Assignment 5: CS 215

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Question 1

Instructions for running the code

1. Unzip, `cd` into `q1/code`, open and run `q1.m`
2. The respective plots for MLE, MAP_1, and MAP_2 will be created and saved to `q1/results/`

ML Estimate

We know that ML estimate is the sample mean = $\boxed{\bar{x}}$

MAP_1 Estimate:

This was derived in class,

$$\boxed{\frac{\sigma_0^2\bar{x} + \sigma^2\mu_0/N}{\sigma_0^2 + \sigma^2/N}}$$

MAP_2 Estimate:

Likelihood:

$$L(x|\mu) = C \cdot \exp \sum_{i=1}^{N} \frac{(x_i - \mu)^2}{2\sigma^2}$$

Prior:

$$\begin{aligned} P(\mu) &= \frac{1}{b-a} \quad \text{If } a \leq \mu \leq b \\ &= 0 \quad \text{otherwise} \end{aligned}$$

Posterior is proportional to Likelihood×Prior

$$\begin{aligned} F(\mu|x) &= \frac{C}{b-a} \cdot \exp \sum_{i=1}^{N} \frac{(\mu - x_i)^2}{2\sigma^2} \\ &= \frac{C'}{b-a} \cdot \exp \frac{(\mu - \bar{x})^2}{2\sigma^2/N} \end{aligned}$$

Thus, the maximum of the posterior is at $\bar{x}$, which is same as the ML estimate.
But, if $\bar{x} < a$, then we know from the prior that this is not possible, thus in this case, the MAP estimate will be a, similarly, if $\bar{x} > b$, the MAP estimate will be b. Thus, the MAP$_2$ estimate can be written compactly as:

$$\boxed{\min\left(\max(\bar{x}, a), b\right)}$$

Boxplots:

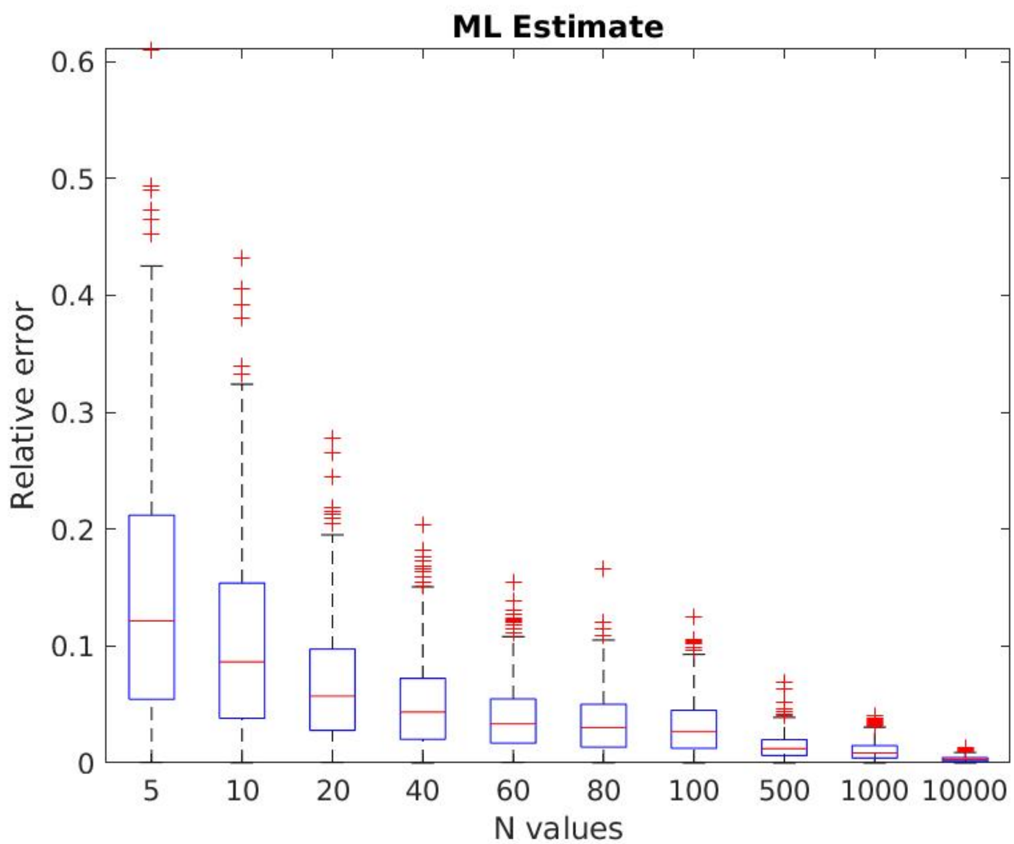

Figure 1: ML Estimate relative error

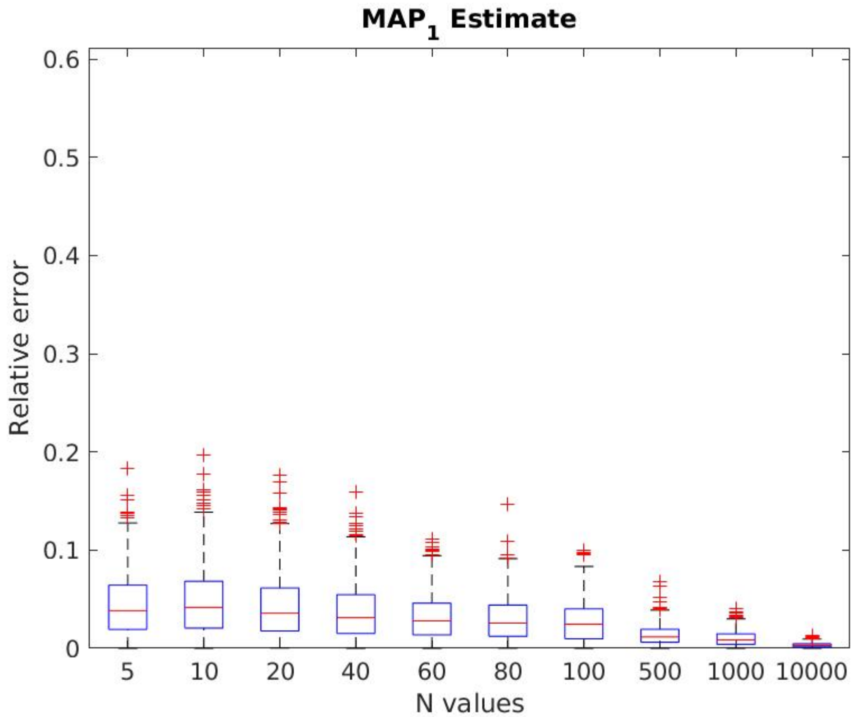

Figure 2: MAP_1 Estimate relative error

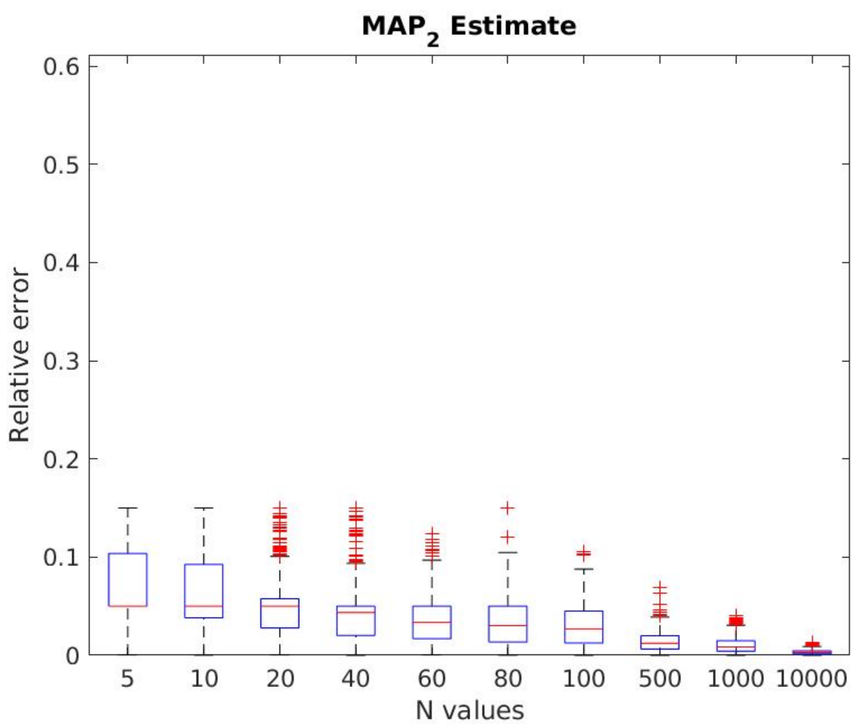

Figure 3: MAP_2 Estimate relative error

Interpretation

i.) The relative errors decrease to zero for all 3 estimates as N increases, this is desirable.
ii.) When N is small, MAP_1 estimate performs better than the others.
For large N, the three estimates perform approximately the same.
Due to the reasons stated above, we prefer the MAP_1 estimate over the other two.