Simple Stereo

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Columbia University

Topic: Camera Calibration, Module: Reconstruction II

First Principles of Computer Vision

Backward Projection: From 2D to 3D

Given a calibrated camera, can we find the 3D scene point from a single 2D image?

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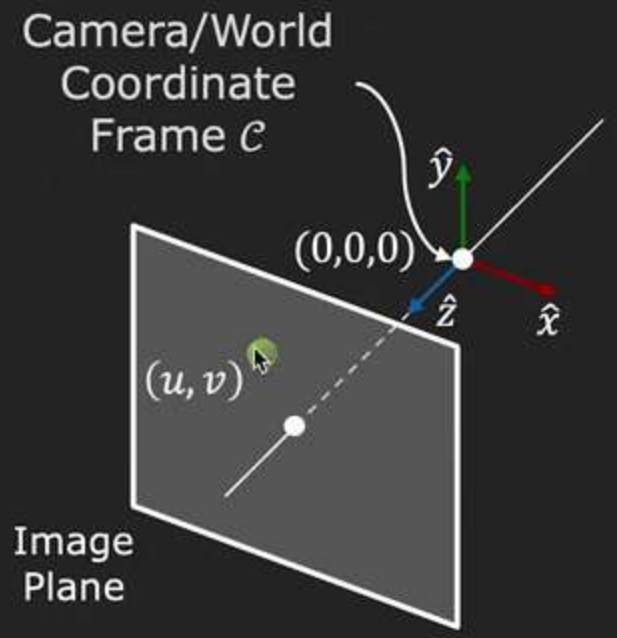

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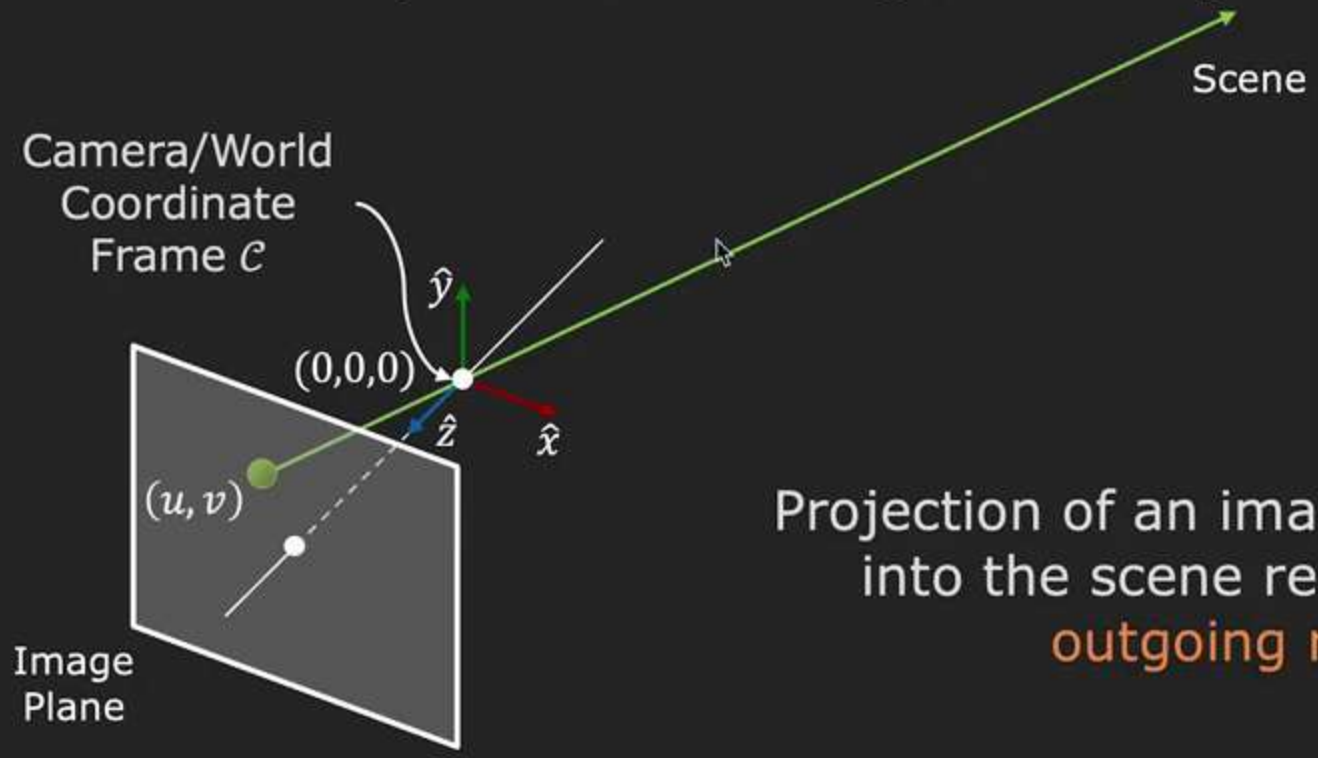
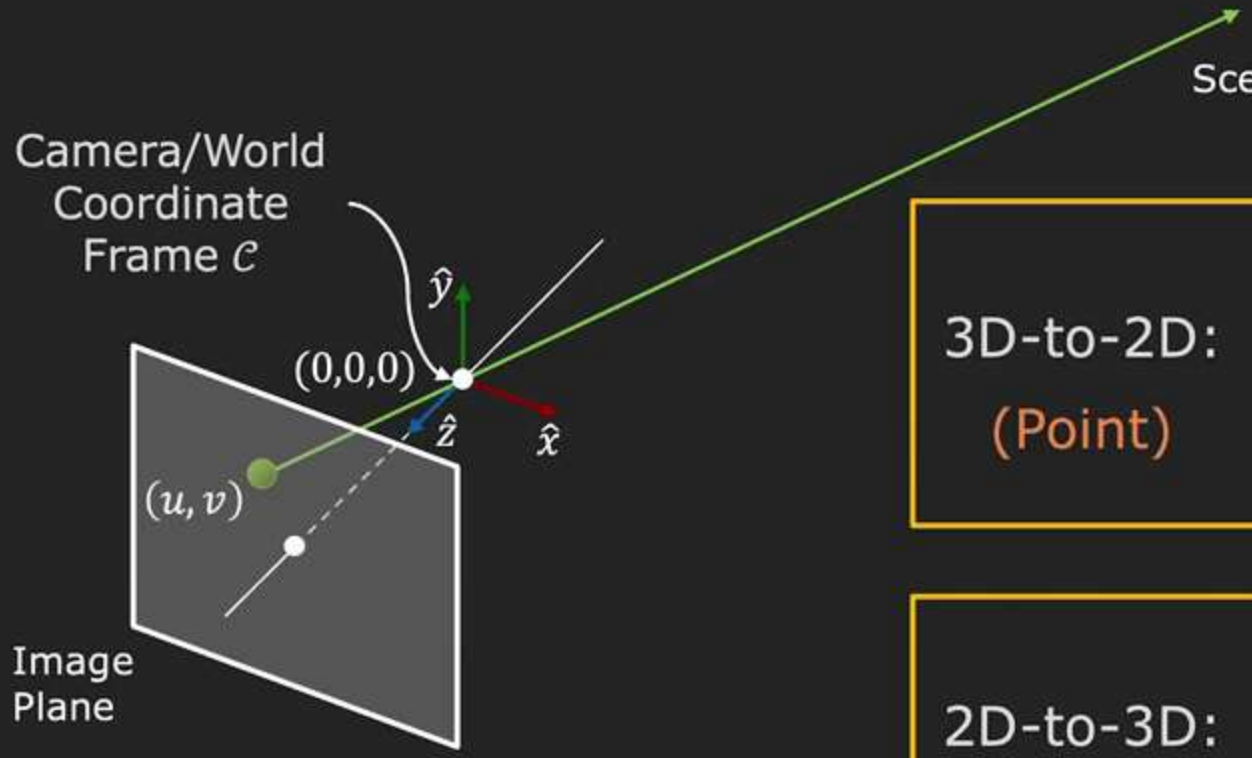


Projection of an image point back into the scene results in an outgoing ray.

Computing 2D-to-3D Outgoing Ray



3D-to-2D: (Point)

$$u = f_x \frac{x_c}{z_c} + o_x$$

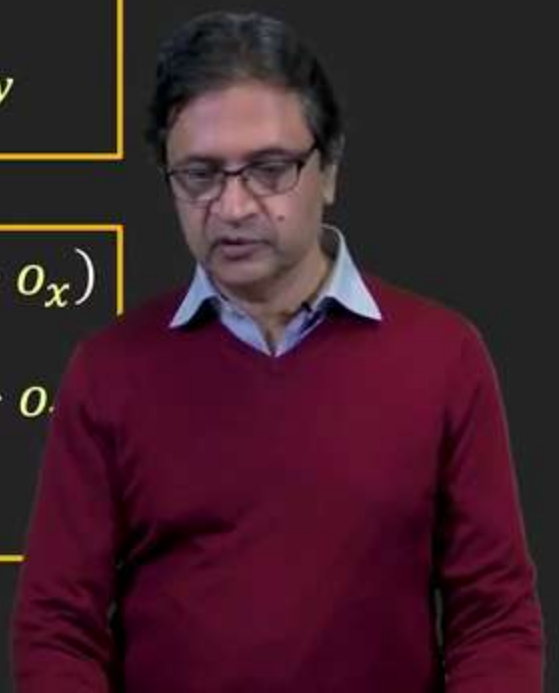
$$v = f_y \frac{y_c}{z_c} + o_y$$

2D-to-3D: (Ray)

