Finding Correspondences

Shree K. Nayar

Columbia University

Topic: Uncalibrated Stereo, Module: Reconstruction II

First Principles of Computer Vision

Simple Stereo: Finding Correspondences

Left Camera Image

Right Camera Image

REVIEW

Simple Stereo: Finding Correspondences

Left Camera Image

Right Camera Image

REVIEW

Simple Stereo: Finding Correspondences

Left Camera Image

Right Camera Image

Corresponding scene points lie on the same horizontal scan-line.
Finding correspondence is a 1D search.

REVIEW

Simple Stereo: Finding Correspondences

Left Camera Image

Right Camera Image

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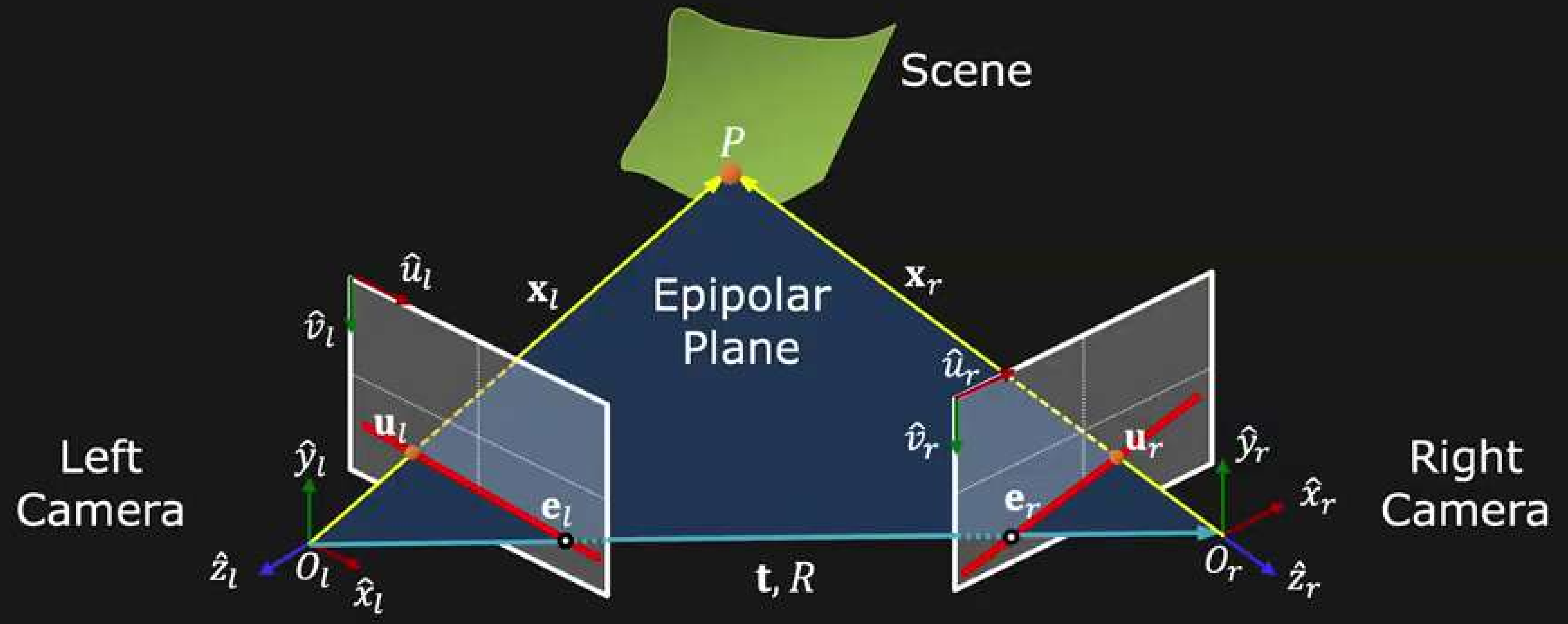
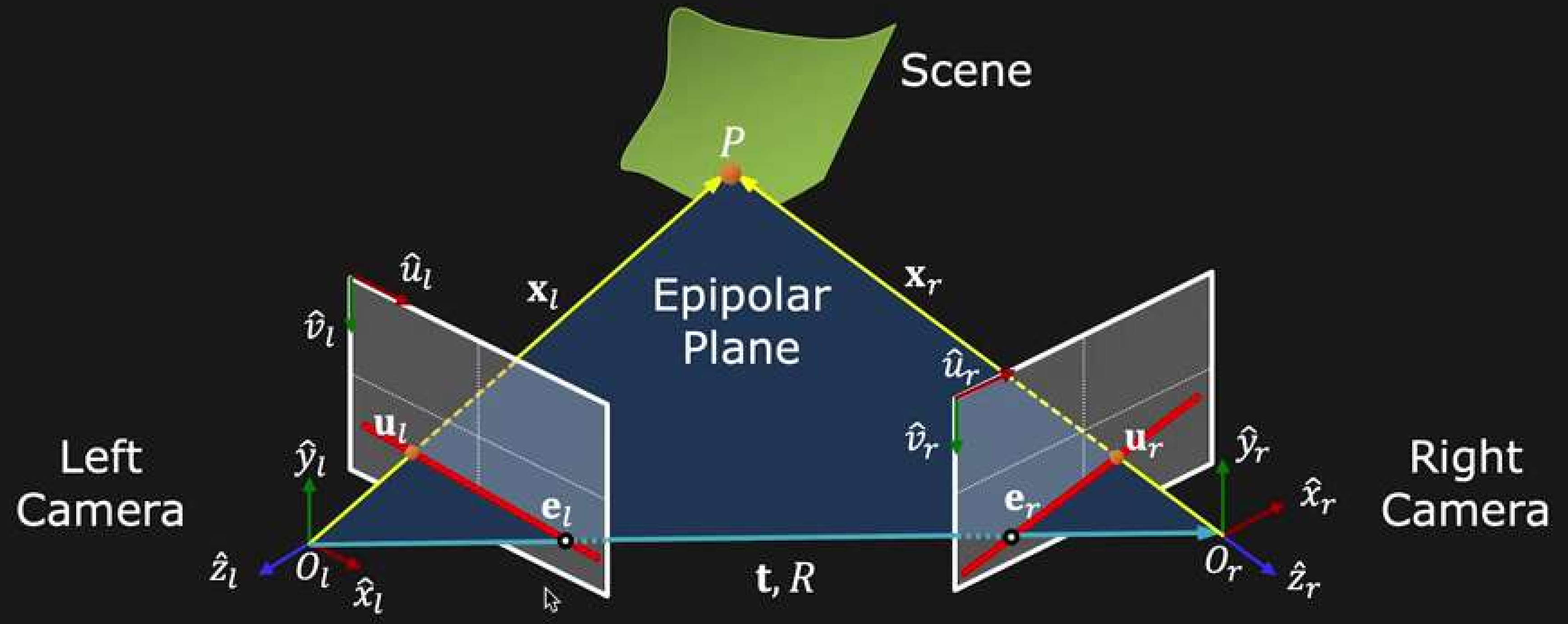
REVIEW

