

A new Markovian approximation of the coalescent with recombination

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Oxford, UK

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Modeling the coalescent

Whole-genome data across population is now readily available.

Why analyze whole genomes (as opposed to a few loci)?

- Informative about recombination
- Scans for outlier regions (e.g. +ve selection)
- More data allows for fine-grained inferences, e.g.
 - Inferring historical population sizes
 - Inferring migration rates through time
 - Inferring recombination rates and hotspots through time

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How to analyze the data?

- **Foundation:** Wright-Fisher model with recombination
- **Exact:** Griffith's ARG, Hudson's ms, Wiuf and Hein's algorithm
- **Approximate likelihood:**
 - Composite likelihood (e.g. Fearnhead & Donnelly 2002)
 - Pairs of segregating sites (e.g. Hudson 2001)
- **Approximating the model:**
 - Li and Stephens 2003: copying model
 - McVean and Cardin 2005: sequentially Markov coalescent (SMC)

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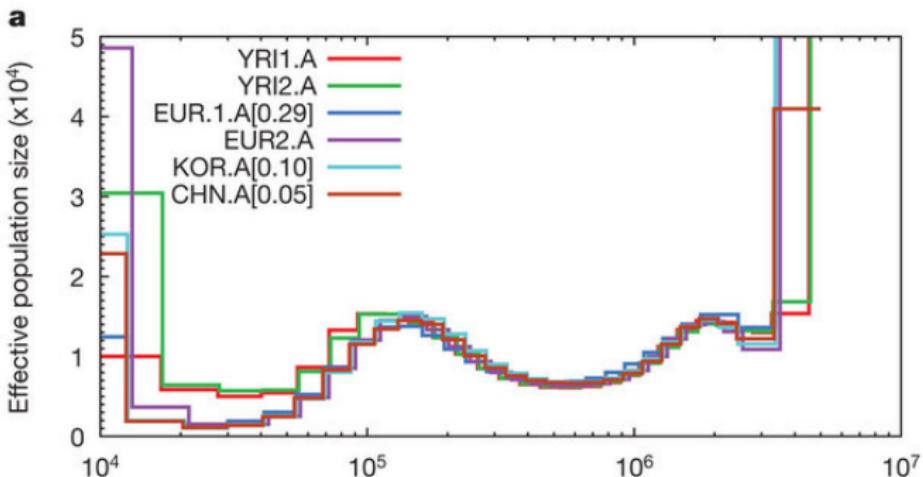
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Modeling the coalescent



Approximating the ARG

Focus here is on **sequential** algorithms:

Algorithm	Sequential algorithm <i>Wiuf & Hein 1999</i>	SMC <i>McVean & Cardin 2005</i>	this talk
Precision	Exact	Approximate	Tunable
Data structure	ARG-like	tree	forest
Model	Non-Markovian	Markovian	Markovian

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Overview of this talk

- Coalescent and ARG
- Wiuf and Hein's sequential algorithm
- Simplifying the algorithm
- Approximating the algorithm
- Performance

Introduction
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Coalescent & ARG
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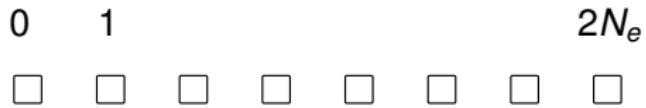
Sequential algorithm
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Approximations
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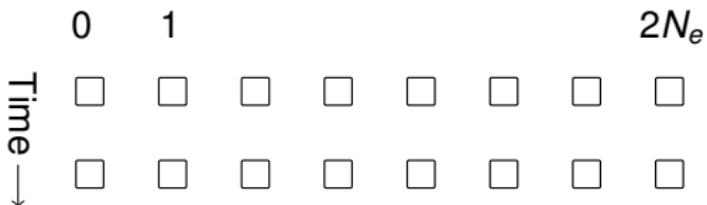
Performance
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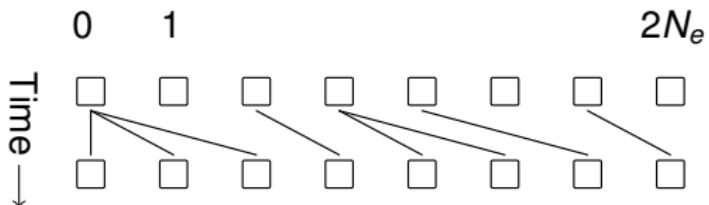
Wright-Fisher model



Wright-Fisher model

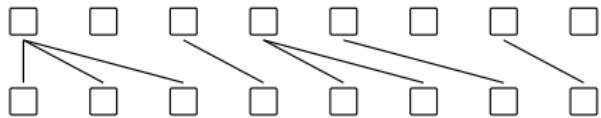


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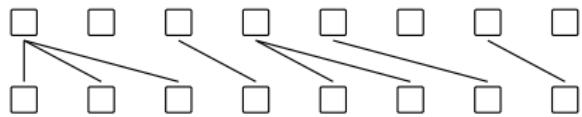
Wright-Fisher model

Time →



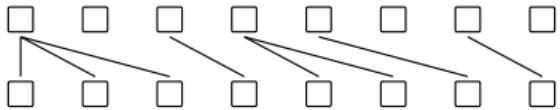
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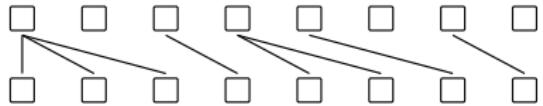
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Sequential algorithm
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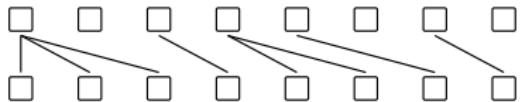
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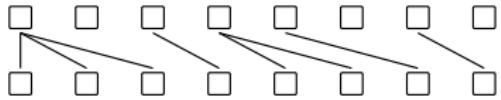
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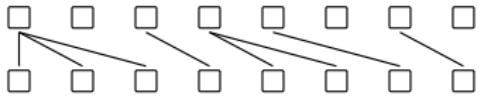
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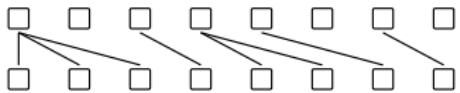
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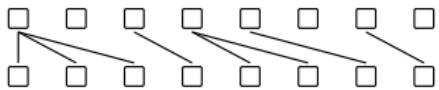
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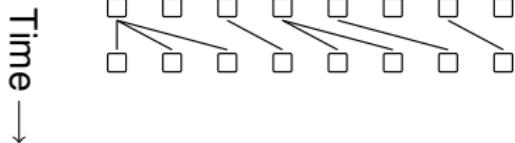


Wright-Fisher model

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Wright-Fisher model



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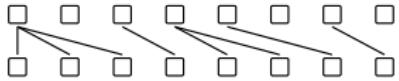
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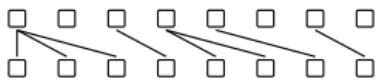
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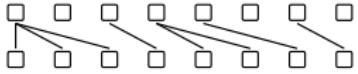
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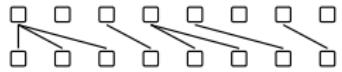
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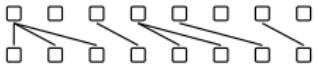
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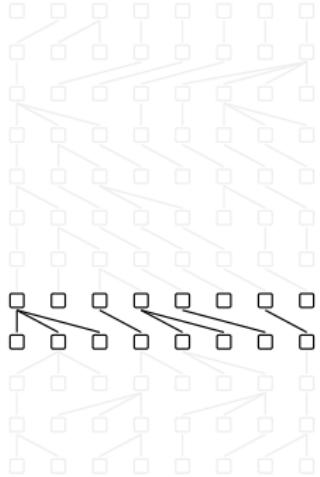
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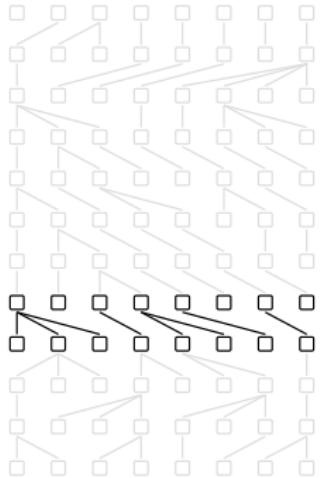
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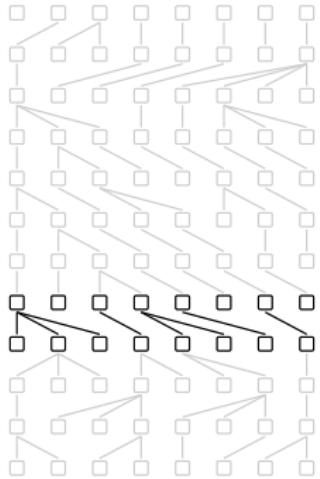
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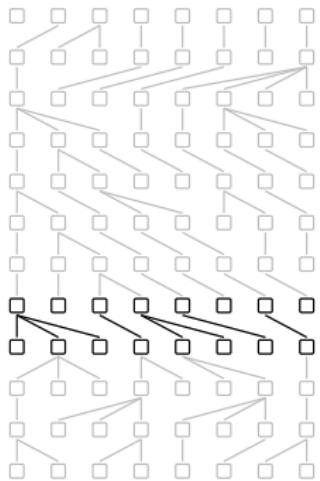
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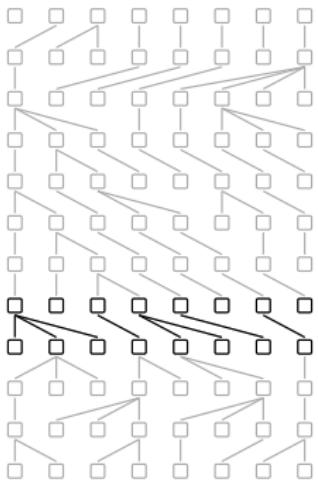
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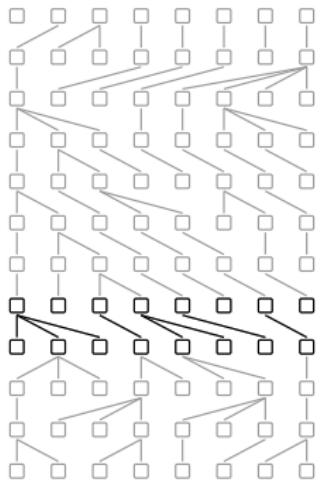
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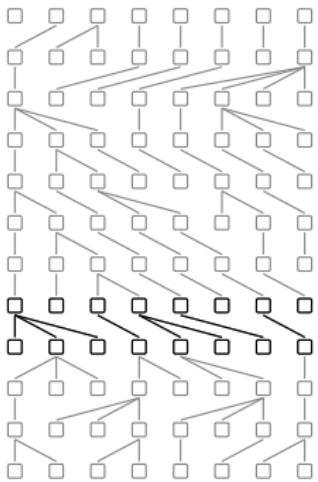
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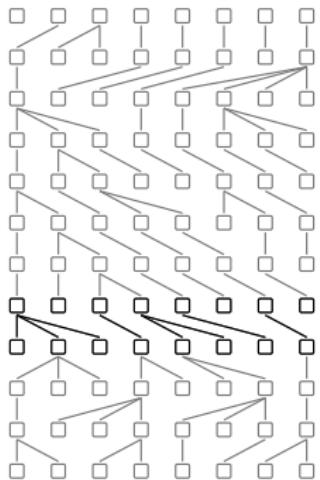
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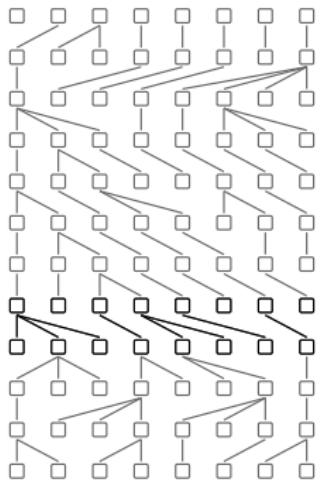
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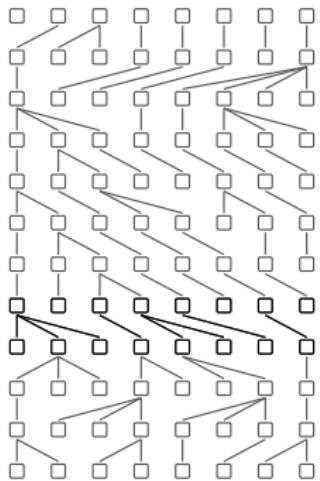
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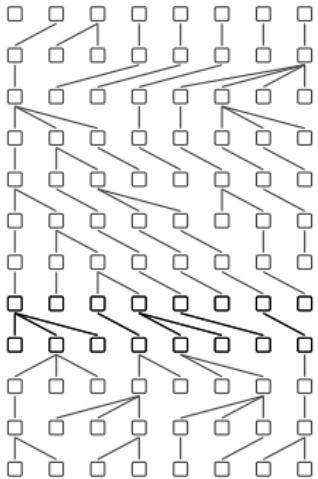
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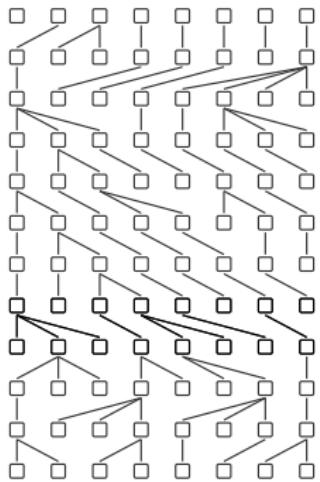
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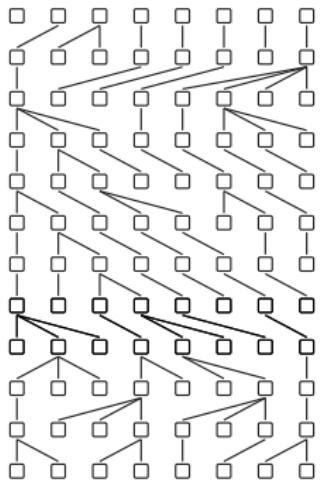
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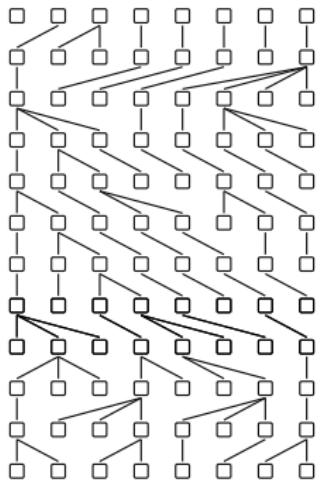
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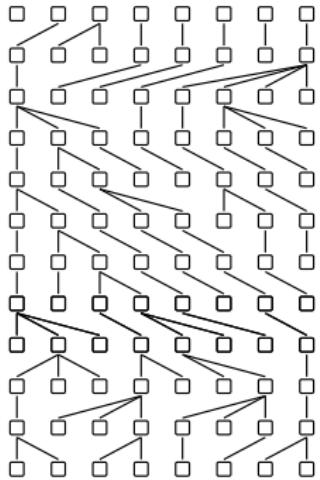
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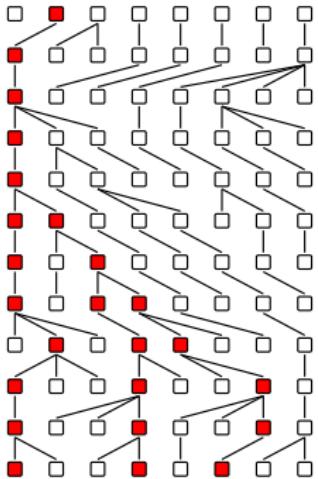
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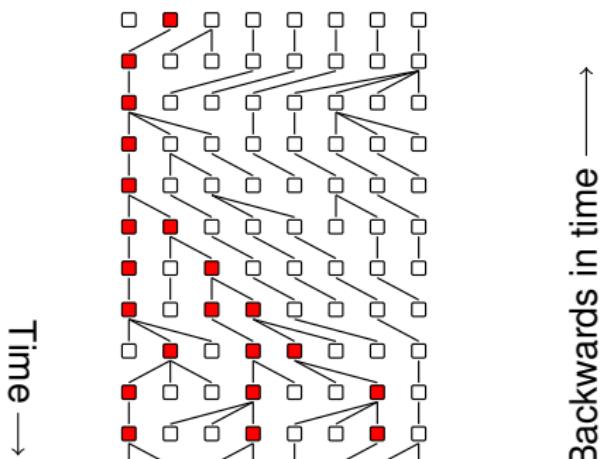


Coalescent: Small sample approximation

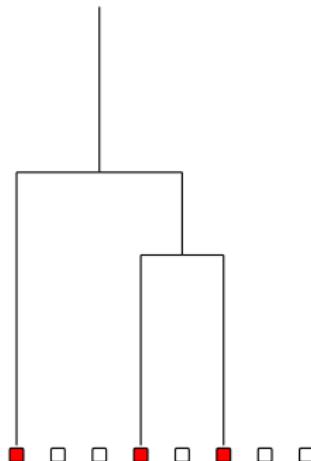
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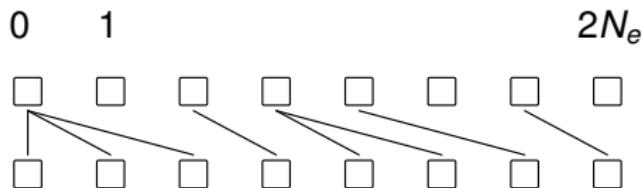


Backwards in time →



$$\delta = \frac{1}{2N_e} \frac{k(k-1)}{2}$$

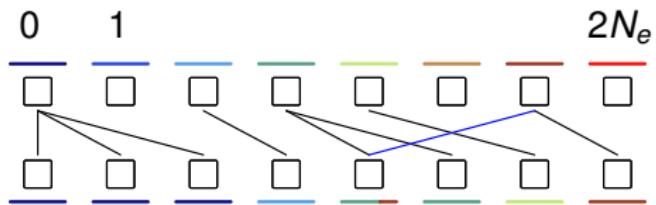
Wright-Fisher with recombination



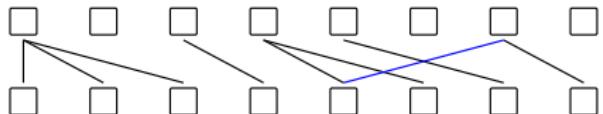
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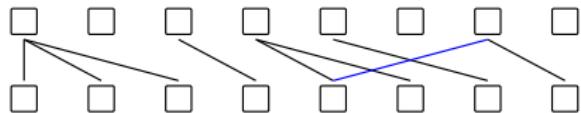
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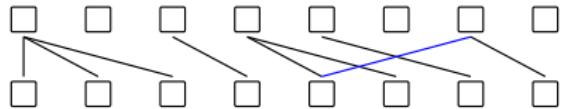
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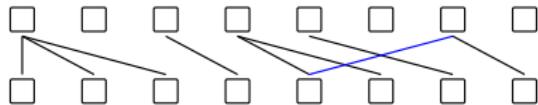
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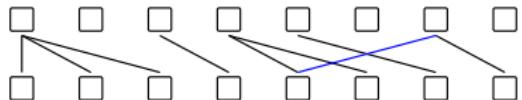
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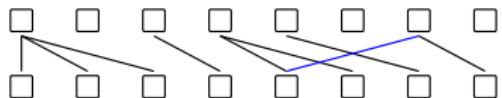
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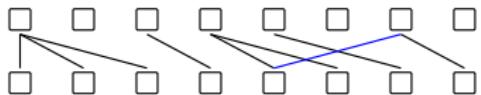
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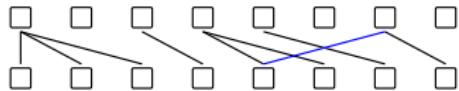
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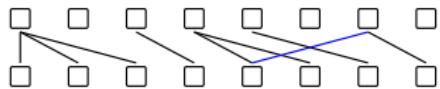
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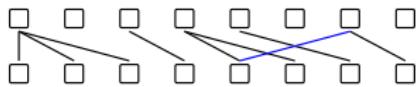
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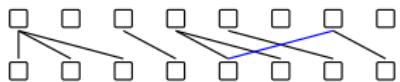
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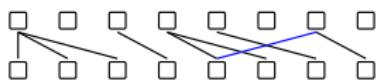
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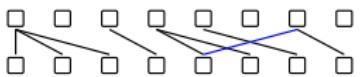
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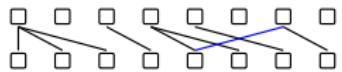
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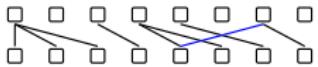
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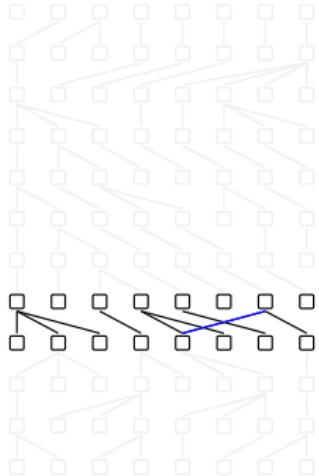
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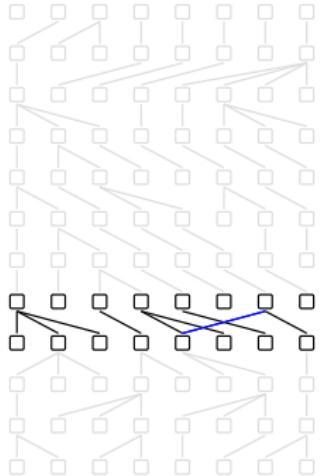
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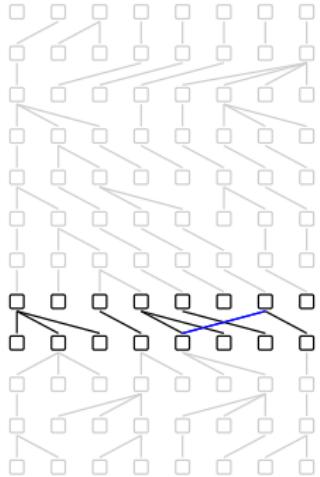
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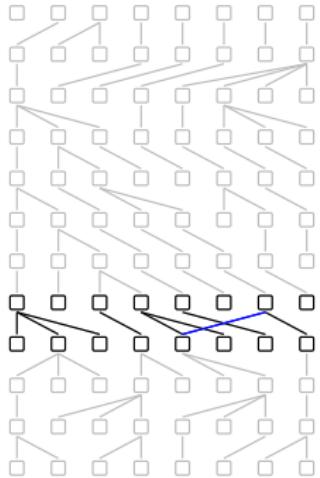
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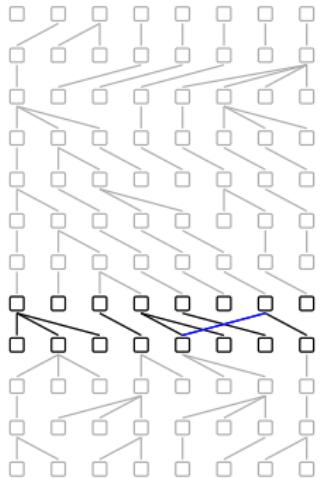
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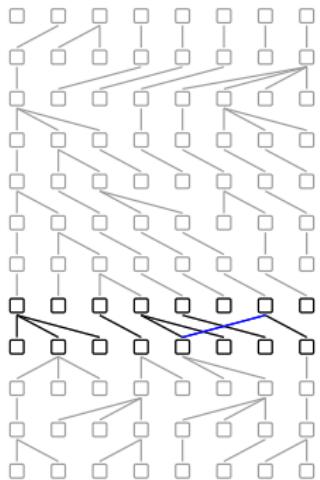
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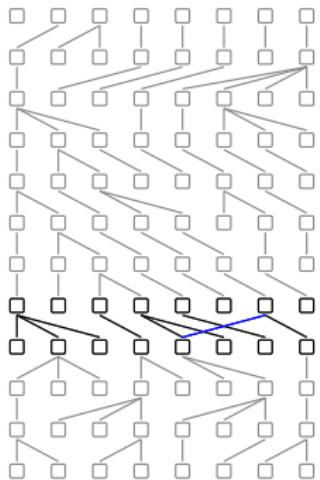
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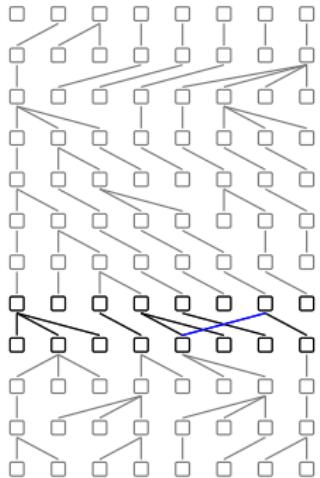
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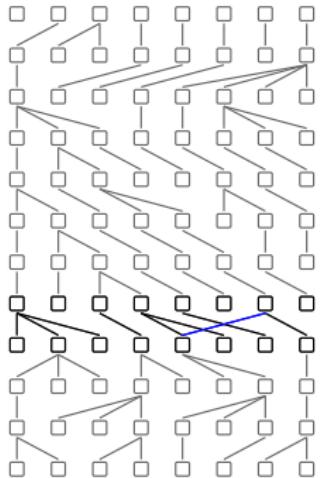
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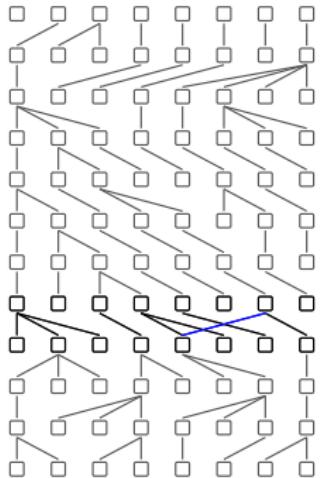
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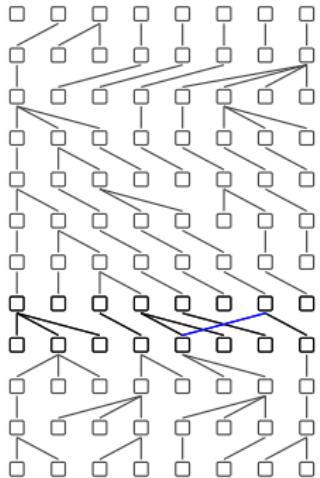
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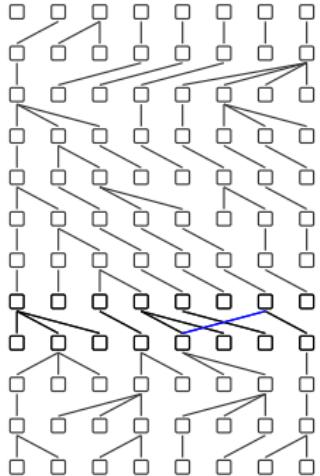
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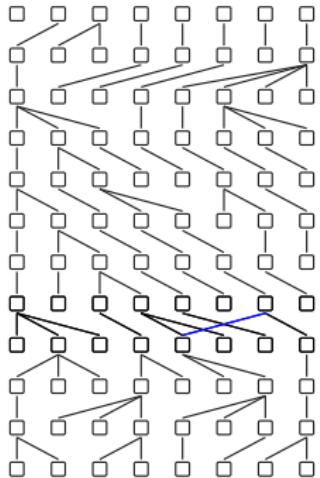
Wright-Fisher with recombination



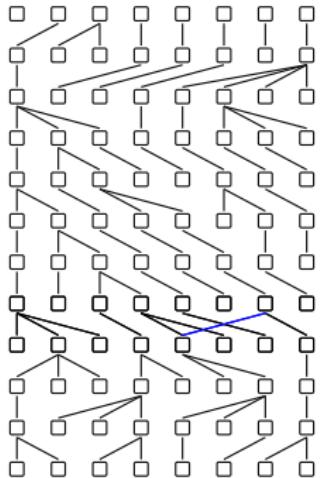
Wright-Fisher with recombination



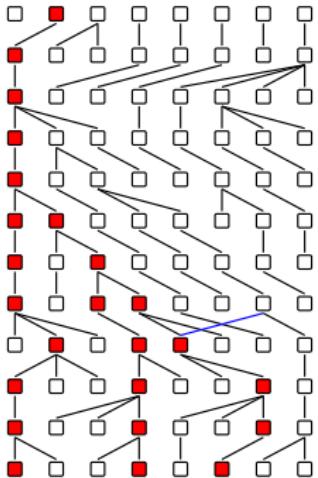
Wright-Fisher with recombination



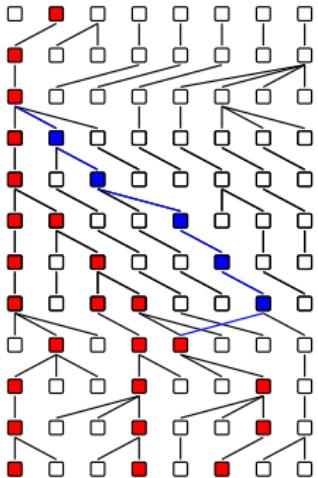
Wright-Fisher with recombination



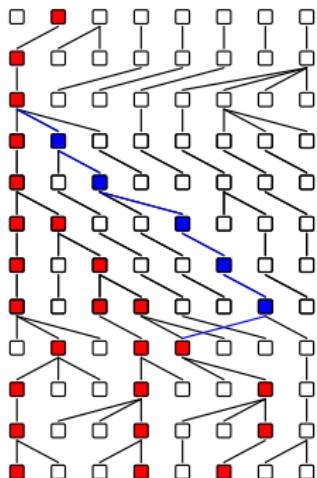
Wright-Fisher with recombination



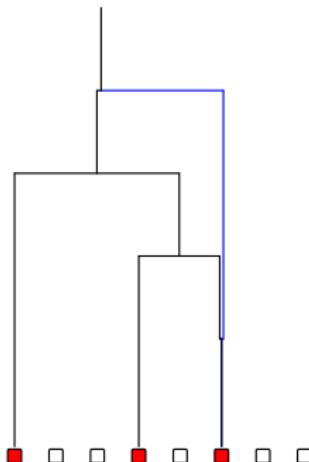
Wright-Fisher with recombination



The Ancestral Recombination Graph



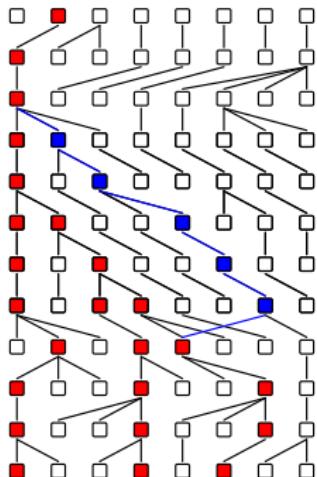
Backwards in time →



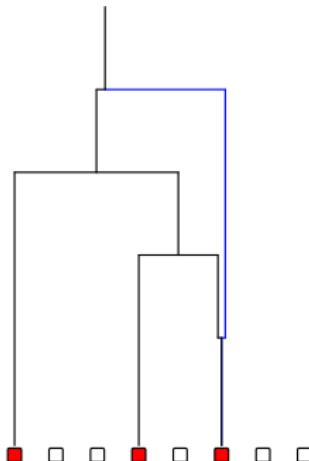
$$\delta = \frac{1}{2N_e} \frac{k(k-1)}{2}$$

$$\lambda = \frac{\rho}{4N_e} k$$

The Ancestral Recombination Graph



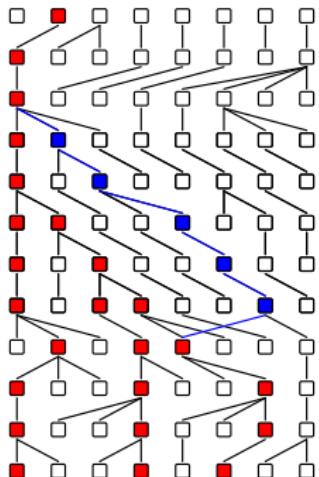
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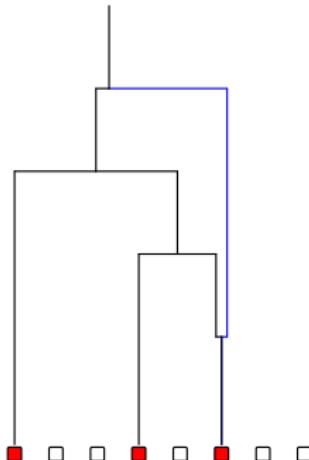
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The Ancestral Recombination Graph



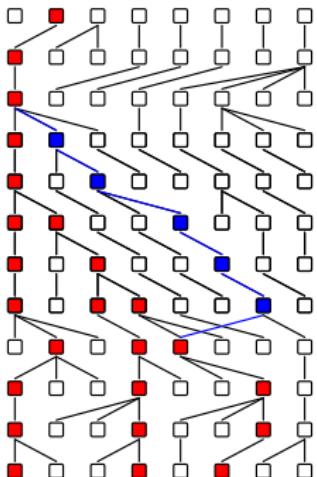
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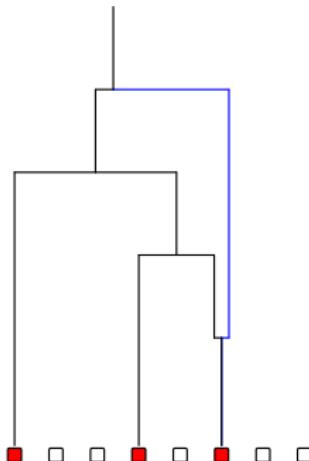
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The Ancestral Recombination Graph



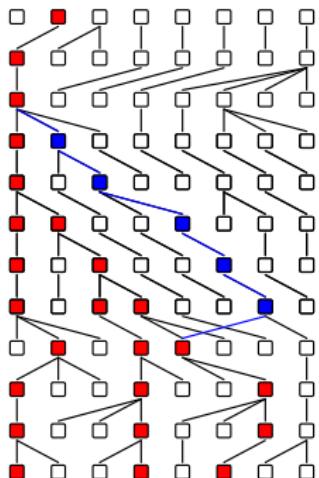
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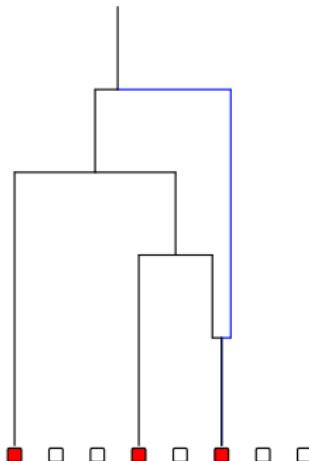
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The Ancestral Recombination Graph



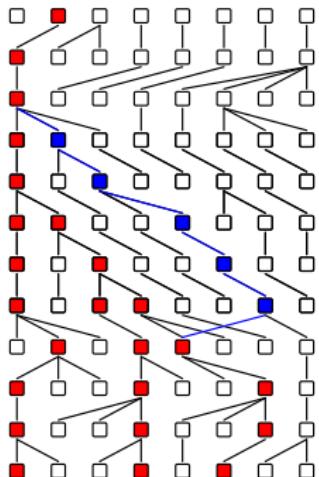
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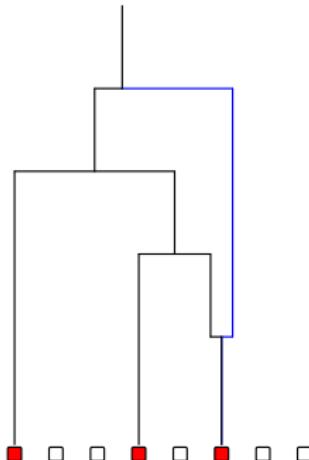
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The Ancestral Recombination Graph



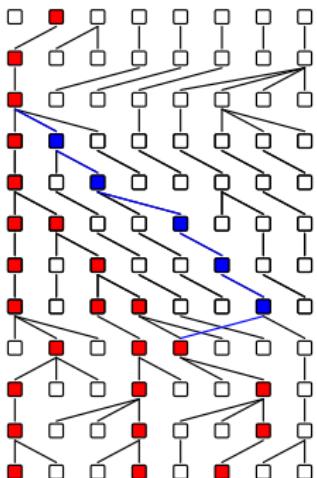
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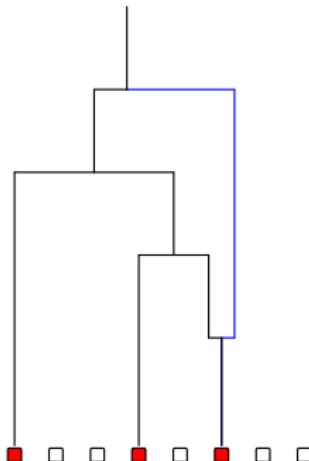
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The Ancestral Recombination Graph



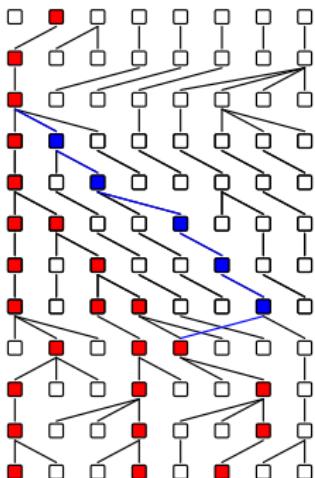
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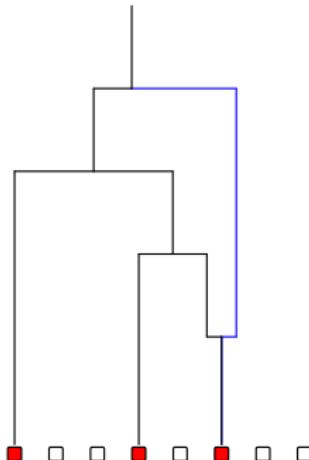
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The Ancestral Recombination Graph



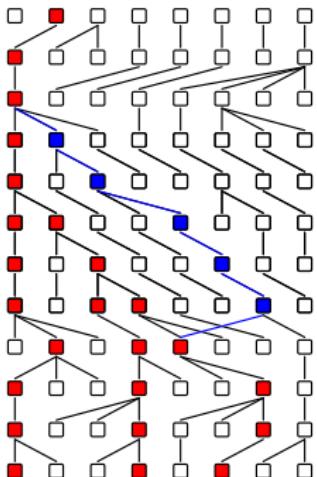
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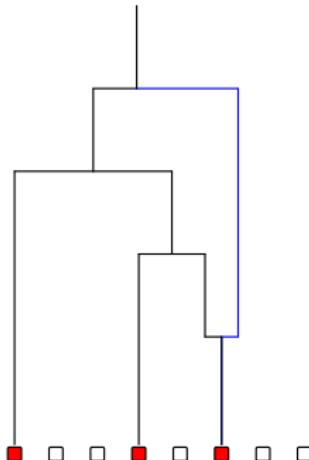
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The Ancestral Recombination Graph



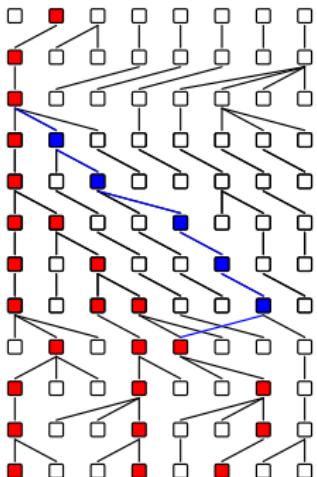
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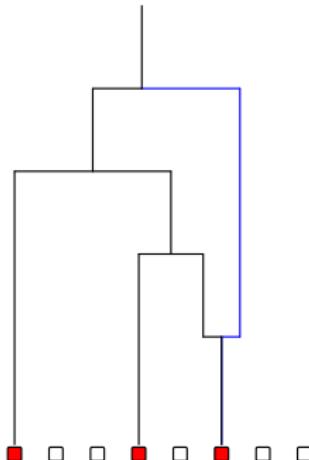
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The Ancestral Recombination Graph



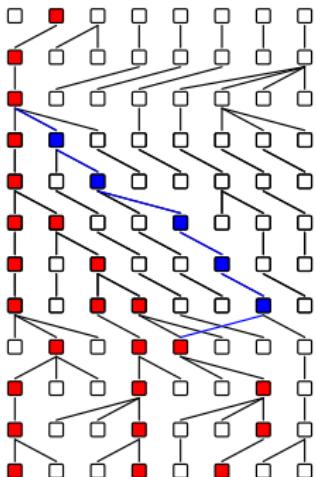
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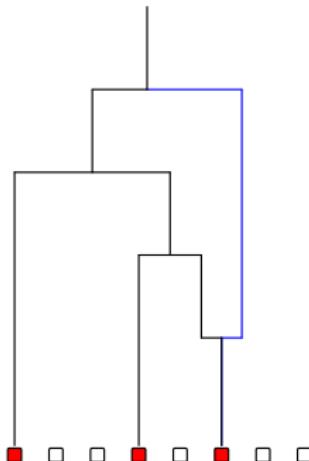
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The Ancestral Recombination Graph



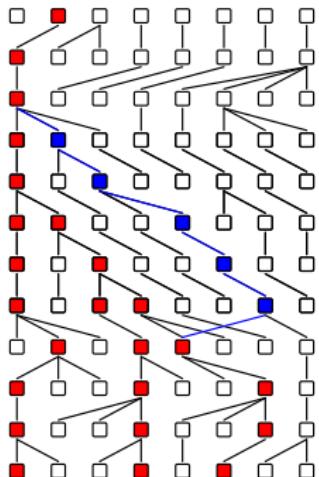
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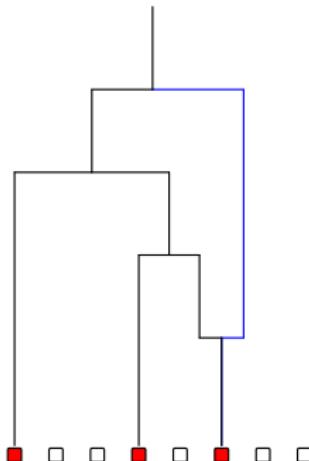
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The Ancestral Recombination Graph



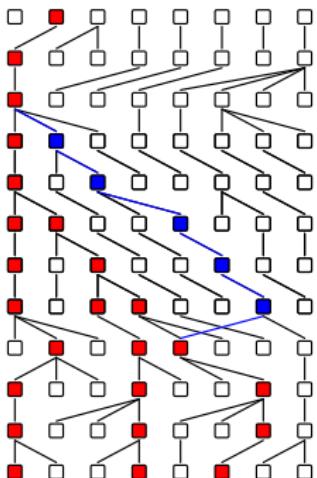
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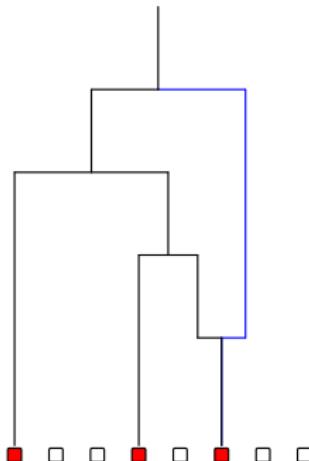
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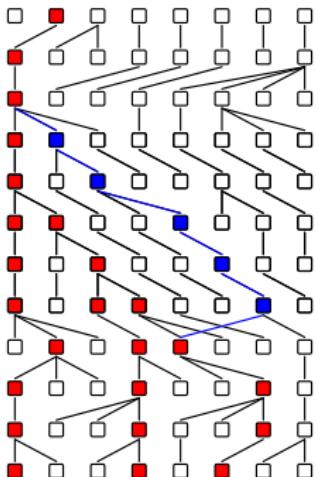
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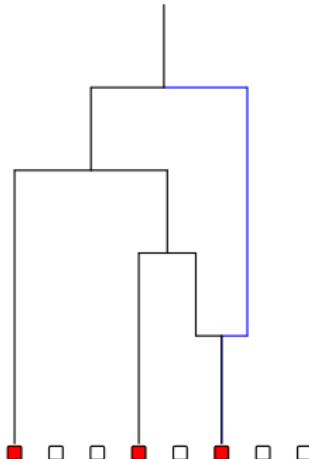
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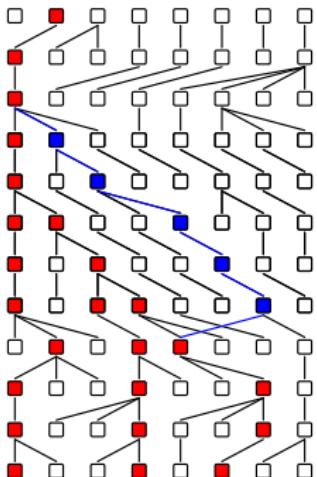
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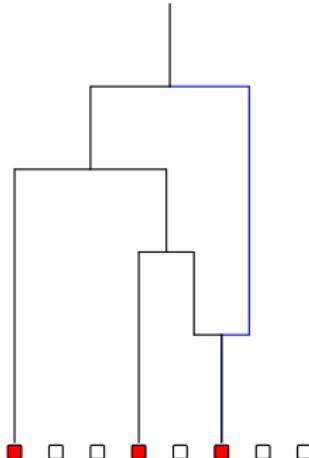
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The Ancestral Recombination Graph



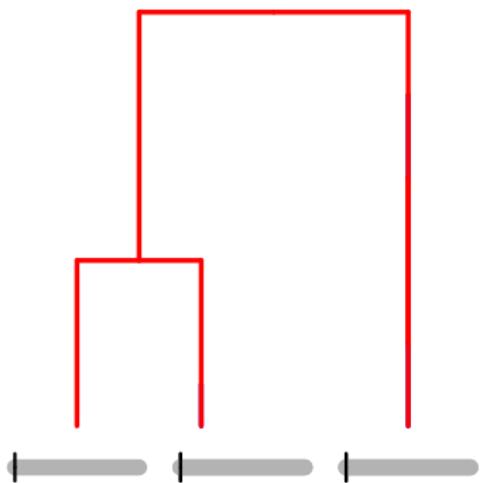
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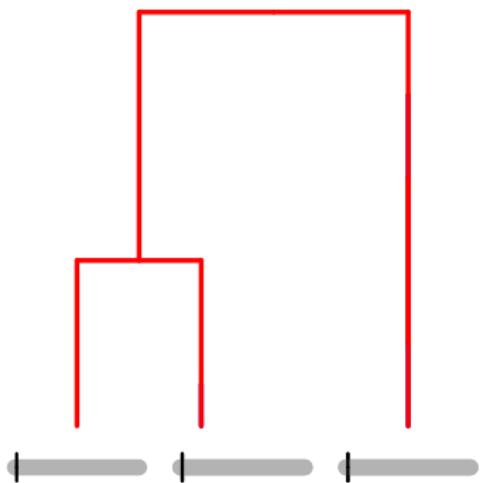
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Wiuf and Hein's sequential algorithm



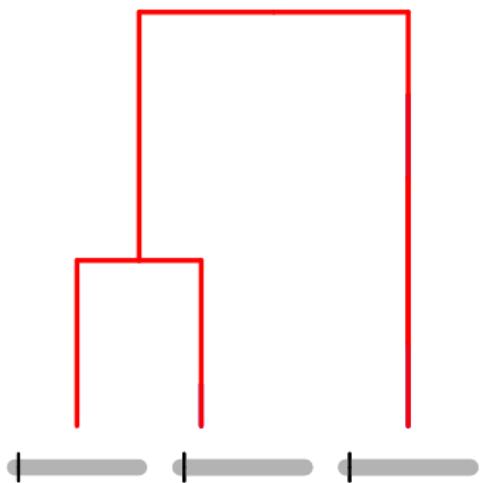
- Extend
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- Extend

Wiuf and Hein's sequential algorithm



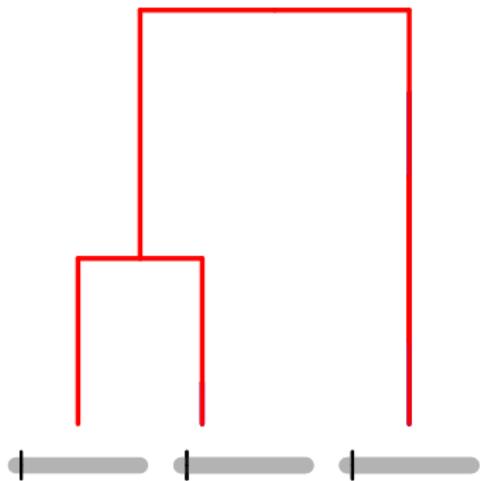
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Wiuf and Hein's sequential algorithm



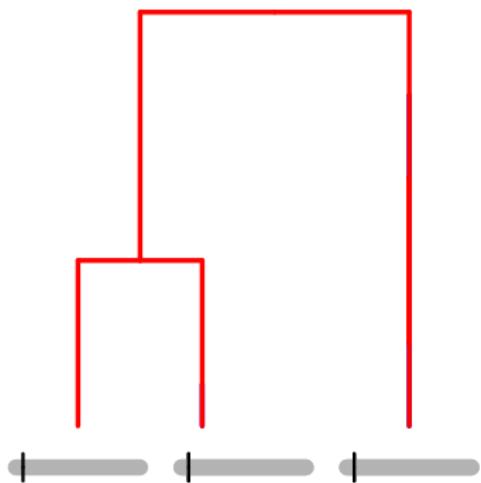
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Wiuf and Hein's sequential algorithm



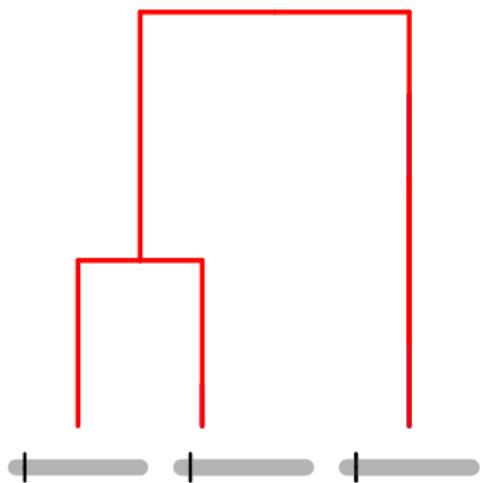
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Wiuf and Hein's sequential algorithm



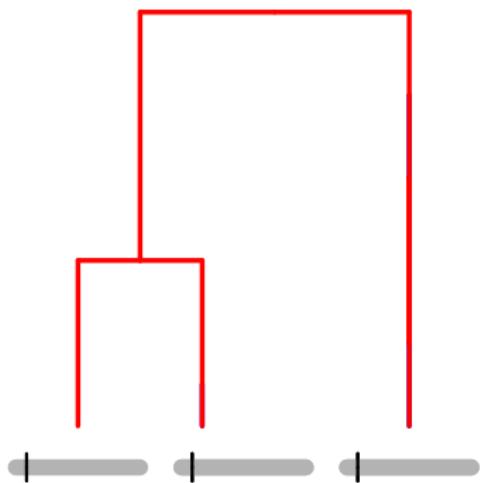
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Wiuf and Hein's sequential algorithm



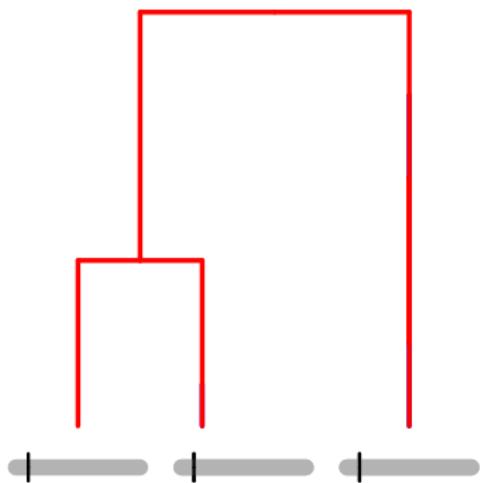
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Wiuf and Hein's sequential algorithm



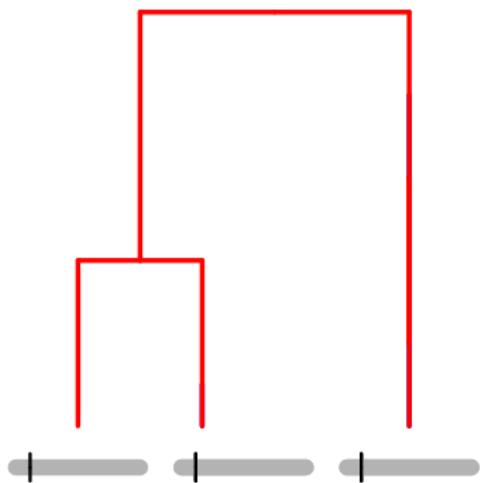
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Wiuf and Hein's sequential algorithm



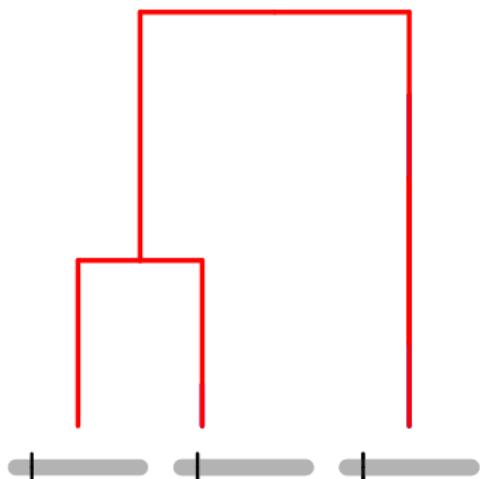
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Wiuf and Hein's sequential algorithm



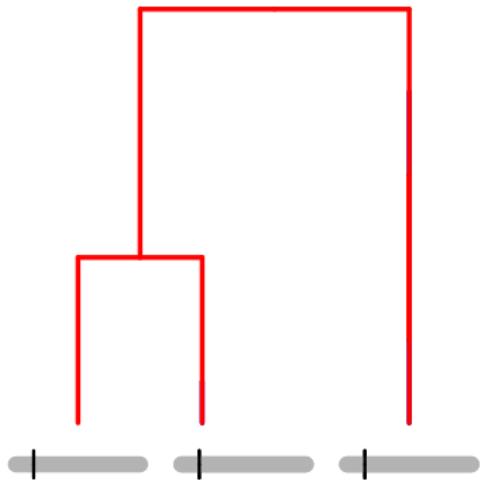
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Wiuf and Hein's sequential algorithm



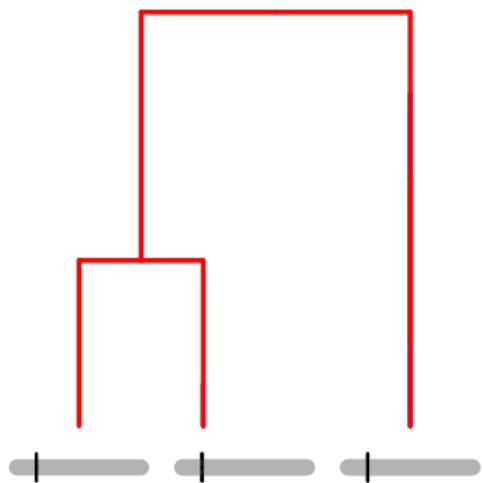
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Wiuf and Hein's sequential algorithm



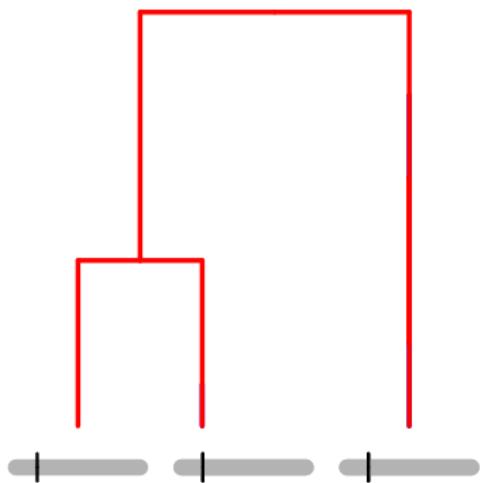
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Wiuf and Hein's sequential algorithm



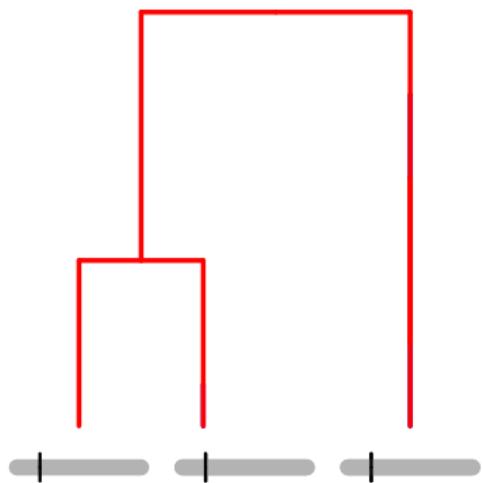
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Wiuf and Hein's sequential algorithm



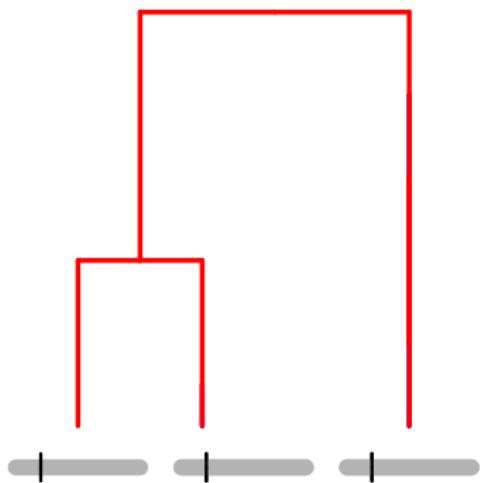
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Wiuf and Hein's sequential algorithm



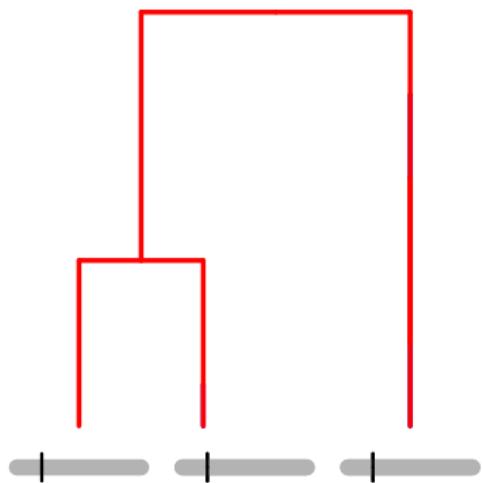
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Wiuf and Hein's sequential algorithm



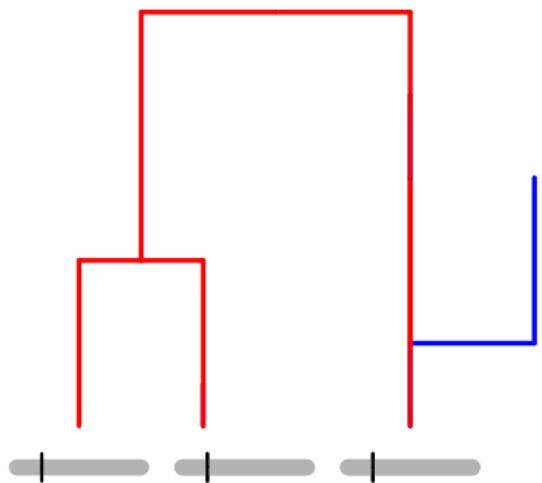
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Wiuf and Hein's sequential algorithm



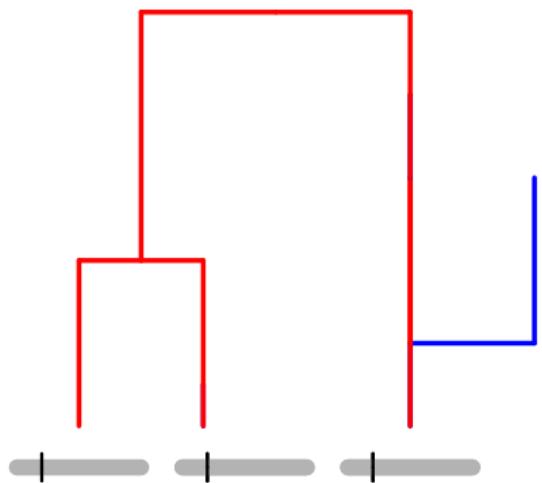
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Wiuf and Hein's sequential algorithm



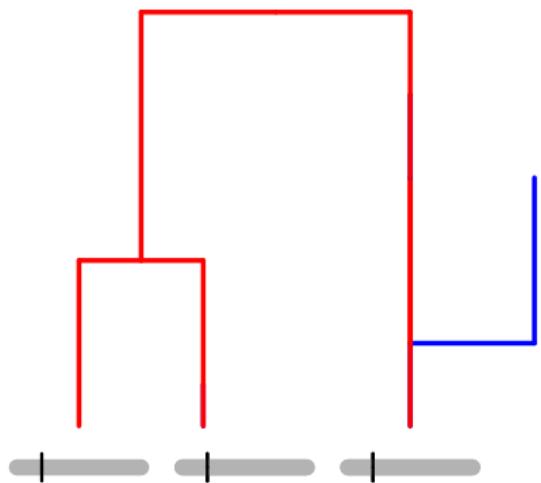
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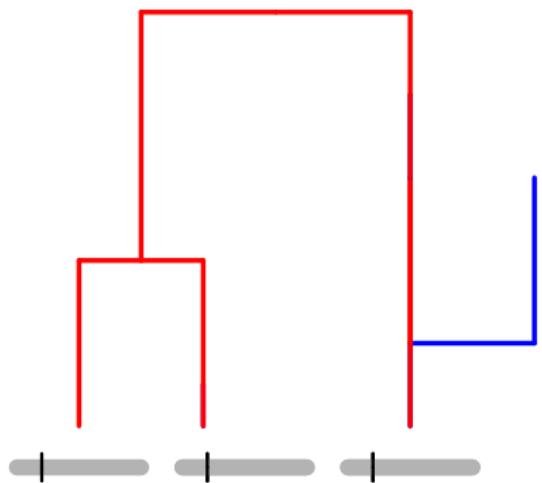
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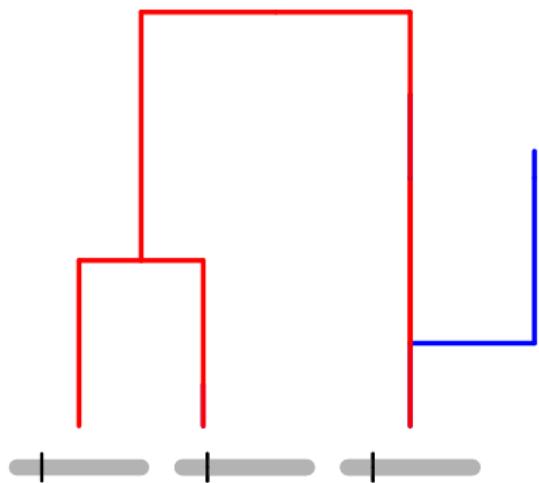
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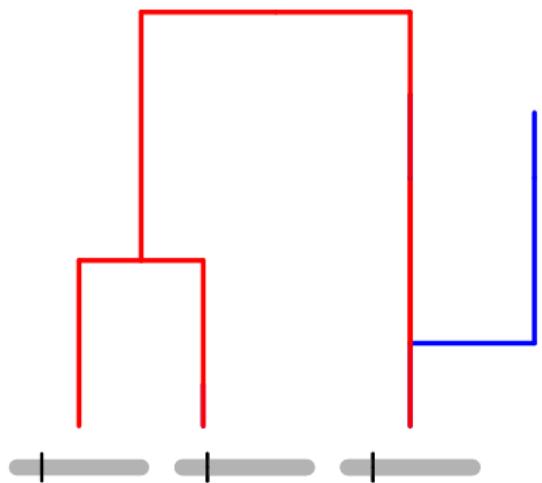
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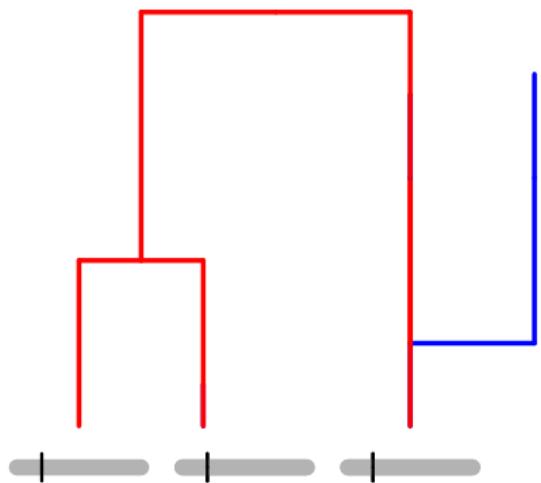
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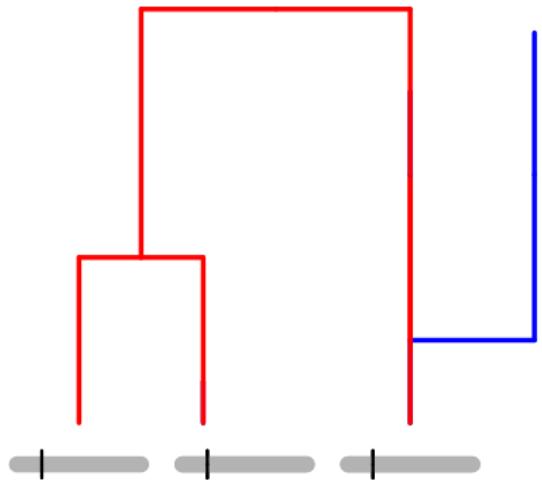
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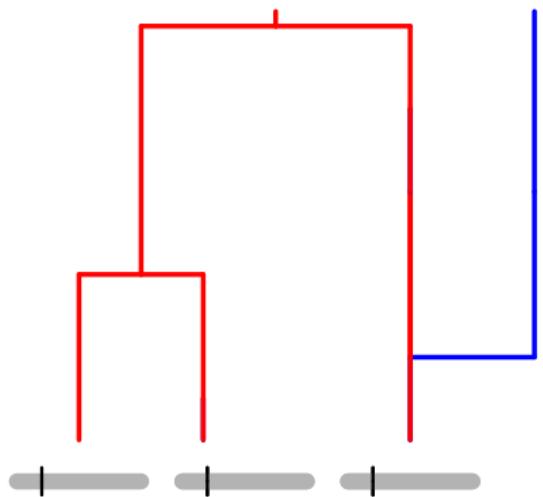
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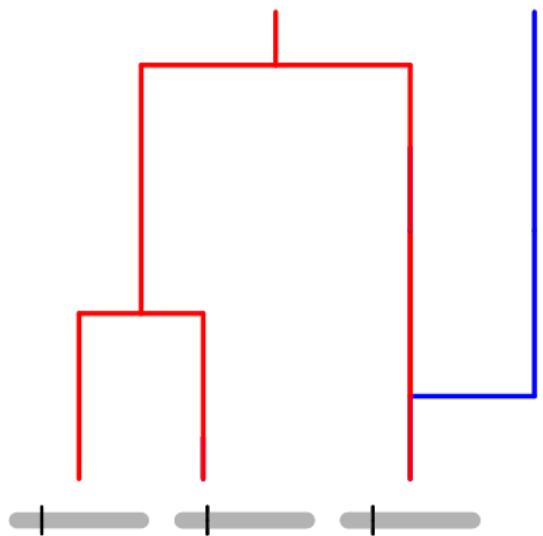
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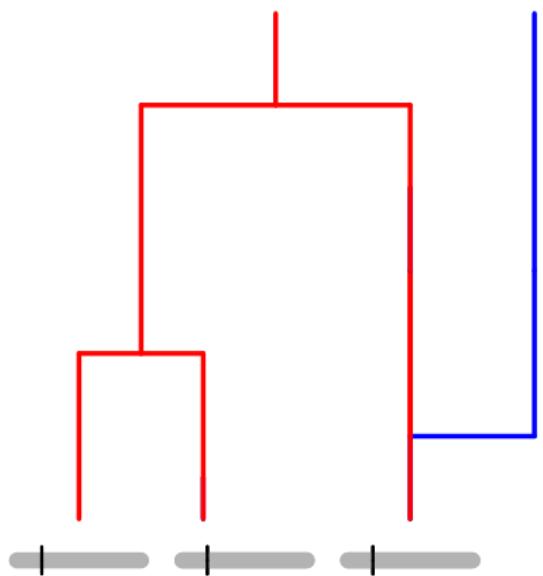
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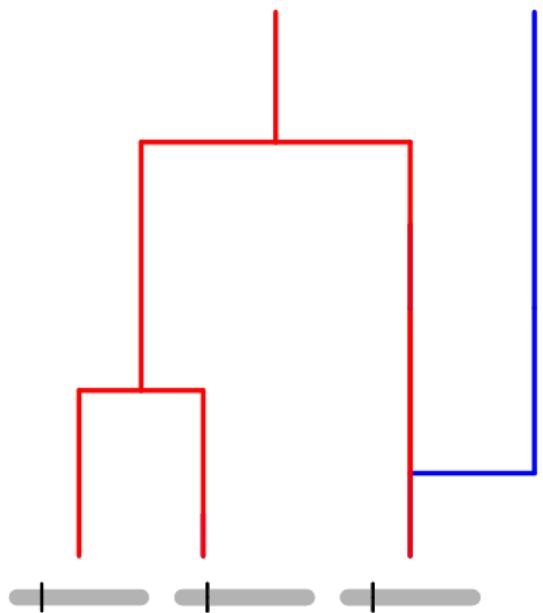
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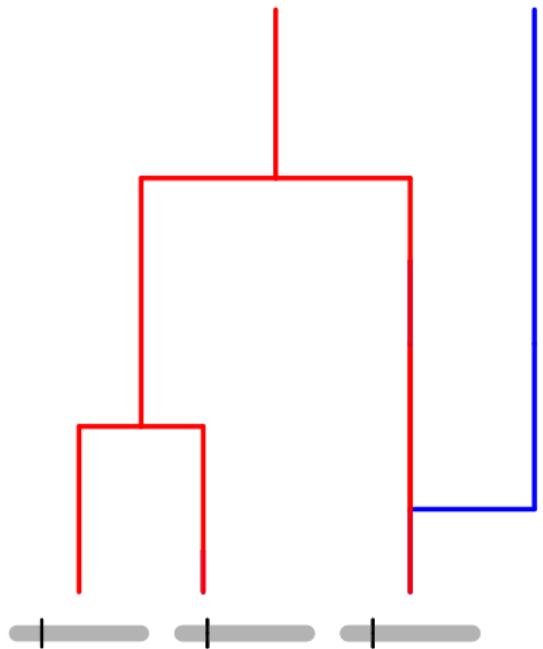
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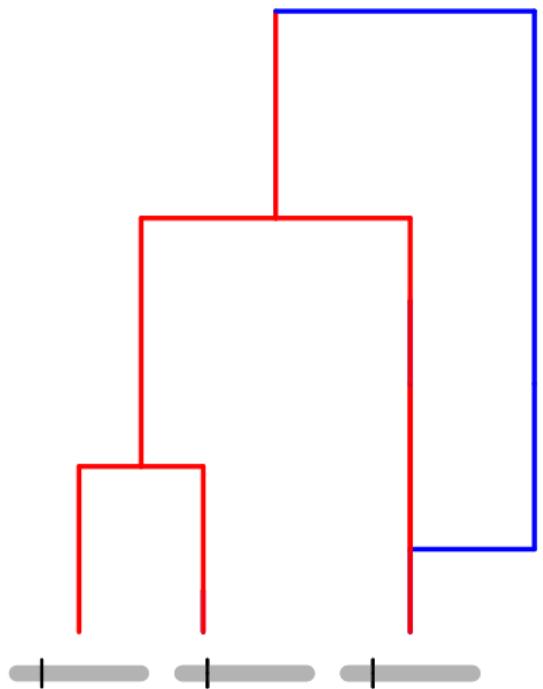
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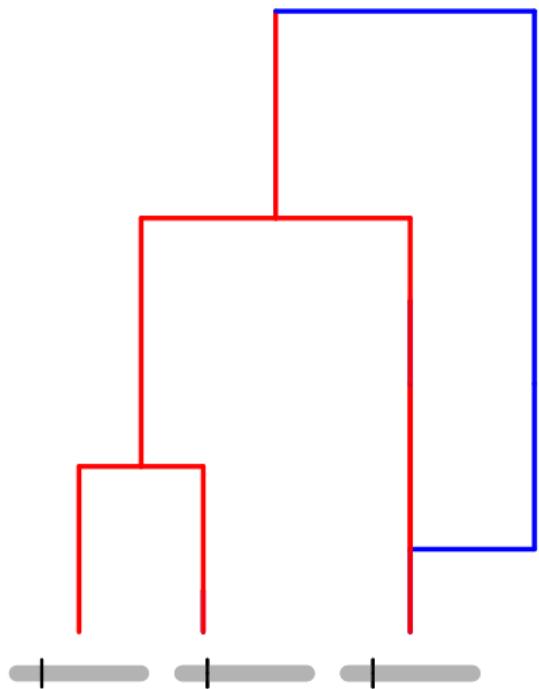
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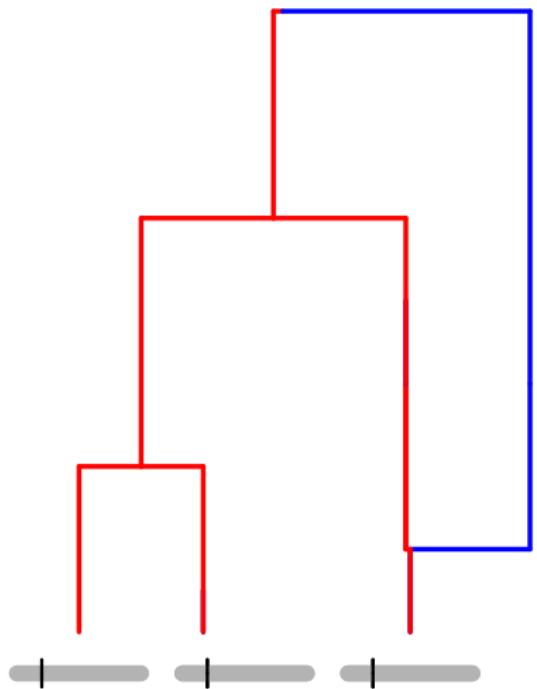
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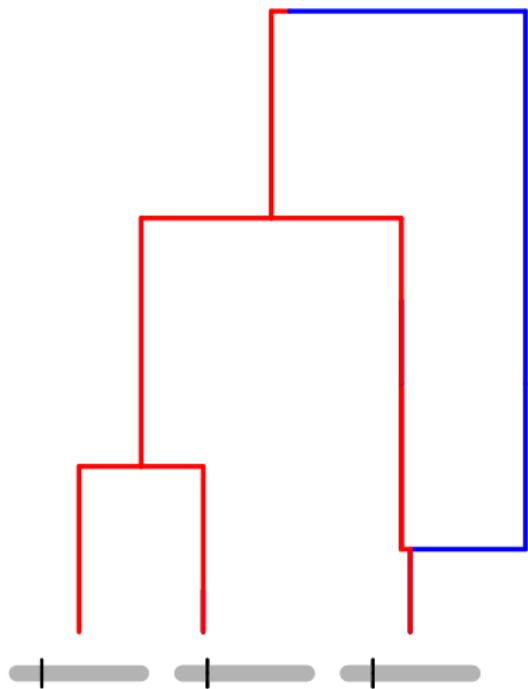
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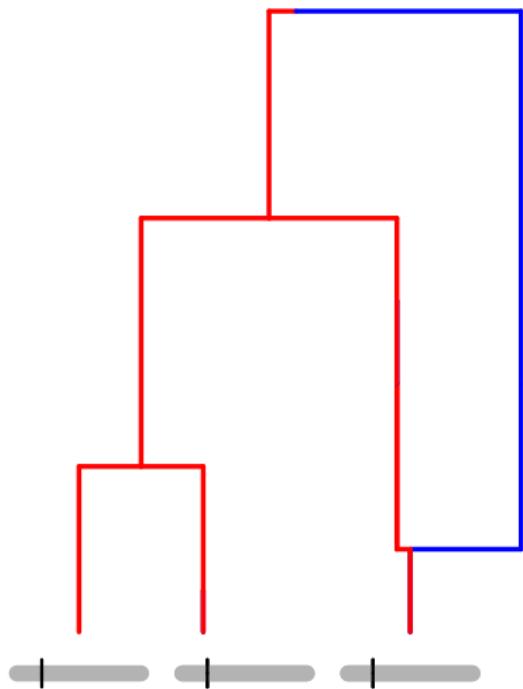
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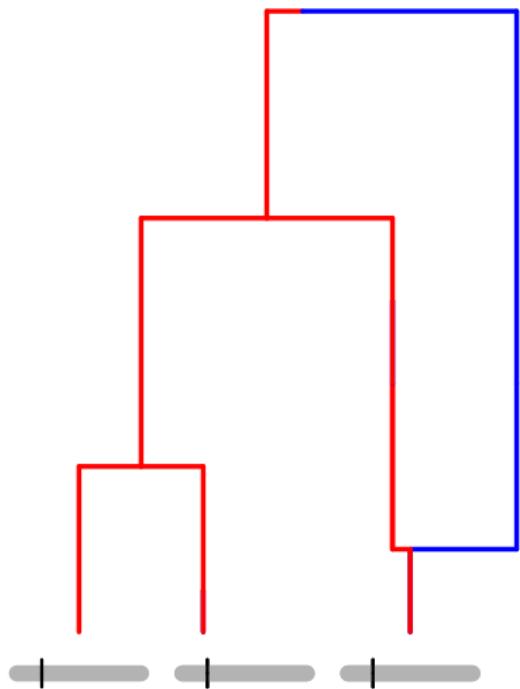
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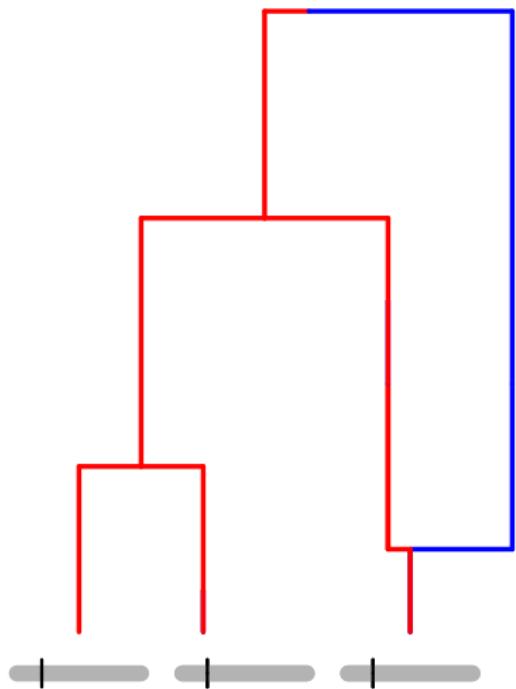
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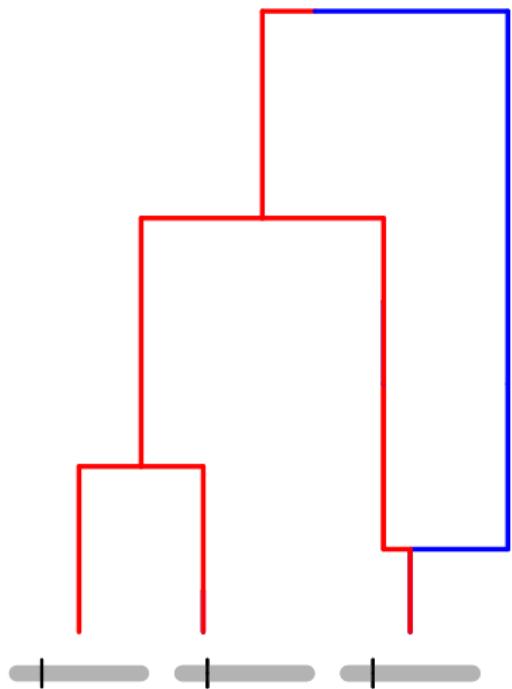
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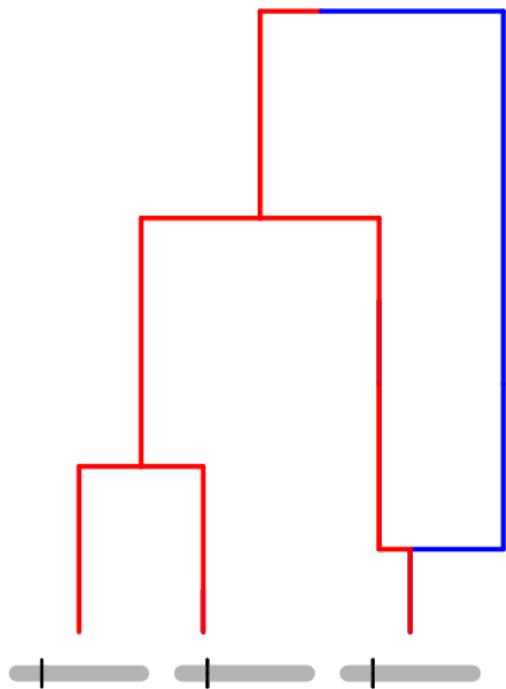
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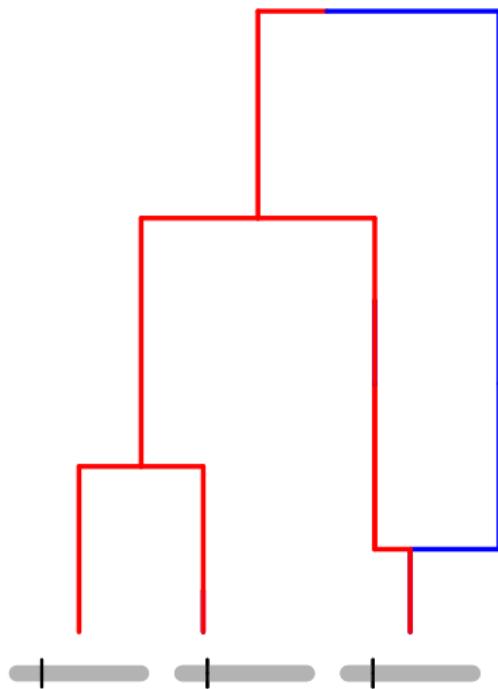
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Wiuf and Hein's sequential algorithm



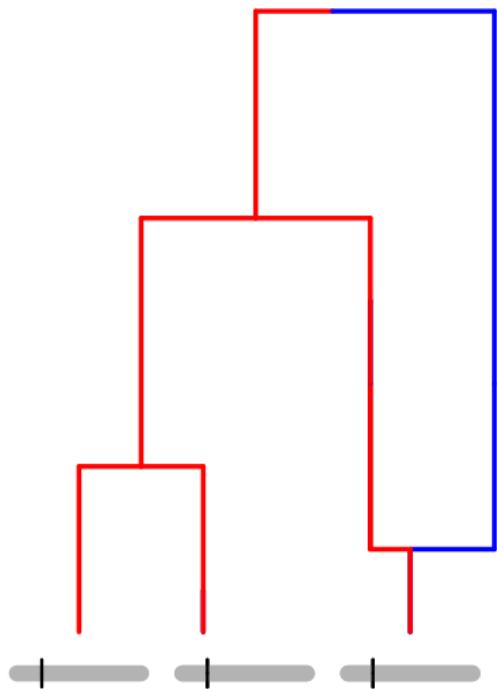
- Extend
- Recombine & coalesce
- Extend

Wiuf and Hein's sequential algorithm



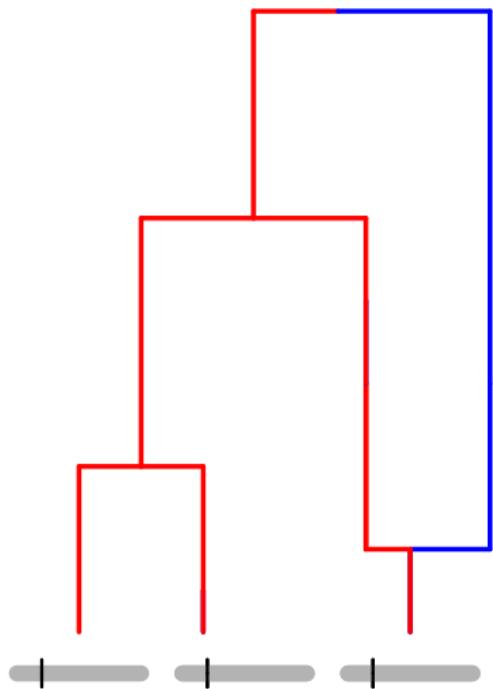
- Extend
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Wiuf and Hein's sequential algorithm



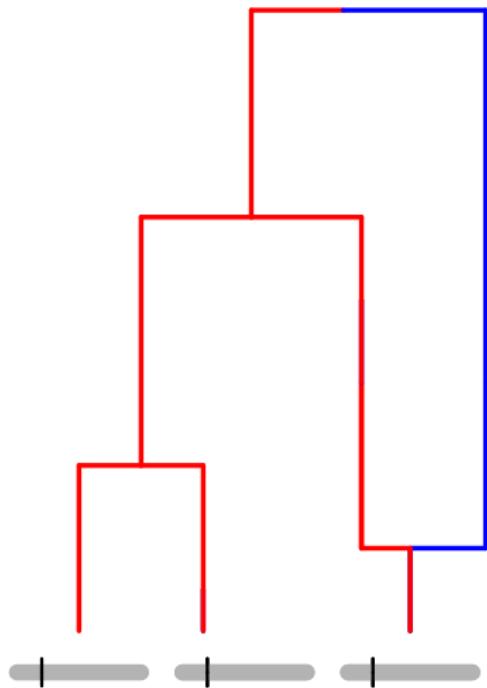
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Wiuf and Hein's sequential algorithm



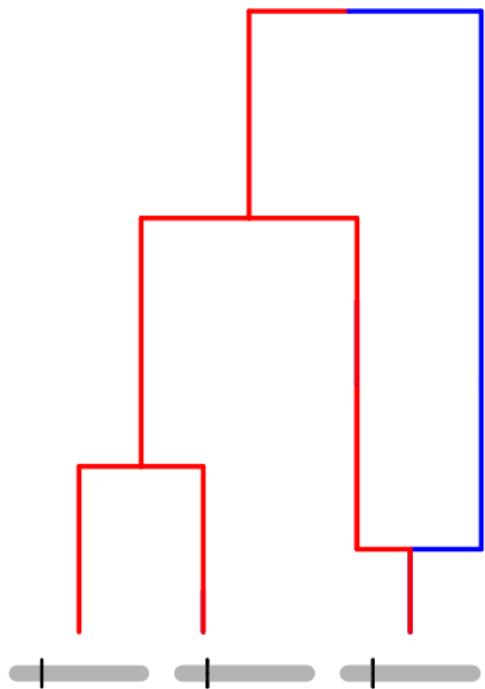
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Wiuf and Hein's sequential algorithm



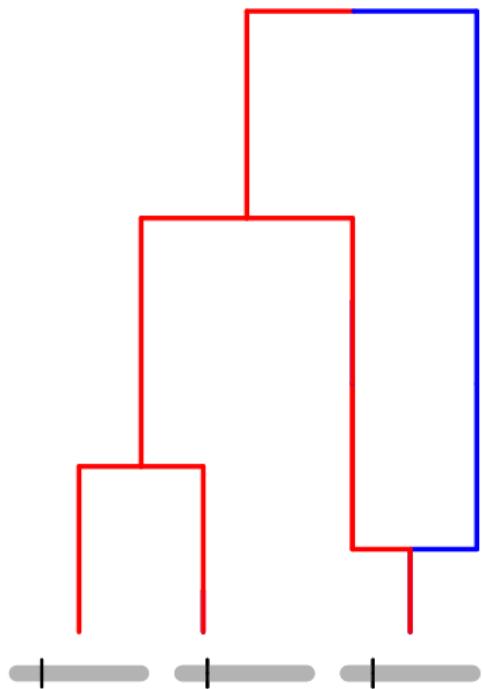
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Wiuf and Hein's sequential algorithm



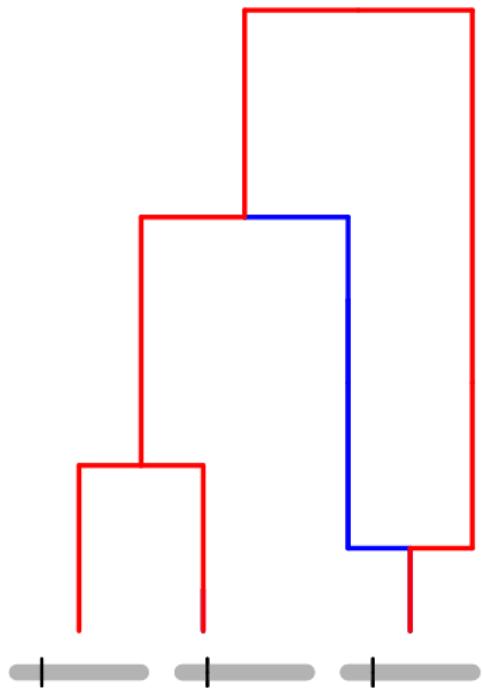
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Wiuf and Hein's sequential algorithm



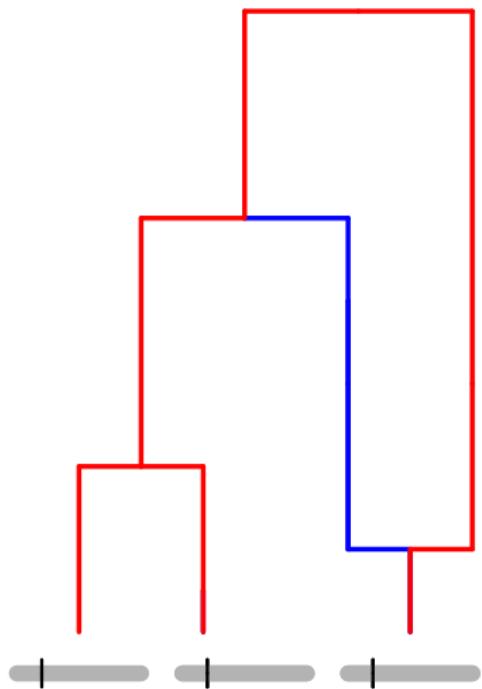
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Wiuf and Hein's sequential algorithm



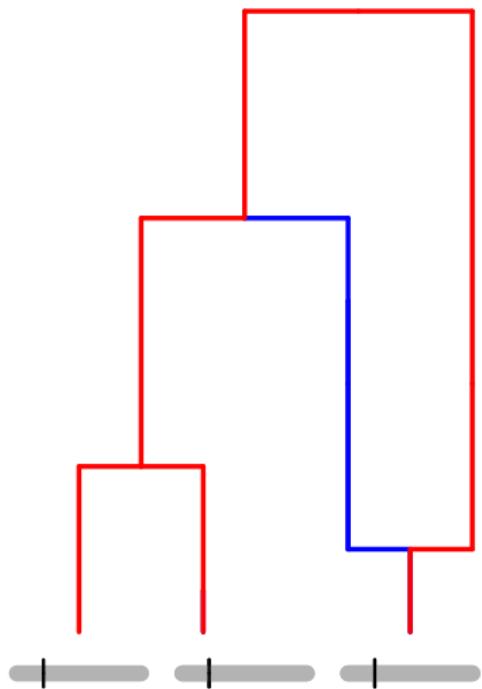
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Wiuf and Hein's sequential algorithm



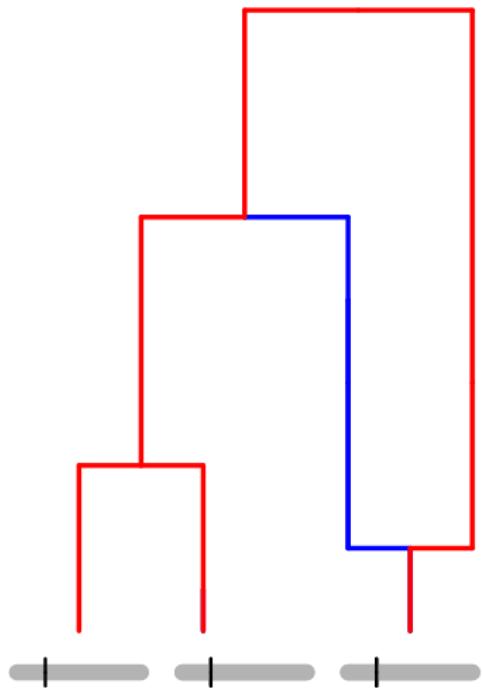
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Wiuf and Hein's sequential algorithm



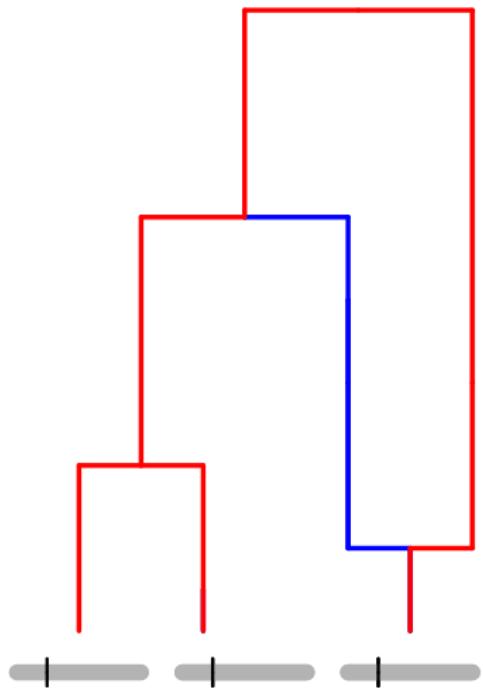
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Wiuf and Hein's sequential algorithm



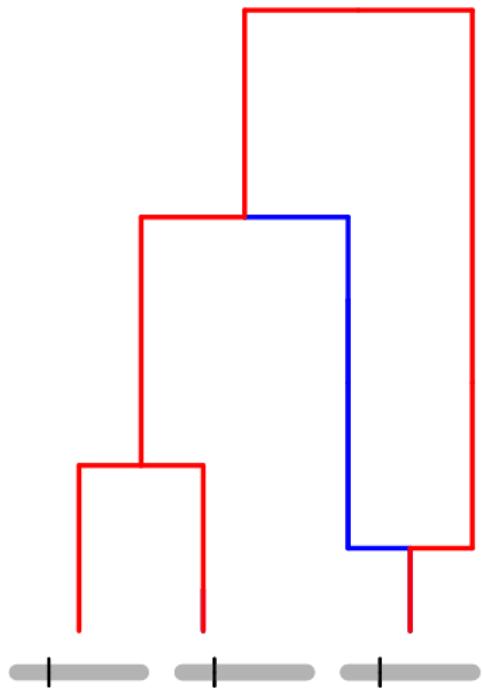
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Wiuf and Hein's sequential algorithm



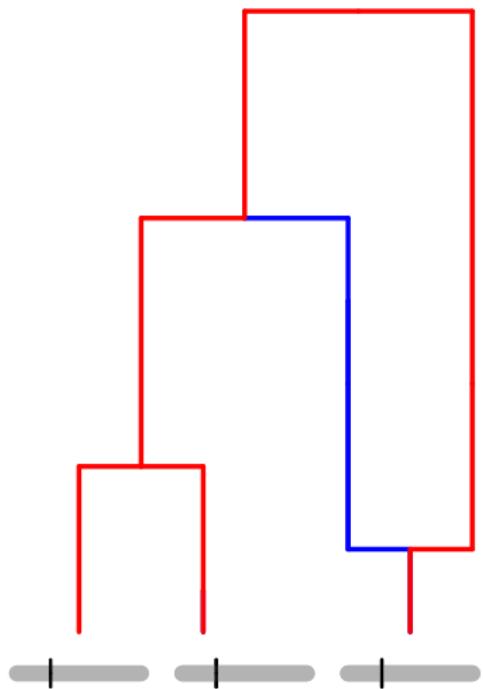
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Wiuf and Hein's sequential algorithm



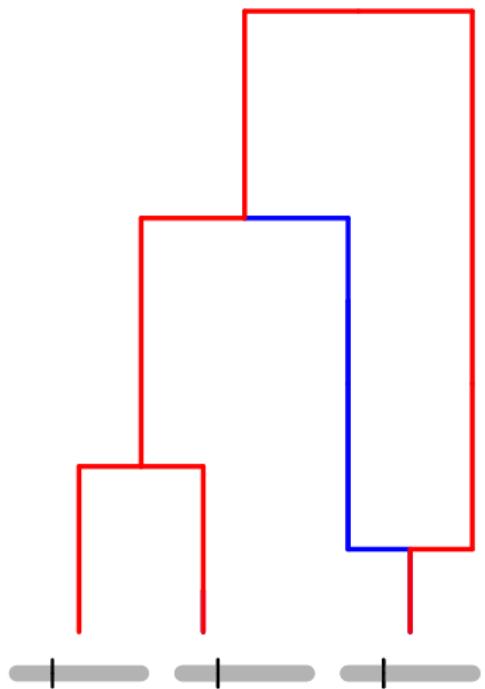
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Wiuf and Hein's sequential algorithm



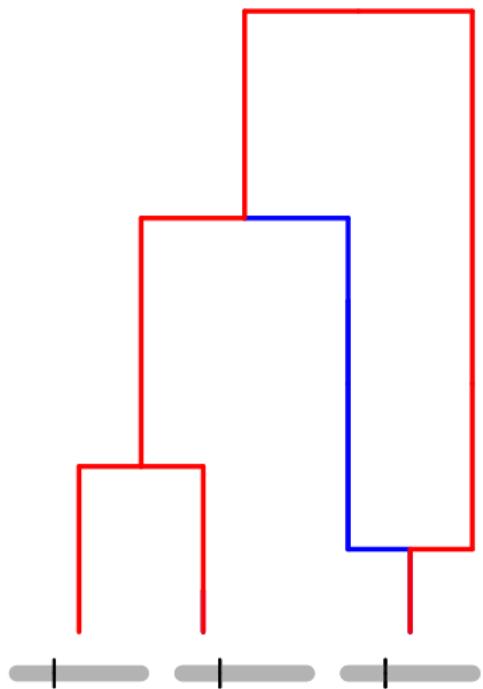
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Wiuf and Hein's sequential algorithm



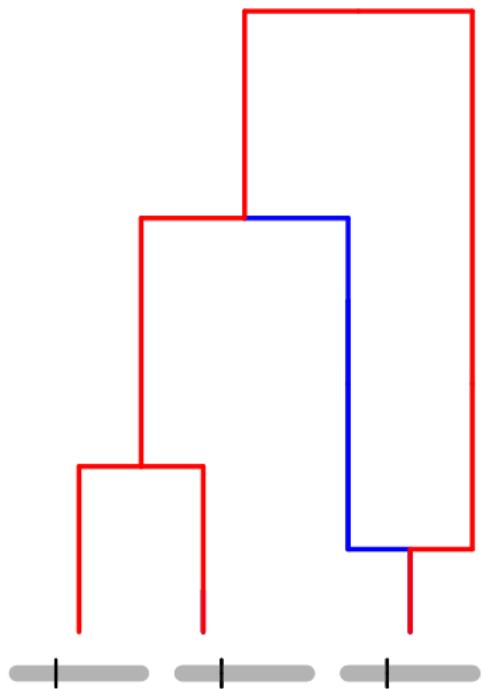
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Wiuf and Hein's sequential algorithm



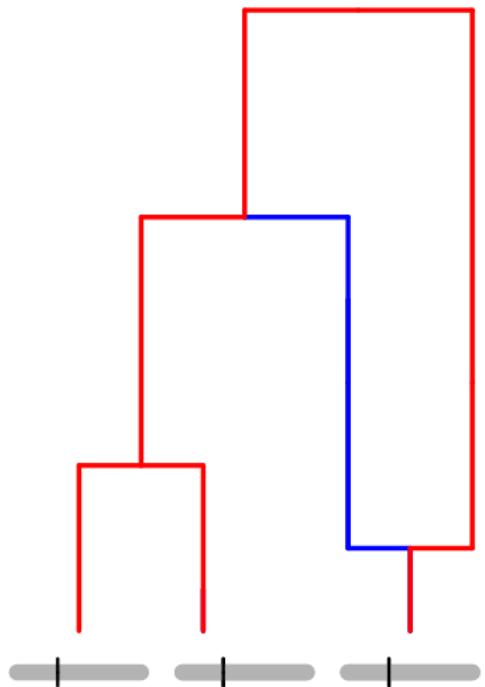
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Wiuf and Hein's sequential algorithm



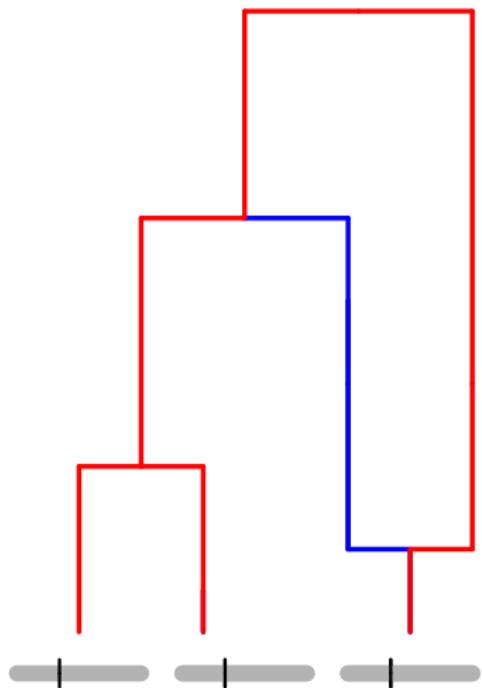
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Wiuf and Hein's sequential algorithm



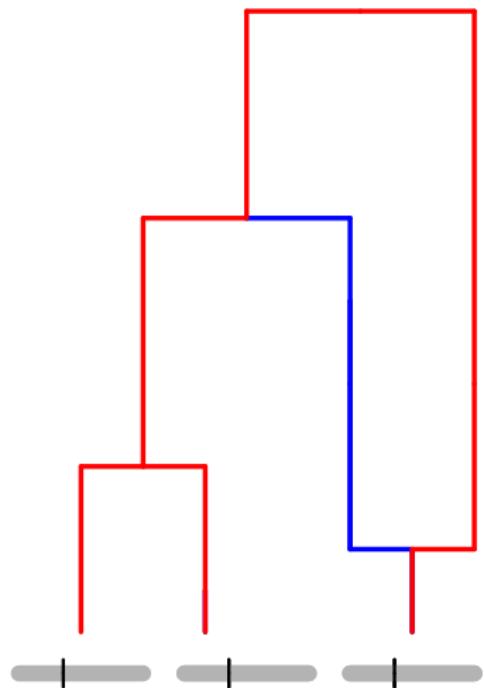
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Wiuf and Hein's sequential algorithm



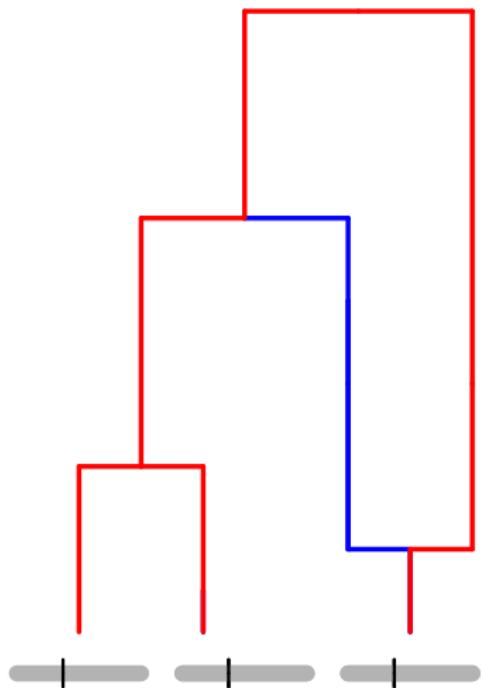
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Wiuf and Hein's sequential algorithm



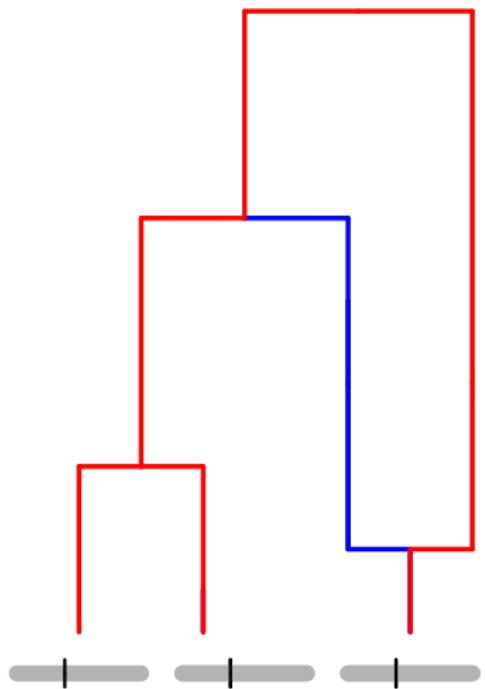
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Wiuf and Hein's sequential algorithm



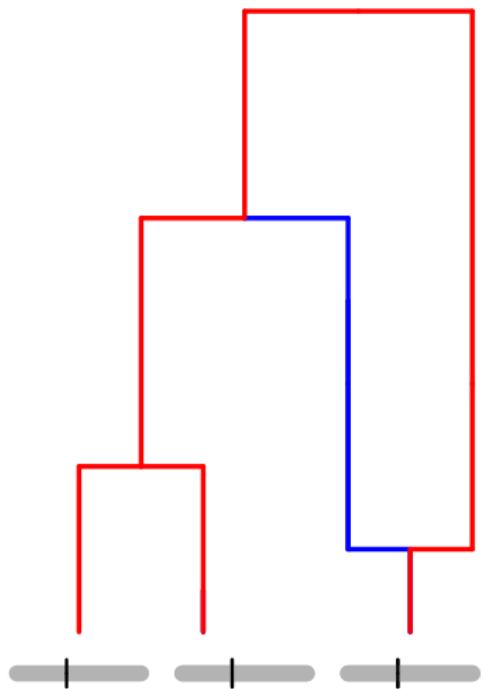
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Wiuf and Hein's sequential algorithm



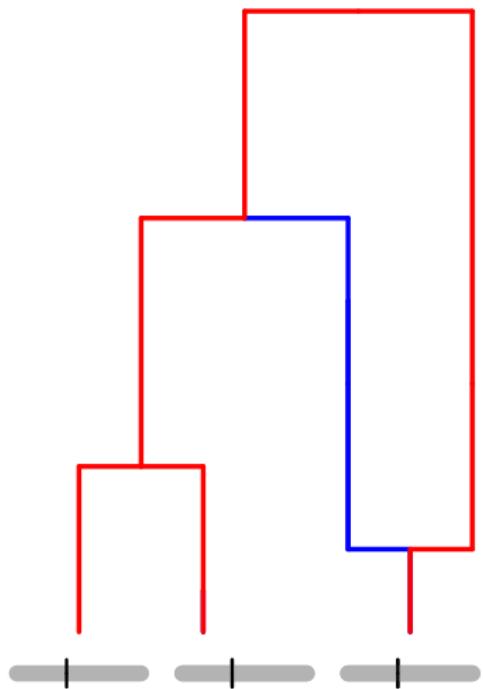
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Wiuf and Hein's sequential algorithm



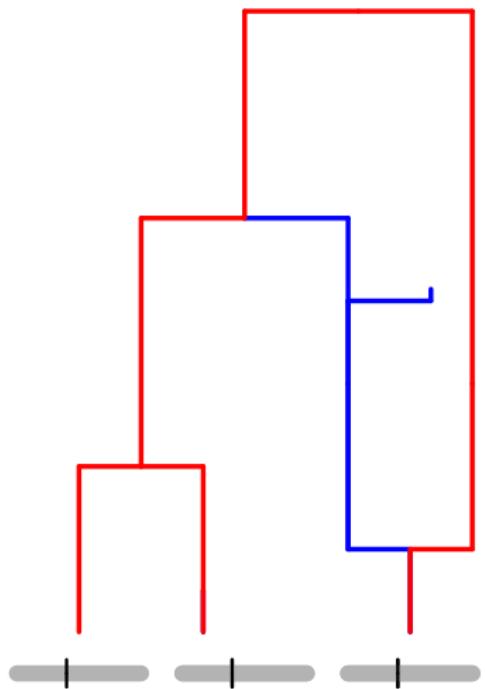
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Wiuf and Hein's sequential algorithm



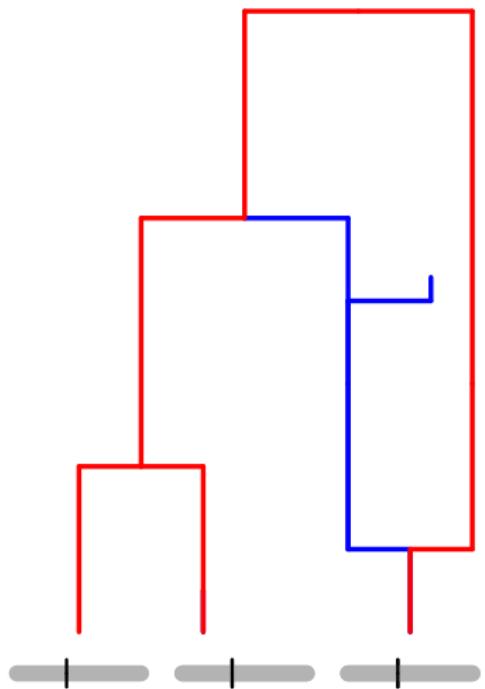
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- **Recombine & coalesce**
- Extend
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Wiuf and Hein's sequential algorithm



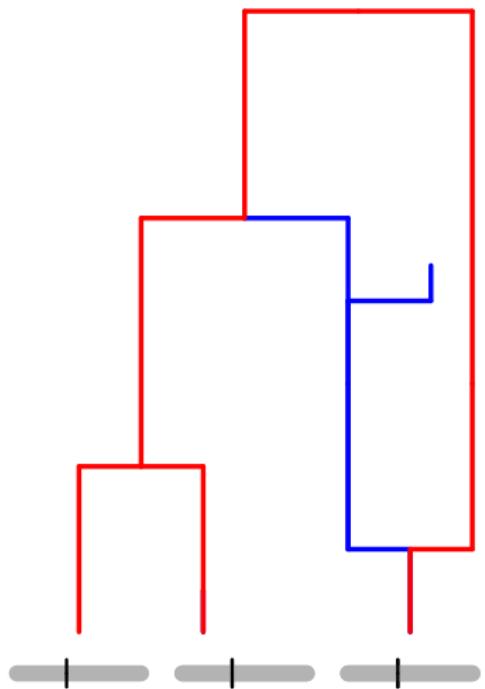
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Wiuf and Hein's sequential algorithm



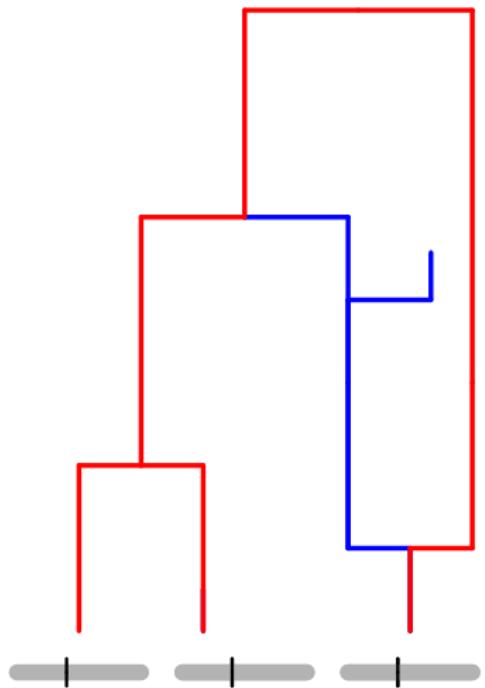
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Wiuf and Hein's sequential algorithm



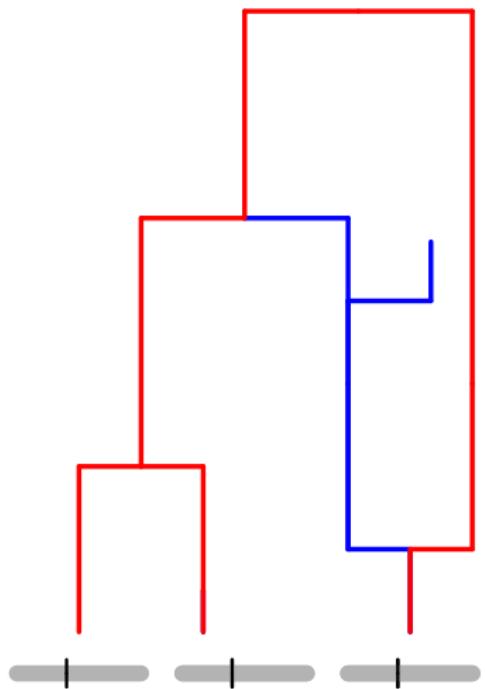
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Wiuf and Hein's sequential algorithm



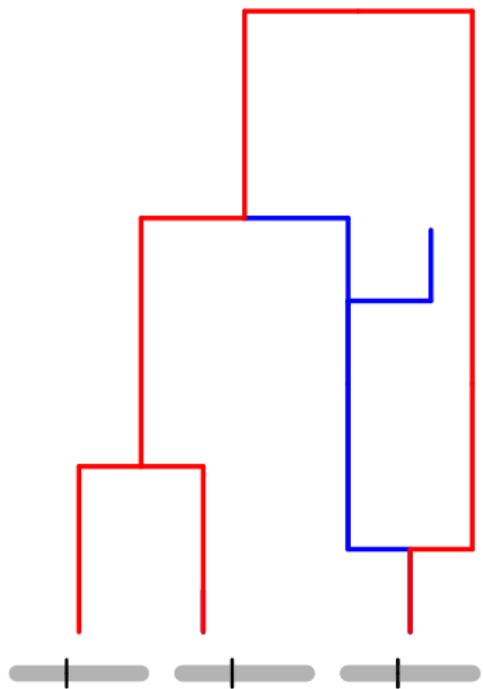
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Wiuf and Hein's sequential algorithm



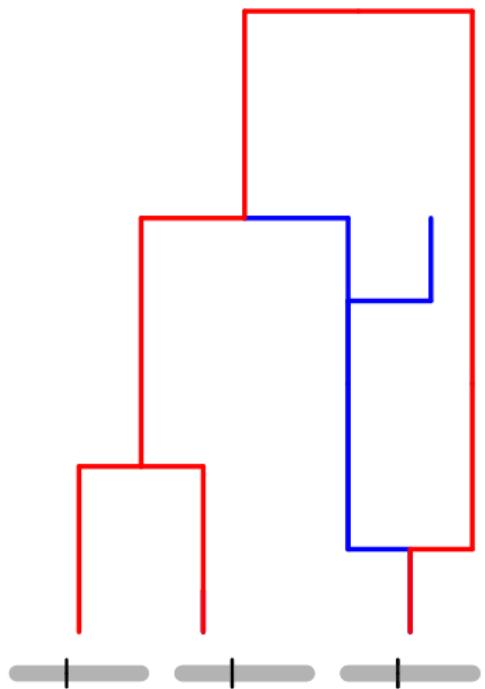
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Wiuf and Hein's sequential algorithm



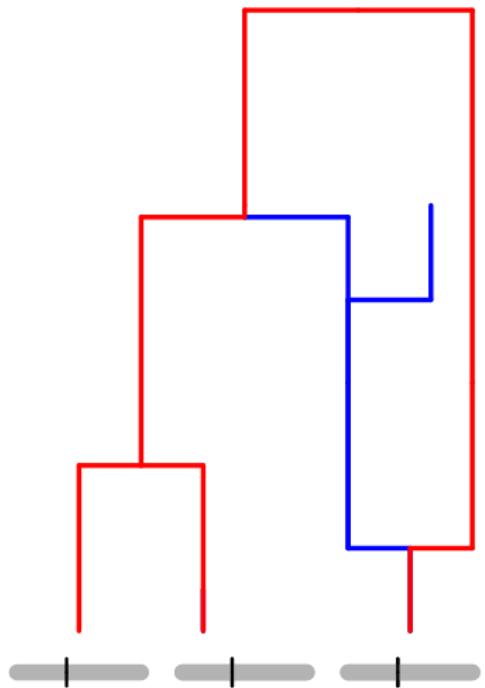
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Wiuf and Hein's sequential algorithm



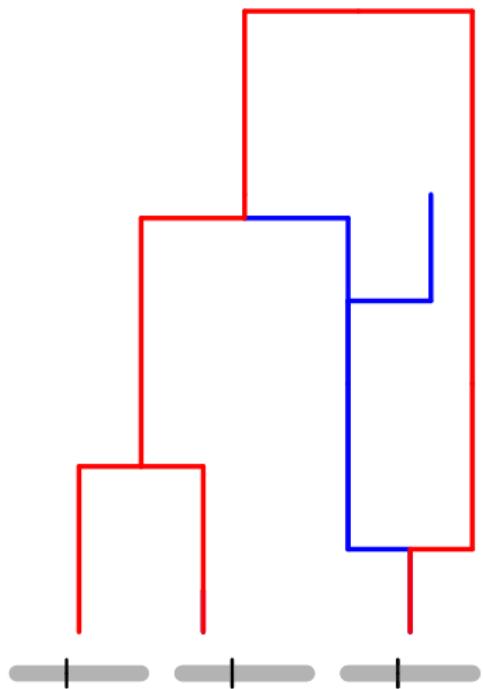
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Wiuf and Hein's sequential algorithm



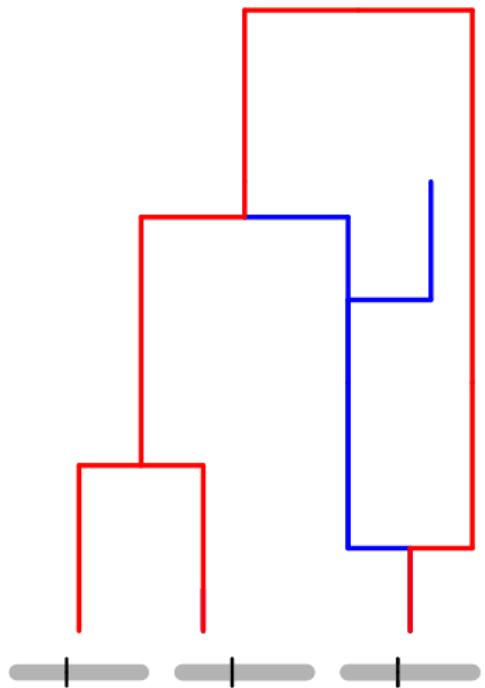
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Wiuf and Hein's sequential algorithm



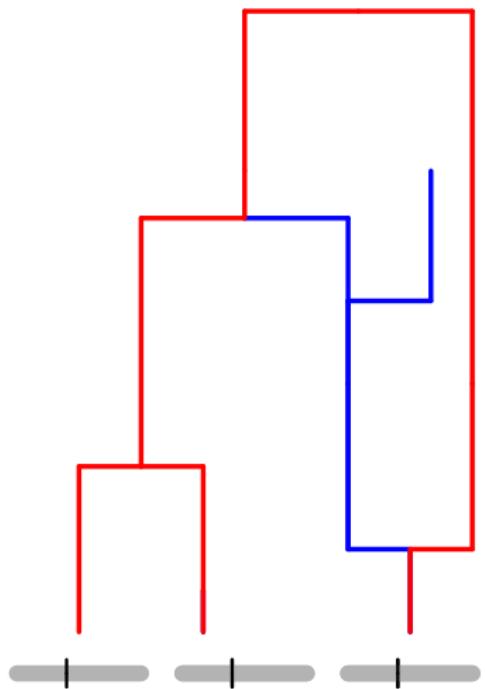
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Wiuf and Hein's sequential algorithm



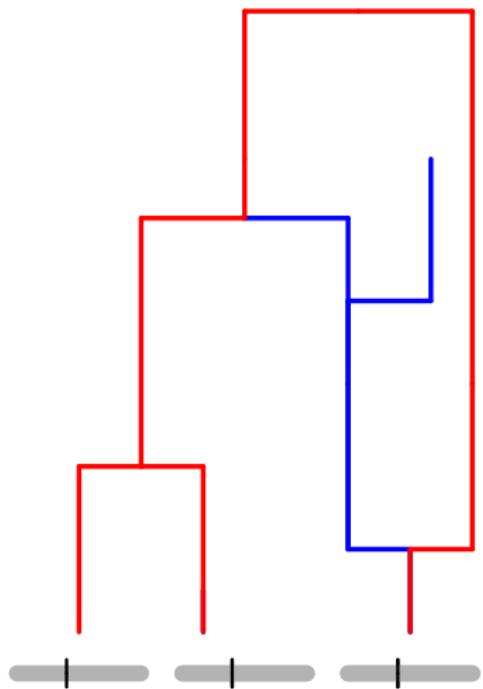
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Wiuf and Hein's sequential algorithm



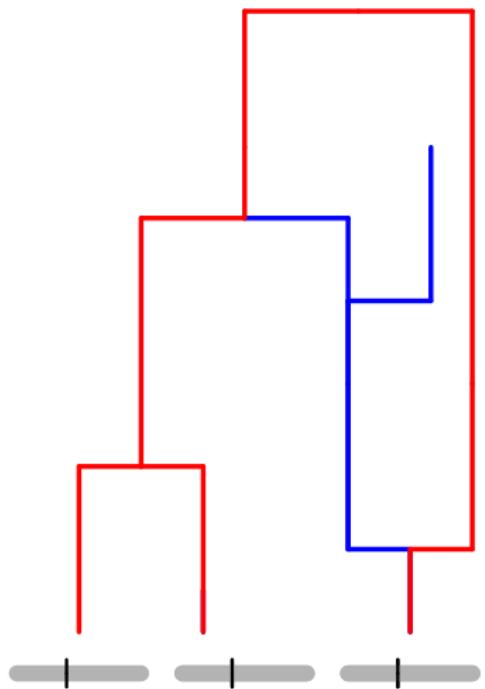
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Wiuf and Hein's sequential algorithm



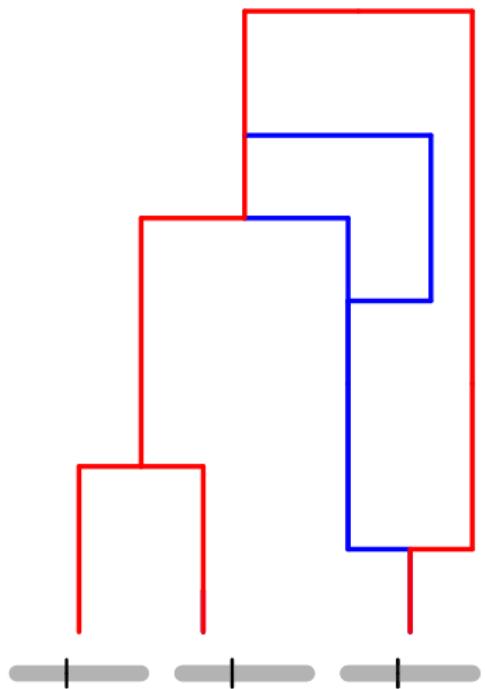
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Wiuf and Hein's sequential algorithm



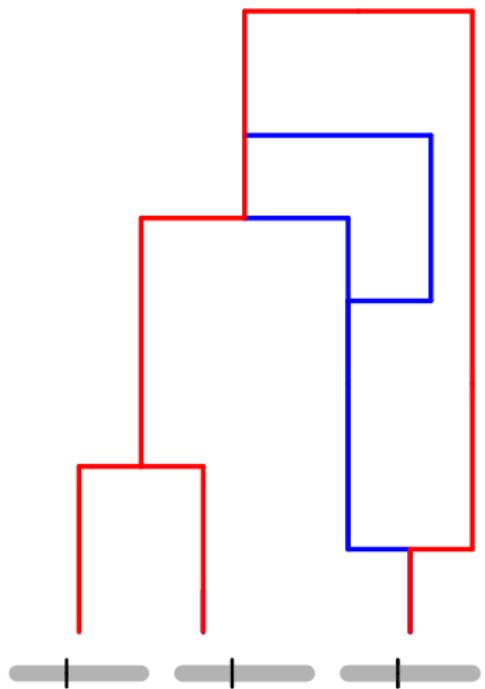
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Wiuf and Hein's sequential algorithm



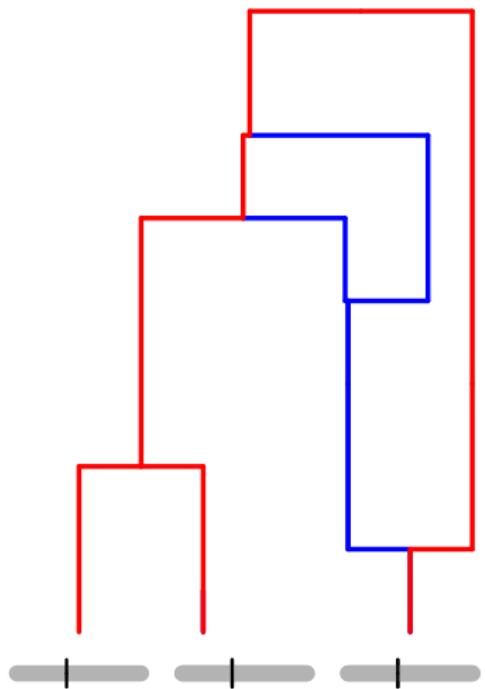
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Wiuf and Hein's sequential algorithm



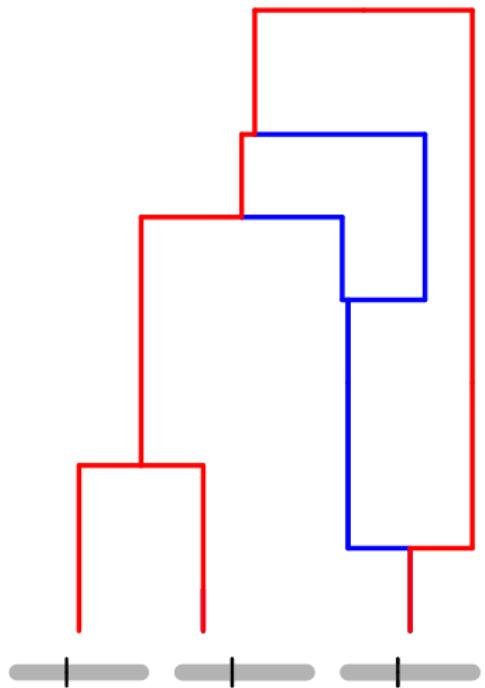
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Wiuf and Hein's sequential algorithm



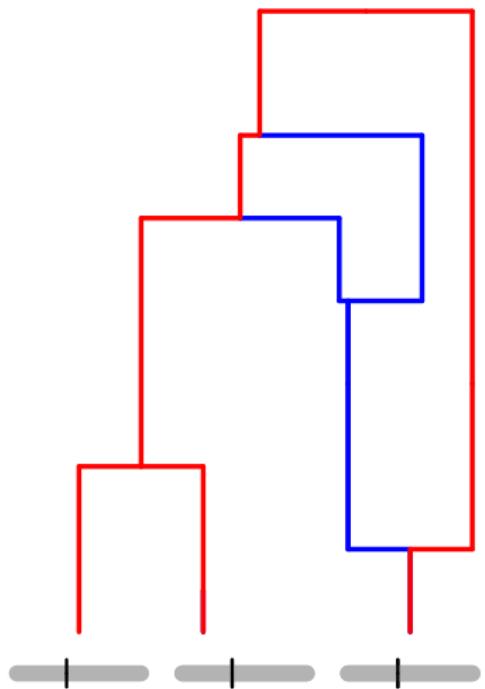
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Wiuf and Hein's sequential algorithm



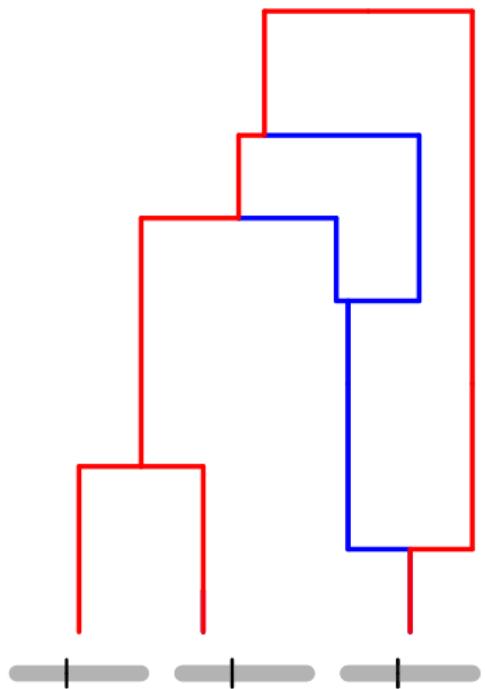
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Wiuf and Hein's sequential algorithm



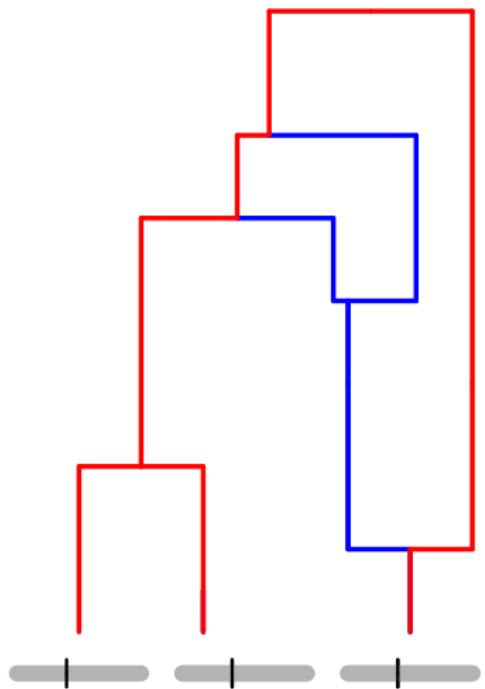
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Wiuf and Hein's sequential algorithm



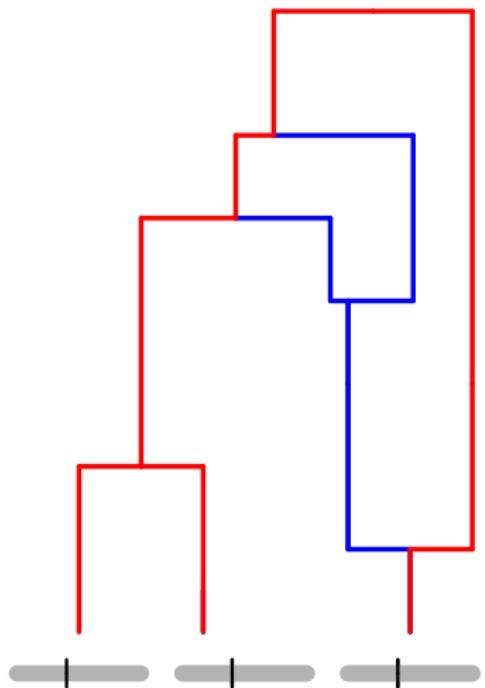
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Wiuf and Hein's sequential algorithm



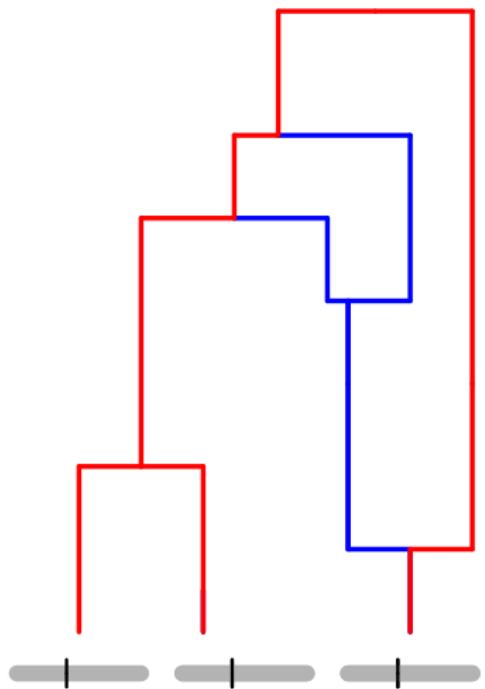
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Wiuf and Hein's sequential algorithm



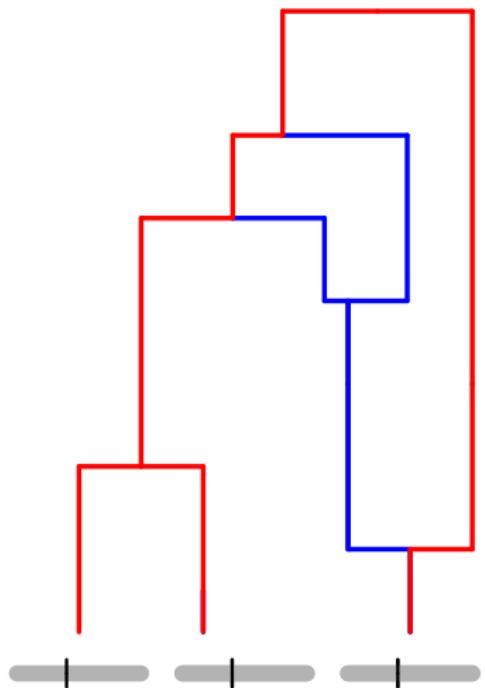
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Wiuf and Hein's sequential algorithm



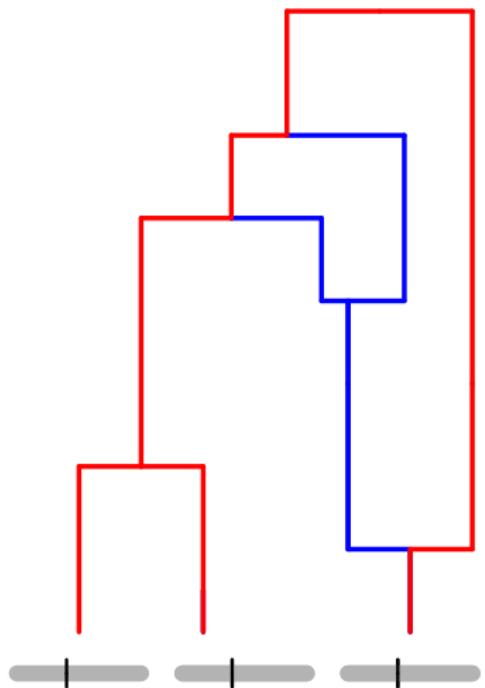
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Wiuf and Hein's sequential algorithm



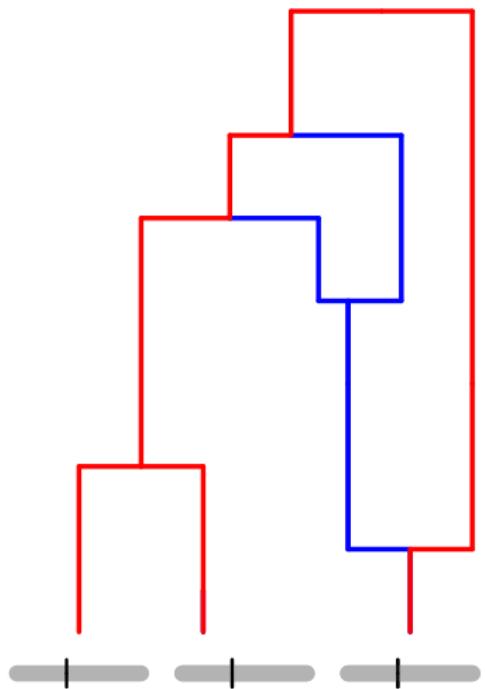
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Wiuf and Hein's sequential algorithm



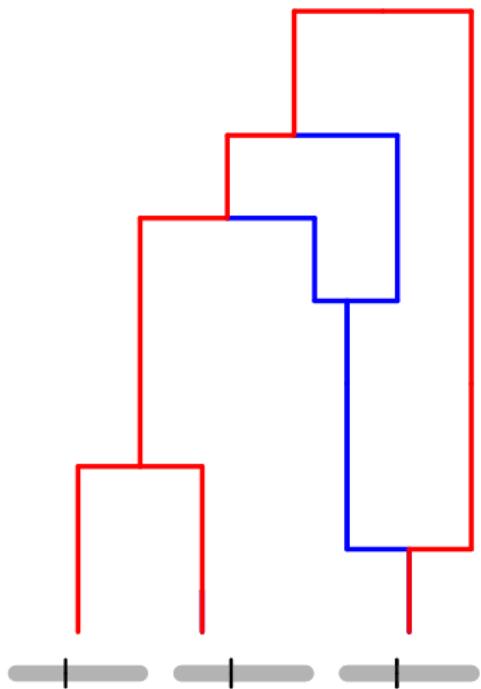
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Wiuf and Hein's sequential algorithm



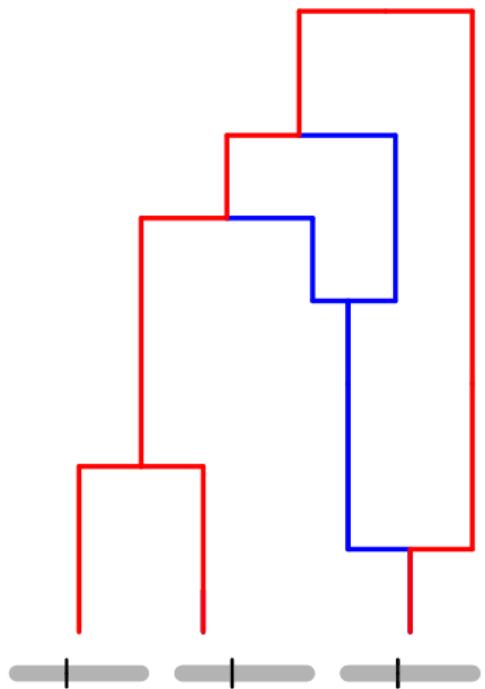
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Wiuf and Hein's sequential algorithm



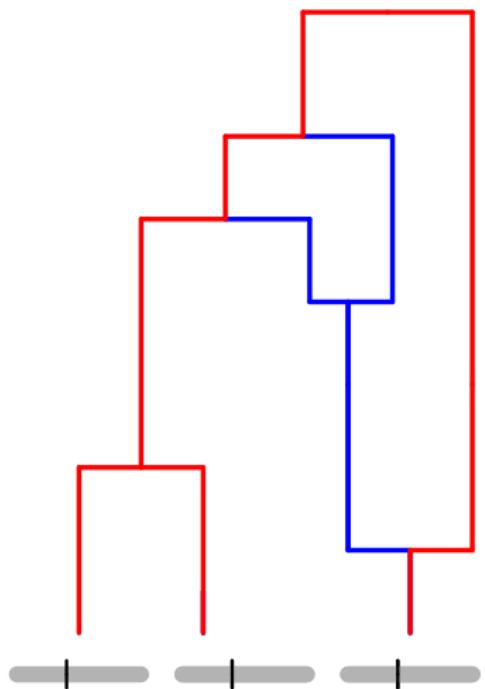
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Wiuf and Hein's sequential algorithm



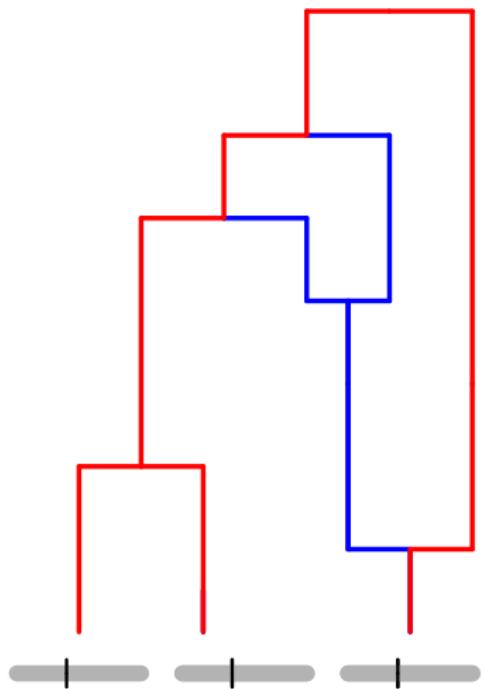
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Wiuf and Hein's sequential algorithm



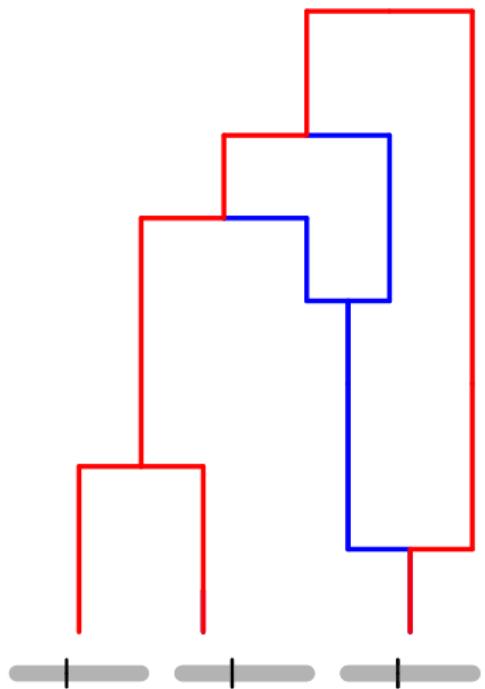
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Wiuf and Hein's sequential algorithm



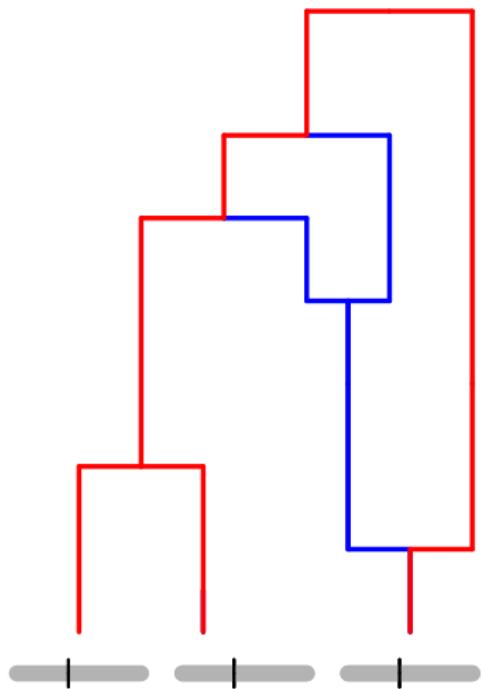
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Wiuf and Hein's sequential algorithm



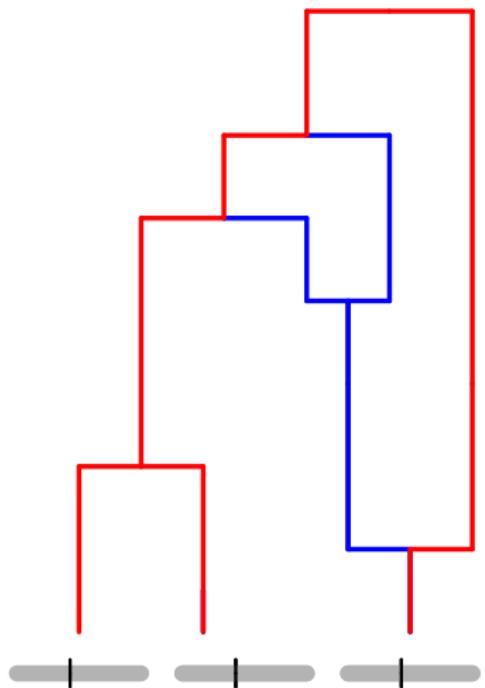
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Wiuf and Hein's sequential algorithm



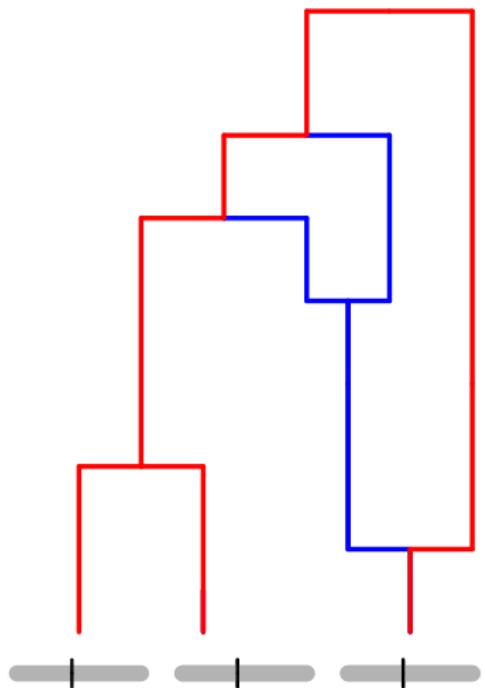
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Wiuf and Hein's sequential algorithm



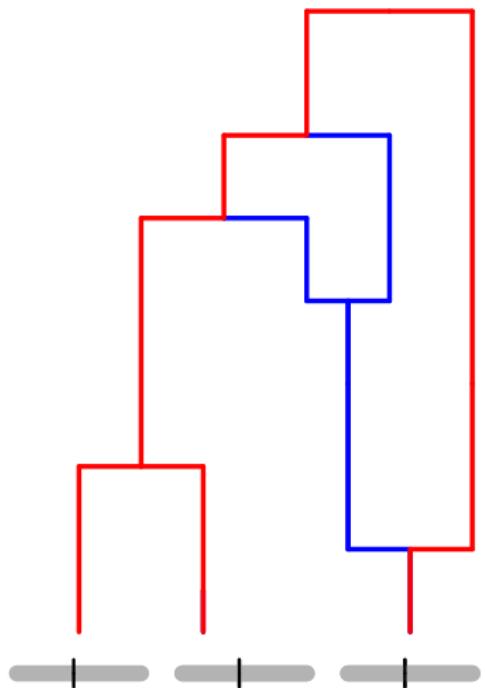
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Wiuf and Hein's sequential algorithm



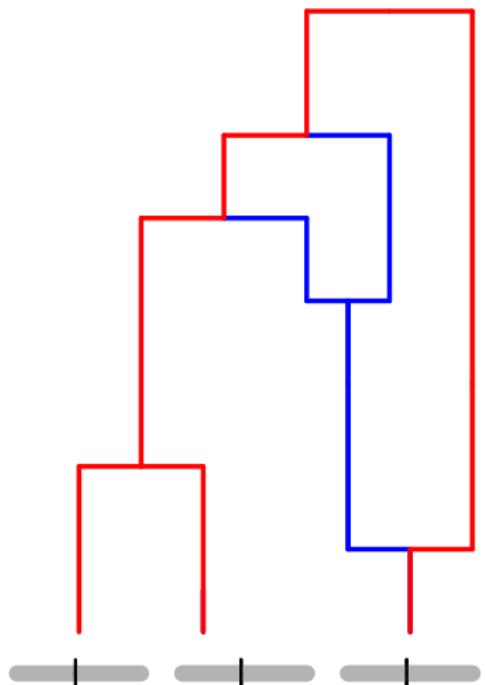
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Wiuf and Hein's sequential algorithm



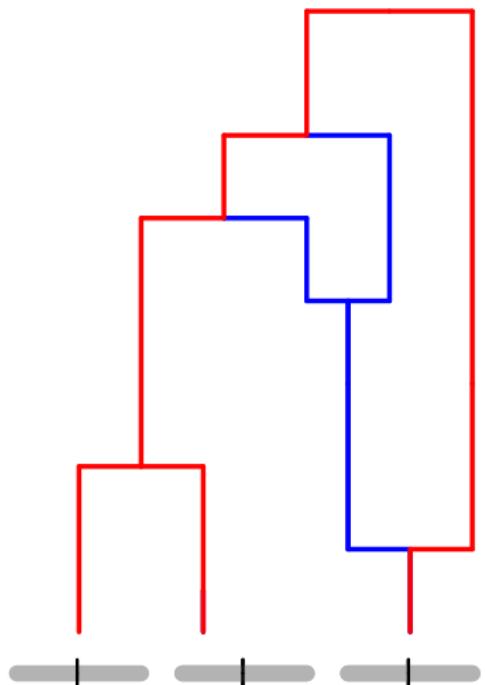
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Wiuf and Hein's sequential algorithm



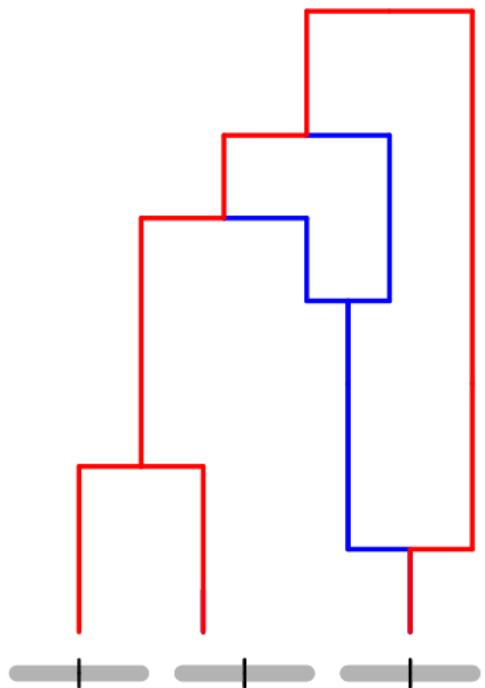
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Wiuf and Hein's sequential algorithm



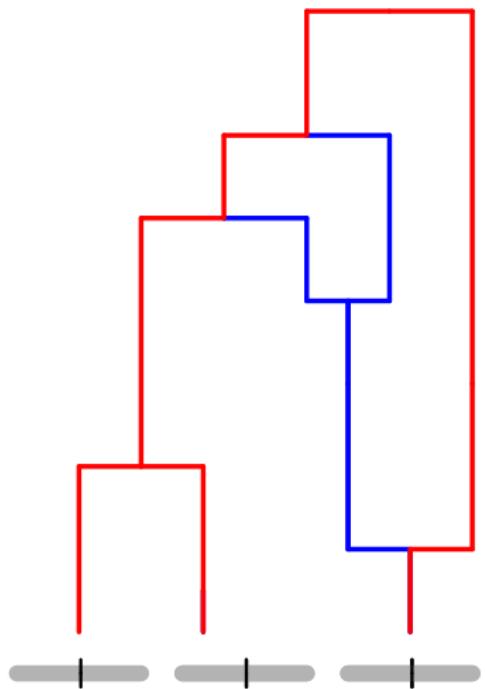
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Wiuf and Hein's sequential algorithm



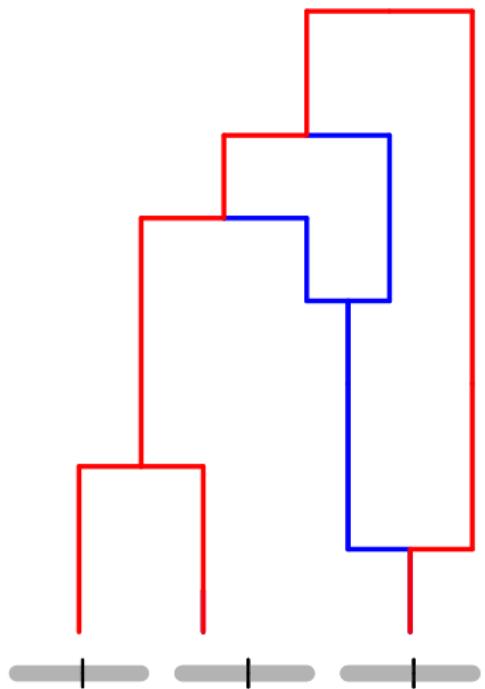
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Wiuf and Hein's sequential algorithm



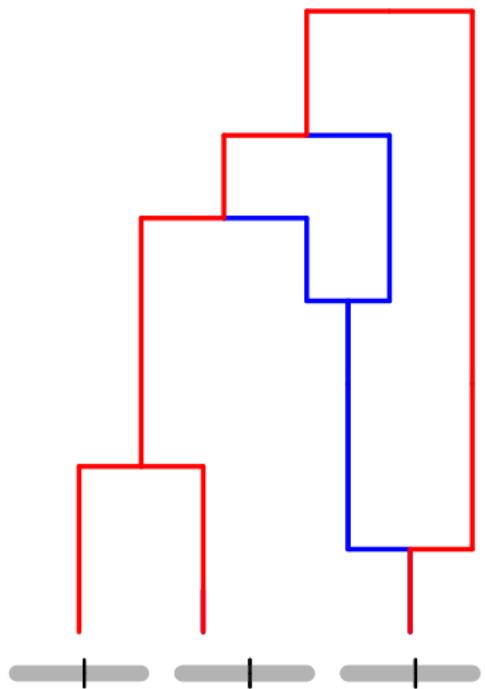
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Wiuf and Hein's sequential algorithm



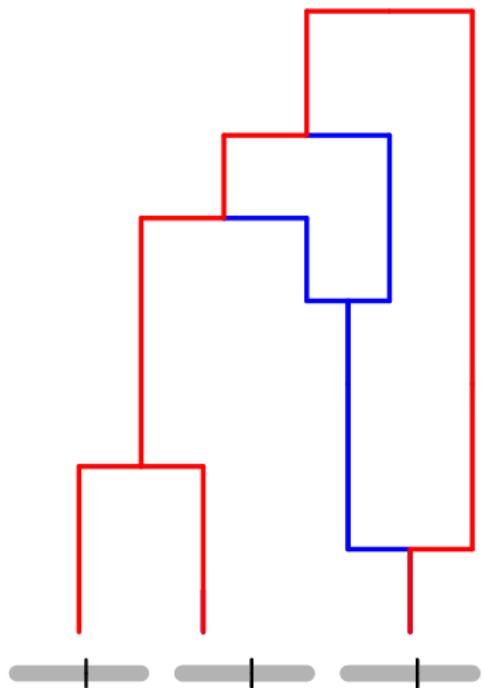
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Wiuf and Hein's sequential algorithm



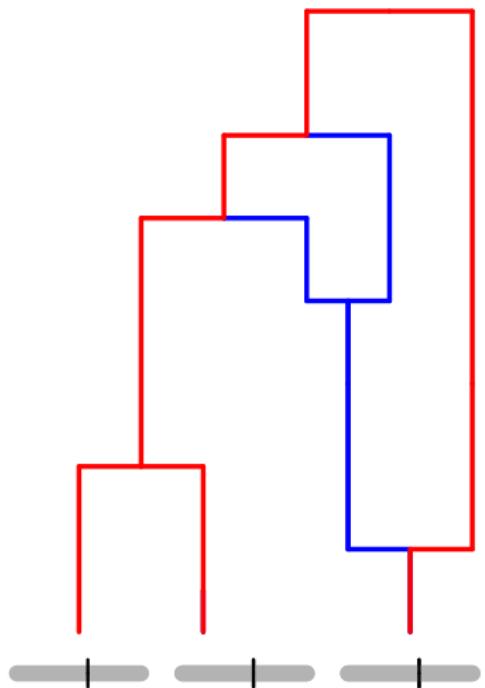
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Wiuf and Hein's sequential algorithm



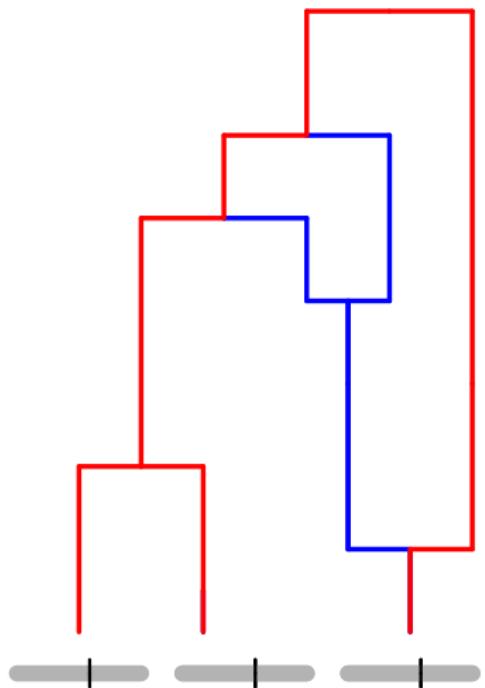
- Extend
- Recombine & coalesce
- Extend

Wiuf and Hein's sequential algorithm



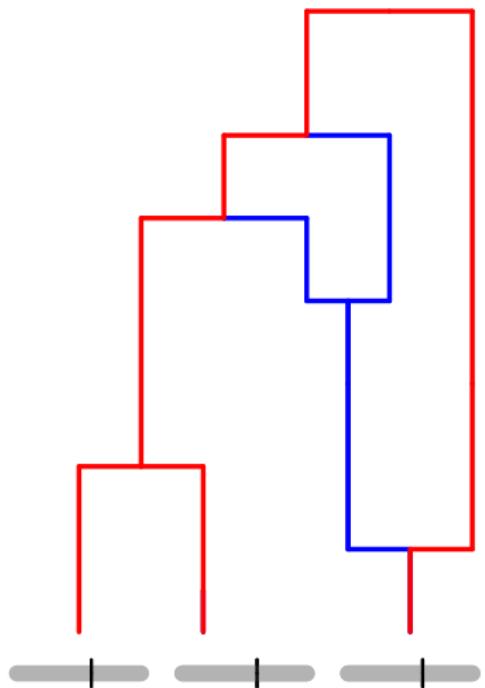
- Extend
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Wiuf and Hein's sequential algorithm



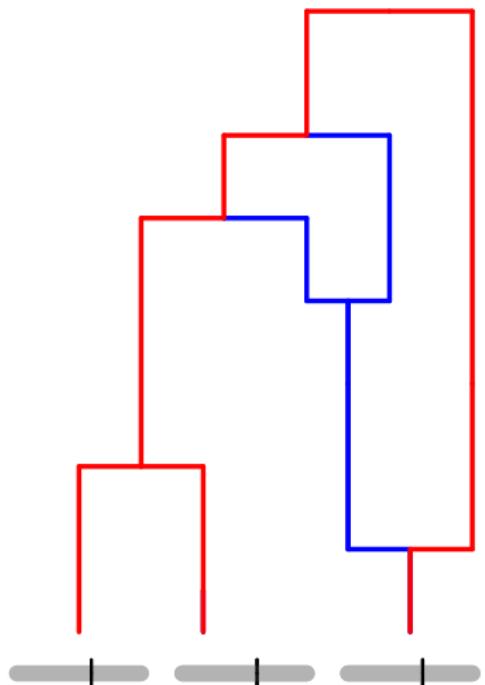
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Wiuf and Hein's sequential algorithm



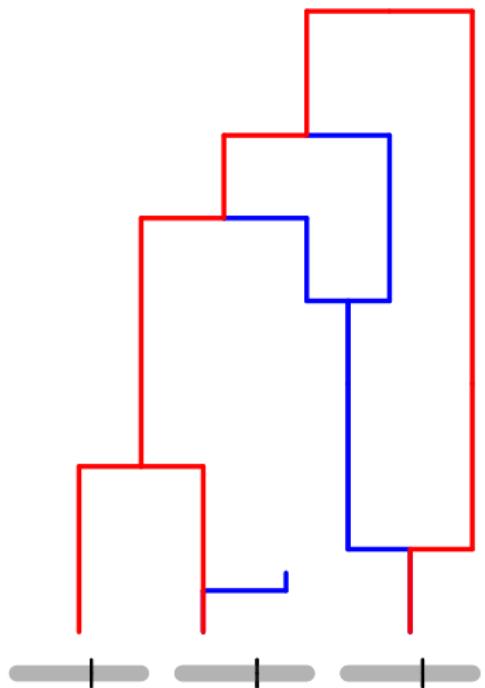
- Extend
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Wiuf and Hein's sequential algorithm



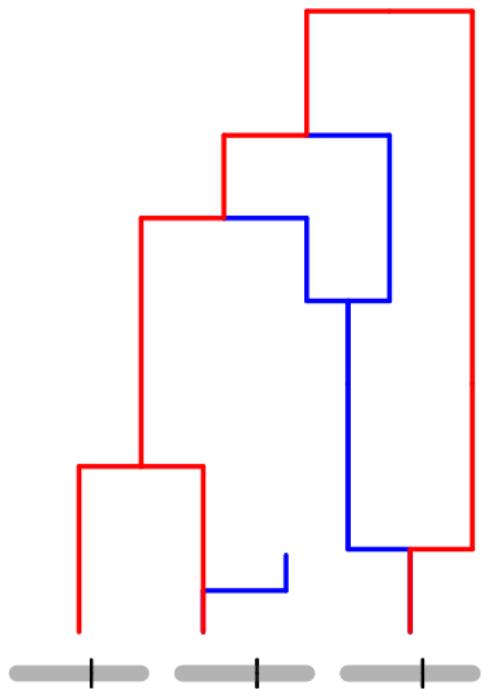
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Wiuf and Hein's sequential algorithm



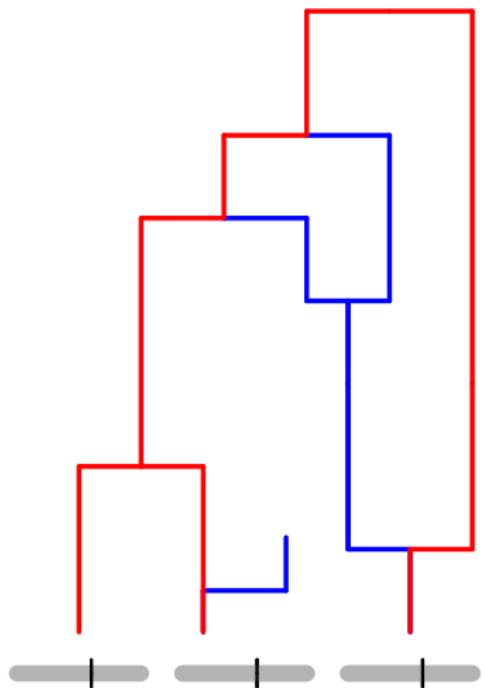
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Wiuf and Hein's sequential algorithm



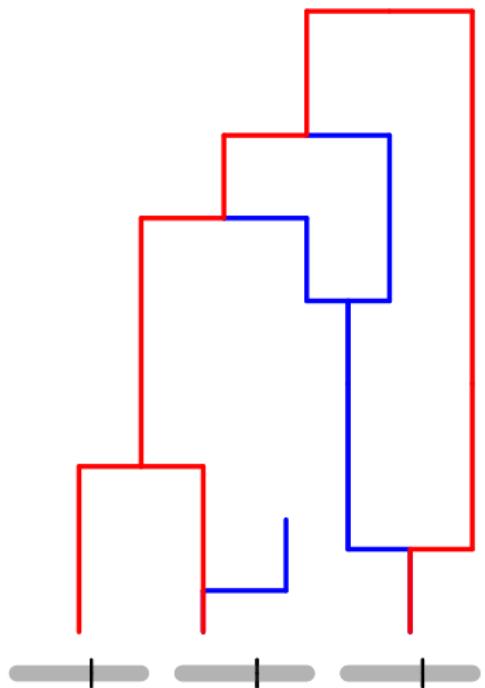
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Wiuf and Hein's sequential algorithm



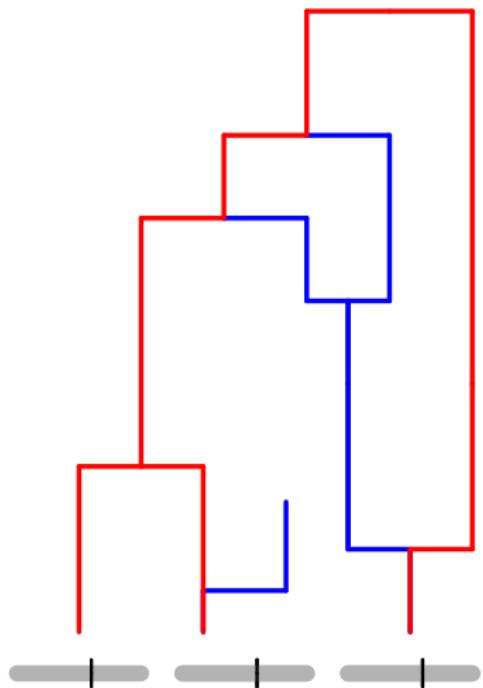
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Wiuf and Hein's sequential algorithm



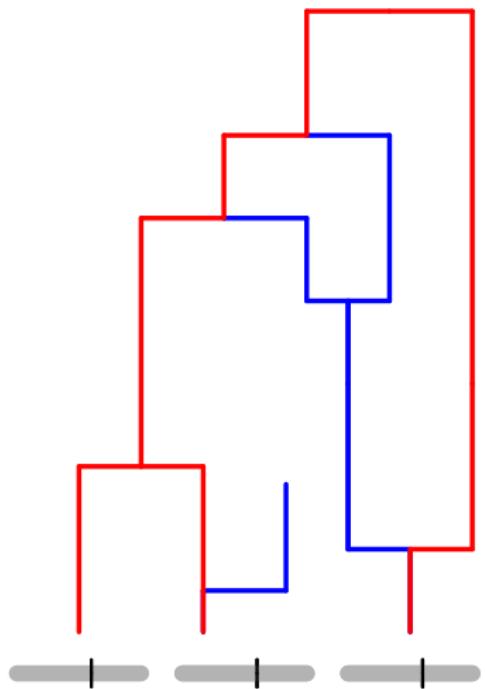
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Wiuf and Hein's sequential algorithm



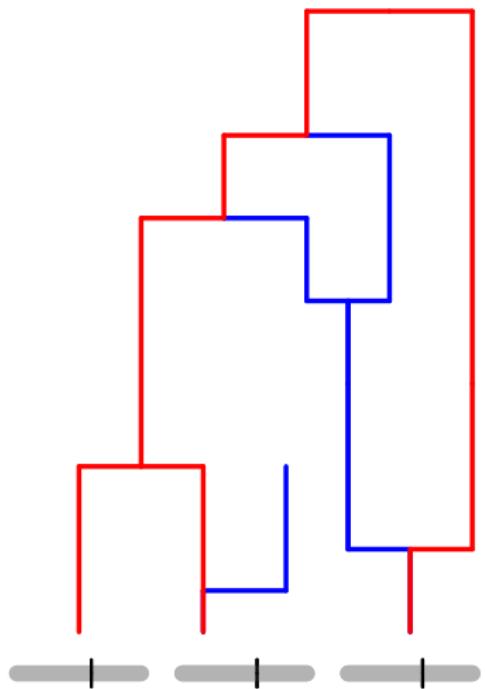
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Wiuf and Hein's sequential algorithm



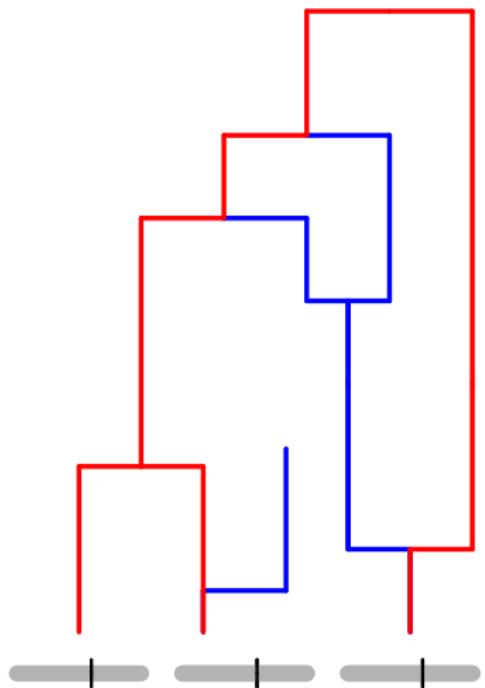
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Wiuf and Hein's sequential algorithm



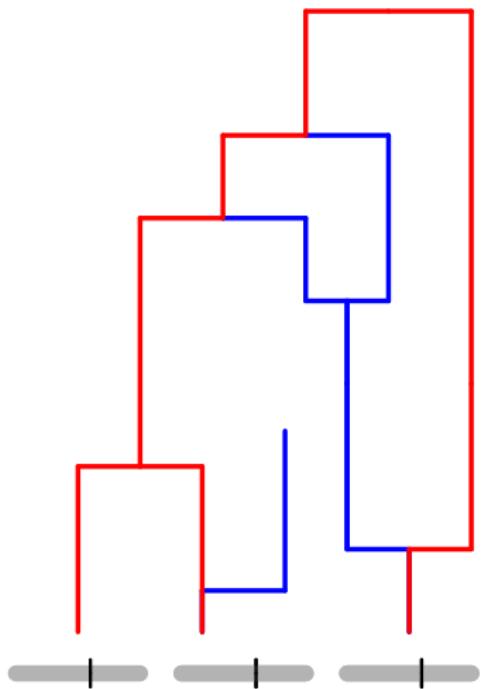
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Wiuf and Hein's sequential algorithm



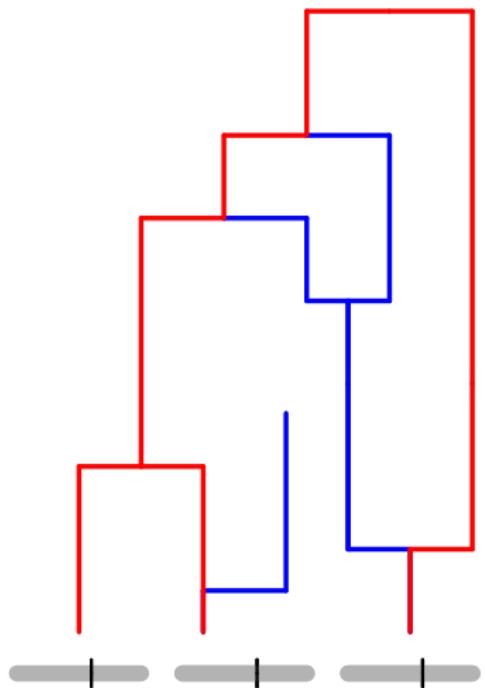
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Wiuf and Hein's sequential algorithm



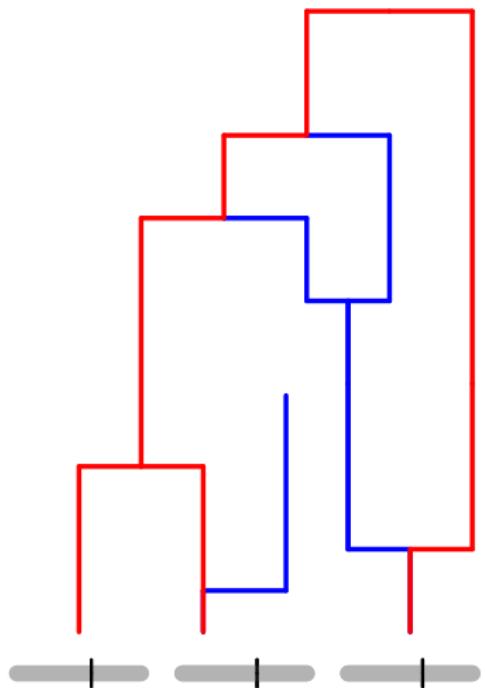
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Wiuf and Hein's sequential algorithm



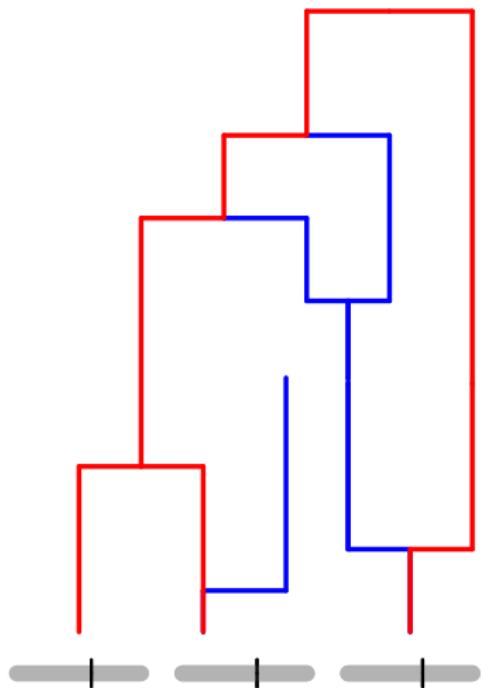
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Wiuf and Hein's sequential algorithm



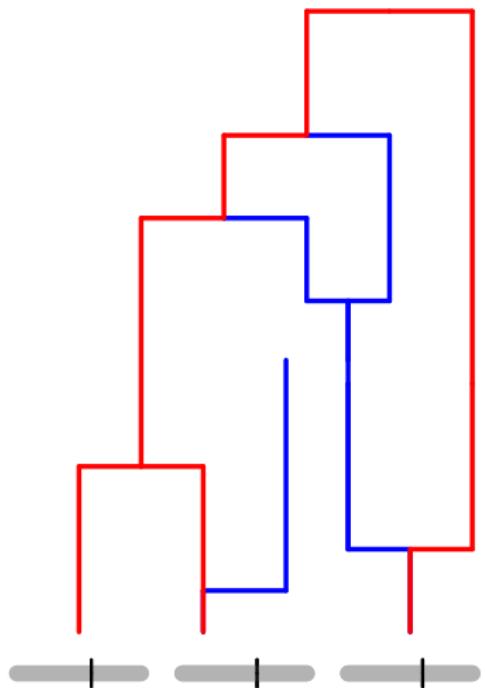
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Wiuf and Hein's sequential algorithm



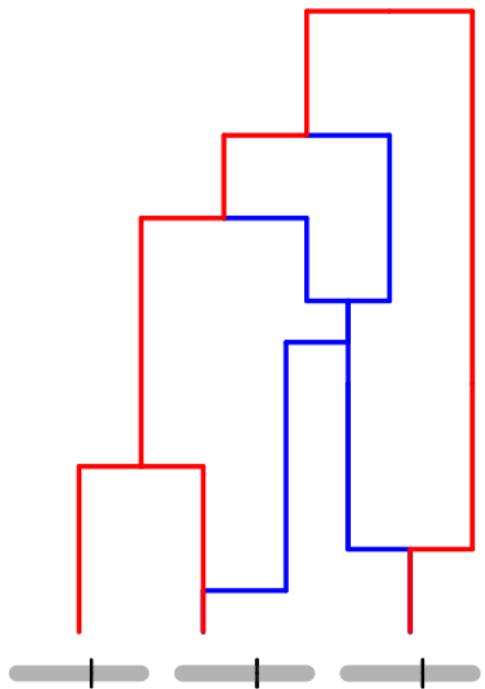
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Wiuf and Hein's sequential algorithm



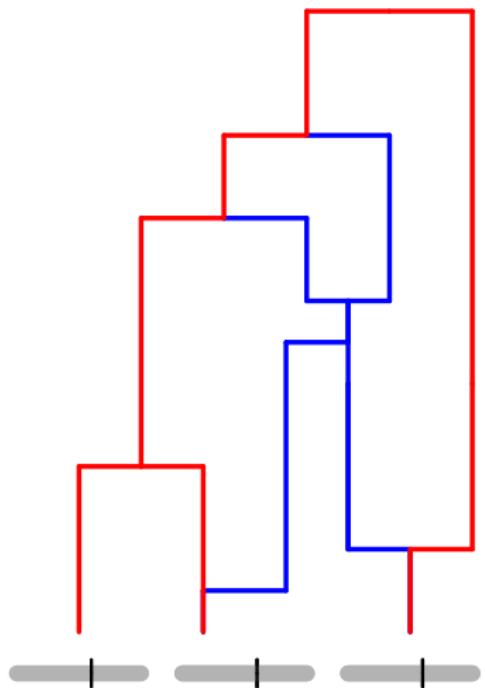
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Wiuf and Hein's sequential algorithm



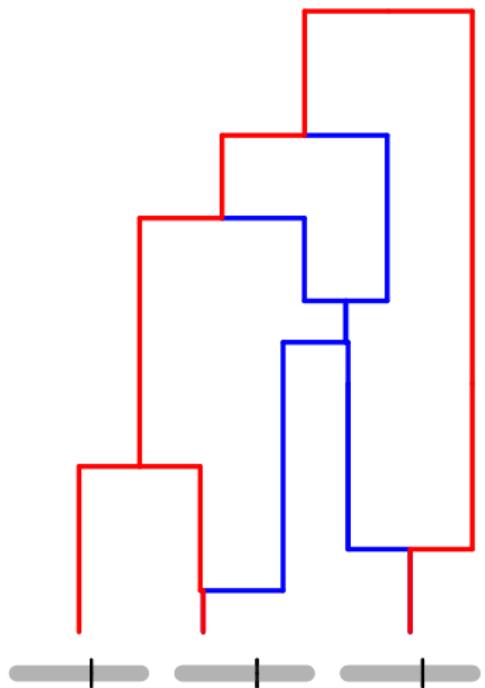
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Wiuf and Hein's sequential algorithm



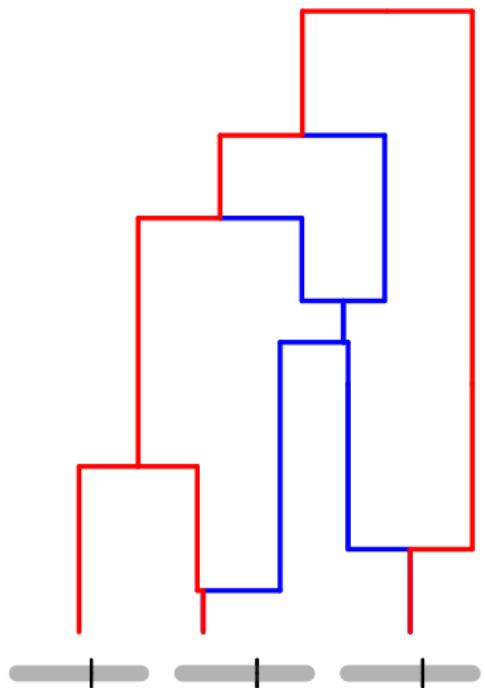
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Wiuf and Hein's sequential algorithm



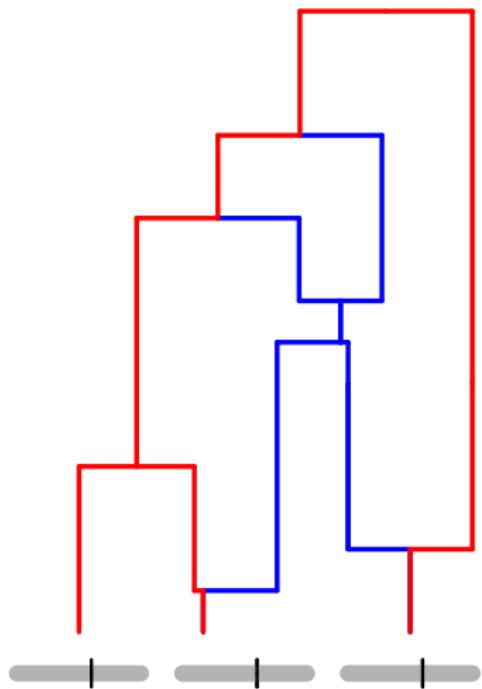
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Wiuf and Hein's sequential algorithm



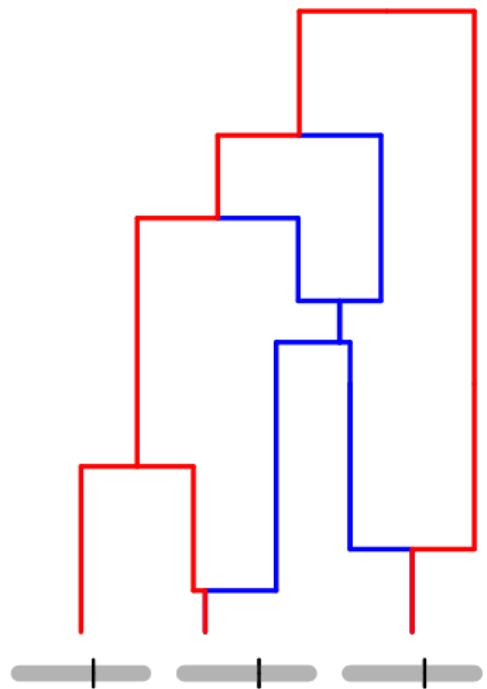
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Wiuf and Hein's sequential algorithm



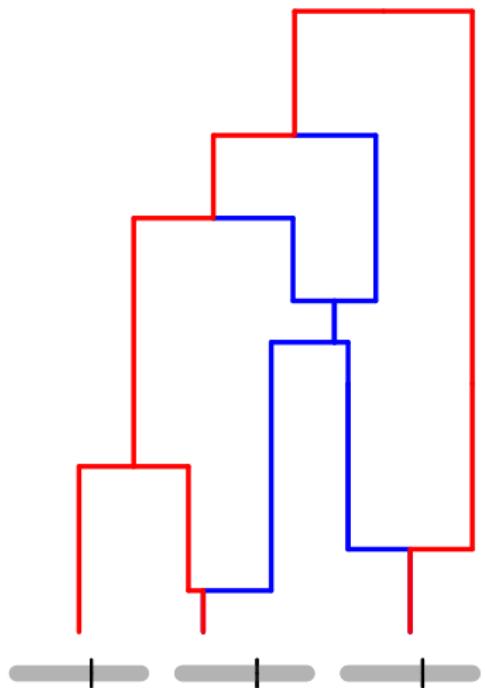
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Wiuf and Hein's sequential algorithm



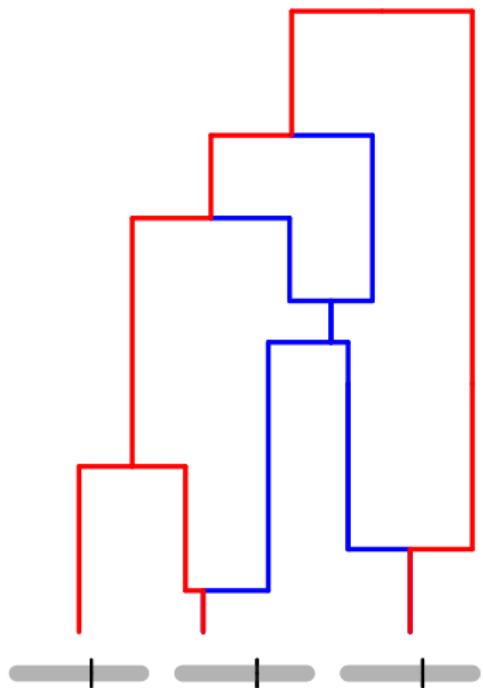
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Wiuf and Hein's sequential algorithm



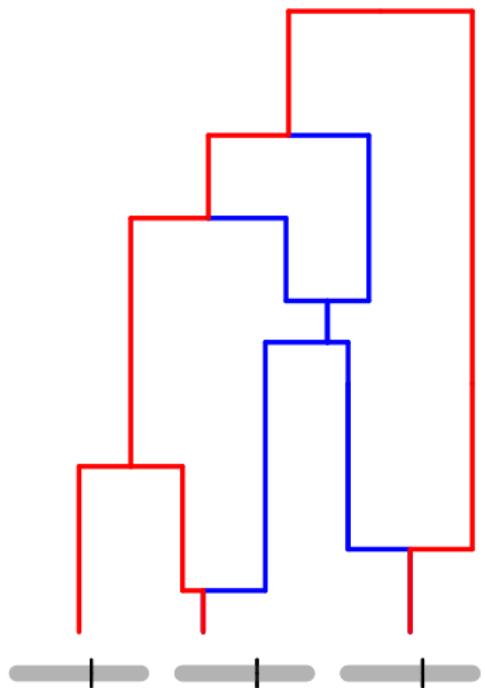
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Wiuf and Hein's sequential algorithm



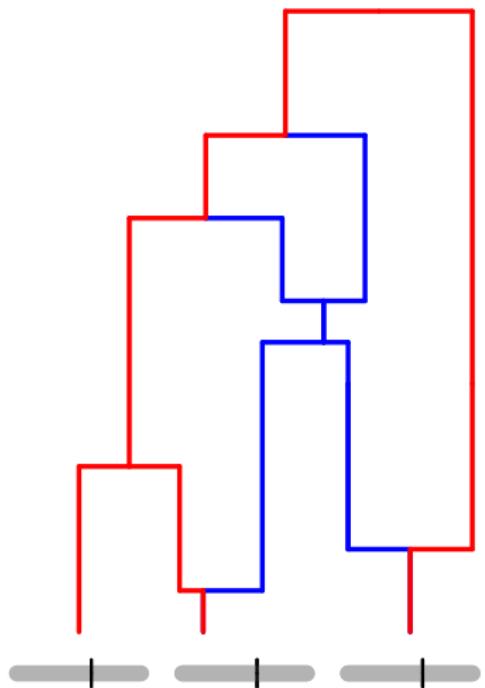
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Wiuf and Hein's sequential algorithm



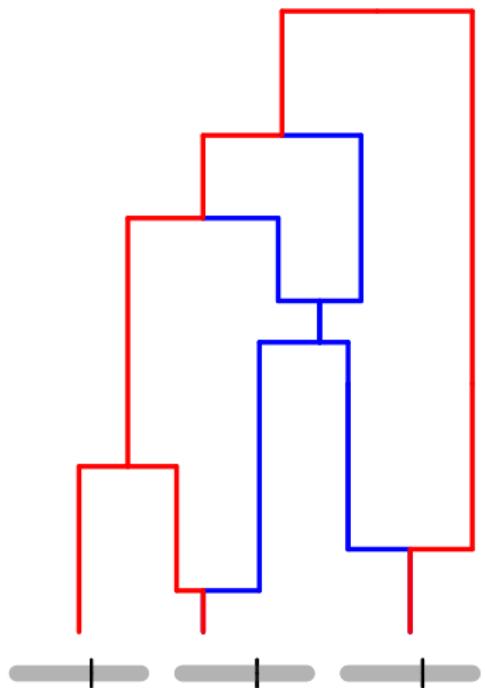
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Wiuf and Hein's sequential algorithm



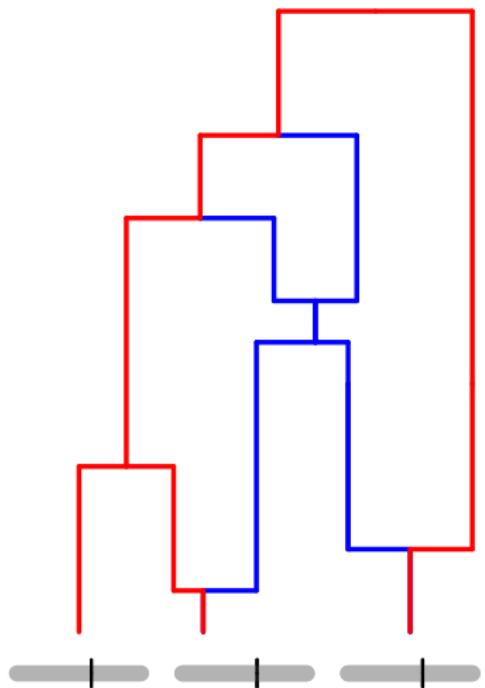
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Wiuf and Hein's sequential algorithm



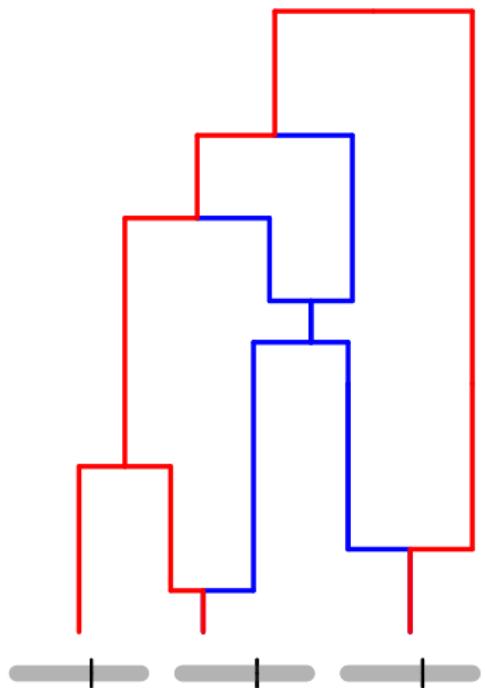
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Wiuf and Hein's sequential algorithm



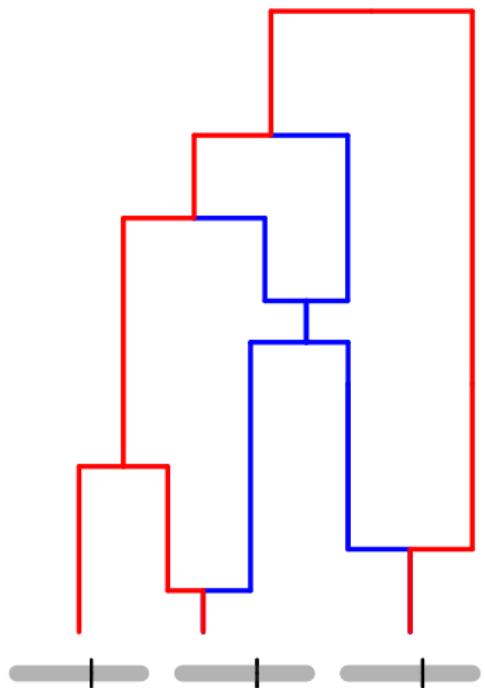
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Wiuf and Hein's sequential algorithm



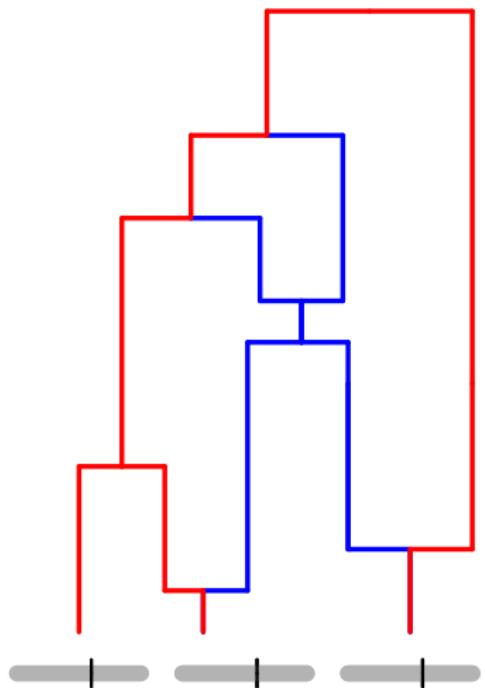
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Wiuf and Hein's sequential algorithm



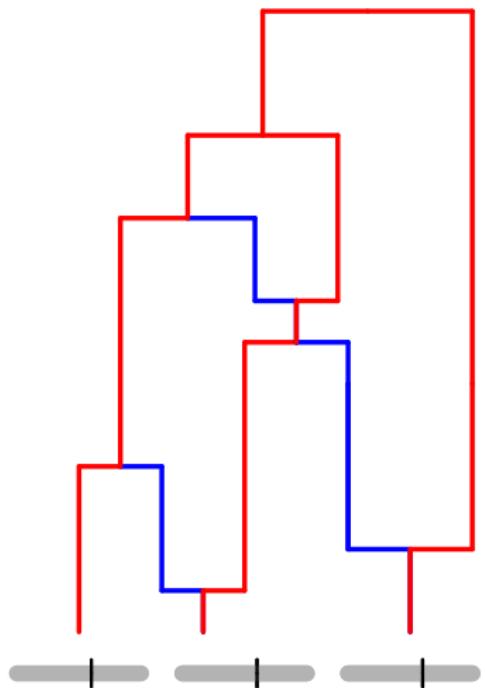
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Wiuf and Hein's sequential algorithm



