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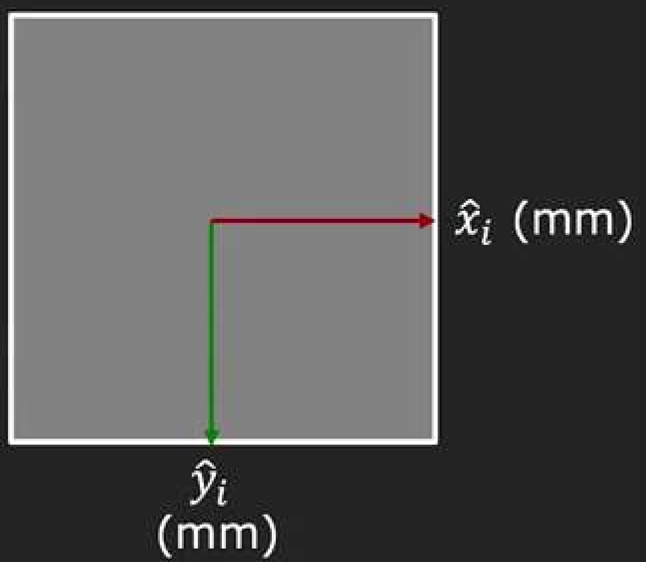

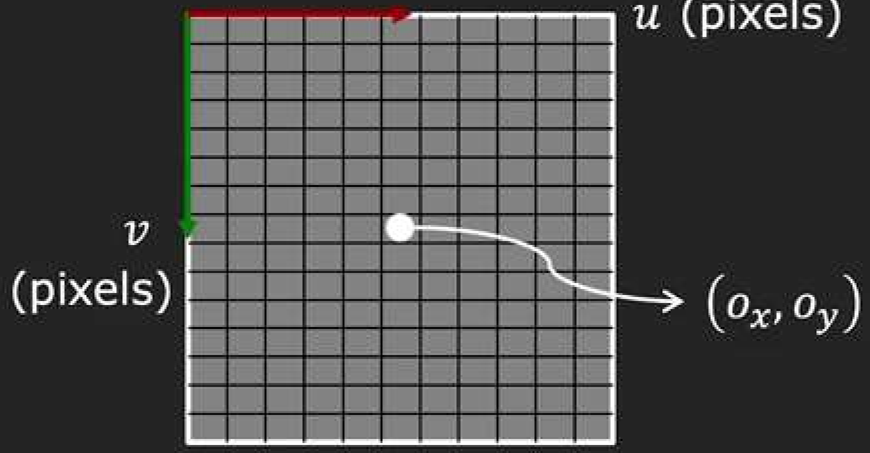

We usually treat the top-left corner of the image sensor as its origin (easier for indexing). If pixel (o_x, o_y) is the Principle Point where the optical axis pierces the sensor, then:

$$u = m_x f \frac{x_c}{z_c} + o_x \qquad v = m_y f \frac{y_c}{z_c} + o_y$$

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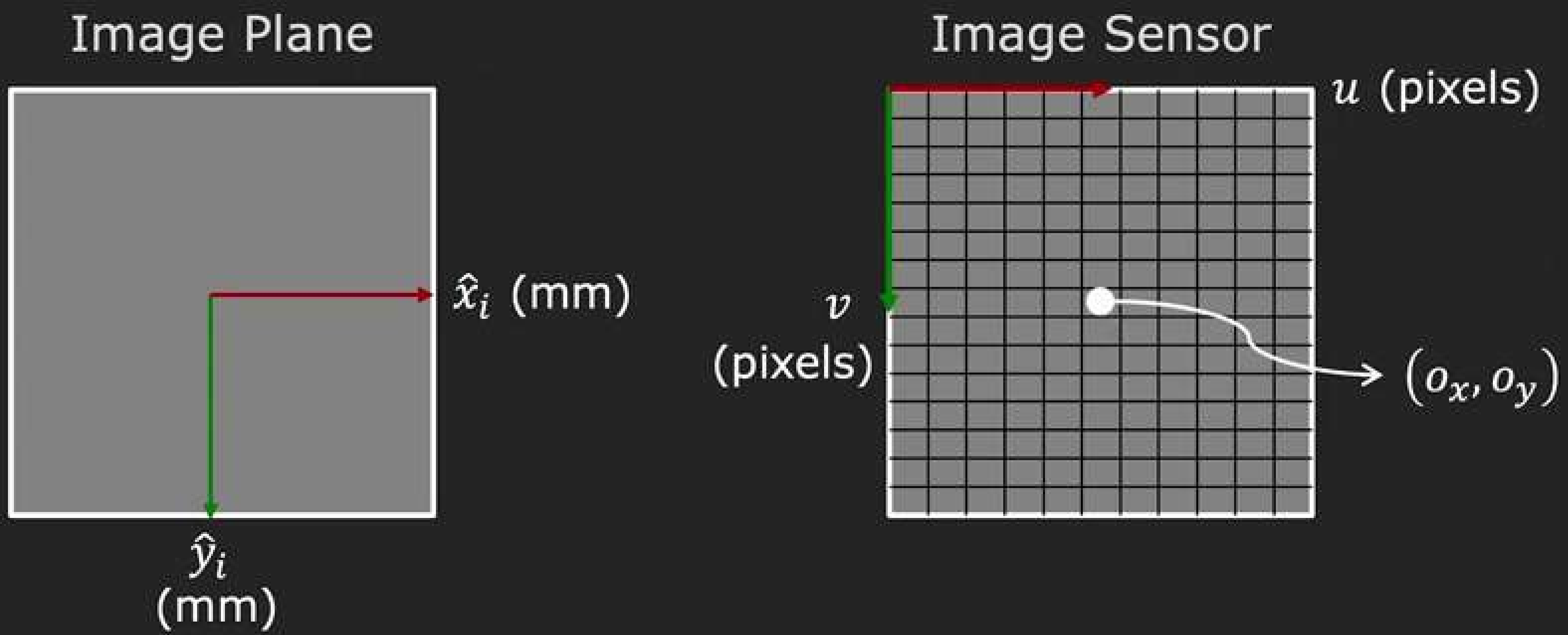