Epipolar Geometry: Epipolar Line



Epipolar Line: Intersection of image plane and epipolar plane.

Epipolar Geometry: Epipolar Line



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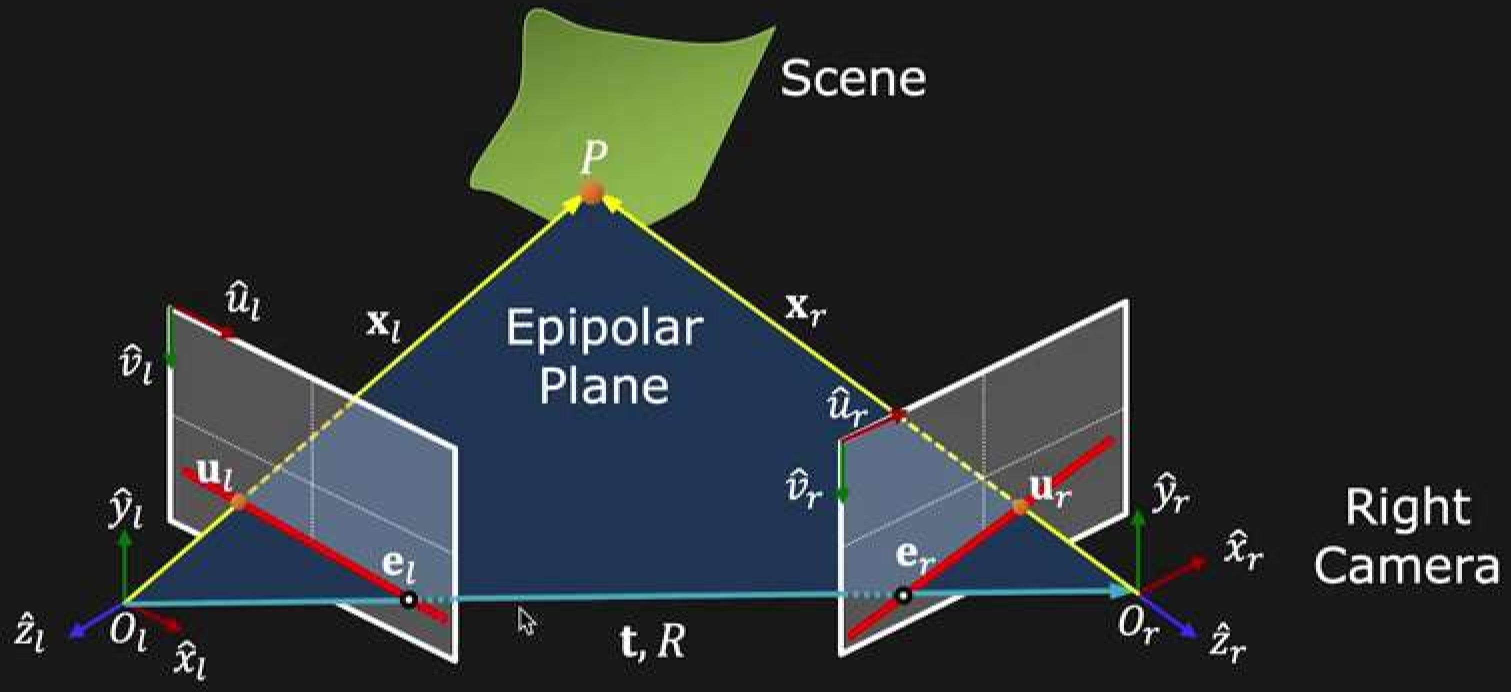