$$x = {}^{z}/_{f_x}\,(u - o_x)$$

$$y = {}^{z}/_{f_y}\,(v - o_y)$$

$$z > 0$$



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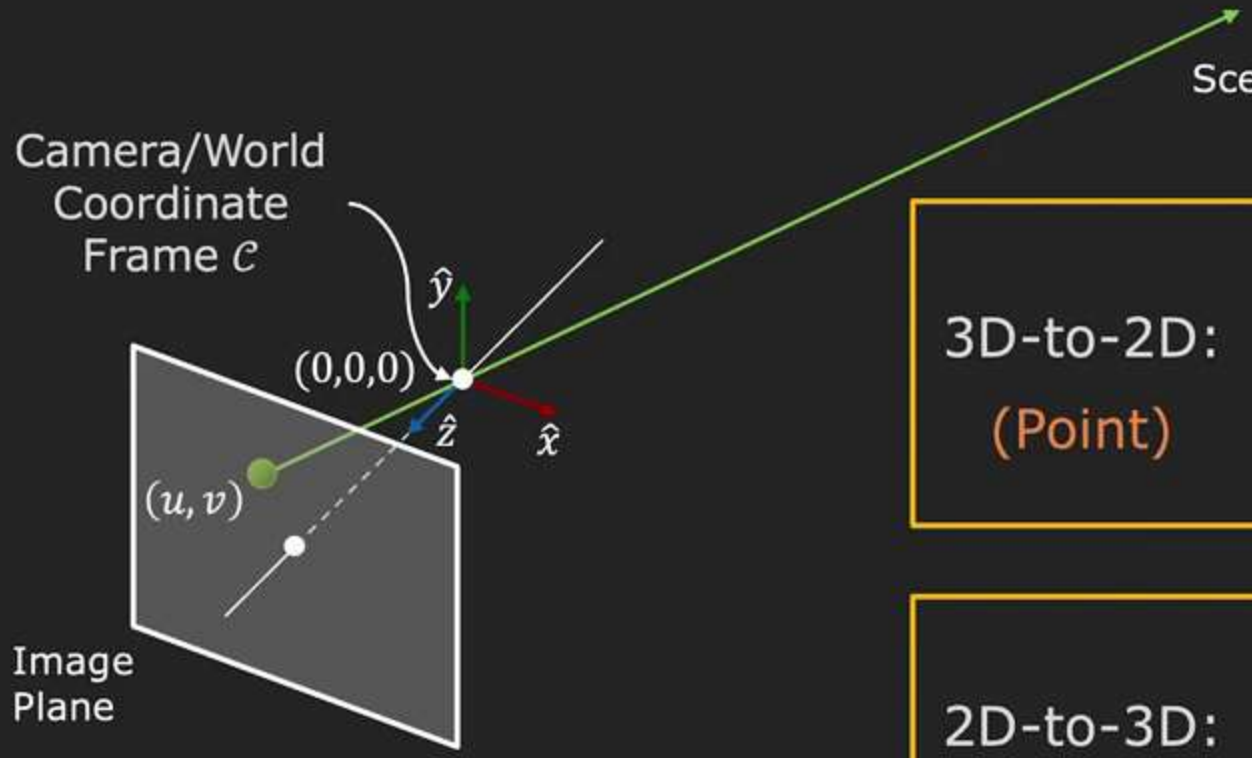

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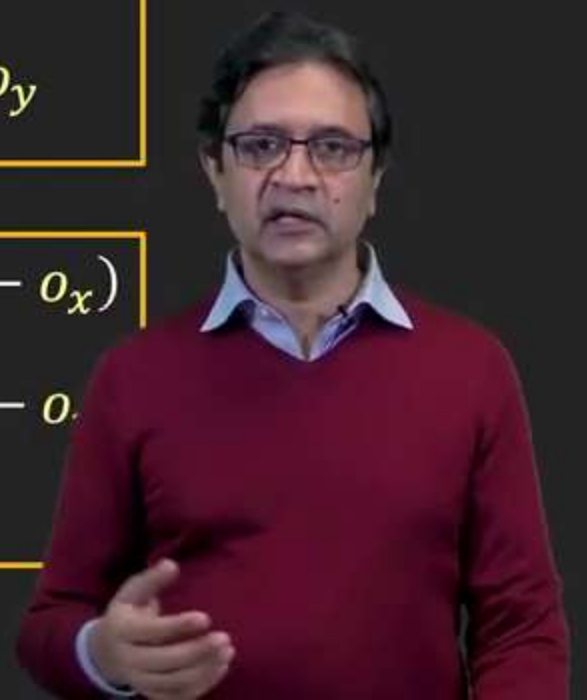
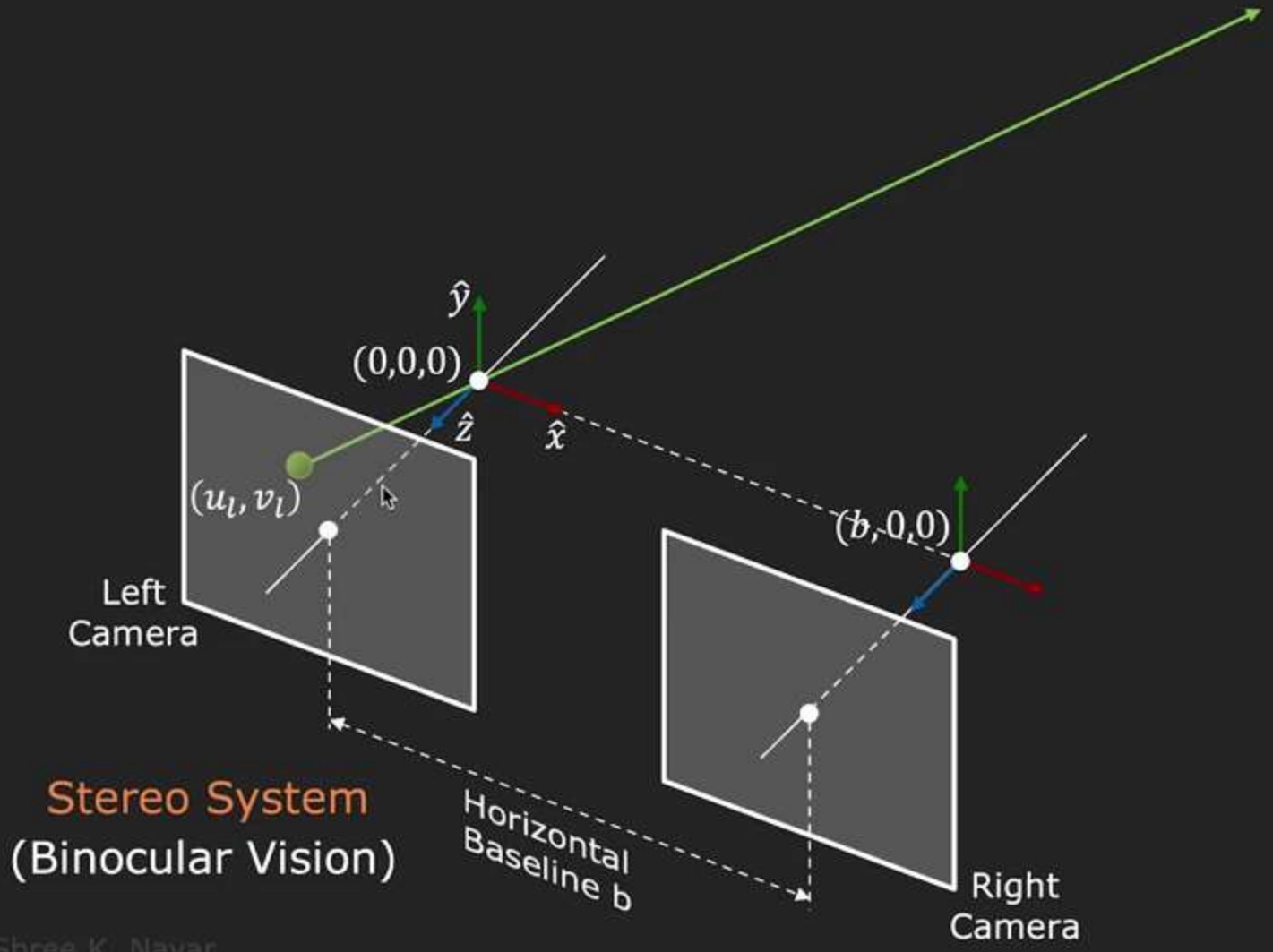


Triangulation using Two Cameras



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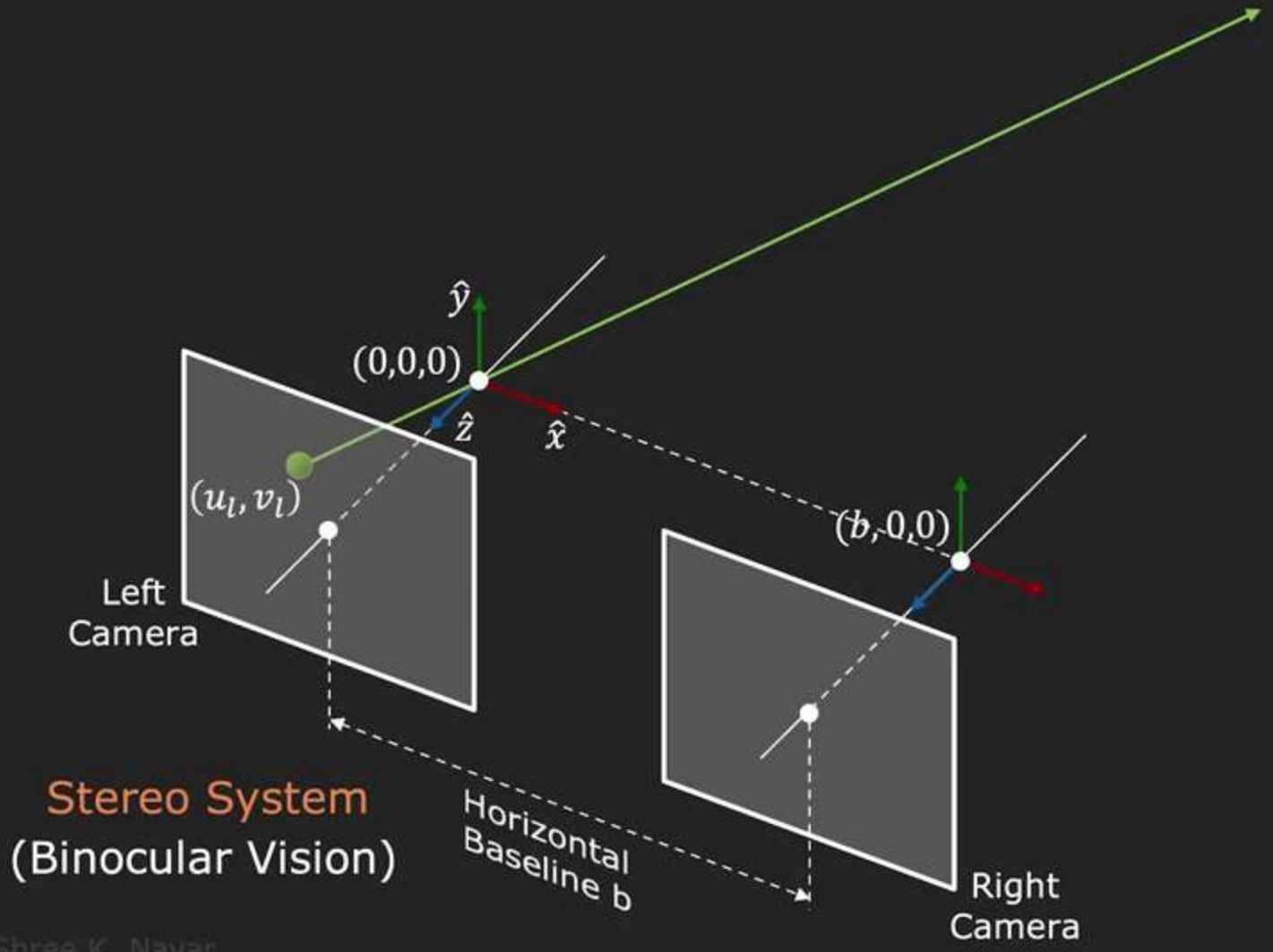