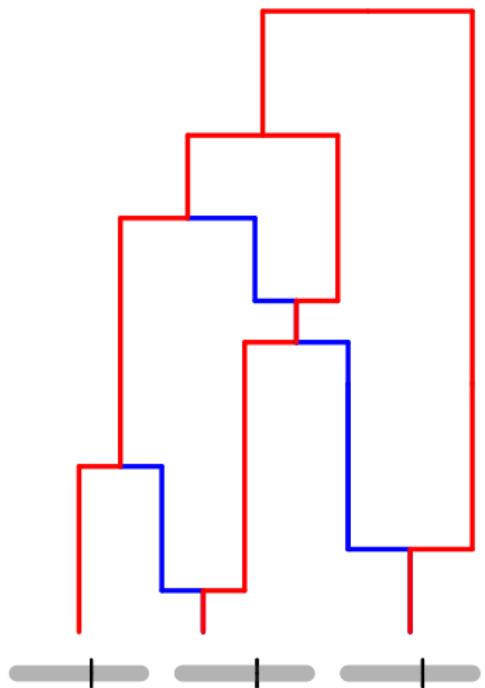
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Wiuf and Hein's sequential algorithm



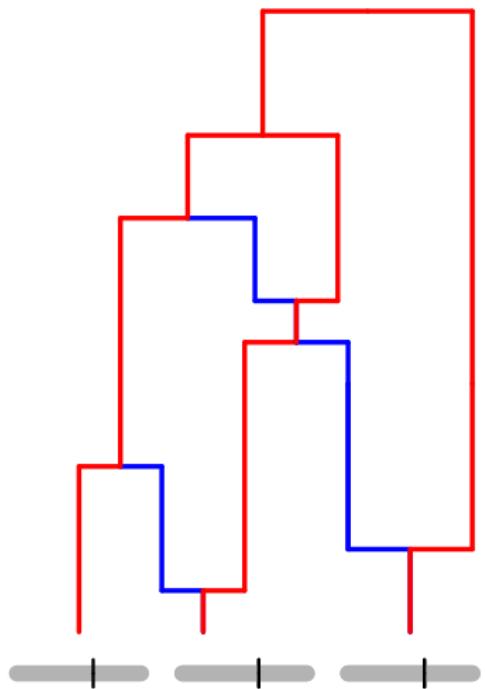
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Wiuf and Hein's sequential algorithm



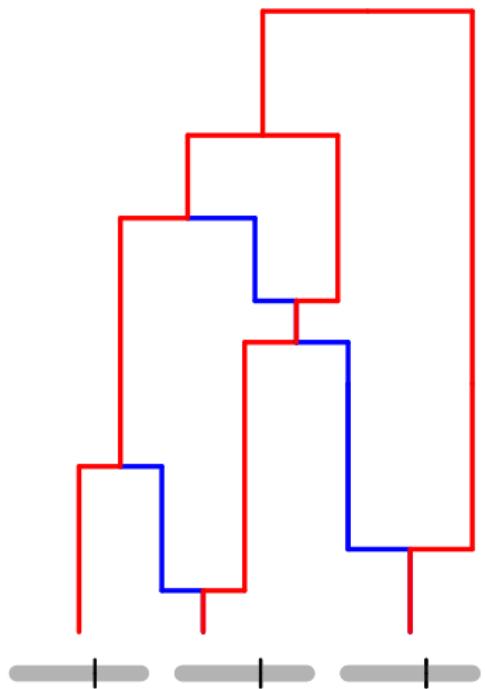
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Wiuf and Hein's sequential algorithm



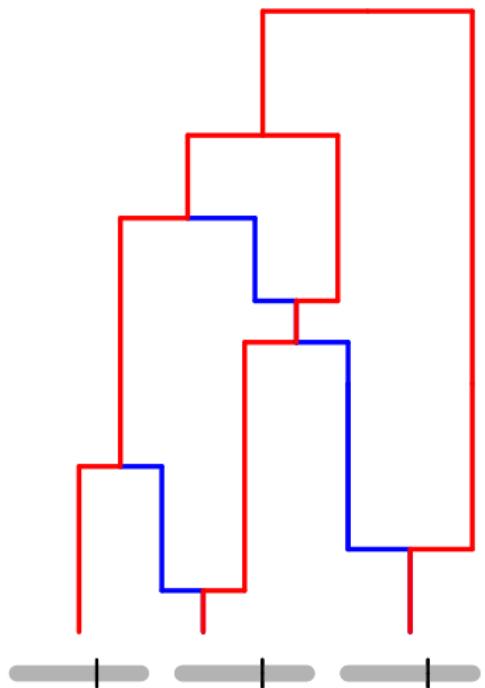
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Wiuf and Hein's sequential algorithm



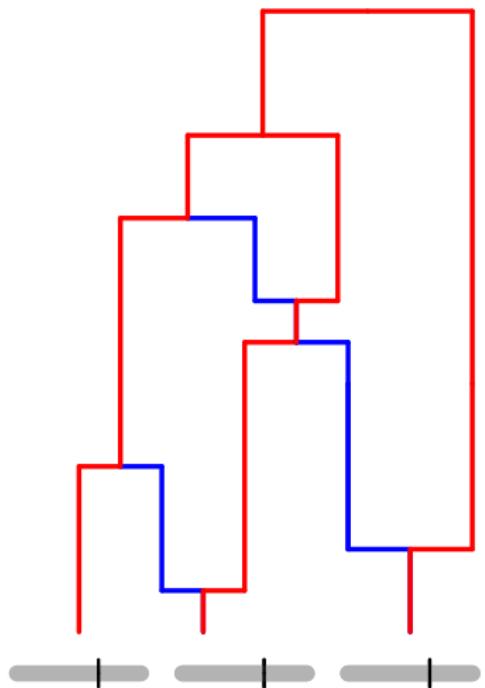
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Wiuf and Hein's sequential algorithm



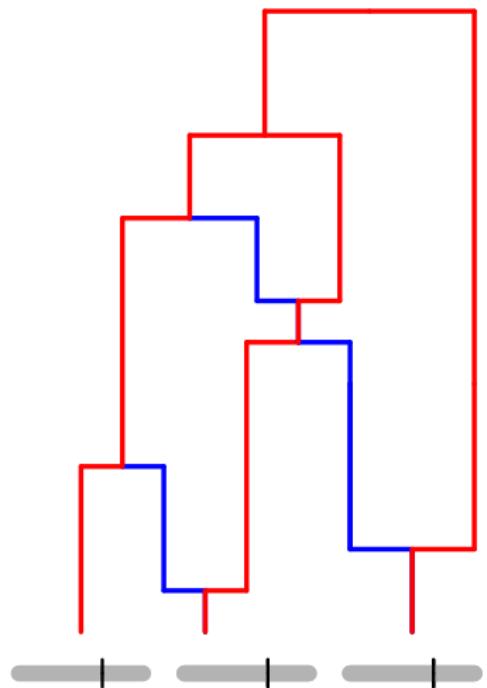
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Wiuf and Hein's sequential algorithm



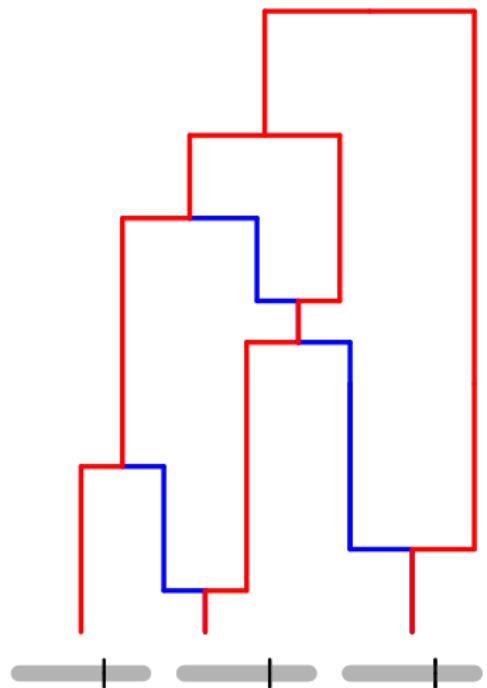
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Wiuf and Hein's sequential algorithm



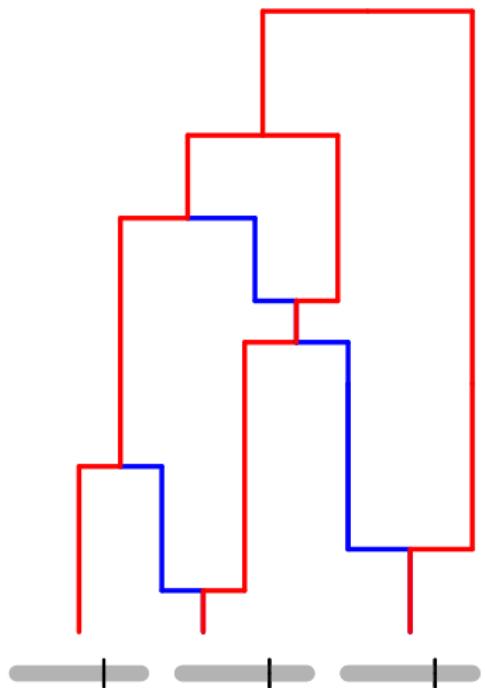
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Wiuf and Hein's sequential algorithm



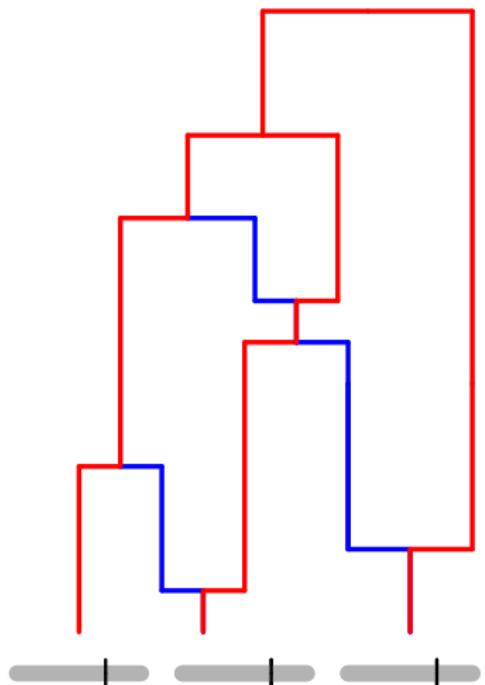
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Wiuf and Hein's sequential algorithm



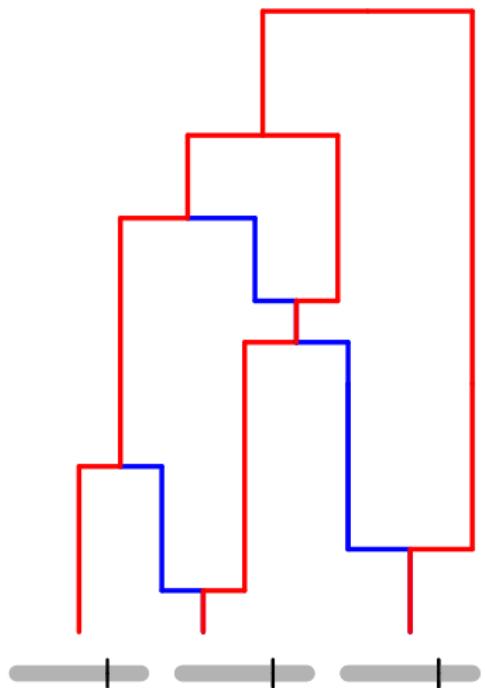
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Wiuf and Hein's sequential algorithm



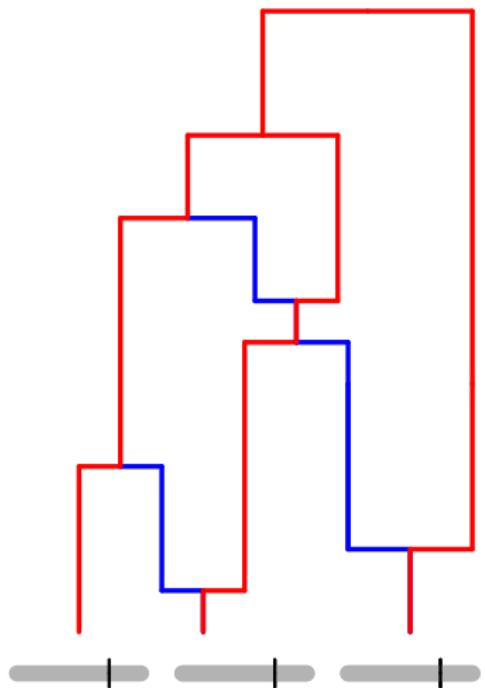
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Wiuf and Hein's sequential algorithm



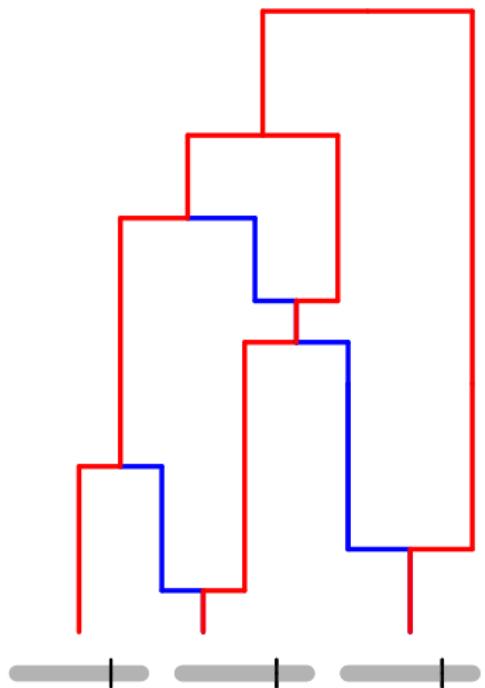
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Wiuf and Hein's sequential algorithm



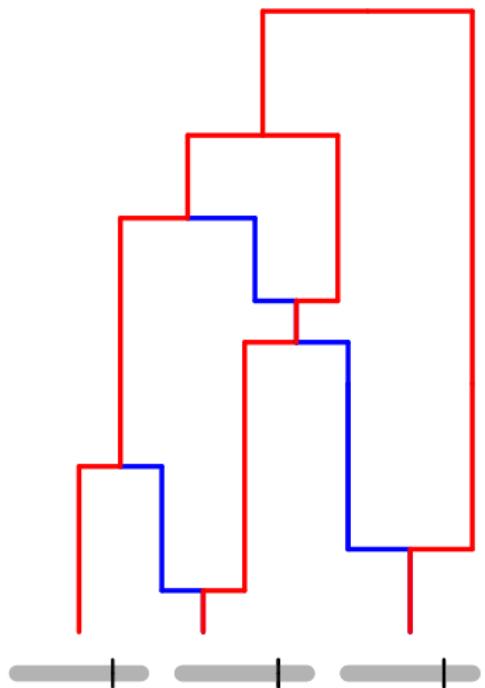
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Wiuf and Hein's sequential algorithm



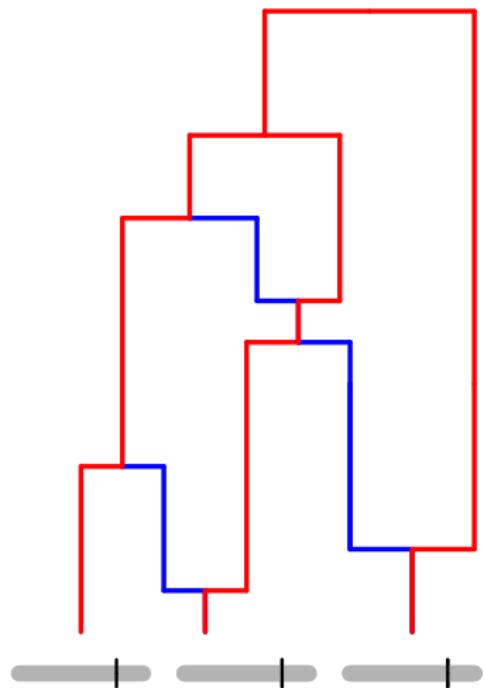
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Wiuf and Hein's sequential algorithm



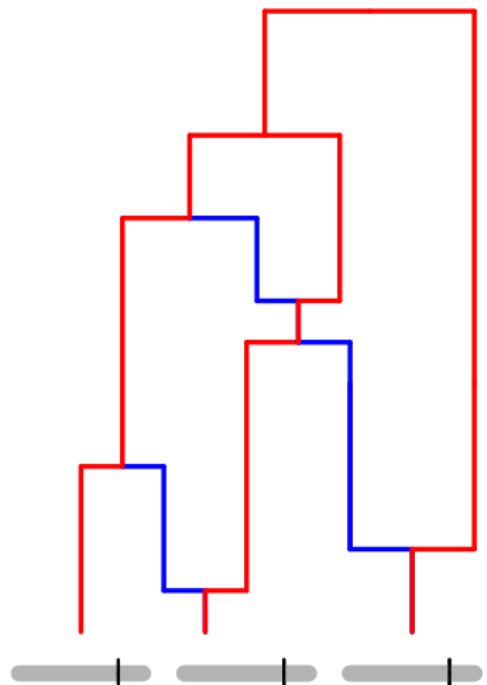
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Wiuf and Hein's sequential algorithm



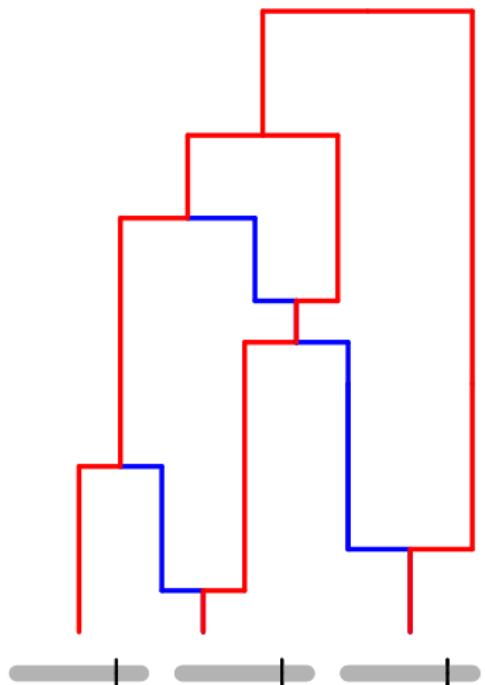
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Wiuf and Hein's sequential algorithm



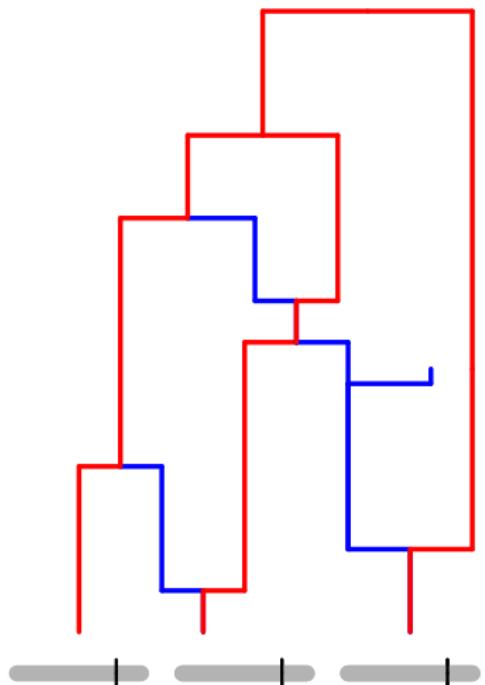
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Wiuf and Hein's sequential algorithm



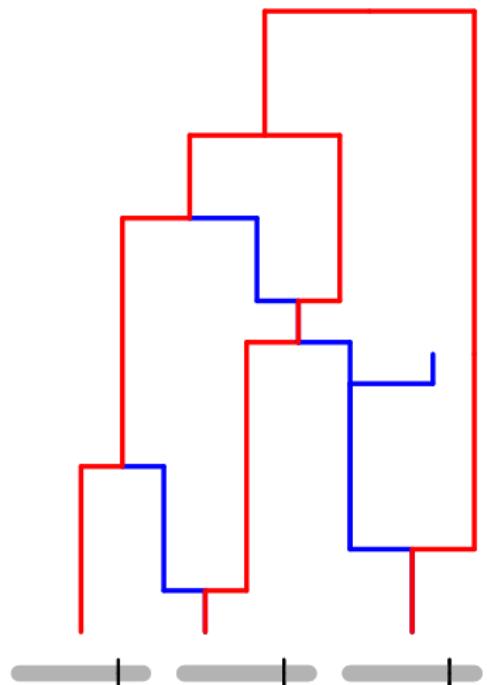
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Wiuf and Hein's sequential algorithm



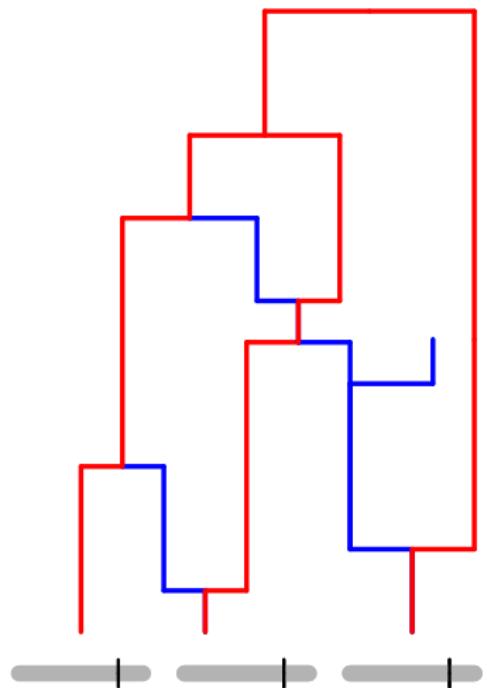
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Wiuf and Hein's sequential algorithm



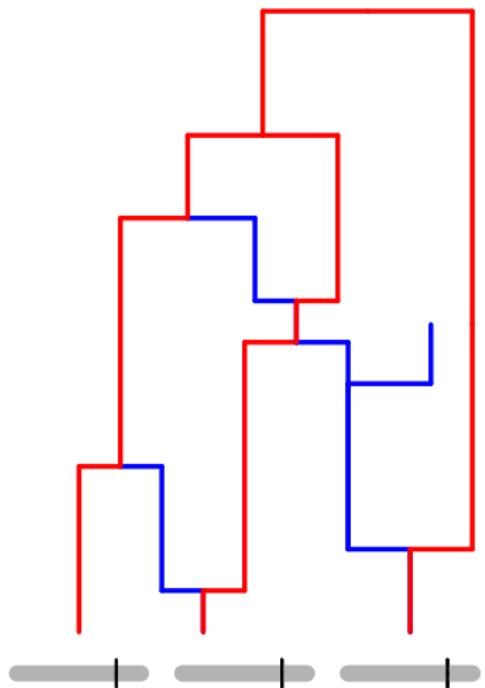
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Wiuf and Hein's sequential algorithm



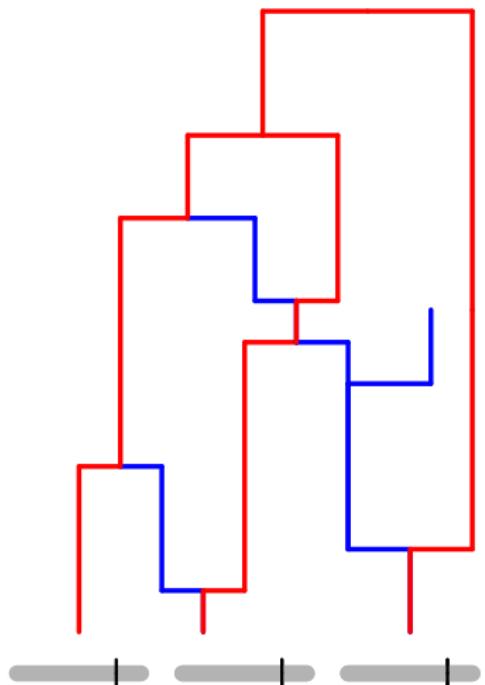
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Wiuf and Hein's sequential algorithm



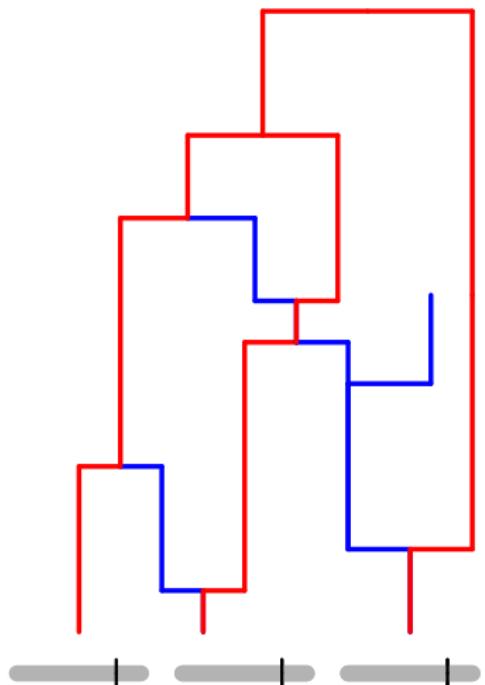
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Wiuf and Hein's sequential algorithm



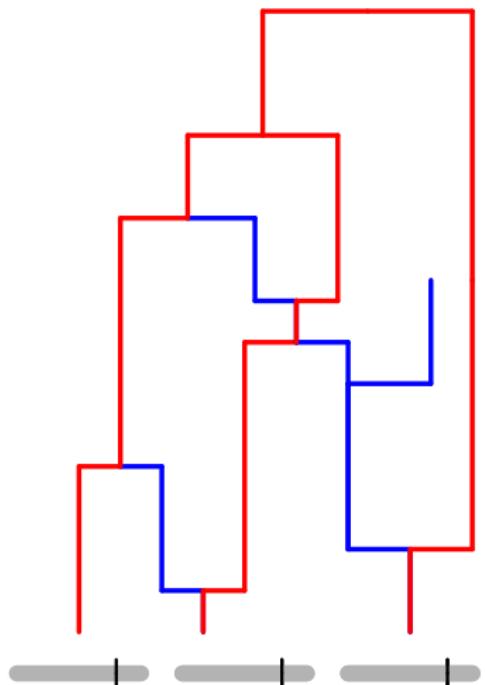
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Wiuf and Hein's sequential algorithm



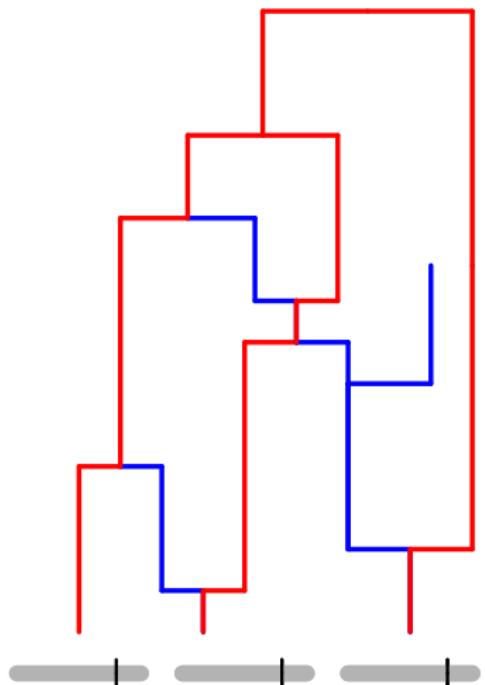
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Wiuf and Hein's sequential algorithm



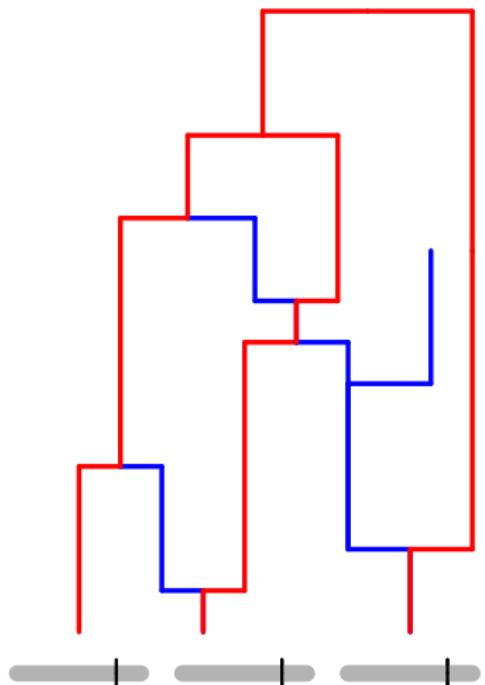
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Wiuf and Hein's sequential algorithm



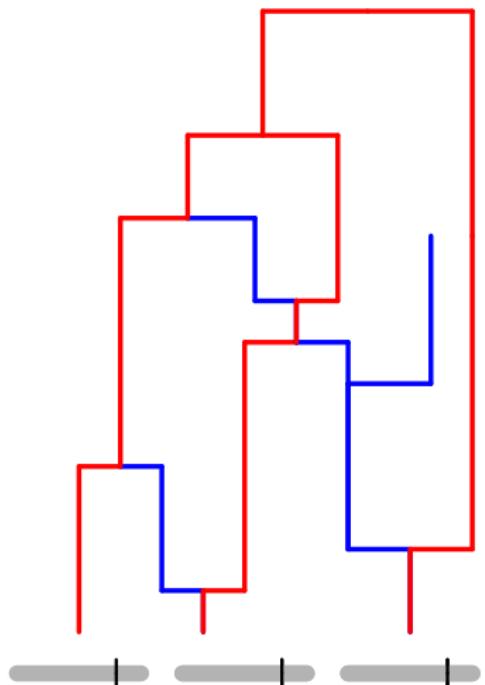
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Wiuf and Hein's sequential algorithm



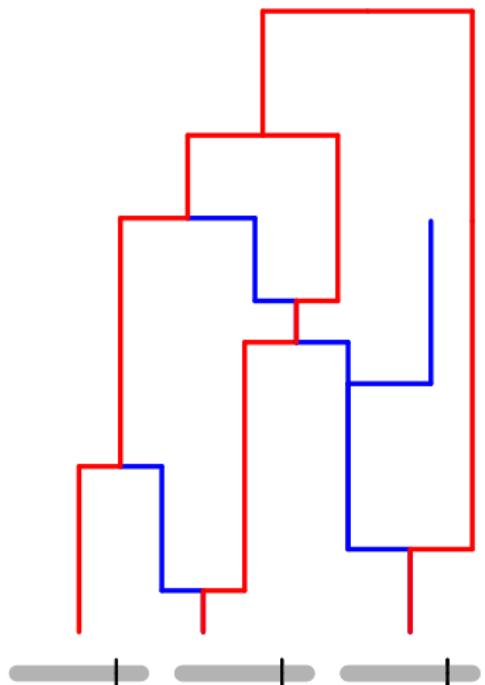
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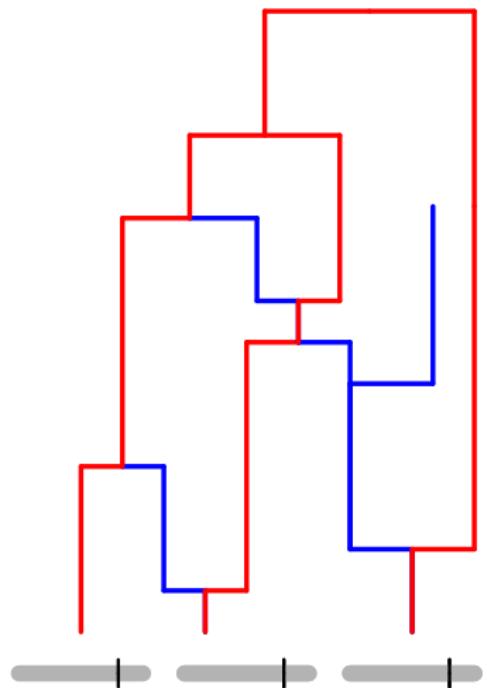
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Wiuf and Hein's sequential algorithm



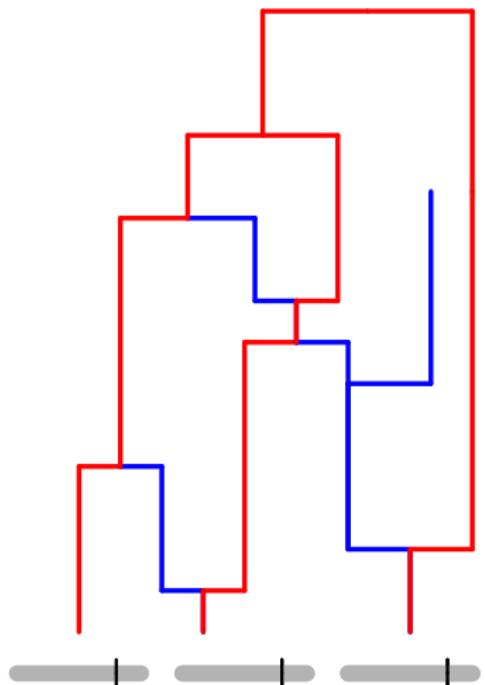
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Wiuf and Hein's sequential algorithm



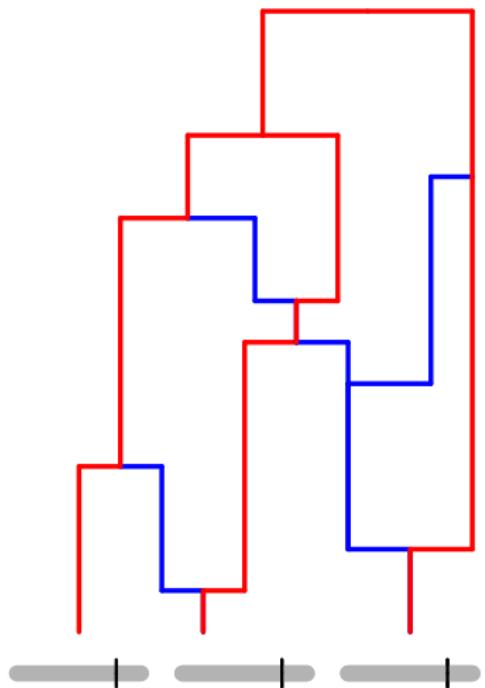
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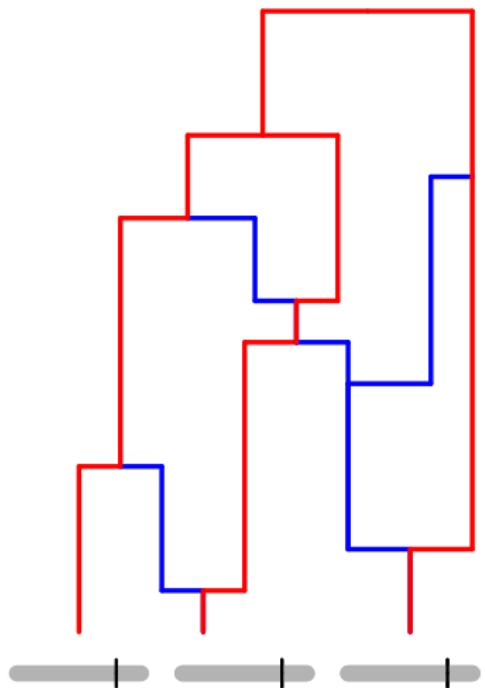
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Wiuf and Hein's sequential algorithm



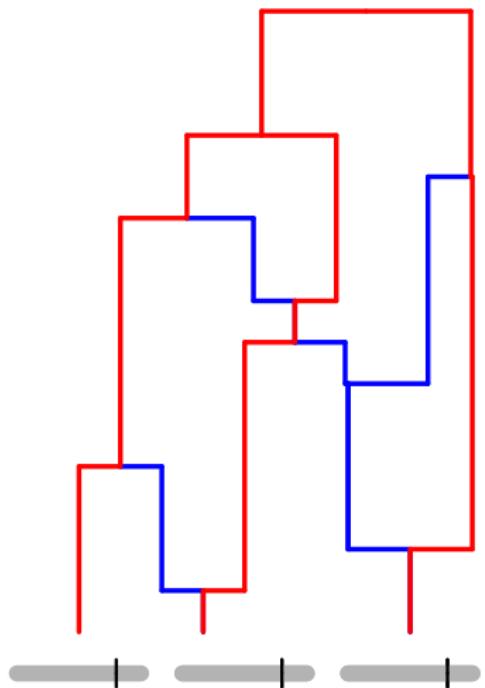
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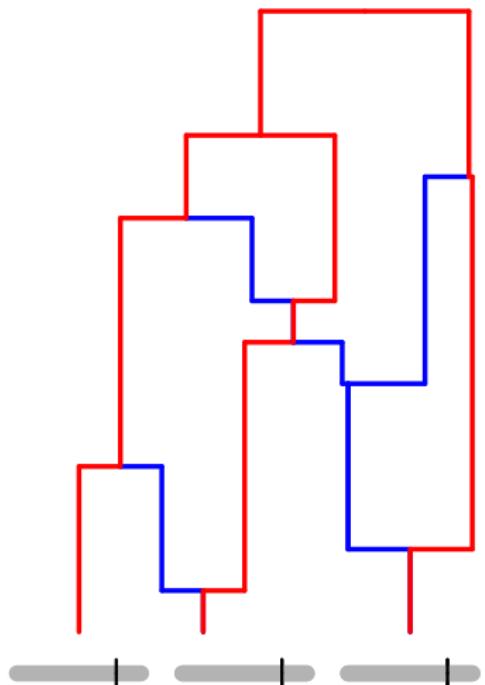
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Wiuf and Hein's sequential algorithm



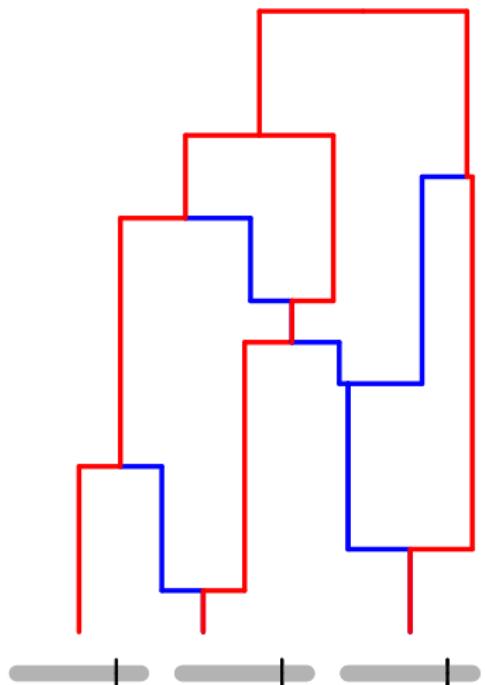
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Wiuf and Hein's sequential algorithm



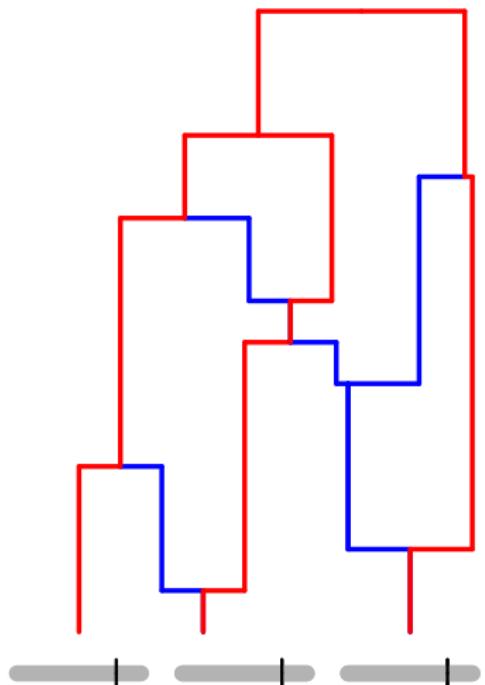
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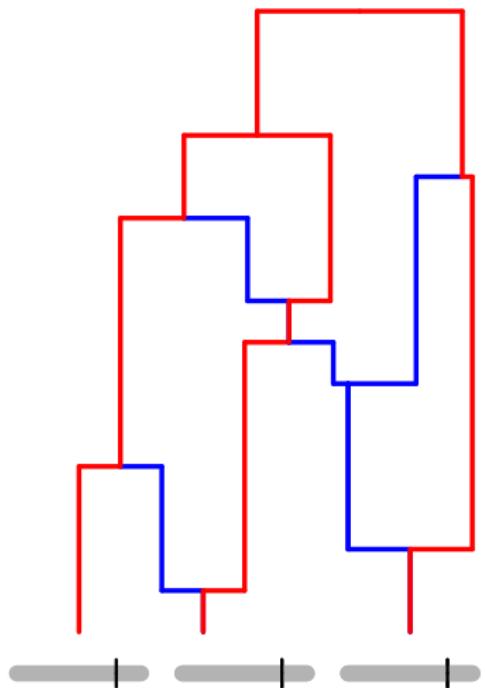
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Wiuf and Hein's sequential algorithm



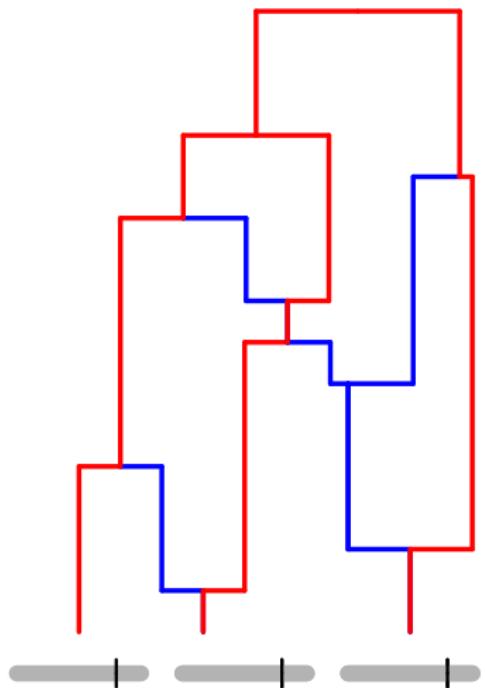
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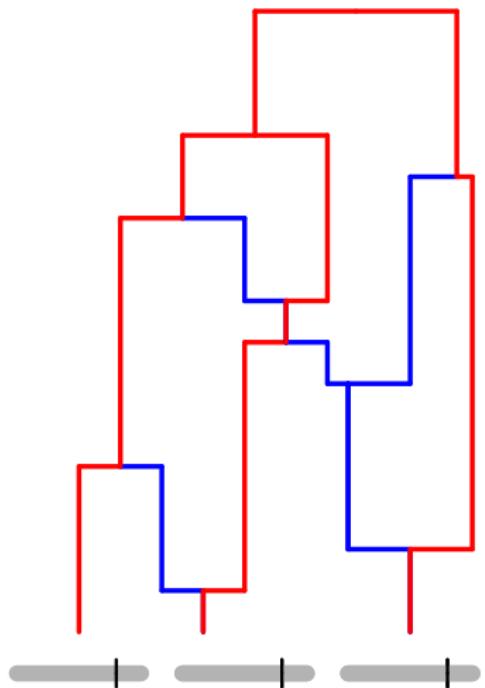
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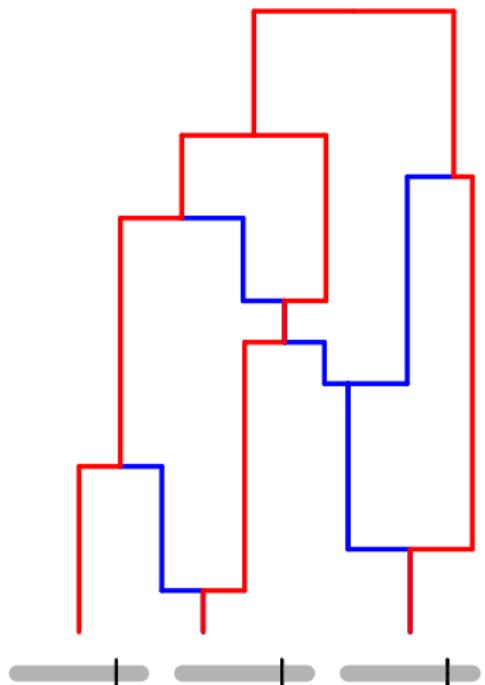
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Wiuf and Hein's sequential algorithm



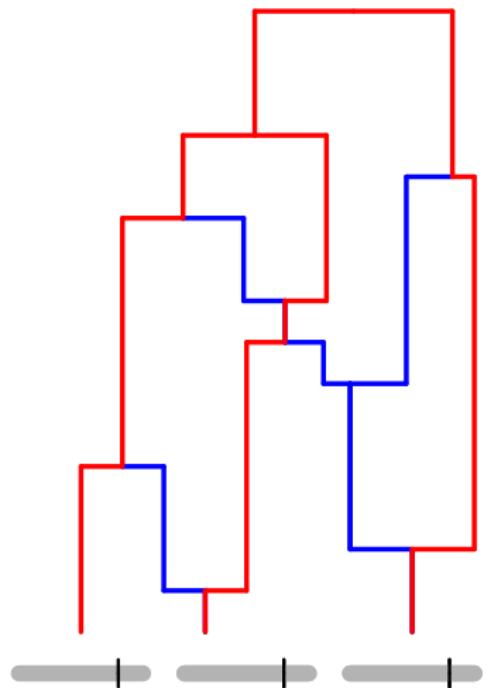
- Extend
- Recombine & coalesce
- Extend

Wiuf and Hein's sequential algorithm



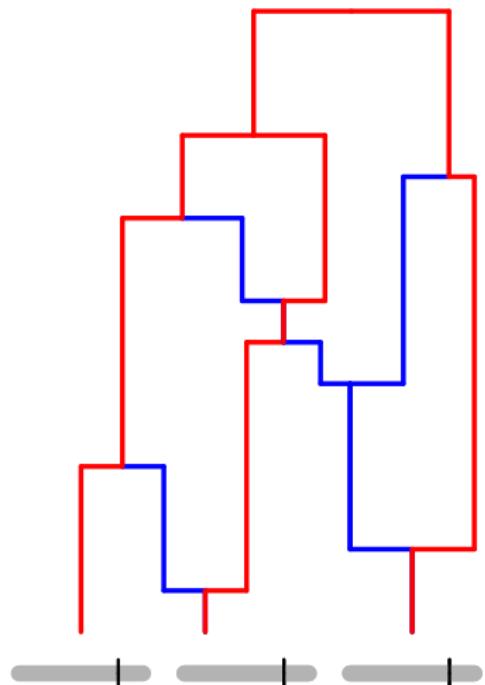
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Wiuf and Hein's sequential algorithm



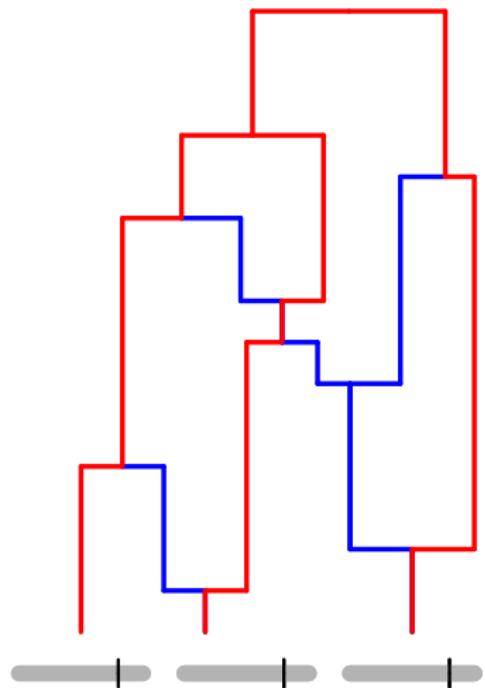
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Wiuf and Hein's sequential algorithm



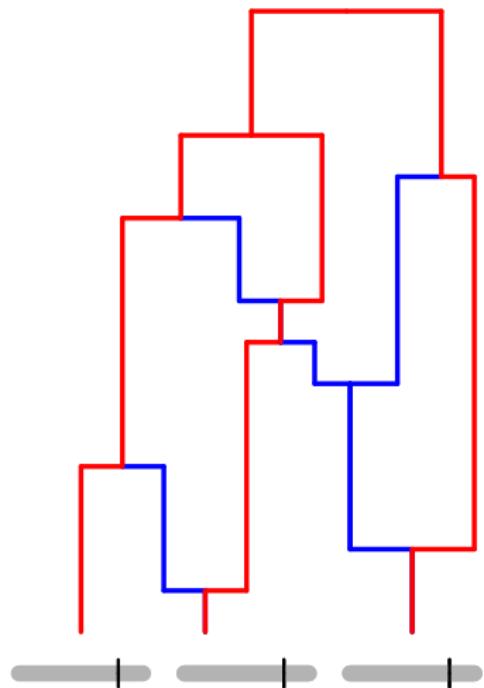
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Wiuf and Hein's sequential algorithm



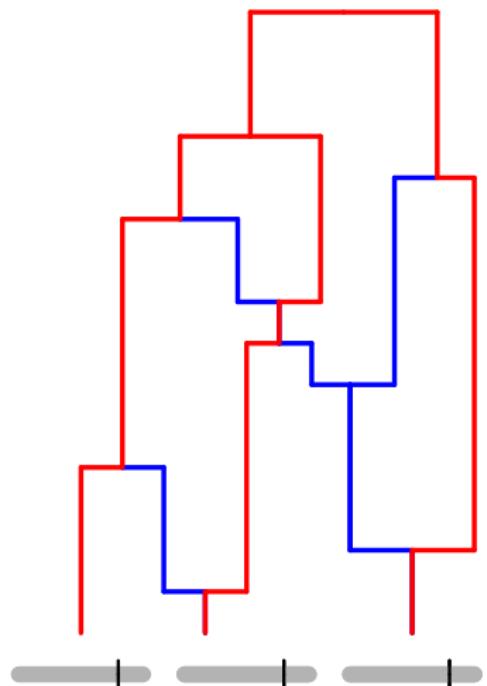
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Wiuf and Hein's sequential algorithm



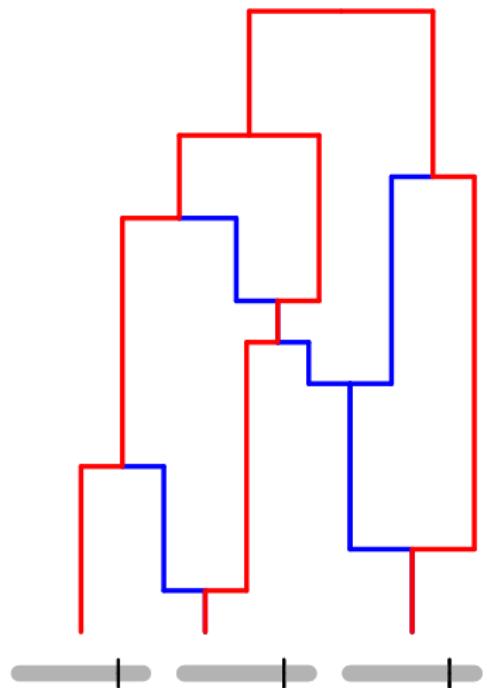
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Wiuf and Hein's sequential algorithm



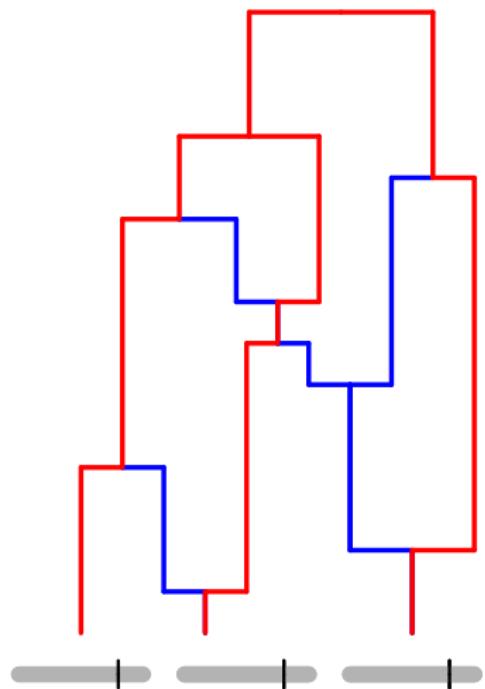
- Extend
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Wiuf and Hein's sequential algorithm



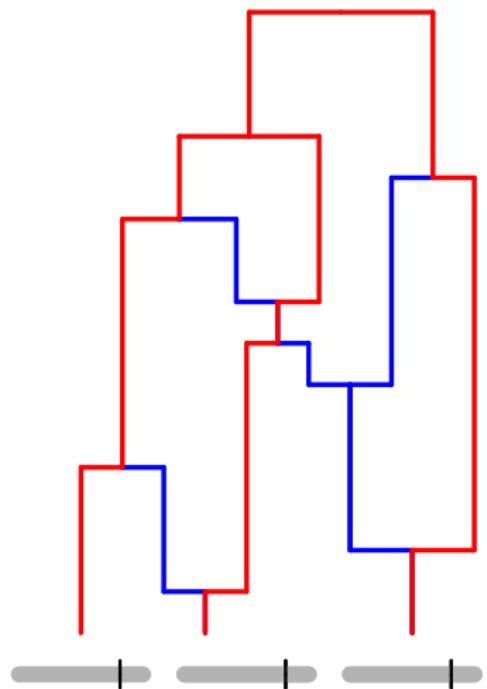
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Wiuf and Hein's sequential algorithm



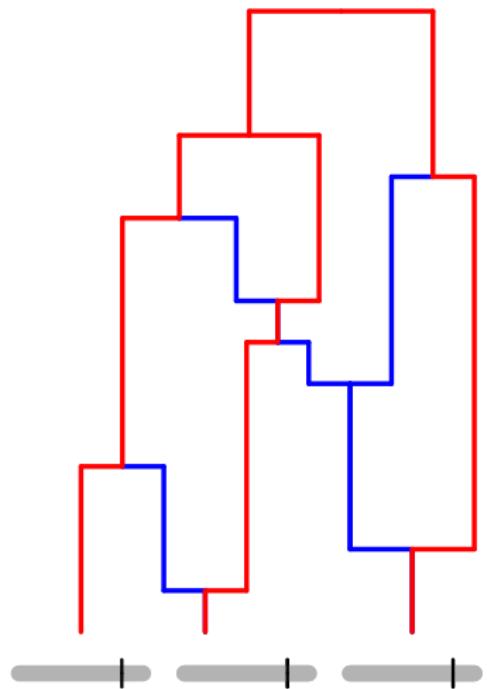
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Wiuf and Hein's sequential algorithm



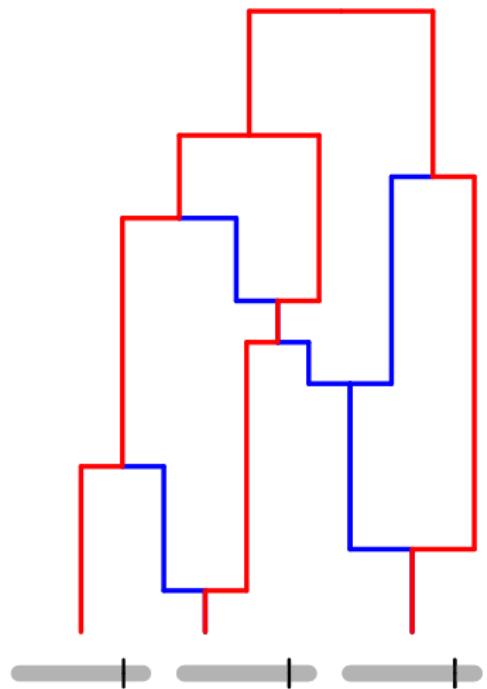
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Wiuf and Hein's sequential algorithm



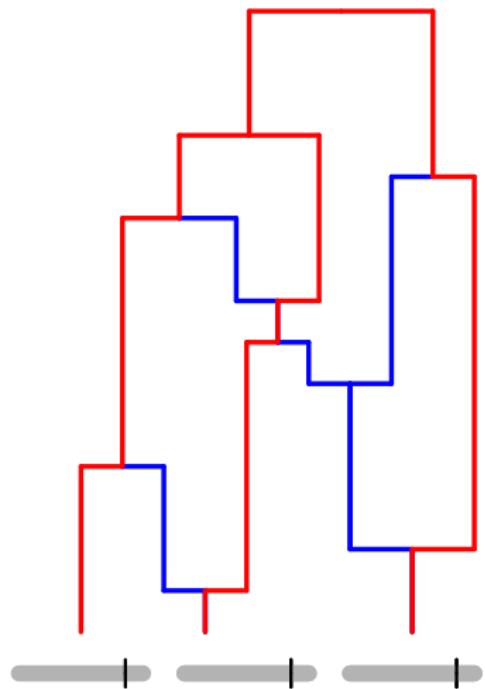
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Wiuf and Hein's sequential algorithm



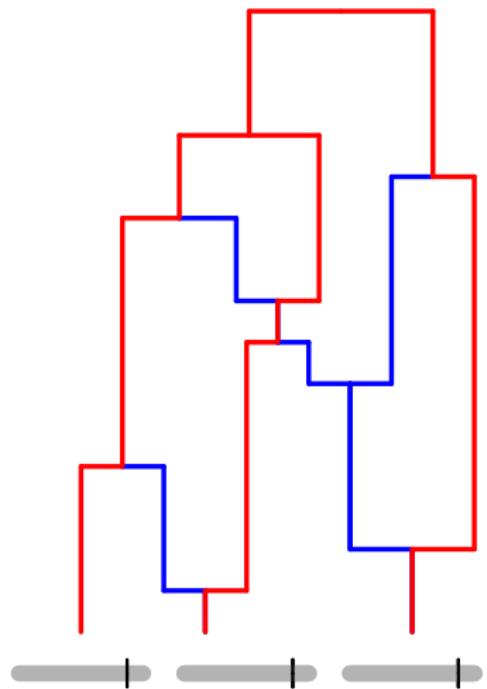
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Wiuf and Hein's sequential algorithm



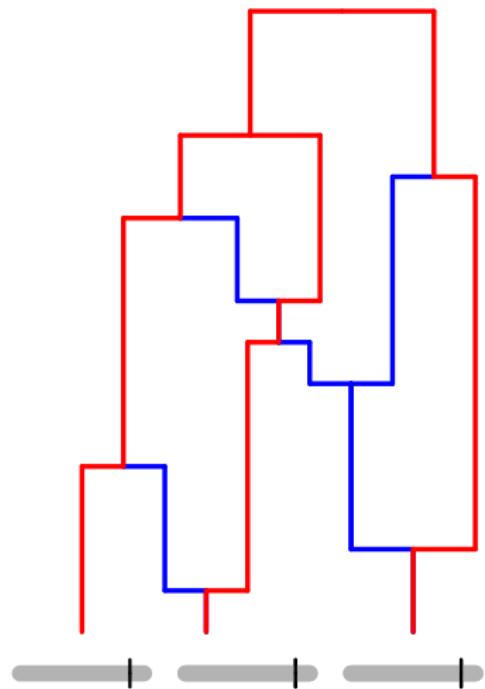
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Wiuf and Hein's sequential algorithm



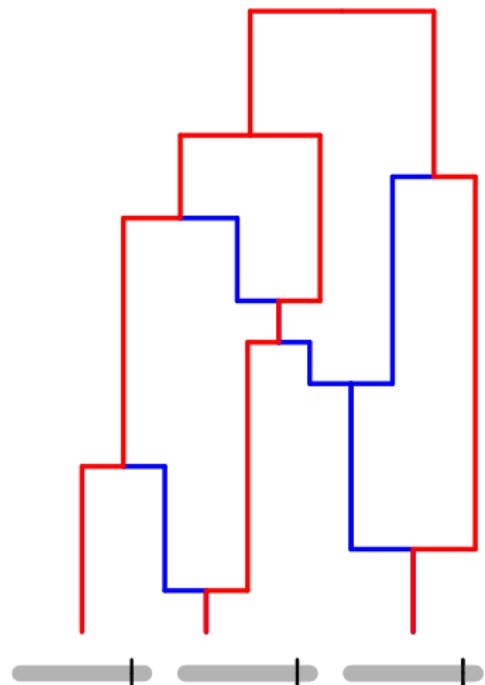
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Wiuf and Hein's sequential algorithm



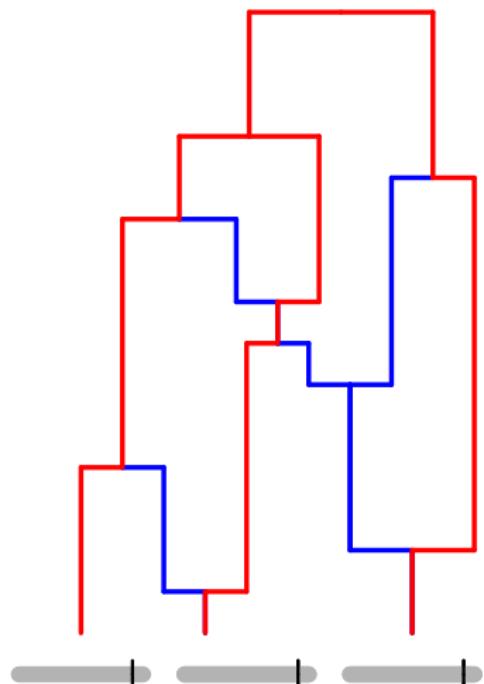
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Wiuf and Hein's sequential algorithm



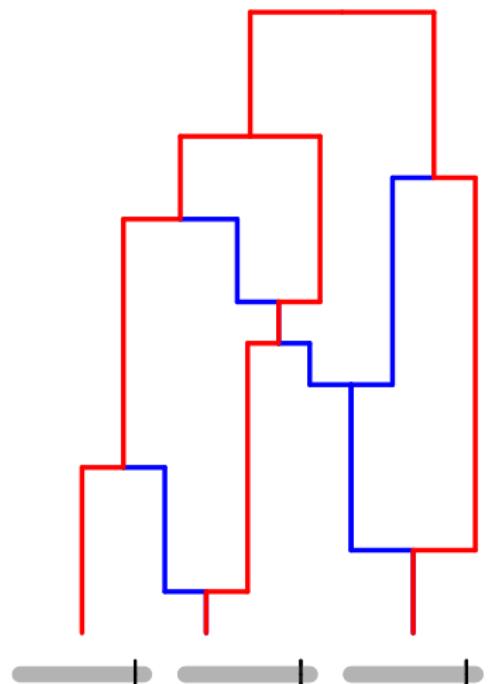
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Wiuf and Hein's sequential algorithm



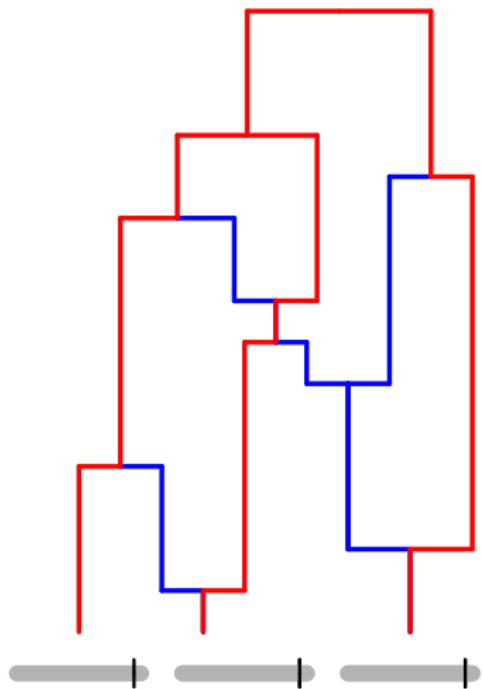
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Wiuf and Hein's sequential algorithm



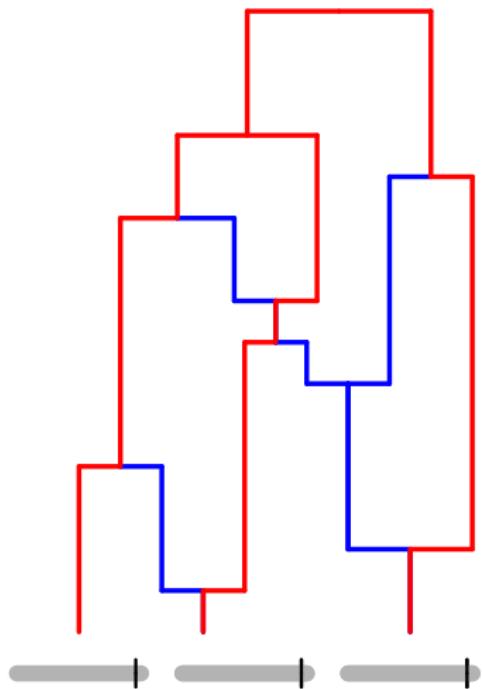
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Wiuf and Hein's sequential algorithm



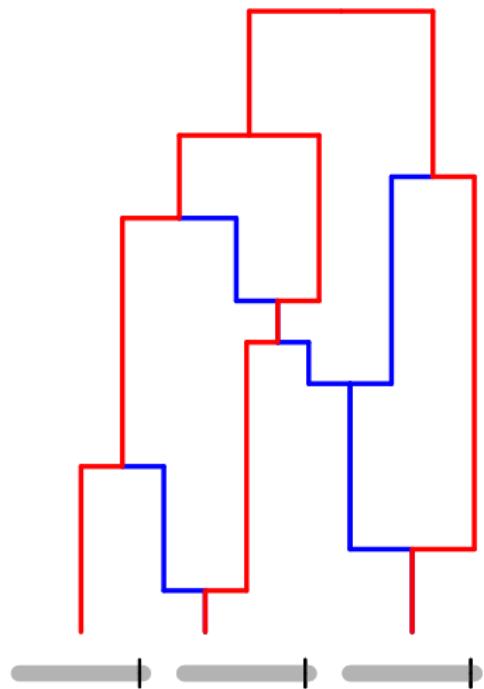
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Wiuf and Hein's sequential algorithm



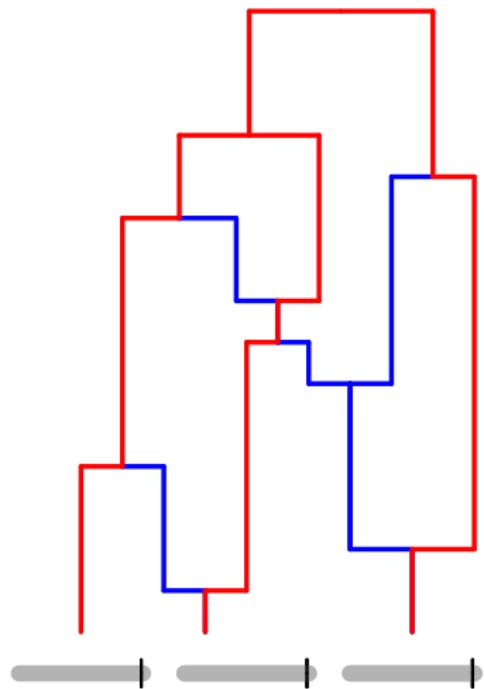
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Wiuf and Hein's sequential algorithm



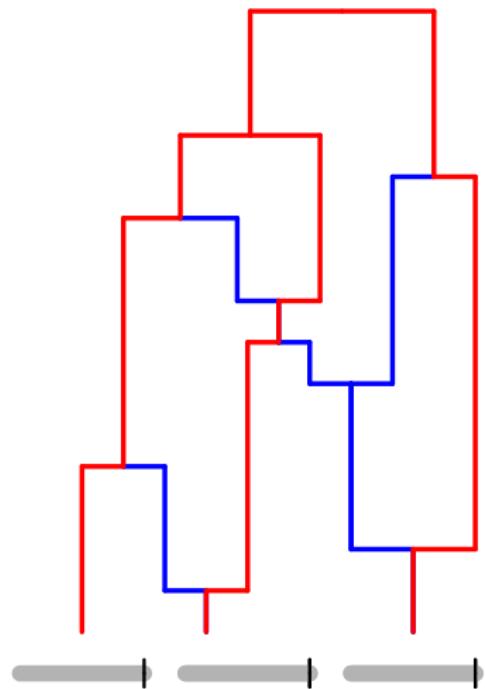
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Wiuf and Hein's sequential algorithm



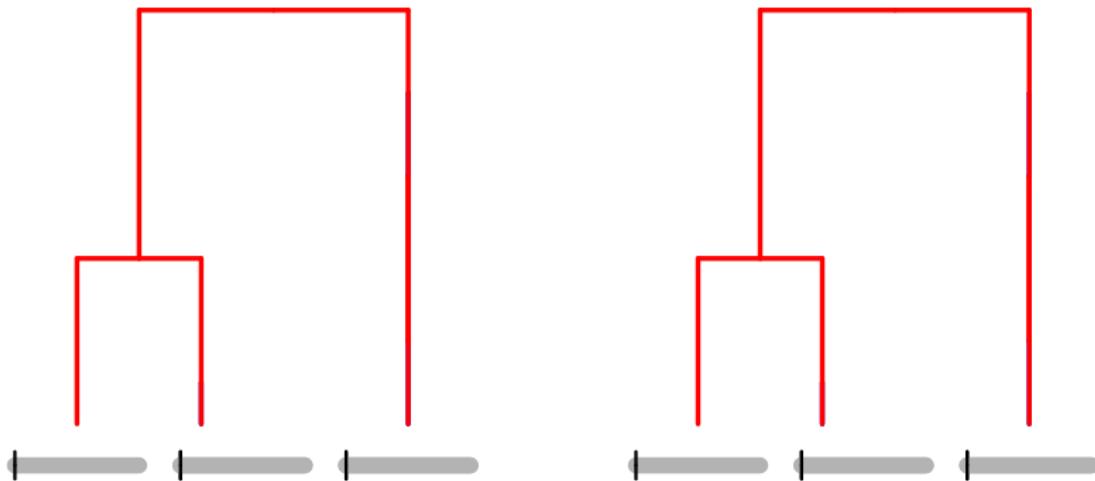
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Wiuf and Hein's sequential algorithm

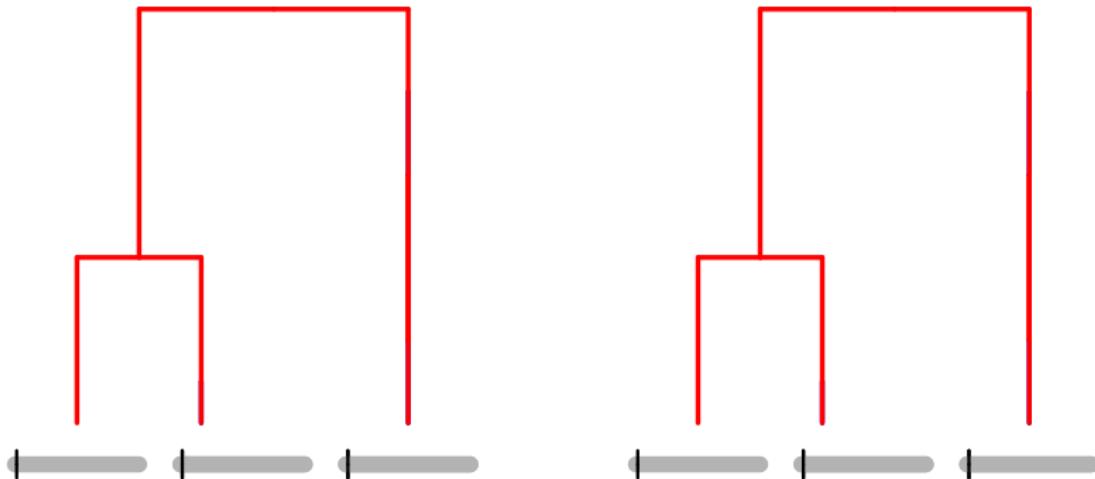


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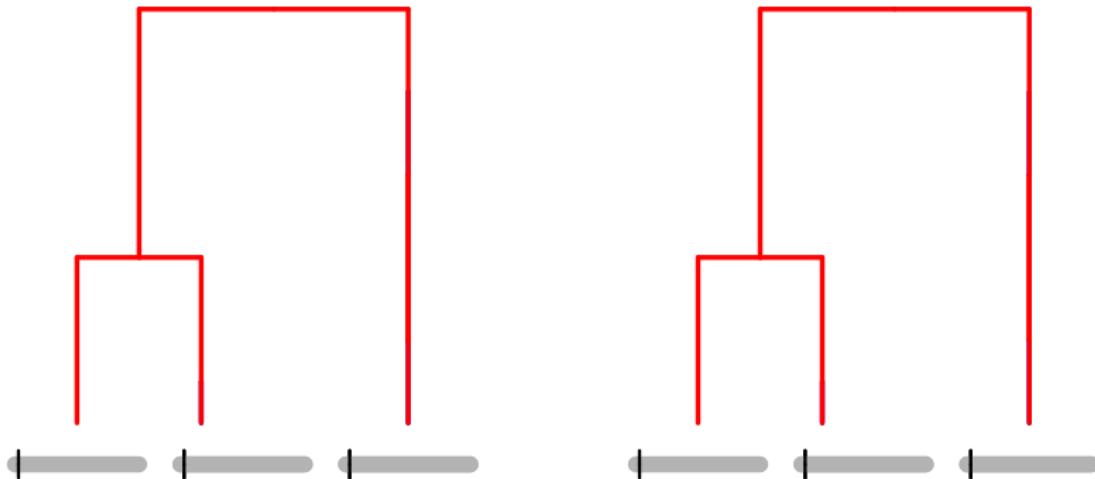
Simplification 1: postpone nonancestral recombinations



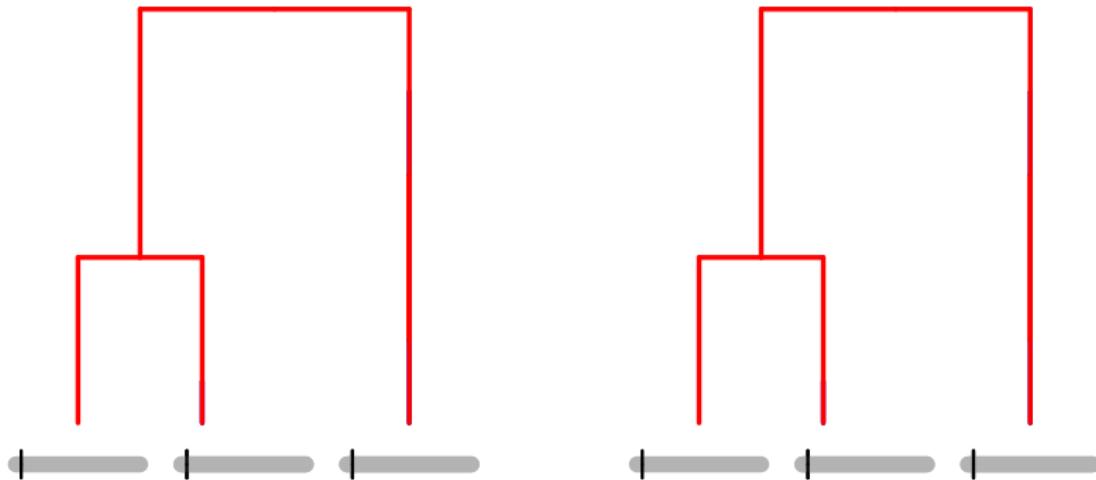
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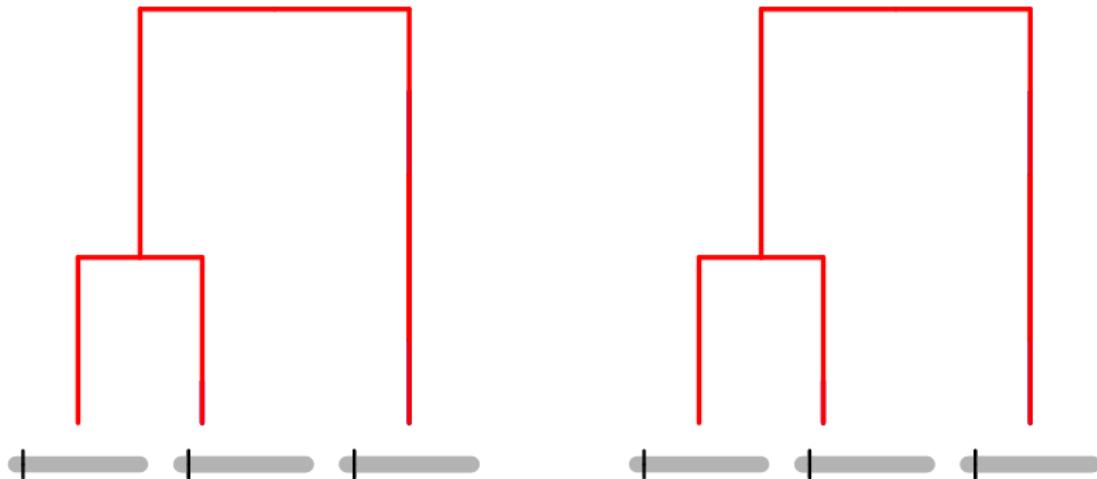
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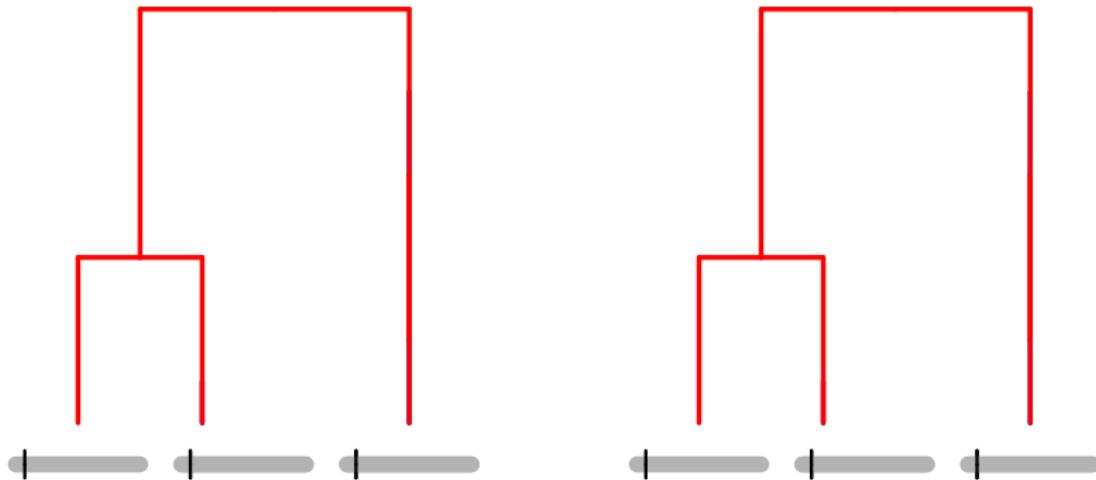
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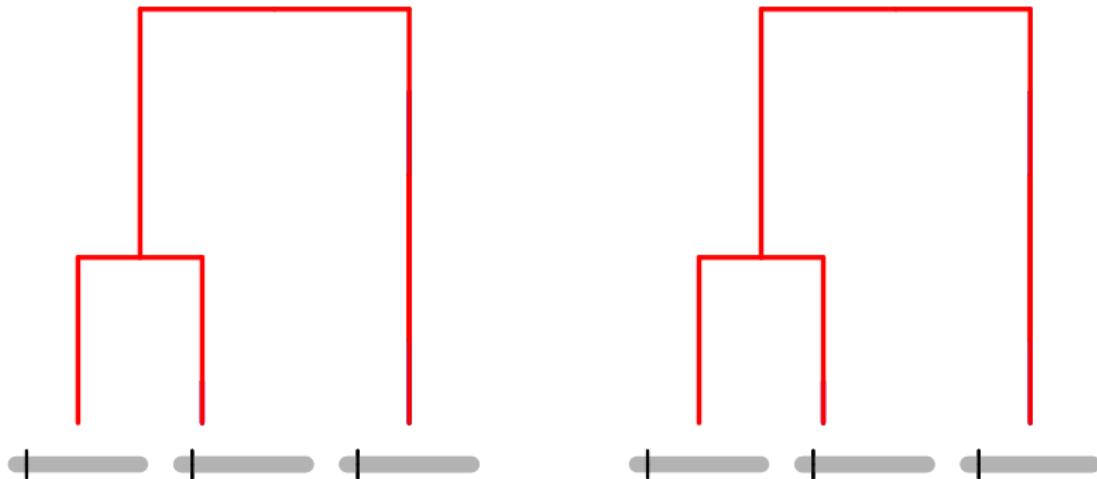
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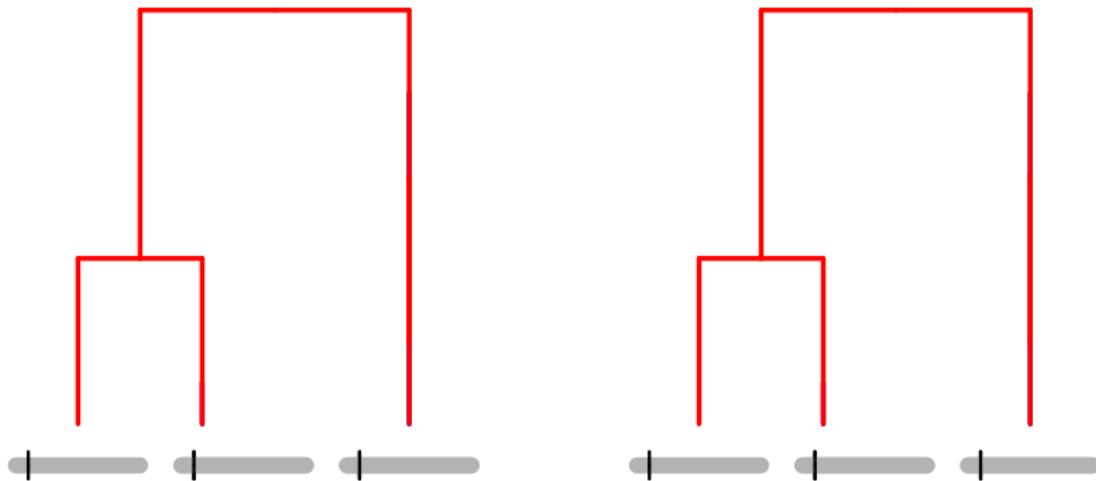
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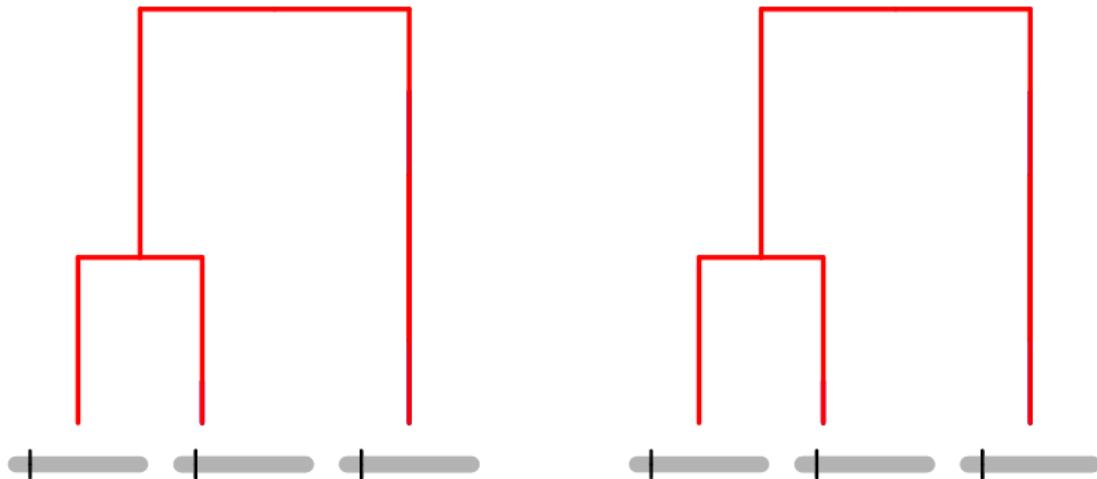
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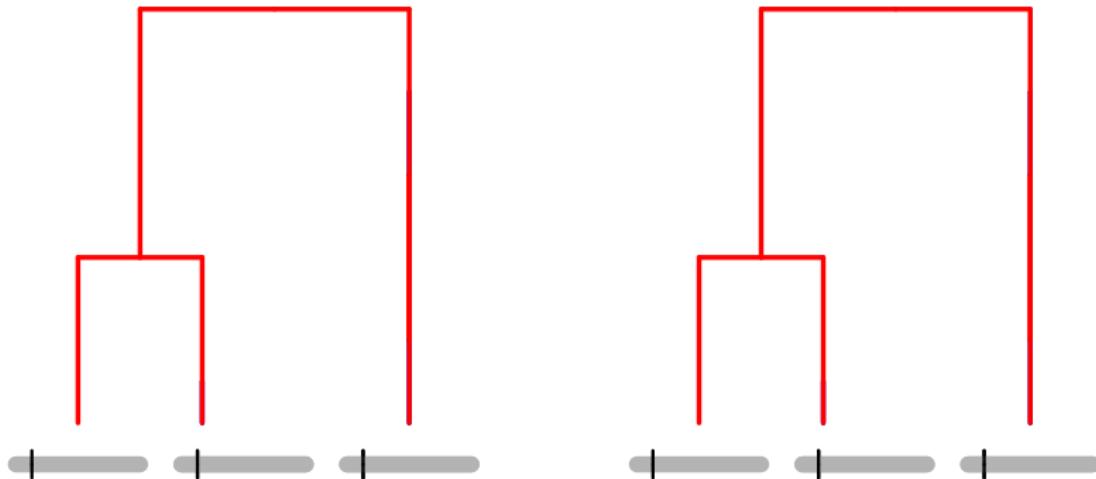
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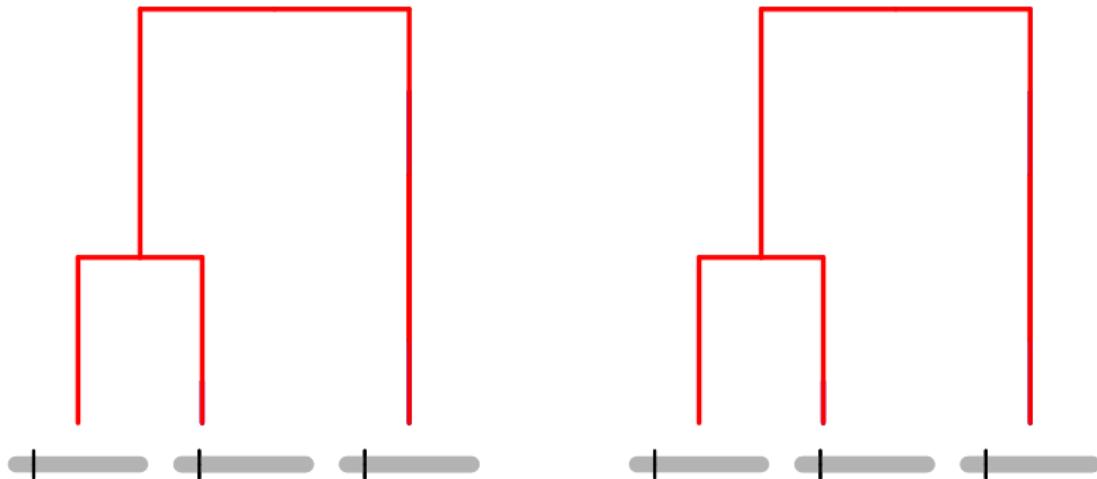
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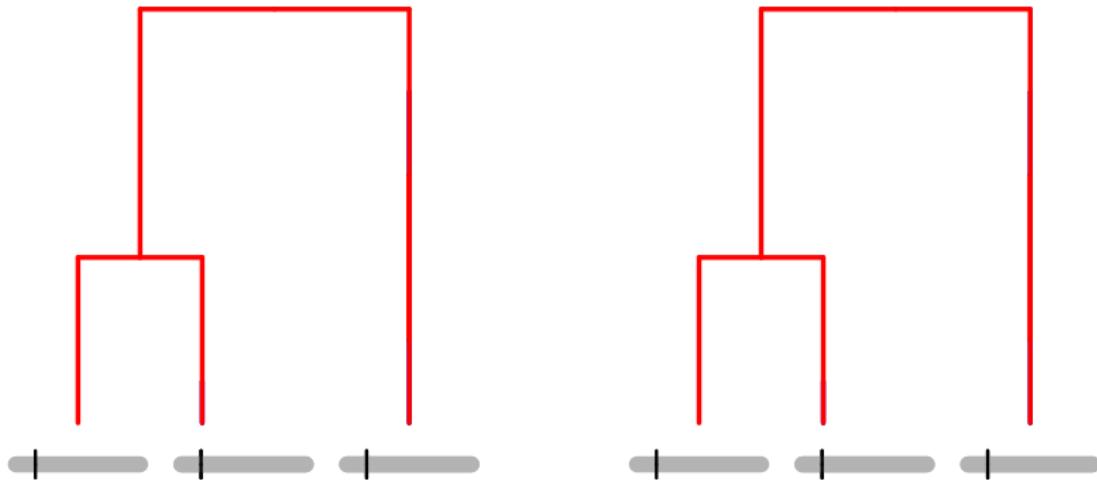
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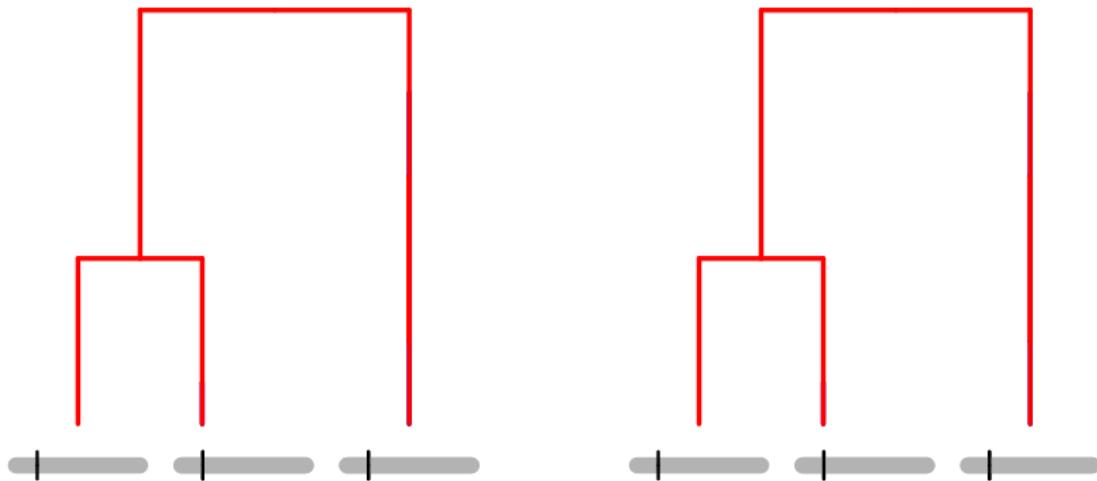
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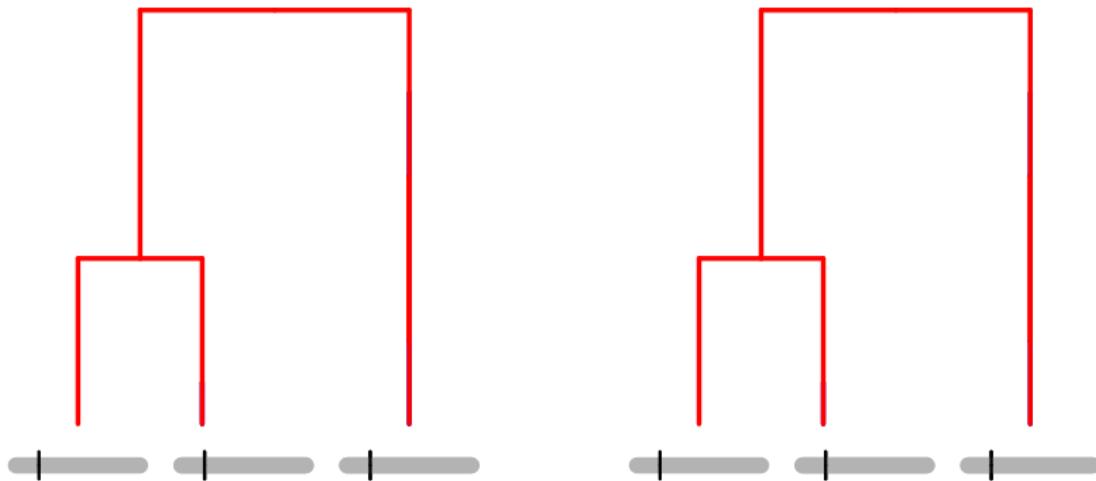
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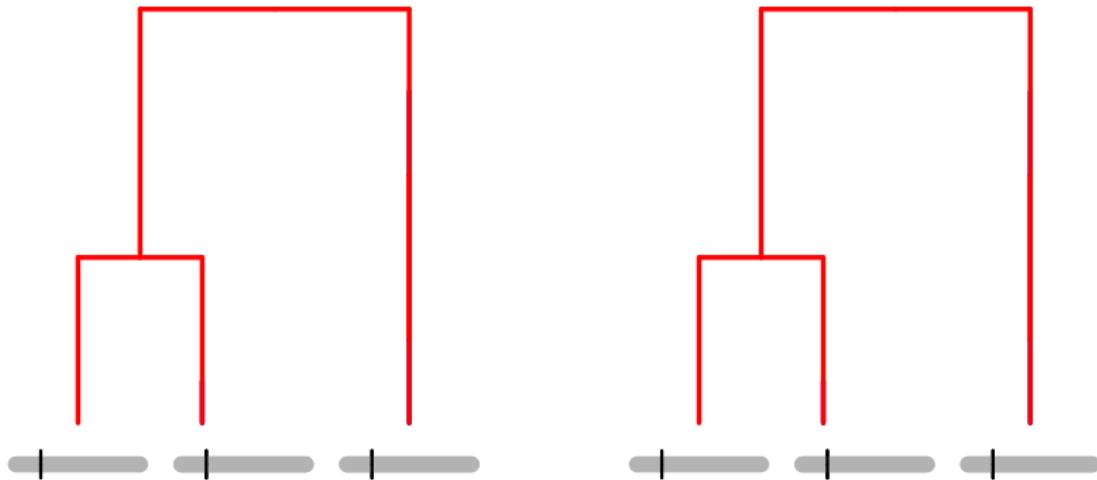
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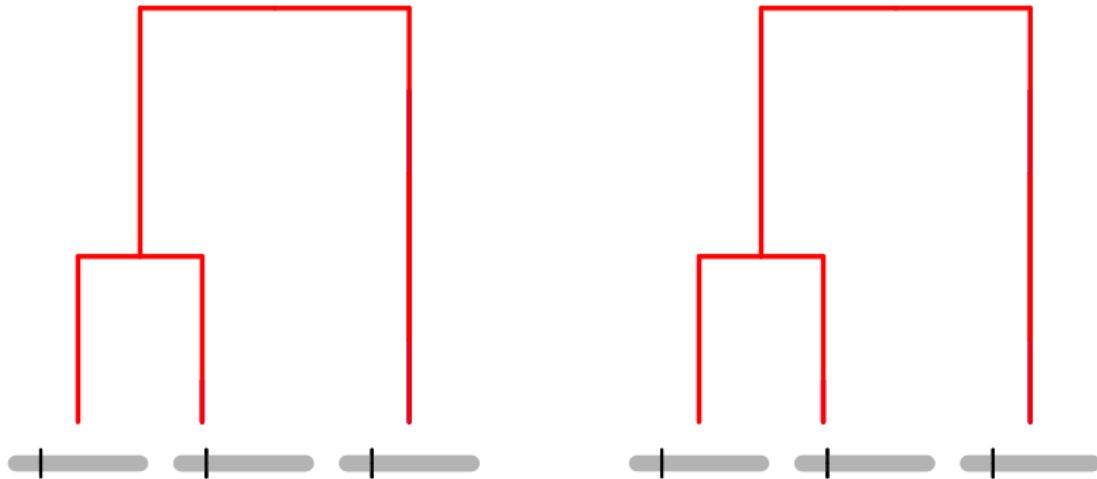
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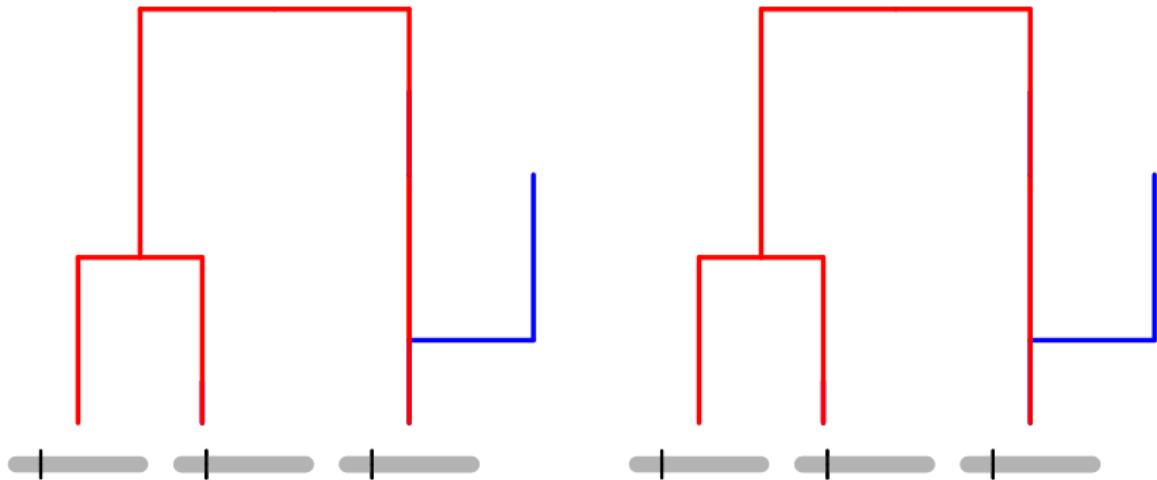
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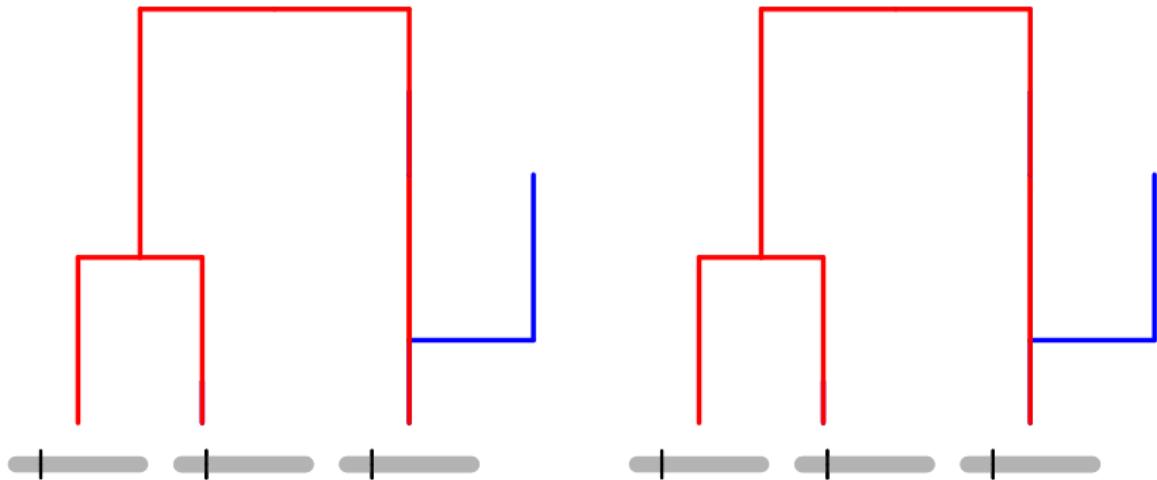
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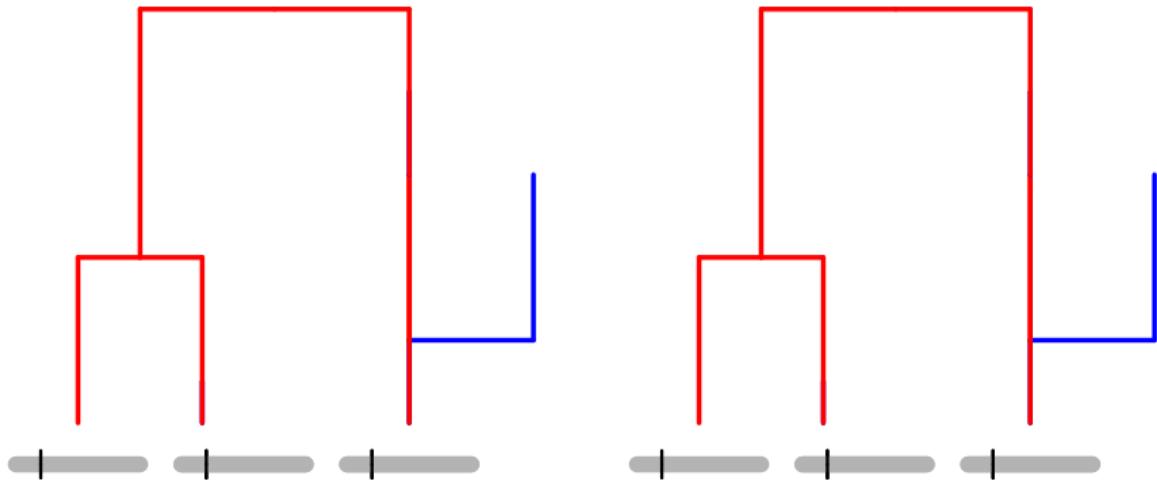
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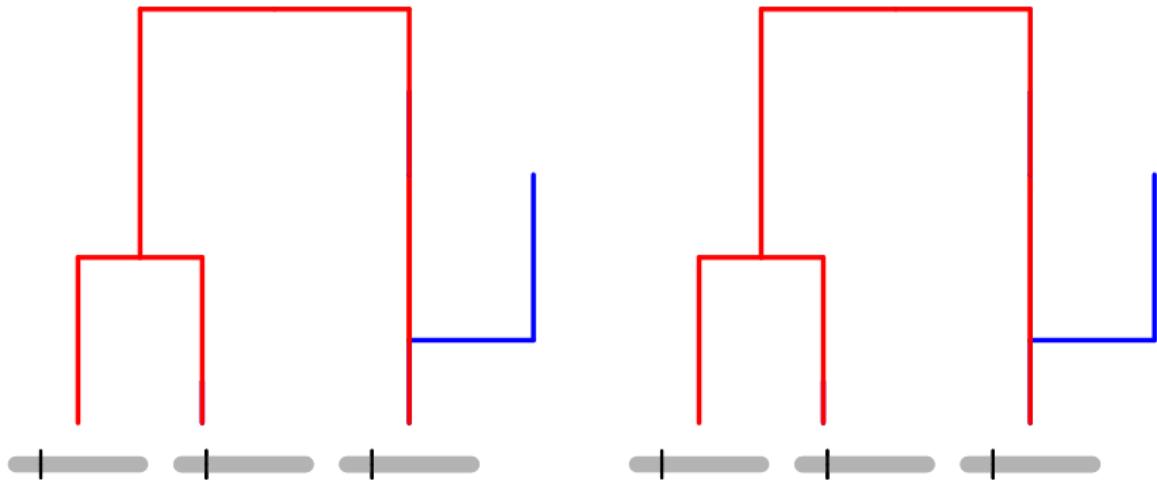
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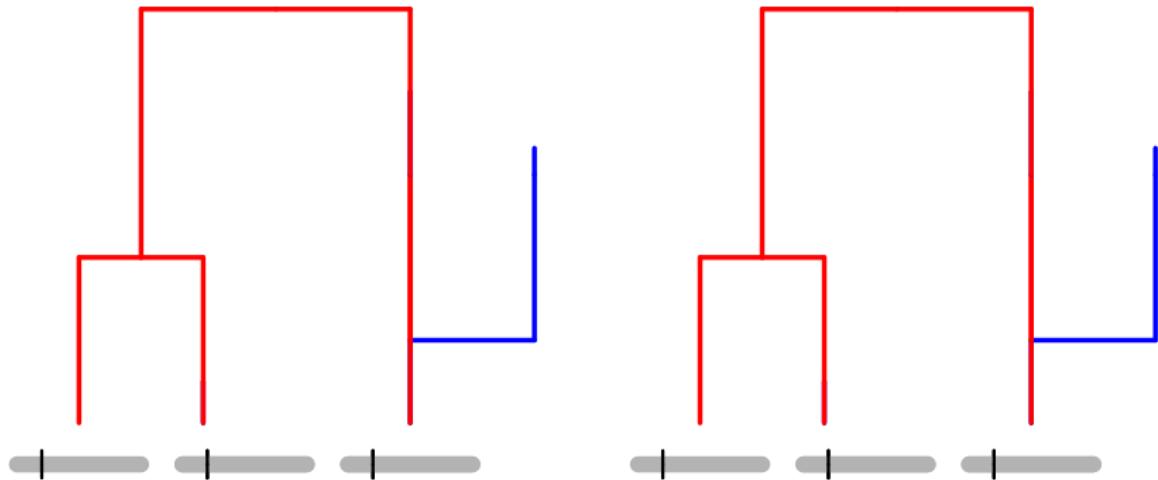
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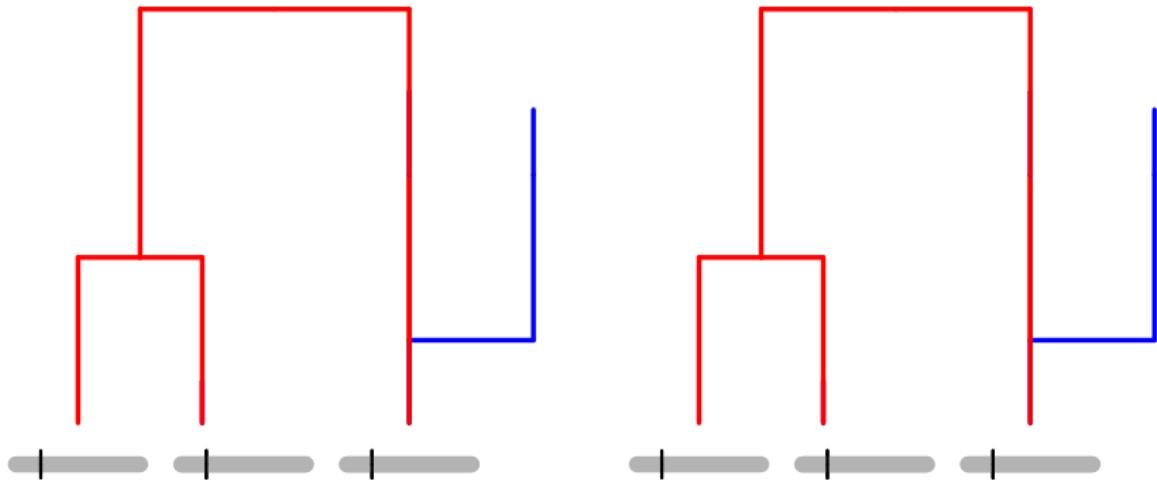
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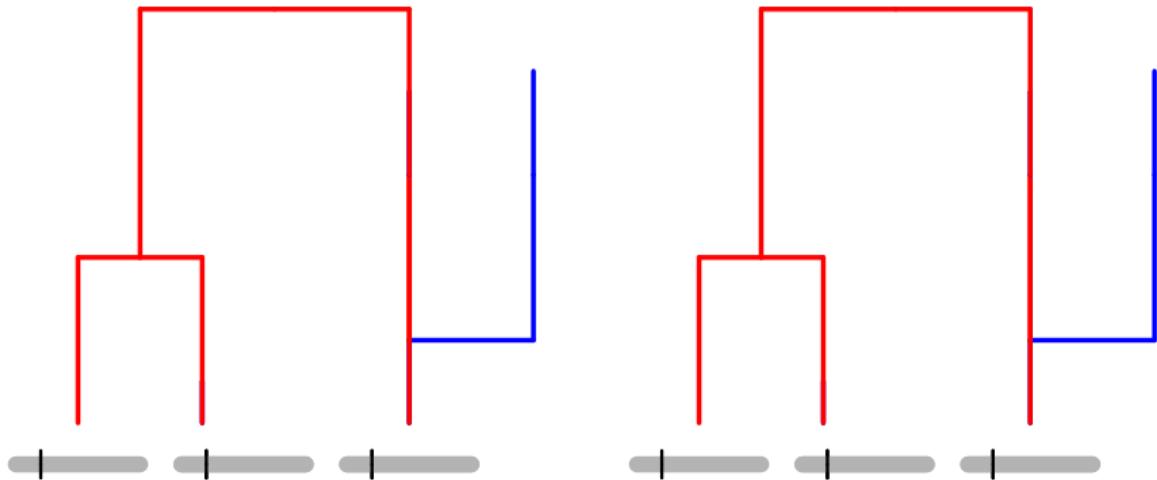
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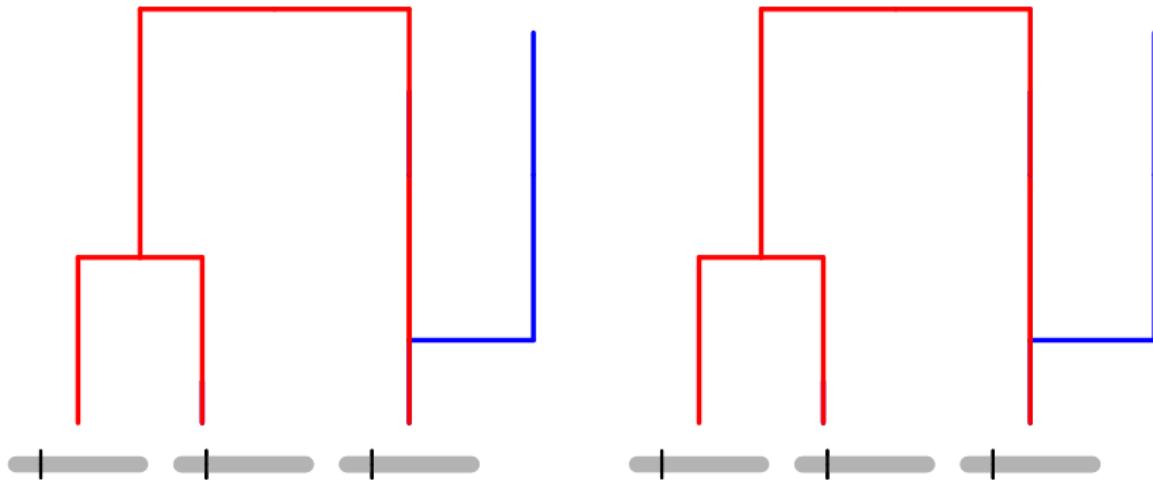
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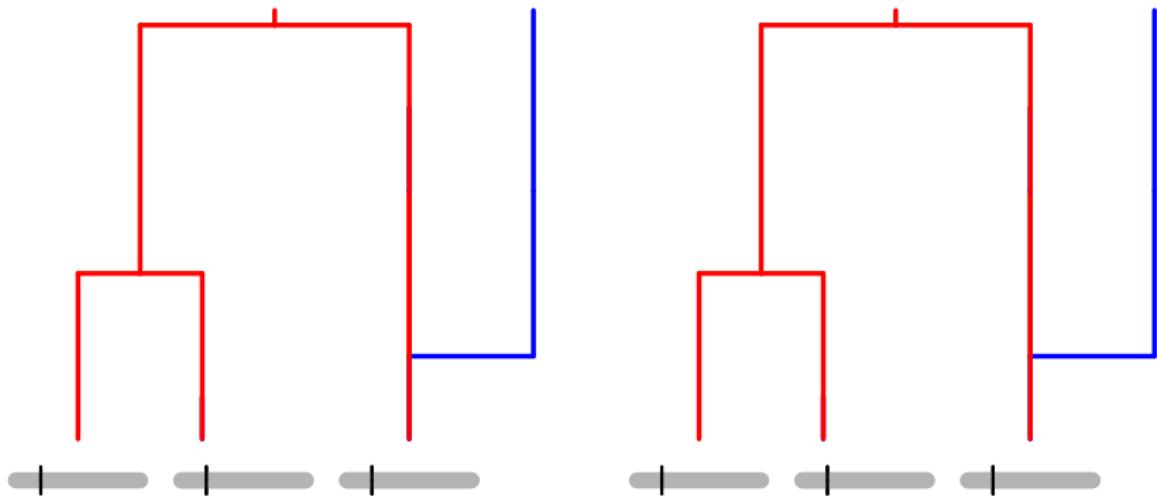
Simplification 1: postpone nonancestral recombinations



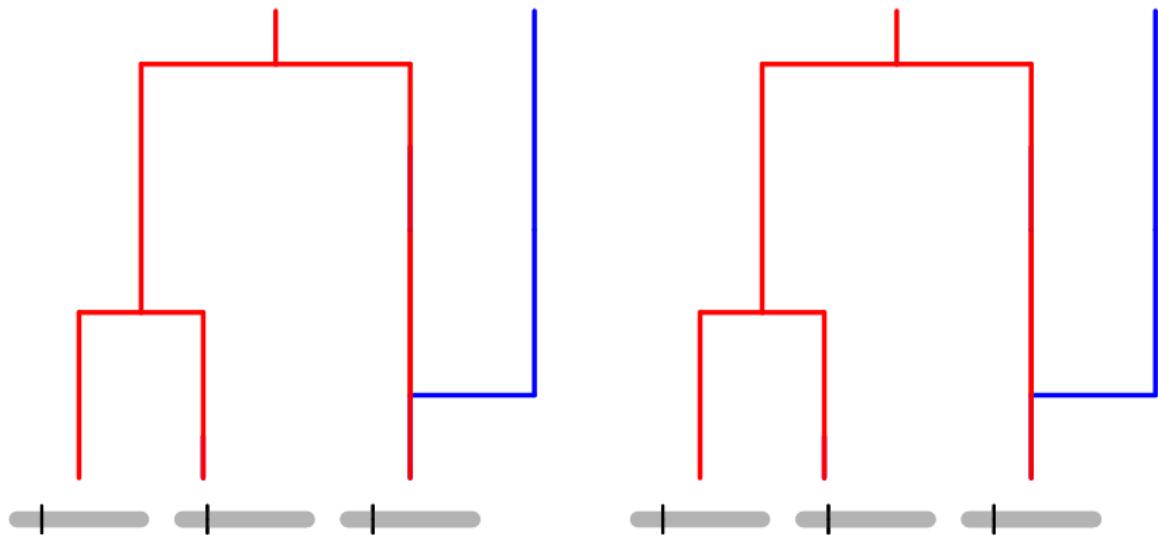
Simplification 1: postpone nonancestral recombinations



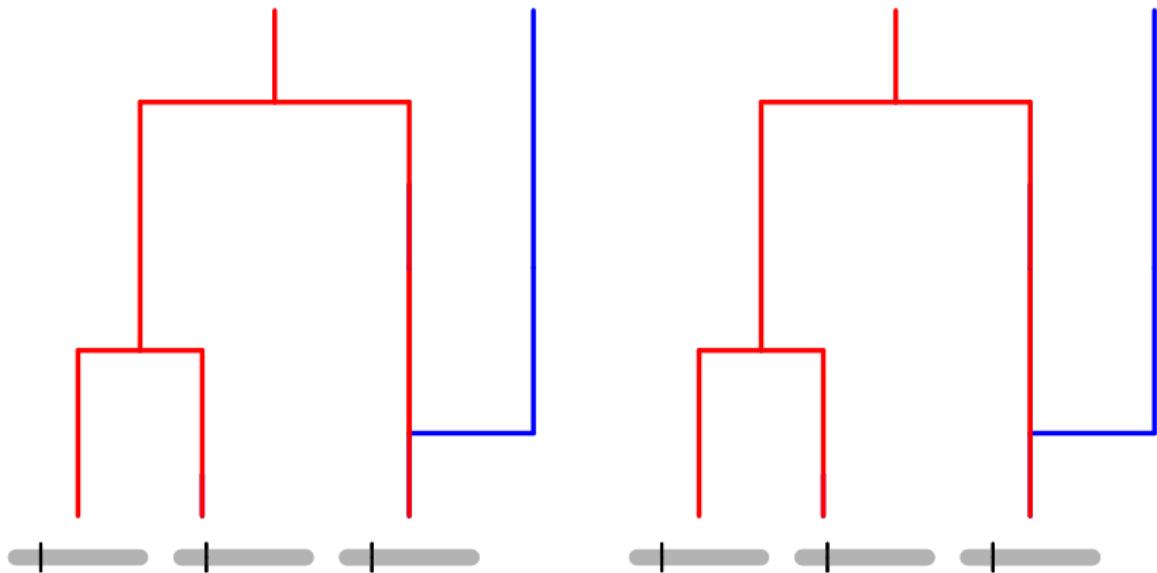
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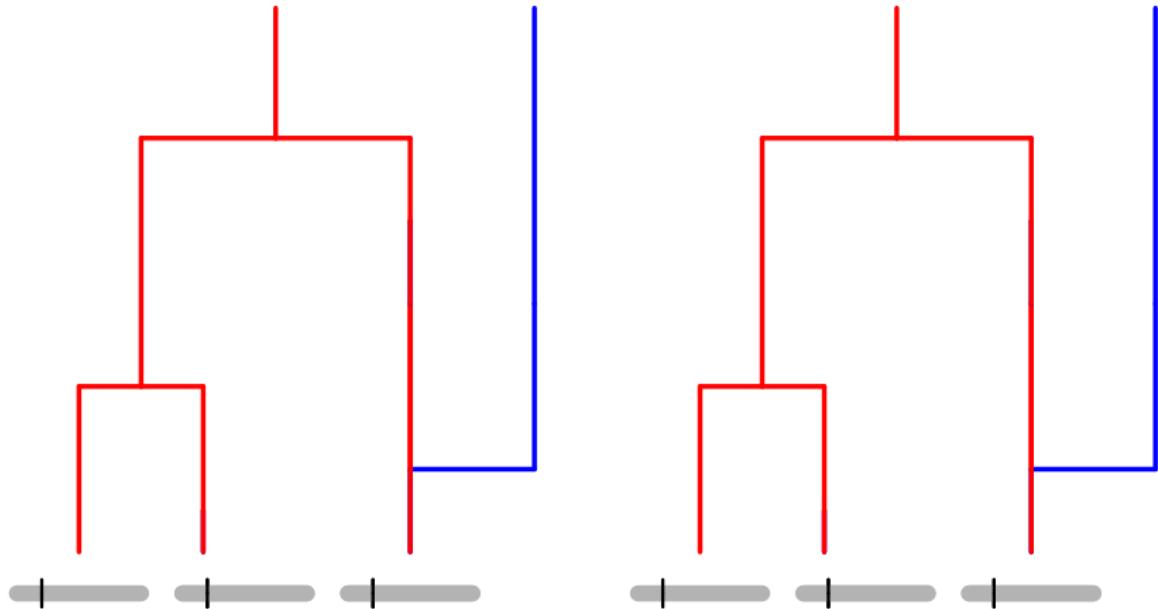
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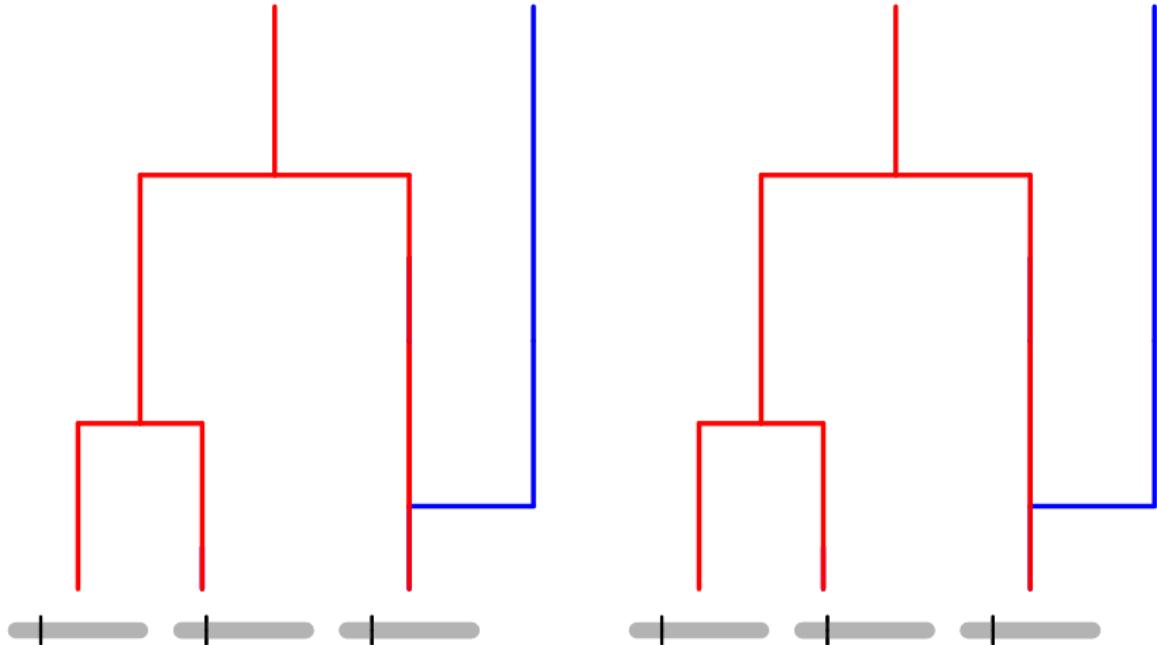
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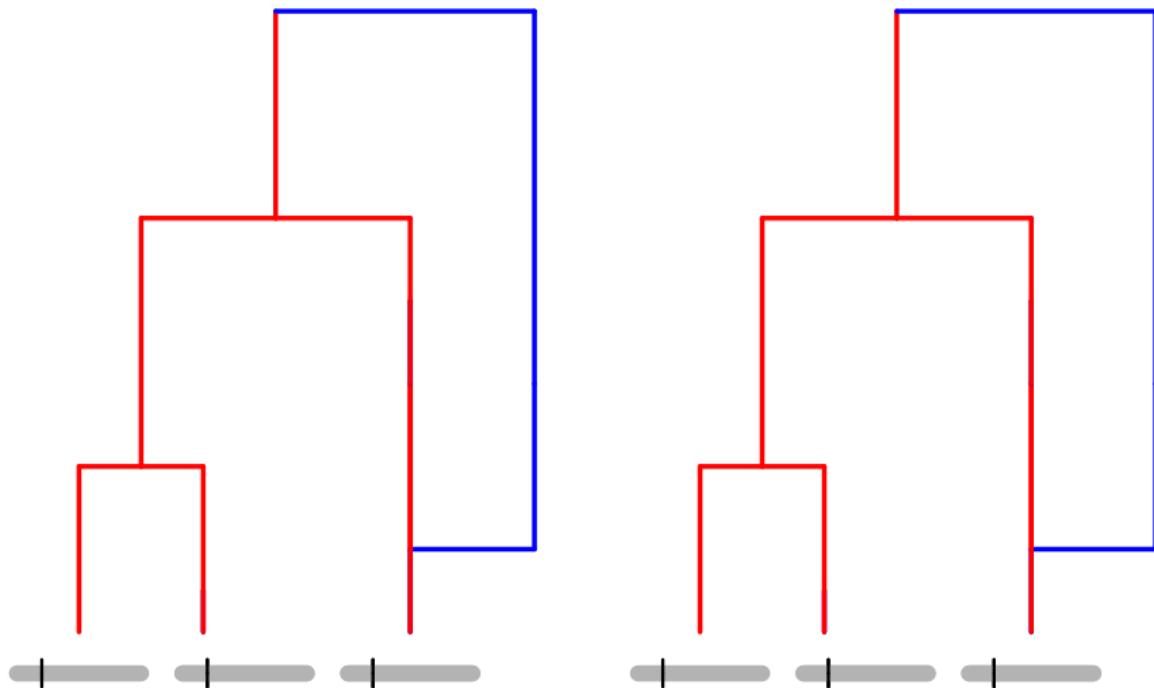
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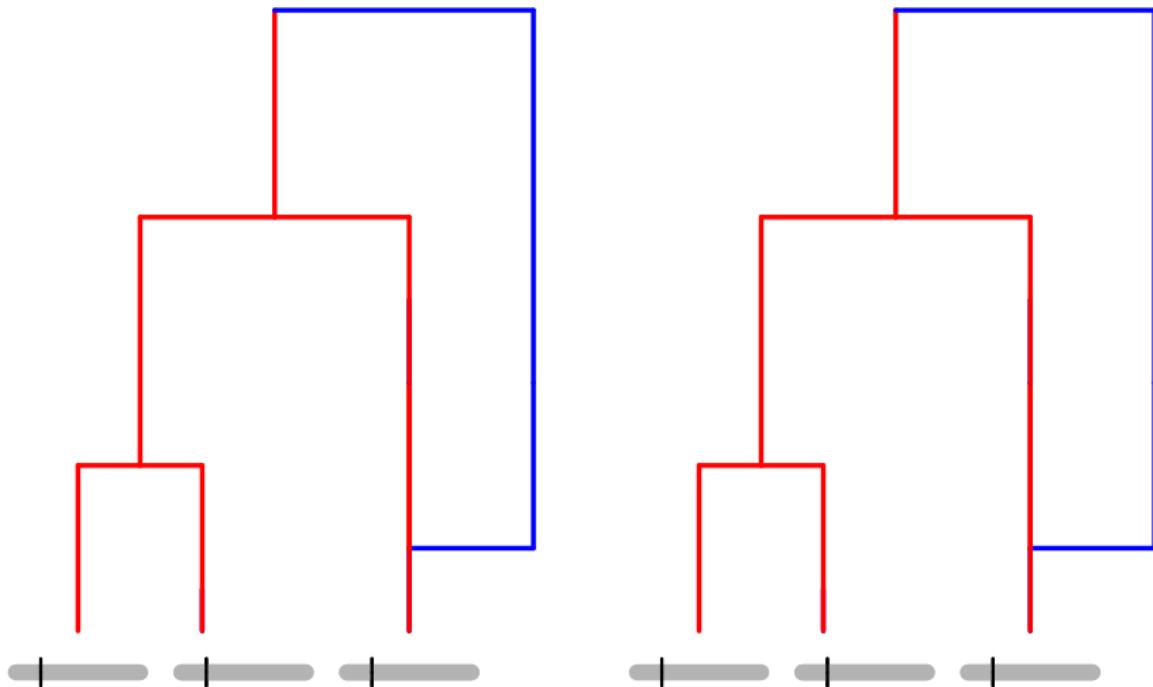
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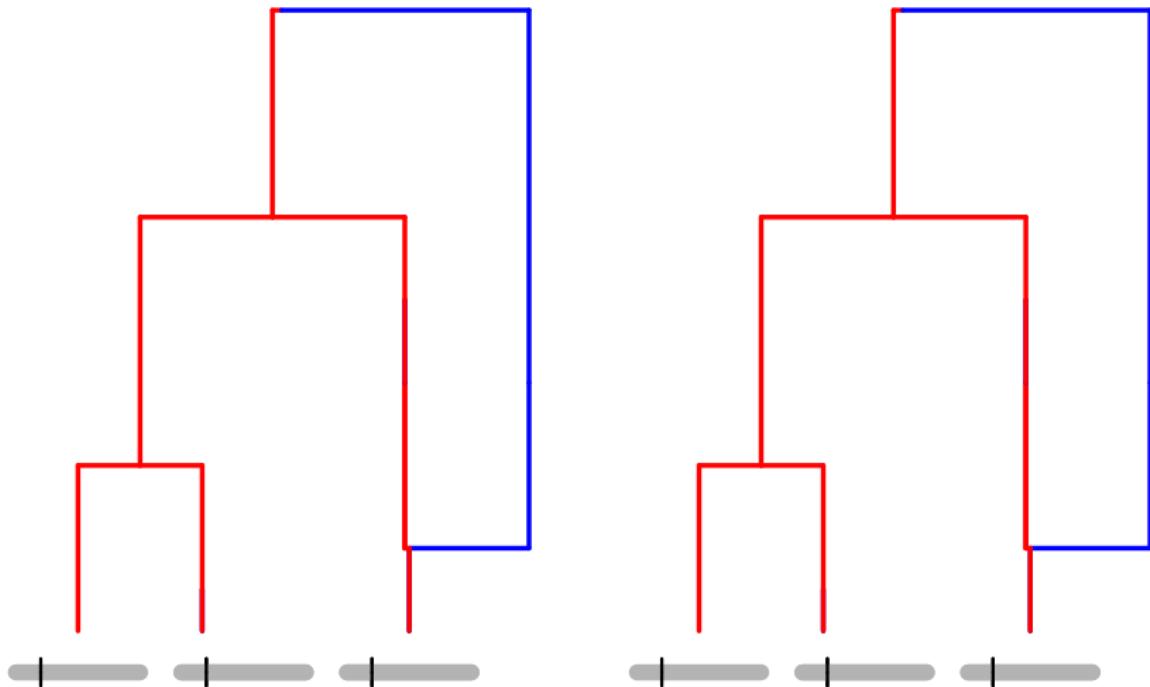
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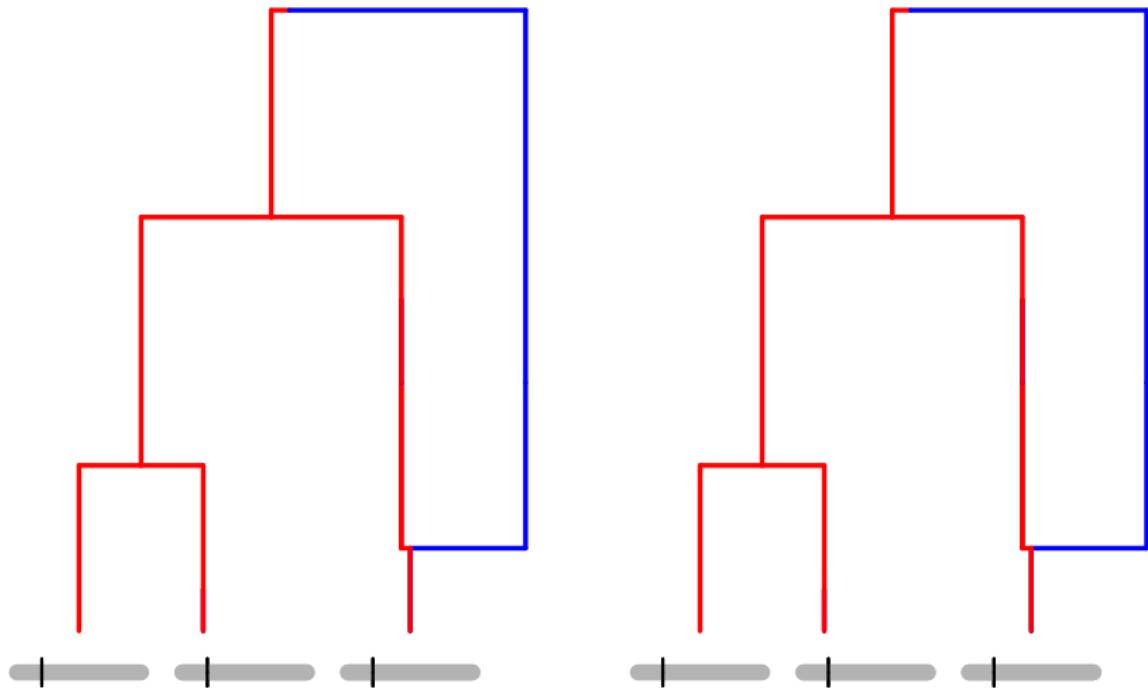
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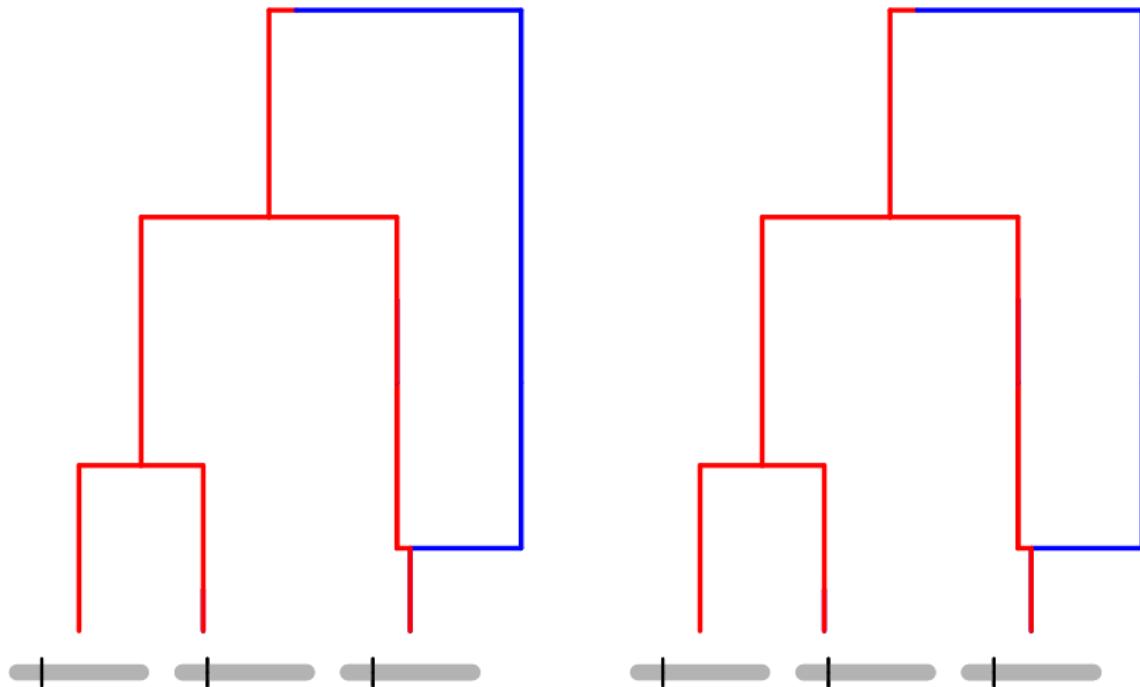
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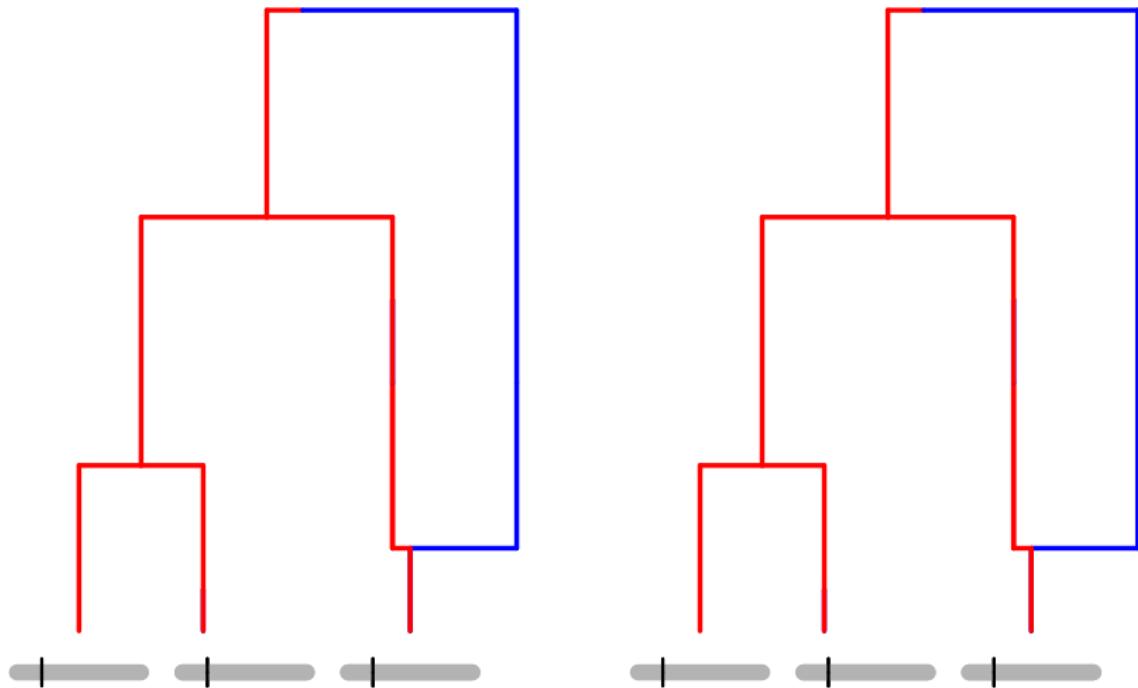
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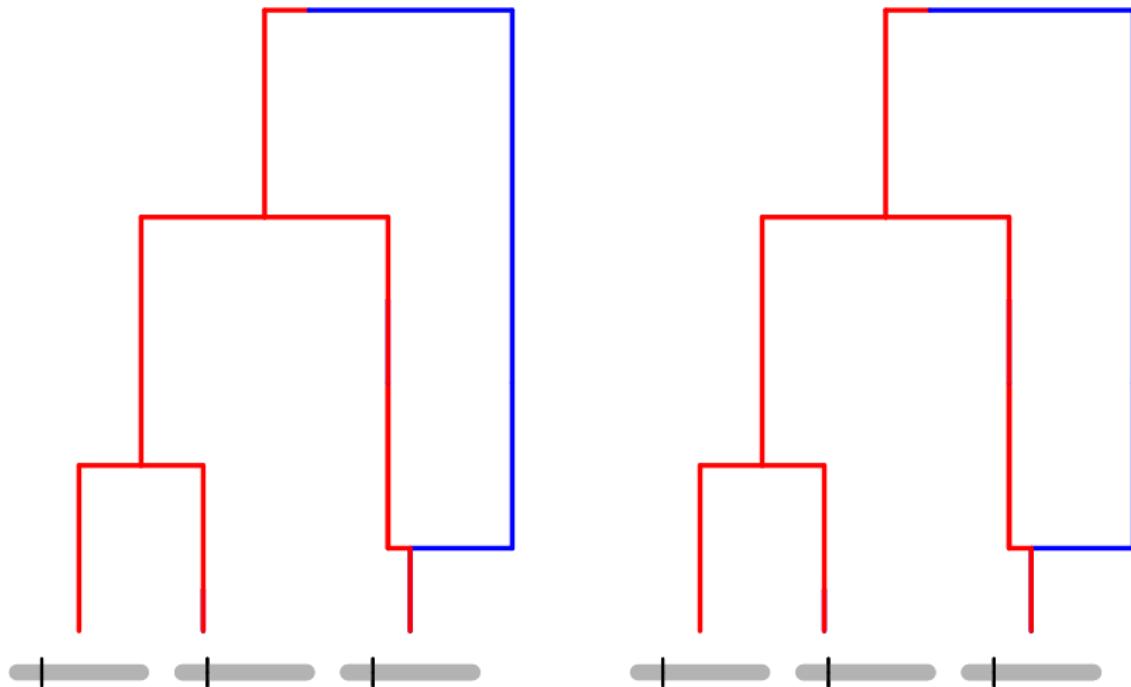
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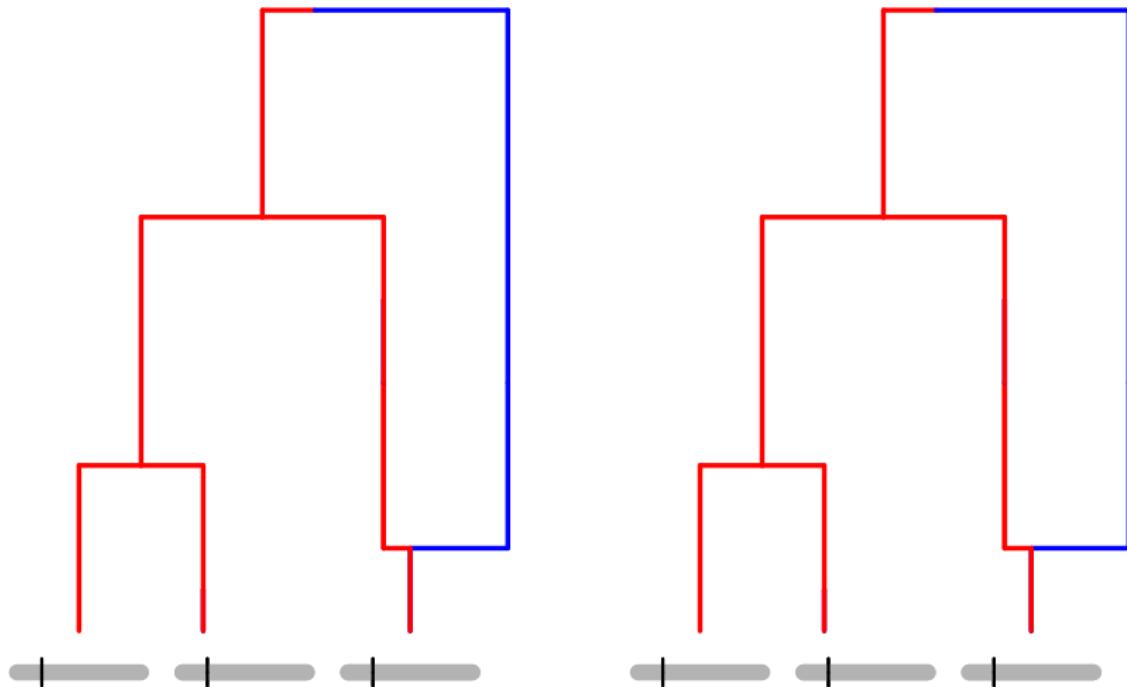
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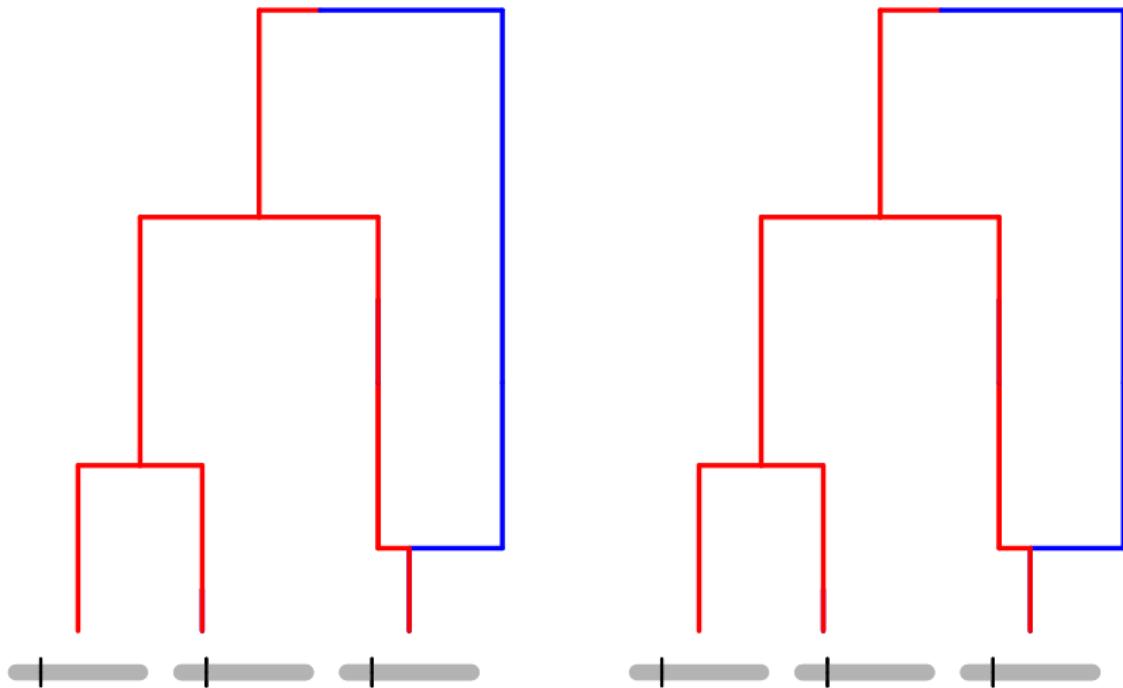
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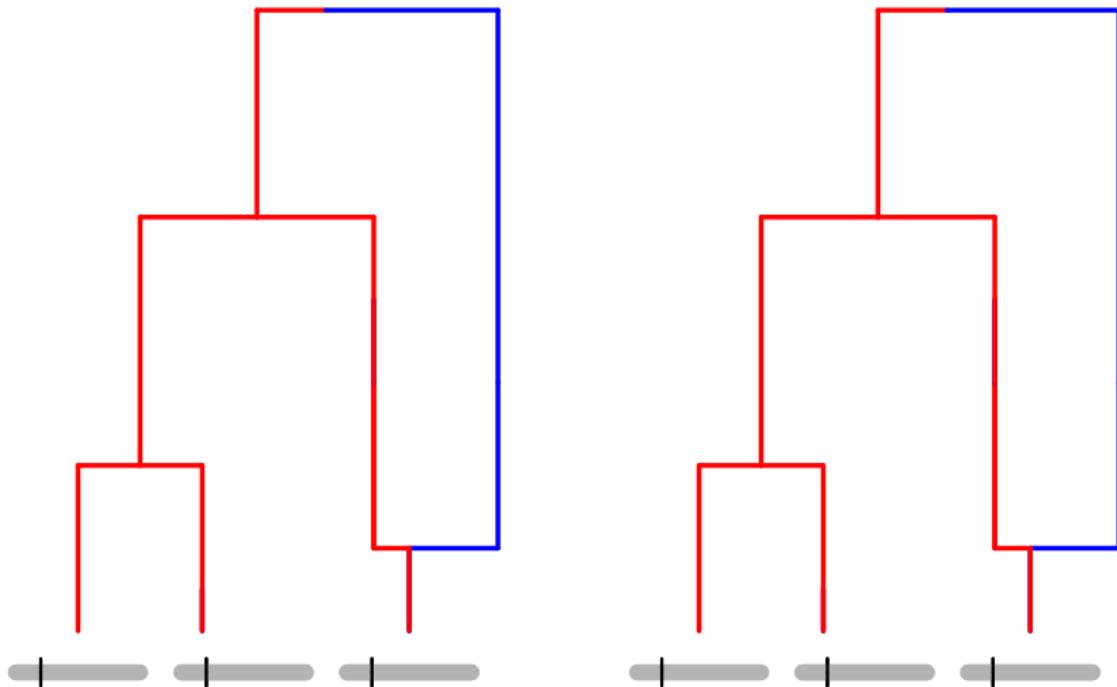
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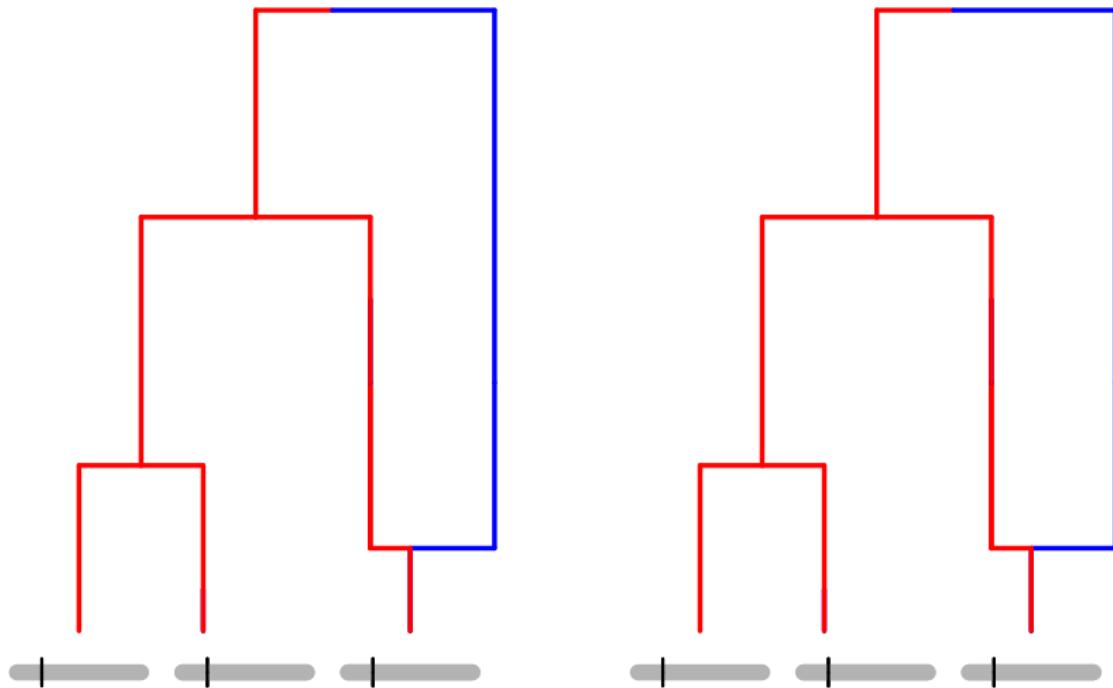
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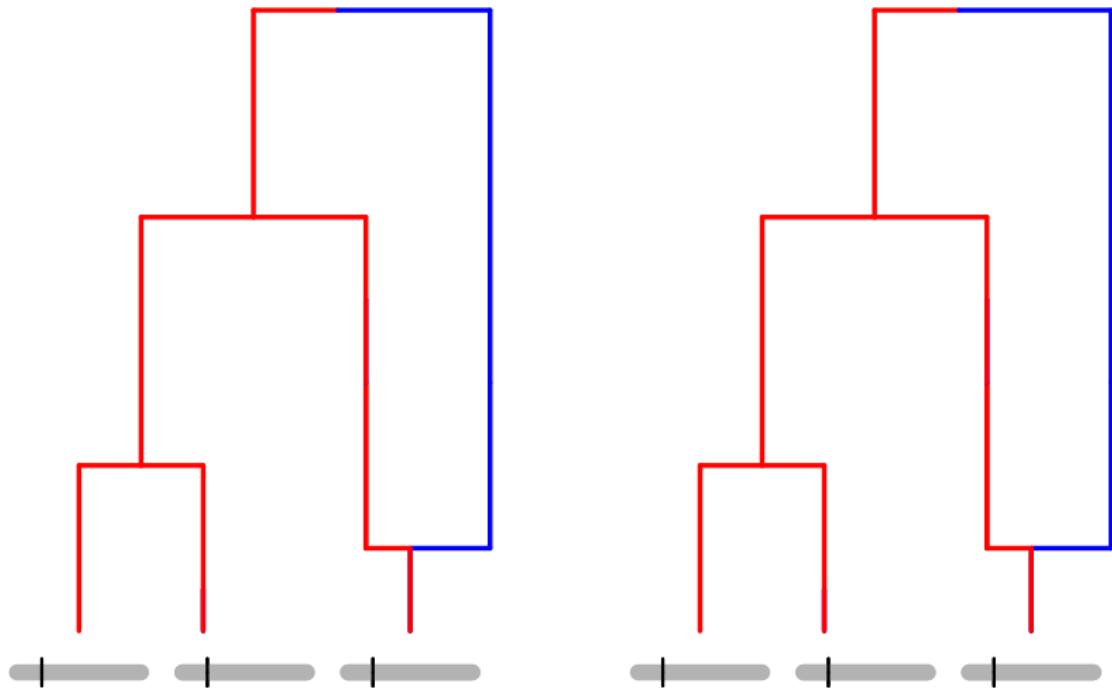
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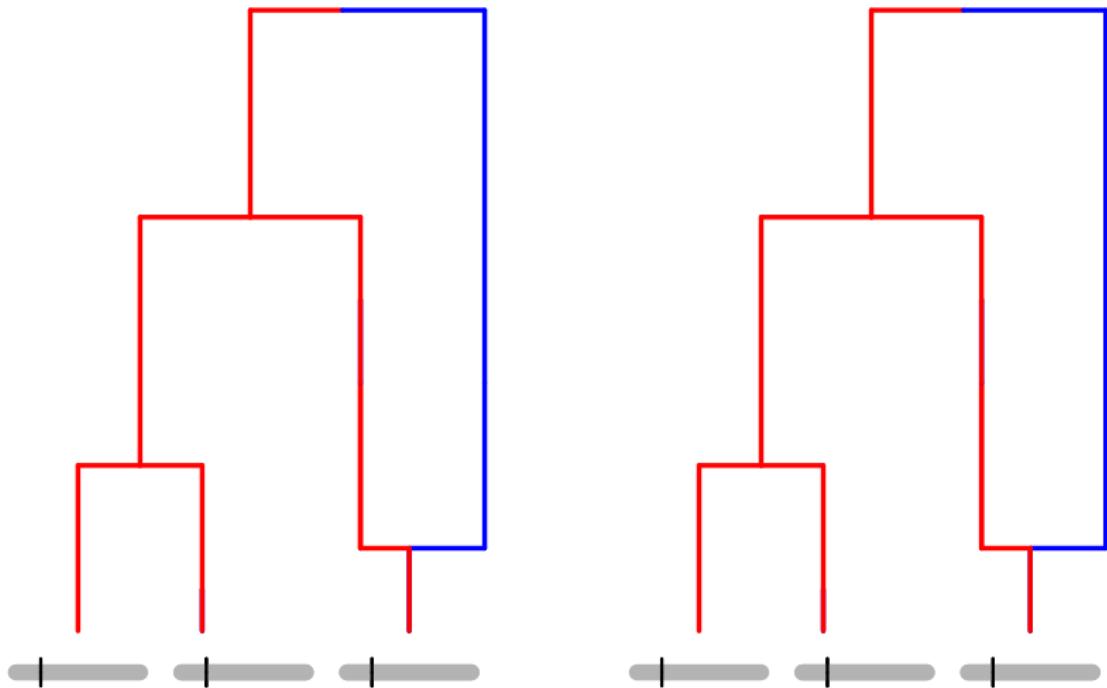
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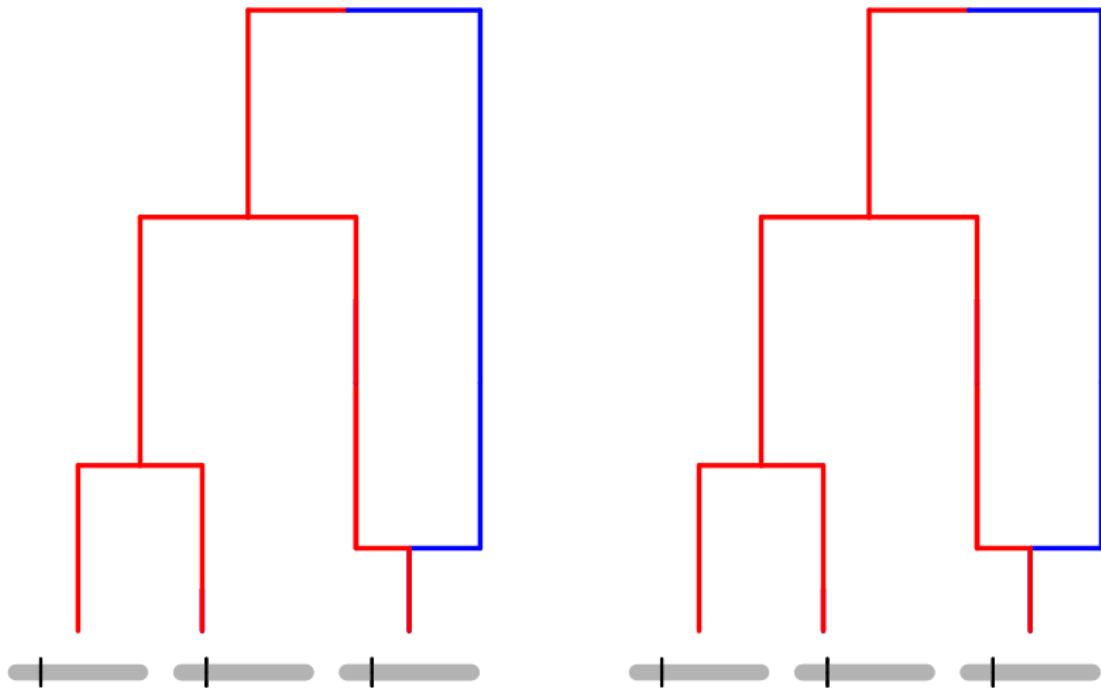
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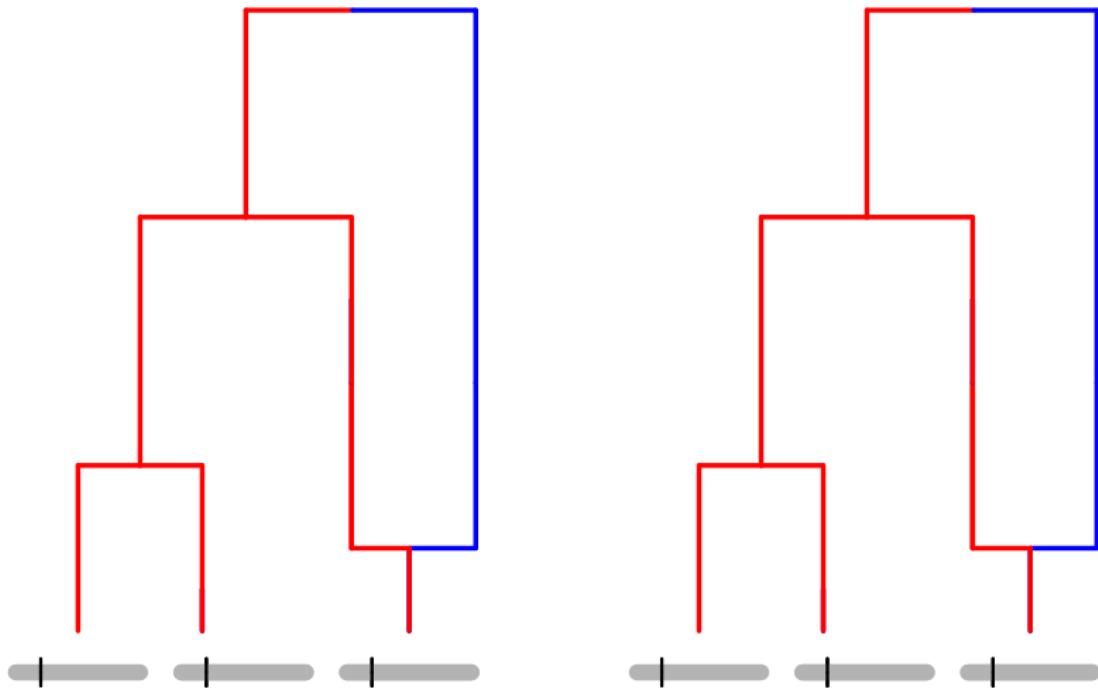
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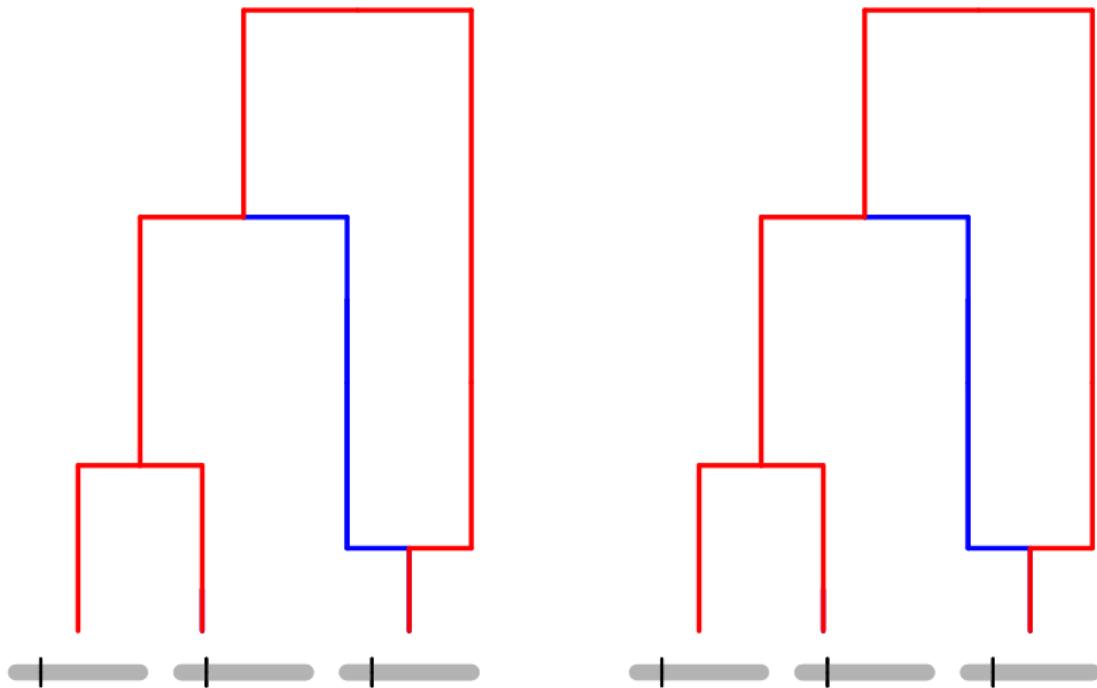
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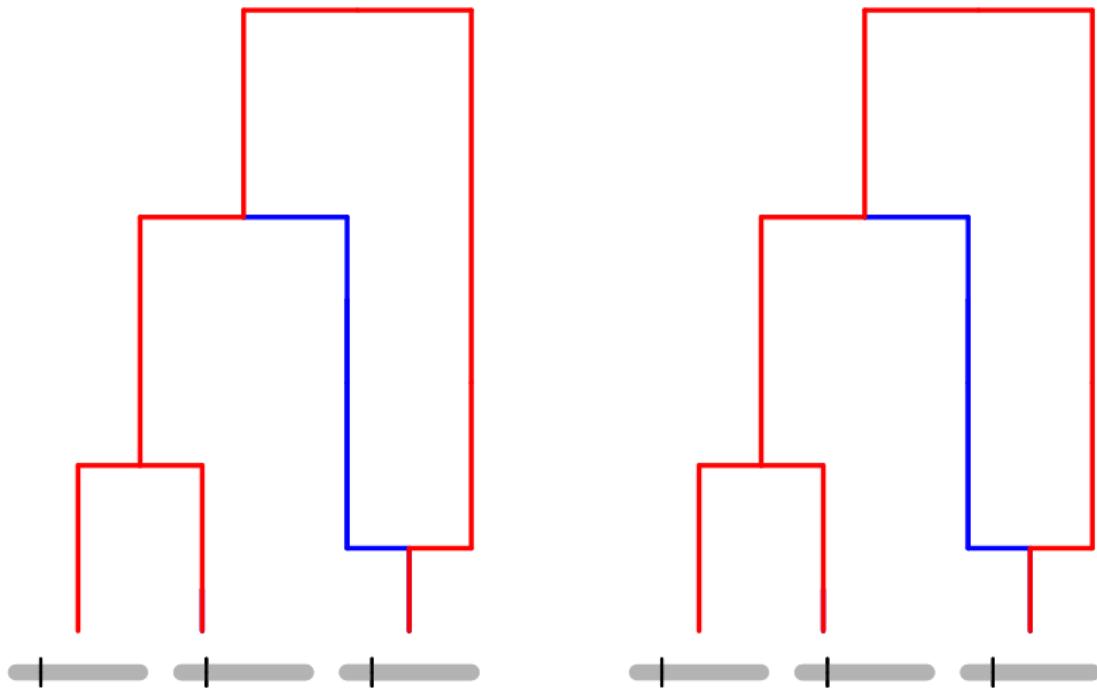
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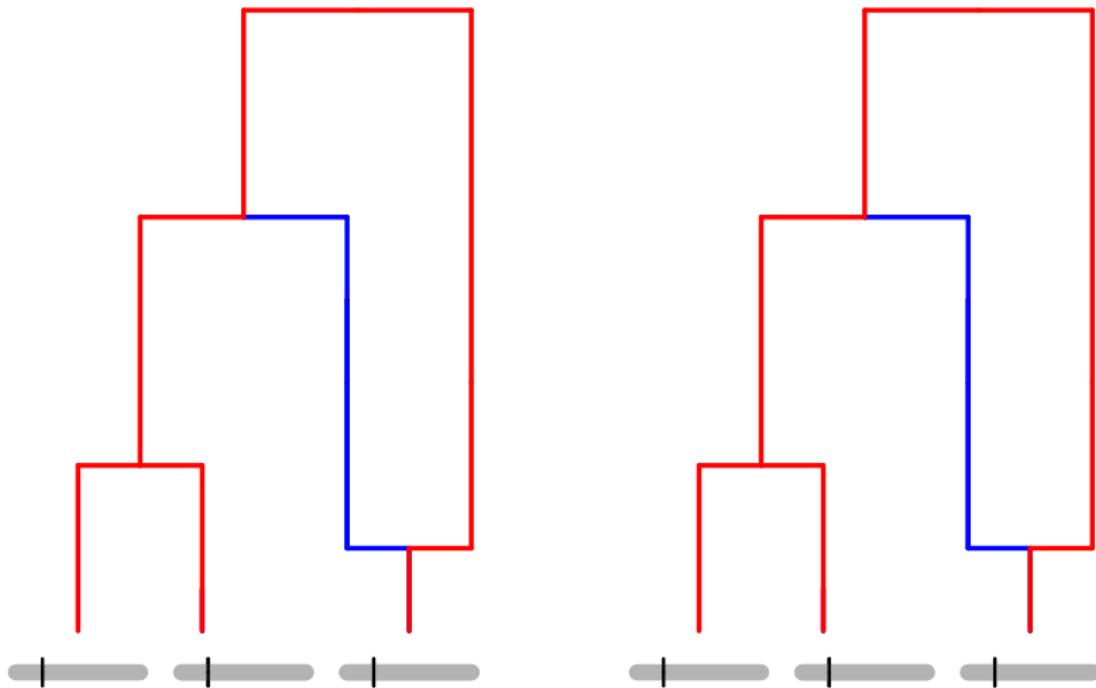
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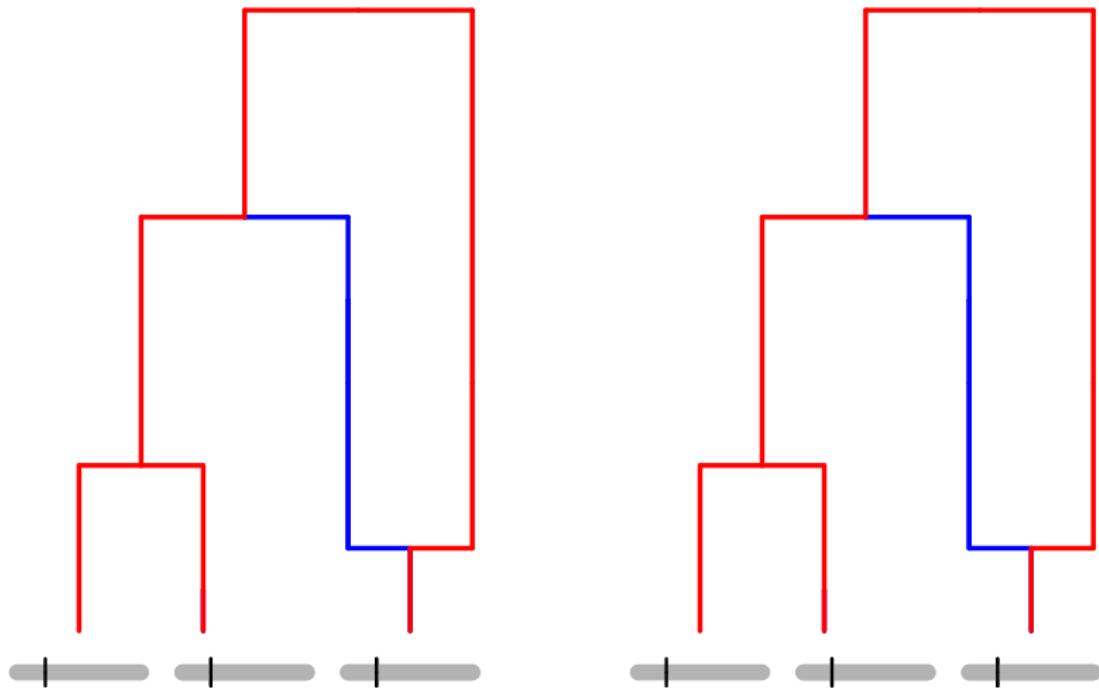
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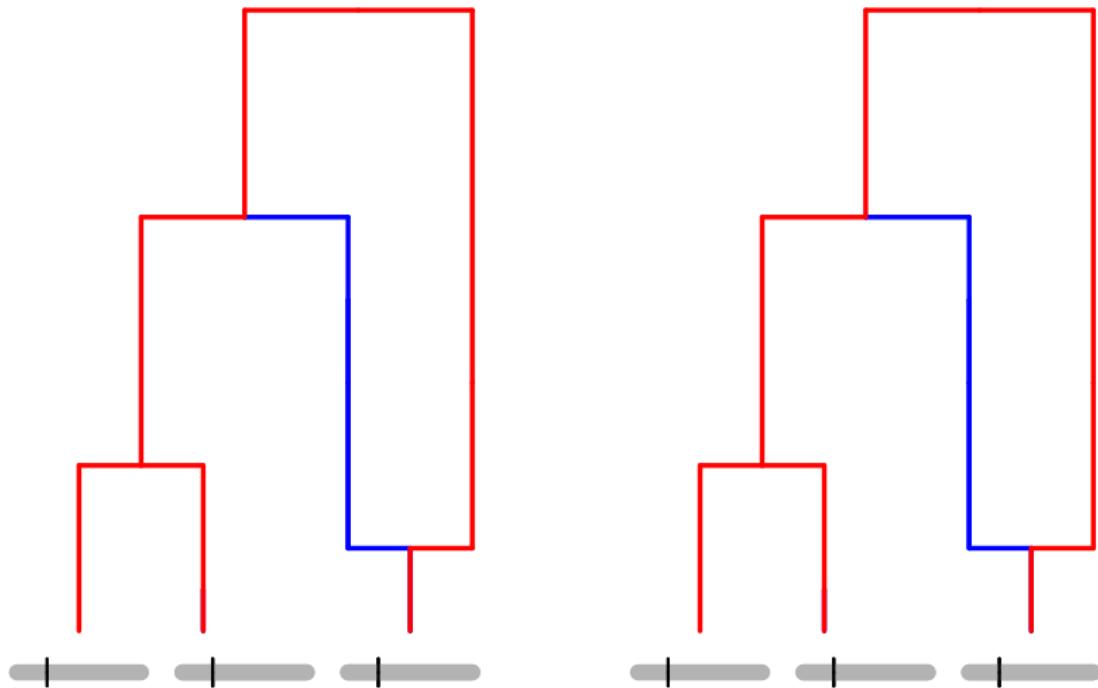
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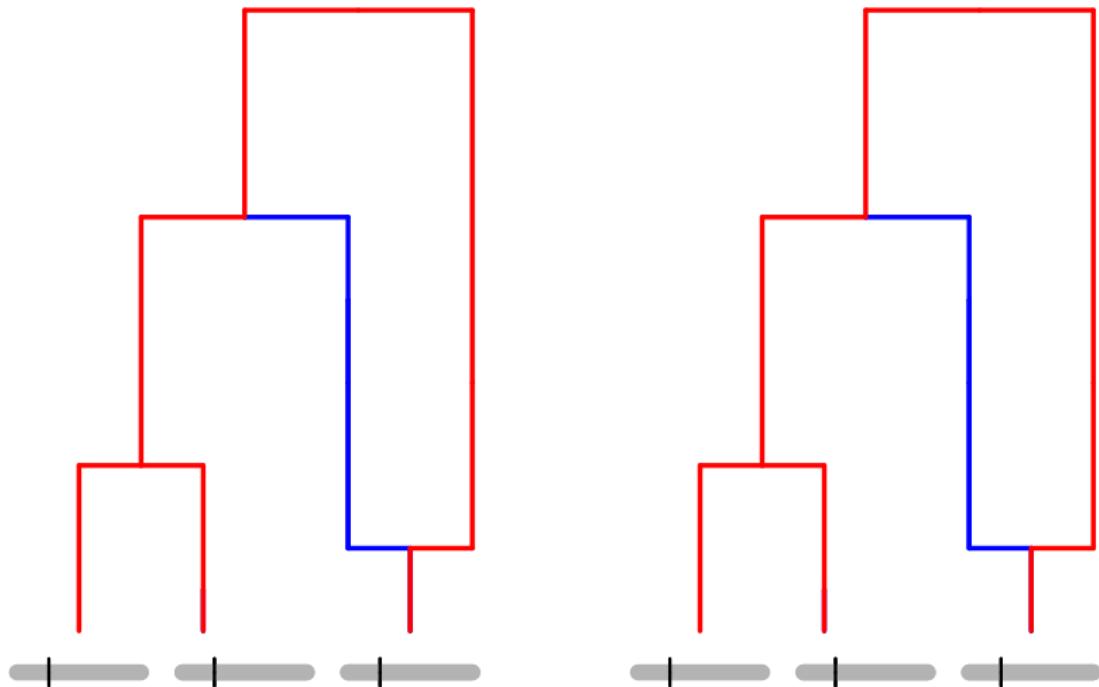
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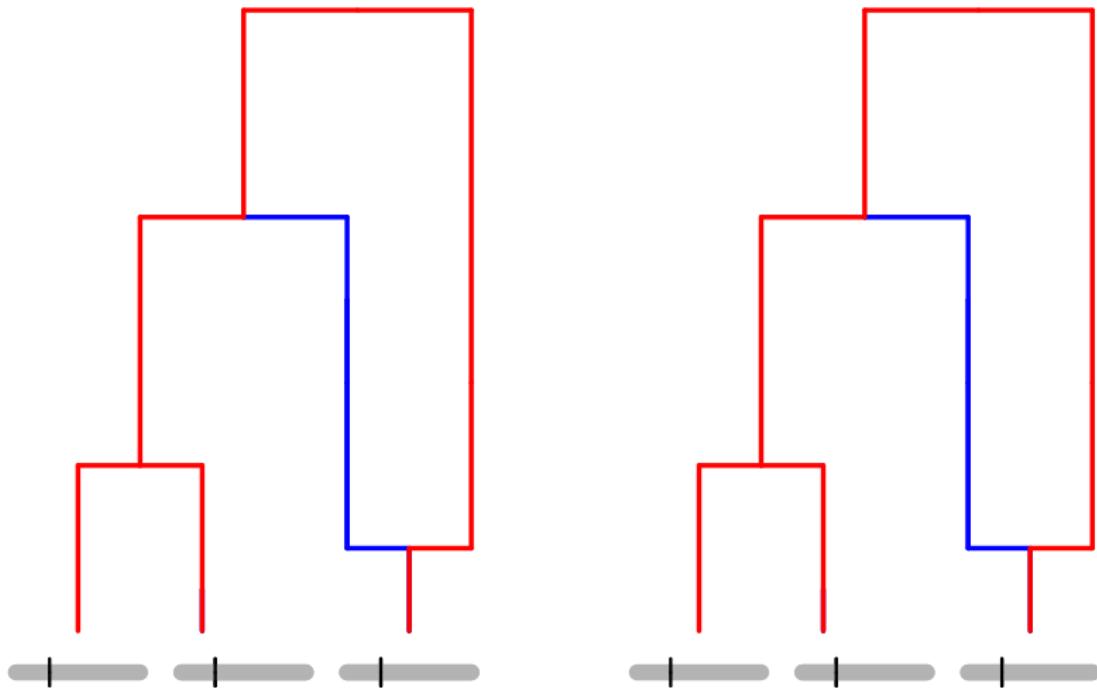
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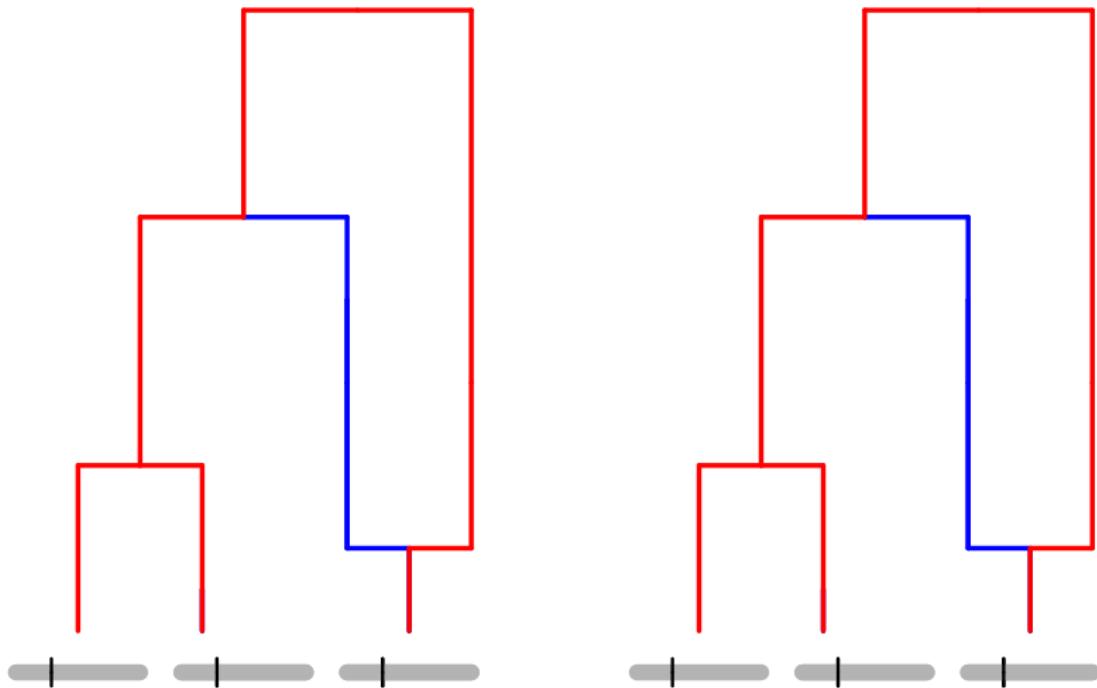
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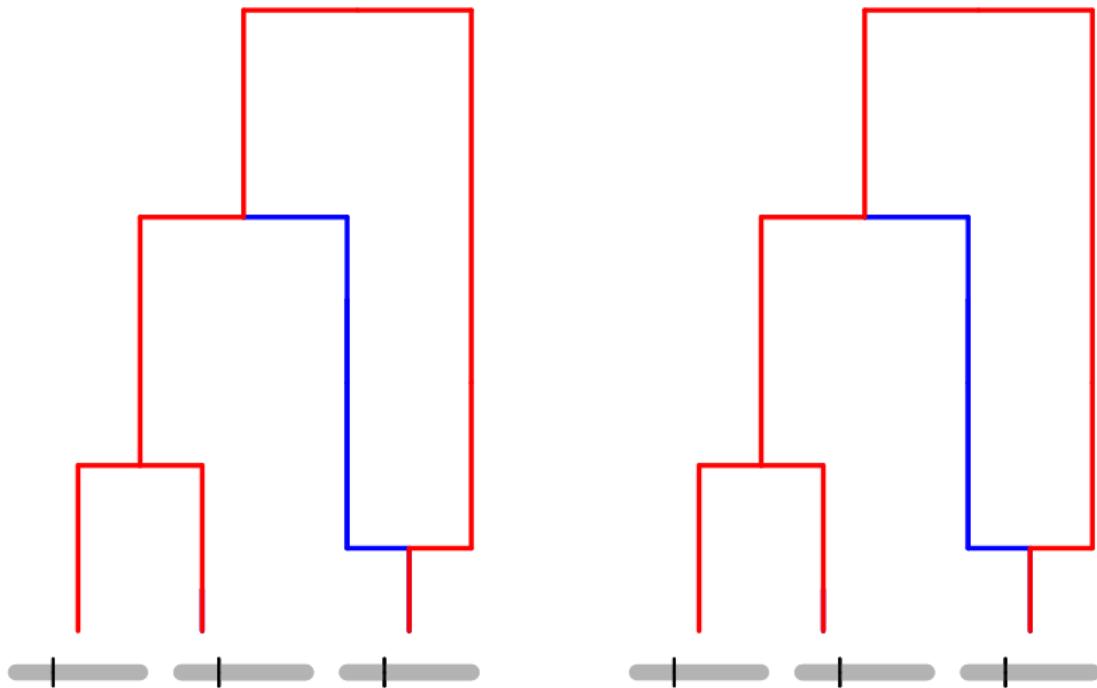
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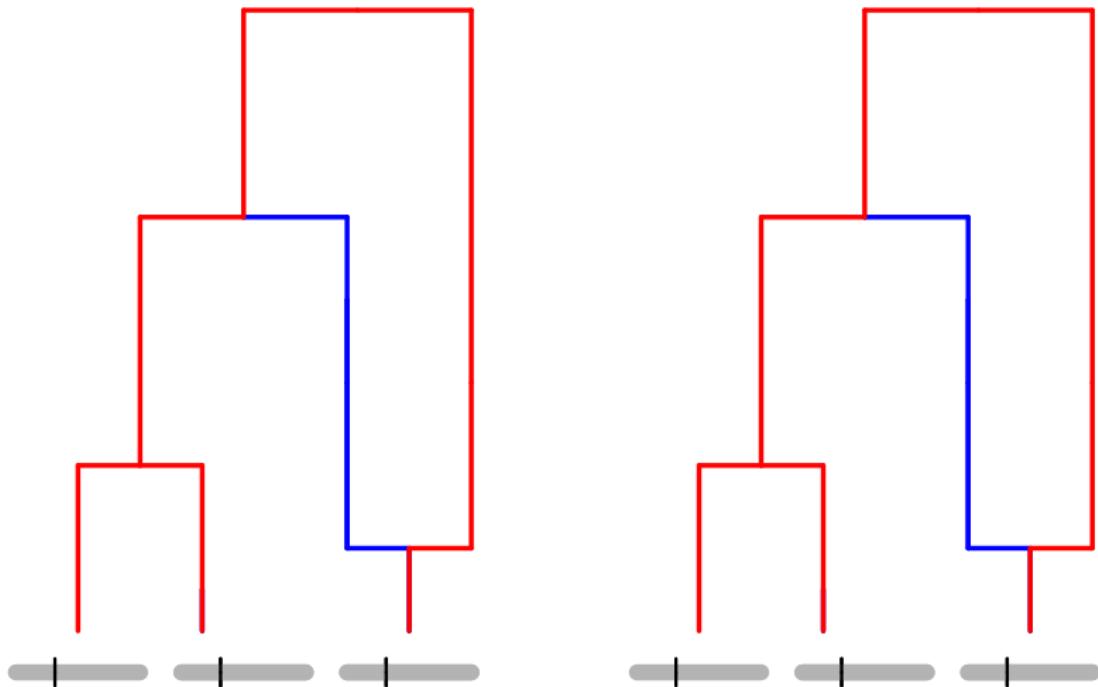
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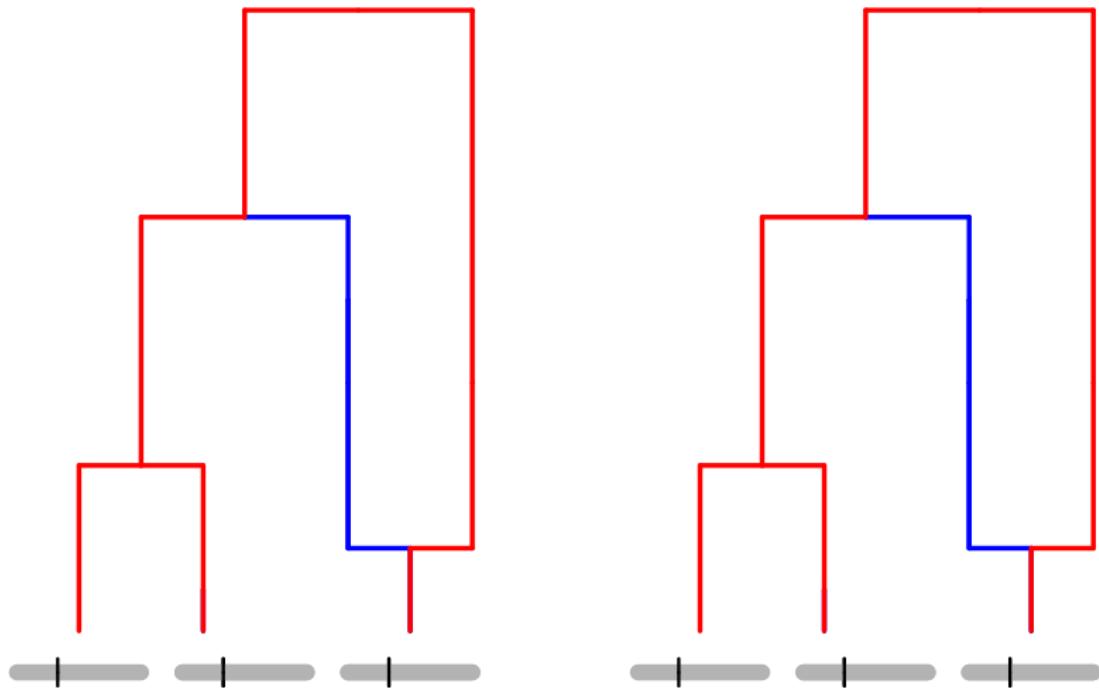
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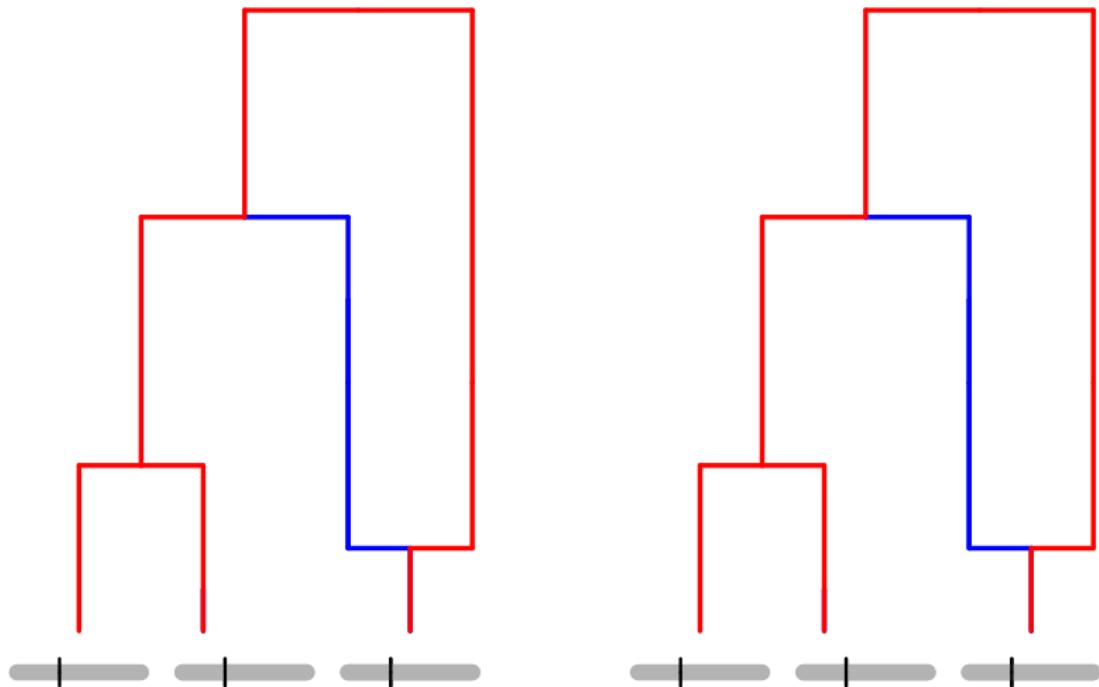
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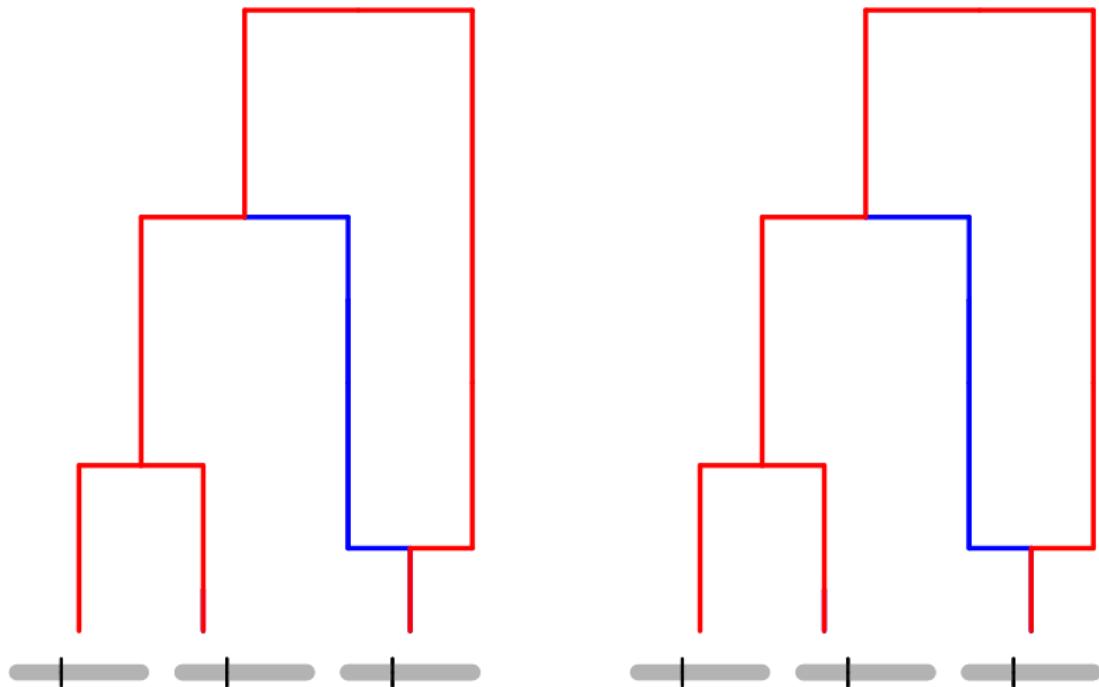
Simplification 1: postpone nonancestral recombinations