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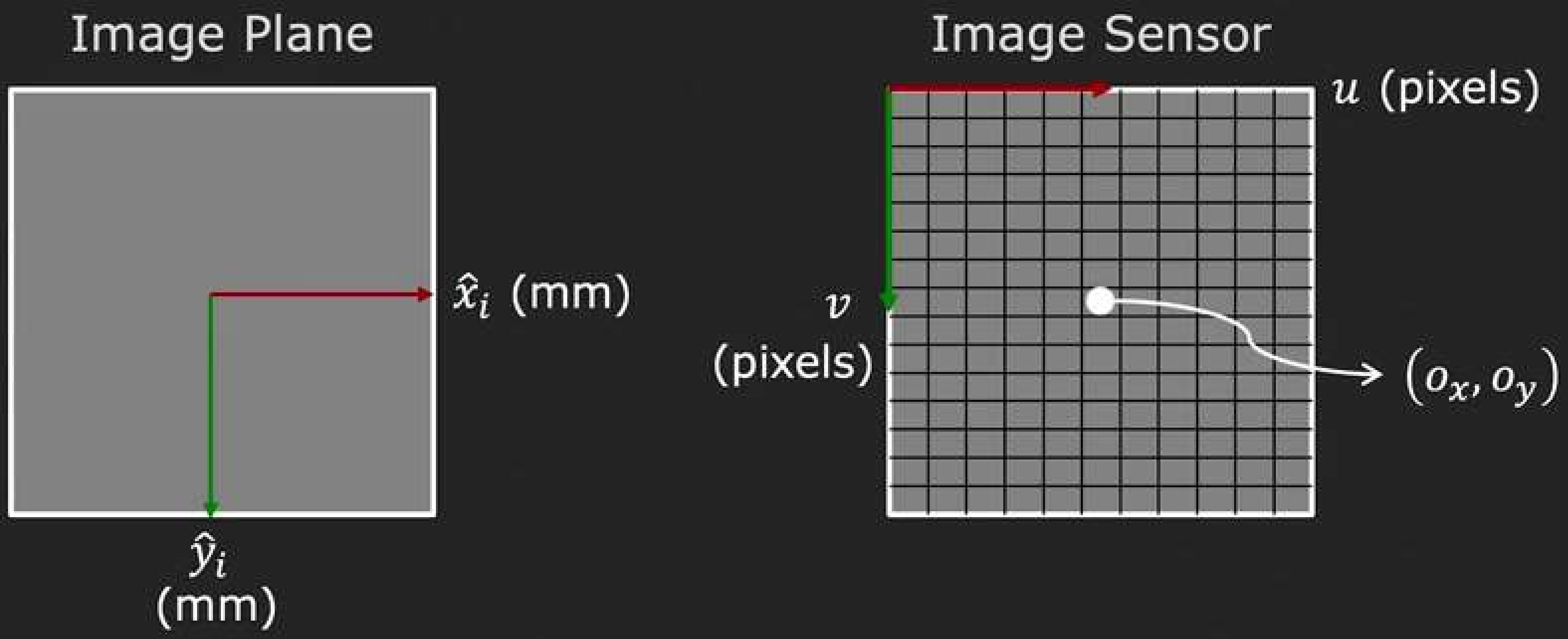

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where: $(f_x, f_y) = (m_x f, m_y f)$ are the focal lengths in pixels in the x and y directions.

Perspective Projection

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where: $(f_x, f_y) = (m_x f, m_y f)$ are the focal lengths in pixels in the x and y directions.

(f_x, f_y, o_x, o_y): Intrinsic parameters of the camera.
They represent the camera's internal geometry.

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Equations for perspective projection are Non-Linear.

It is convenient to express them as linear equations.

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Homogenous Coordinates

The homogenous representation of a 2D point $\mathbf{u} = (u, v)$ is a 3D point $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v}, \tilde{w})$. The third coordinate $\tilde{w} \neq 0$ is fictitious such that:

$$u = \frac{\tilde{u}}{\tilde{w}} \qquad v = \frac{\tilde{v}}{\tilde{w}}$$

$$\boxed{\mathbf{u} \equiv \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{w}u \\ \tilde{w}v \\ \tilde{w} \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \tilde{\mathbf{u}}}$$

REVIEW

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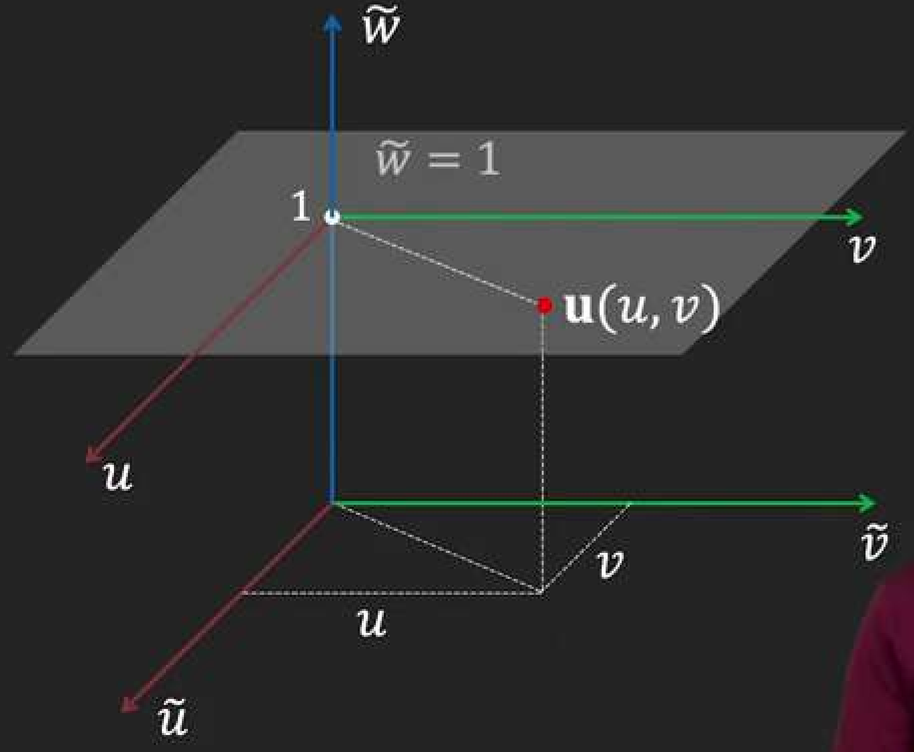

