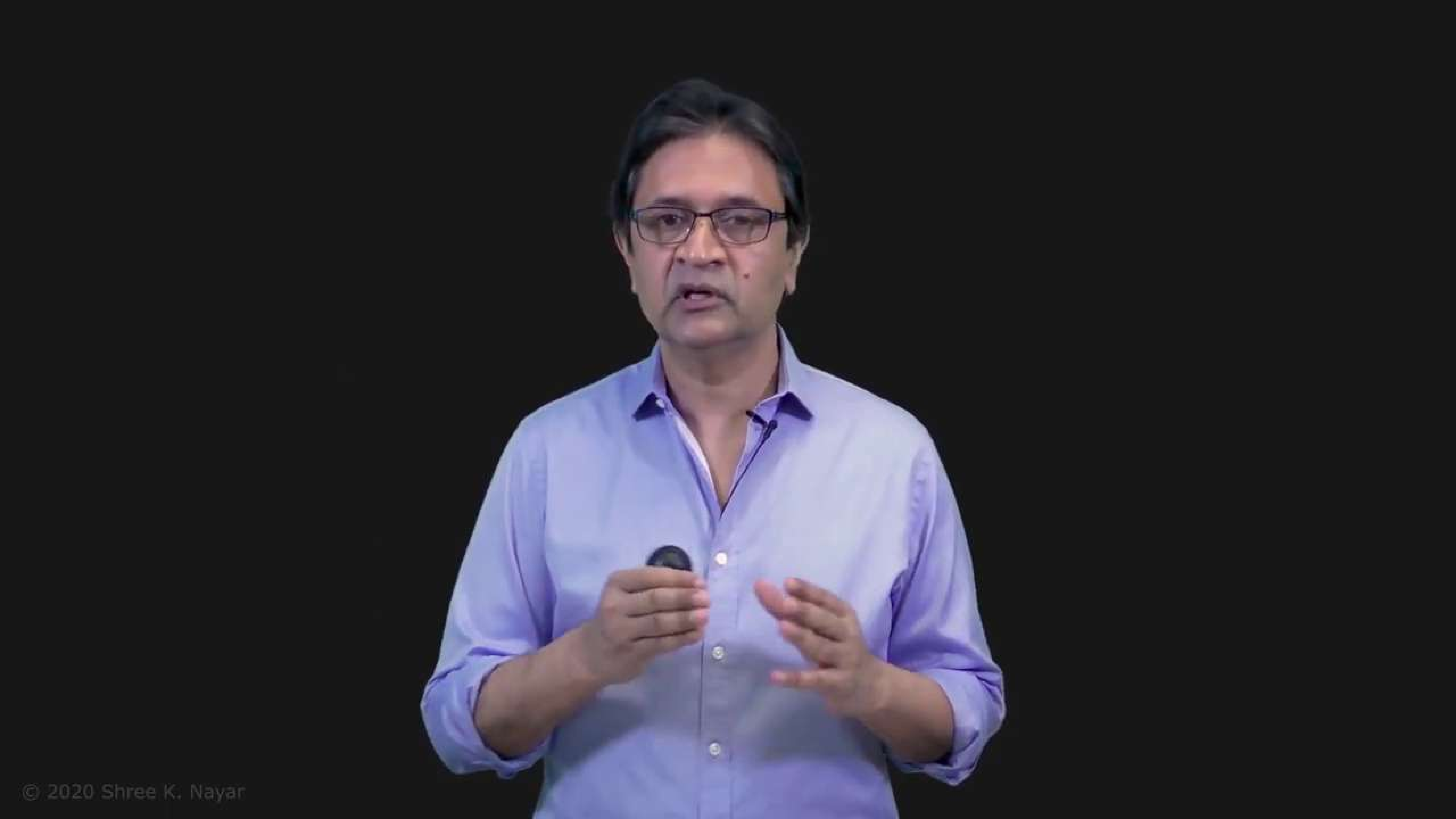
Motion Field and Optical Flow

Shree K. Nayar

Columbia University

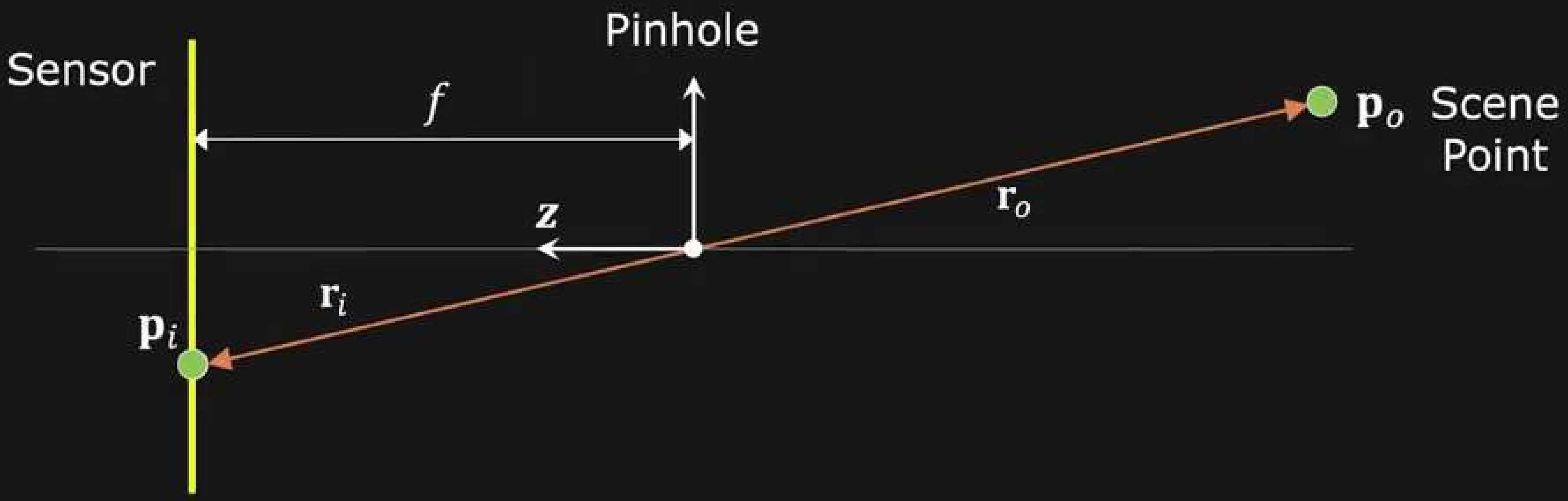
Topic: Motion and Optical Flow, Module: Reconstruction II

First Principles of Computer Vision



Motion Field

Image velocity of a point that is moving in the scene



[Horn 1981]

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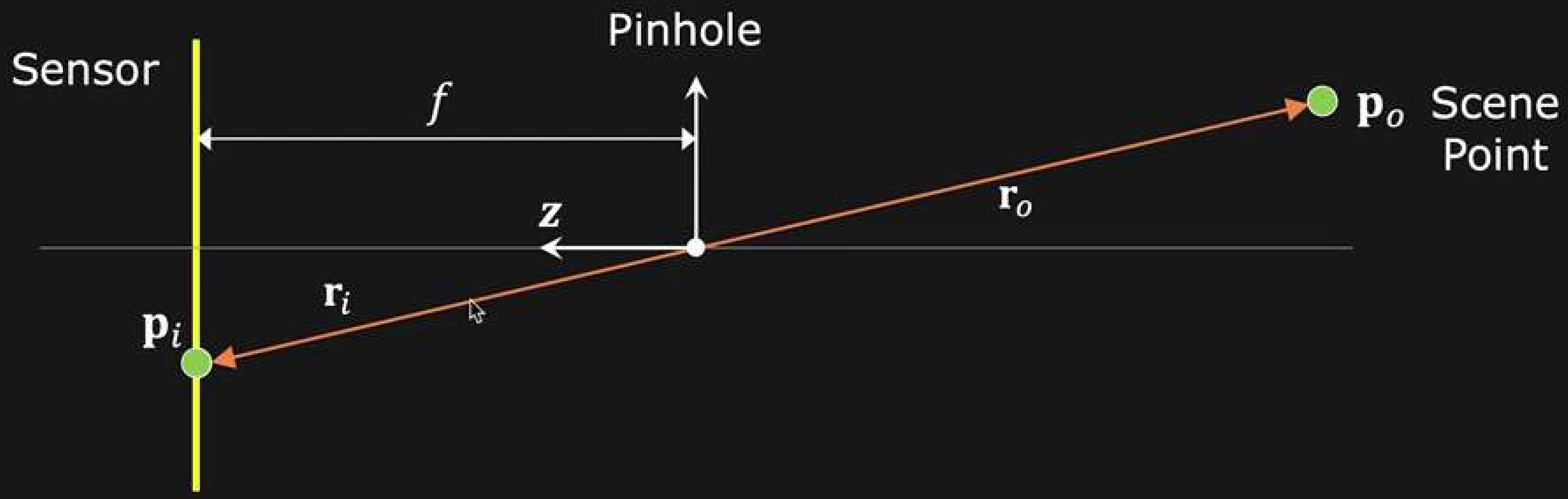

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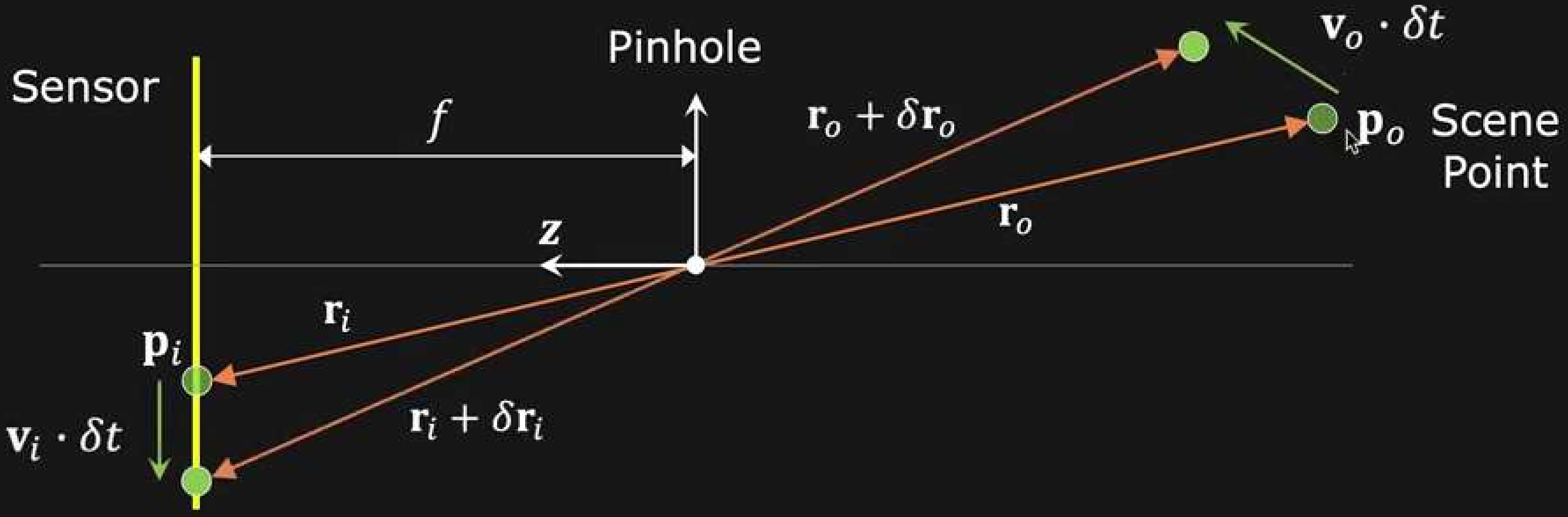