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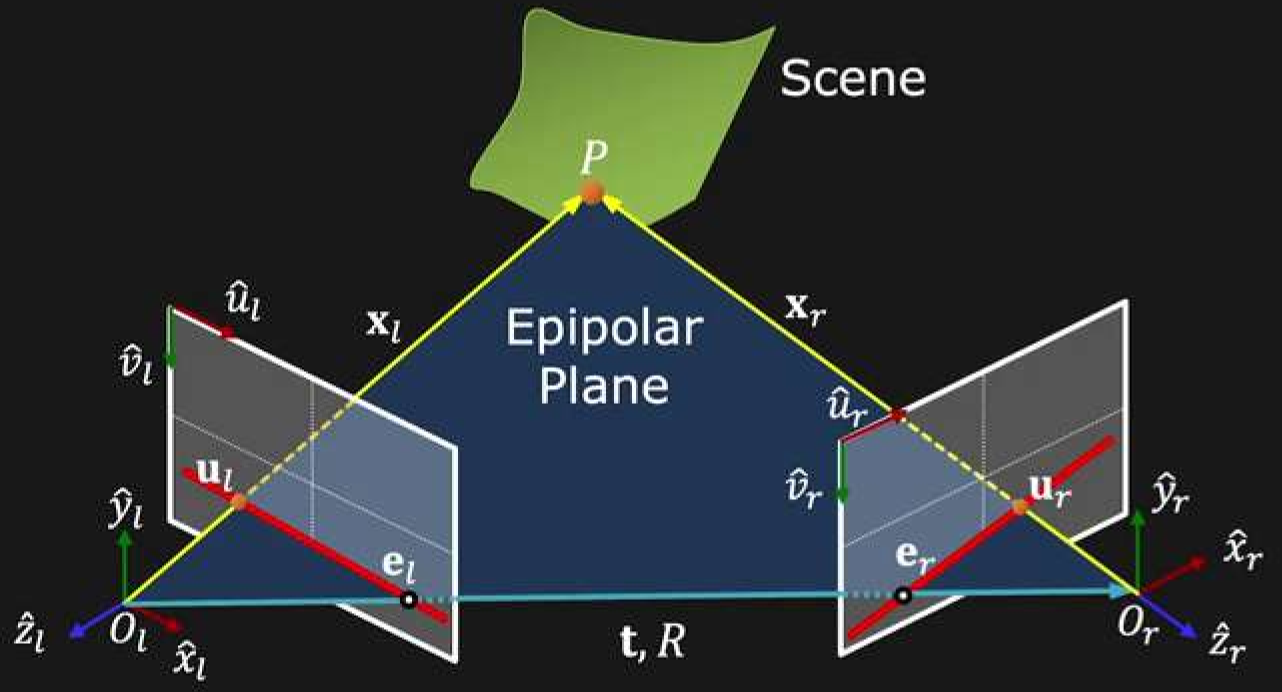
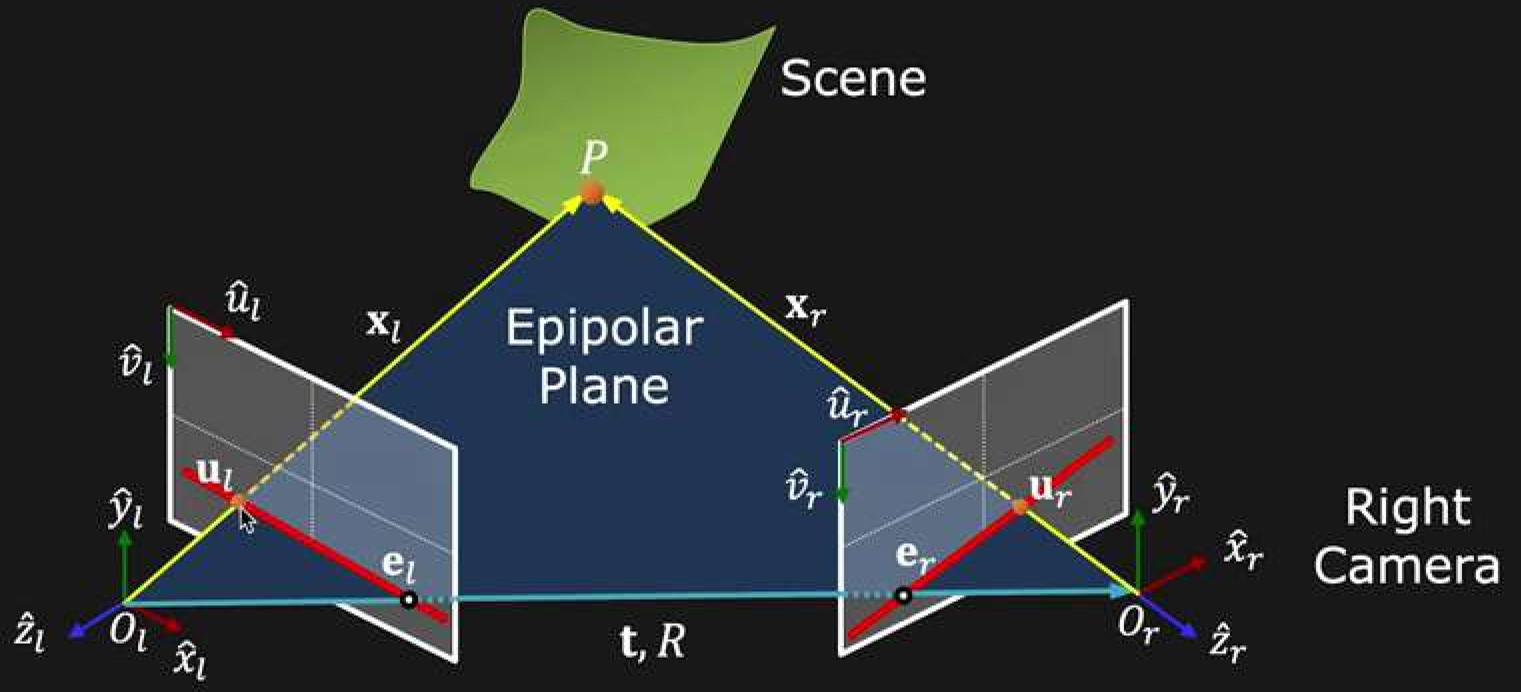