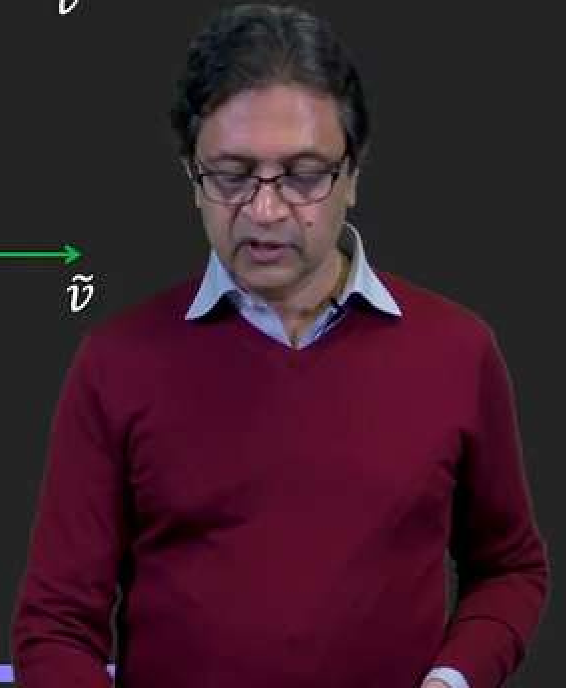

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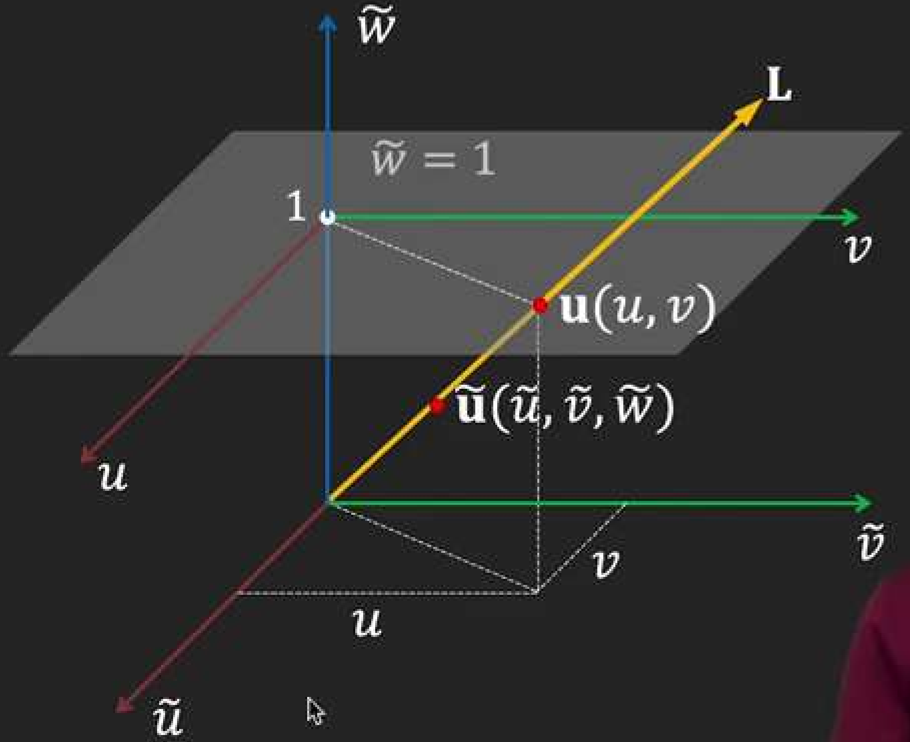

Every point on line $\mathbf{L}$ (except origin) repres
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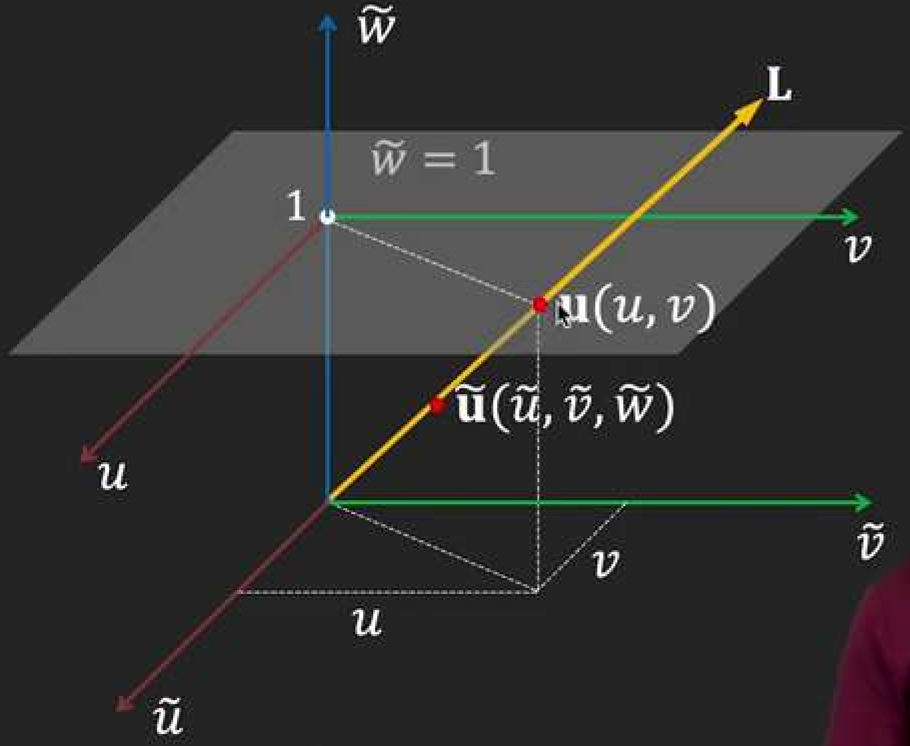

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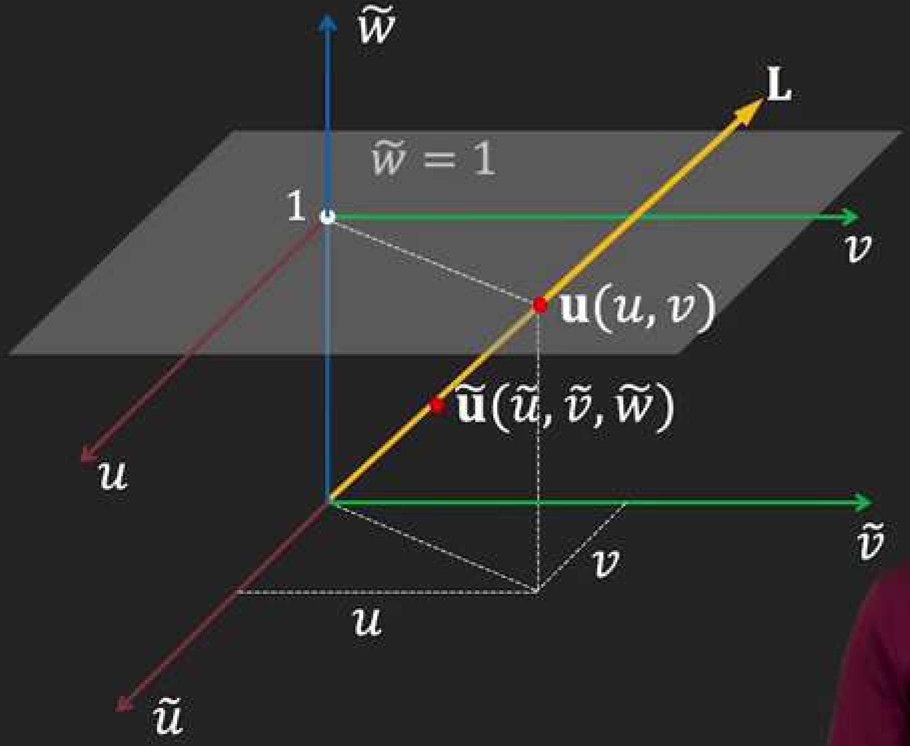