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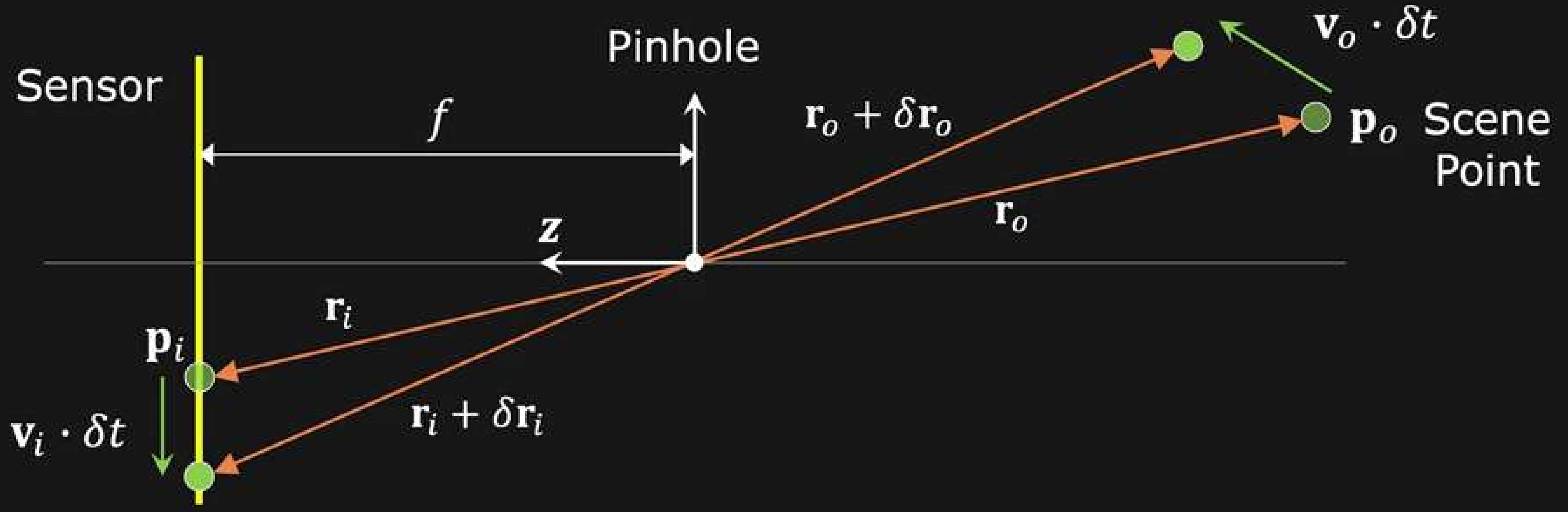

Scene Point Velocity: $\mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$

[Horn 1981]

Motion Field

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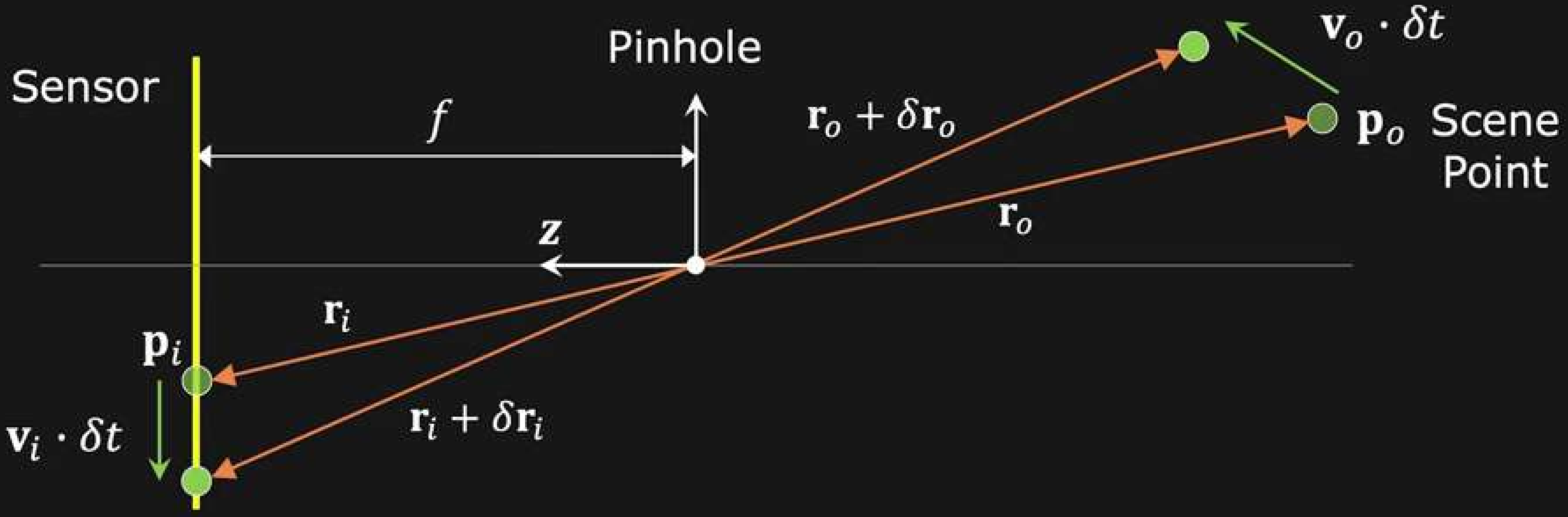

Image Point Velocity: $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$
(Motion Field)

Scene Point Velocity: $\mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$

[Horn 1981]

Motion Field

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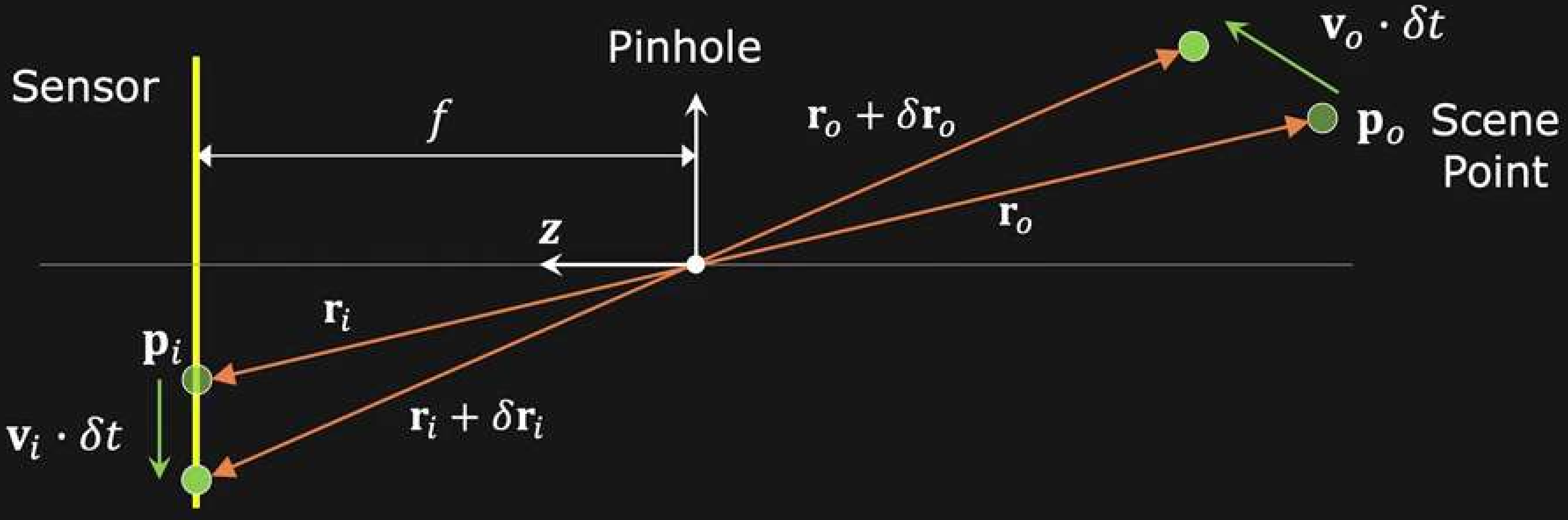

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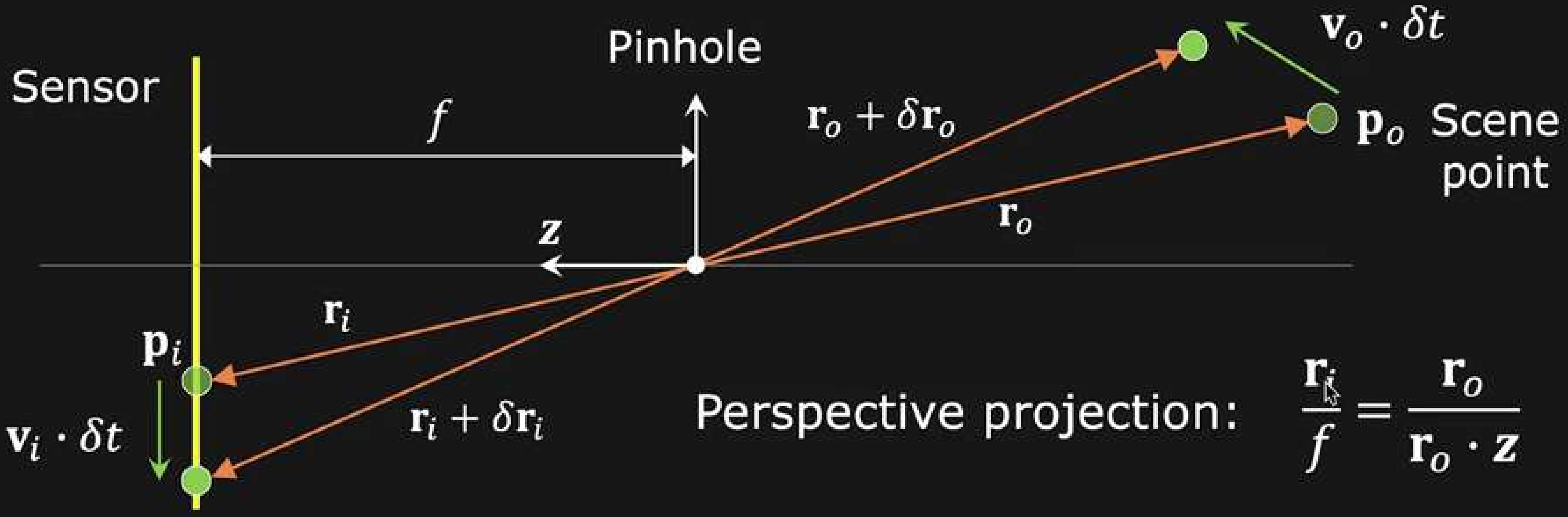