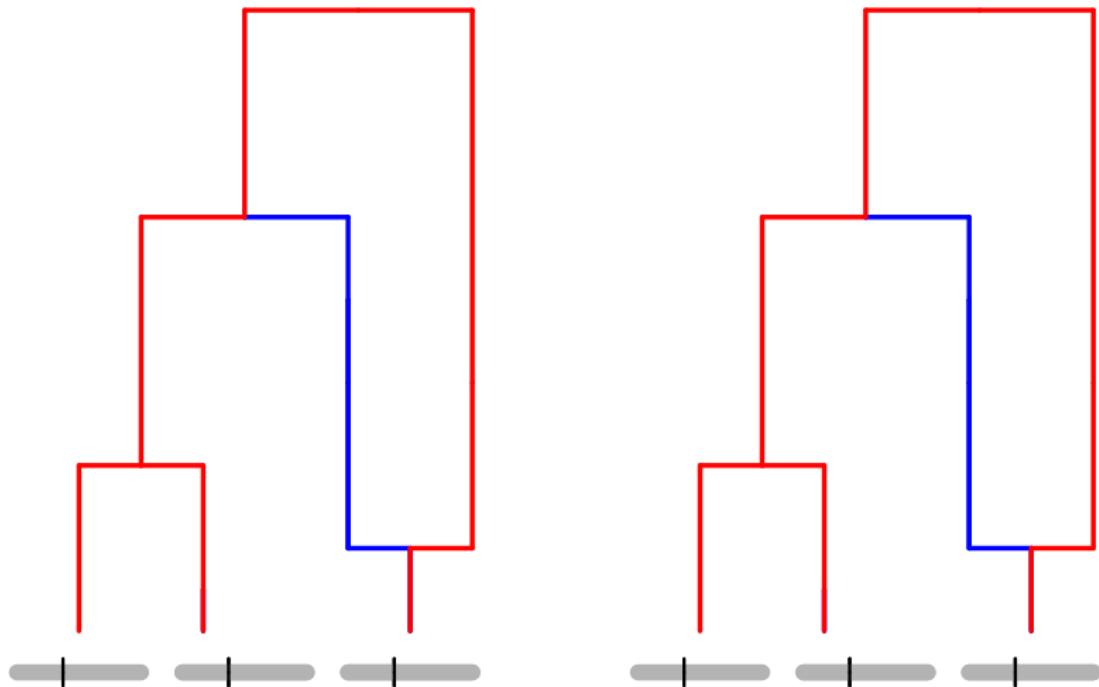
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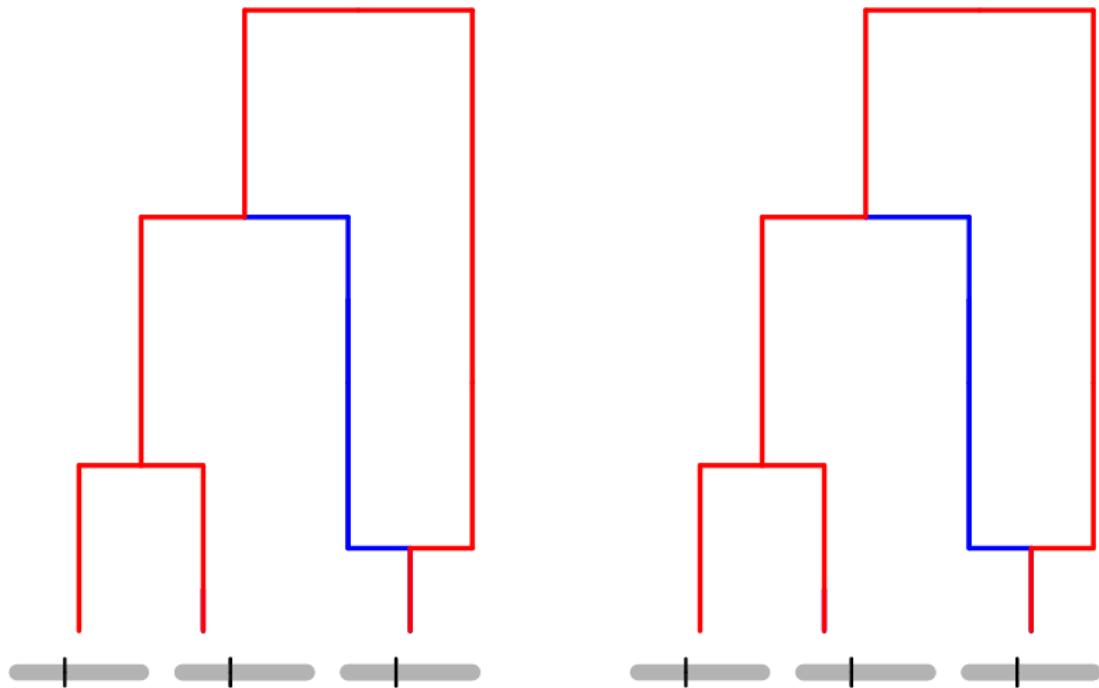
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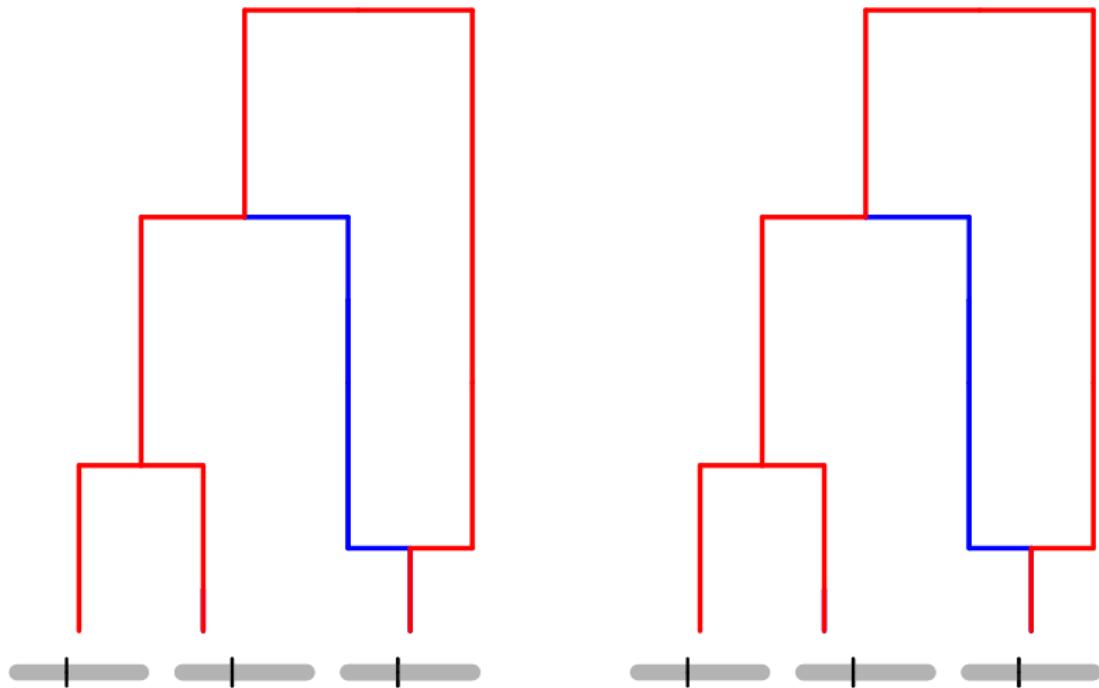
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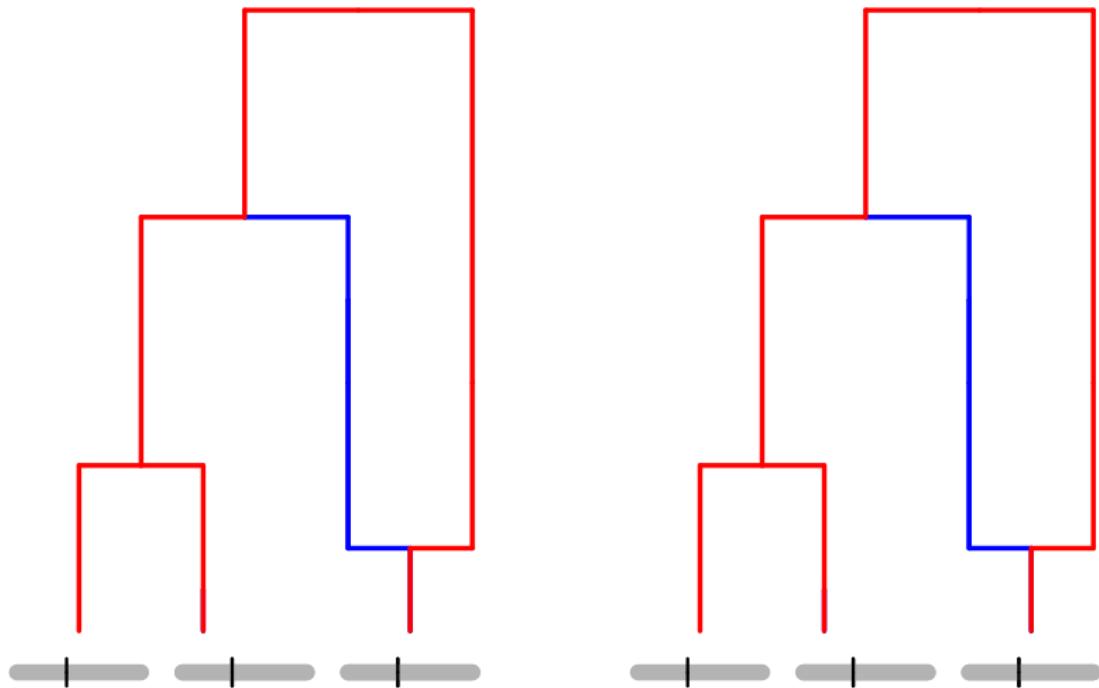
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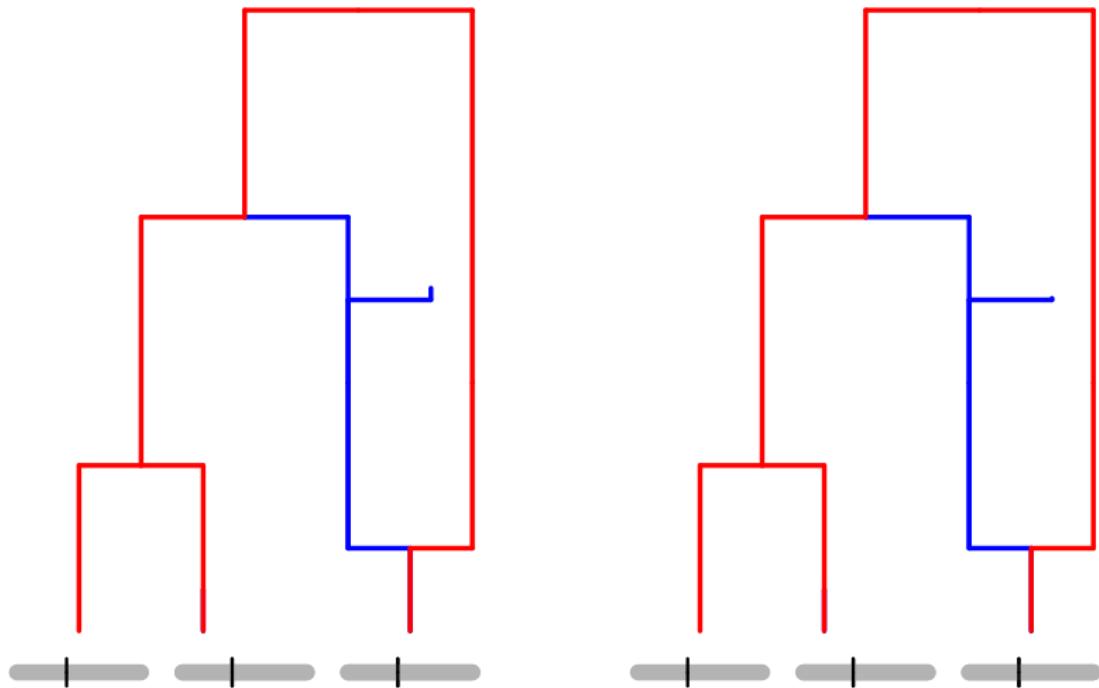
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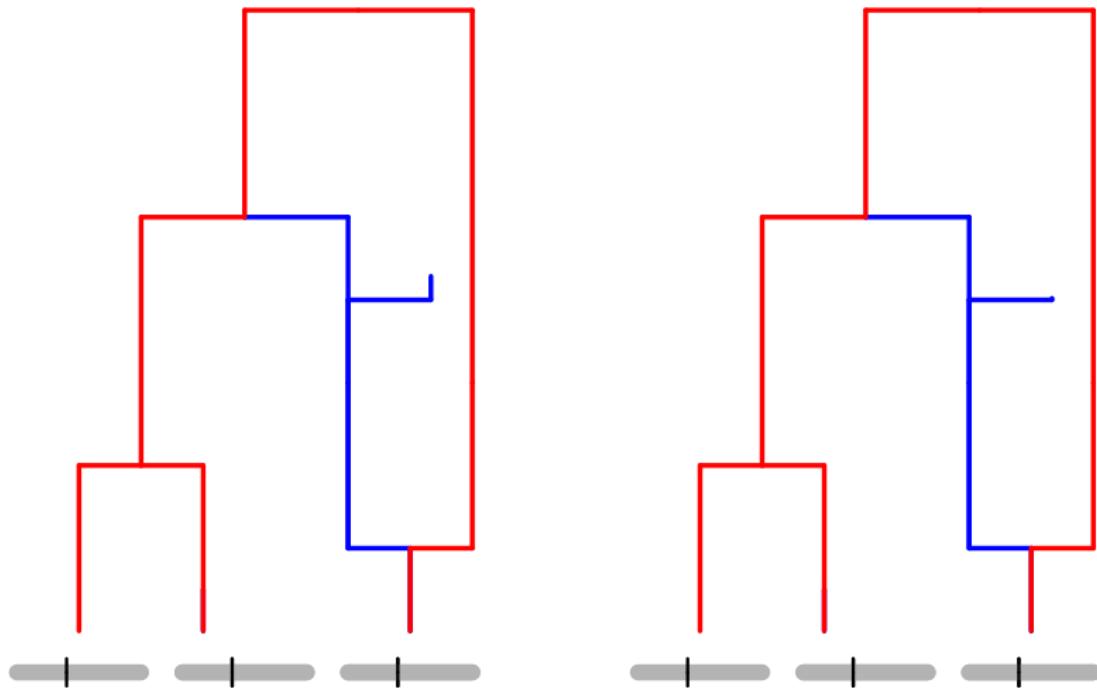
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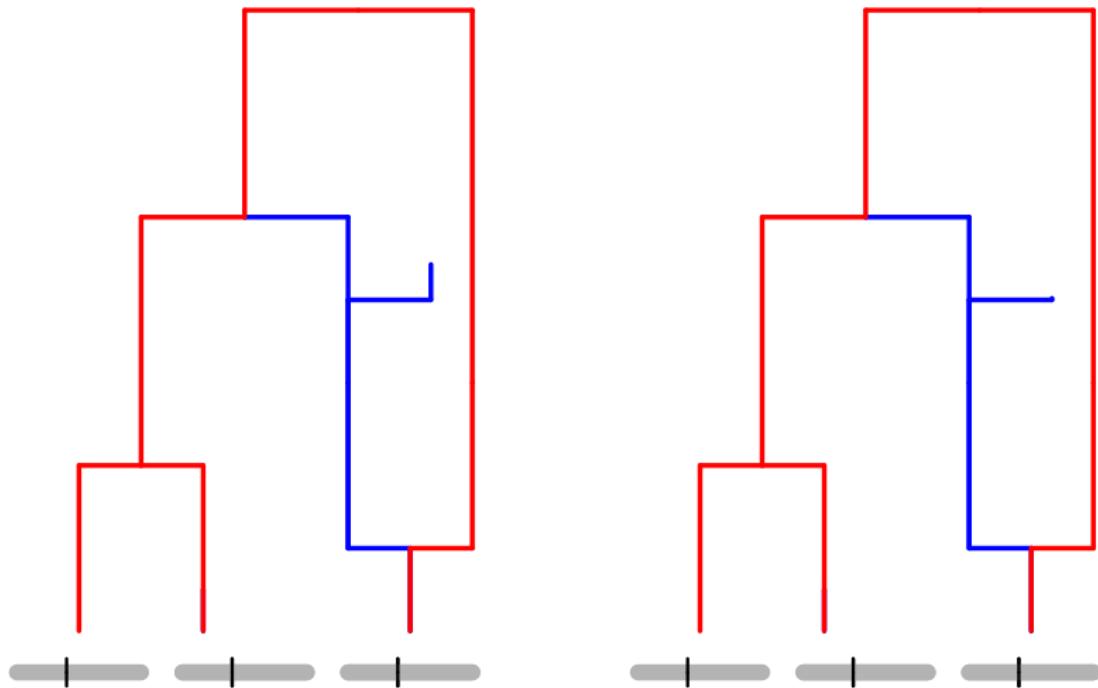
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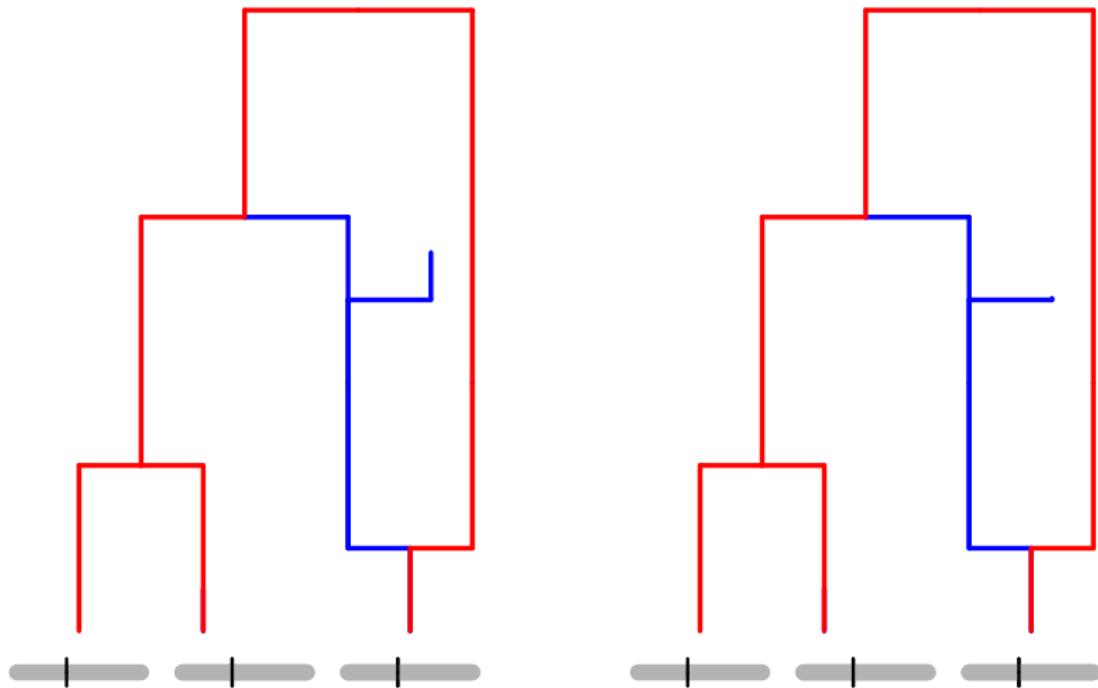
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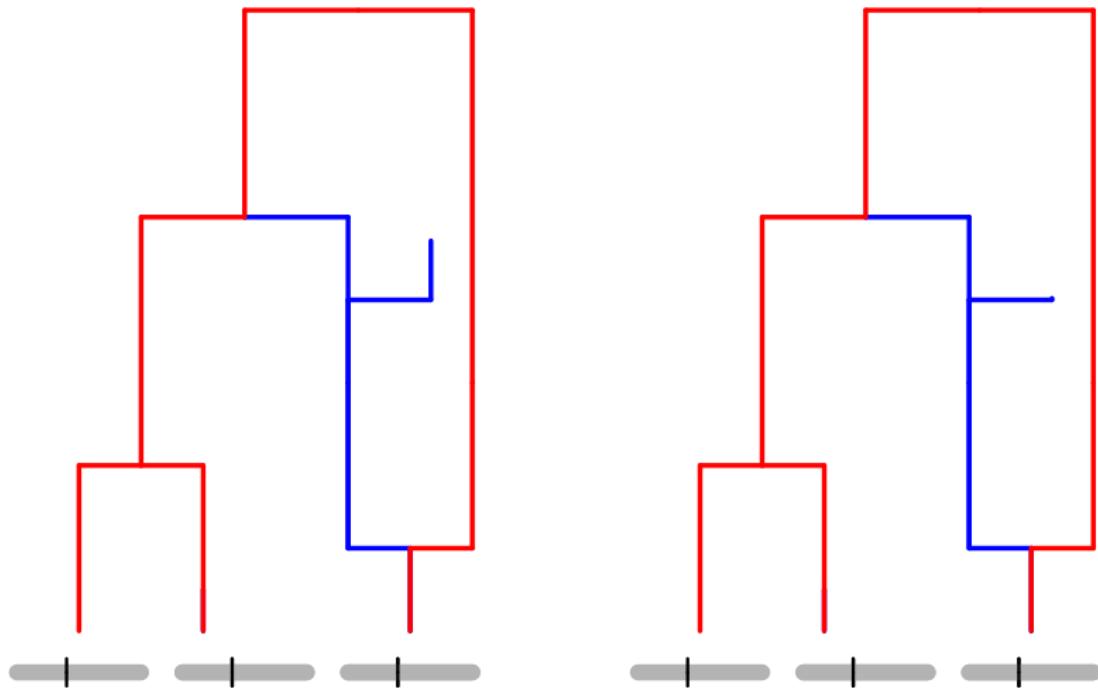
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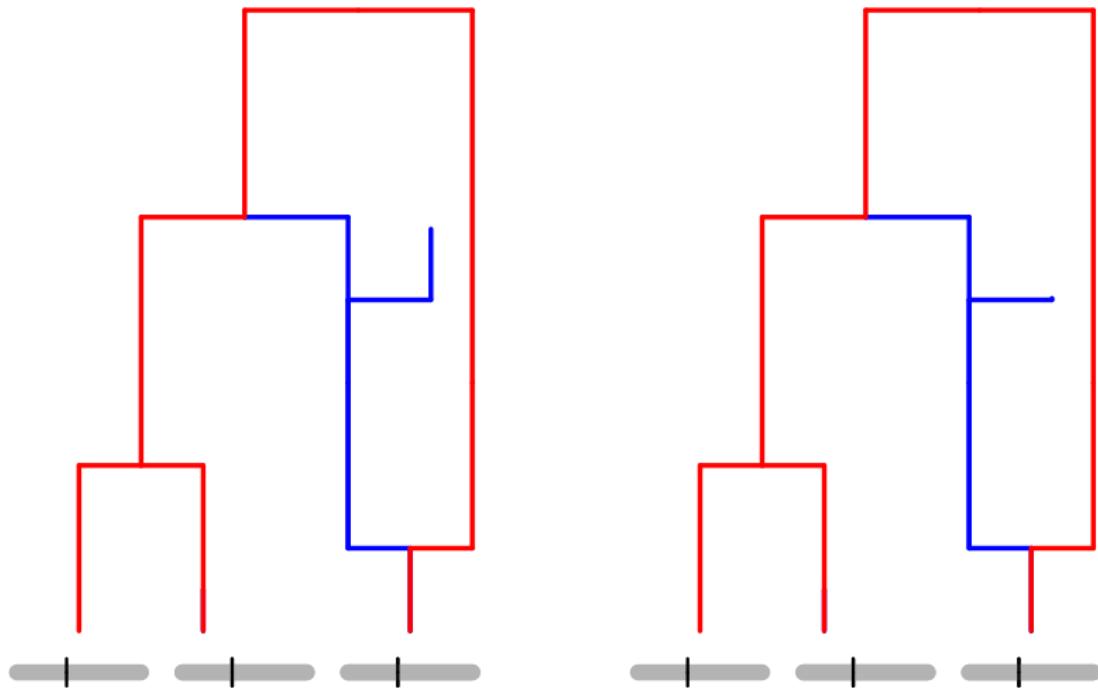
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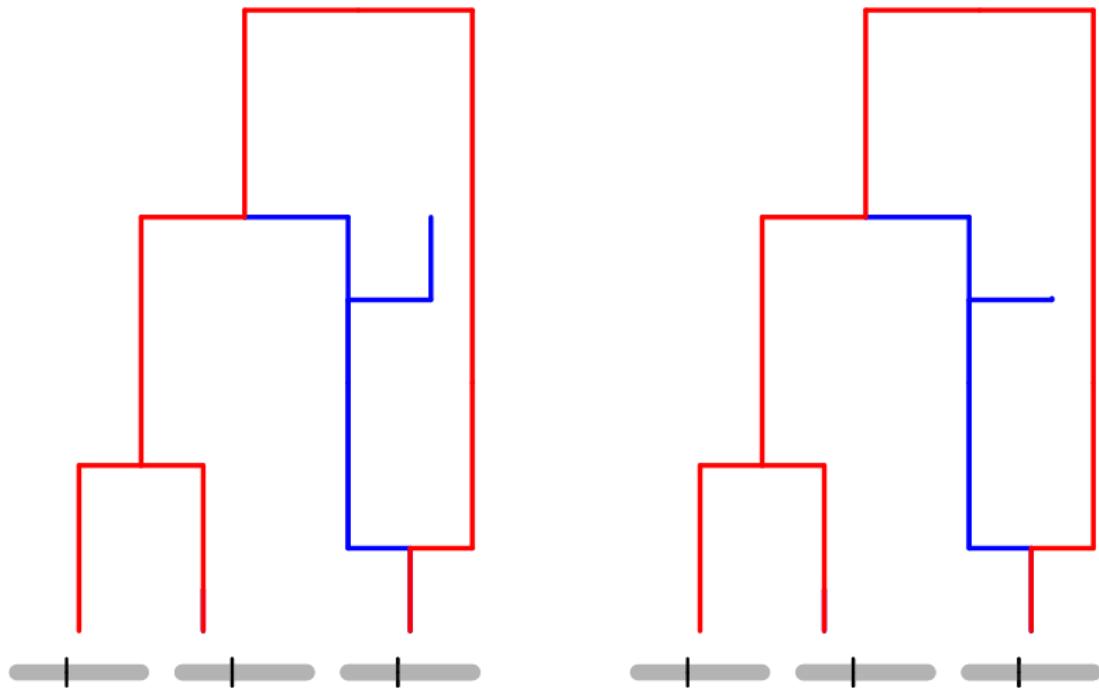
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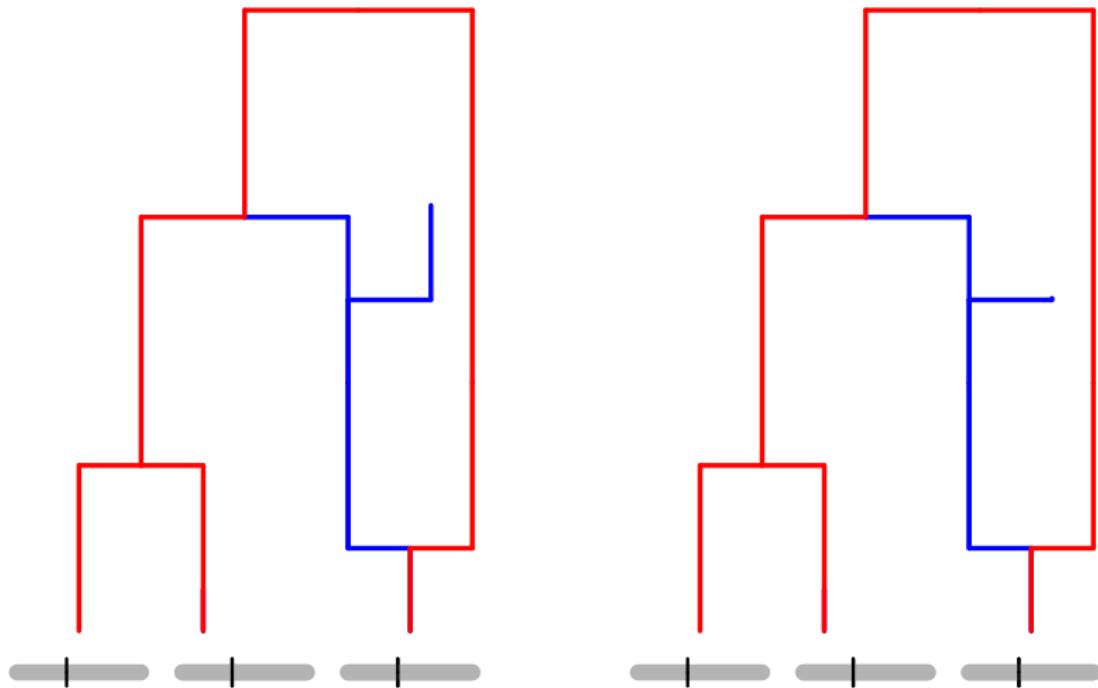
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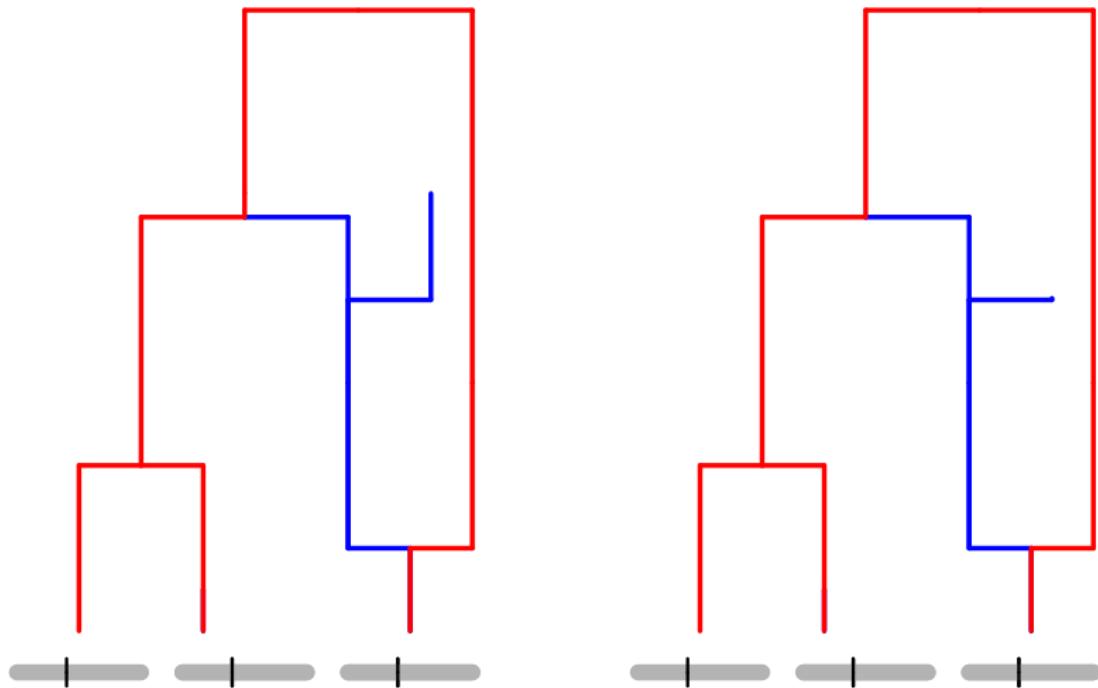
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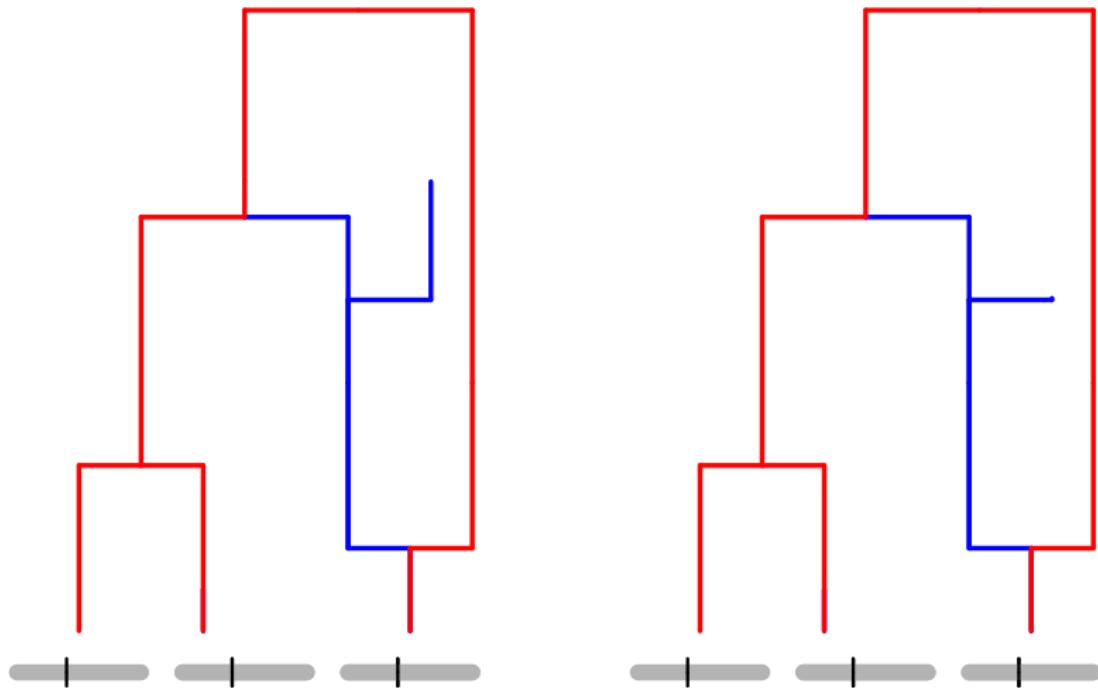
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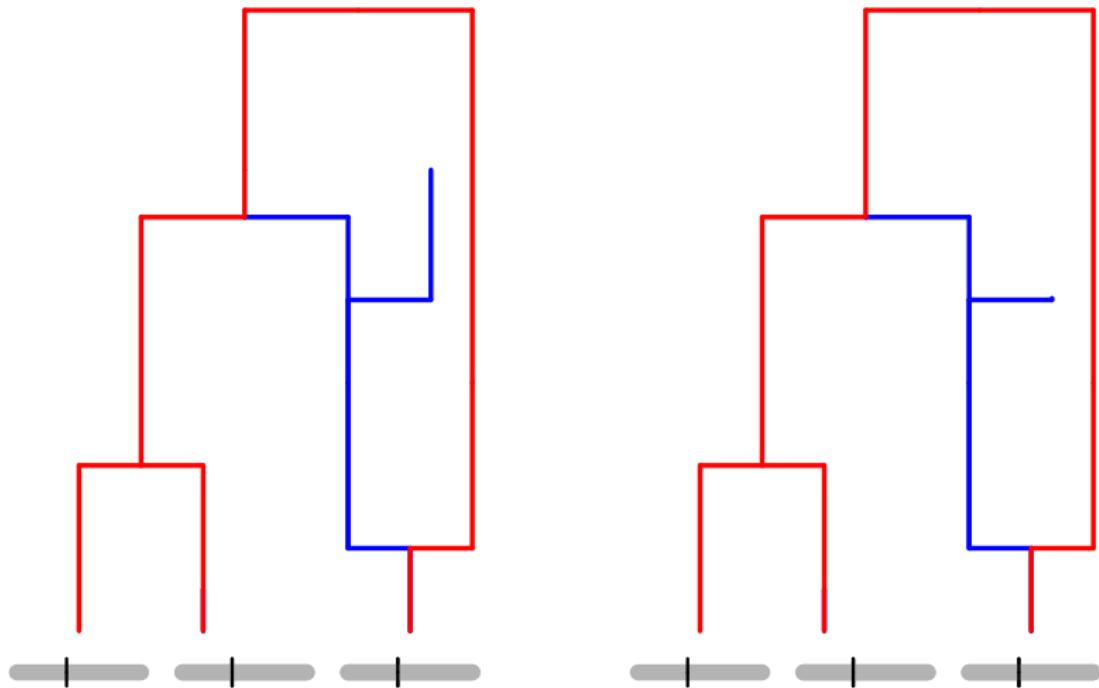
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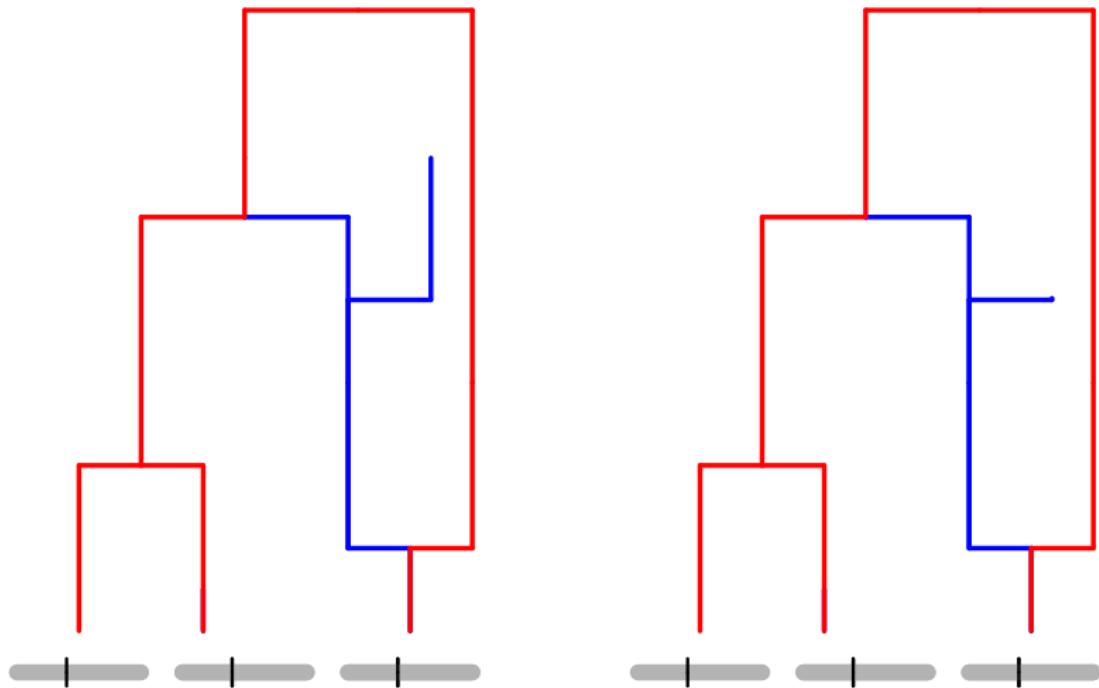
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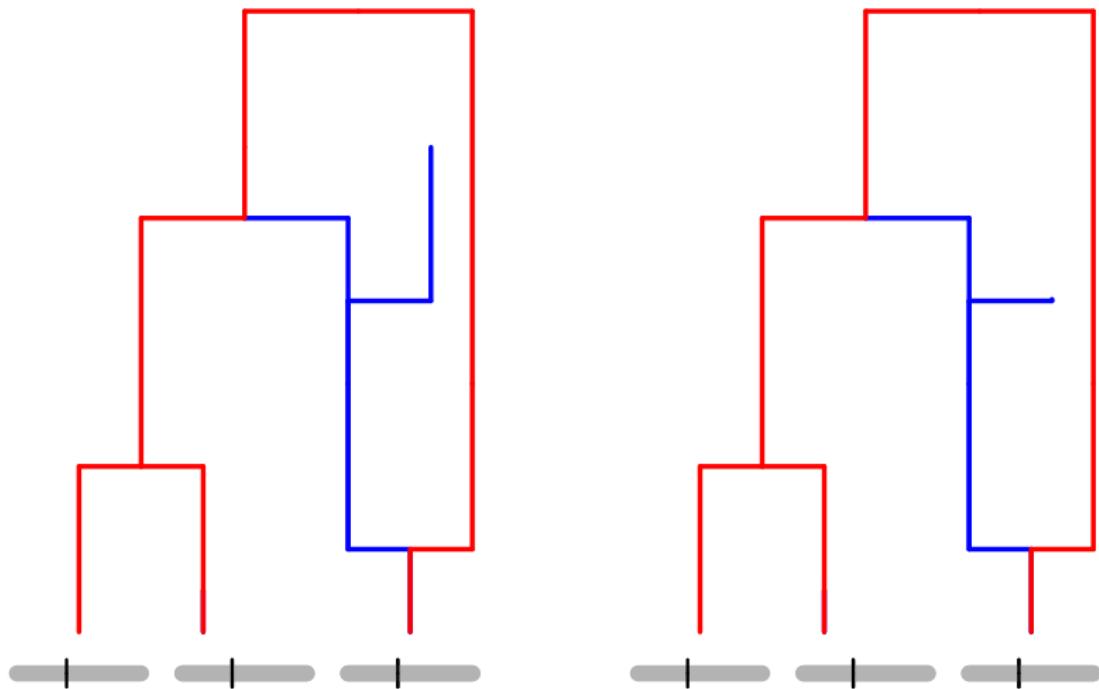
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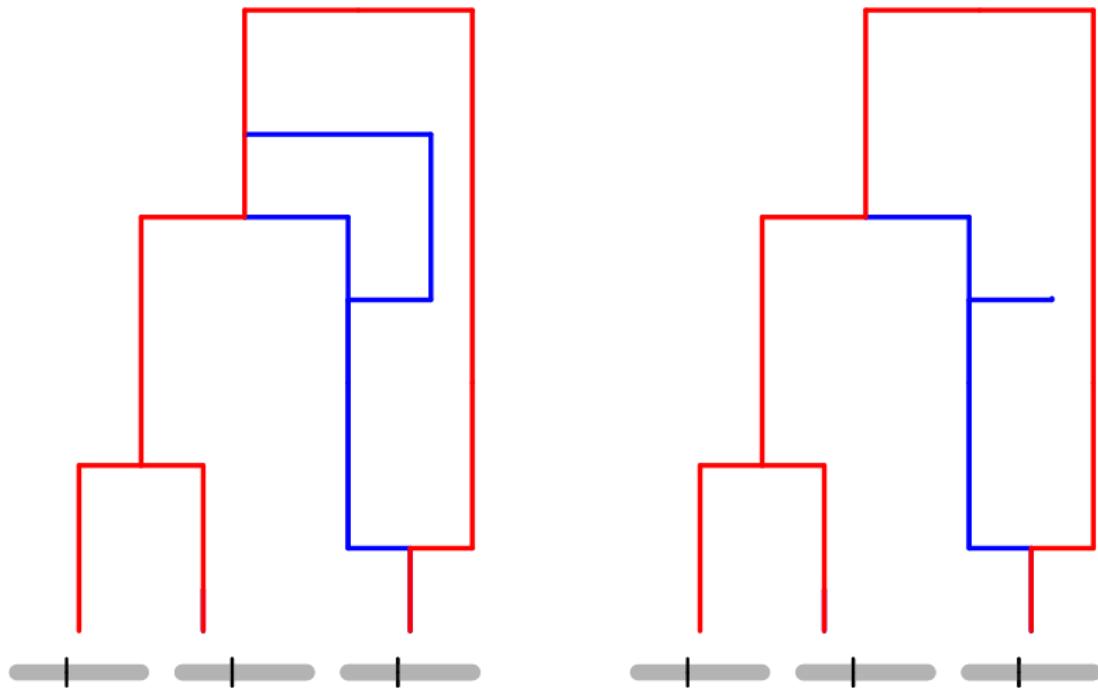
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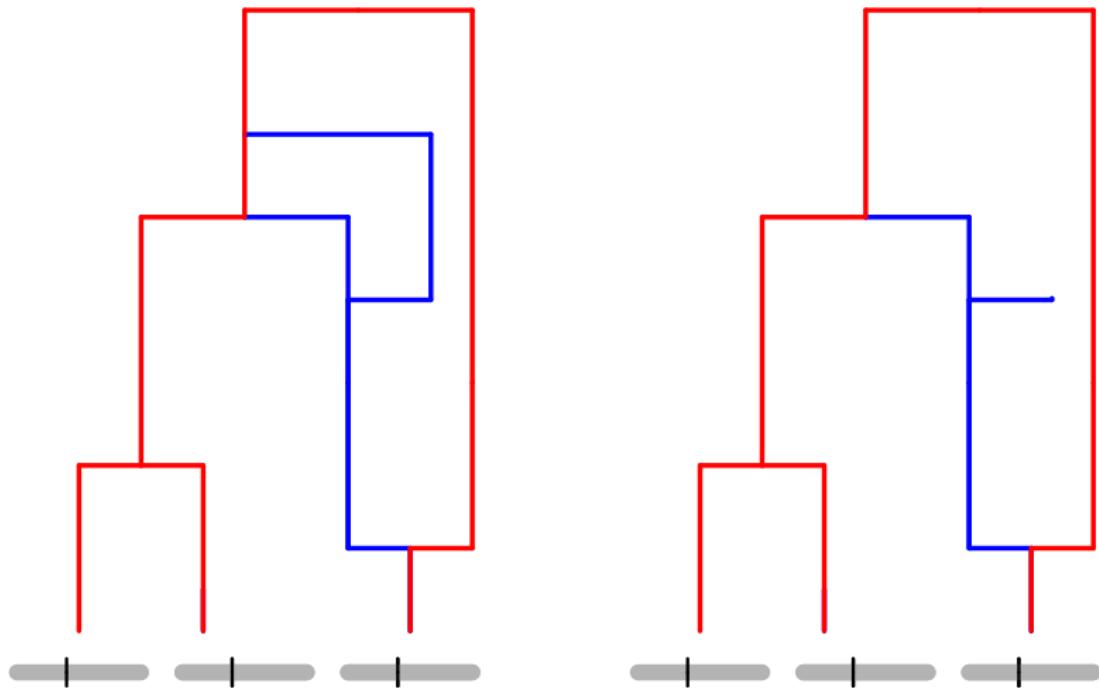
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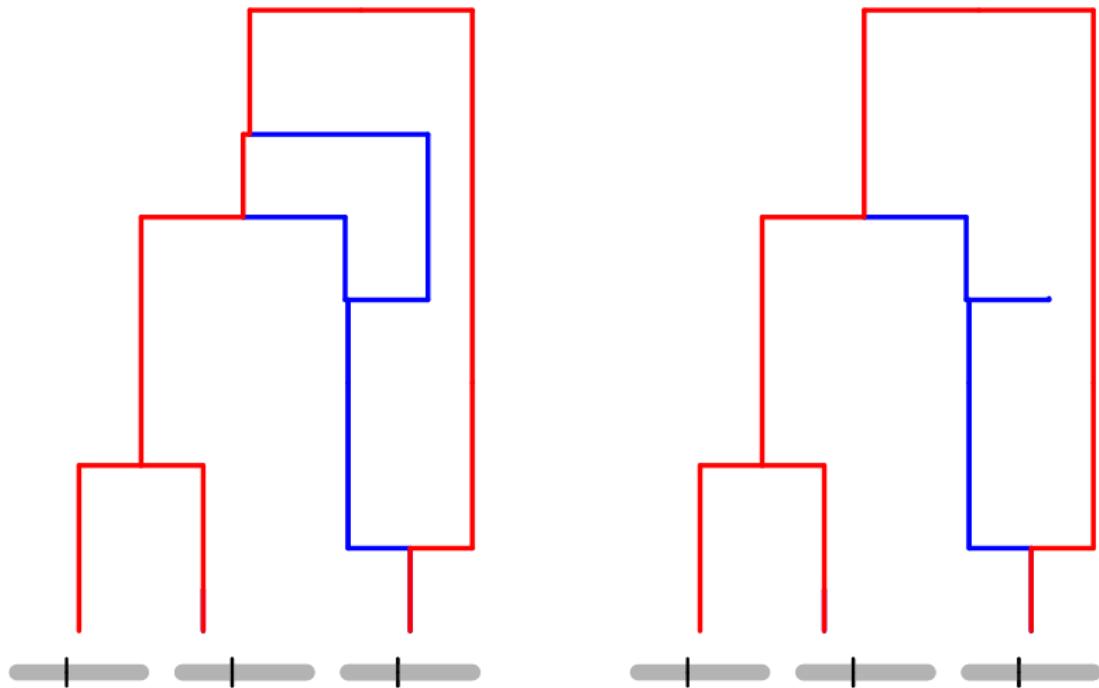
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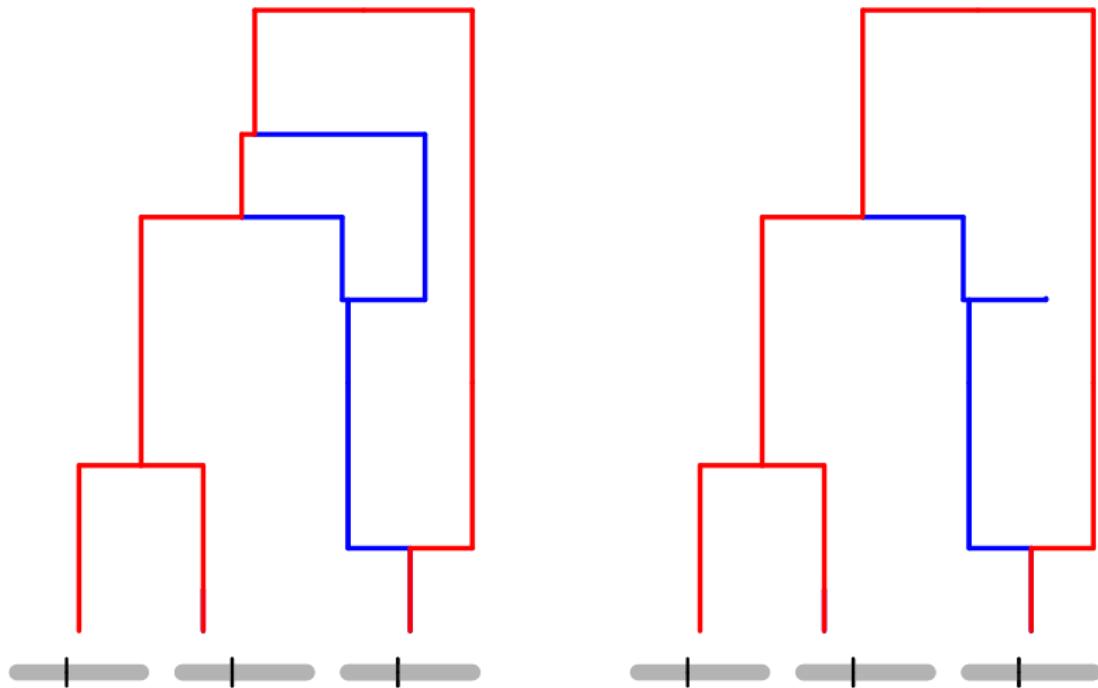
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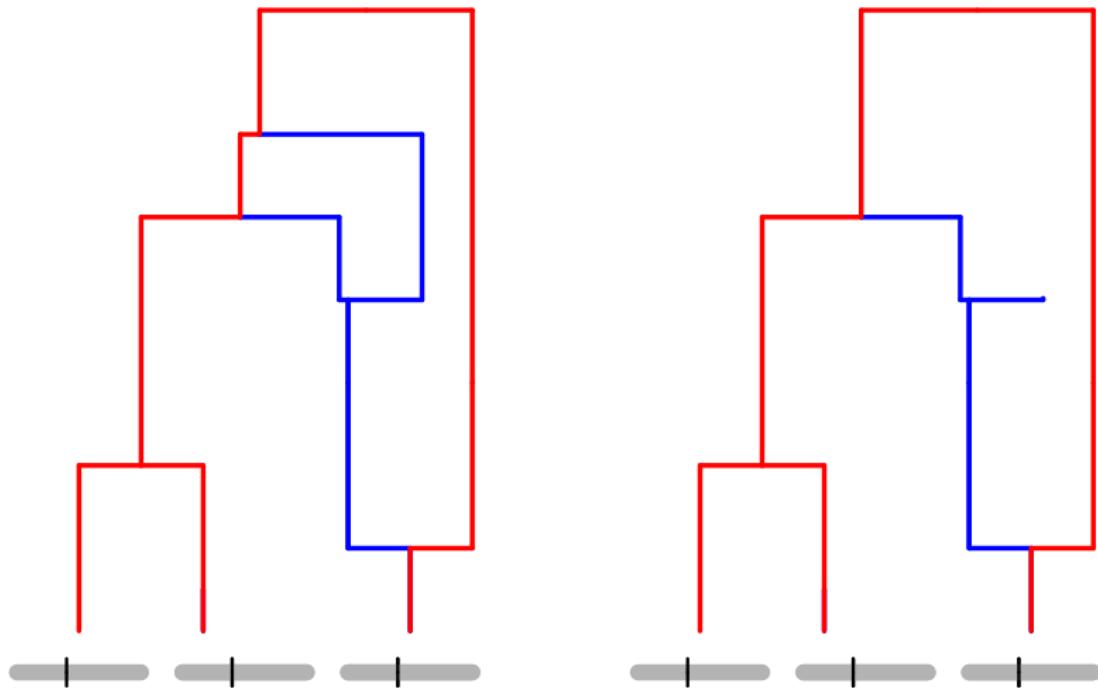
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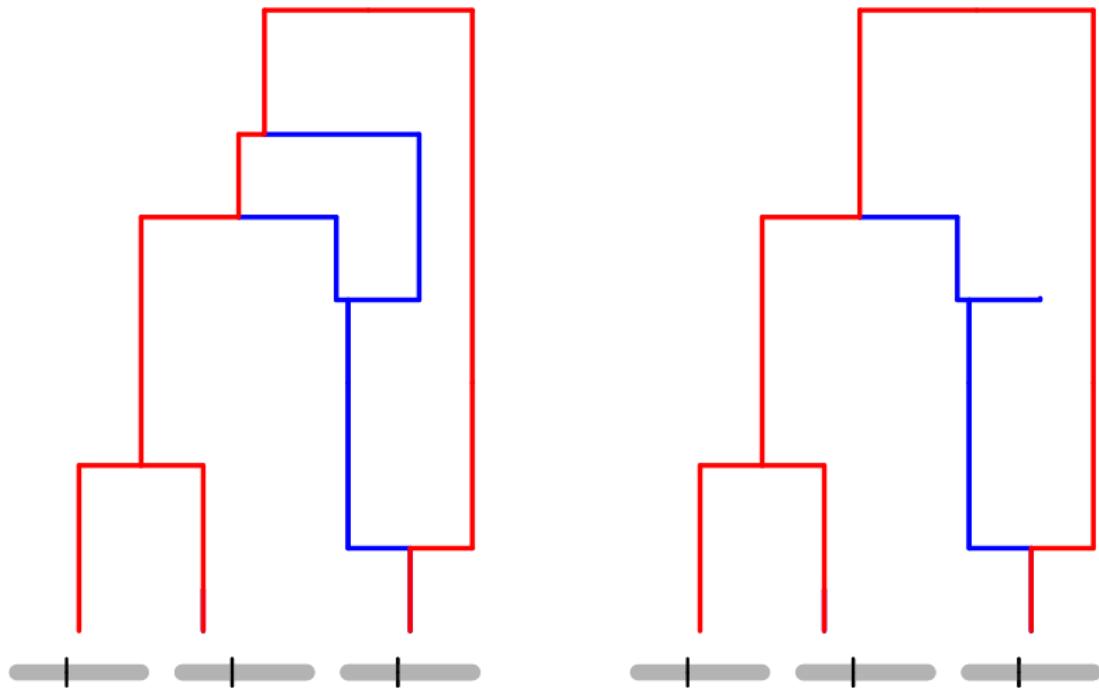
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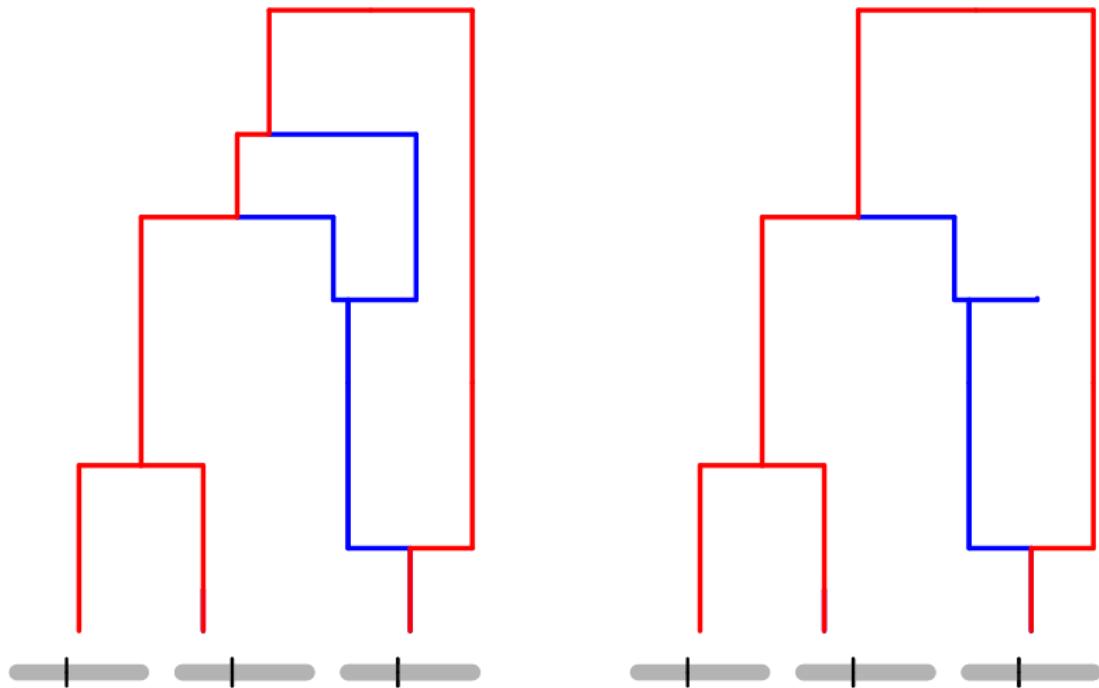
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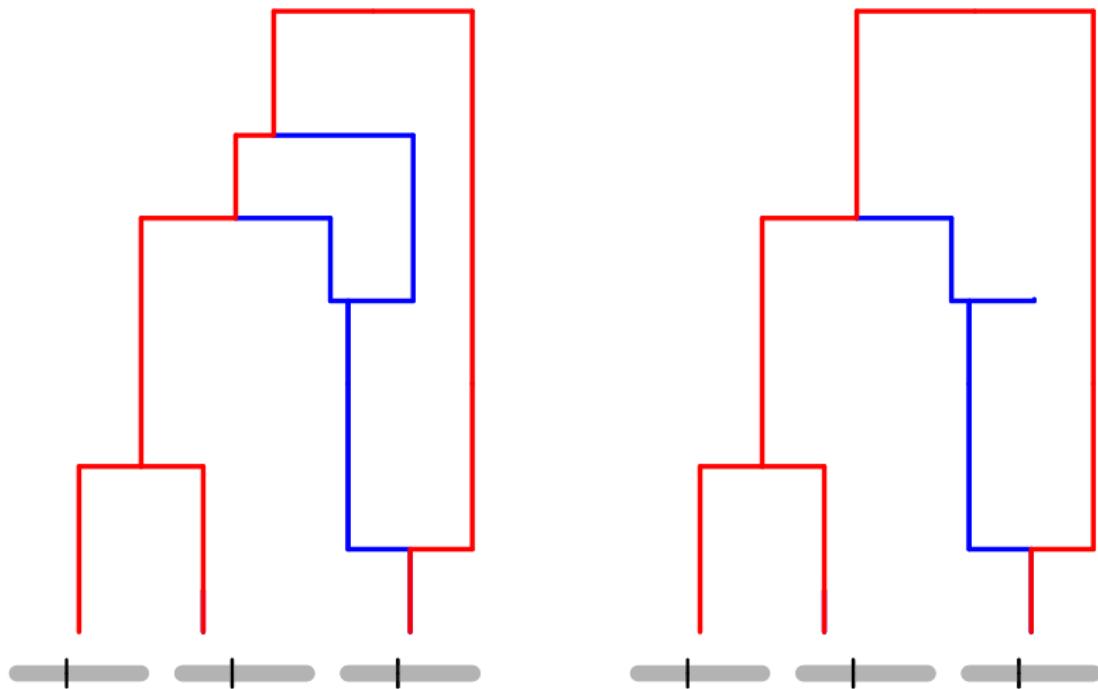
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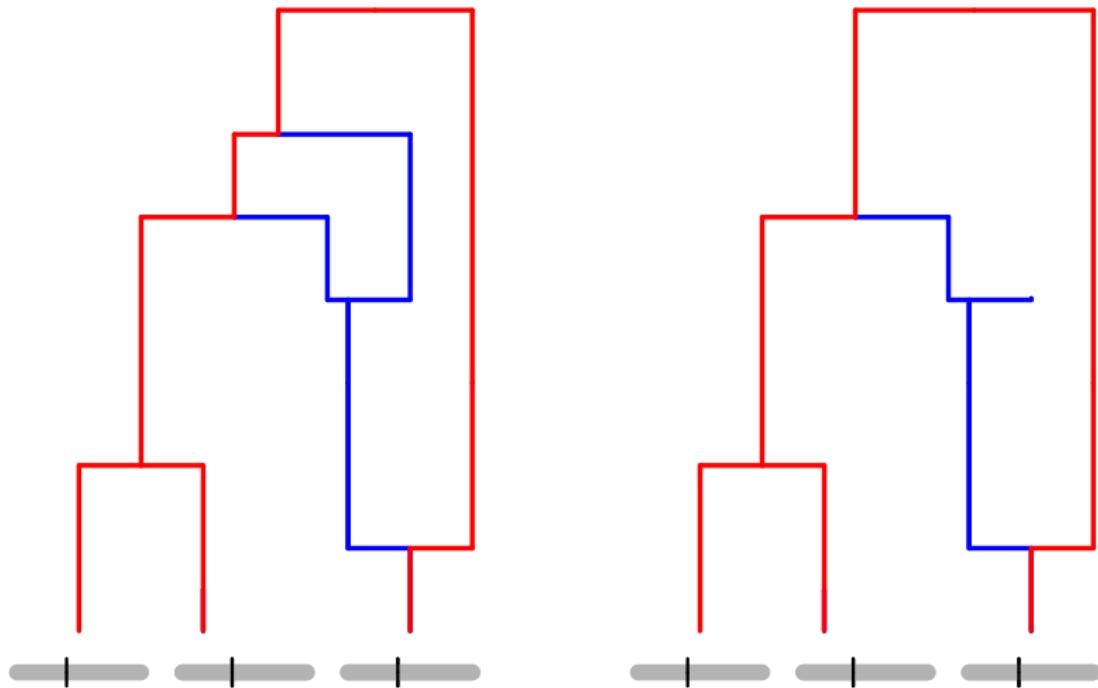
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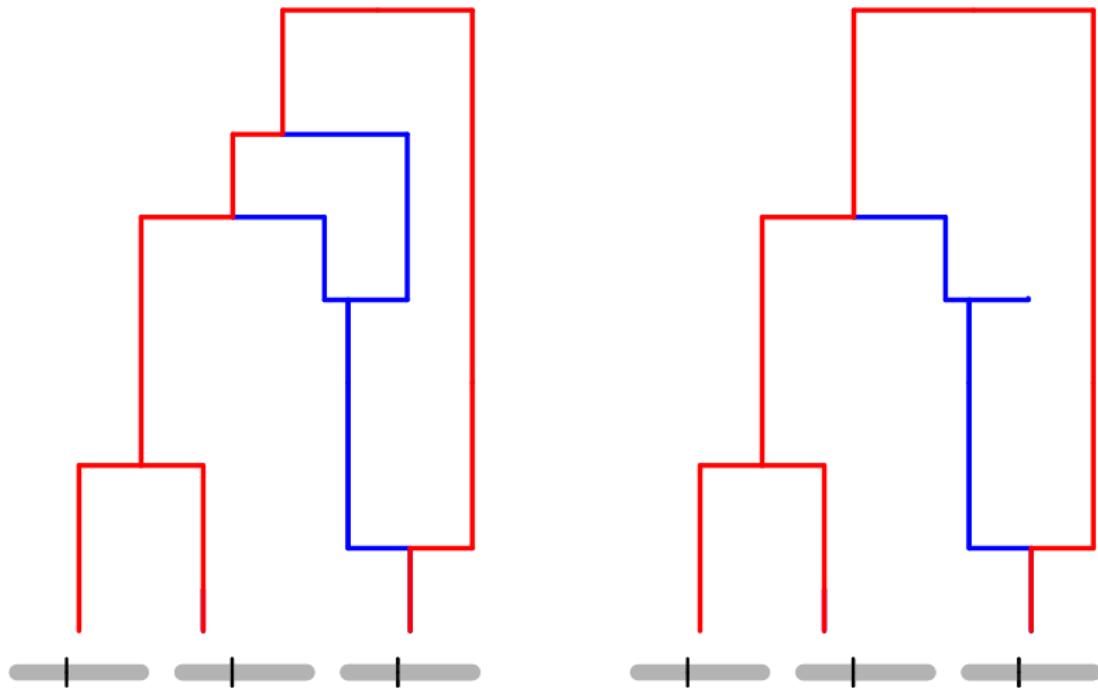
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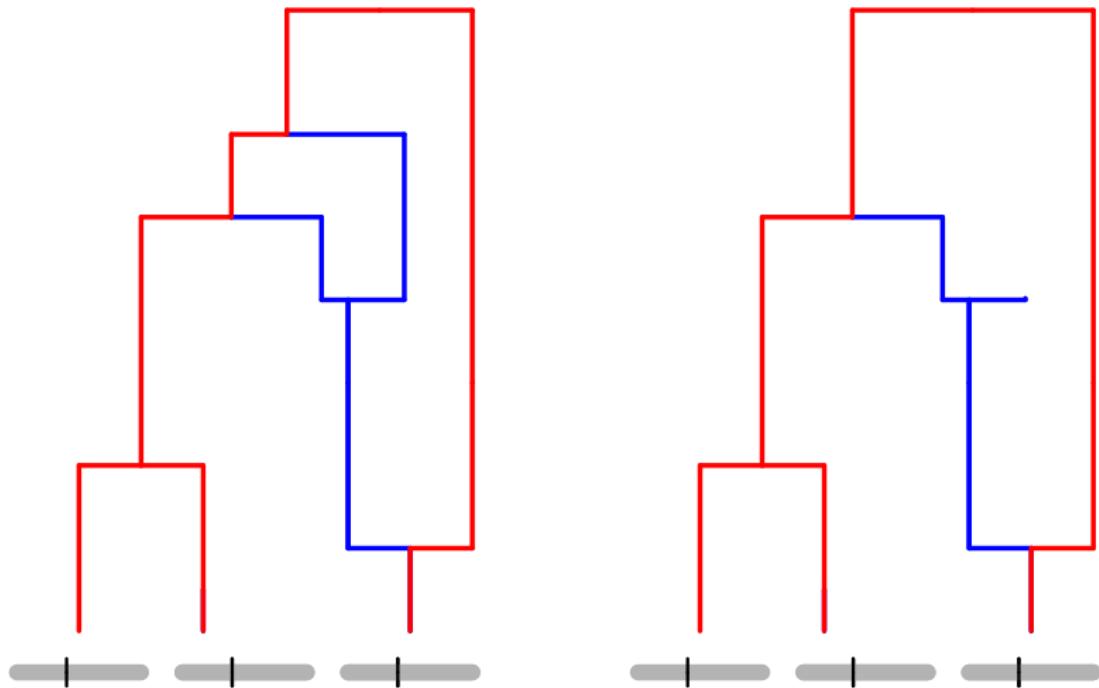
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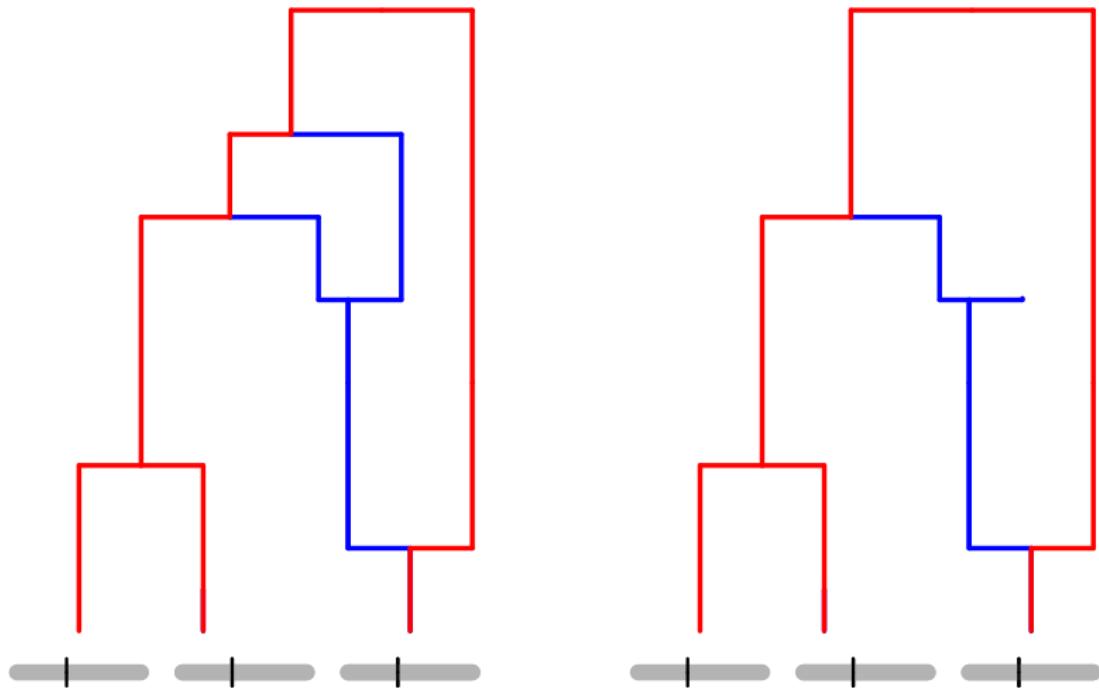
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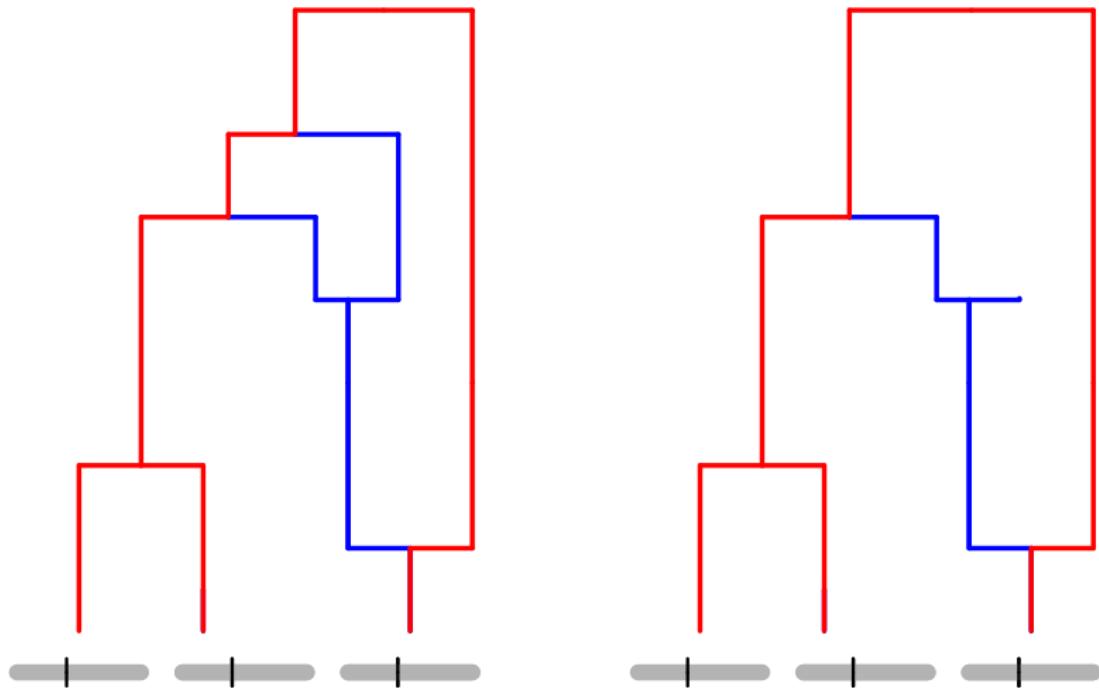
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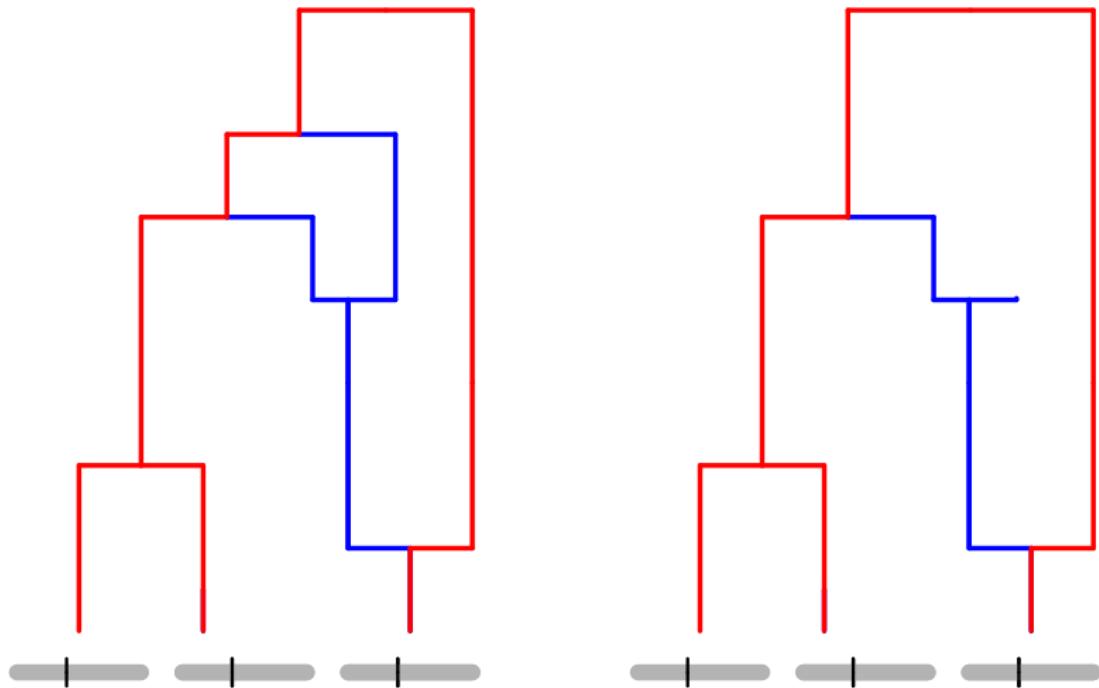
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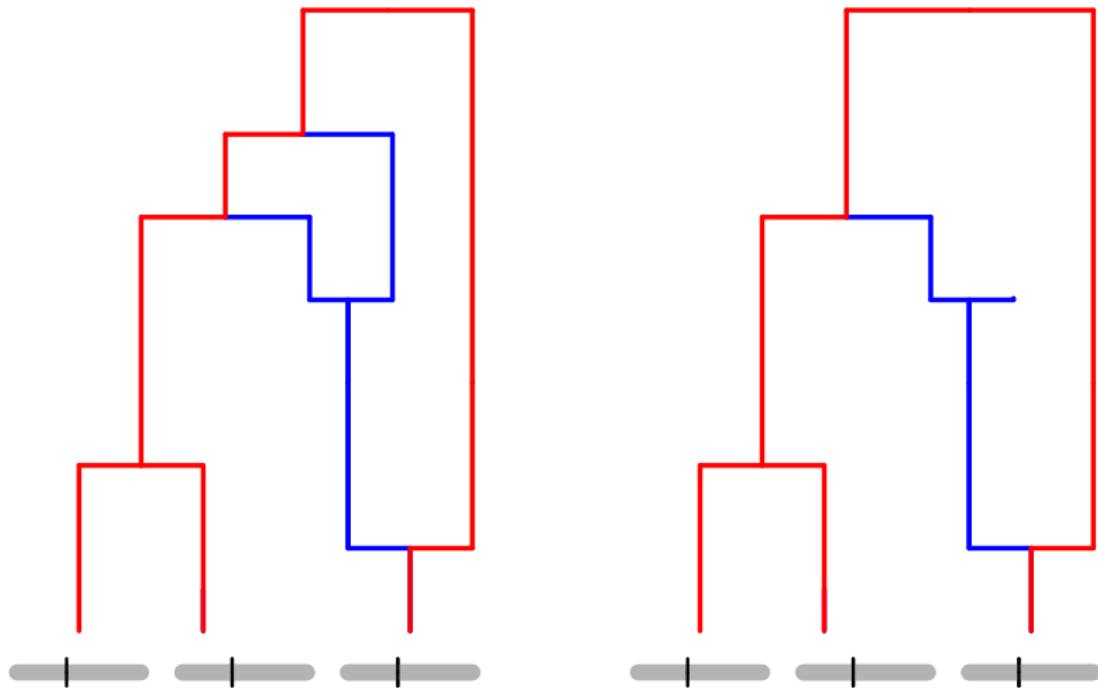
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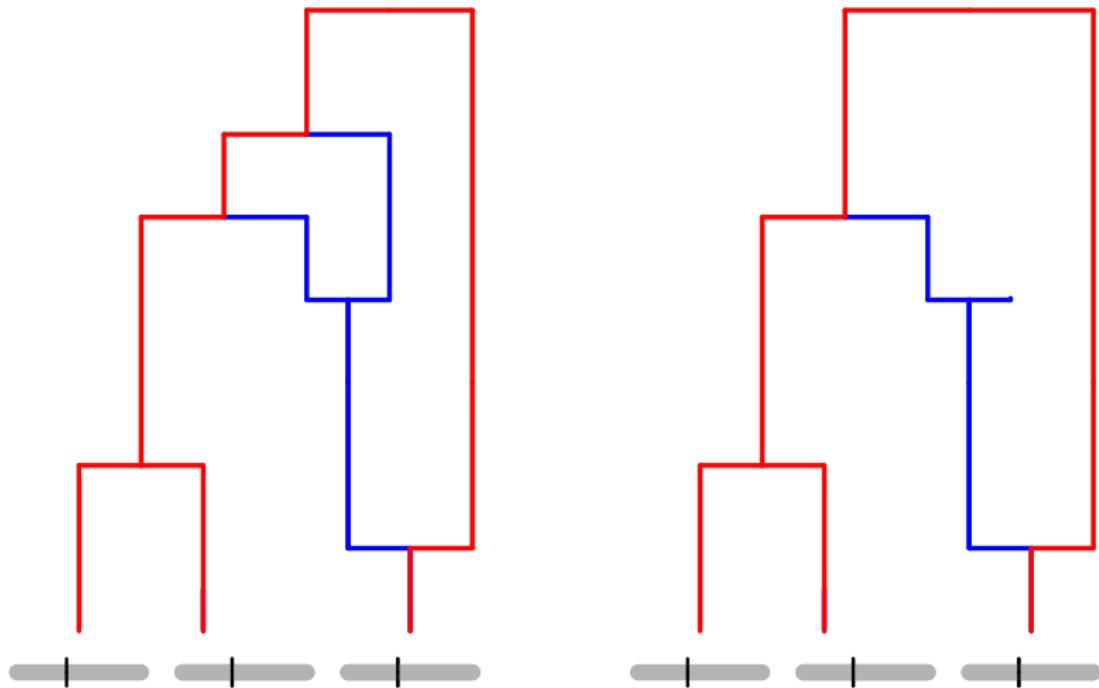
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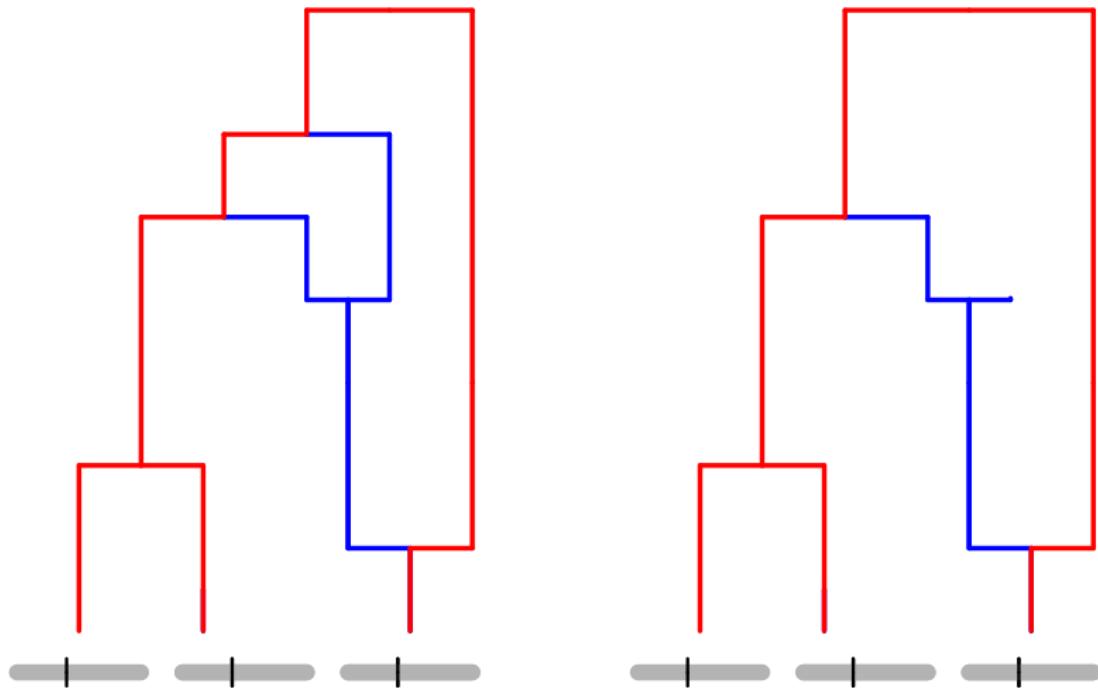
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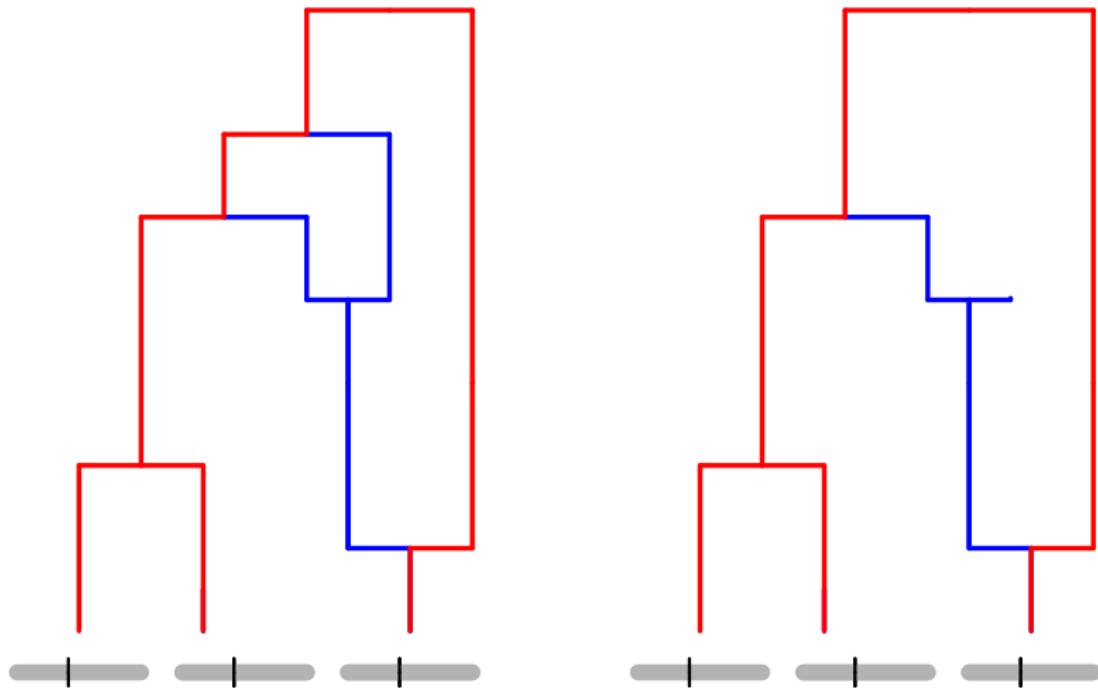
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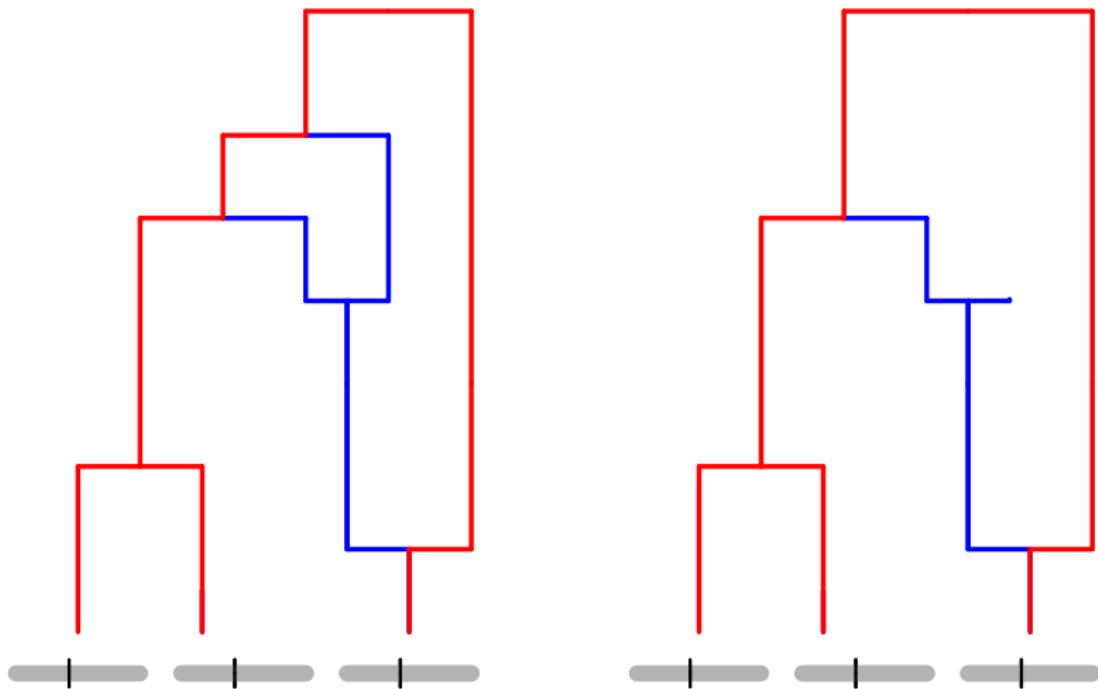
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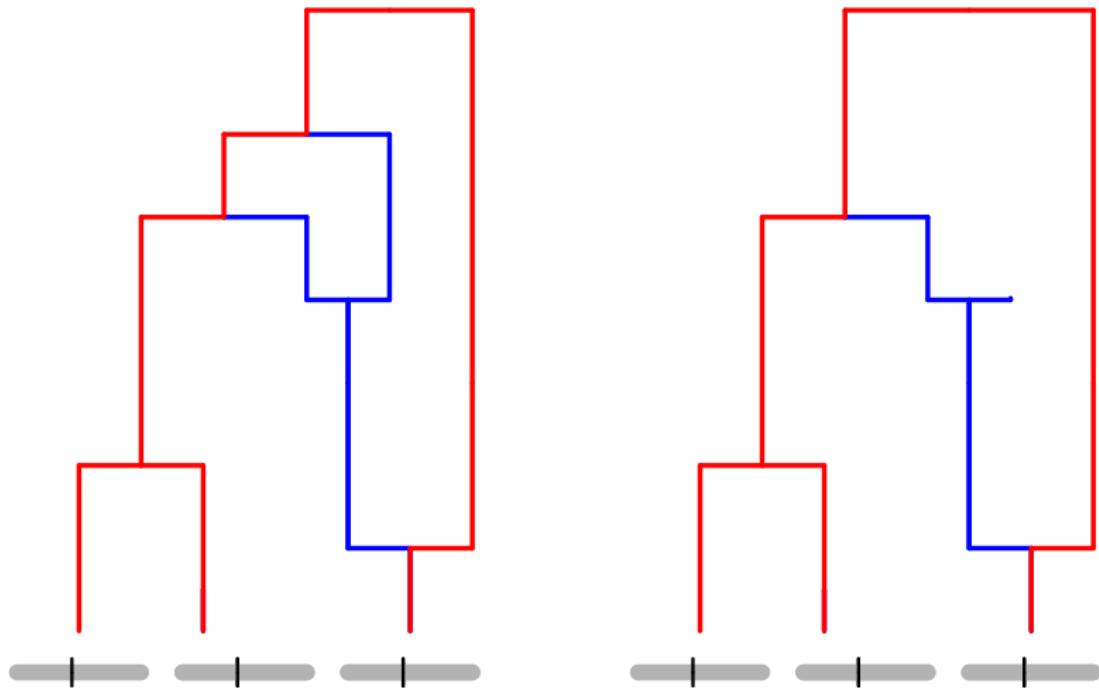
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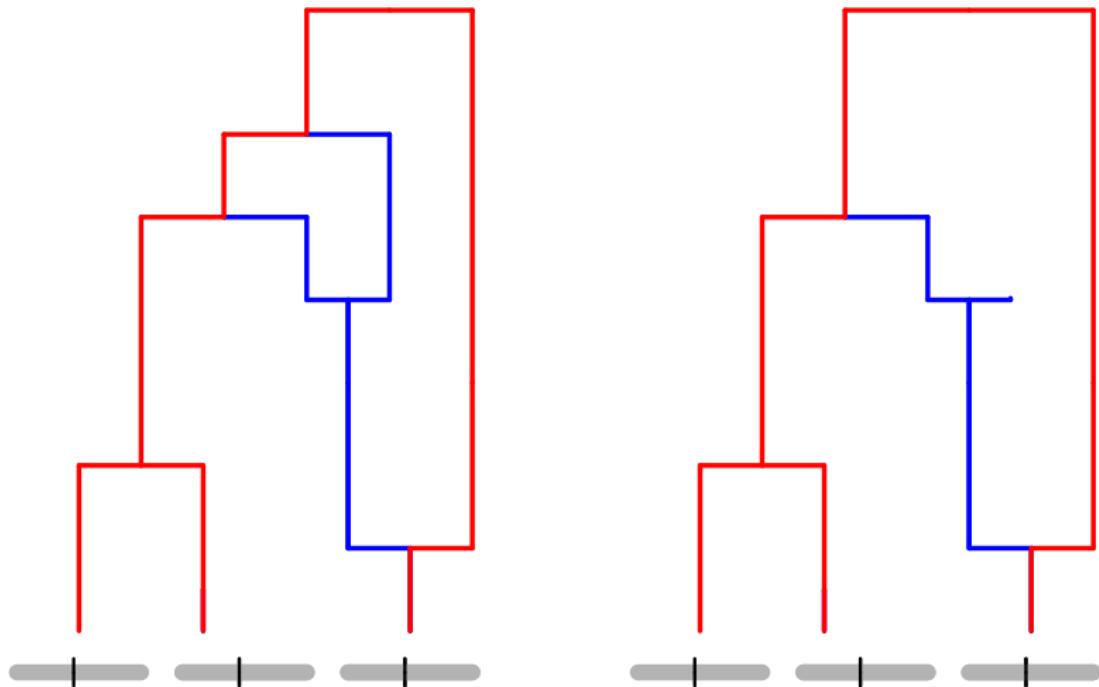
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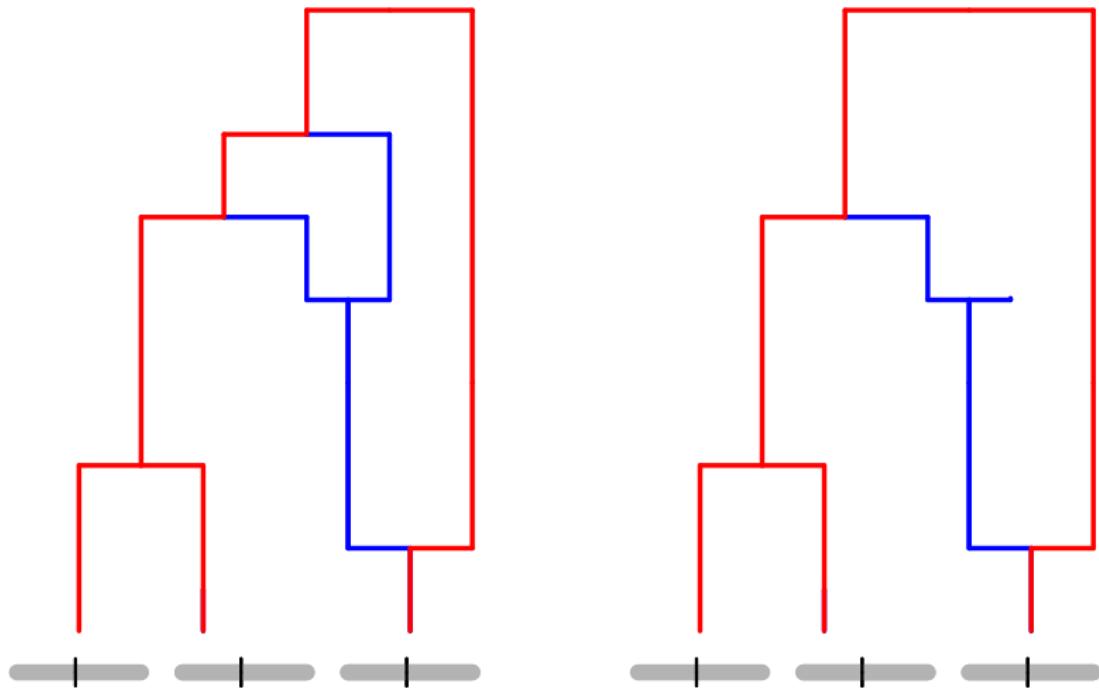
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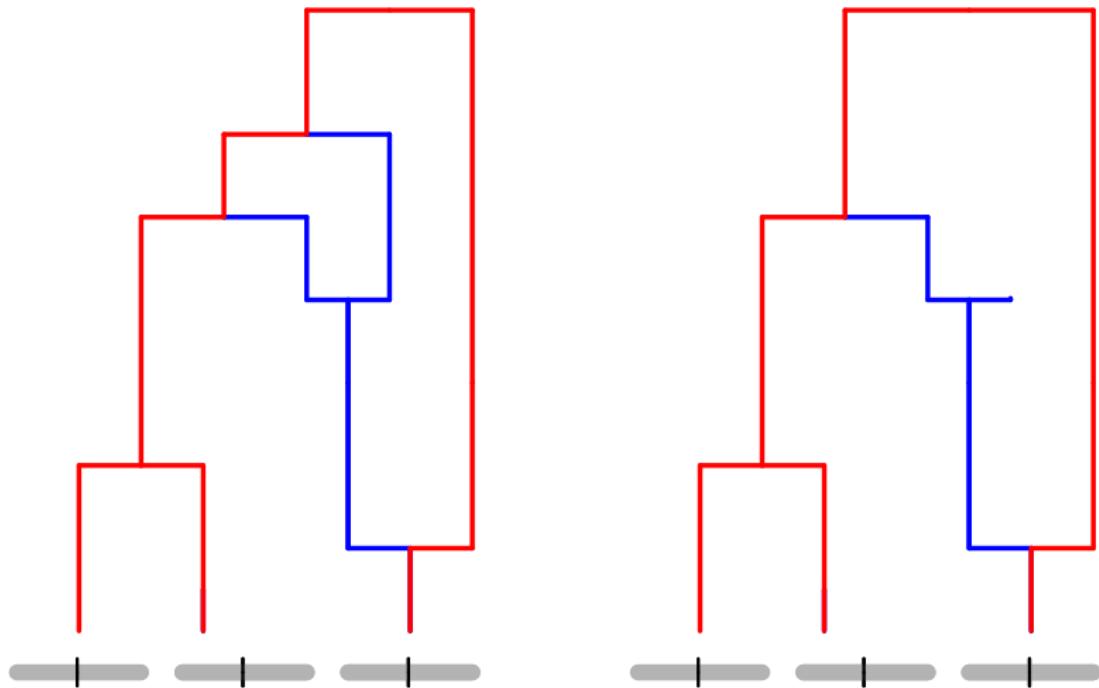
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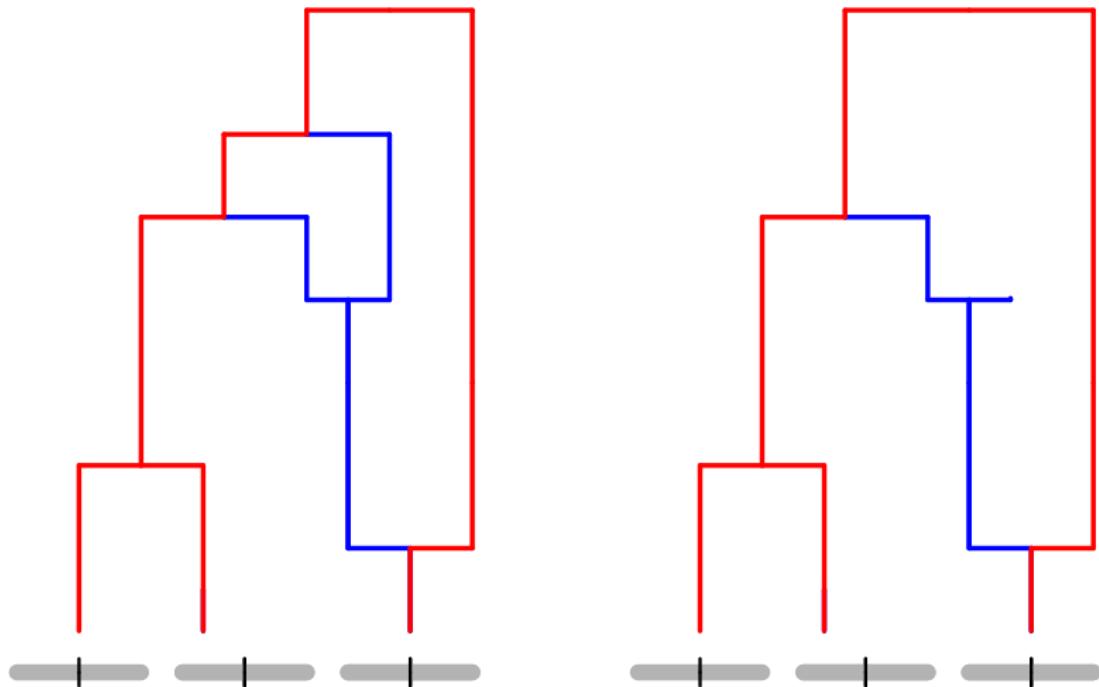
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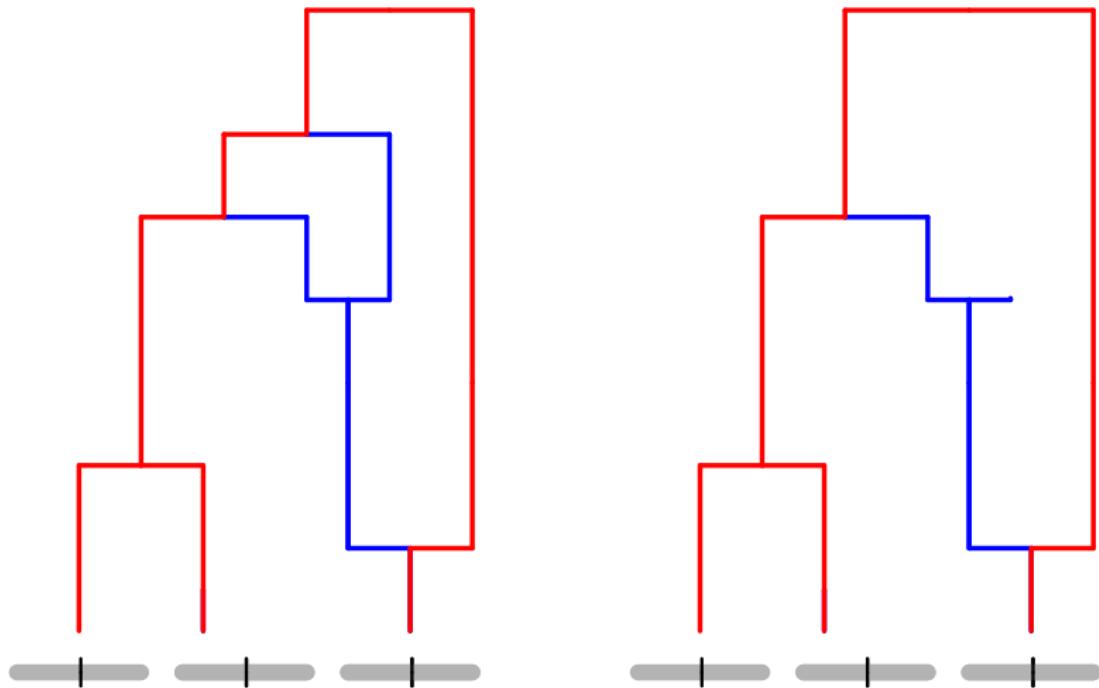
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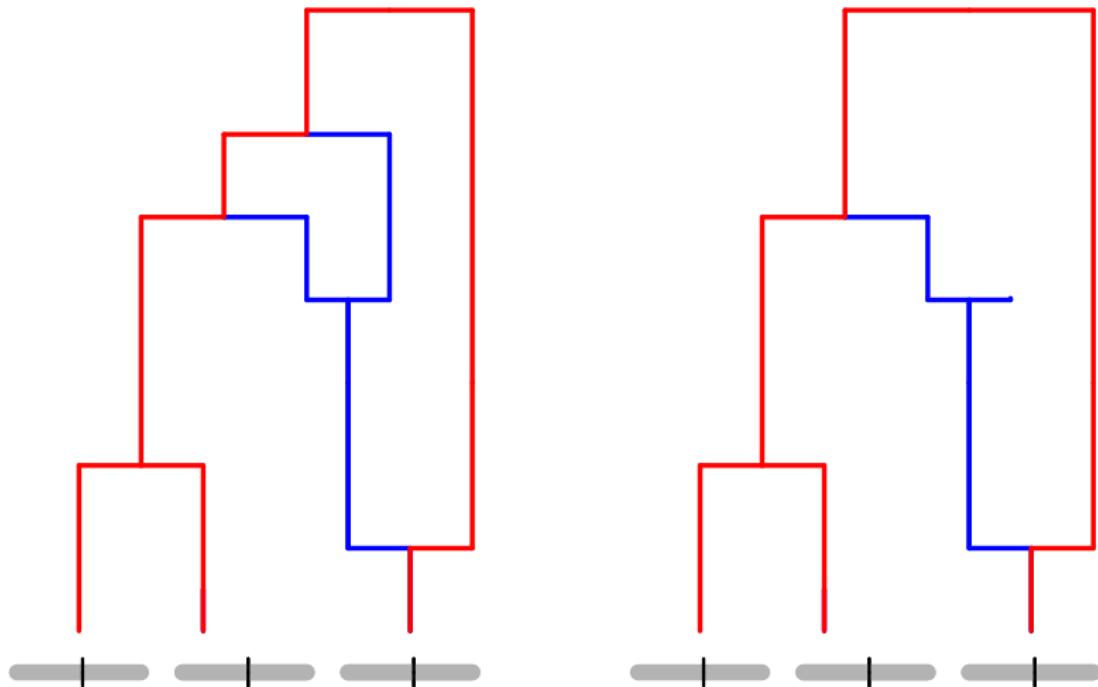
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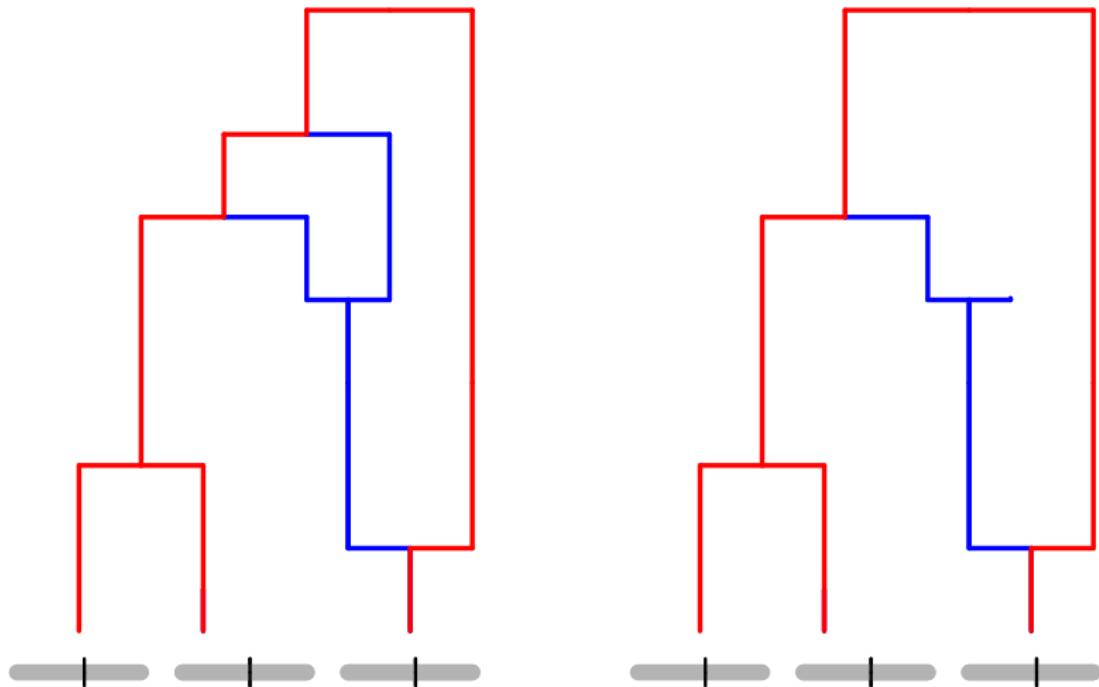
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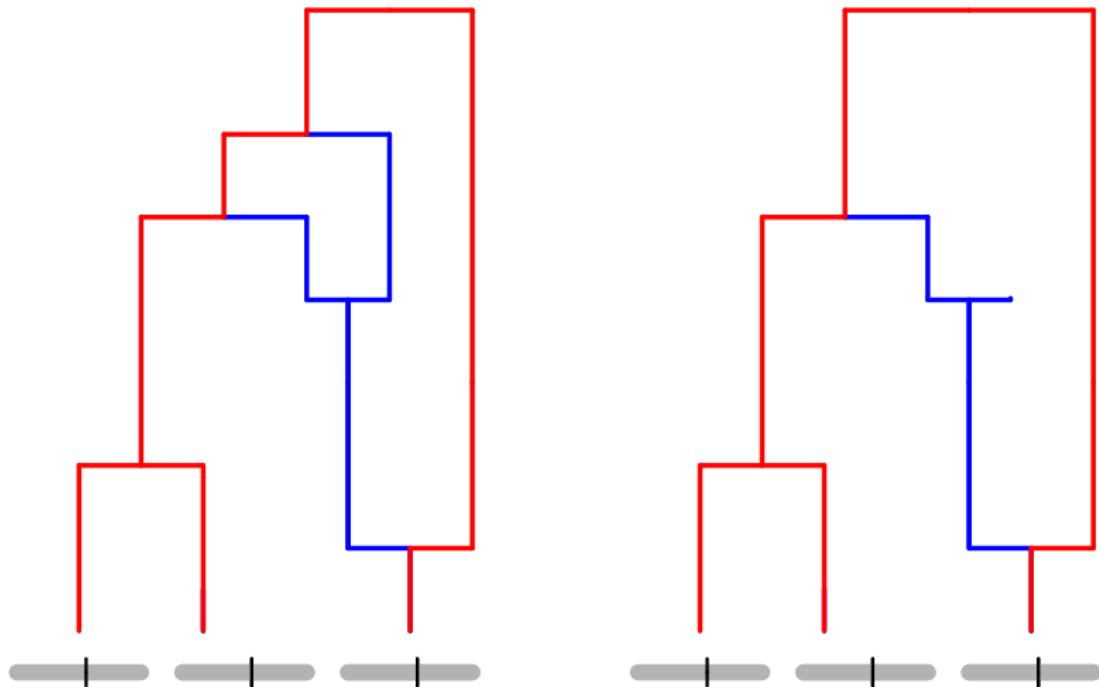
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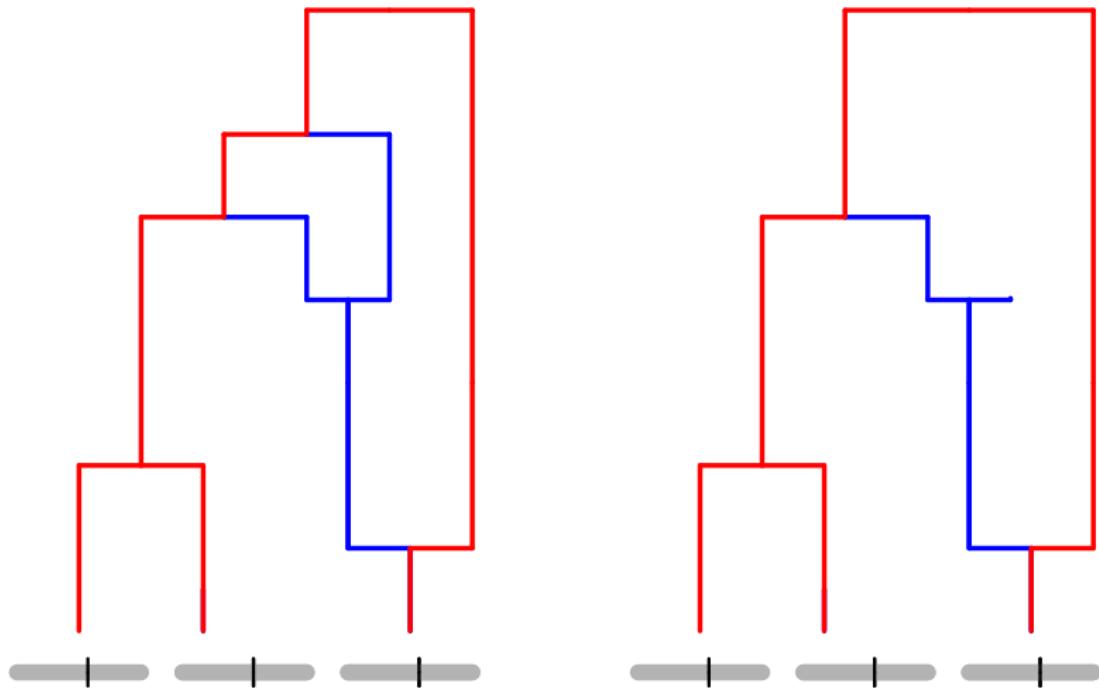
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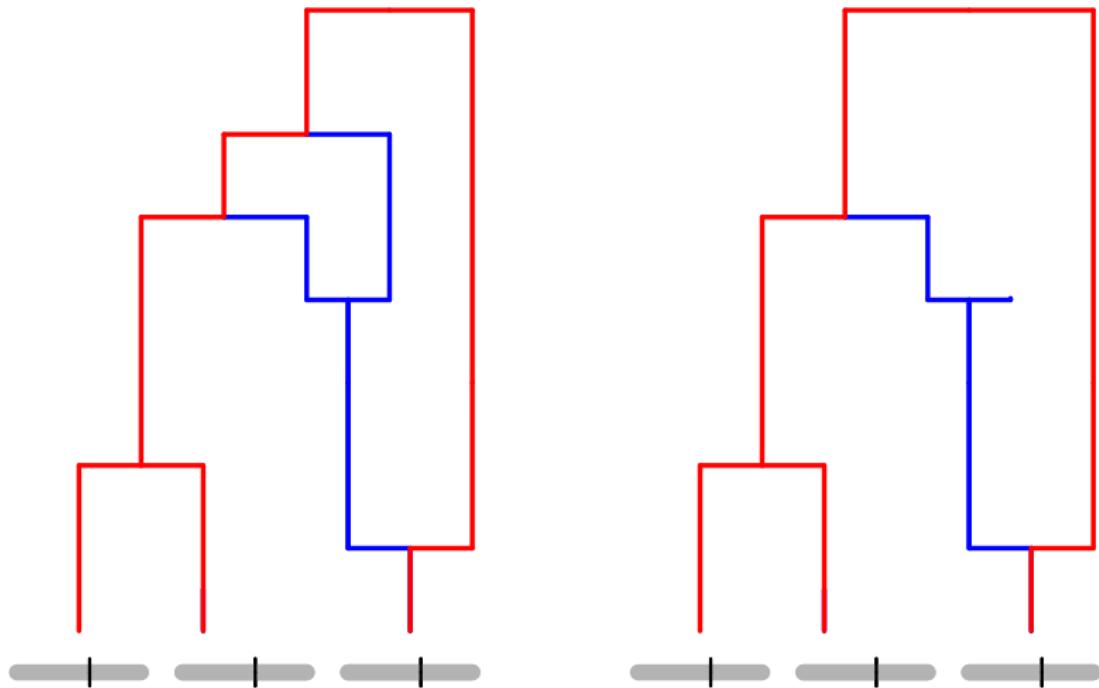
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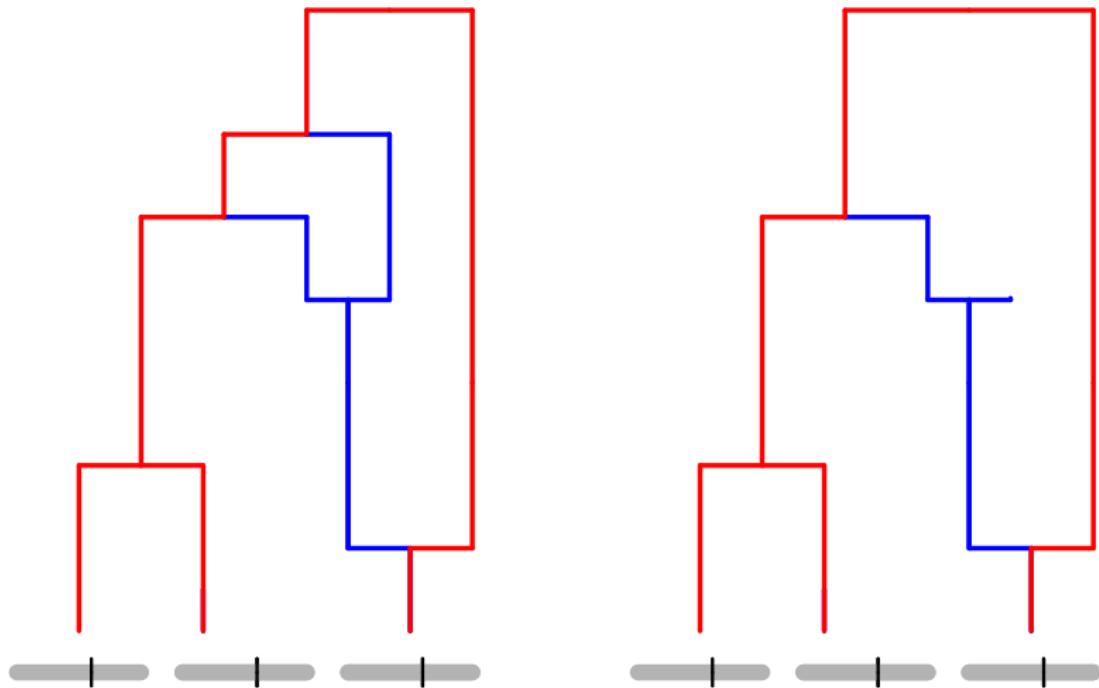
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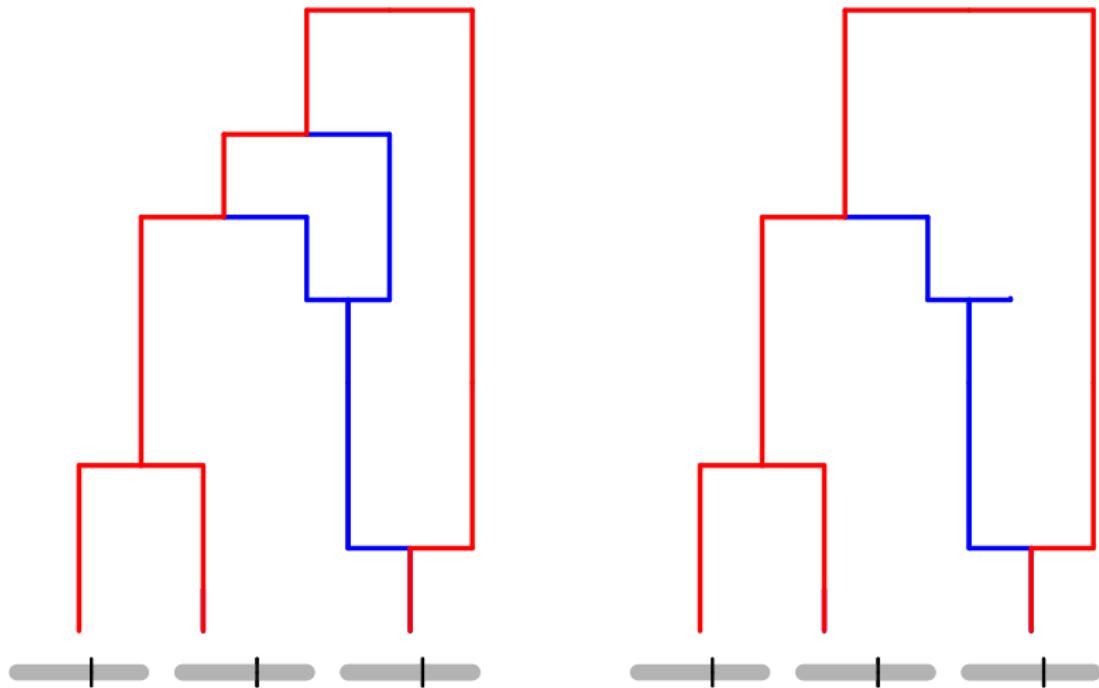
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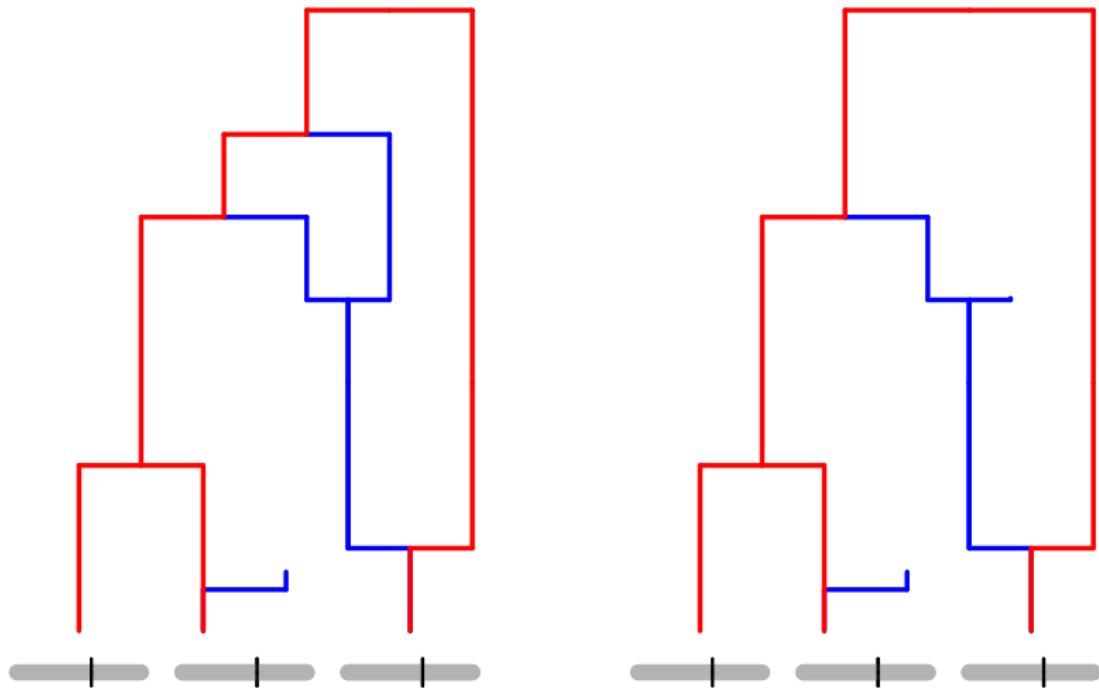
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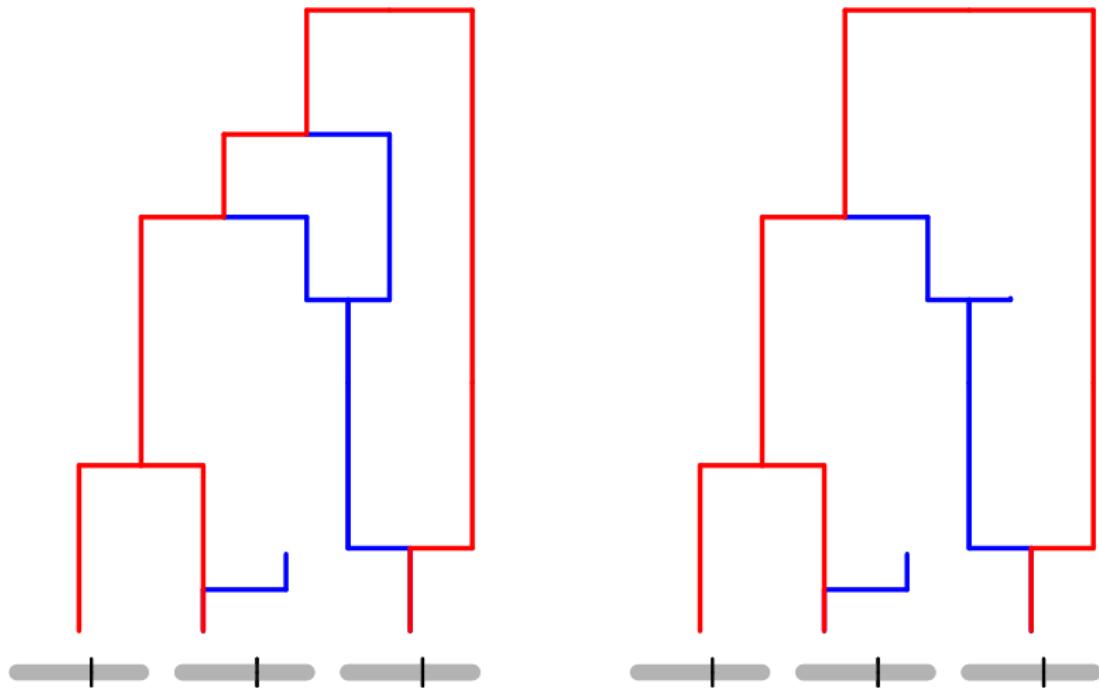
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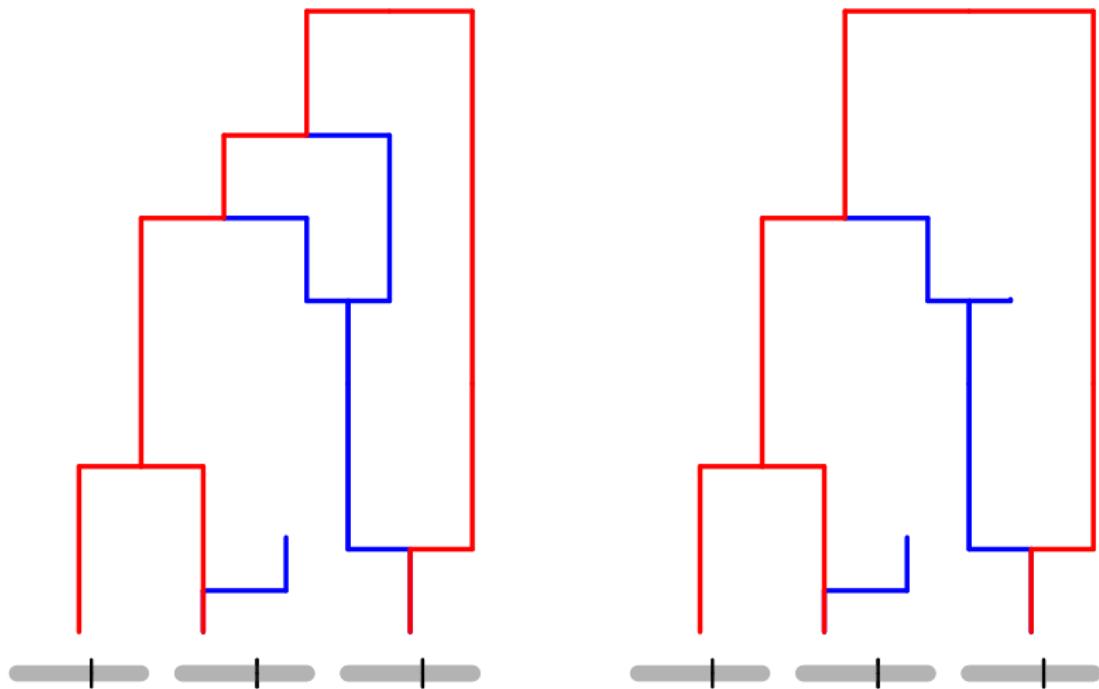
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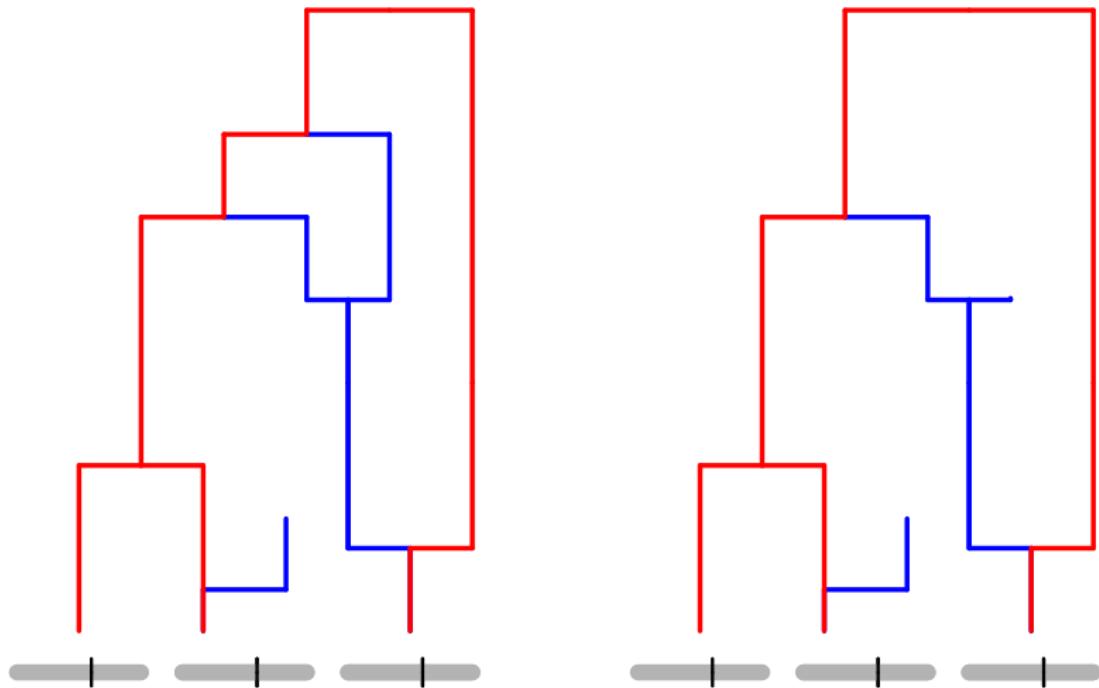
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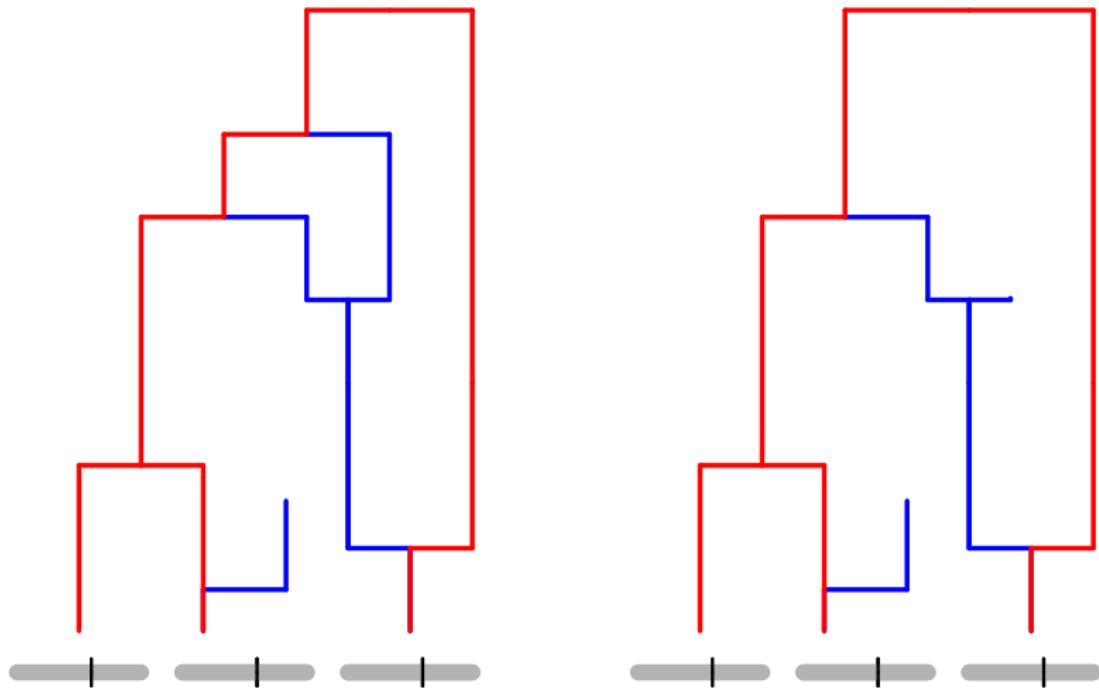
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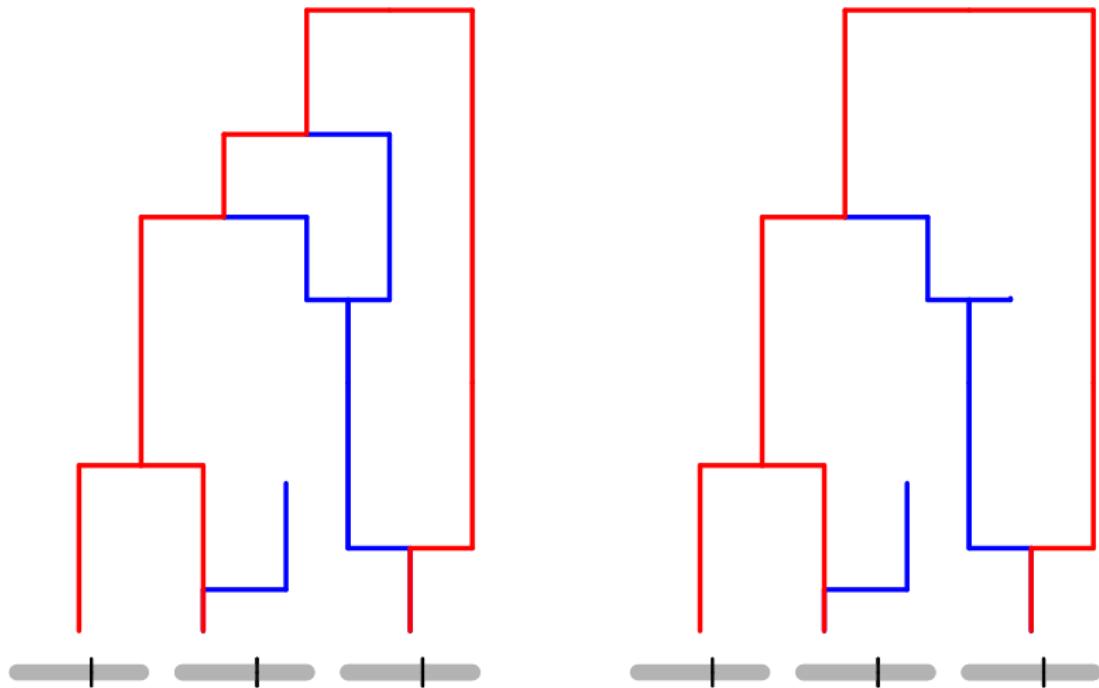
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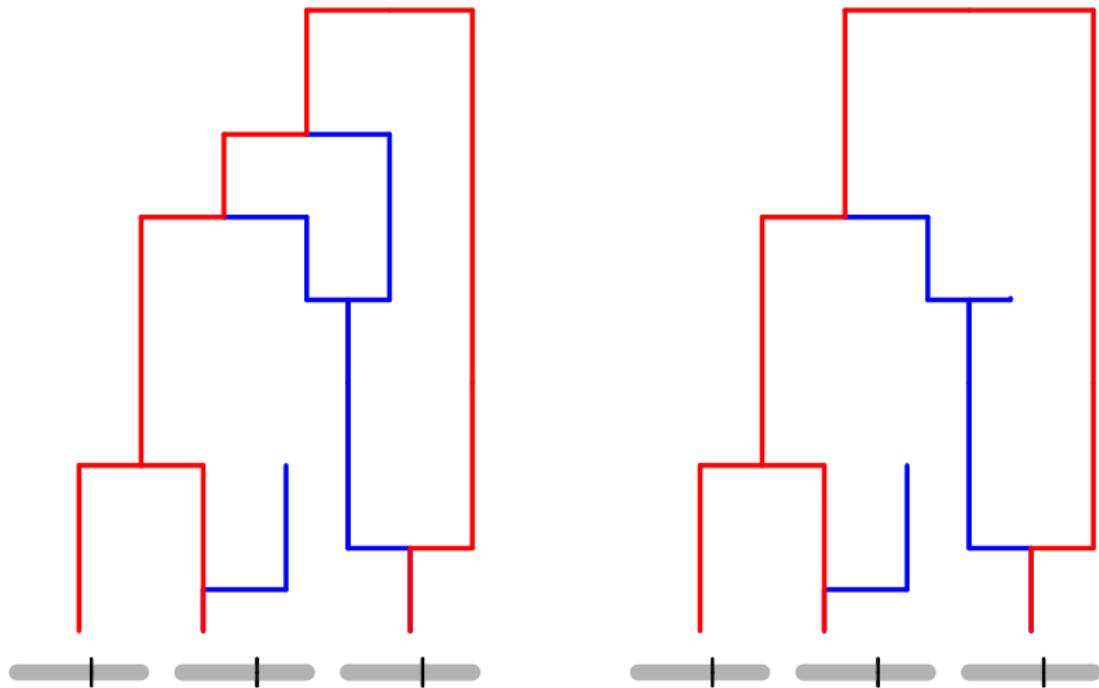
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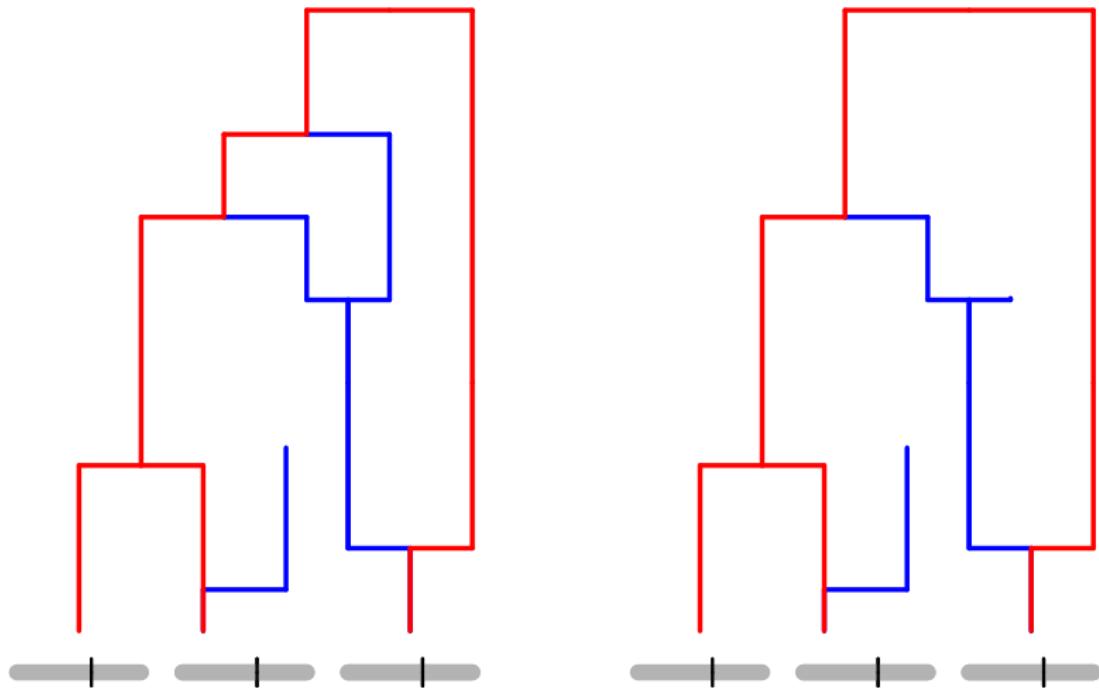
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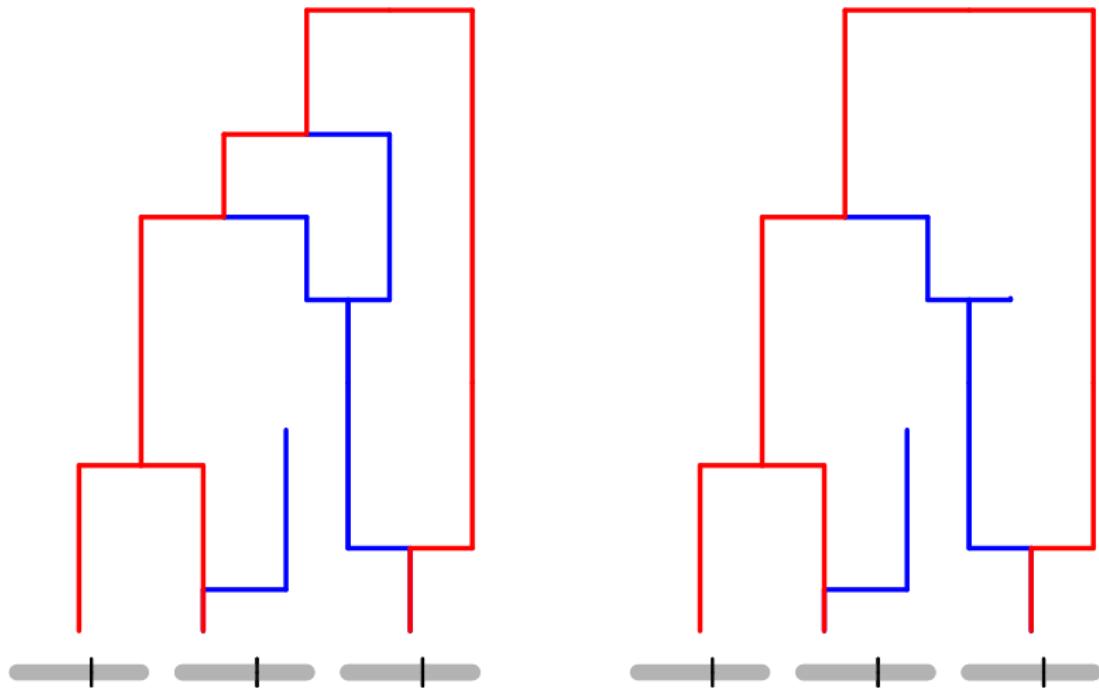
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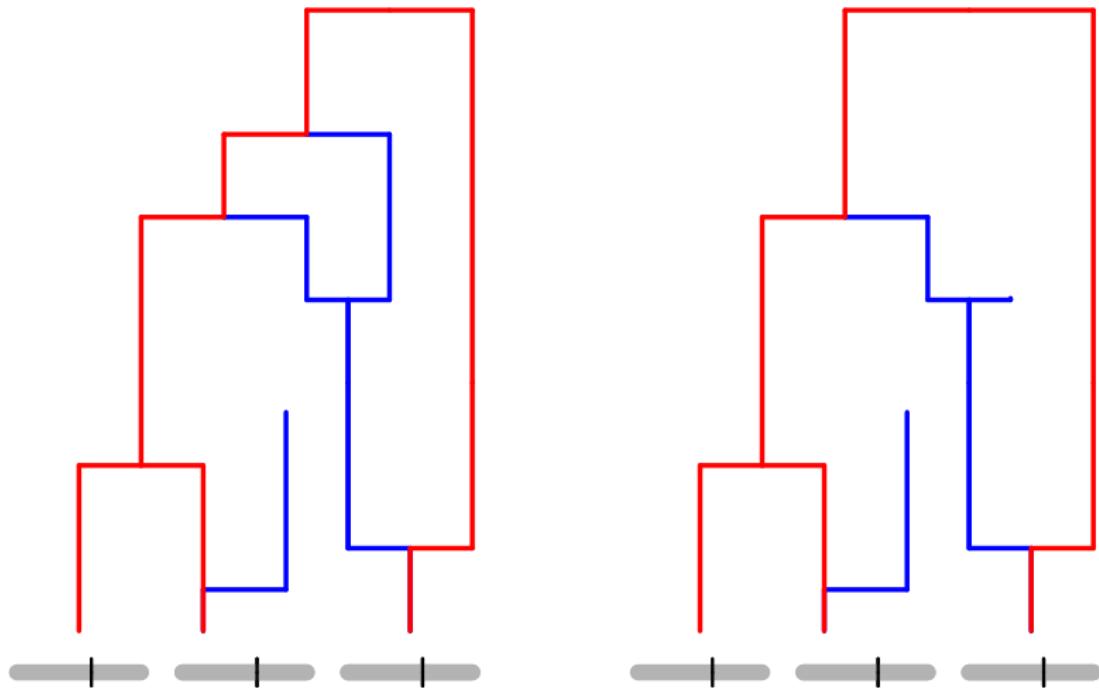
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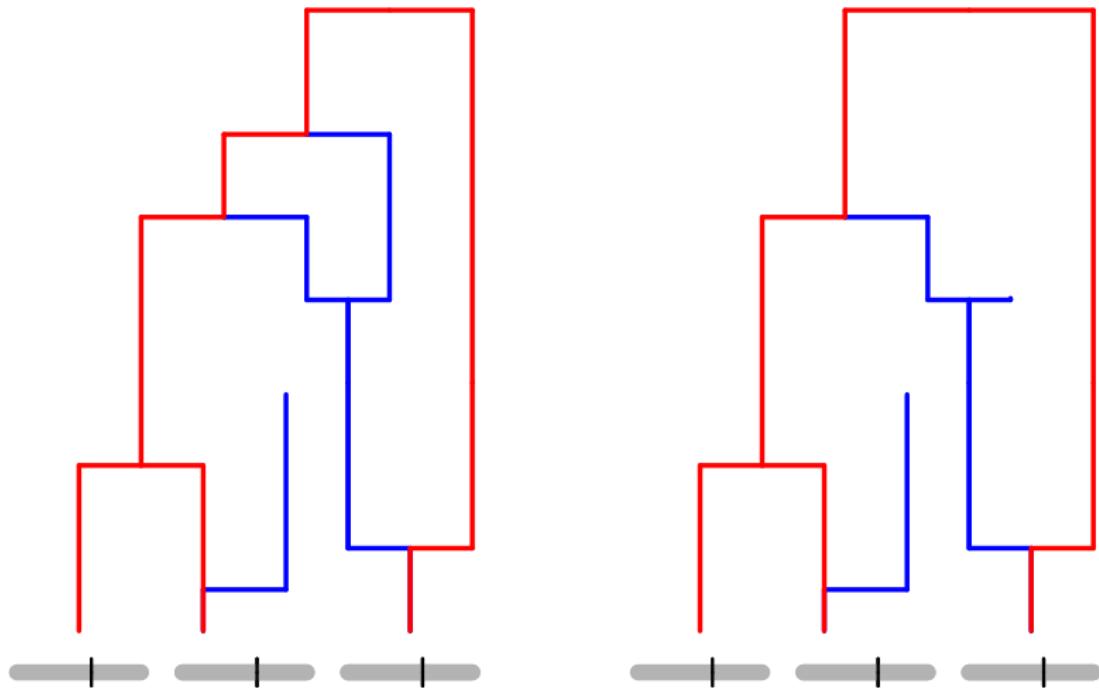
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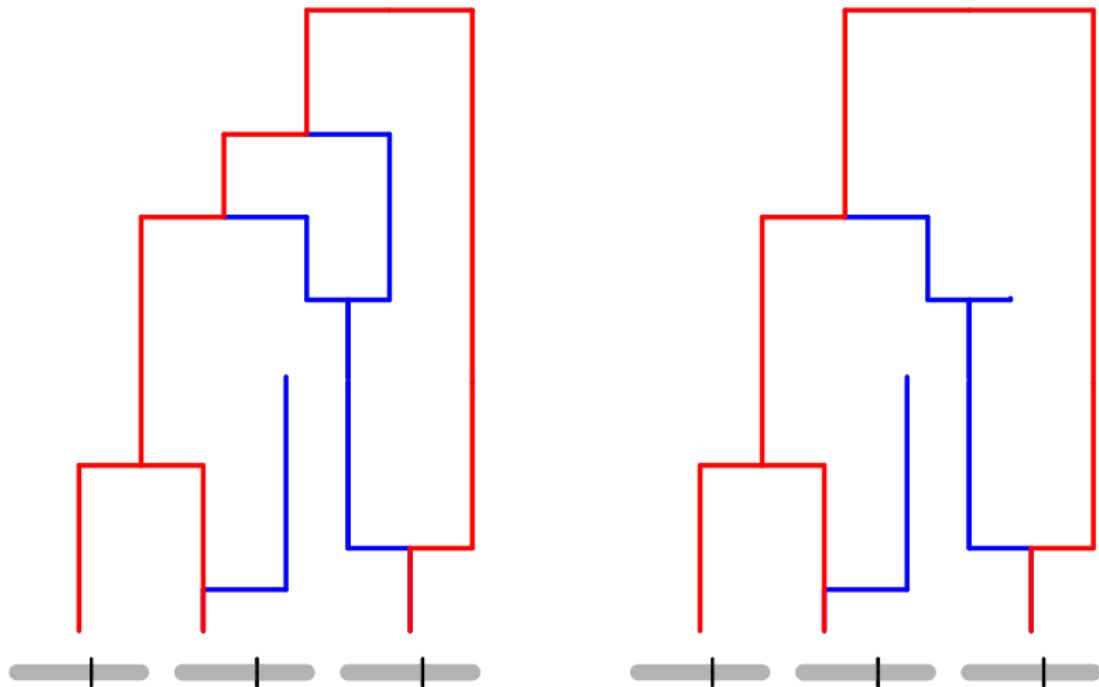
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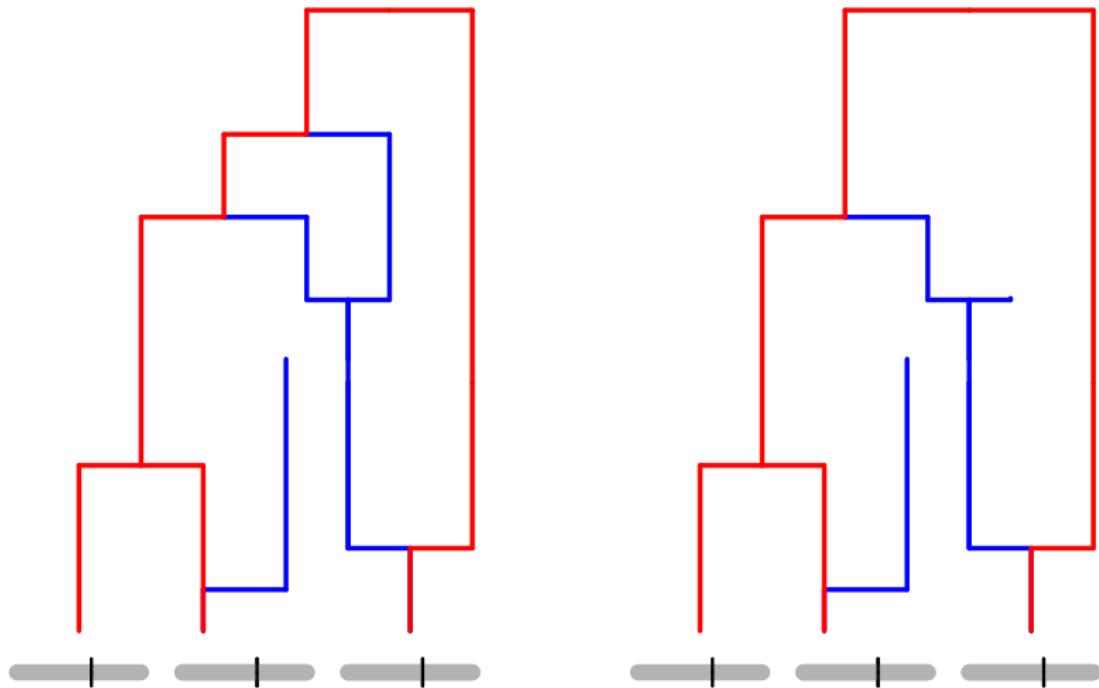
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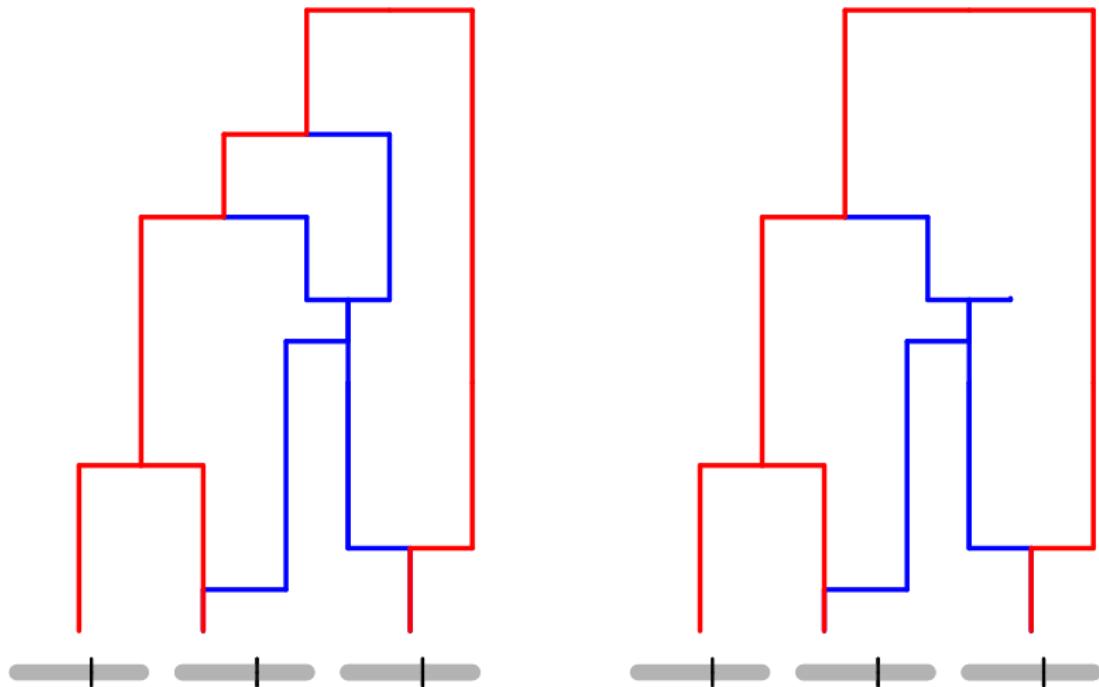
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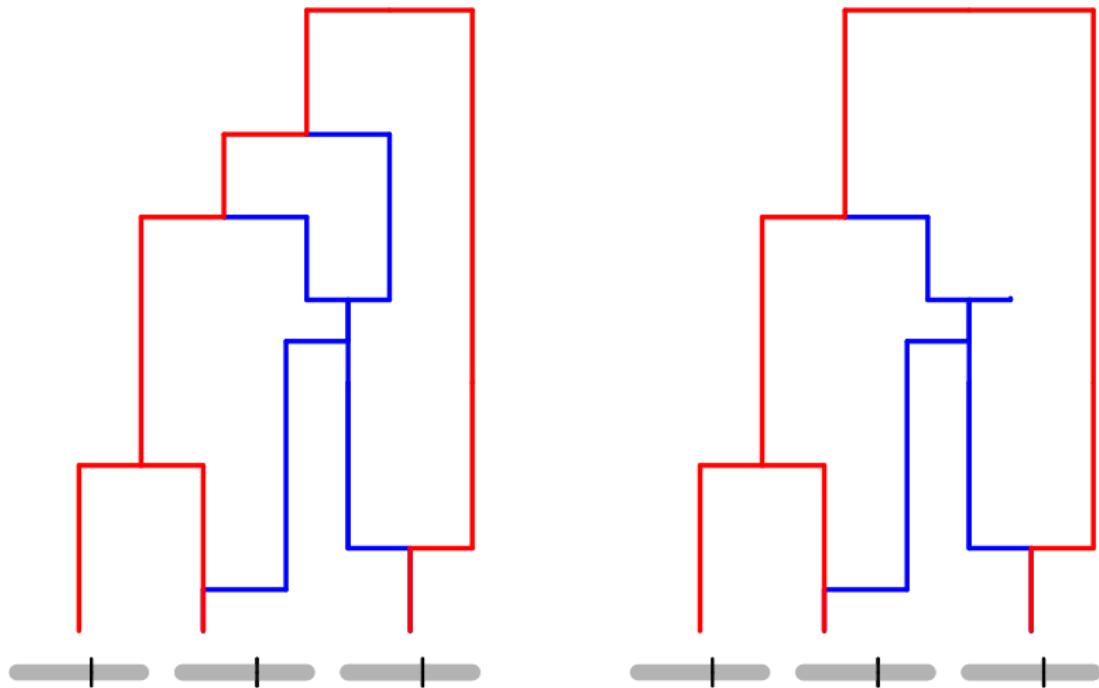
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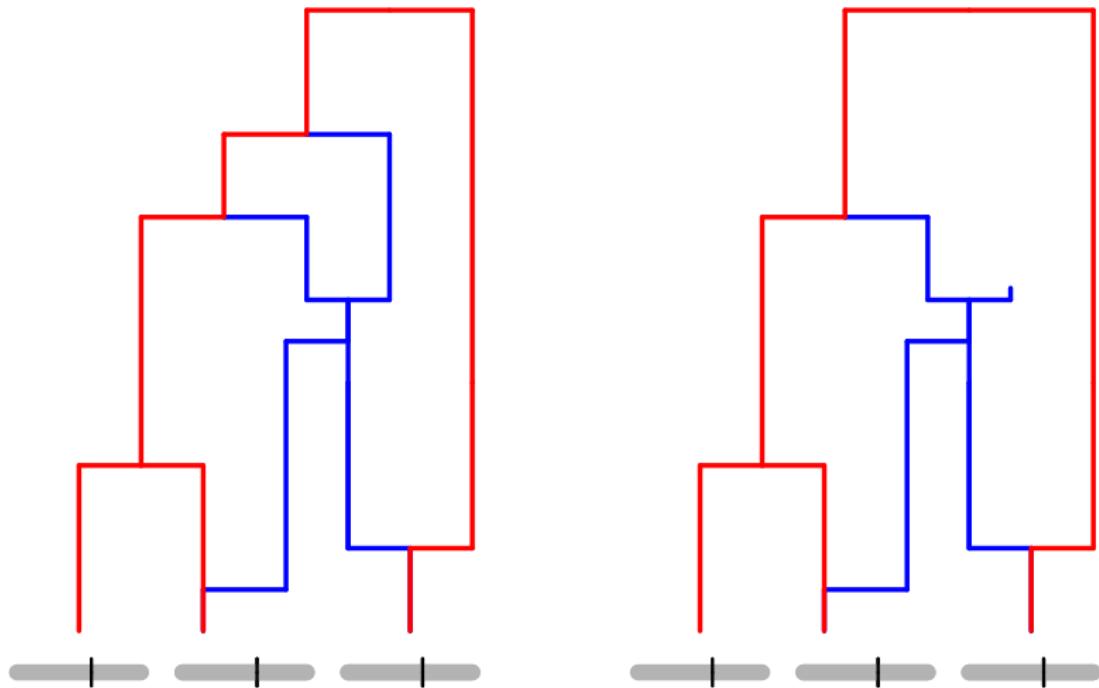
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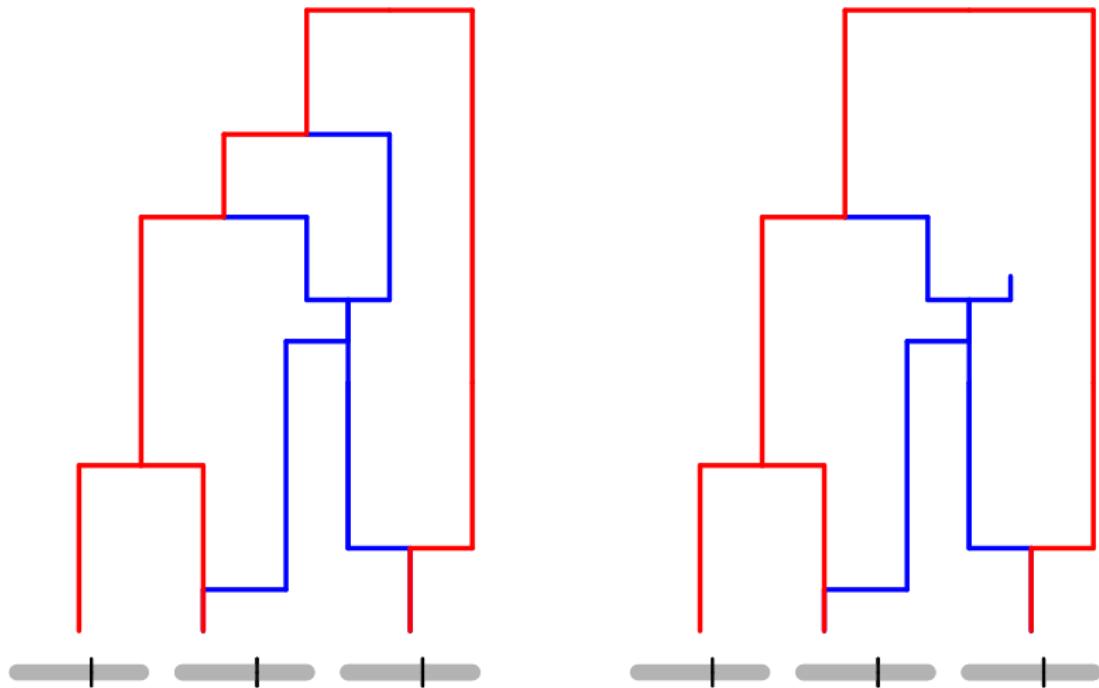
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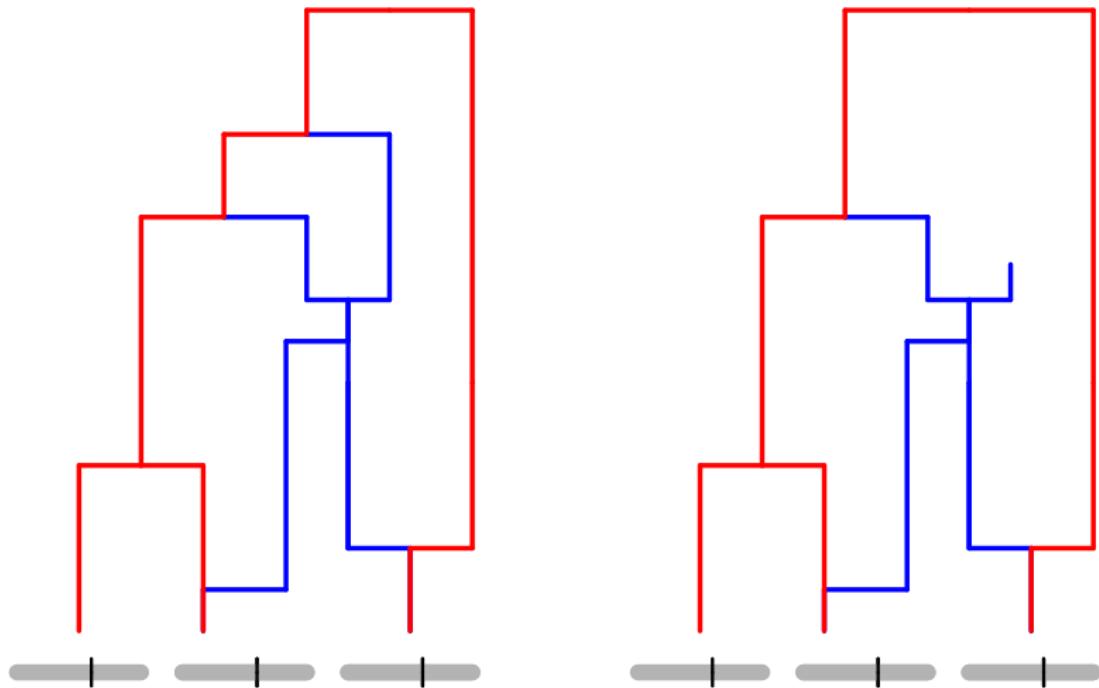
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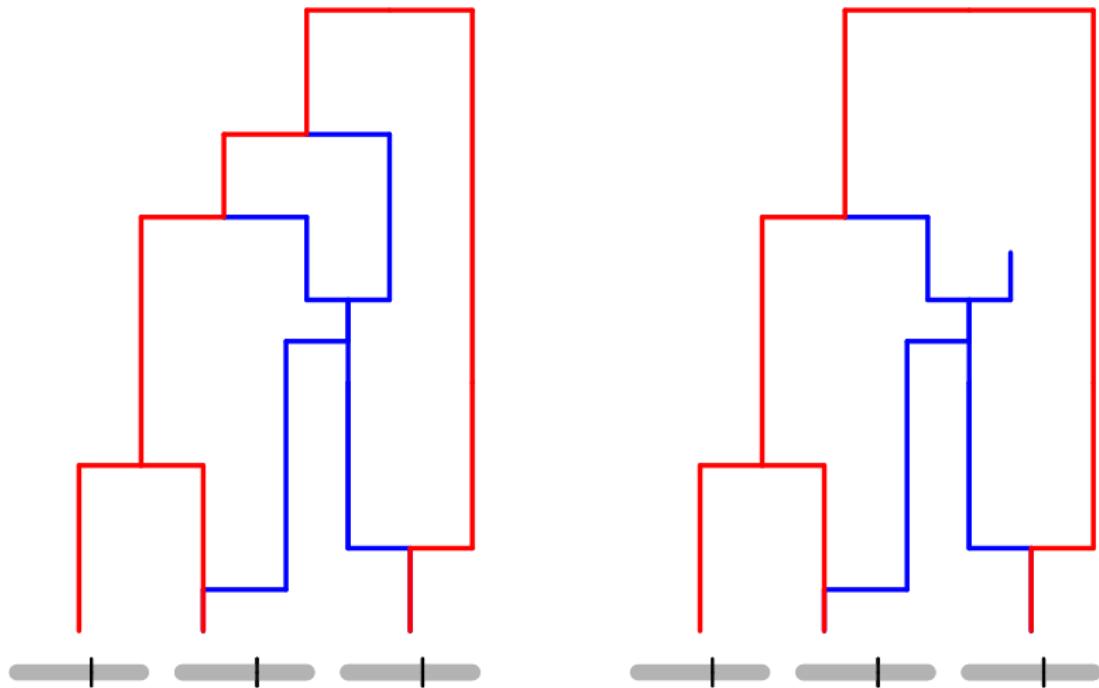
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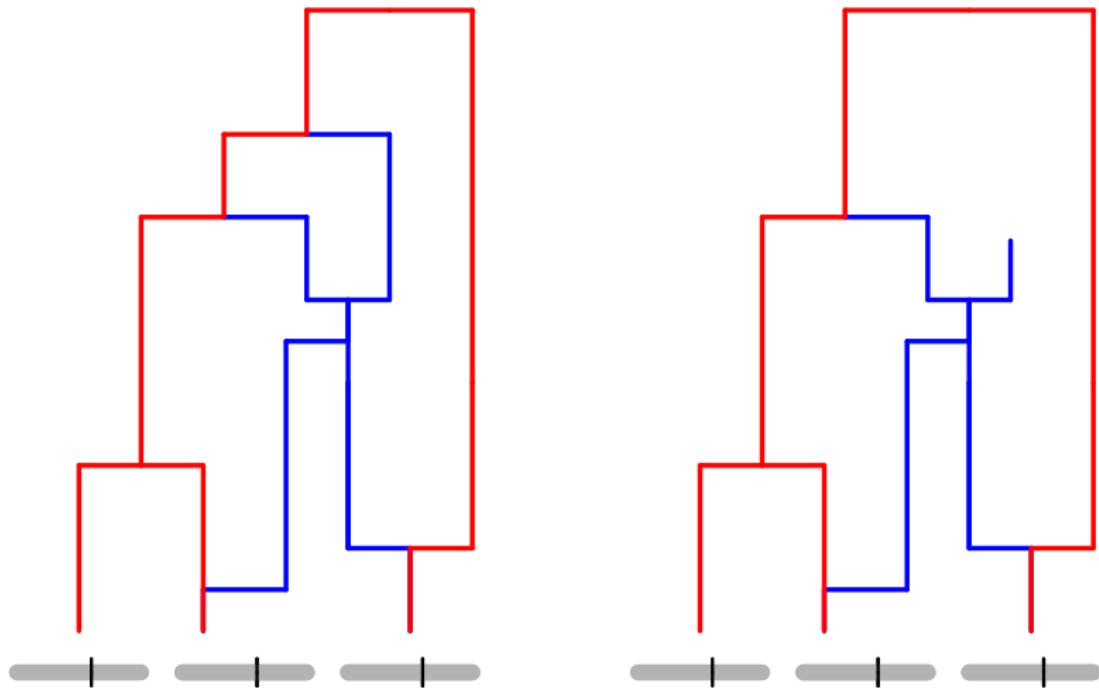
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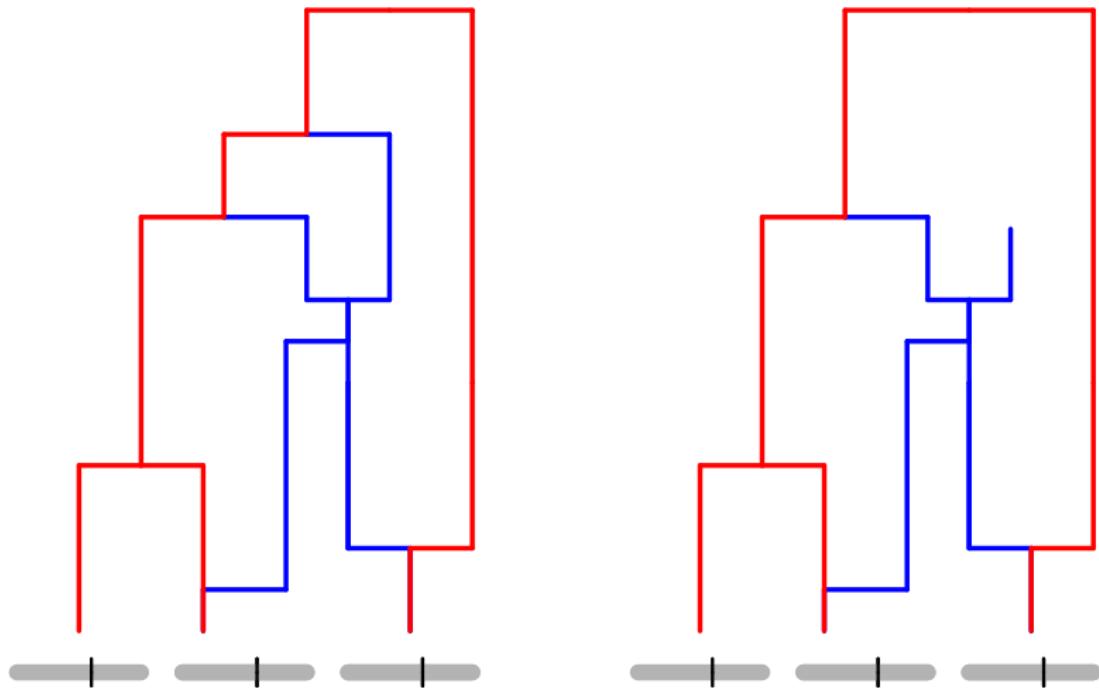
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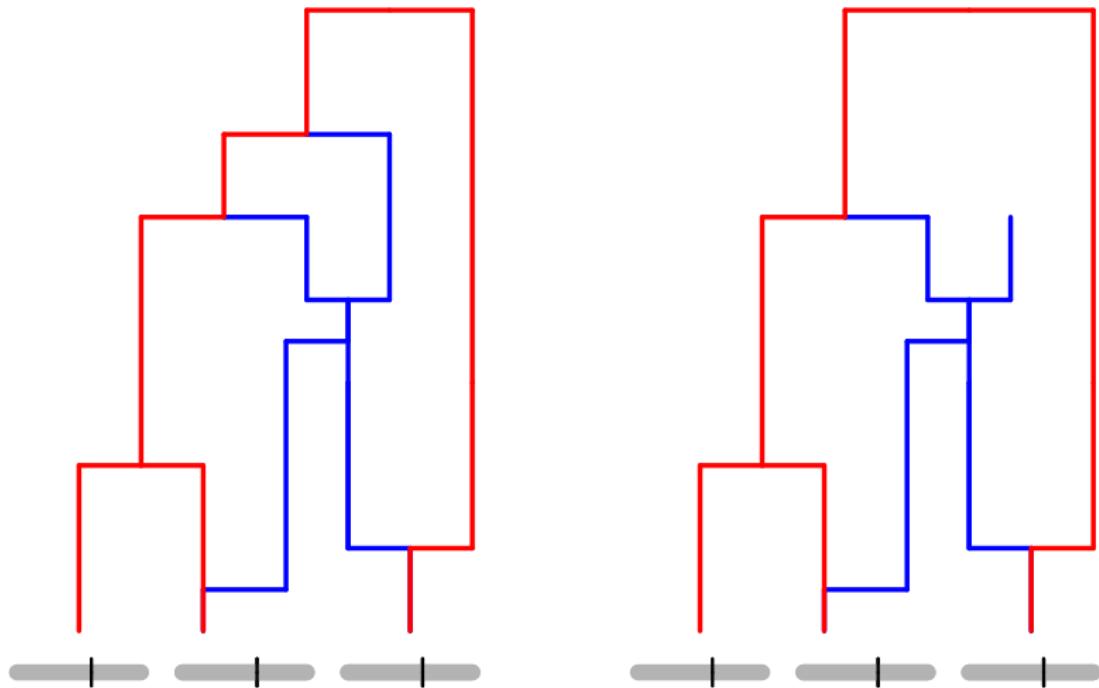
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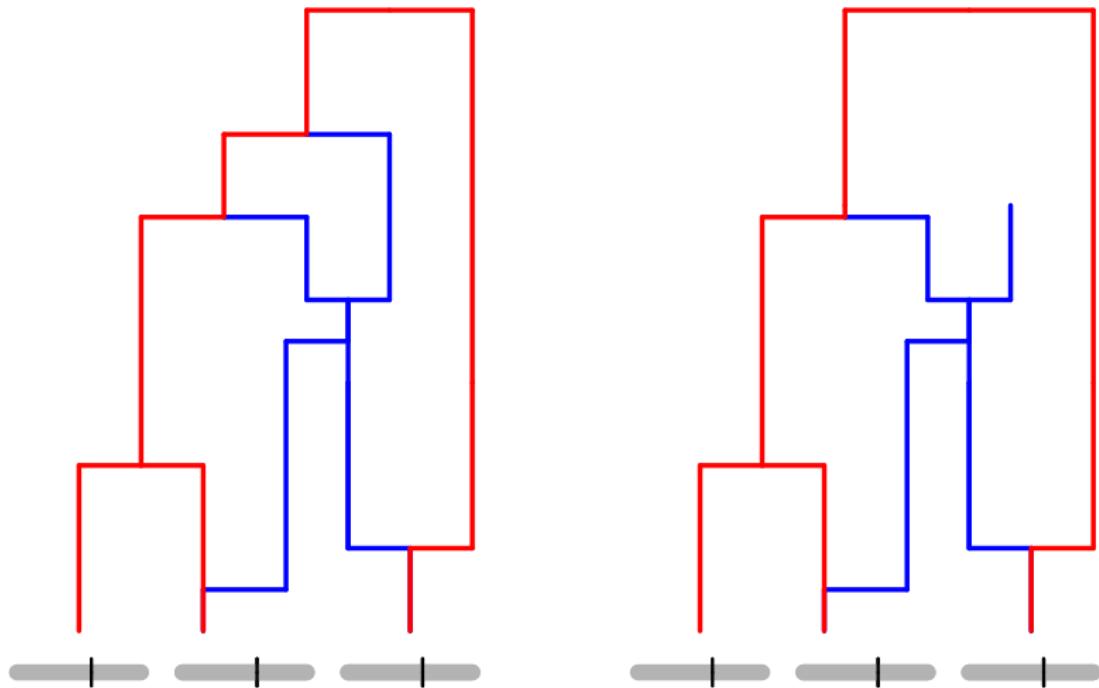
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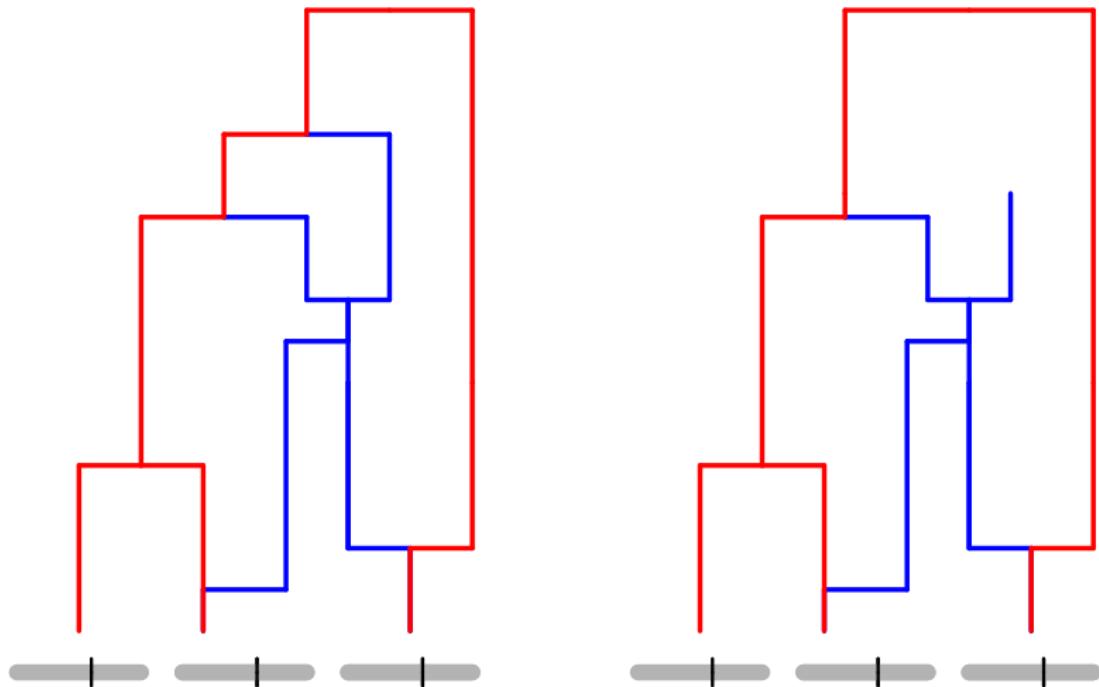
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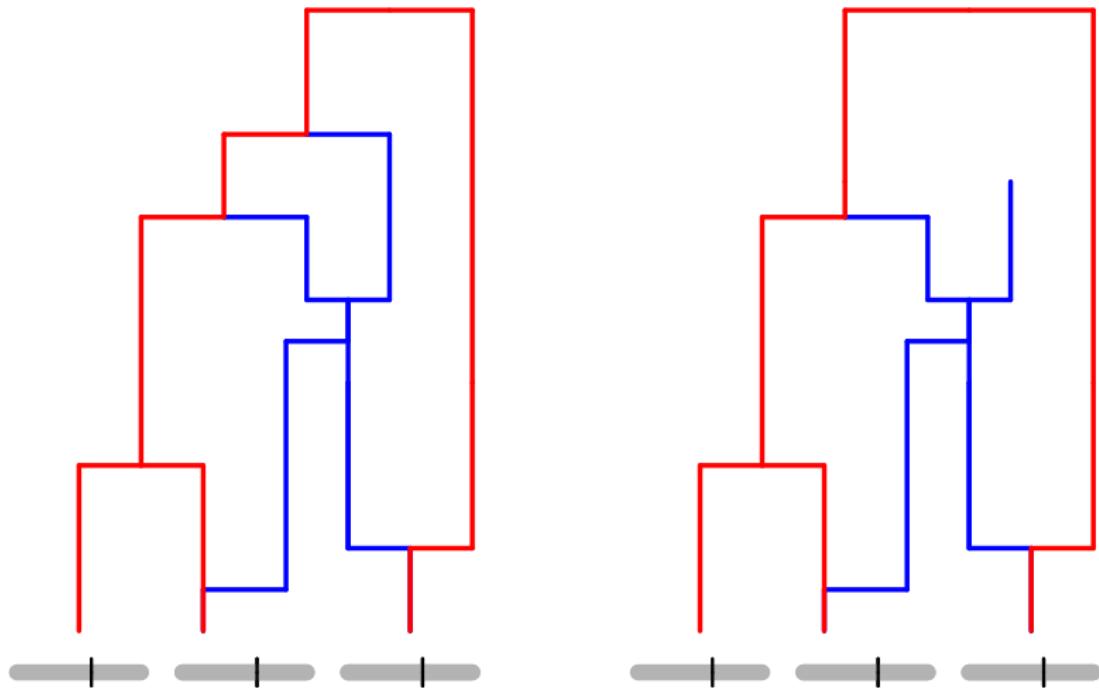
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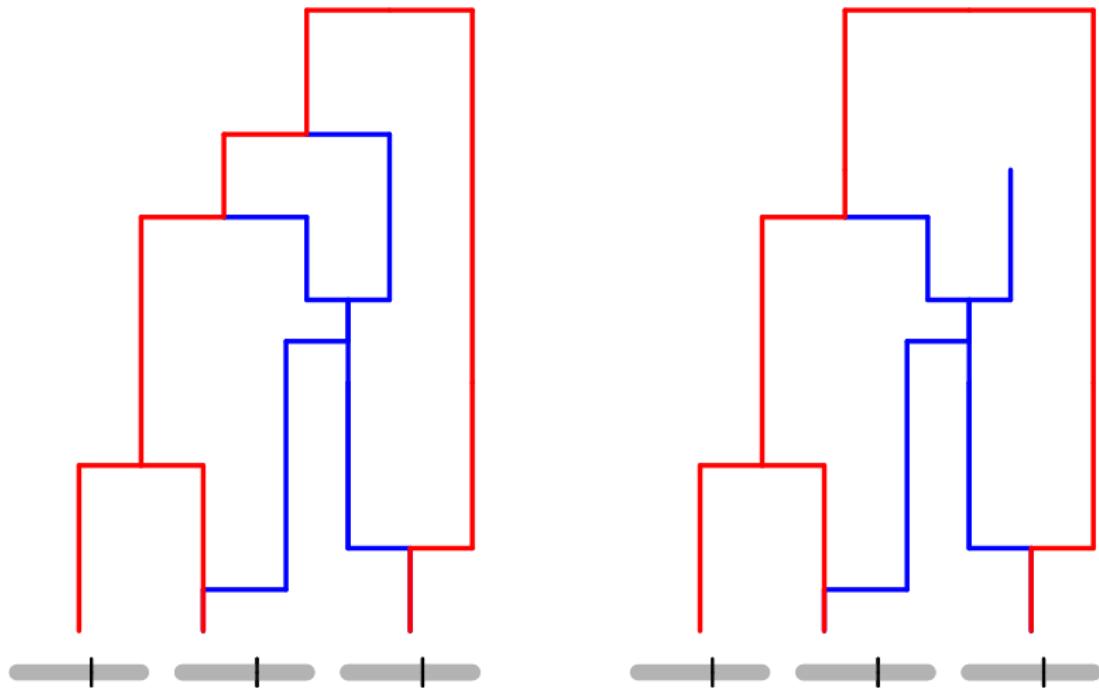
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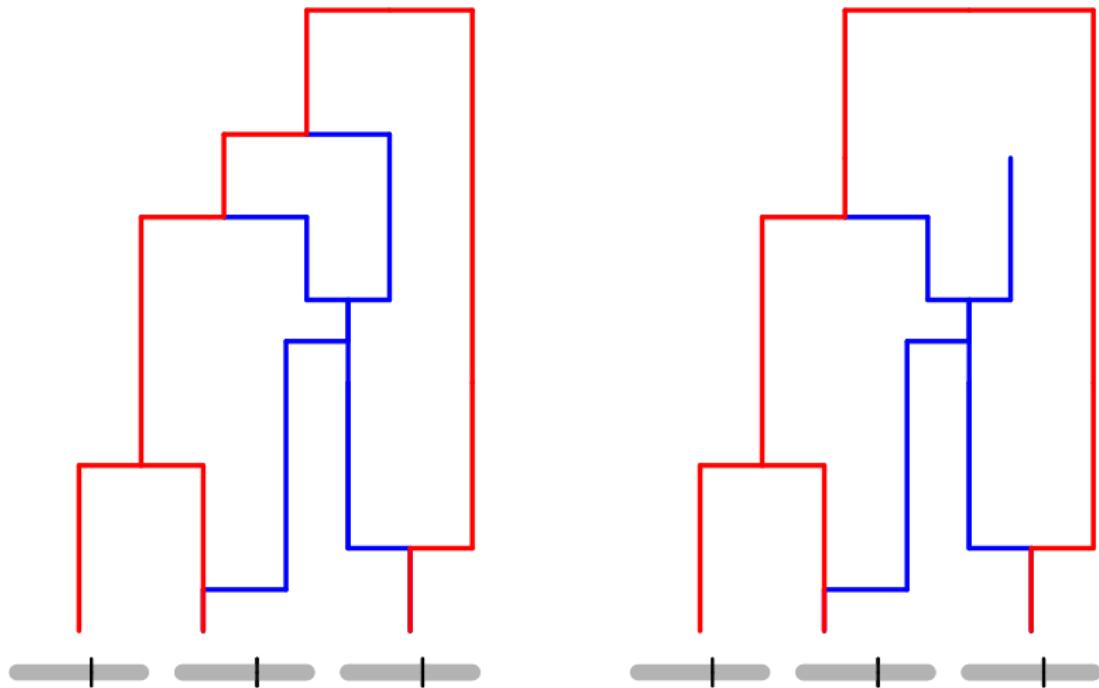
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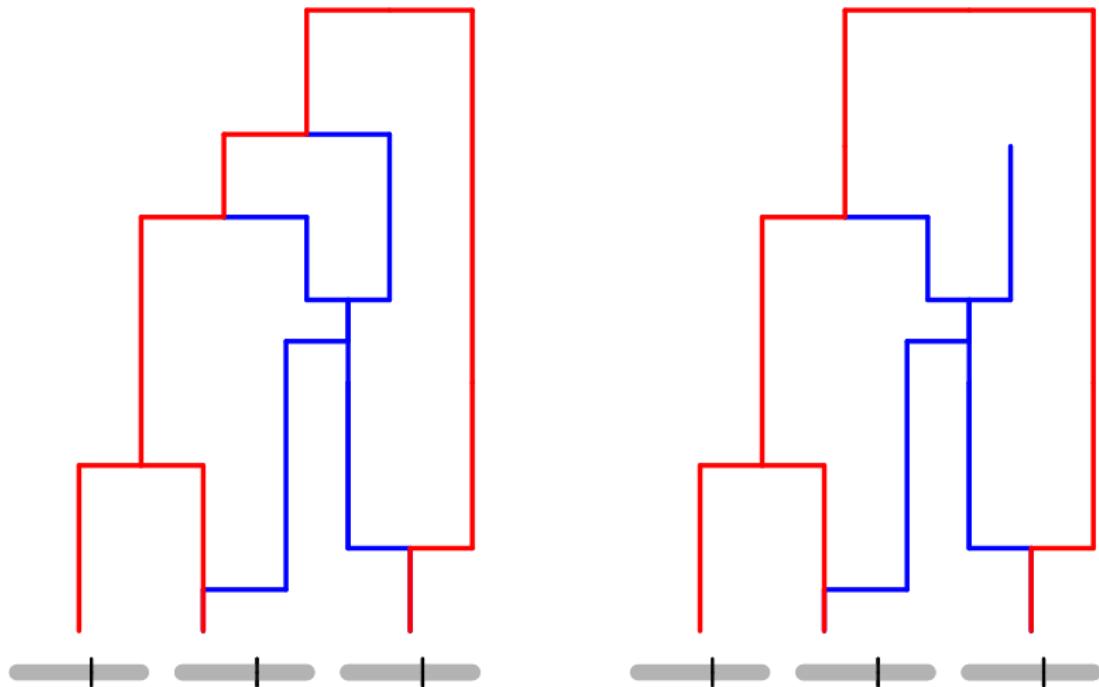
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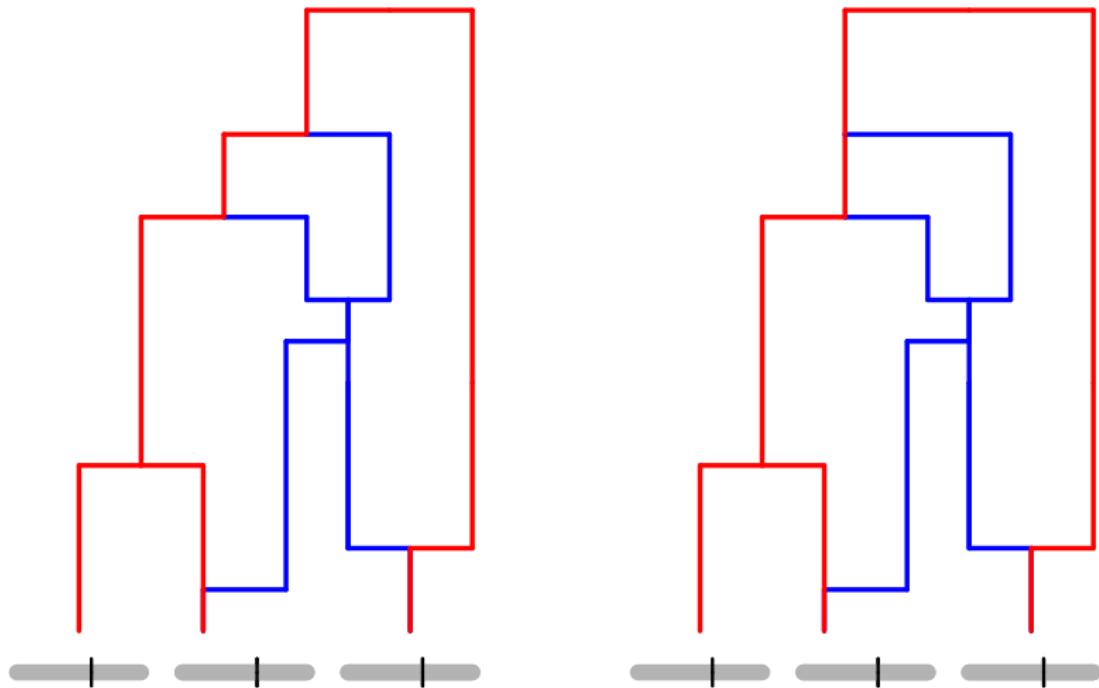
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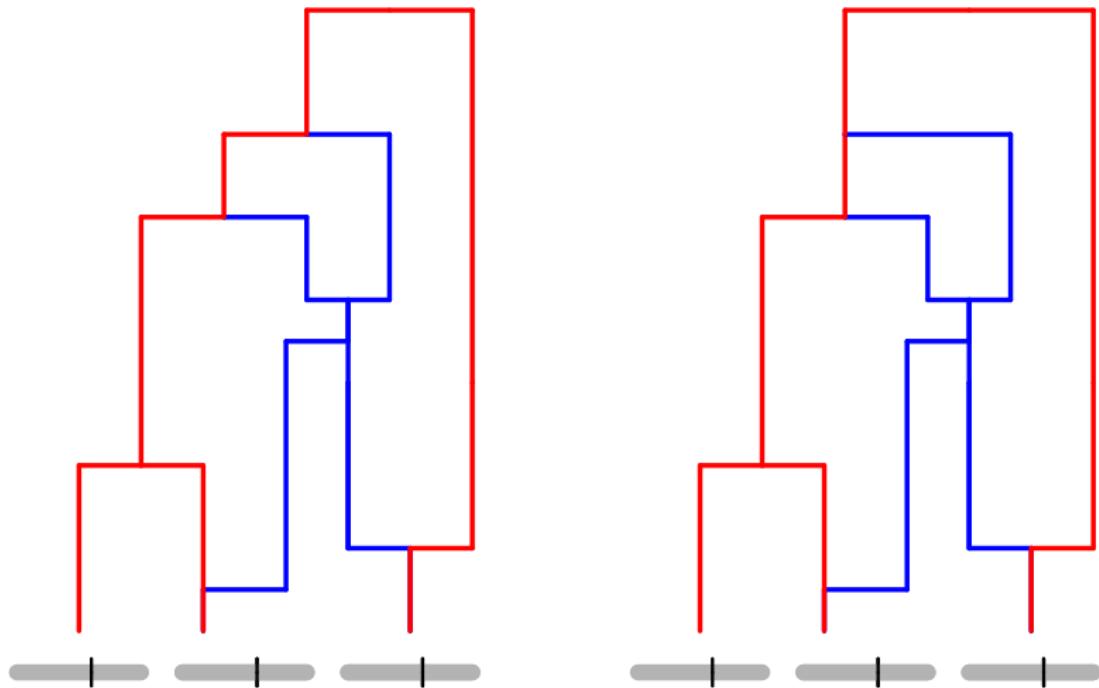
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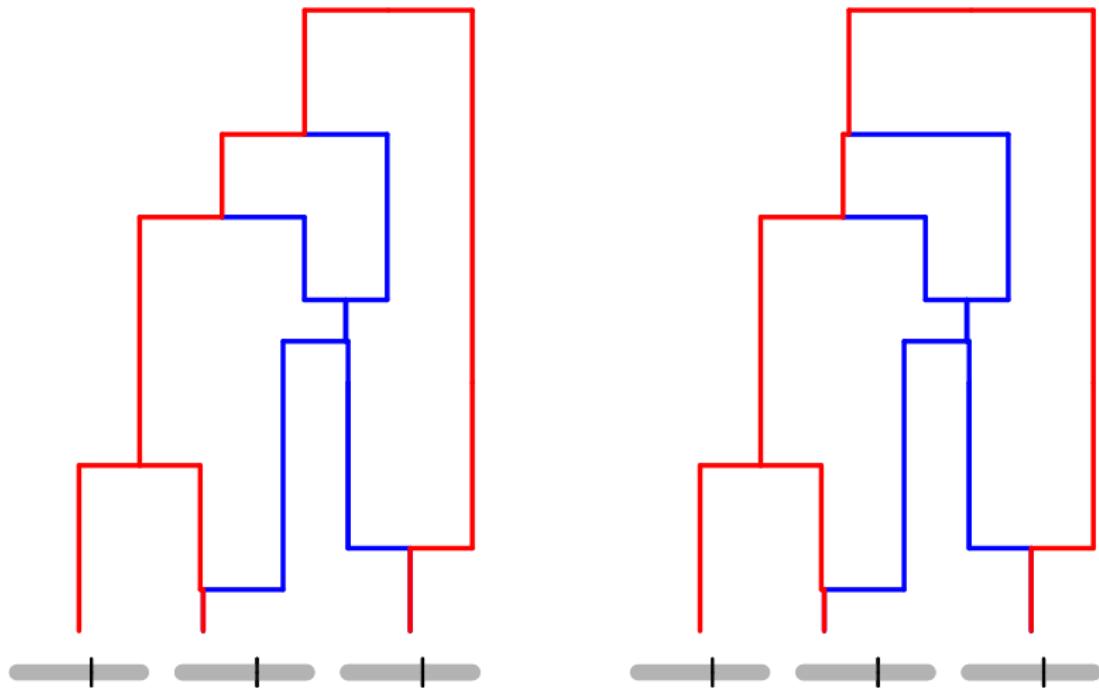
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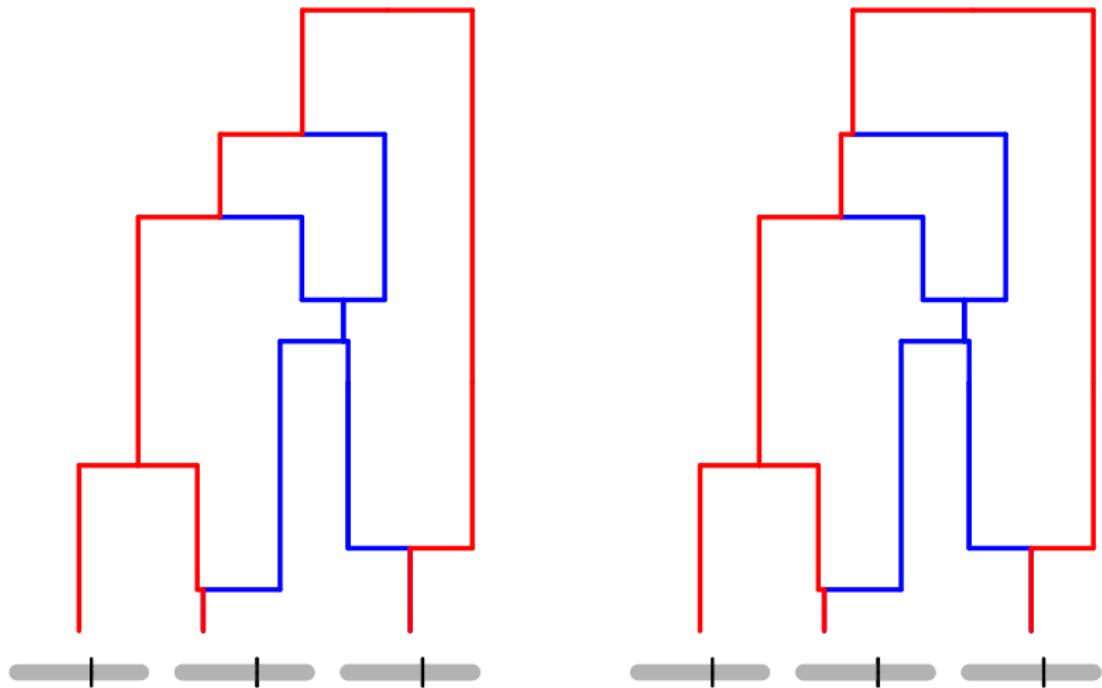
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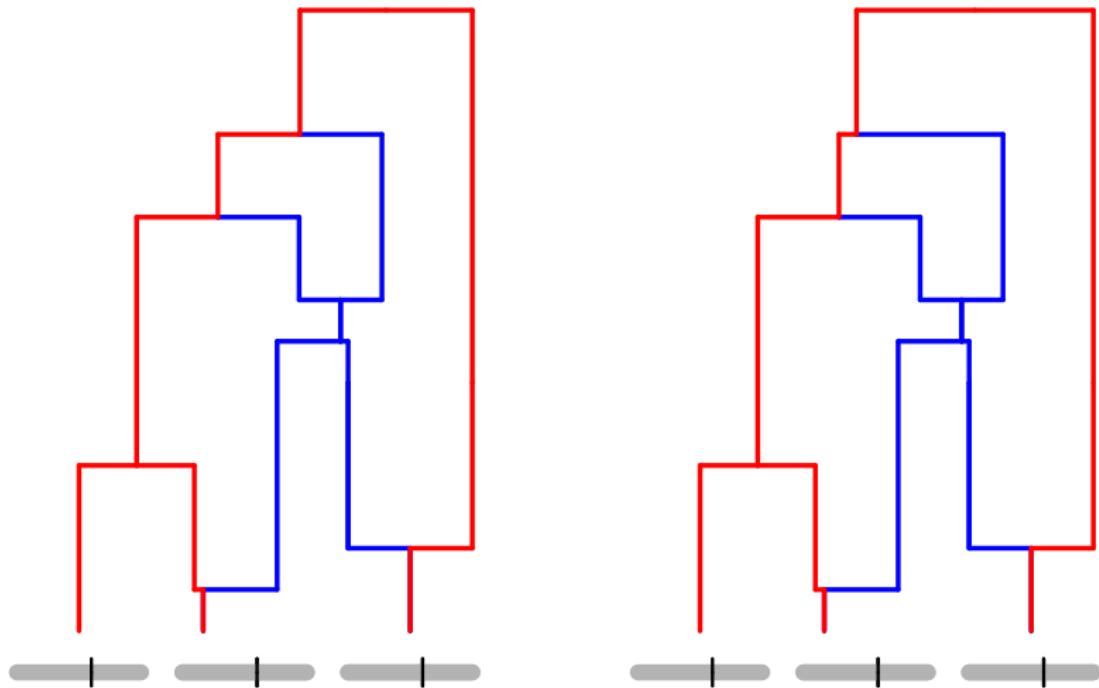
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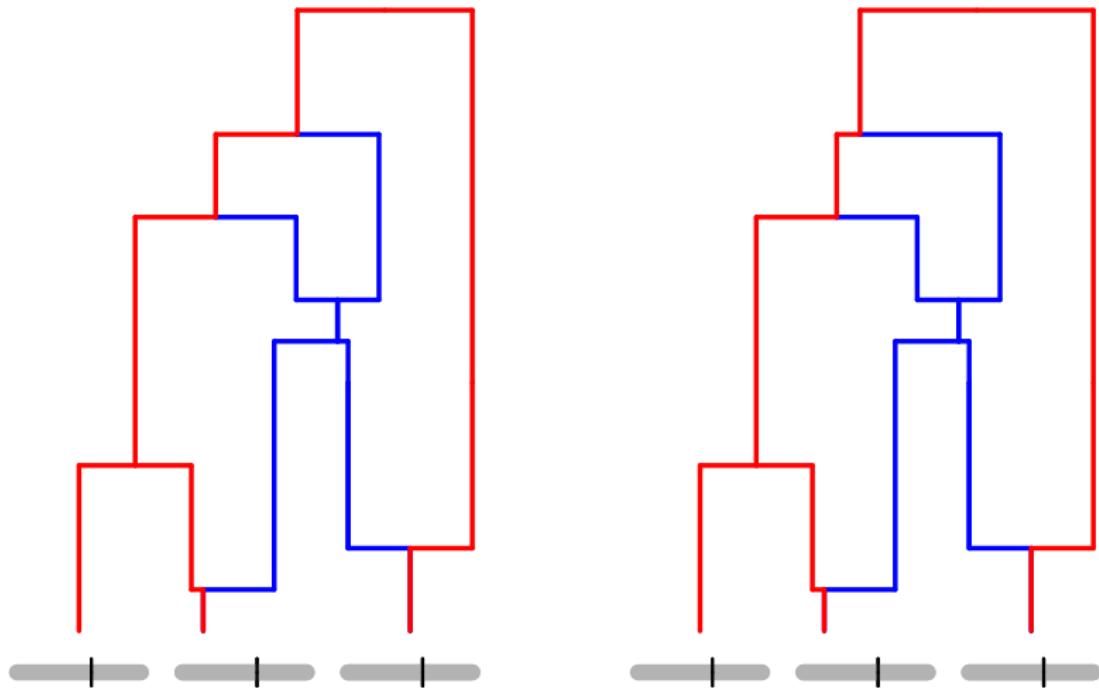
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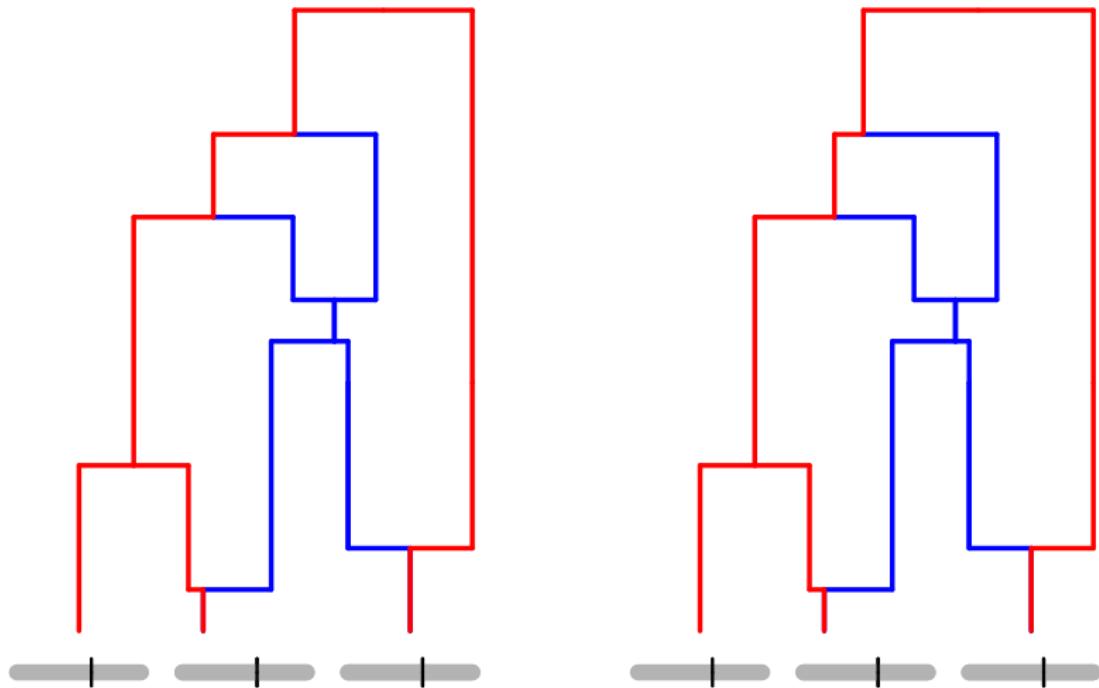
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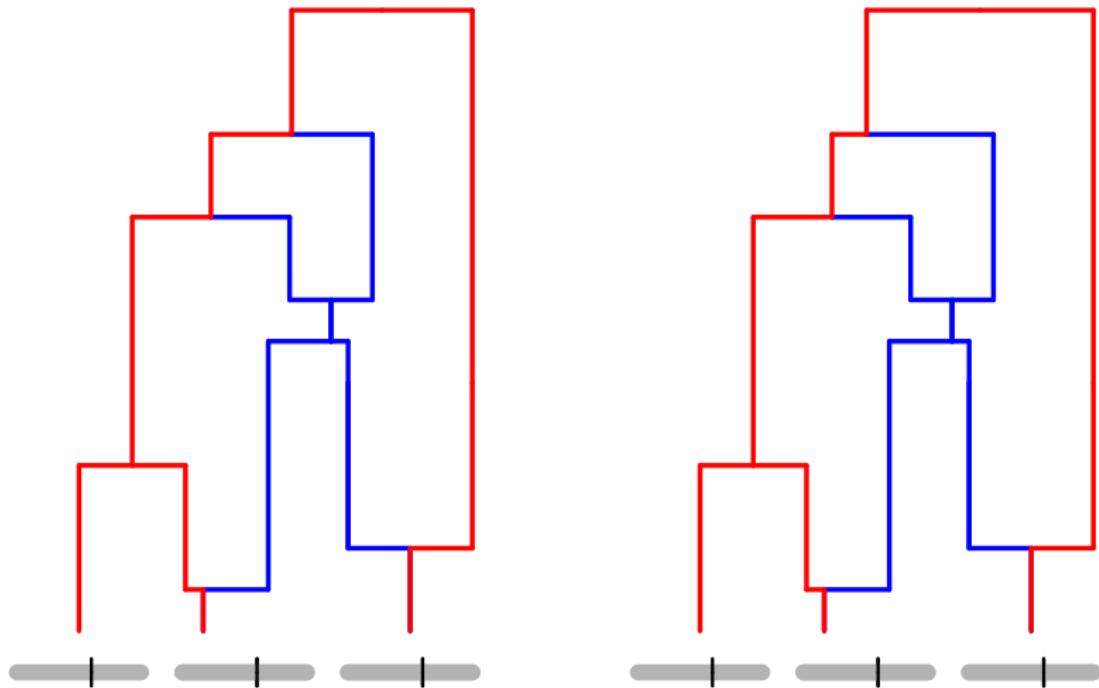
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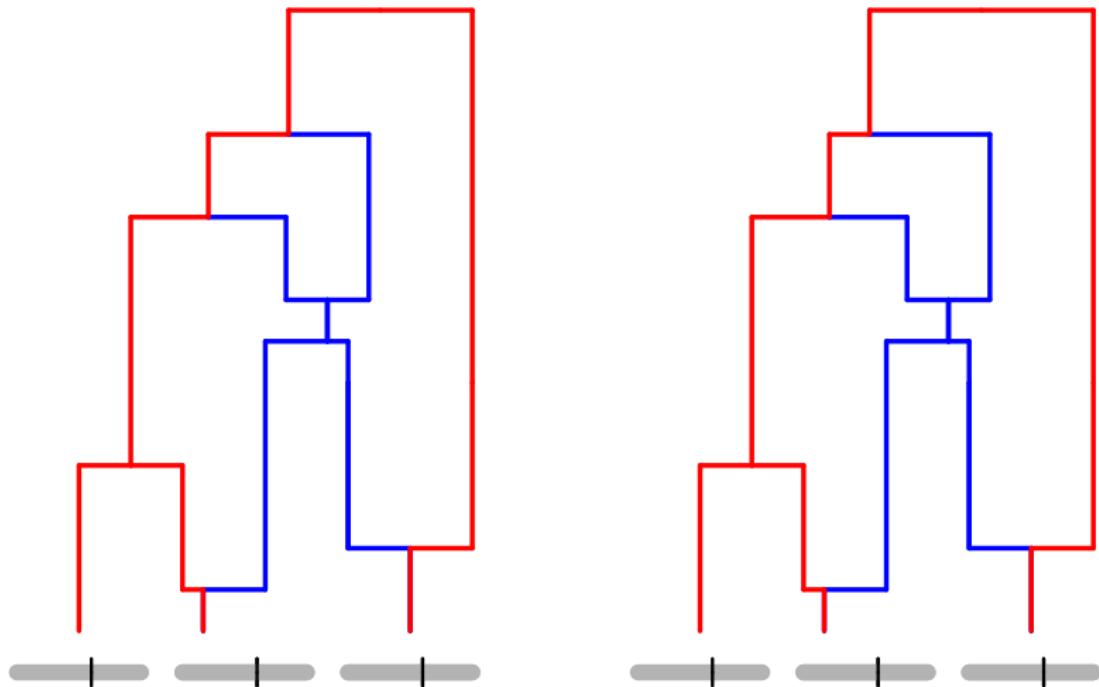
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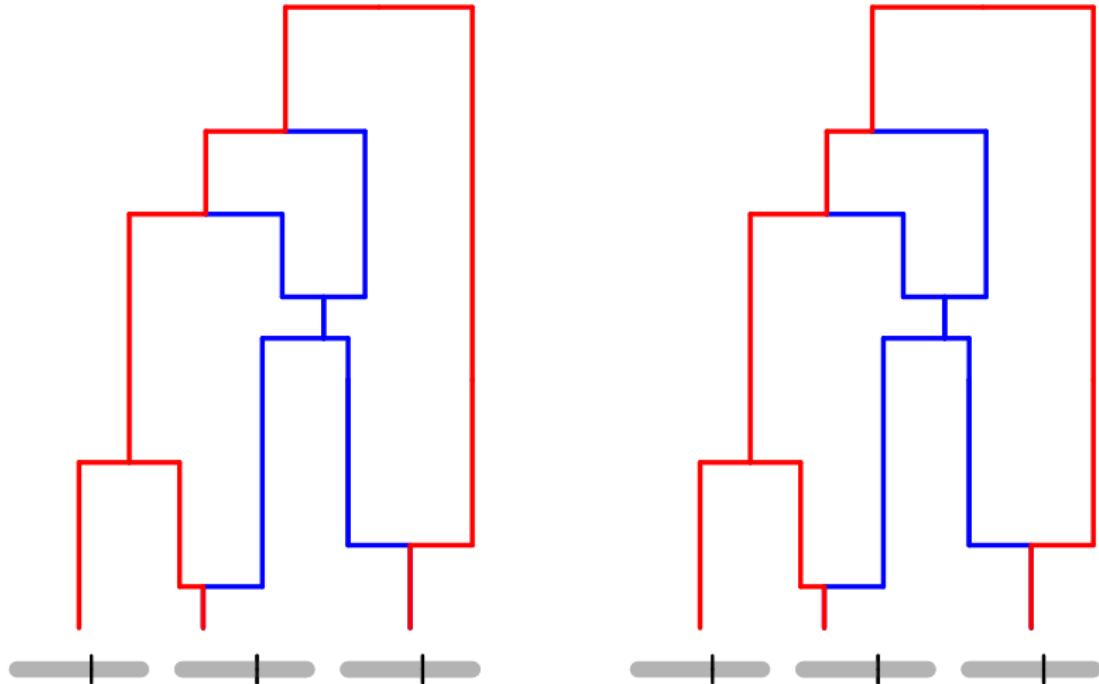
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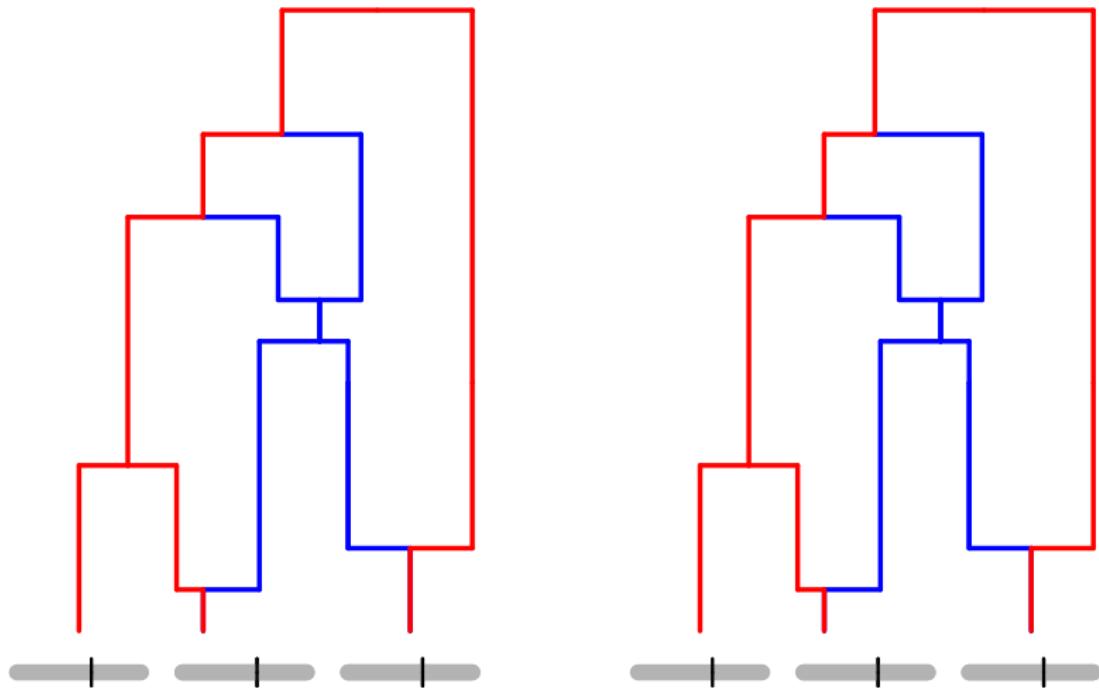
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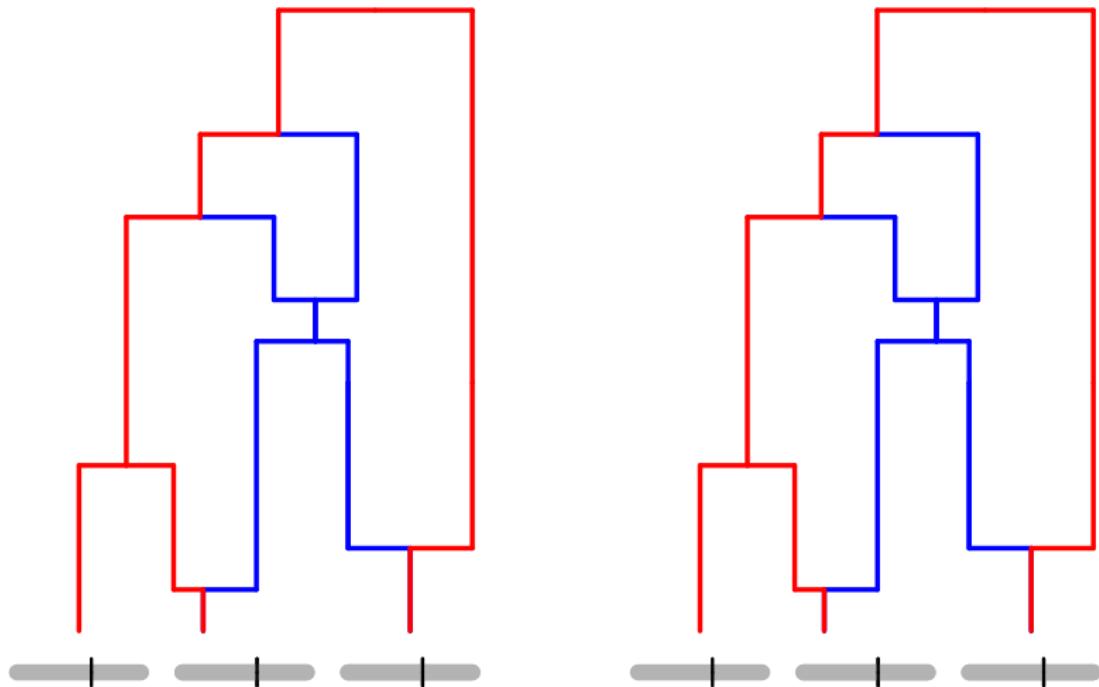
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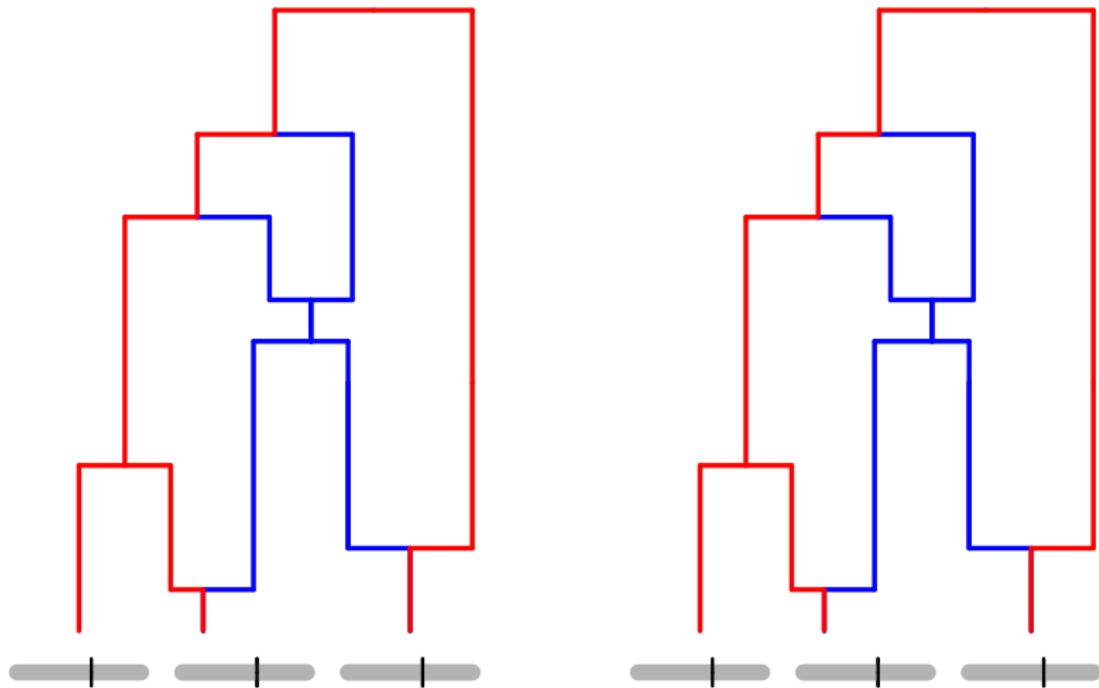
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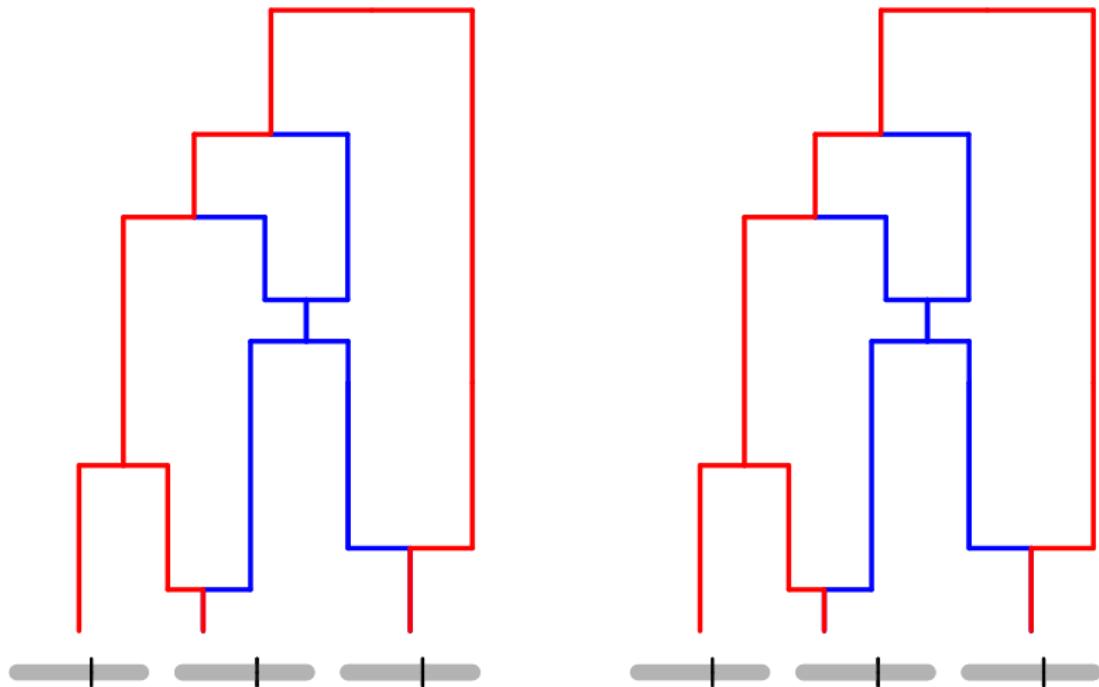
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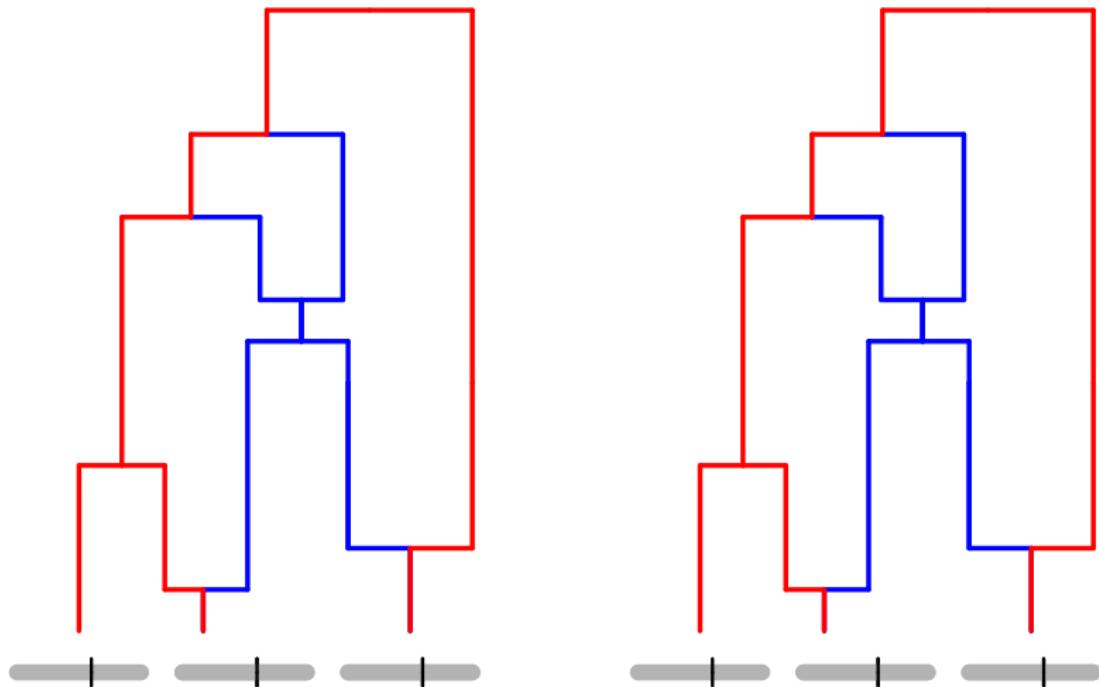
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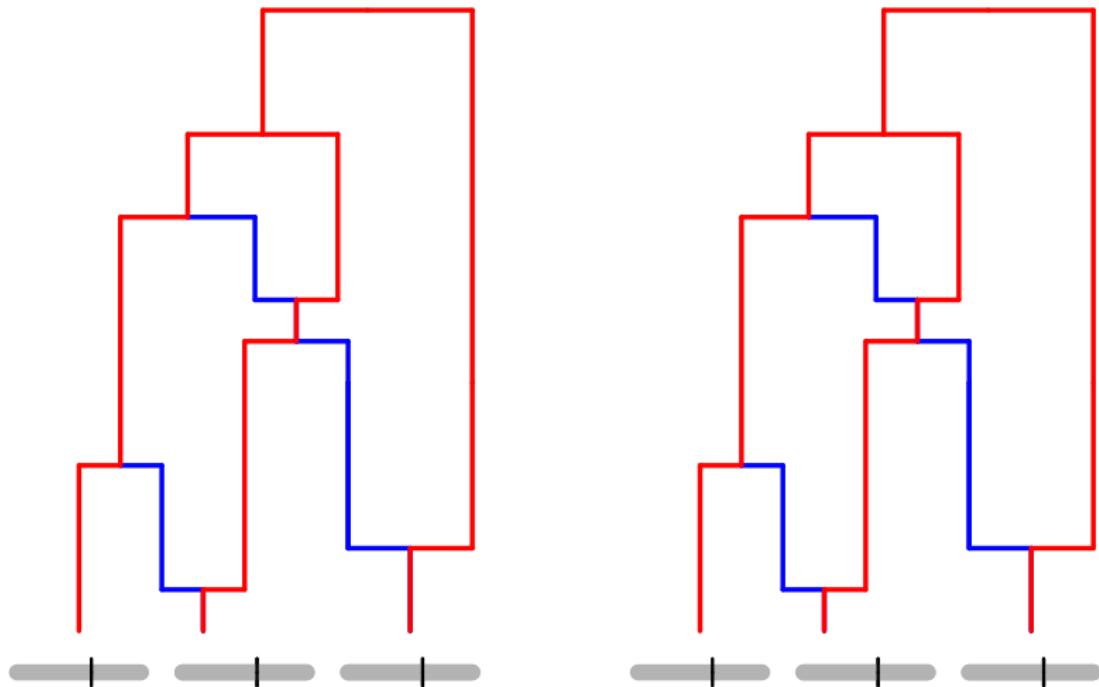
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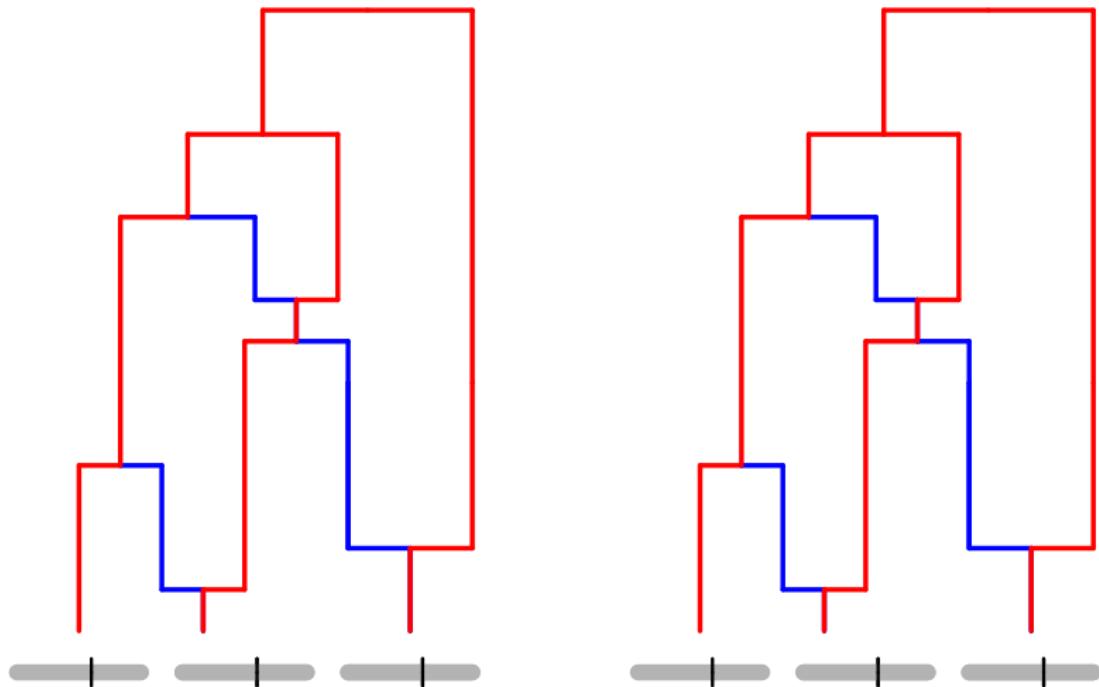
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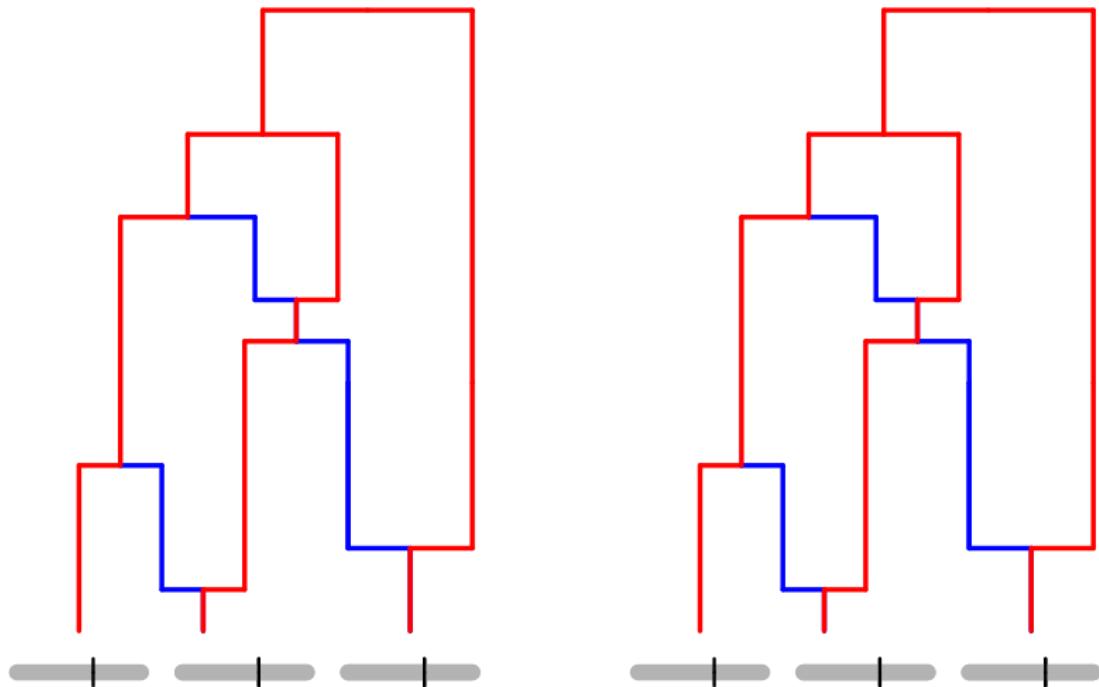
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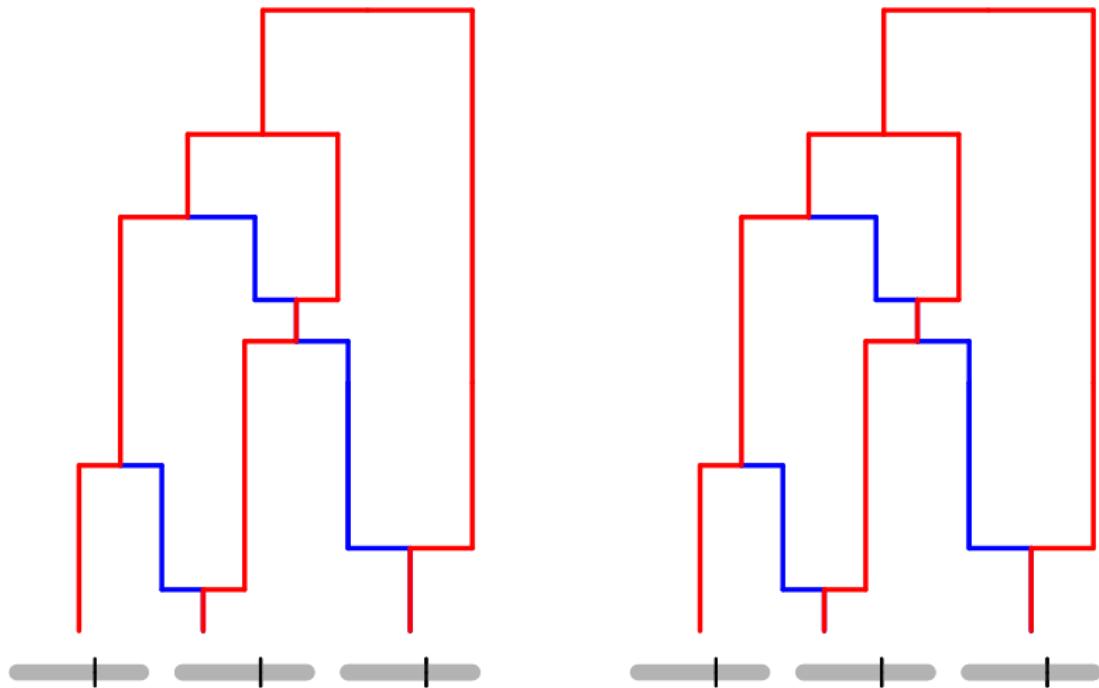
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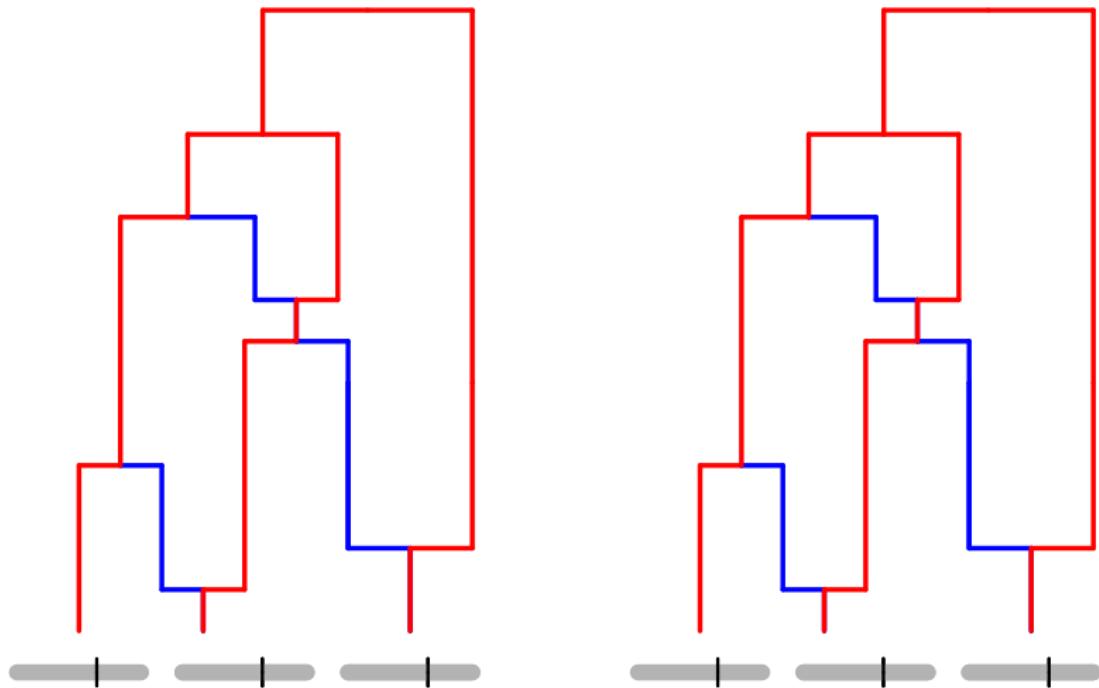
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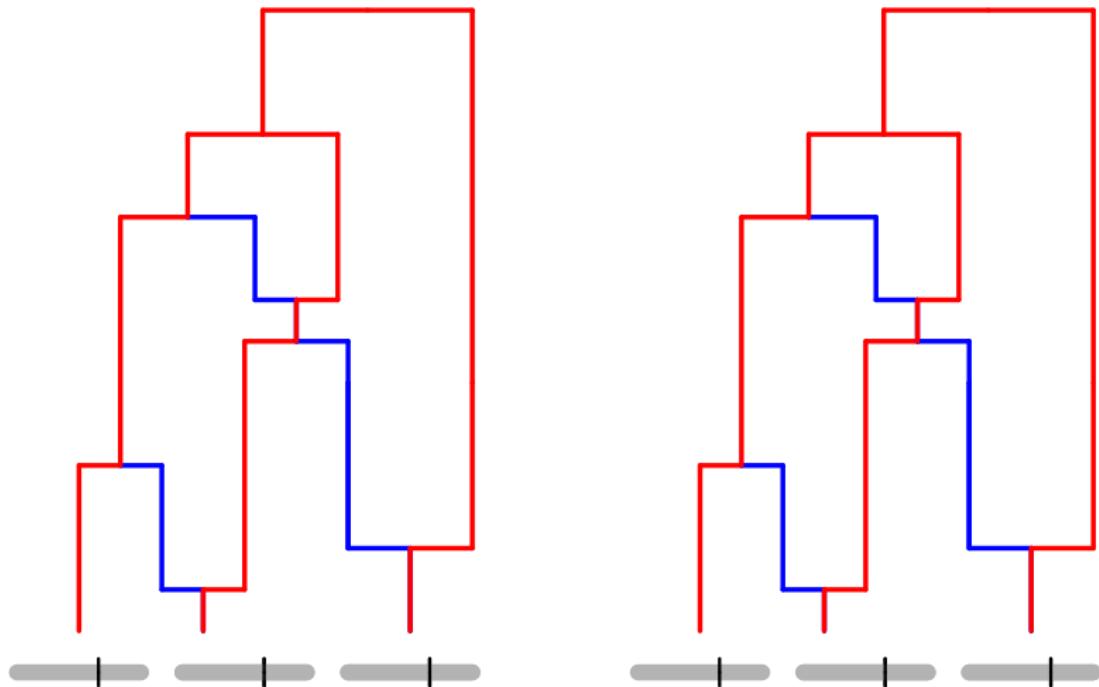
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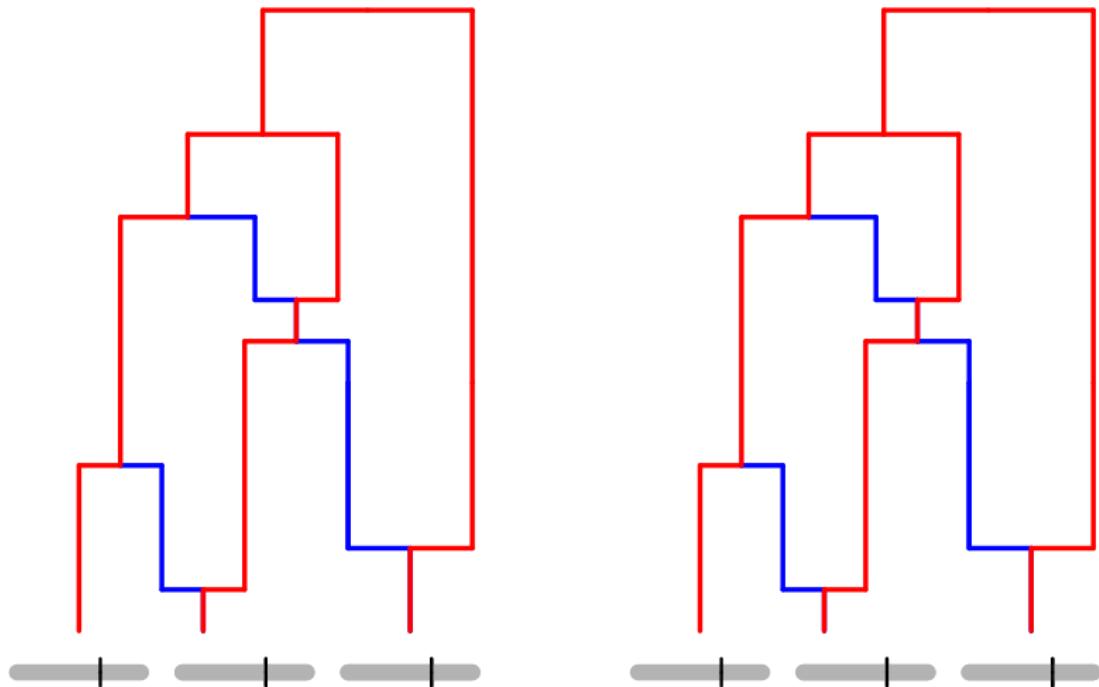
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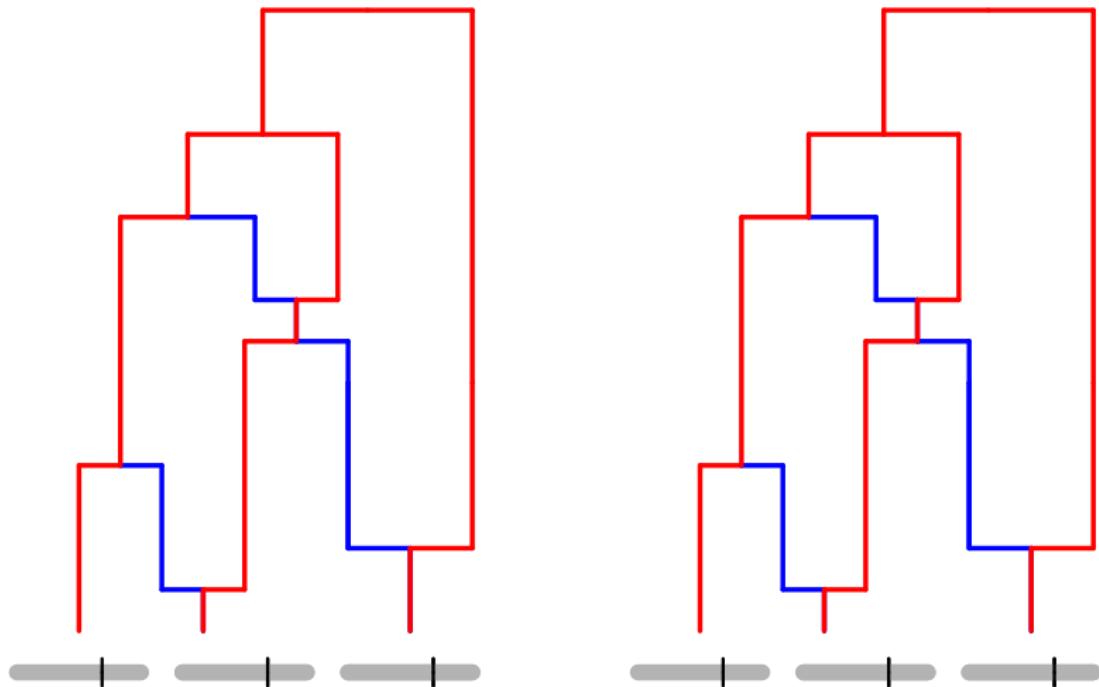
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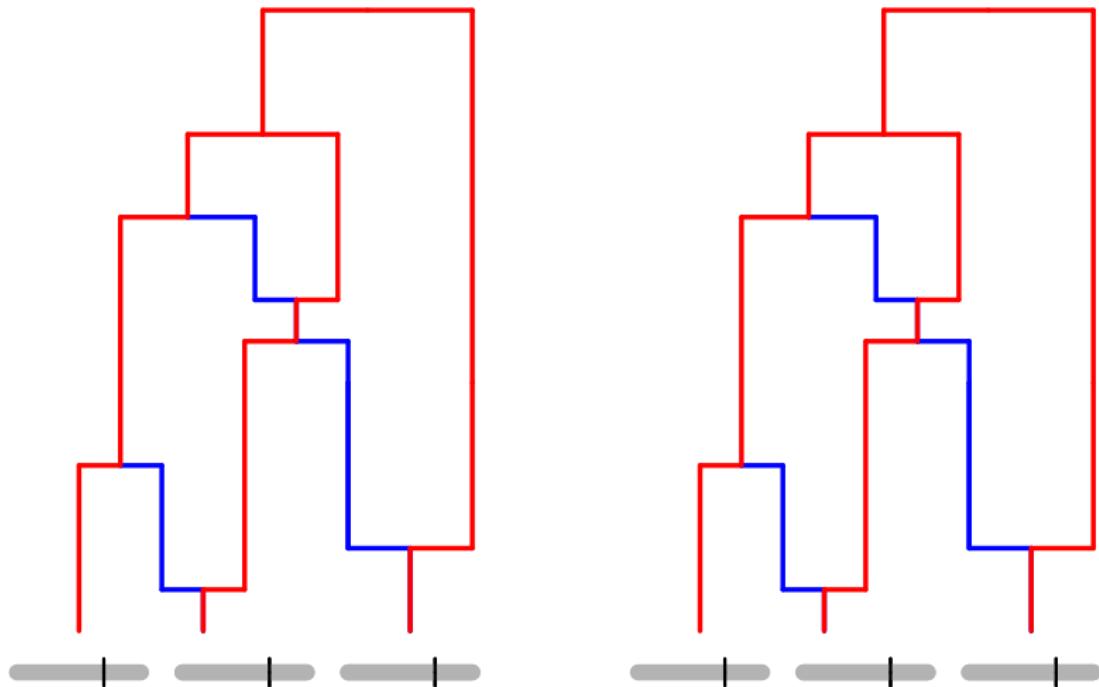
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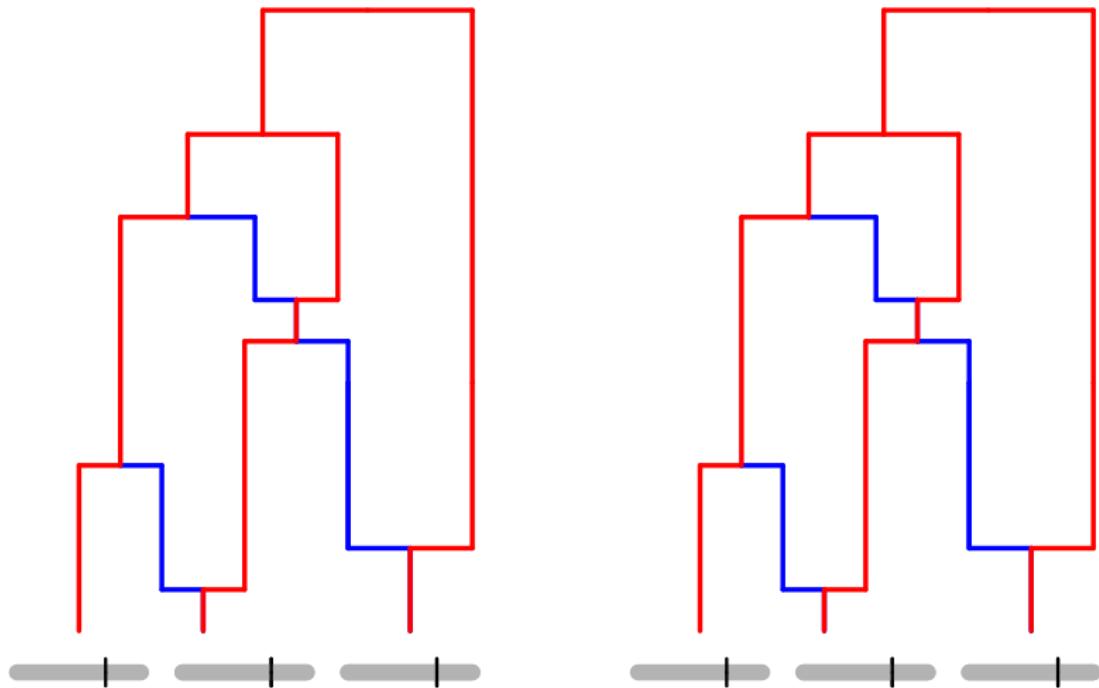
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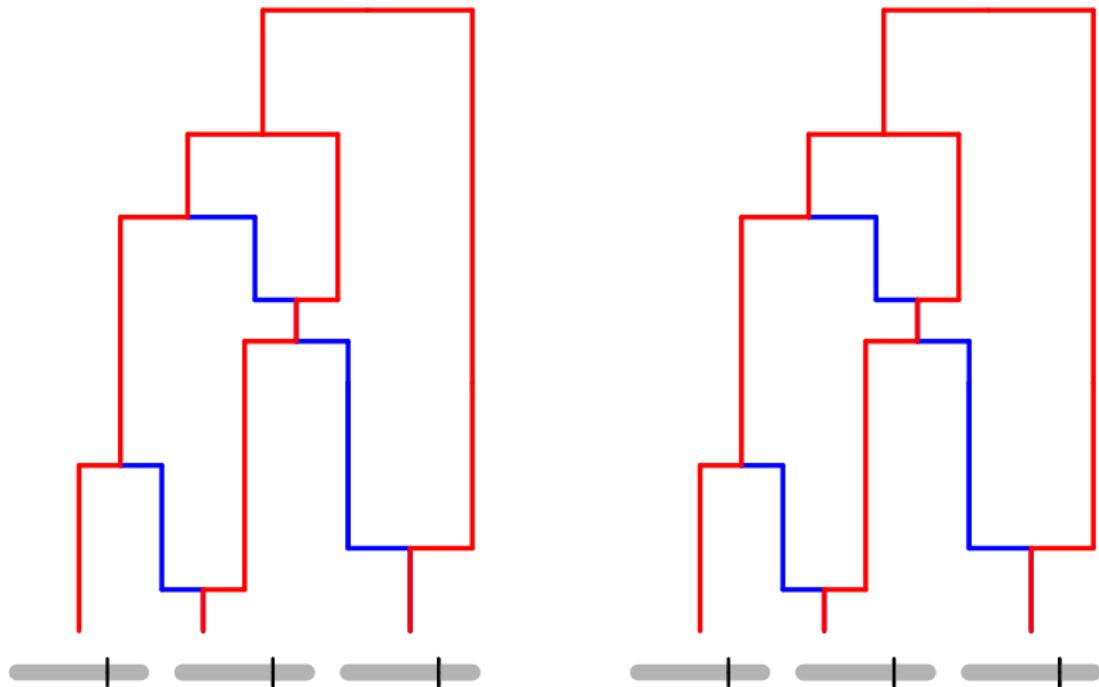
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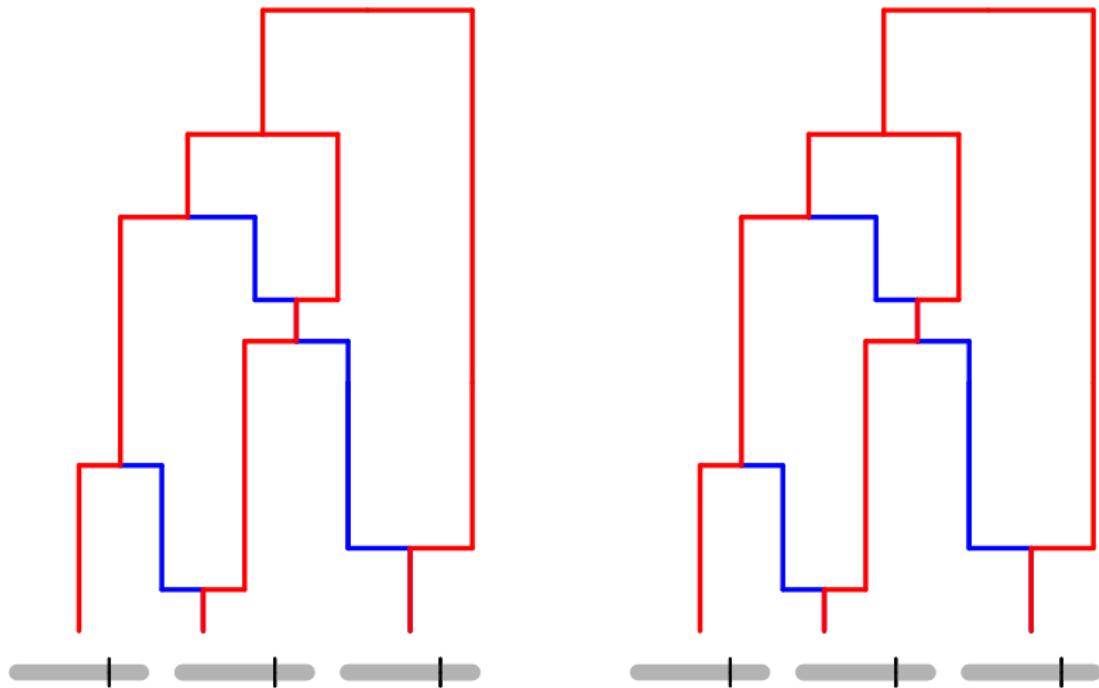
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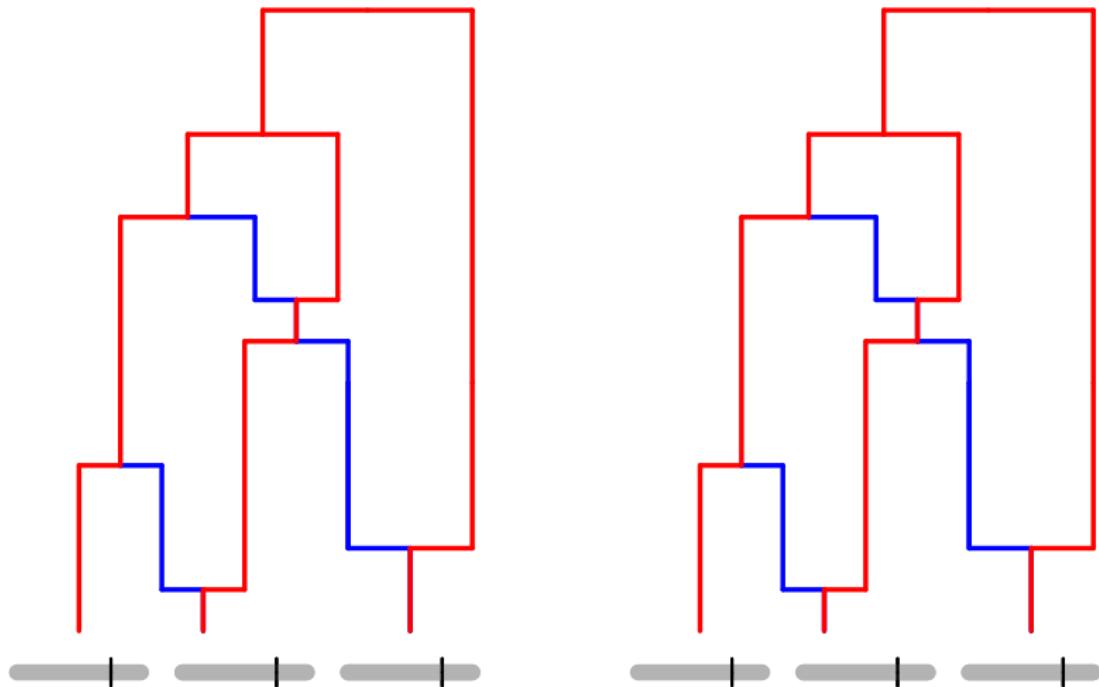
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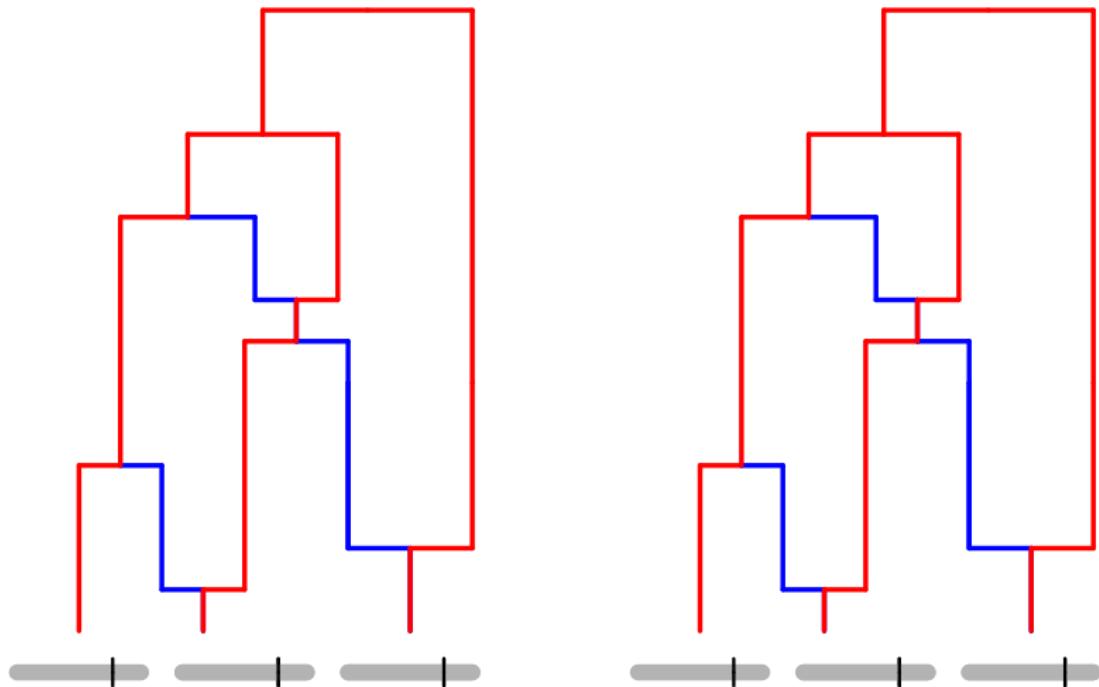
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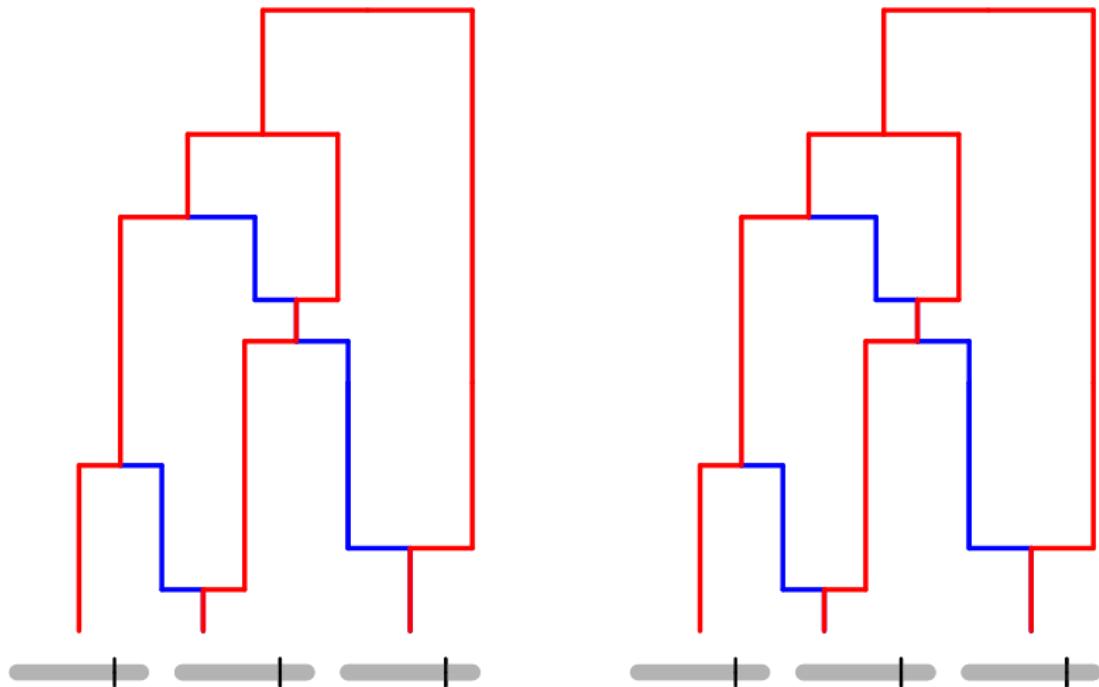
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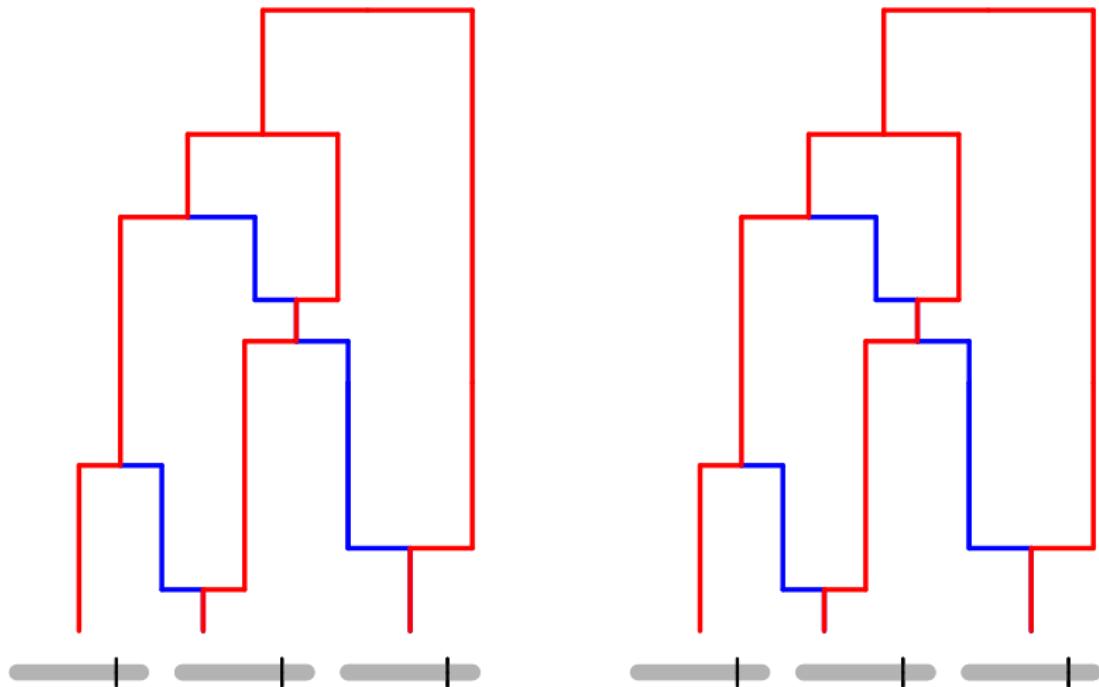
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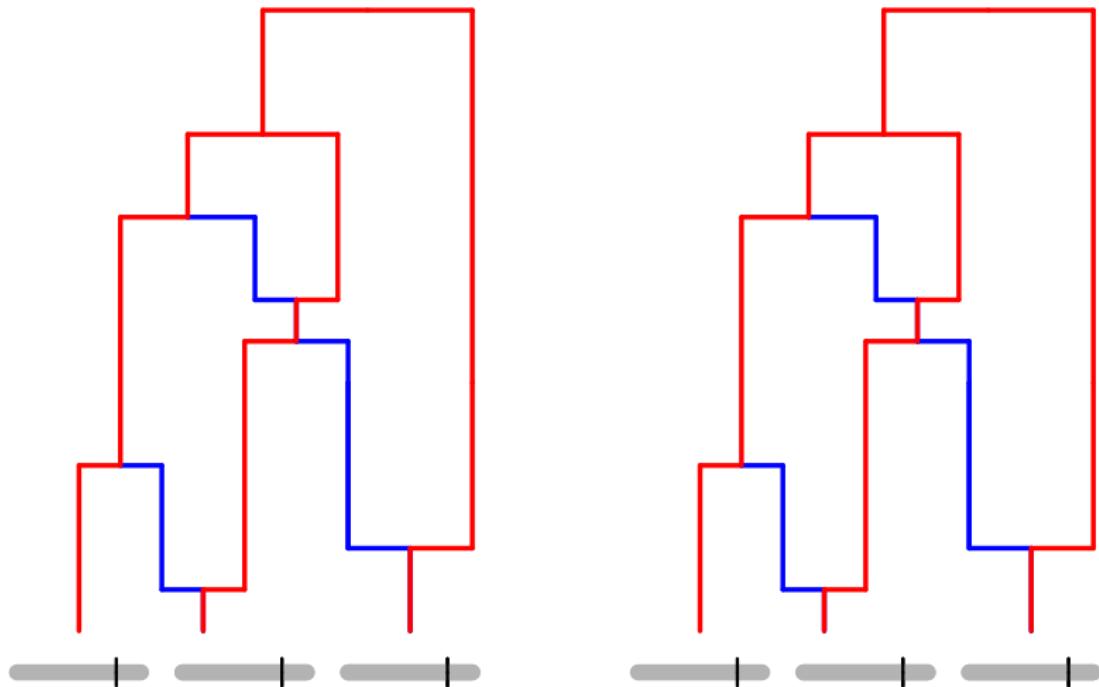
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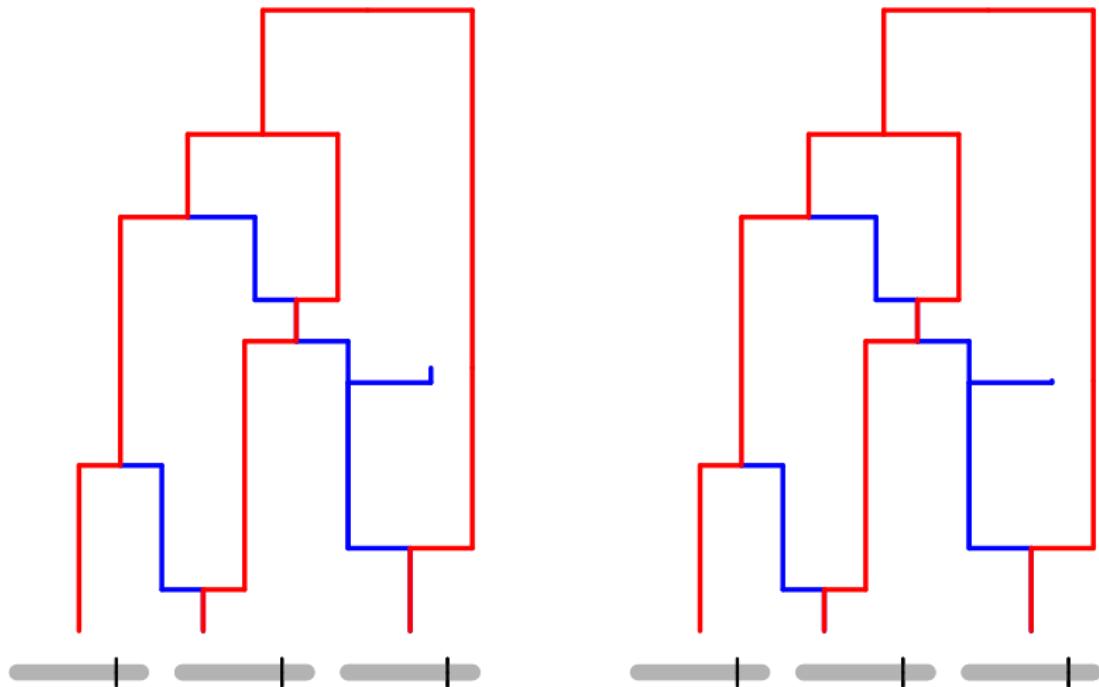
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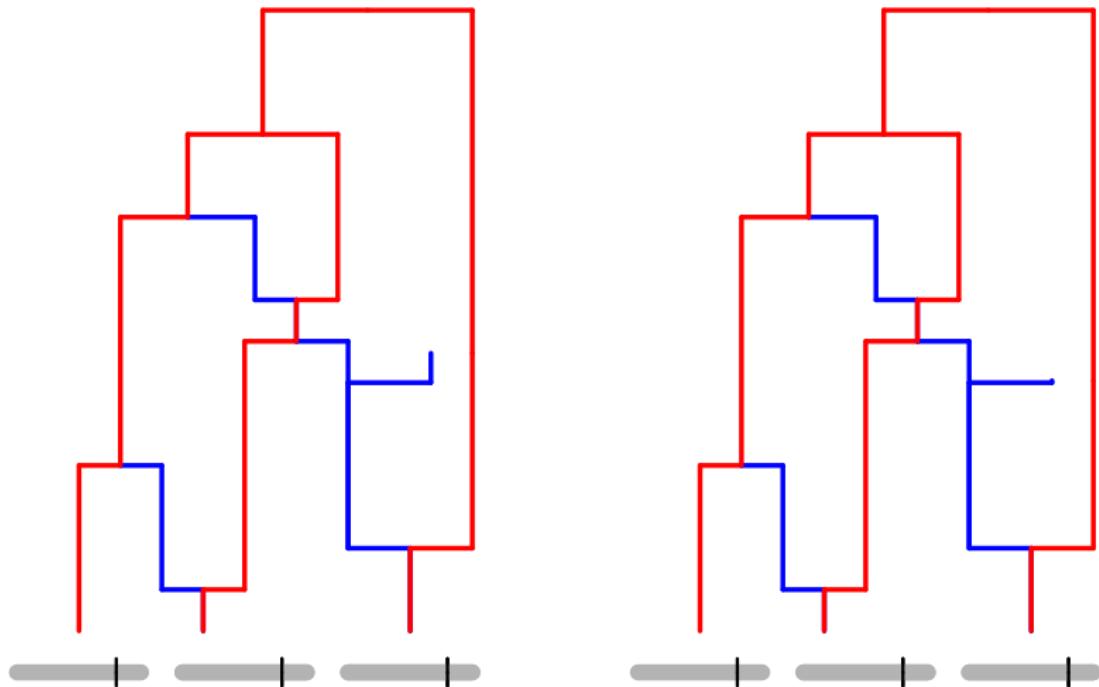
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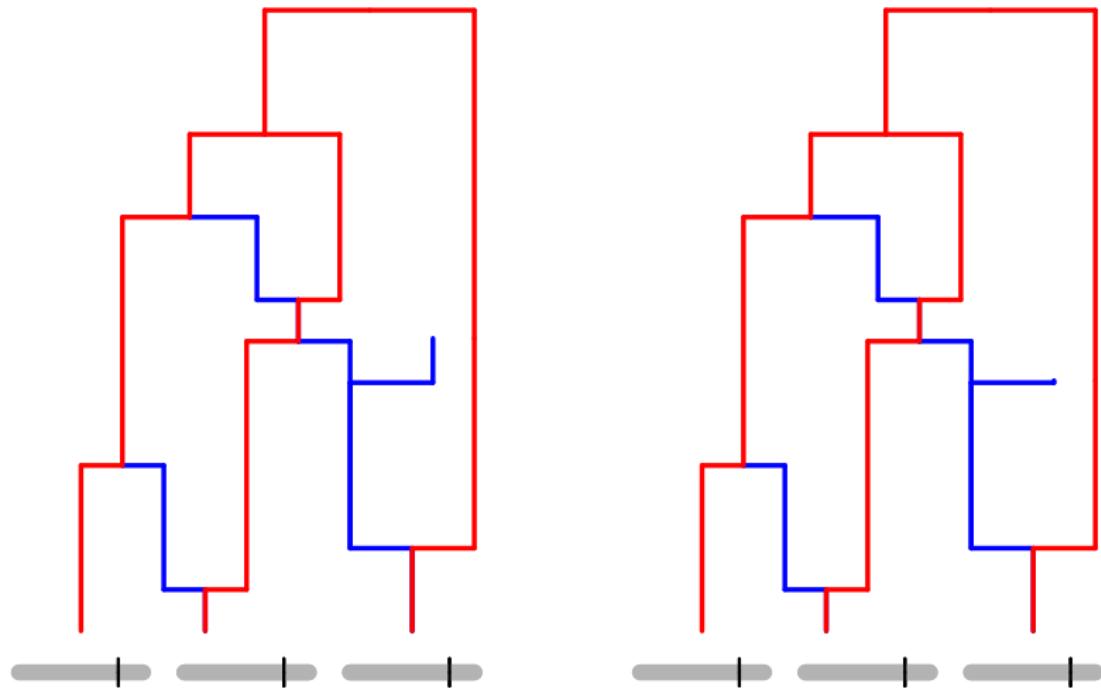
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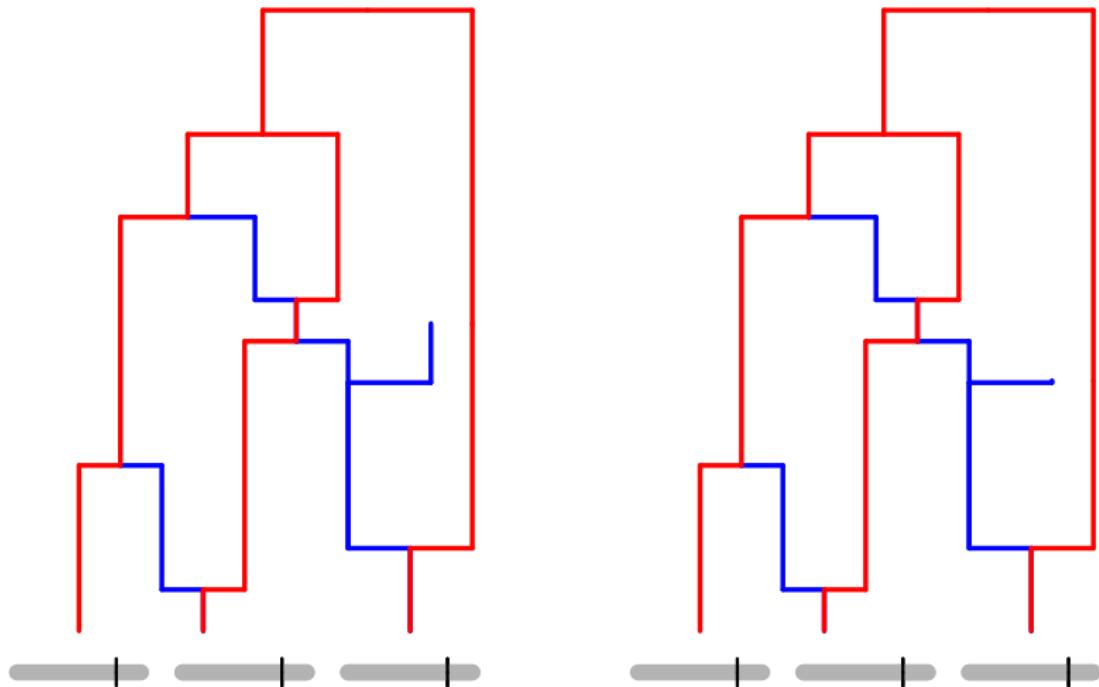
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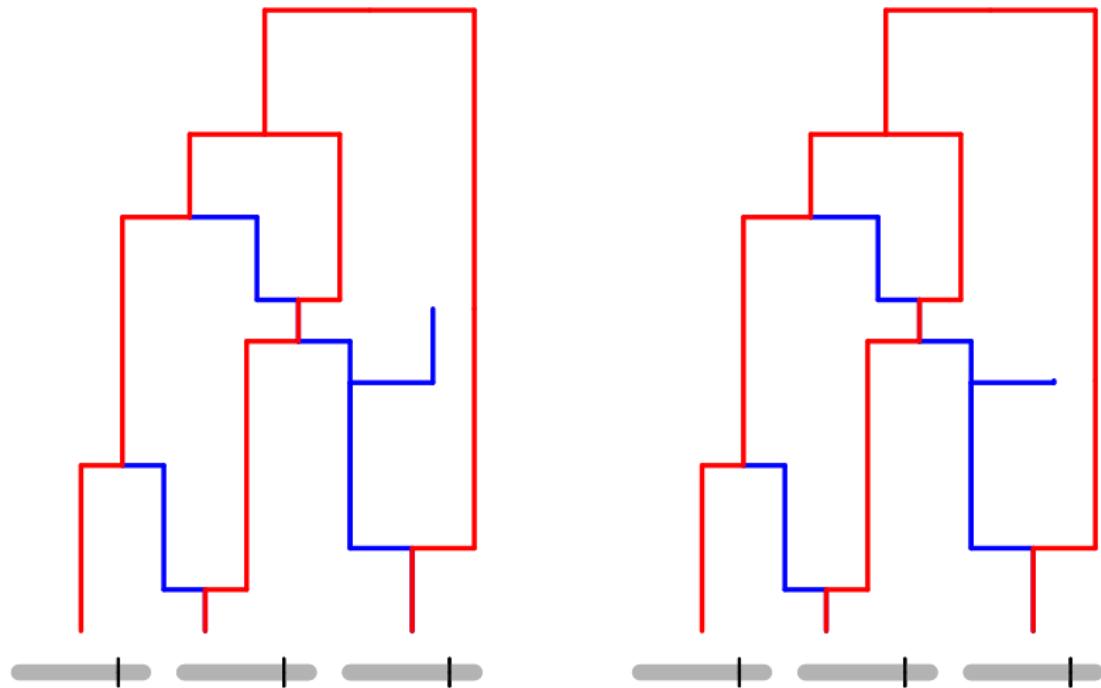
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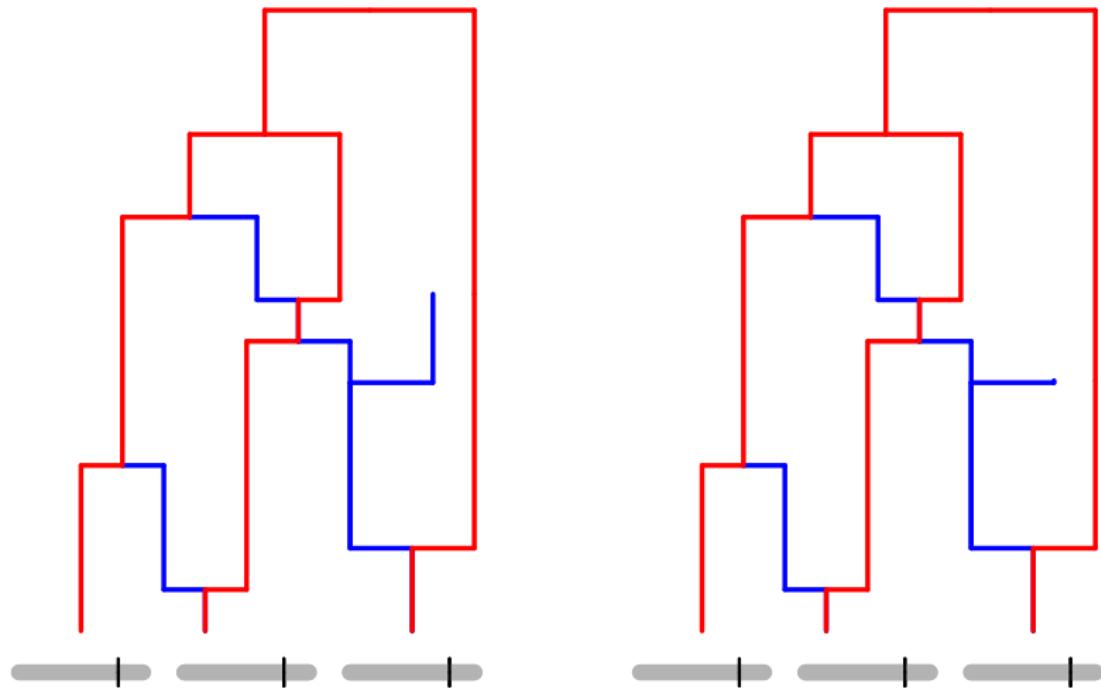
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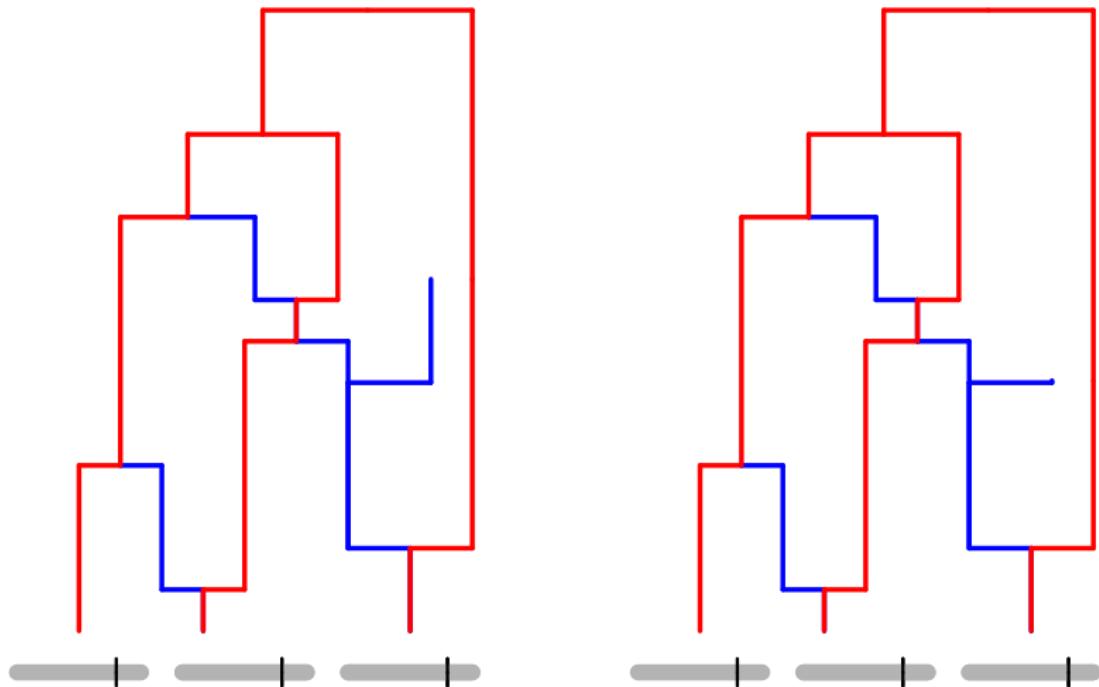
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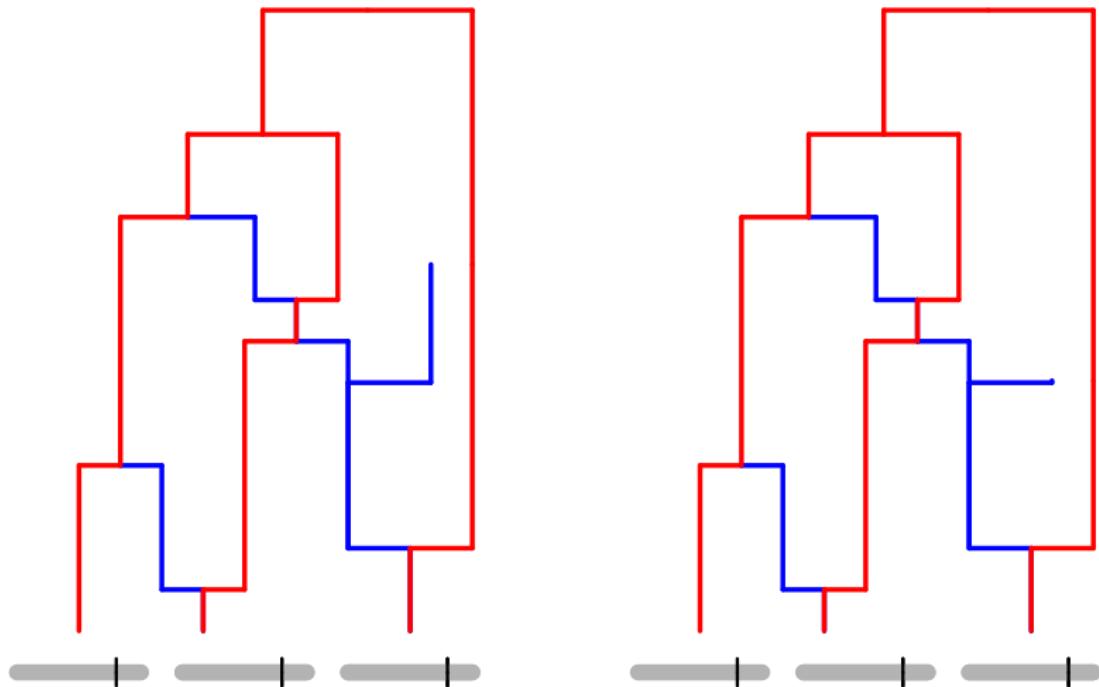
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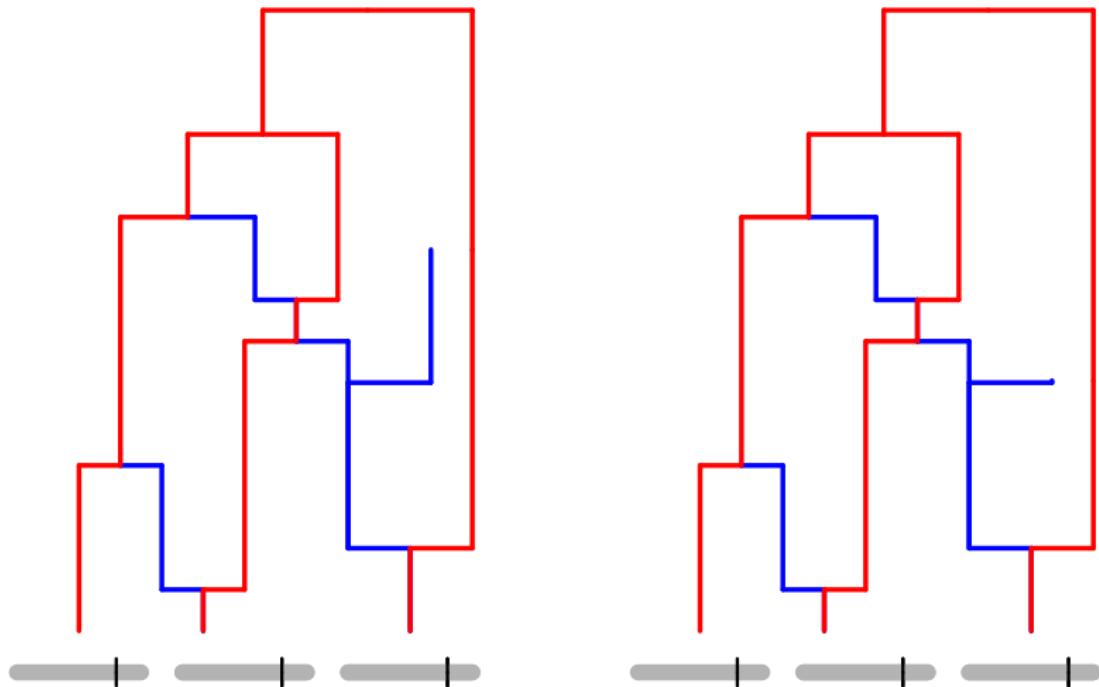
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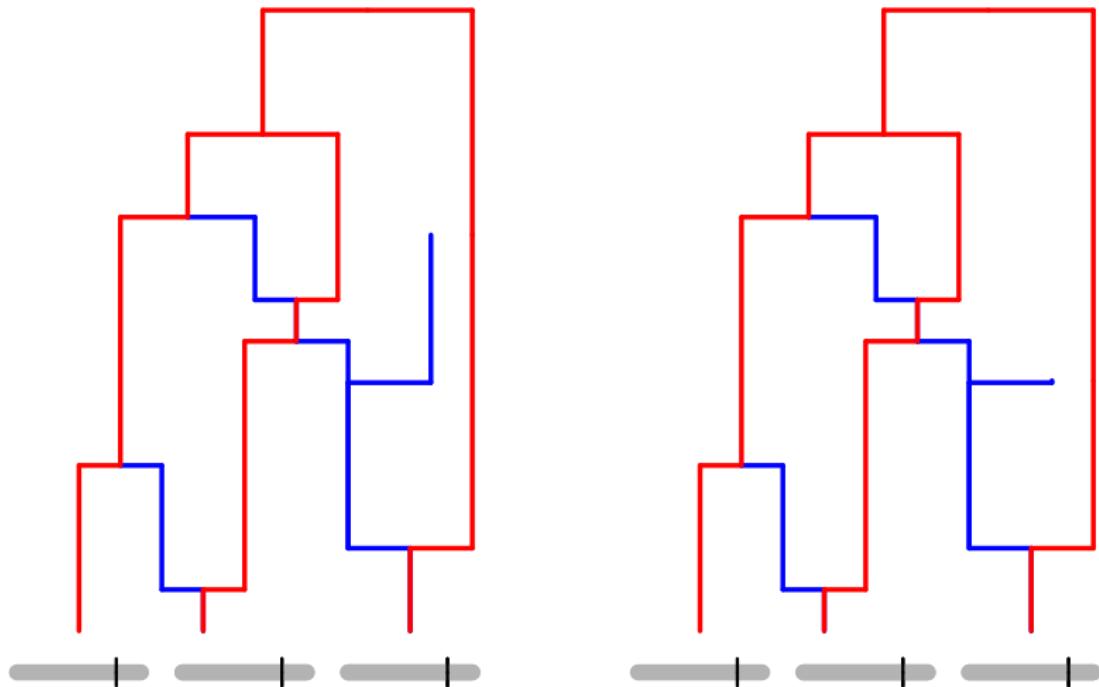
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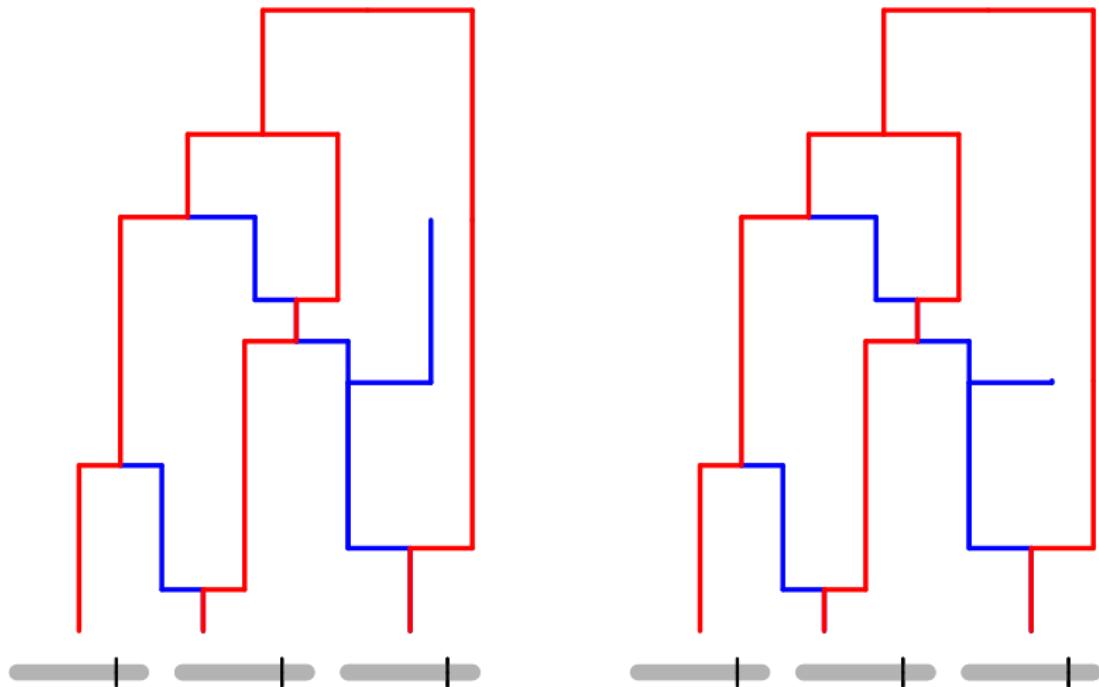
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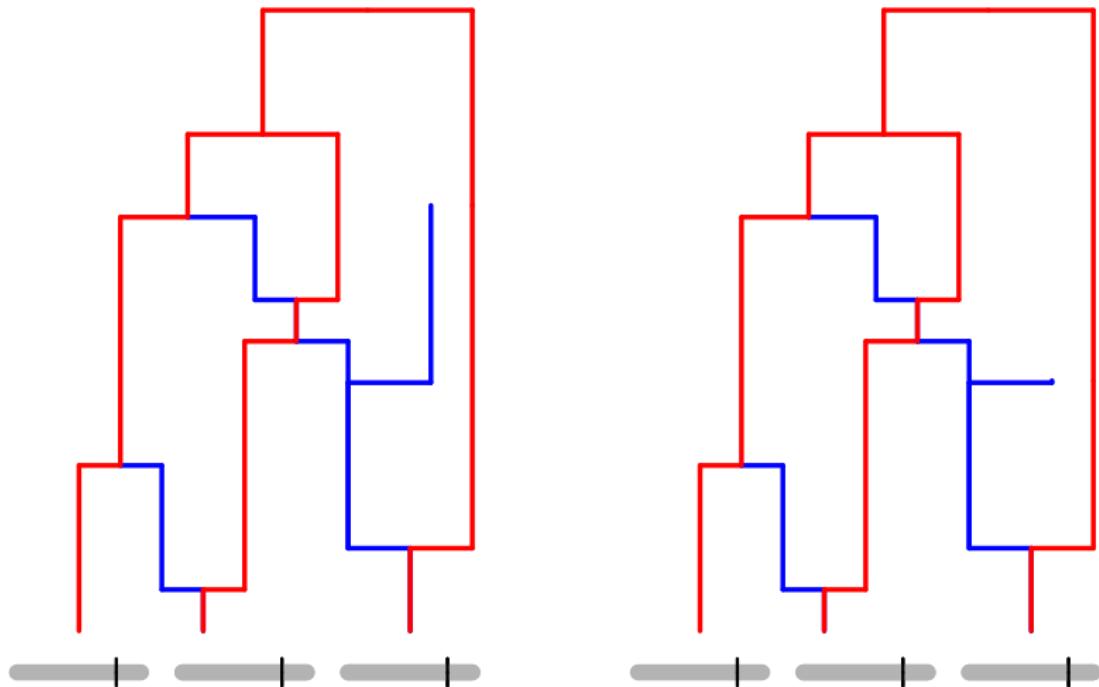
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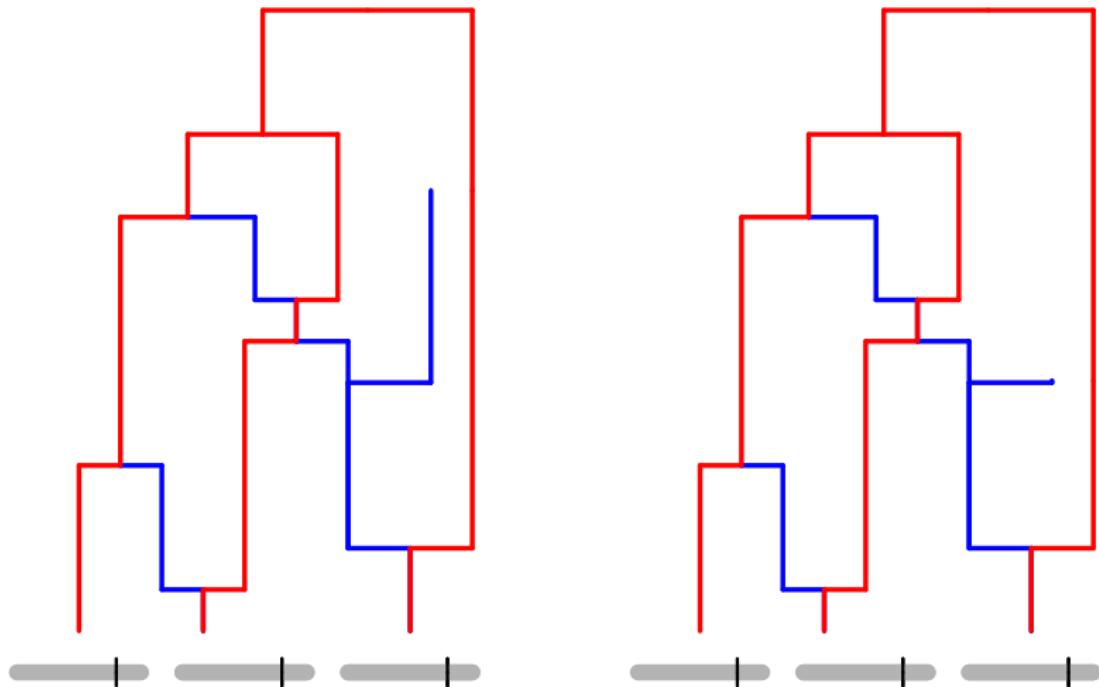
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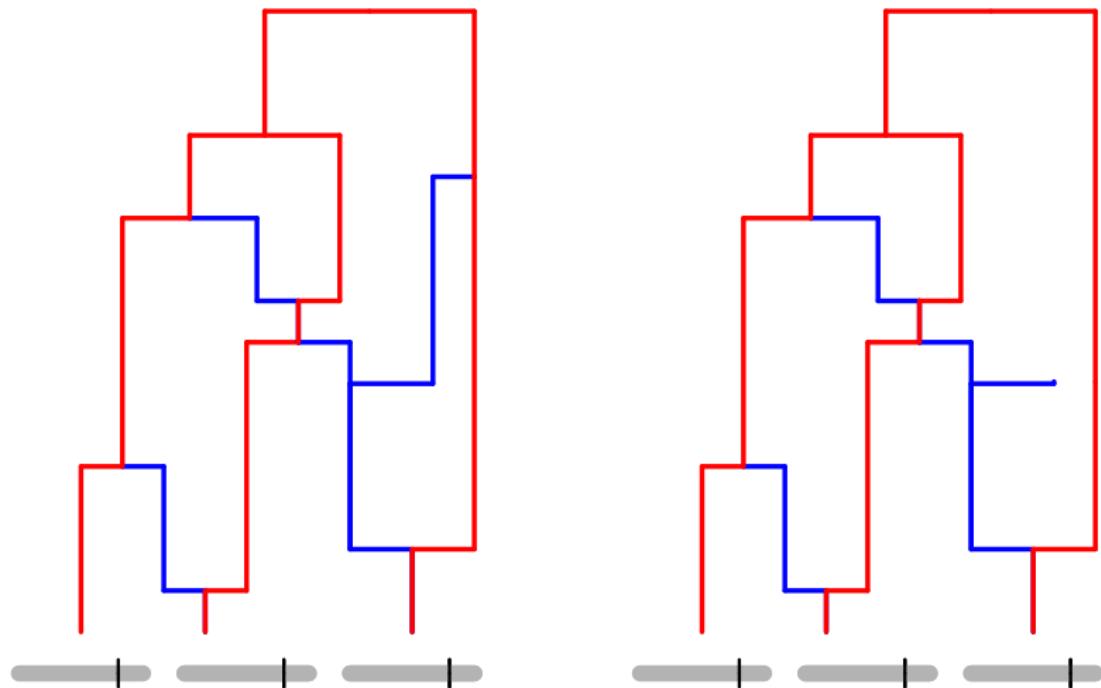
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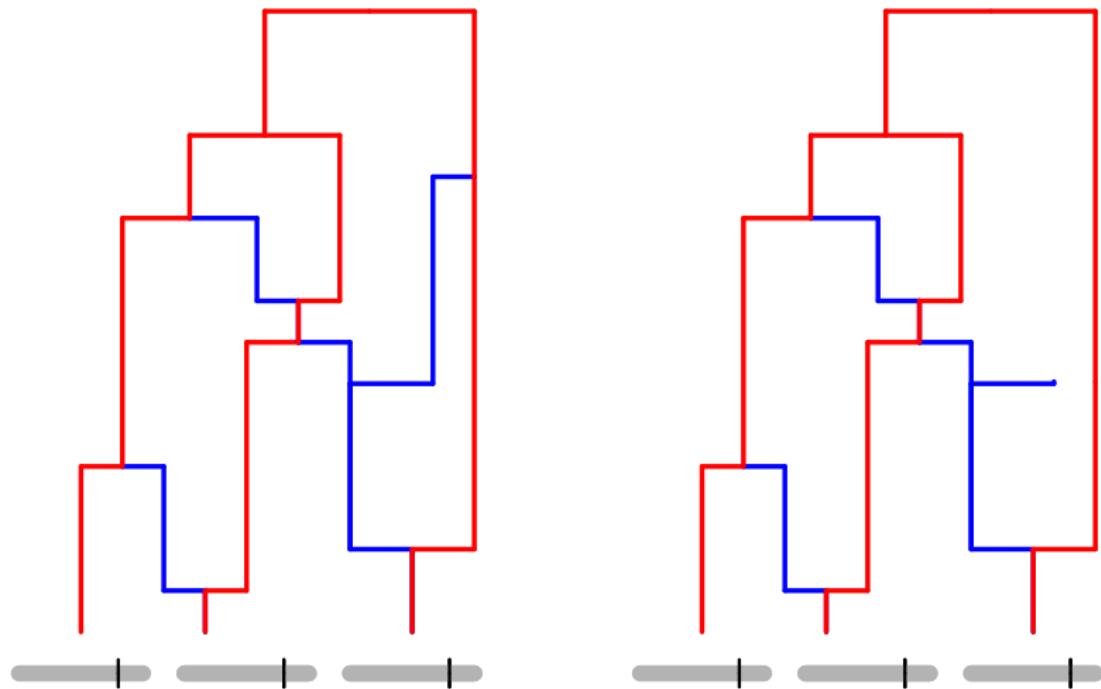
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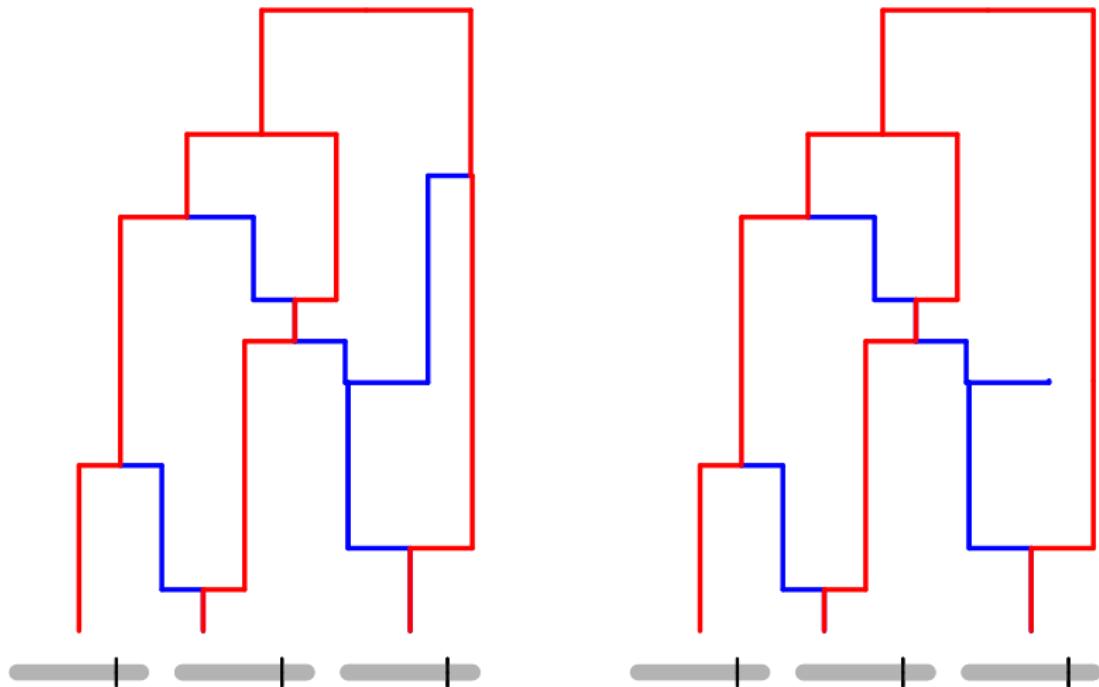
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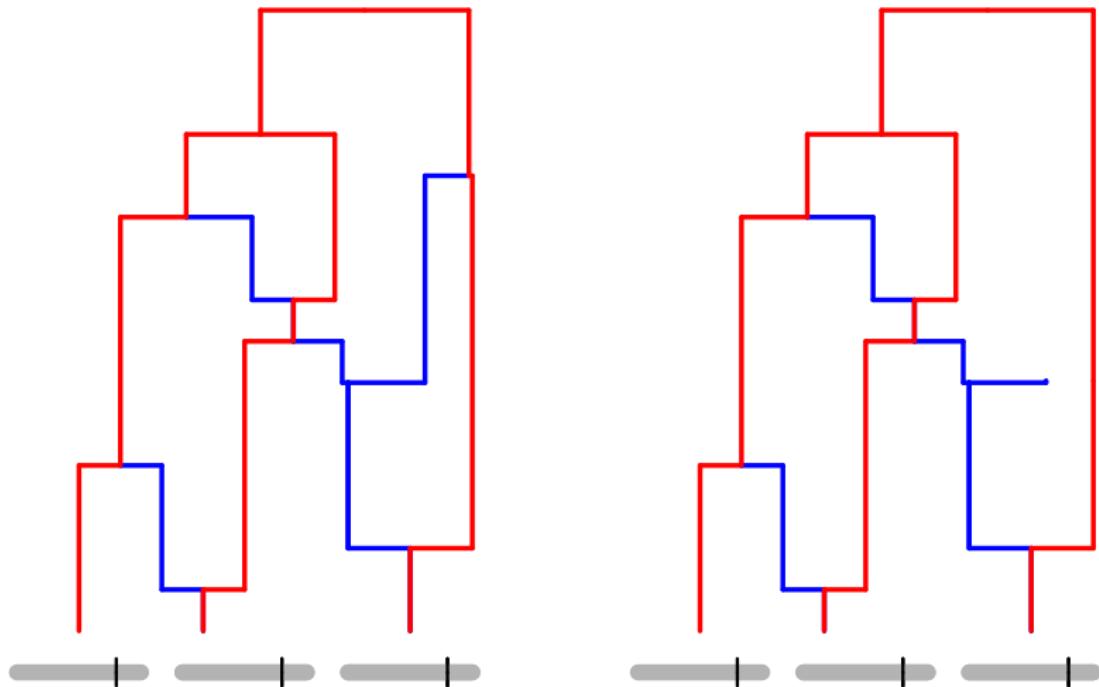
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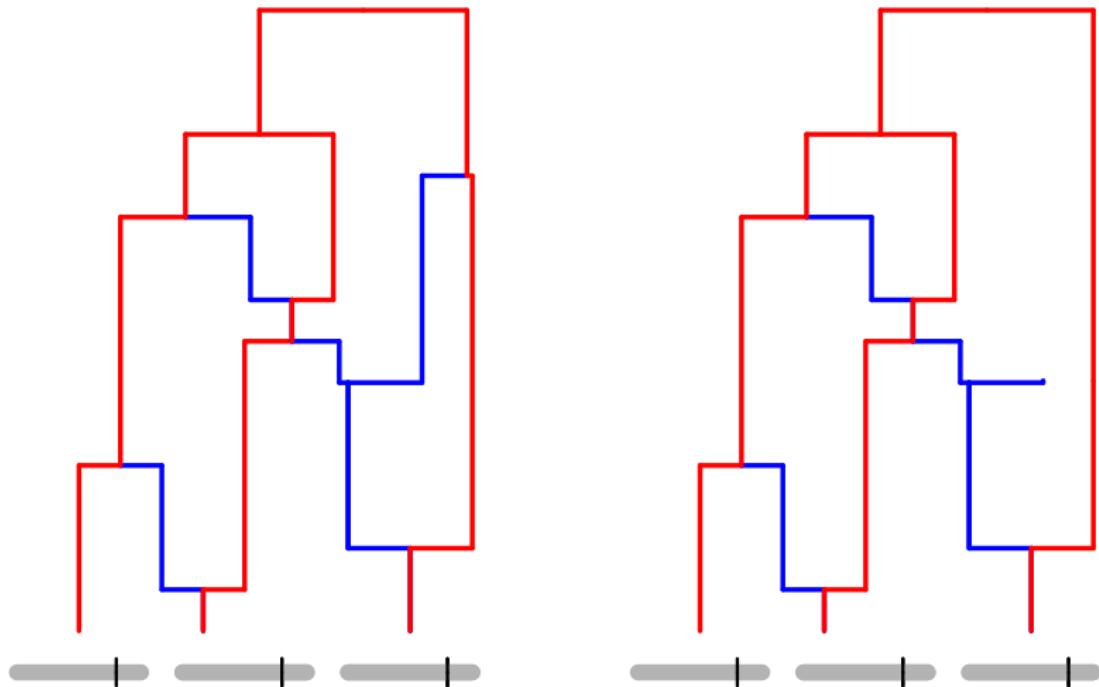
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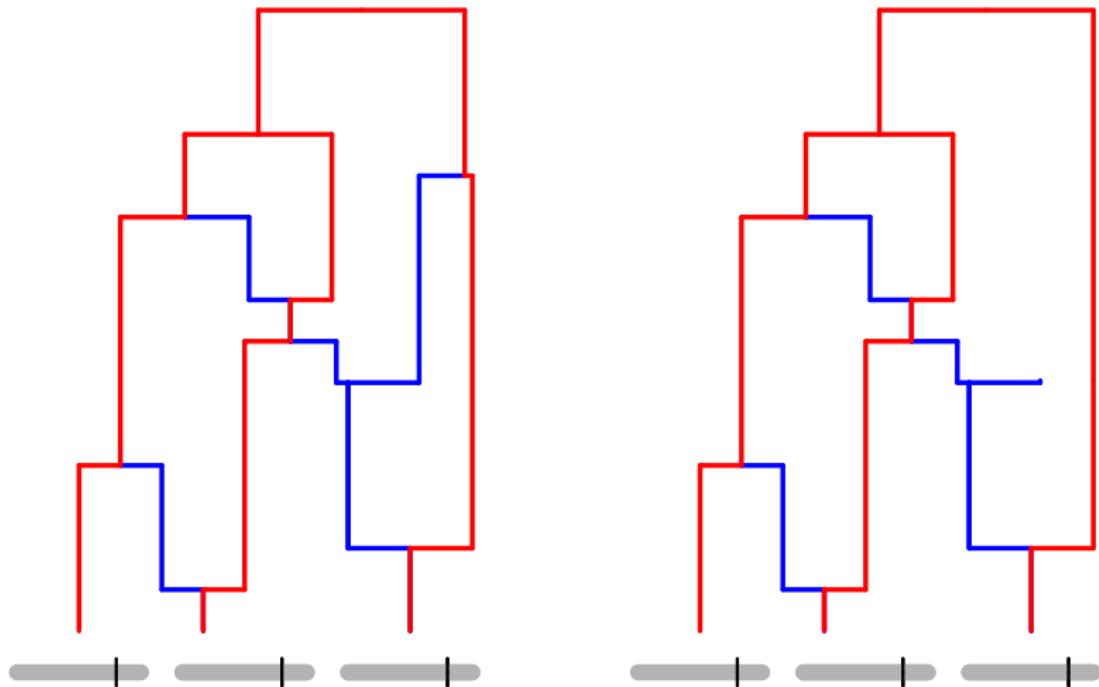
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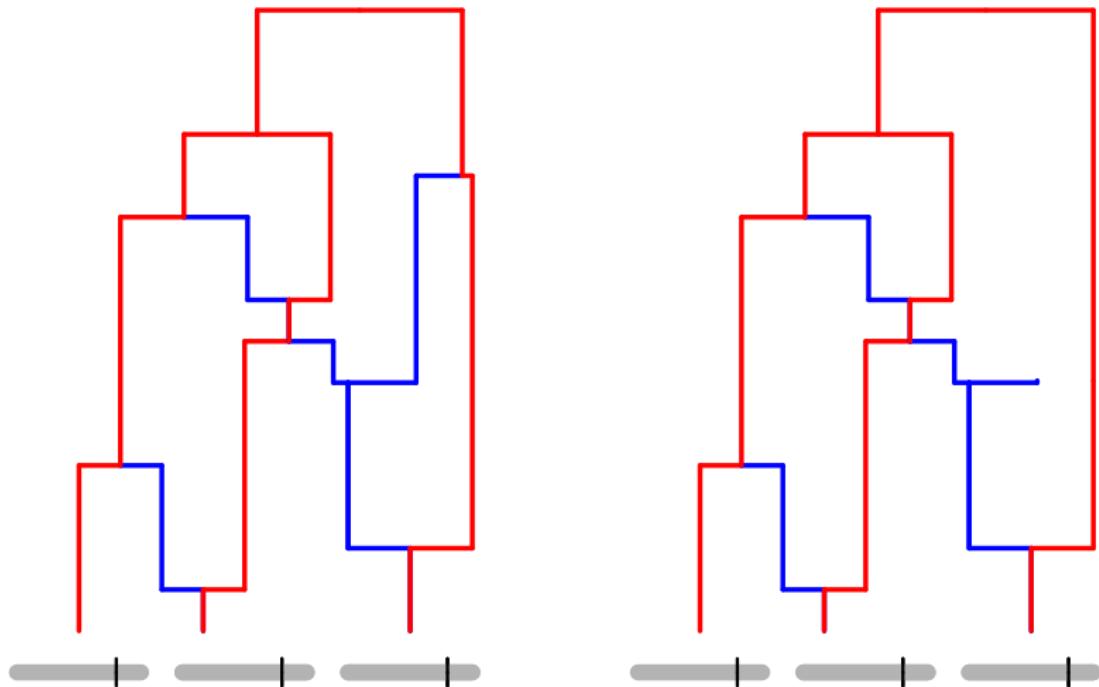
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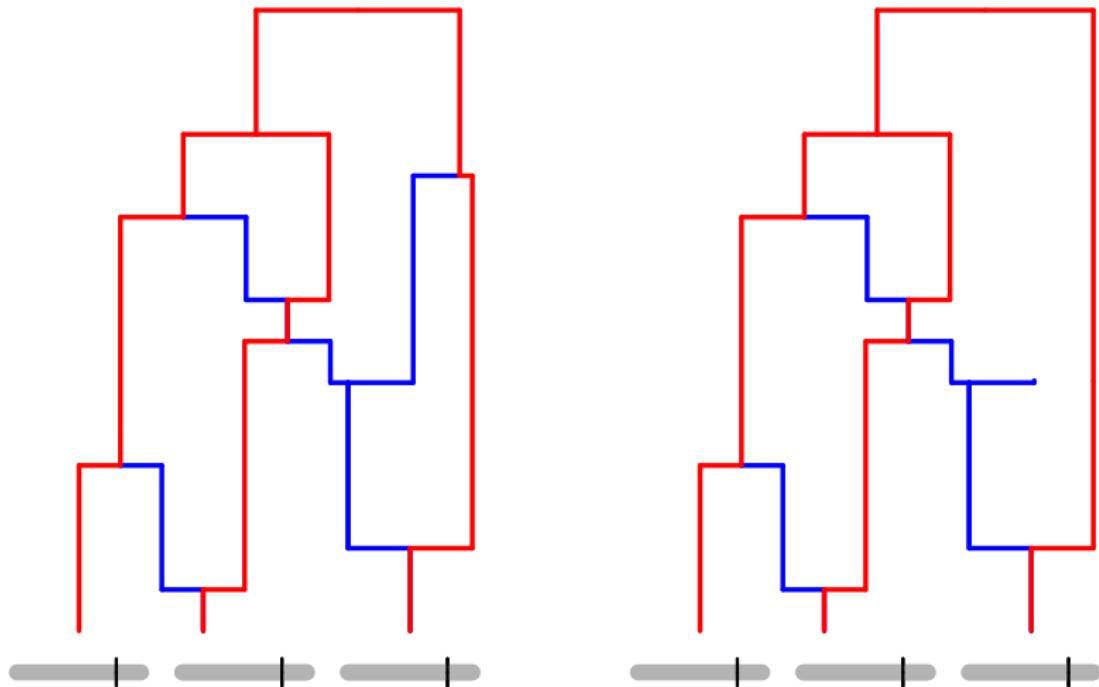
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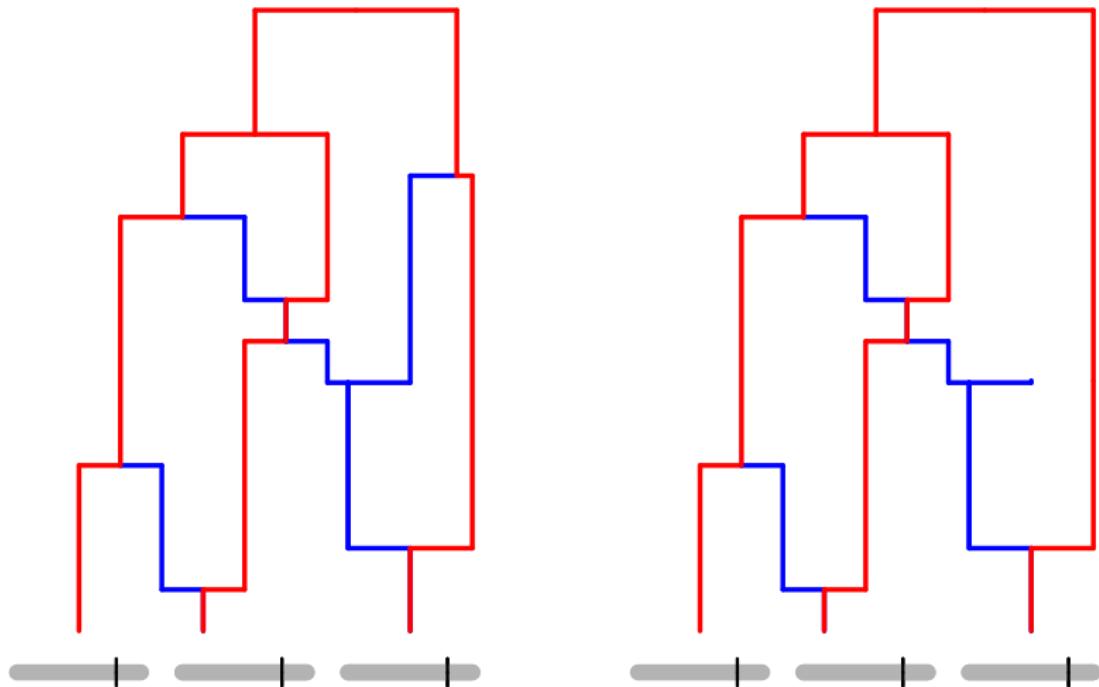
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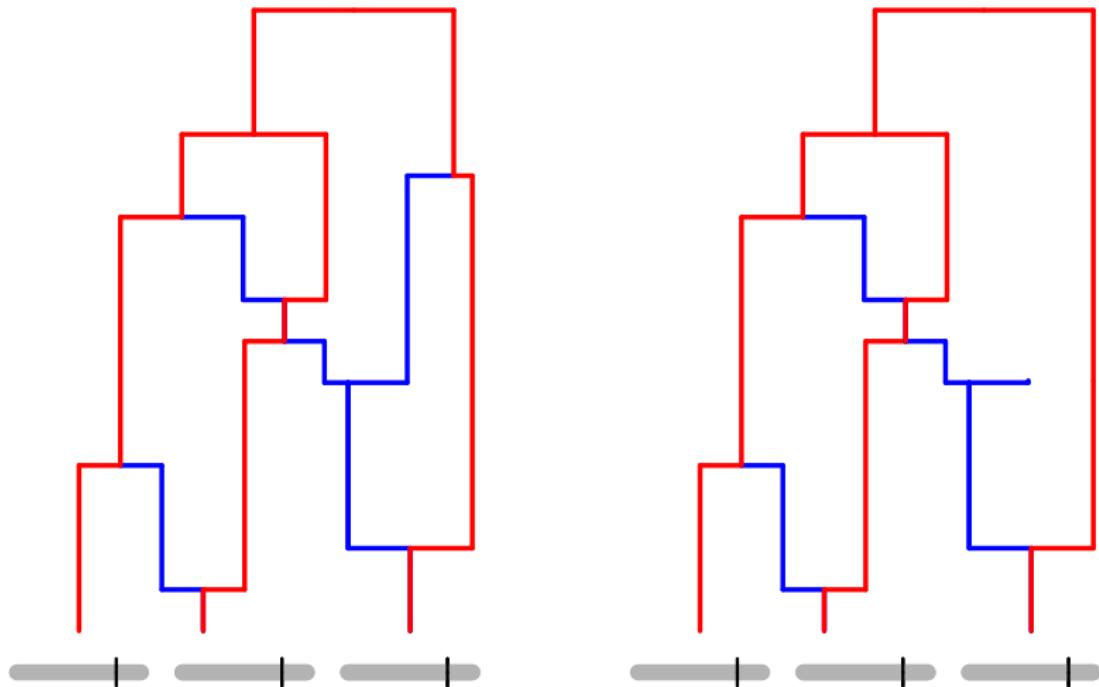
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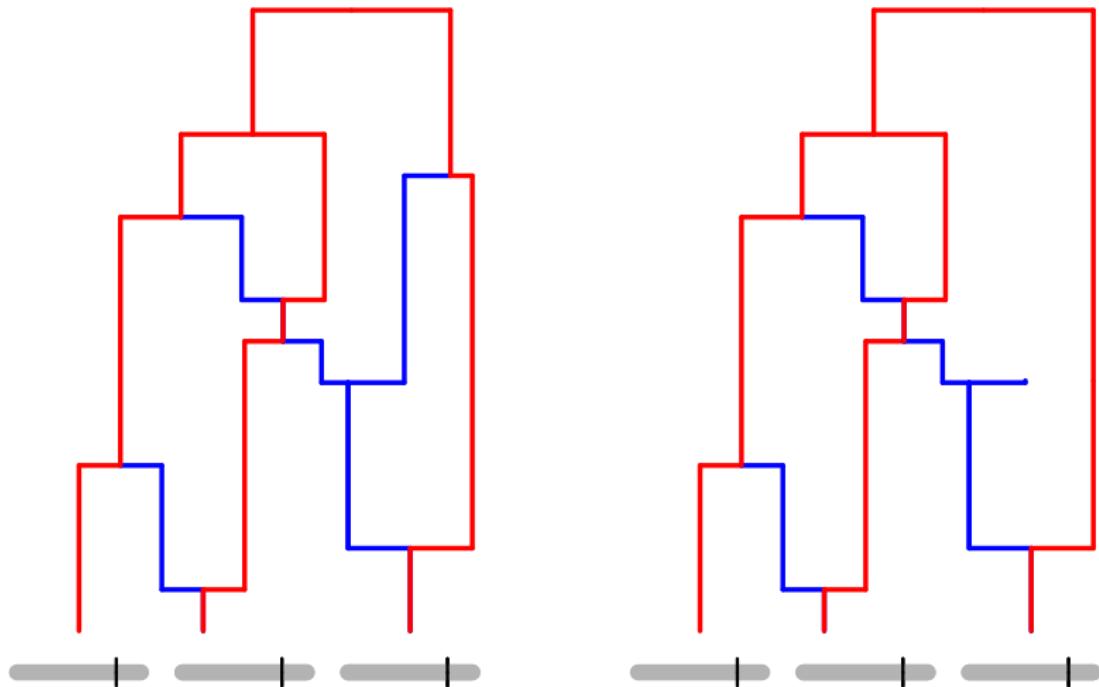
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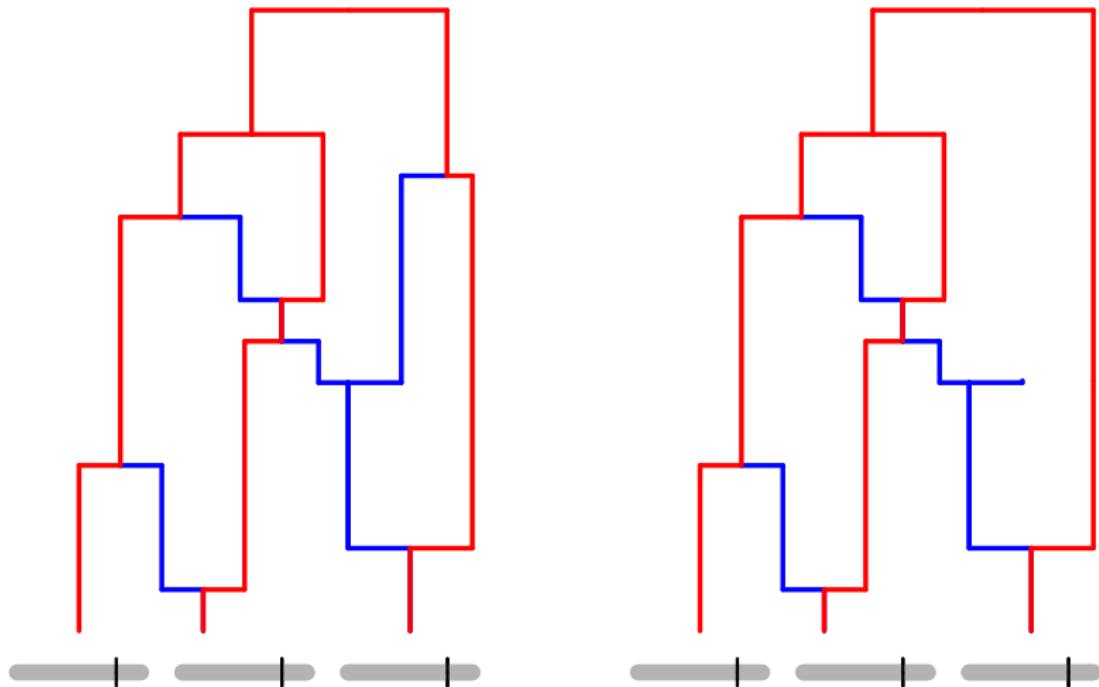
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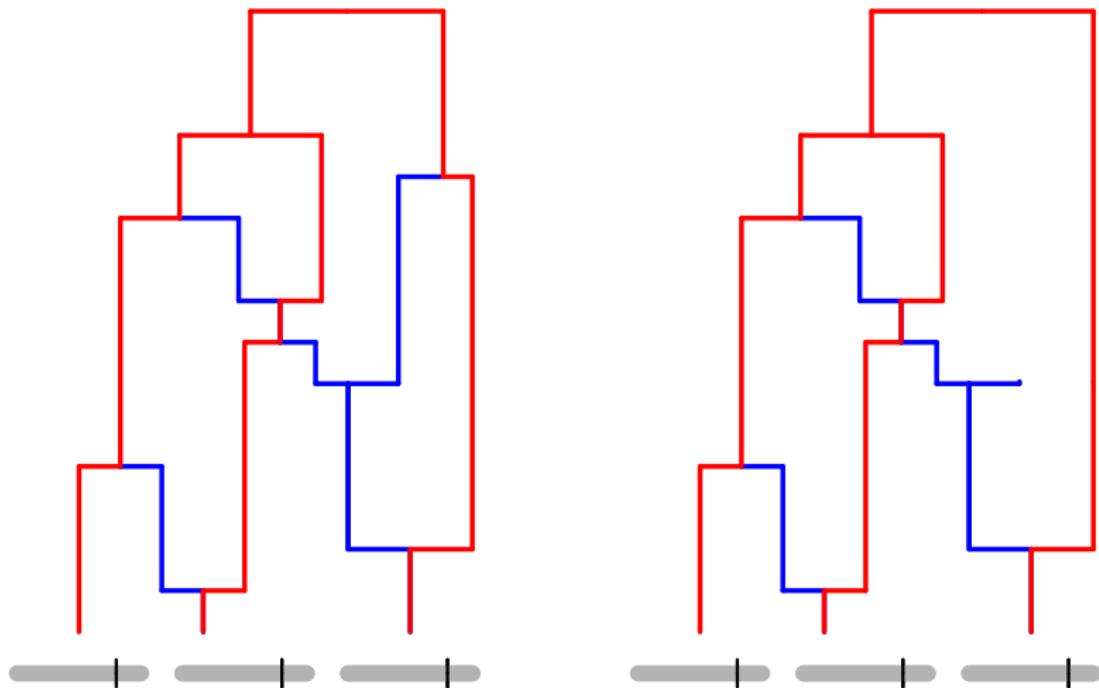
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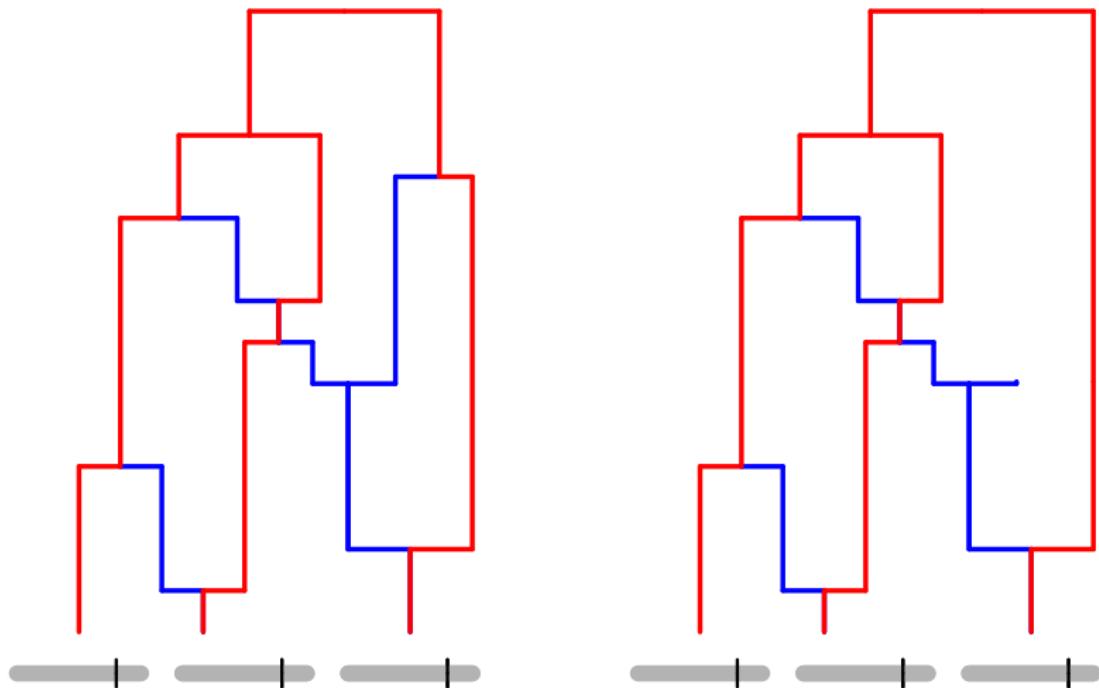
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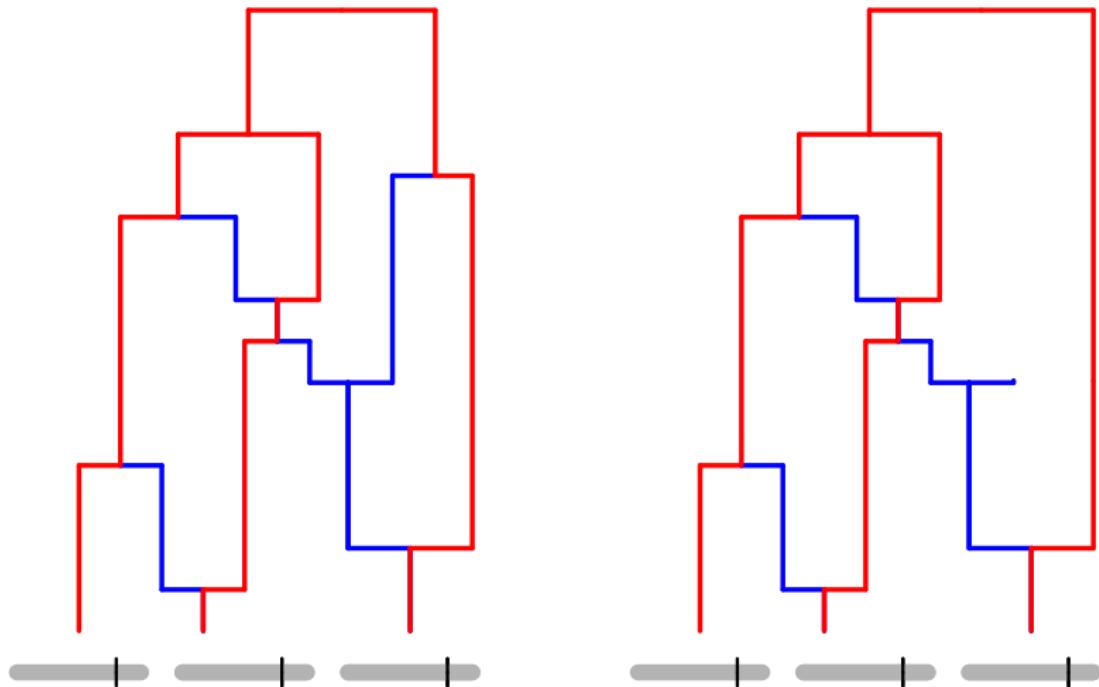
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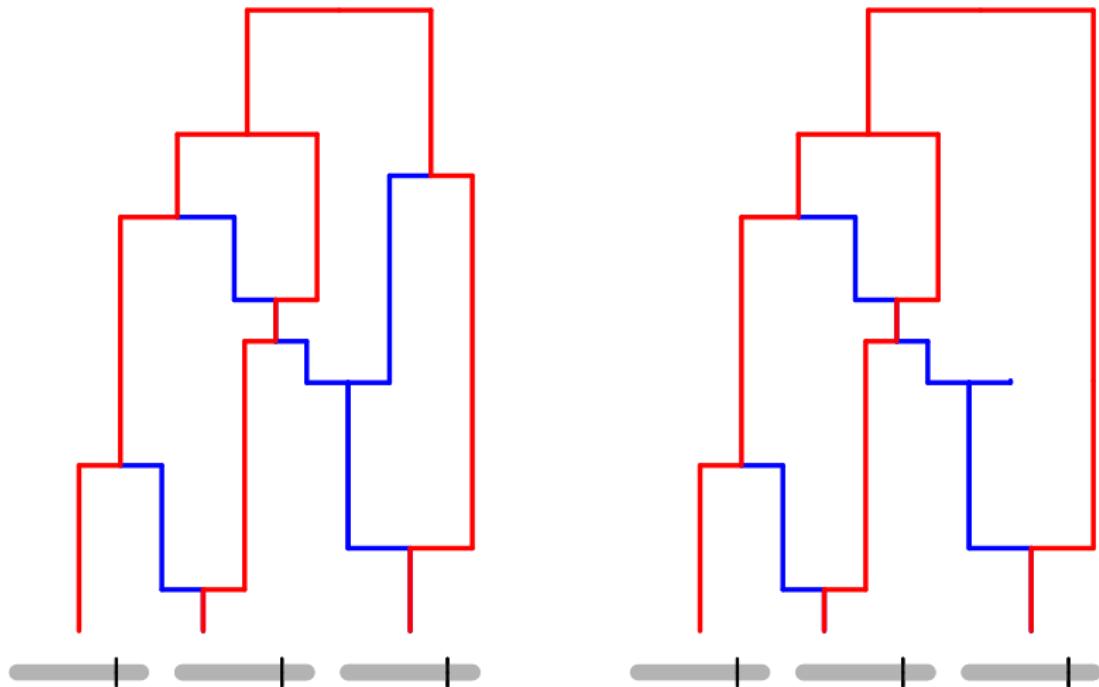
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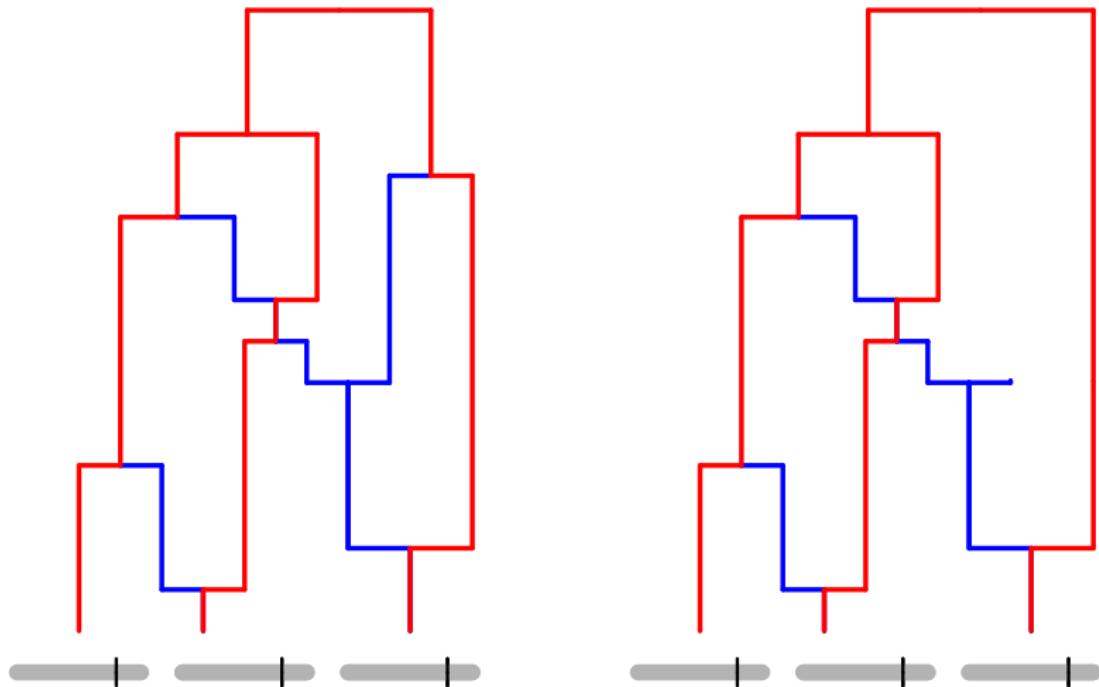
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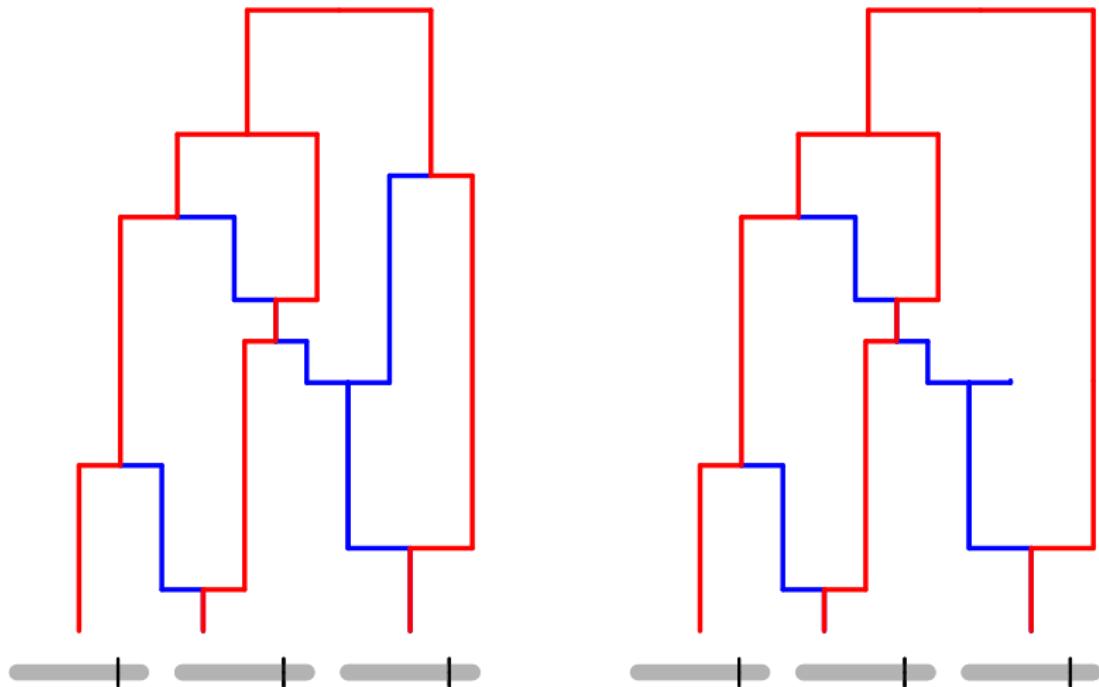
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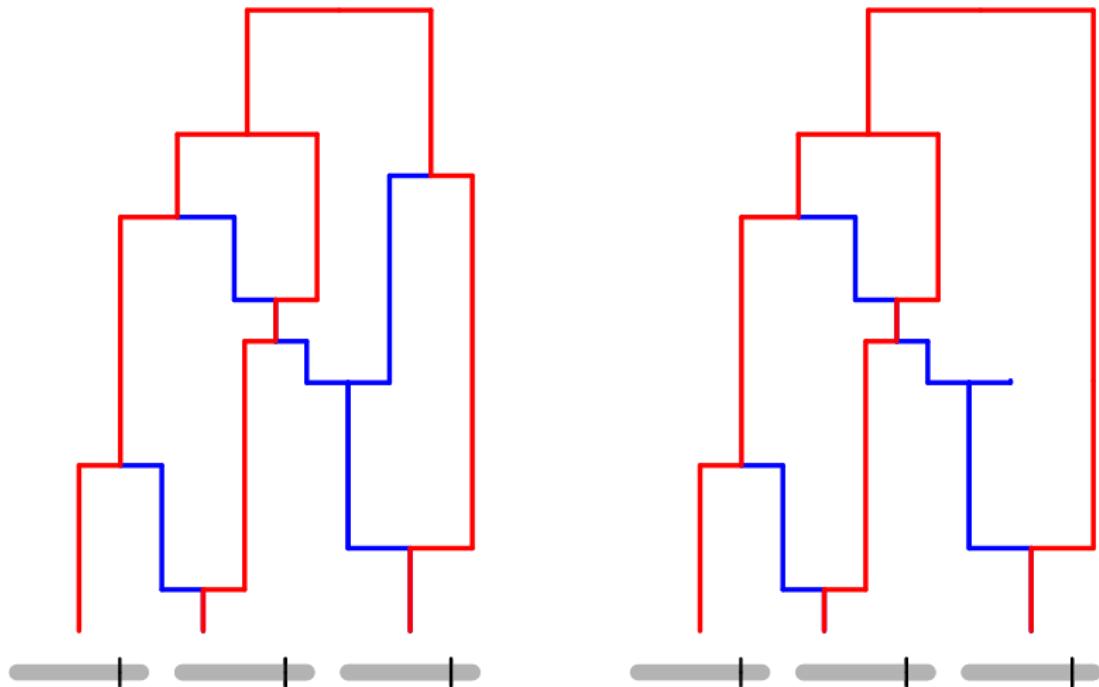
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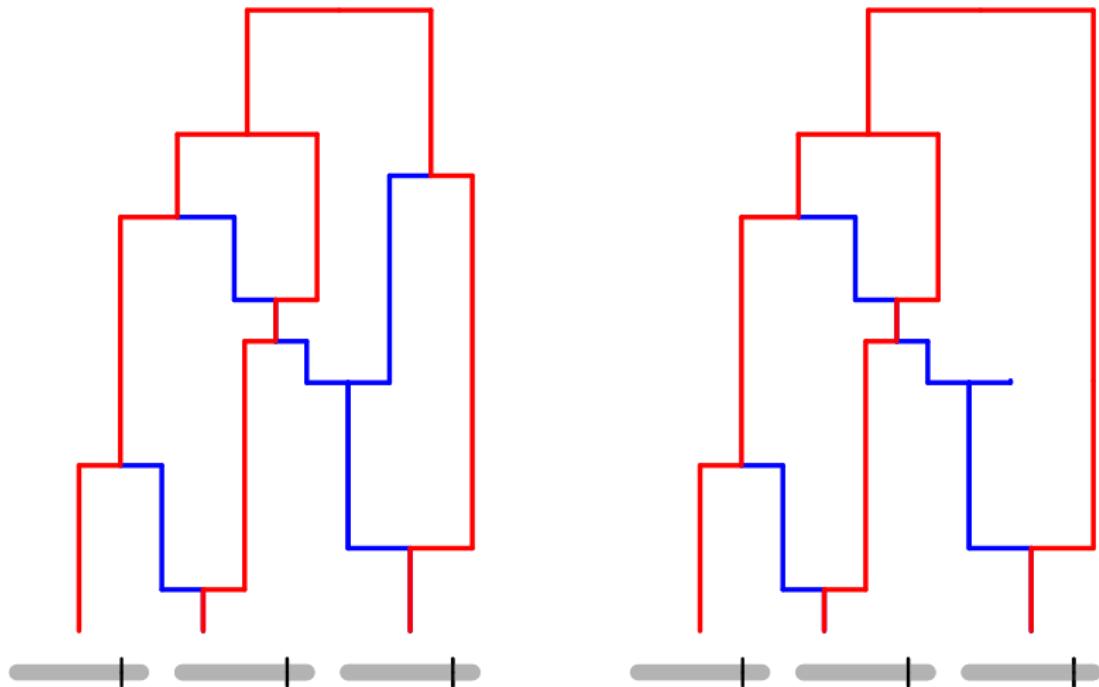
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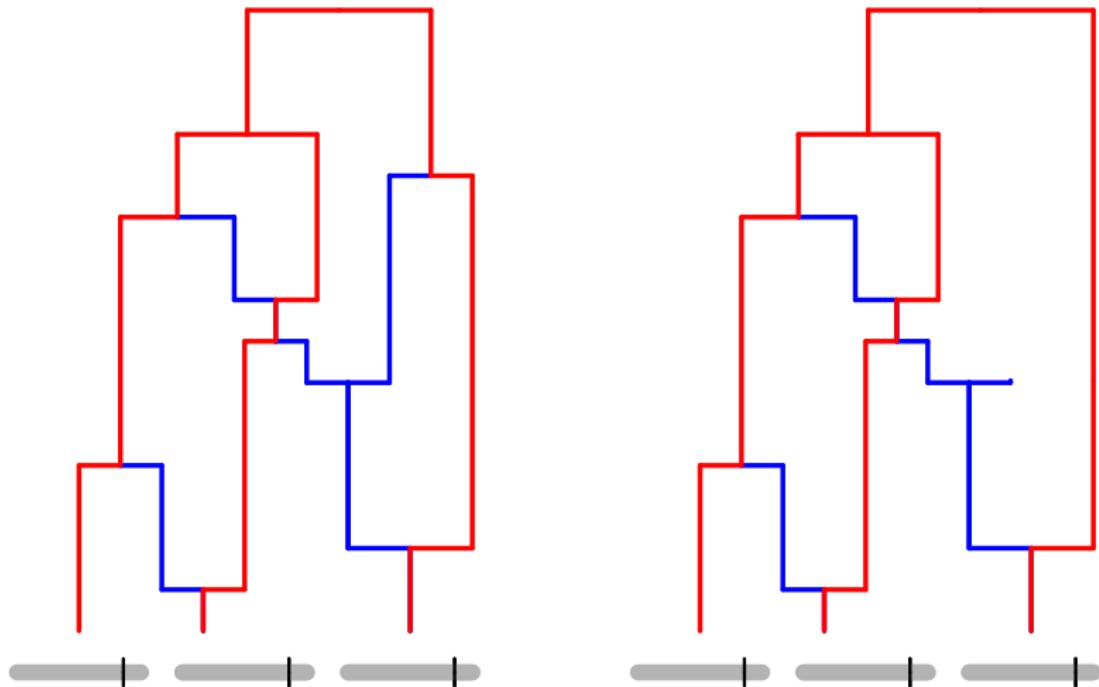
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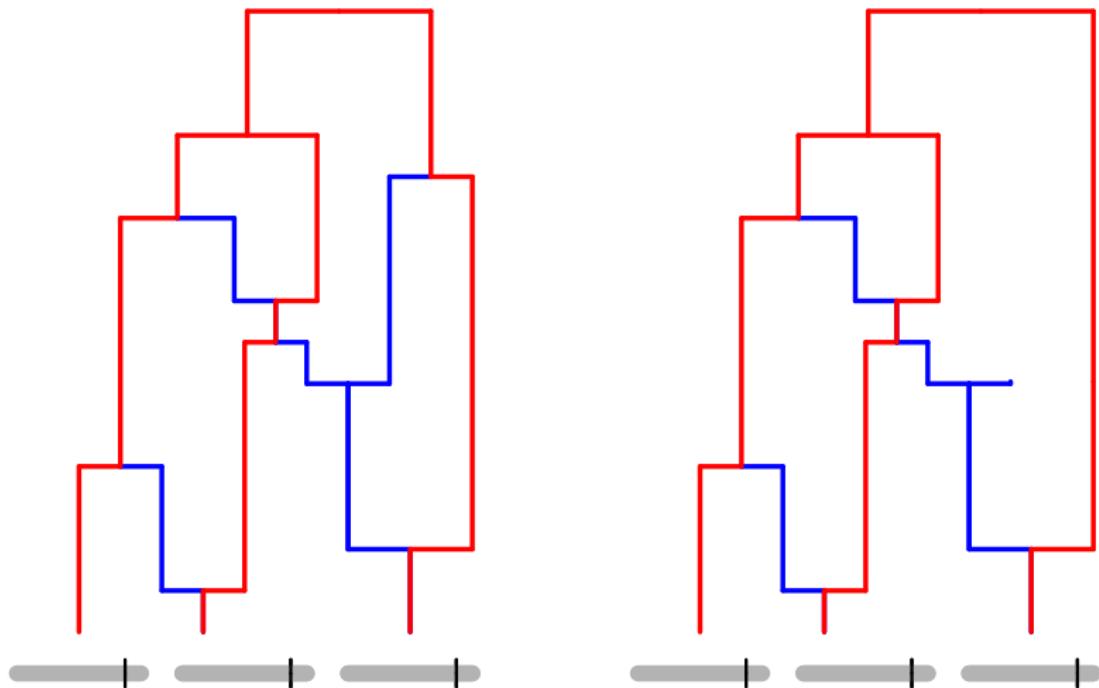
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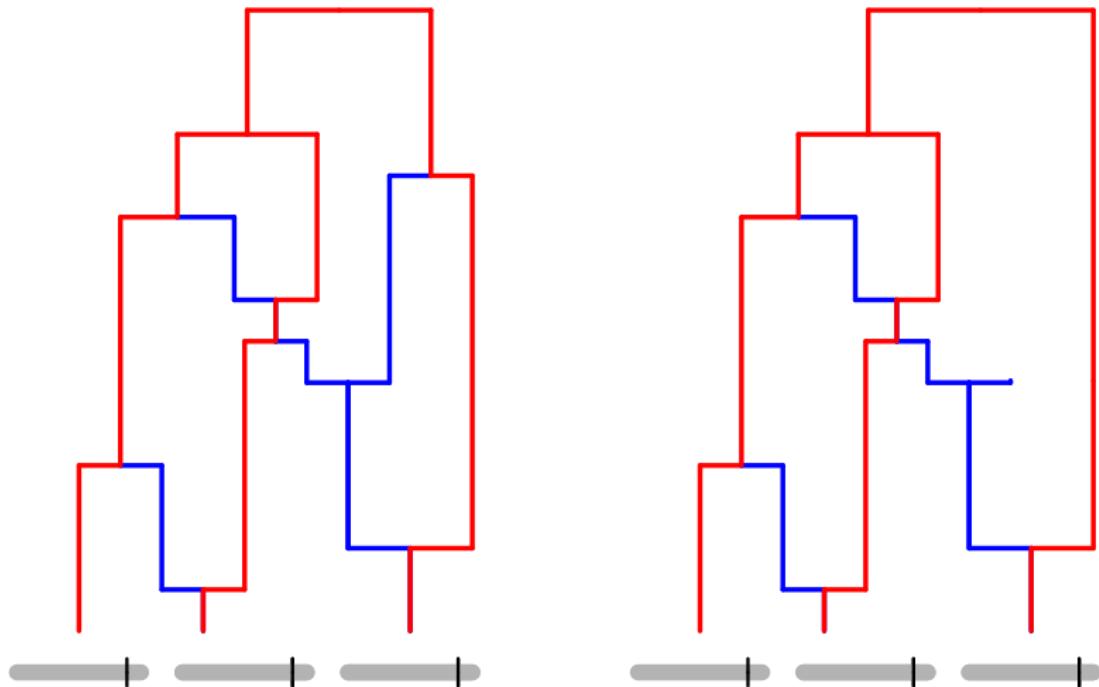
Simplification 1: postpone nonancestral recombinations