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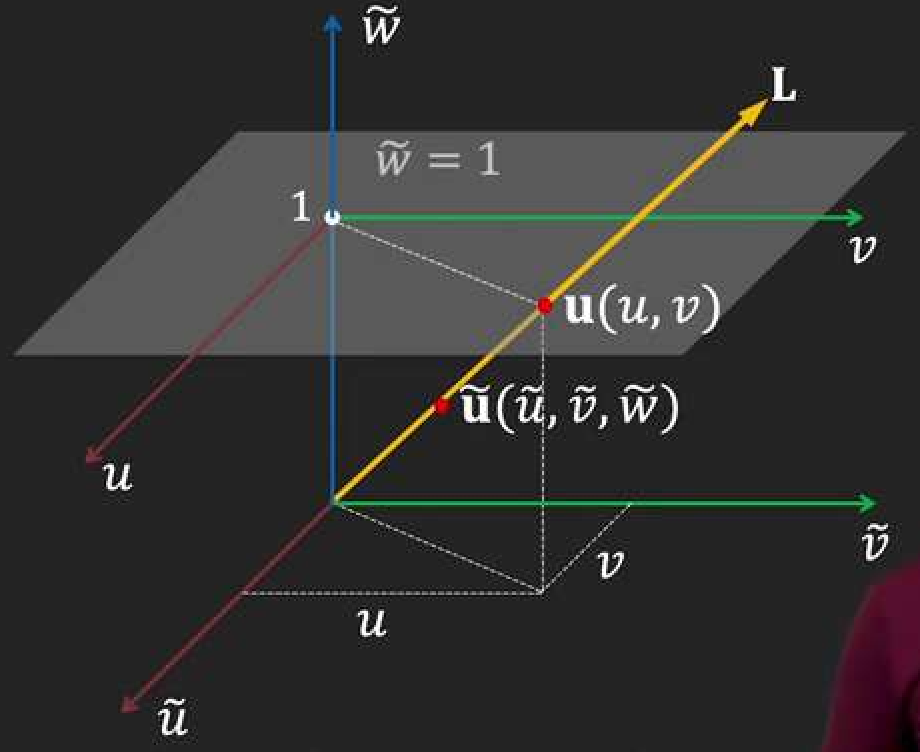

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$$\boxed{\mathbf{x} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{w}x \\ \tilde{w}y \\ \tilde{w}z \\ \tilde{w} \end{bmatrix} \equiv \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \tilde{\mathbf{x}}}$$

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Perspective projection equations:

$$u = f_x \frac{x_c}{z_c} + o_x \qquad v = f_y \frac{y_c}{z_c} + o_y$$

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Intrinsic Matrix

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

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$$K = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

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Upper Right Triangular Matrix

Intrinsic Matrix

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$$K = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

Upper Right Triangular Matrix

Intrinsic Matrix:

$$M_{int} = [K|0] = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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Forward Imaging Model: 3D to 2D

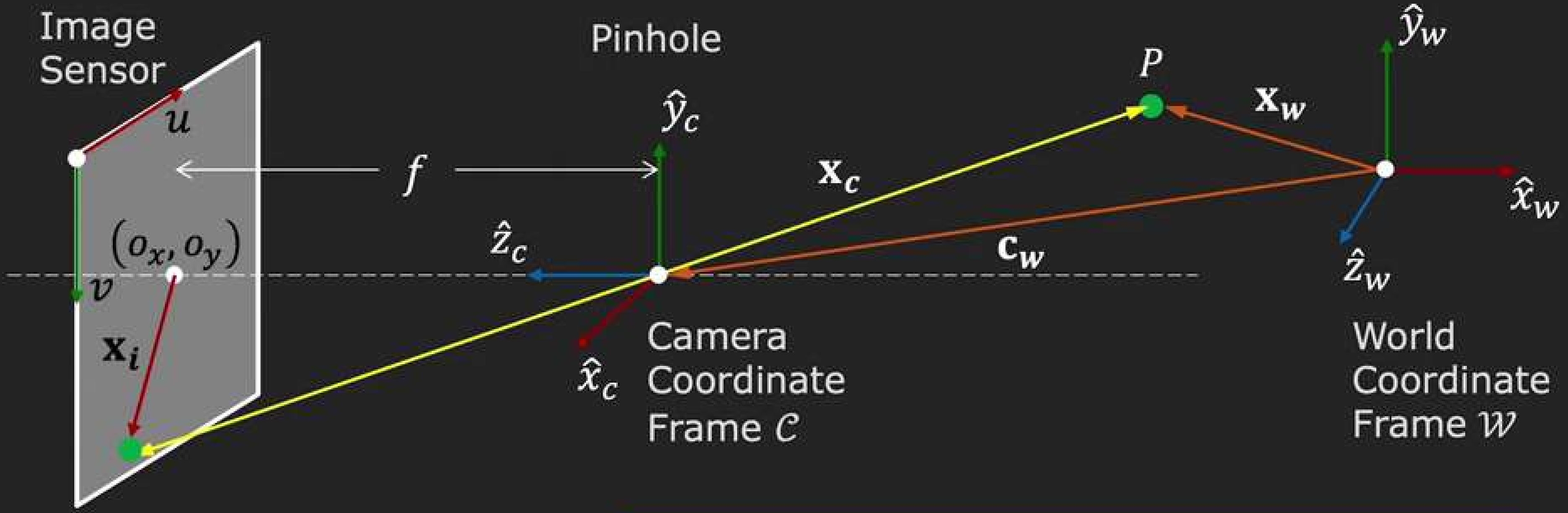


Image Coordinates

$$\mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$\xleftarrow{M_{int}}$

Perspective Projection

Camera Coordinates

$$\mathbf{x}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

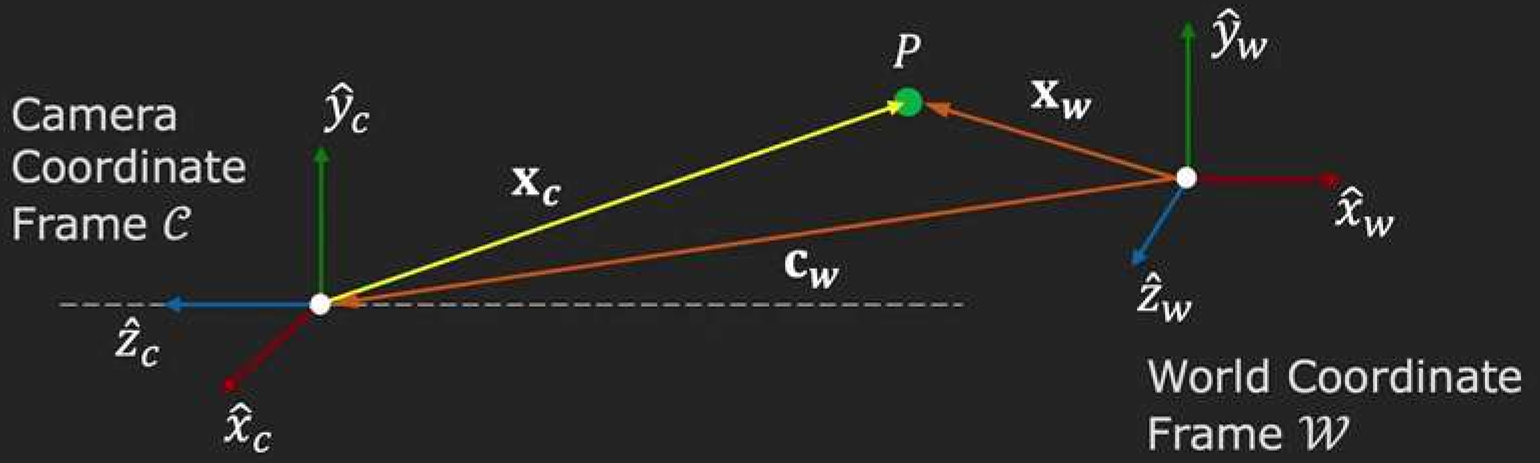
$\xleftarrow{?}$

Coordinate Transformation

World Coordinates

$$\mathbf{x}_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Extrinsic Parameters



Position $\mathbf{c}_w$ and Orientation R of the camera in the world coordinate frame $\mathcal{W}$ are the camera's Extrinsic Parameters.