Epipolar Line: Intersection of image plane and epipolar plane.

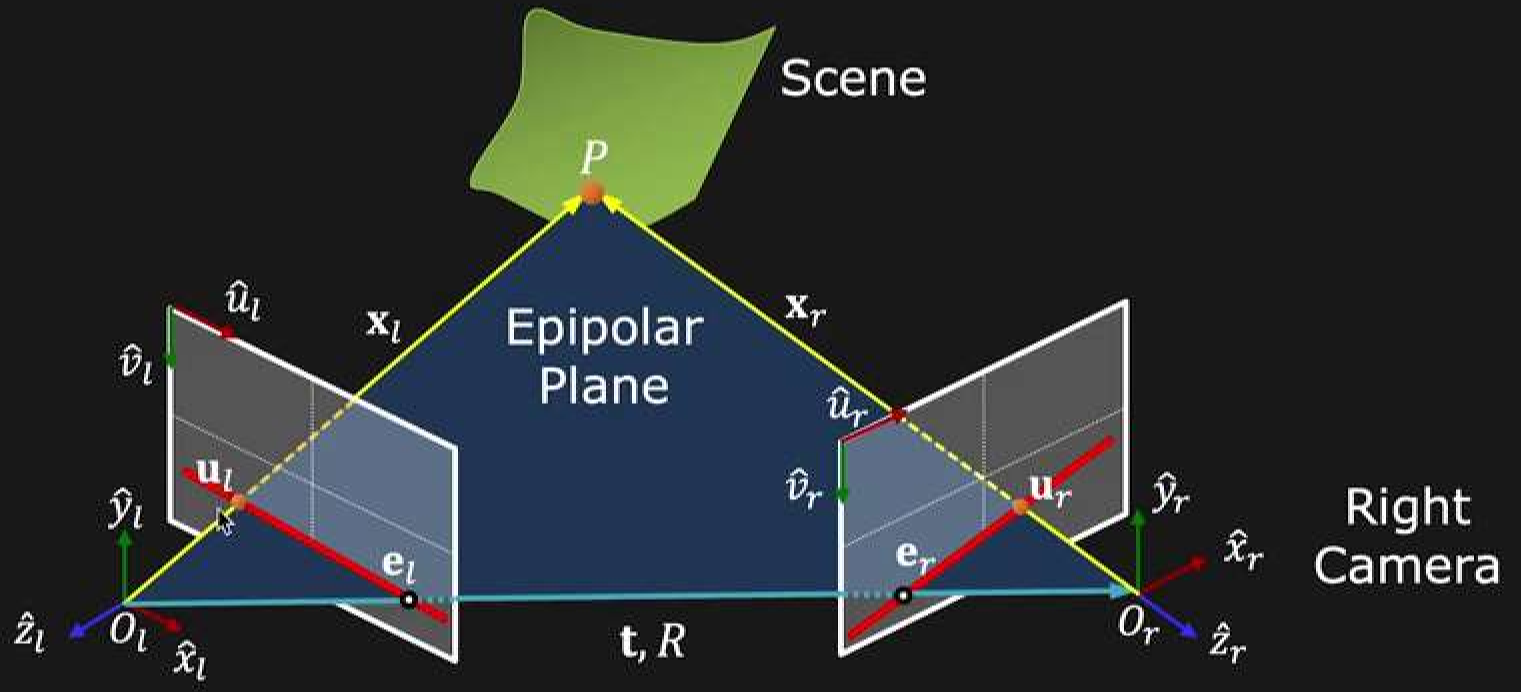
Every scene point has two corresponding epipolar lines, one each on the two image planes.

Epipolar Geometry: Epipolar Line



Given a point in one image, the corresponding point in the other image must lie on the epipolar line.