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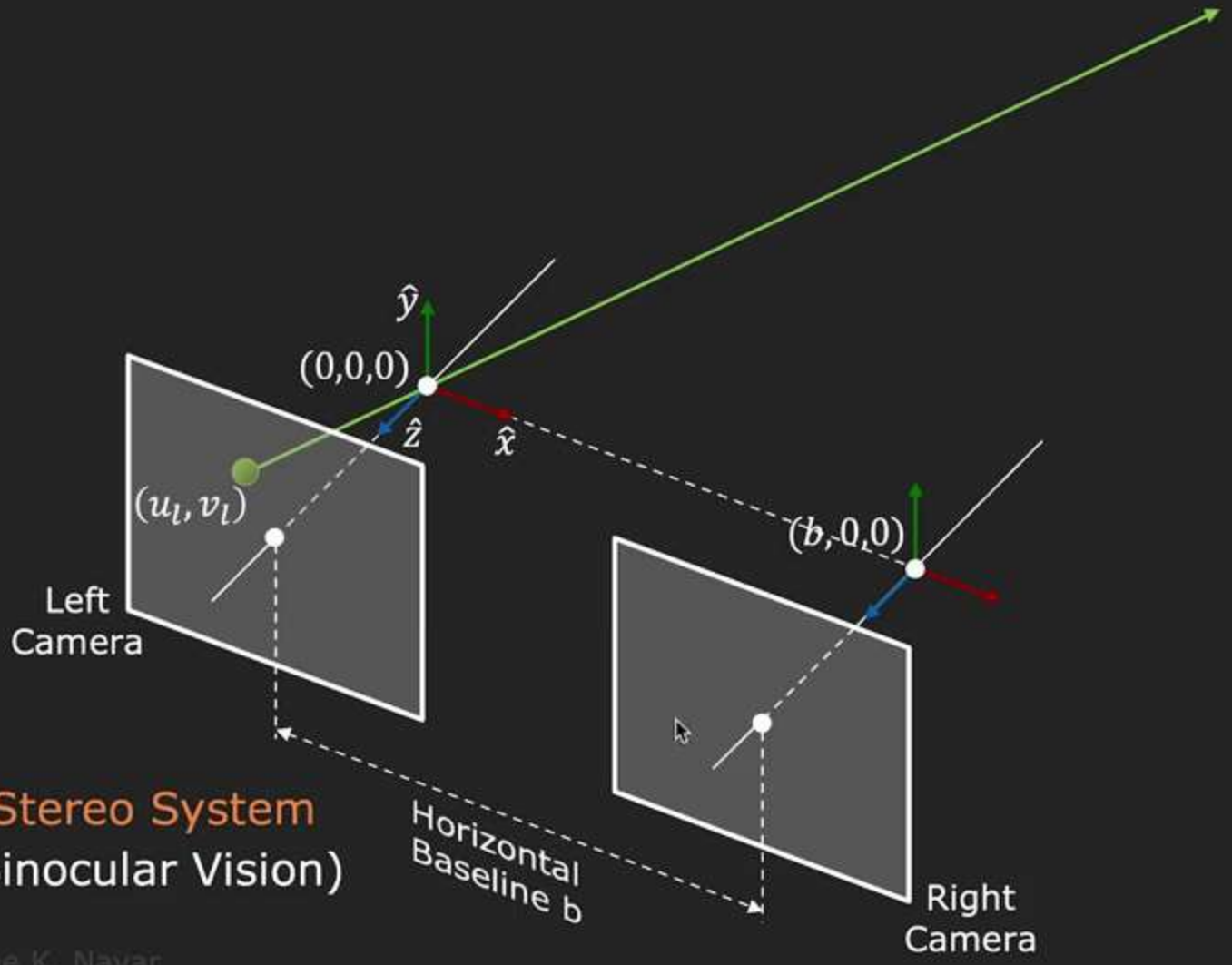

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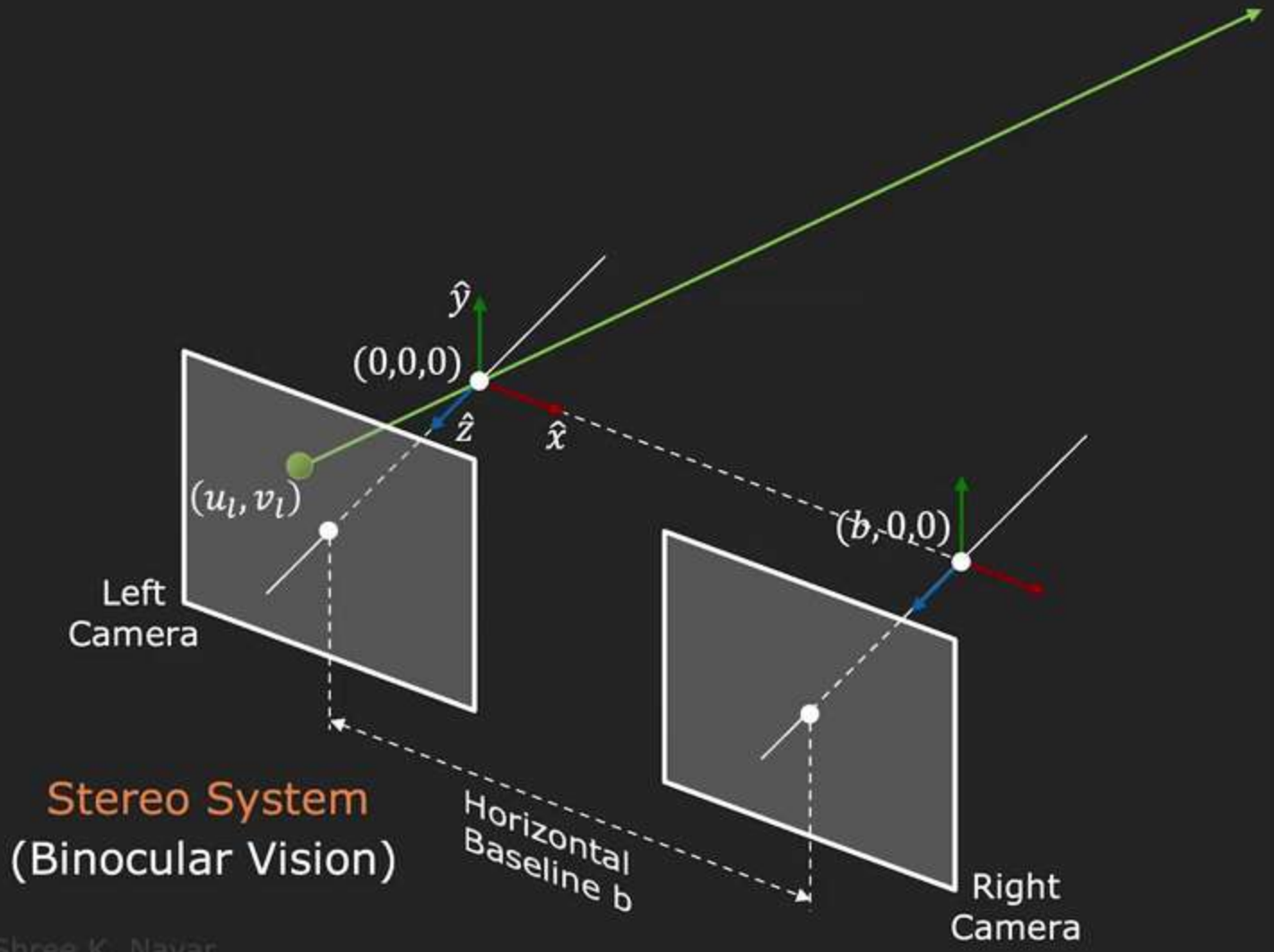