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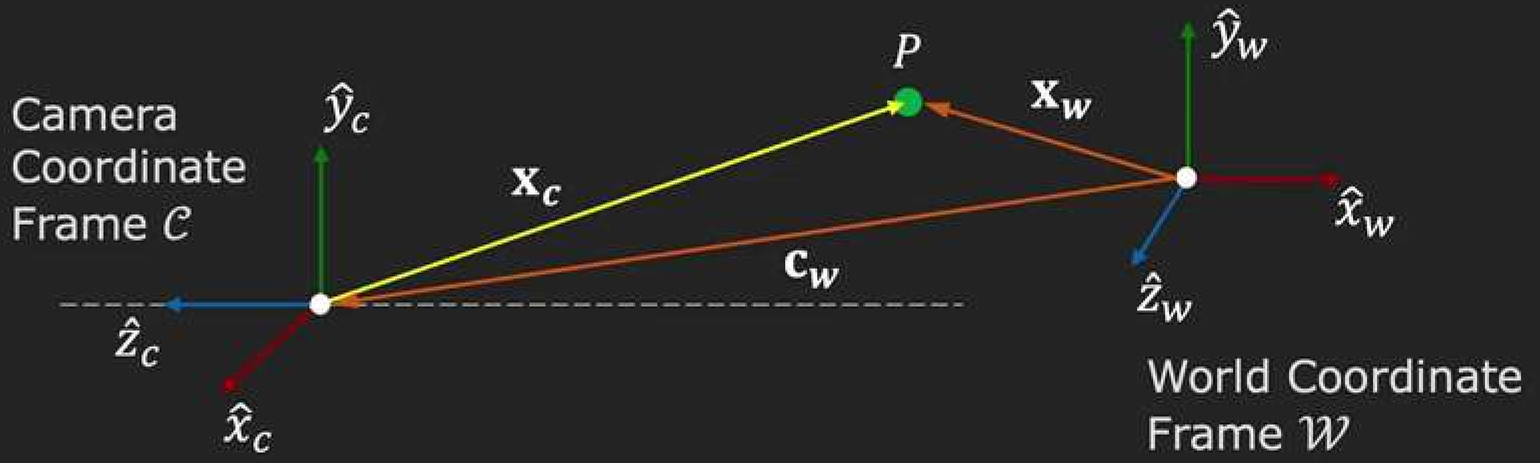

Position $\mathbf{c}_w$ and Orientation R of the camera in the world coordinate frame $\mathcal{W}$ are the camera's Extrinsic Parameters.

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

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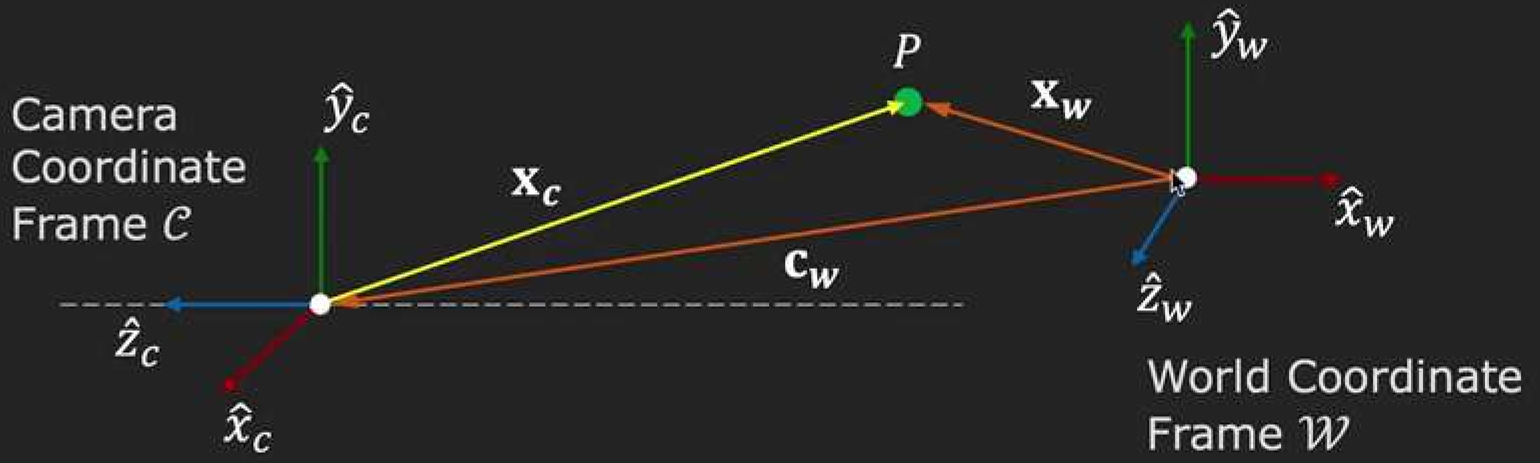

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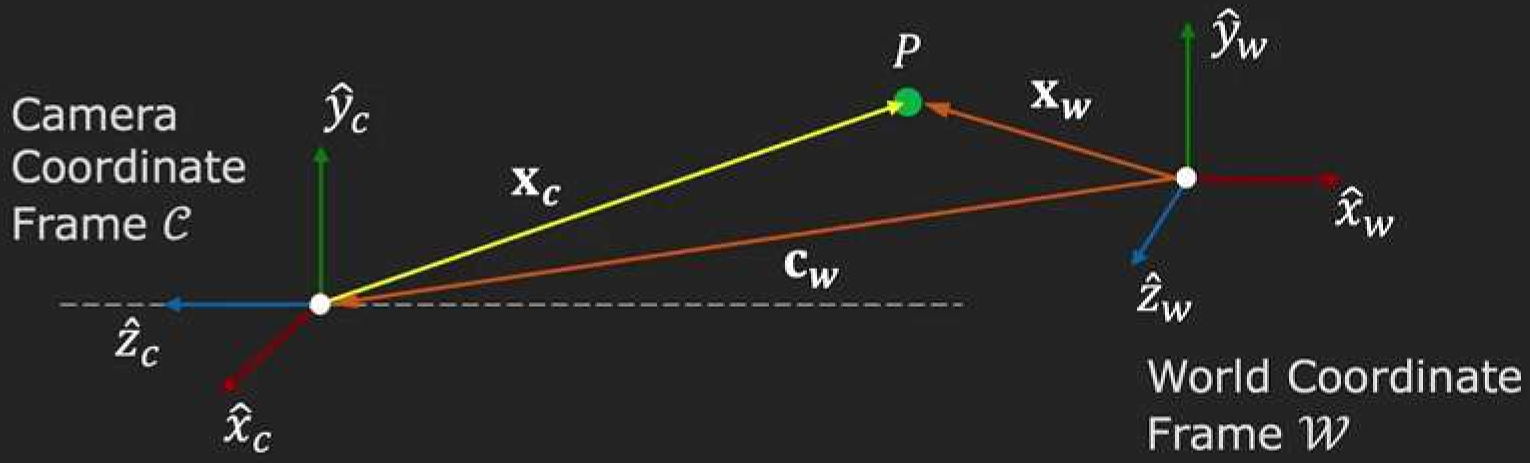
→ Row 1: Direction of $\hat{x}_c$ in world coordinate frame