Epipolar Geometry: Epipolar Line



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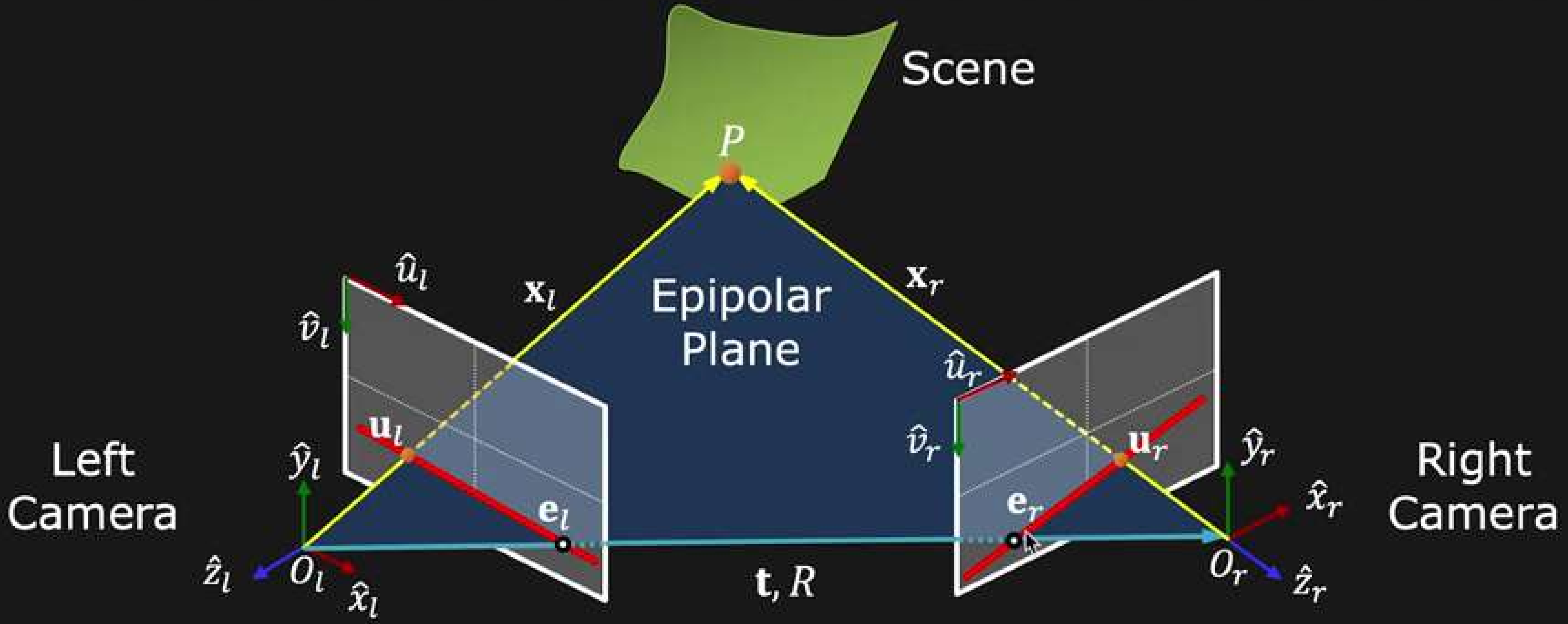

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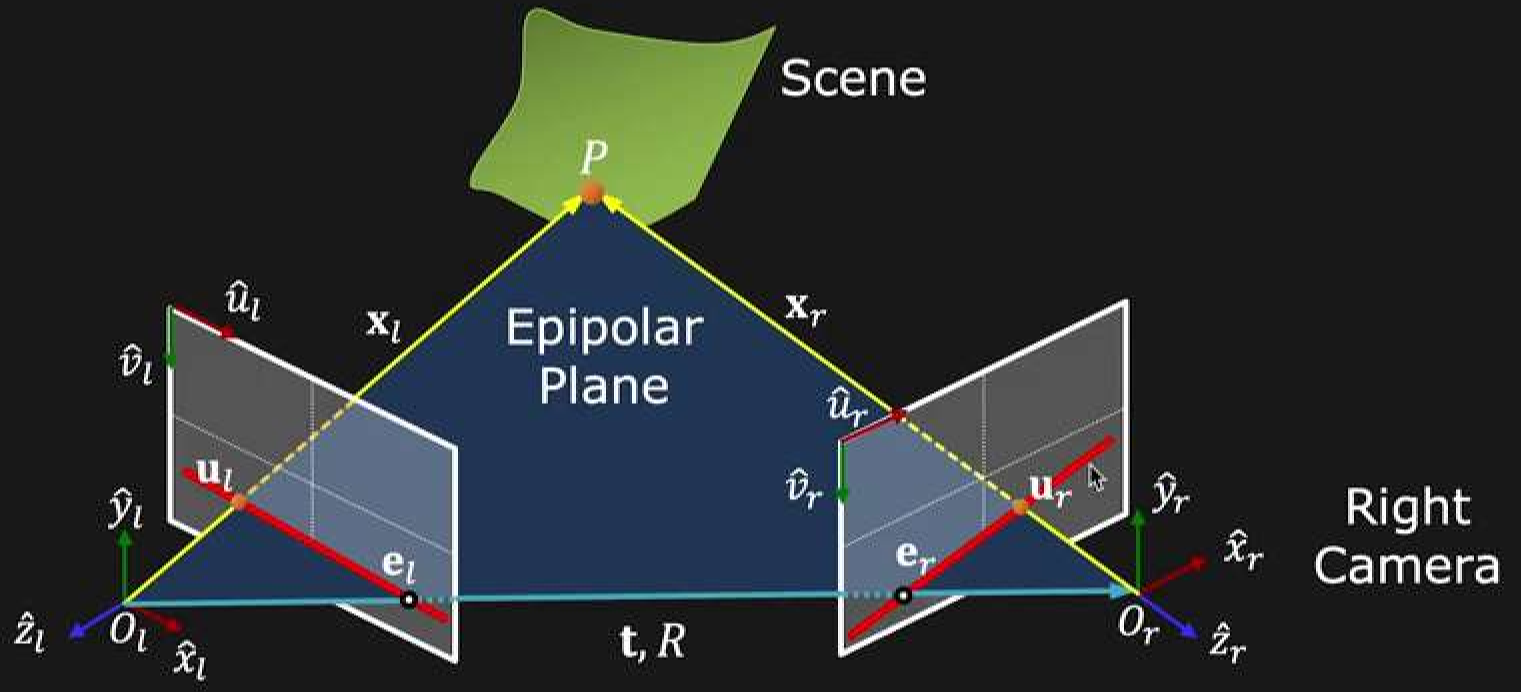