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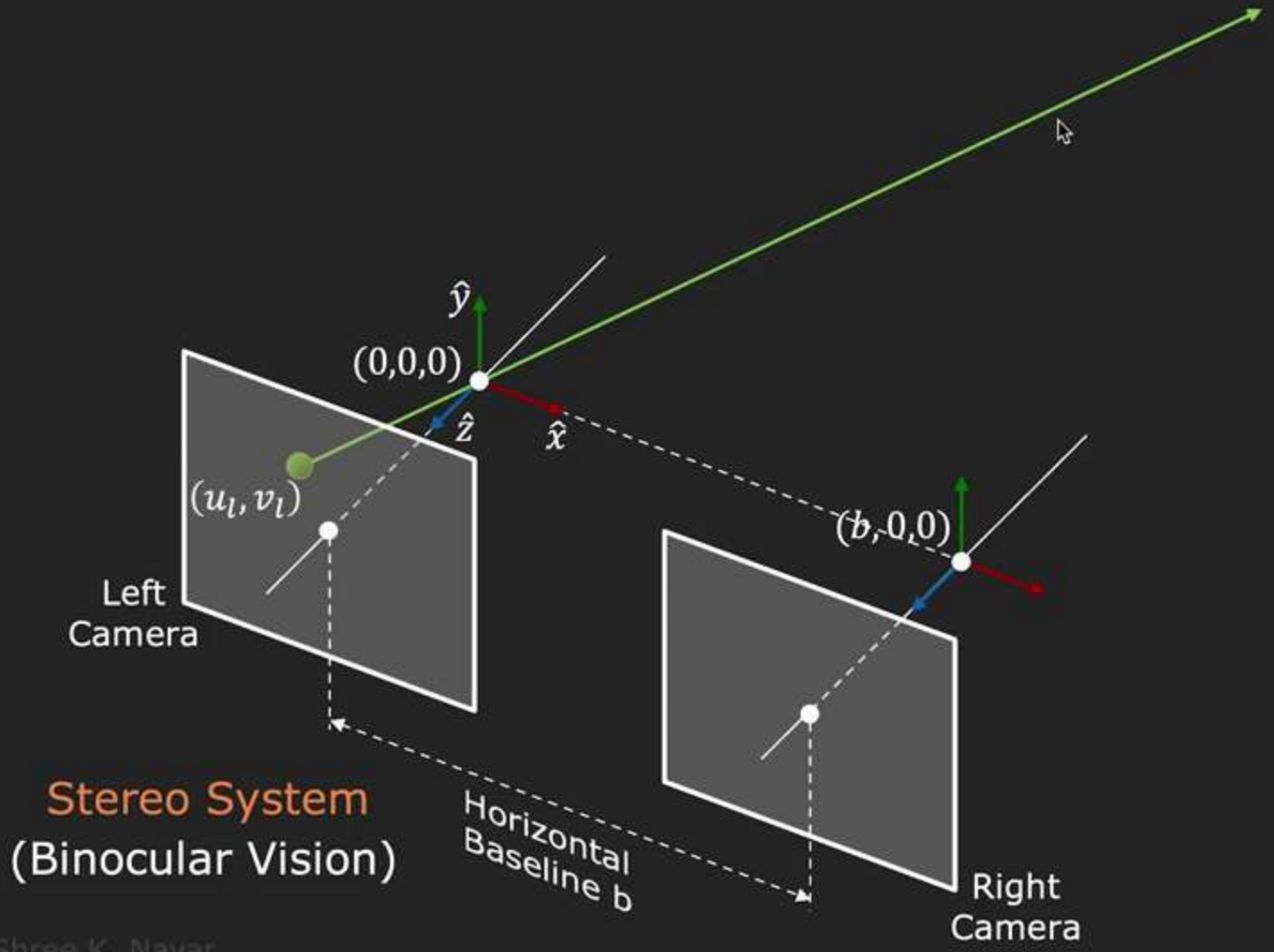

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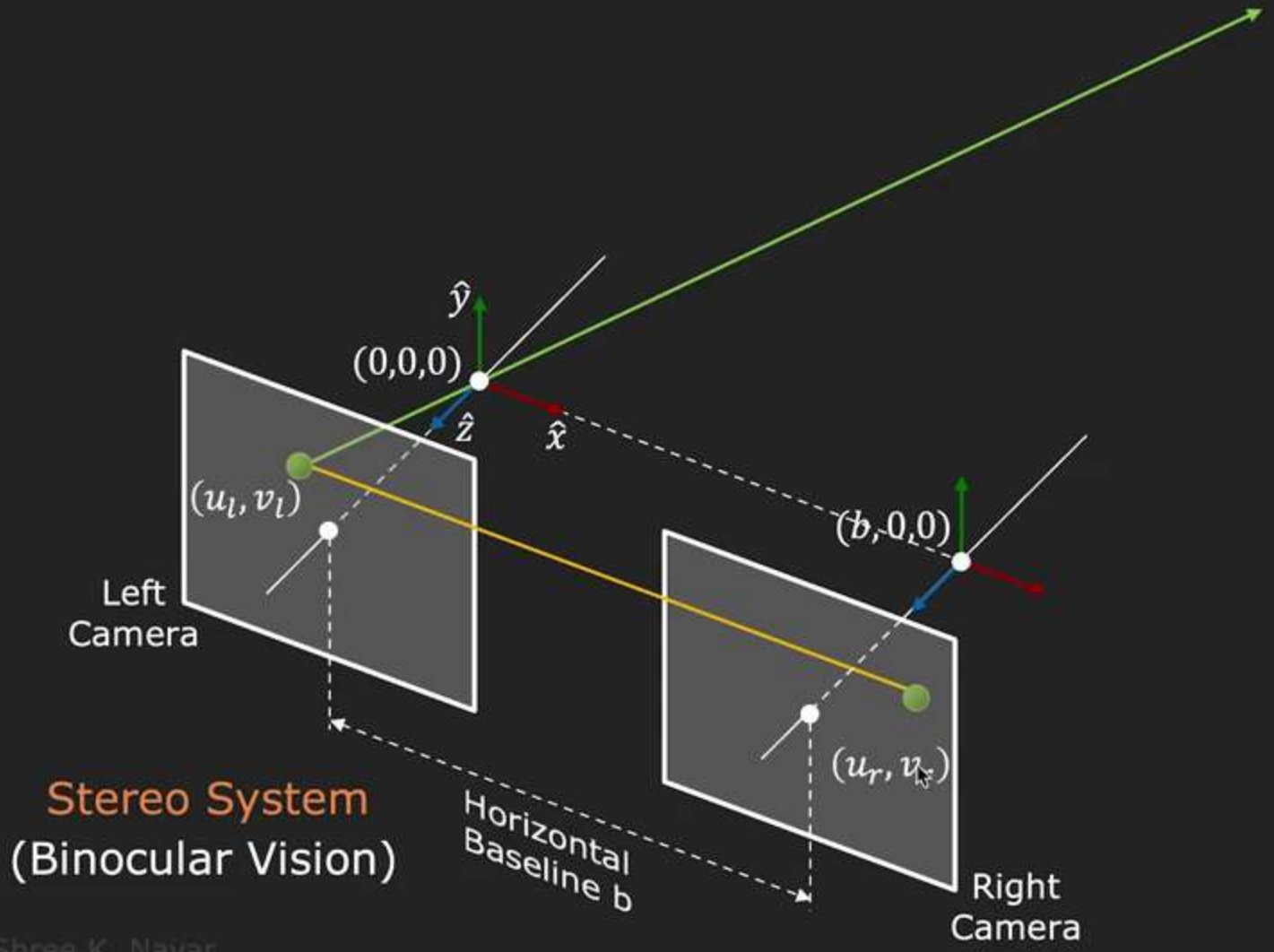
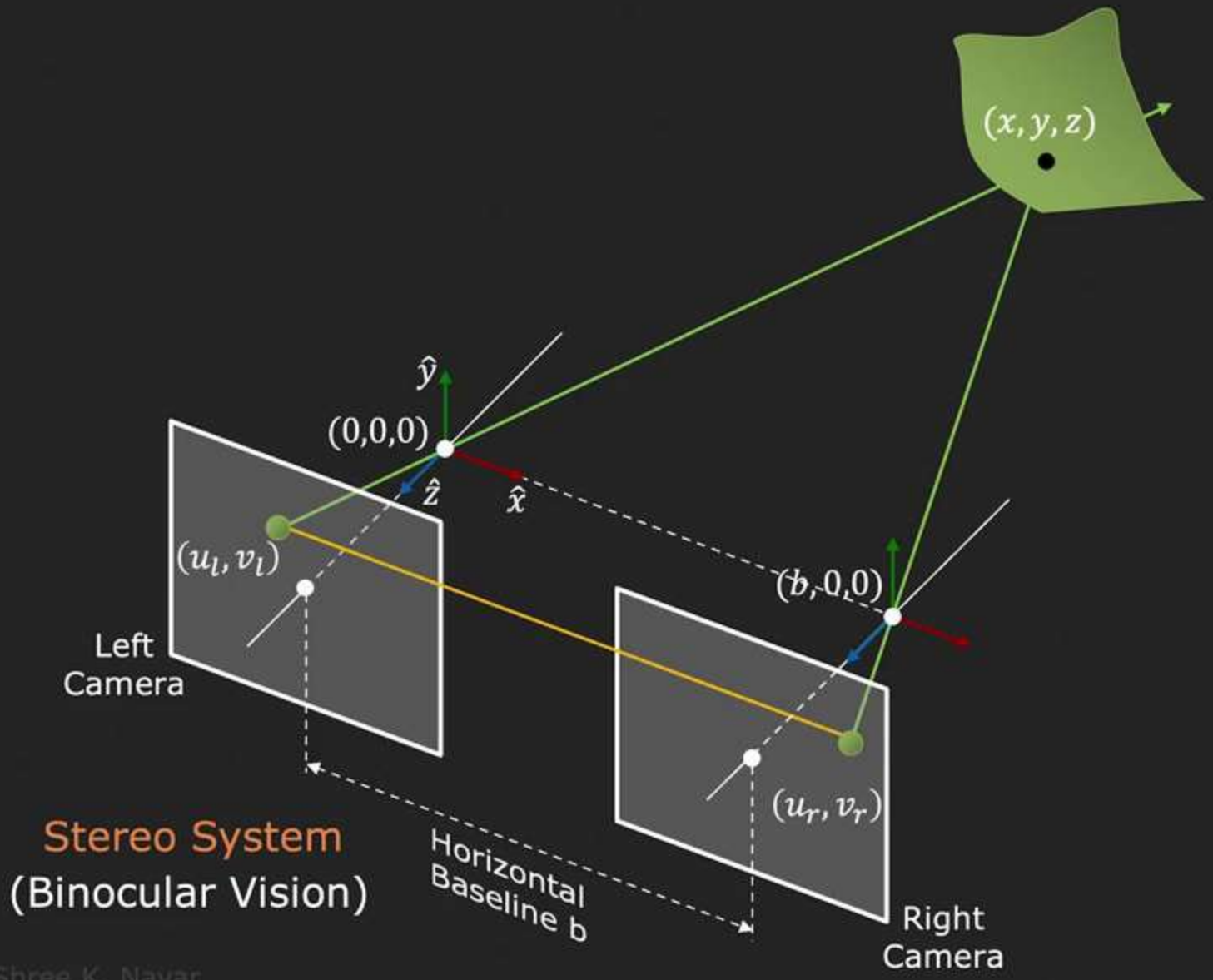


Stereo System
(Binocular Vision)

