Estimating Fundamental Matrix

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Topic: Uncalibrated Stereo, Module: Reconstruction II

First Principles of Computer Vision

Stereo Calibration Procedure

Find a set of corresponding features in left and right images
(e.g. using SIFT or hand-picked)

Left image

Right image

Stereo Calibration Procedure

Find a set of corresponding features in left and right images (e.g. using SIFT or hand-picked)

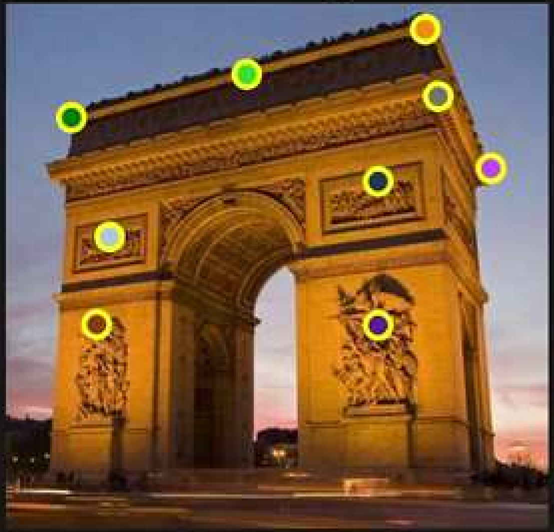
Left image

Right image

Stereo Calibration Procedure

Find a set of corresponding features in left and right images (e.g. using SIFT or hand-picked)

Left image

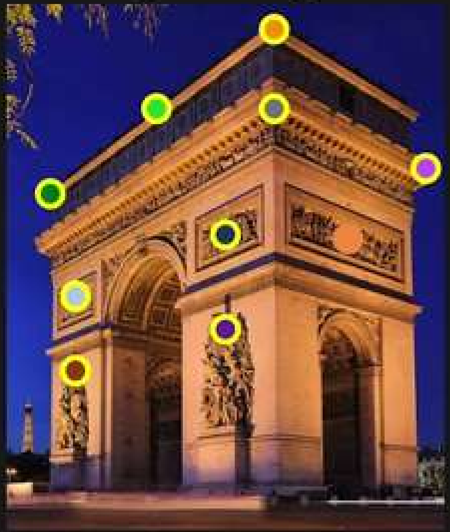


$(\boldsymbol{u}_l^{(1)}, \boldsymbol{v}_l^{(1)})$

$\vdots$

$(\boldsymbol{u}_l^{(m)}, \boldsymbol{v}_l^{(m)})$

Right image



$(\boldsymbol{u}_r^{(1)}, \boldsymbol{v}_r^{(1)})$

$\vdots$

$(\boldsymbol{u}_r^{(m)}, \boldsymbol{v}_r^{(m)})$

Stereo Calibration Procedure

Step A: For each correspondence i, write out epipolar constraint.

$$\begin{bmatrix} u_l^{(i)} & v_l^{(i)} & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r^{(i)} \\ v_r^{(i)} \\ 1 \end{bmatrix} = 0$$

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Expand the matrix to get linear equation:

$$\left(f_{11}u_r^{(i)} + f_{12}v_r^{(i)} + f_{13}\right)u_l^{(i)} + \left(f_{21}u_r^{(i)} + f_{22}v_r^{(i)} + f_{23}\right)v_l^{(i)} + f_{31}u_r^{(i)} + f_{32}v_r^{(i)} + f_{33}$$

Stereo Calibration Procedure

Step B: Rearrange terms to form a linear system.

$$
\underbrace{\begin{bmatrix}
u_l^{(1)}u_r^{(1)} & u_l^{(1)}v_r^{(1)} & u_l^{(1)} & v_l^{(1)}u_r^{(1)} & v_l^{(1)}v_r^{(1)} & v_l^{(1)} & u_r^{(1)} & v_r^{(1)} & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
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\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
u_l^{(m)}u_r^{(m)} & u_l^{(m)}v_r^{(m)} & u_l^{(m)} & v_l^{(m)}u_r^{(m)} & v_l^{(m)}v_r^{(m)} & v_l^{(m)} & u_l^{(m)} & u_r^{(m)} & 1
\end{bmatrix}}_{\substack{A \\ \text{(Known)}}}
\underbrace{\begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\substack{\mathbf{f} \\ \text{(Unknown)}}}
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\end{bmatrix}}_{A}
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=
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$$