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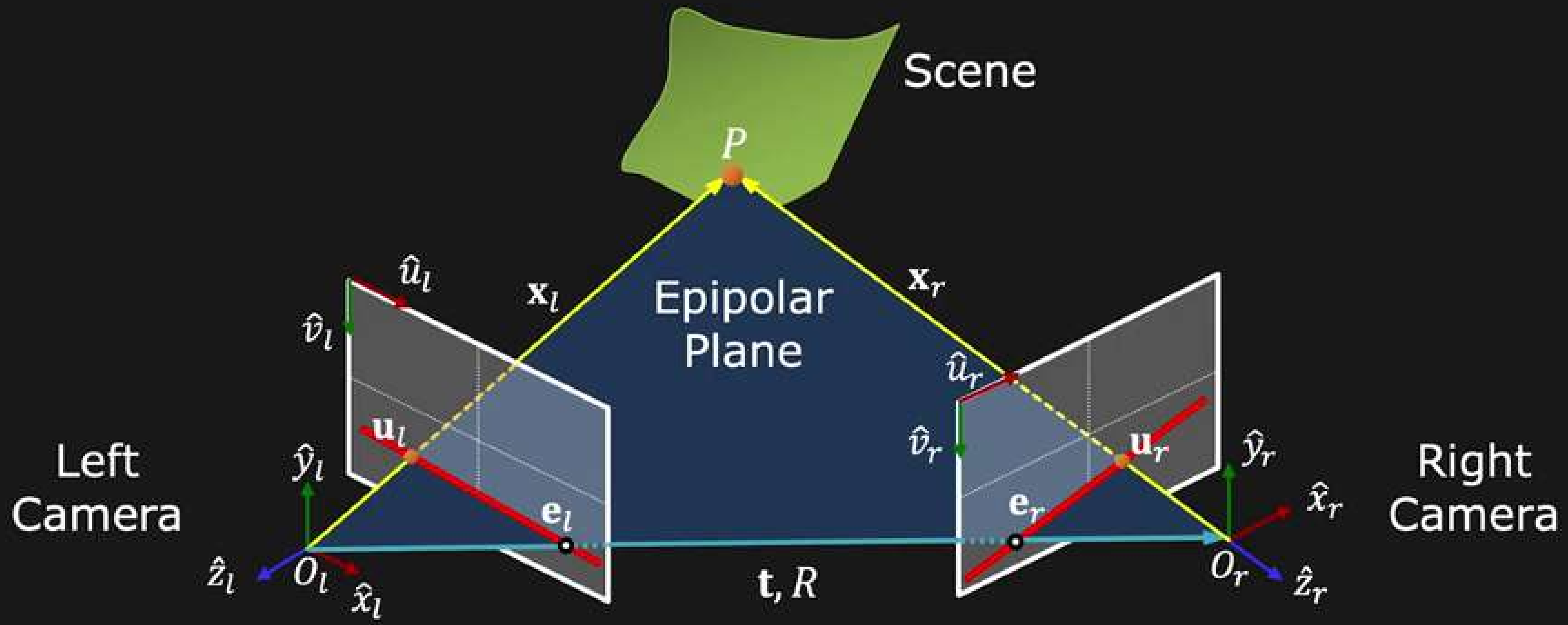

Given a point in one image, the corresponding point in the other image must lie on the epipolar line.

Finding correspondence reduces to a 1D search.

Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

Find: Equation of Epipolar line in the right image

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Epipolar Constraint Equation:

$$[u_l \quad v_l \quad 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

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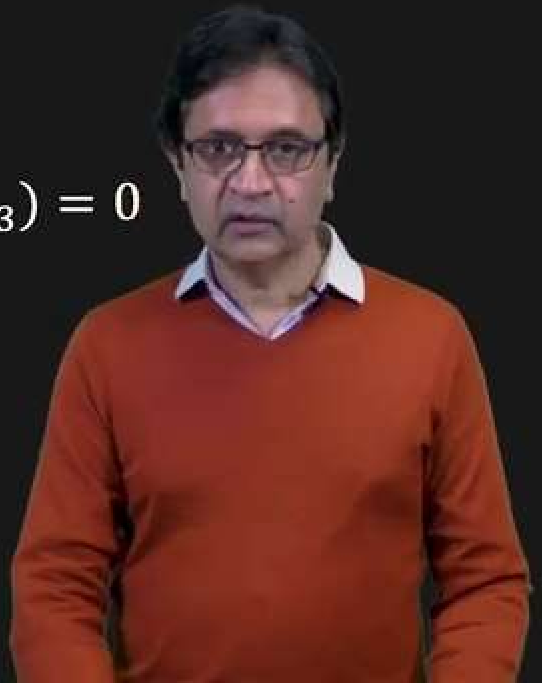
Expanding the matrix equation gives:

$$(f_{11}u_l + f_{21}v_l + f_{31})u_r + (f_{12}u_l + f_{22}v_l + f_{32})v_r + (f_{13}u_l + f_{23}v_l + f_{33}) = 0$$