Simplification 1: postpone nonancestral recombinations



Simplification 1: postpone nonancestral recombinations



Introduction
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Coalescent & ARG
ooooooooo

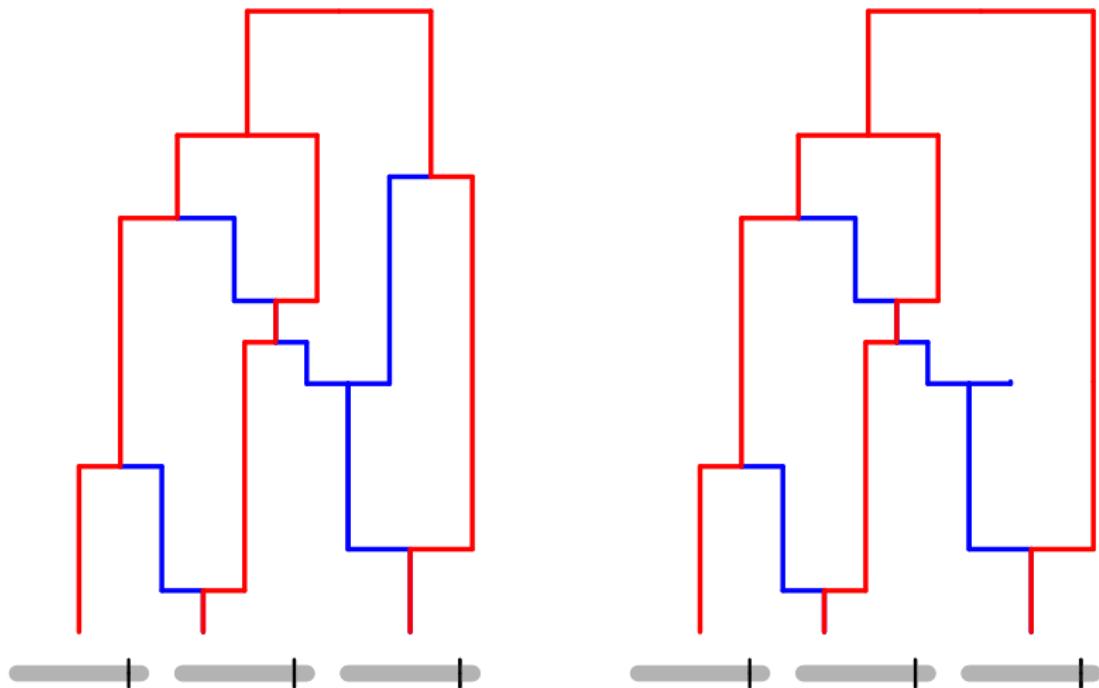
Sequential algorithm
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Approximations
○

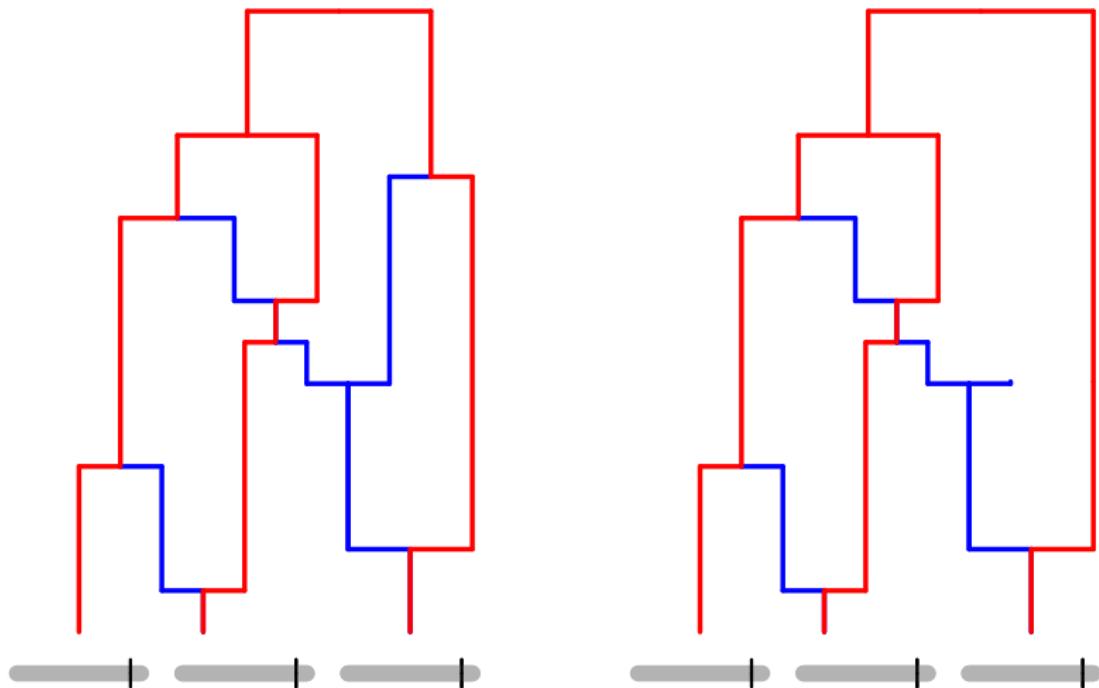
Performance
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Summary
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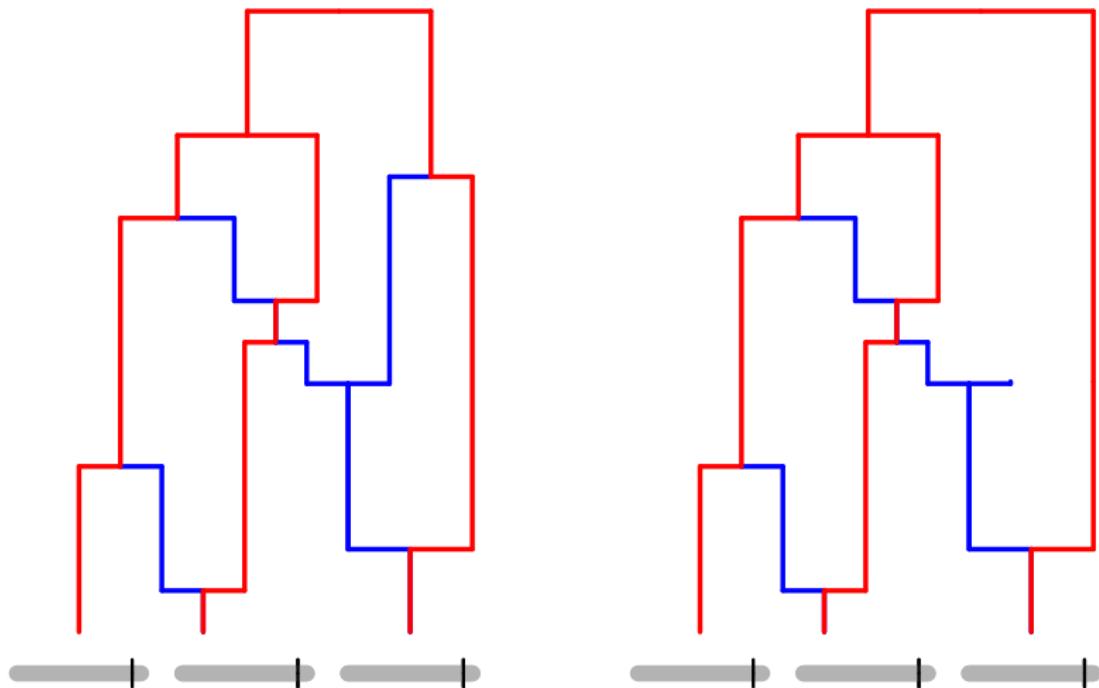
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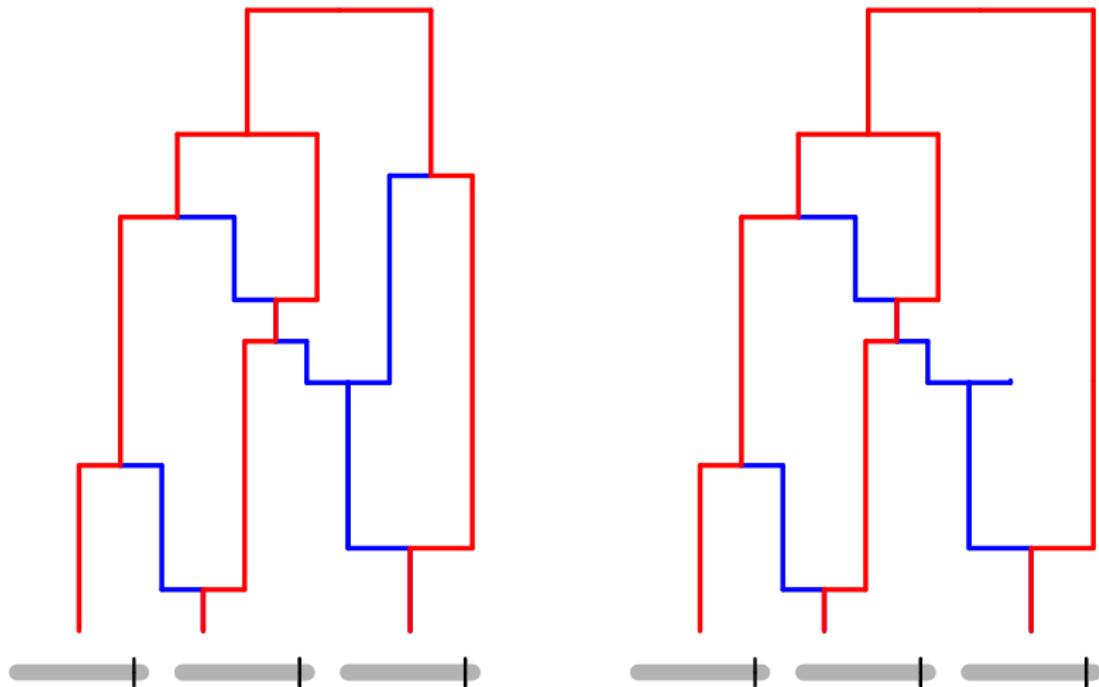
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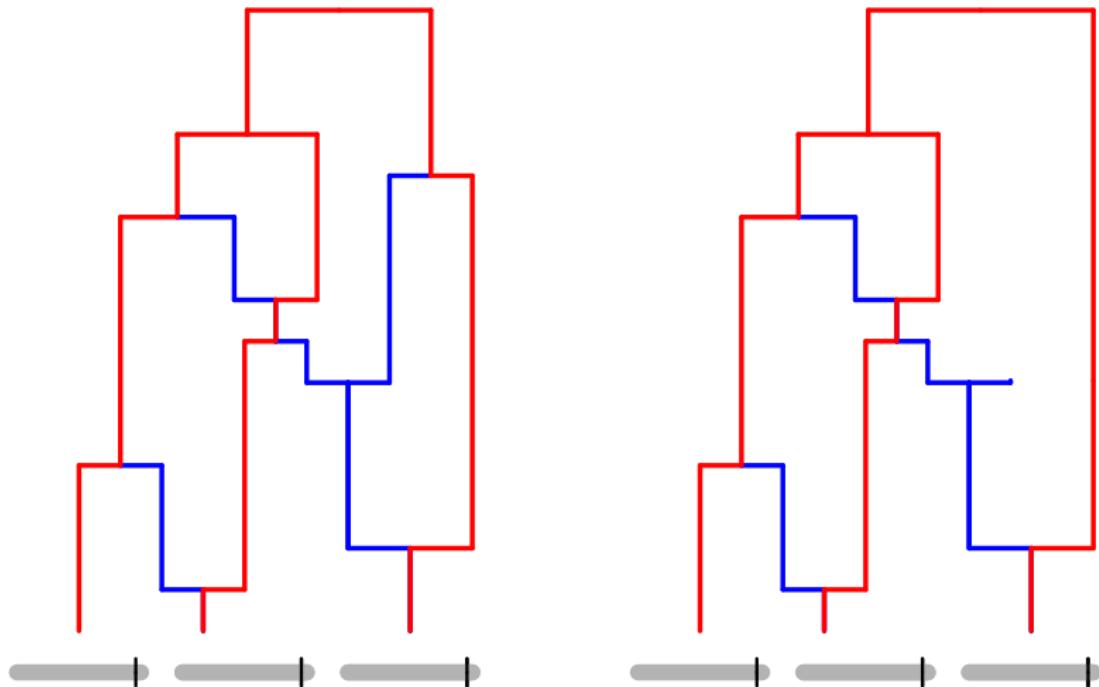
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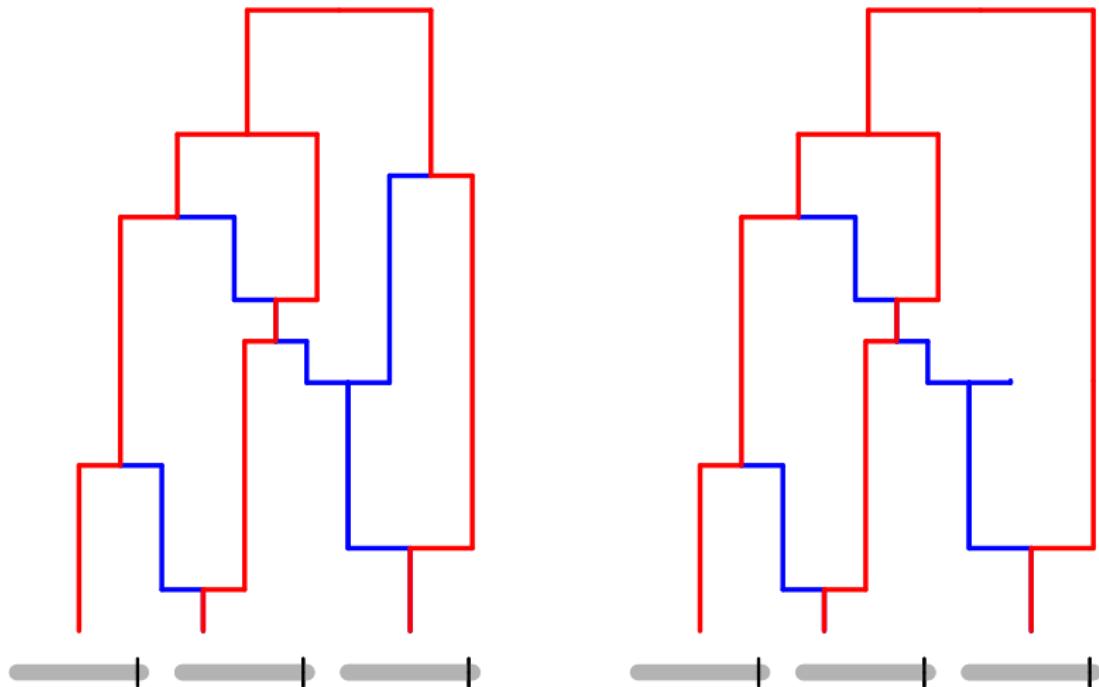
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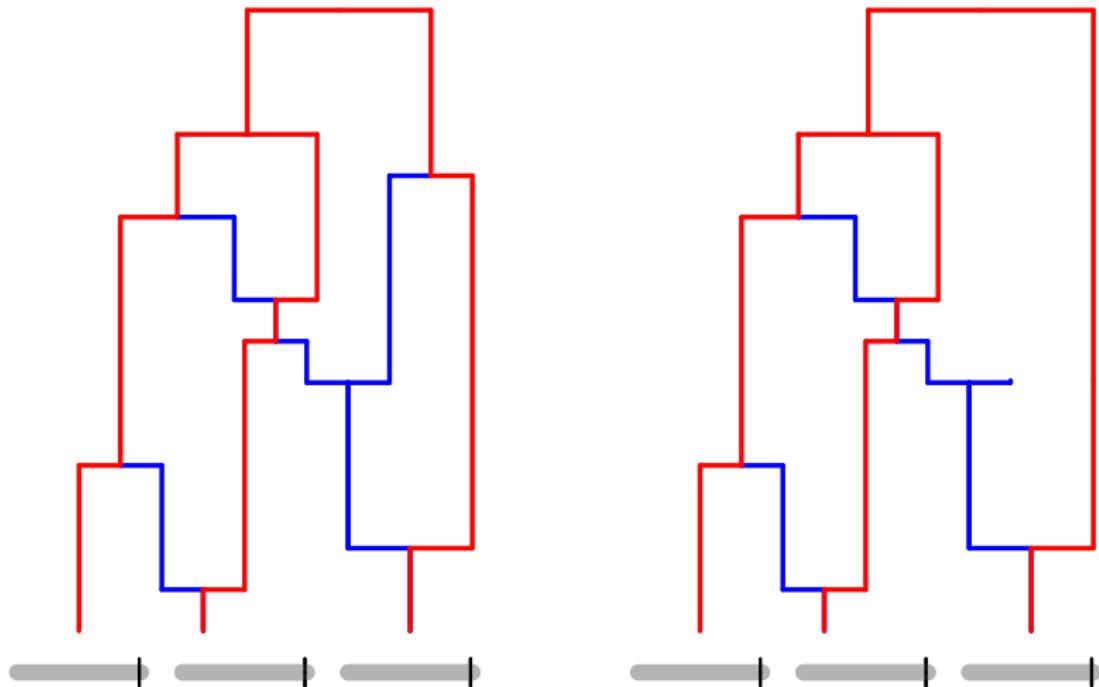
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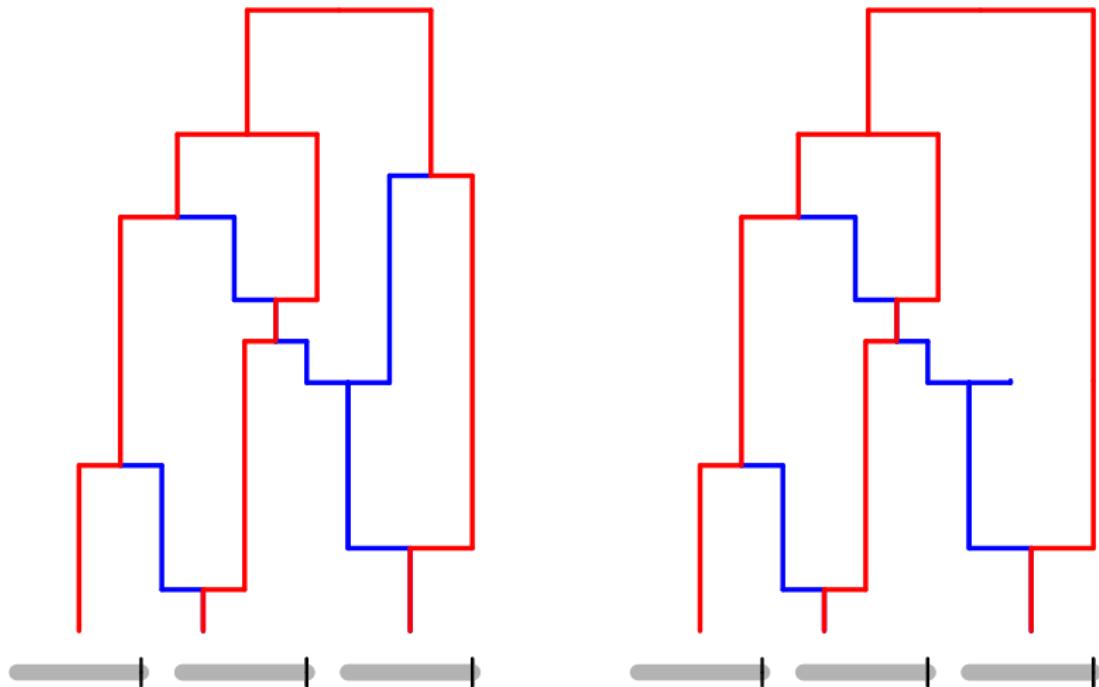
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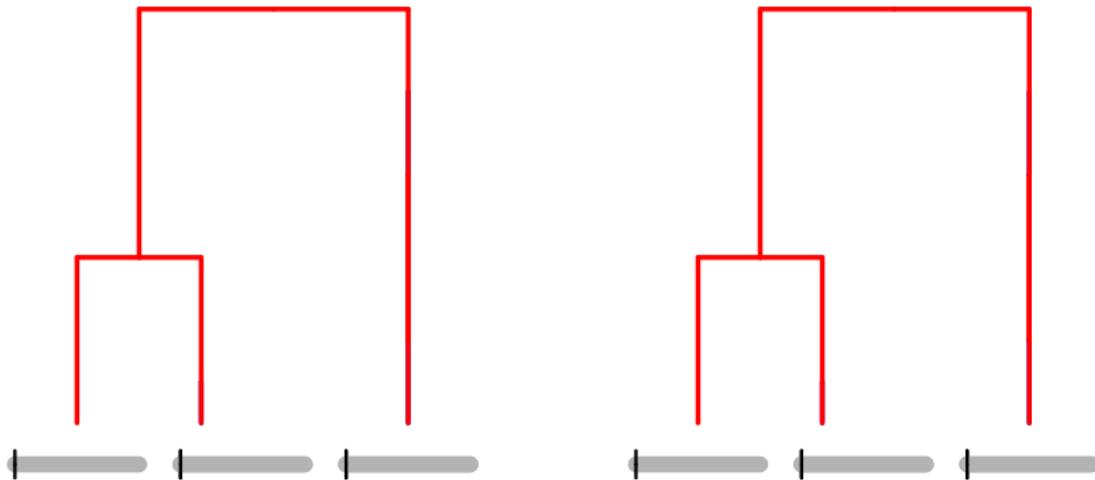
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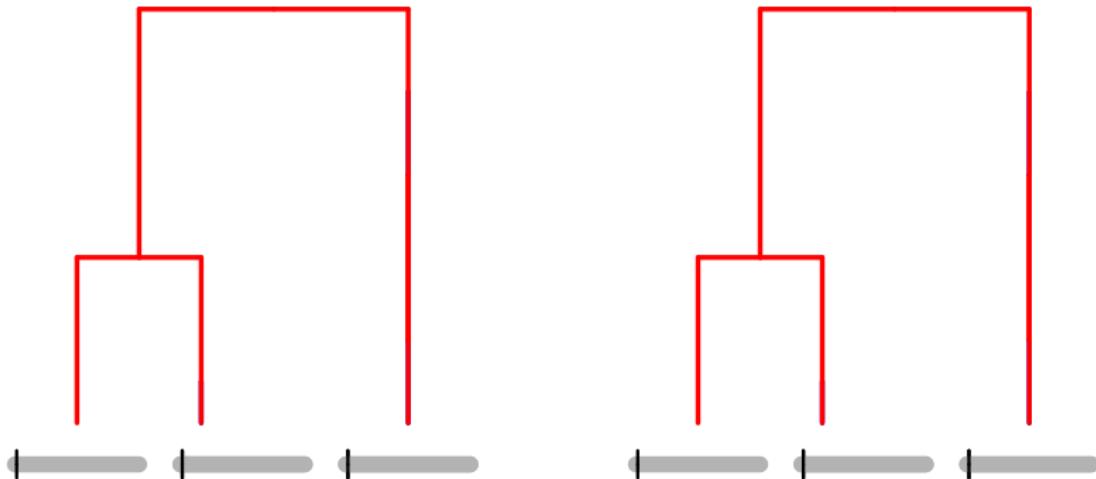
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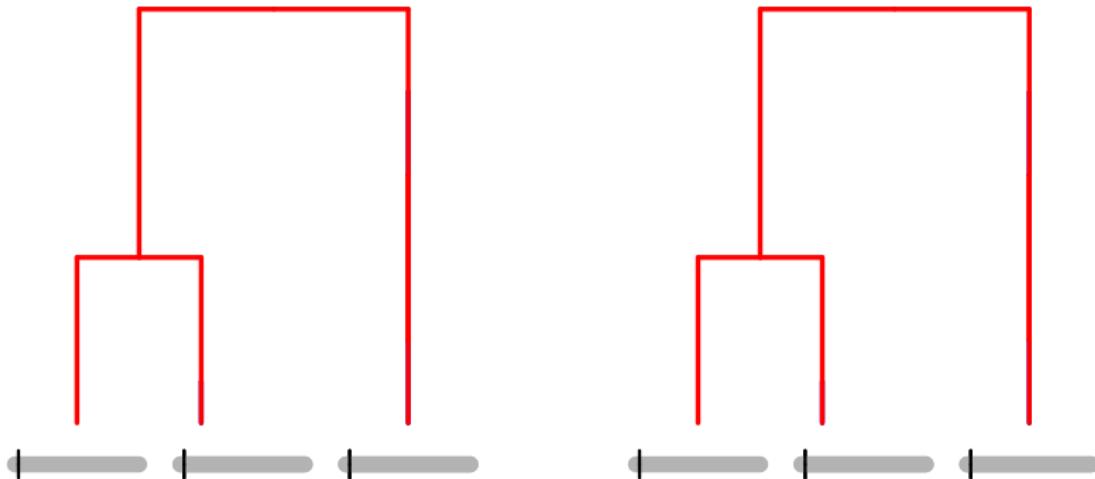
Simplification 2: forget nonlocal genealogies