Triangulation using Two Cameras



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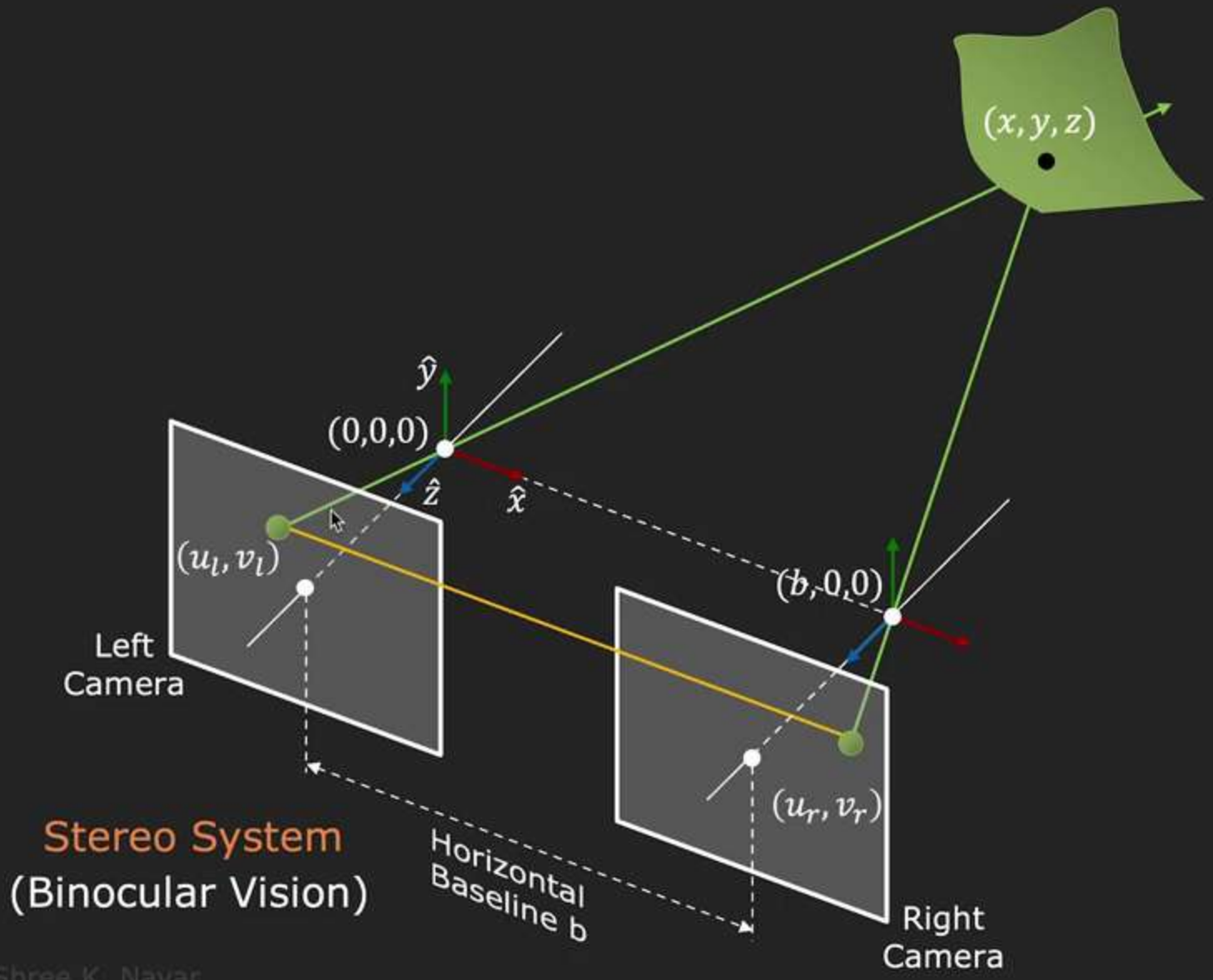

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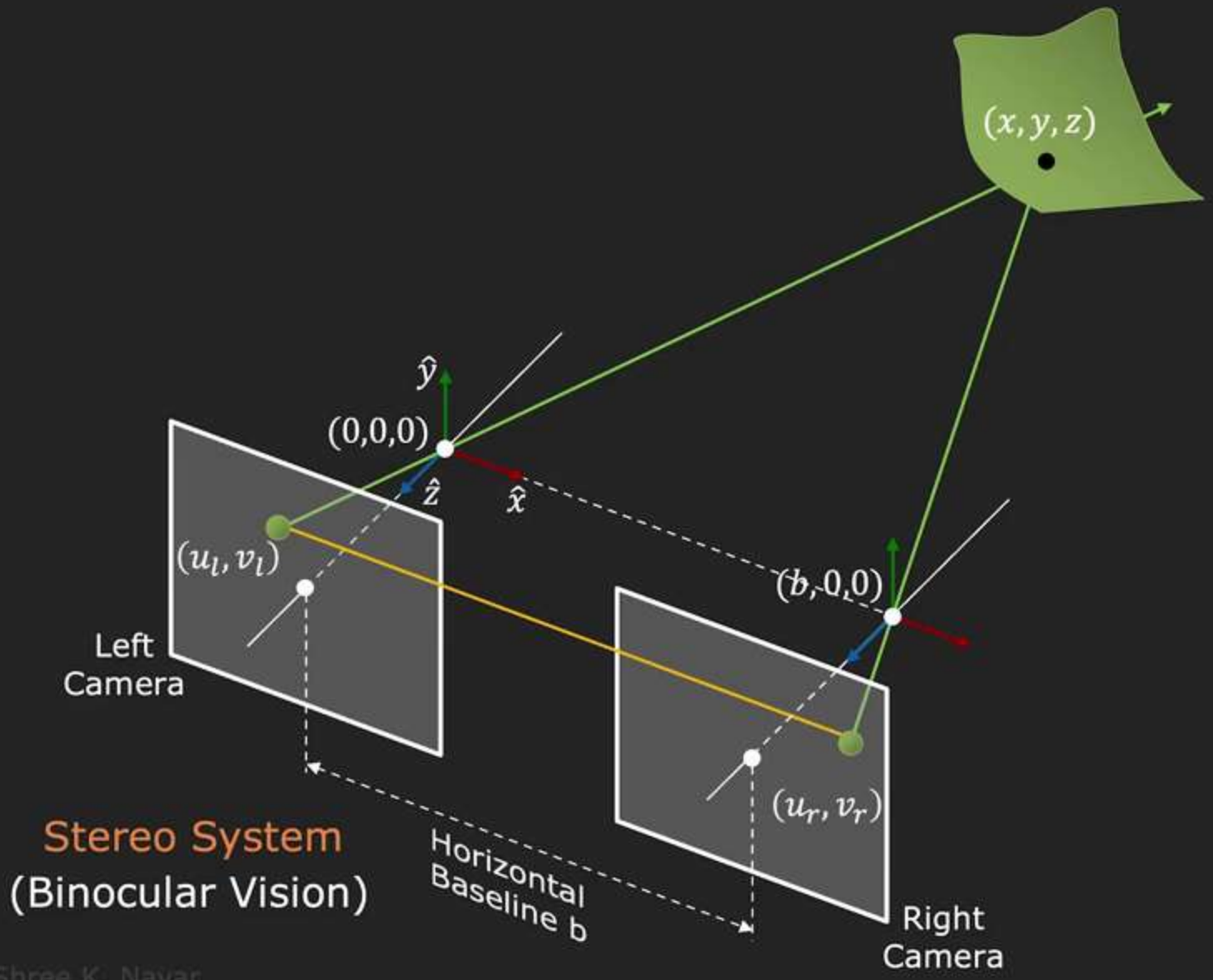

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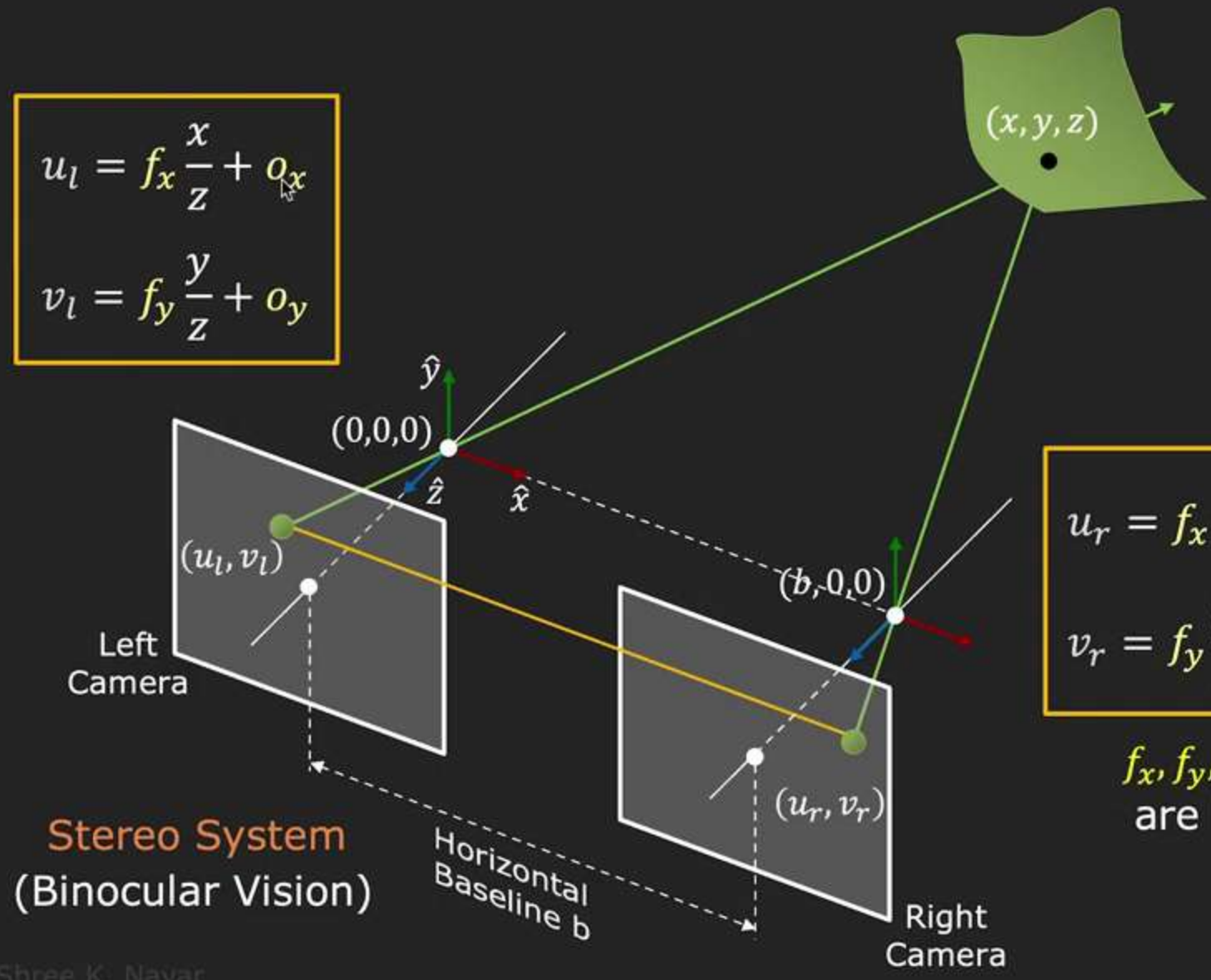

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$$u_r = f_x \frac{x - b}{z} + o_x$$

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f_x, f_y, b, o_x, o_y are known.