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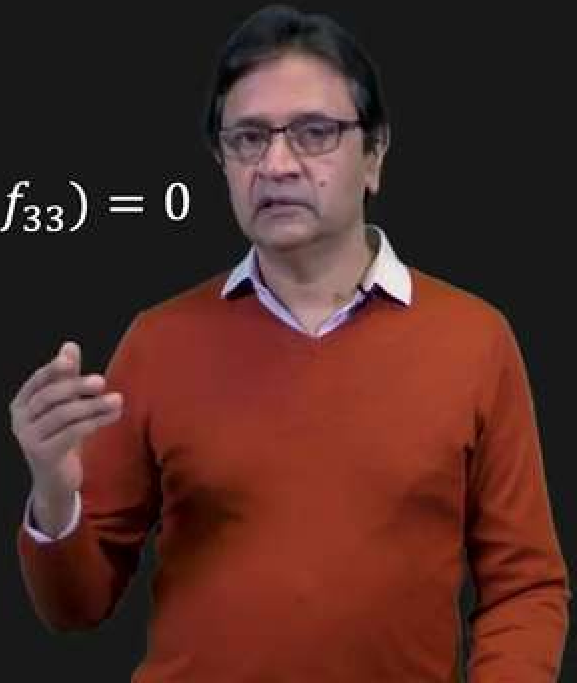
Equation for right epipolar line: $\boxed{a_l u_r + b_l v_r + c_l = 0}$



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Similarly we can calculate epipolar line in left image for a point in right image.

Finding Epipolar Lines: Example

Given the Fundamental matrix,

$$F = \begin{bmatrix} -.003 & -.028 & 13.19 \\ -.003 & -.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

Left Image

Right Image

Finding Epipolar Lines: Example

Given the Fundamental matrix,

$$F = \begin{bmatrix} -.003 & -.028 & 13.19 \\ -.003 & -.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