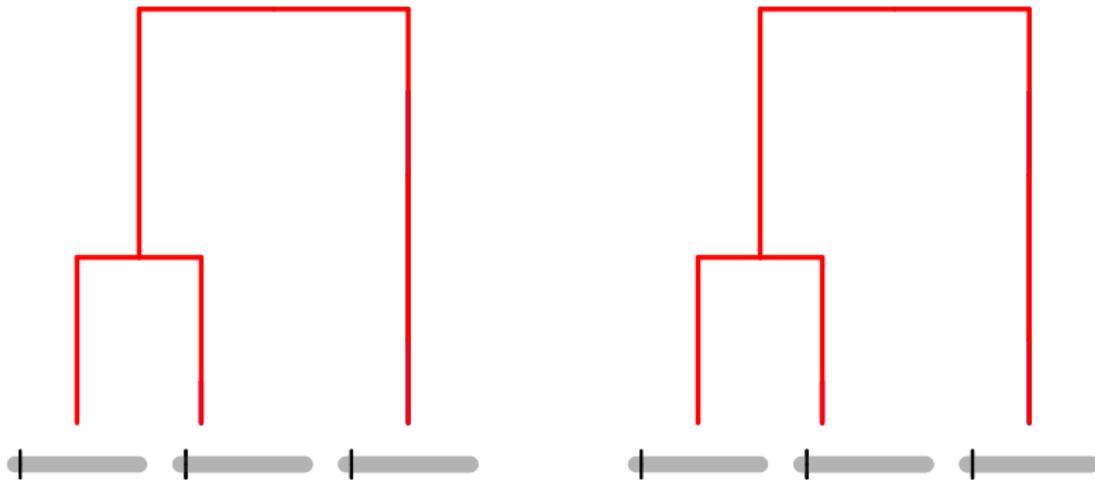
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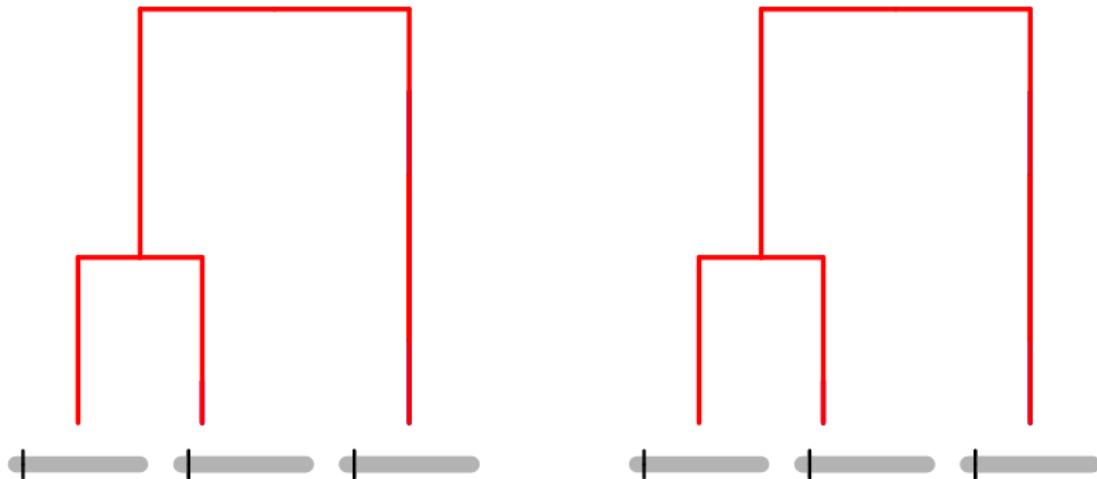
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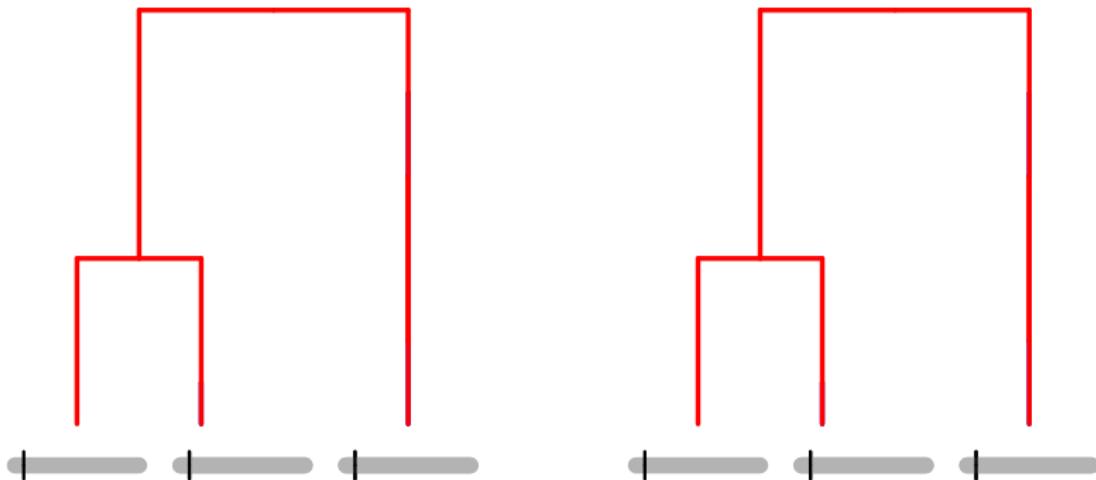
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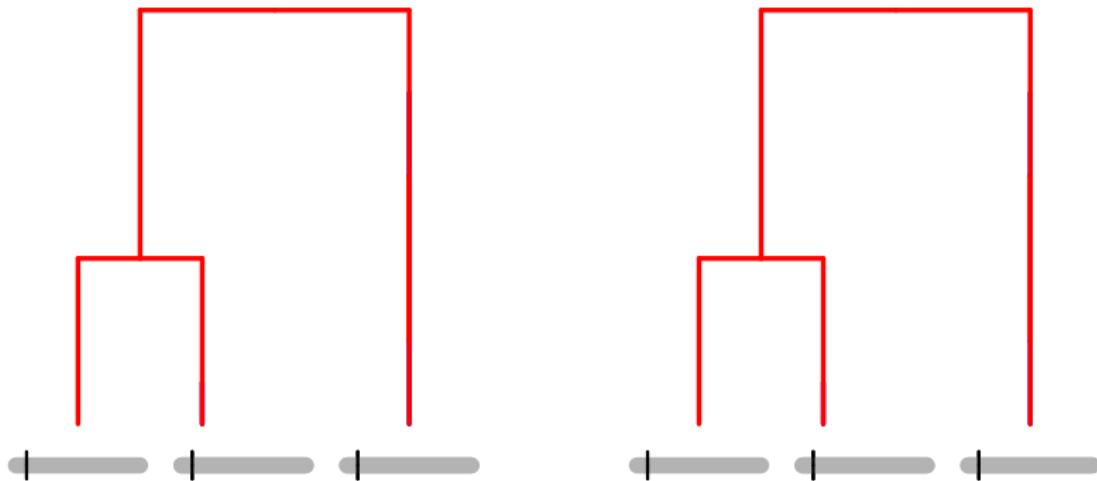
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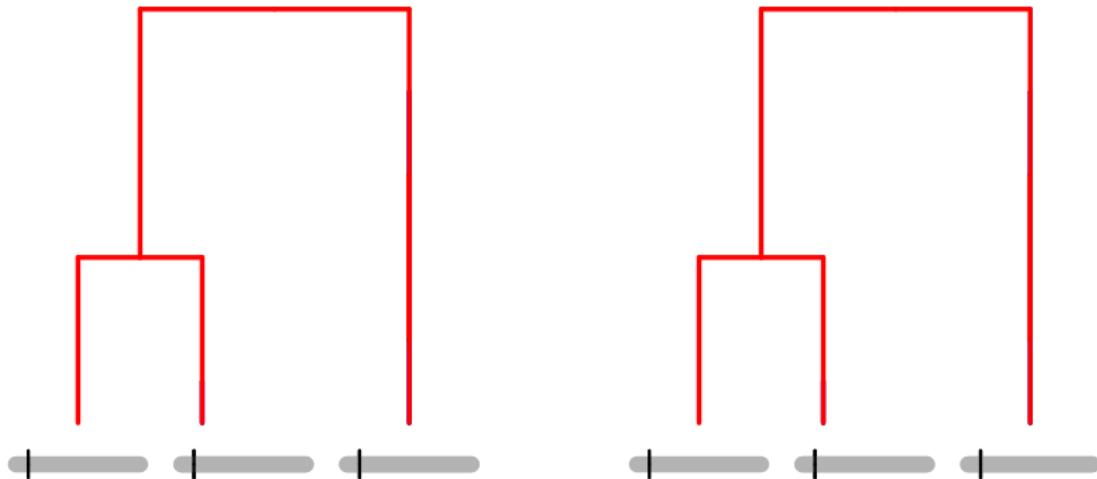
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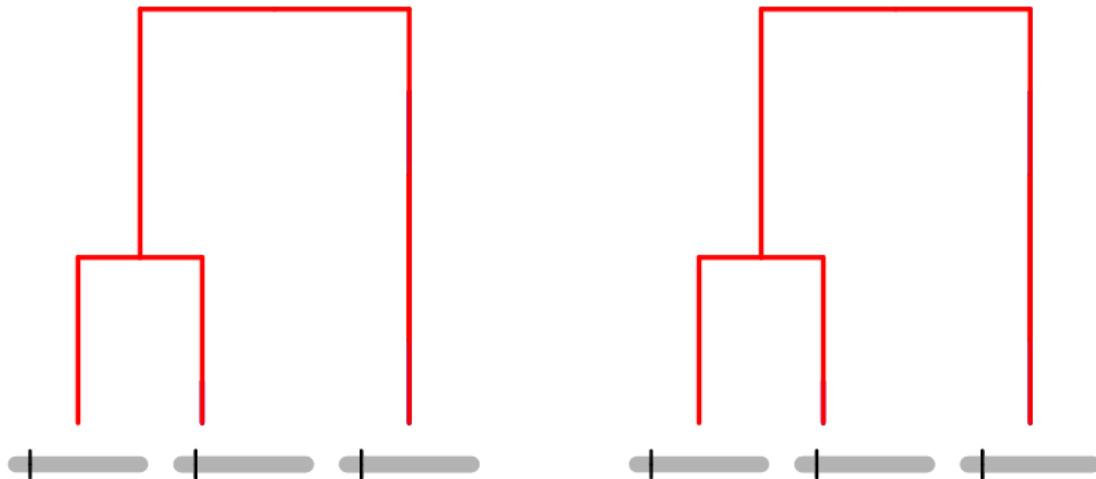
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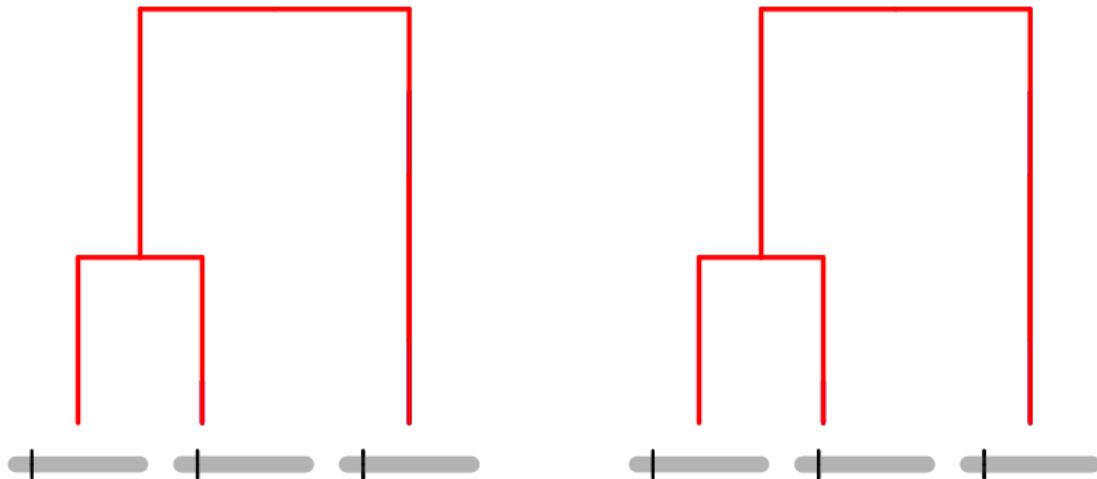
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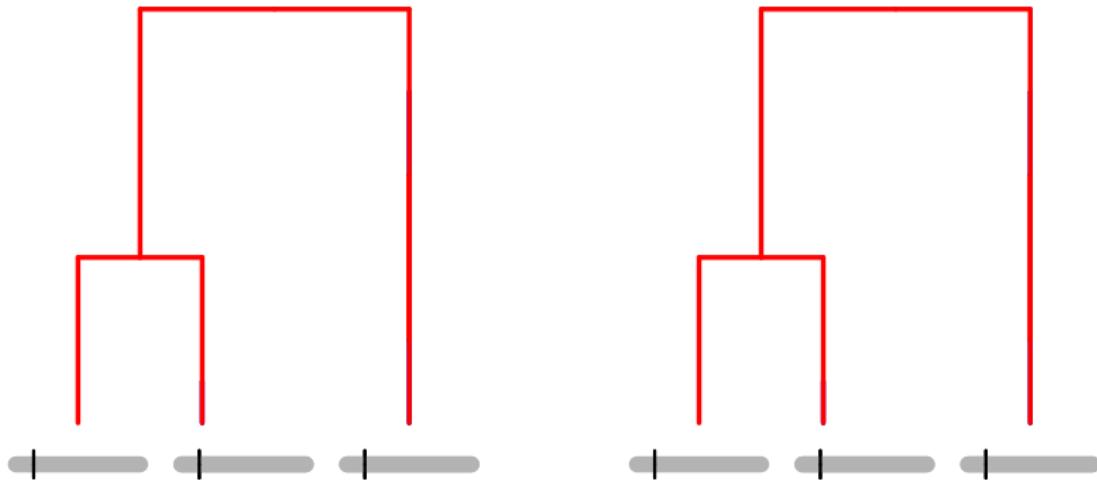
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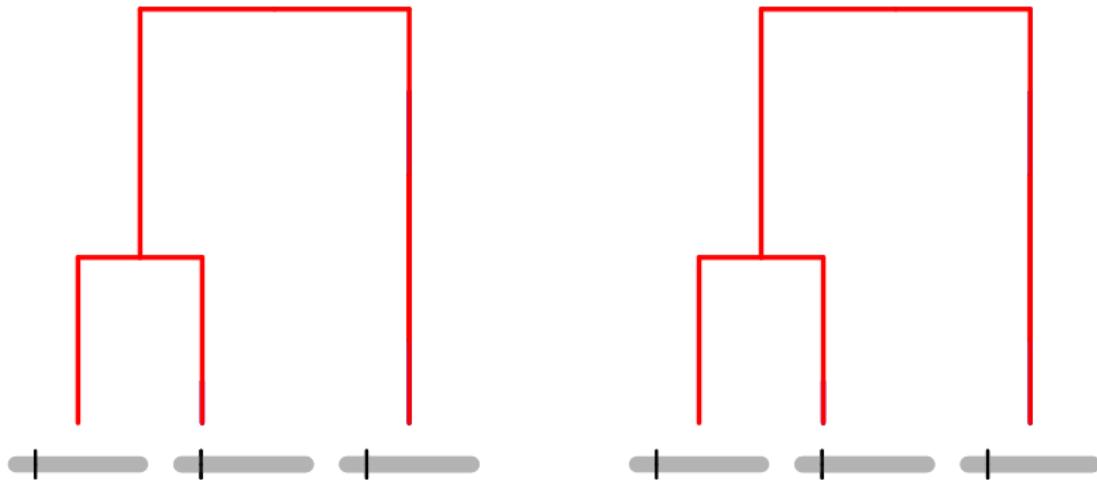
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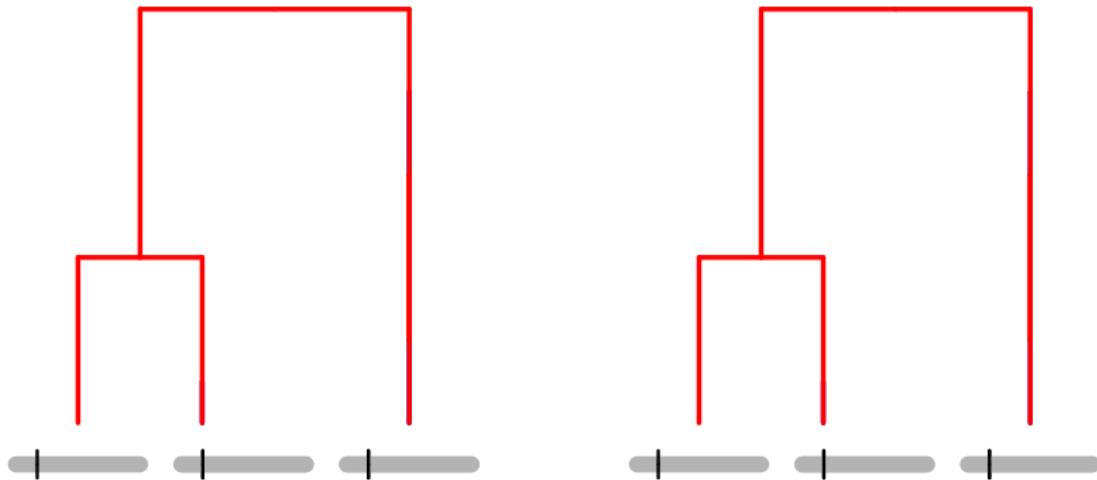
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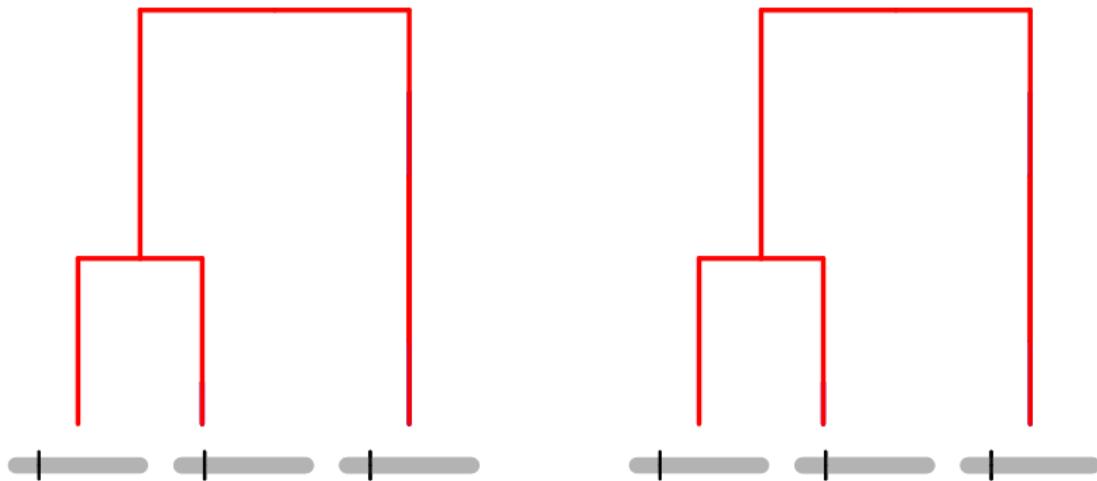
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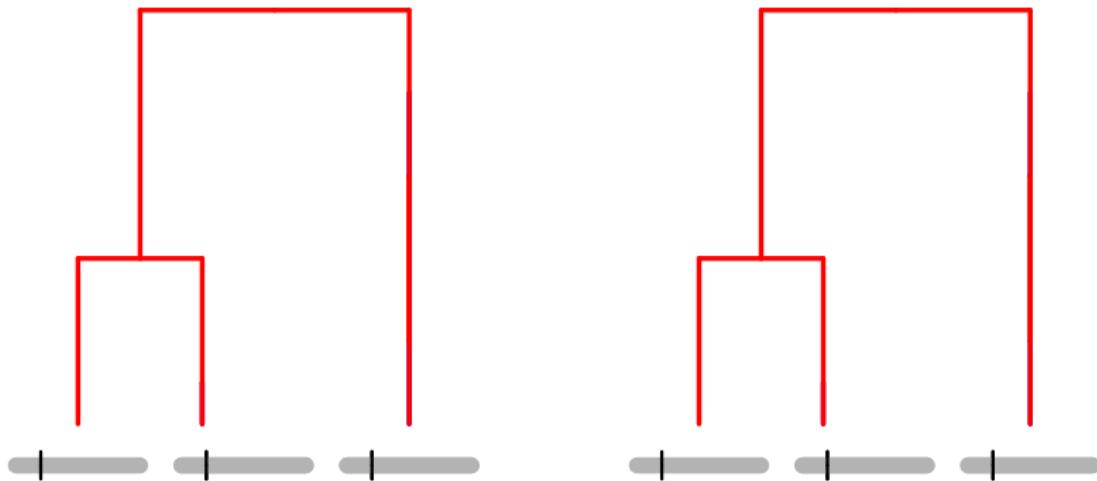
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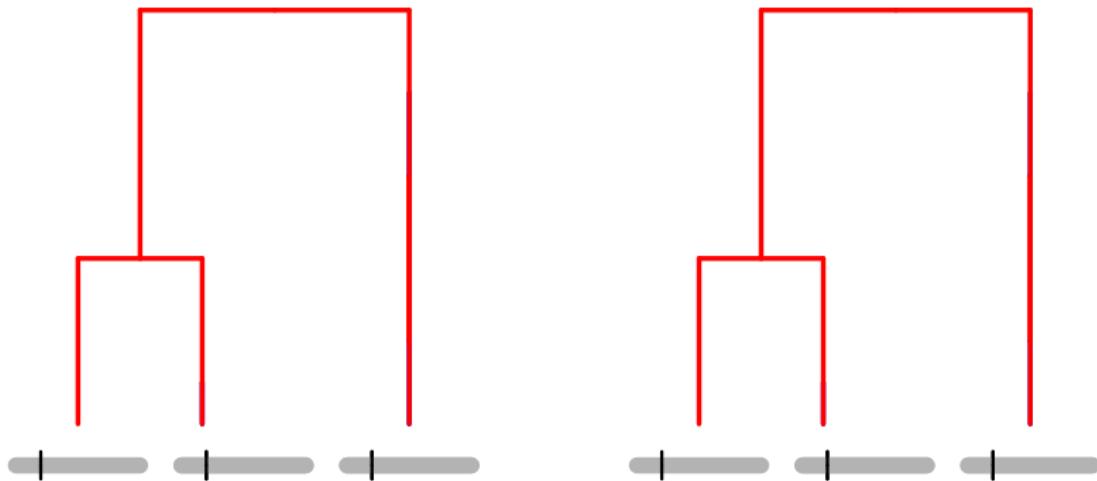
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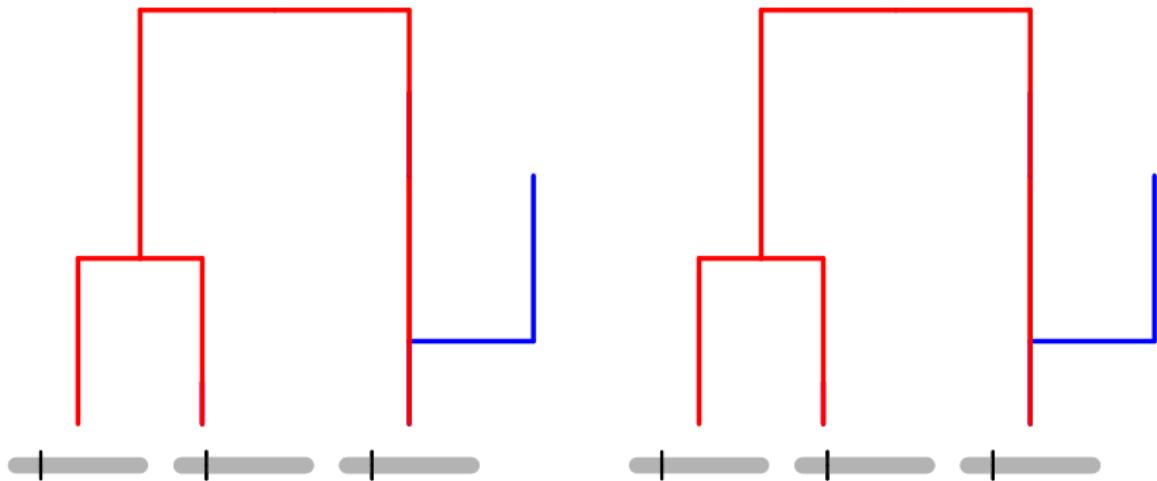
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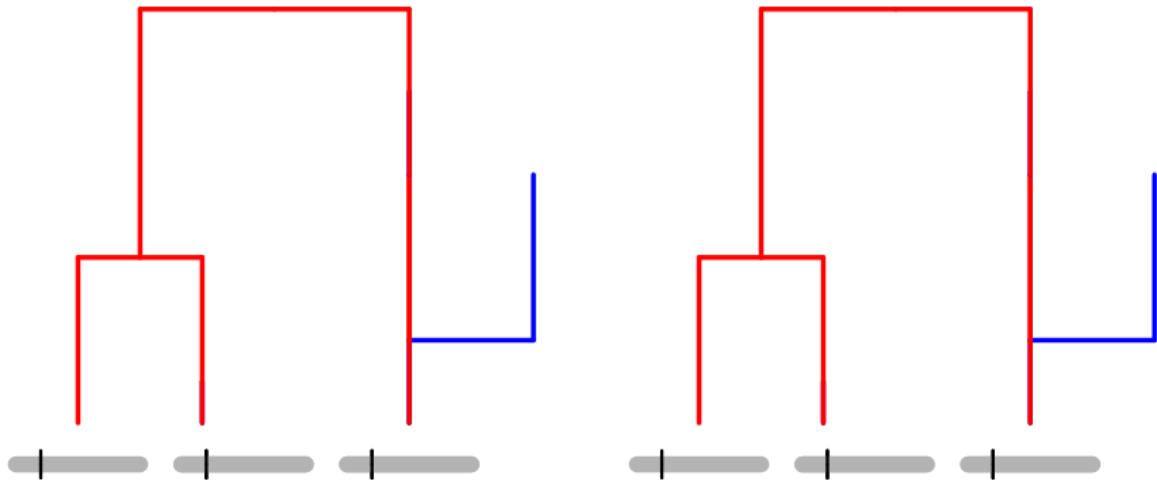
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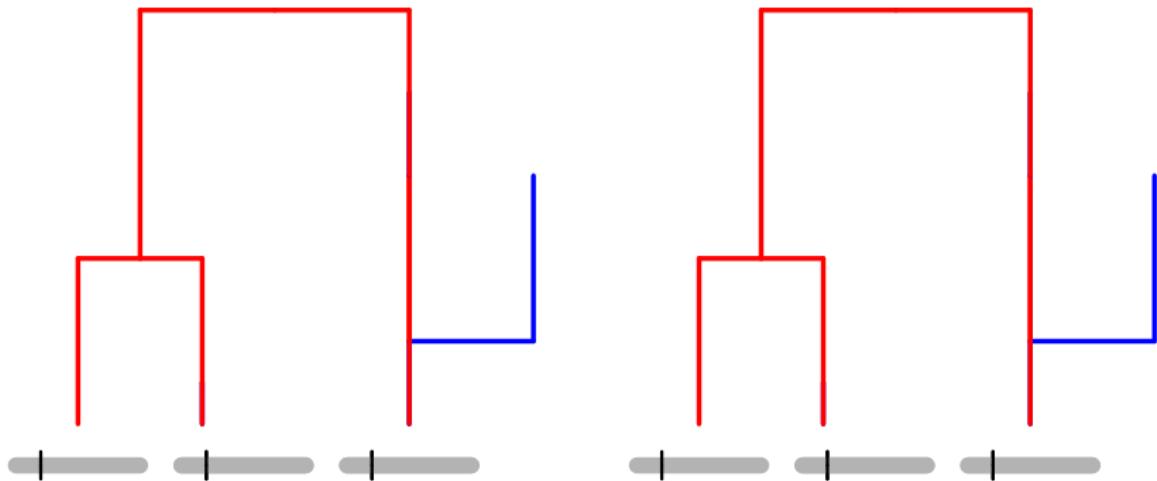
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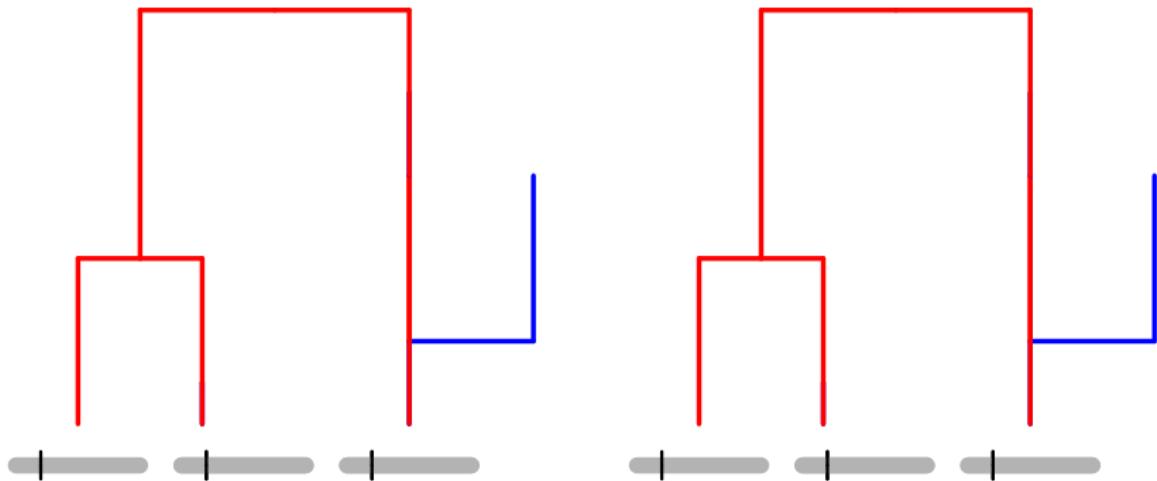
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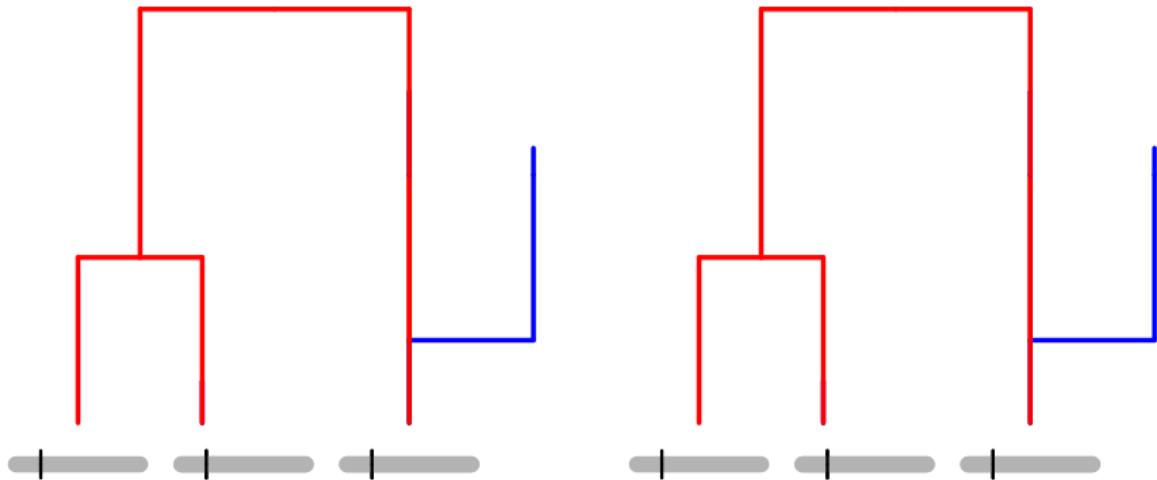
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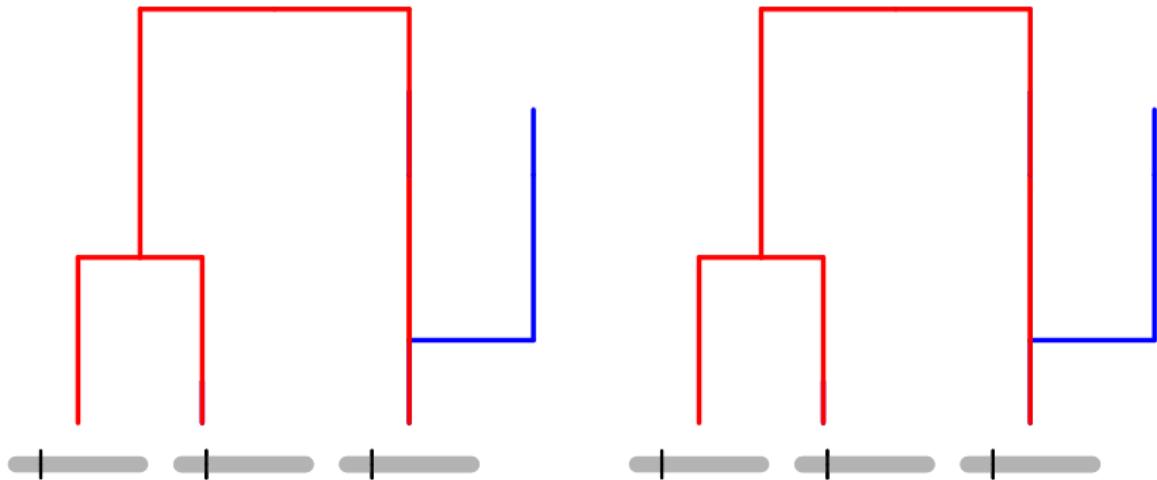
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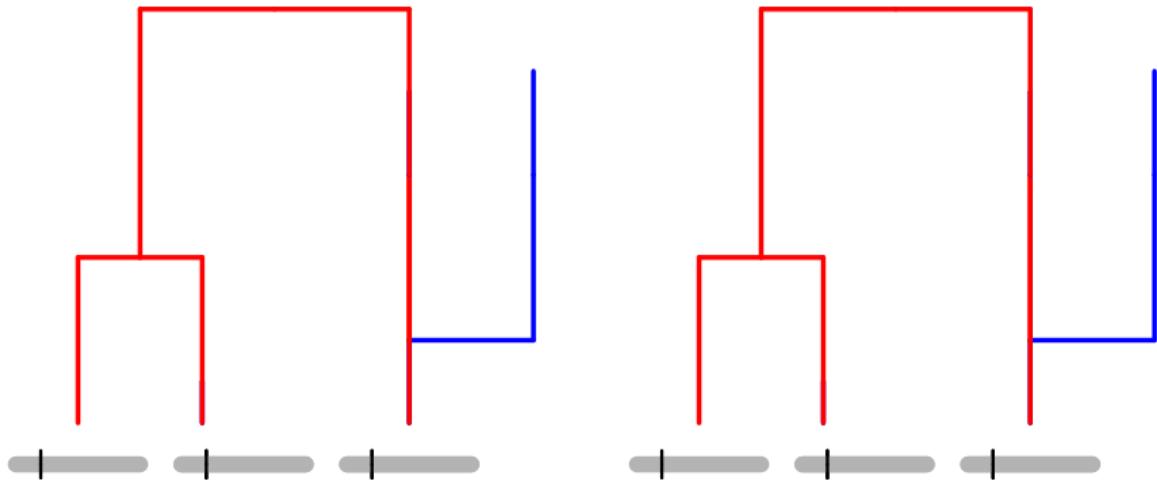
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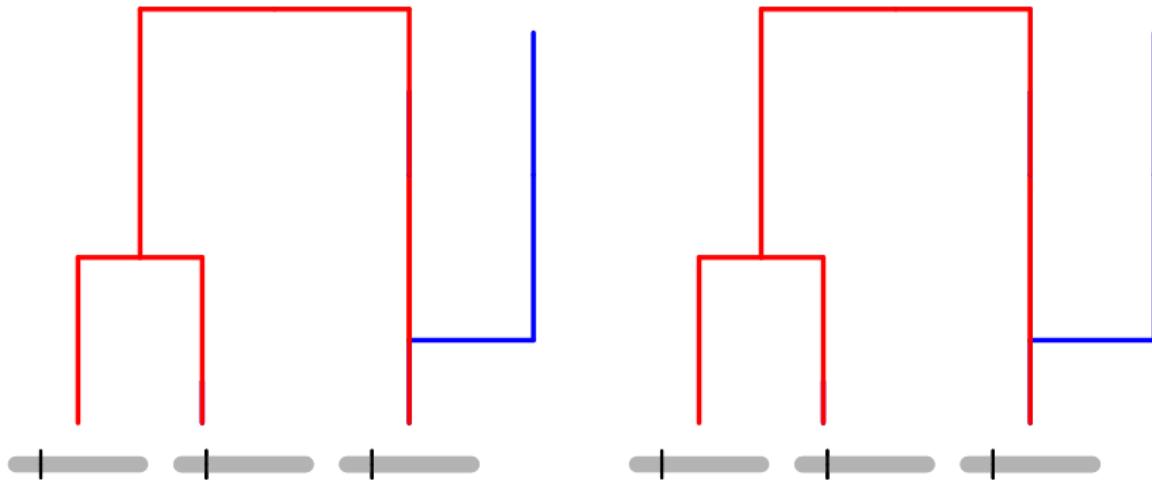
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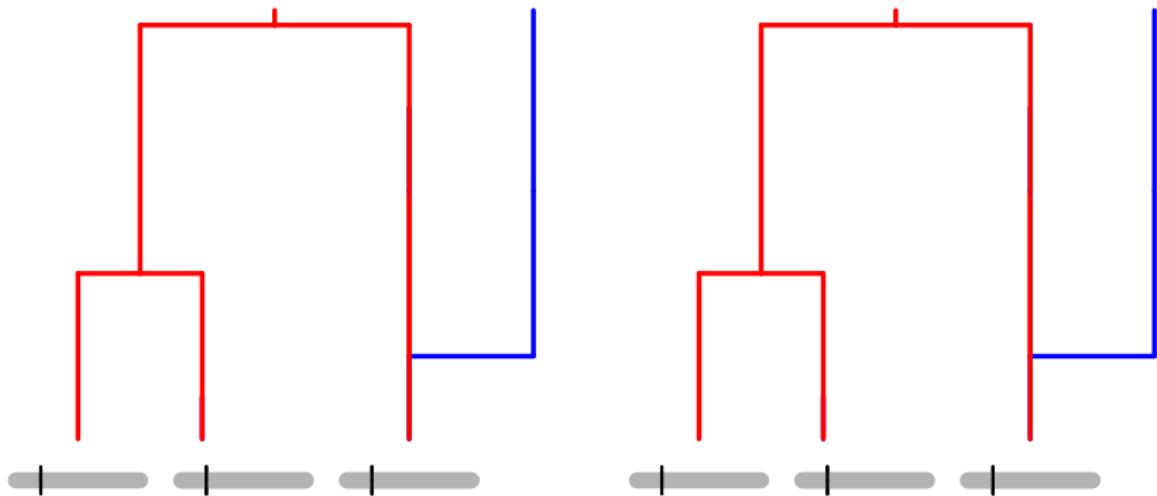
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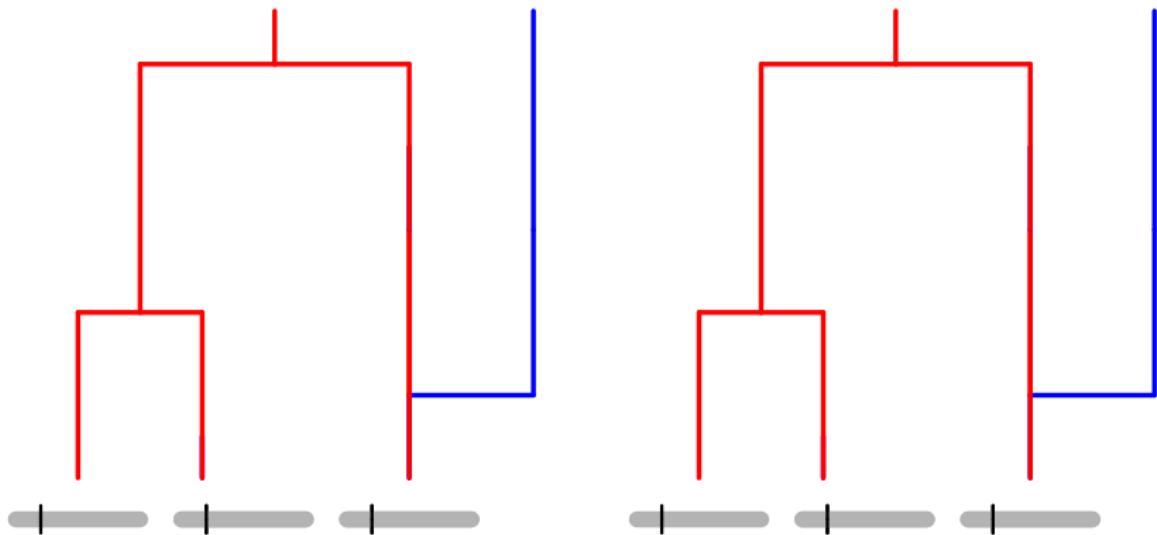
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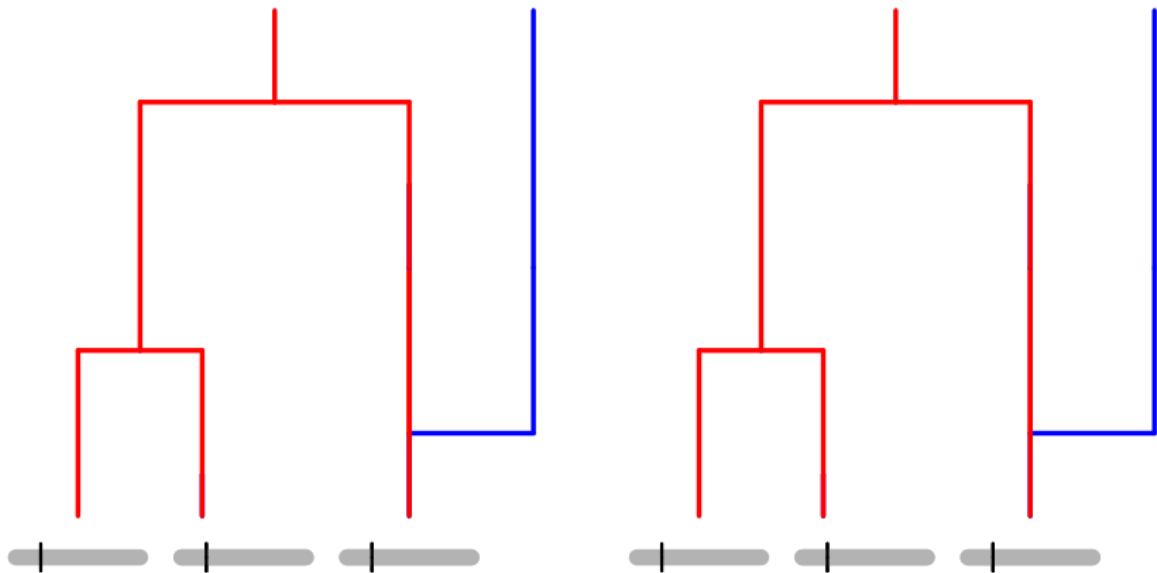
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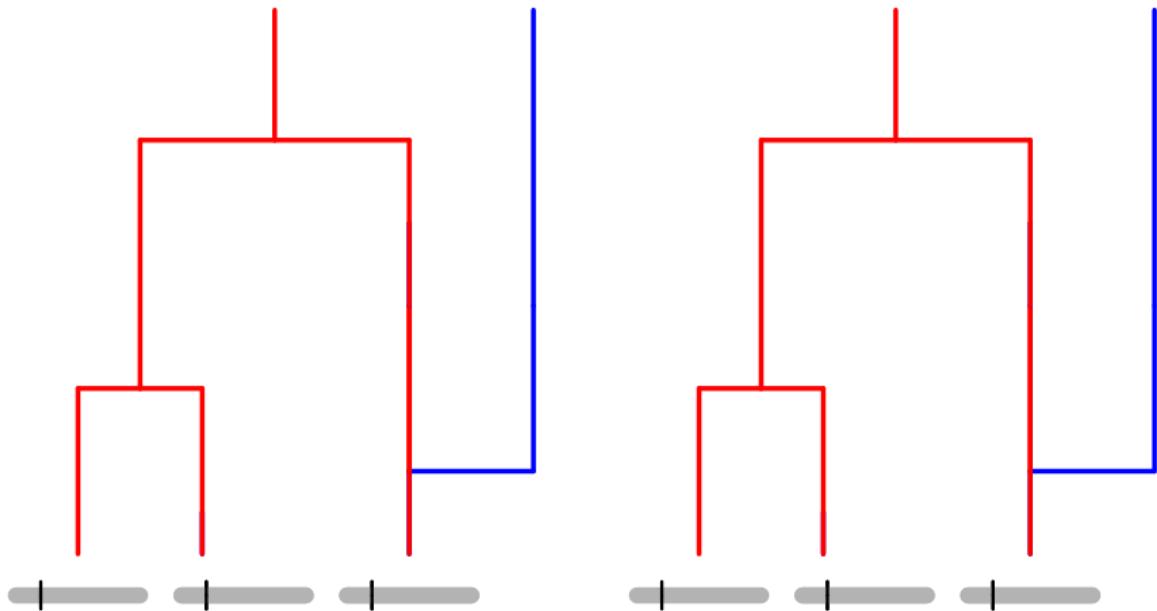
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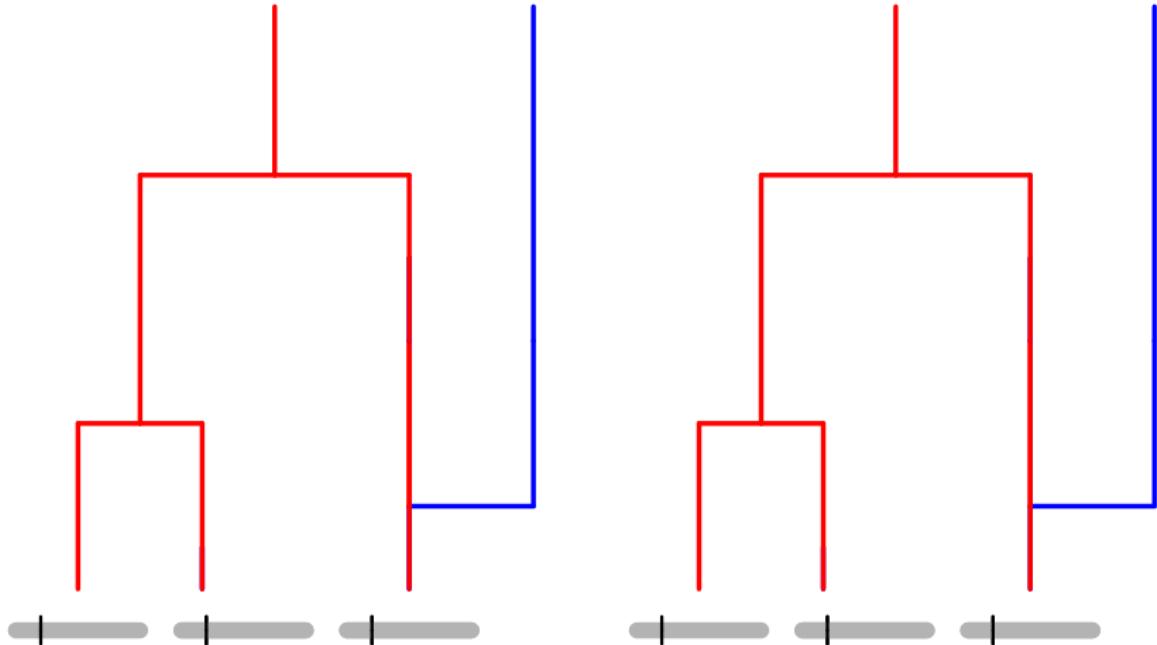
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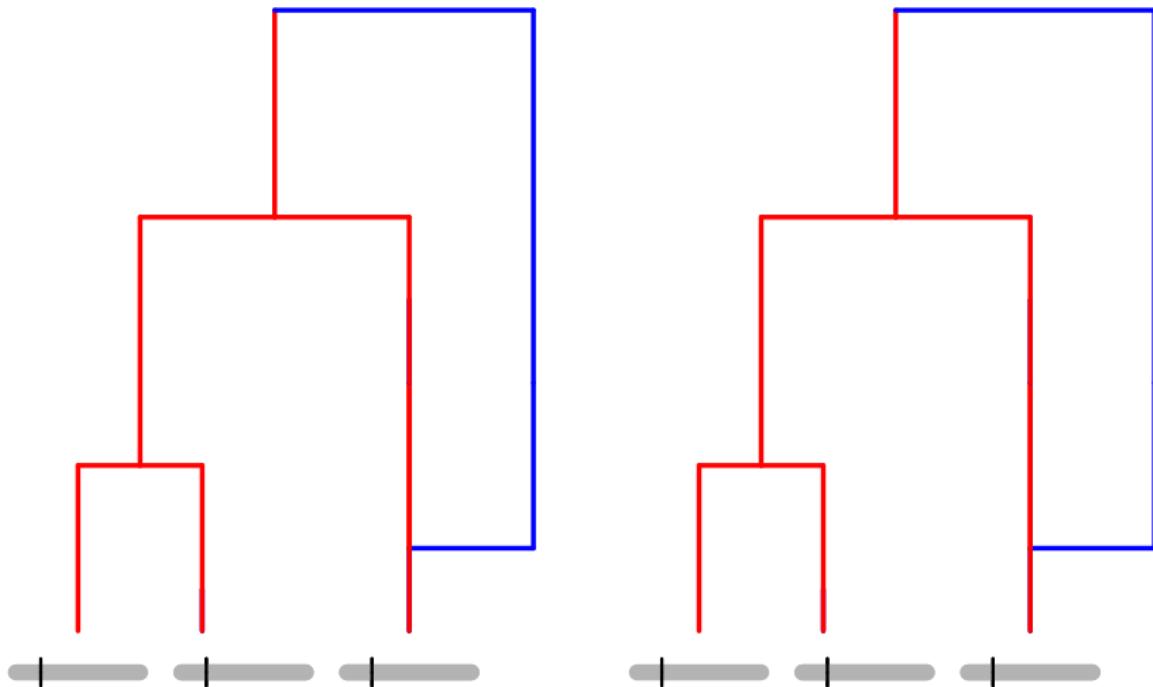
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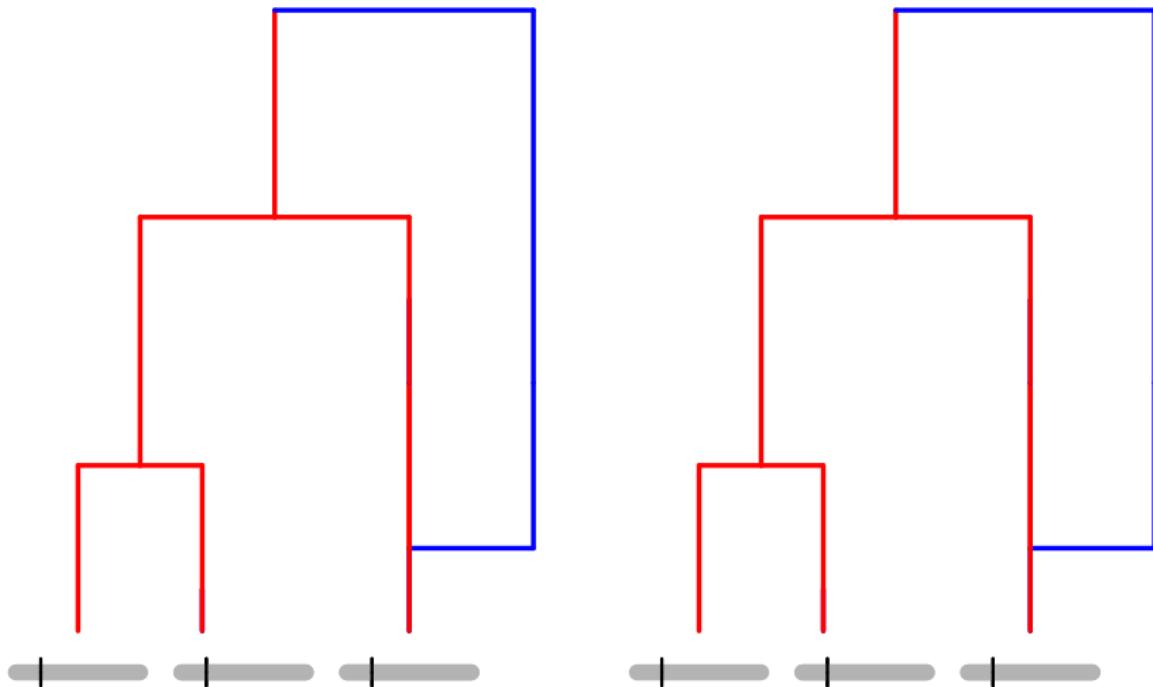
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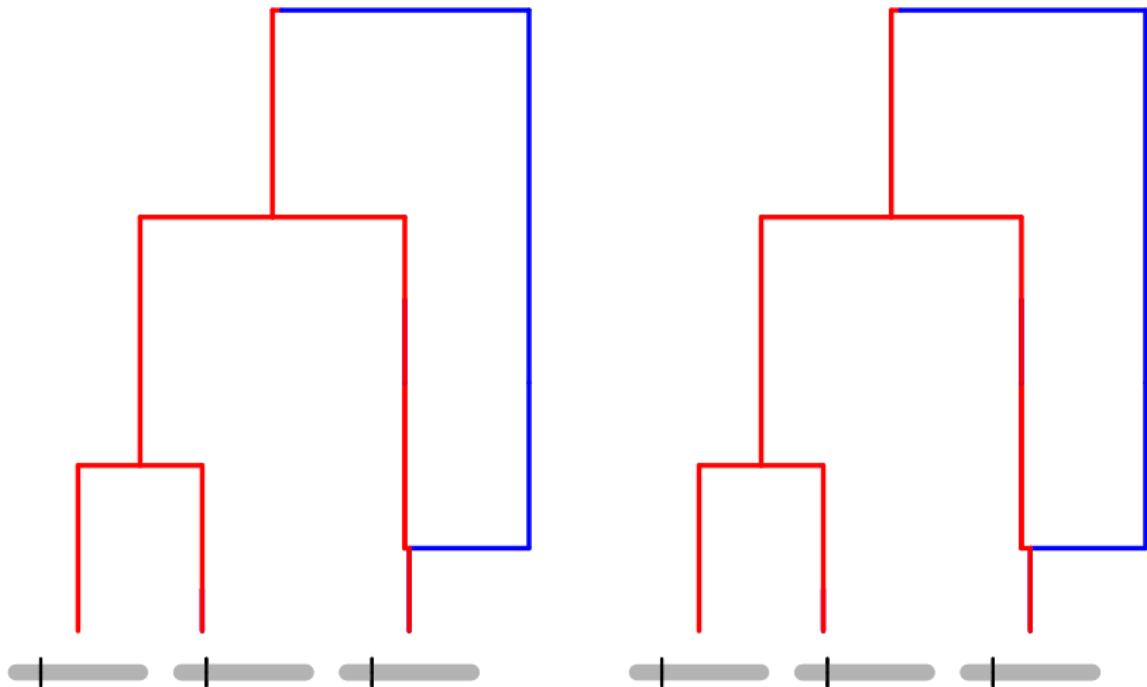
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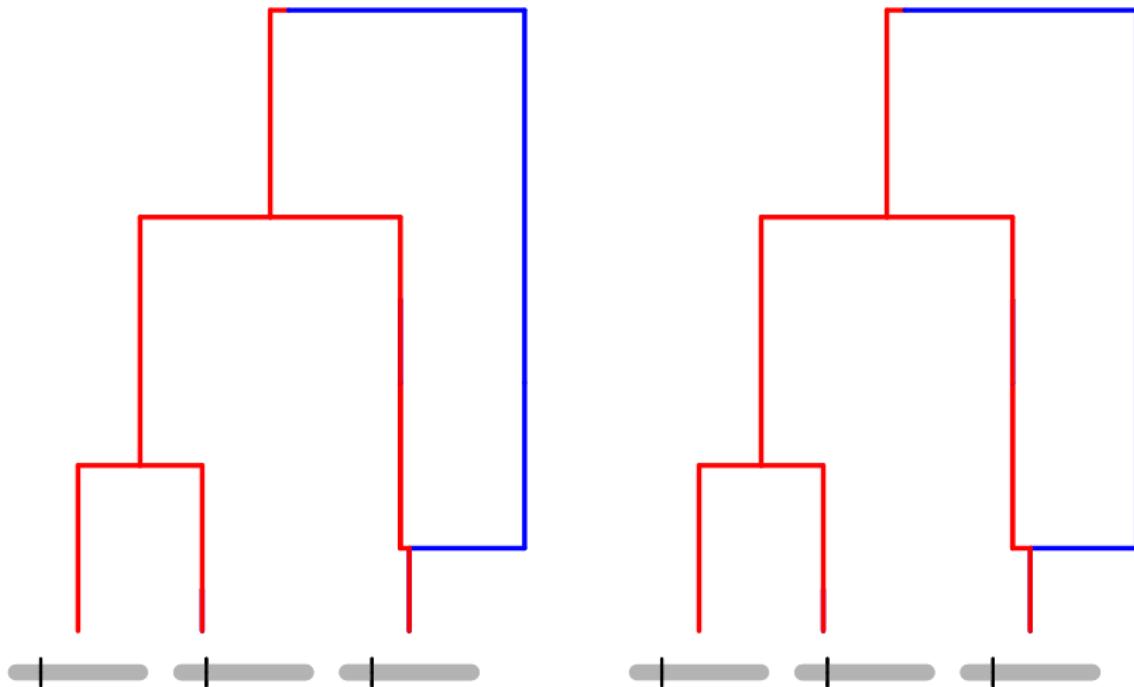
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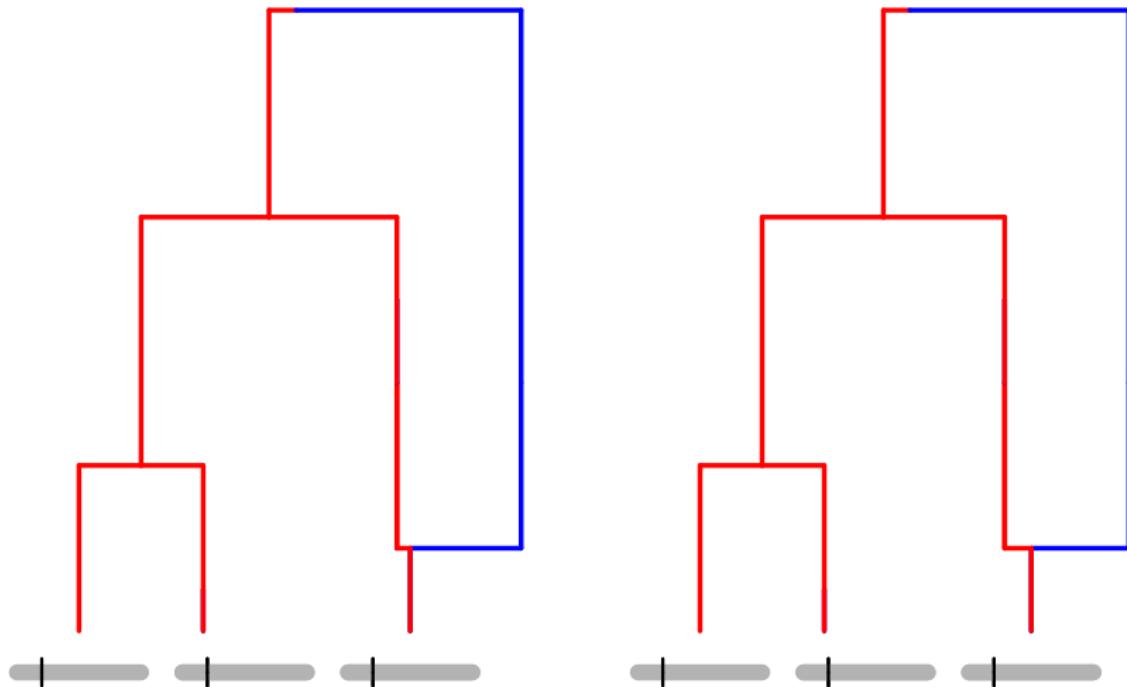
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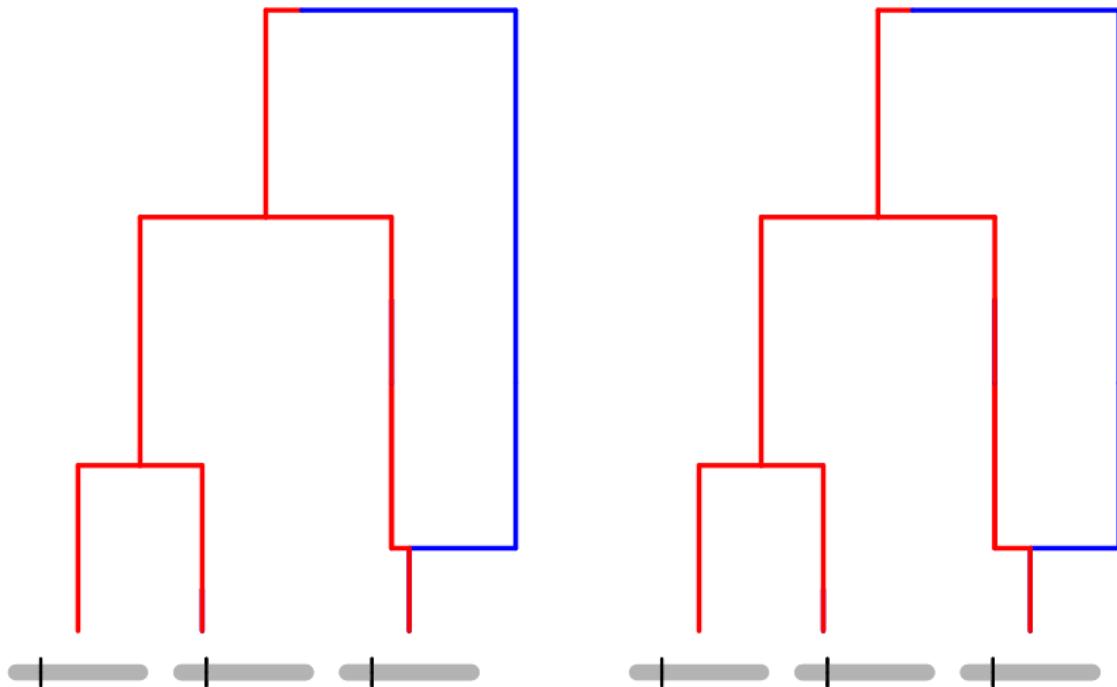
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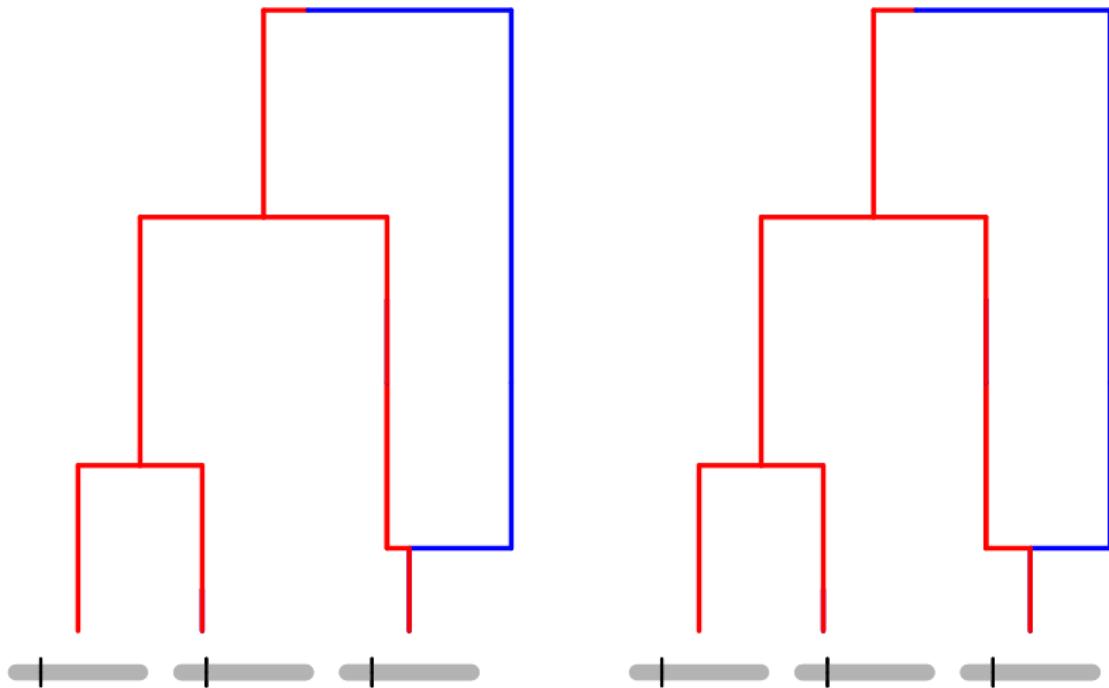
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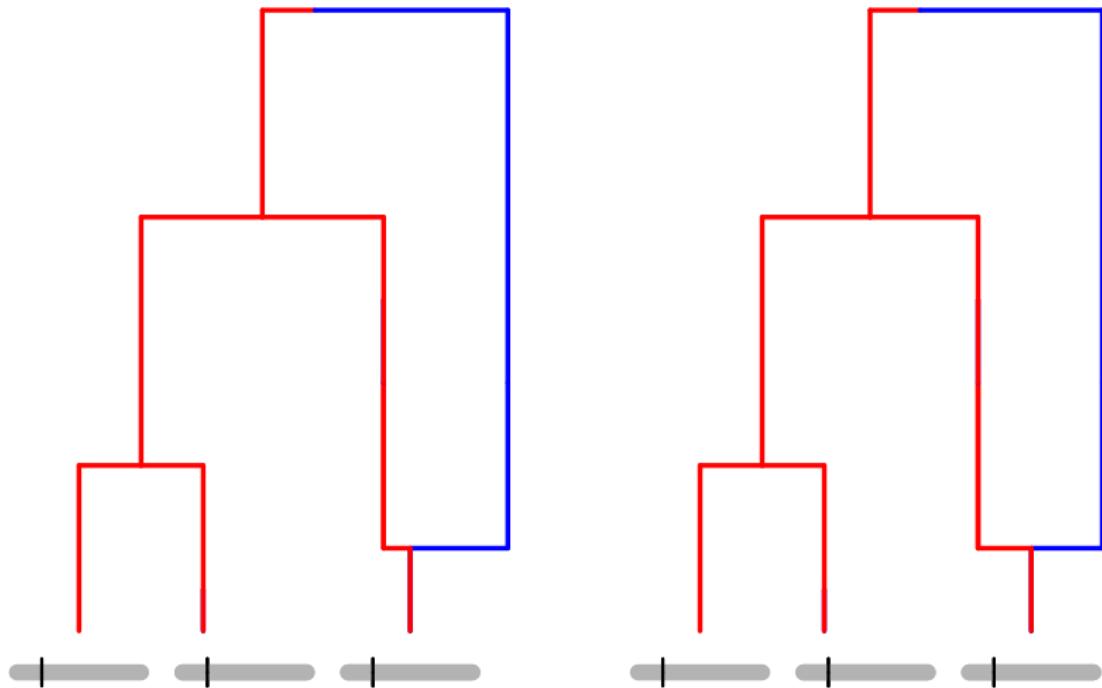
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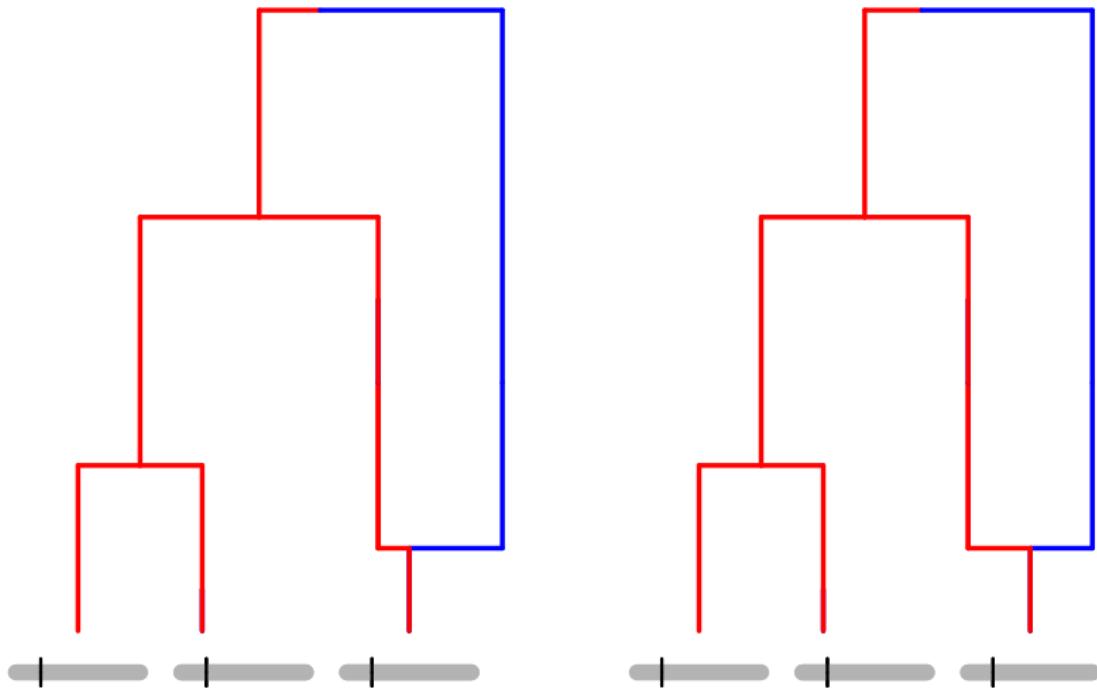
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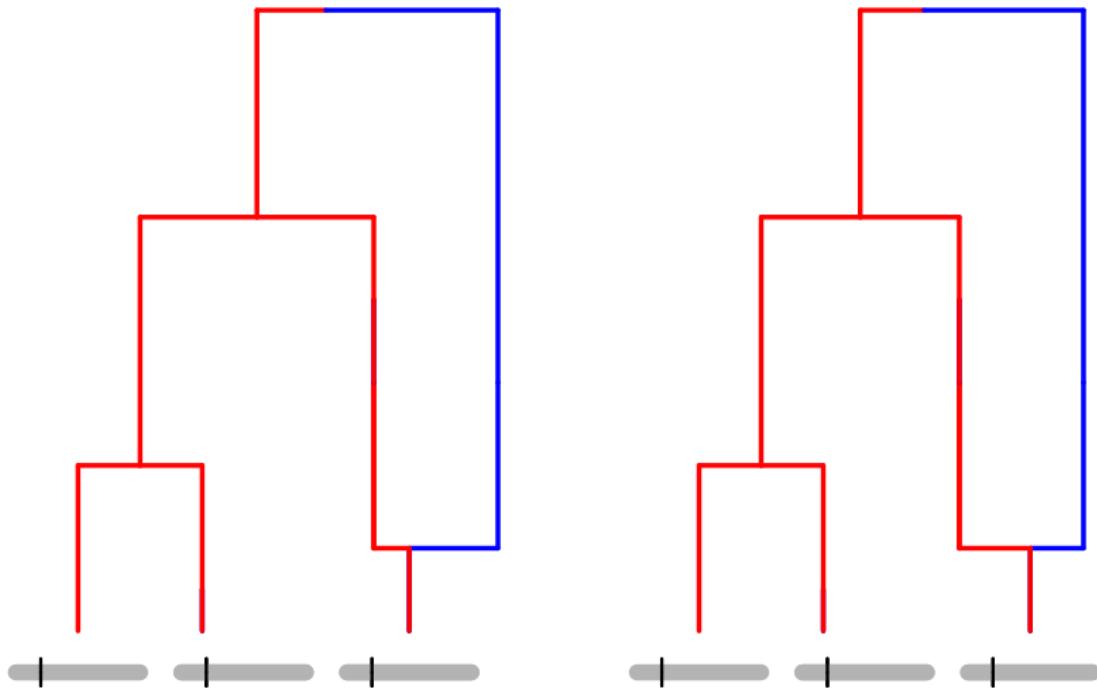
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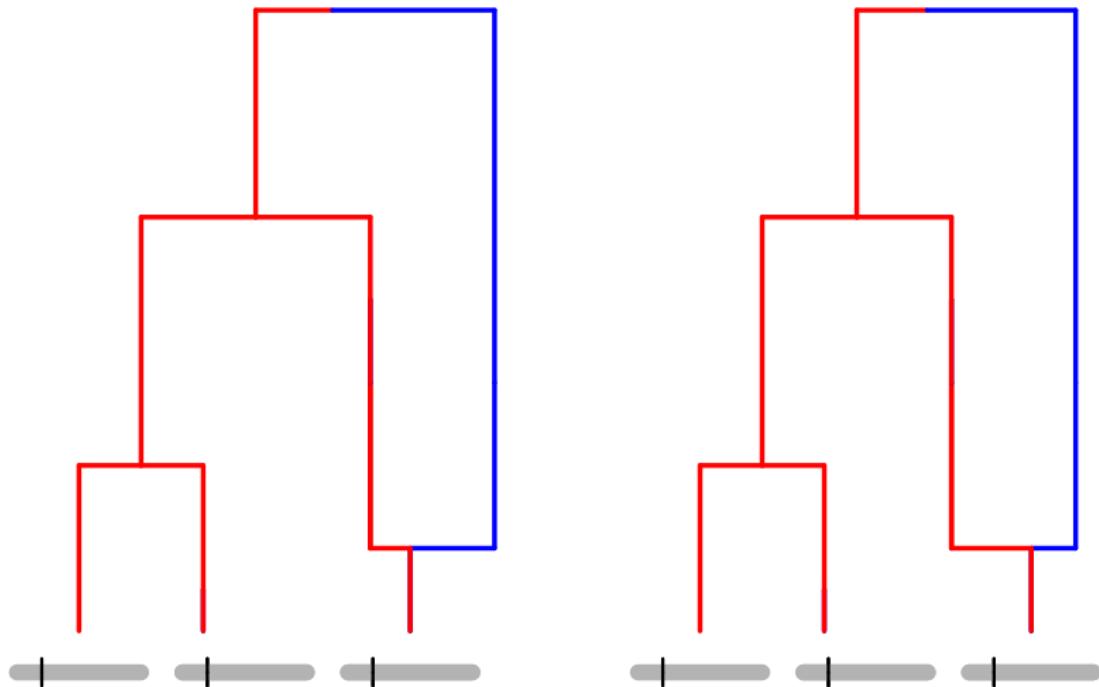
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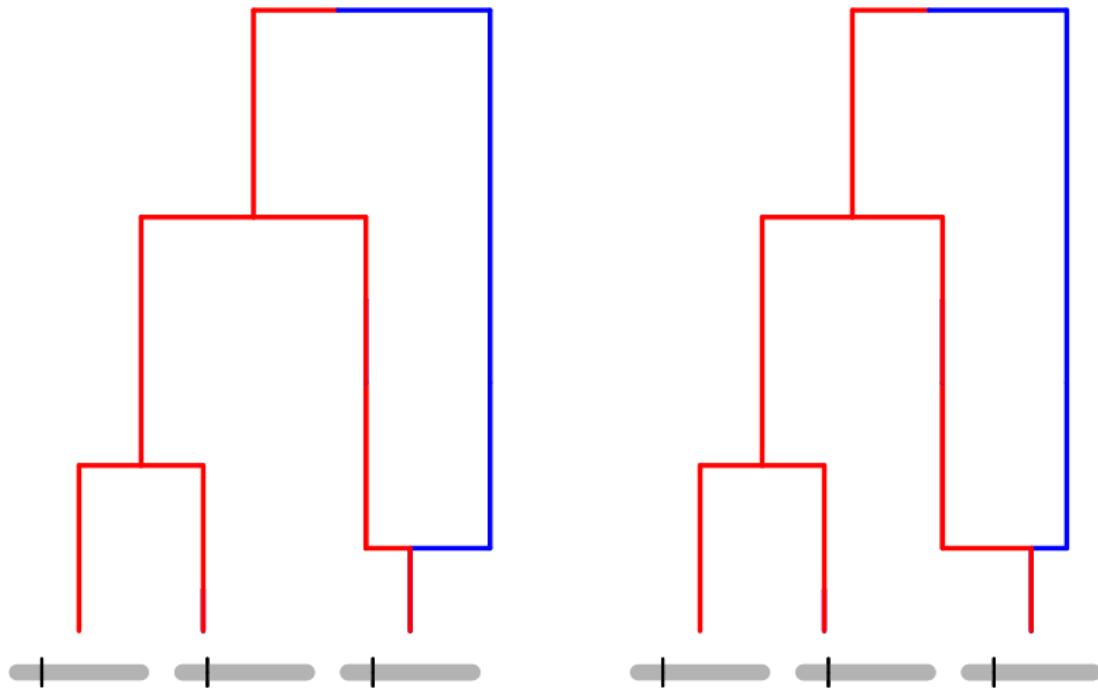
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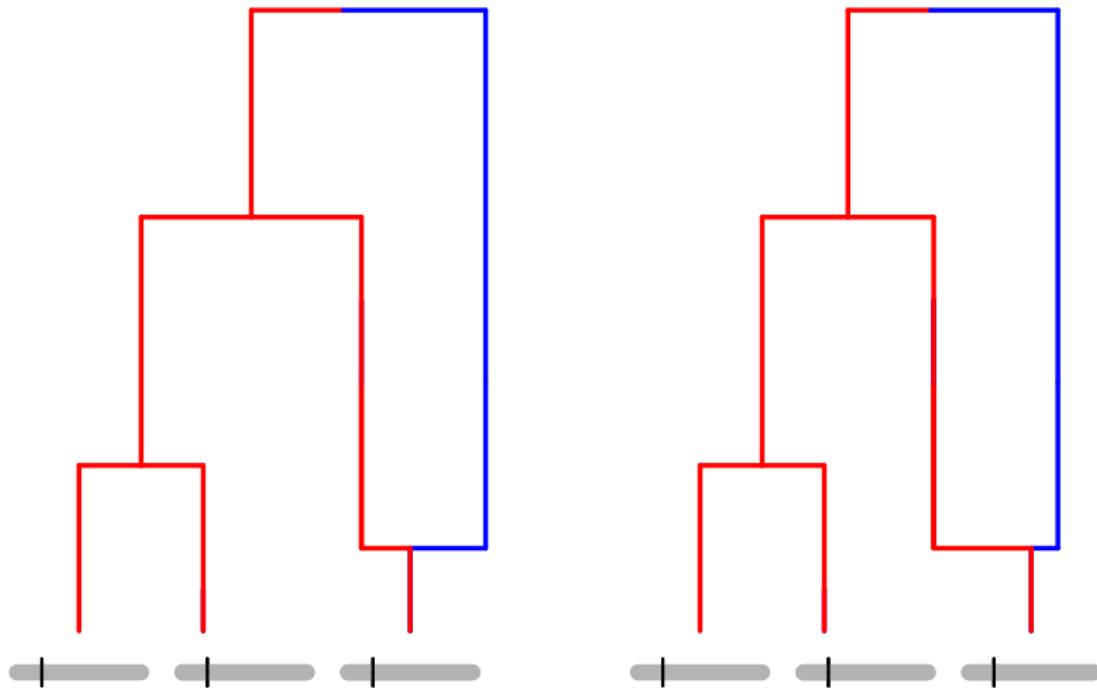
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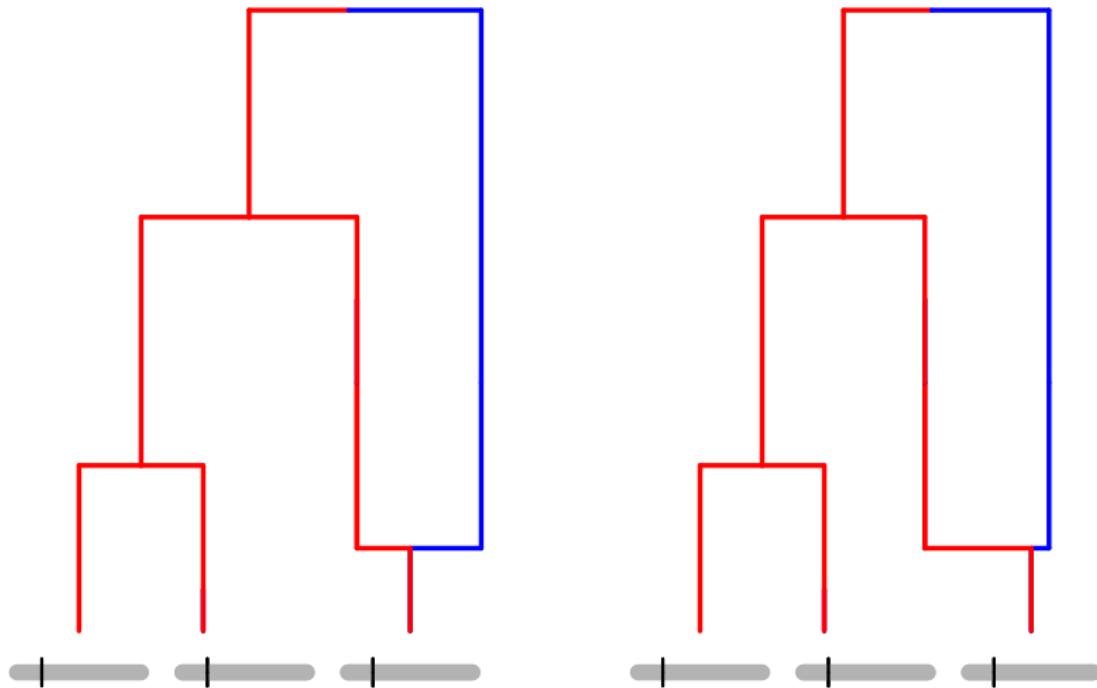
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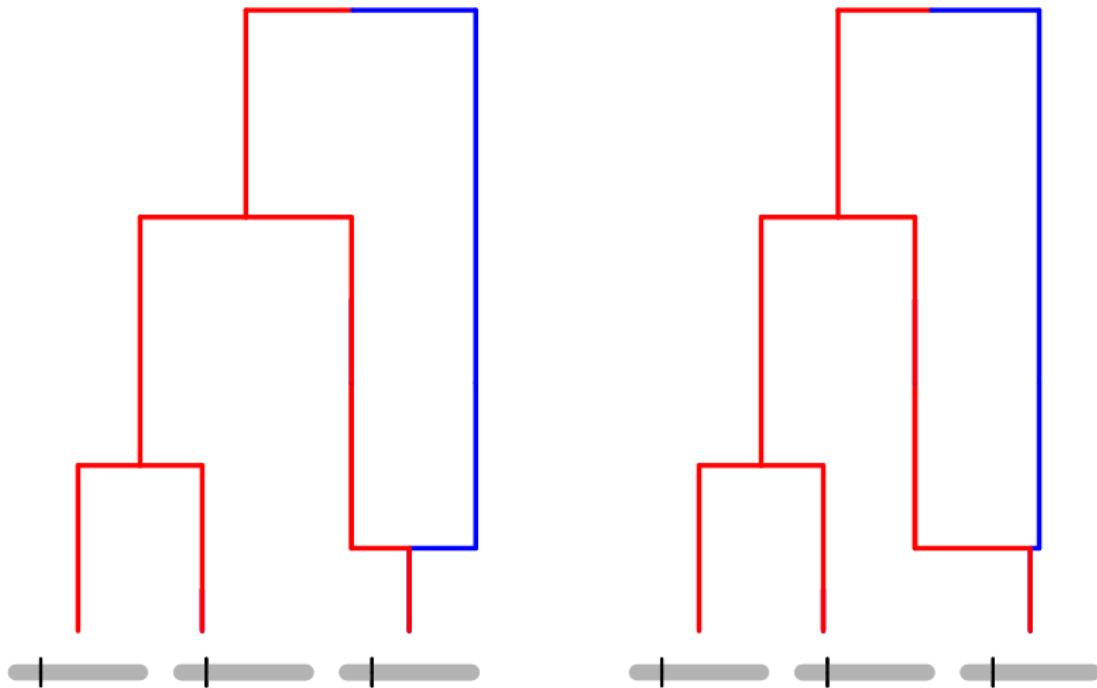
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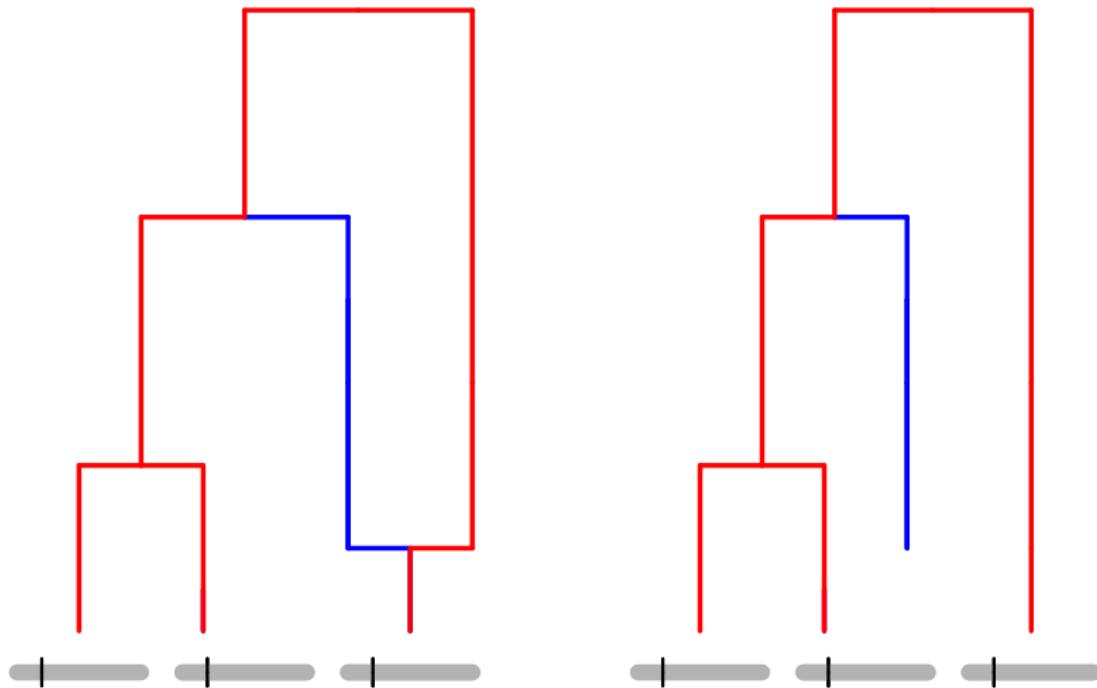
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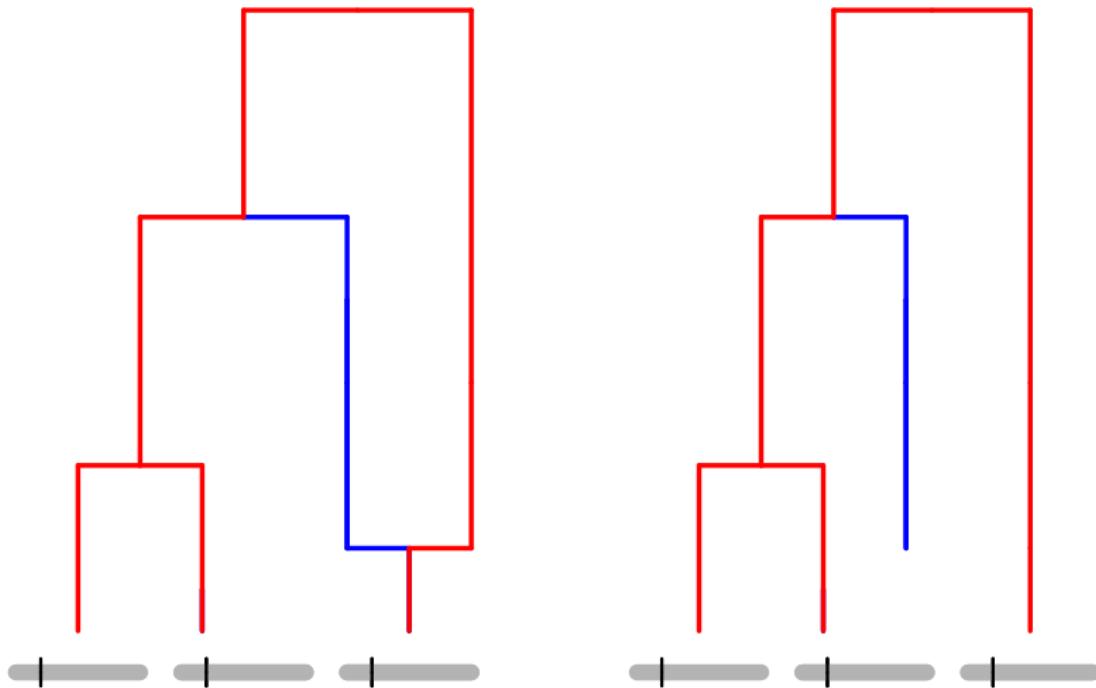
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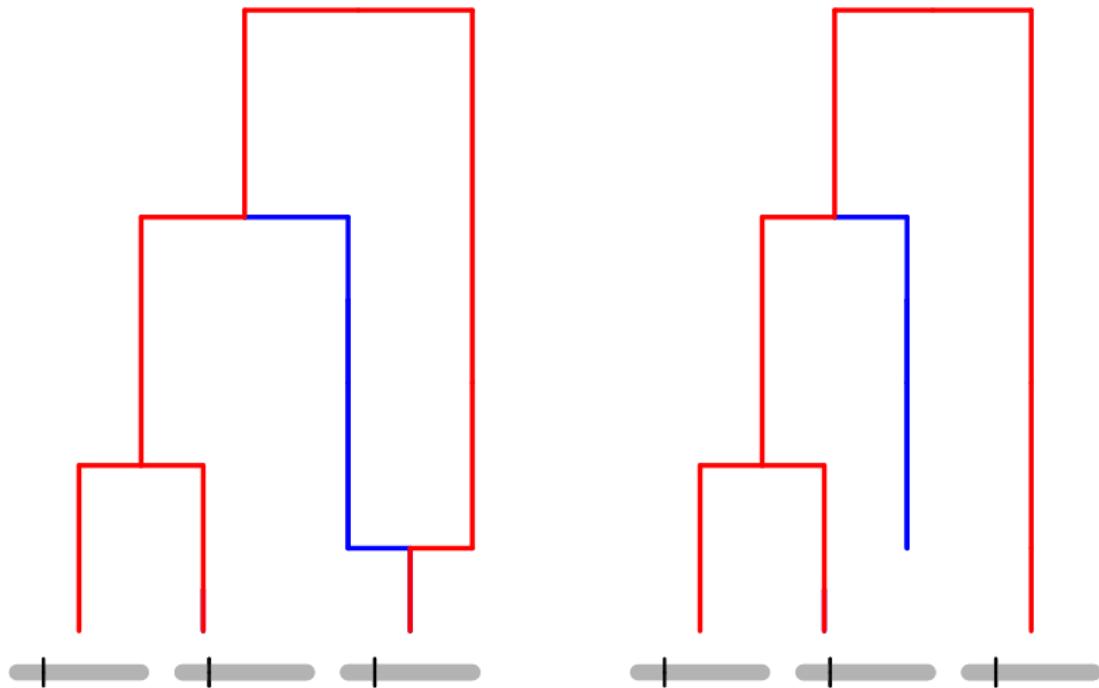
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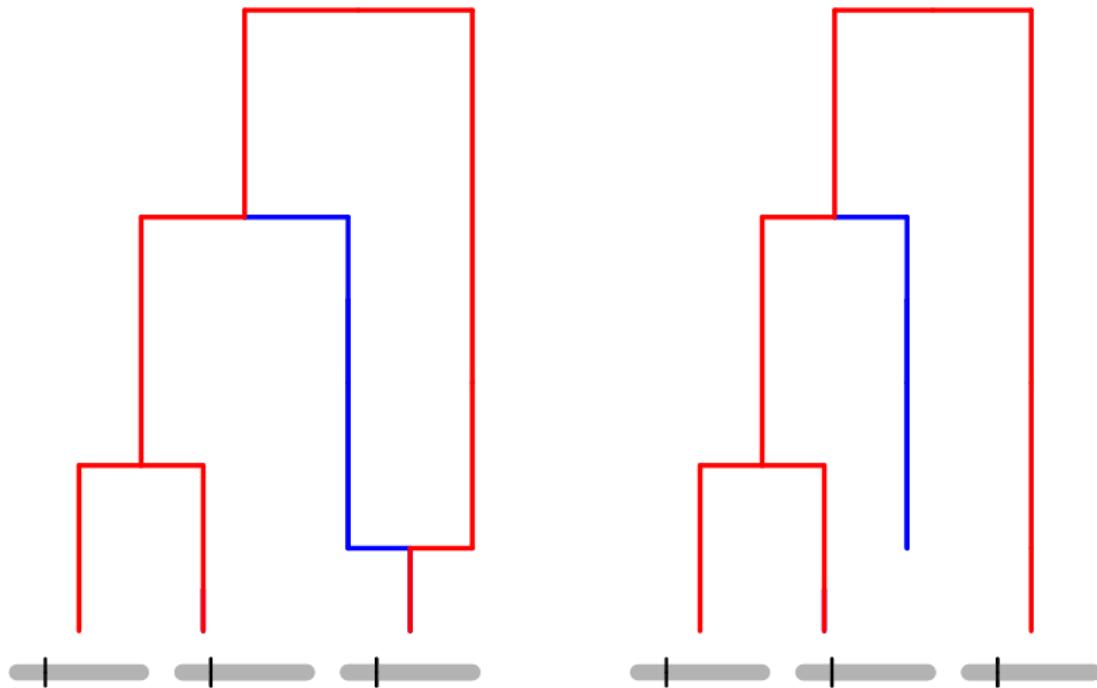
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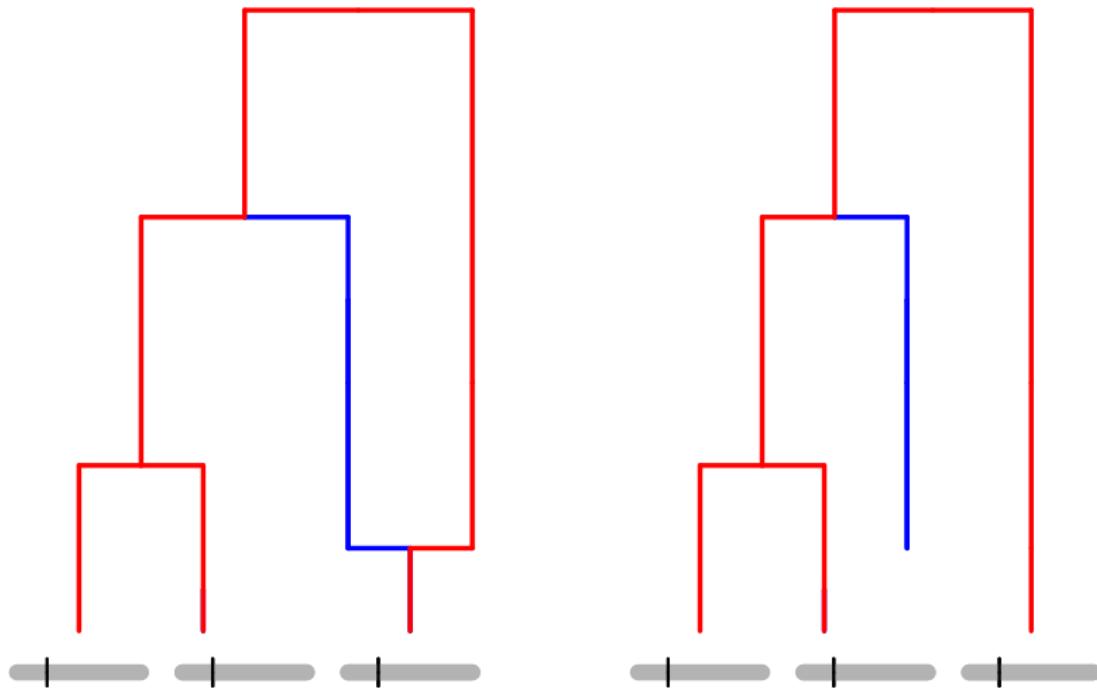
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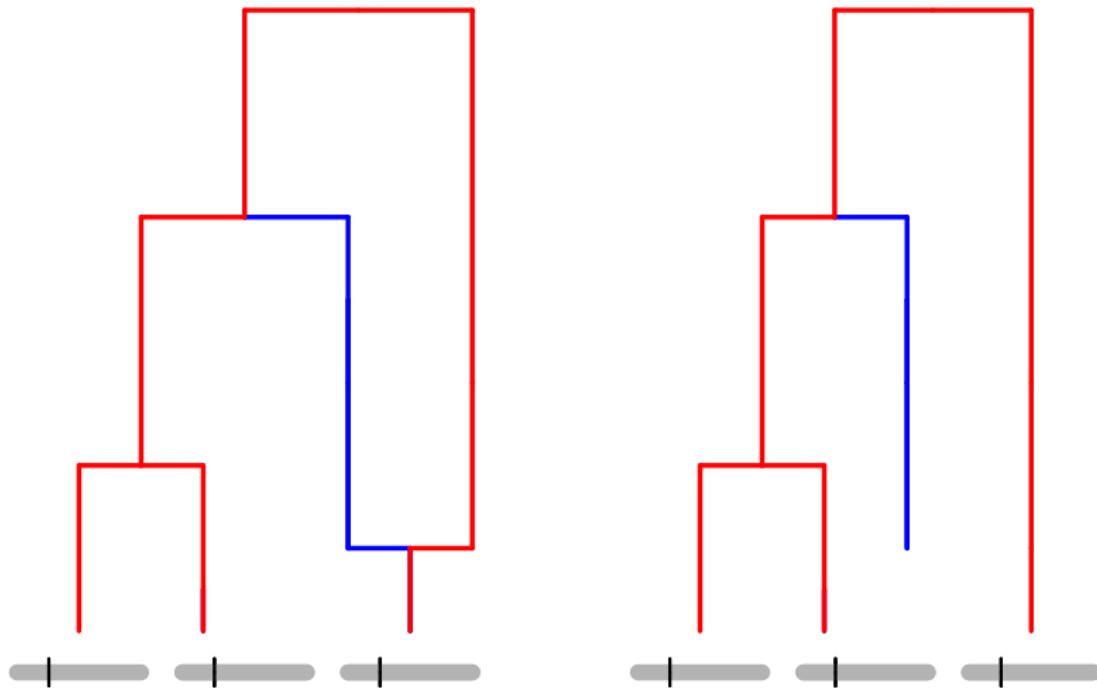
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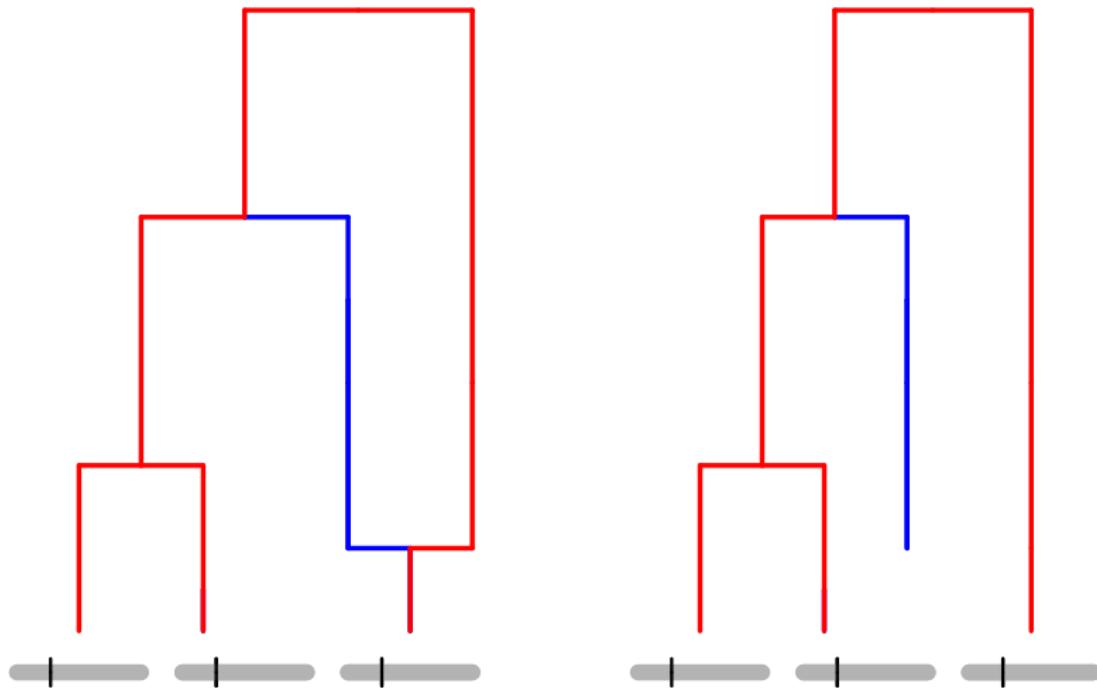
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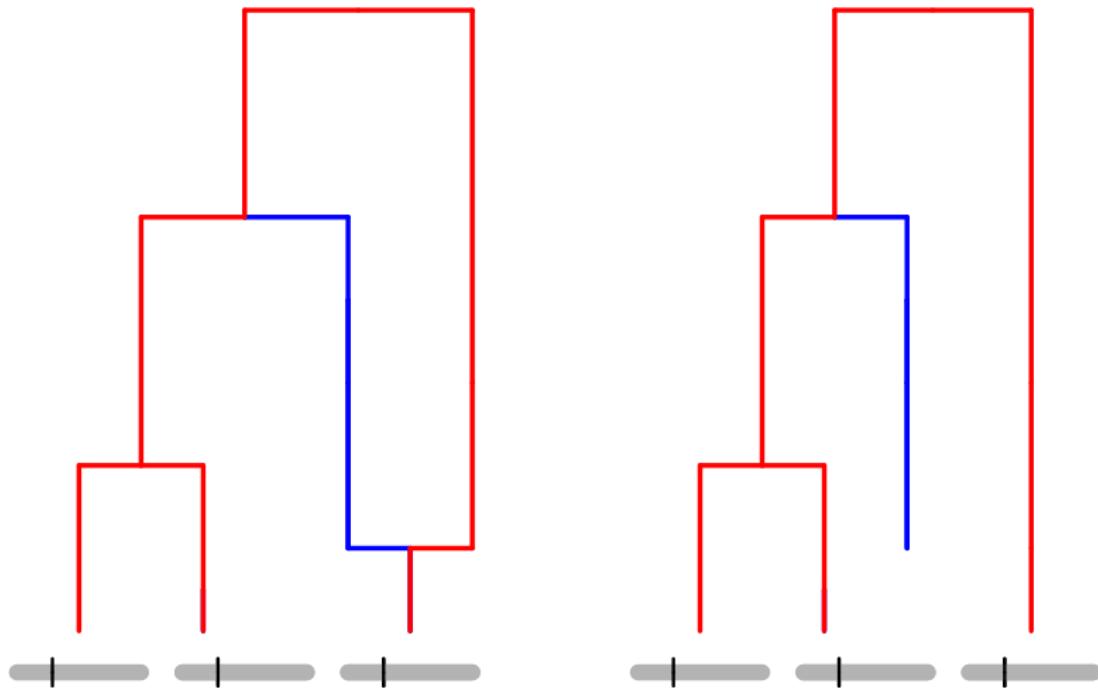
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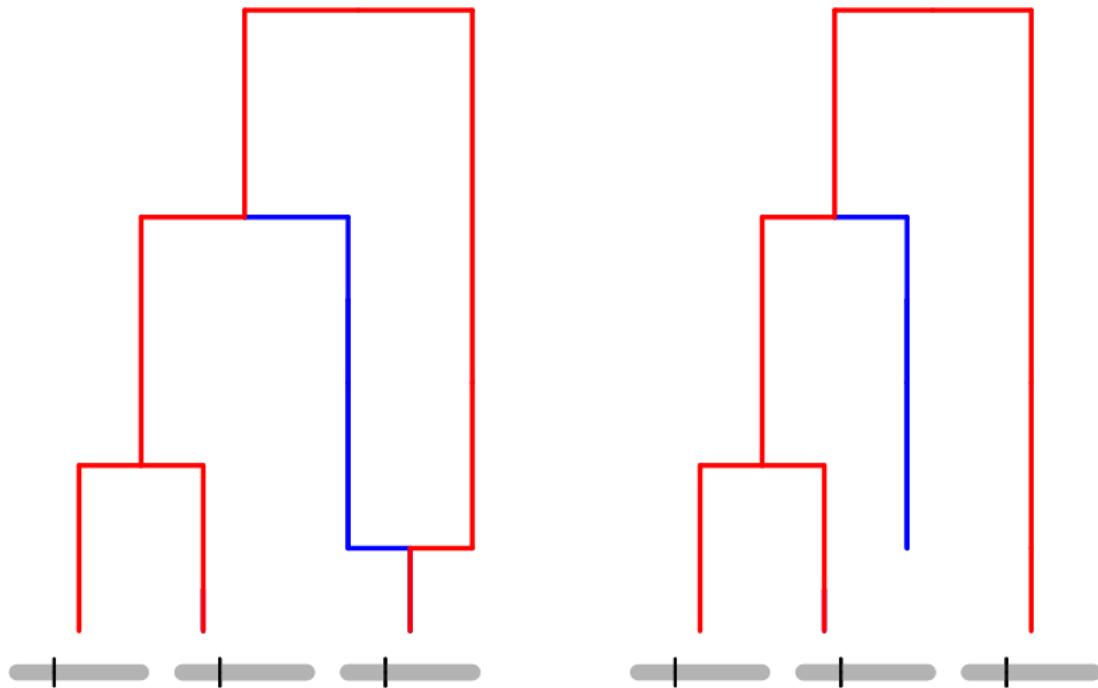
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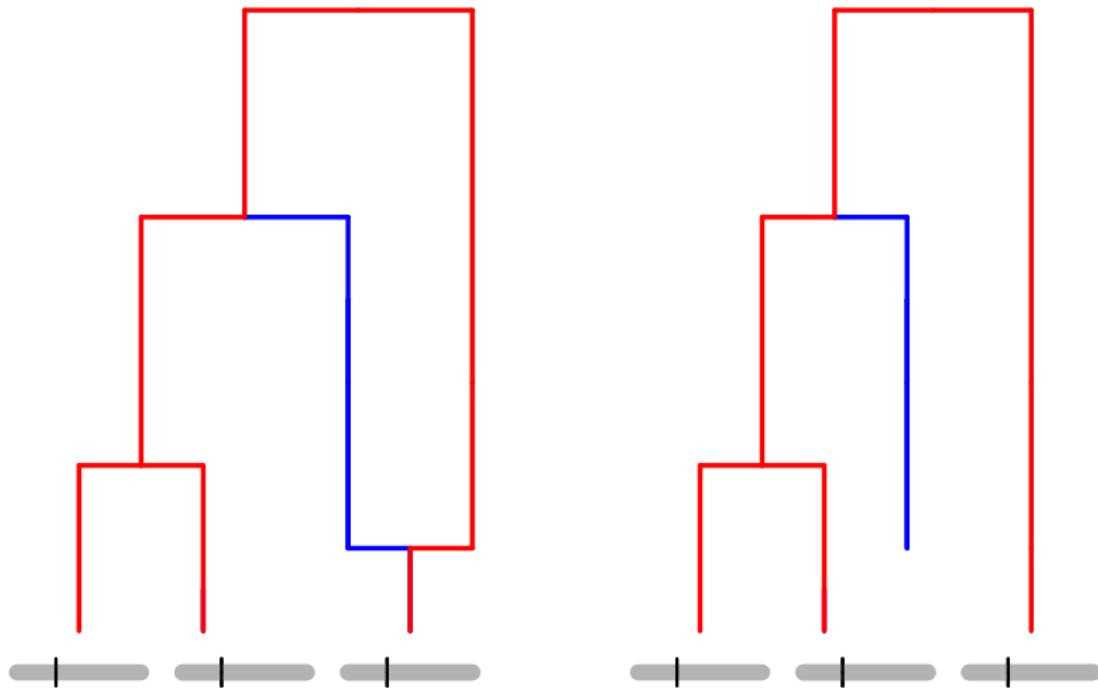
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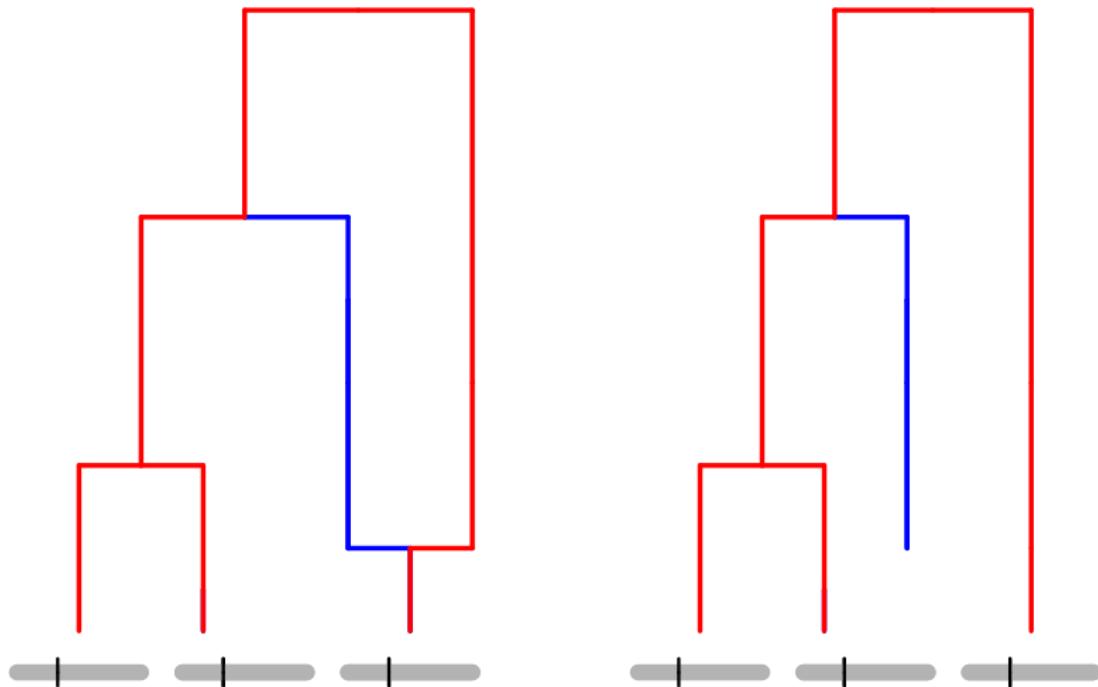
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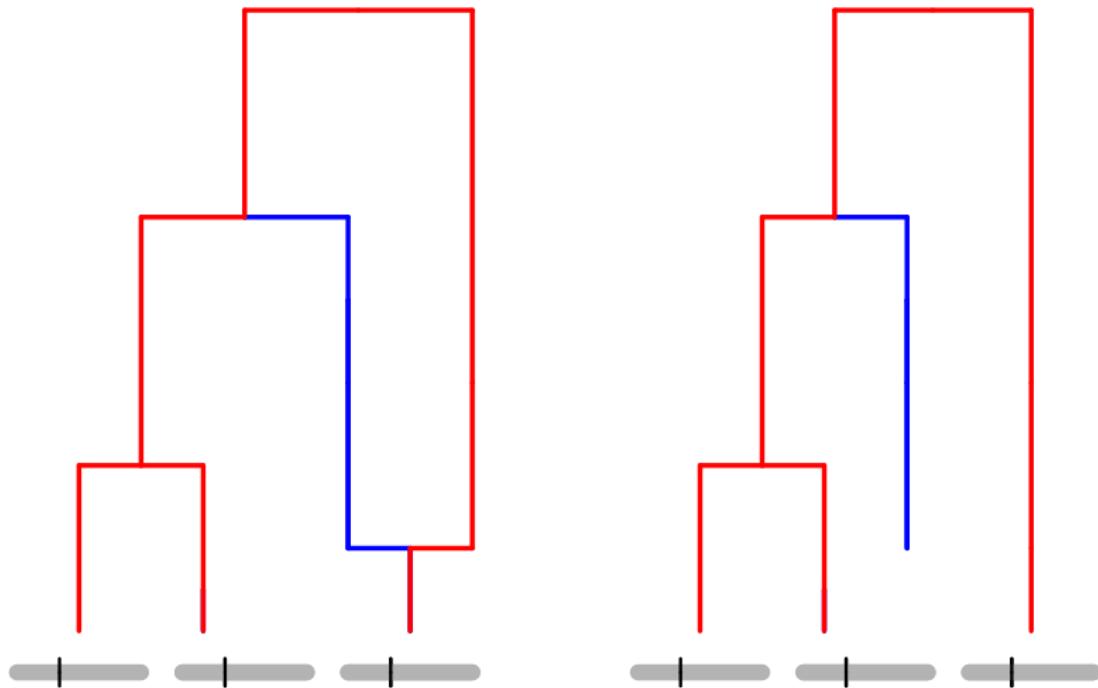
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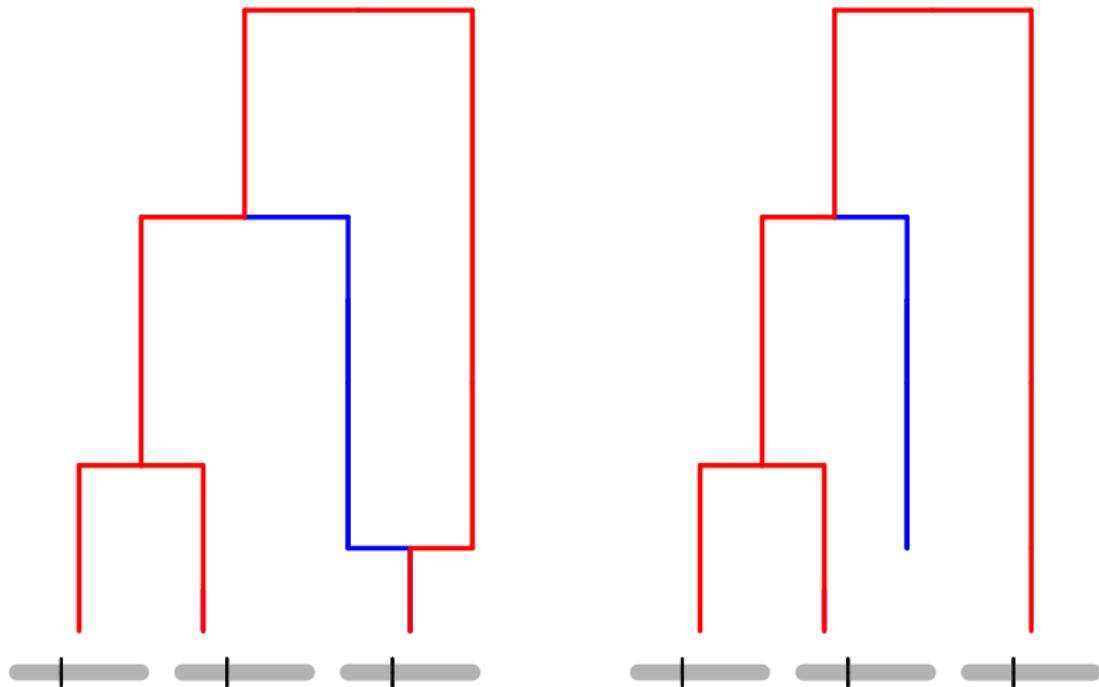
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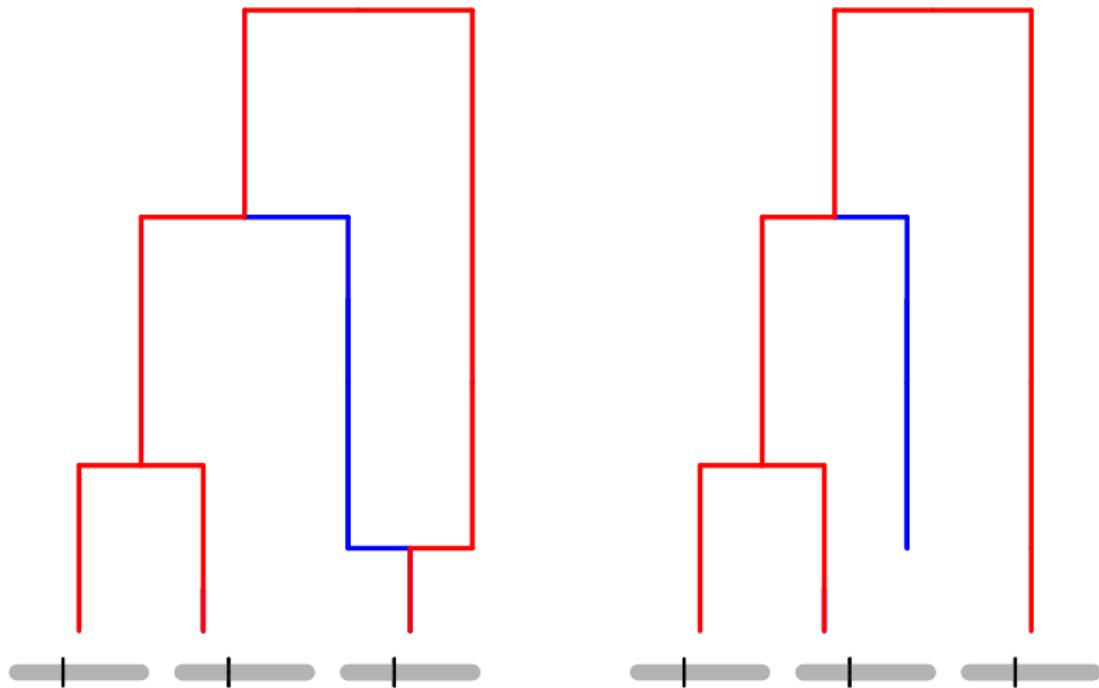
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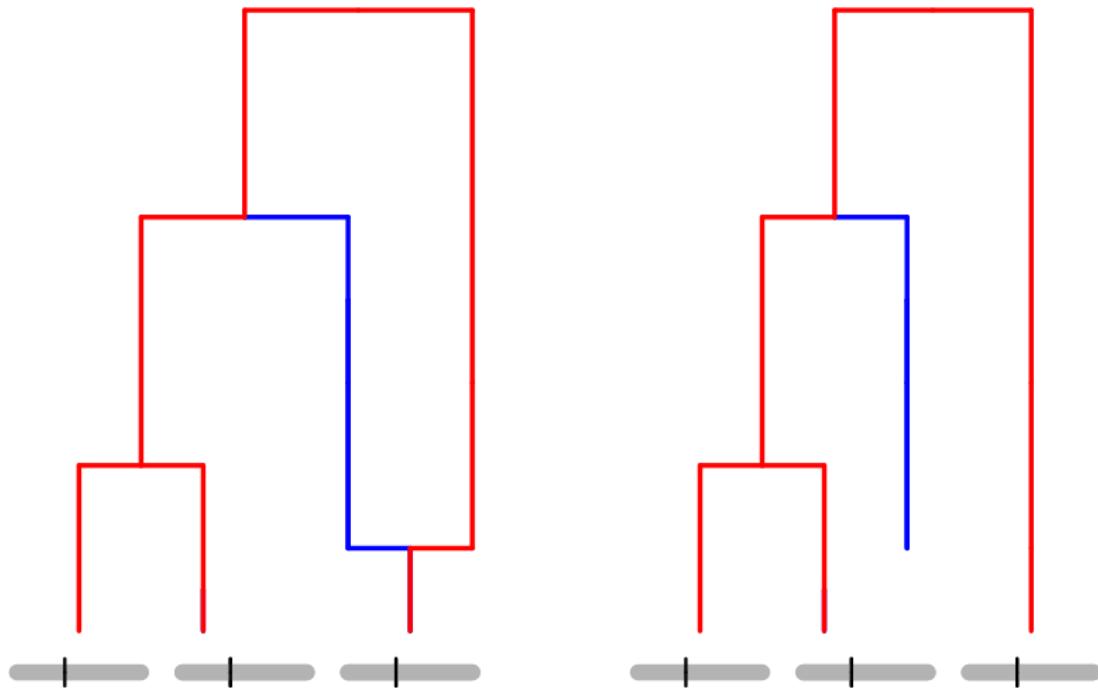
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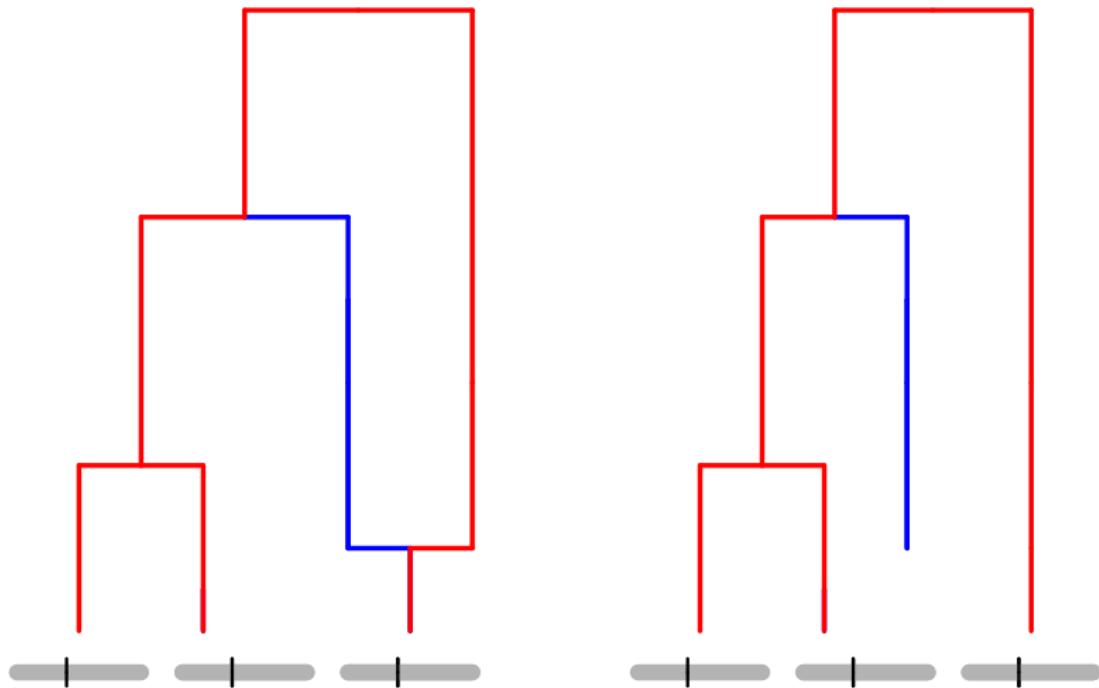
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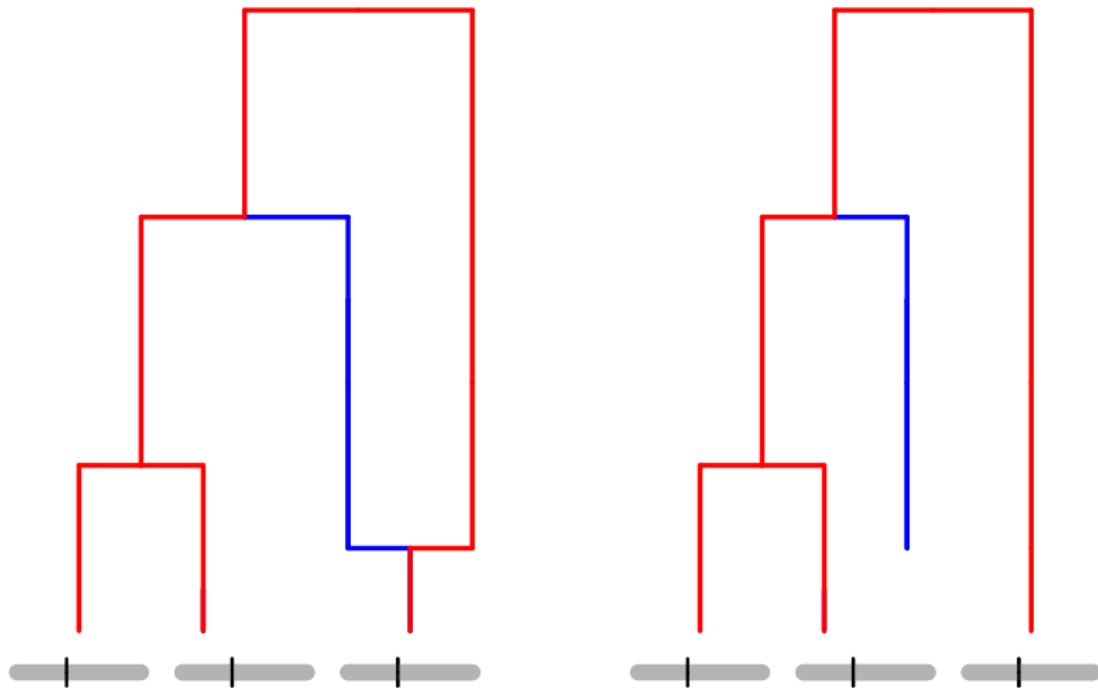
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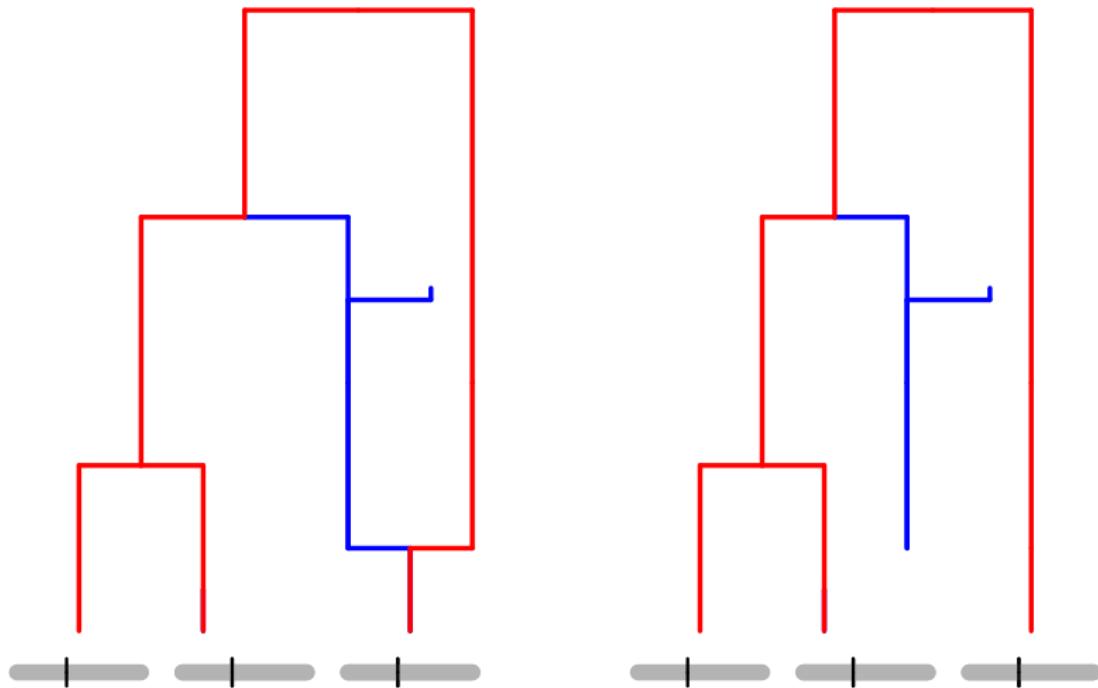
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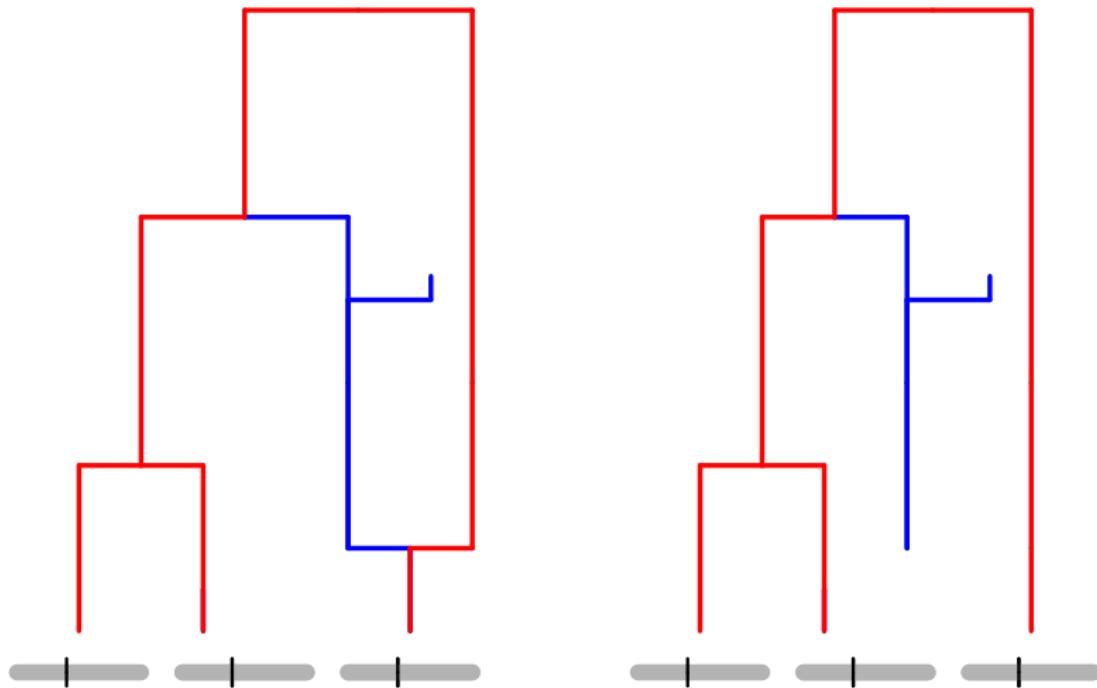
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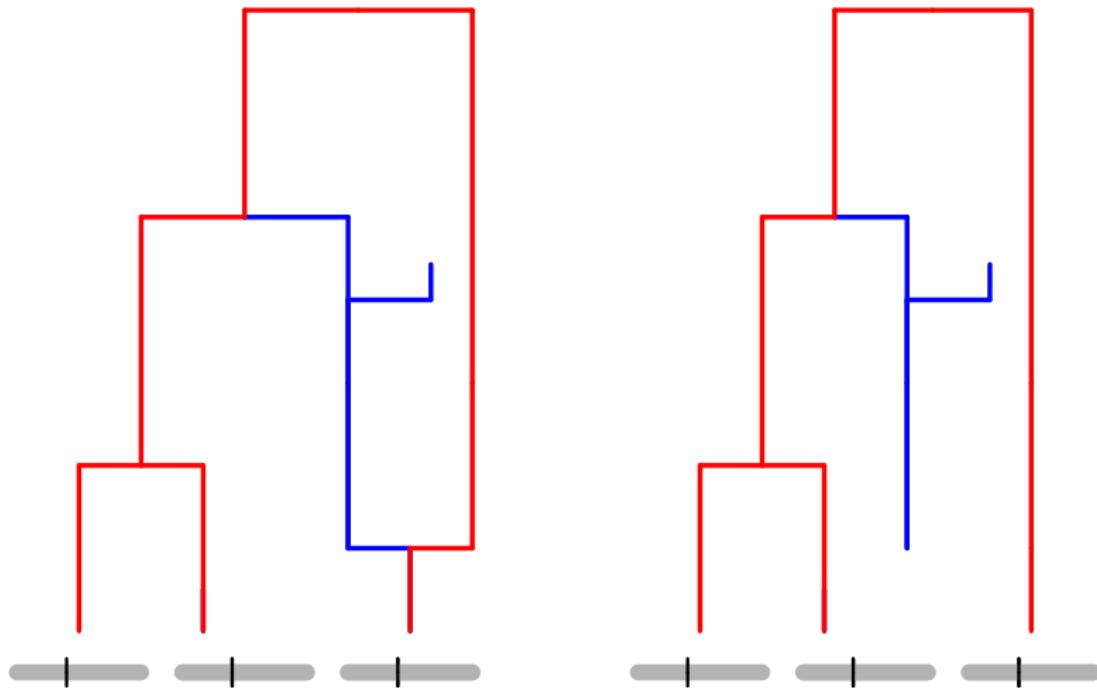
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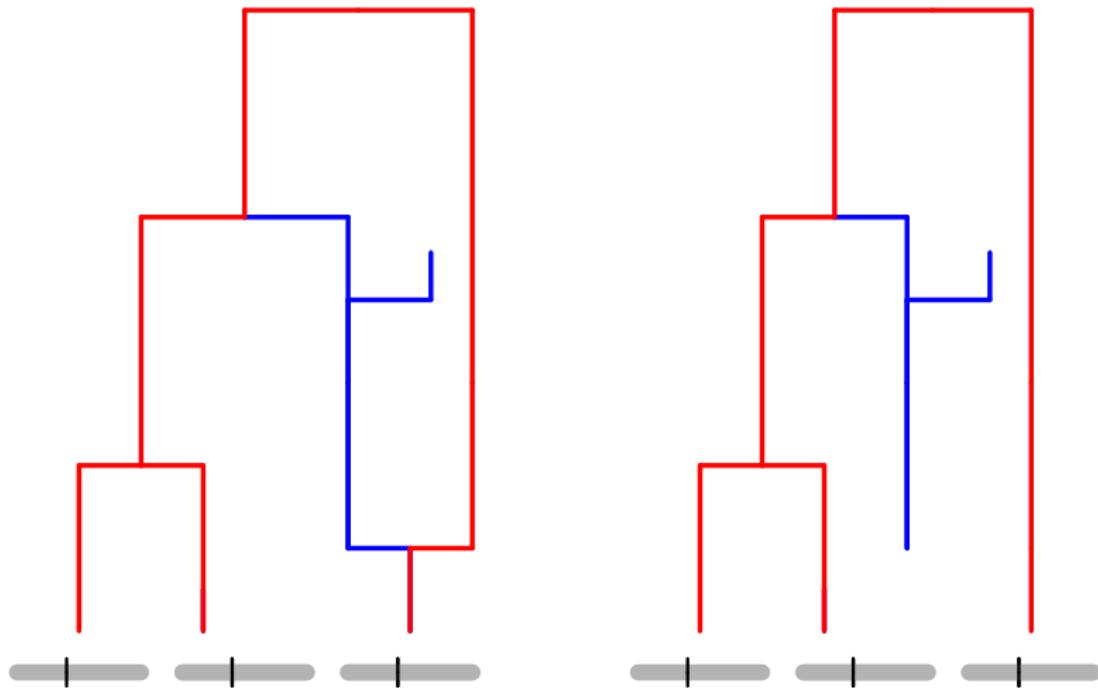
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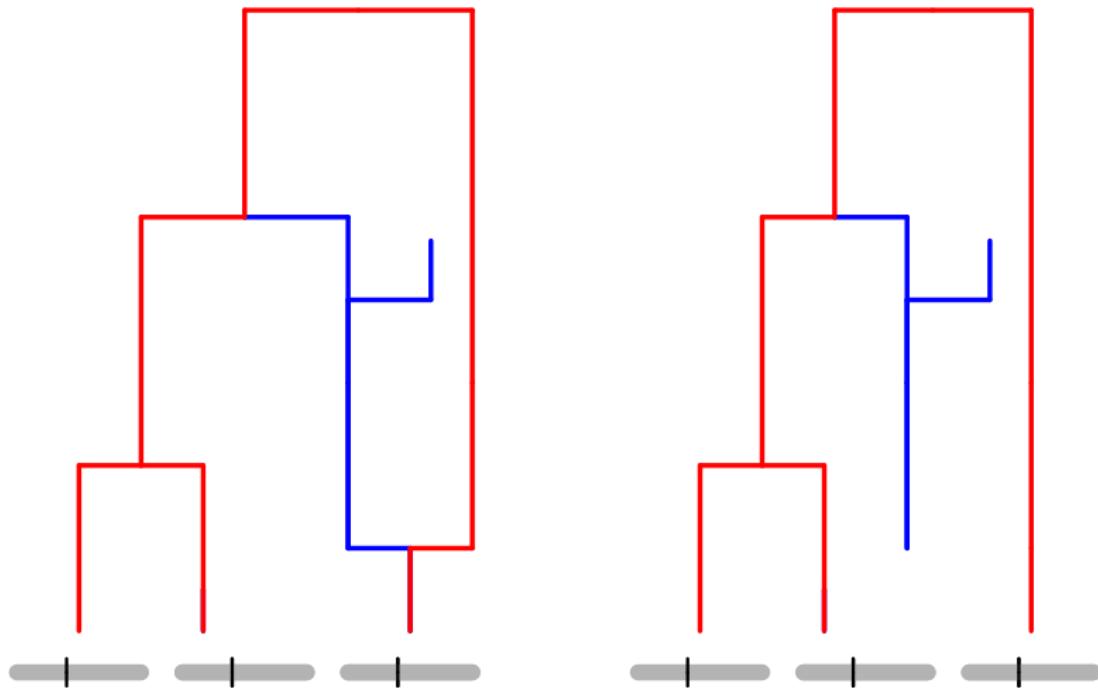
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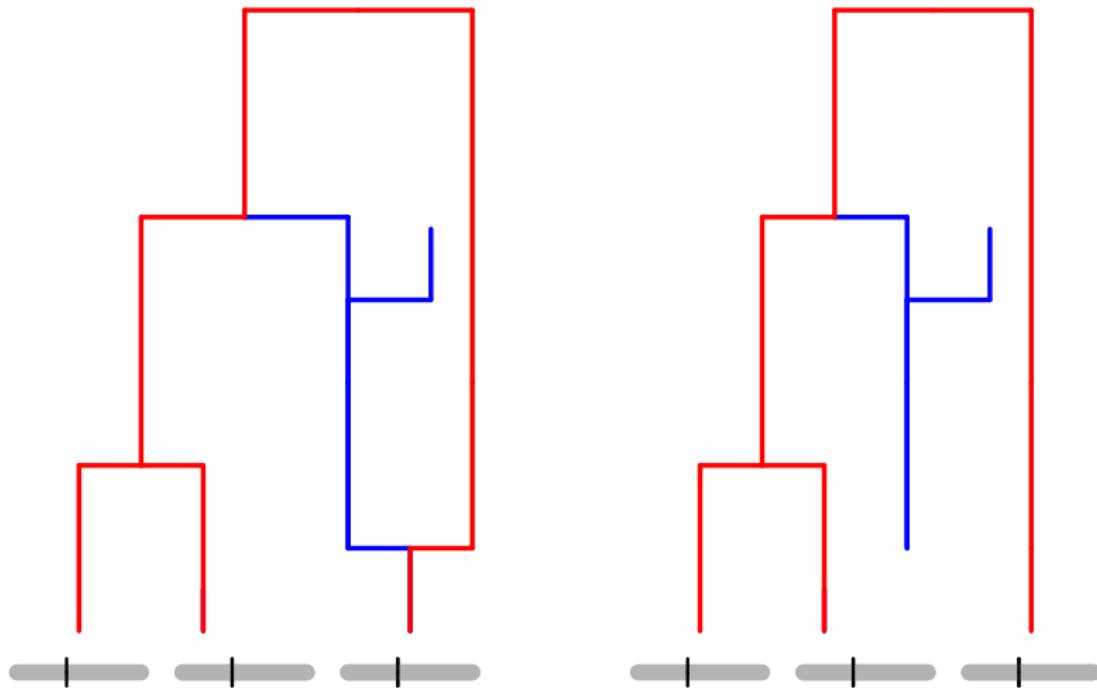
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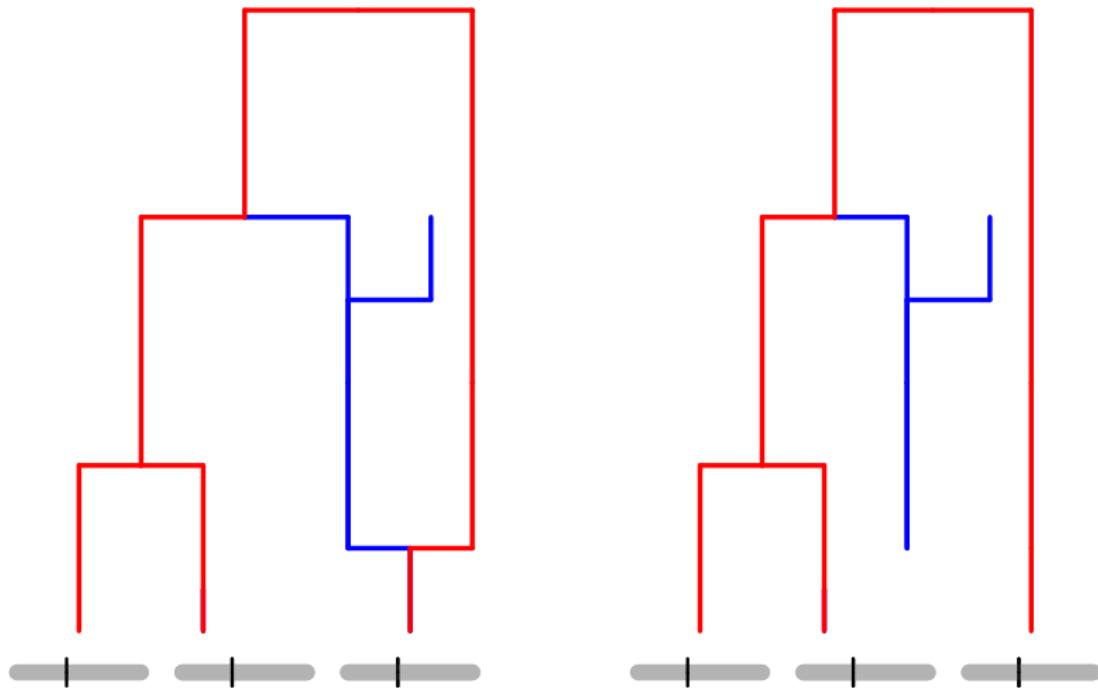
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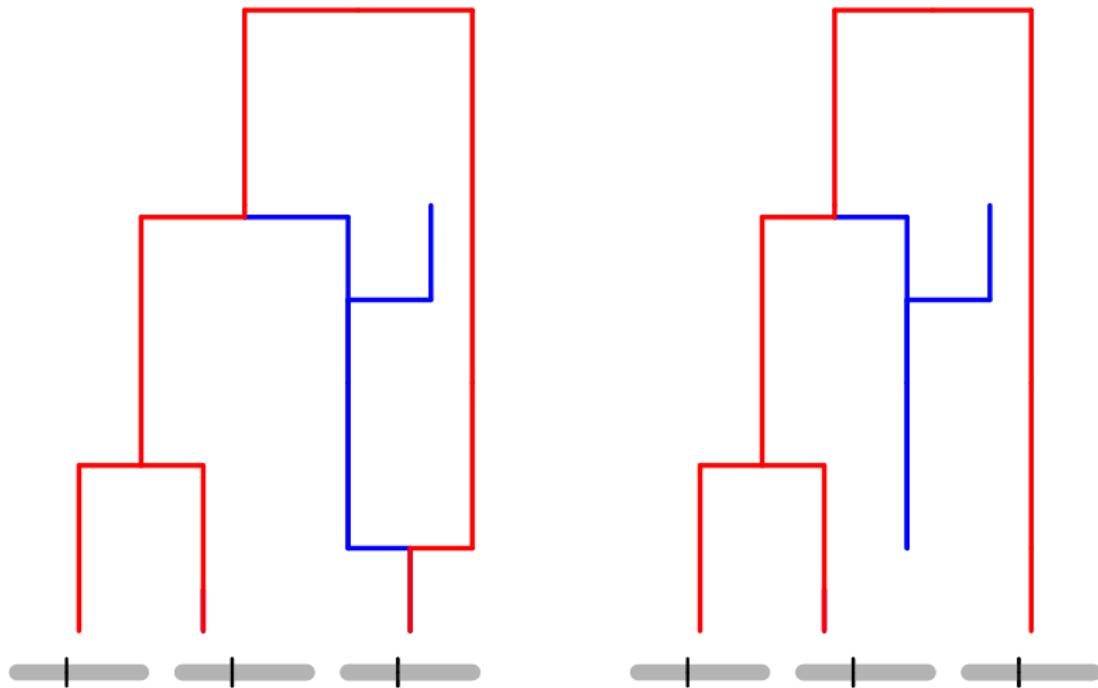
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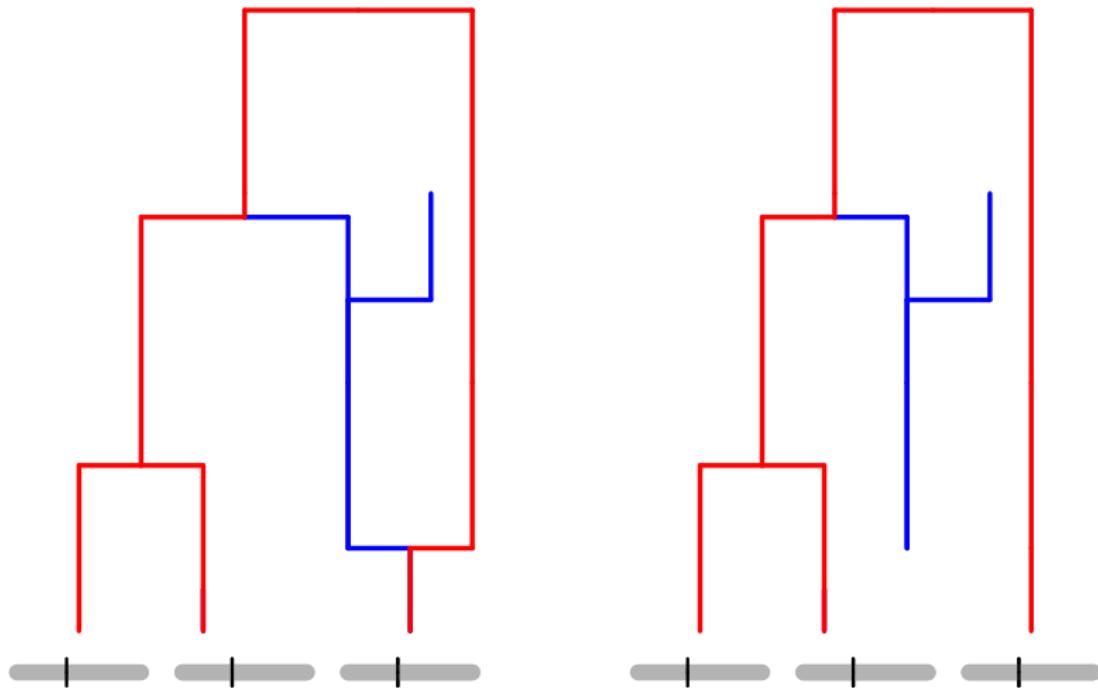
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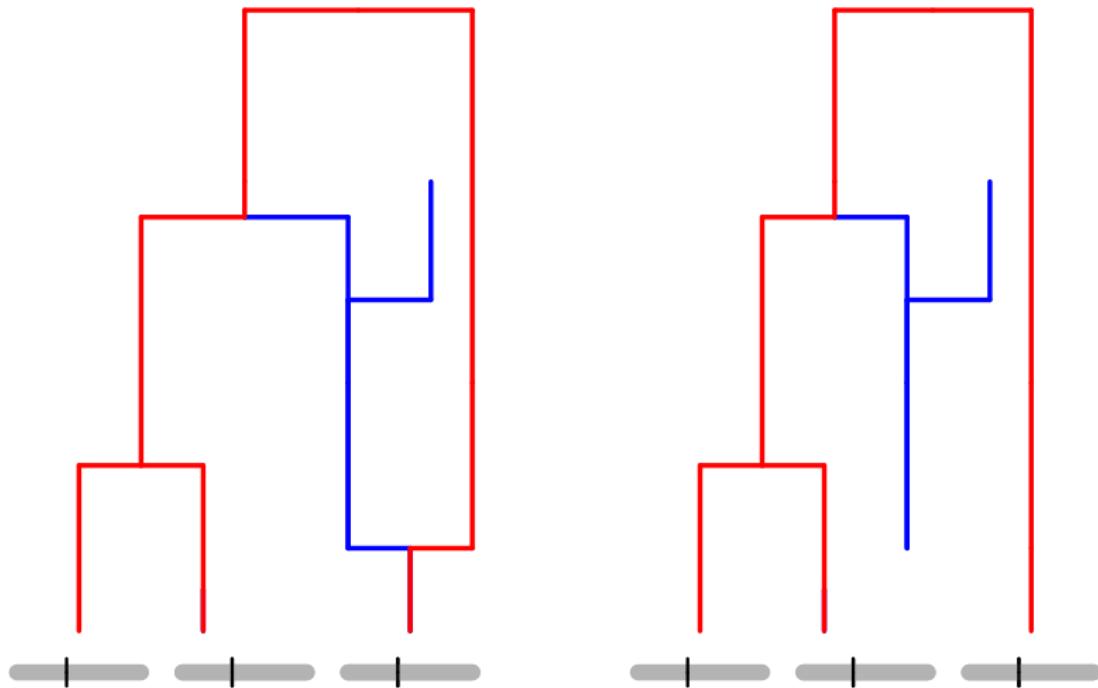
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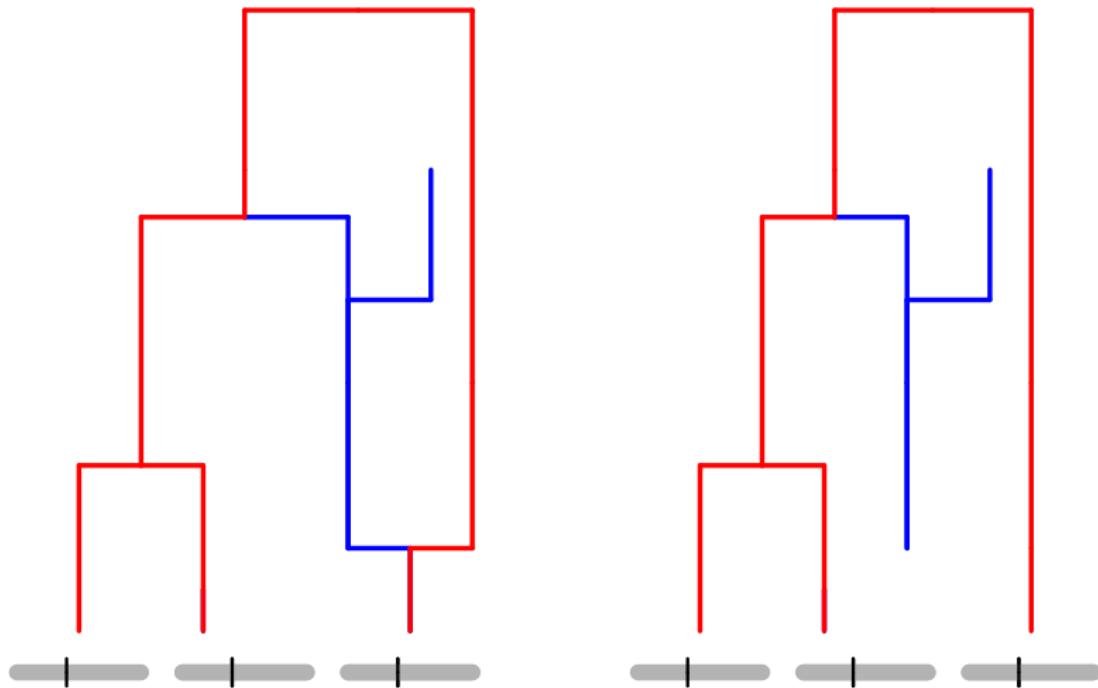
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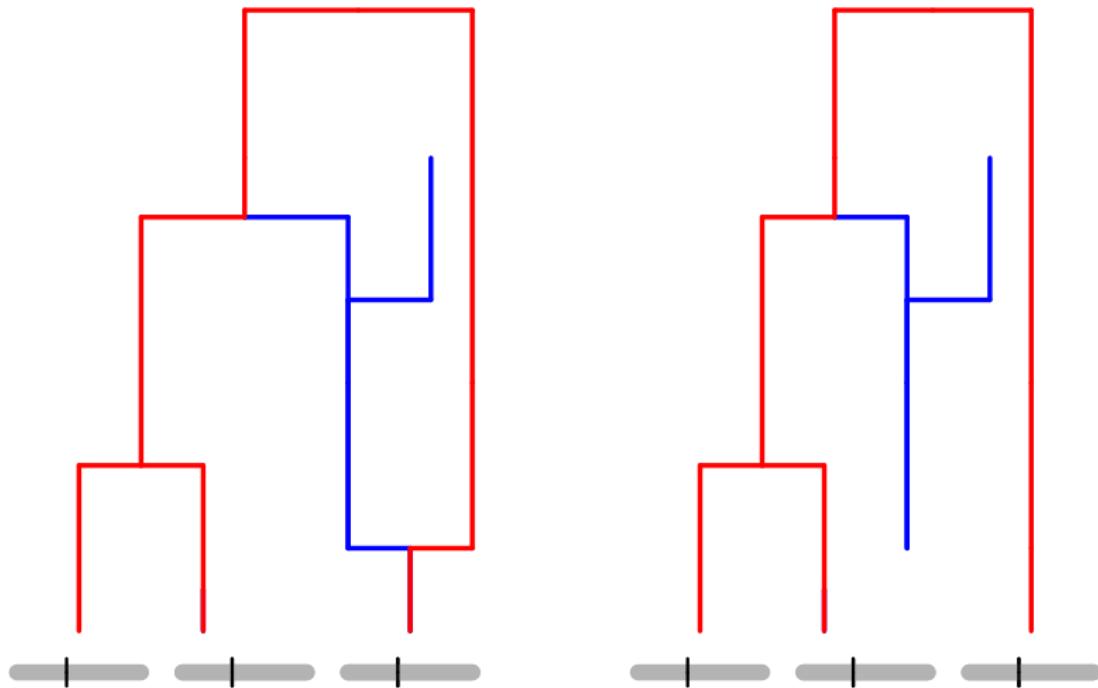
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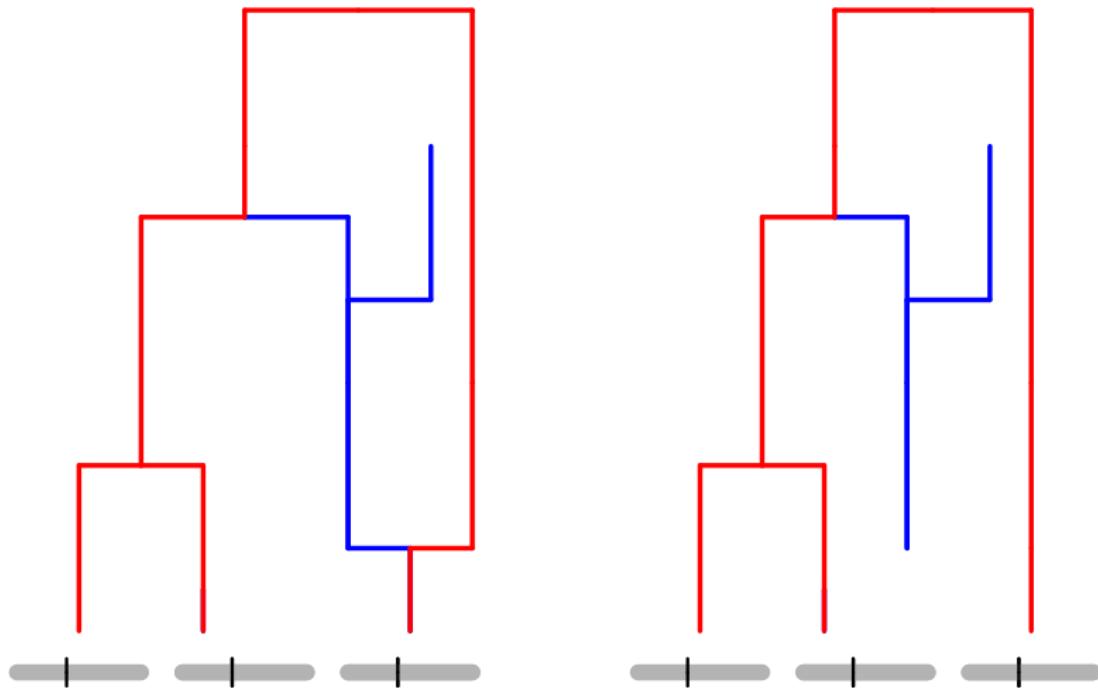
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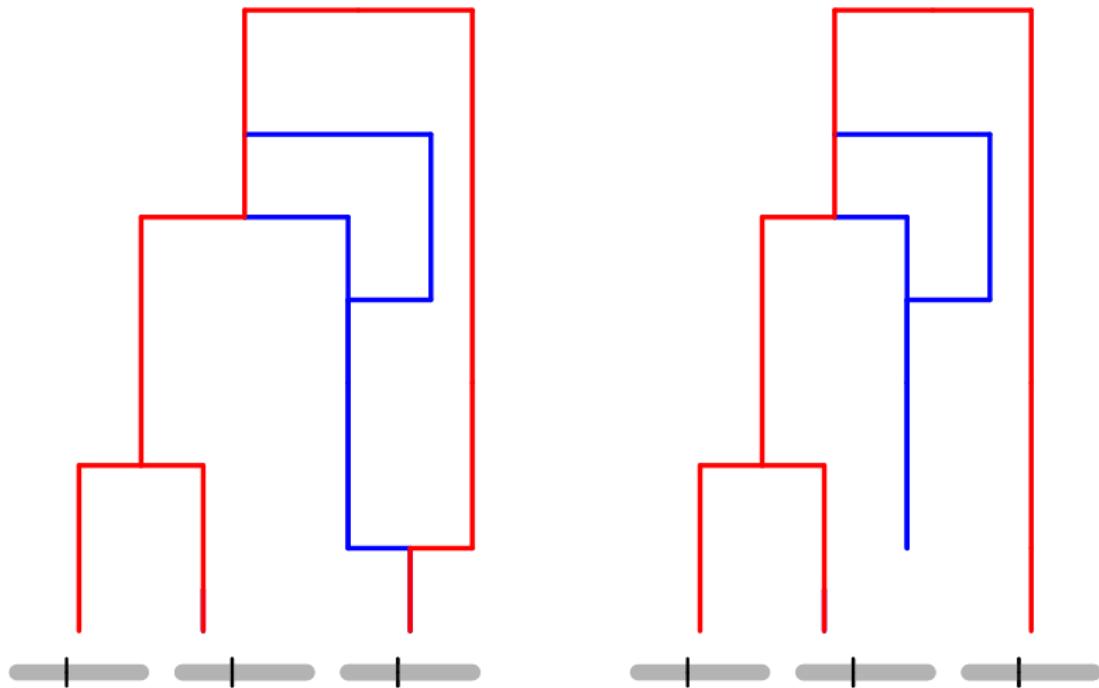
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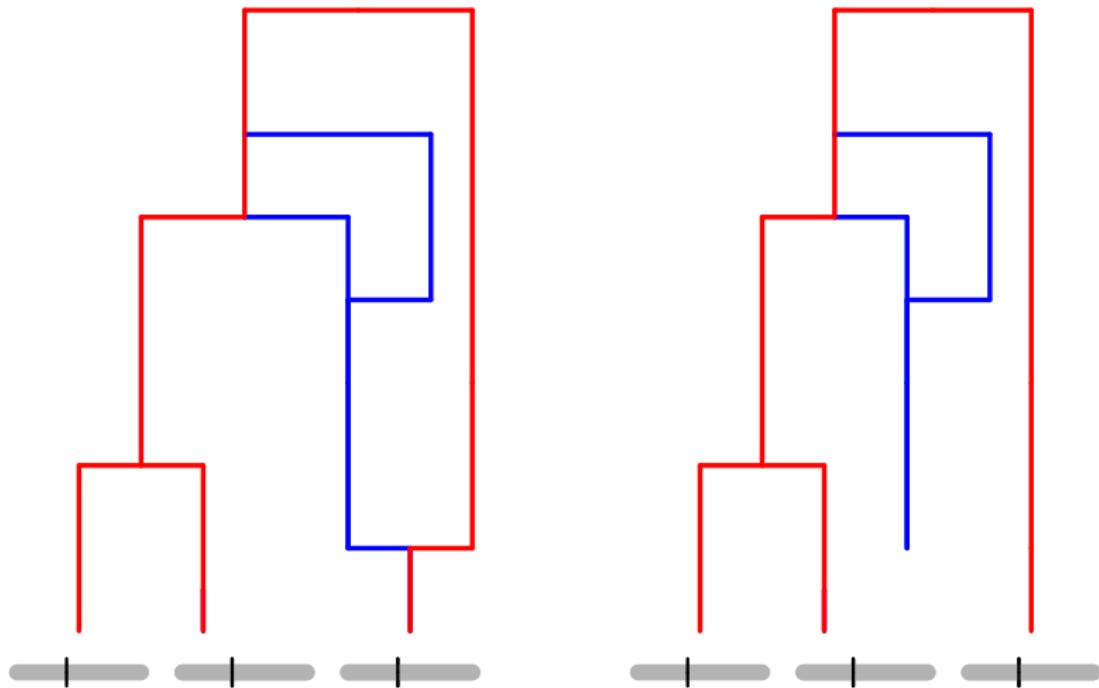
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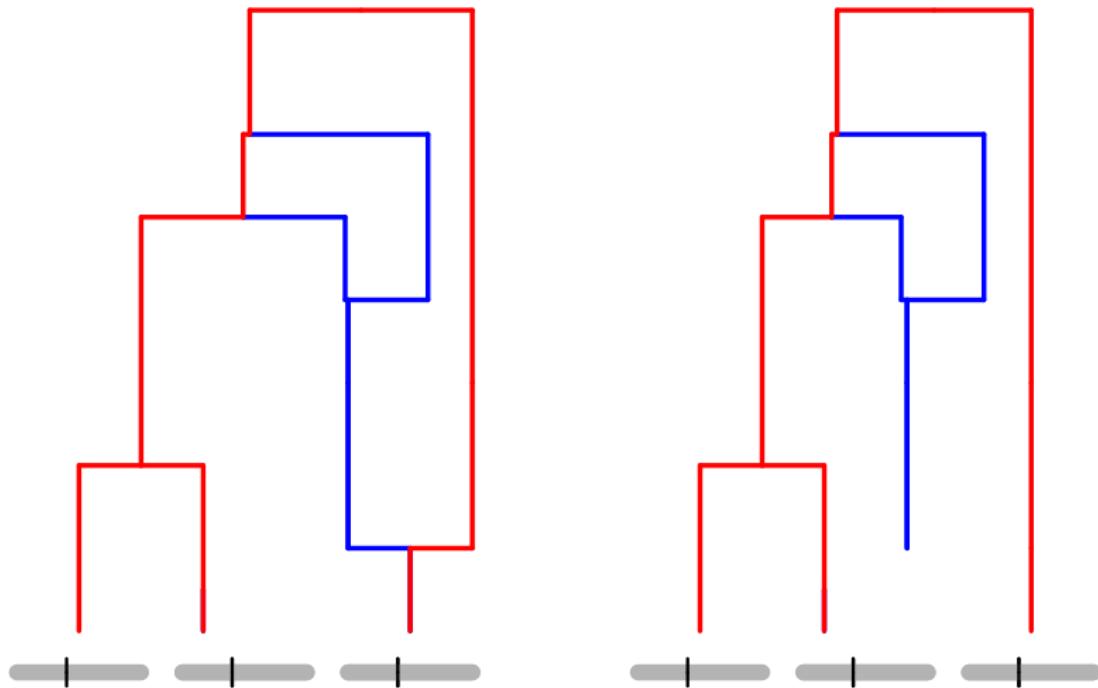
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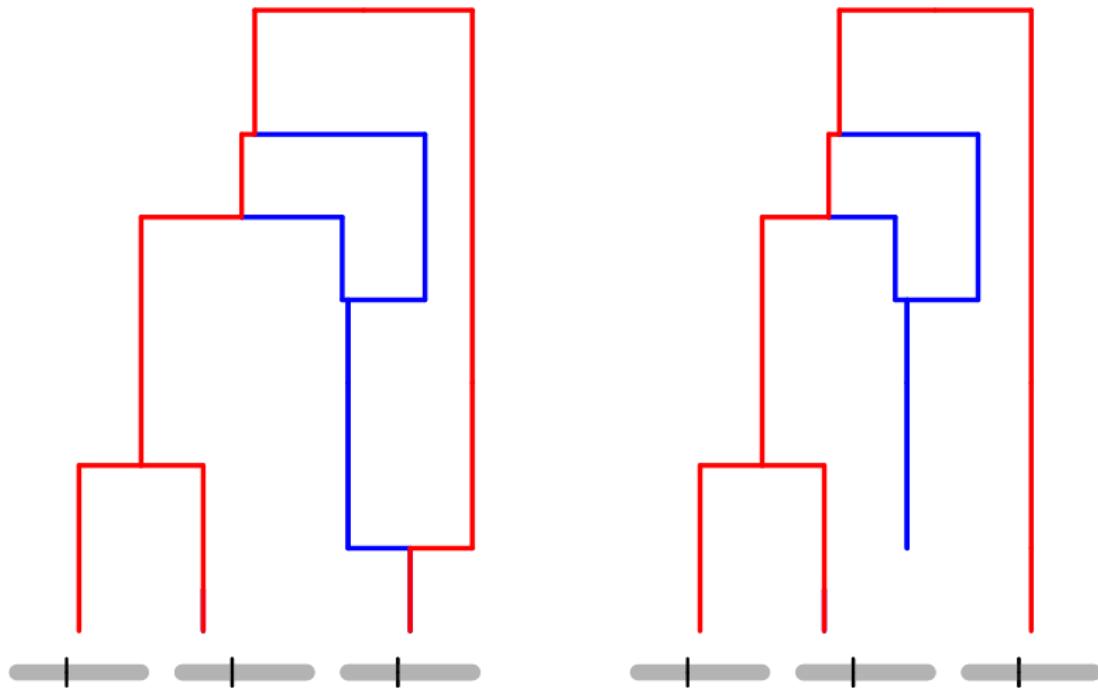
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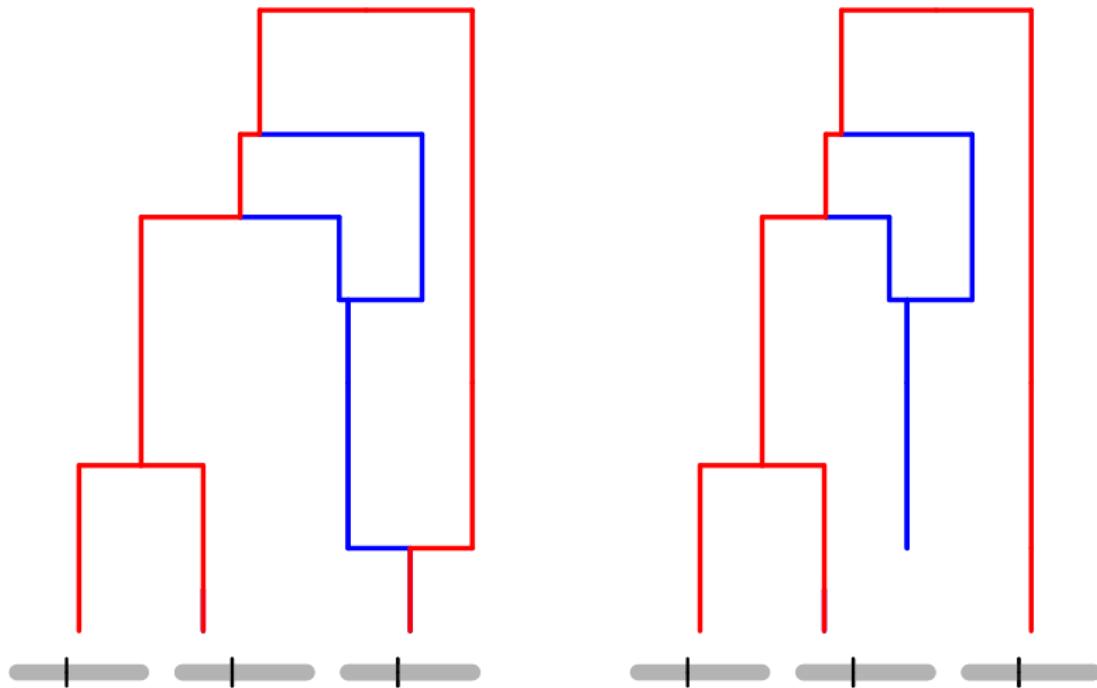
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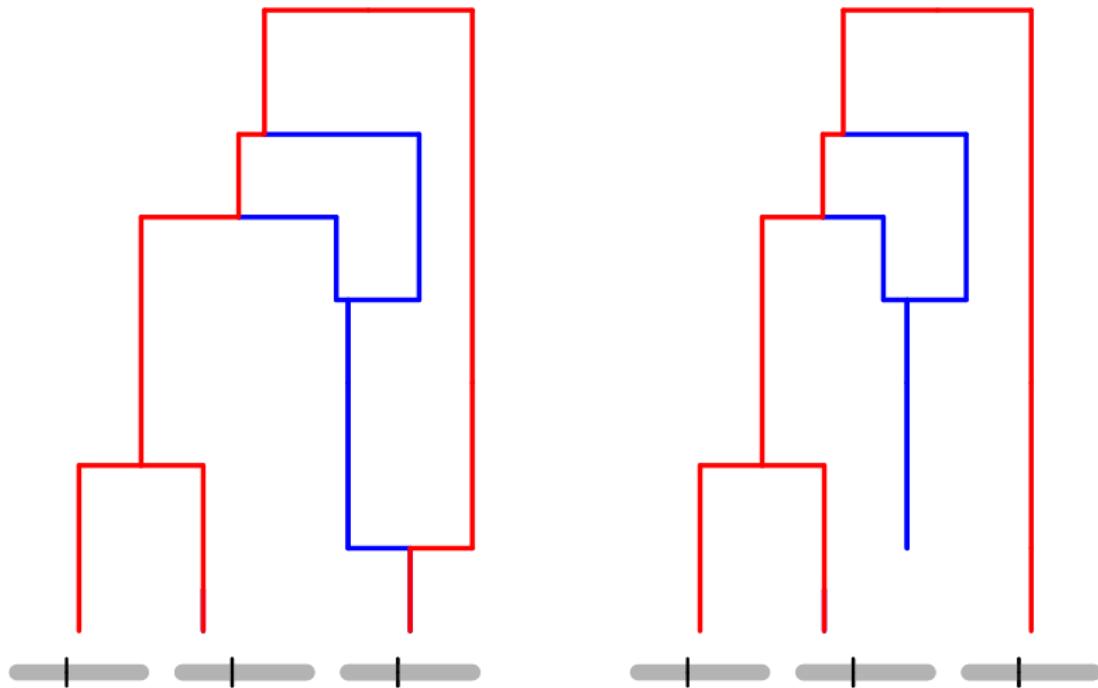
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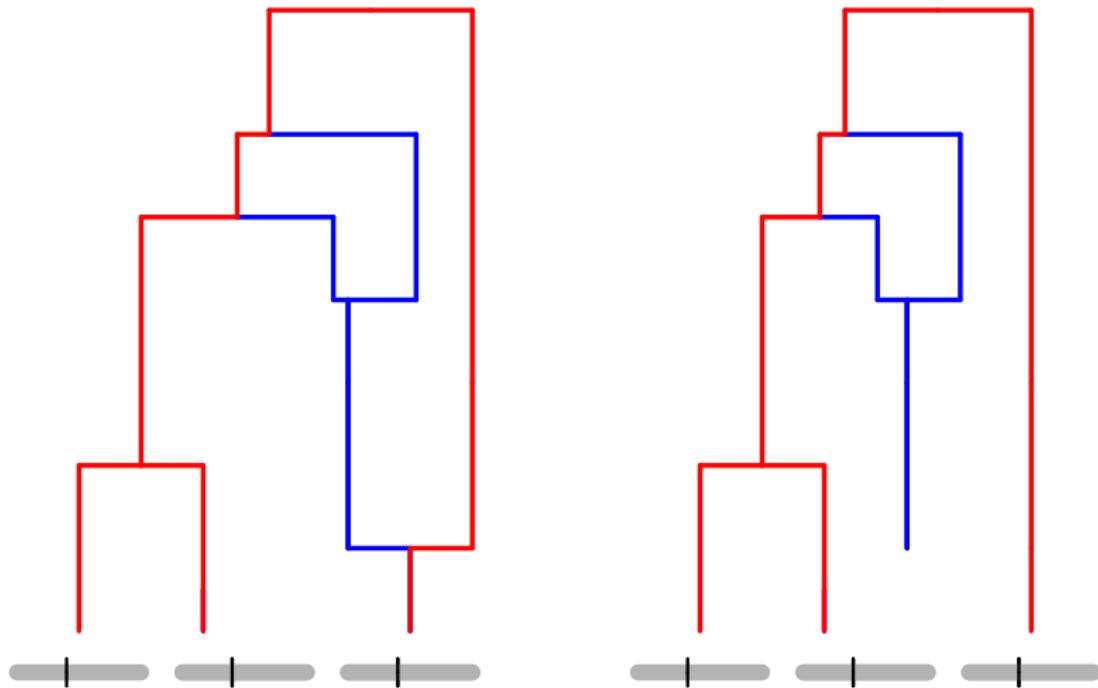
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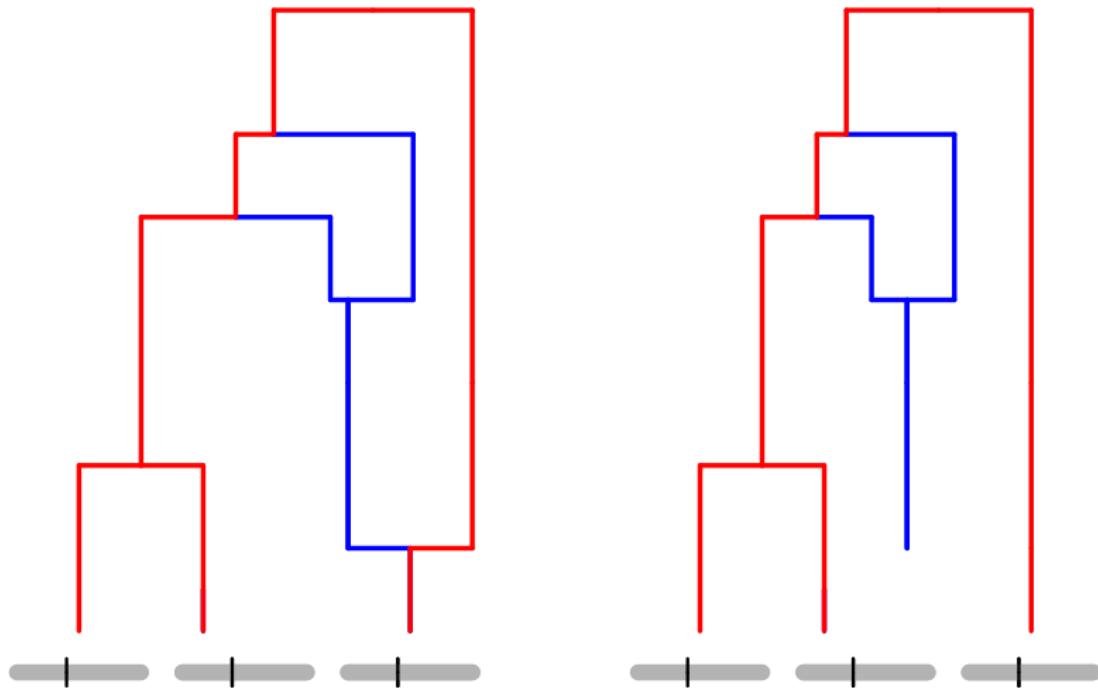
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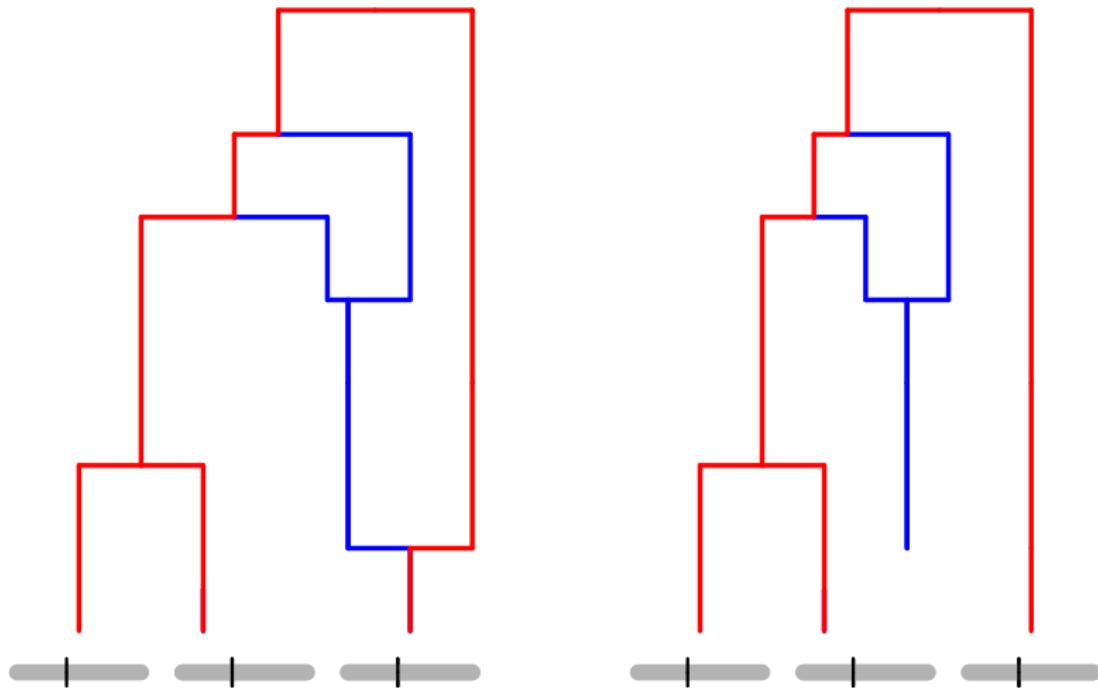
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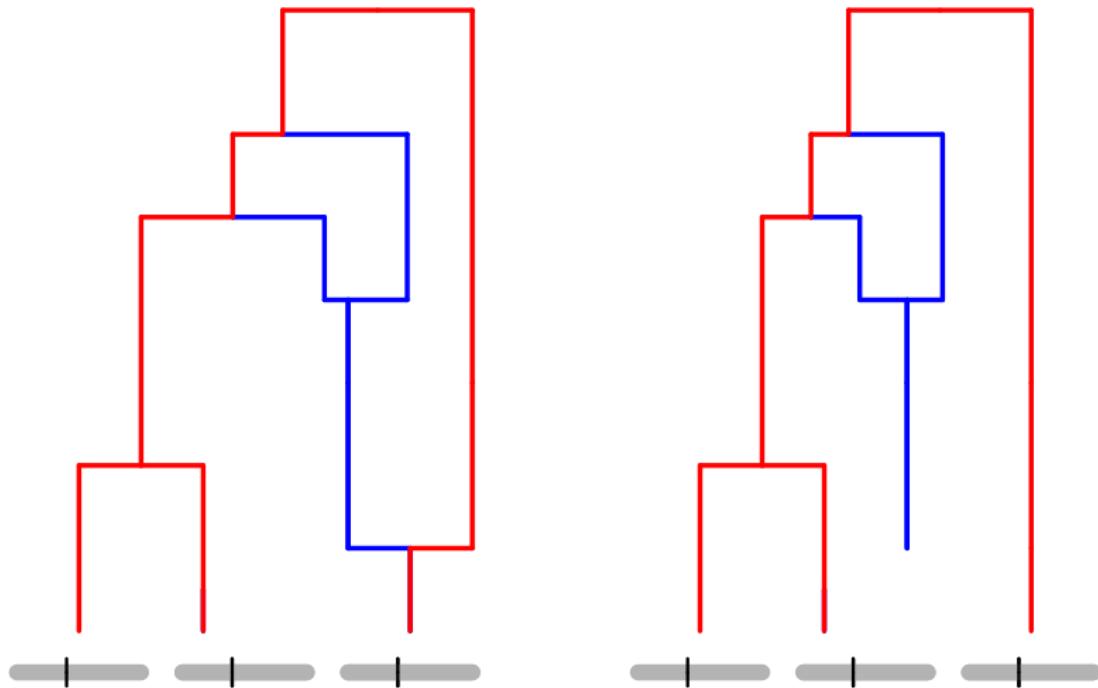
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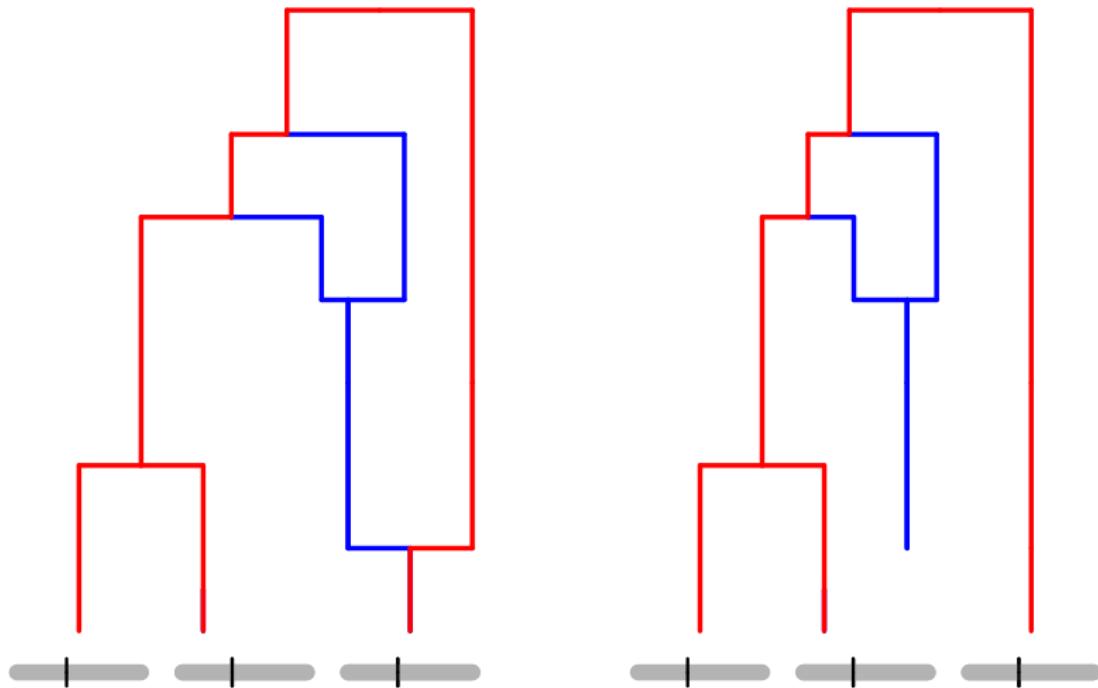
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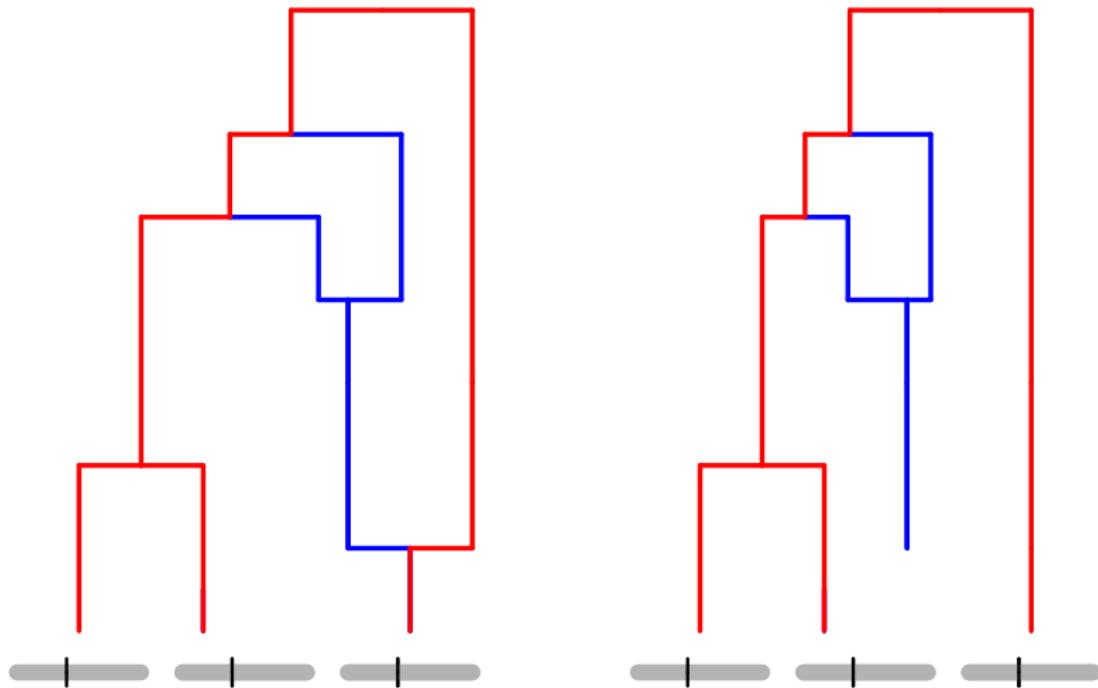
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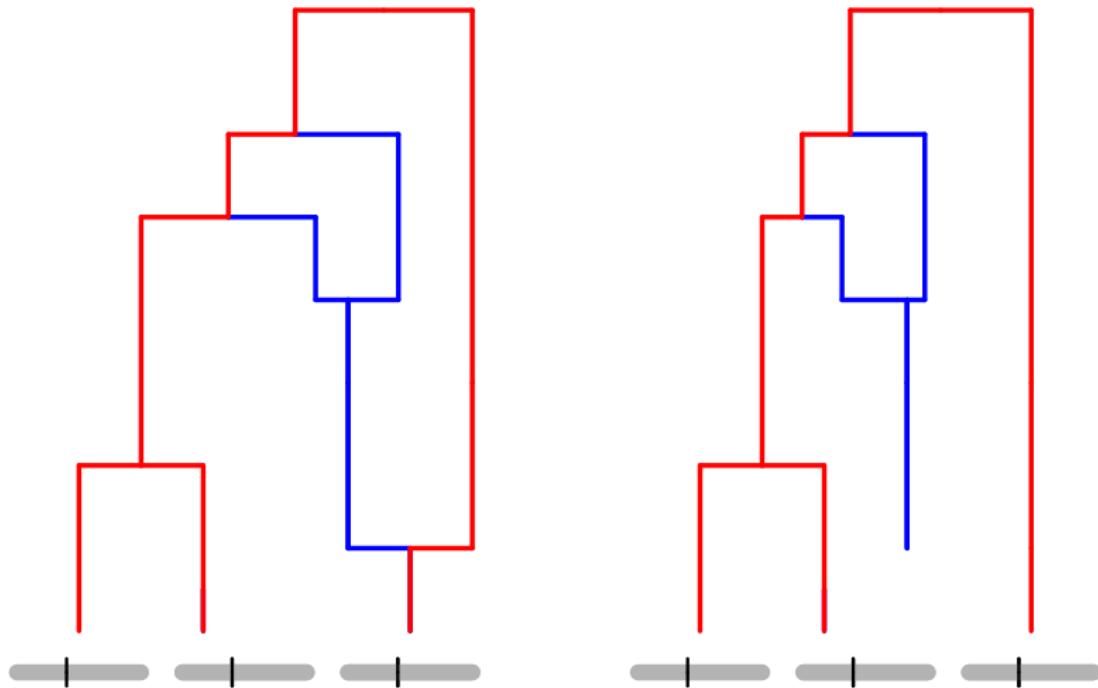