Perspective projection: $\dfrac{\mathbf{r}_i}{f} = \dfrac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$

Image Point Velocity: $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt}$

(Motion Field)

[Horn 1981]

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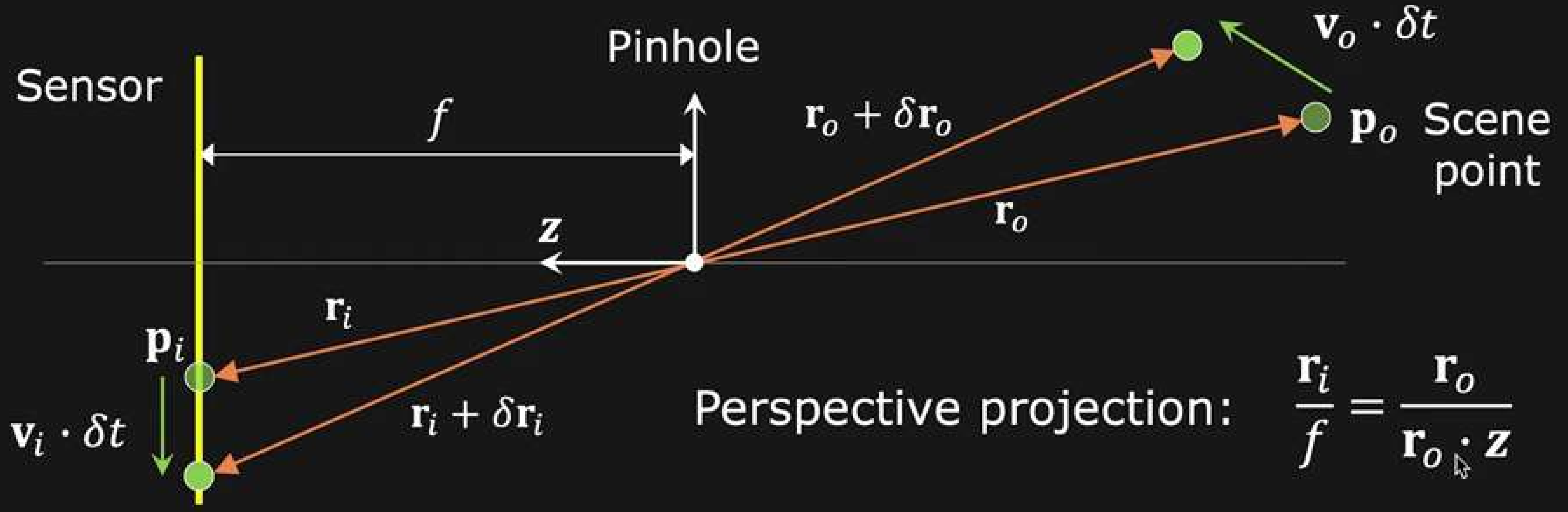

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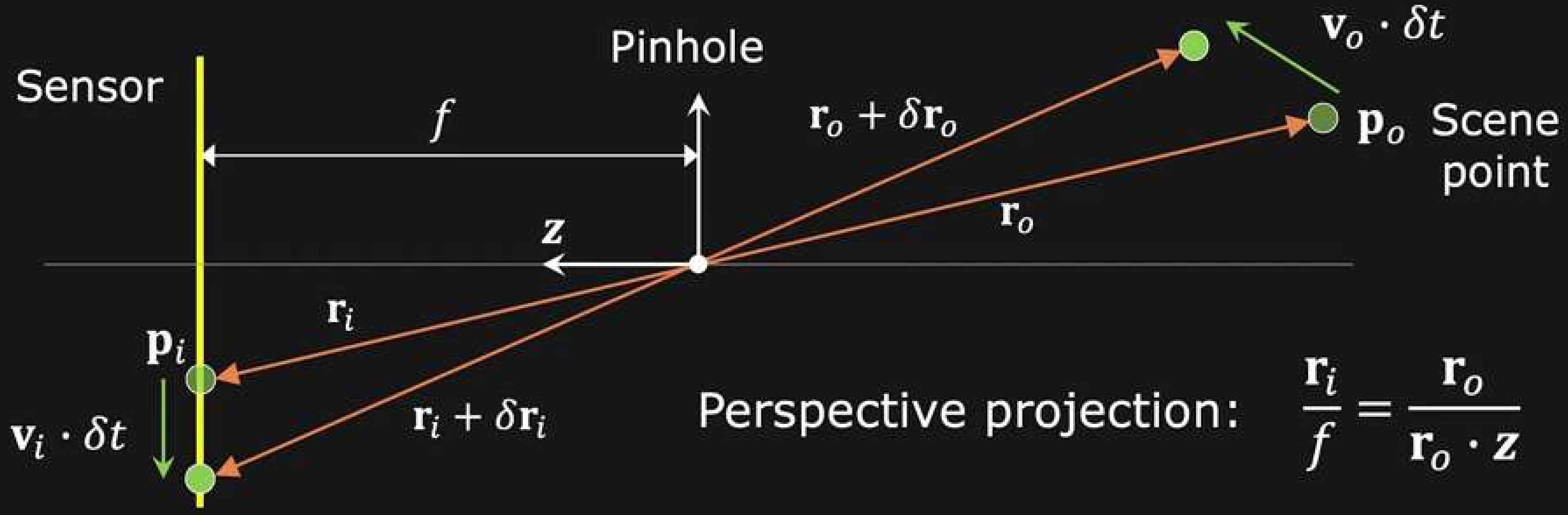

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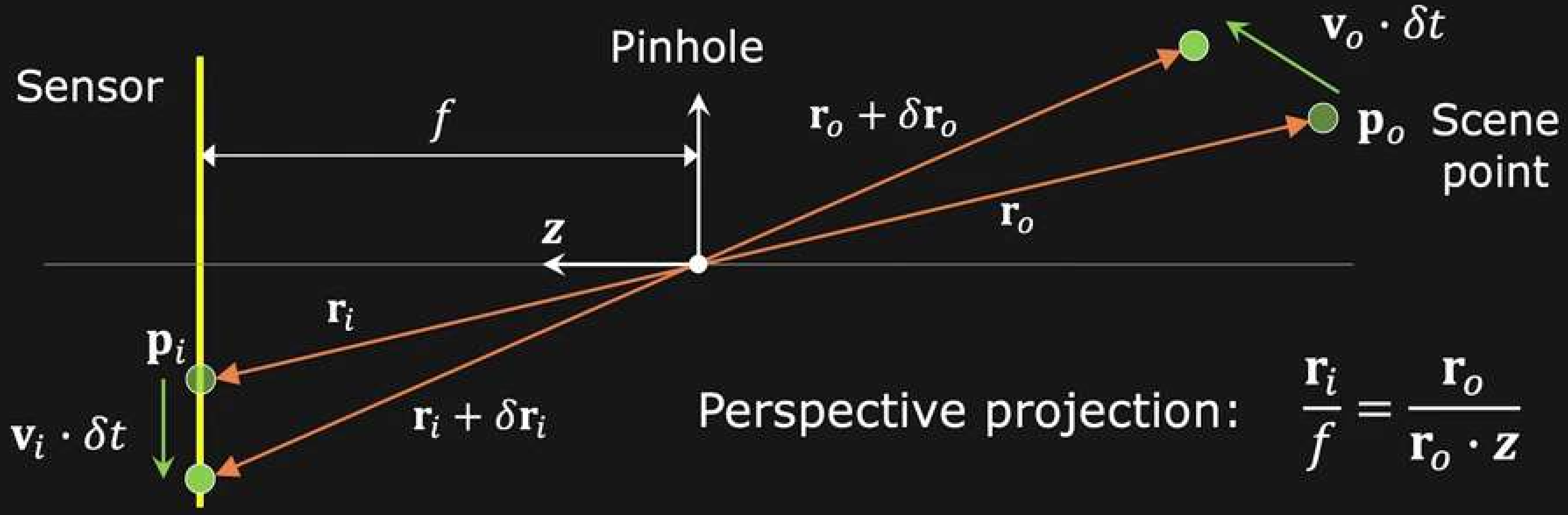

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Image Point Velocity: $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt} = f\,\dfrac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$

(Motion Field)

[Horn 1981]

Motion Field

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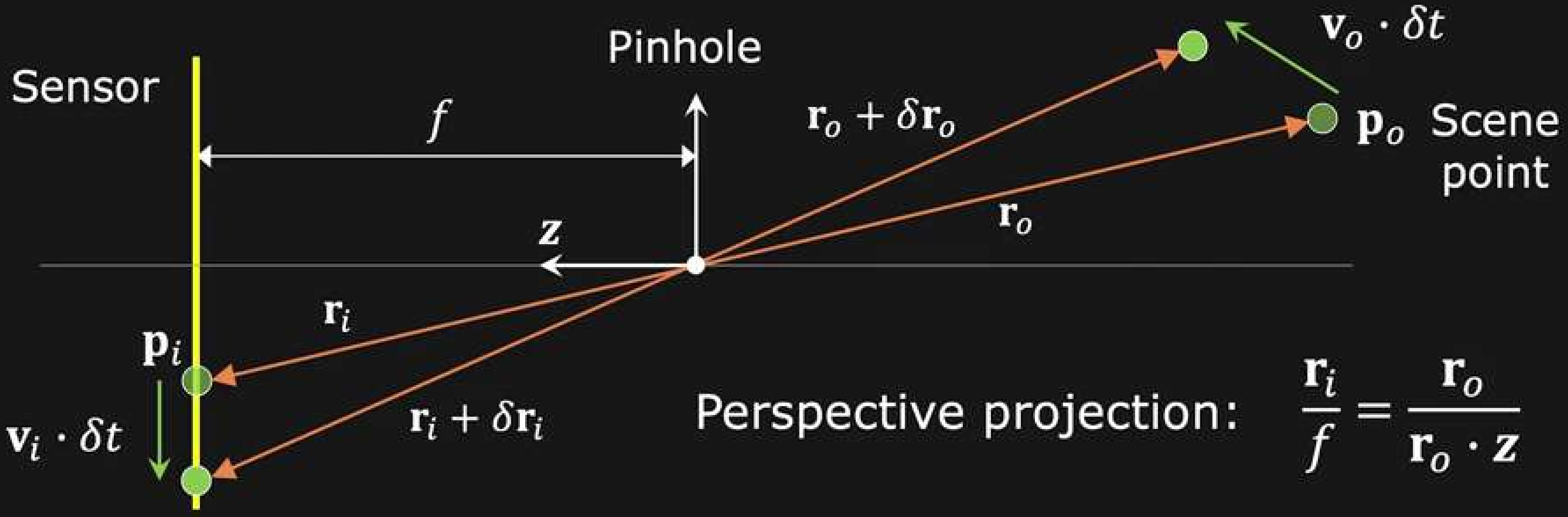