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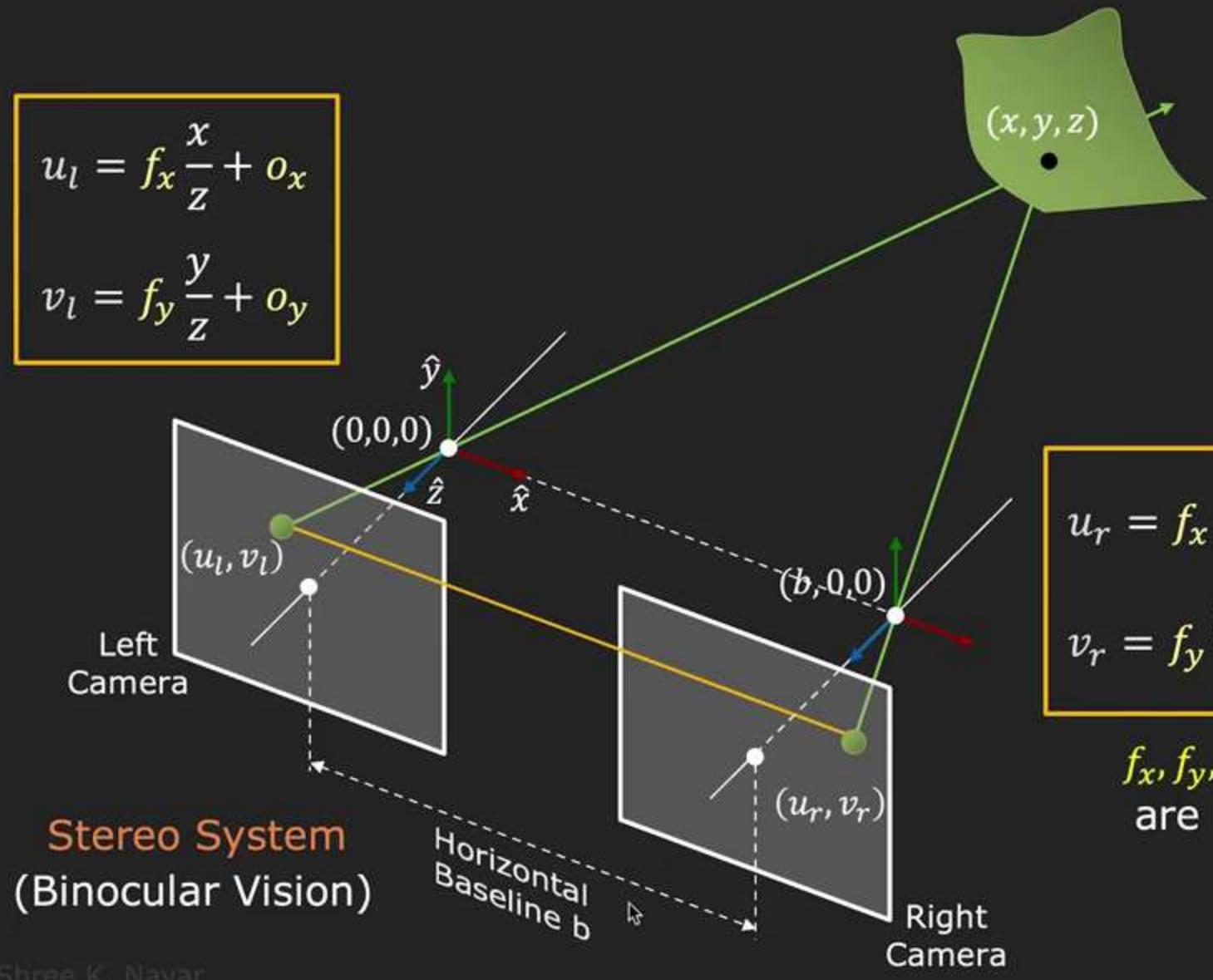

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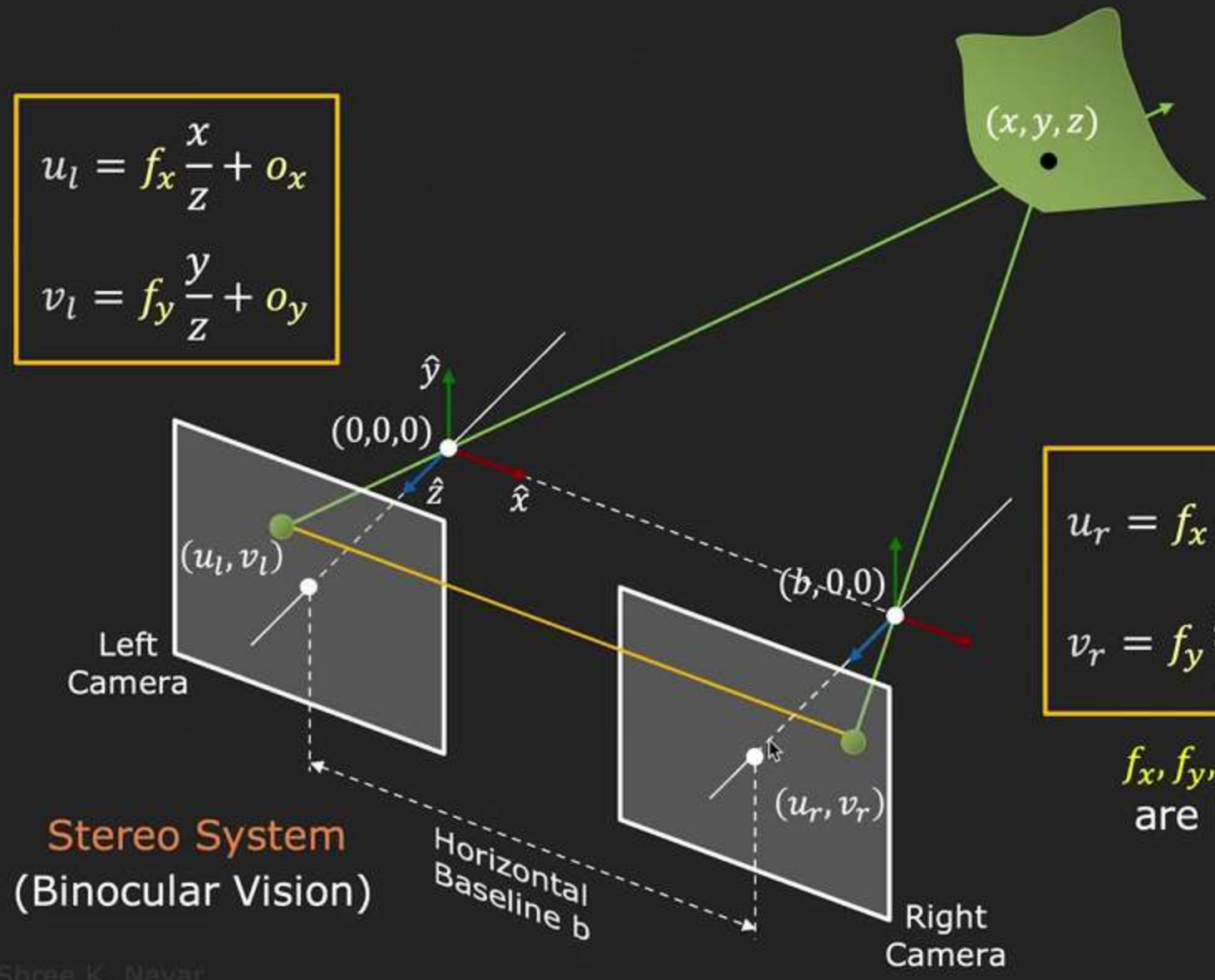

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Simple Stereo: Depth and Disparity

From perspective projection:

$$(u_l, v_l) = \left(f_x \frac{x}{z} + o_x, f_y \frac{y}{z} + o_y\right) \qquad (u_r, v_r) = \left(f_x \frac{x-b}{z} + o_x, f_y \frac{y}{z} + o_y\right)$$

Solving for (x, y, z):

$$x = \frac{b(u_l - o_x)}{(u_l - u_r)} \qquad y = \frac{b f_x (v_l - o_y)}{f_y (u_l - u_r)} \qquad \boxed{z = \frac{b f_x}{(u_l - u_r)}}$$

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A Simple Stereo Camera

I.2

Fujifilm FinePix REAL 3D W3

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Stereo Matching: Finding Disparities

Goal: Find the disparity between left and right stereo pairs.