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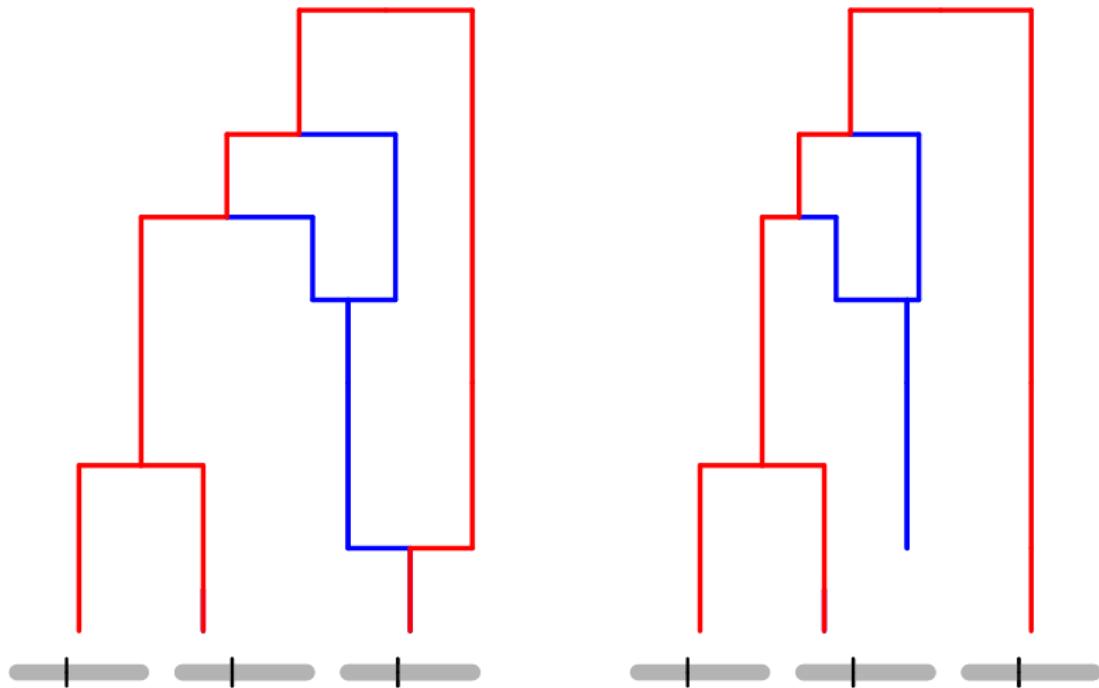
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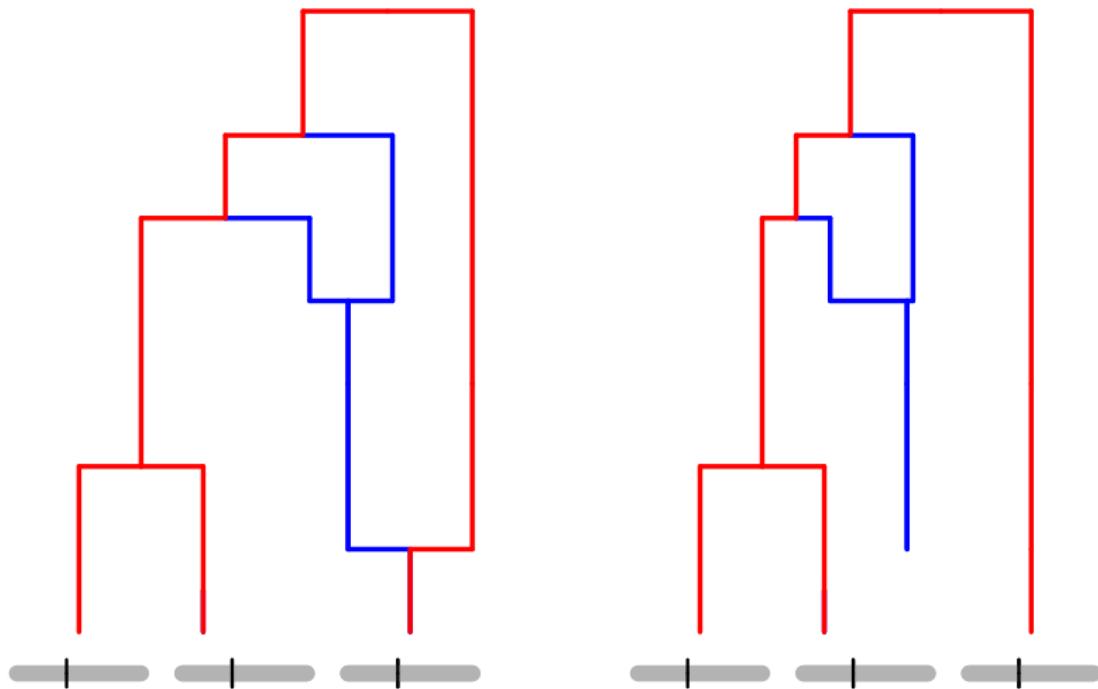
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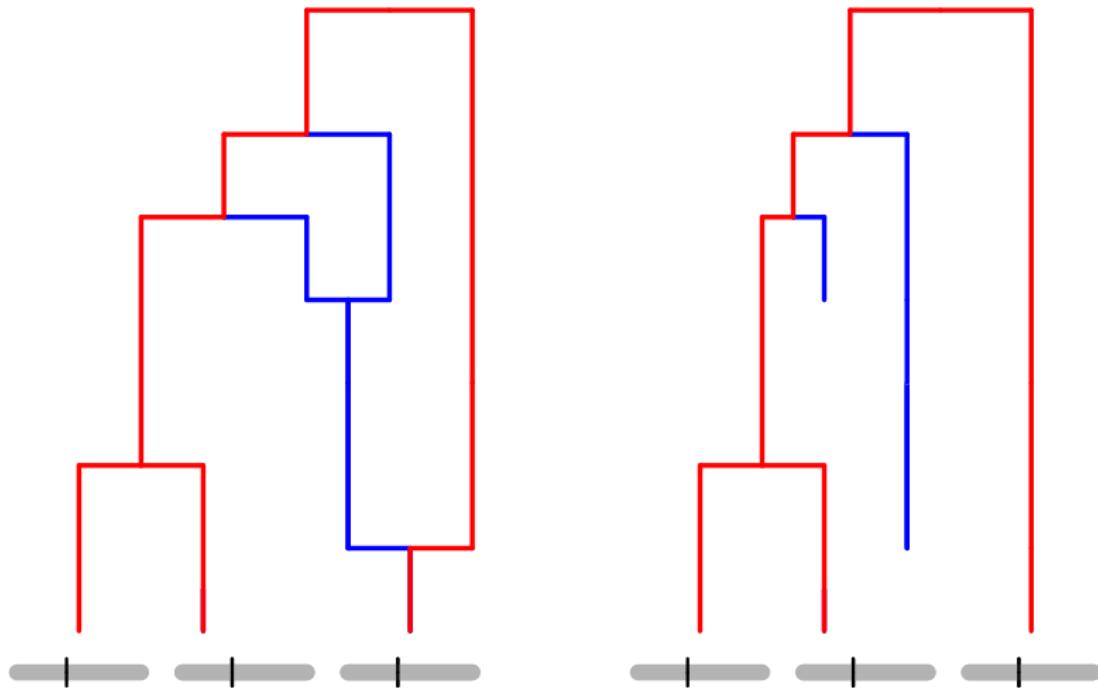
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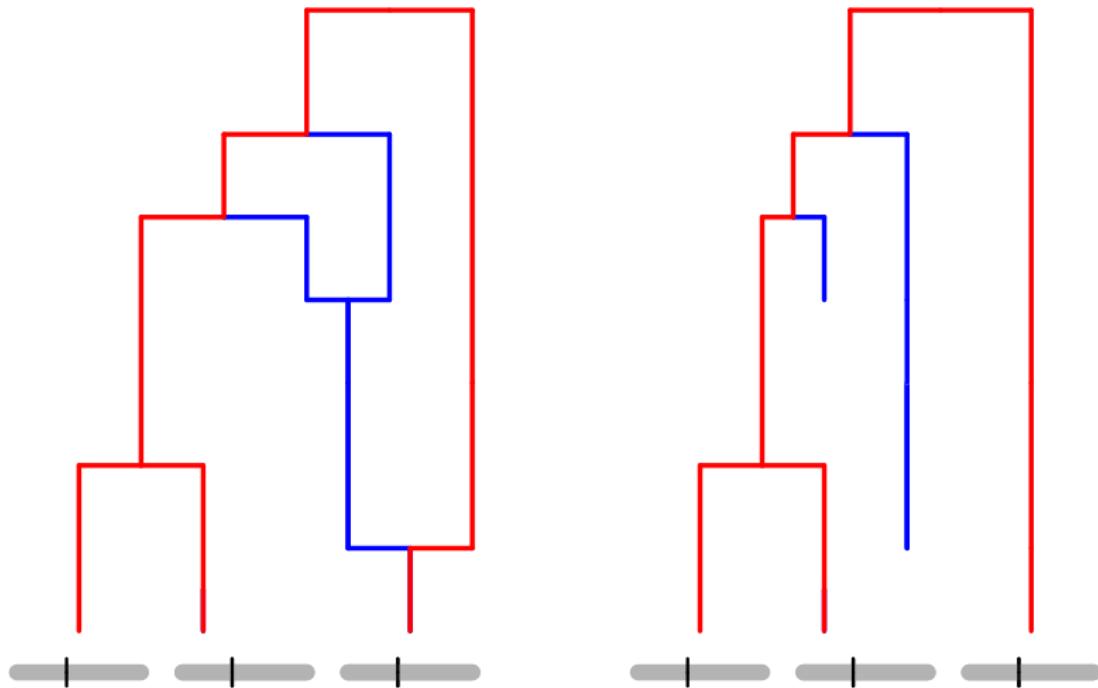
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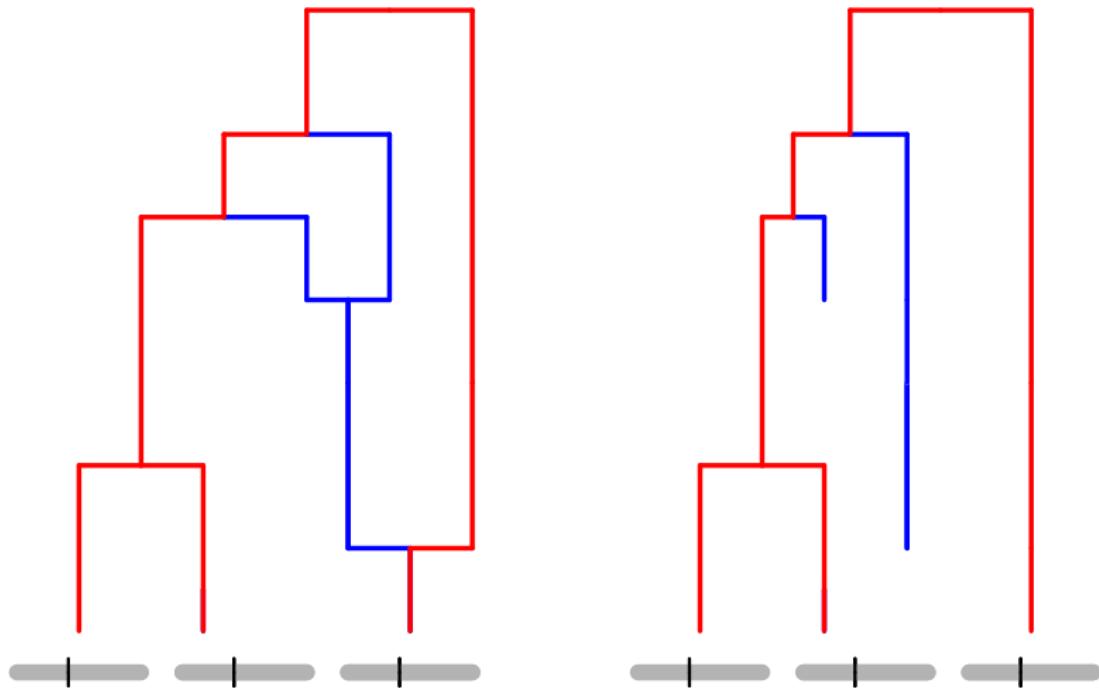
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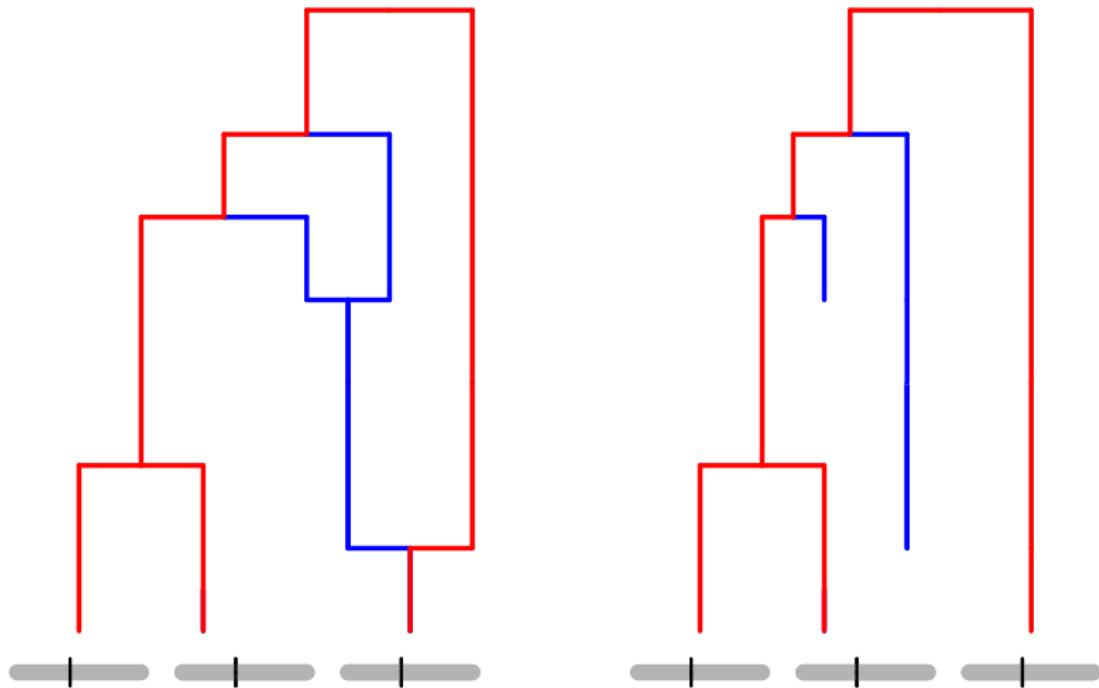
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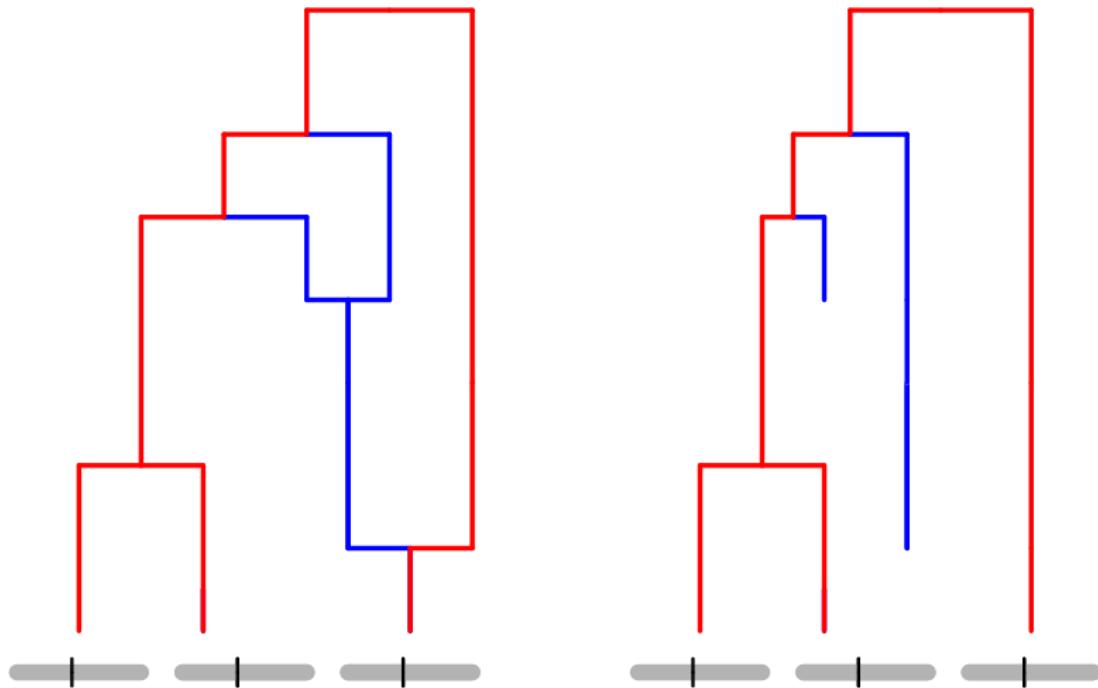
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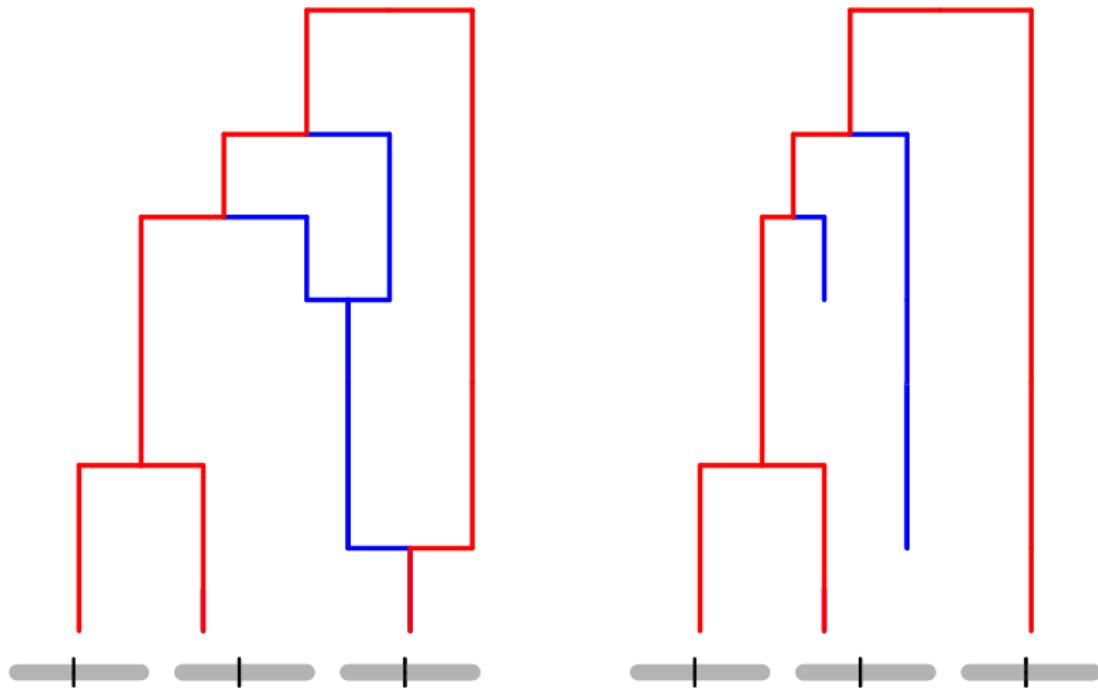
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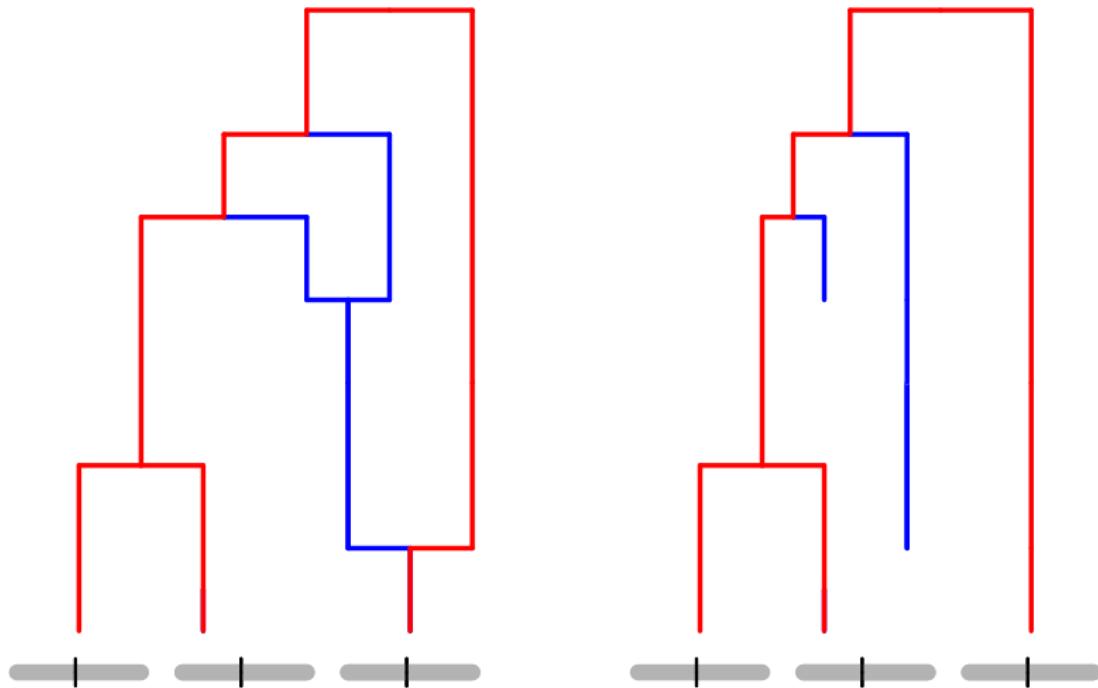
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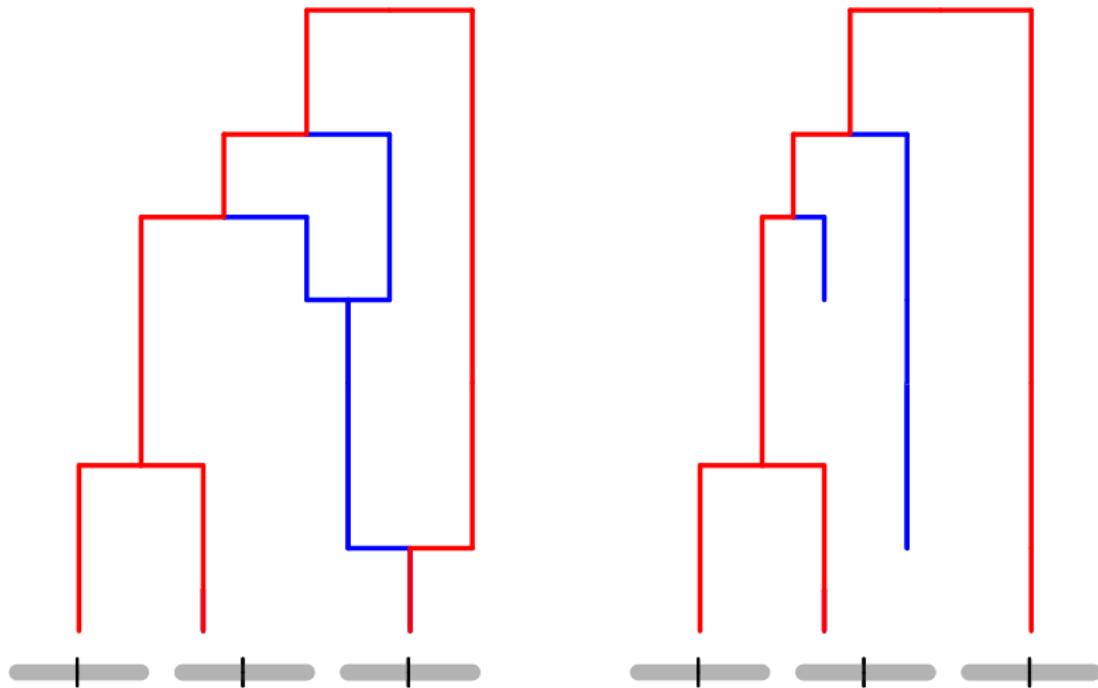
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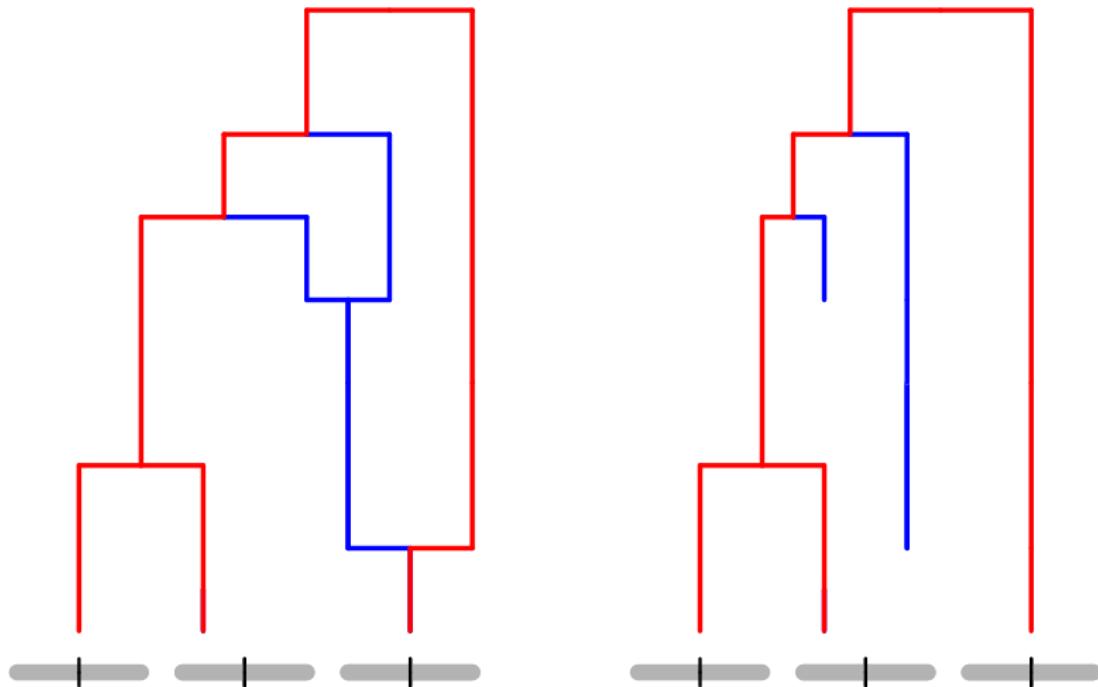
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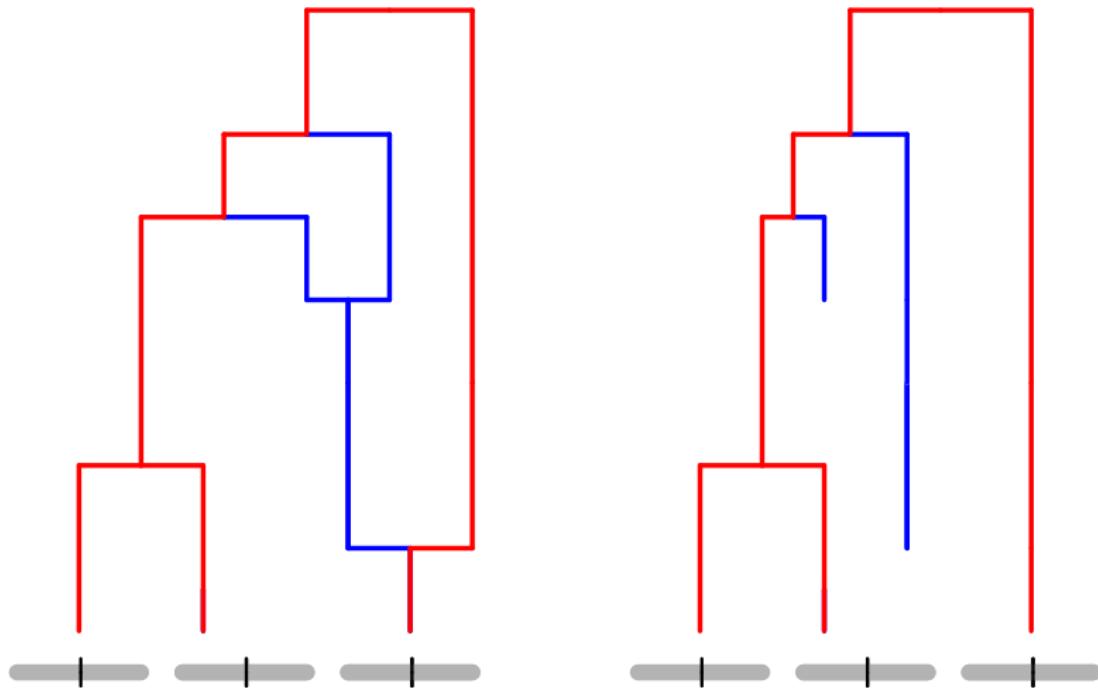
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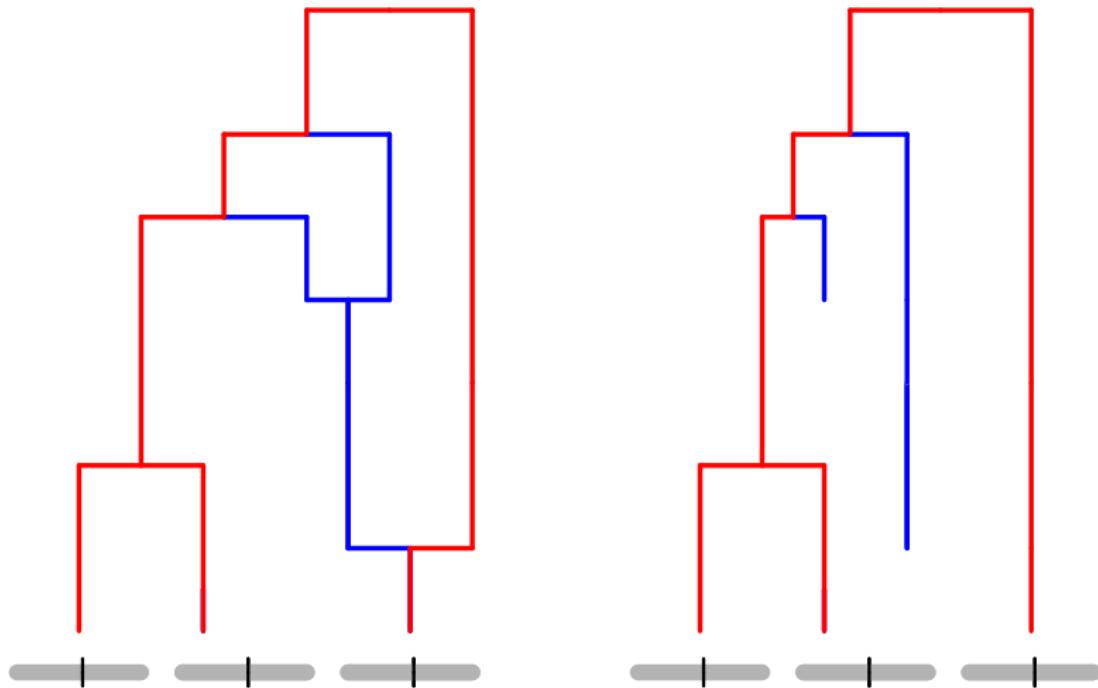
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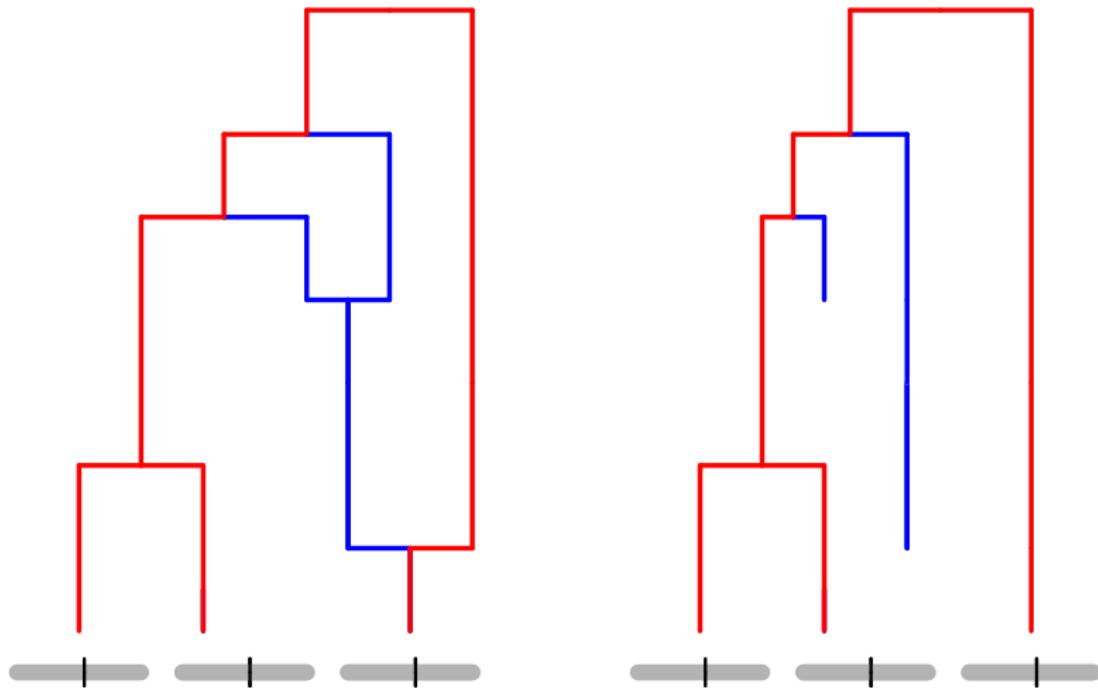
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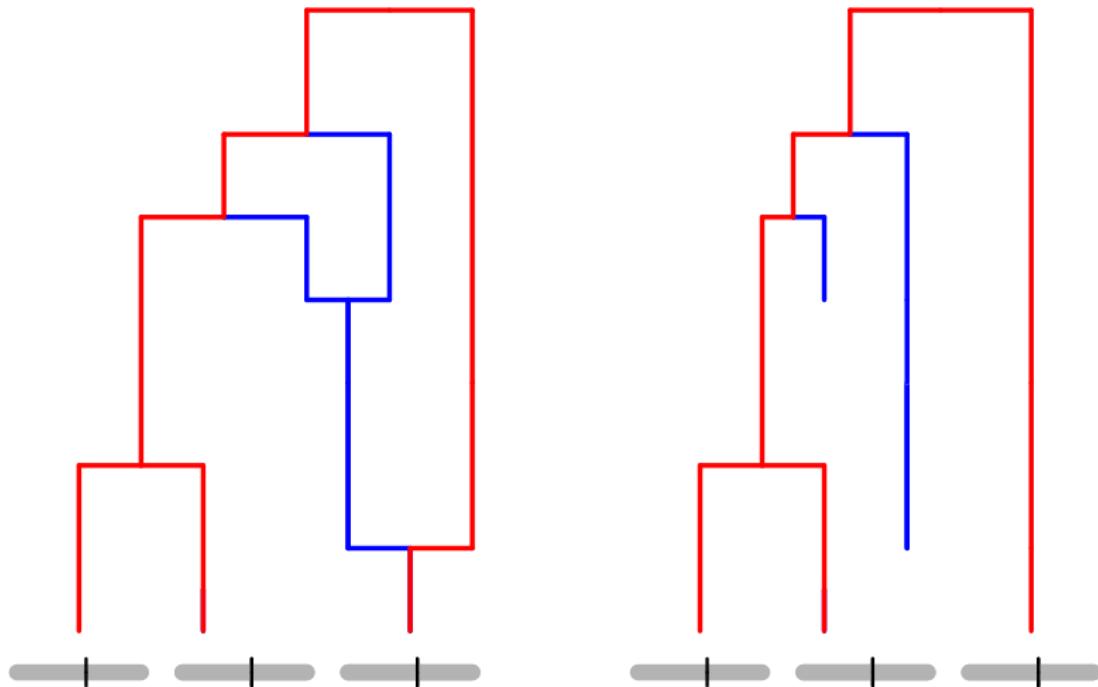
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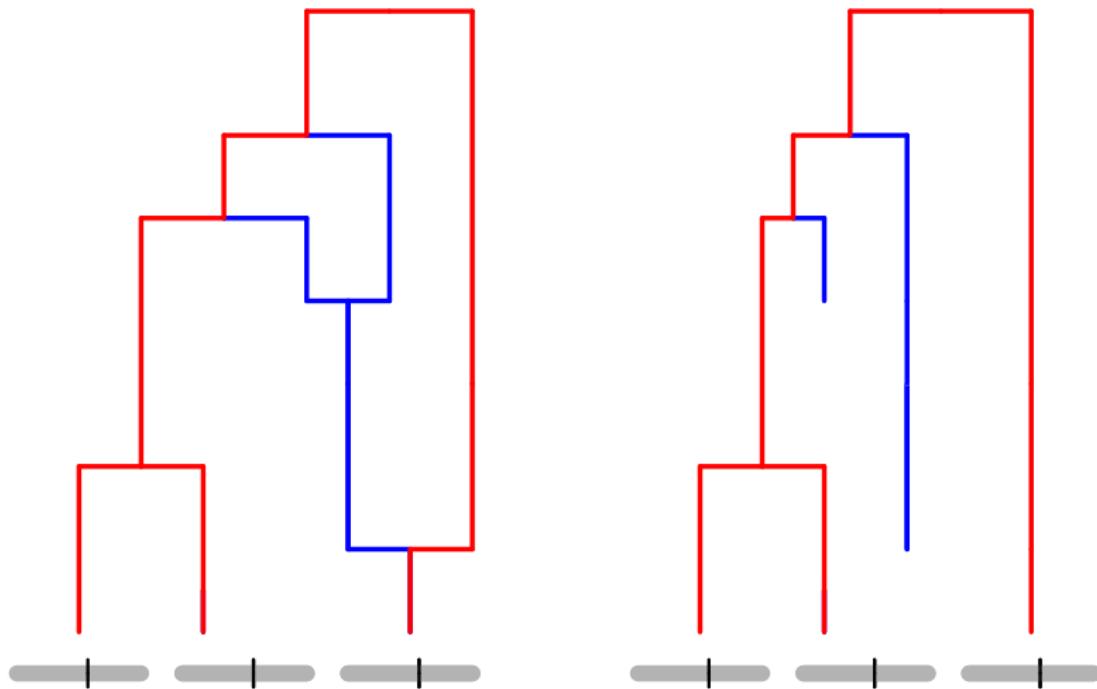
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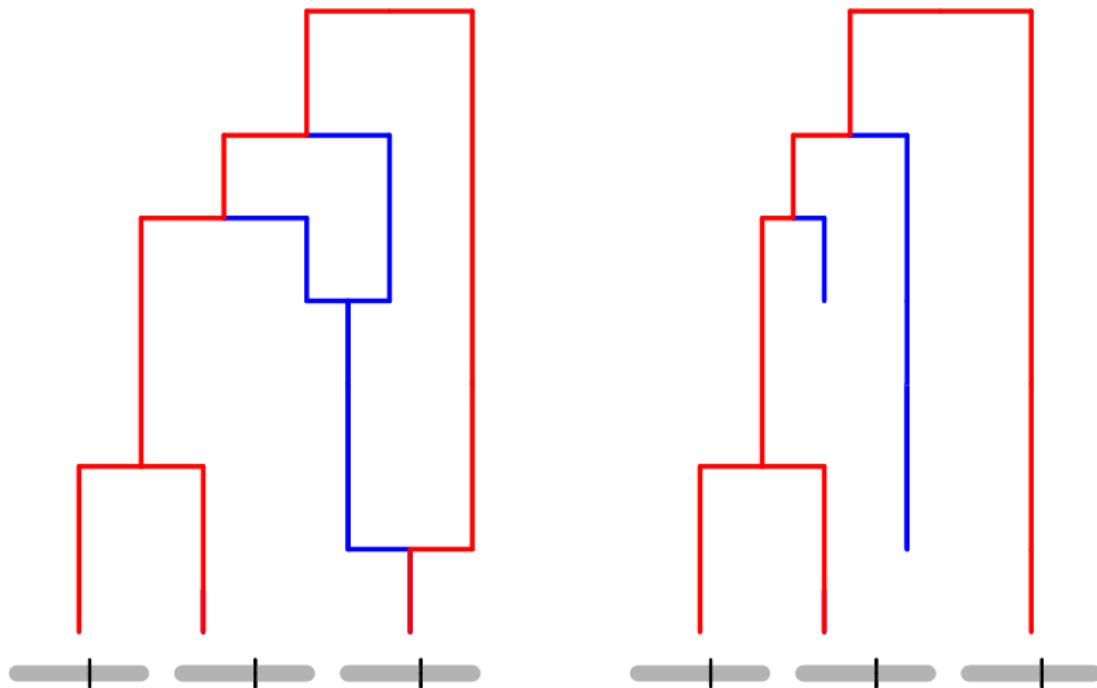
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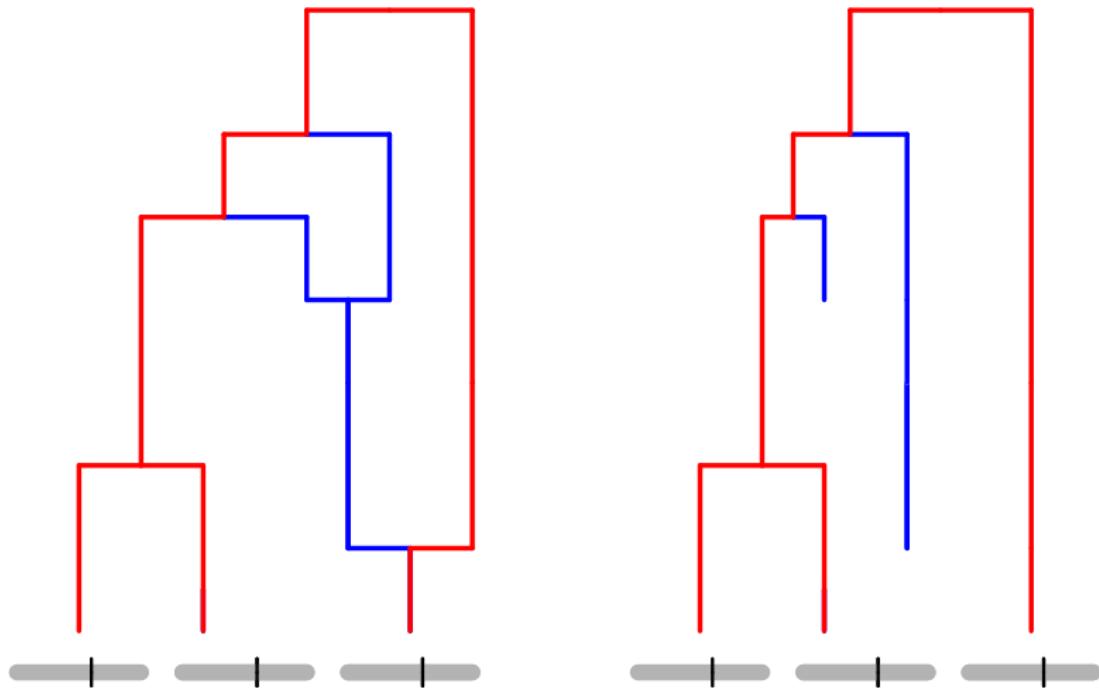
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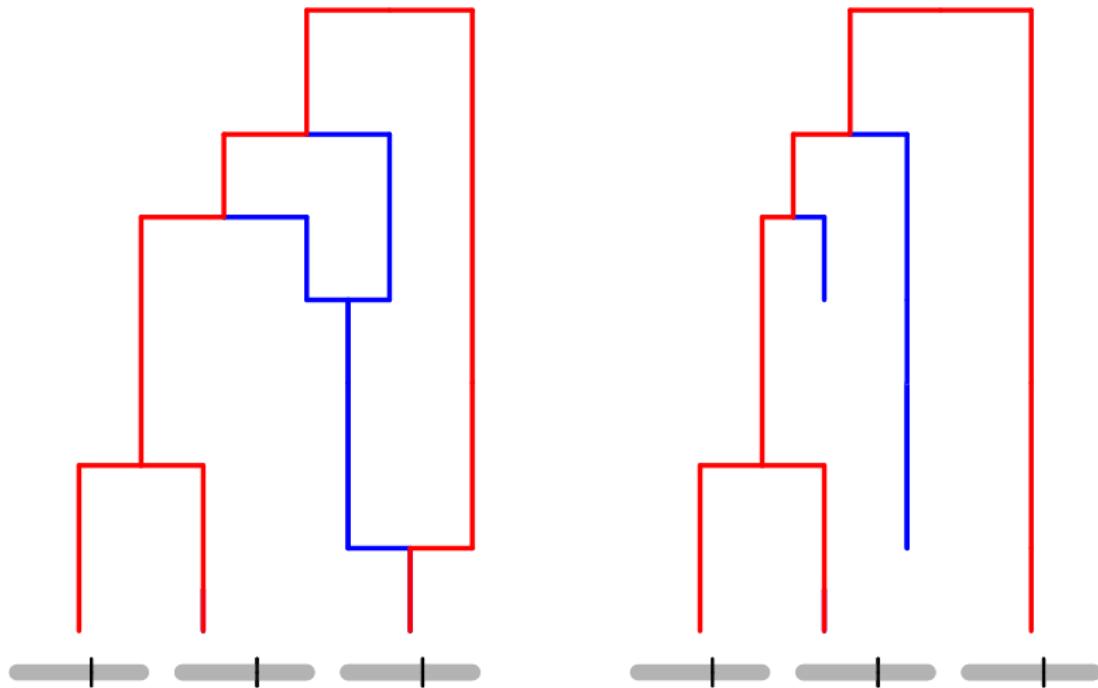
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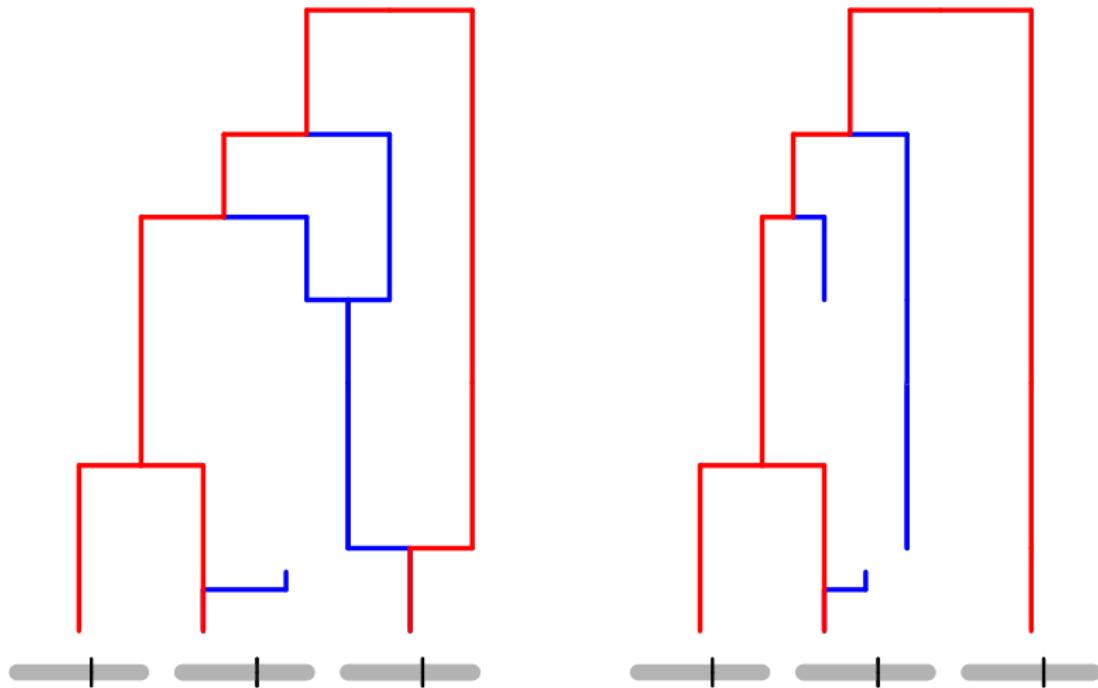
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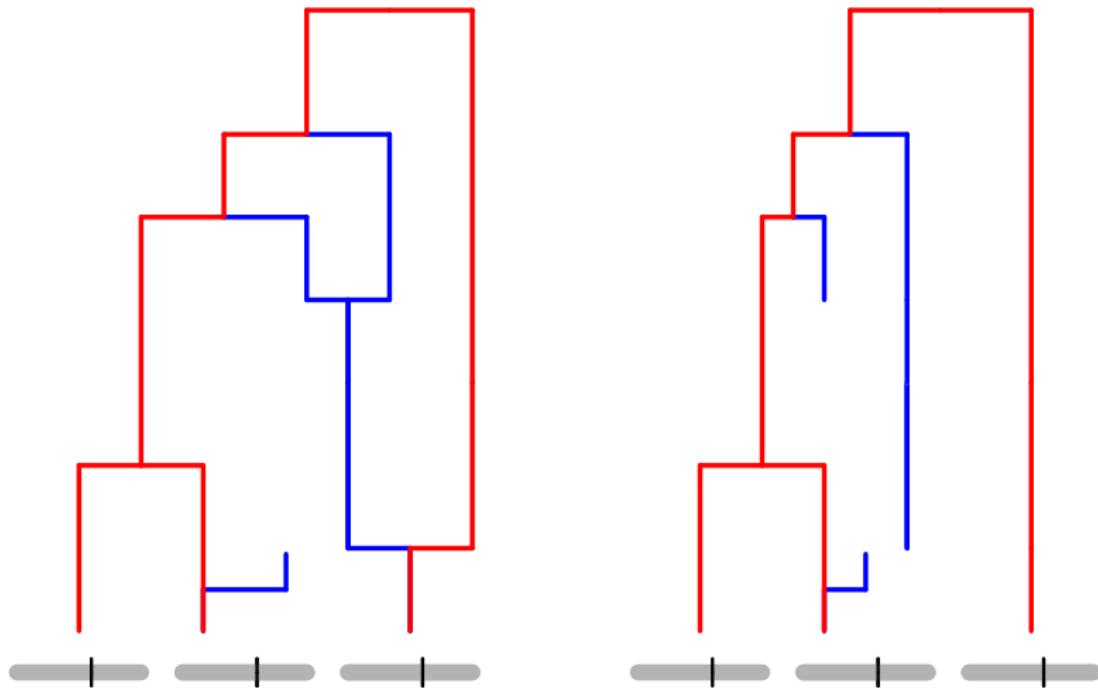
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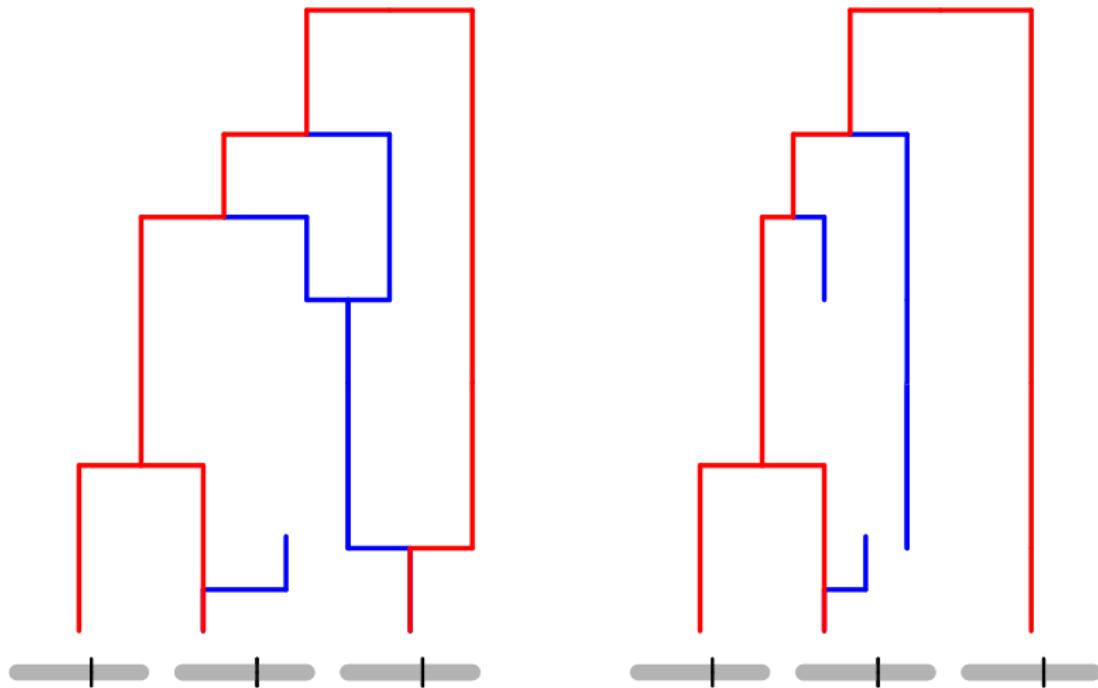
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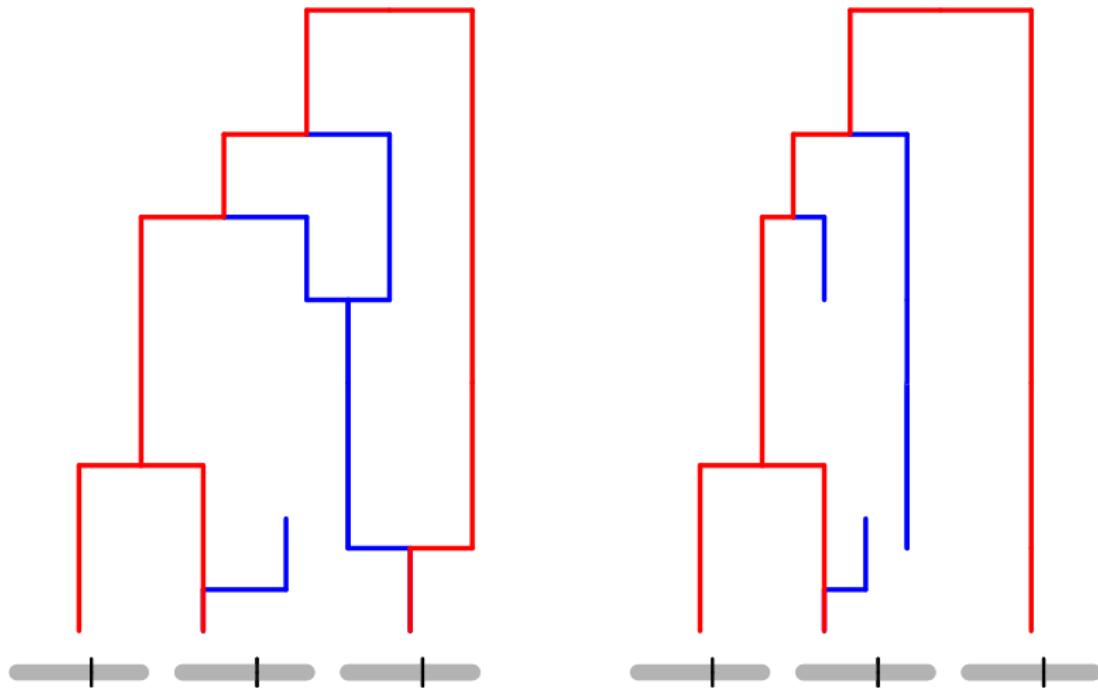
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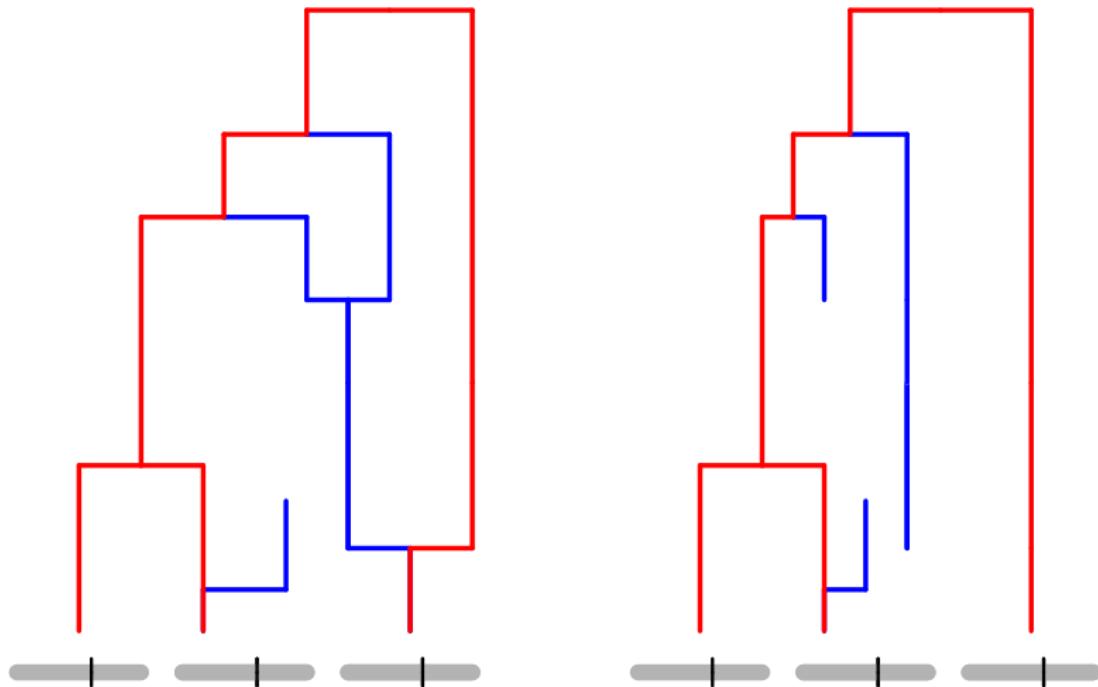
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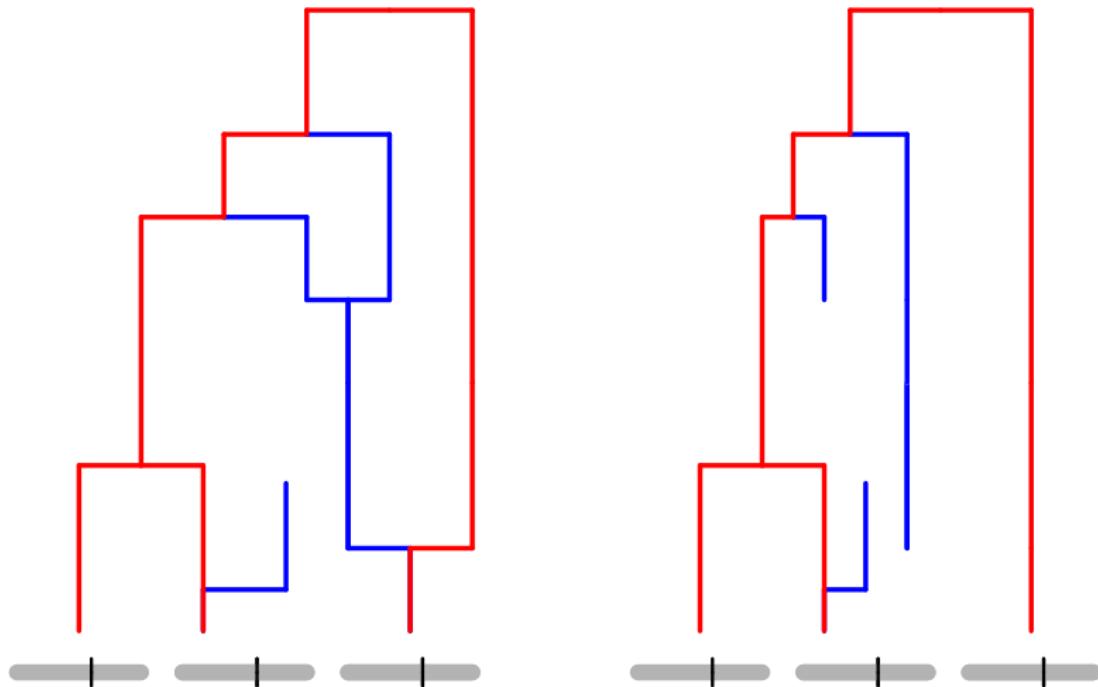
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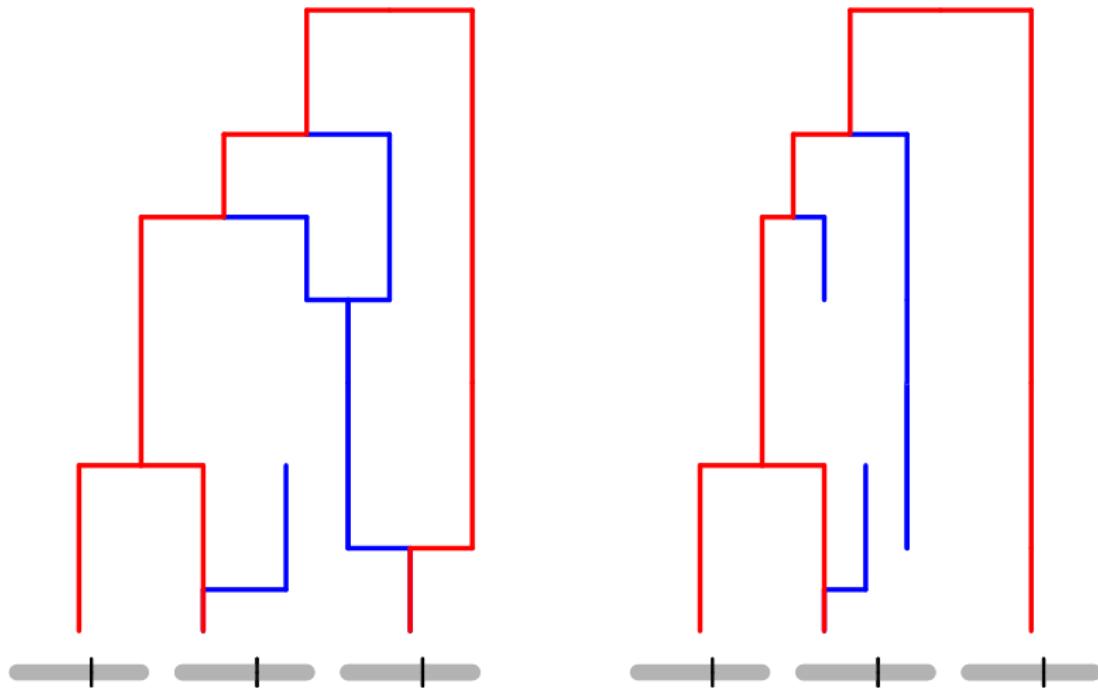
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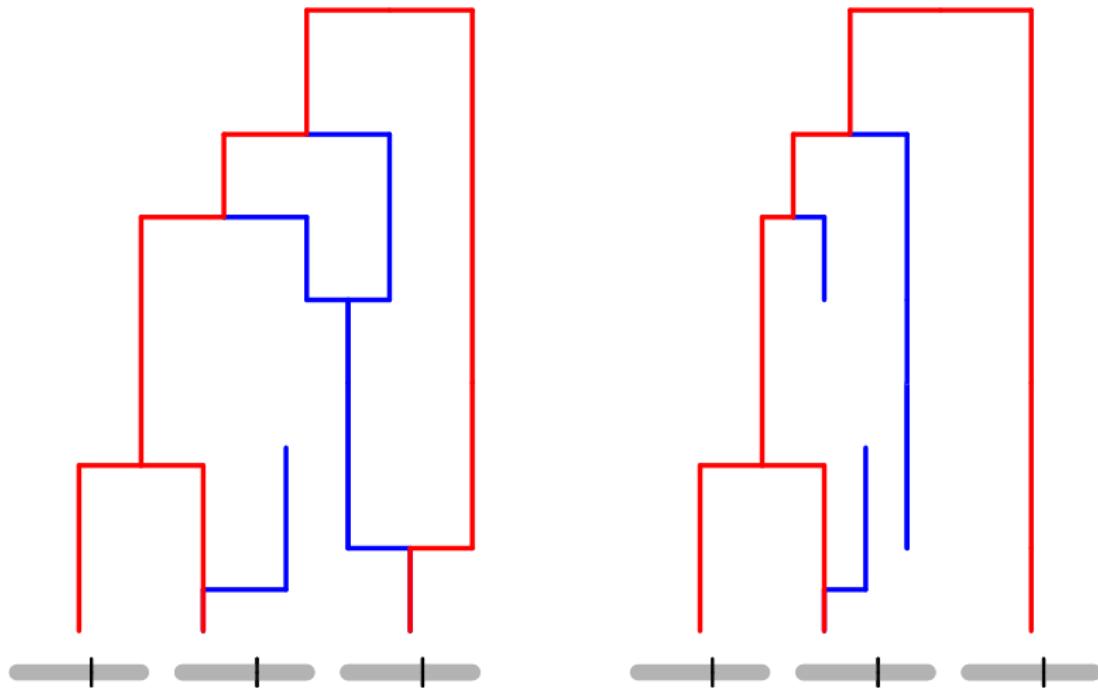
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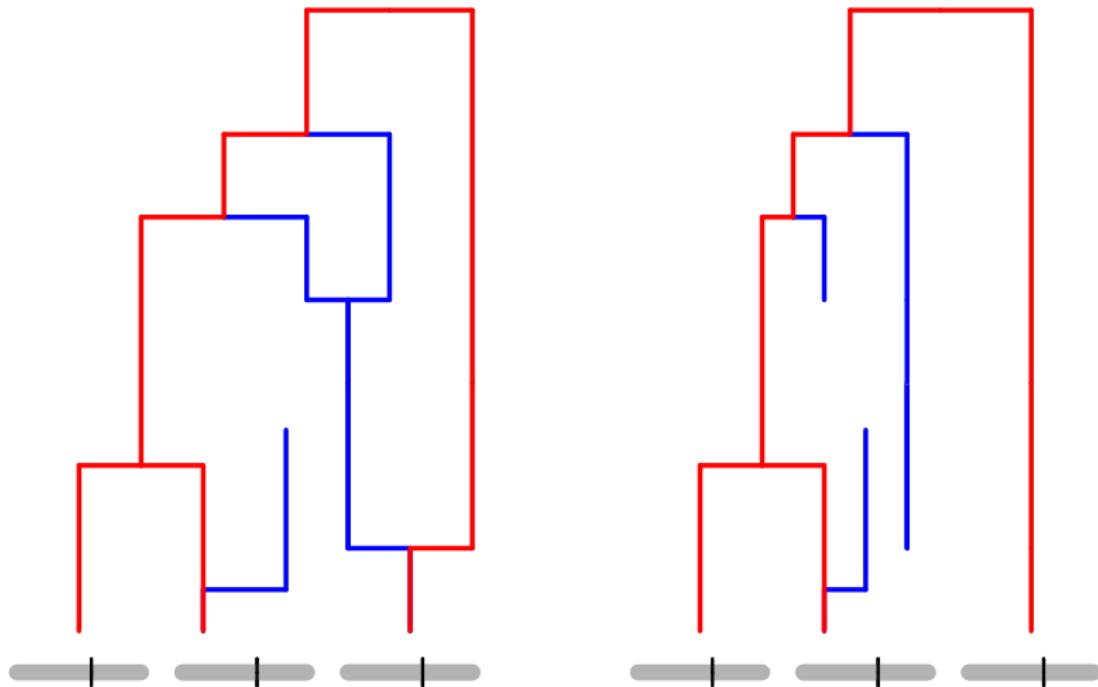
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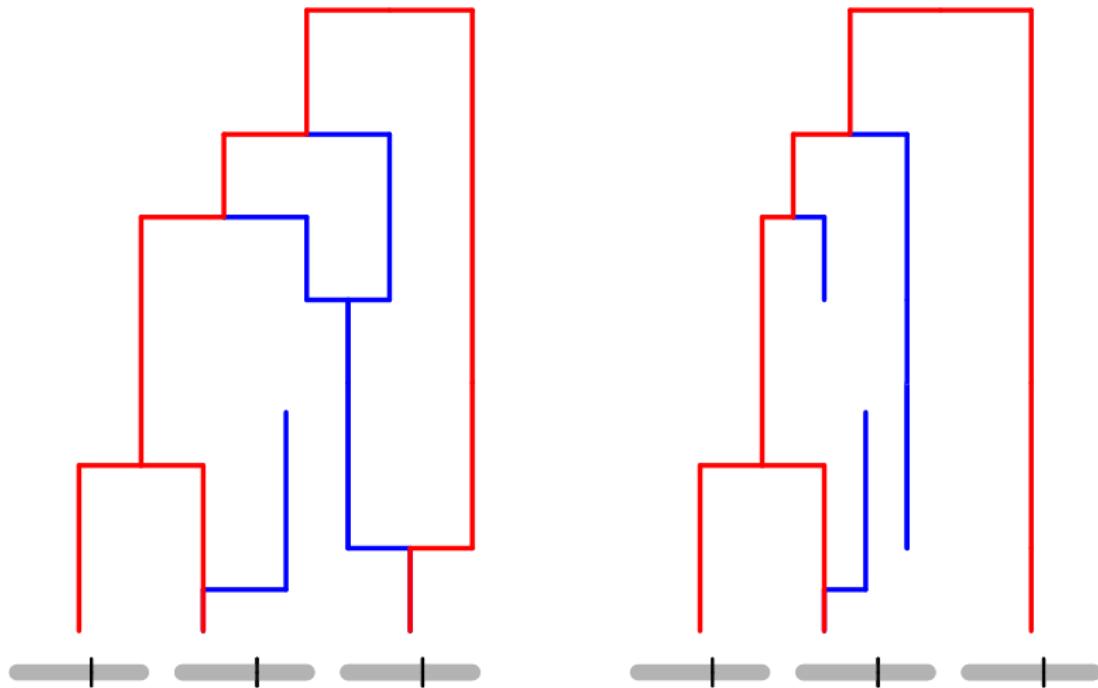
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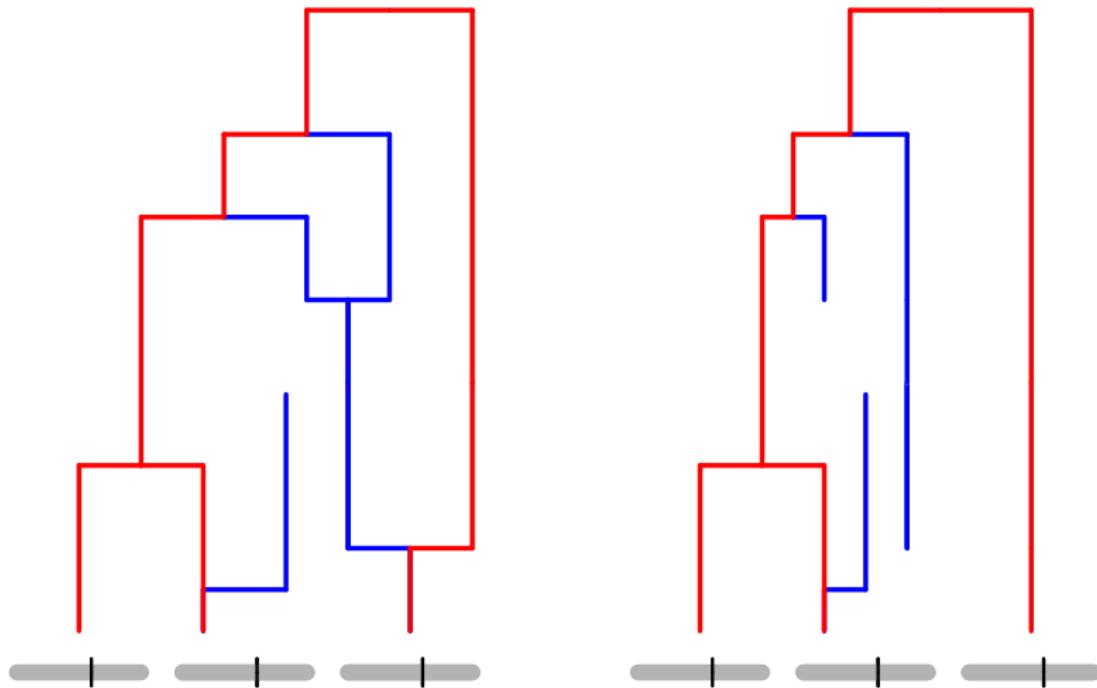
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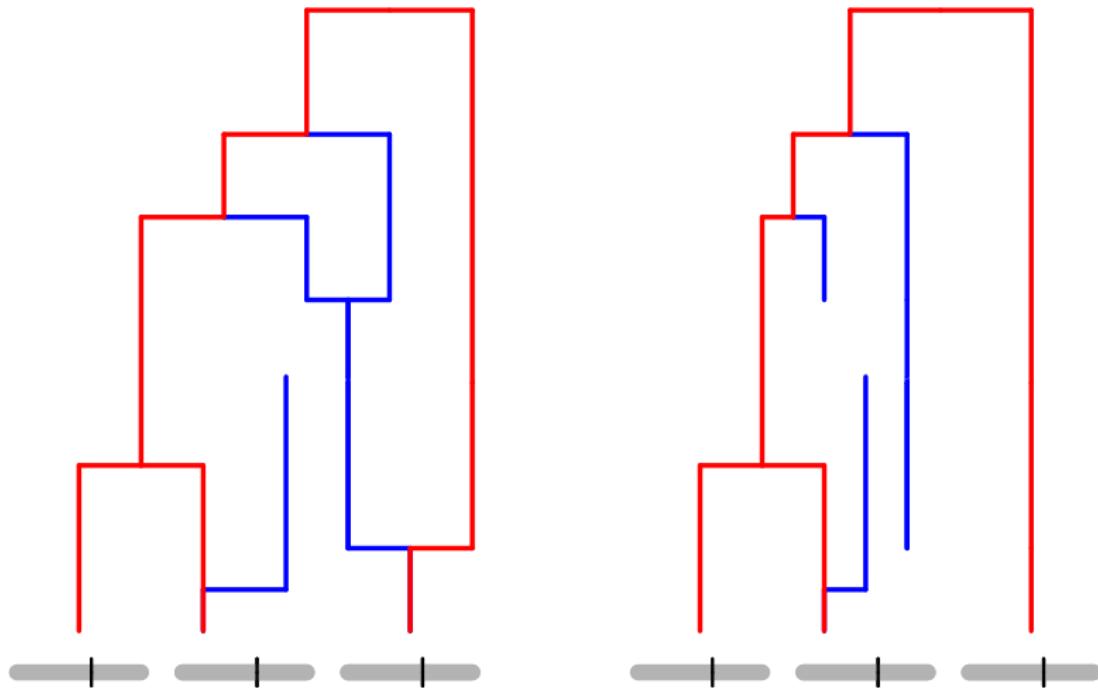
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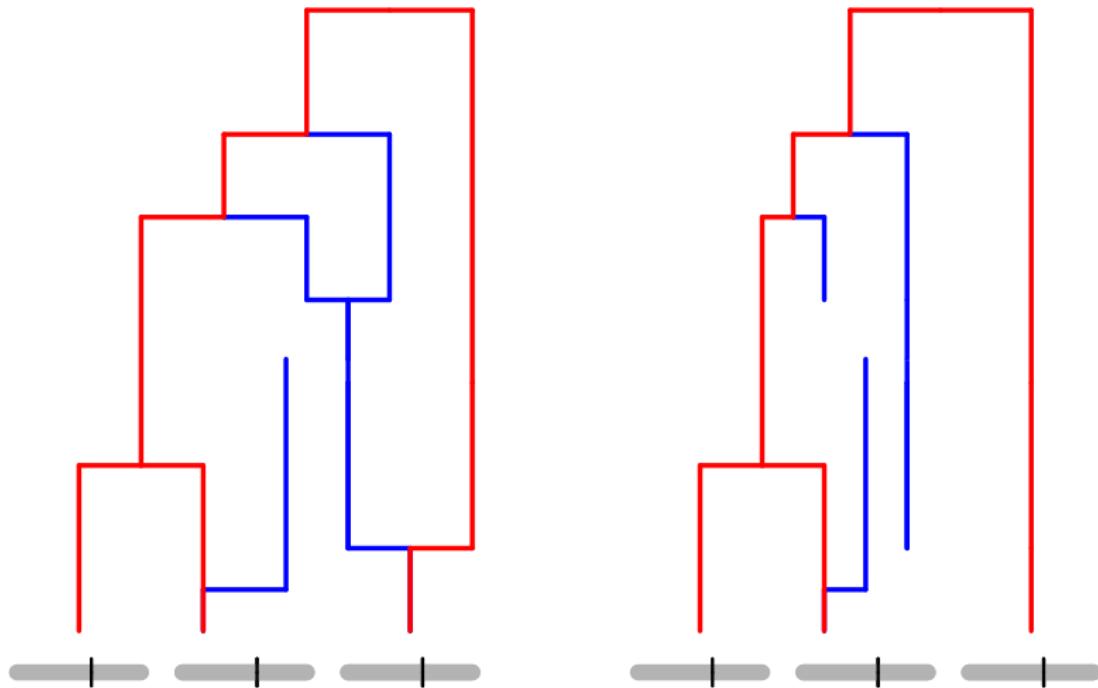
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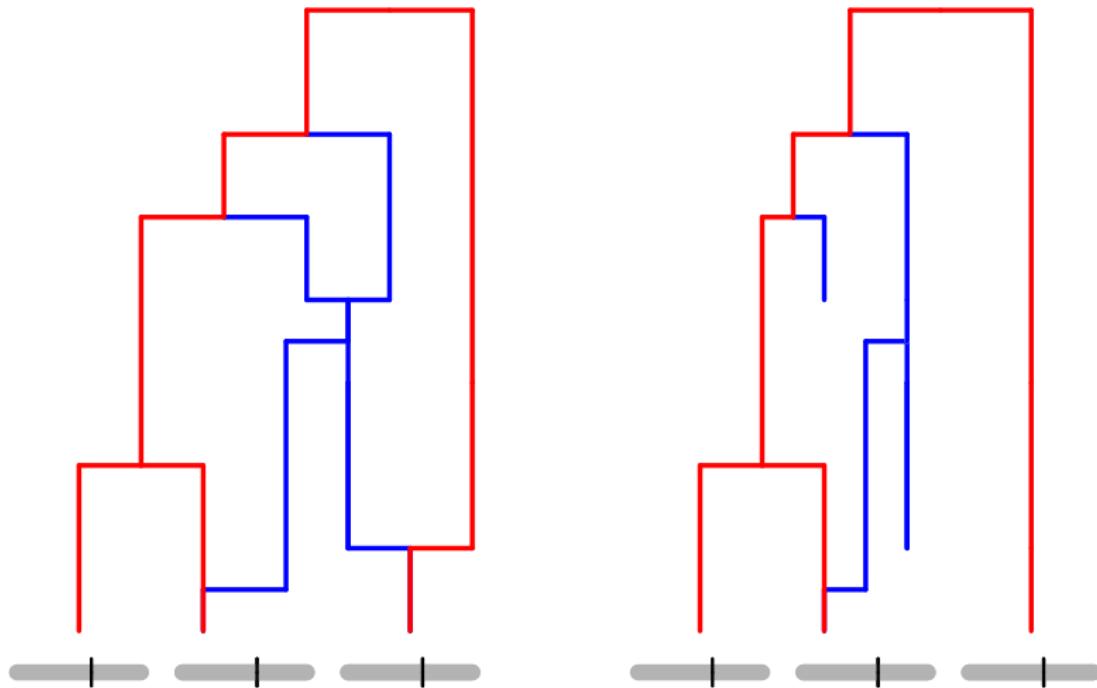
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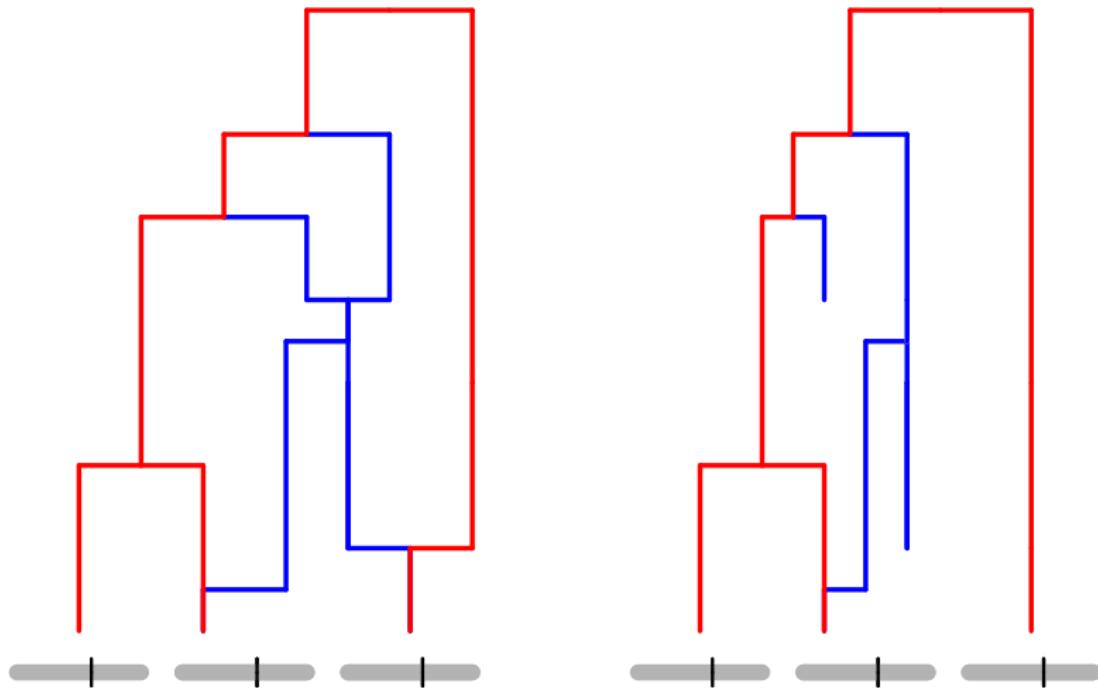
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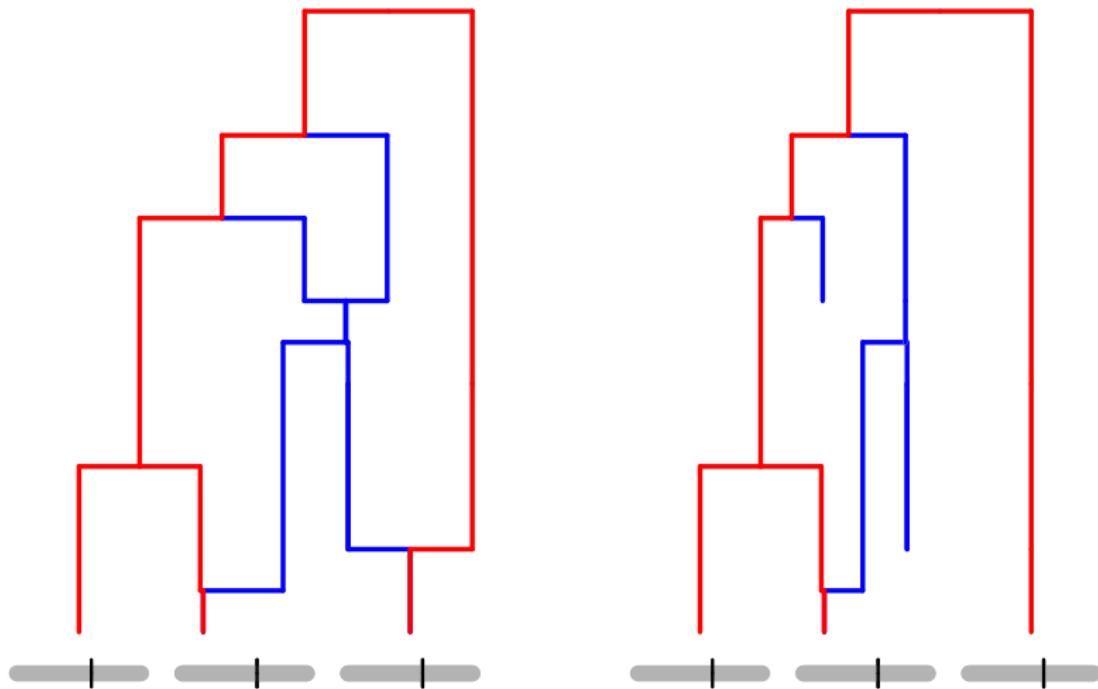
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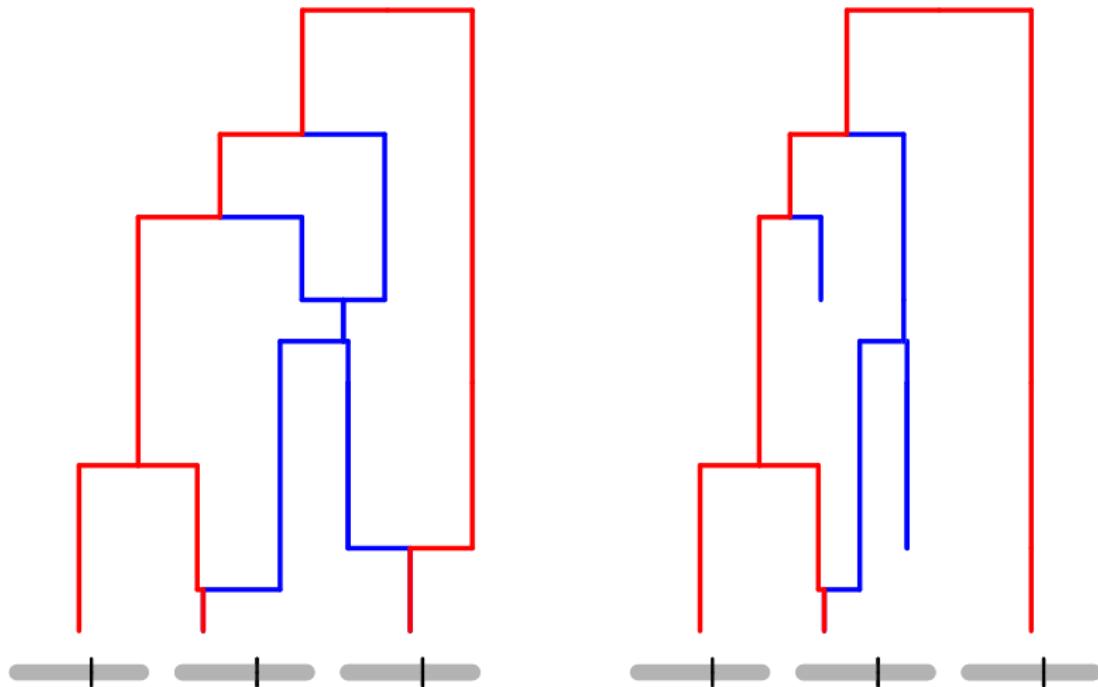
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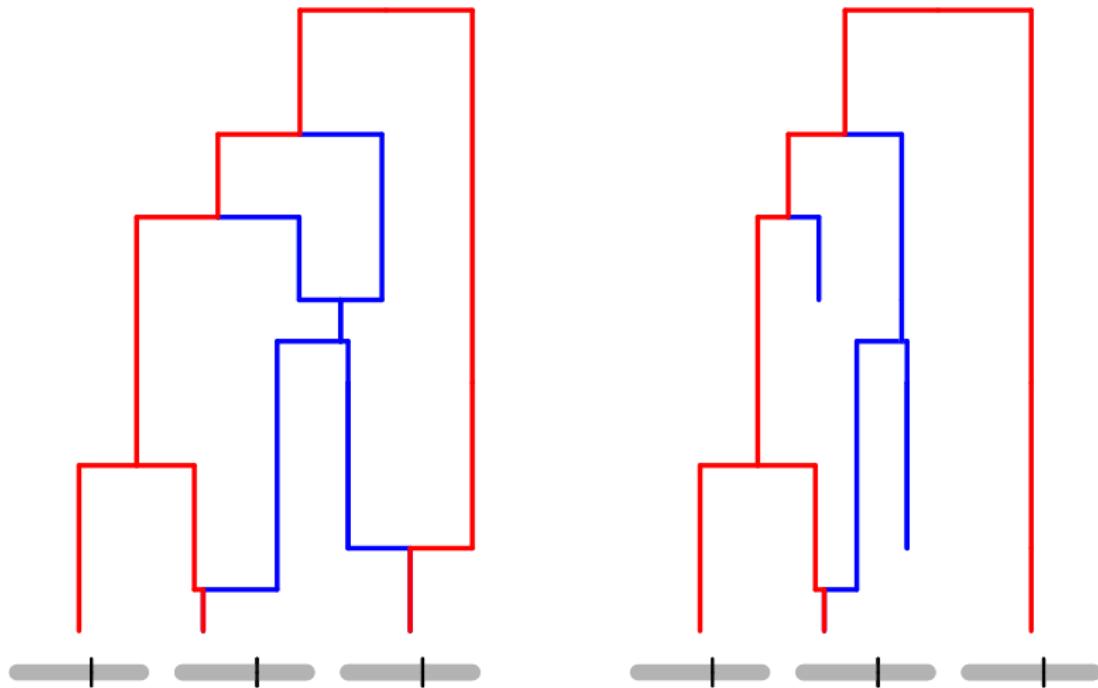
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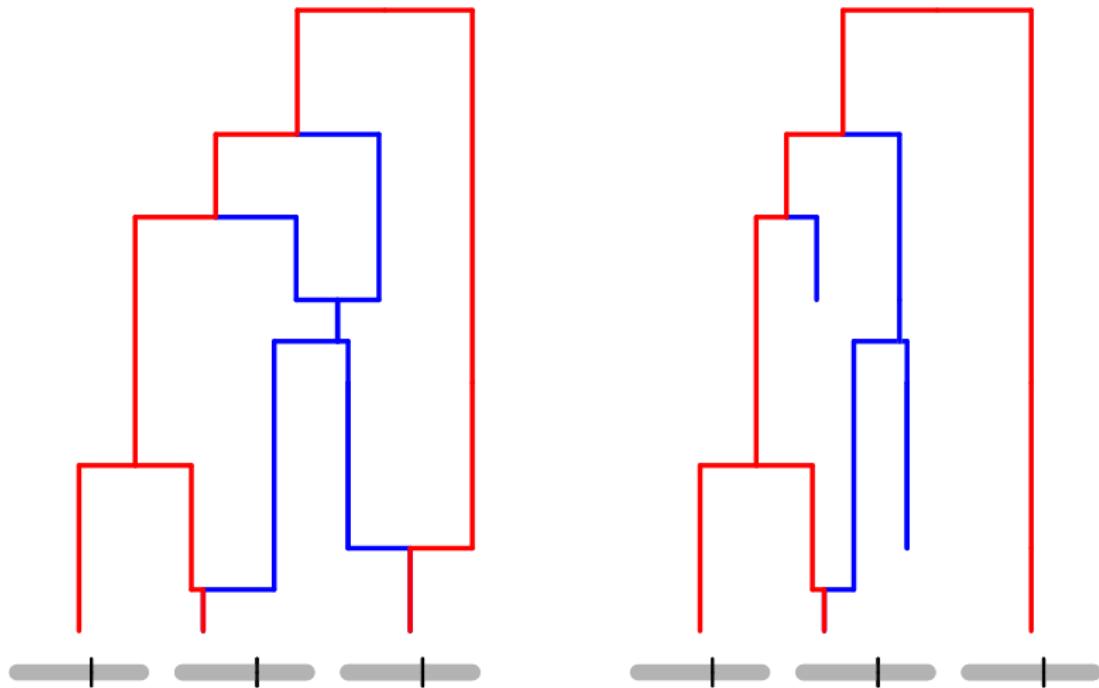
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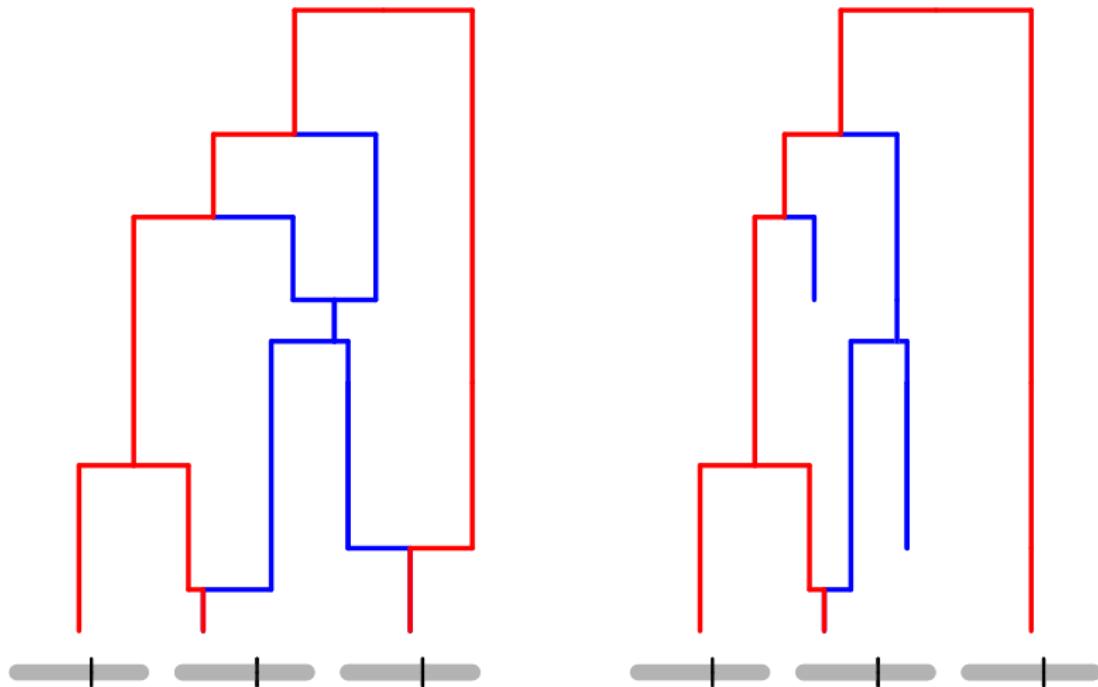
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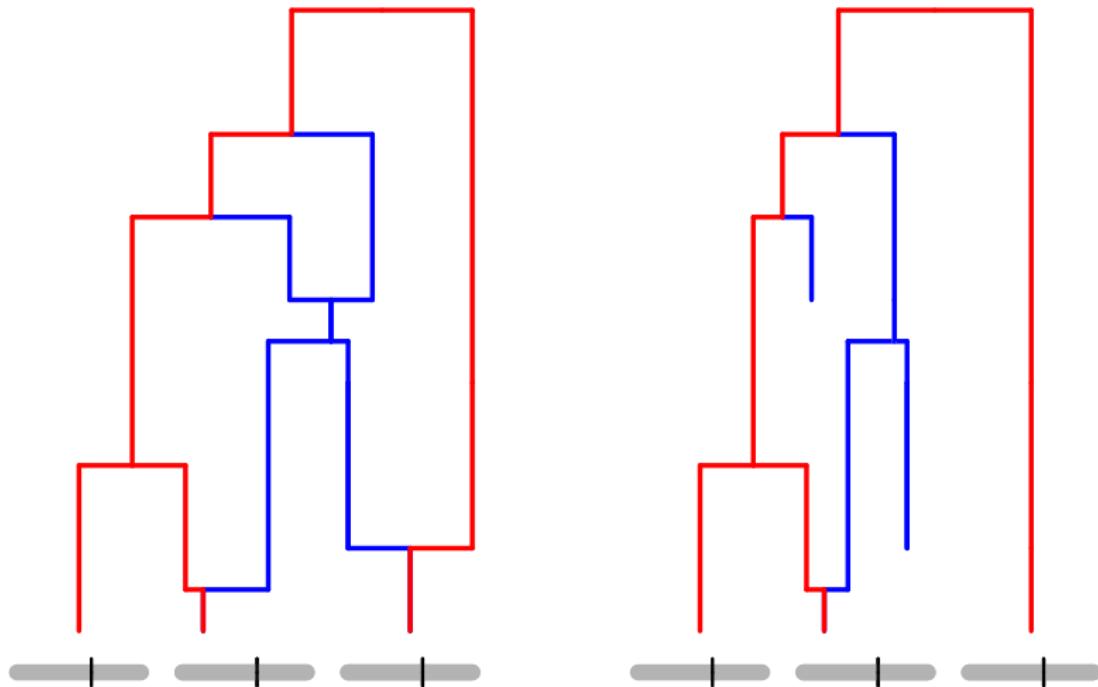
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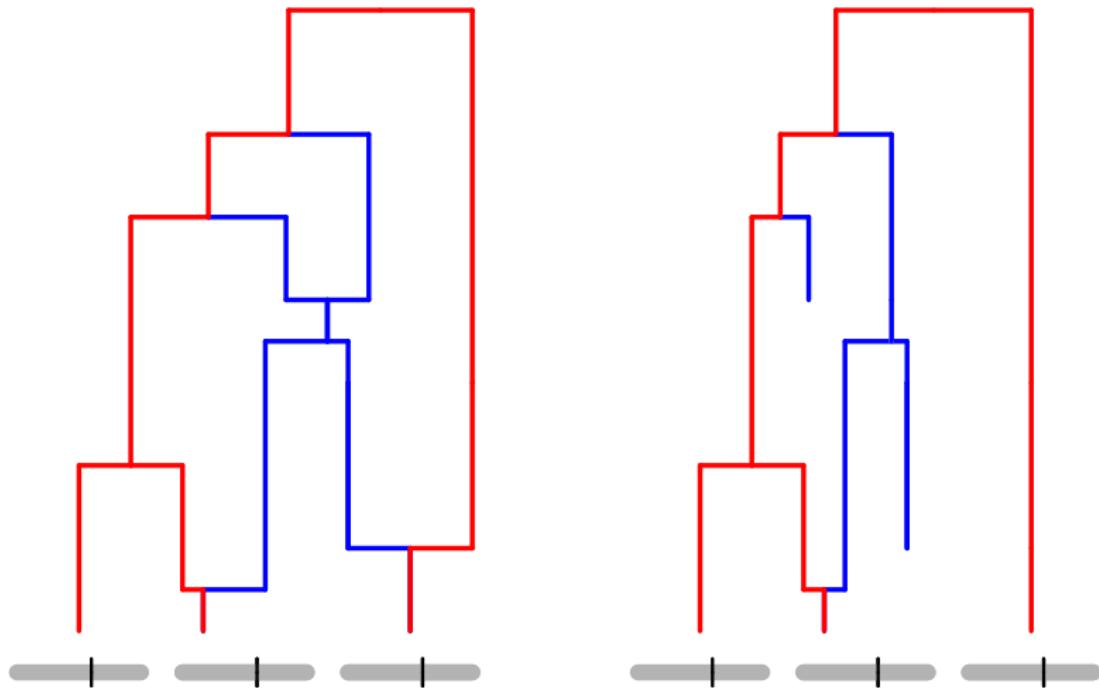
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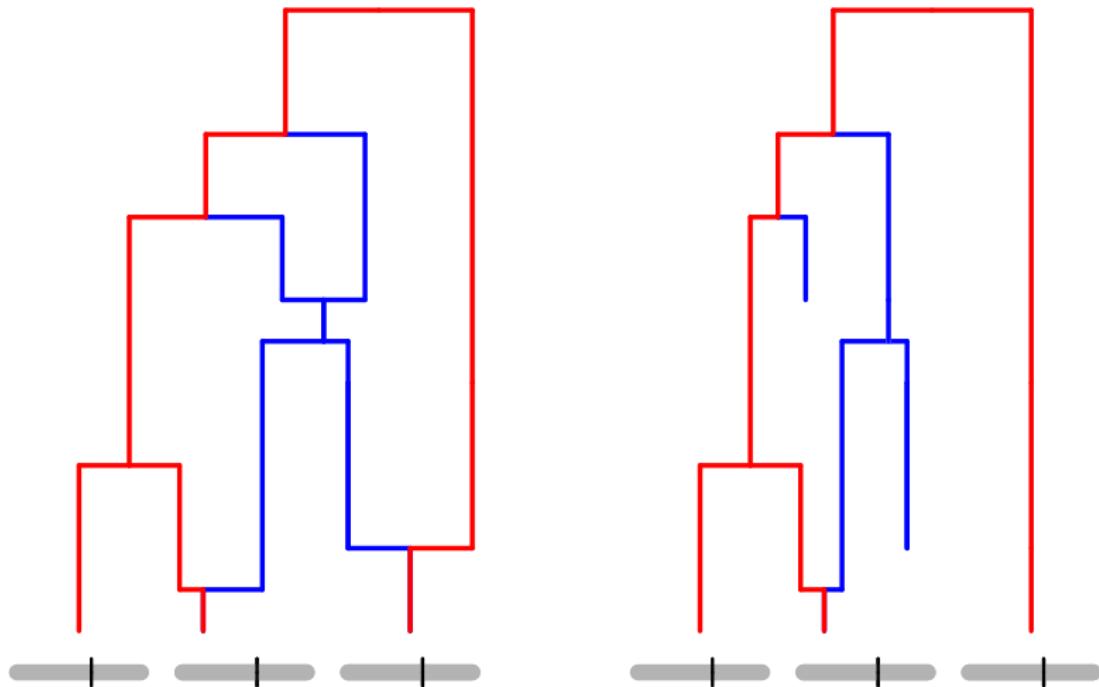
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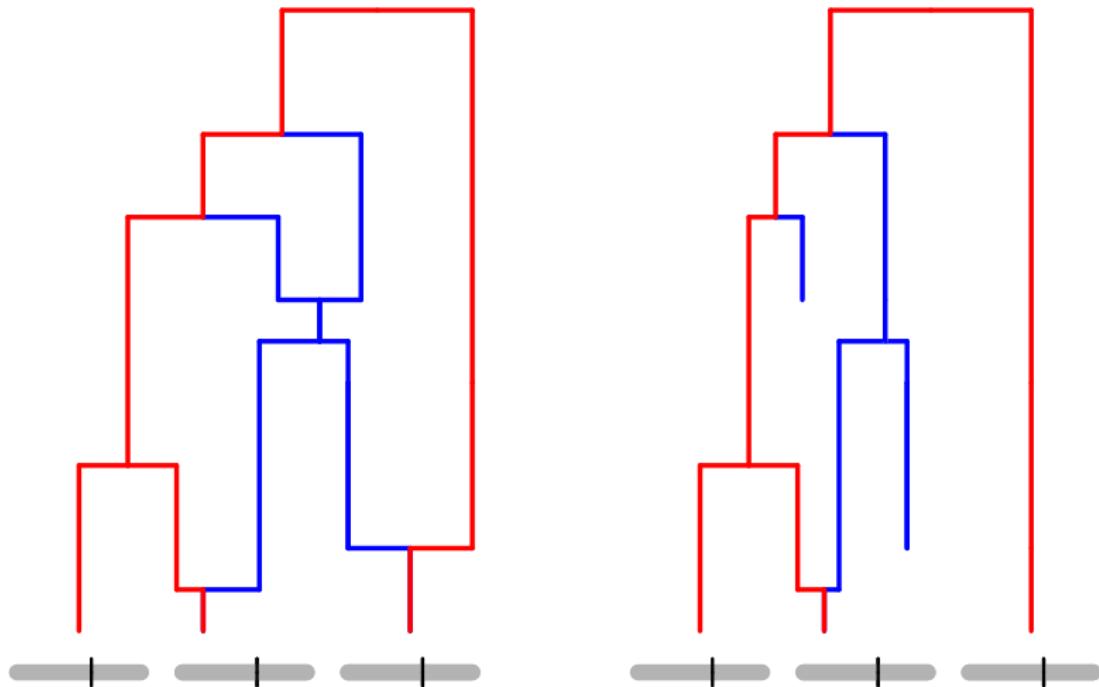
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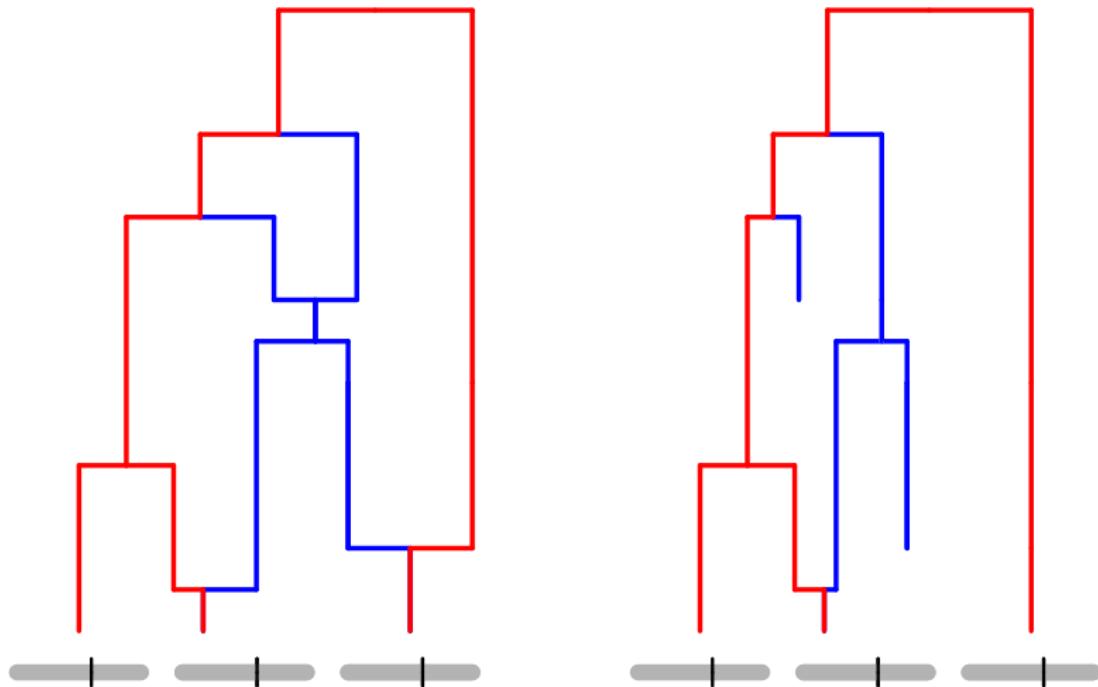
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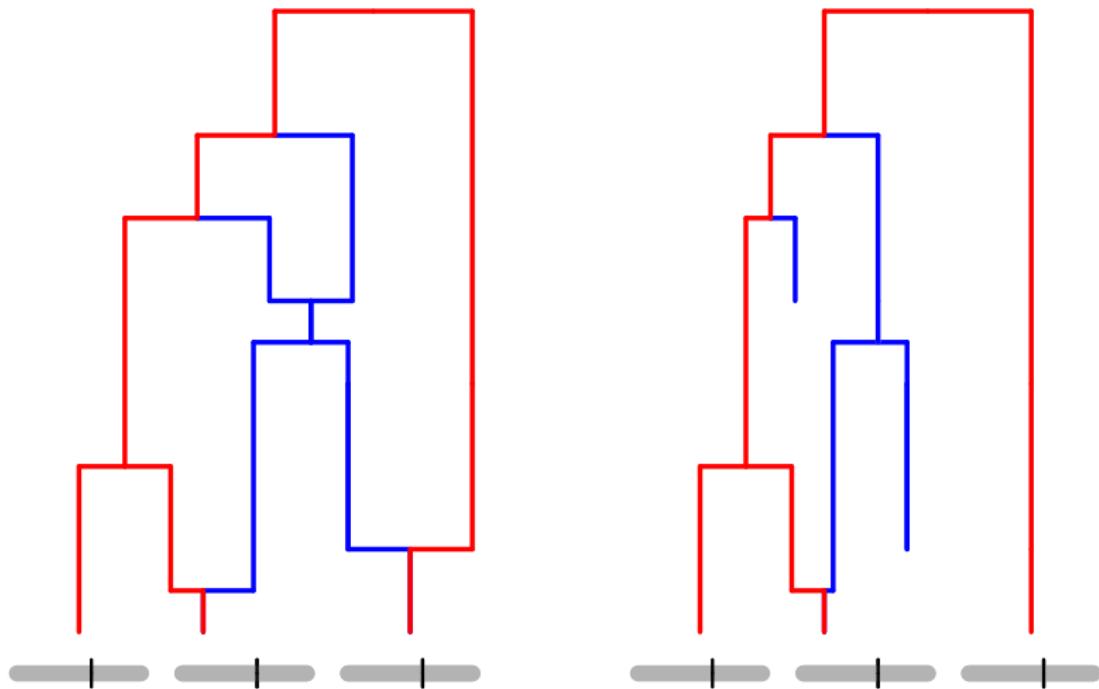
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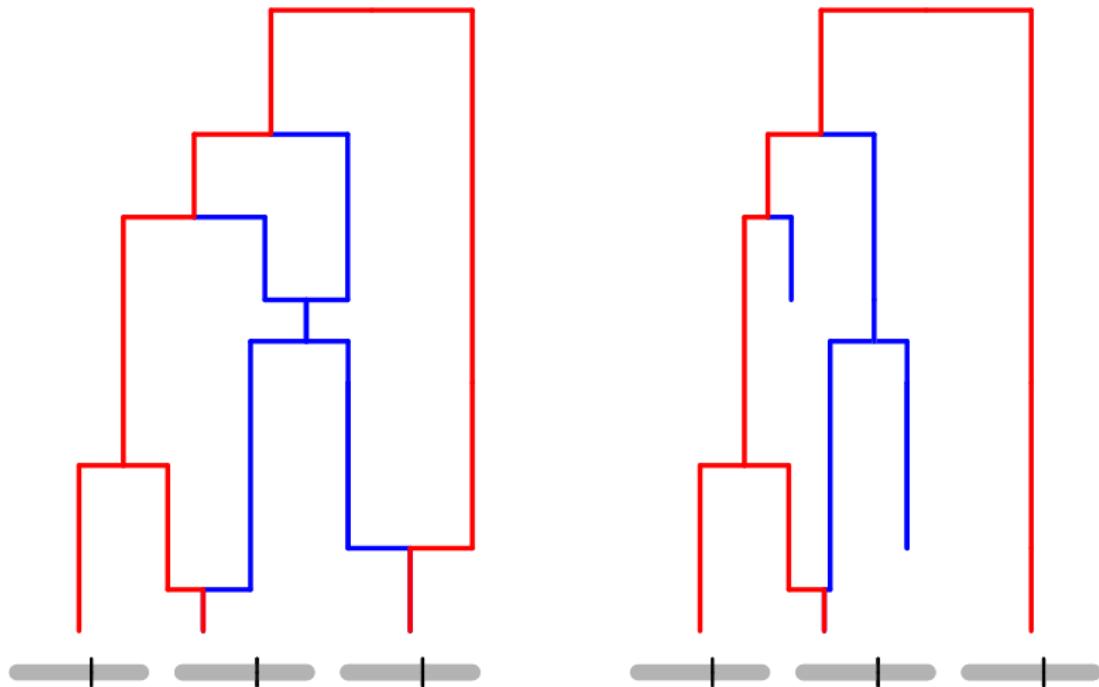
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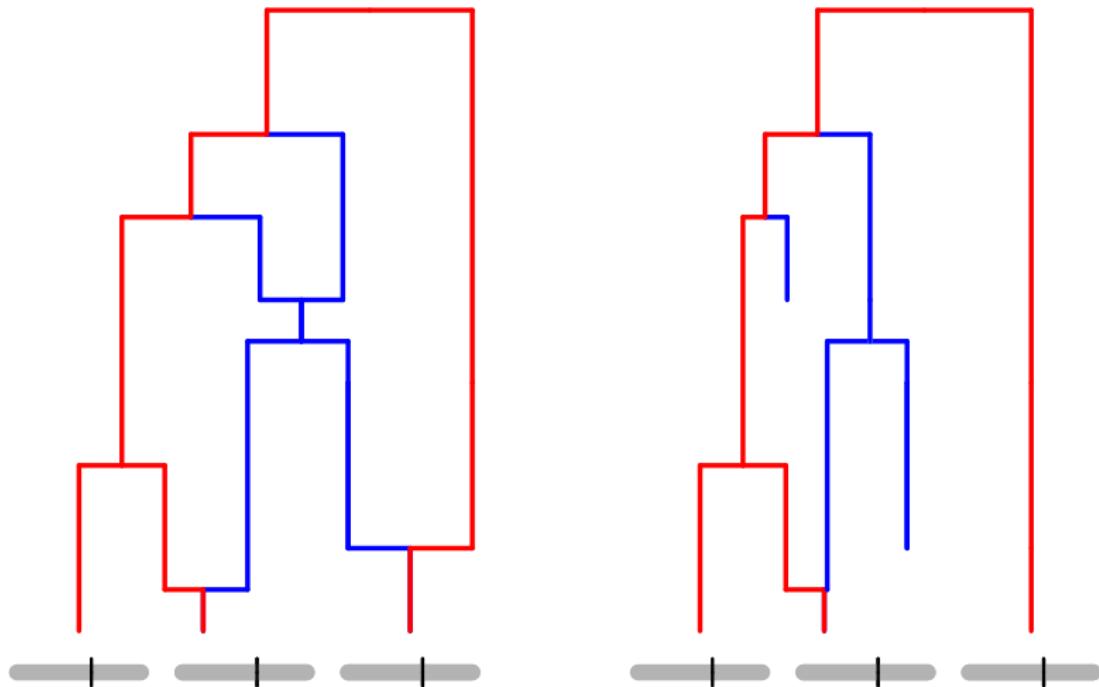
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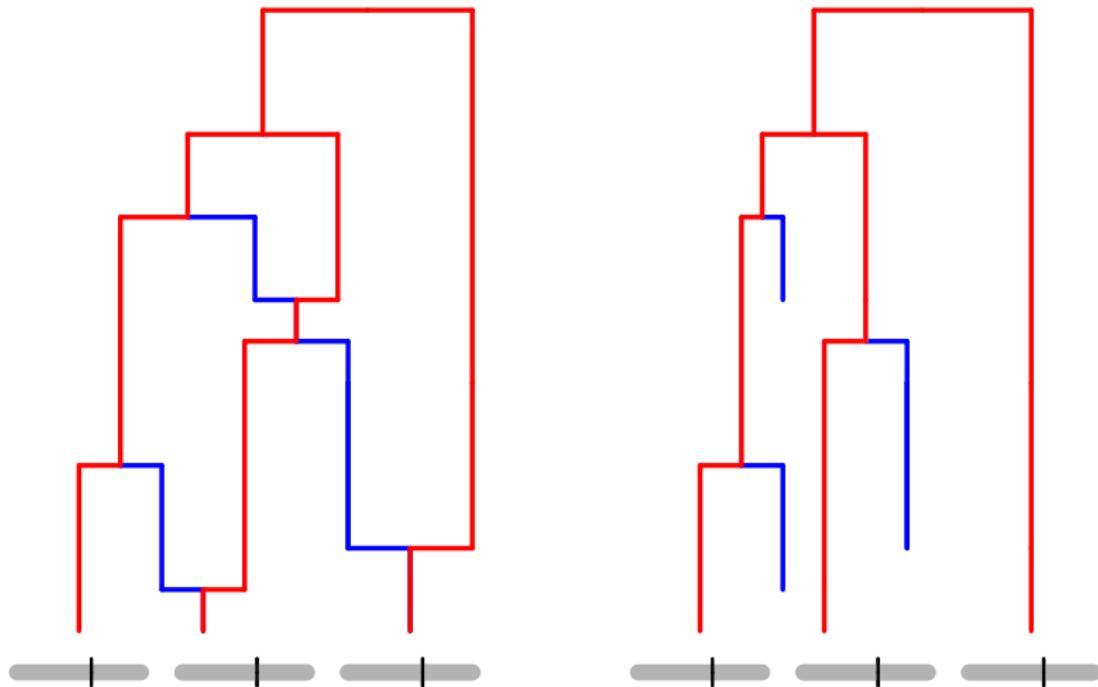
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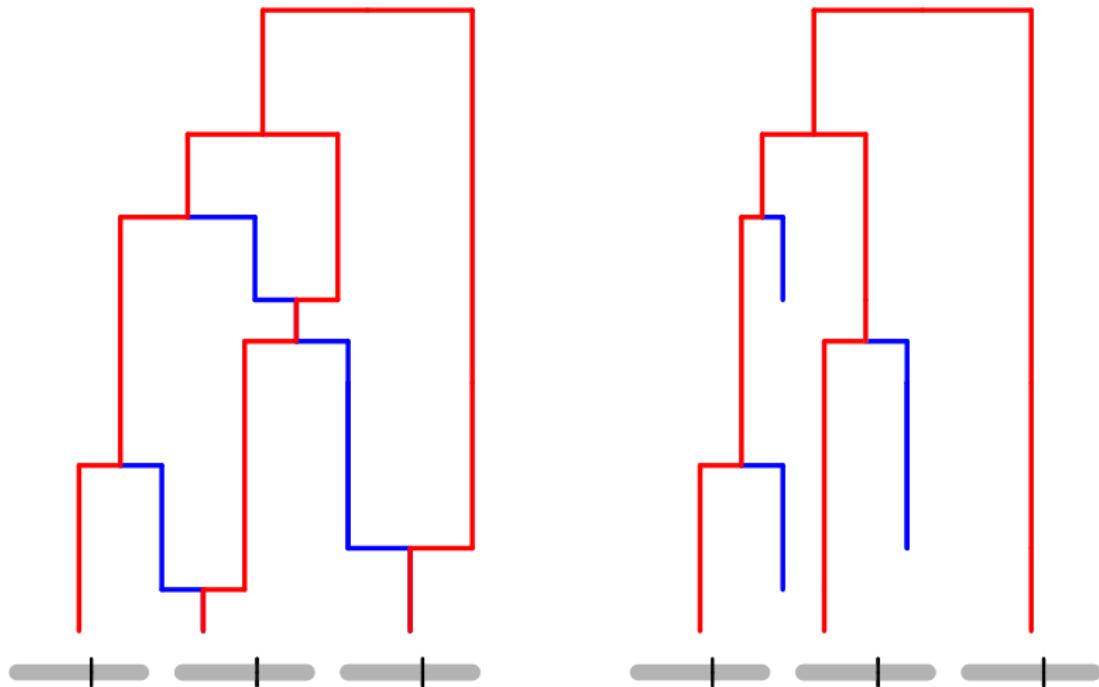
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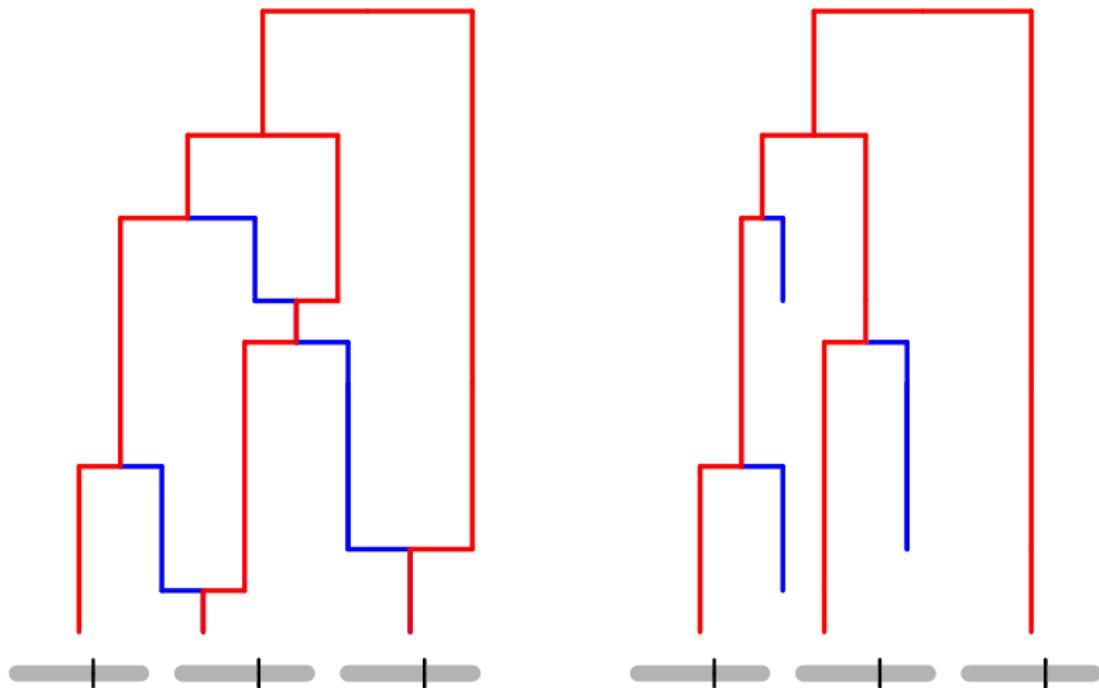
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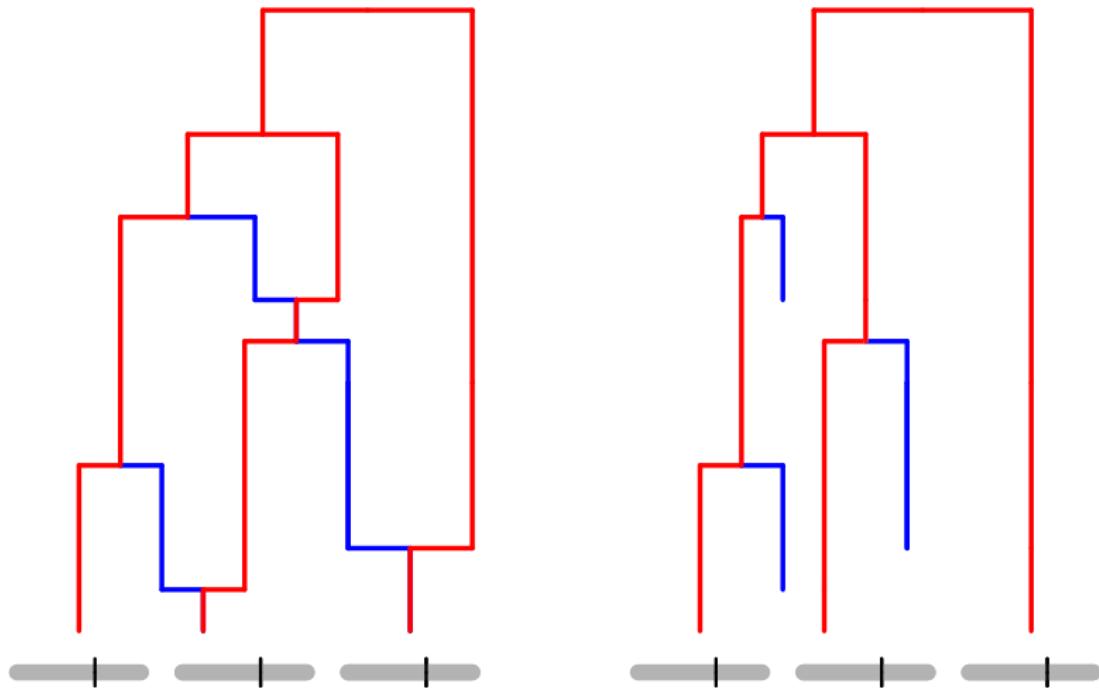
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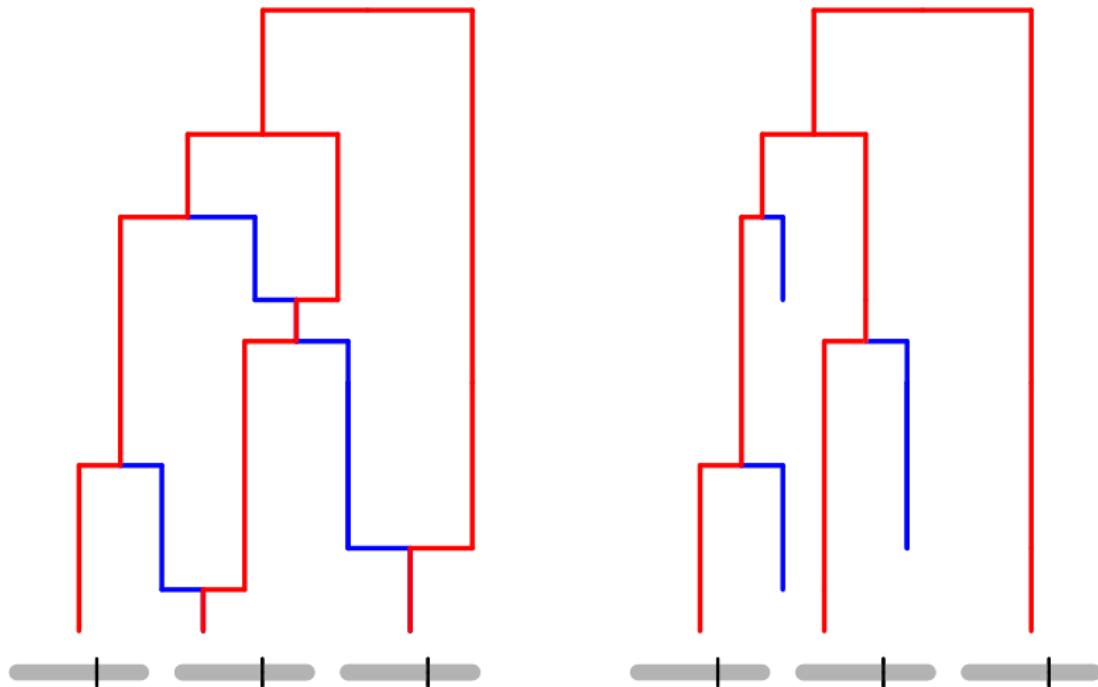
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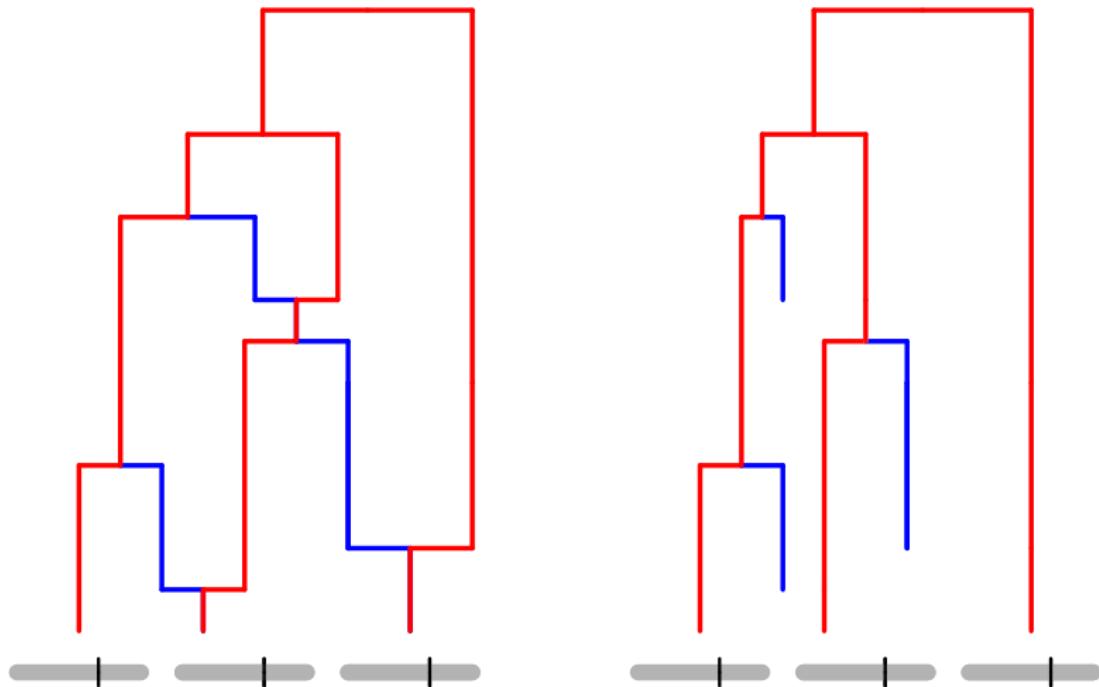
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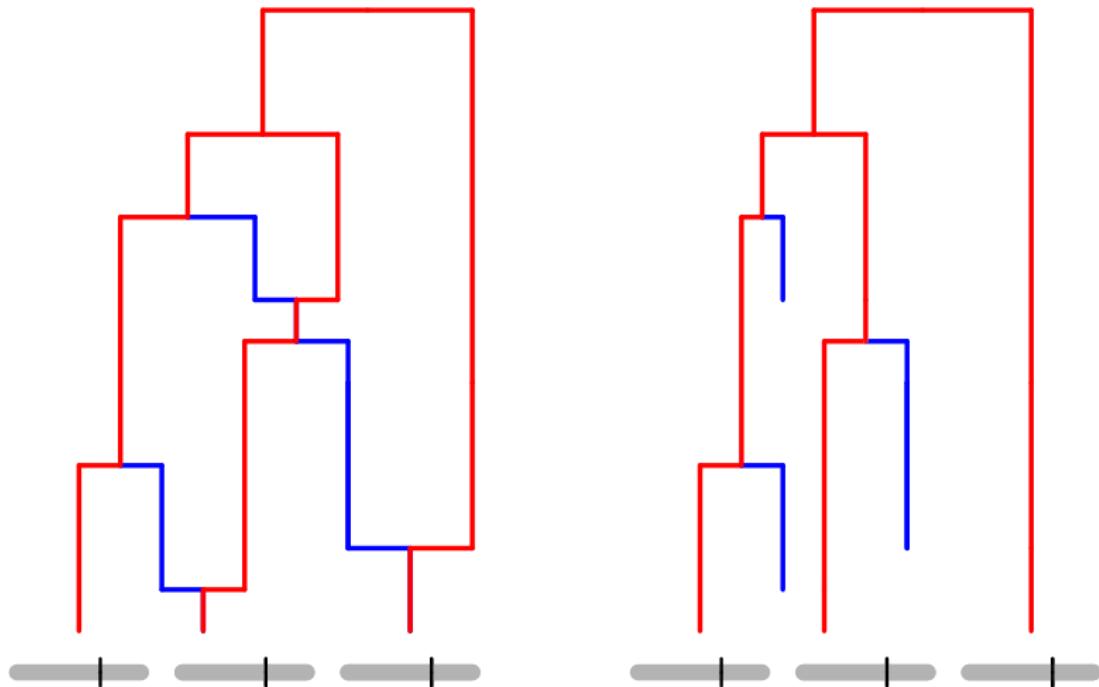
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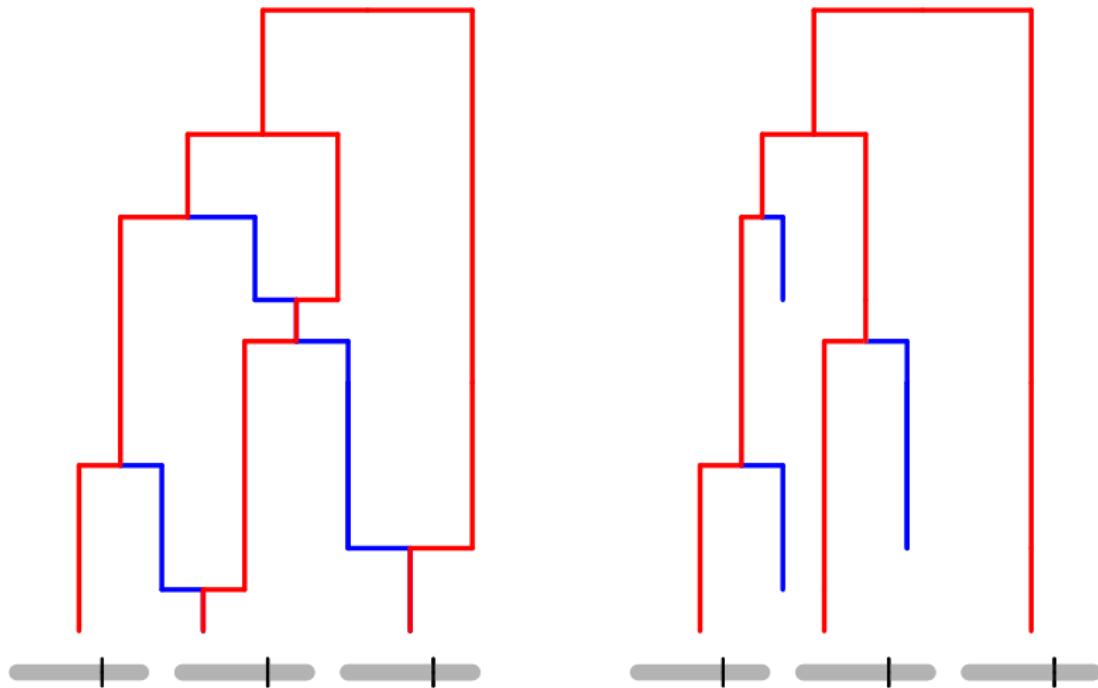
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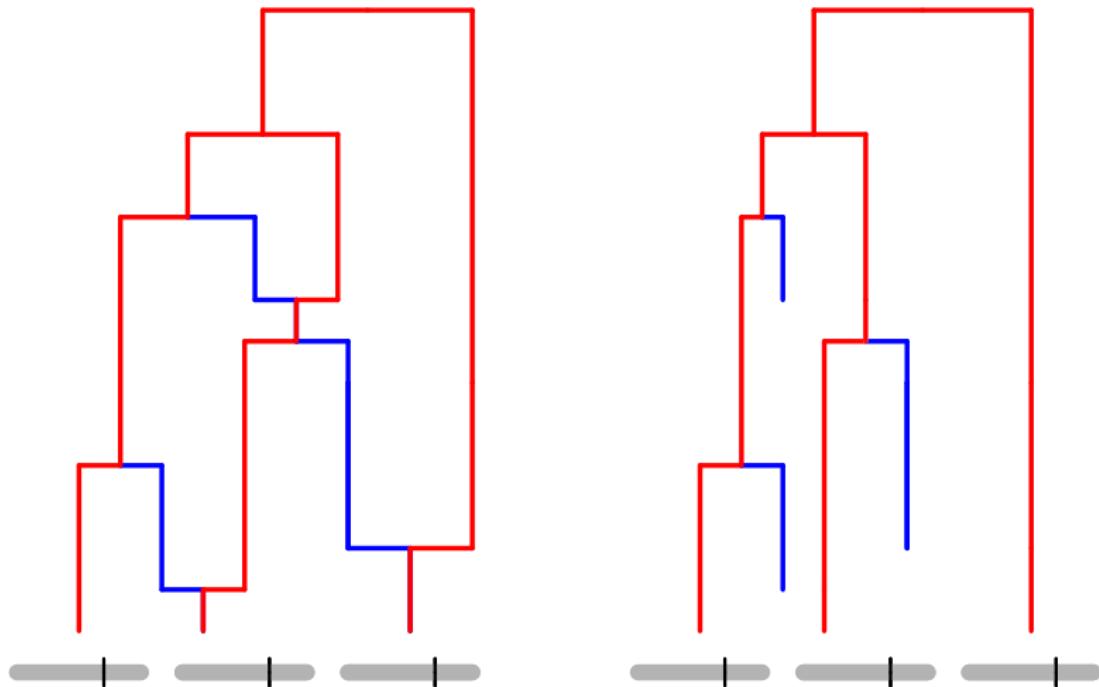
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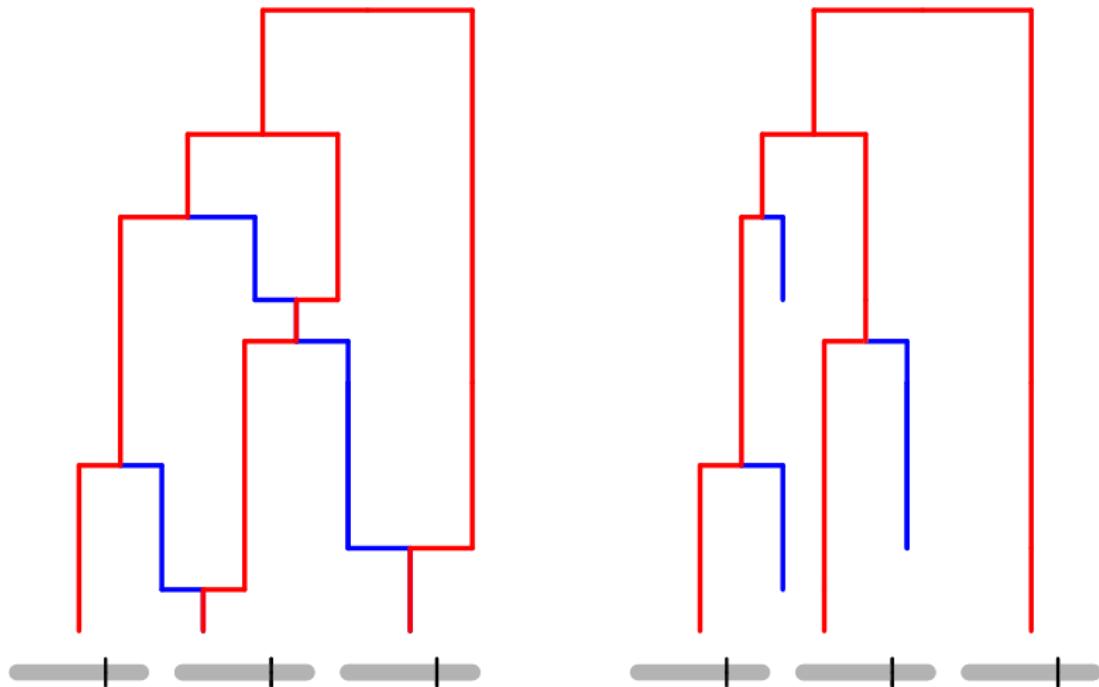
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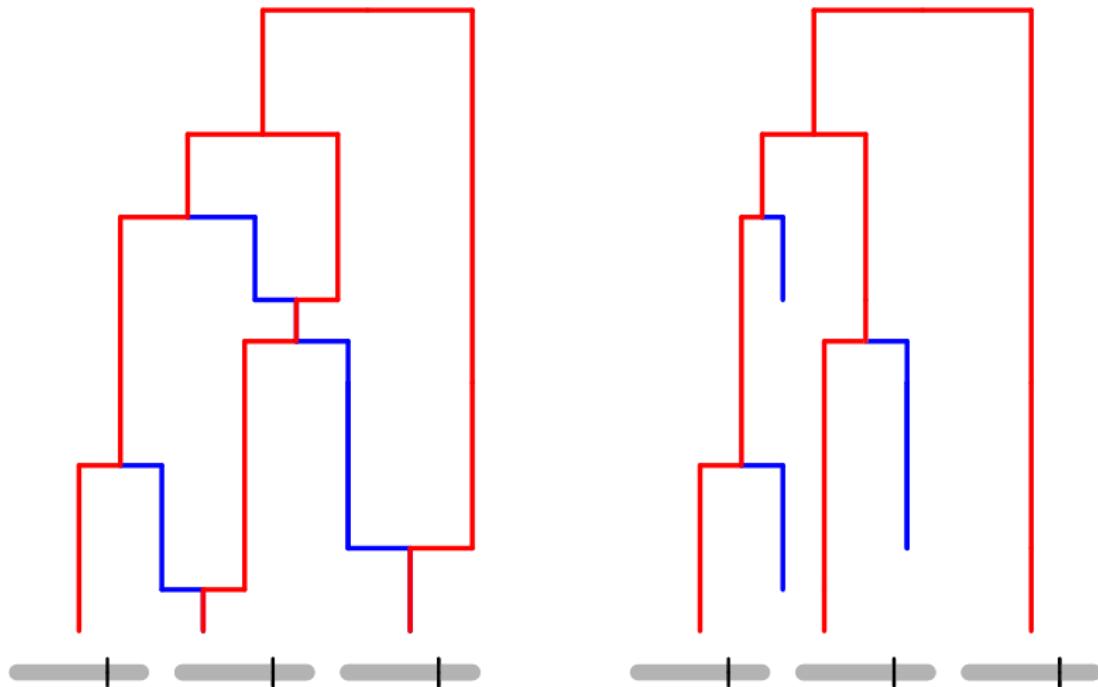
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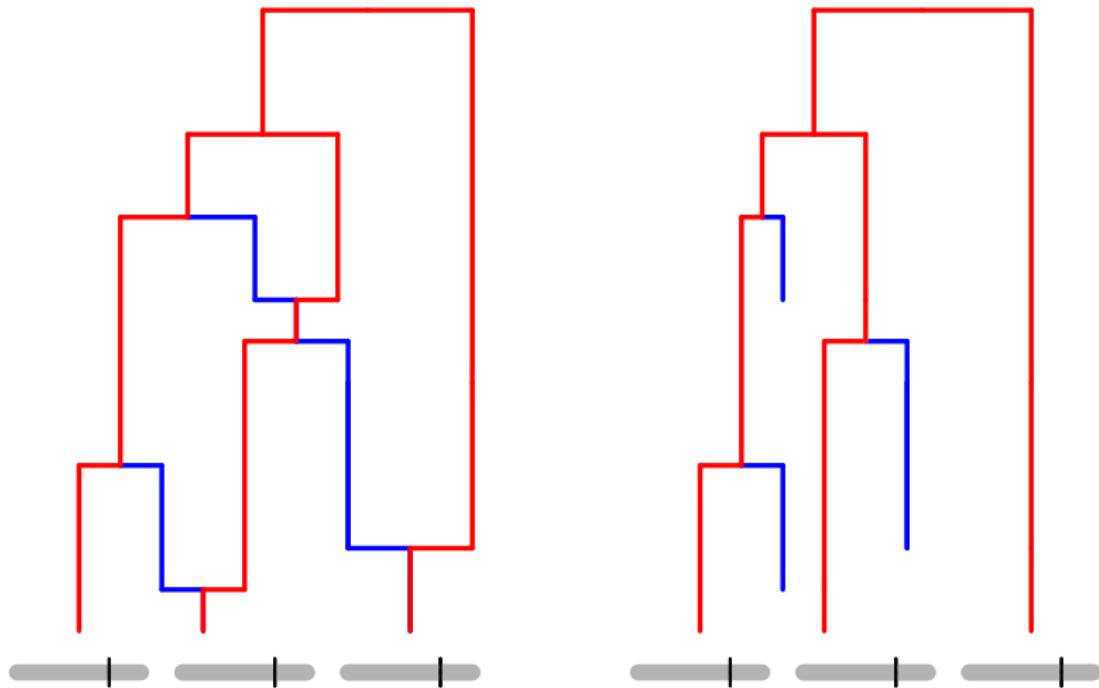
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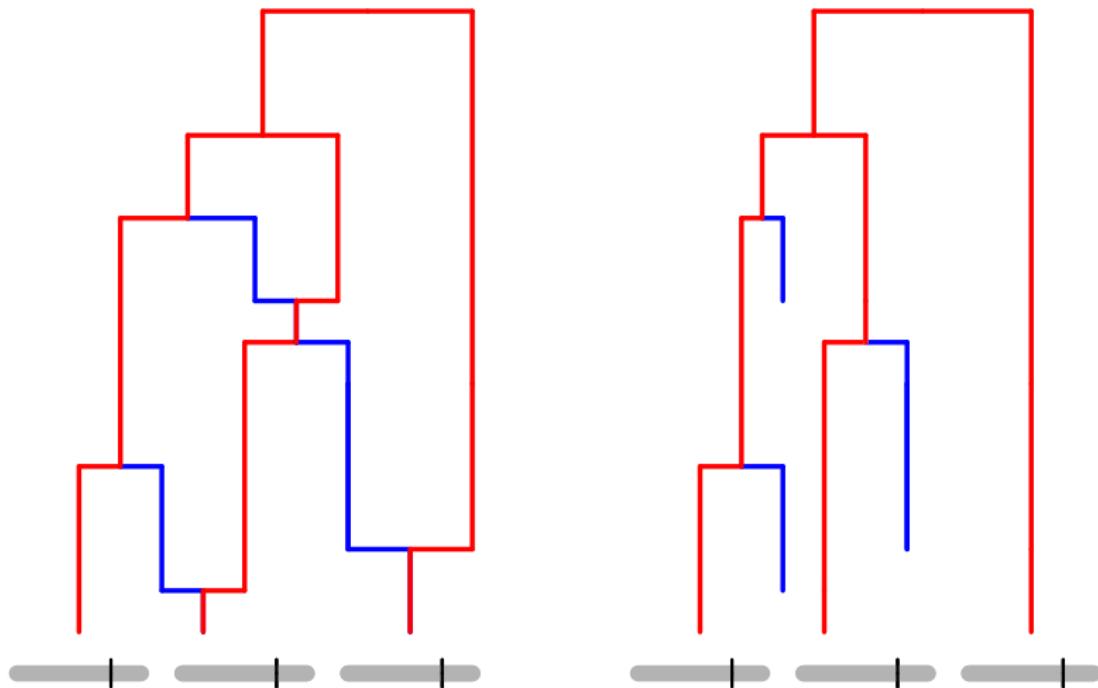
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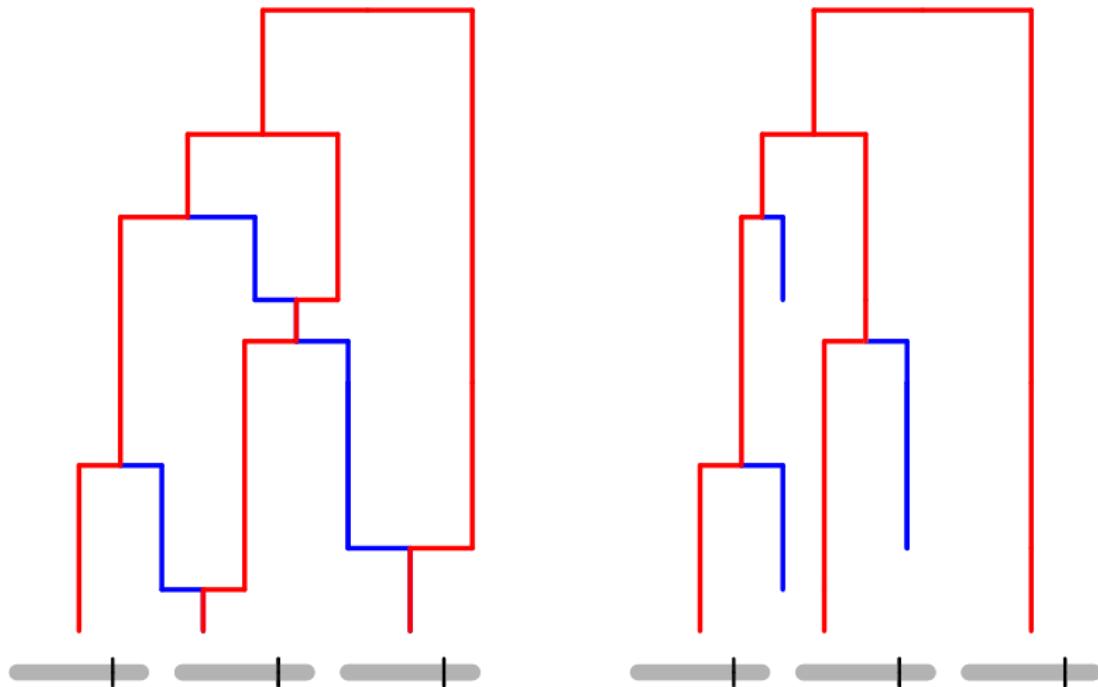
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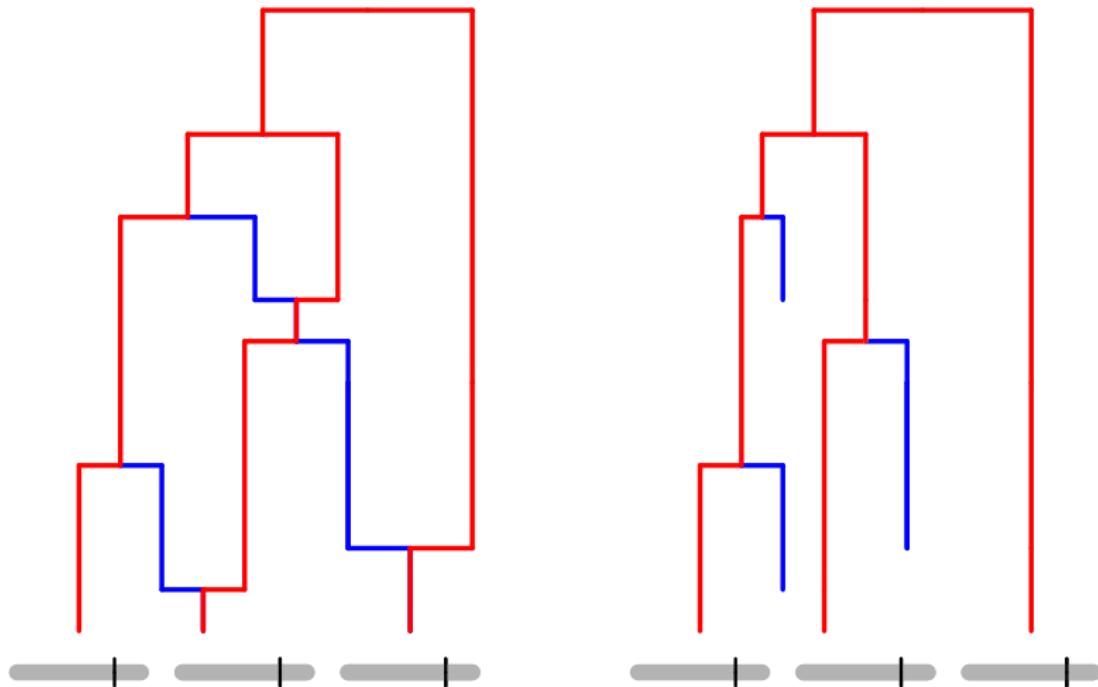
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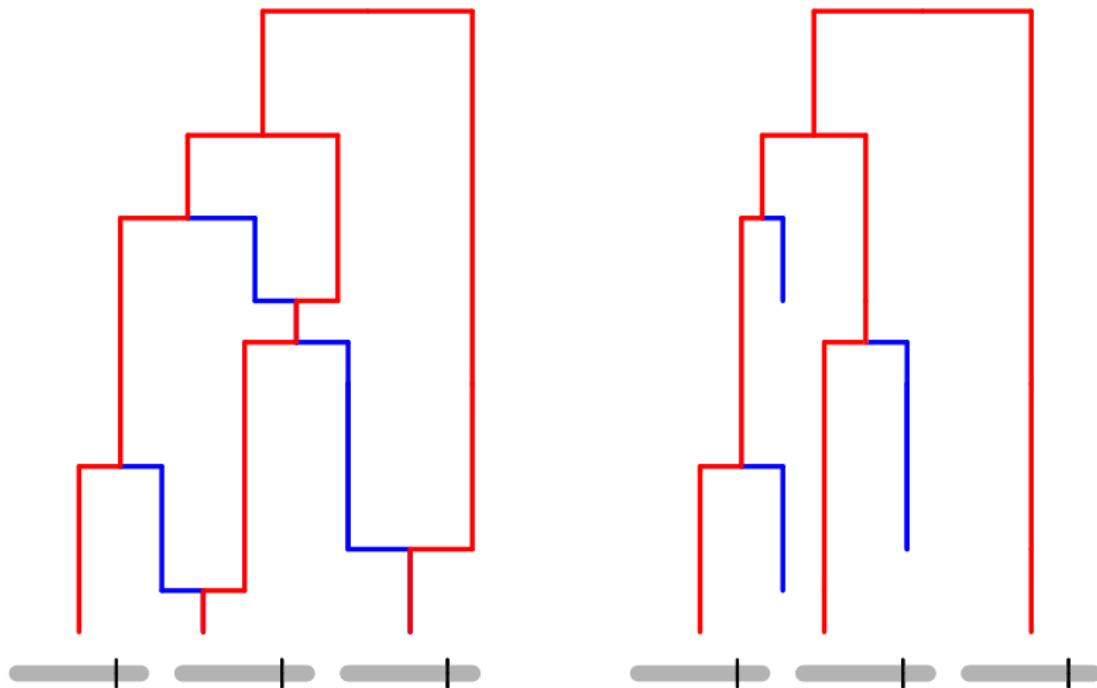
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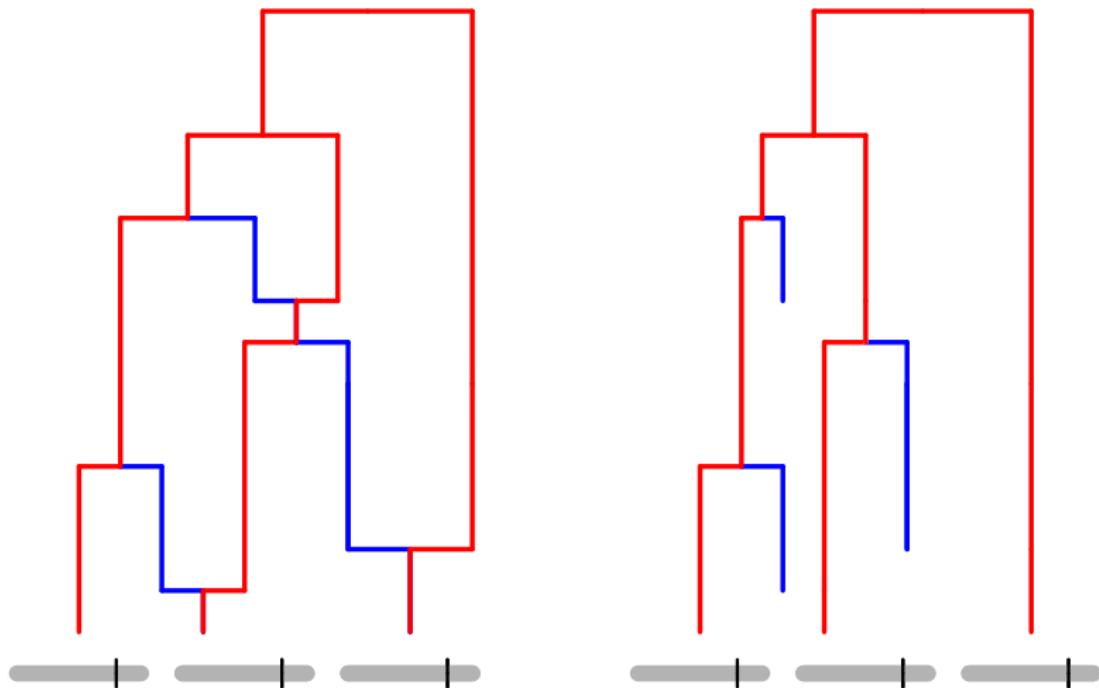
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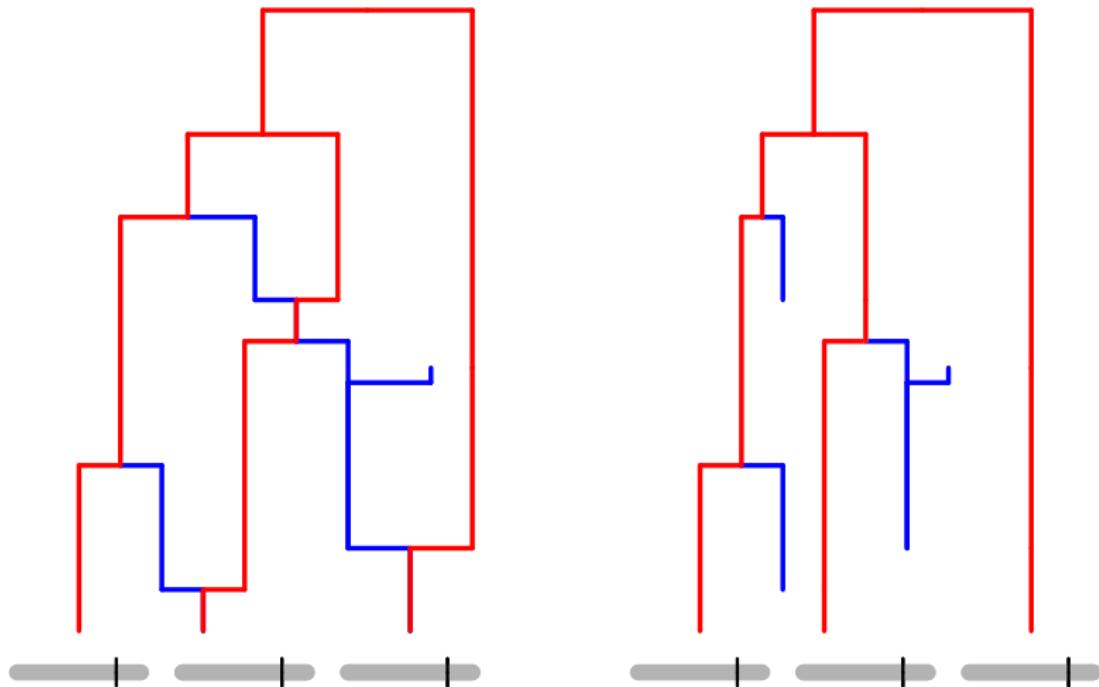
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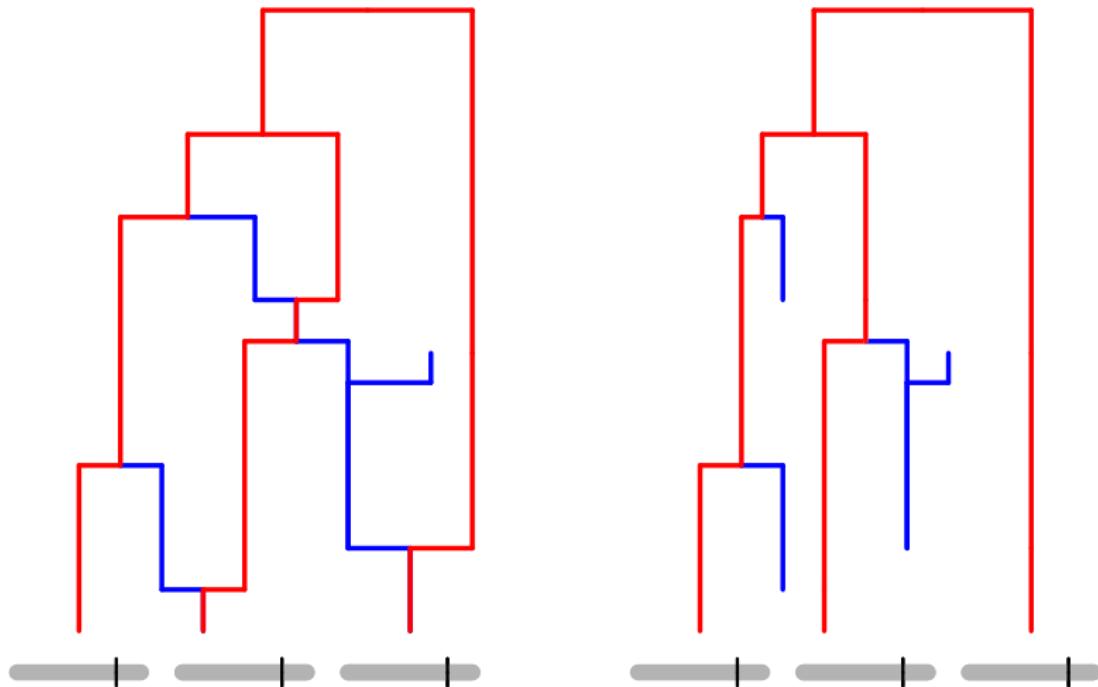
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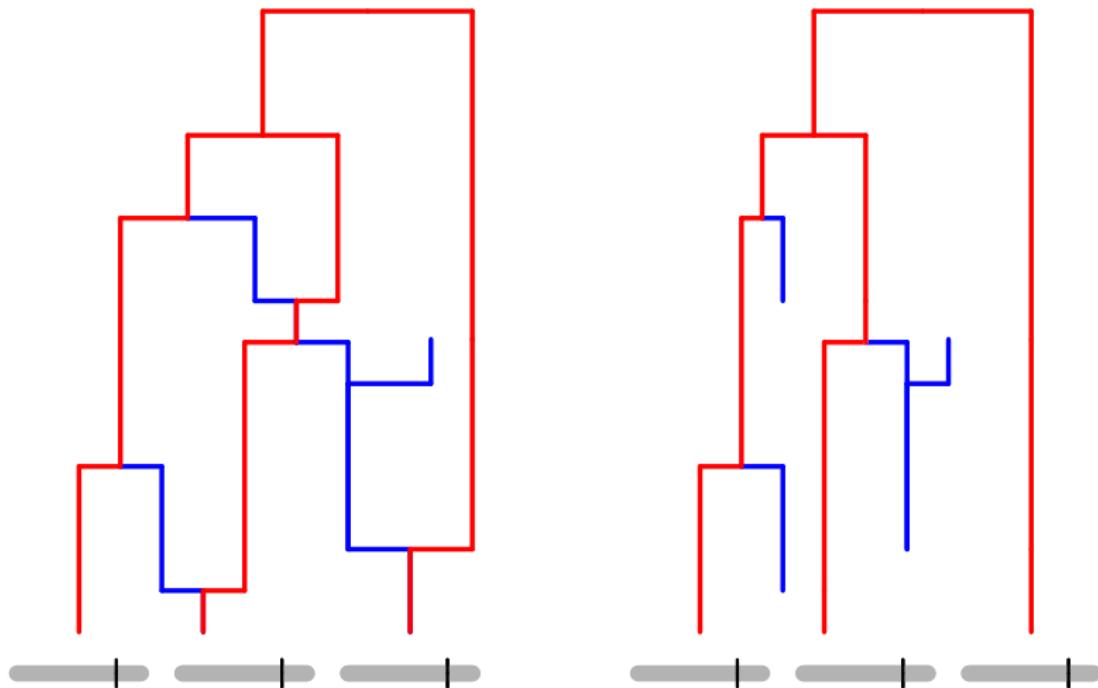
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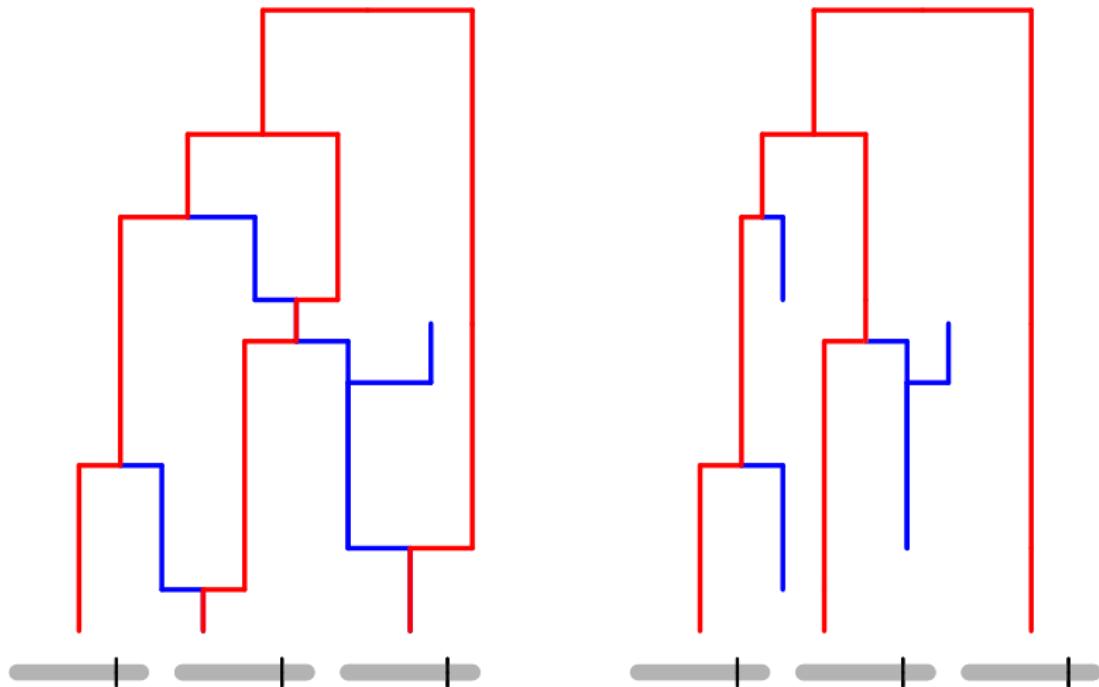
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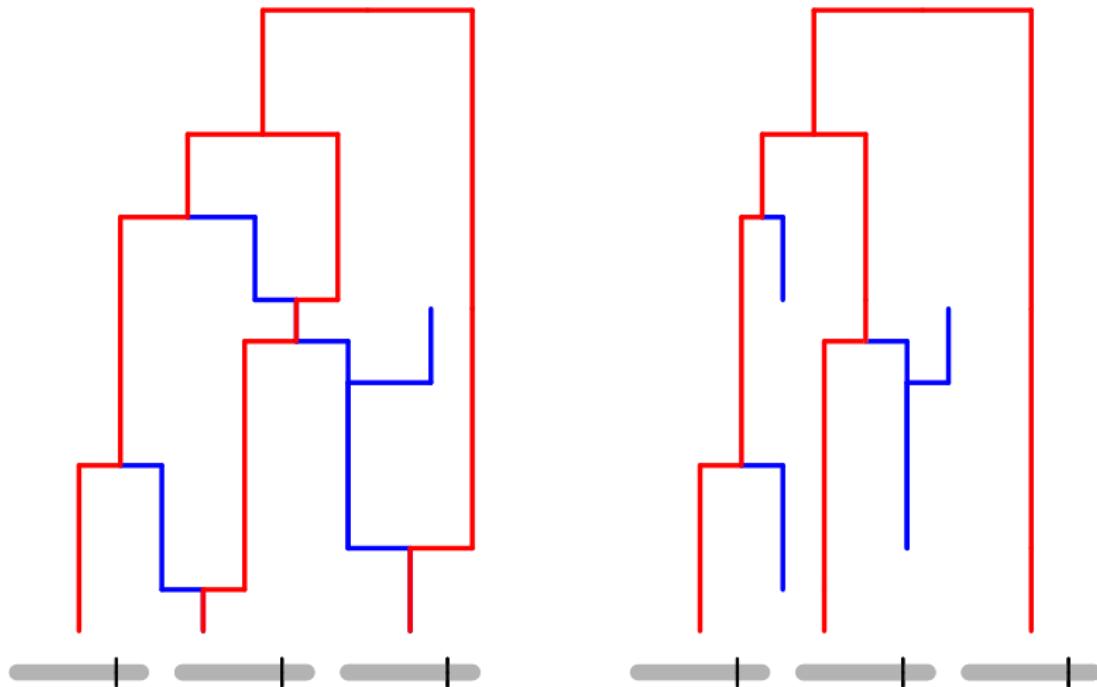
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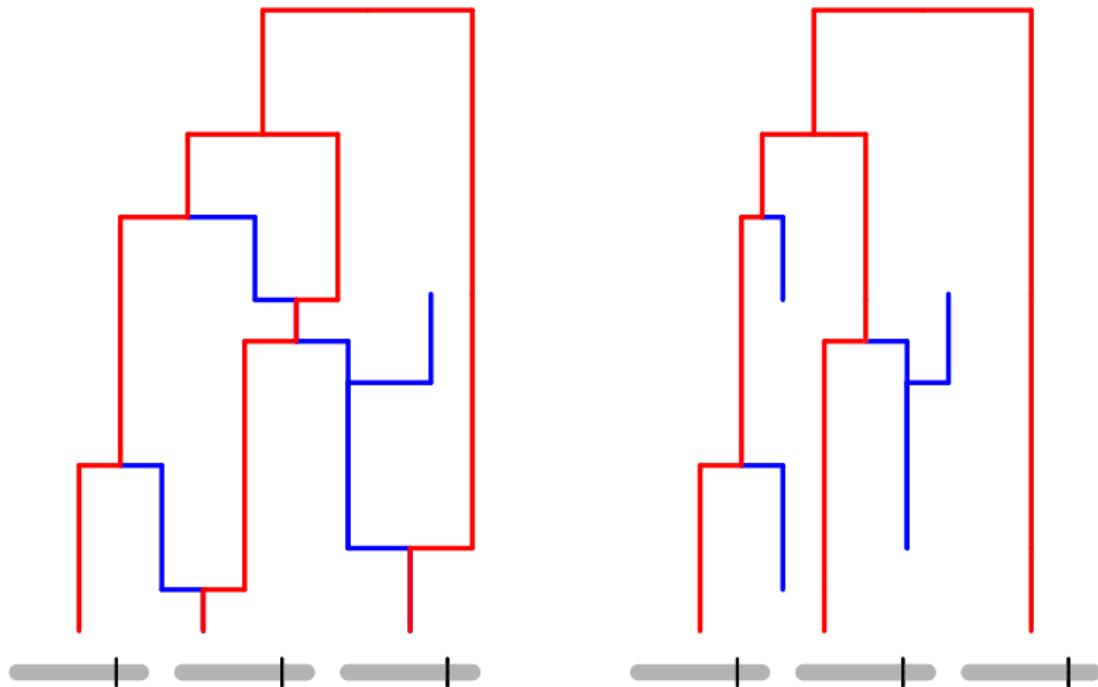
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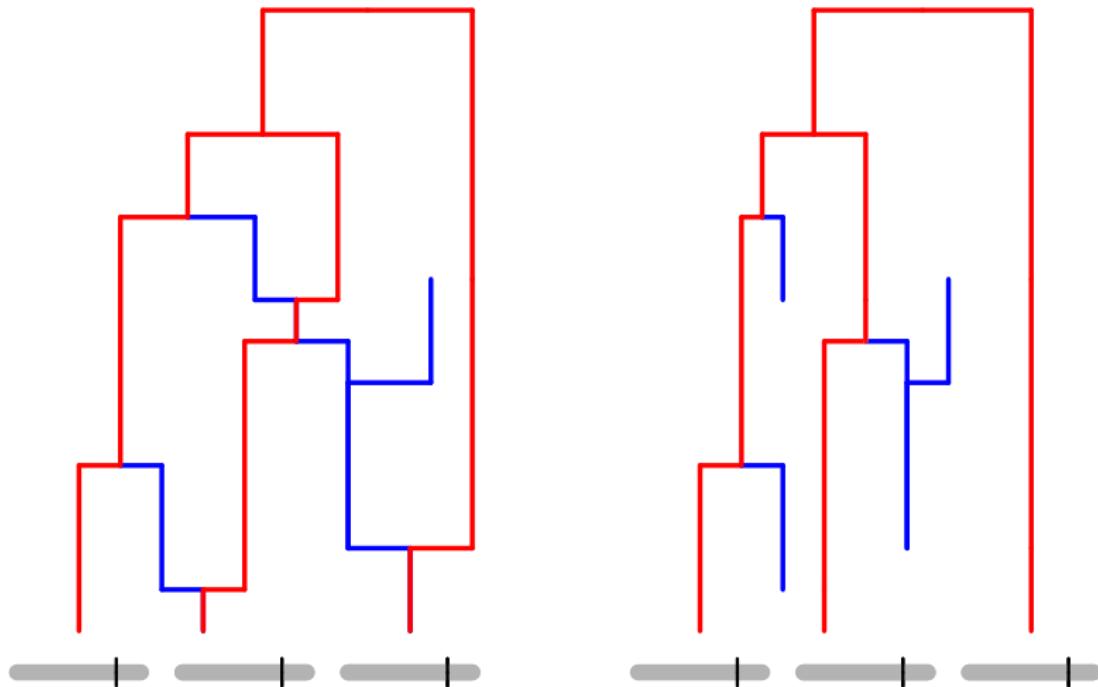
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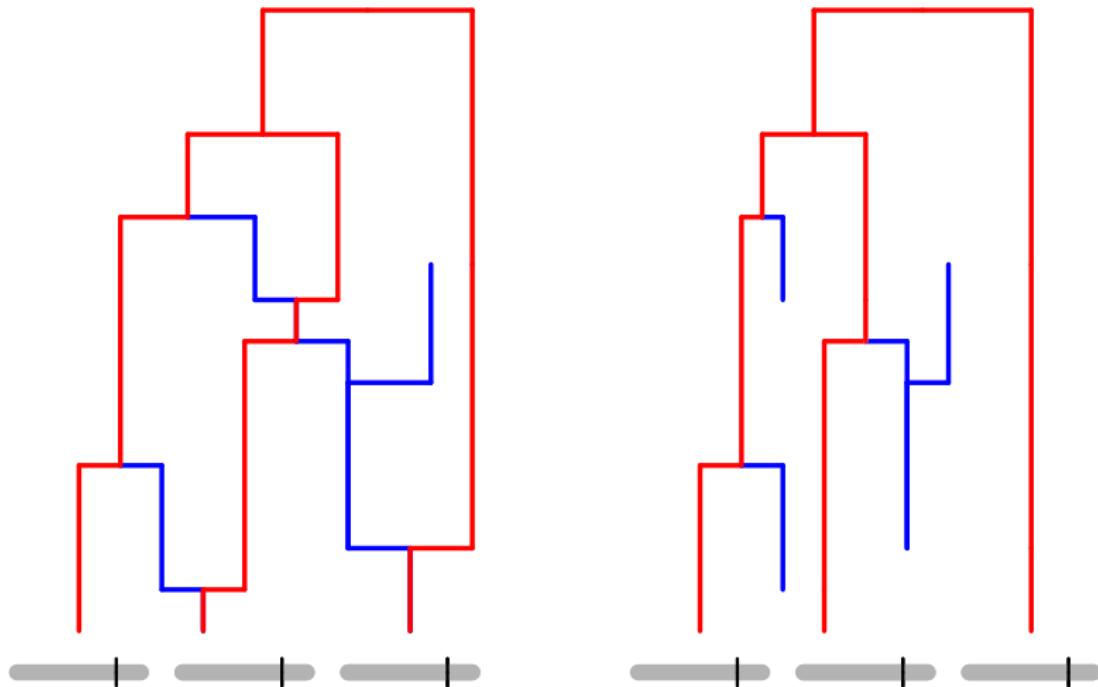
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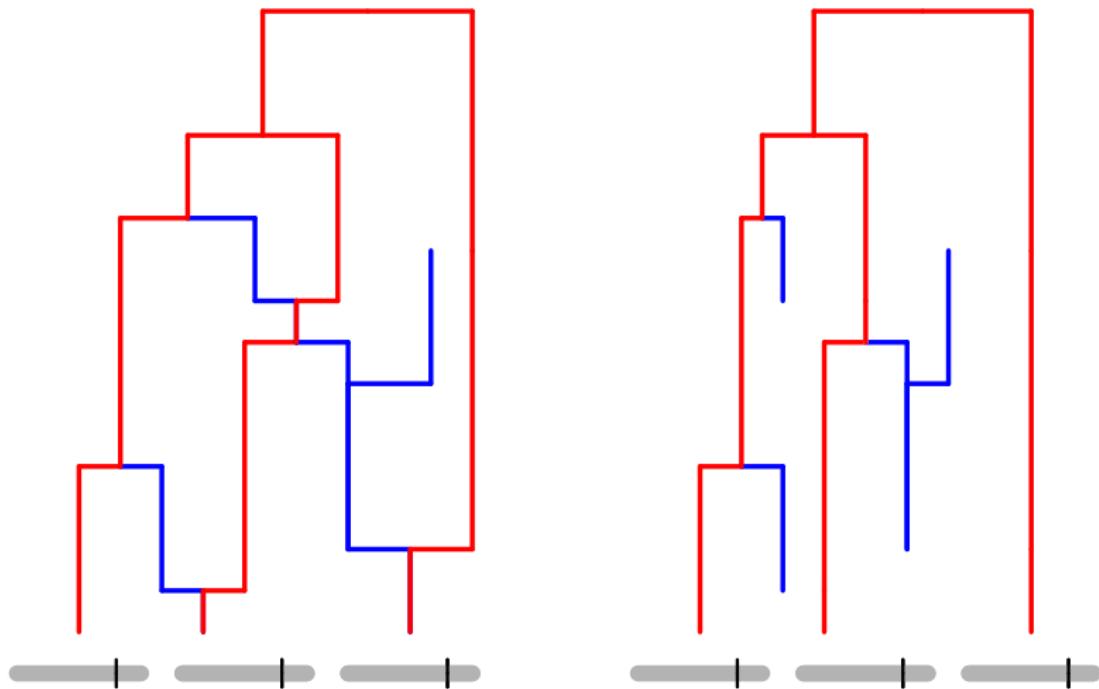
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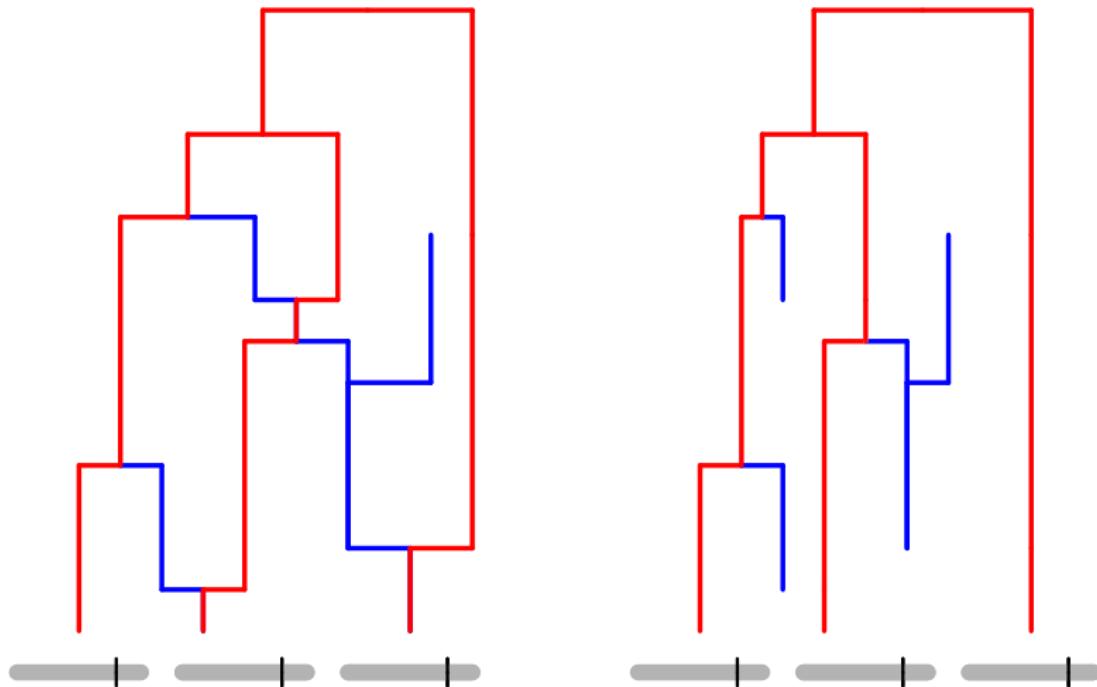
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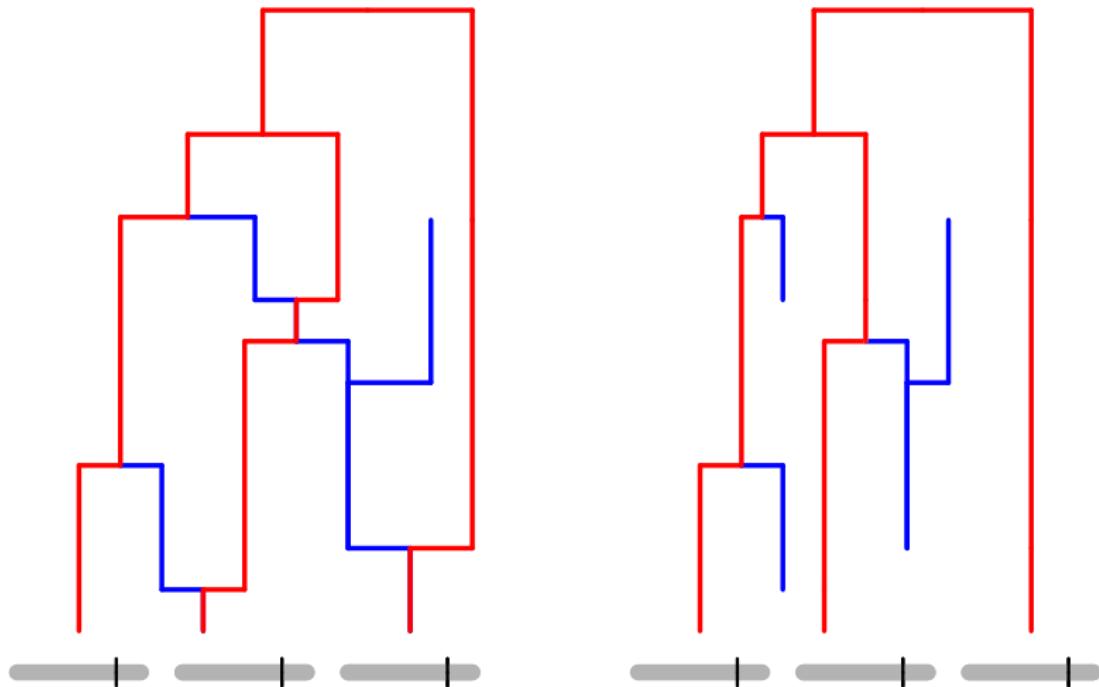
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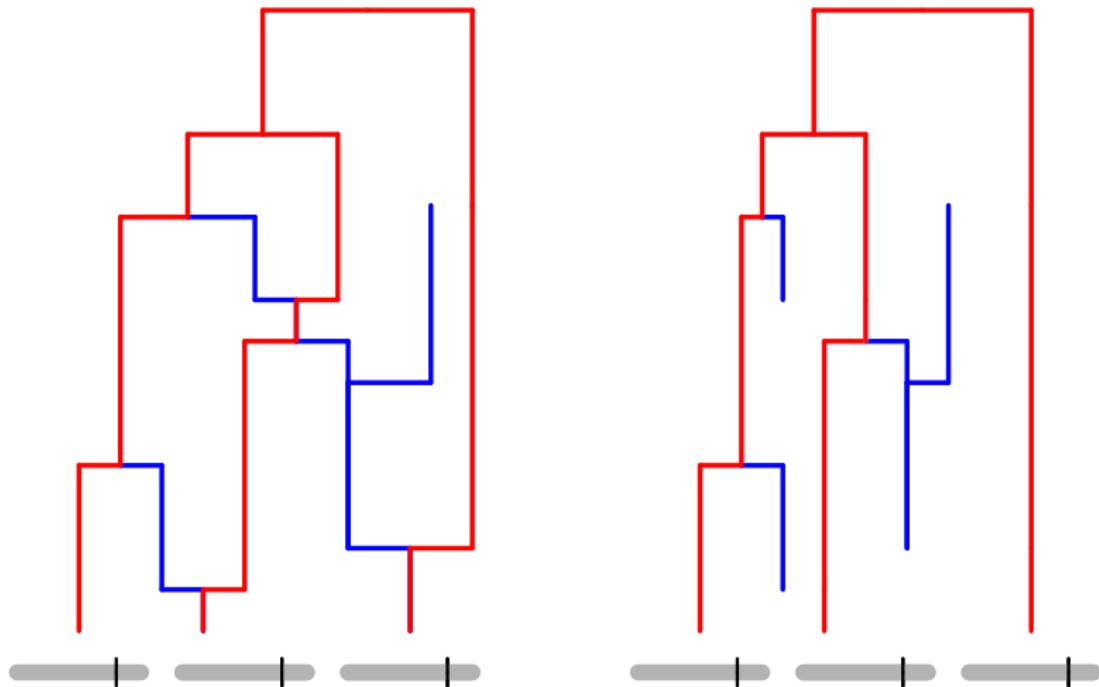
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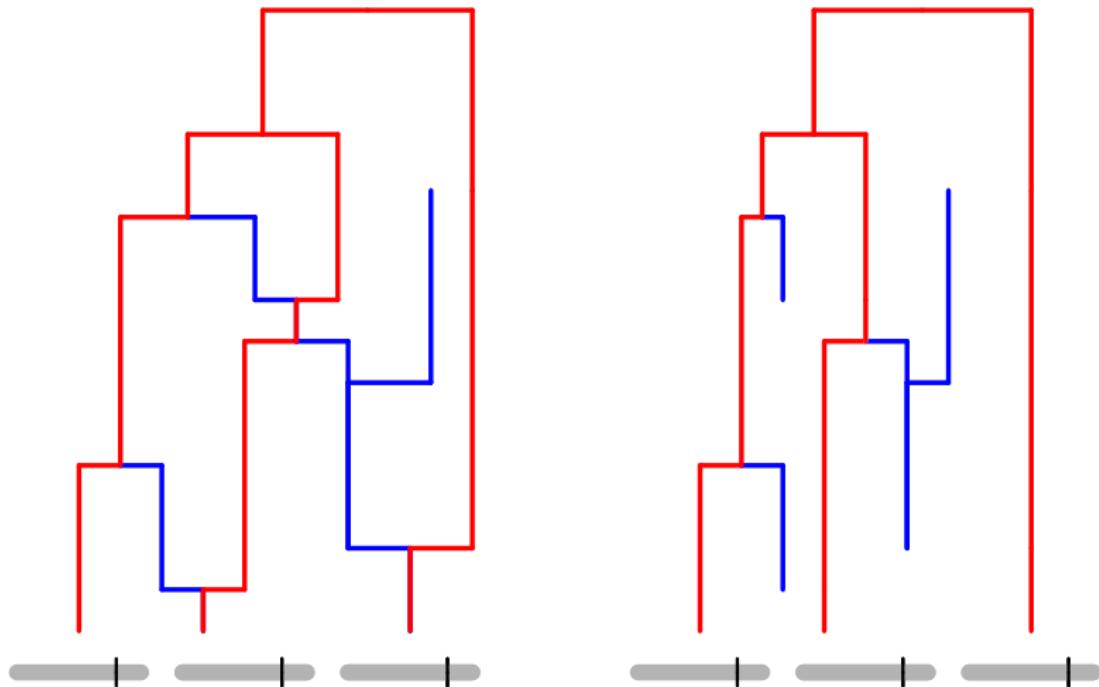
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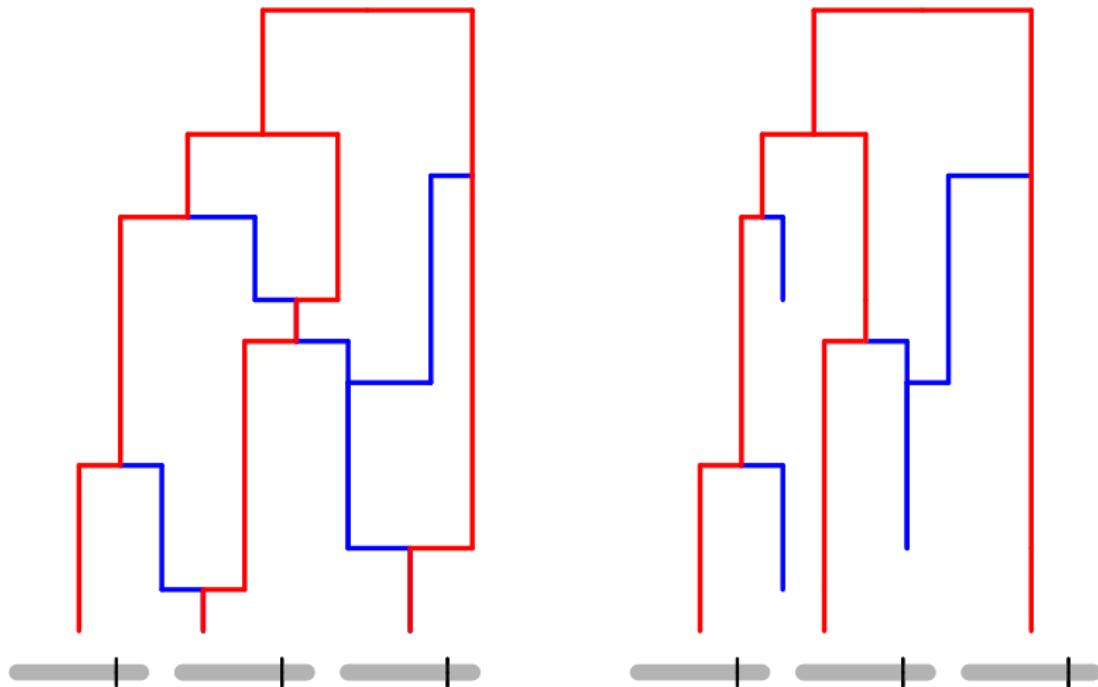
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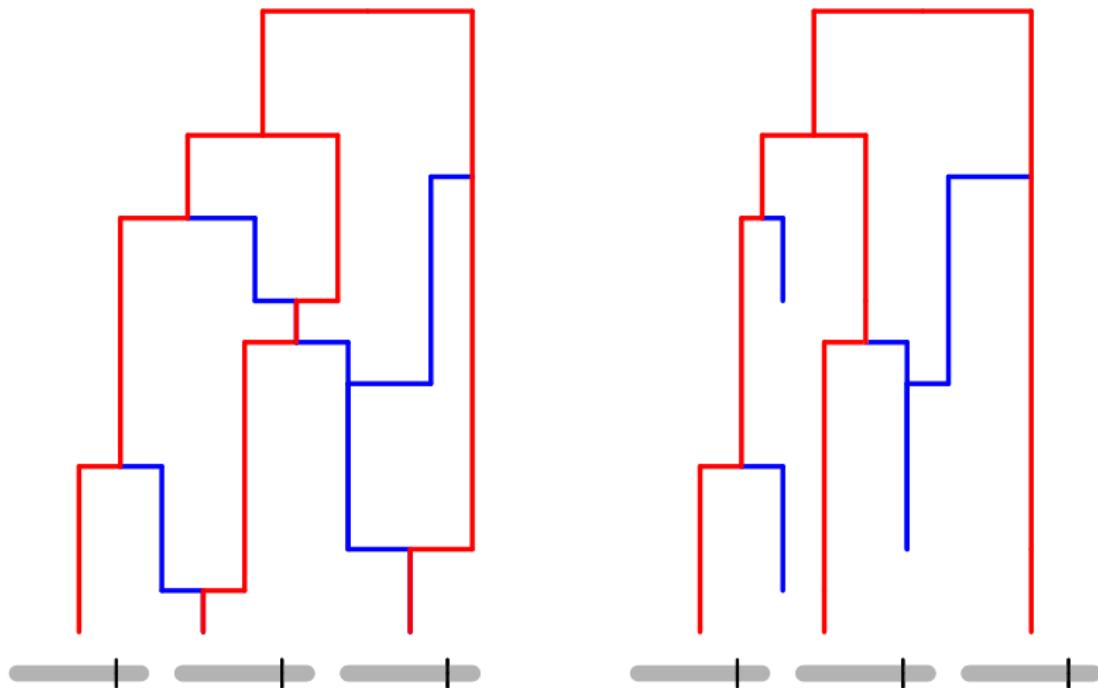
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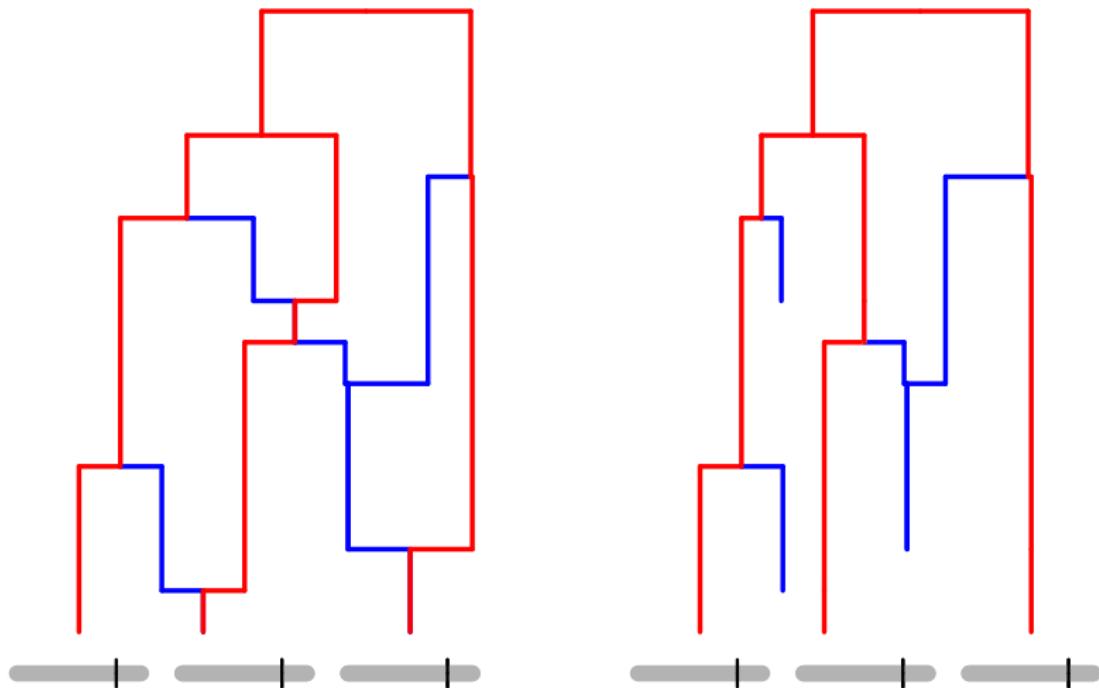
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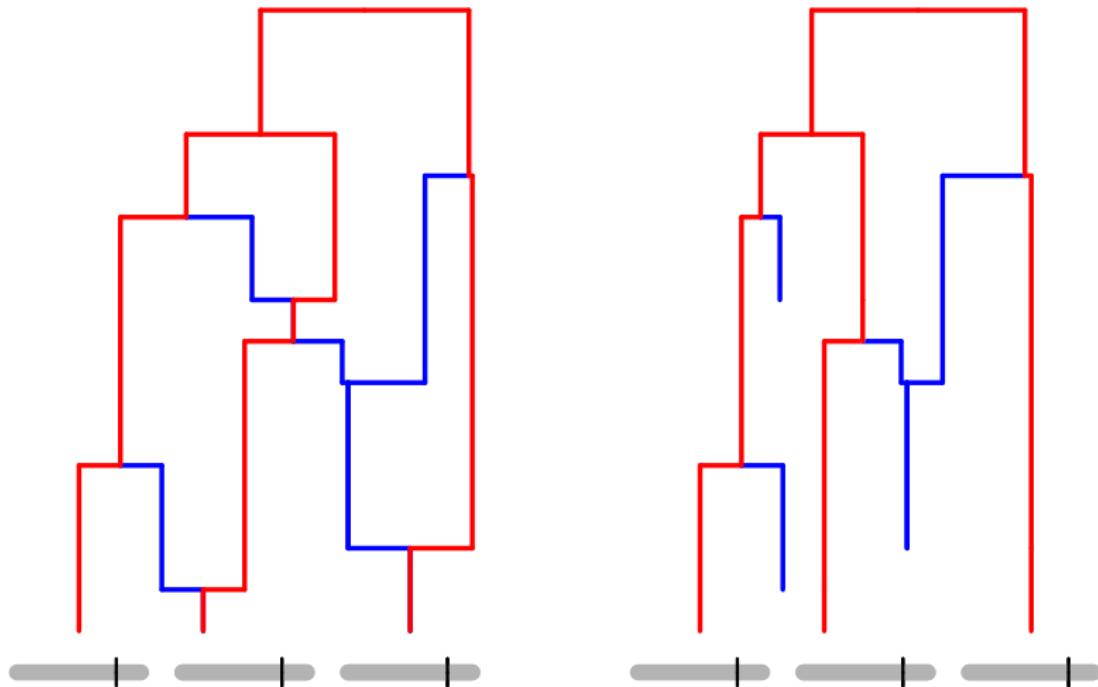
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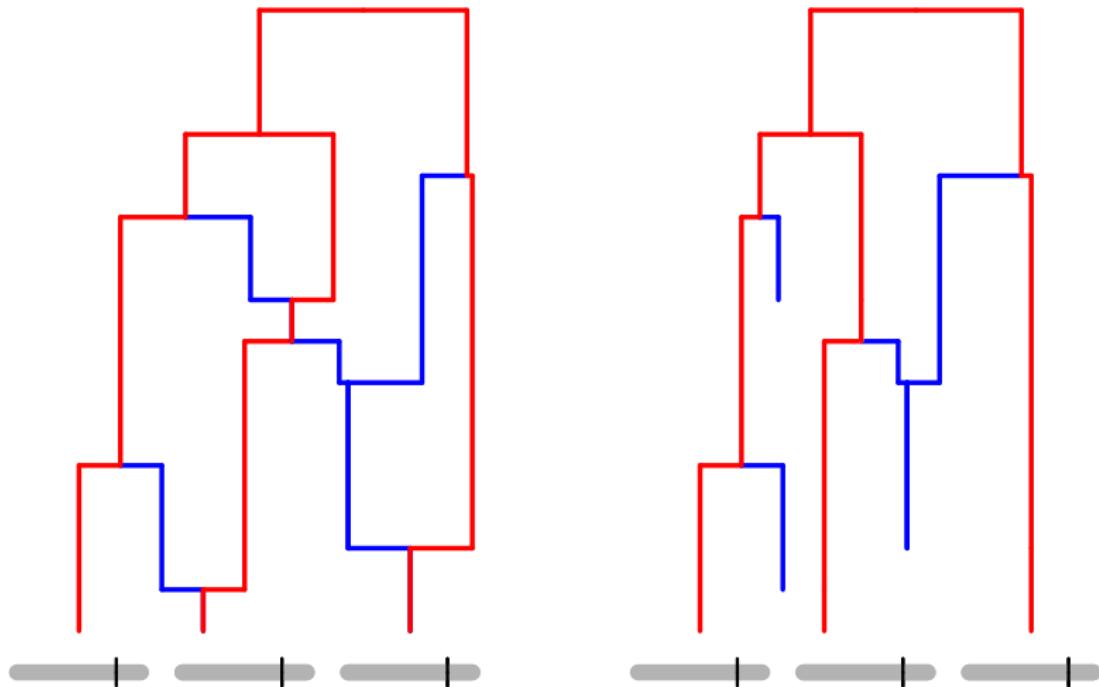
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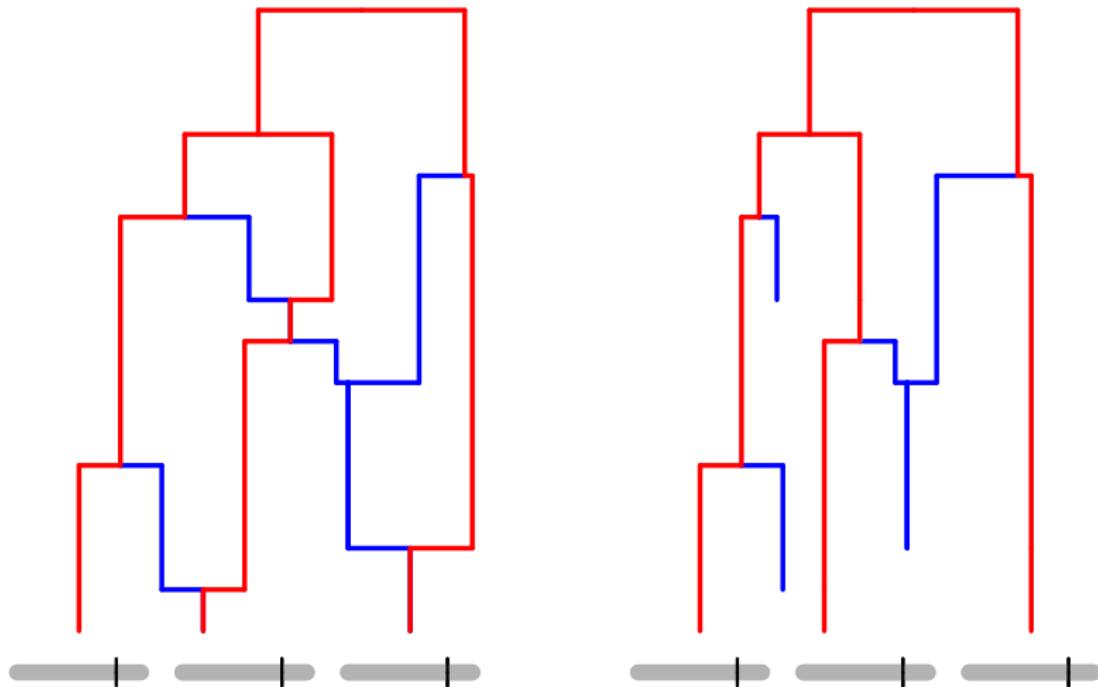
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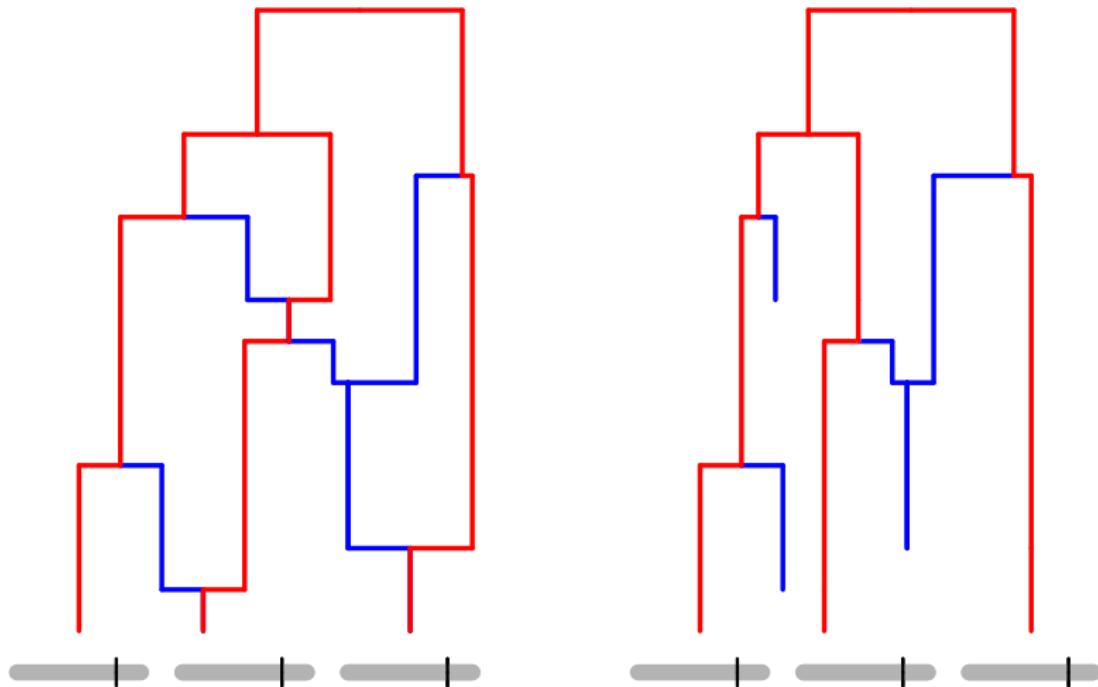