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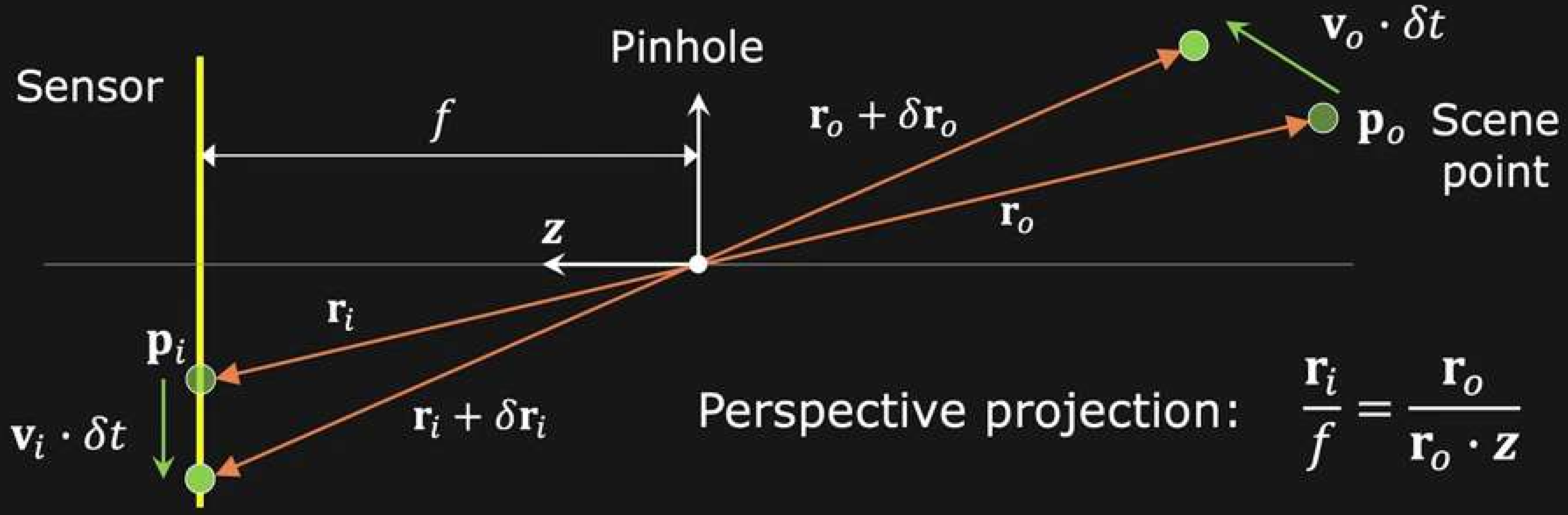

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(Motion Field)

$$\boxed{\mathbf{v}_i = f\,\frac{(\mathbf{r}_o \times \mathbf{v}_0) \times \mathbf{z}}{(\mathbf{r}_o \cdot \mathbf{z})^2}}$$

[Horn 1981]

Motion Field

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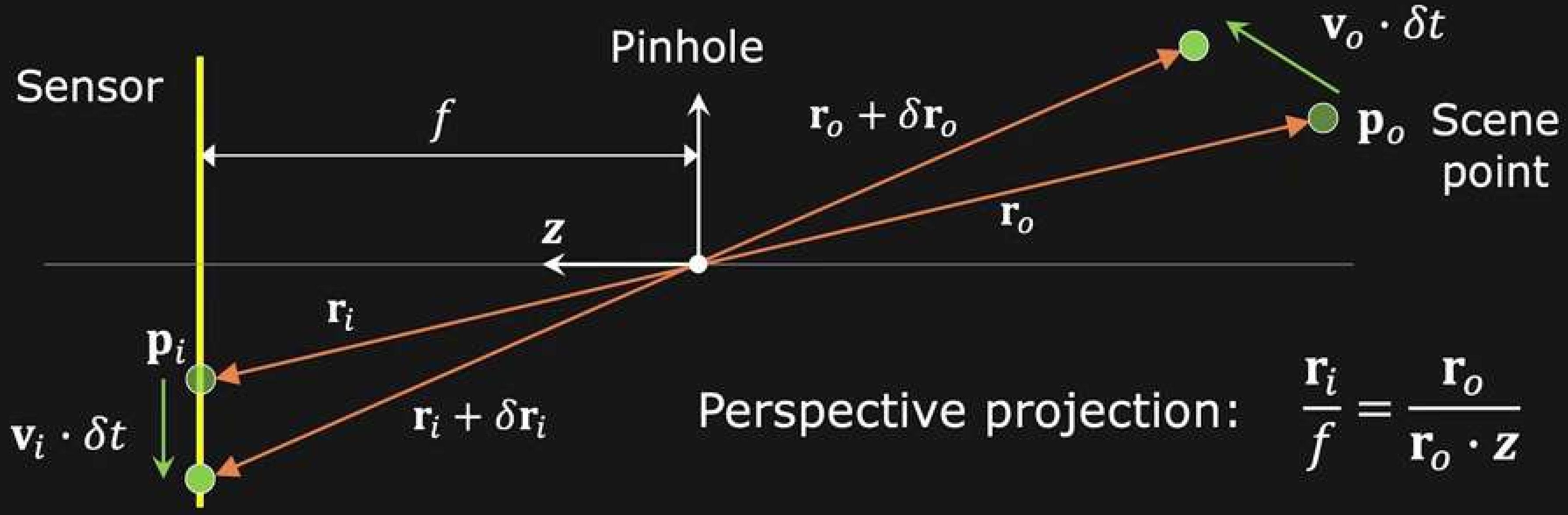

Perspective projection: $\dfrac{\mathbf{r}_i}{f} = \dfrac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$

Image Point Velocity: $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt} = f \dfrac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$

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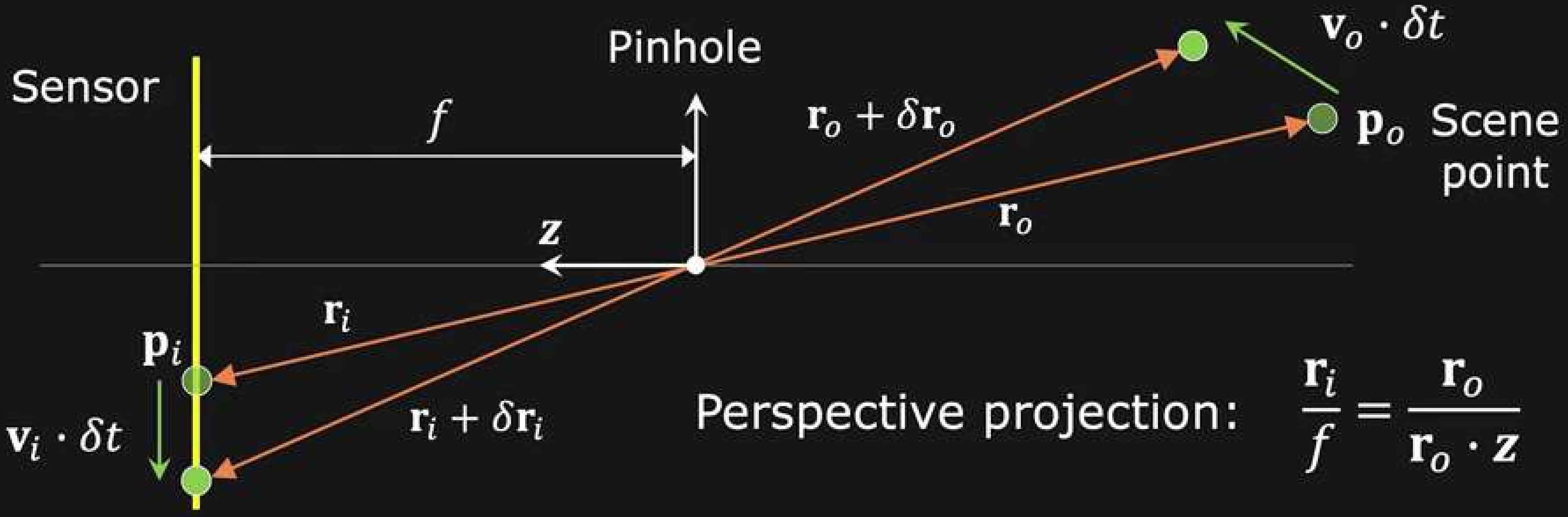

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[Horn 1981]

Optical Flow

Motion of brightness patterns in the image

Image Sequence
(2 frames)