Extrinsic Parameters



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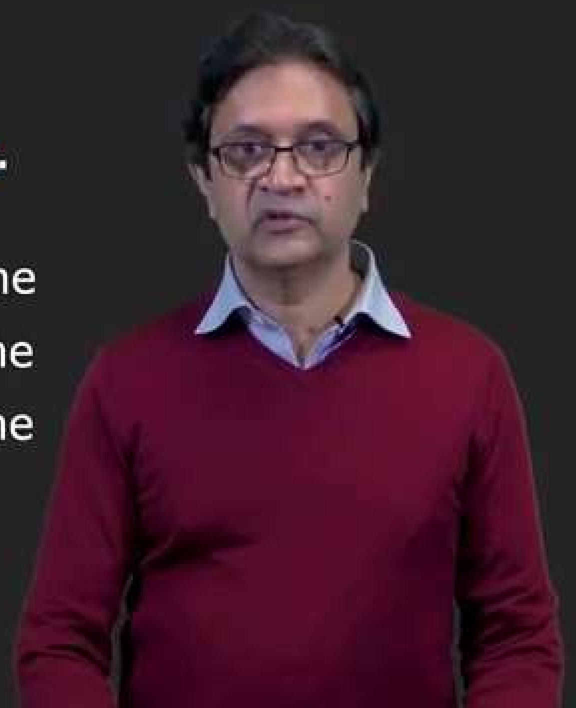
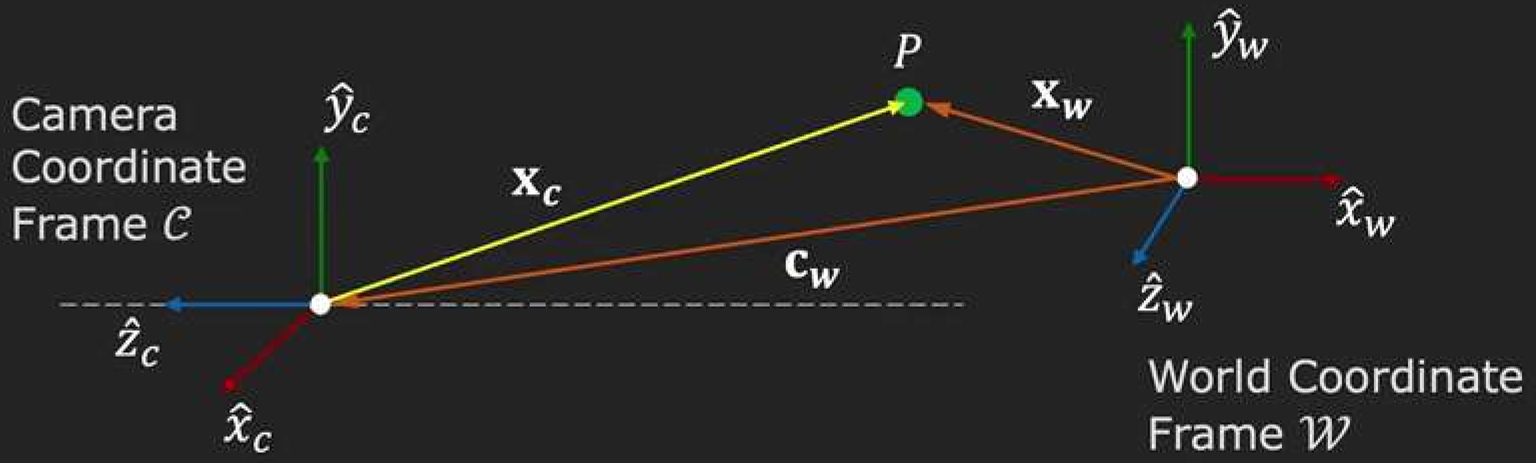


Extrinsic Parameters



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Orientation/Rotation Matrix R is Orthonormal

Orthonormal Vectors and Matrices

Orthonormal Vectors: Two vectors $\mathbf{u}$ and $\mathbf{v}$ are orthonormal if and only if:

$$dot(\mathbf{u}, \mathbf{v}) = \mathbf{u}^T \mathbf{v} = 0$$

(Orthogonality)

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Example: The x-, y- and z-axes of $\mathbb{R}^3$ Euclidean space

Orthonormal Matrix: A square matrix R whose row (or column) vectors are orthonormal. For such a matrix:

$$R^{-1} = R^T \qquad R^T R = R R^T = I$$

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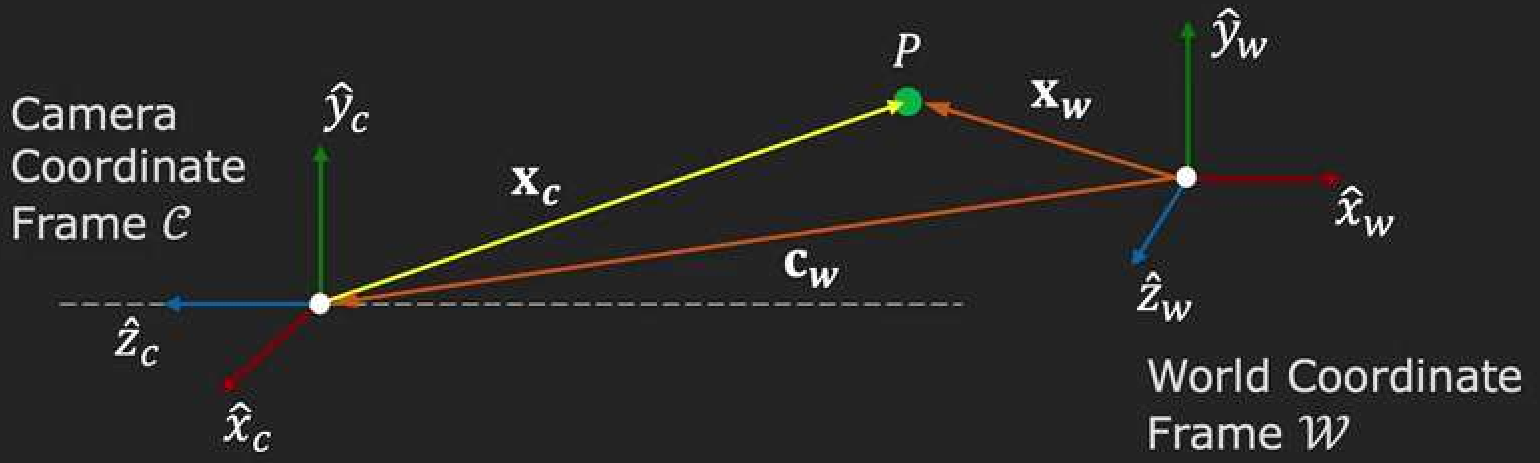
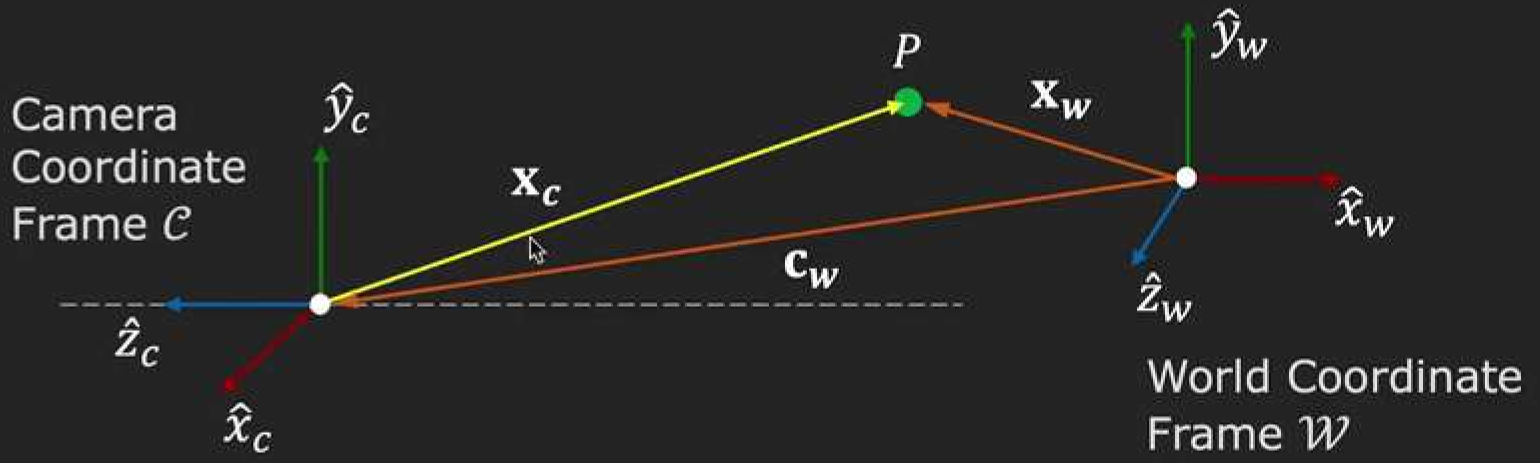
World-to-Camera Transformation



Given the extrinsic parameters $(R, \mathbf{c}_w)$ of the camera, the camera-centric location of the point P in the world coordinate frame is:

World-to-Camera Transformation



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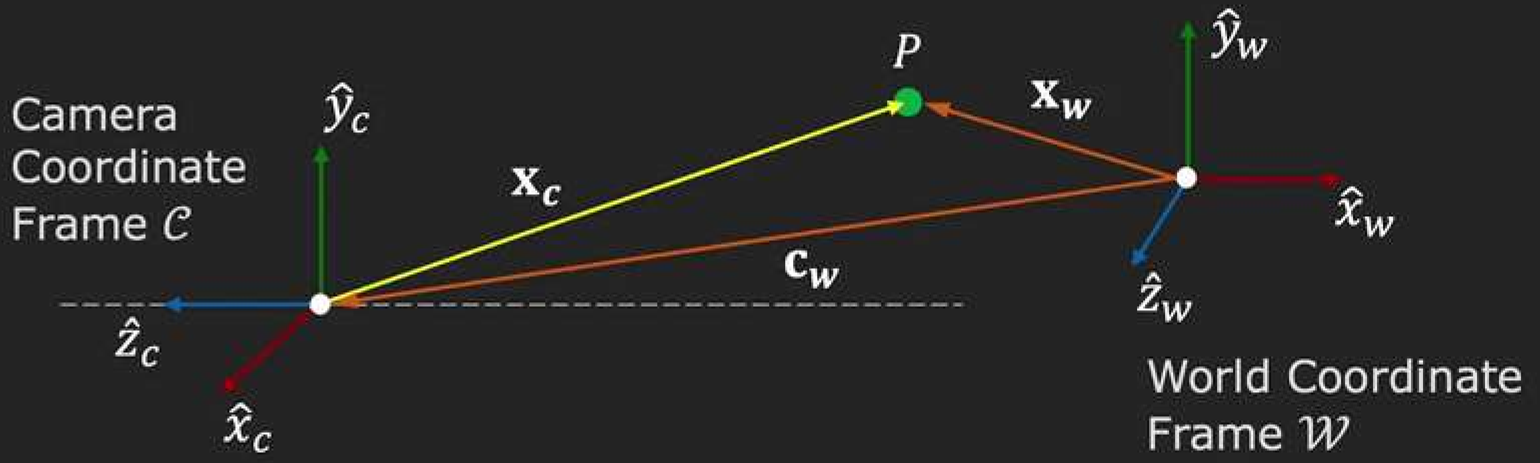

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$$\mathbf{x}_c = R(\mathbf{x}_w - \mathbf{c}_w)$$

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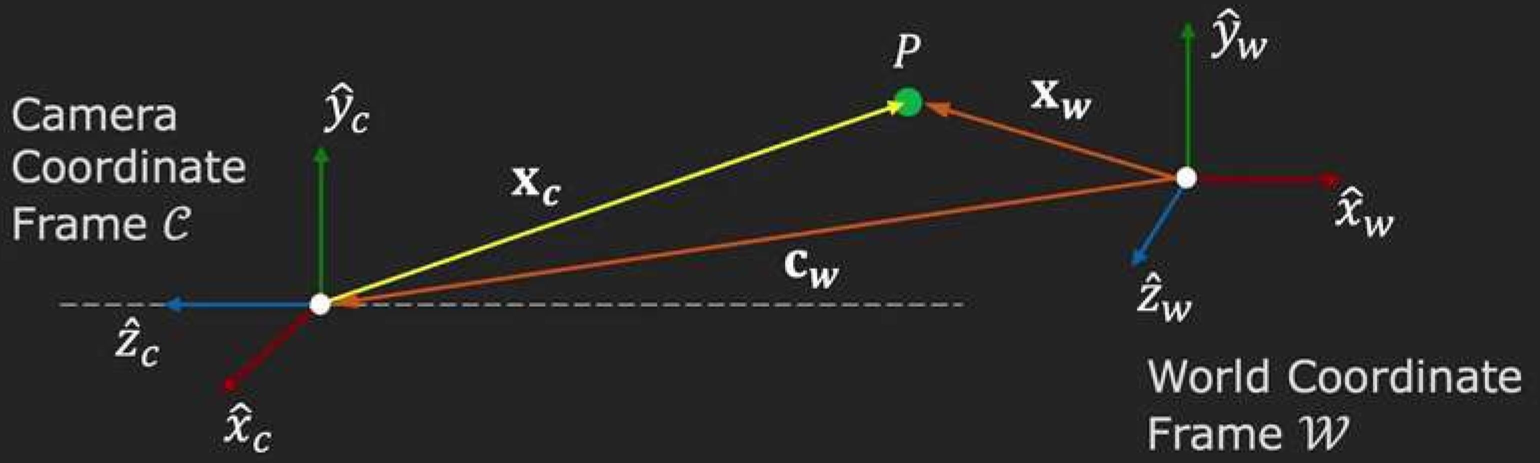