A
(Known)

$\mathbf{f}$
(Unknown)

The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

$$\begin{bmatrix} u_l & v_l & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = \begin{bmatrix} u_l & v_l & 1 \end{bmatrix} \begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

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Set Fundamental Matrix to some arbitrary scale.

$$\boxed{\|\mathbf{f}\|^2 = 1}$$

Solving for F

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Constrained linear least squares problem

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Like solving Projection Matrix during Camera Calibration.

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Rearrange solution $\mathbf{f}$ to form the fundamental matrix F.

Extracting Rotation and Translation

Step D: Compute essential matrix E from known left and right intrinsic camera matrices and fundamental matrix F.

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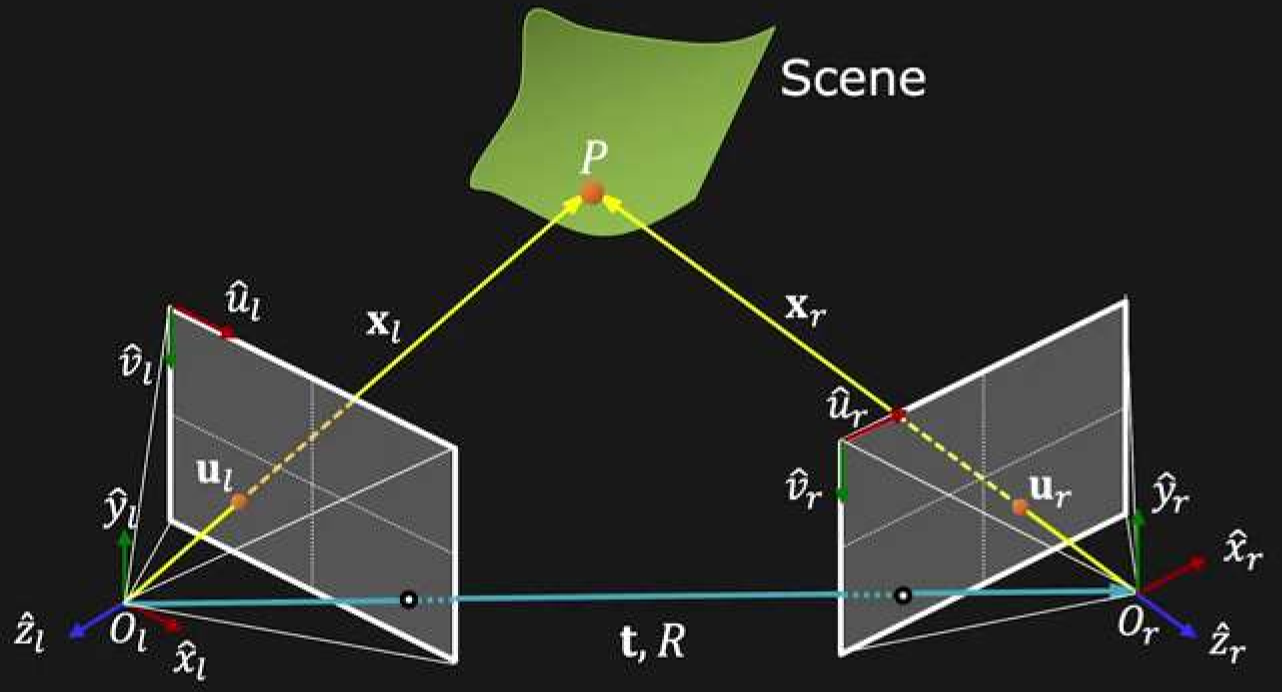
Uncalibrated Stereo



1. Assume Camera Matrix K is known for each camera
2. Find a few Reliable Corresponding Points
3. Find Relative Camera Position $\mathbf{t}$ and Orientation R
4. Find Dense Correspondence
5. Compute Depth using Triangulation