Optical Flow

Motion of brightness patterns in the image

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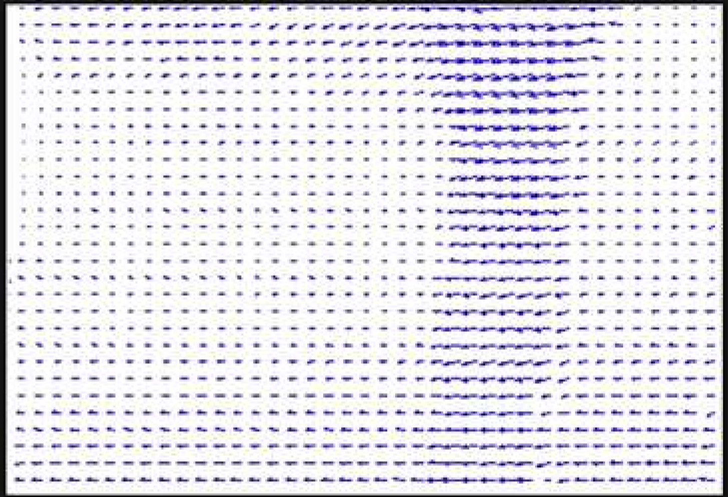
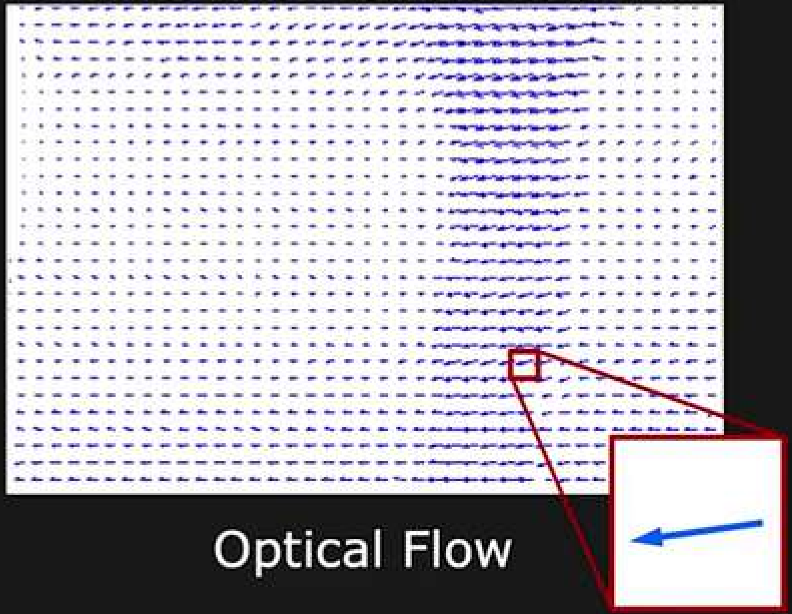


Optical Flow

Optical Flow

Motion of brightness patterns in the image

Image Sequence
(2 frames)



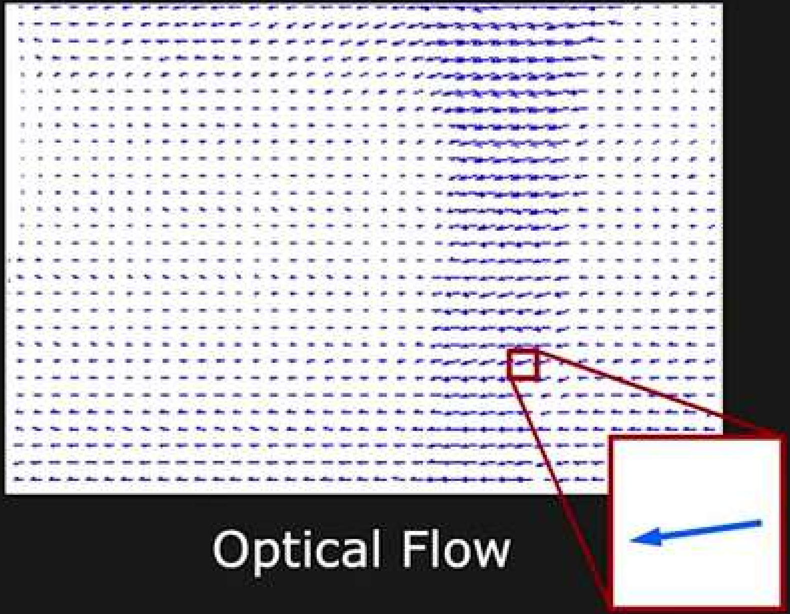
Optical Flow

Velocity of
brightness patte

Optical Flow

Motion of brightness patterns in the image

Image Sequence
(2 frames)

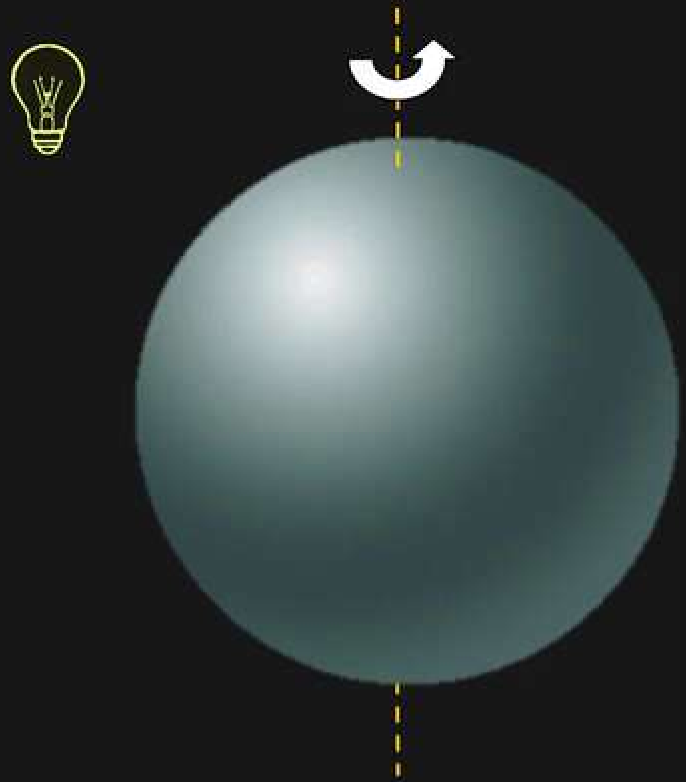


Optical Flow

Velocity of
brightness patt

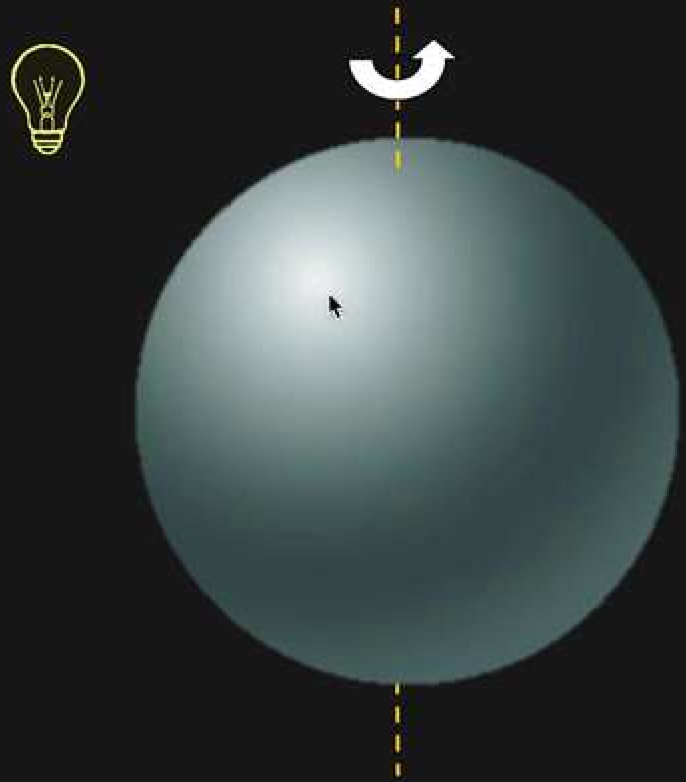
Ideally, Optical Flow = Motion Field

When is Optical Flow ≠ Motion Field?



Spinning Sphere
Stationary Light Source

When is Optical Flow ≠ Motion Field?



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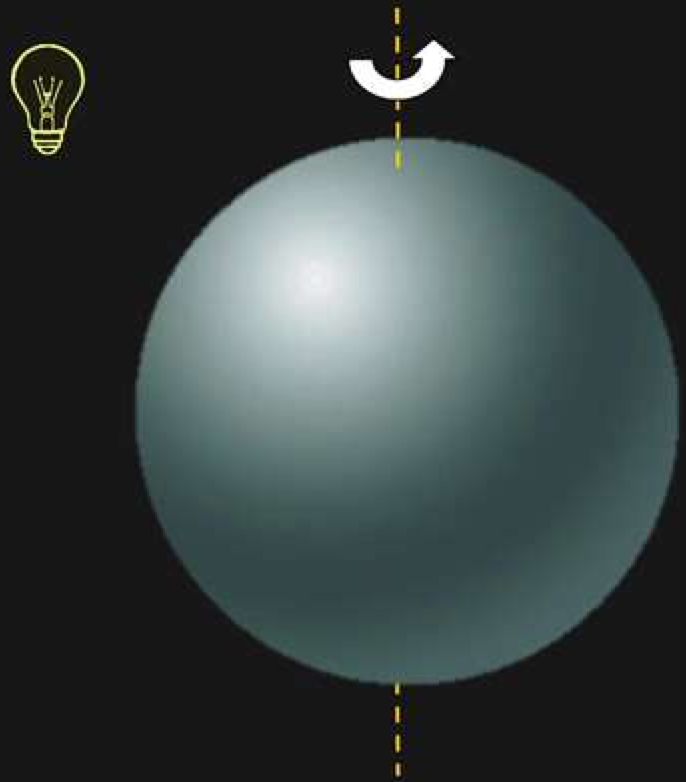

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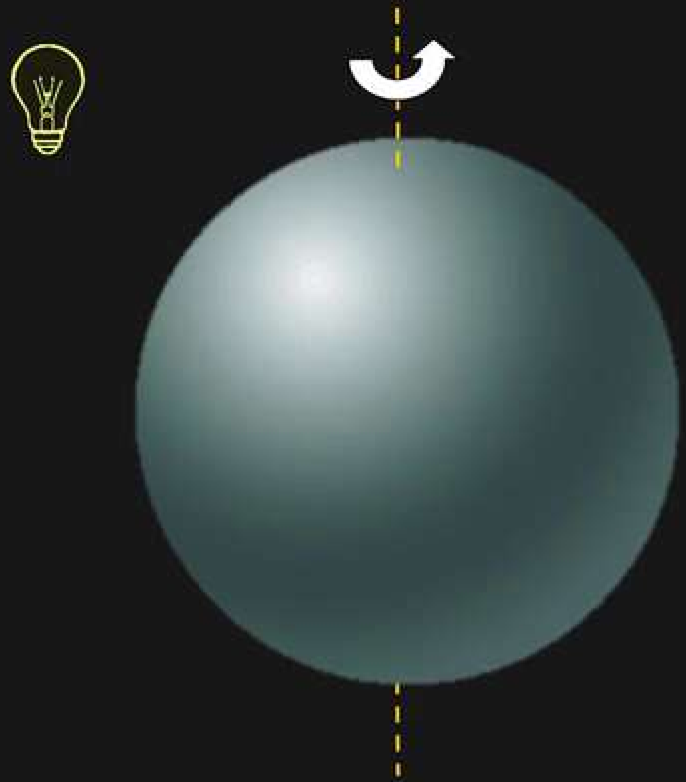

Spinning Sphere
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Motion Field exists
But no Optical Flow