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$$\mathbf{x}_c = R(\mathbf{x}_w - \mathbf{c}_w) = R\mathbf{x}_w - R\mathbf{c}_w = R\mathbf{x}_w + \mathbf{t} \qquad \boxed{\mathbf{t} = -R\mathbf{c}_w}$$

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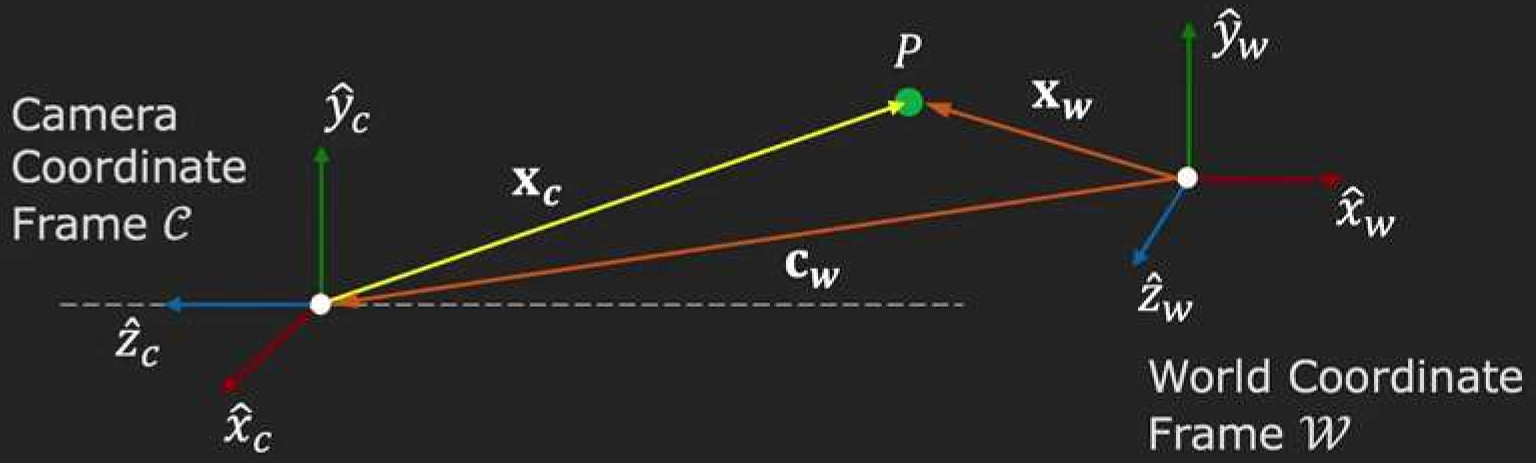

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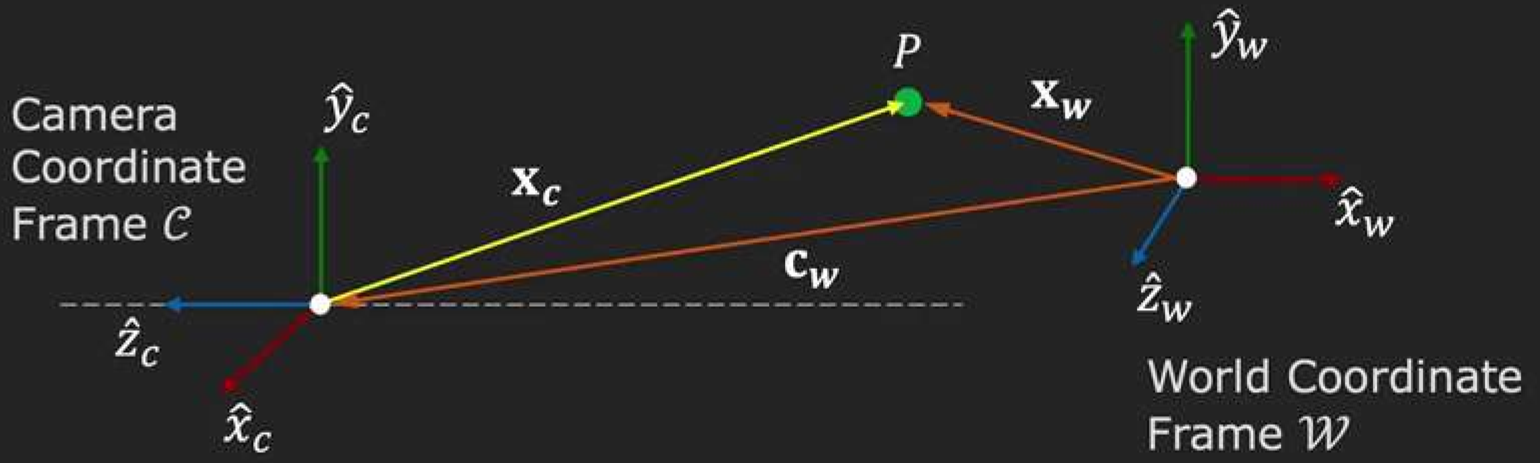

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Extrinsic Matrix

Rewriting using homogenous coordinates:

$$\boxed{\tilde{\mathbf{x}}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}}$$

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Extrinsic Matrix: $M_{ext} = \begin{bmatrix} R_{3\times 3} & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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$$\boxed{\tilde{\mathbf{x}}_c = M_{ext} \tilde{\mathbf{x}}_w}$$

Projection Matrix P

Camera to Pixel

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{u}} = M_{int}\, \tilde{\mathbf{x}}_c$$

World to Camera

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

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World to Camera

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{x}}_c = M_{ext}\tilde{\mathbf{x}}_w$$

Projection Matrix P

Camera to Pixel

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{u}} = M_{int} \, \tilde{\mathbf{x}}_c$$

World to Camera

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Projection Matrix P

Camera to Pixel

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$$\tilde{\mathbf{x}}_c = M_{ext} \tilde{\mathbf{x}}_w$$

Combining the above two equations, we get the full projection matrix P:

$$\boxed{\tilde{\mathbf{u}} = M_{int} M_{ext}\, \tilde{\mathbf{x}}_w = P\, \tilde{\mathbf{x}}_w}$$

Projection Matrix P

Camera to Pixel

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0\\ 0 & f_y & o_y & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_c\\ y_c\\ z_c\\ 1\end{bmatrix}$$

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World to Camera

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