

# Intrinsic and Extrinsic Matrices

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Columbia University

Topic: Camera Calibration, Module: Reconstruction II  
First Principles of Computer Vision

# Extracting Intrinsic/Extrinsic Parameters

We know that:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M_{int}} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_{ext}}$$

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Given that  $K$  is an **Upper Right Triangular** matrix and  $R$  is an **Orthonormal** matrix, it is possible to uniquely “decouple”  $K$  and  $R$  from their product using “**QR factorization**”.

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## Other Intrinsic Parameters

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Pinholes do not exhibit image distortions. But, lenses do!



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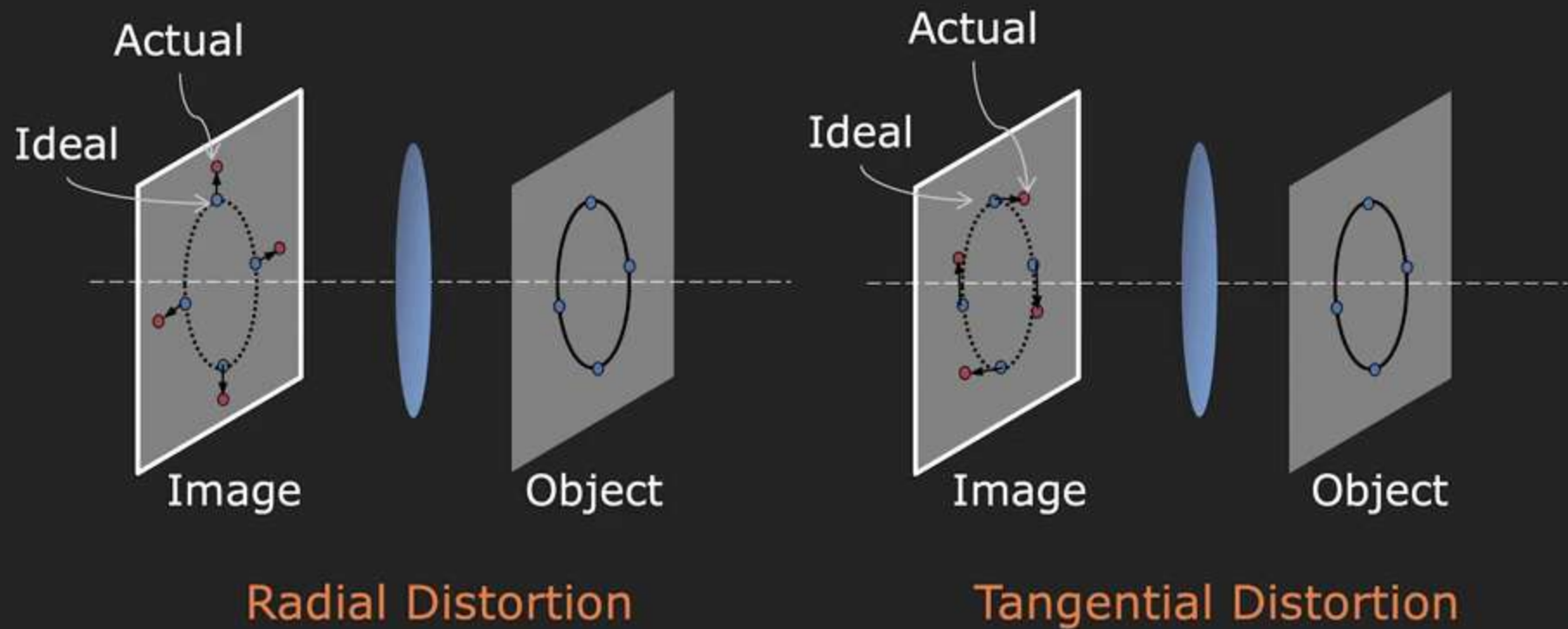
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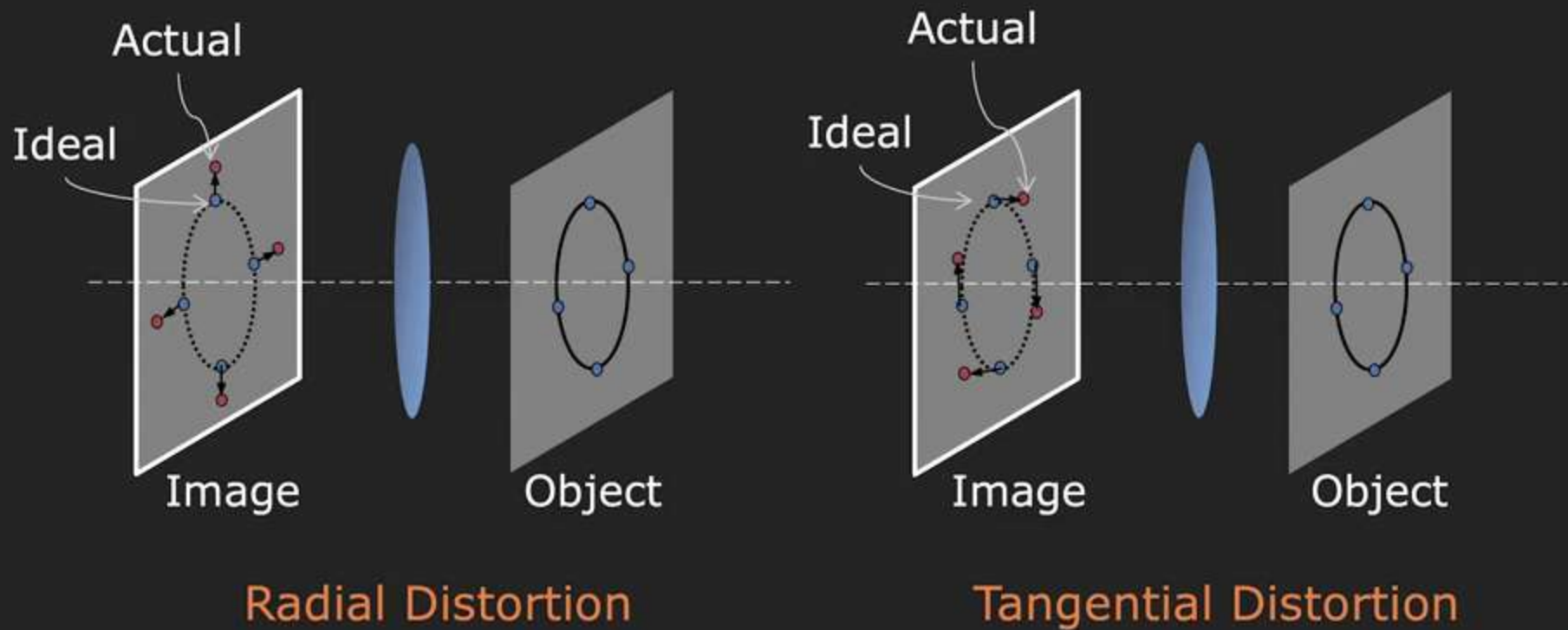
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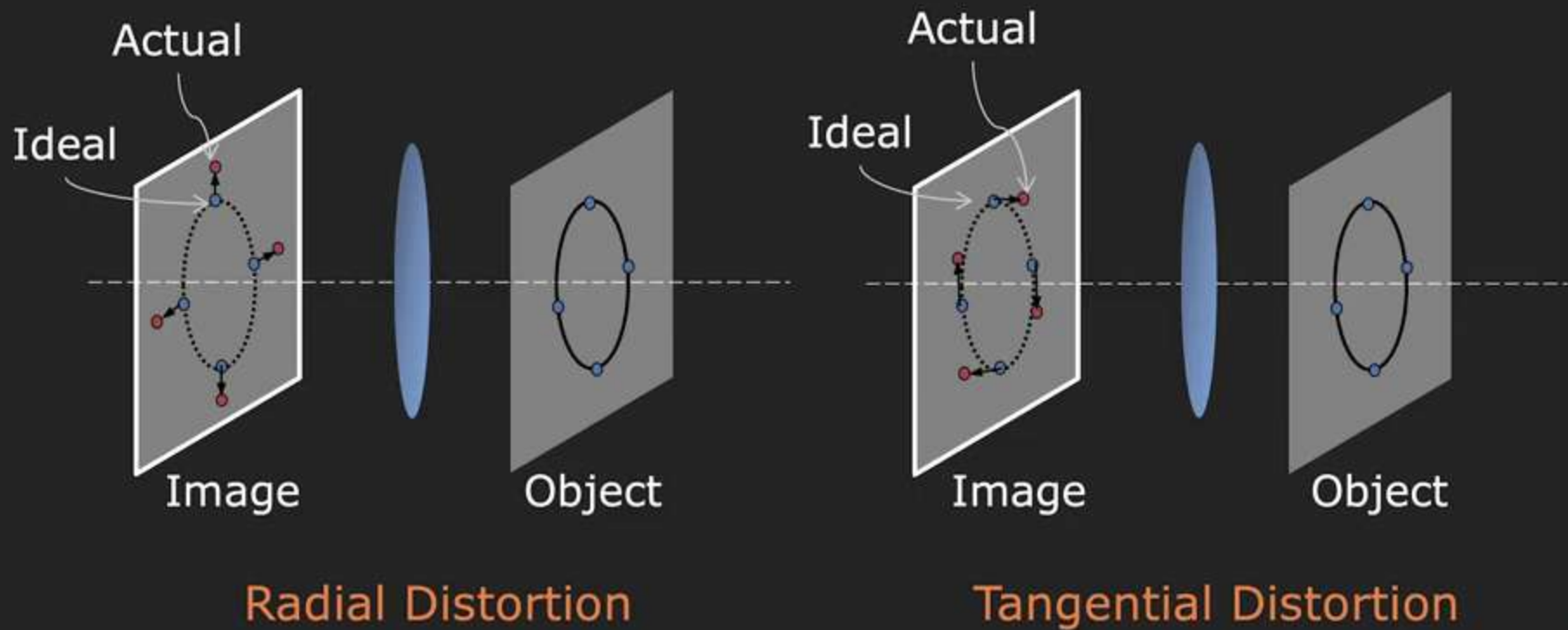
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