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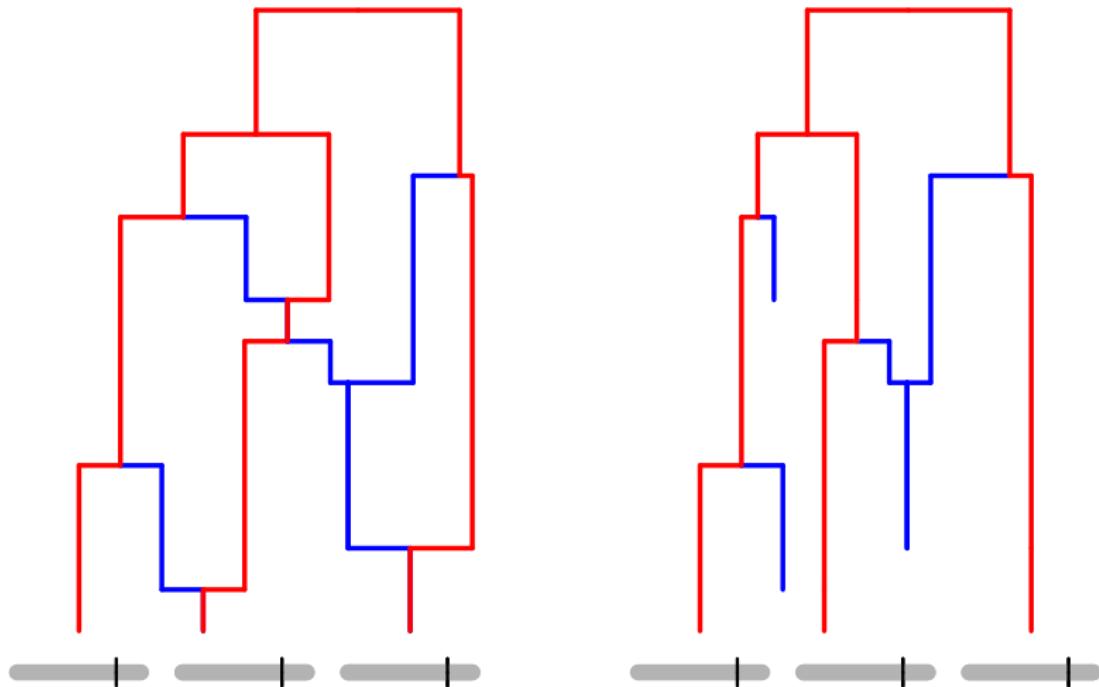
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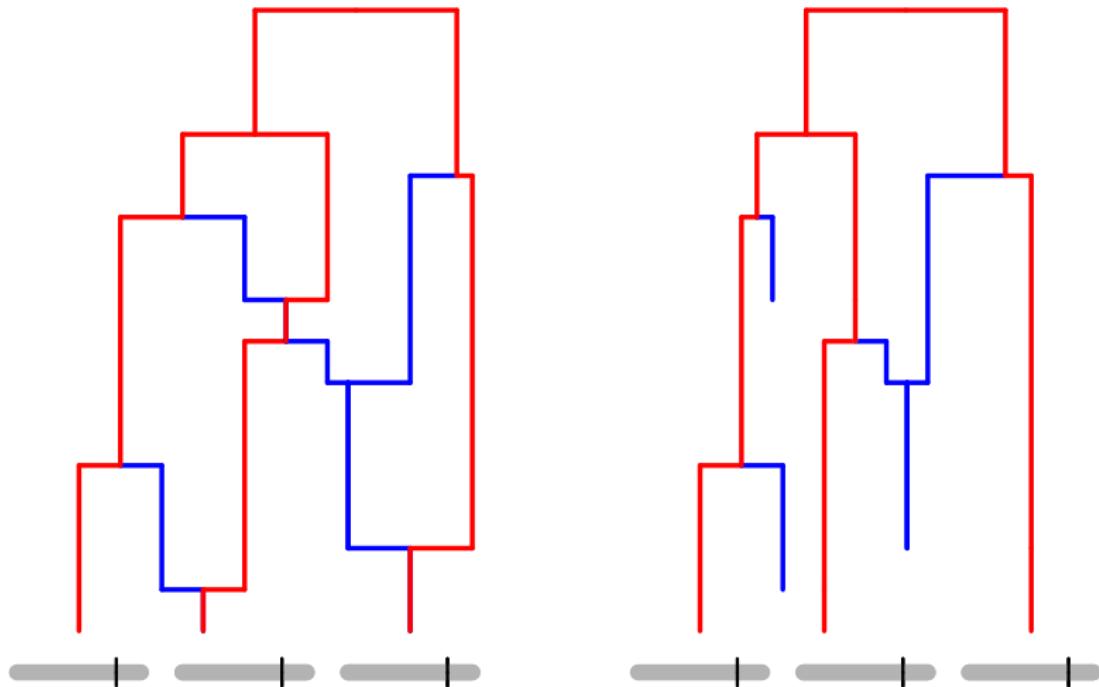
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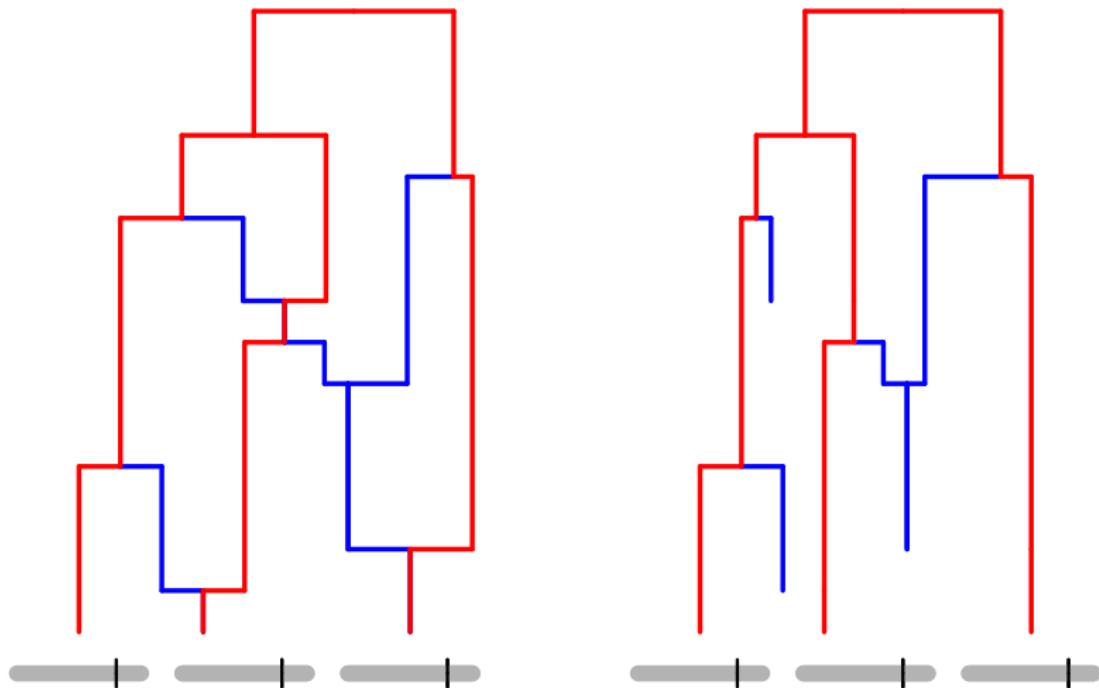
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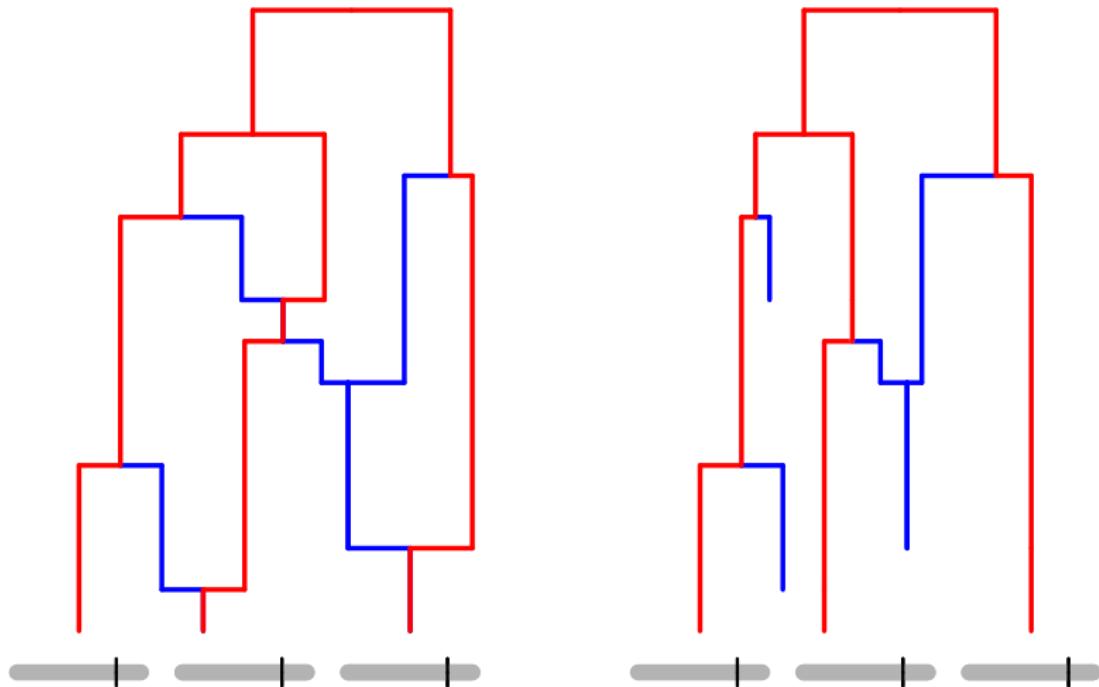
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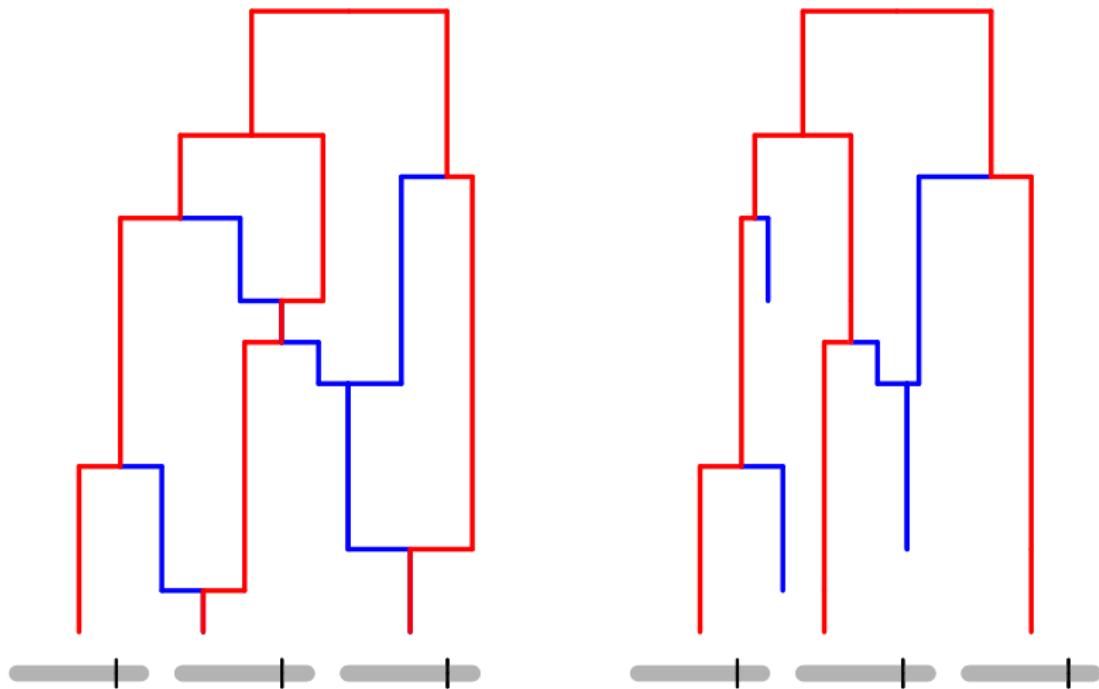
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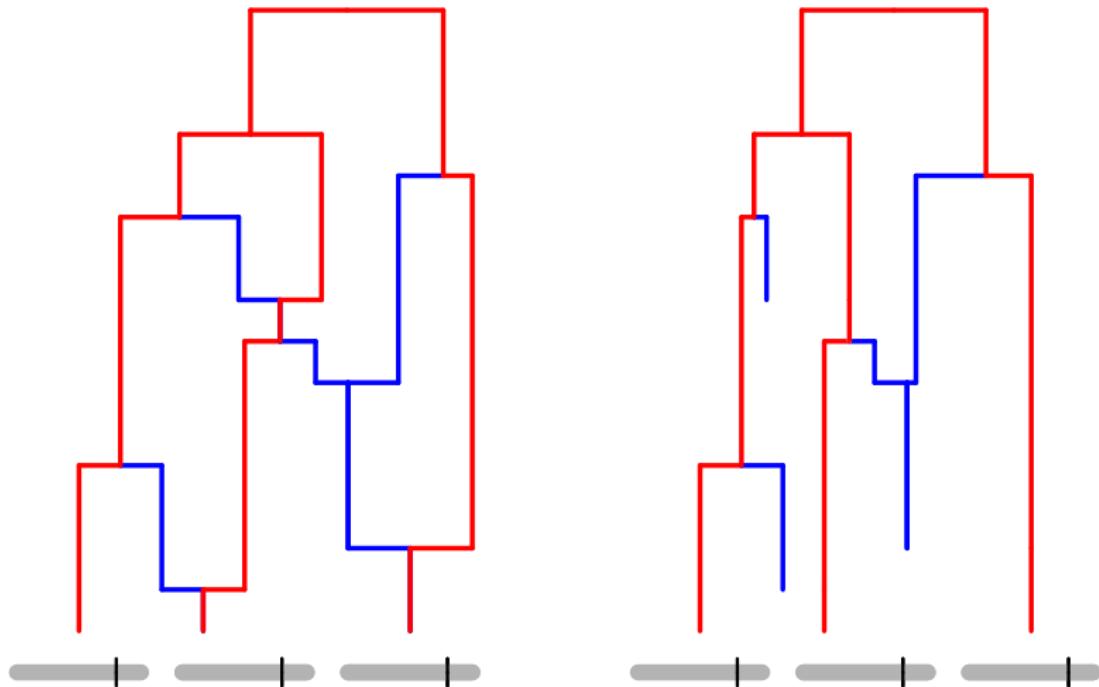
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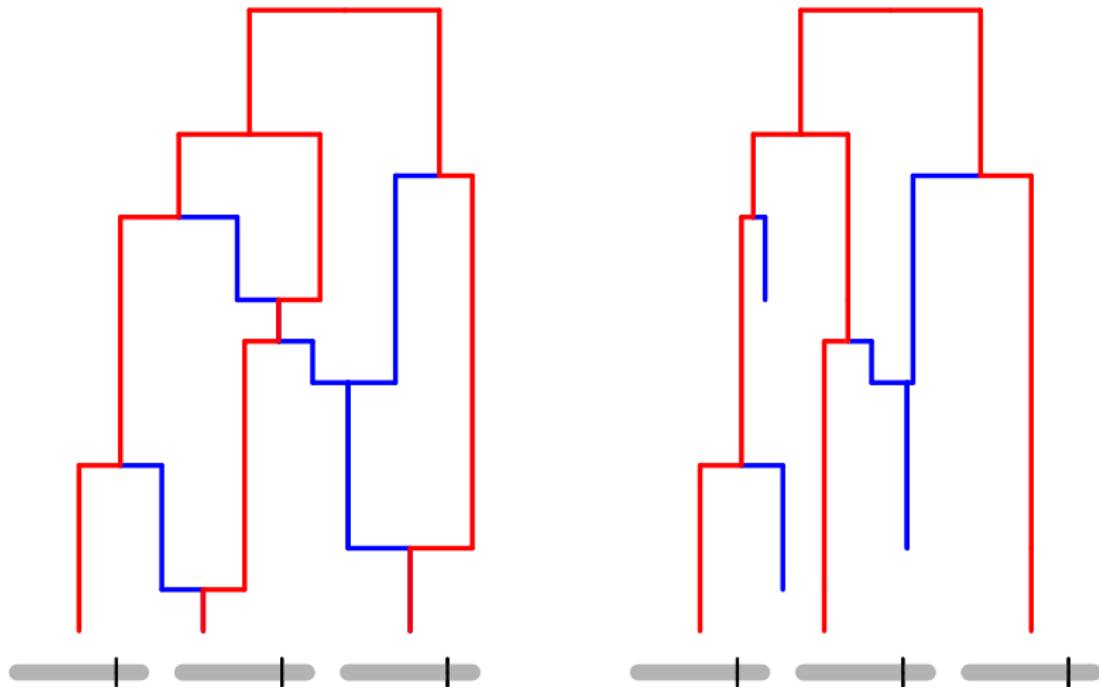
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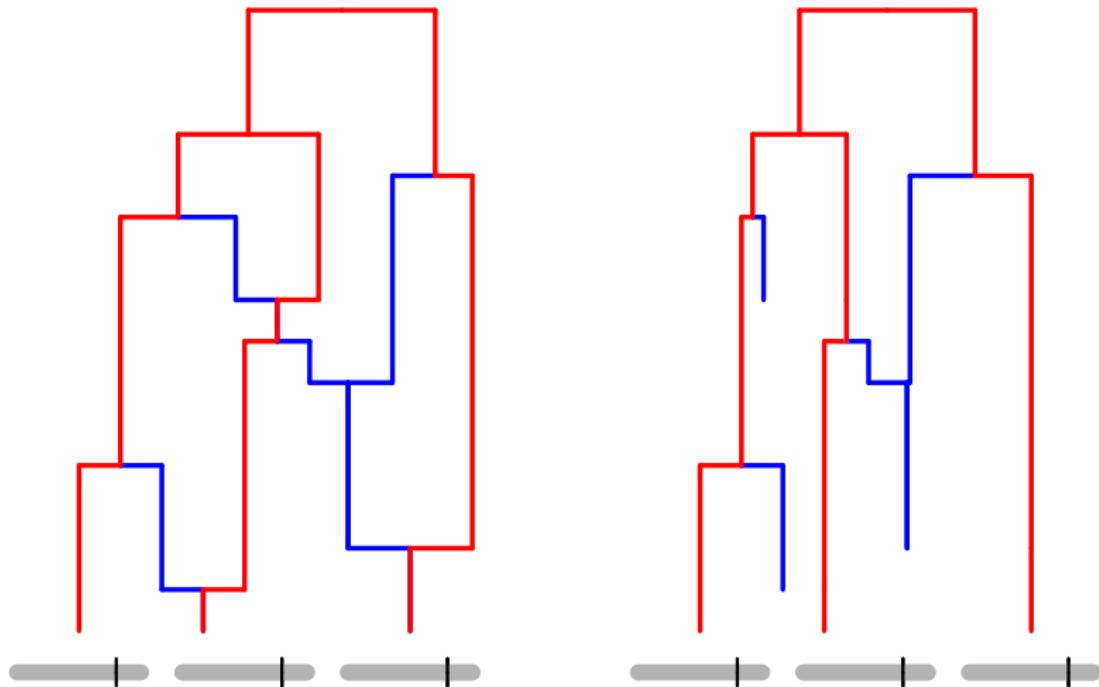
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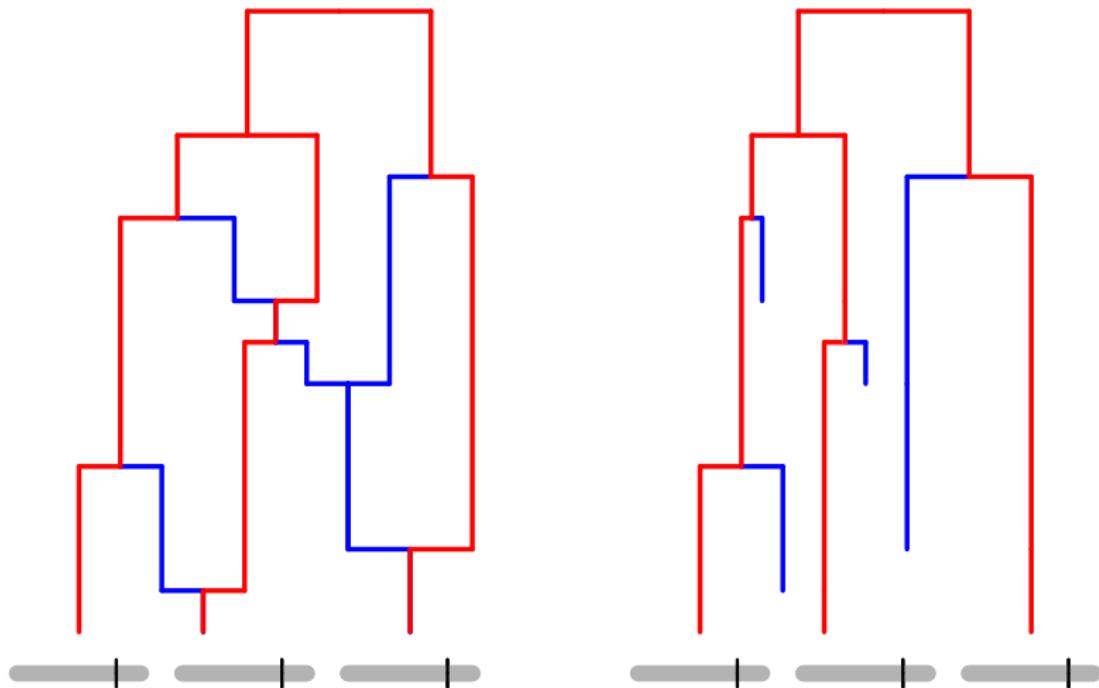
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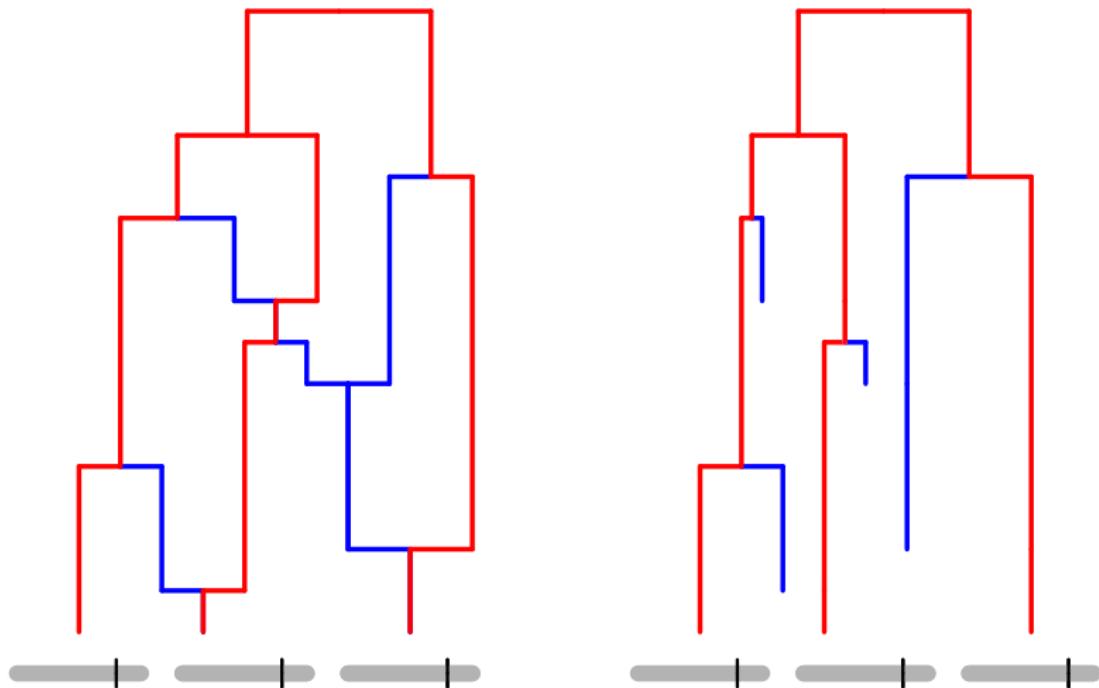
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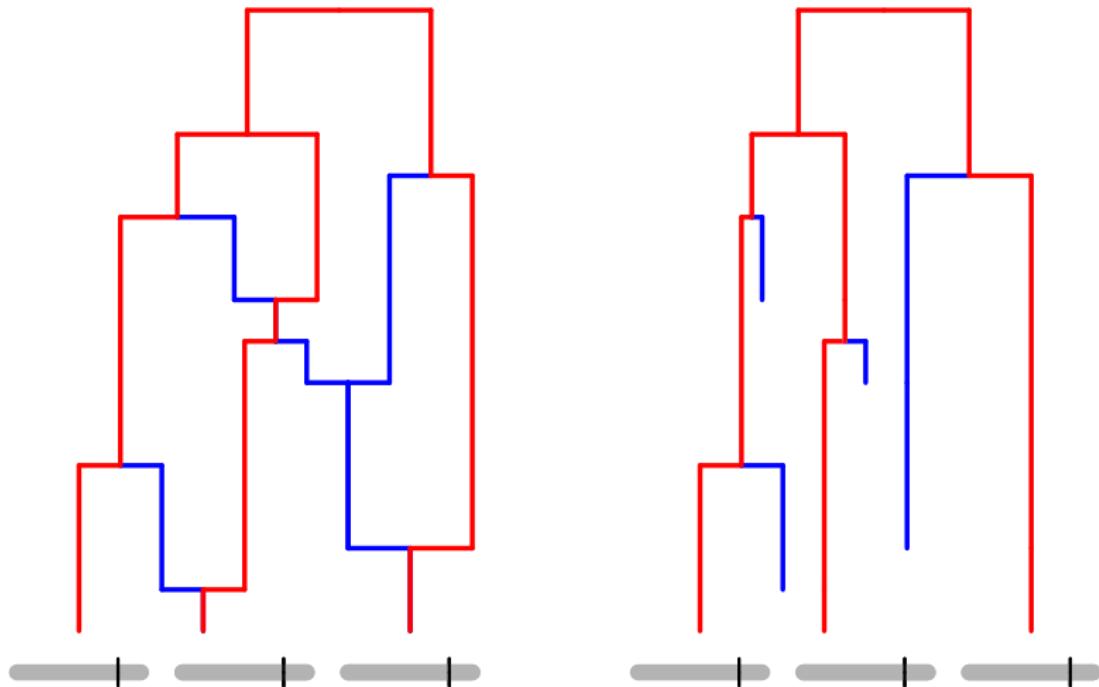
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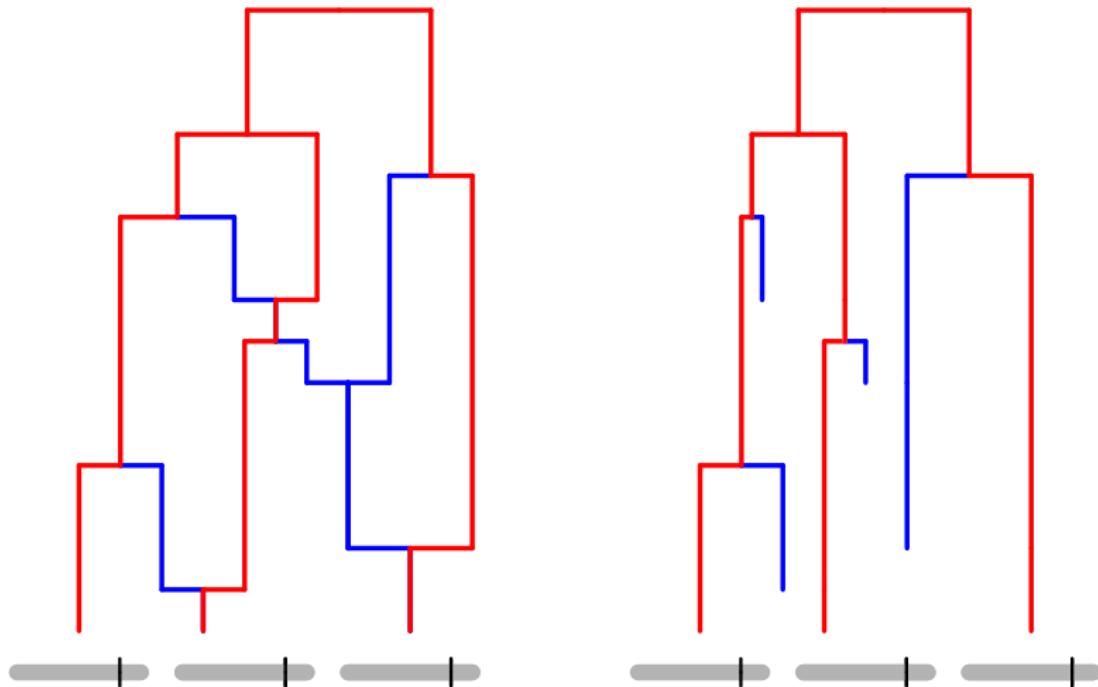
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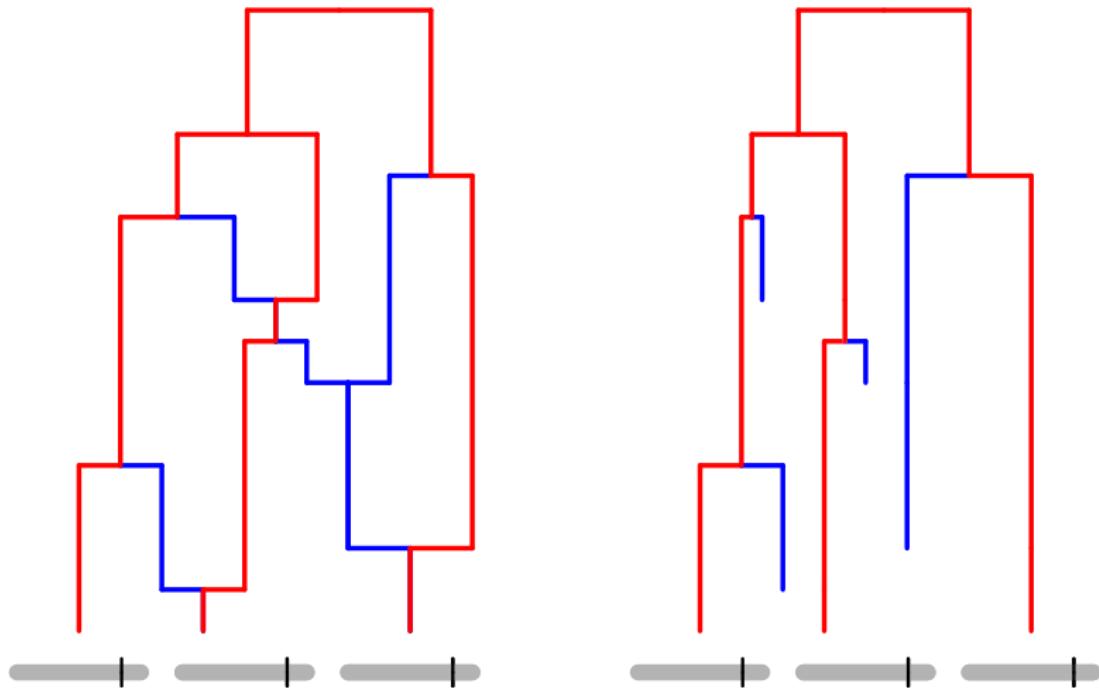
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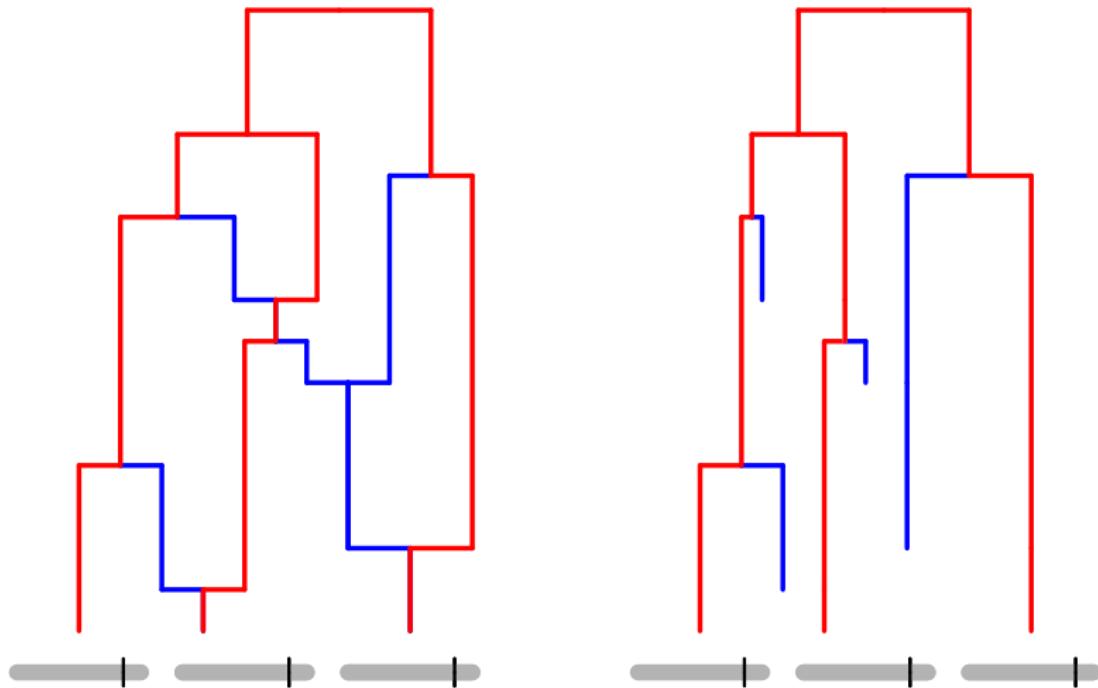
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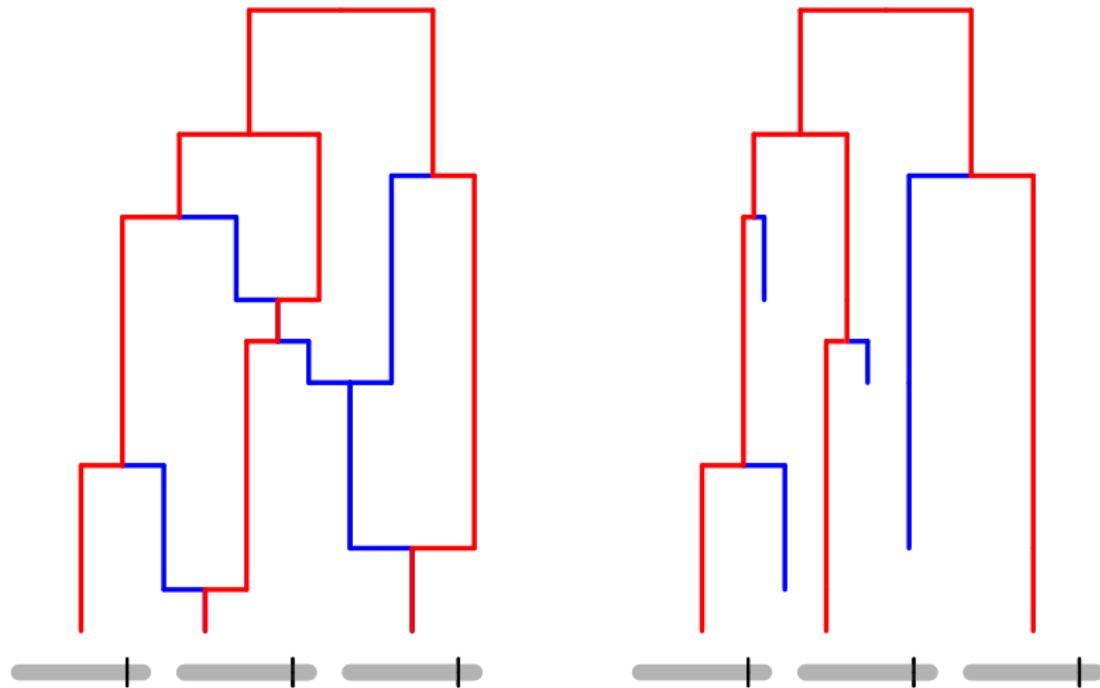
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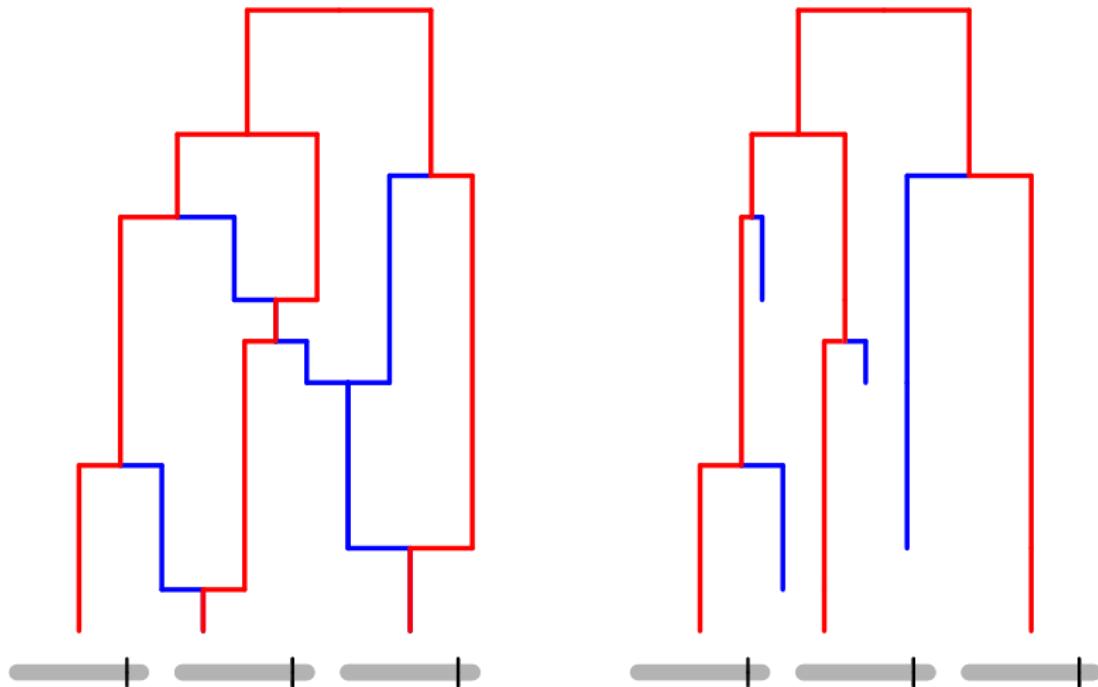
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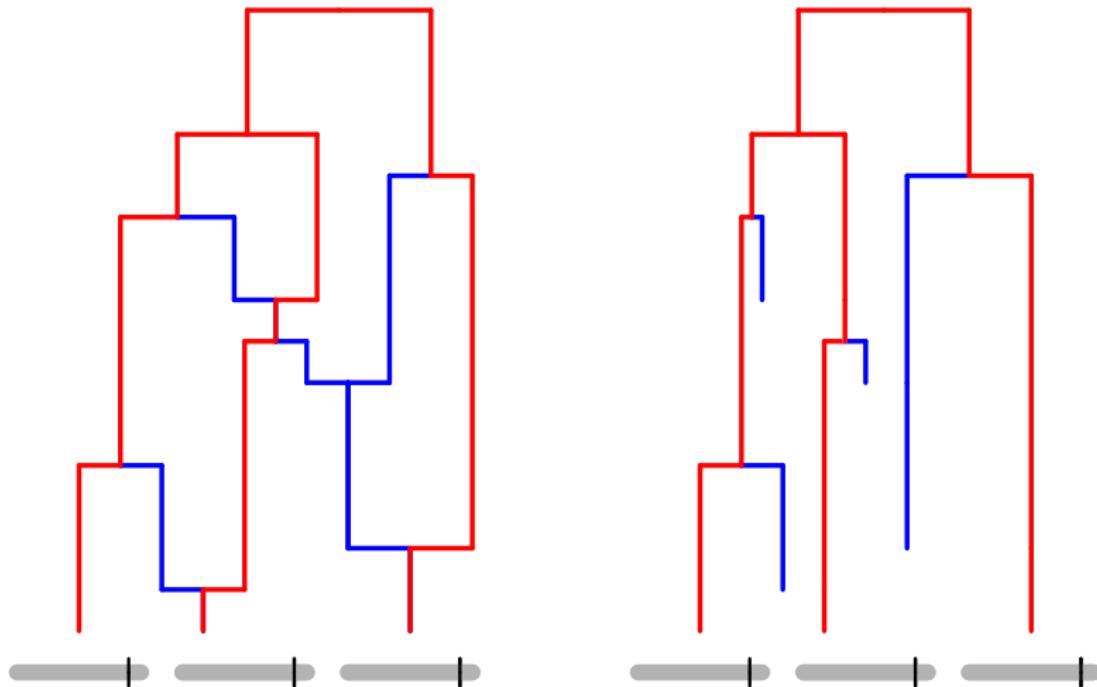
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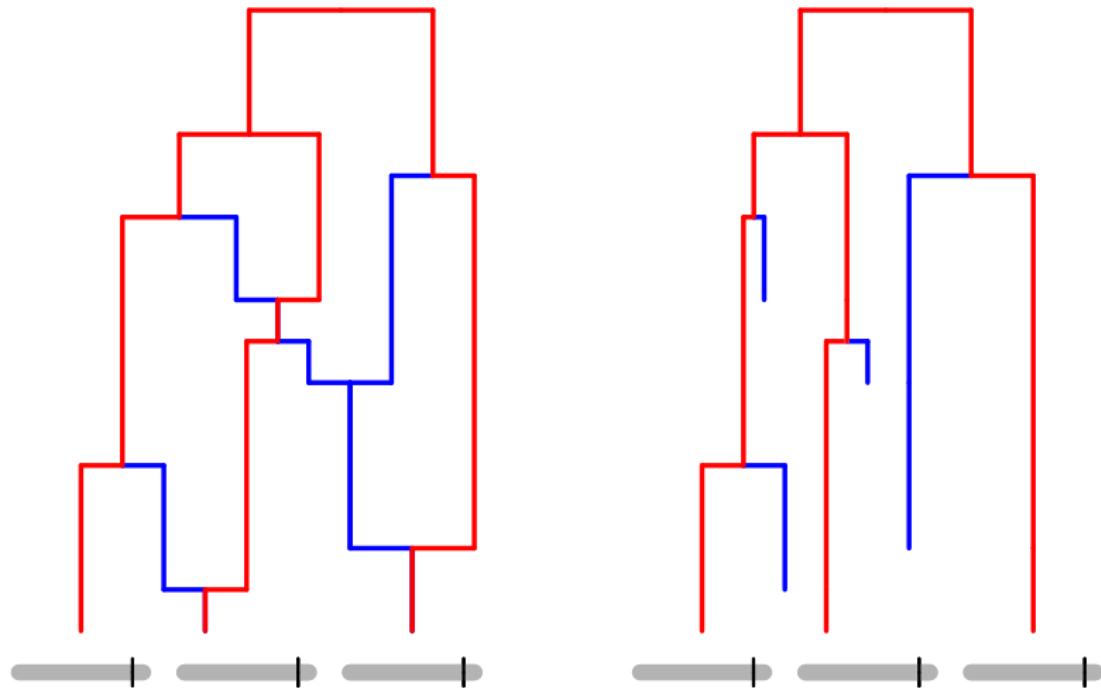
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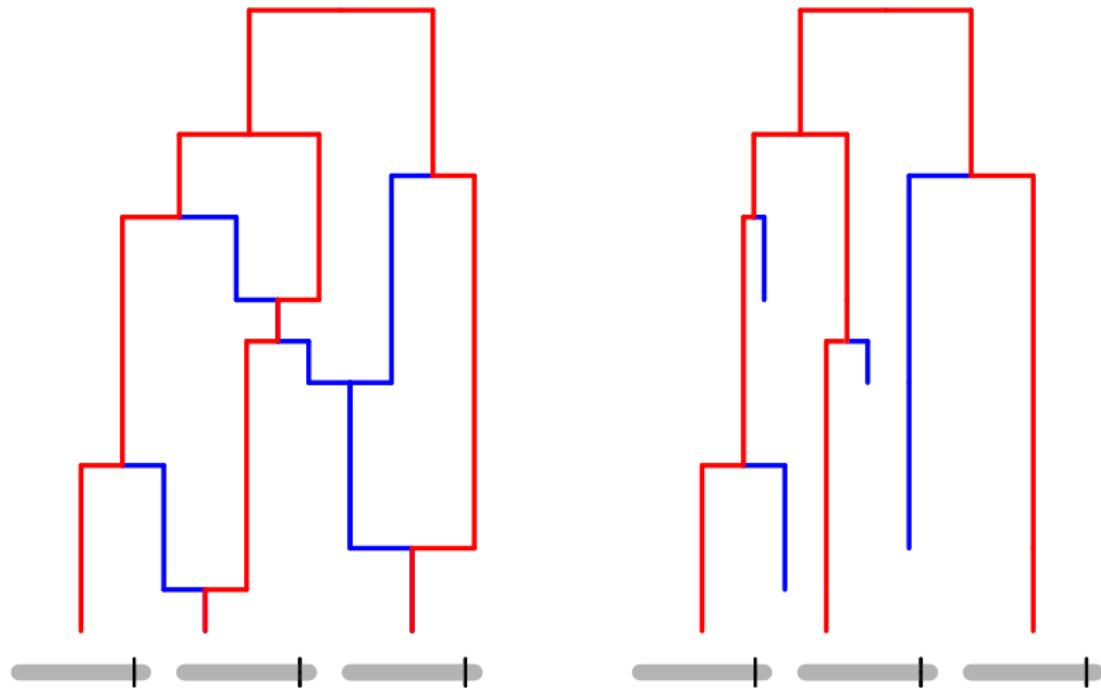
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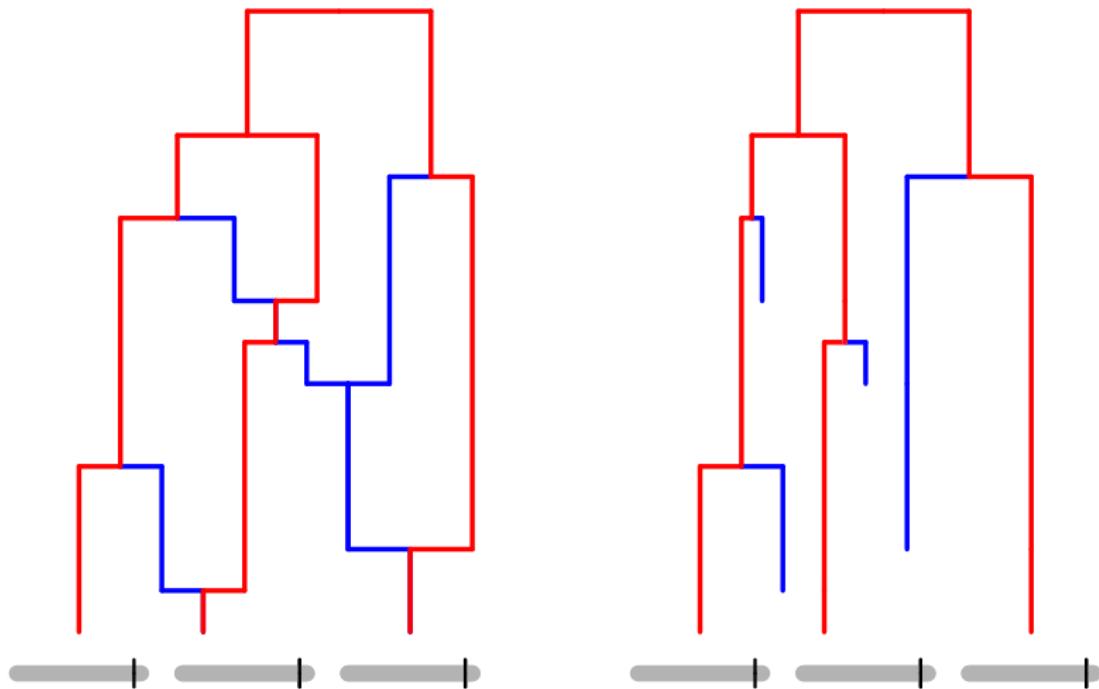
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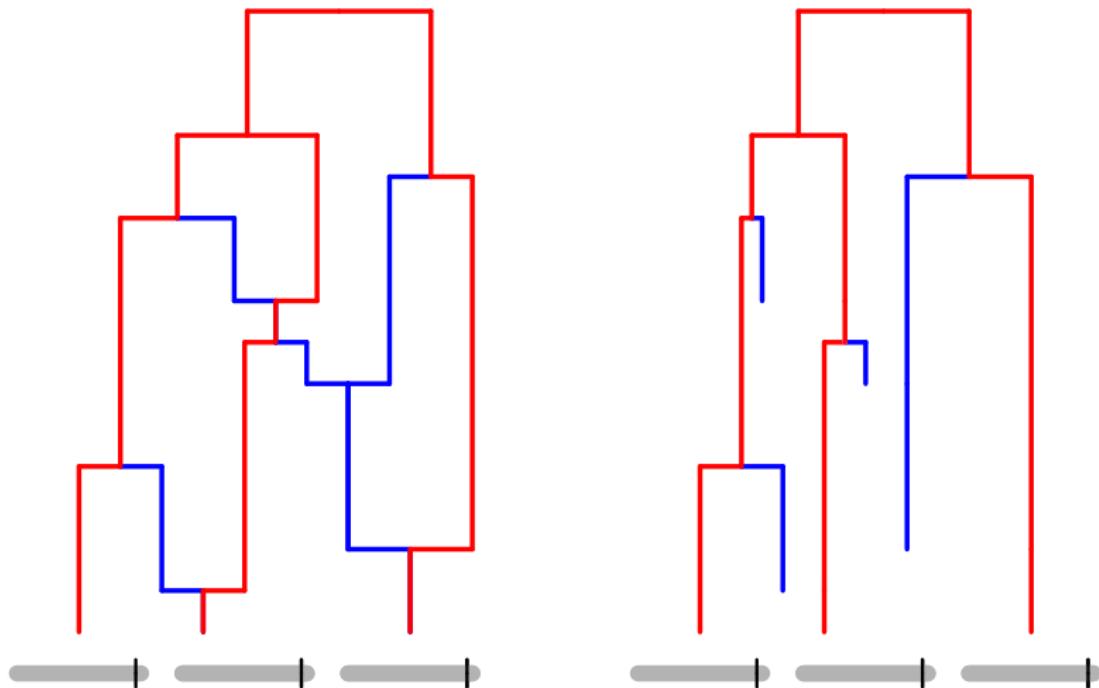
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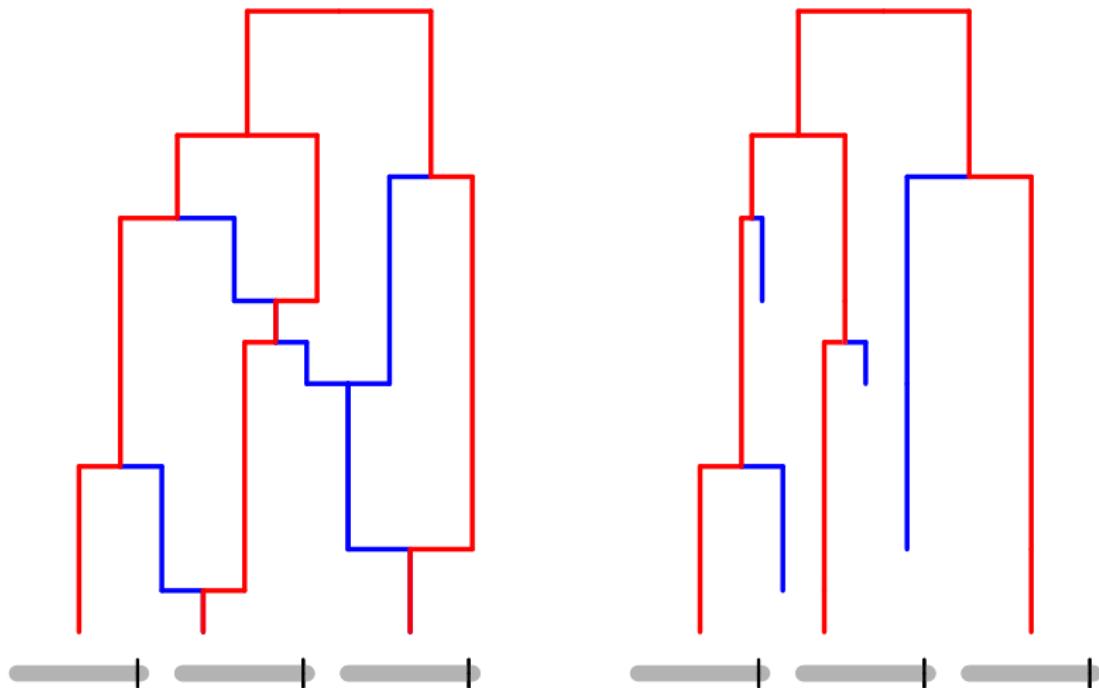
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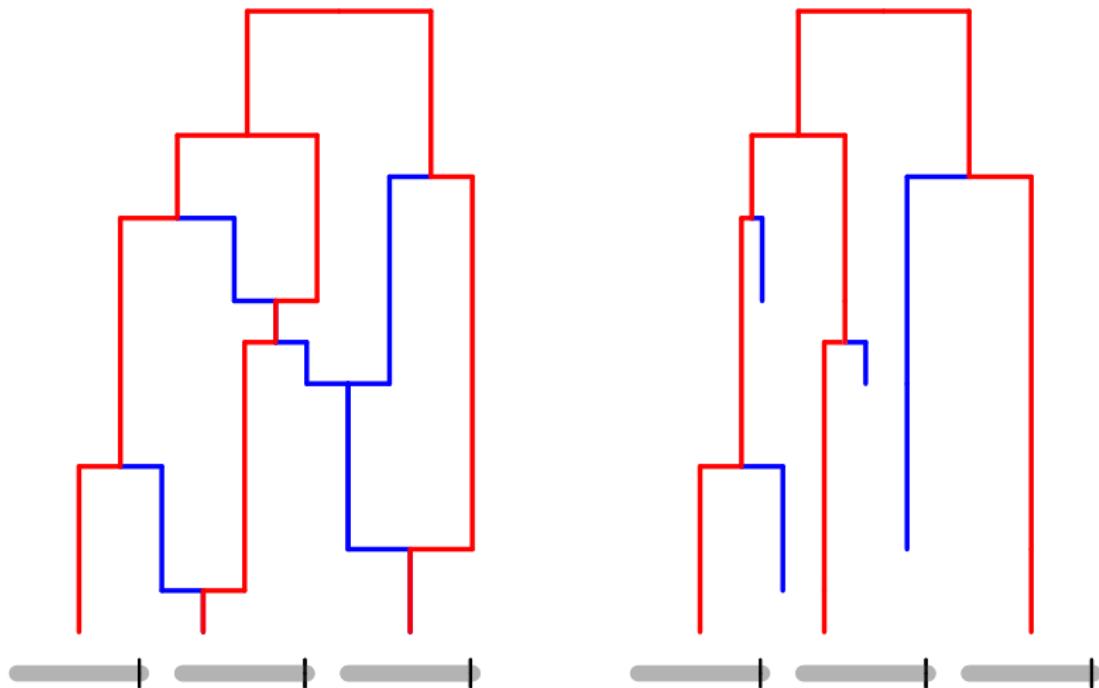
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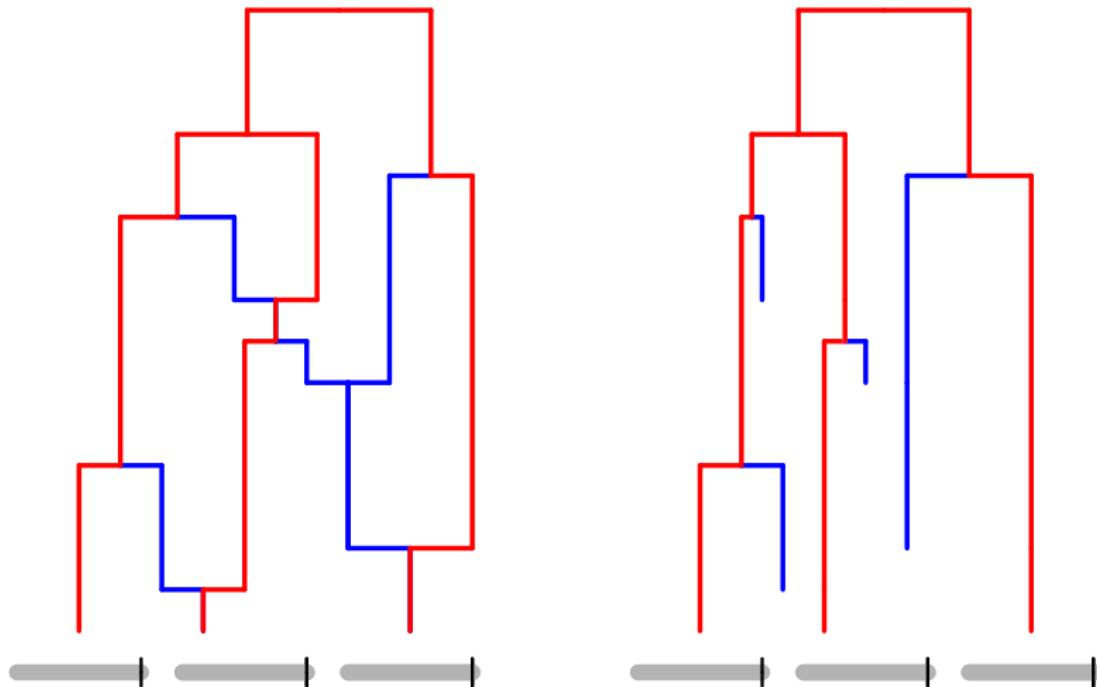
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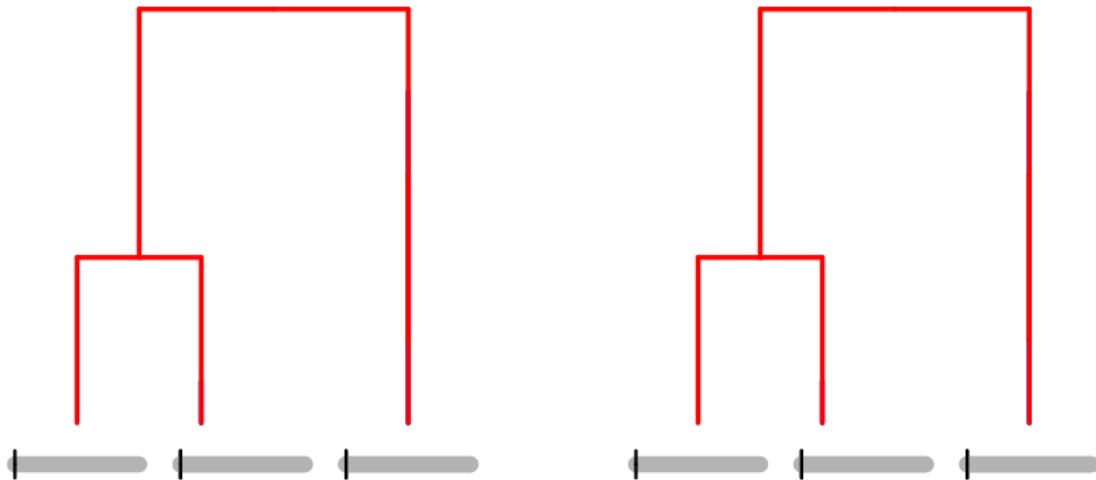
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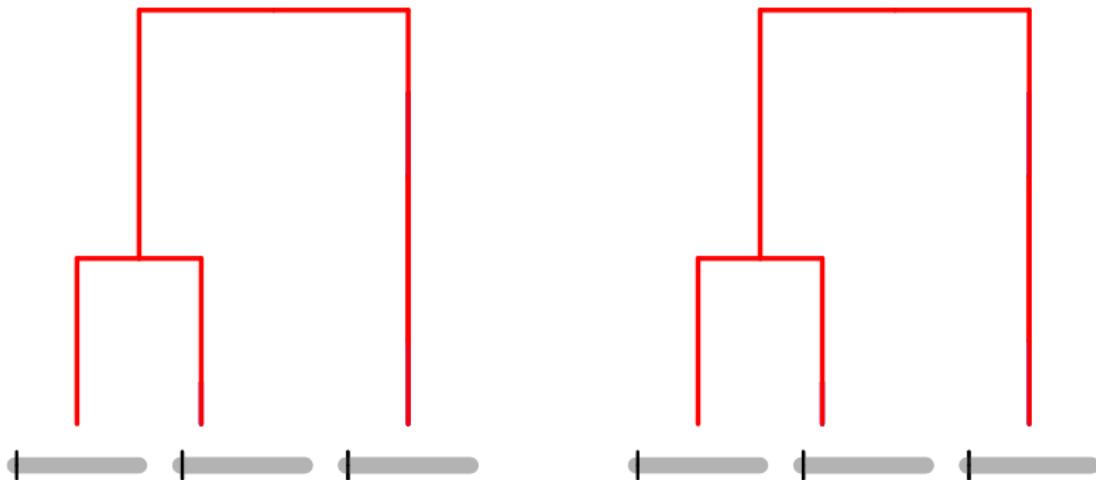
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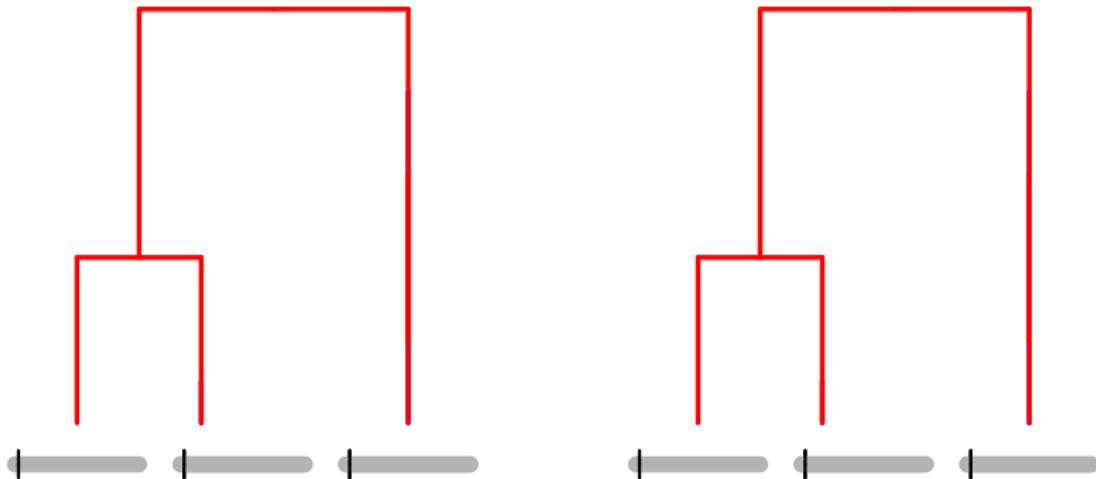
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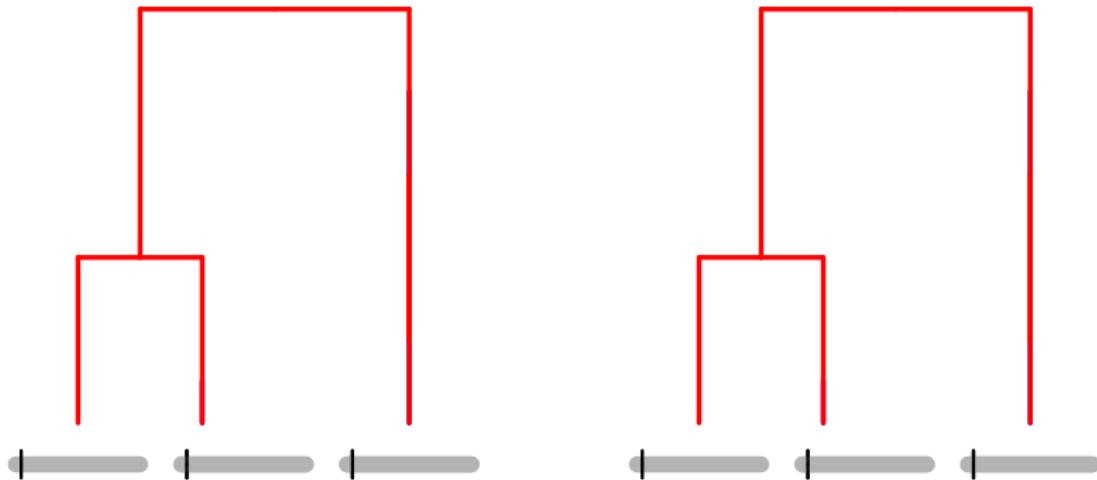
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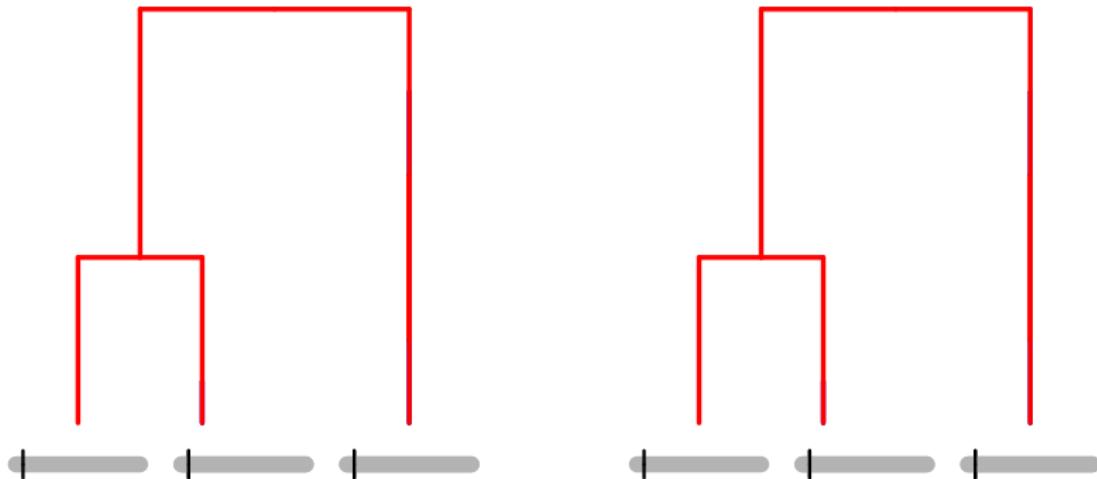
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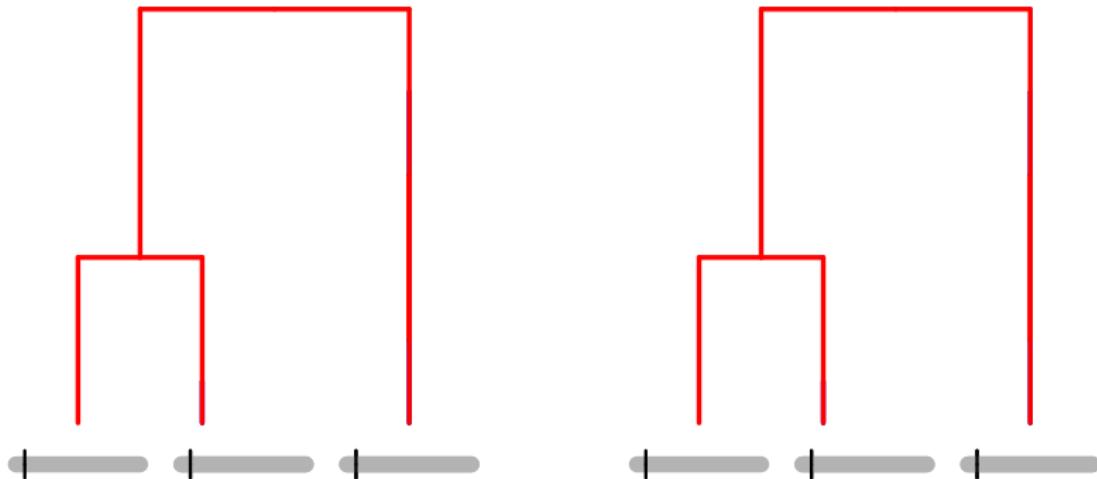
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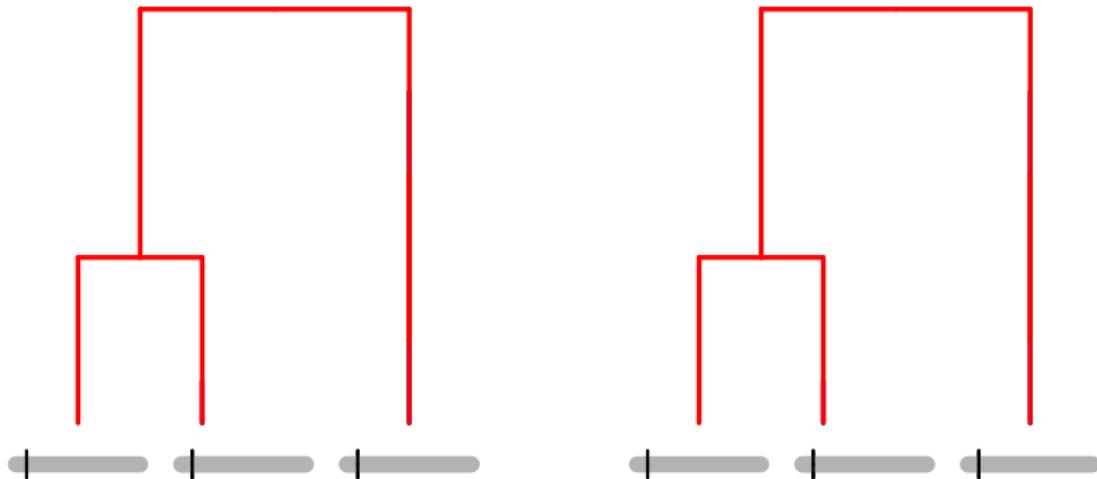
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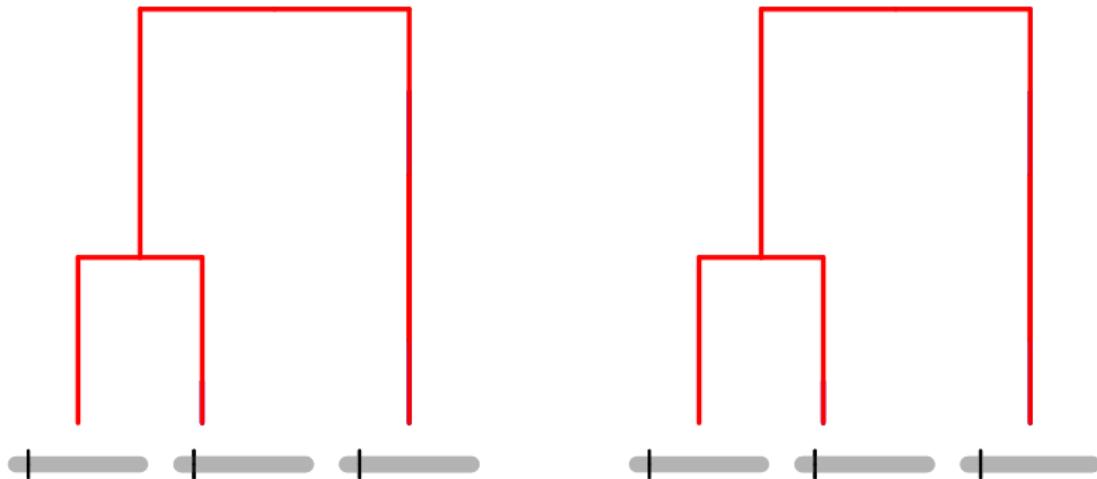
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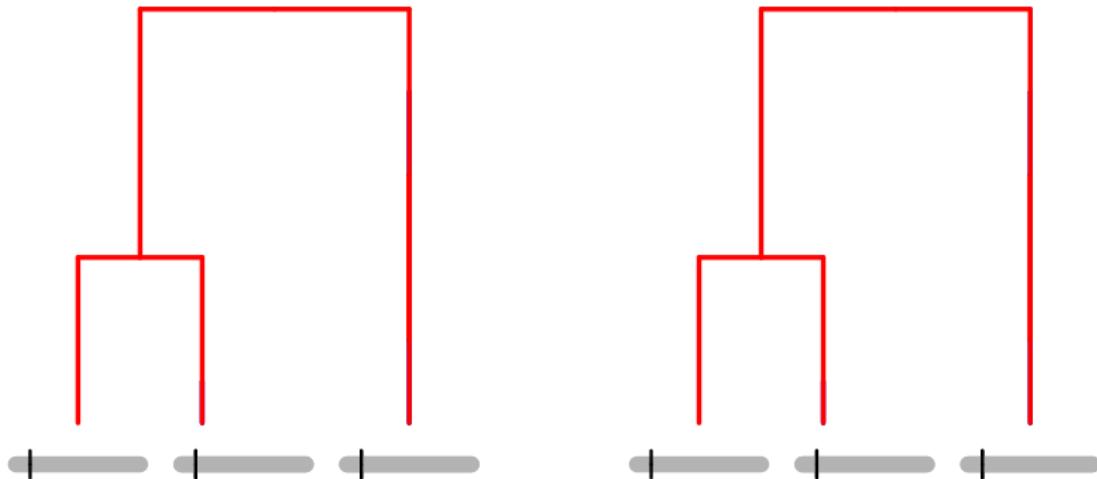
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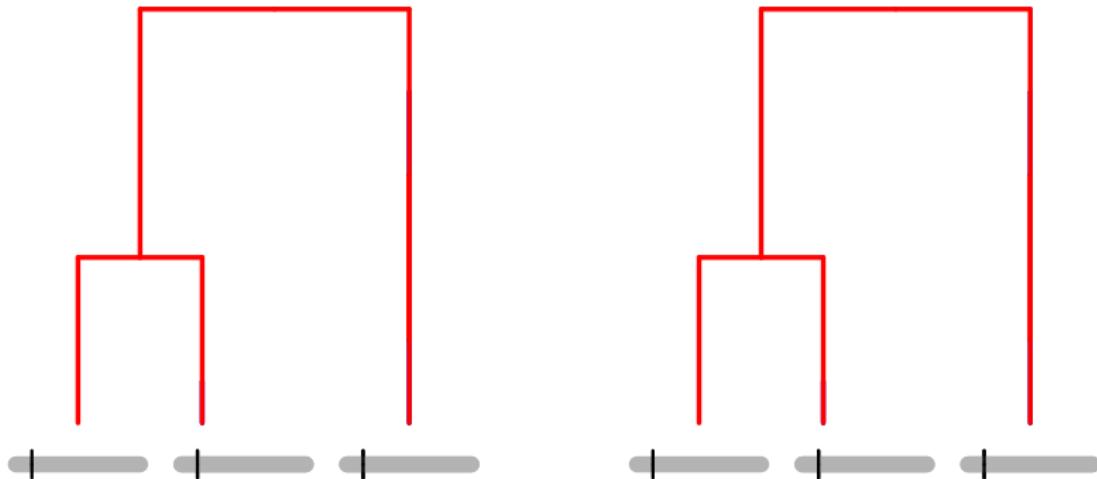
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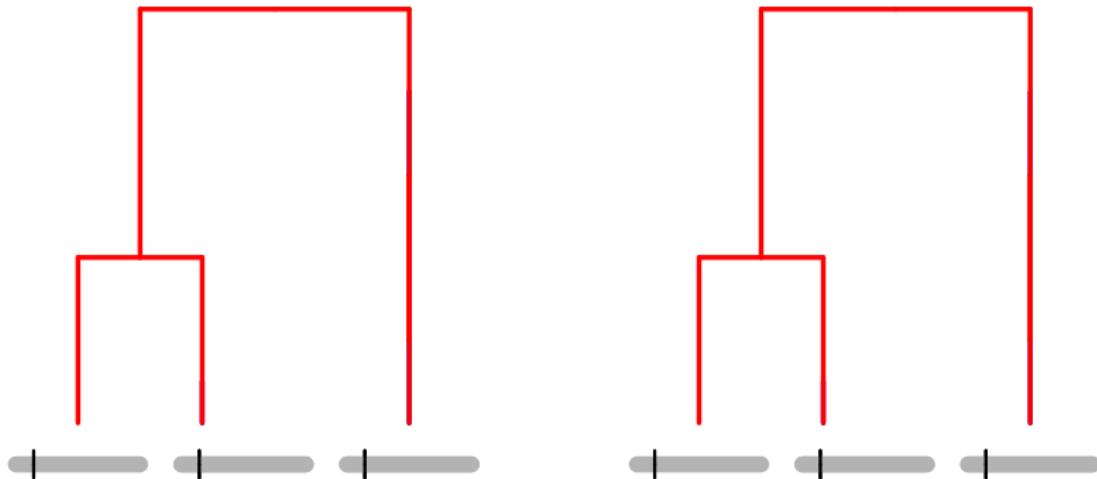
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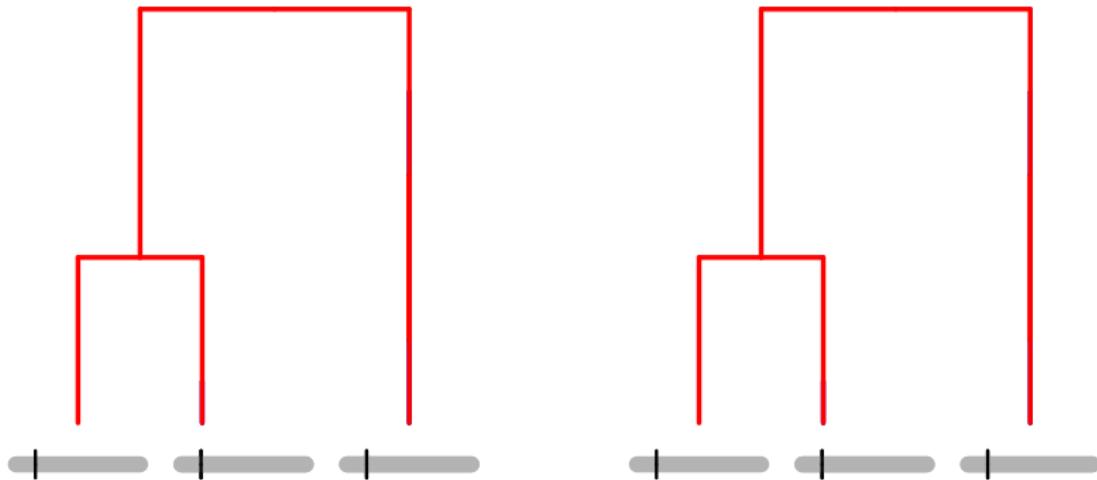
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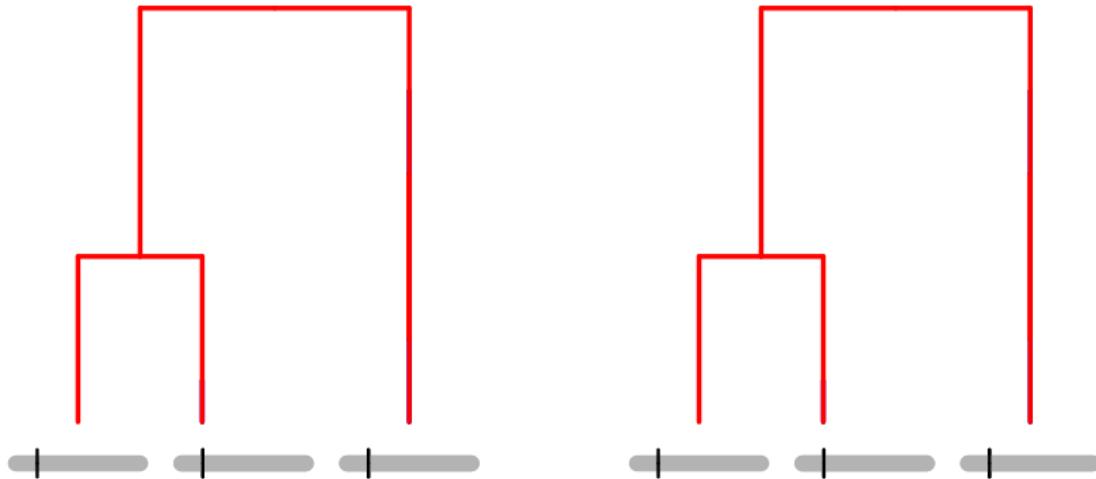
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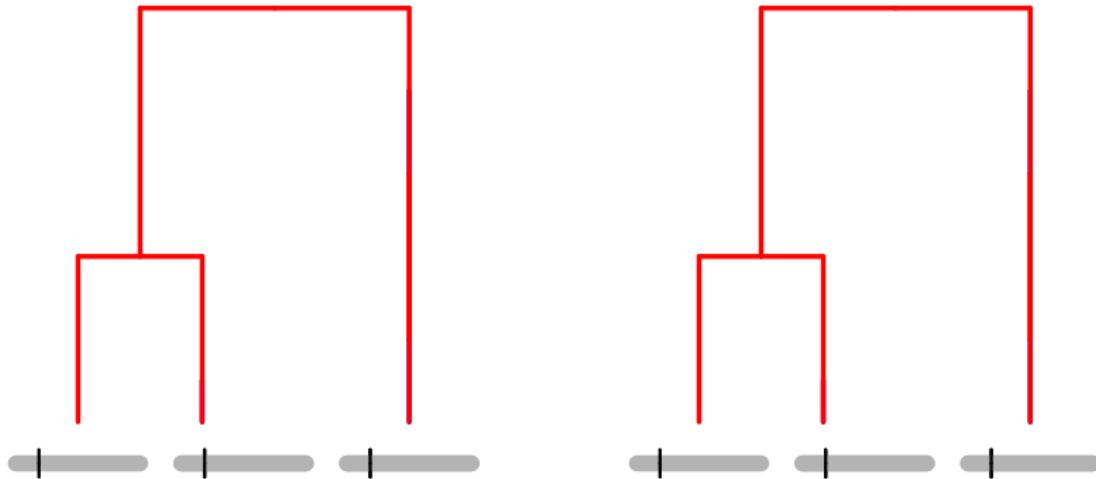
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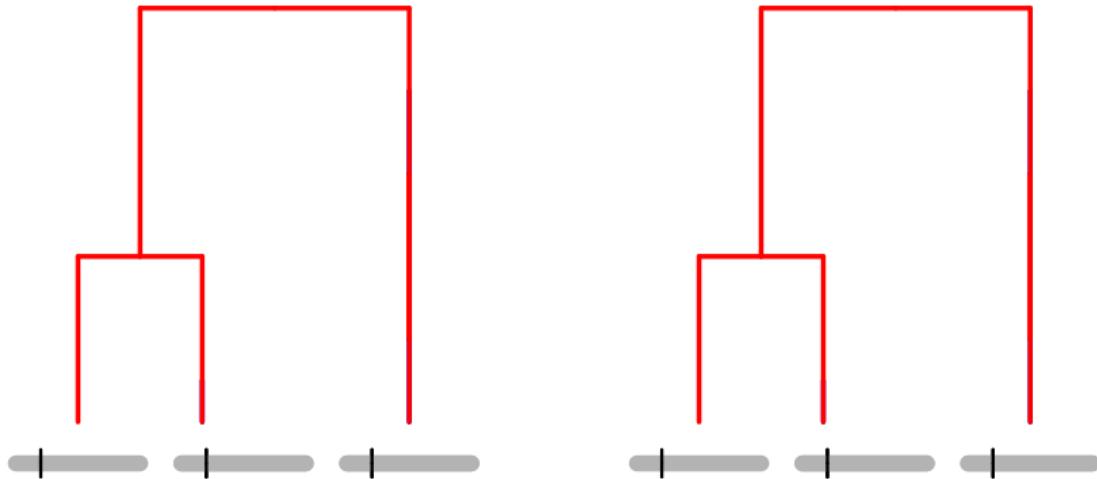
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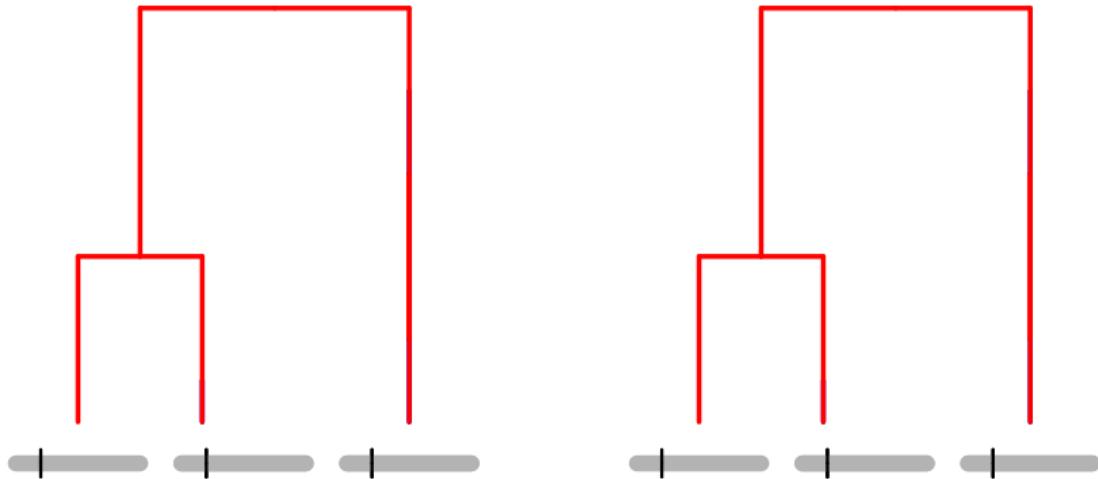
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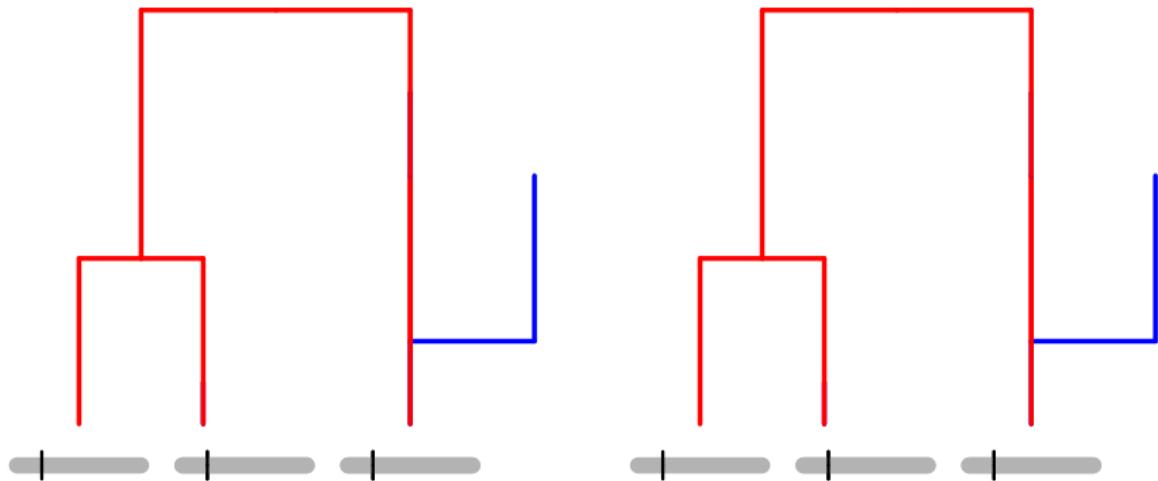
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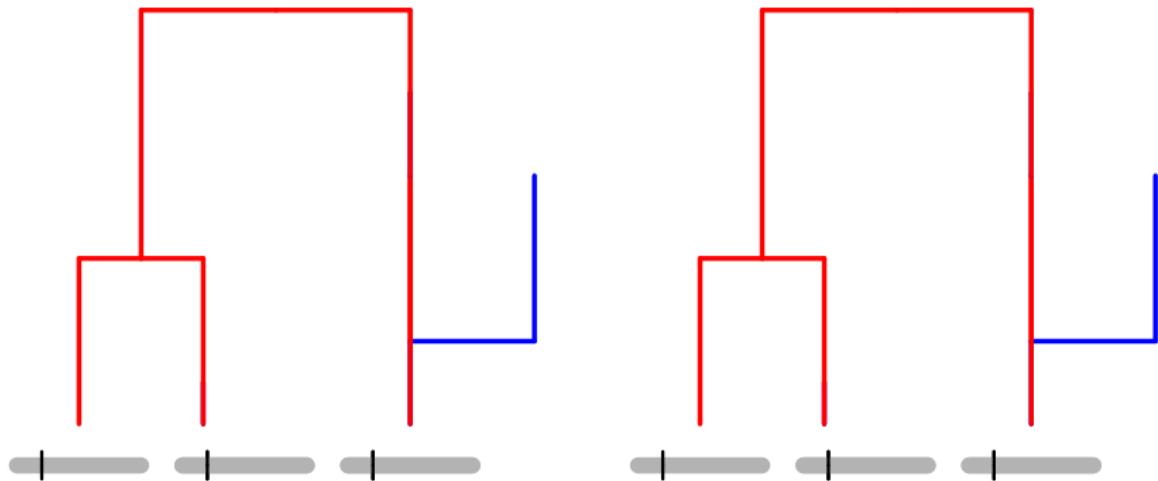
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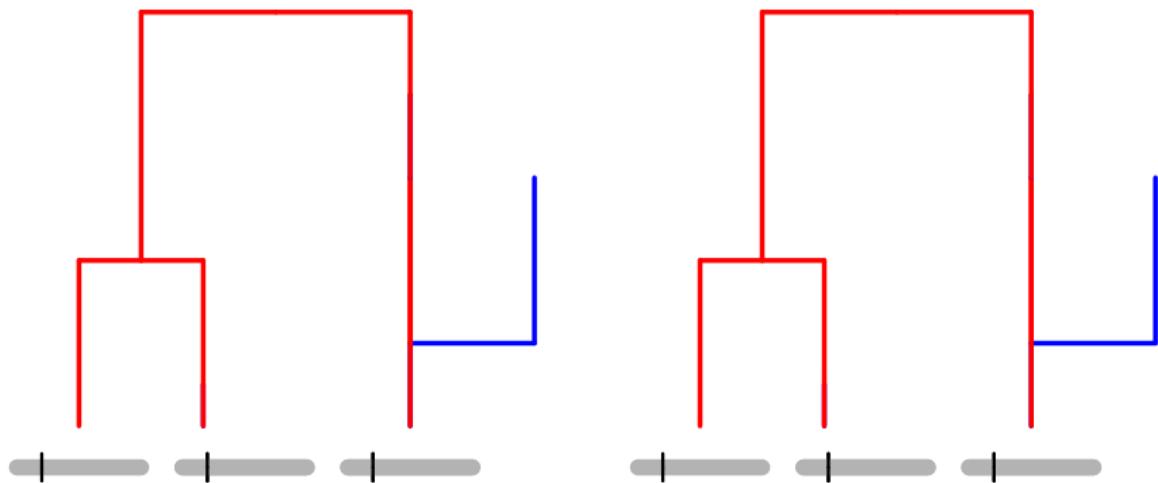
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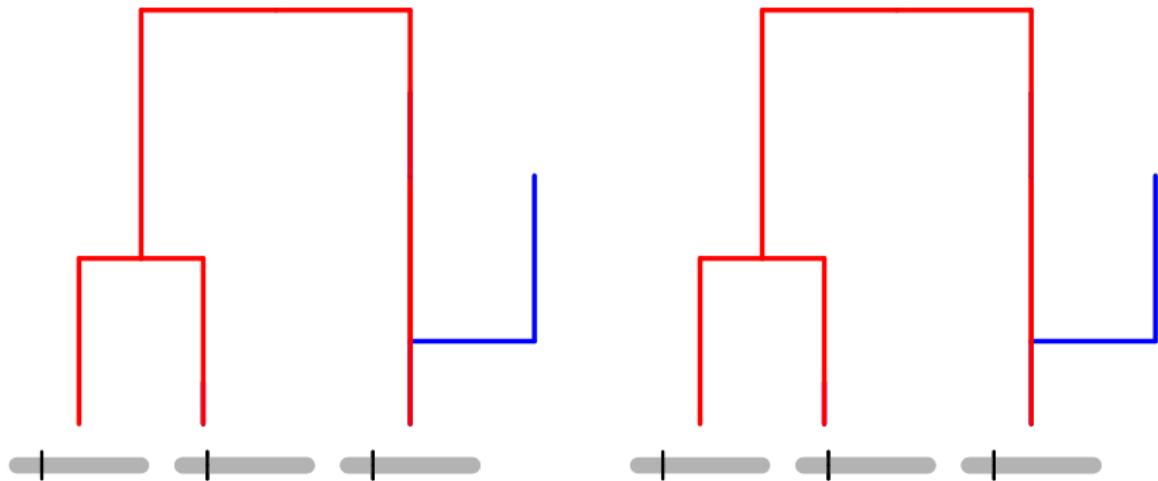
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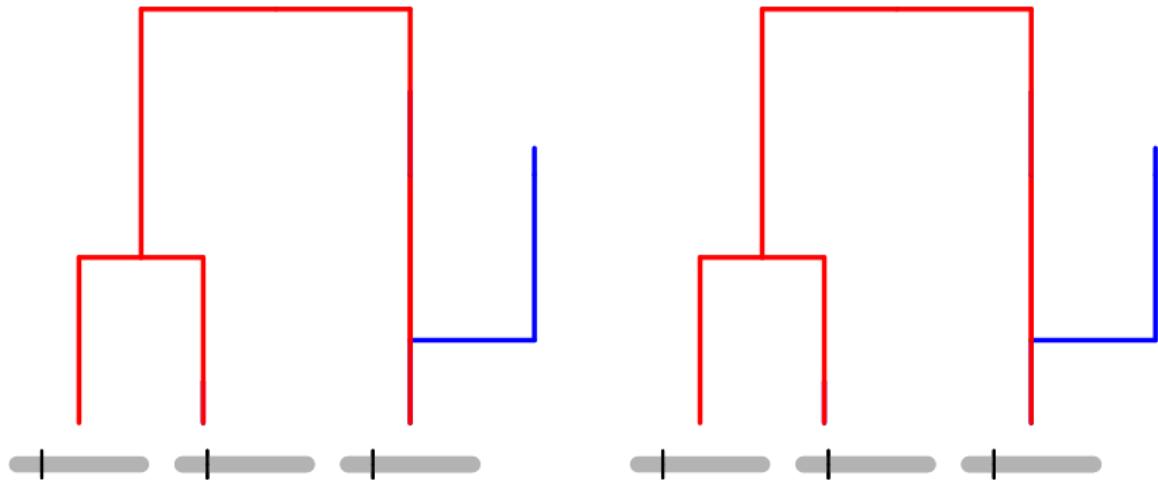
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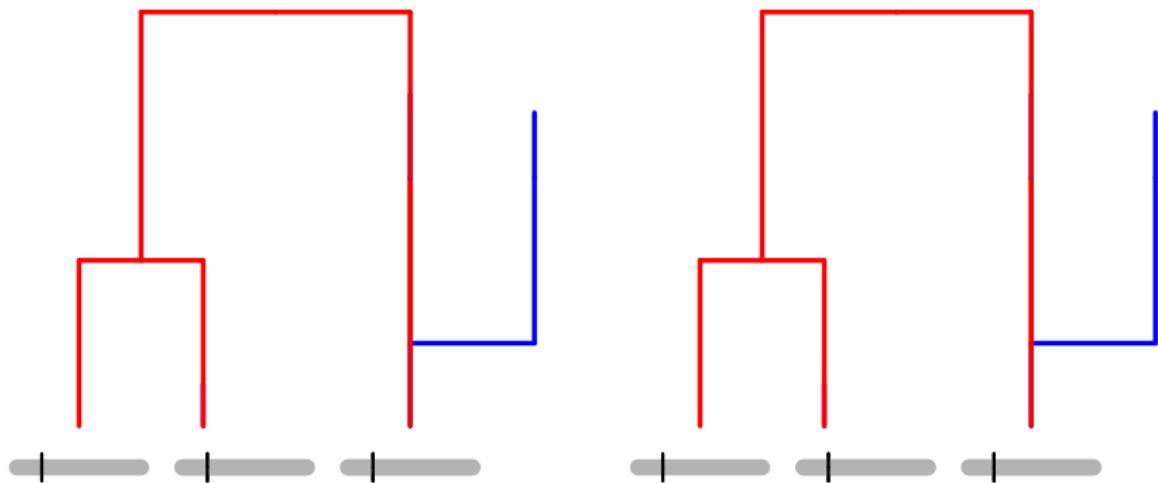
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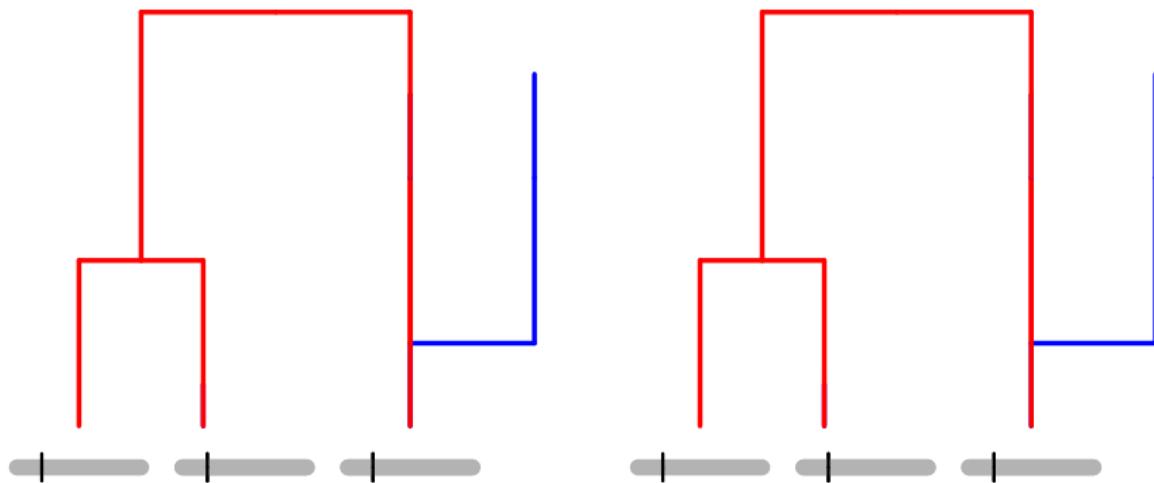
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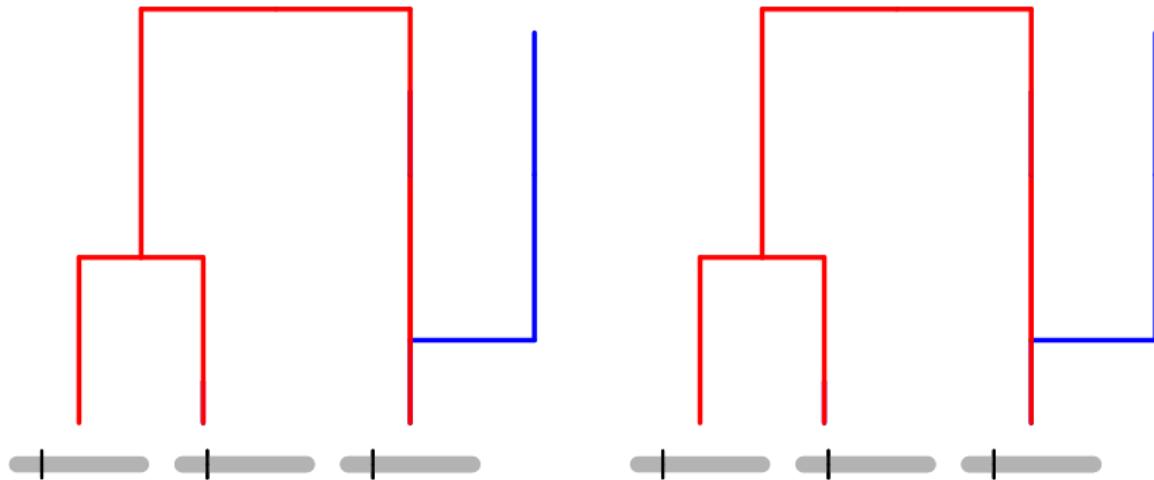
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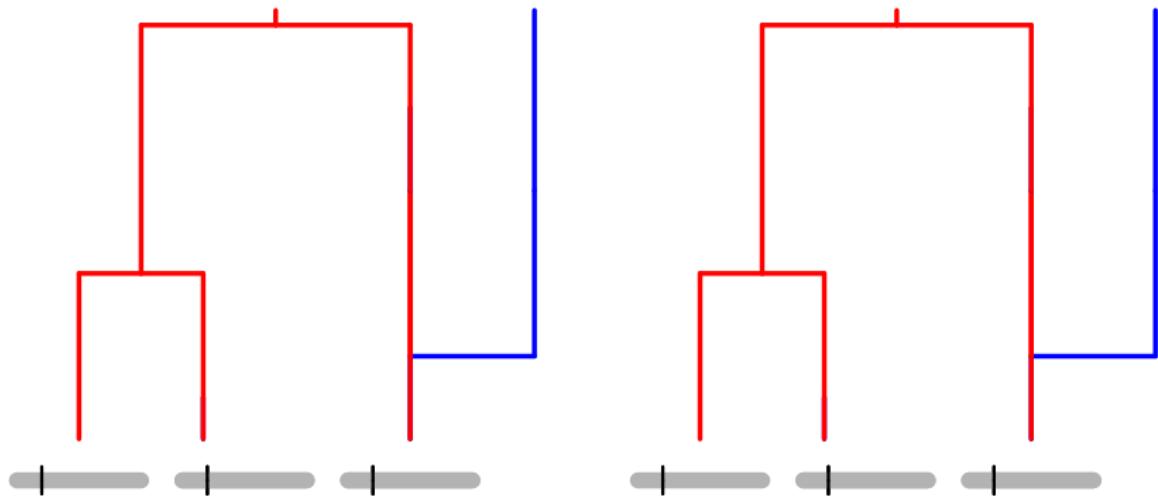
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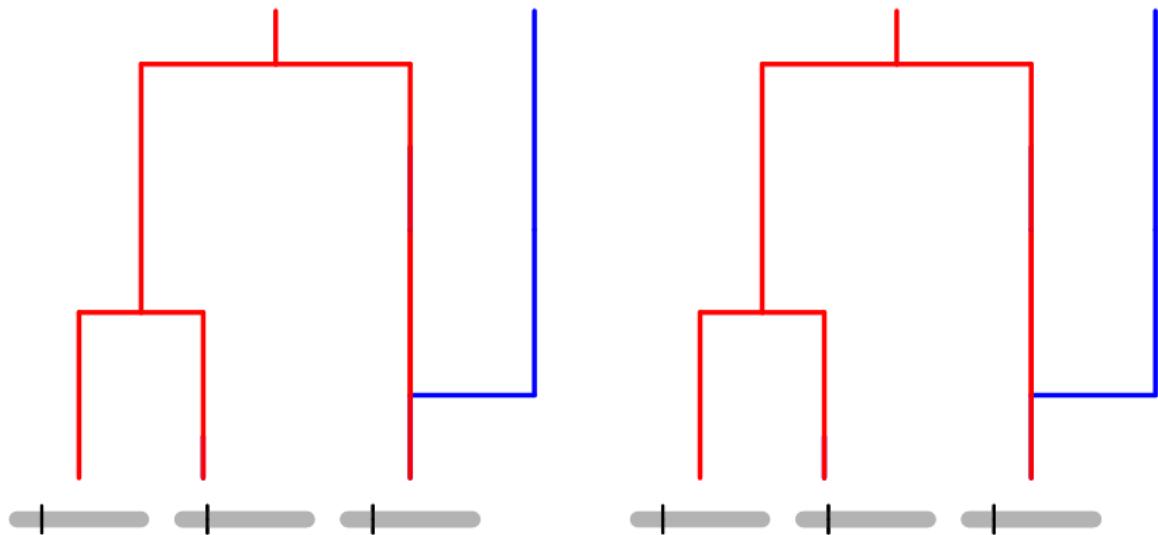
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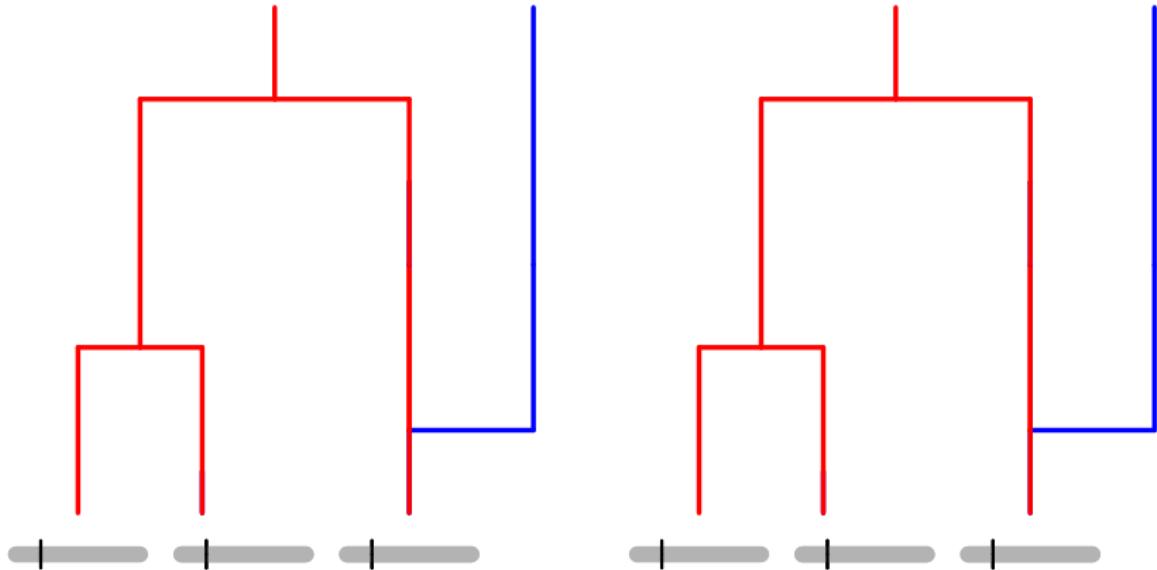
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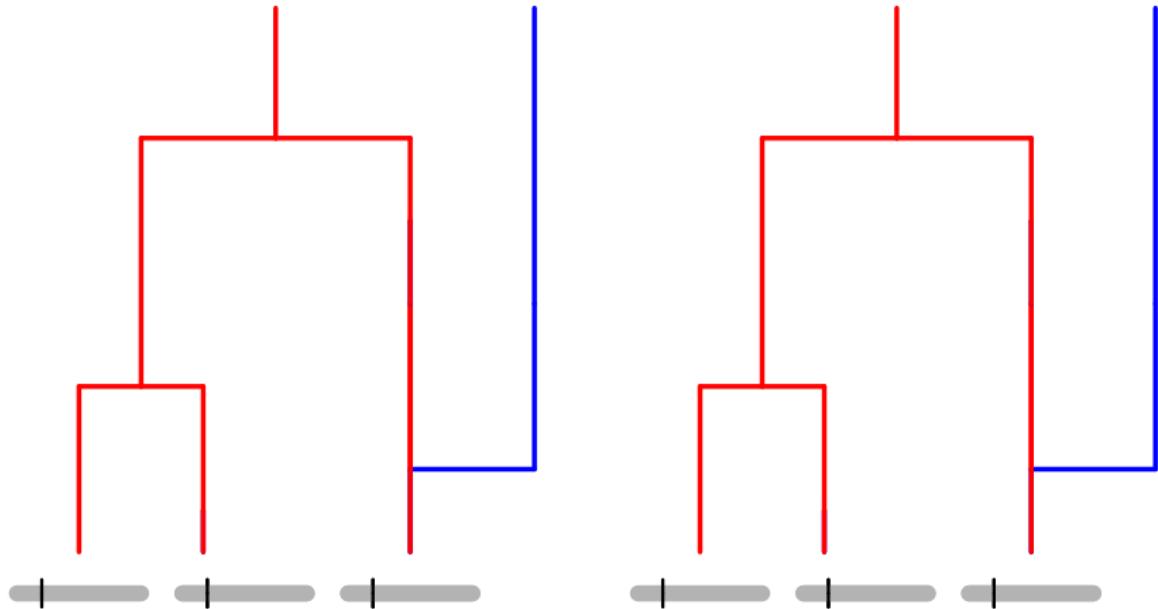
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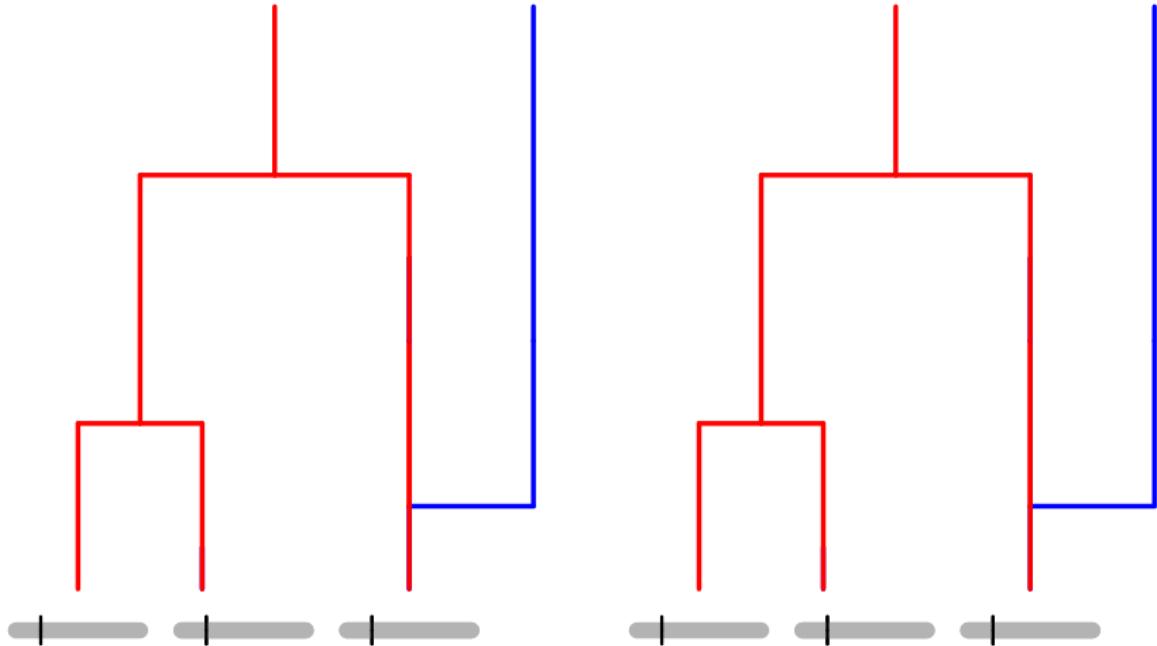
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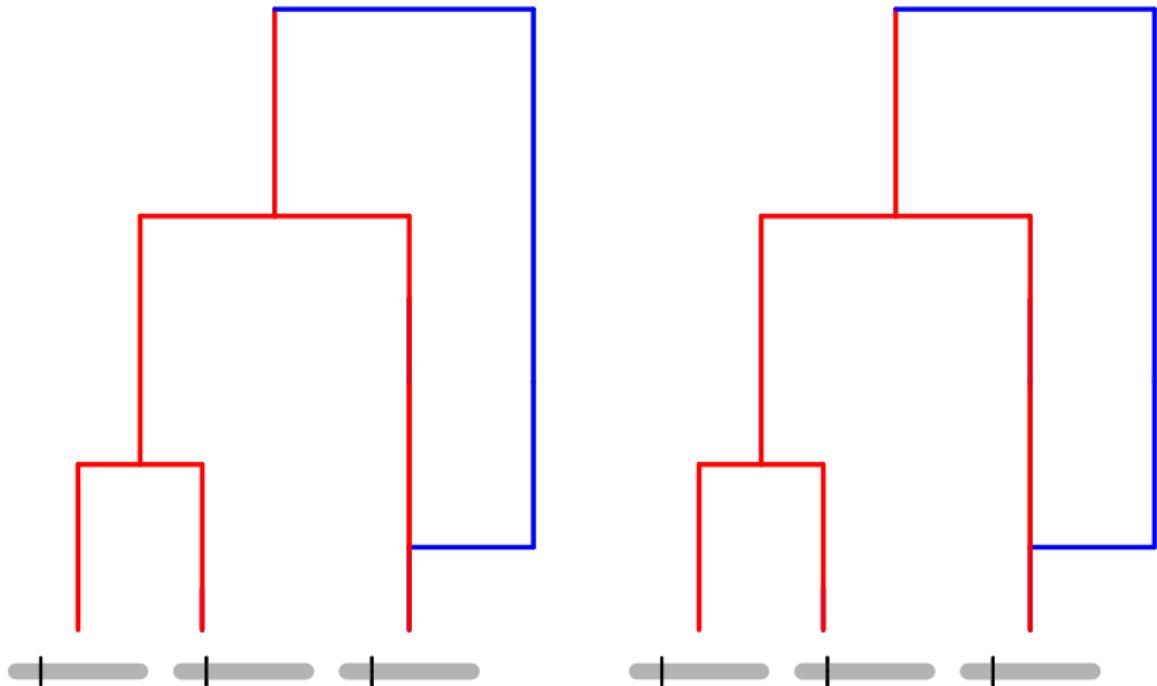
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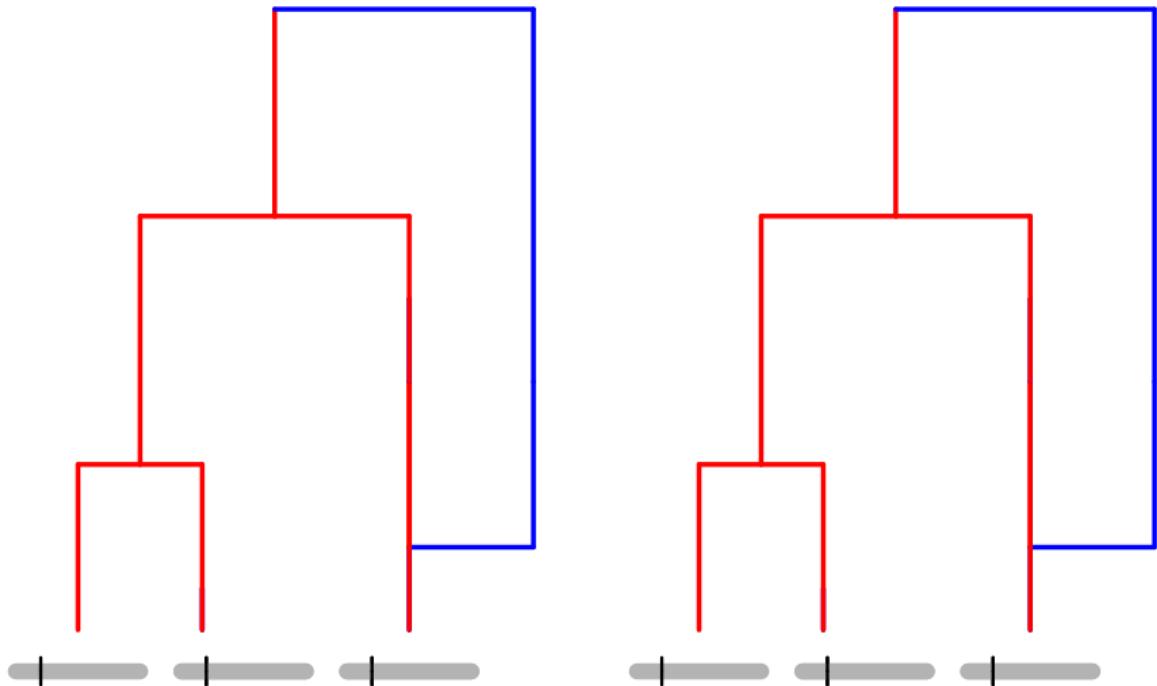
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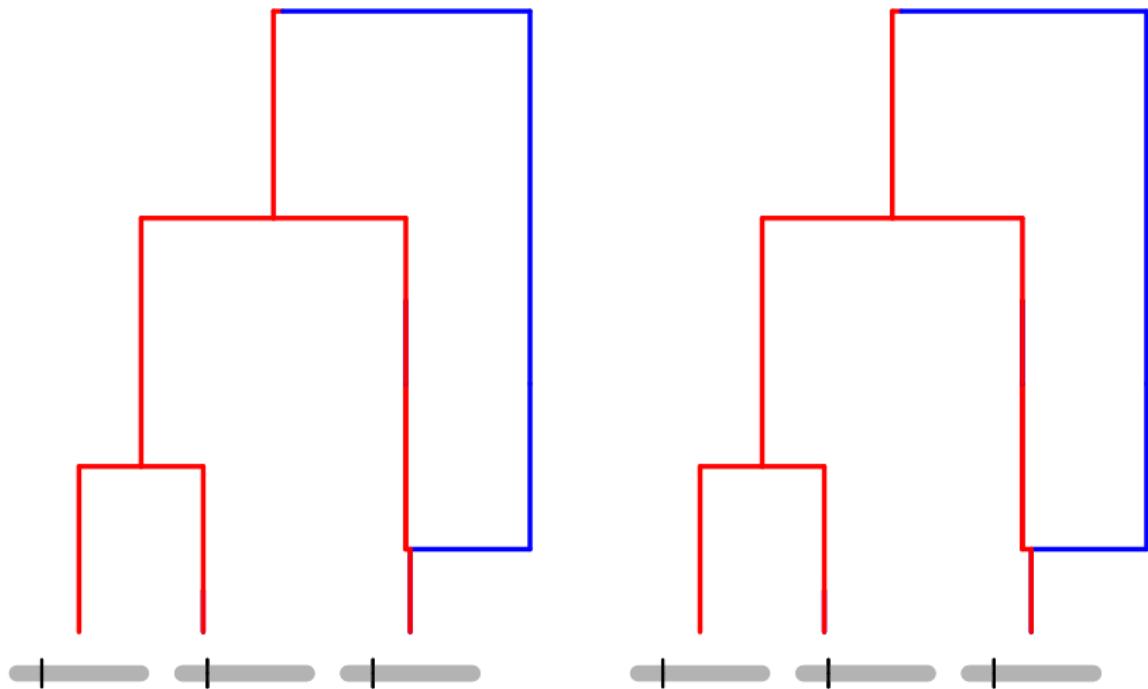
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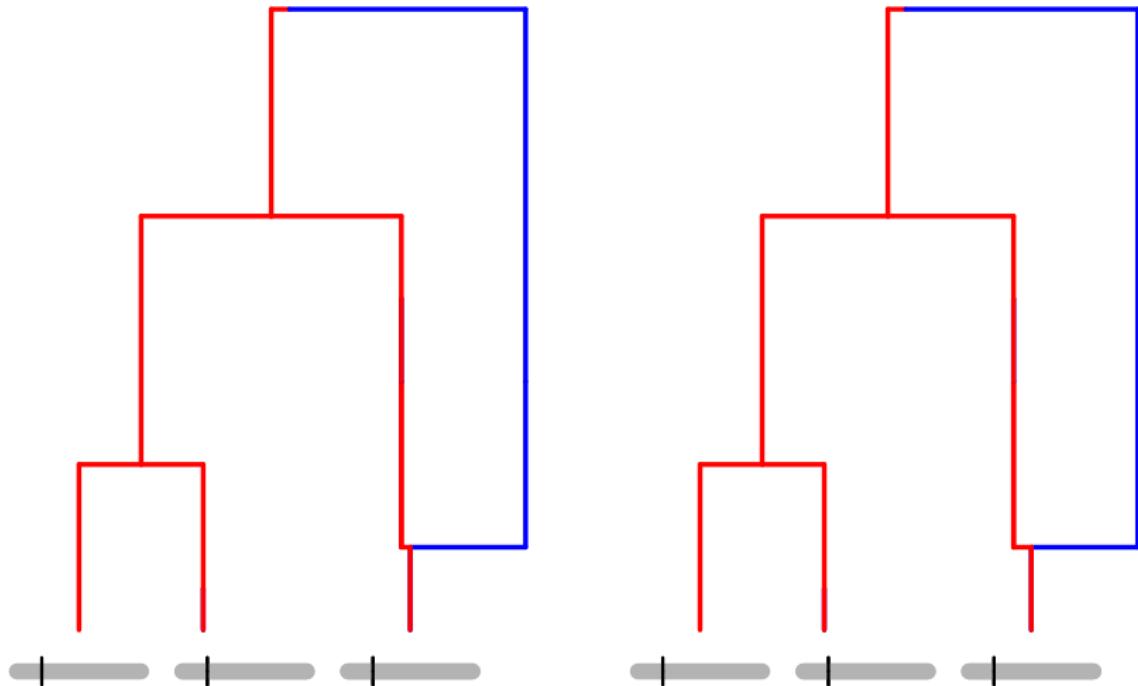
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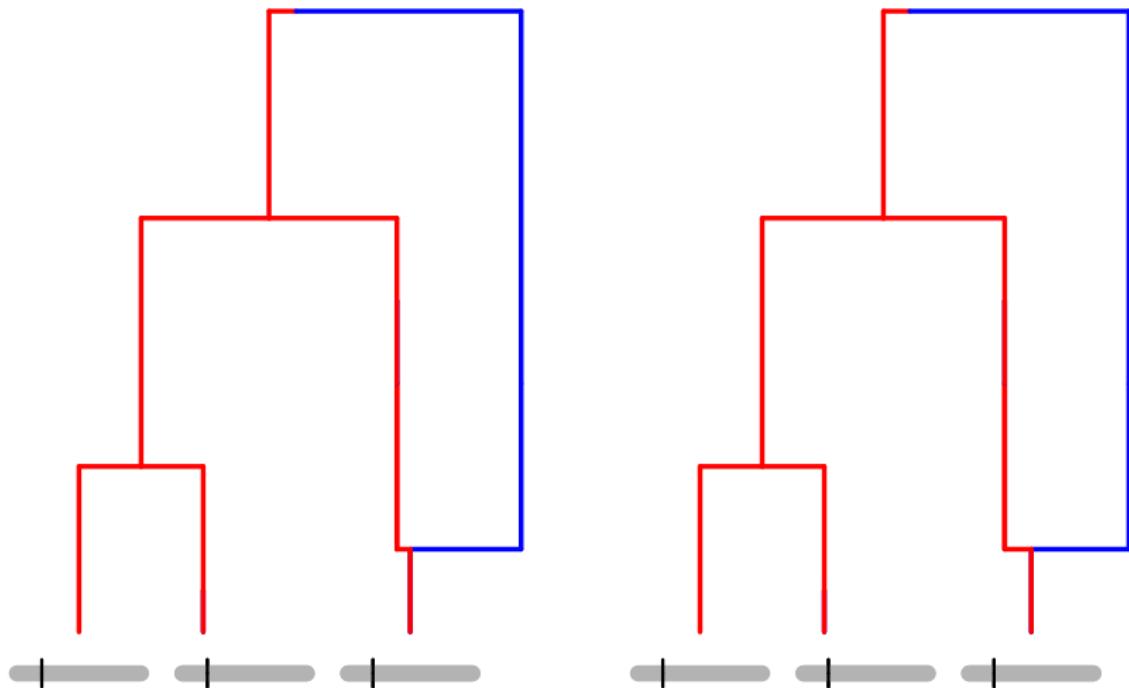
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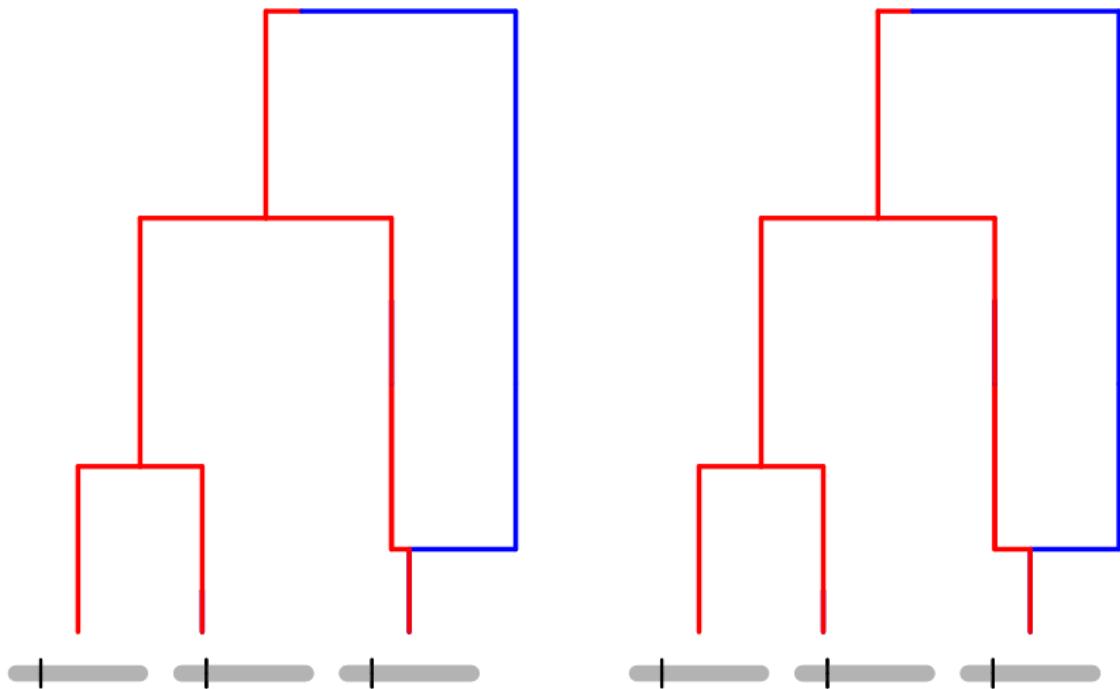
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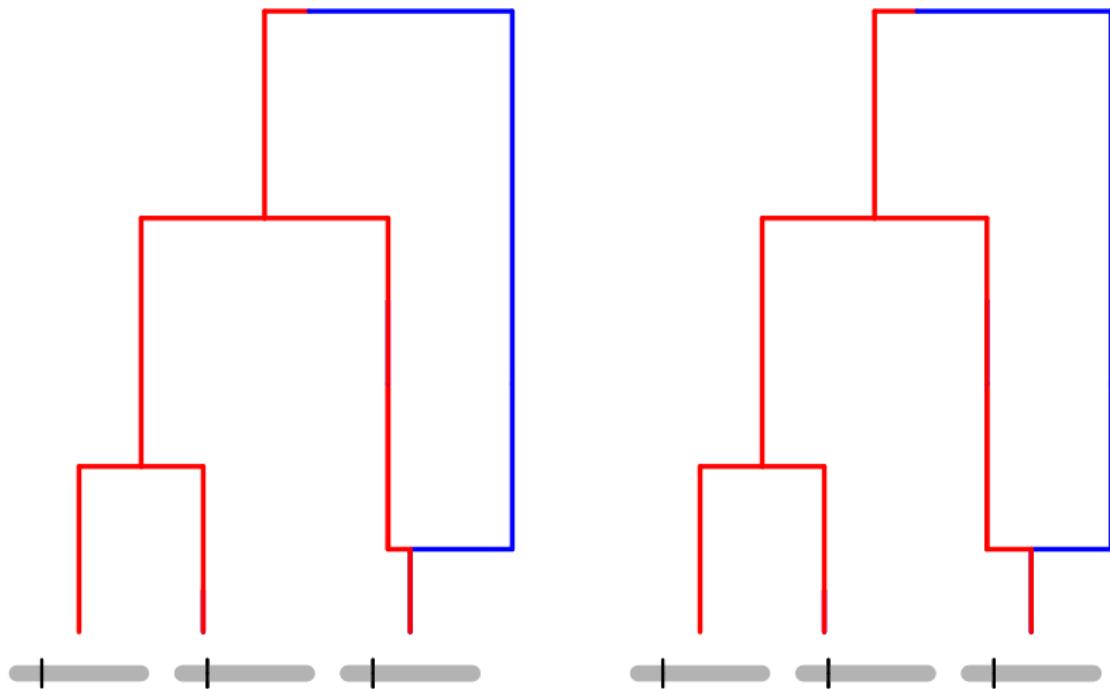
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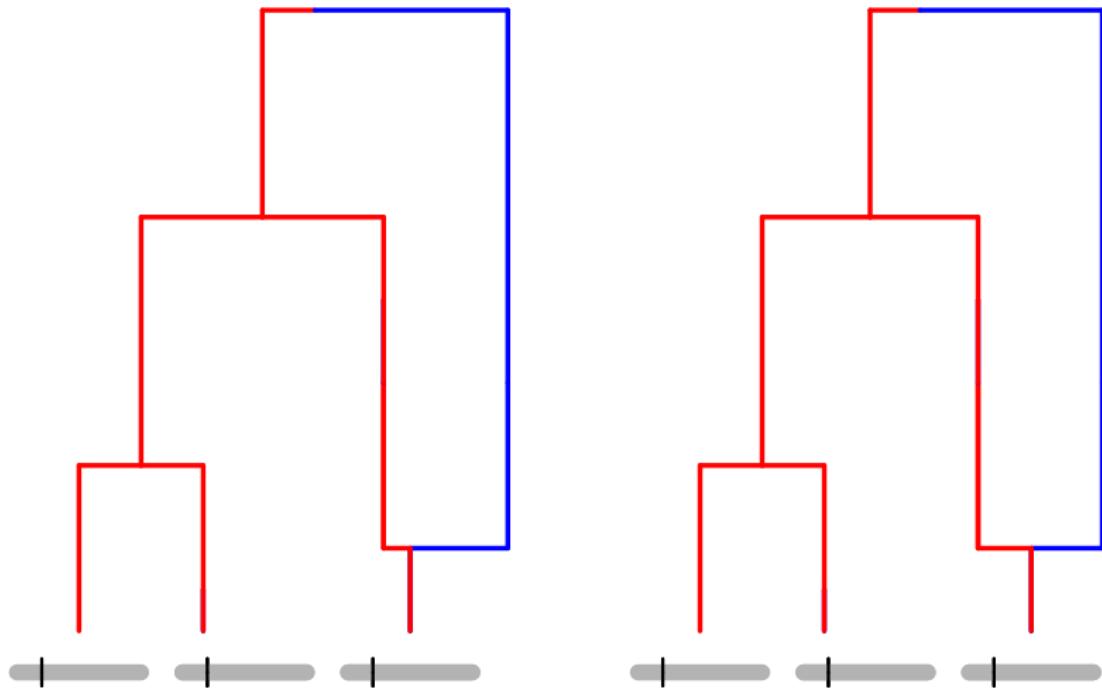
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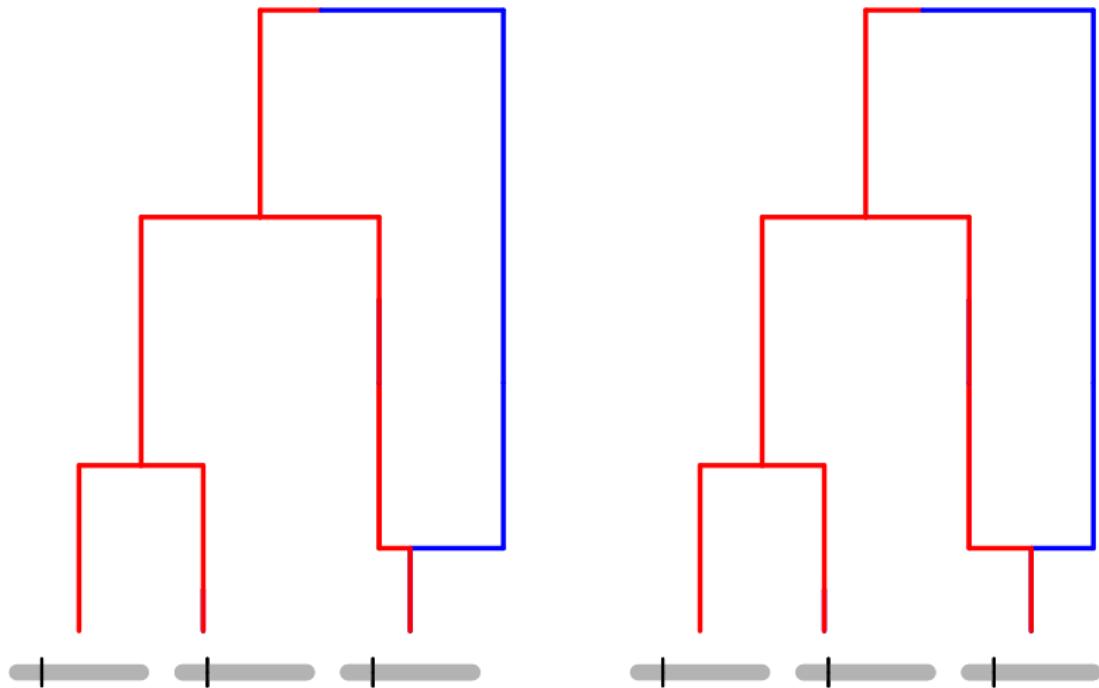
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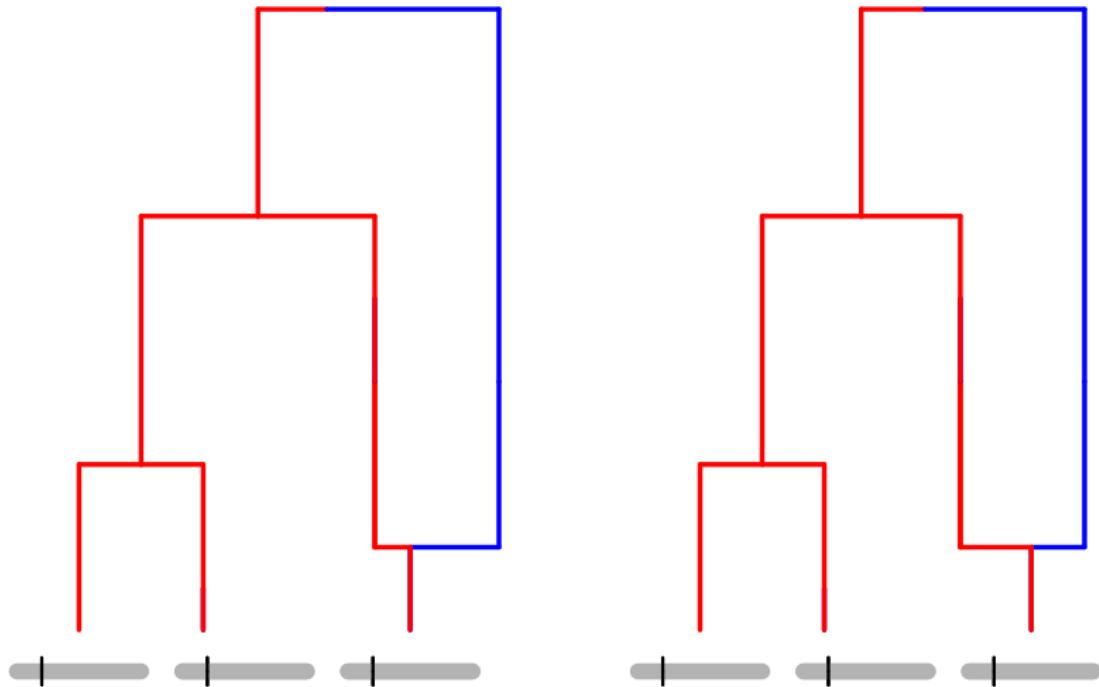
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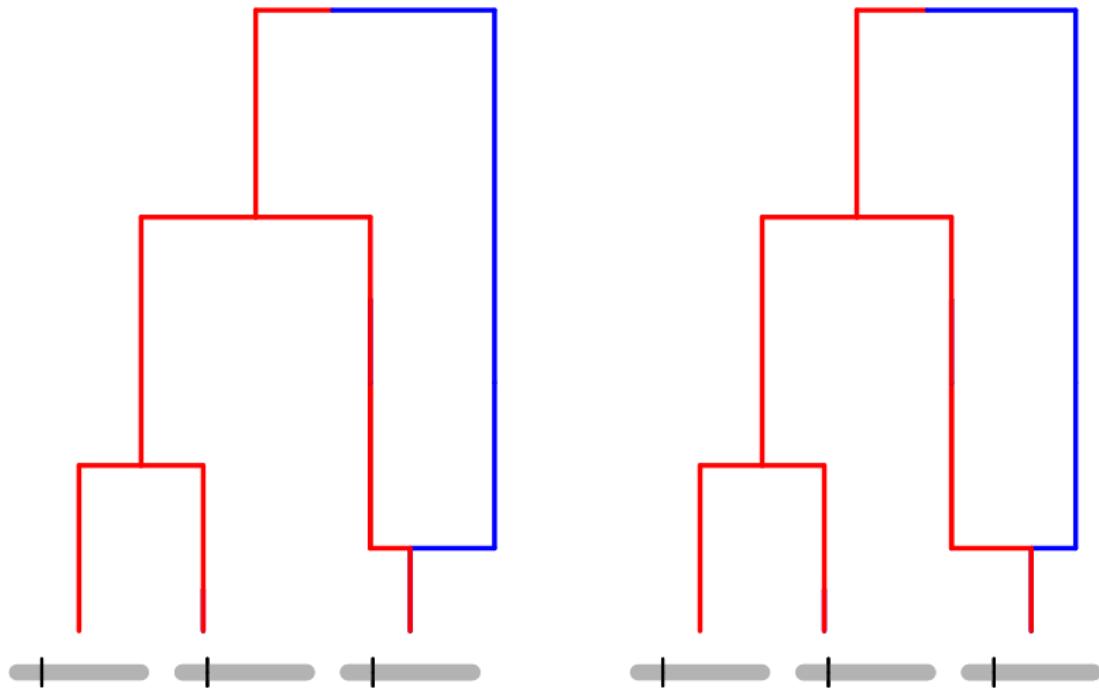
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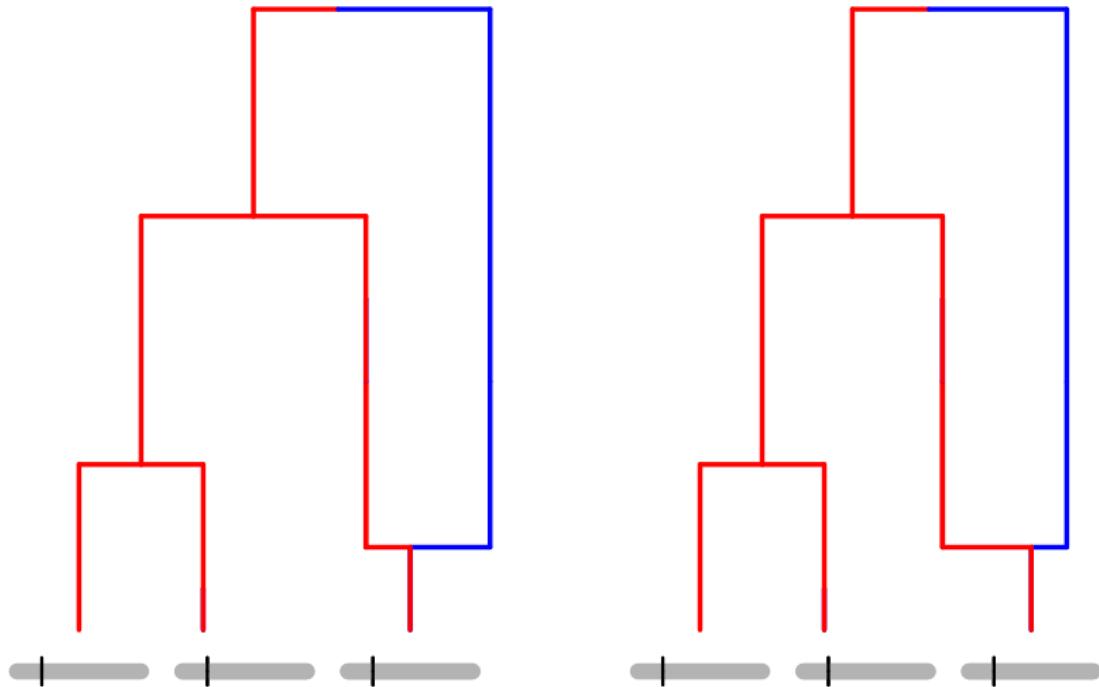
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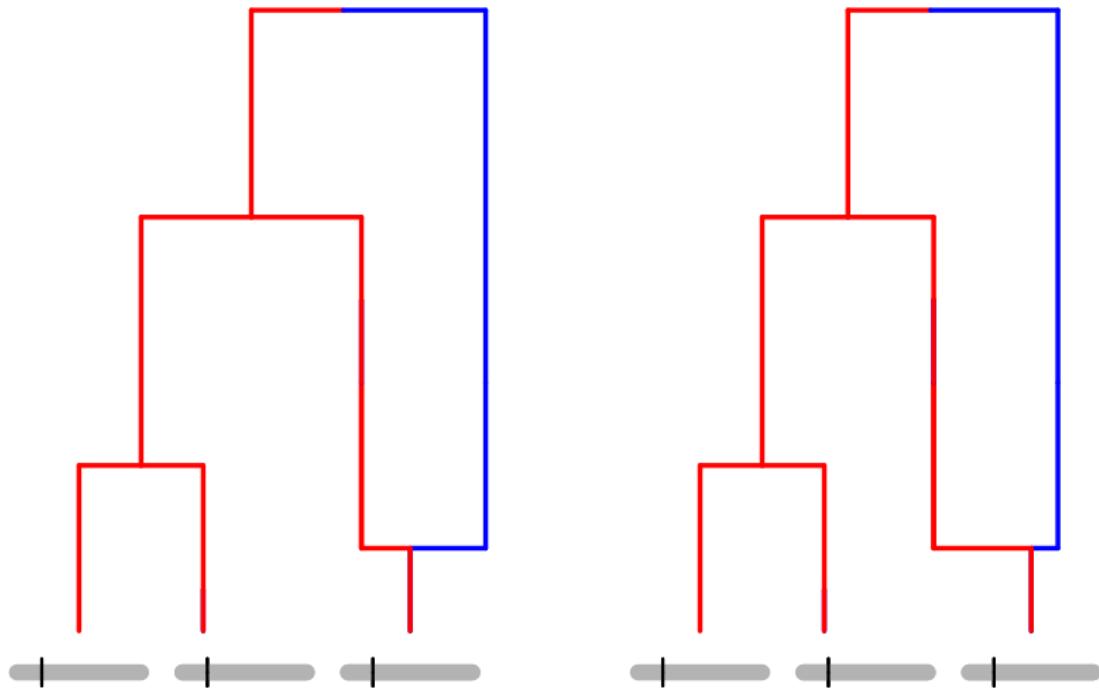
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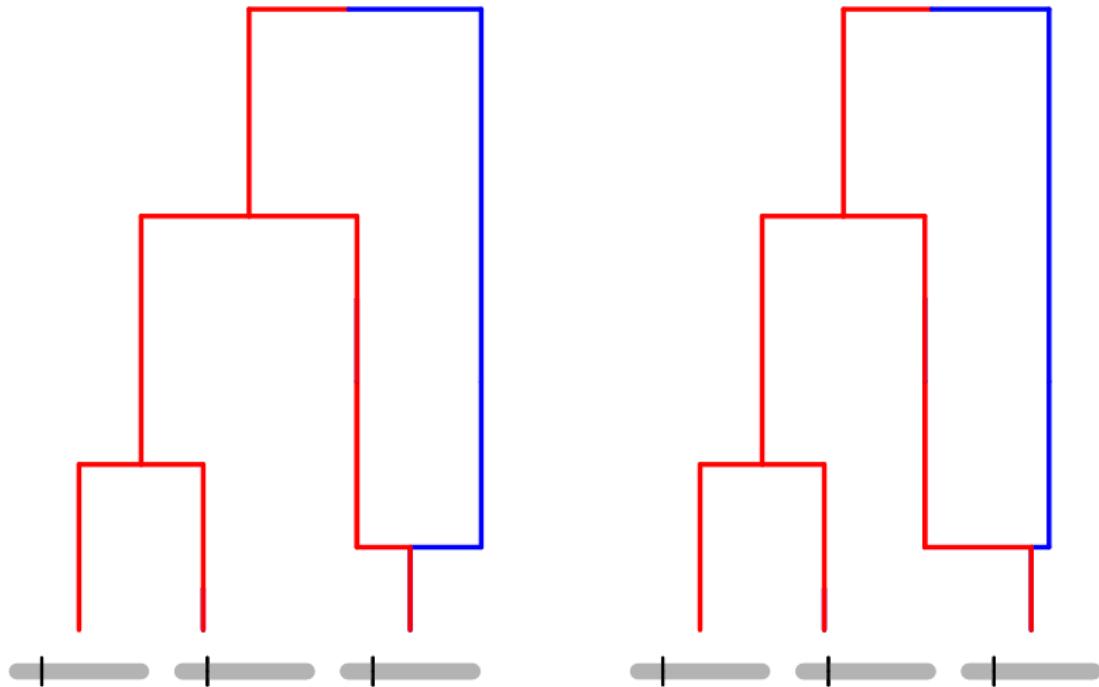
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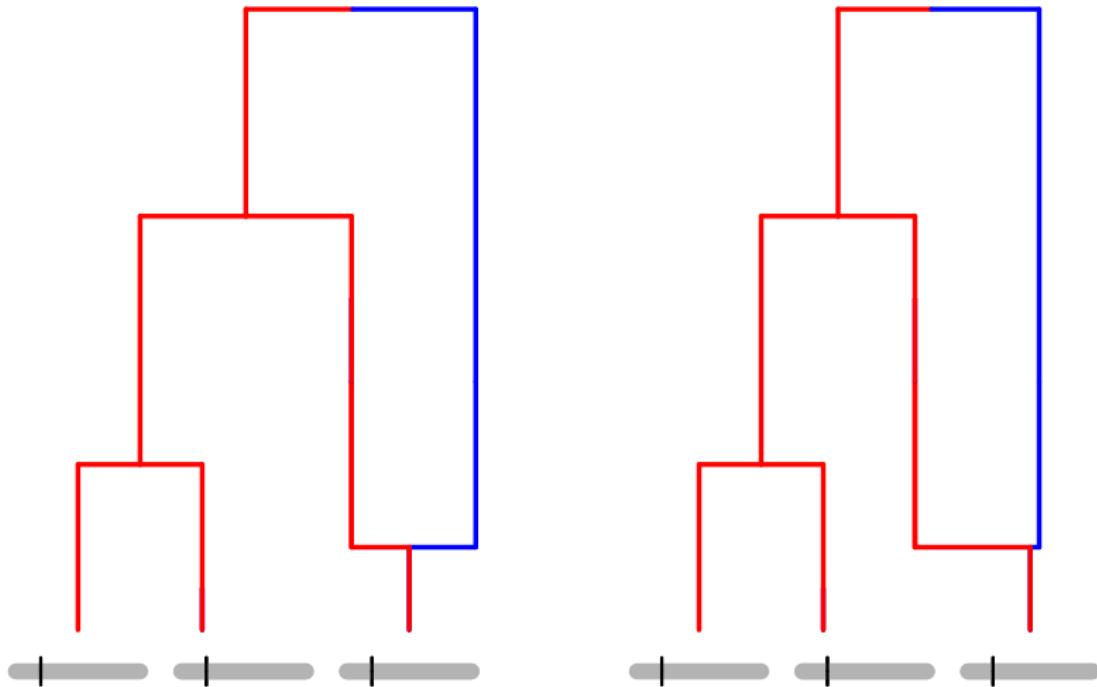
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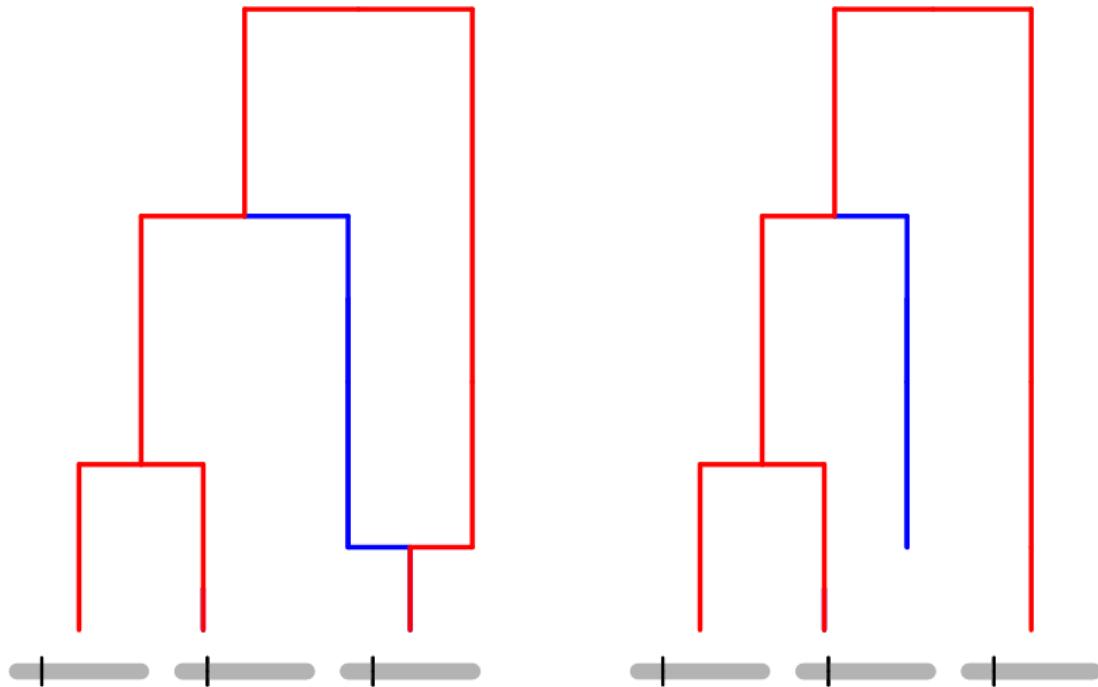
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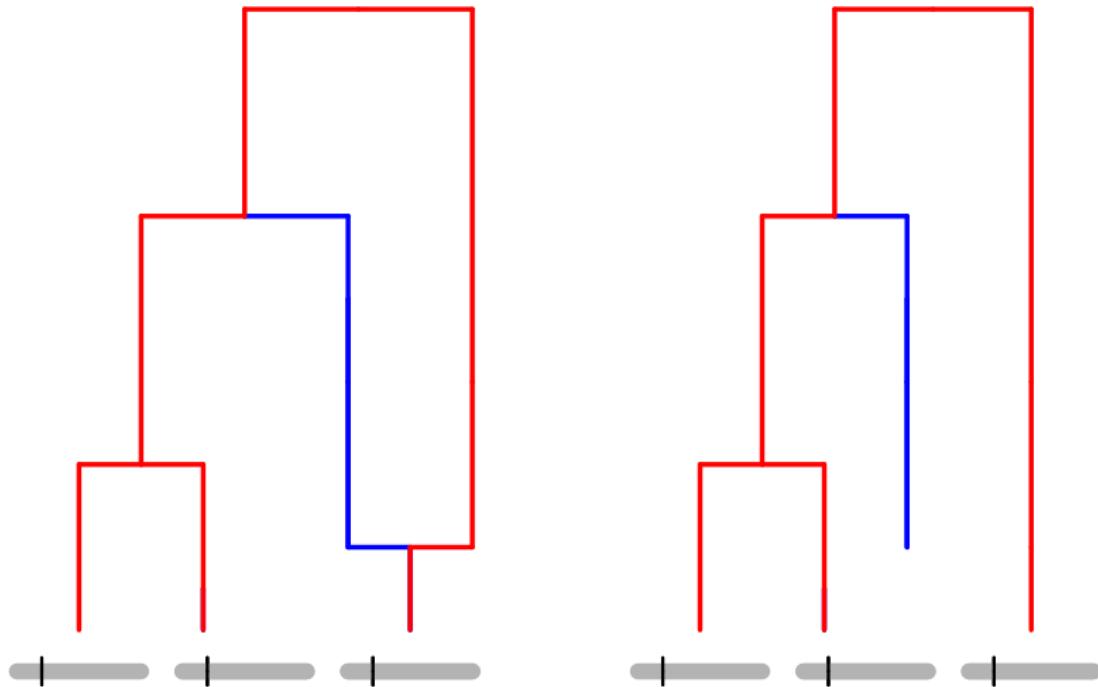
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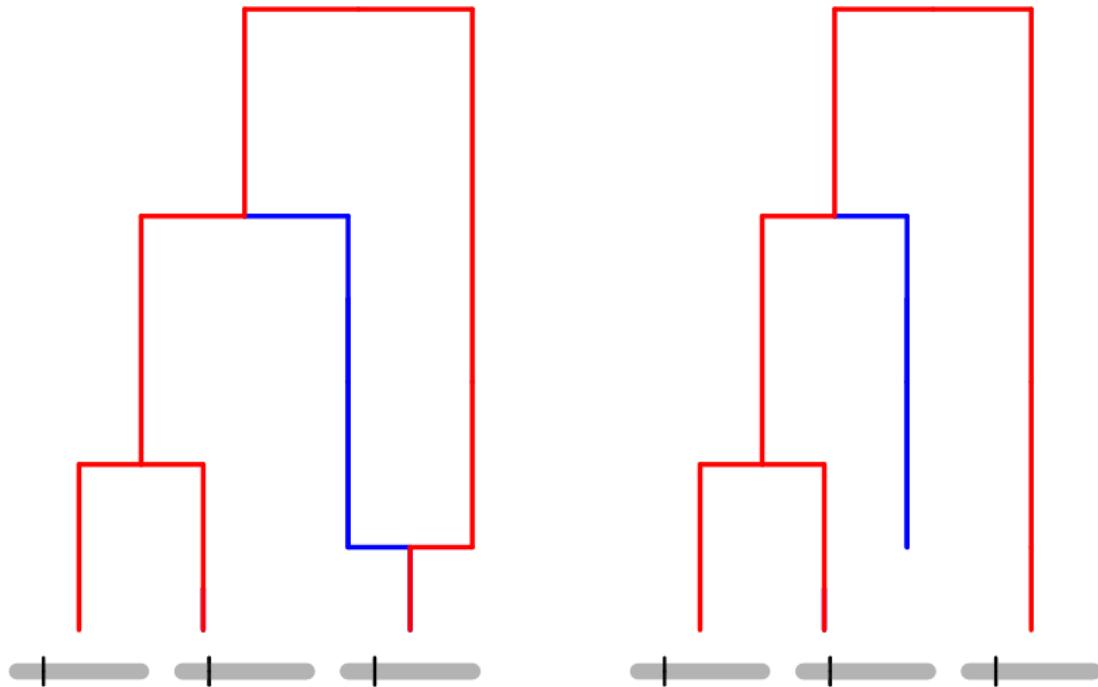
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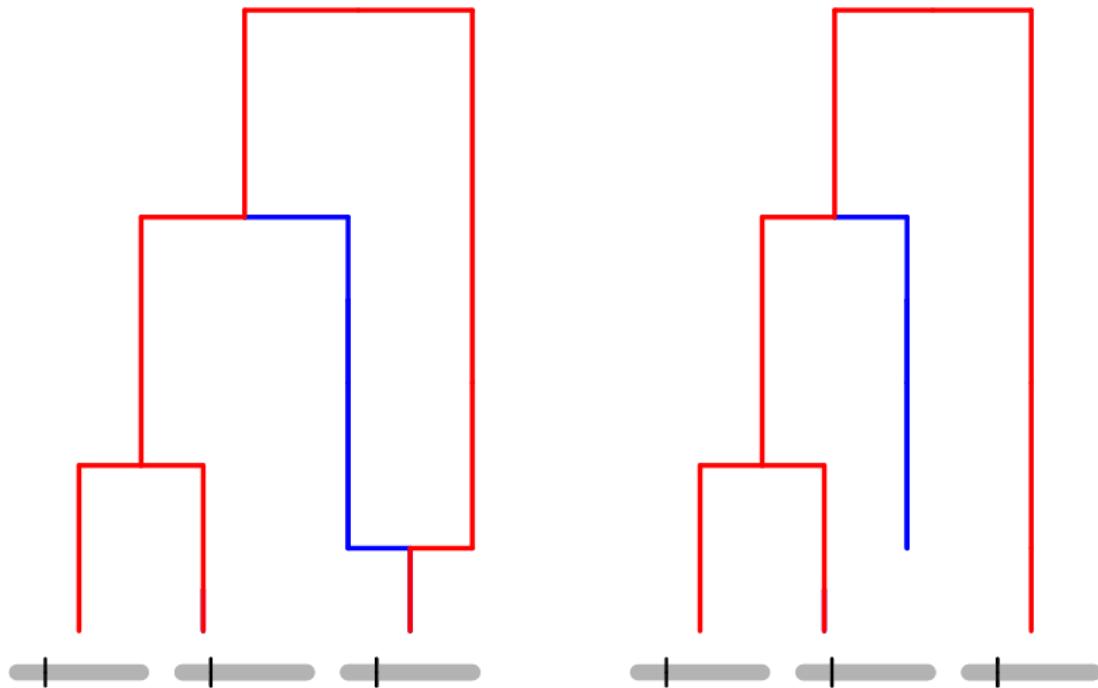
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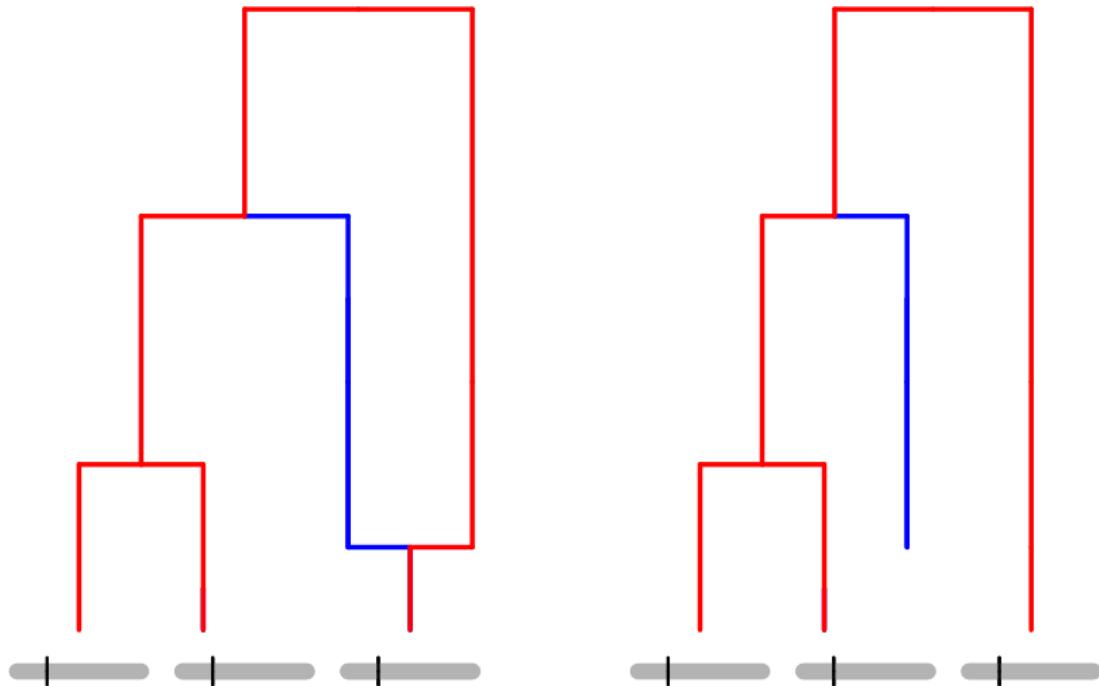
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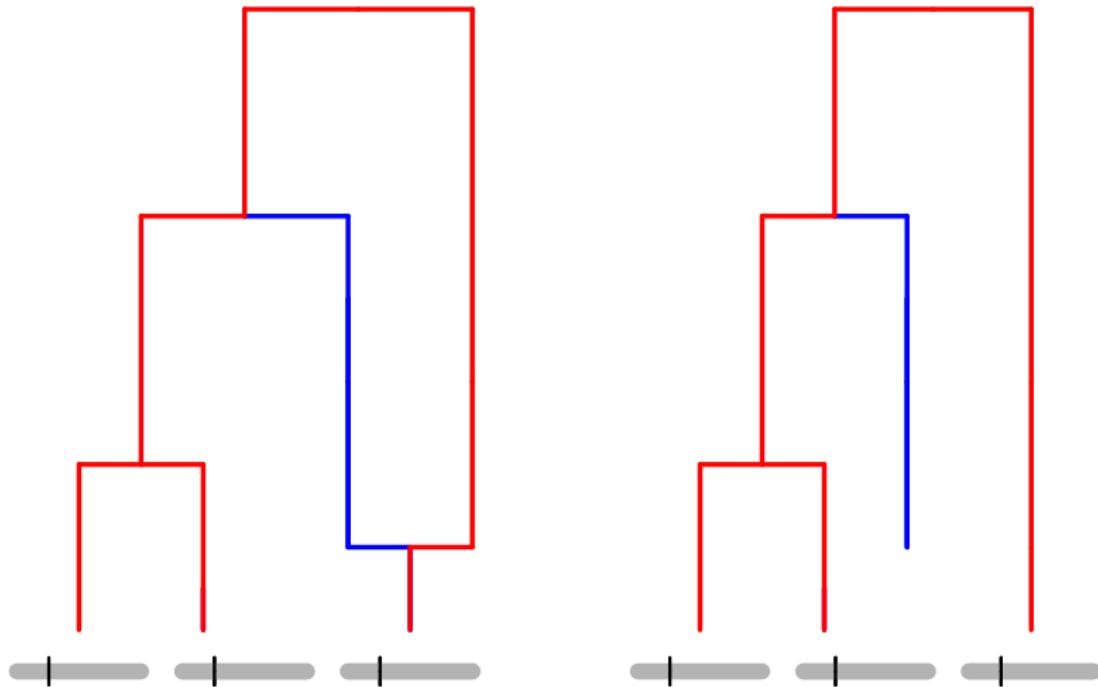
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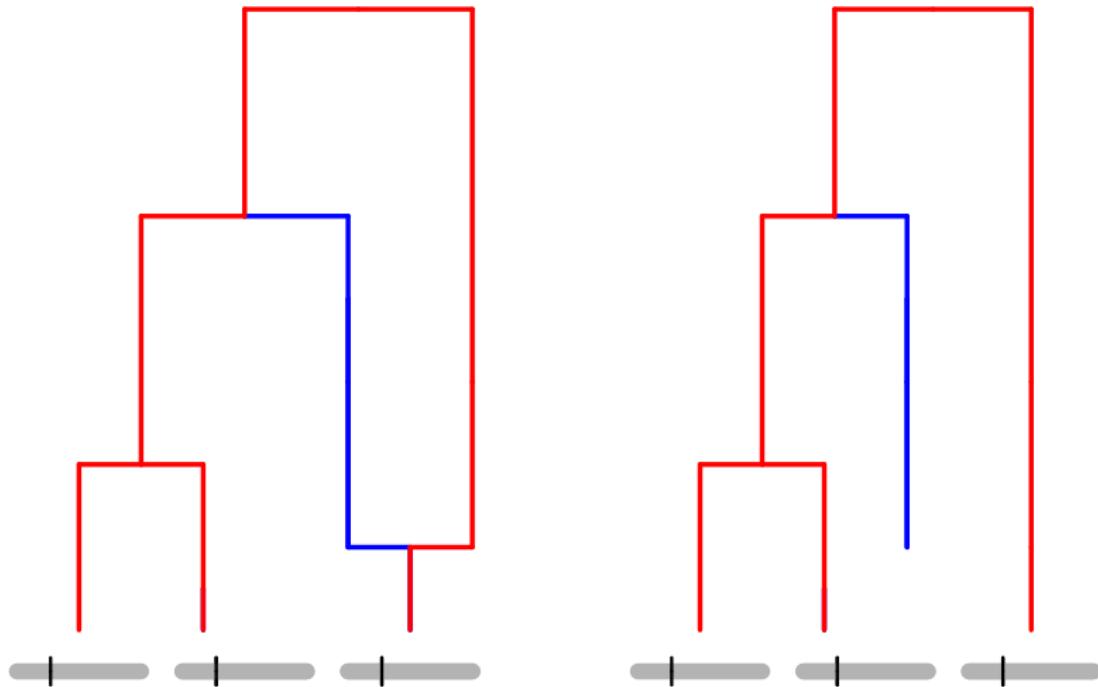
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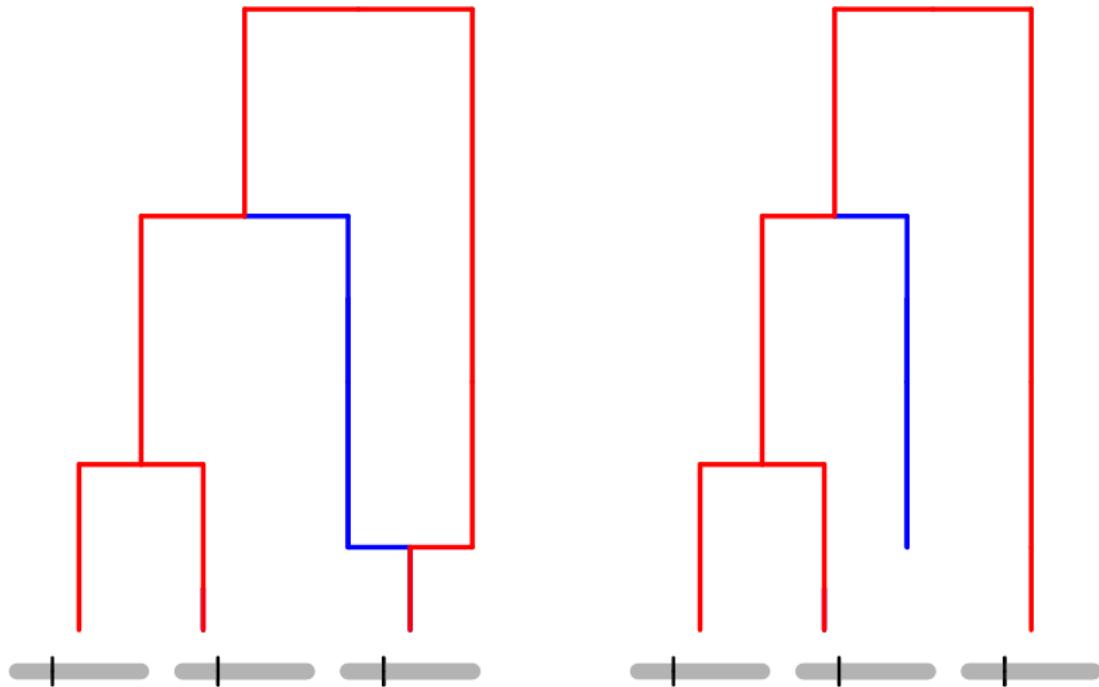
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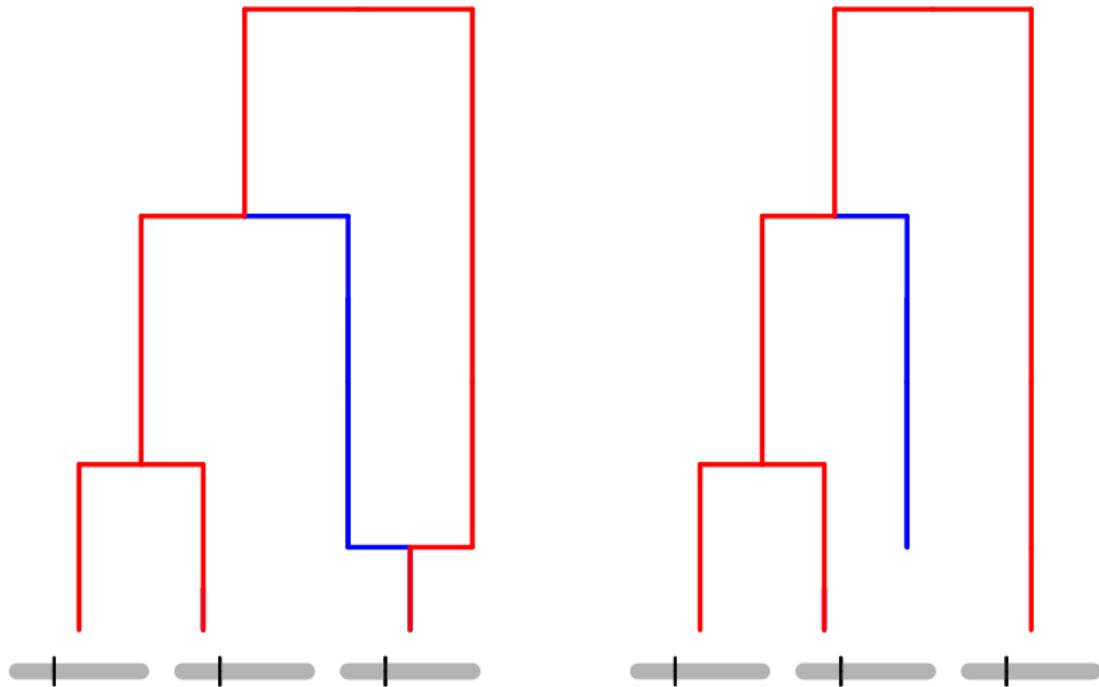
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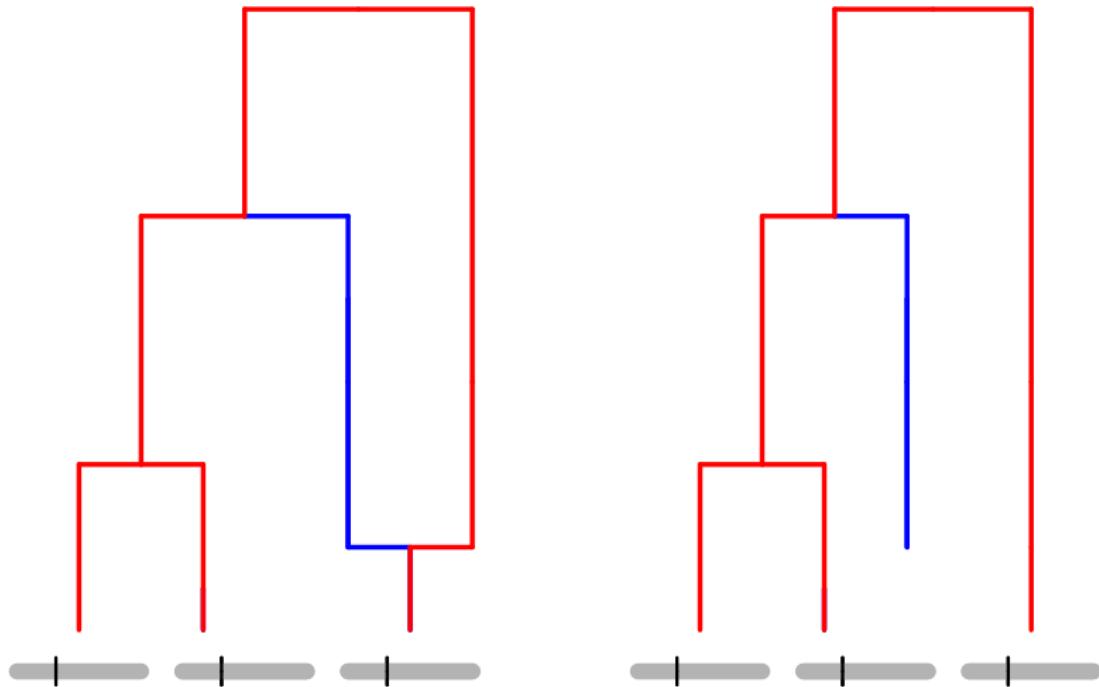
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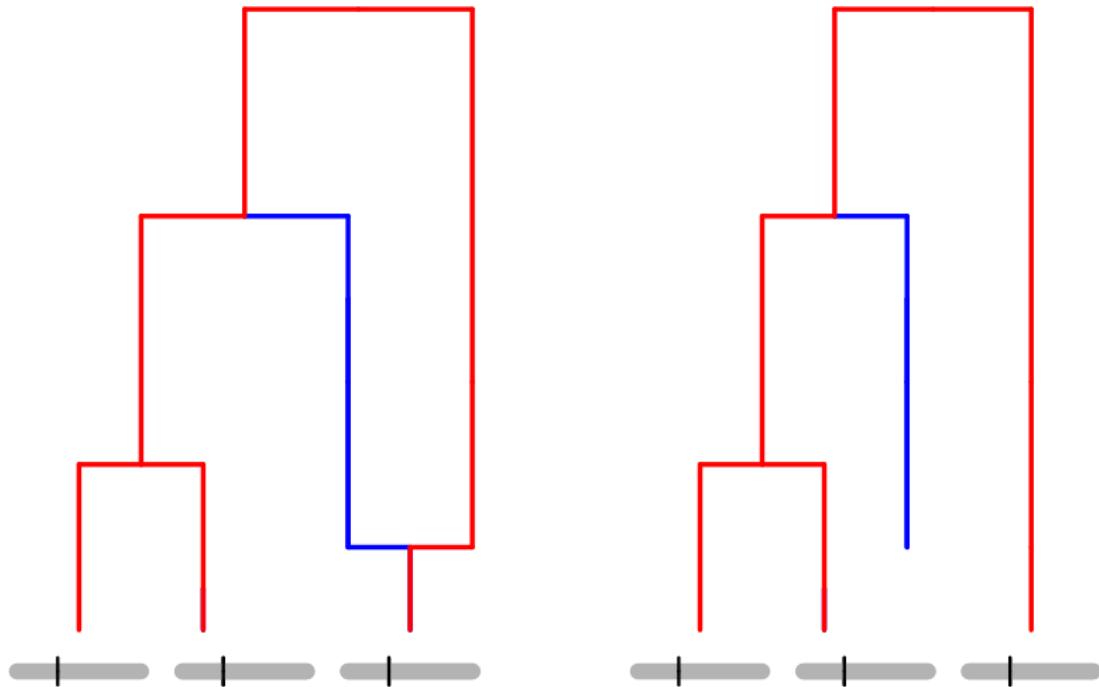
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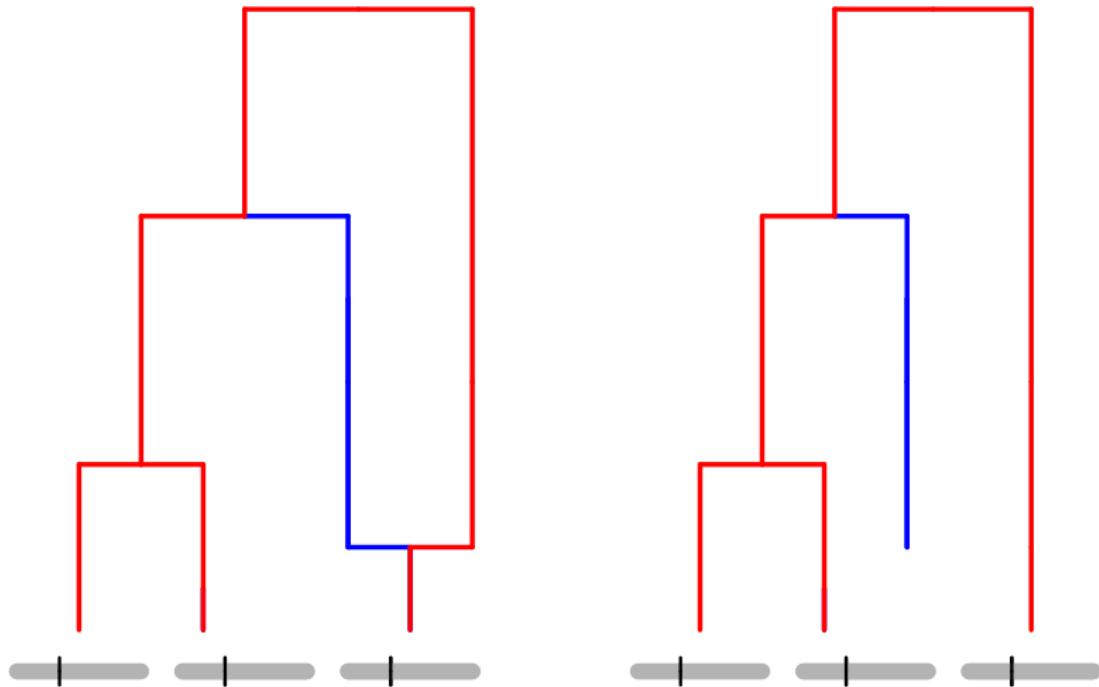
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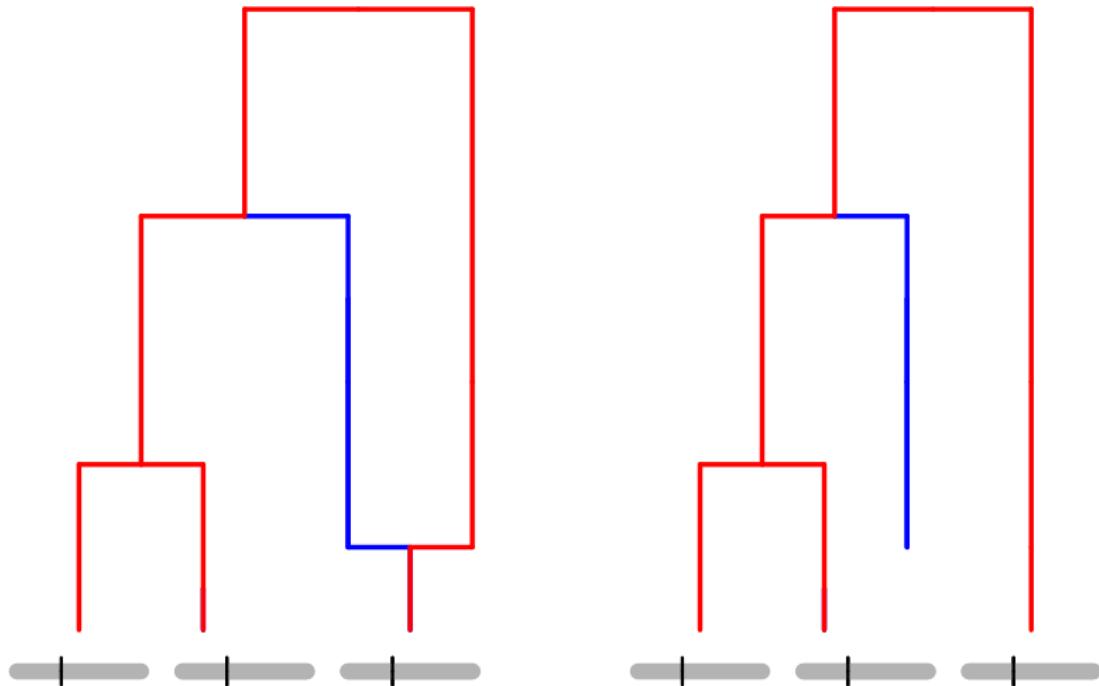
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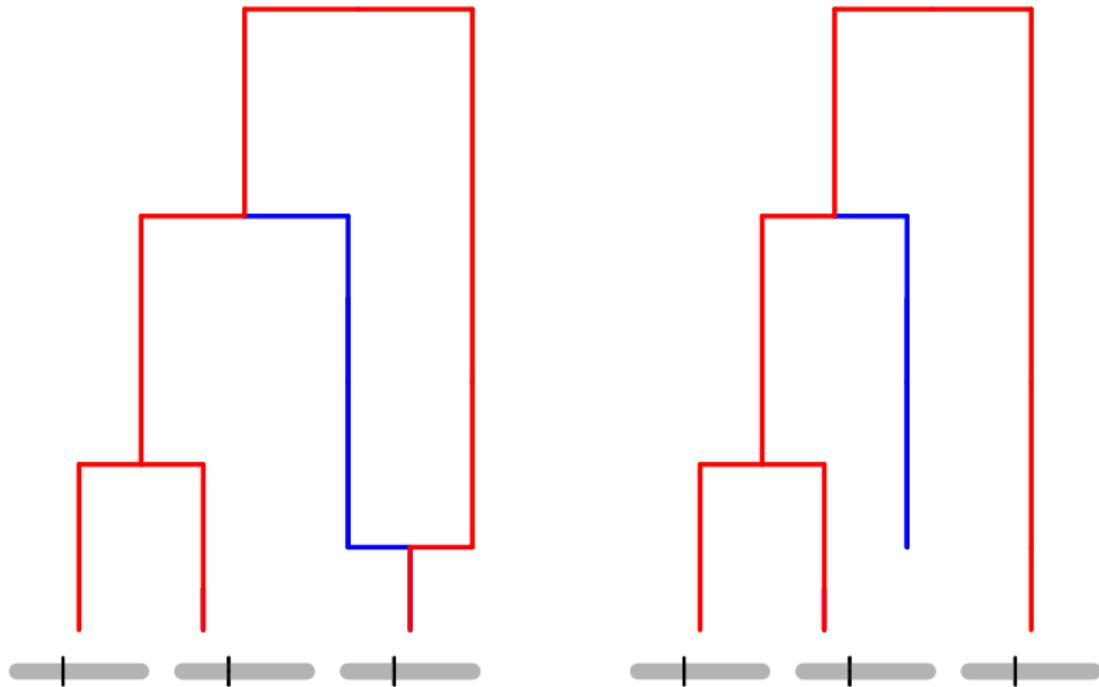
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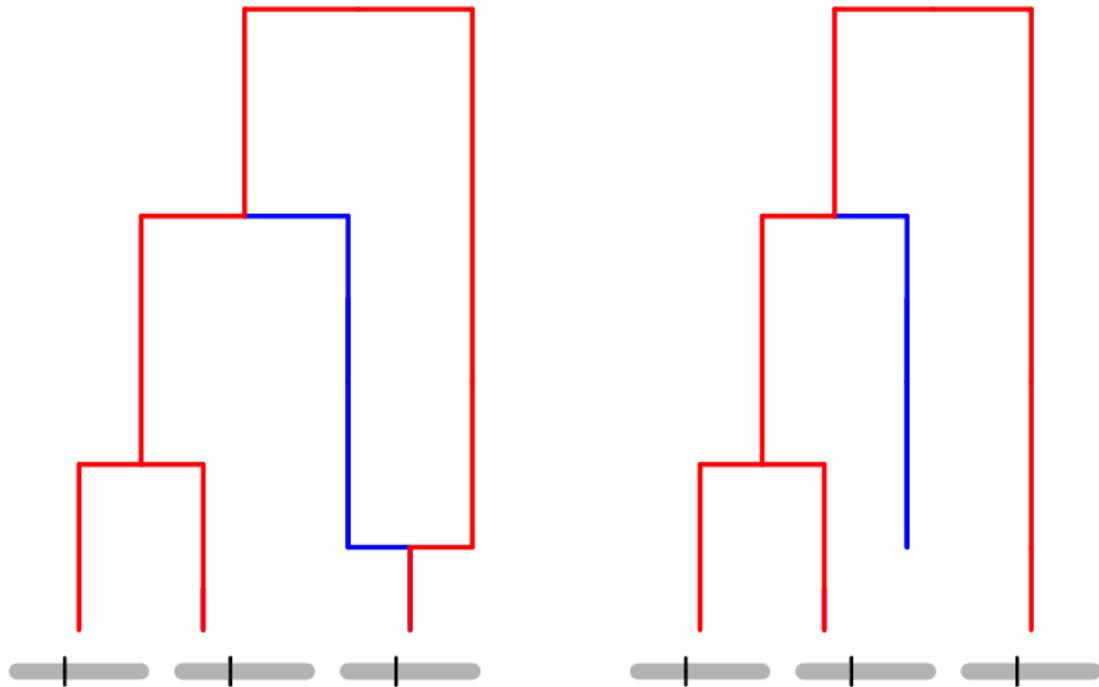