Left/Right Camera Images

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Corresponding scene points lie on the same horizontal scan line

Window Based Methods

Determine Disparity using Template Matching

Template Window T

Left Camera Image E_l

Right Camera Image E_r

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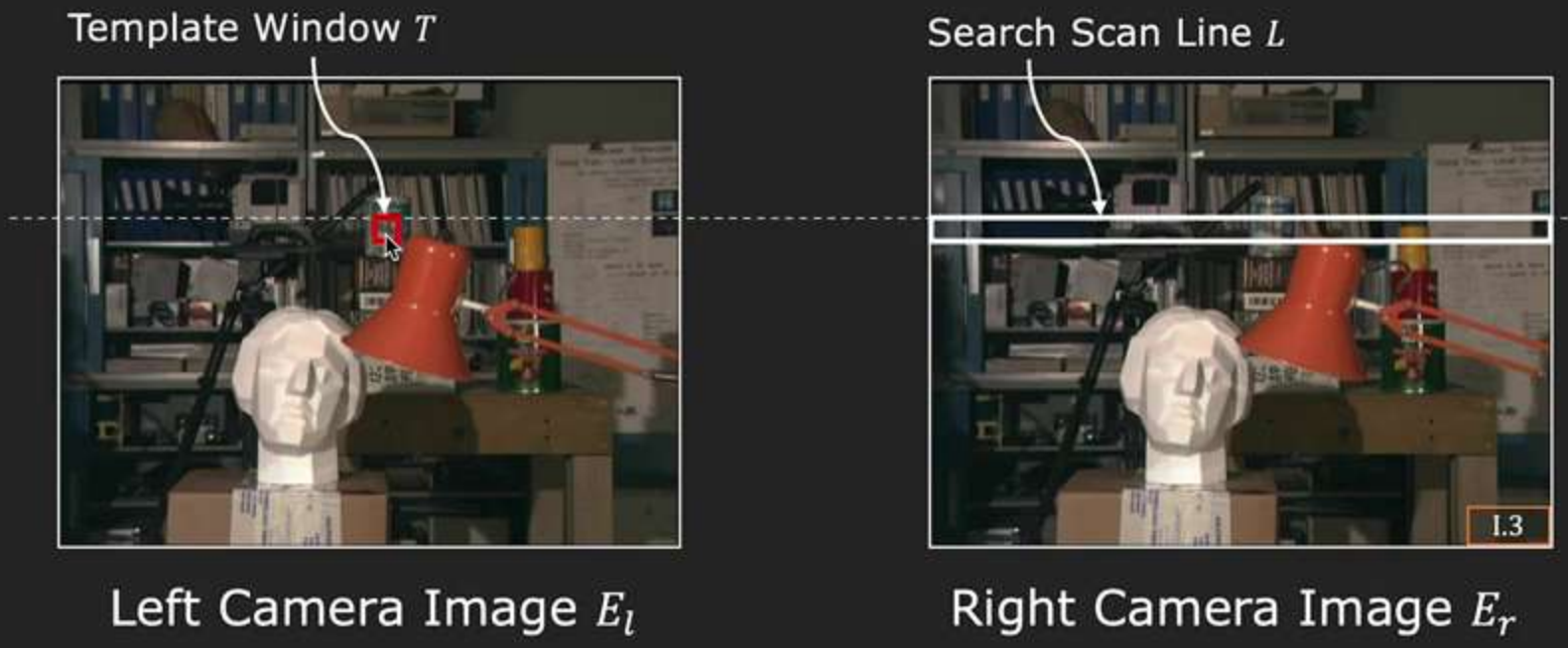

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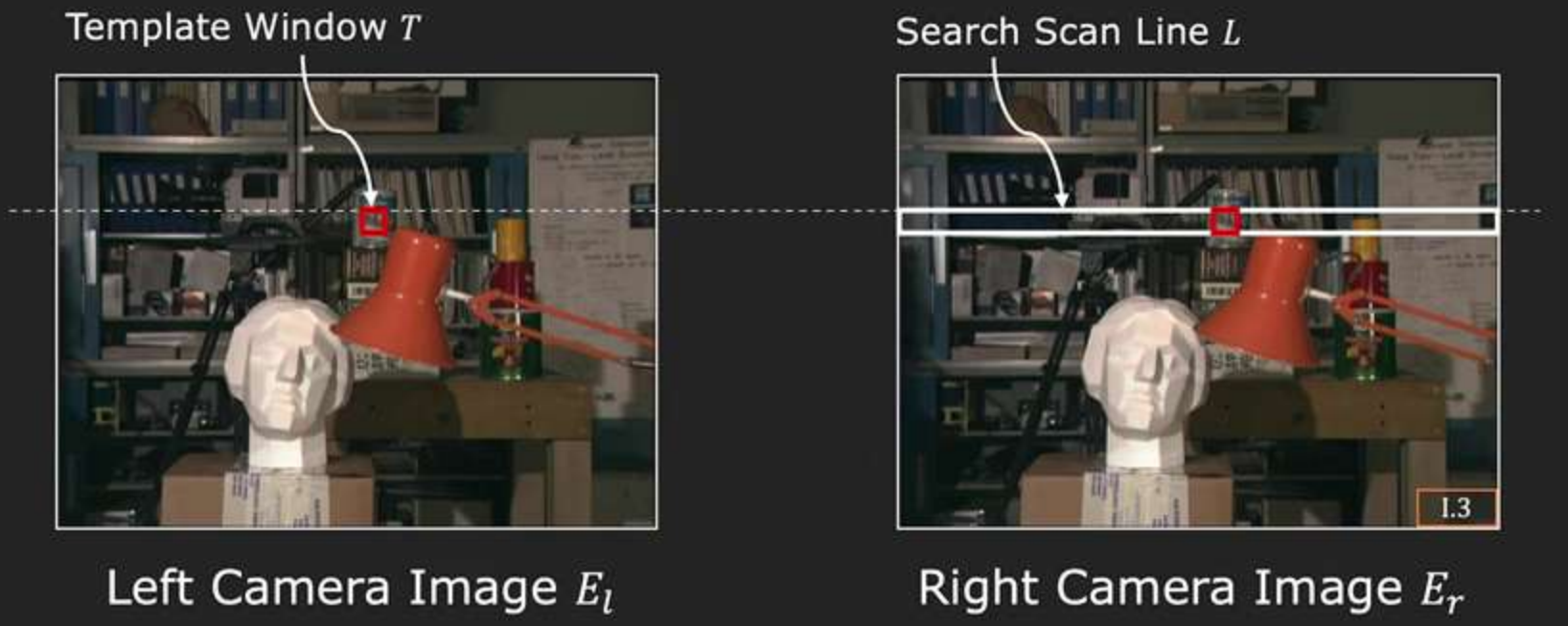

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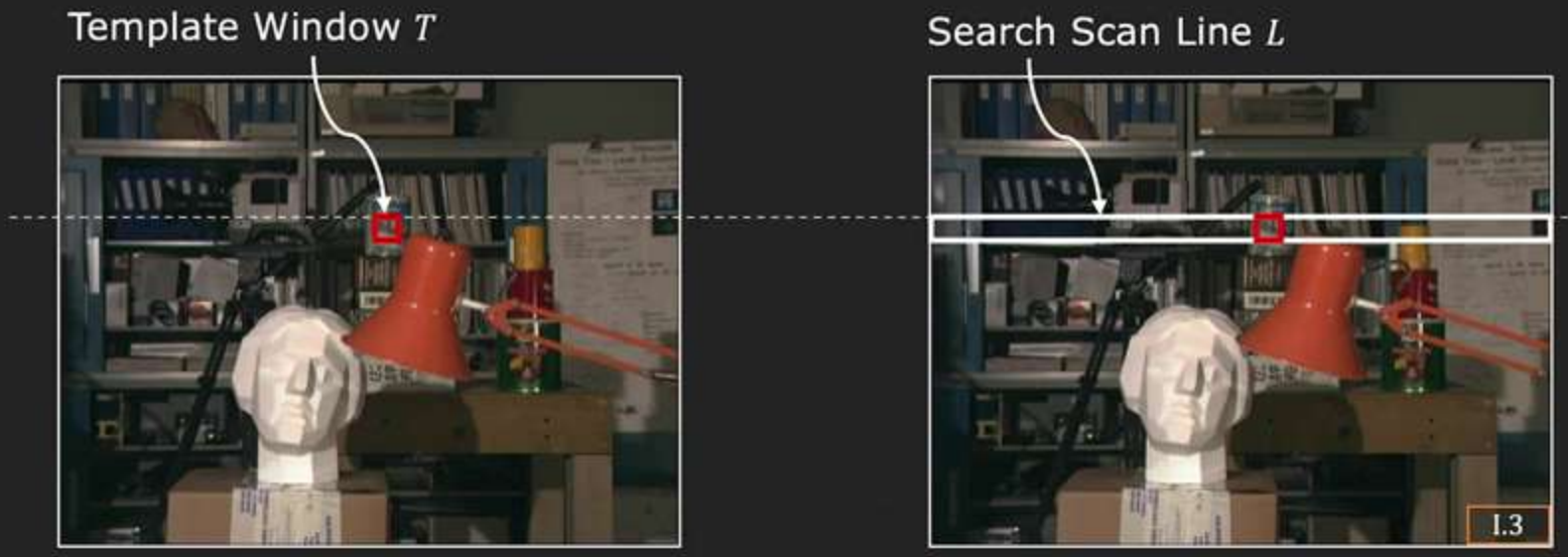

Left Camera Image E_l

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Disparity: $d = u_l - u_r$

Depth: $z = \frac{bf_x}{(u_l - u_r)}$

Similarity Metrics for Template Matching

Find pixel $(k, l) \in L$ with Minimum Sum of Absolute Differences:

$$SAD(k,l) = \sum_{(i,j)\in T} |E_l(i,j) - E_r(i+k, j+l)|$$

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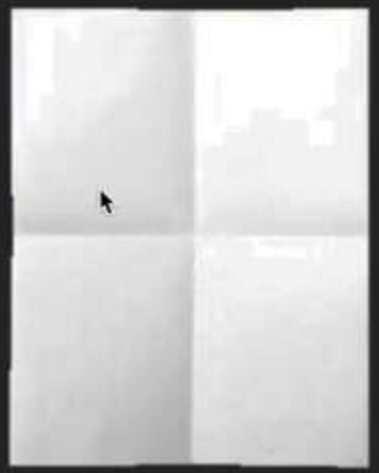

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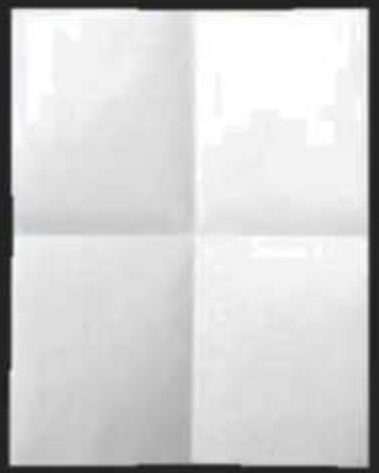

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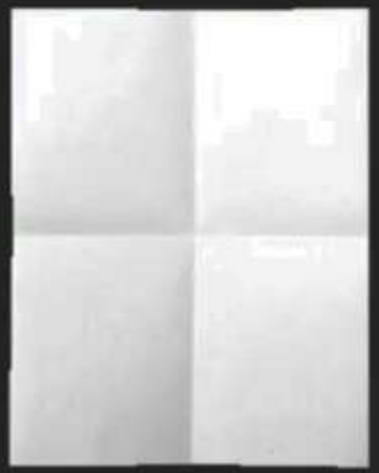