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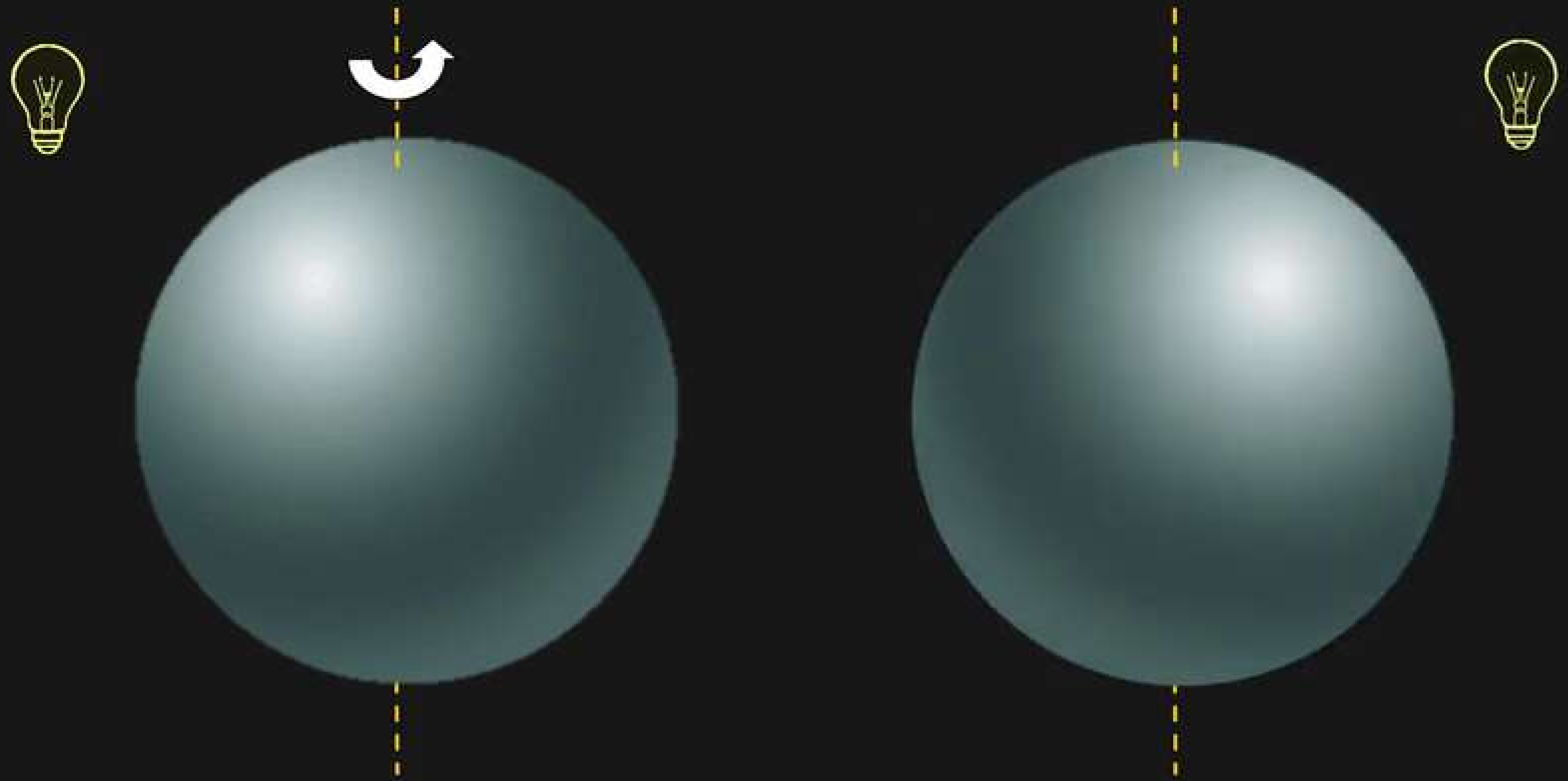

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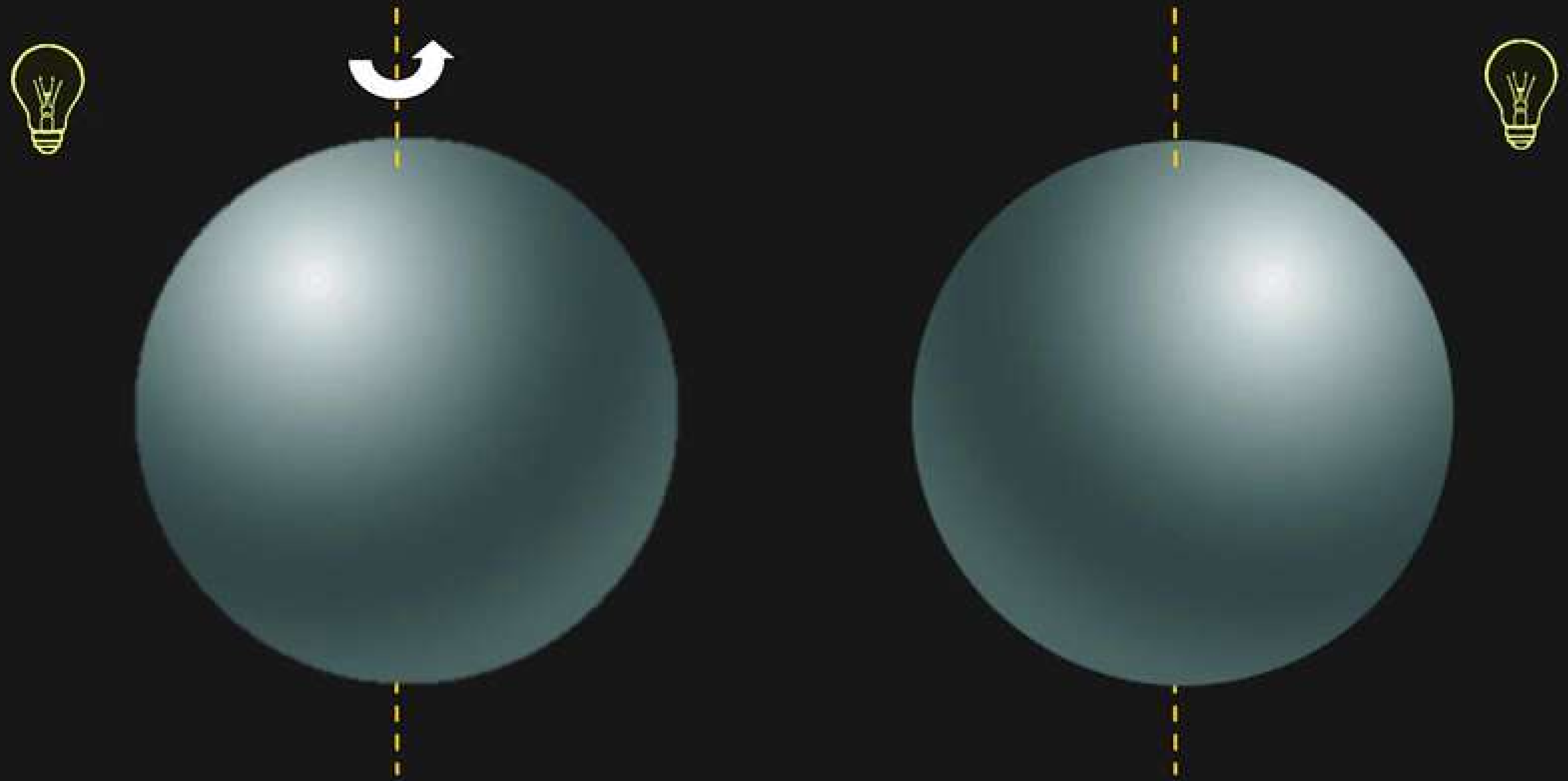

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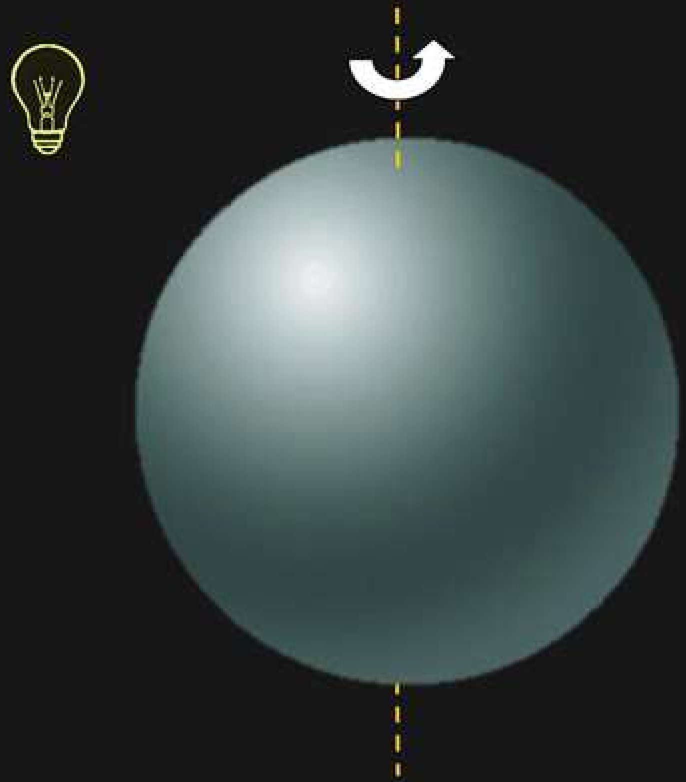

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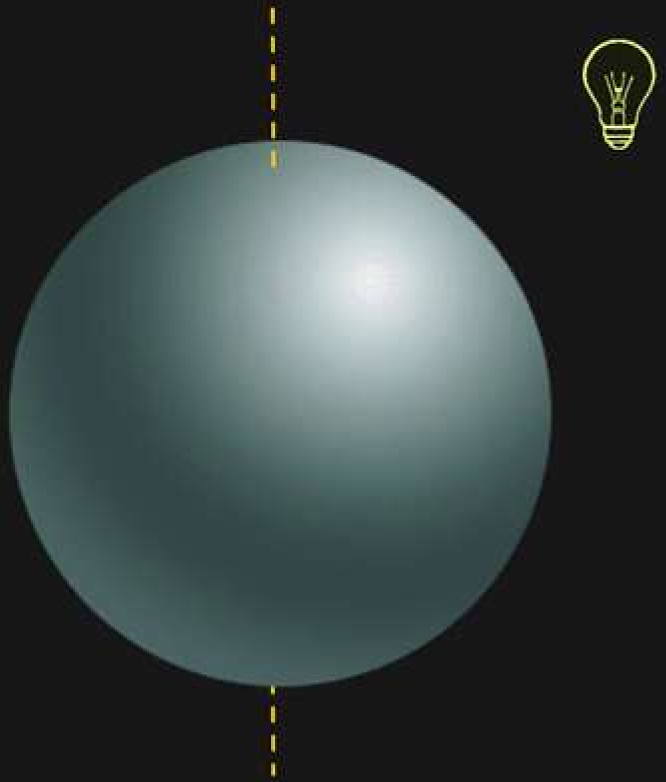