and the left image point

$$\tilde{\boldsymbol{u}}_l = \begin{bmatrix} 343 \\ 221 \\ 1 \end{bmatrix}$$

Left Image

Right Image

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Left Image

Right Image

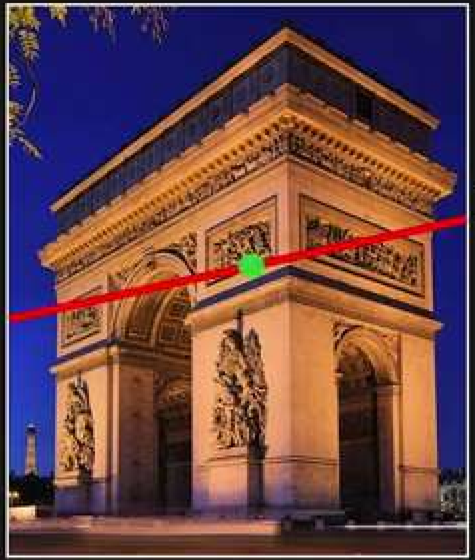
Epipolar Line

The equation for the epipolar line in the right image is

$$.03u_r + .99v_r - 265 = 0$$

Finding Correspondence

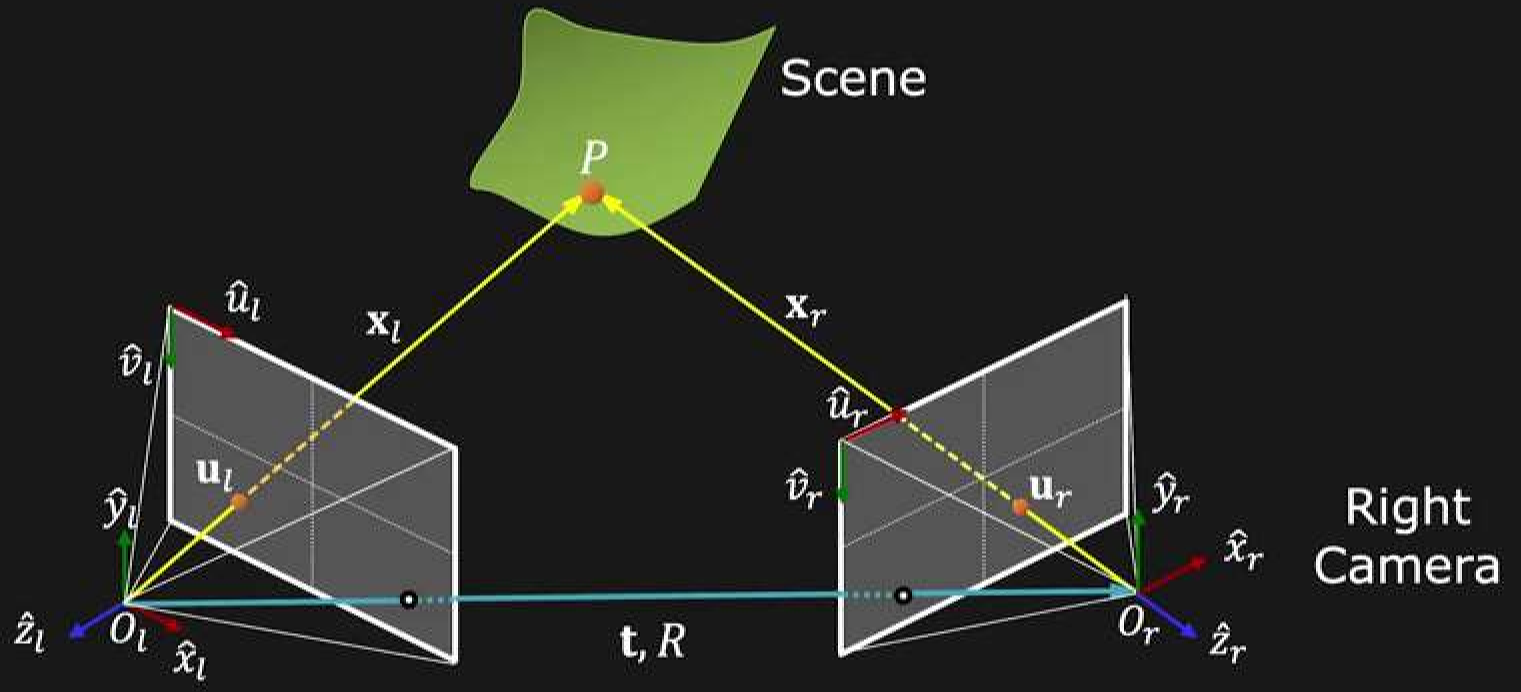
Left Image



Right Image

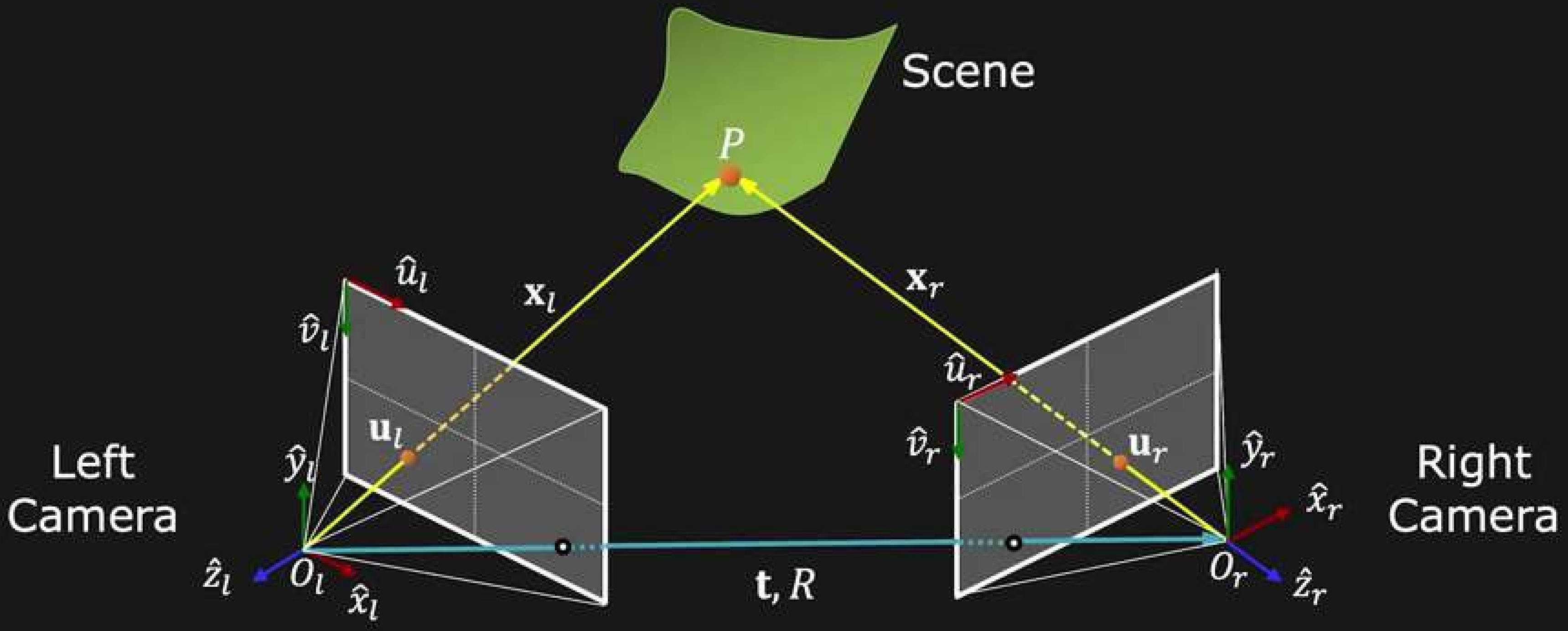
Corresponding scene points lie on the epipolar lines.
Finding correspondence is a 1D search.

Uncalibrated Stereo



- 1. Assume Camera Matrix K is known for each camera
- 2. Find a few Reliable Corresponding Points
- 3. Find Relative Camera Position $\mathbf{t}$ and Orientation R
- 4. Find Dense Correspondence
- 5. Compute Depth using Triangulation

Uncalibrated Stereo



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