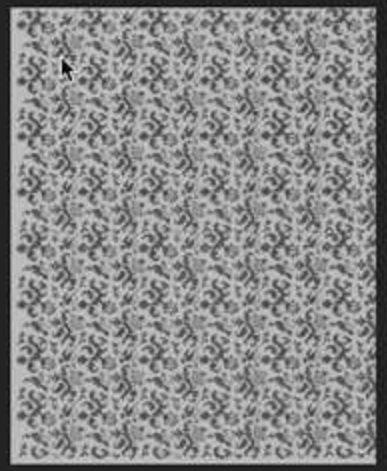

Issues with Stereo Matching

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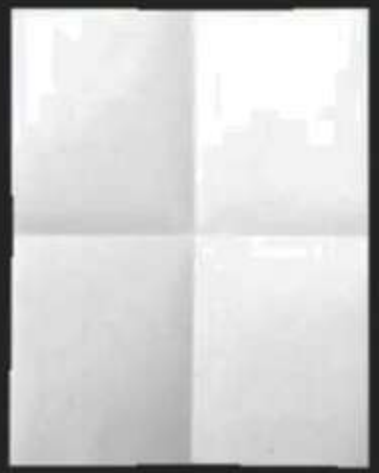

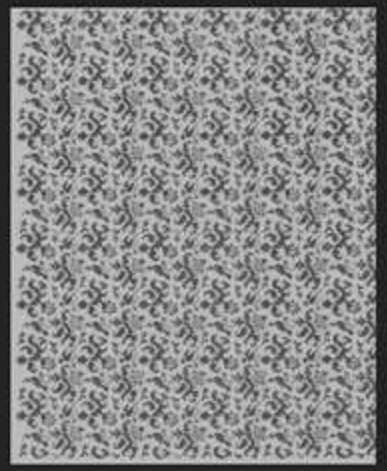

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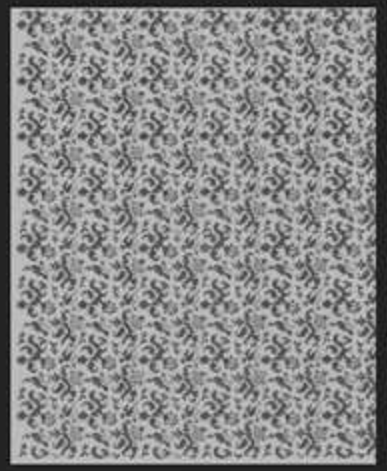
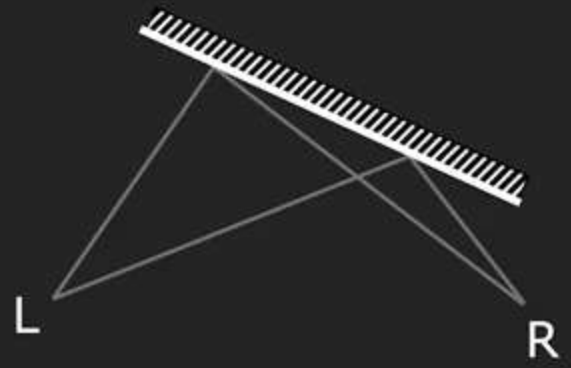


- Foreshortening effect makes matching challenging



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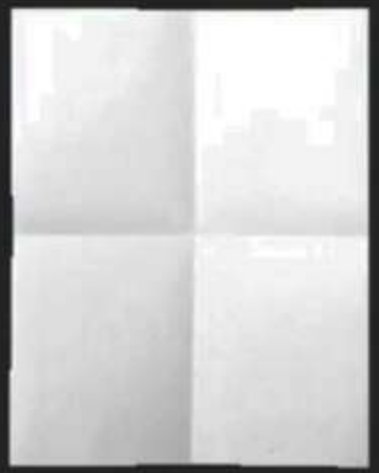

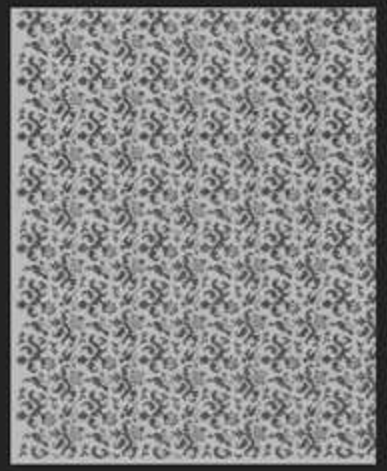

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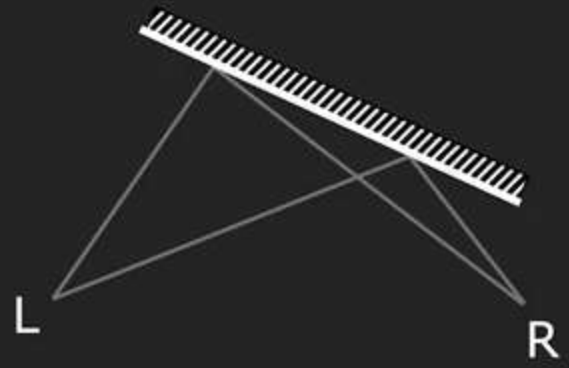

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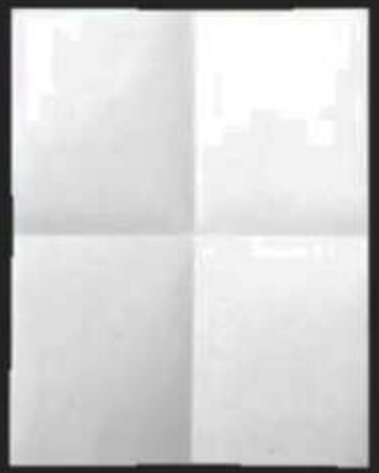

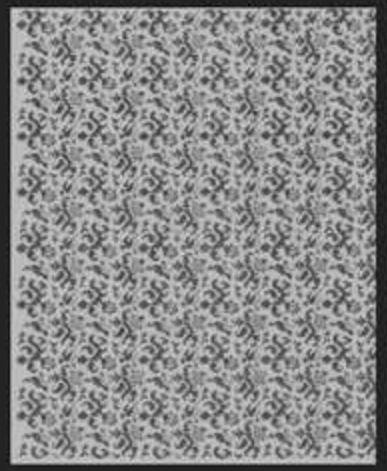

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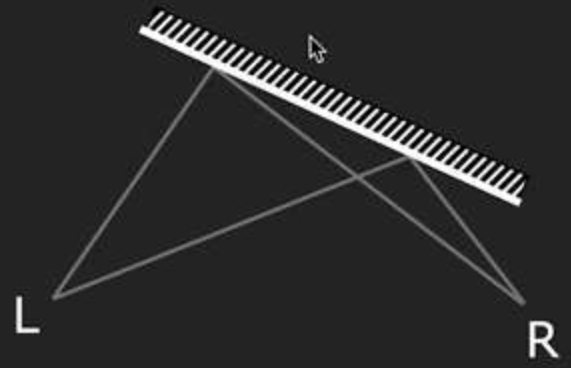

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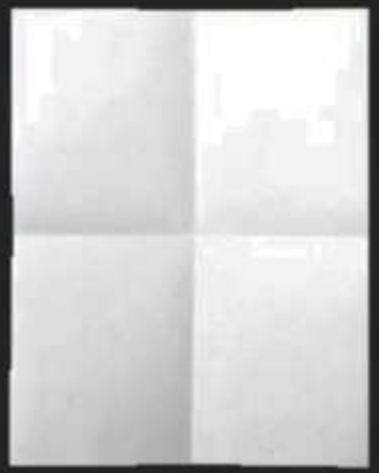

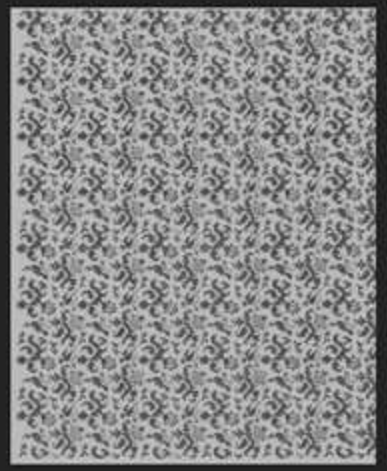

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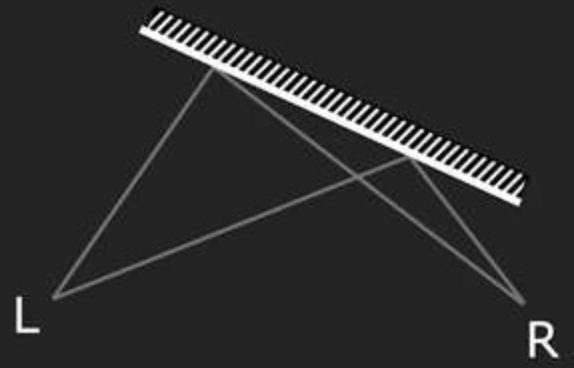

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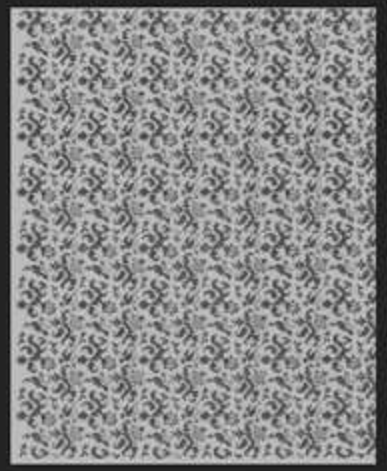

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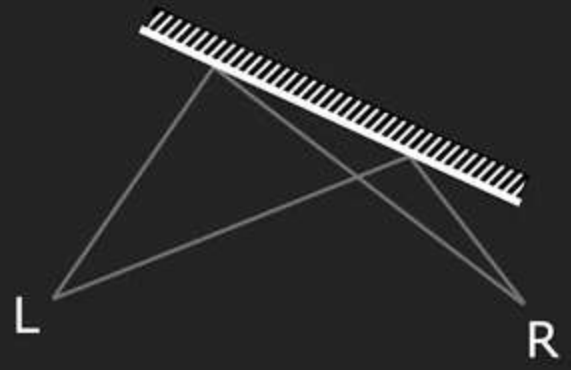

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Window size = 5 pixels
(Sensitive to noise)

Window size = 30 pixels
(Poor localization)

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Adaptive Window Method Solution: For each point, match using windows of multiple sizes and use the disparity that is a result of the best similarity measure (minimize SSD per pixel).

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Window Based Methods: Results

Left Image

Right Image

Ground Truth

SSD (Window size=21)

SSD – Adaptive Window

State of the Art

http://vision.middlebury.edu/stereo

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