Stationary Sphere
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No Motion Field exists
But there is Optical Flow

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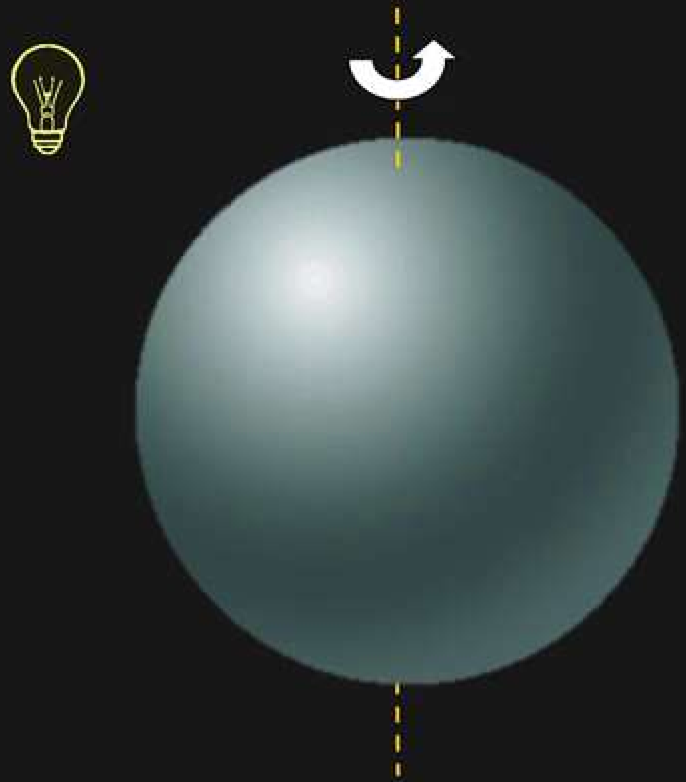

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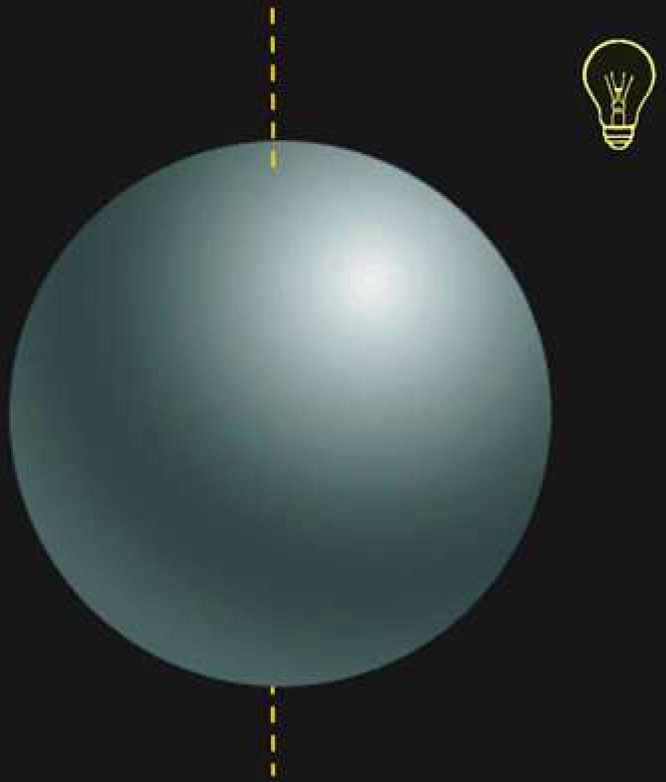

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When is Optical Flow ≠ Motion Field?

Barber Pole
Illusion

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Barber Pole
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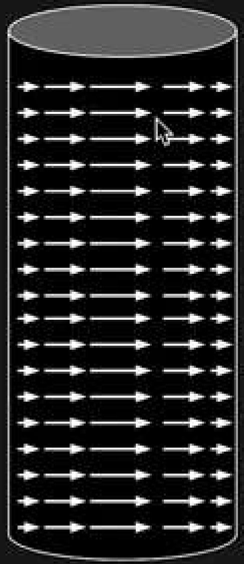

Motion Field

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Barber Pole
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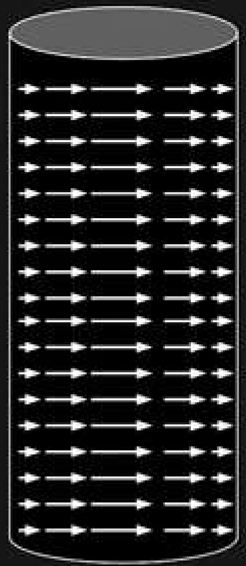

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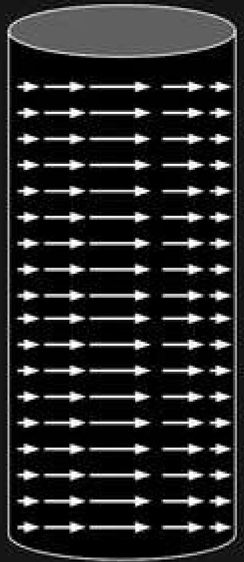

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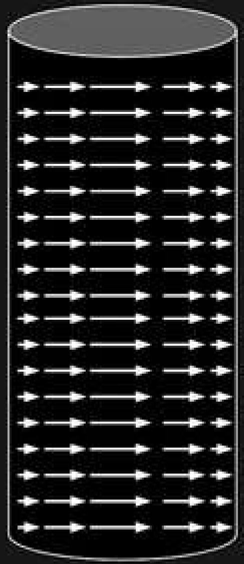

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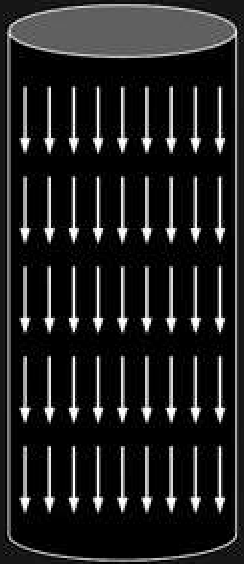

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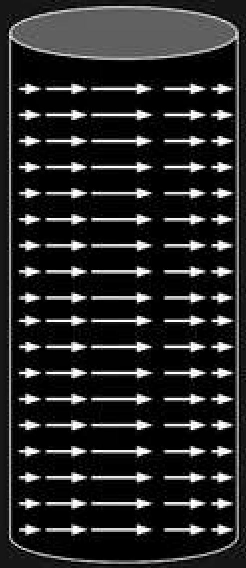

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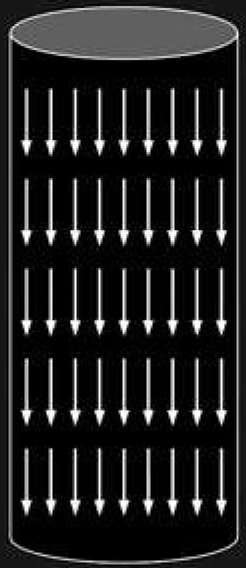
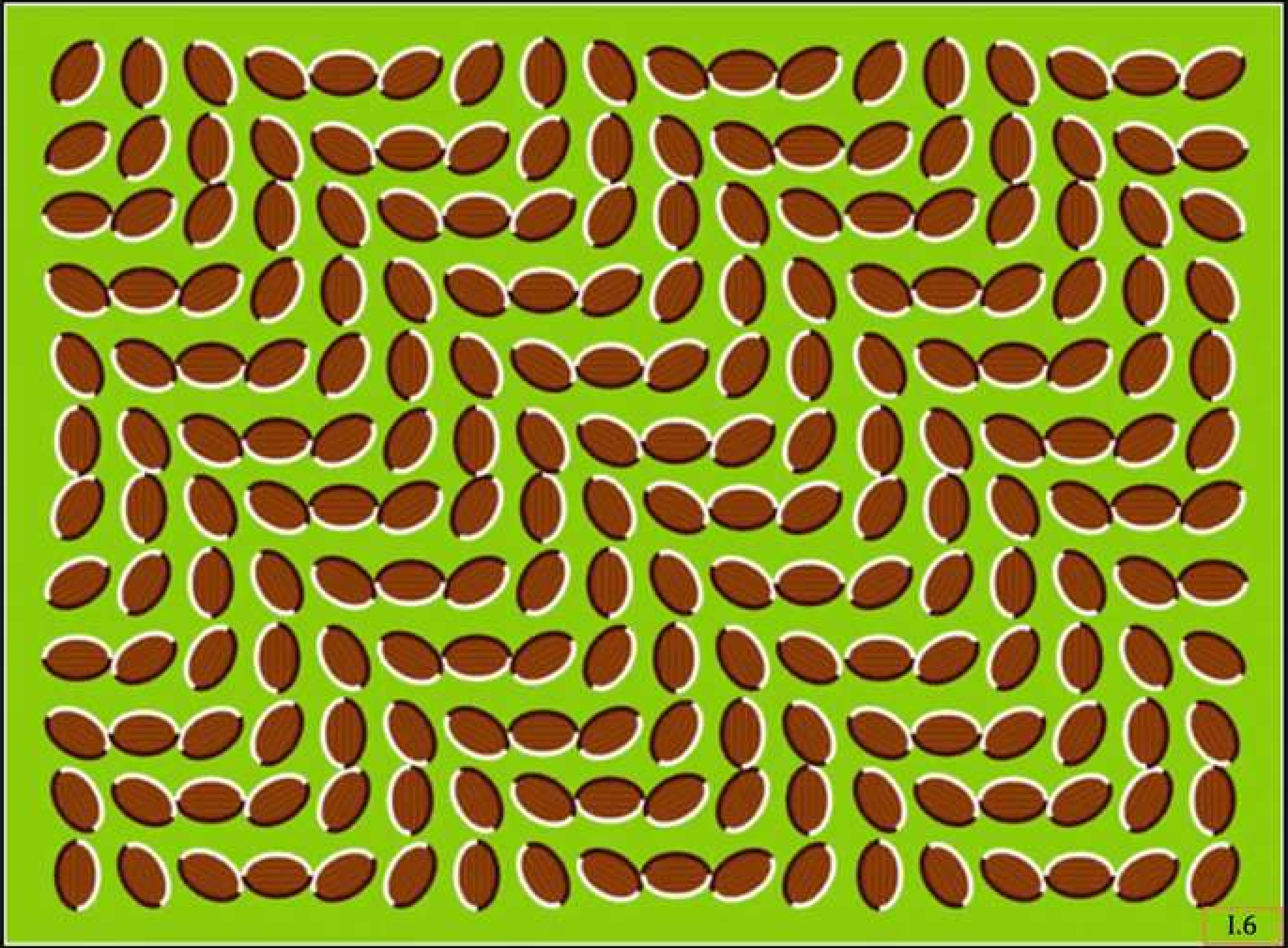


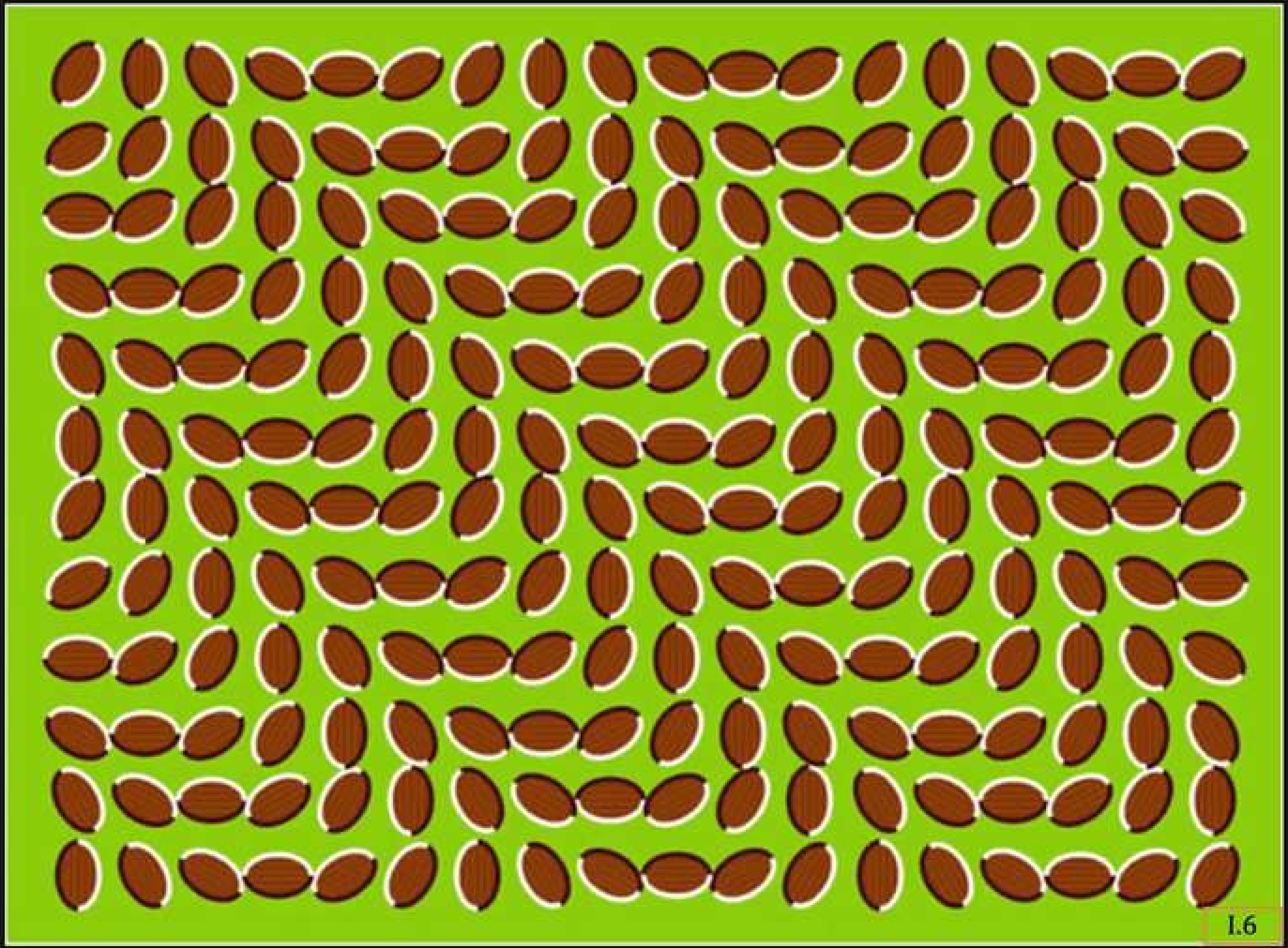
Optical Flow

Motion Illusions



Donguri Wave Illusion

Motion Illusions



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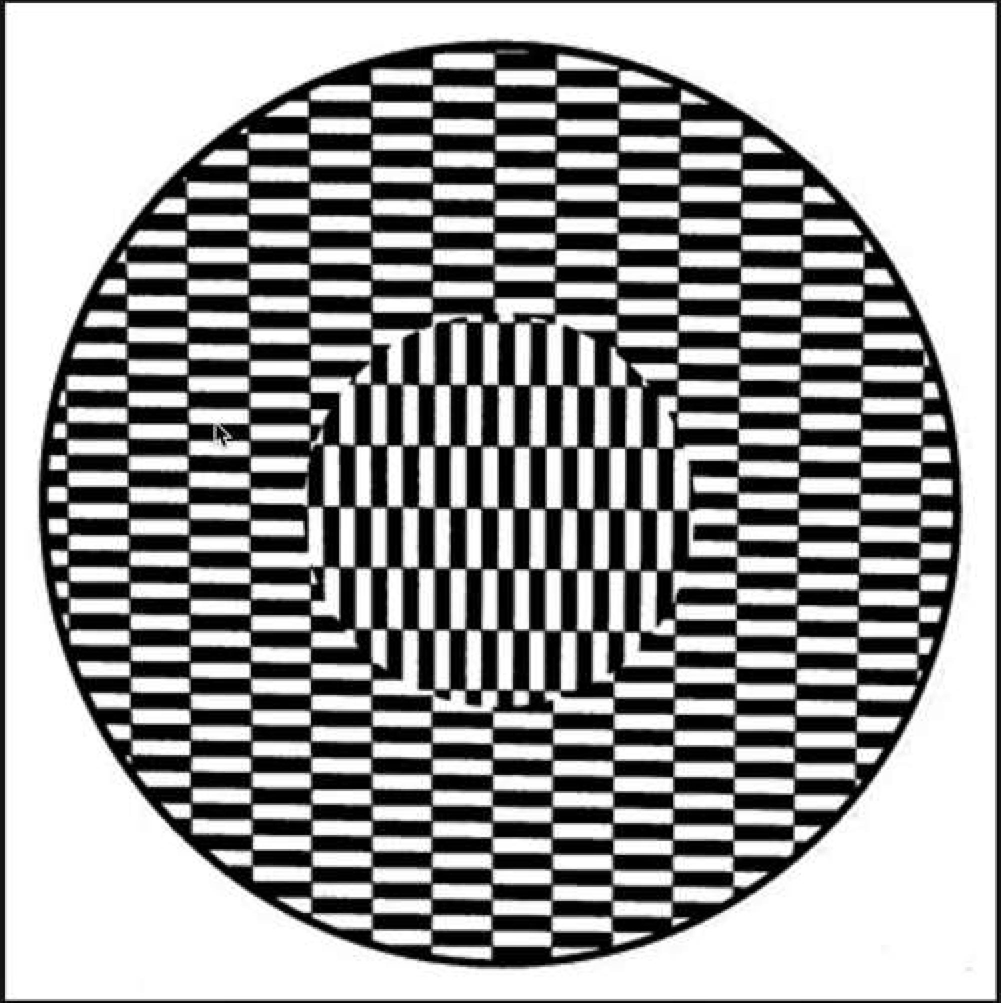
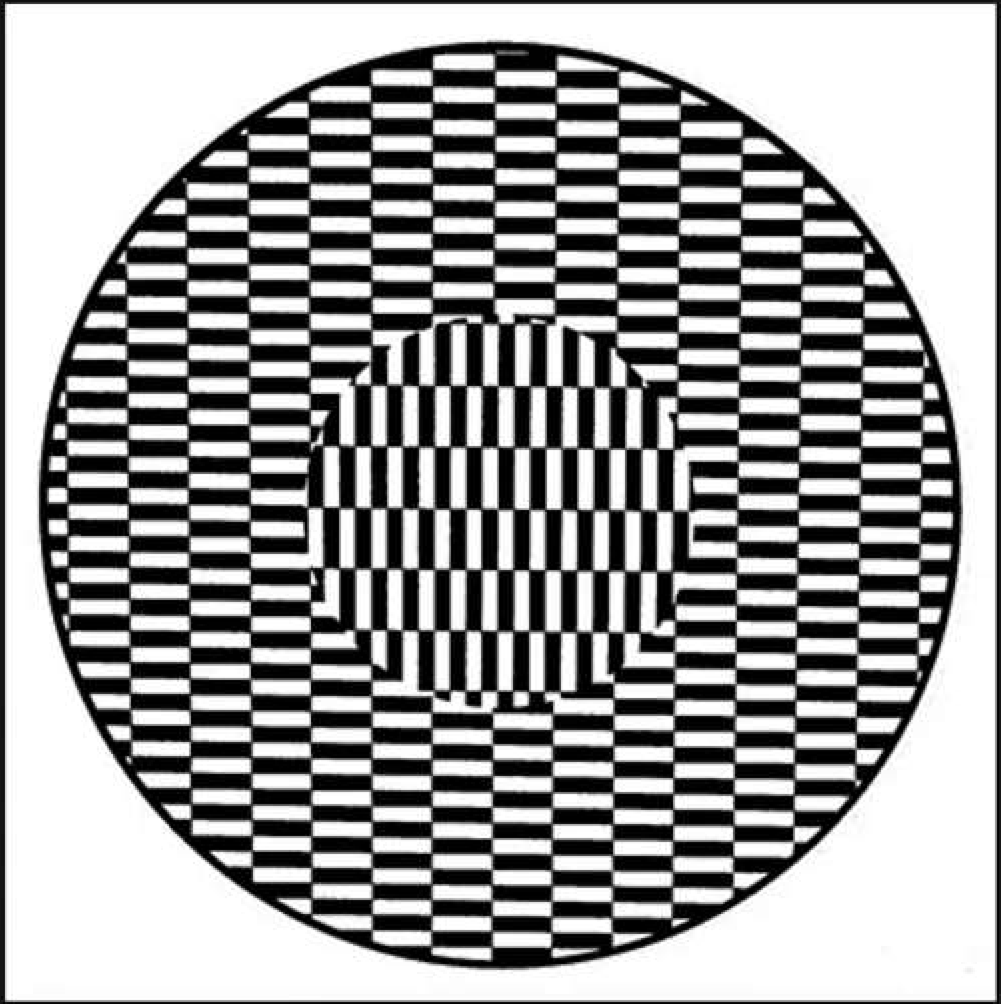


Ouchi Pattern

Motion Illusions



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