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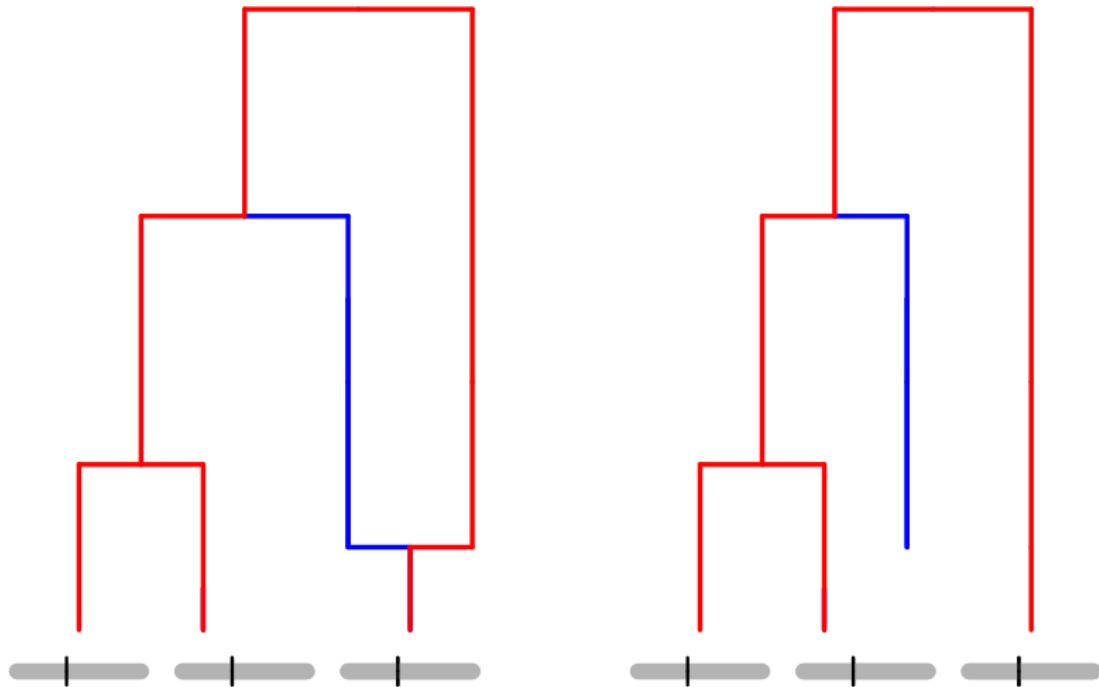
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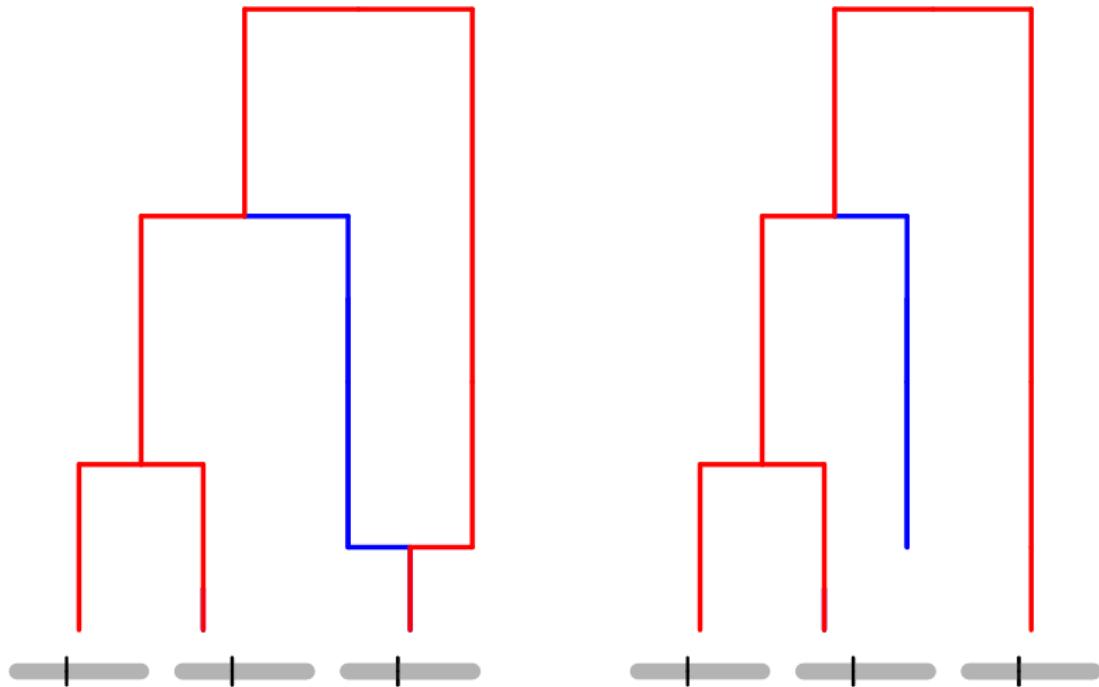
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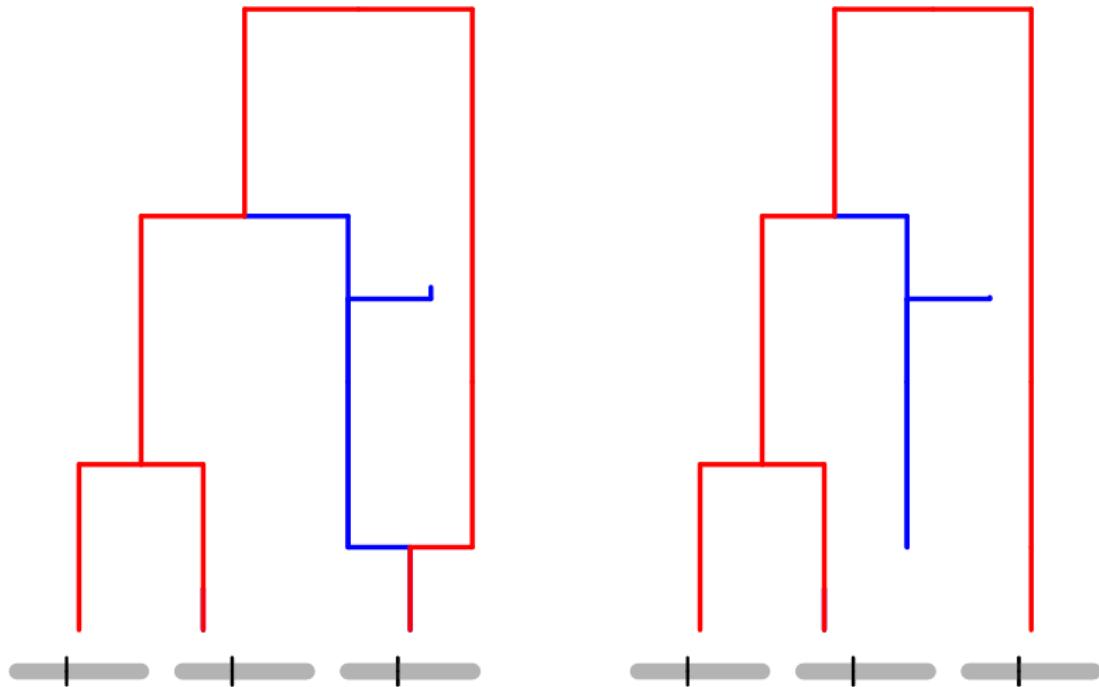
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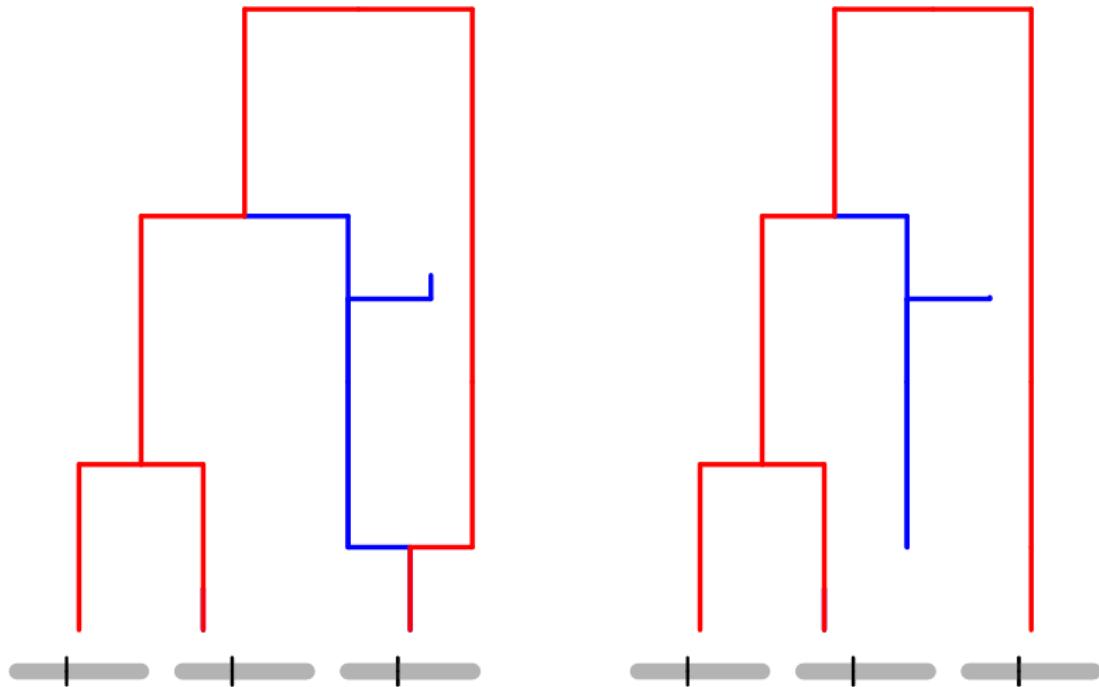
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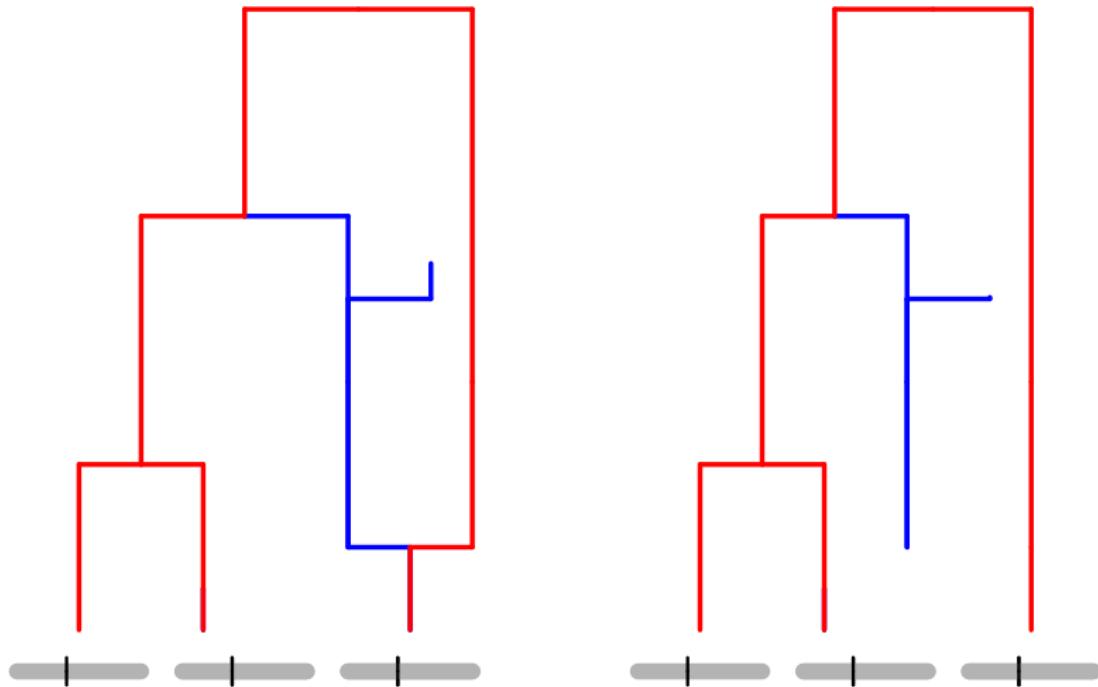
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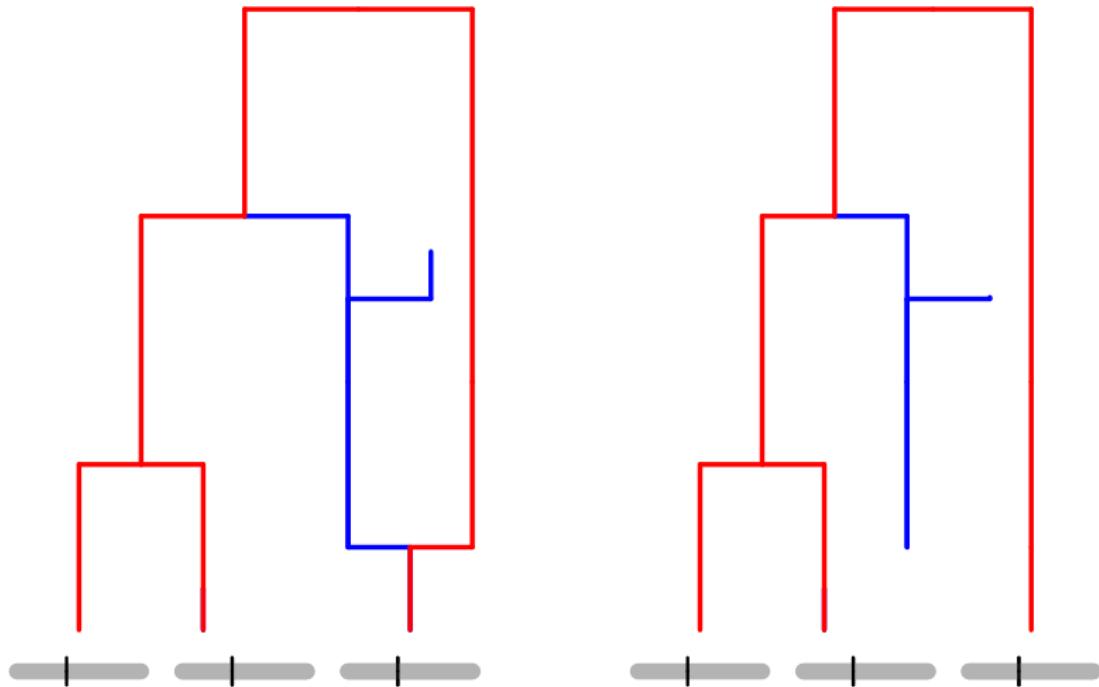
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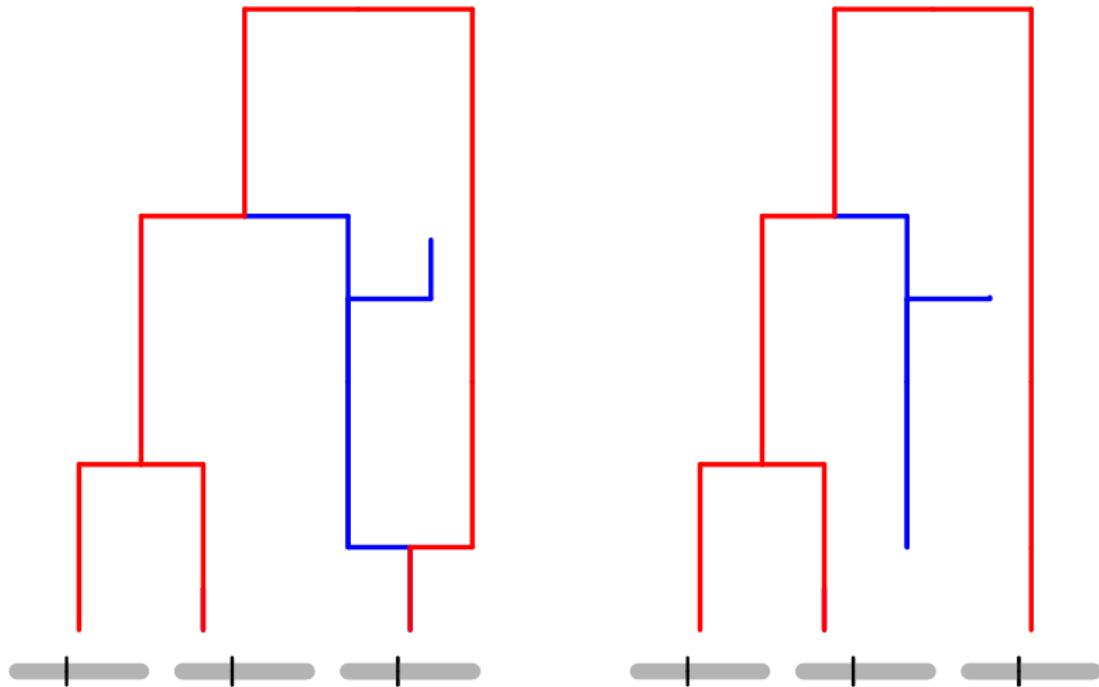
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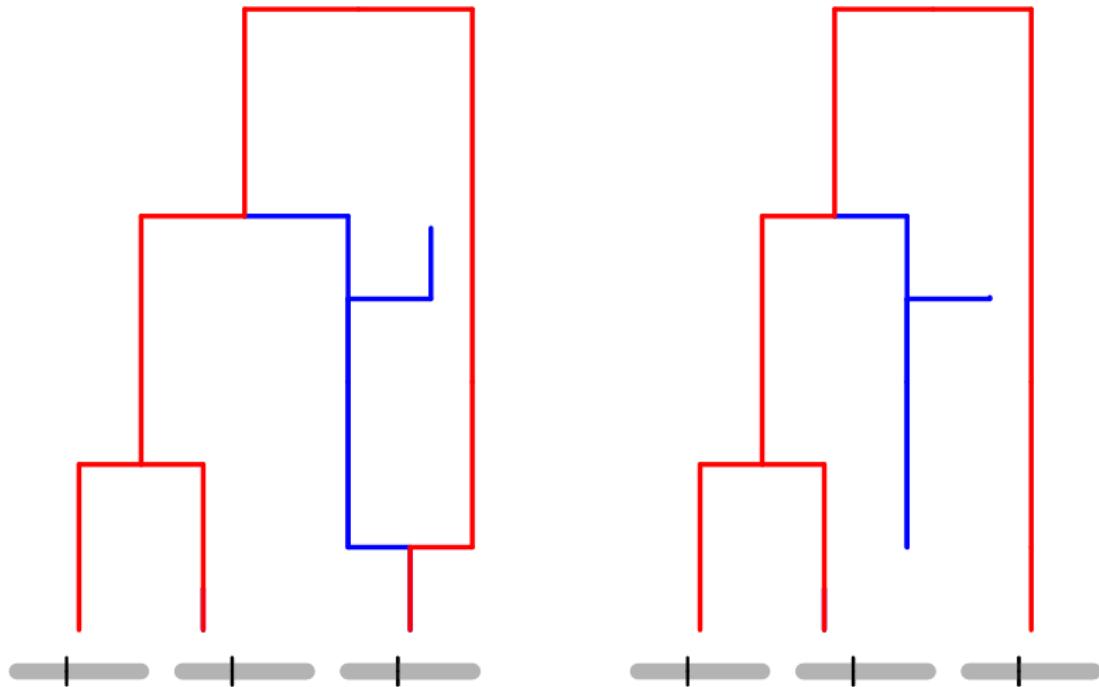
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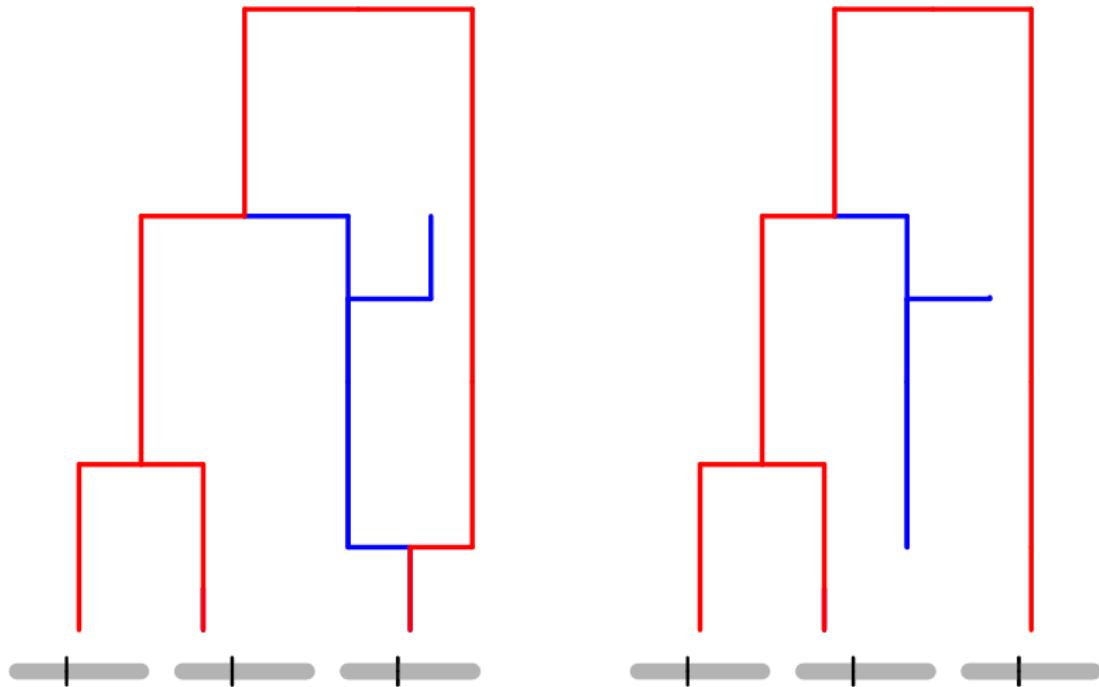
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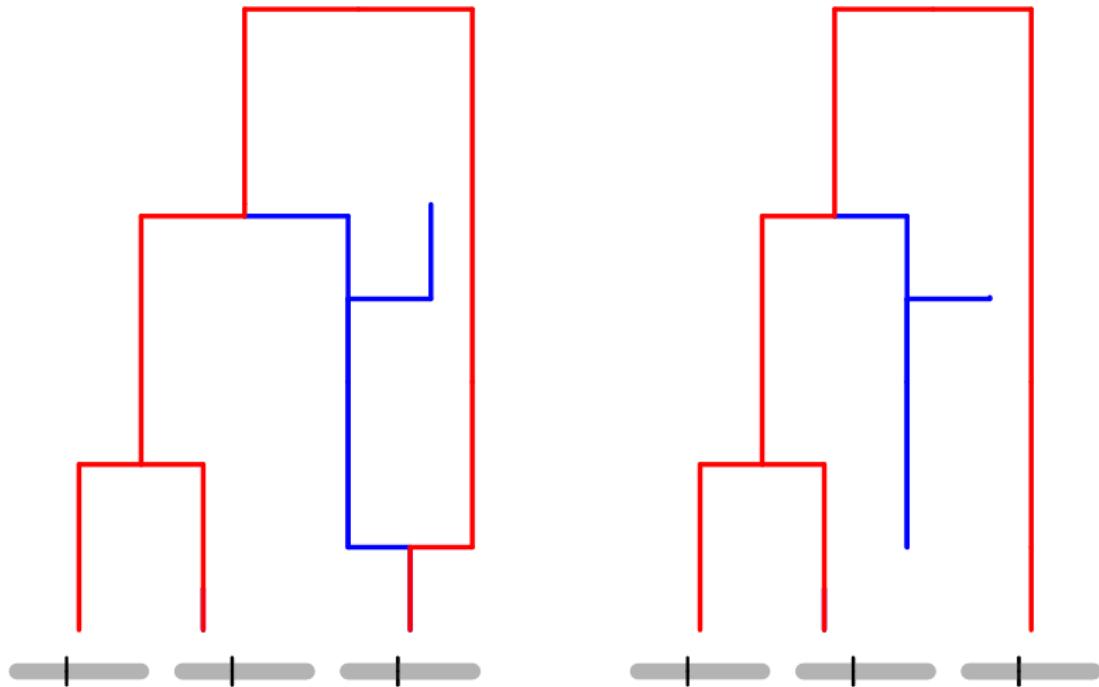
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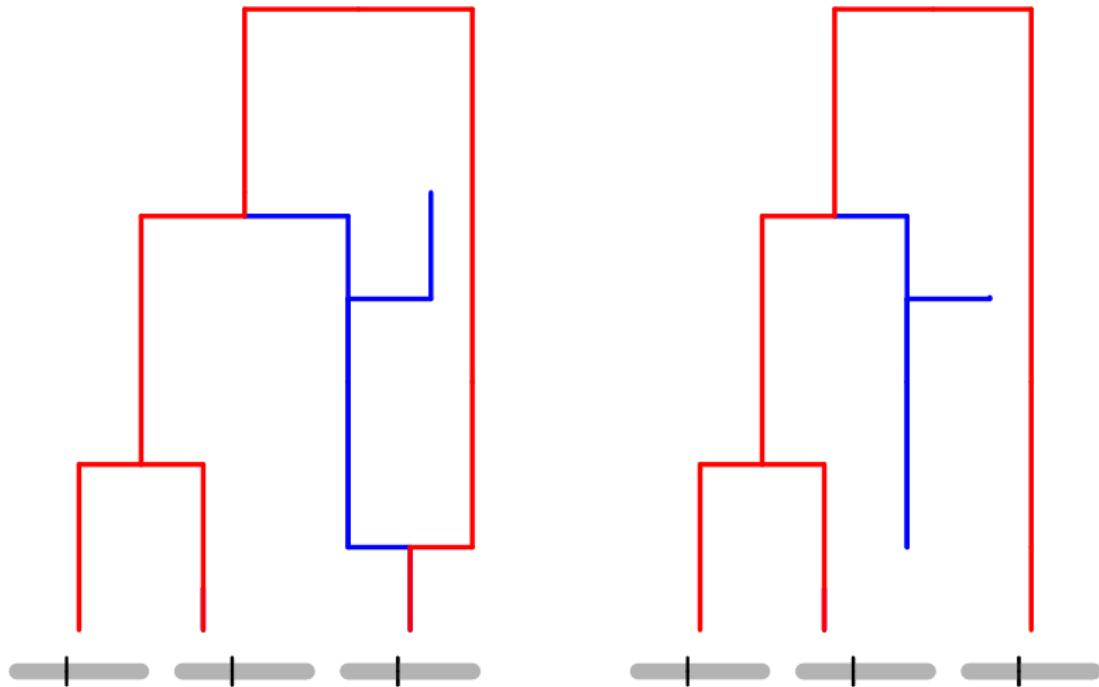
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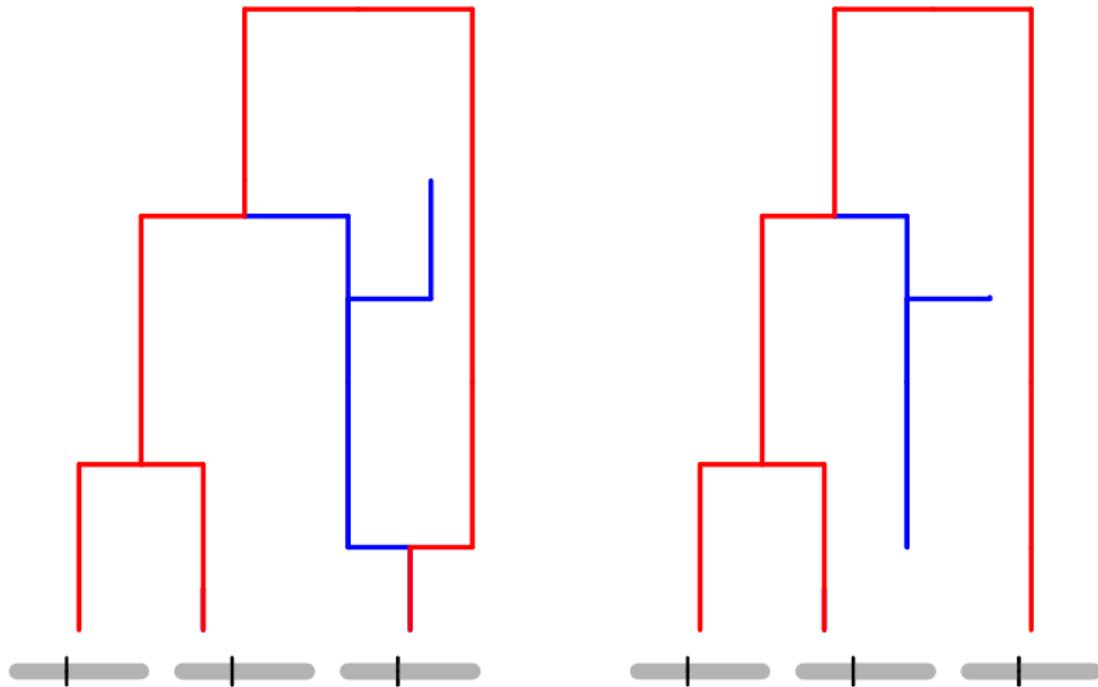
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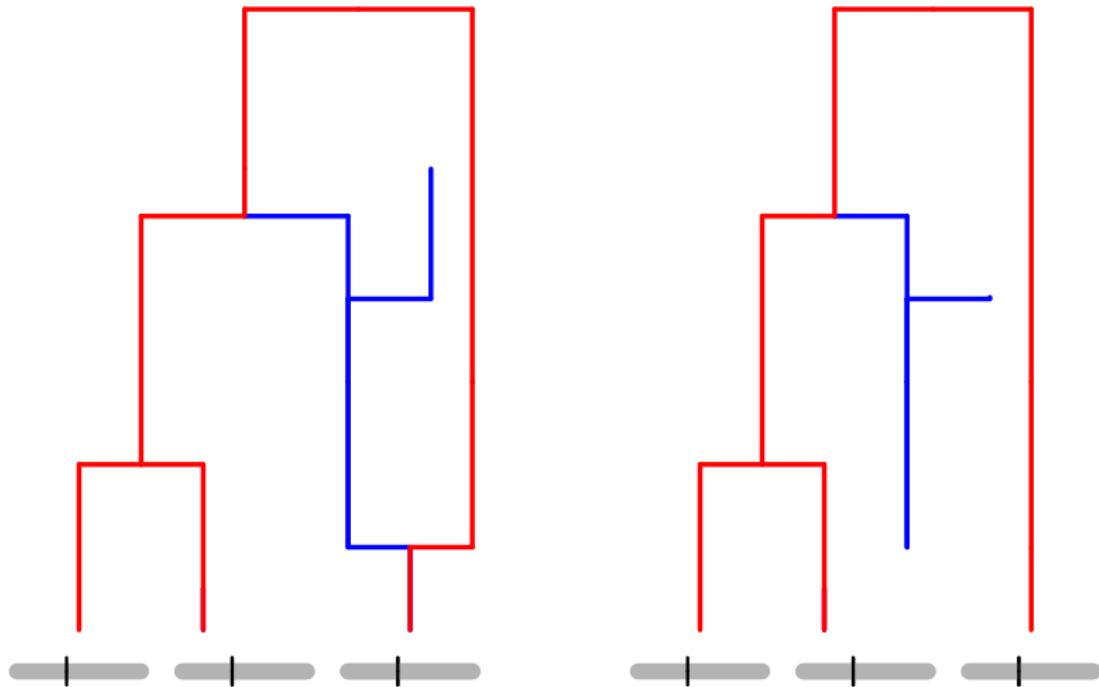
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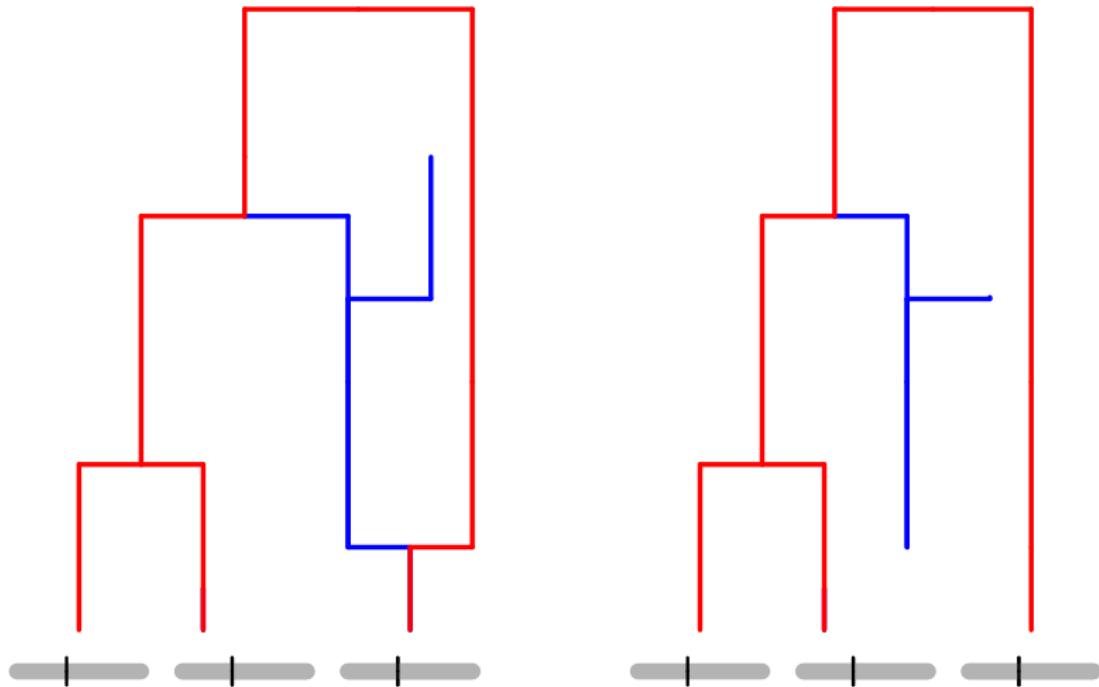
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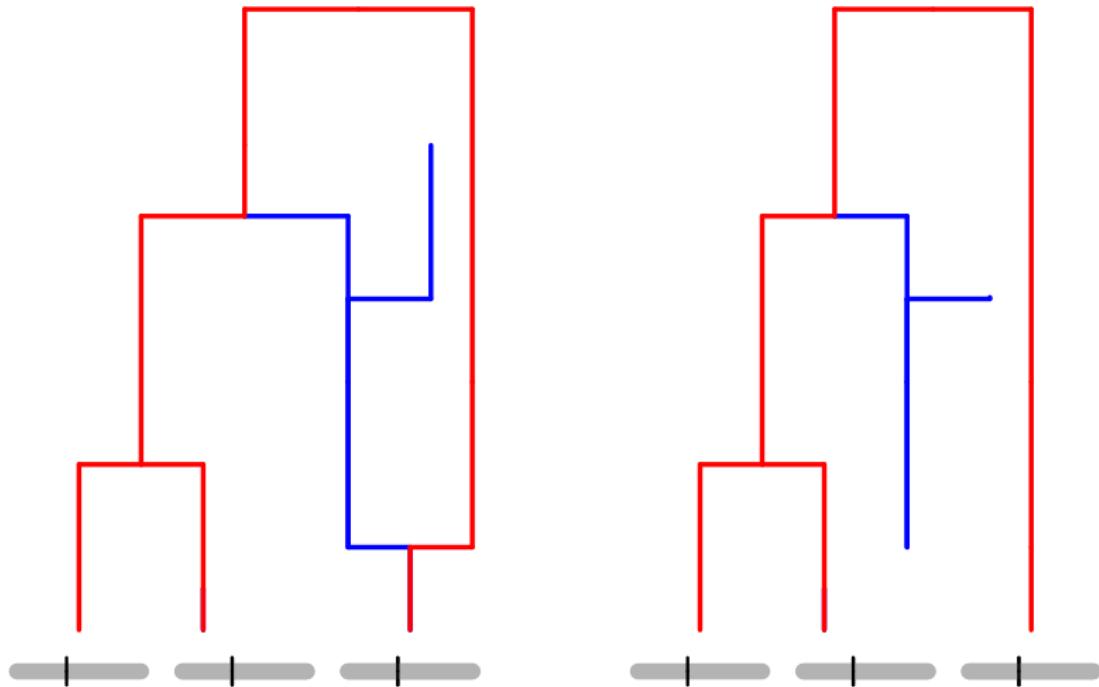
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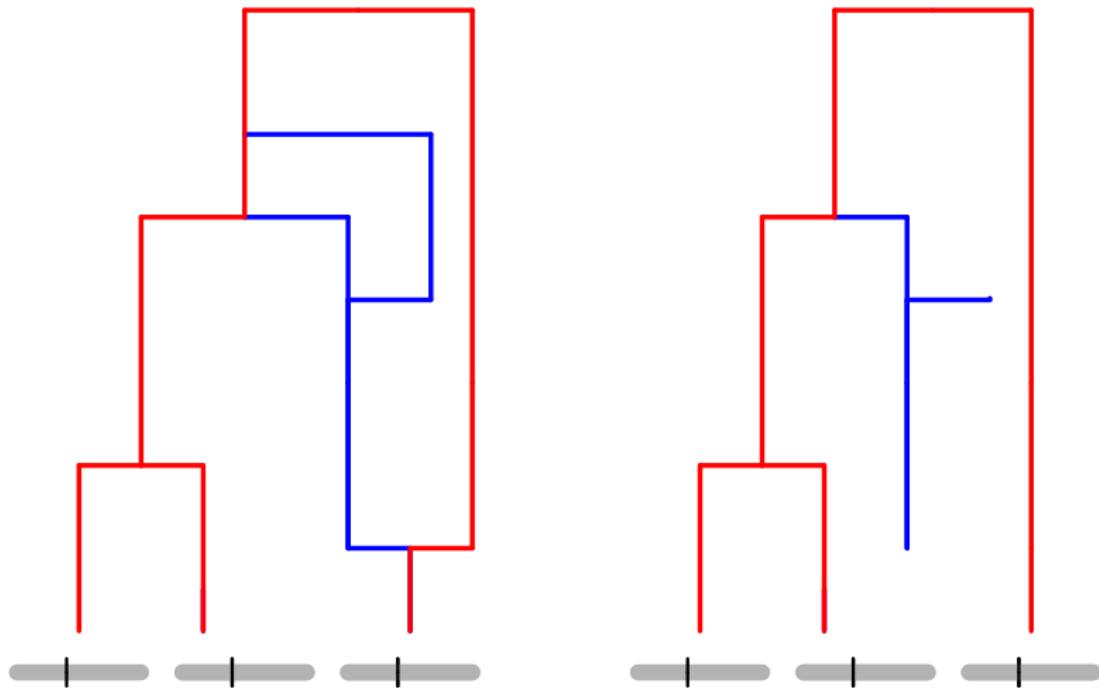
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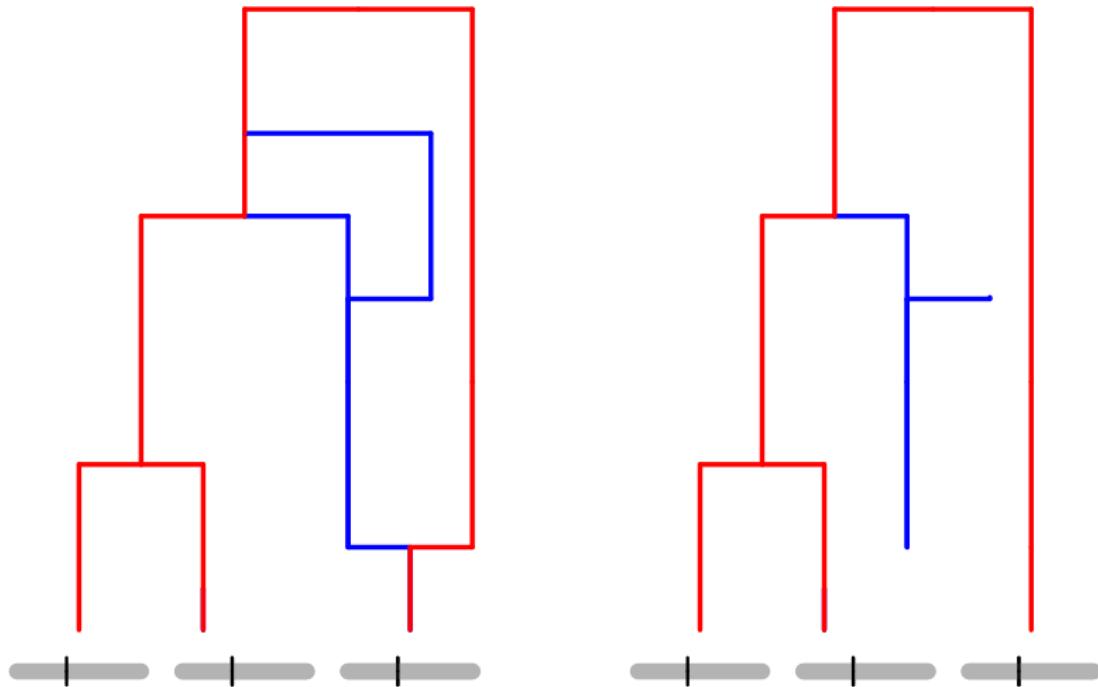
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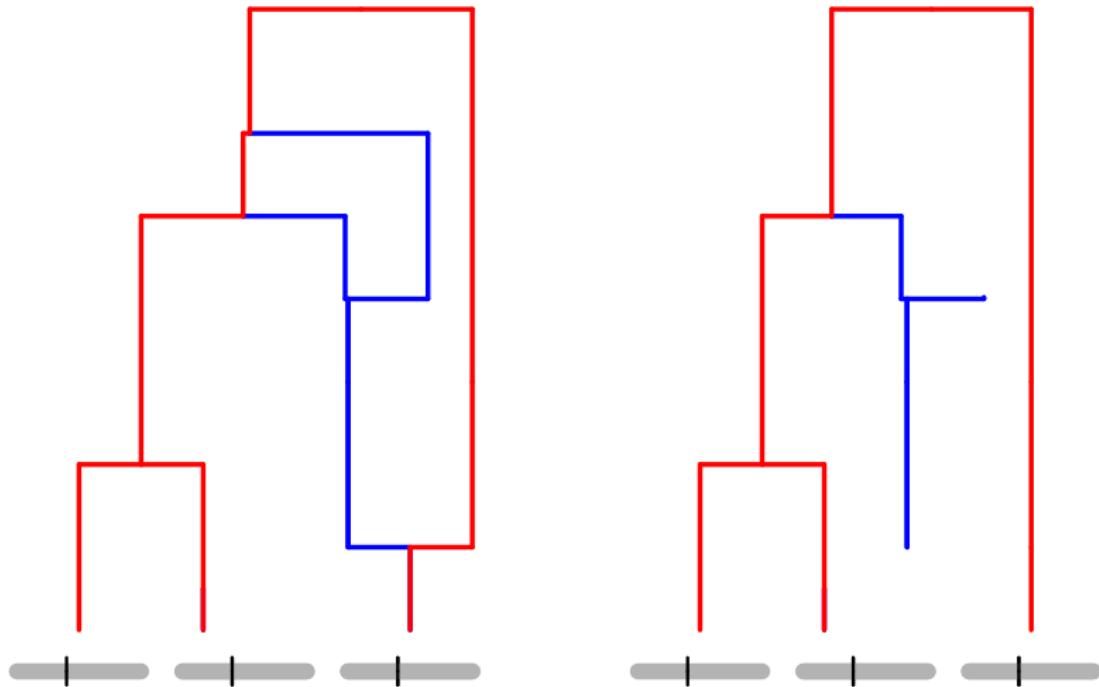
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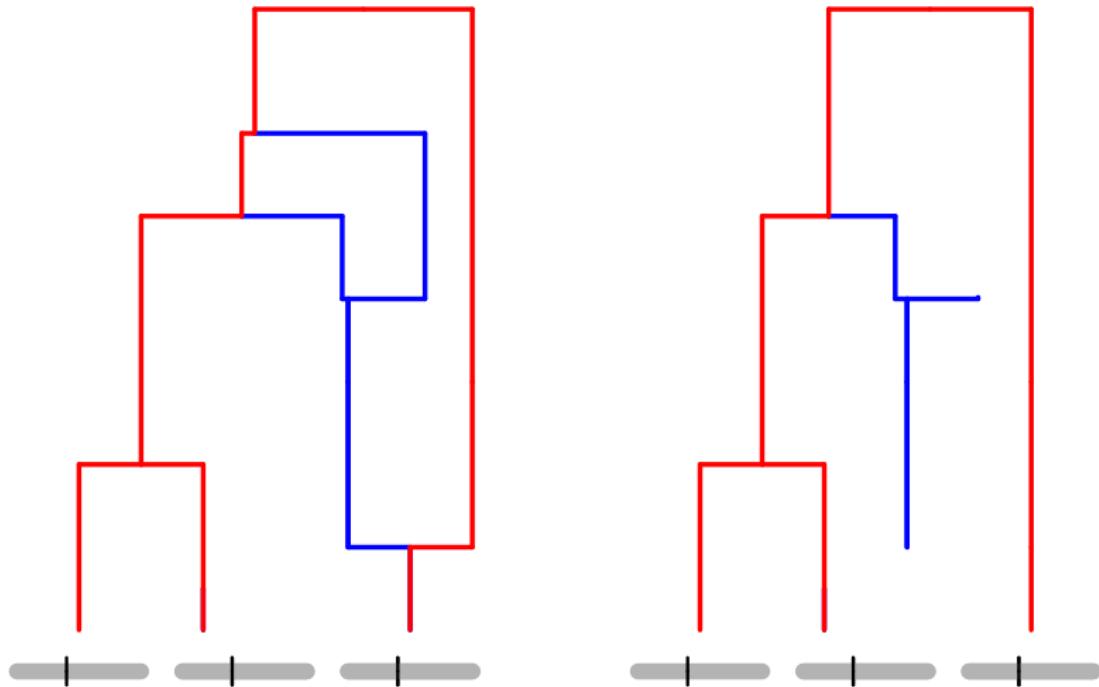
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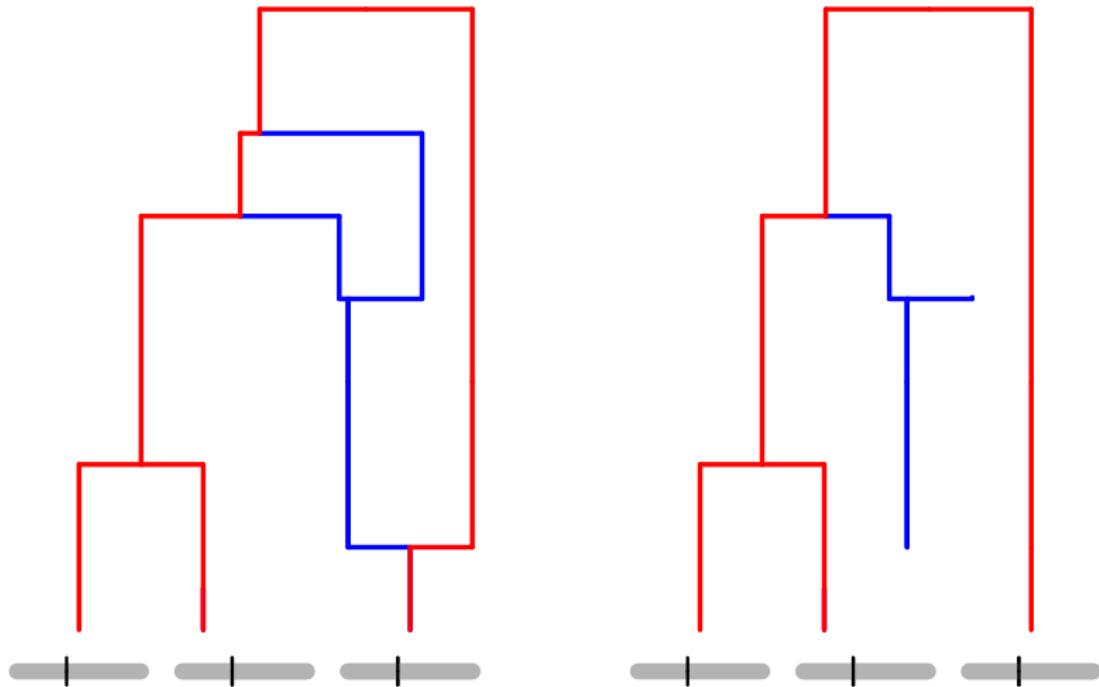
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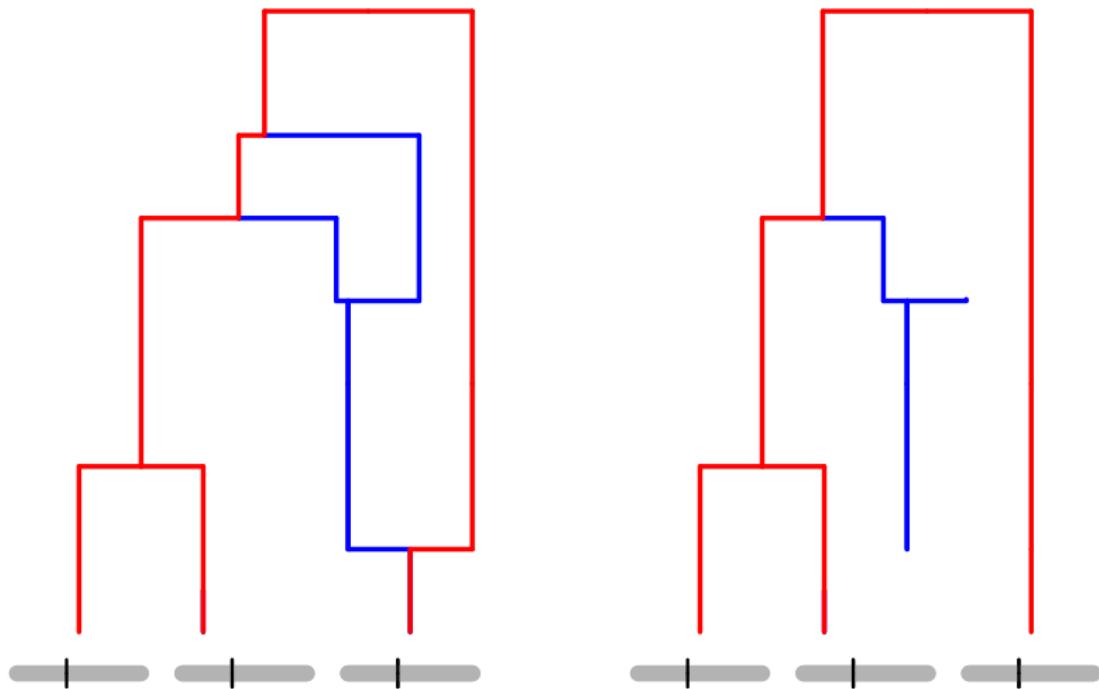
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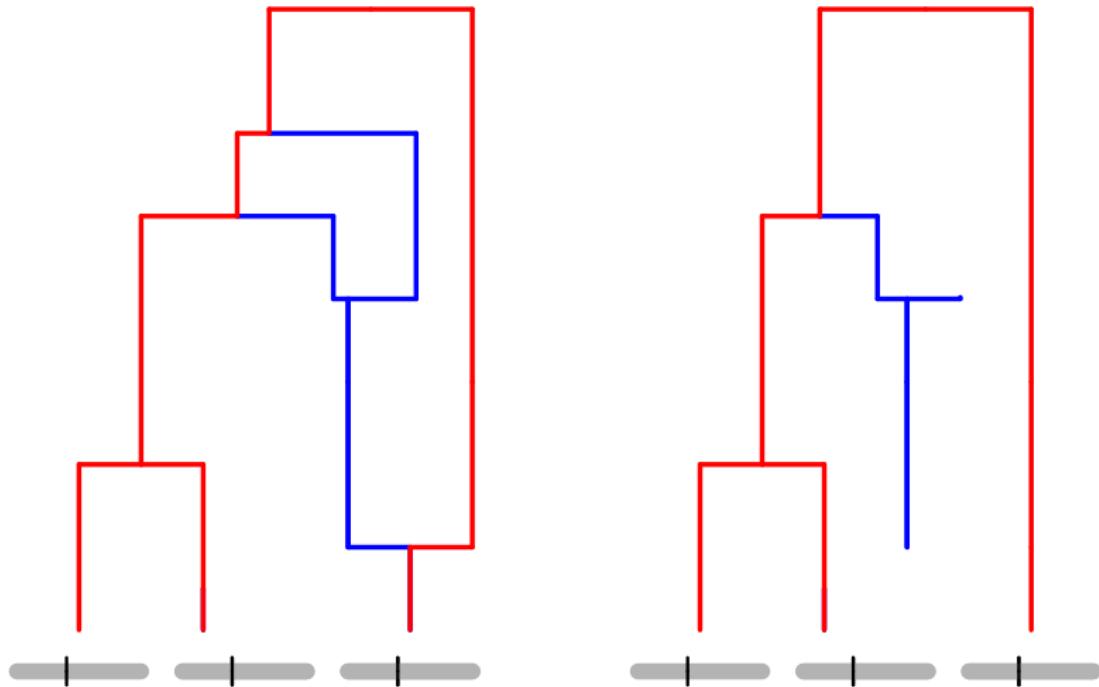
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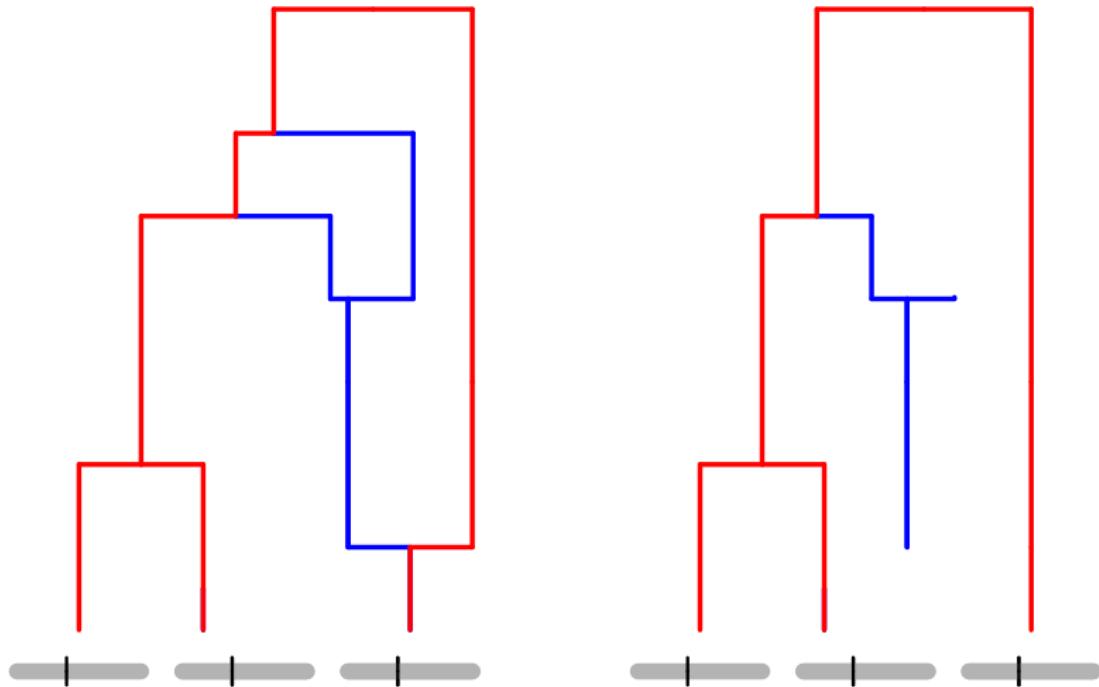
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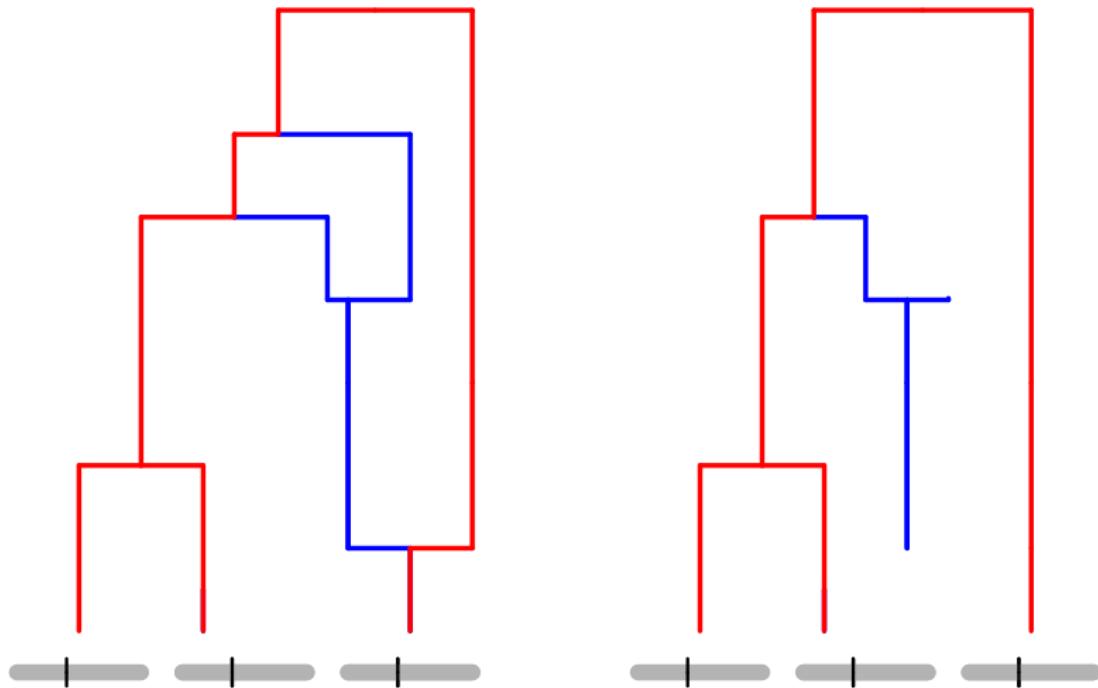
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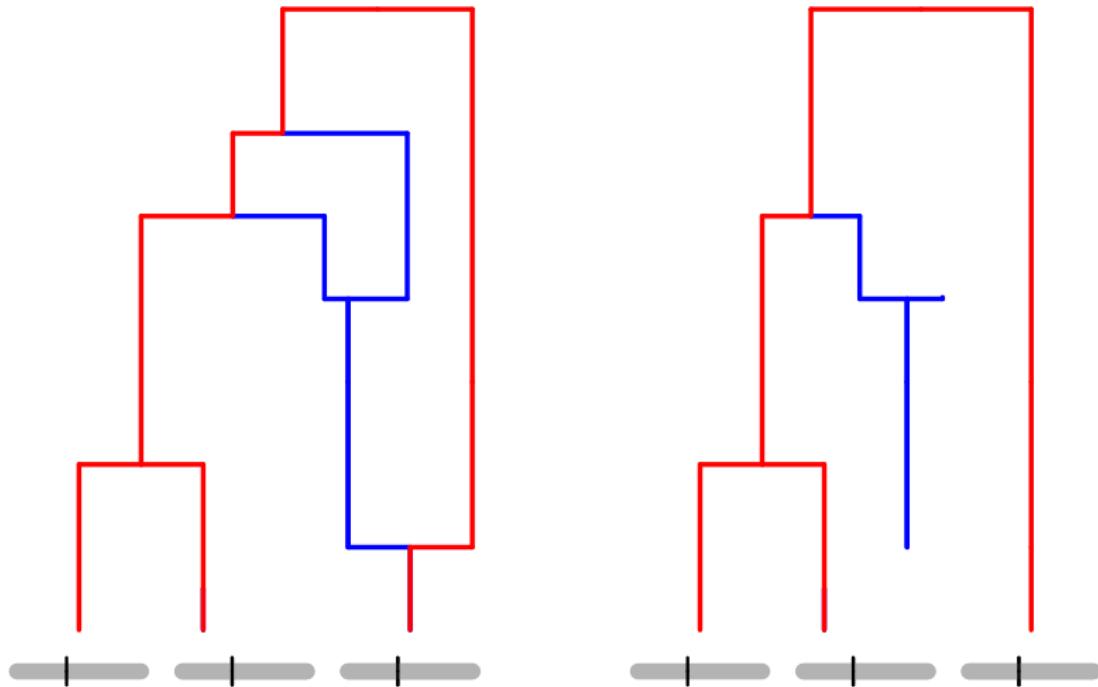
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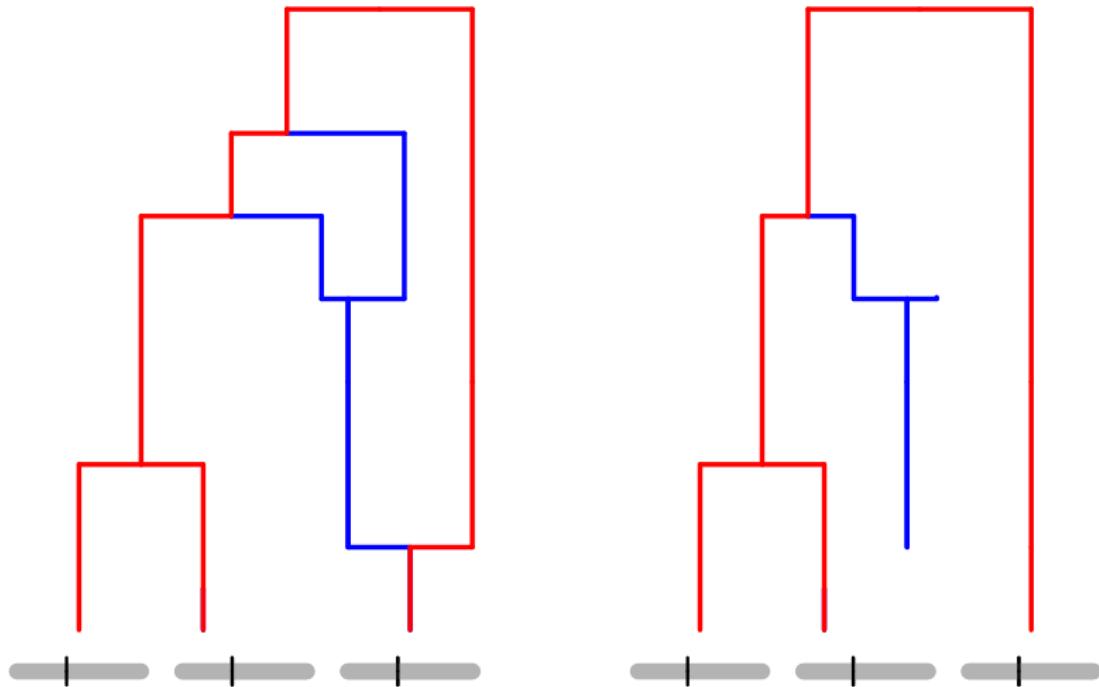
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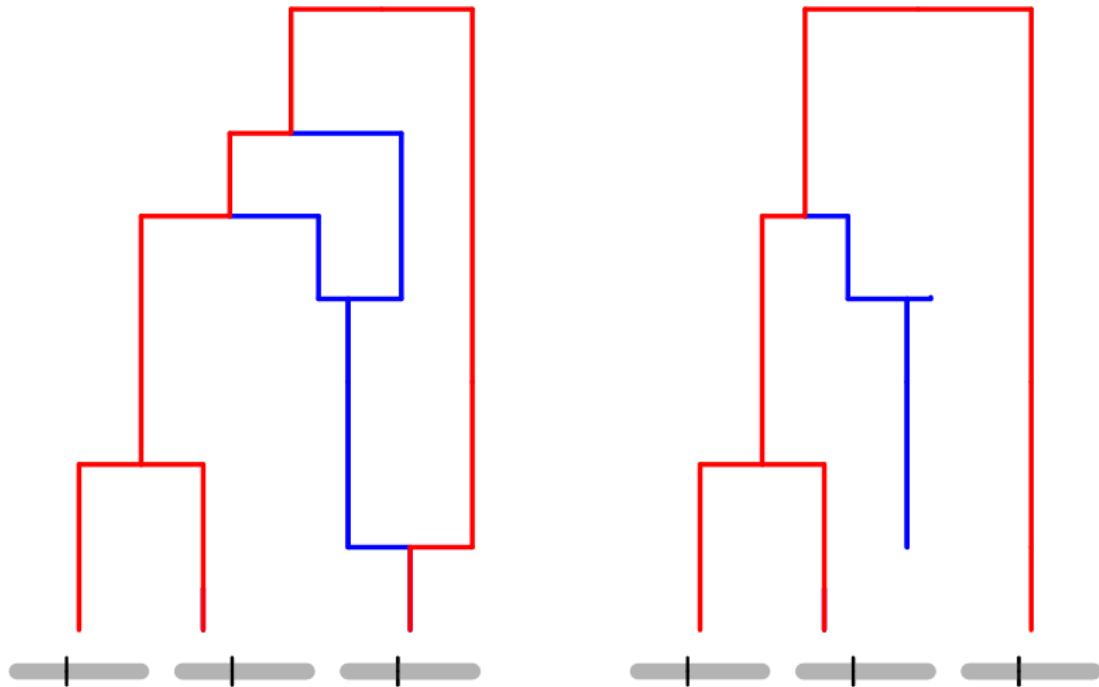
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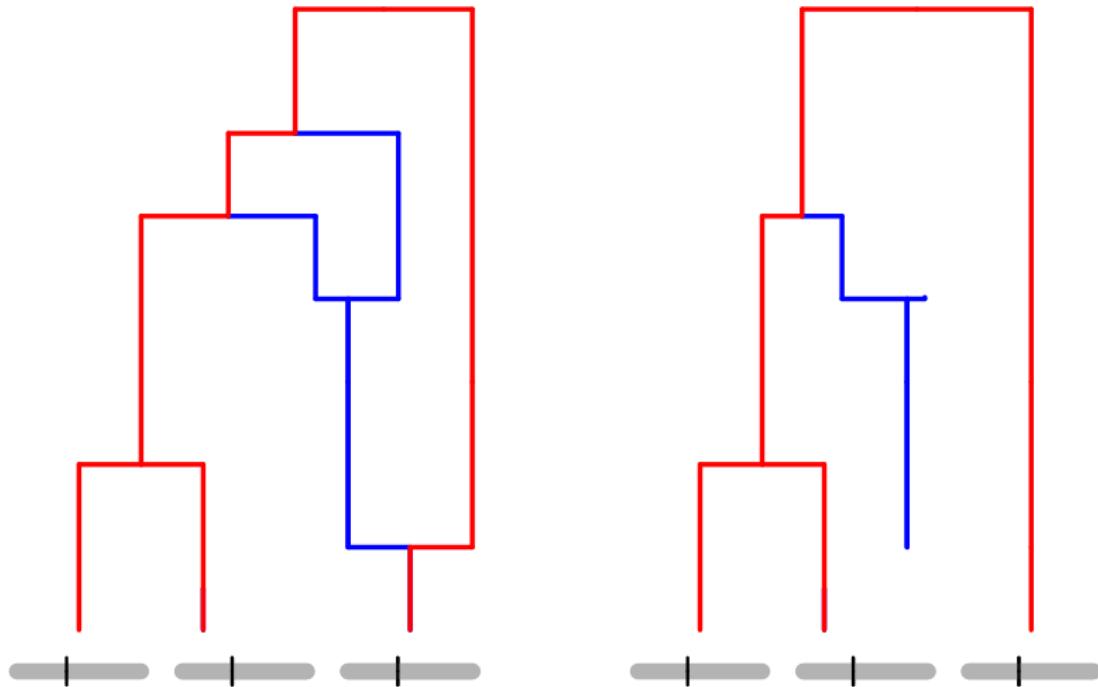
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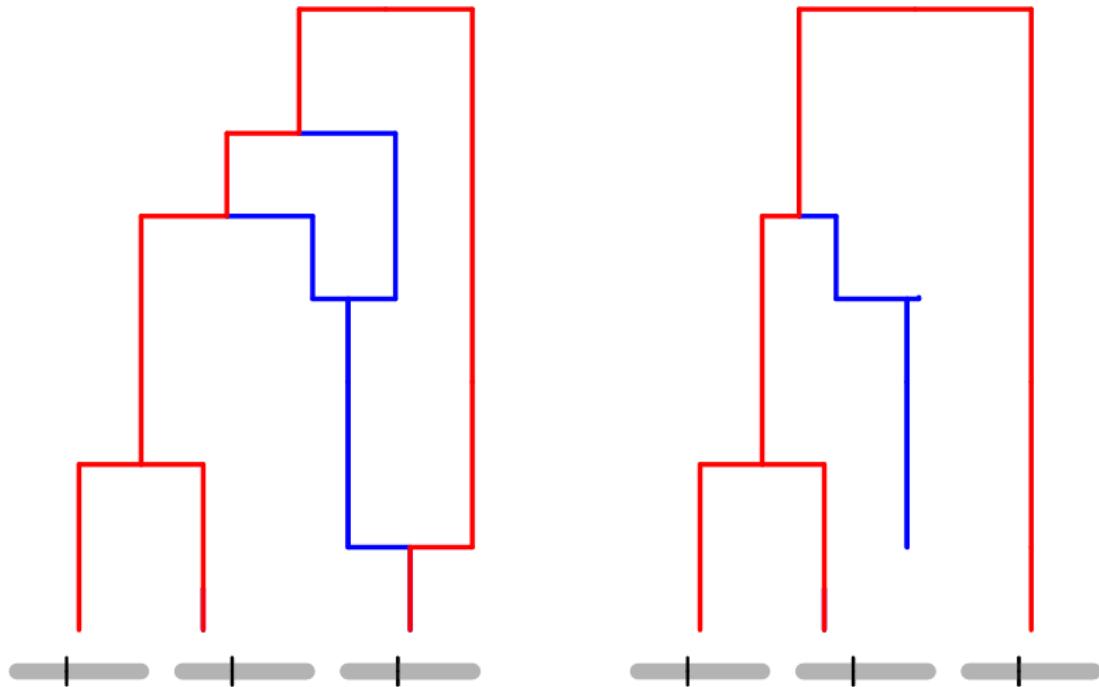
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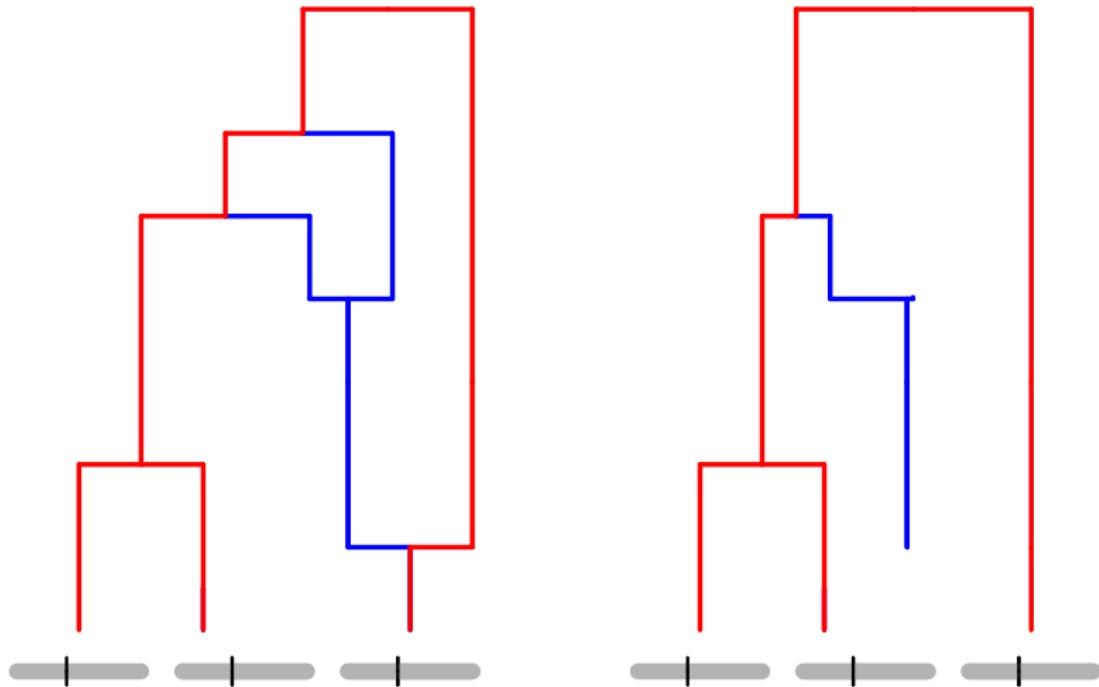
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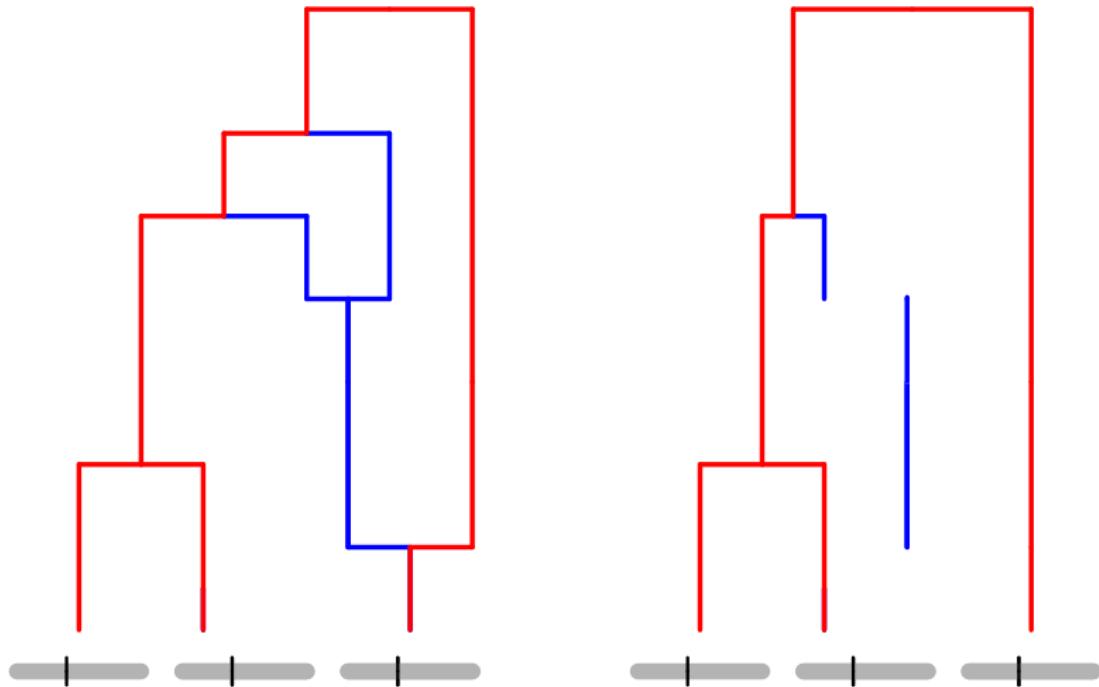
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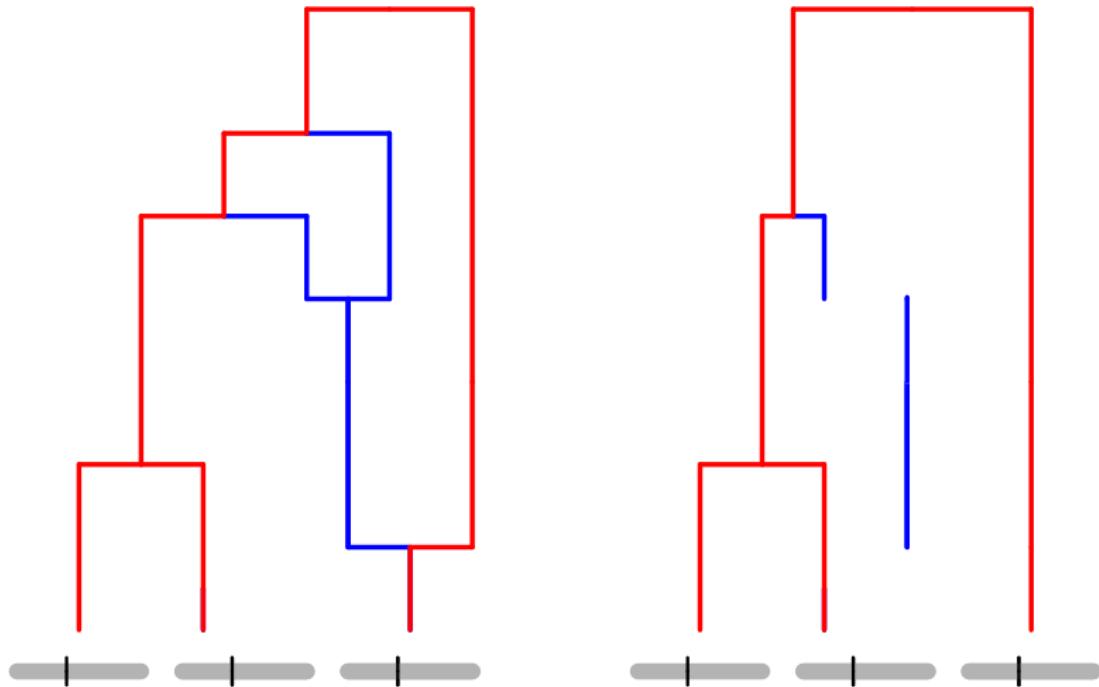
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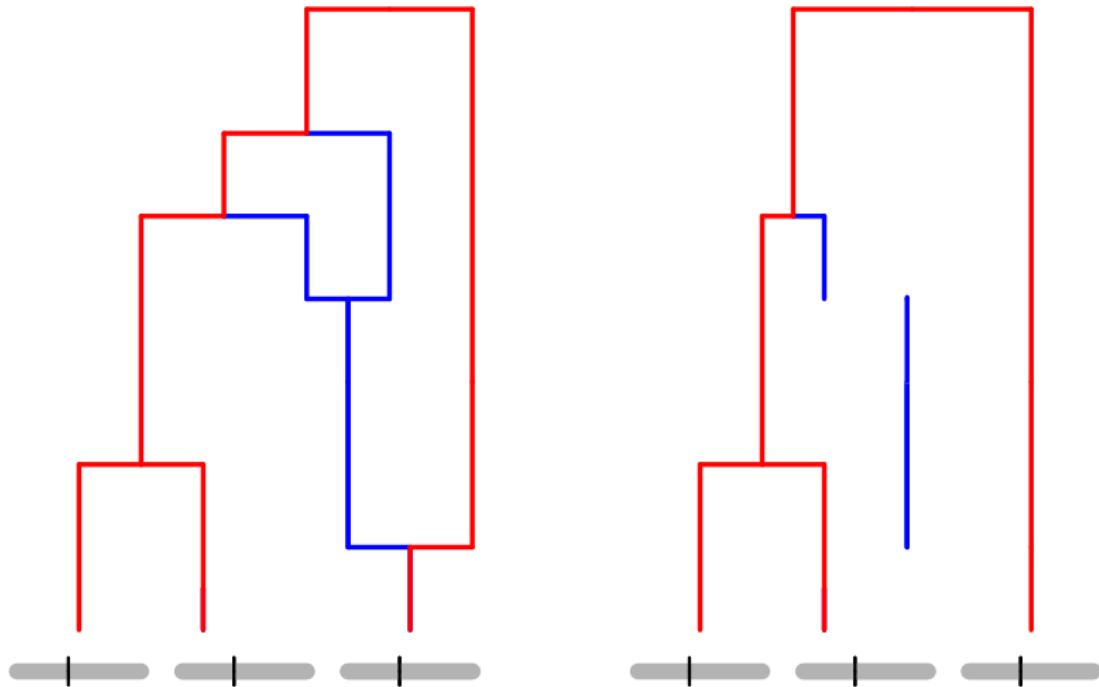
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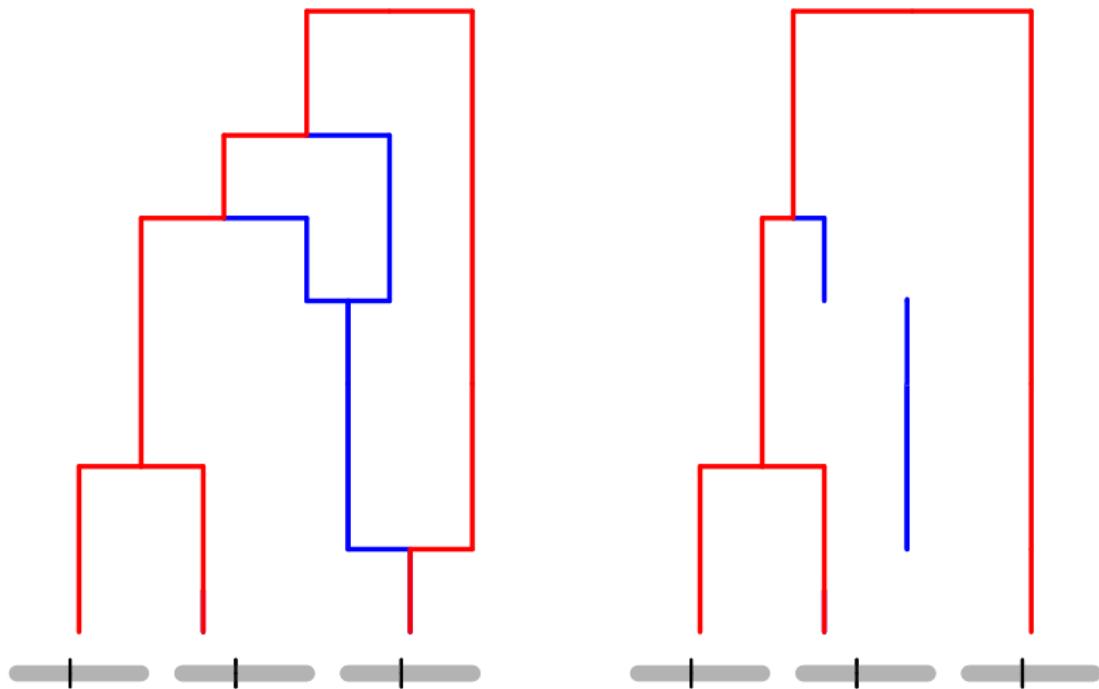
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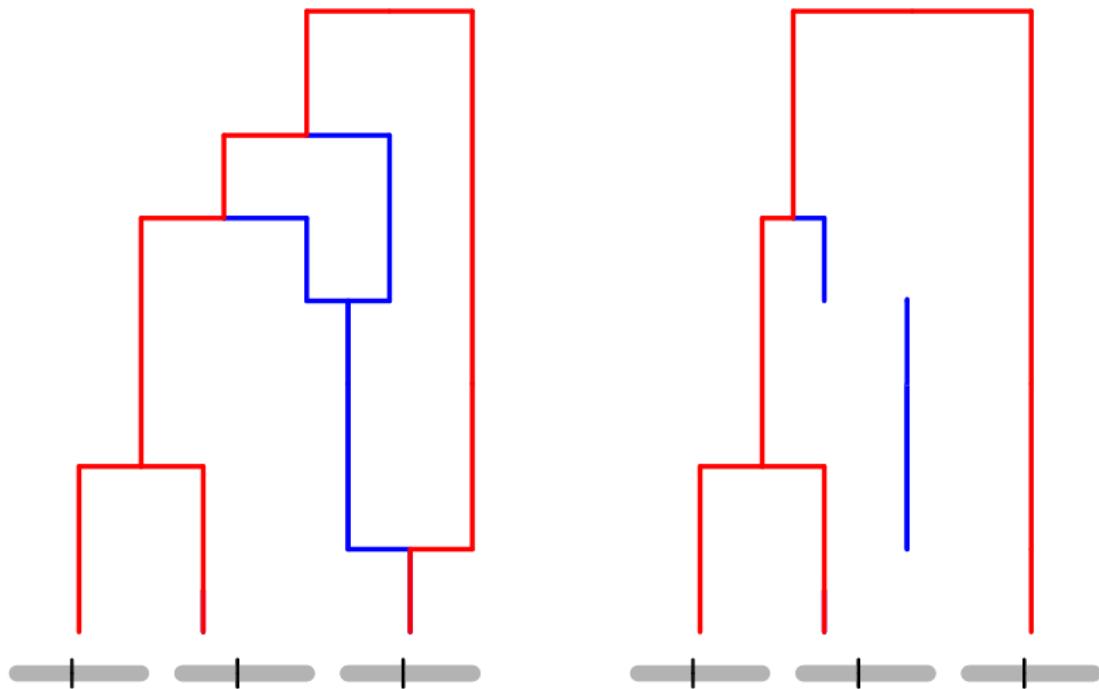
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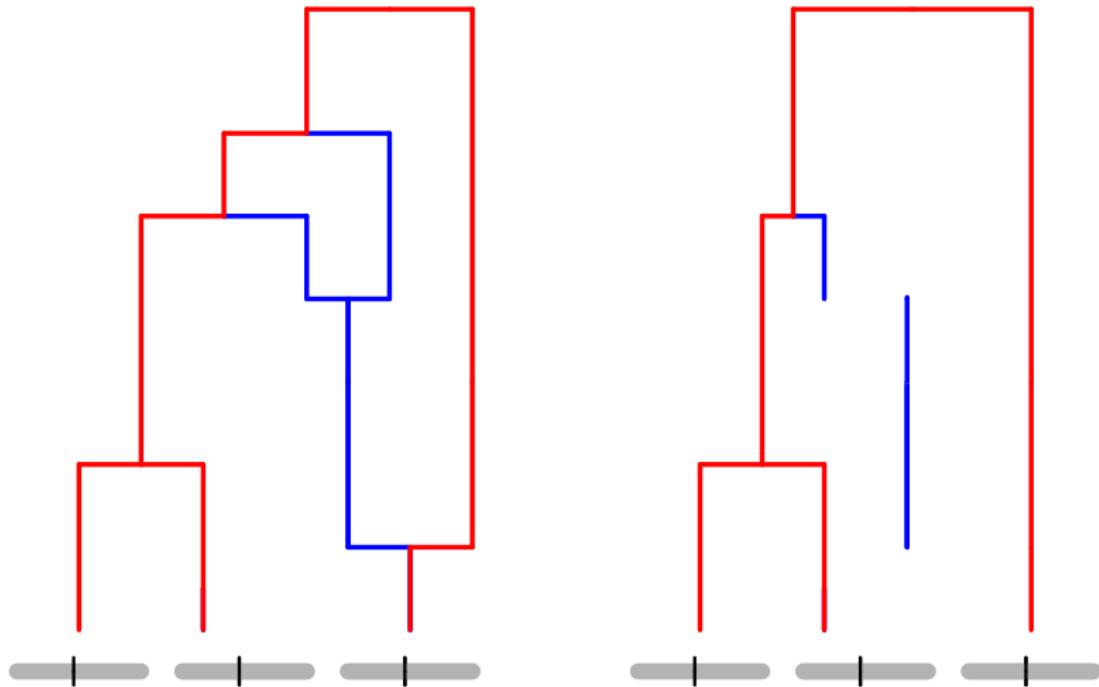
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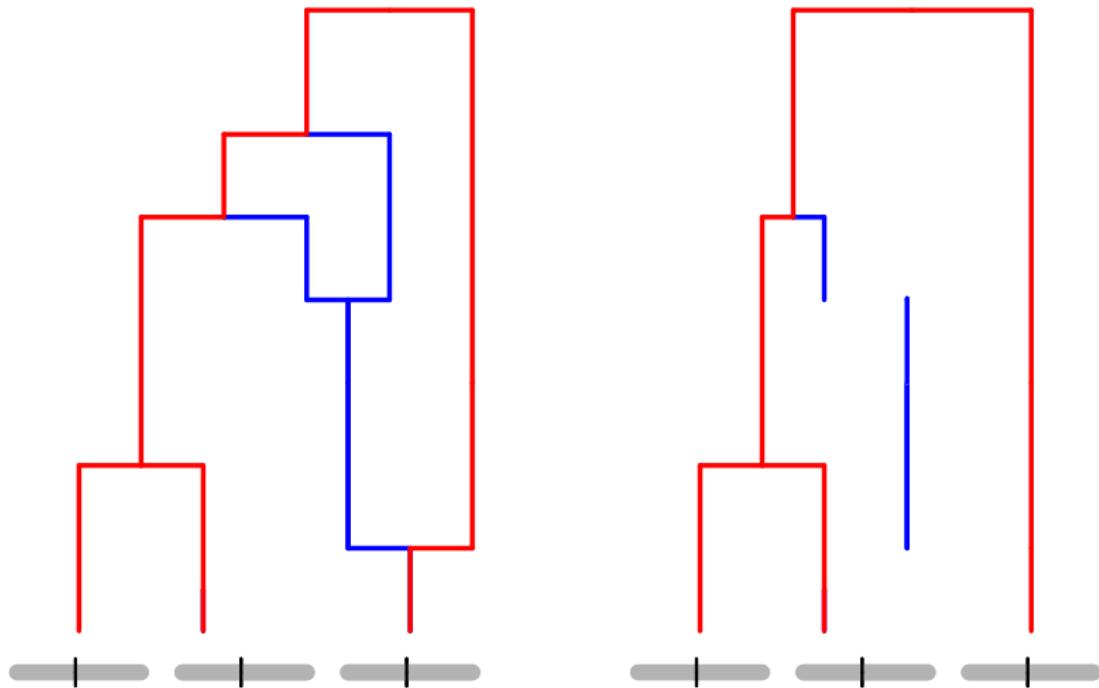
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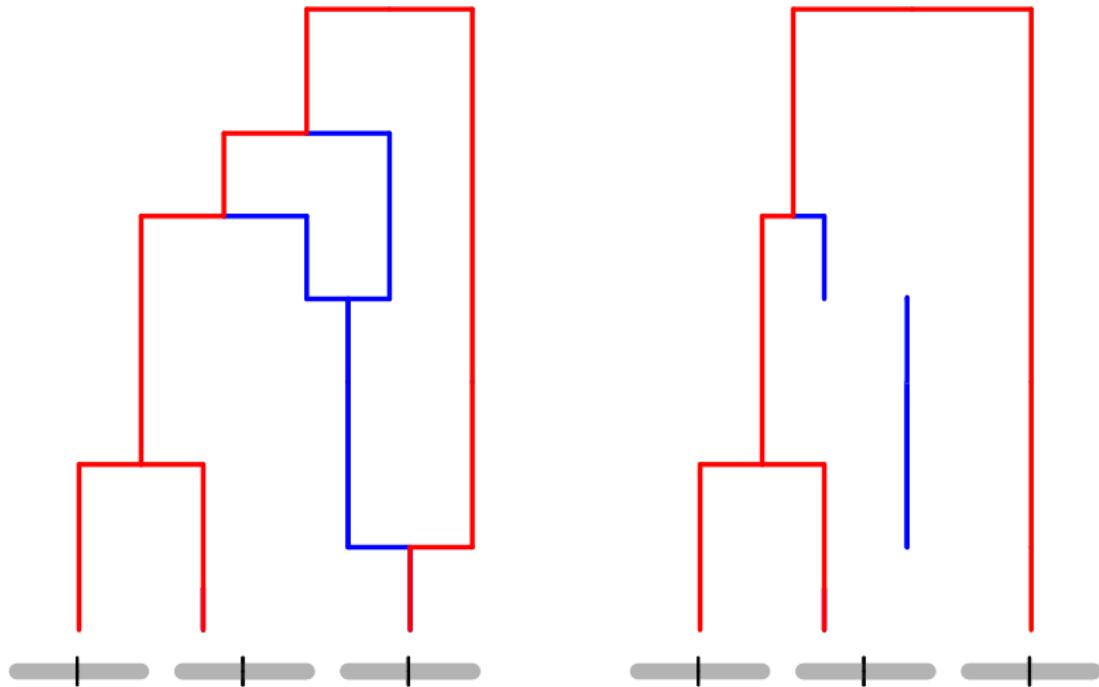
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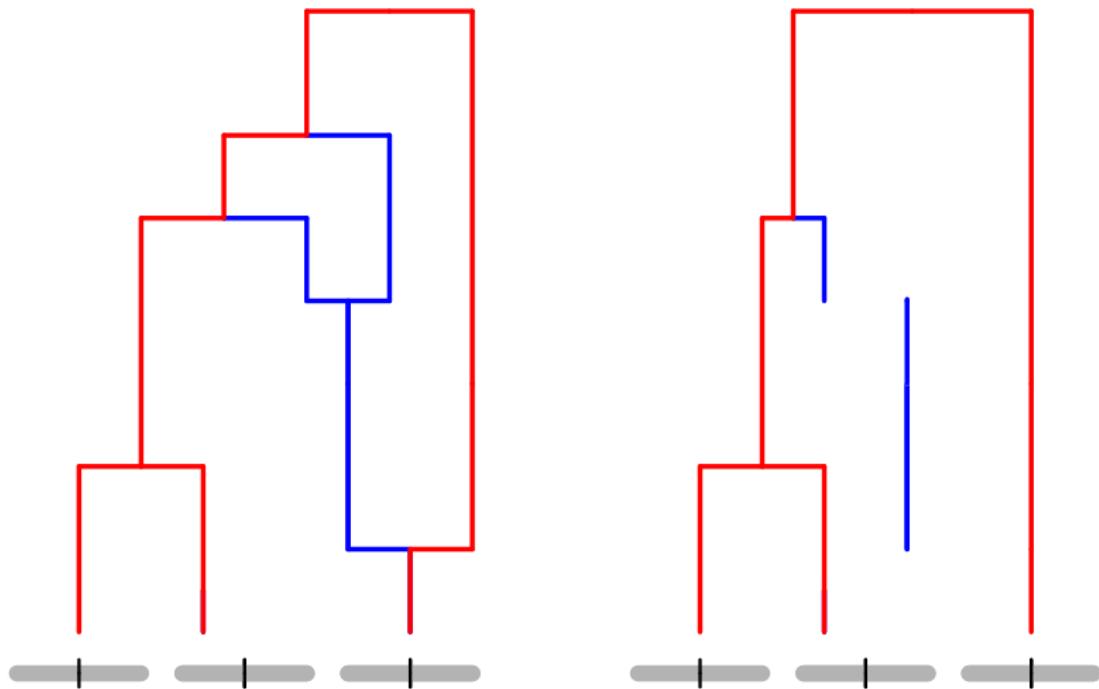
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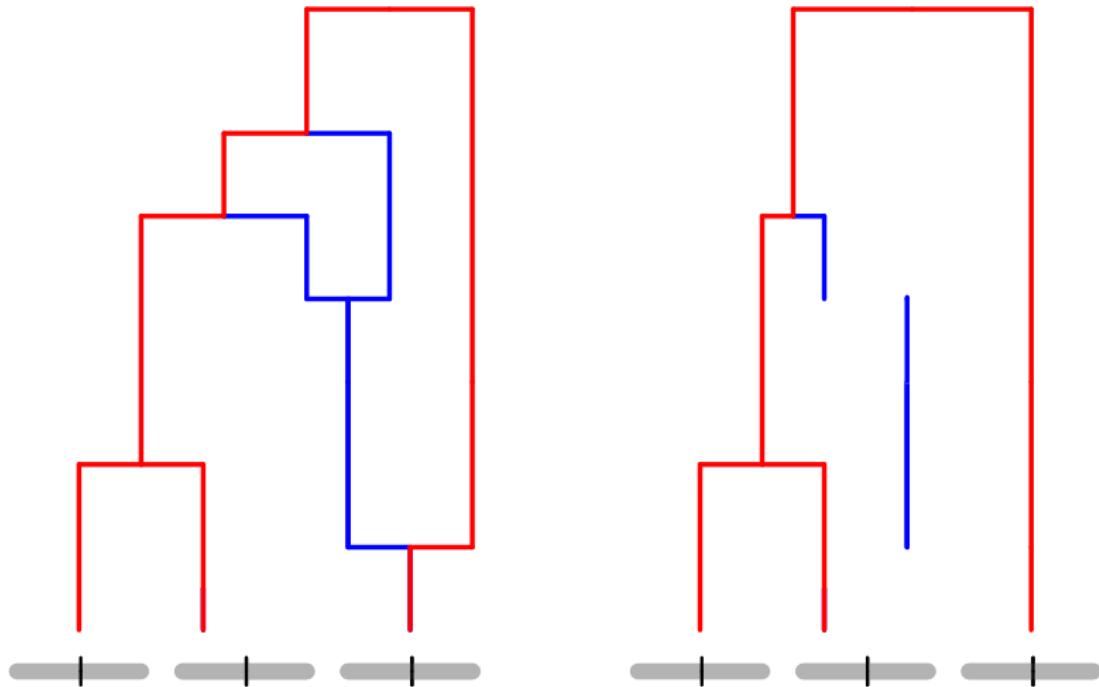
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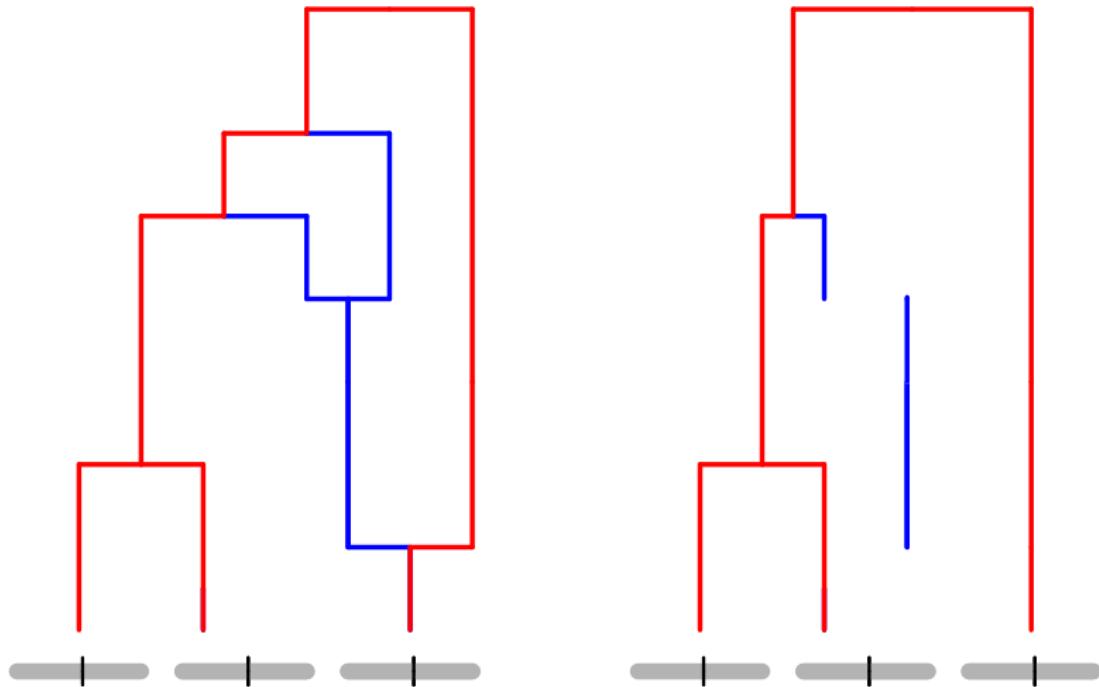
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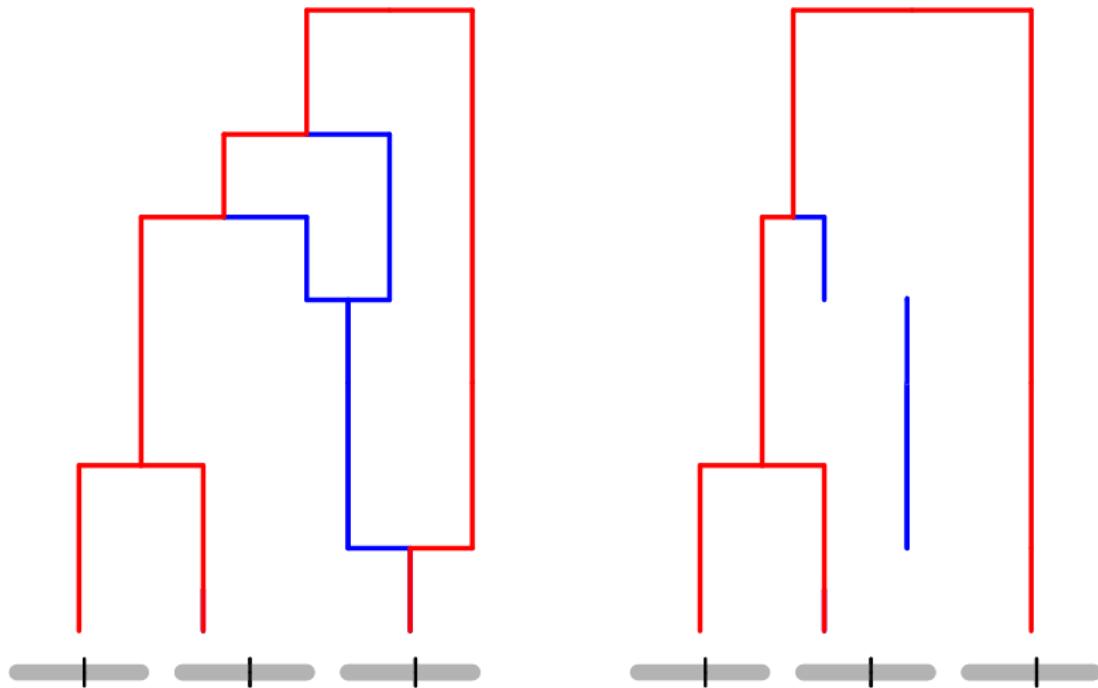
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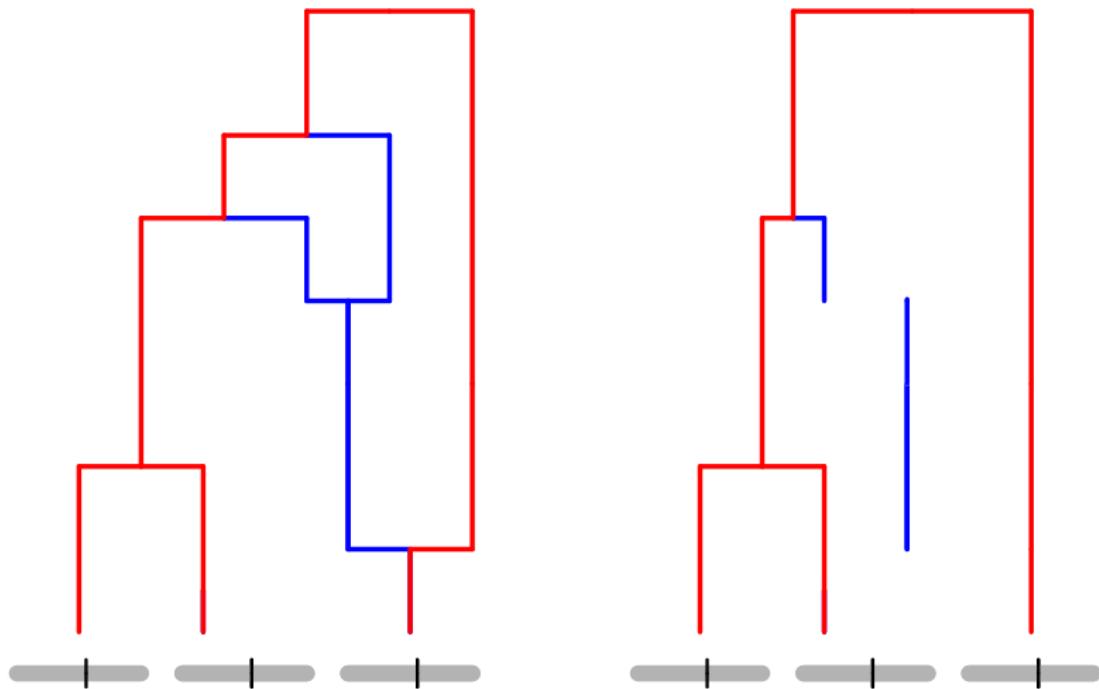
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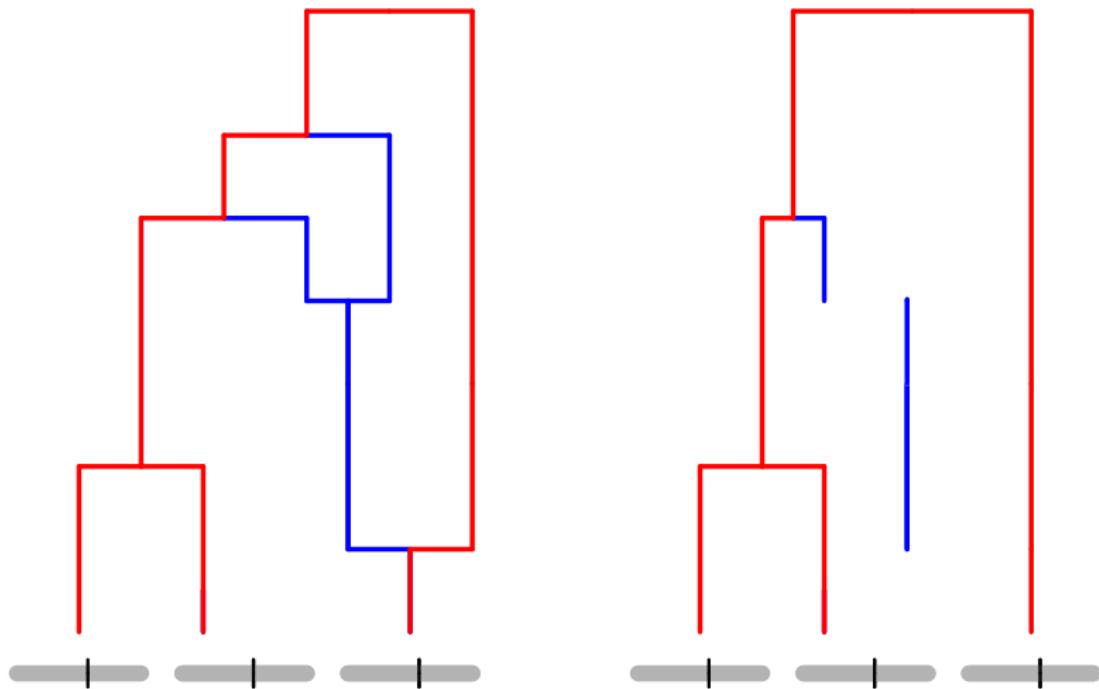
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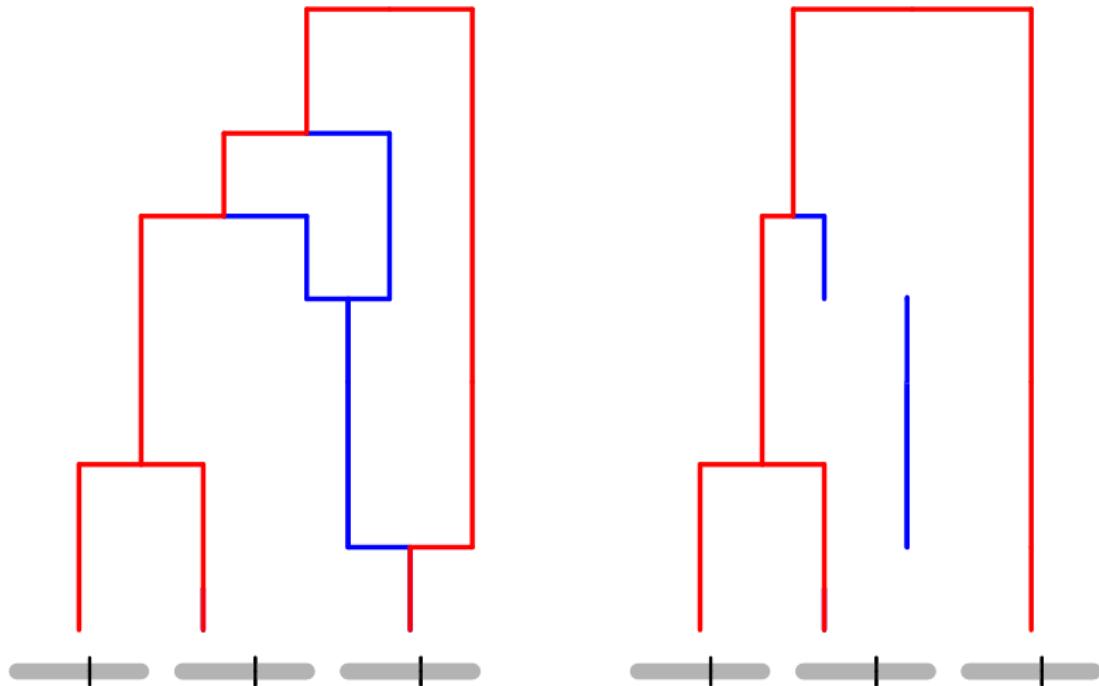
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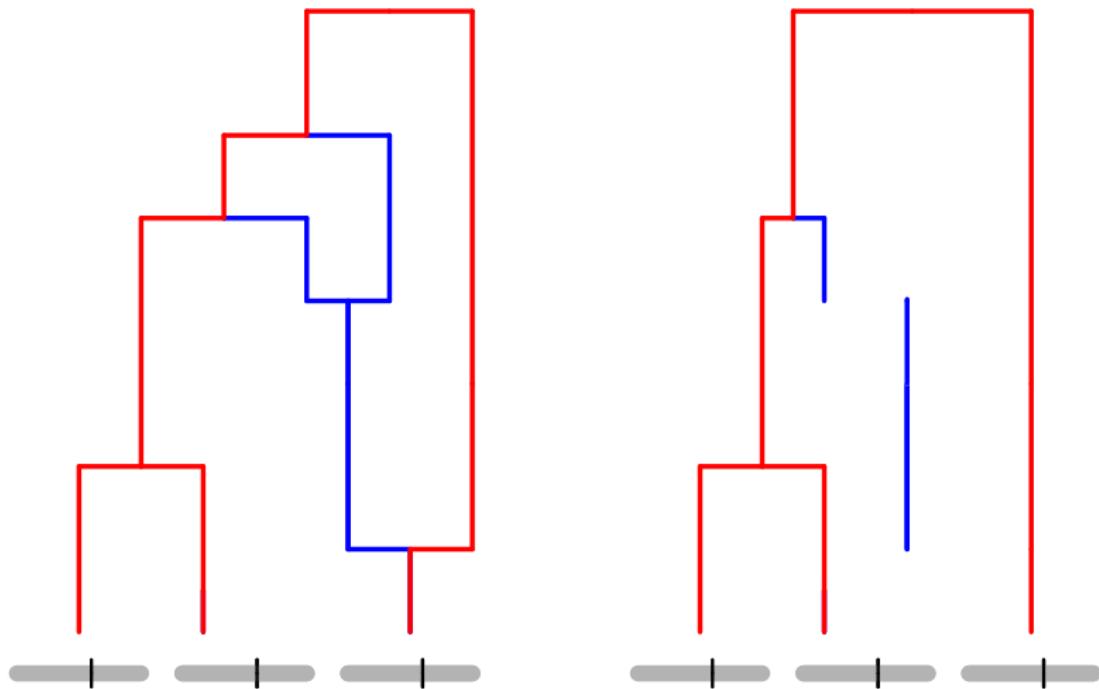
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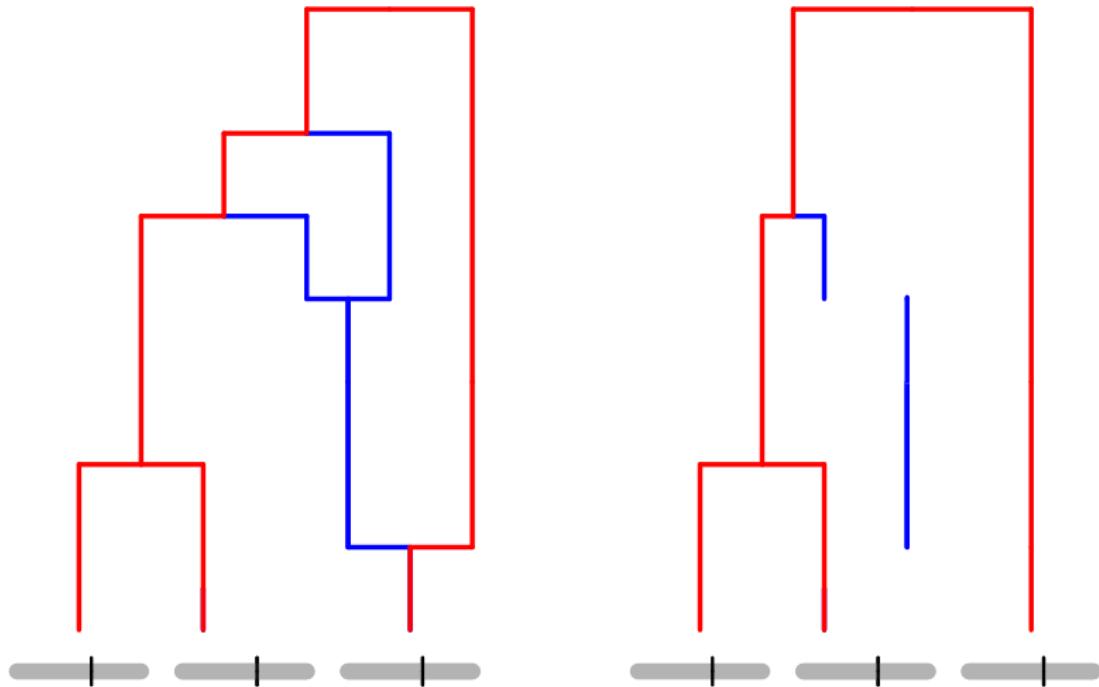
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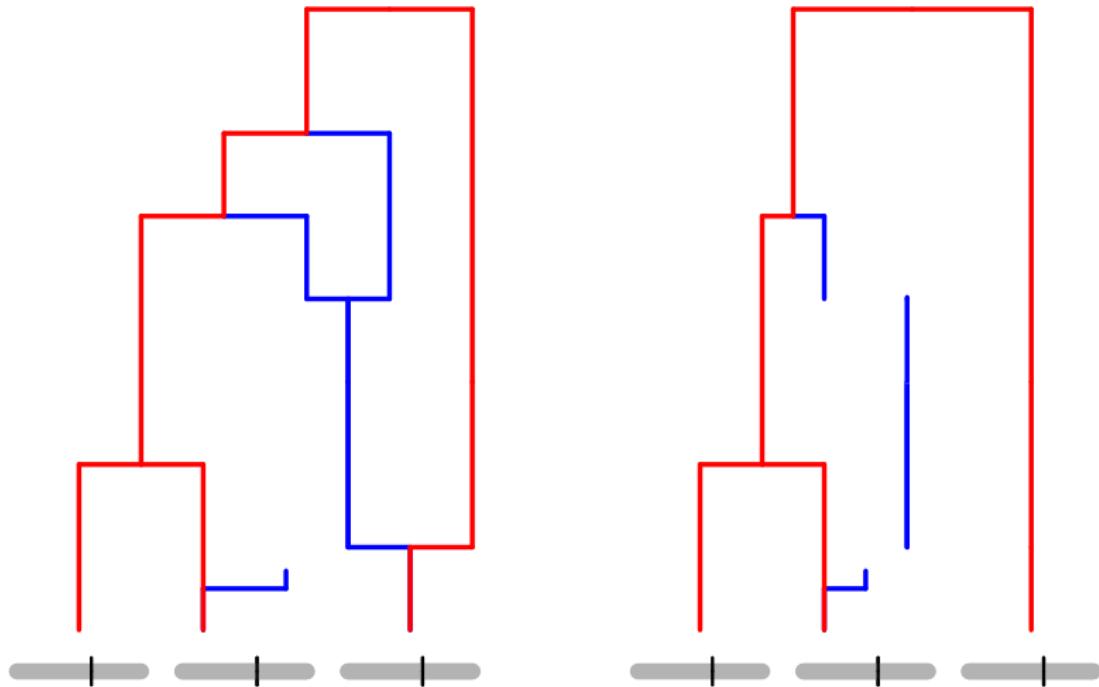
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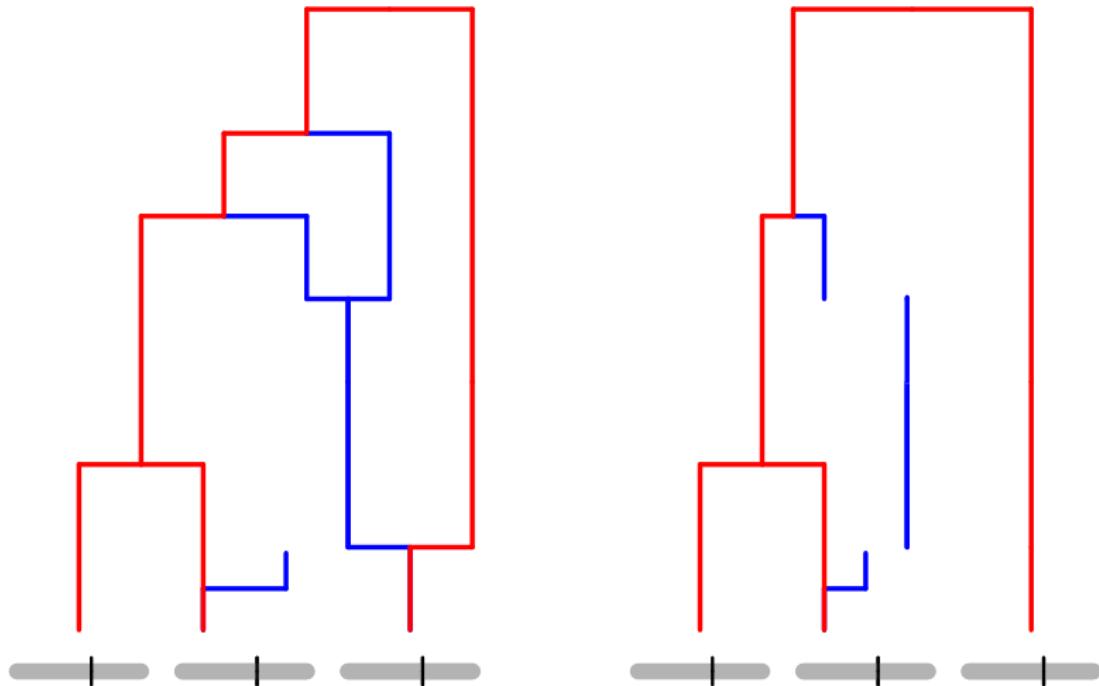
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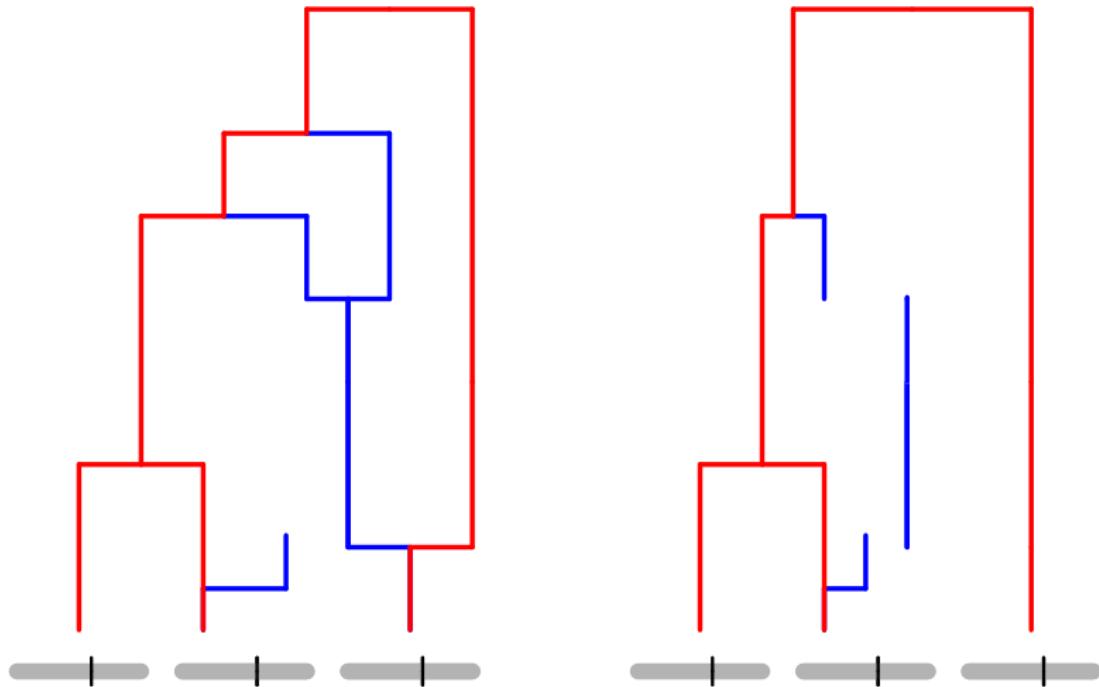
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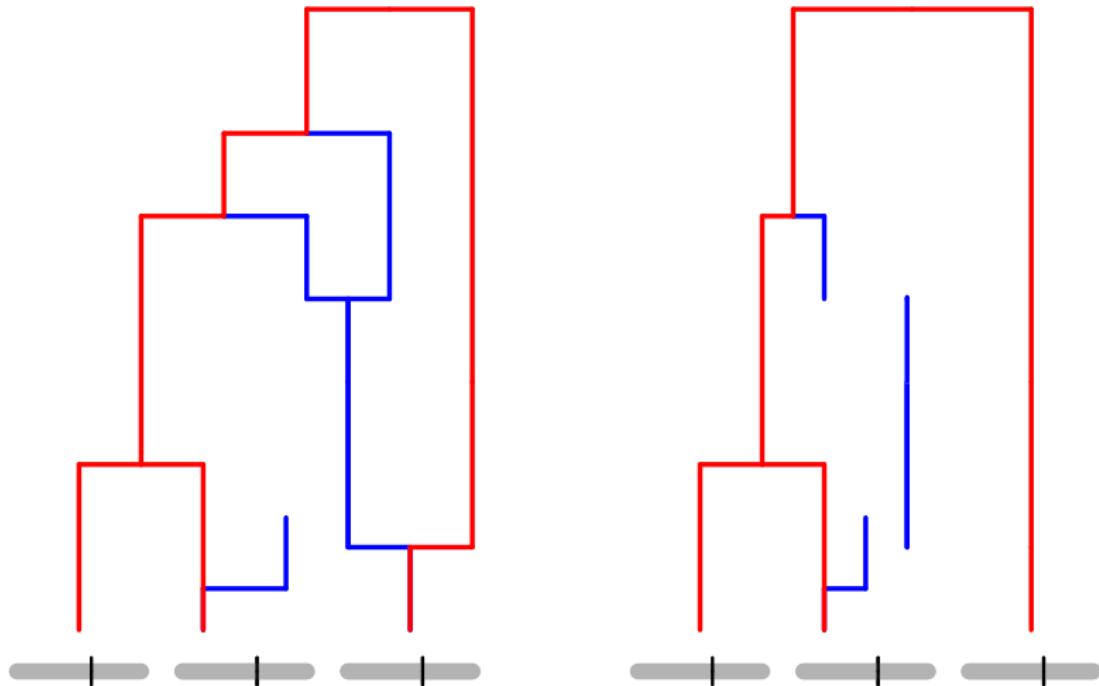
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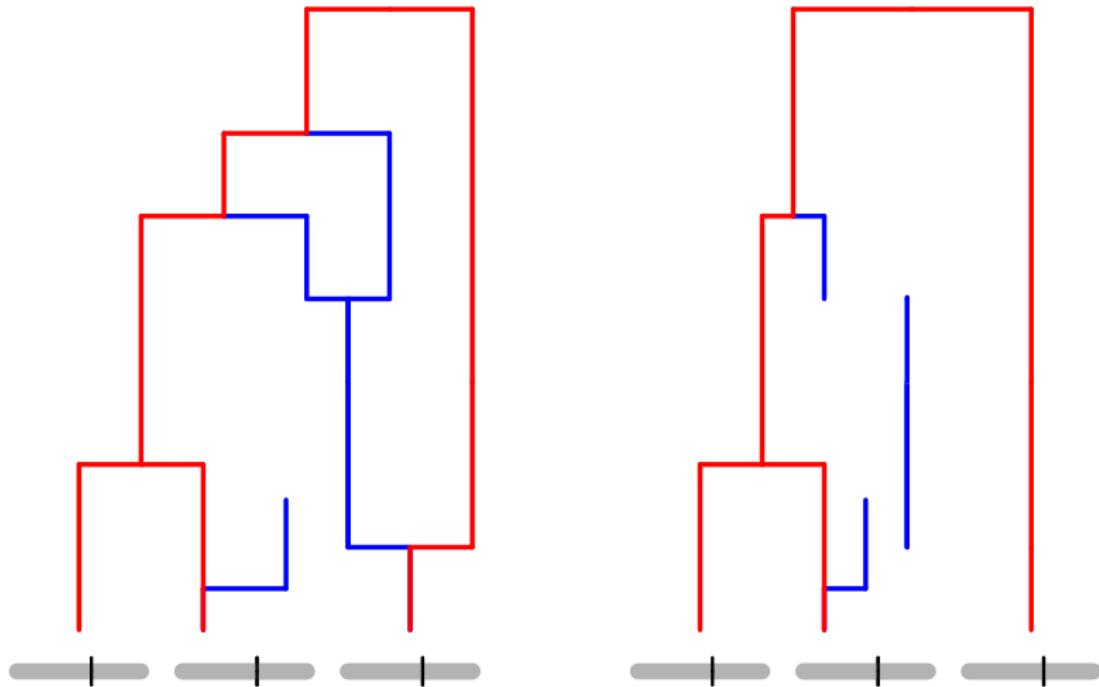
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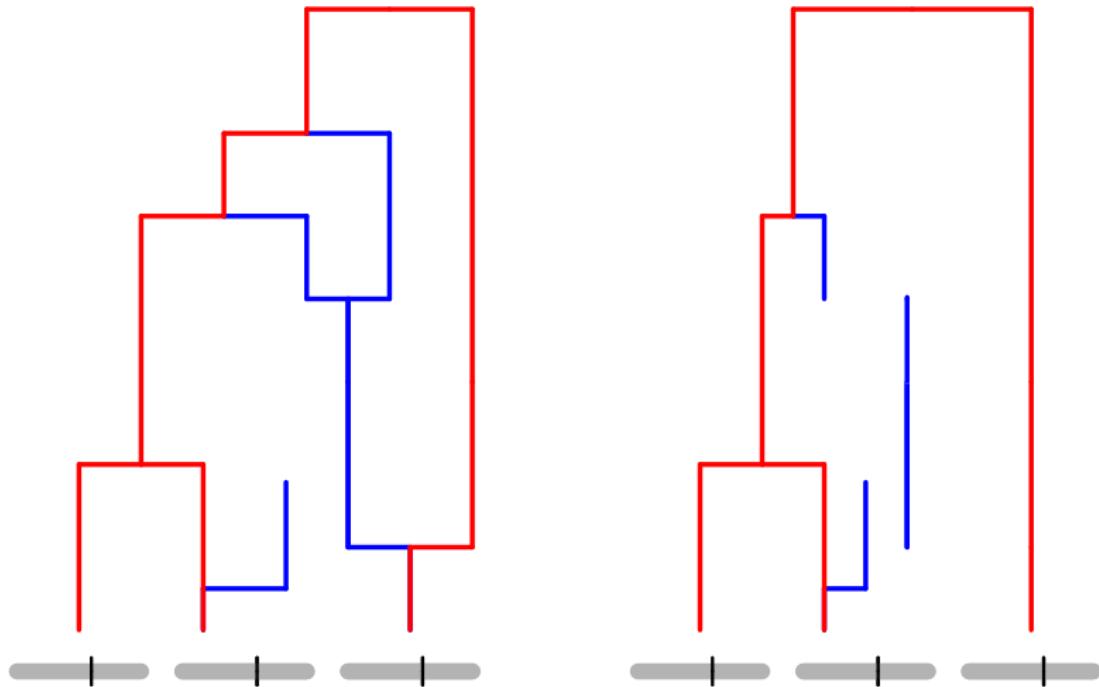
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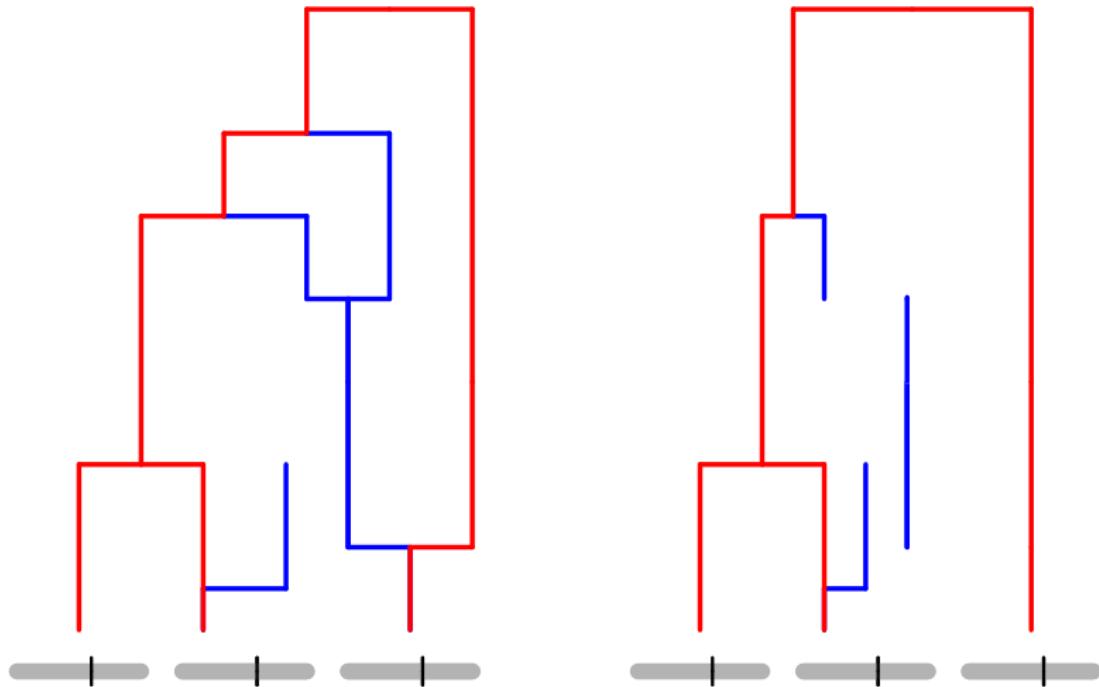
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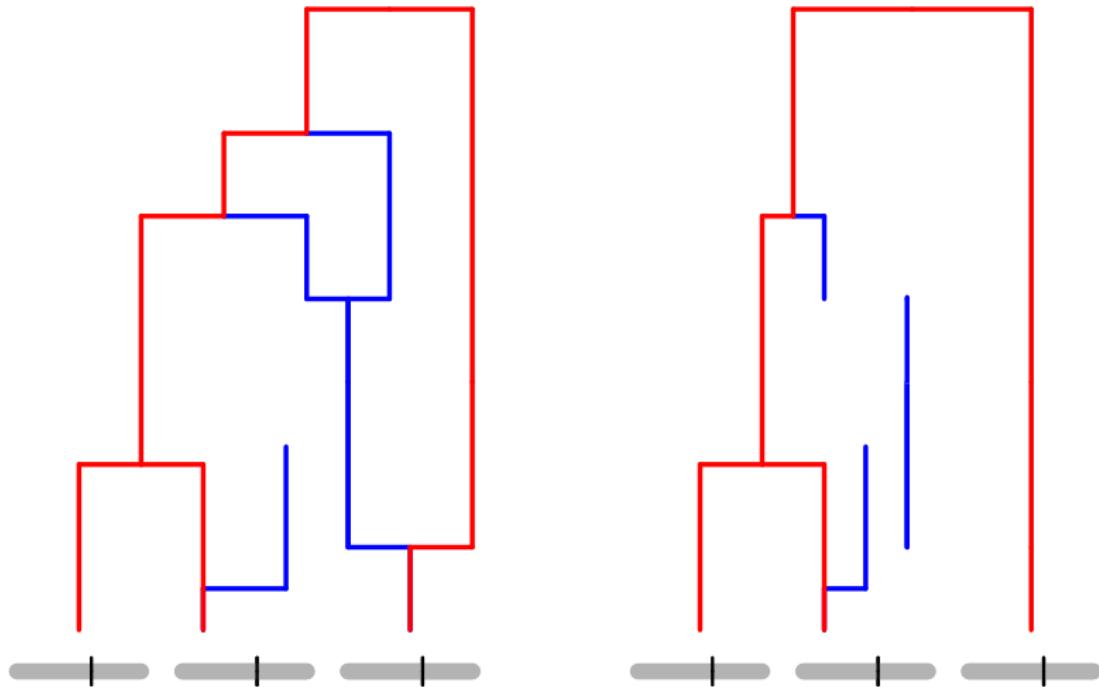
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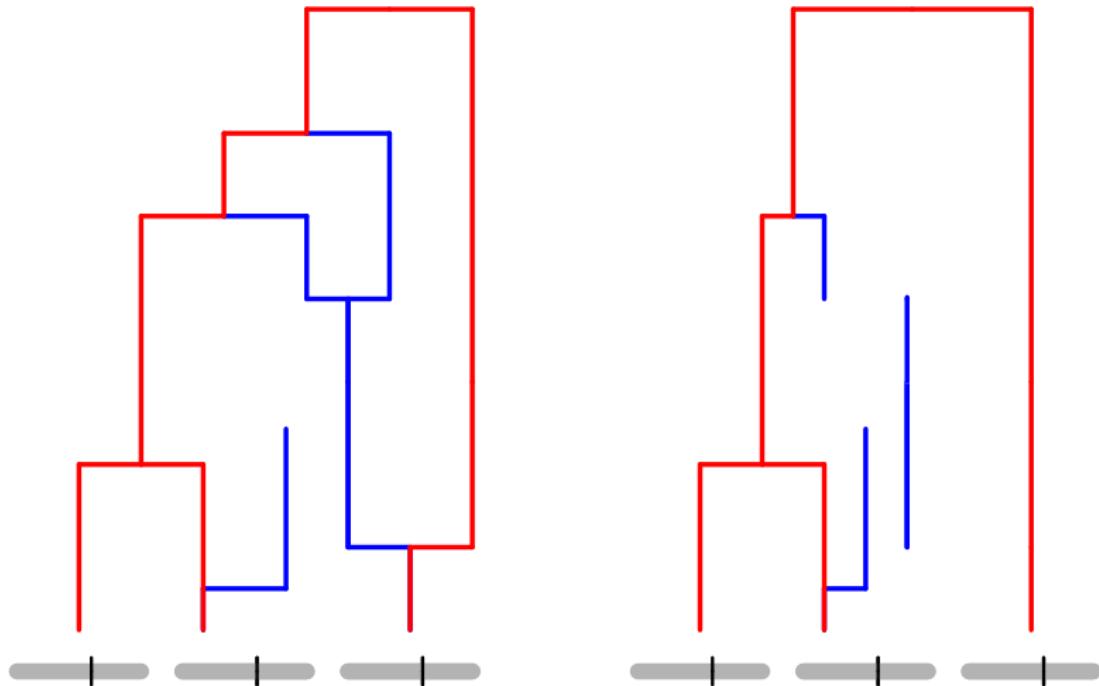
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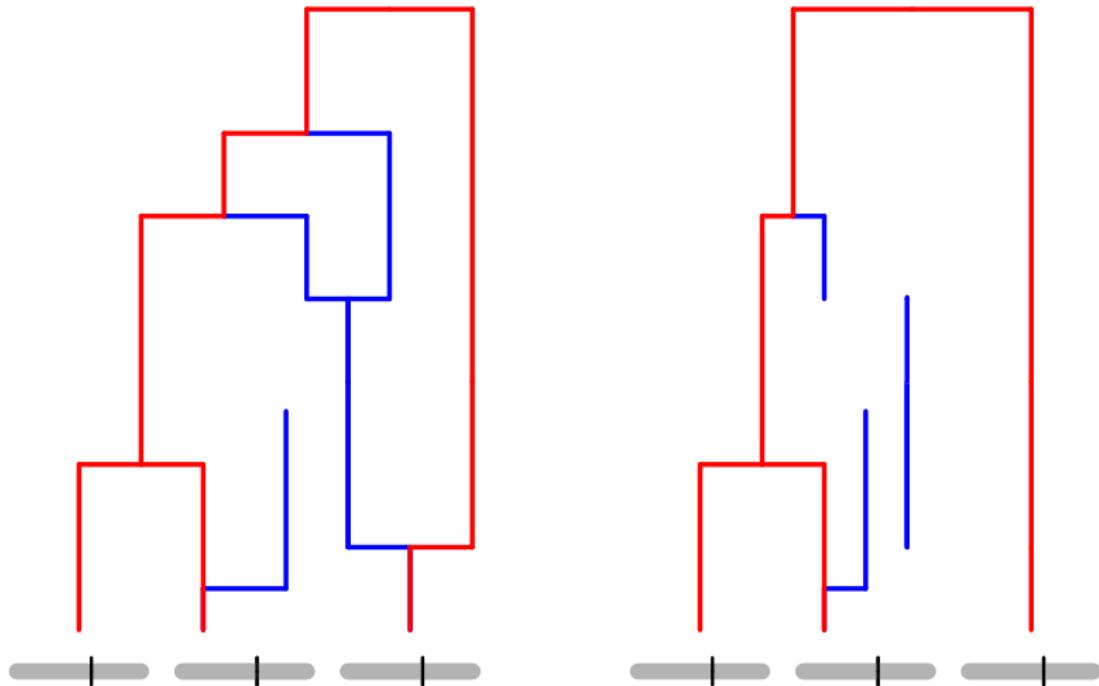
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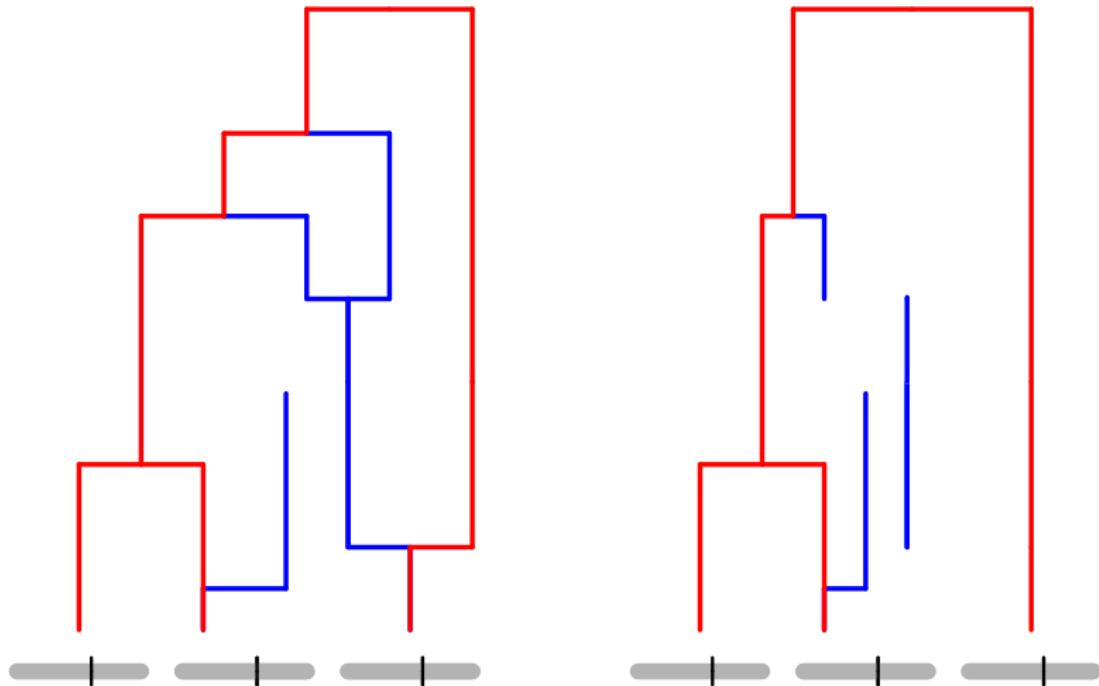
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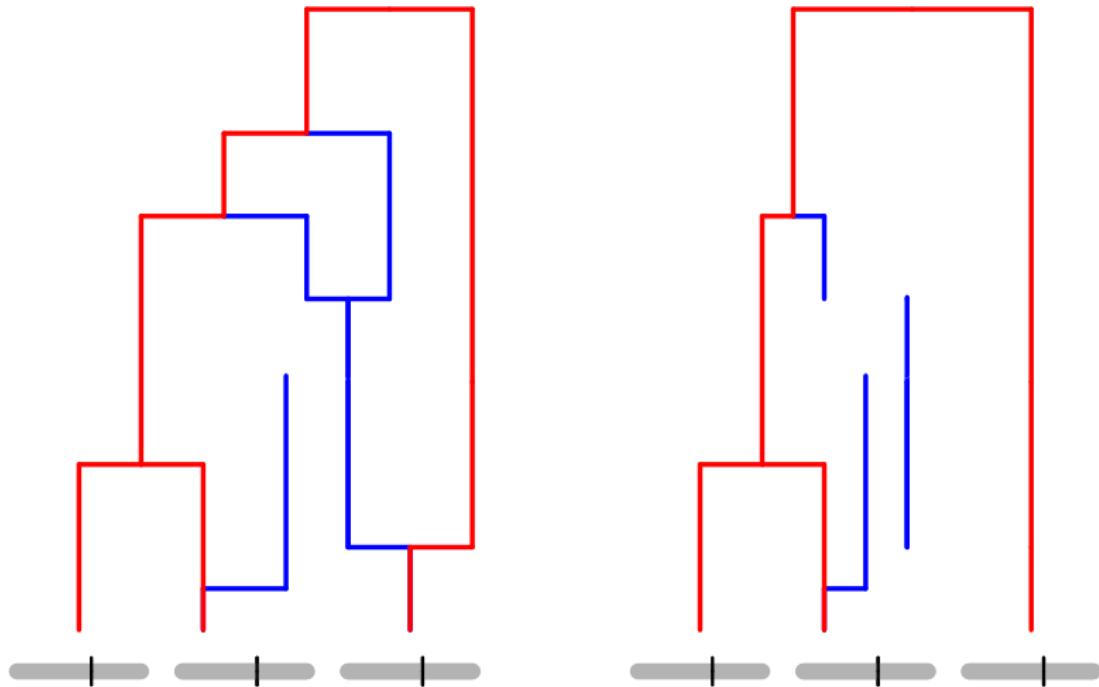
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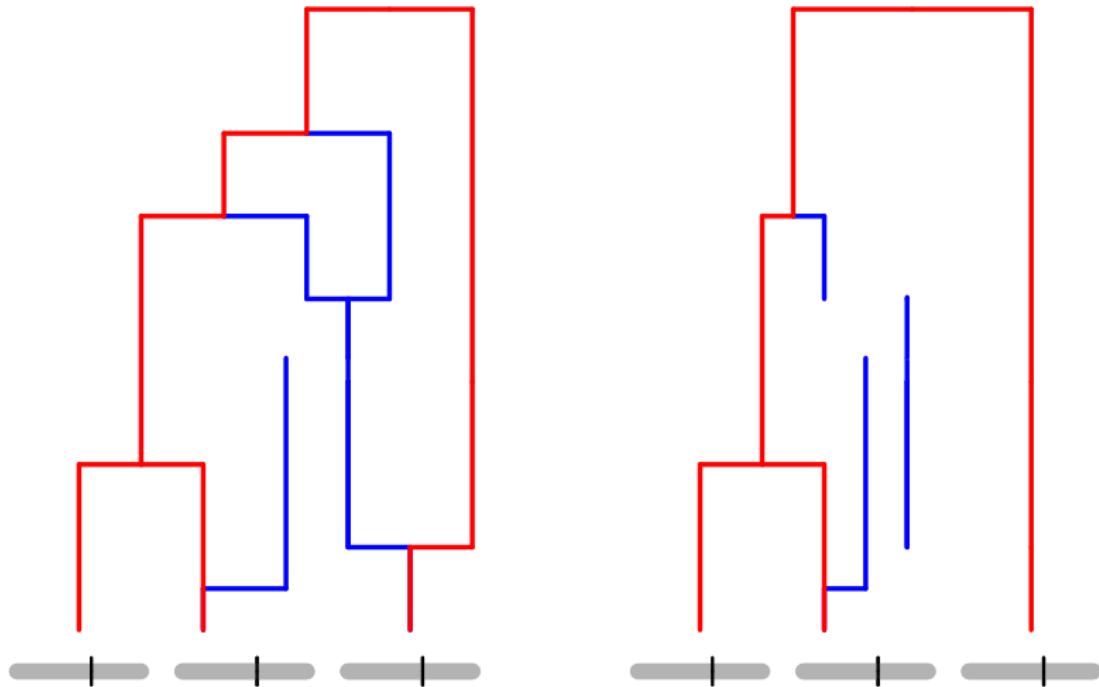
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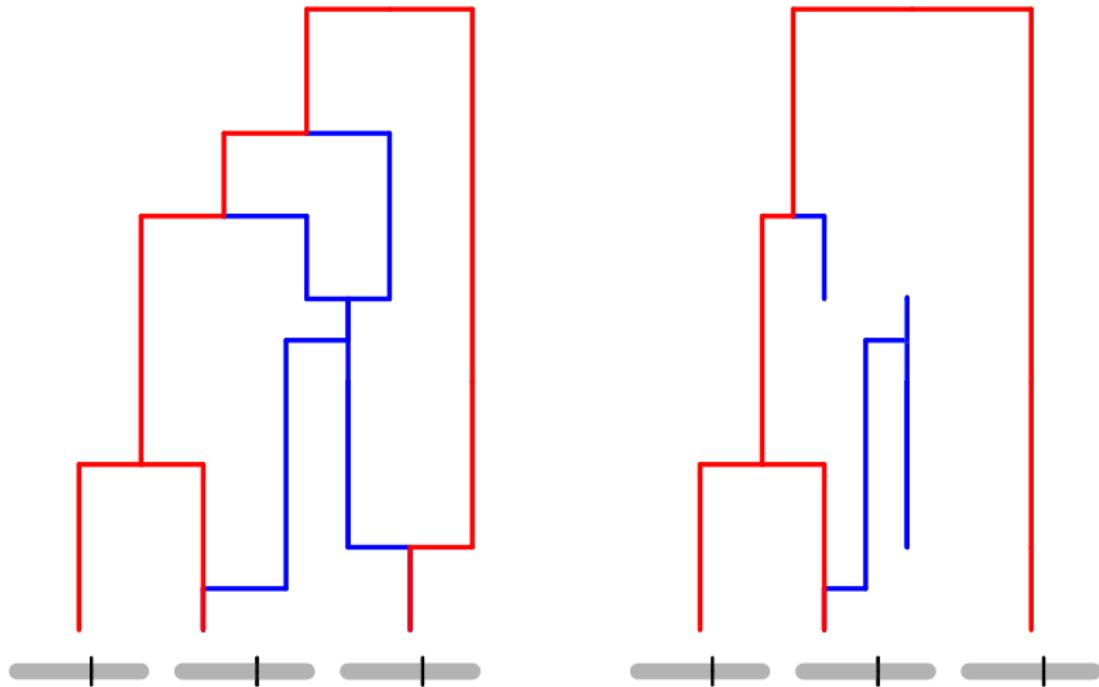
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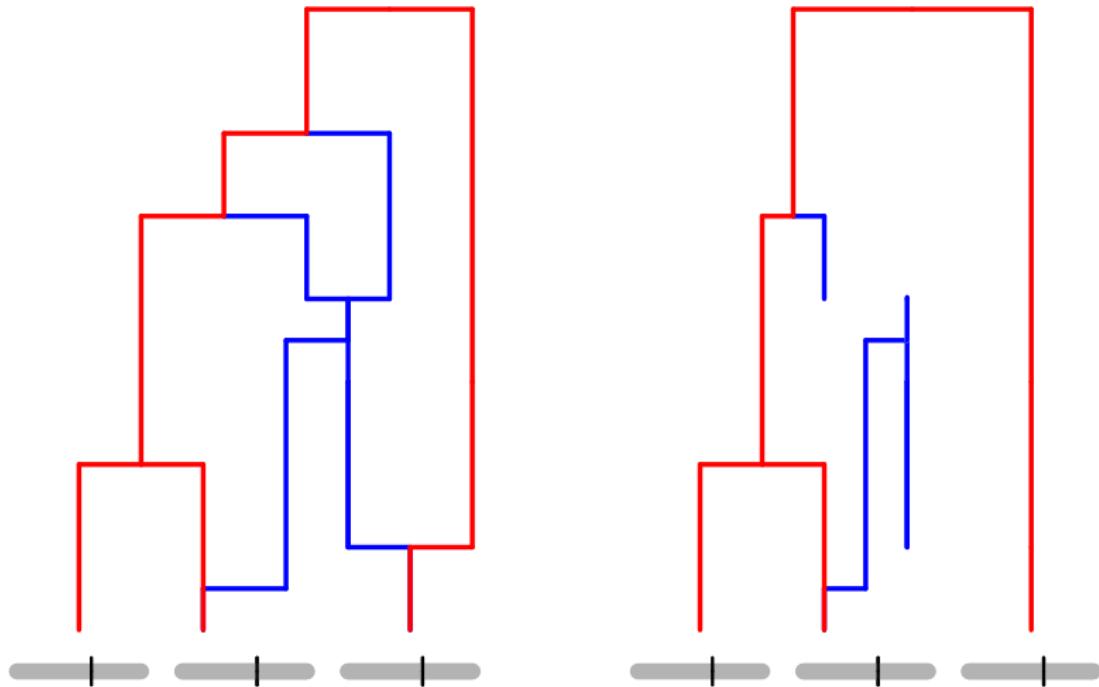
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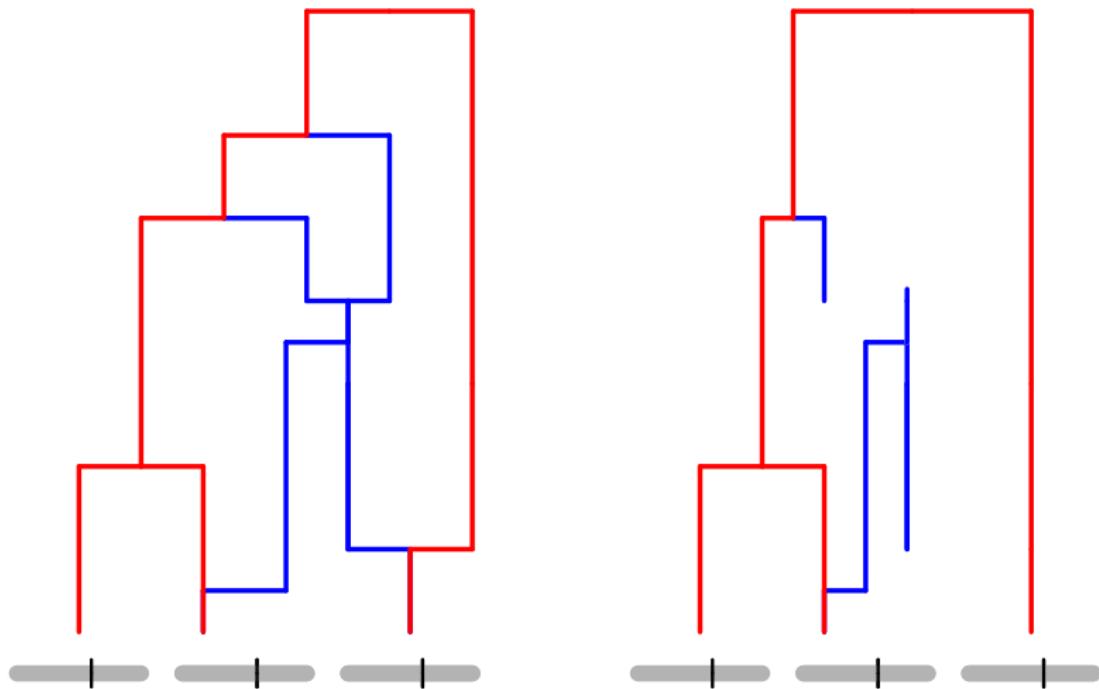
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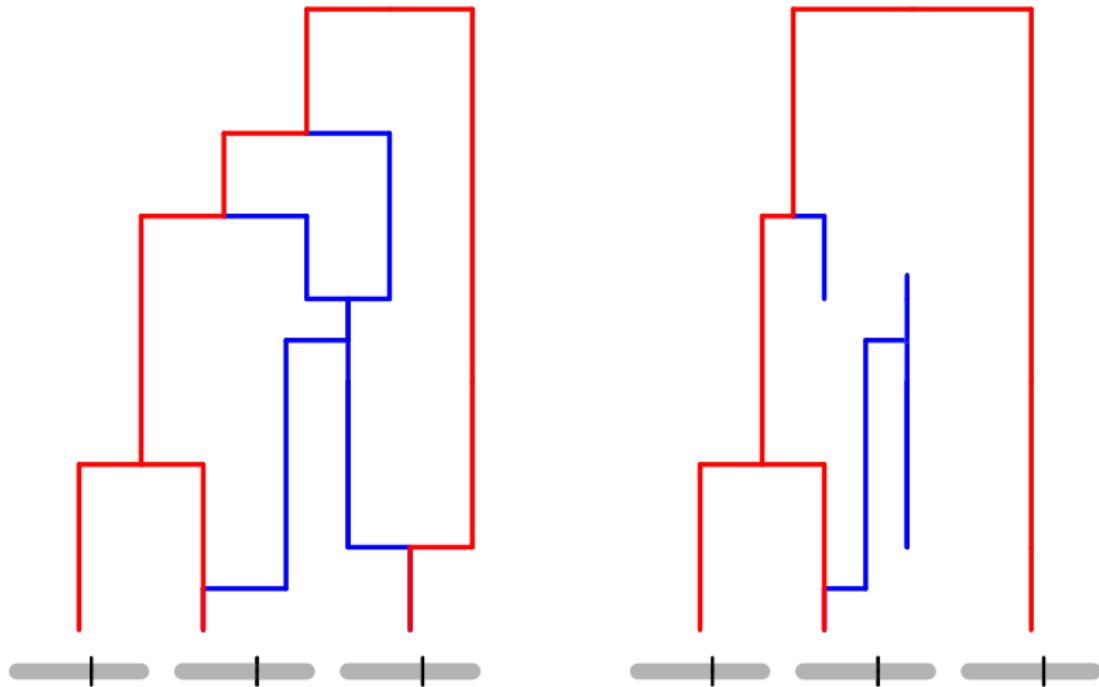
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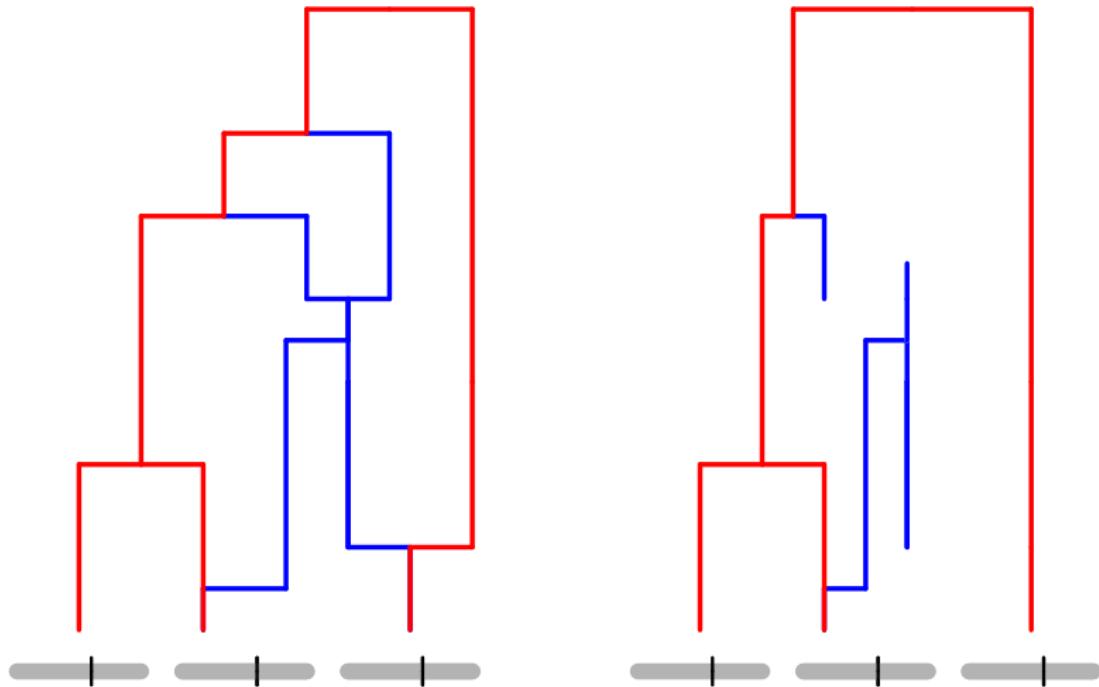
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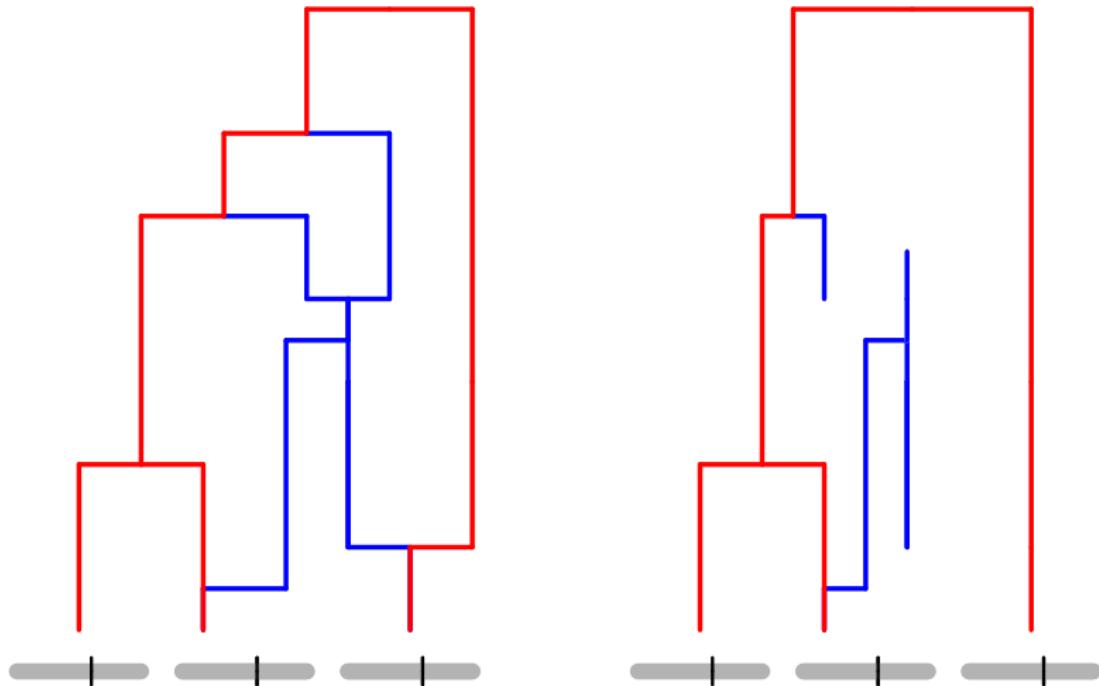
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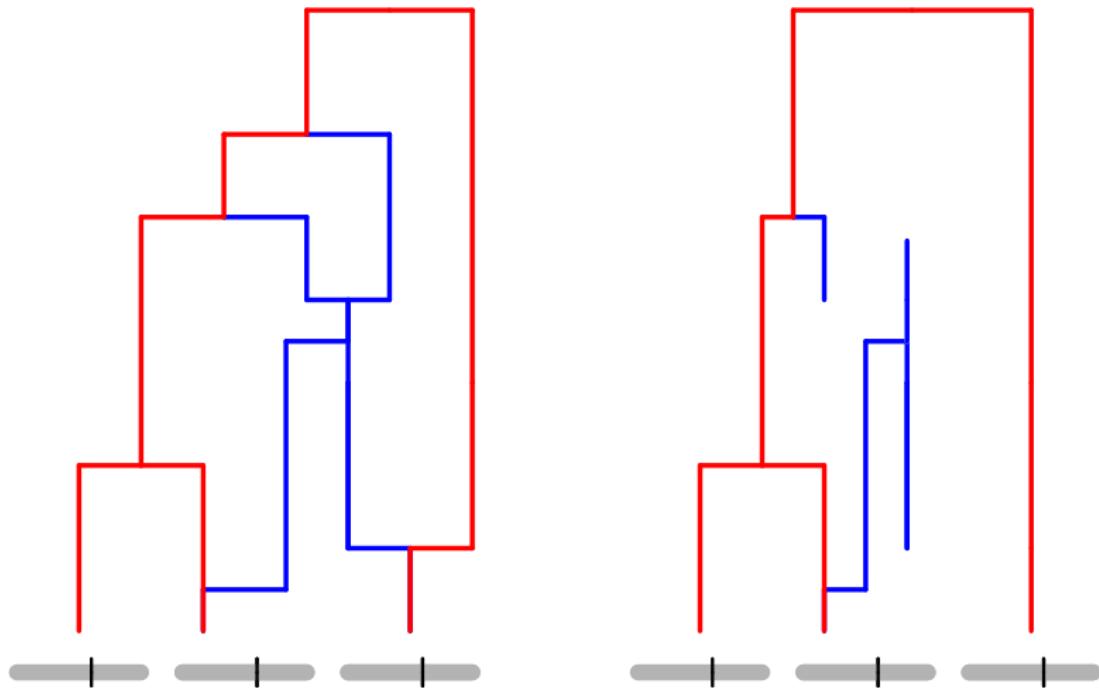
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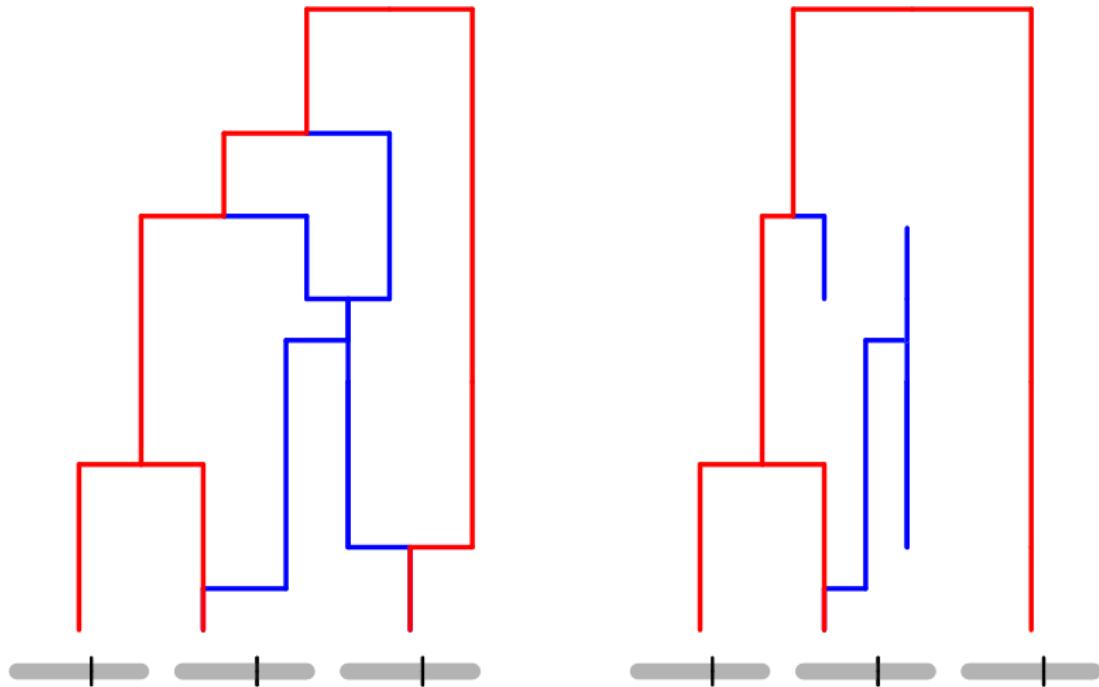
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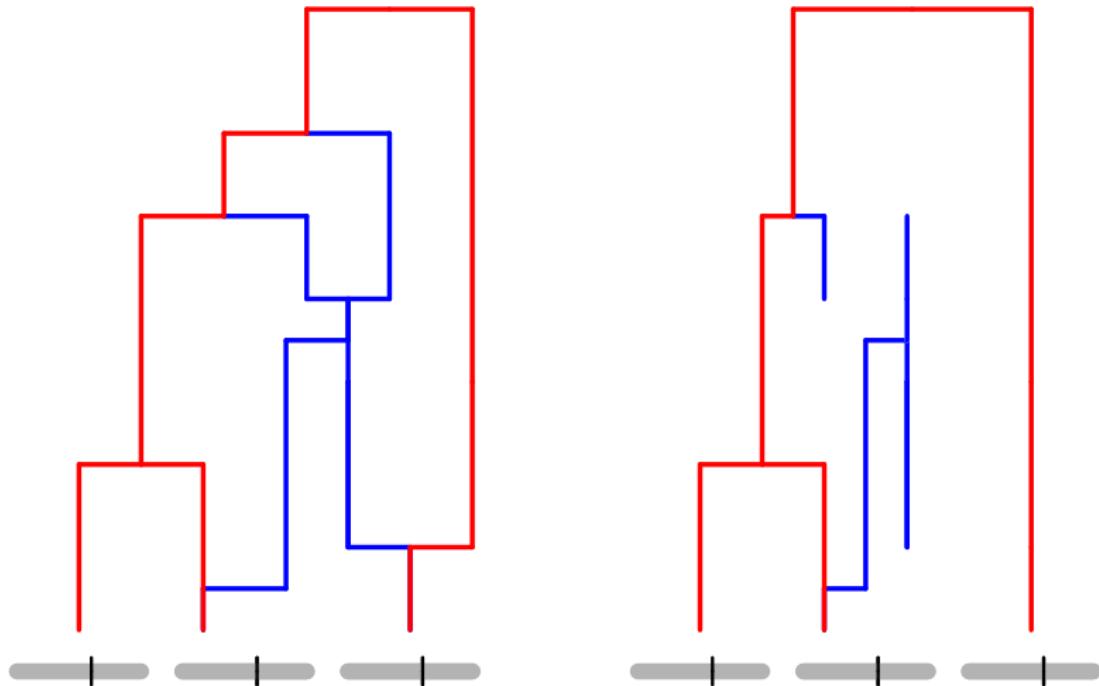
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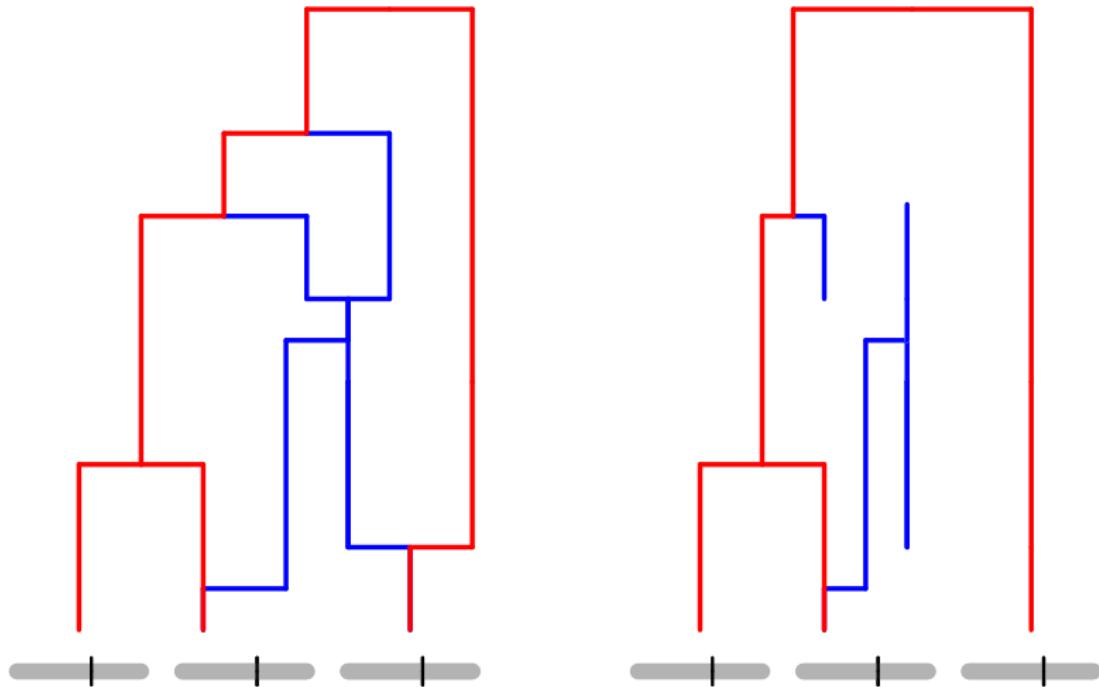
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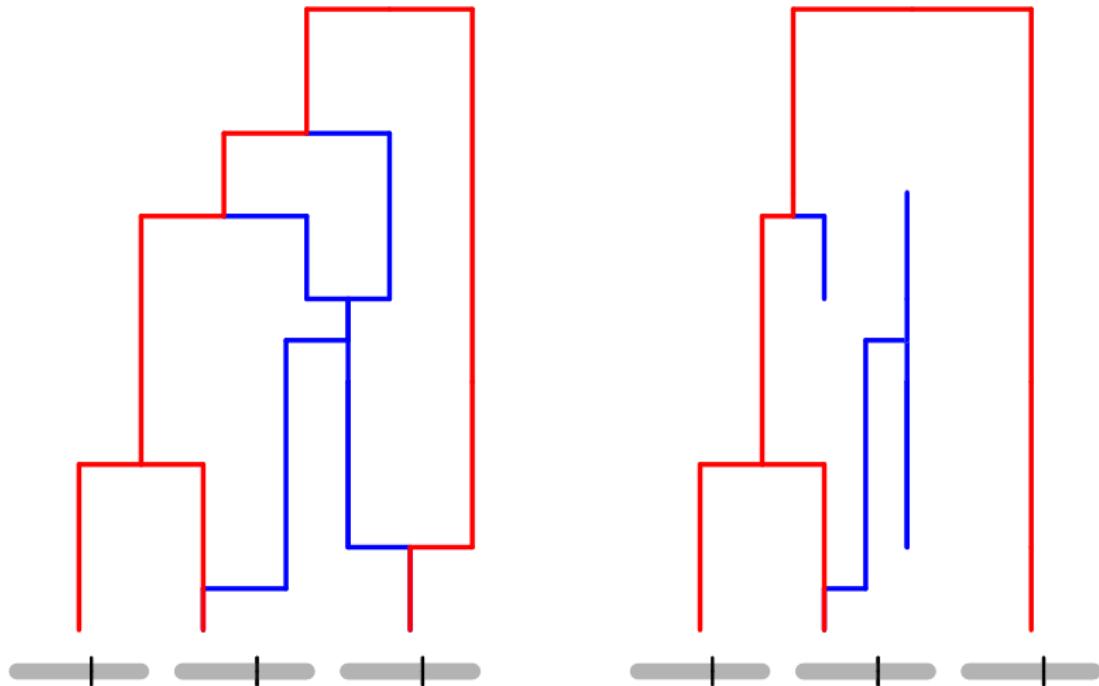
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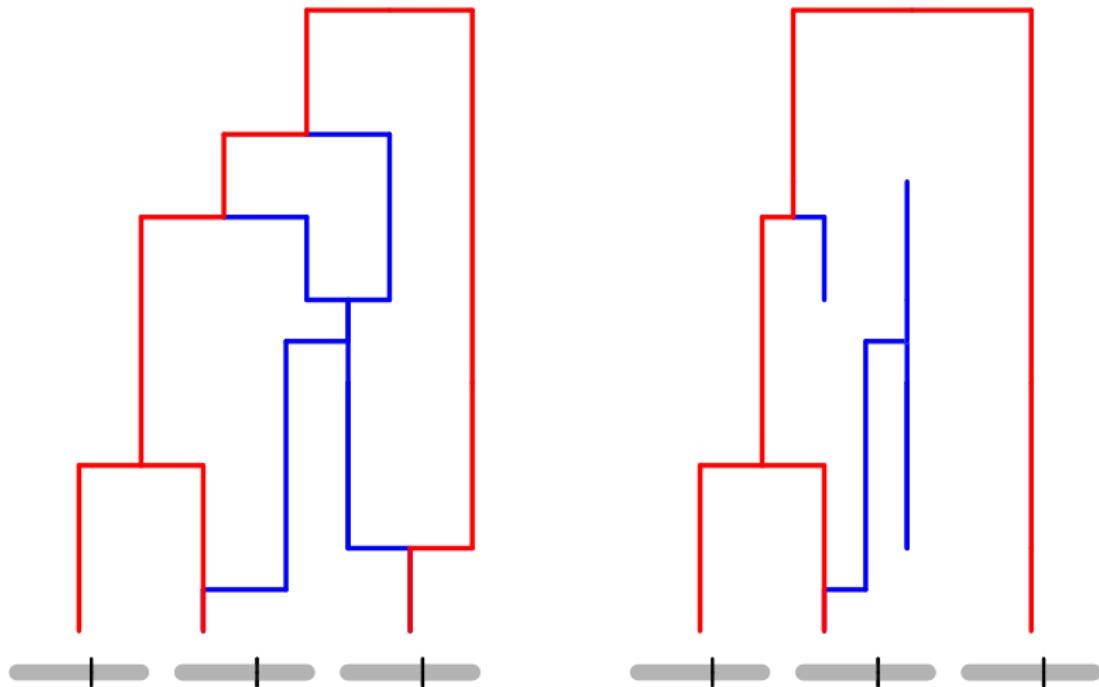
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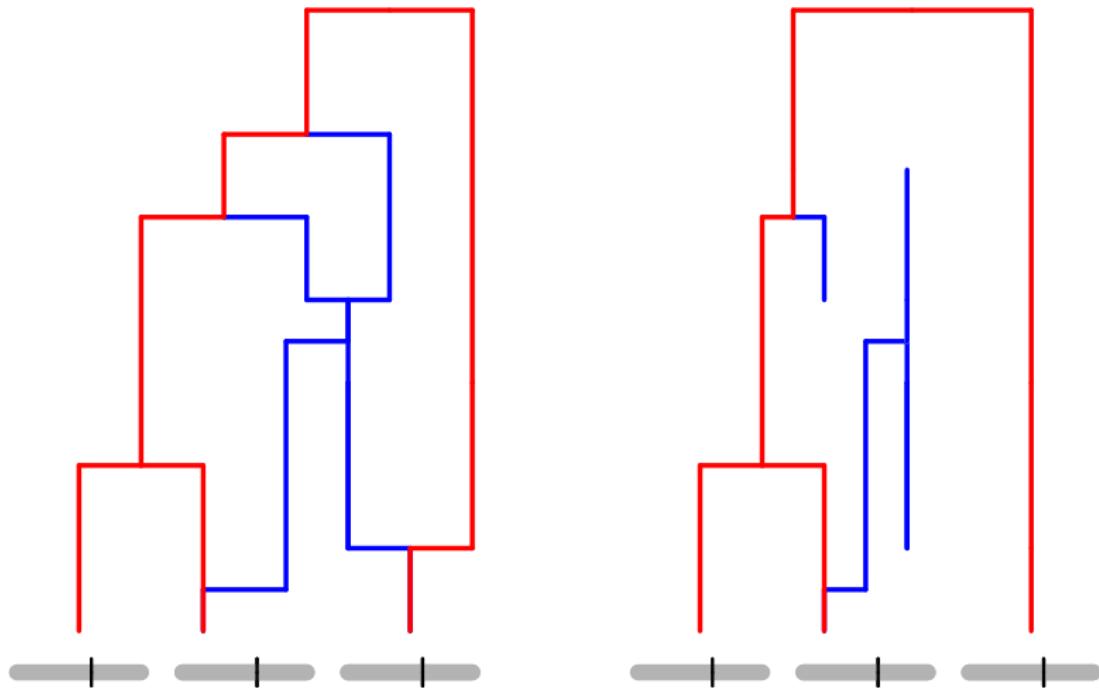
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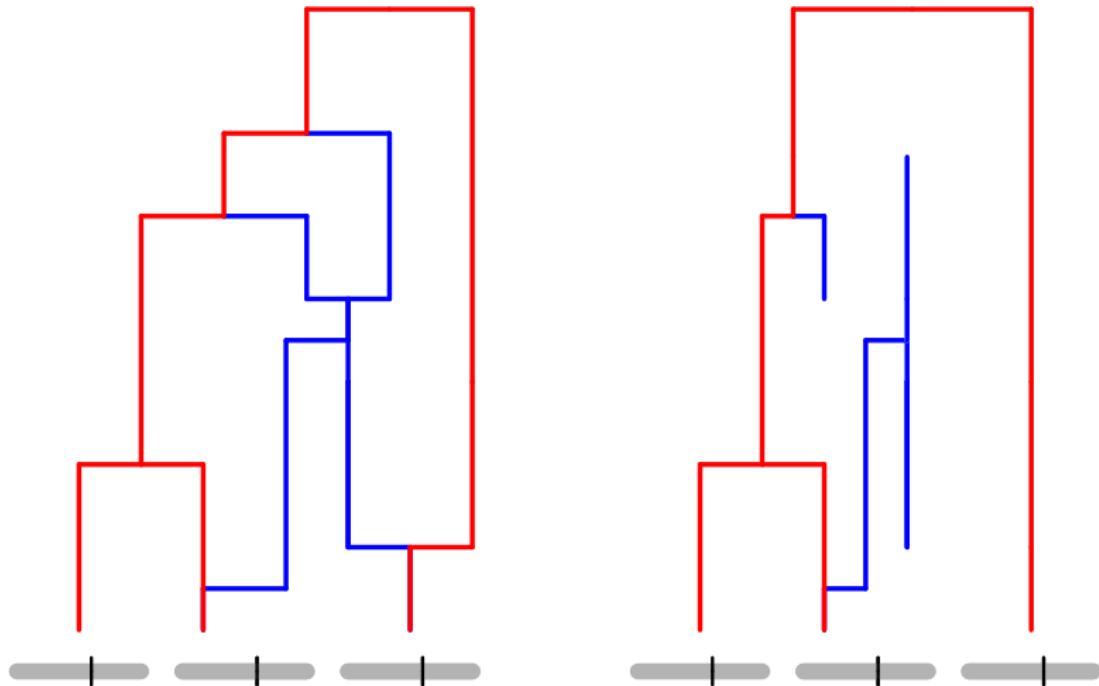
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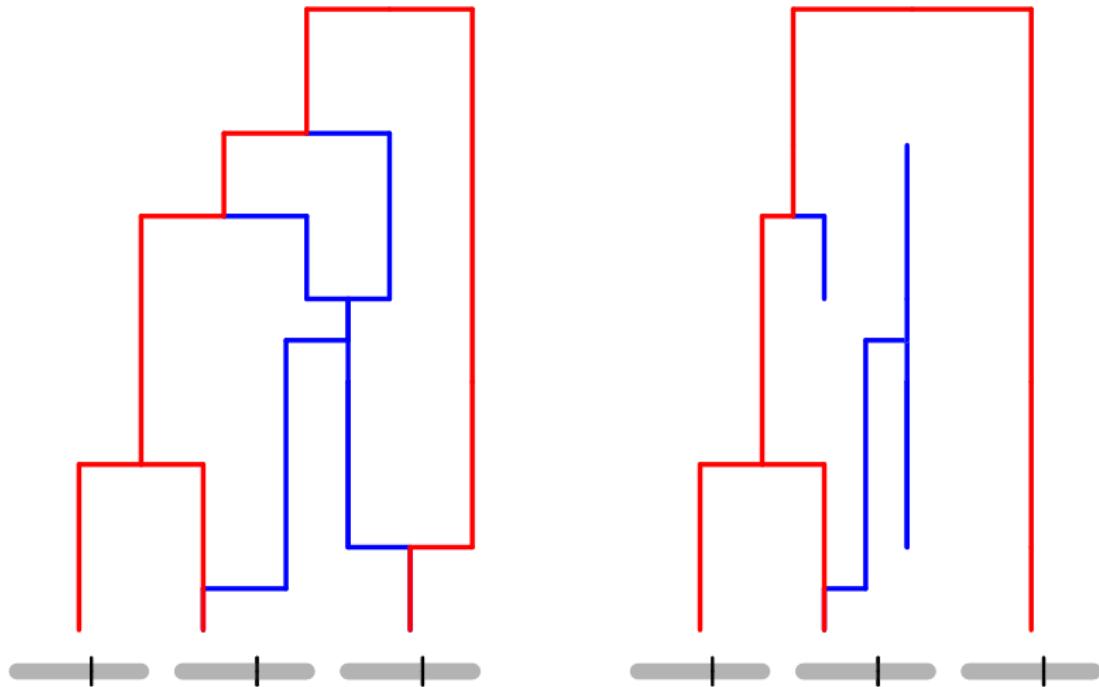
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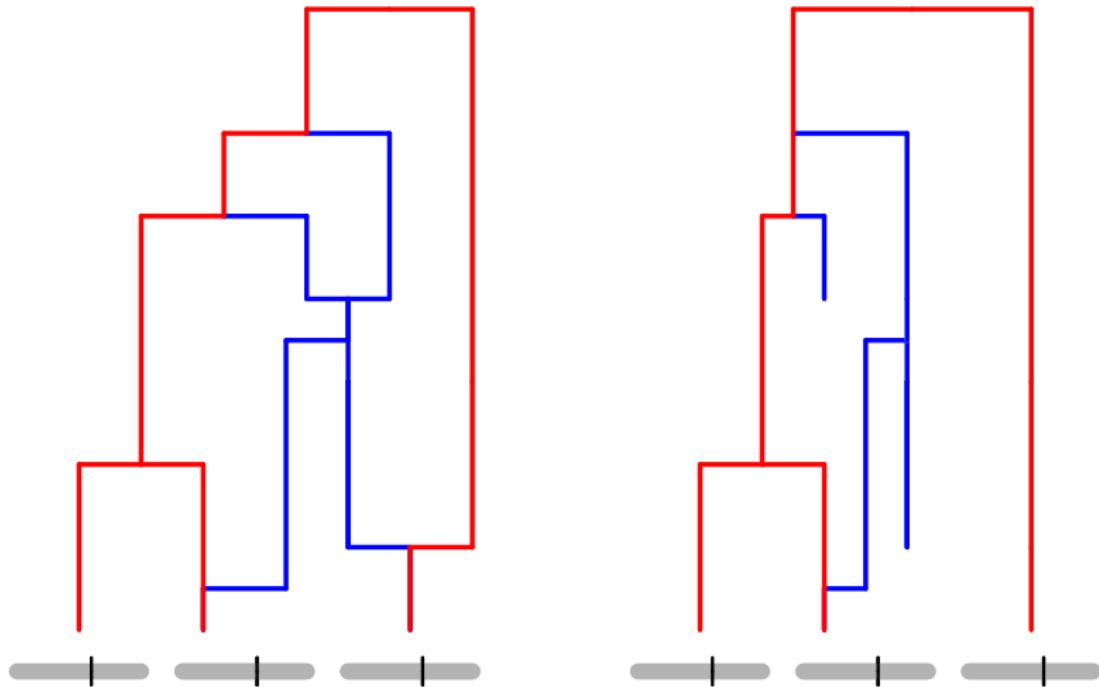
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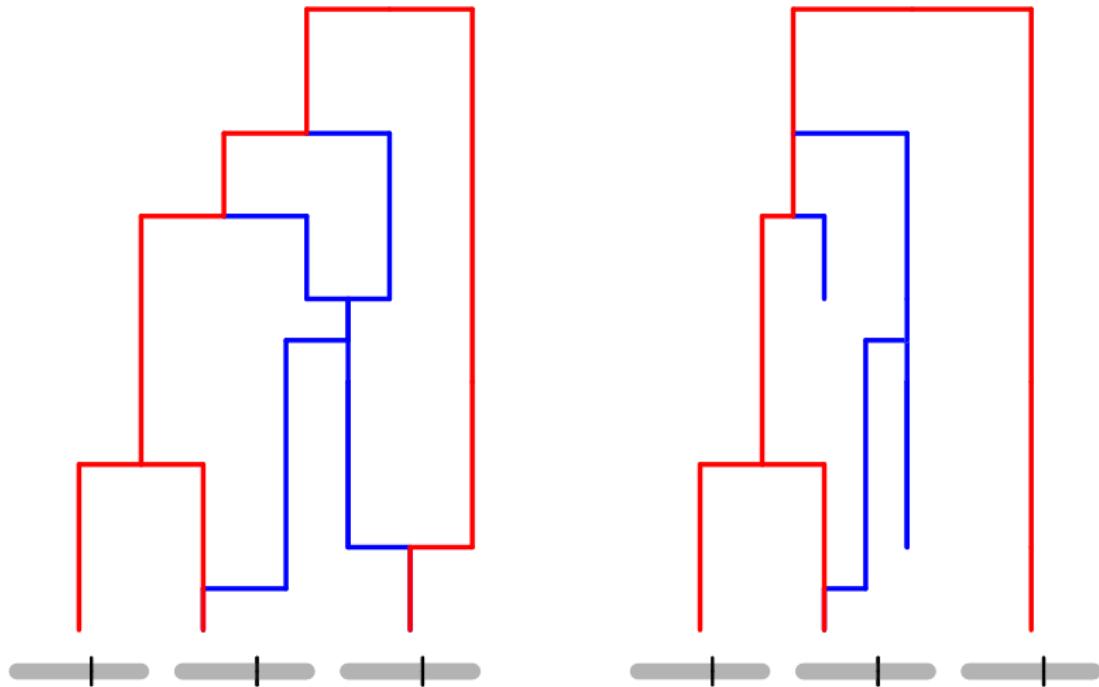
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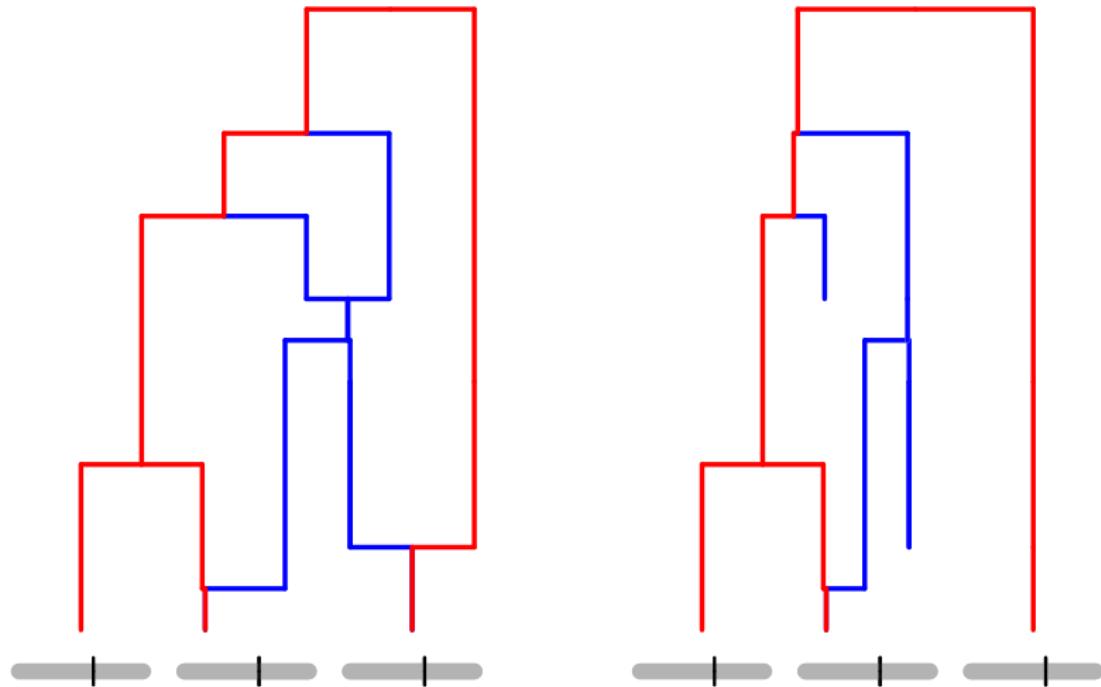
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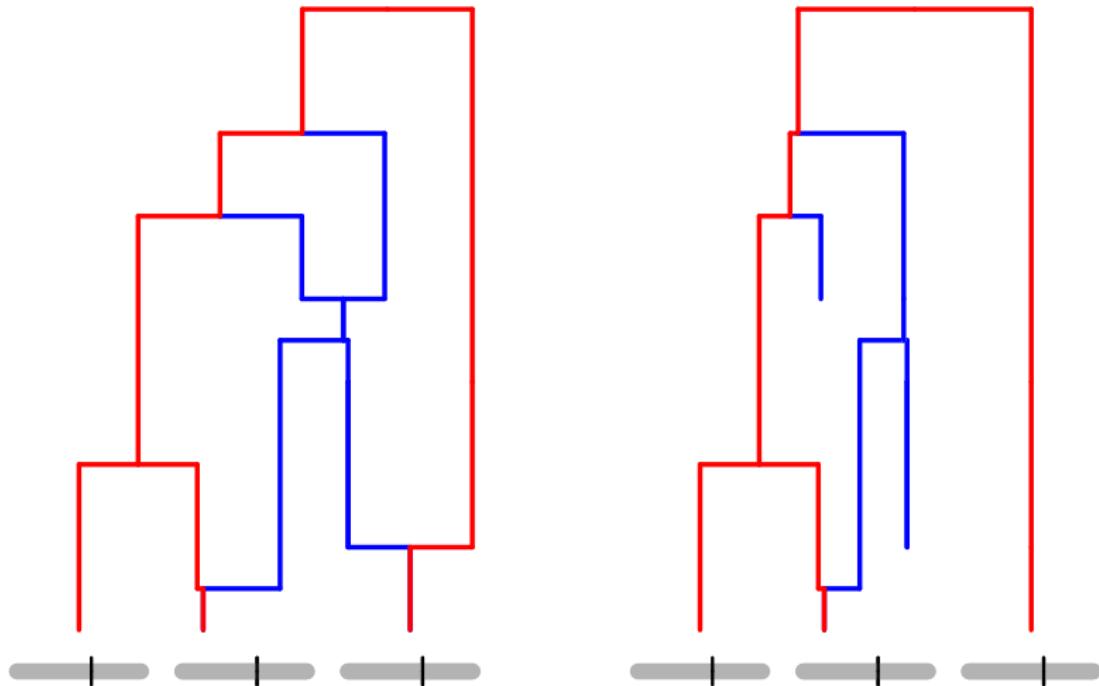
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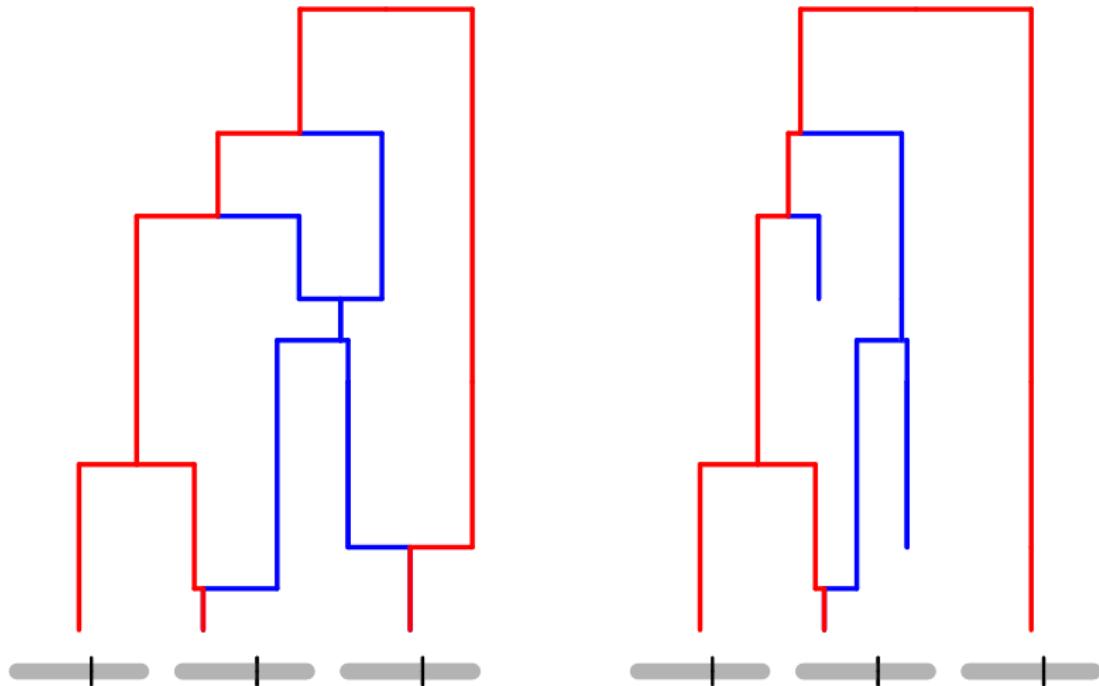
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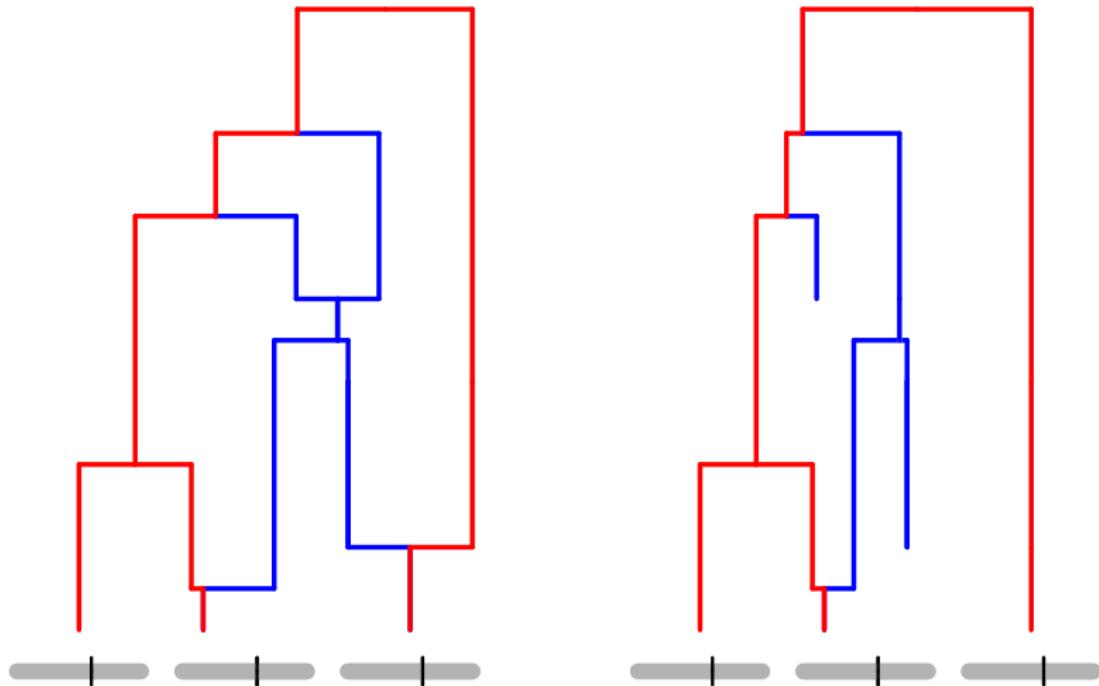
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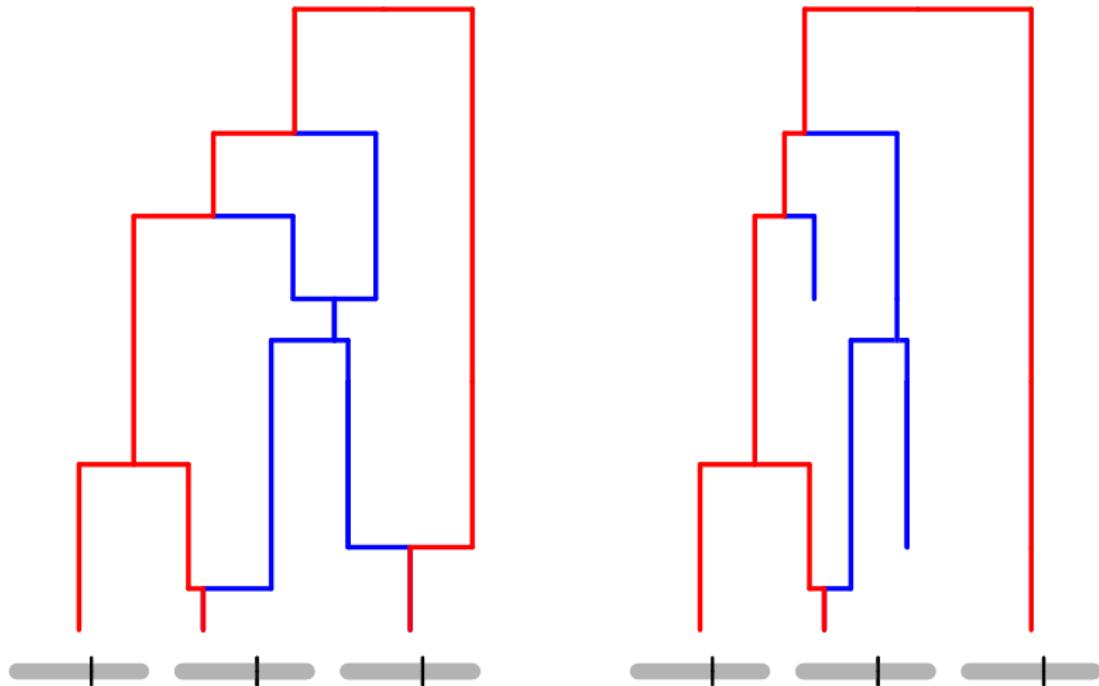
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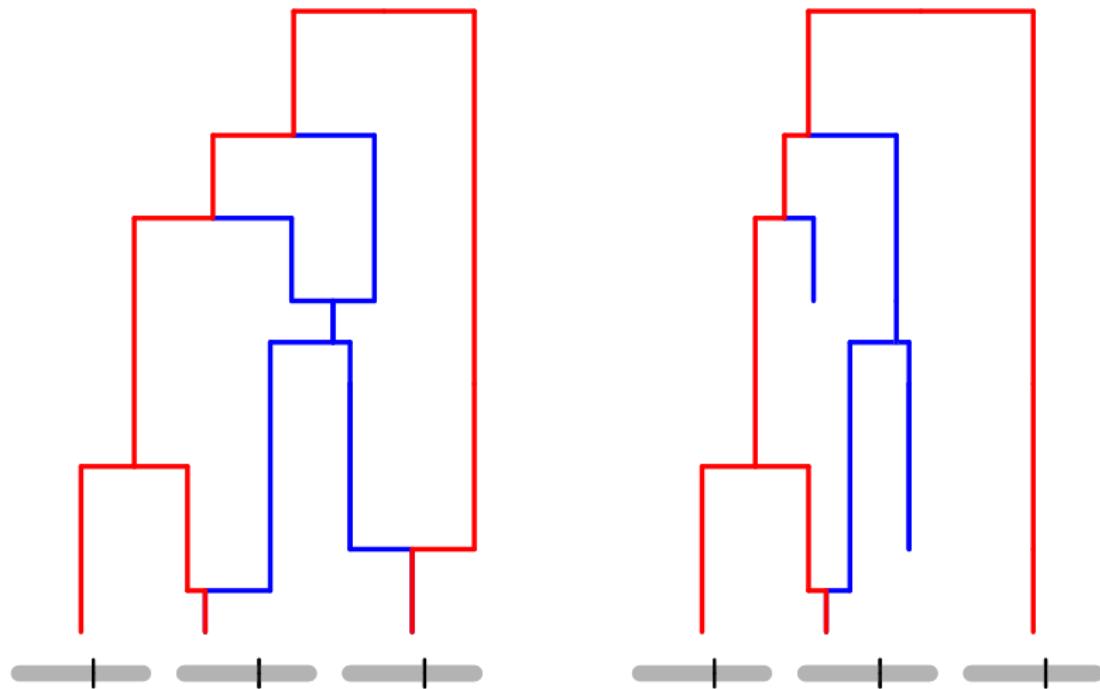
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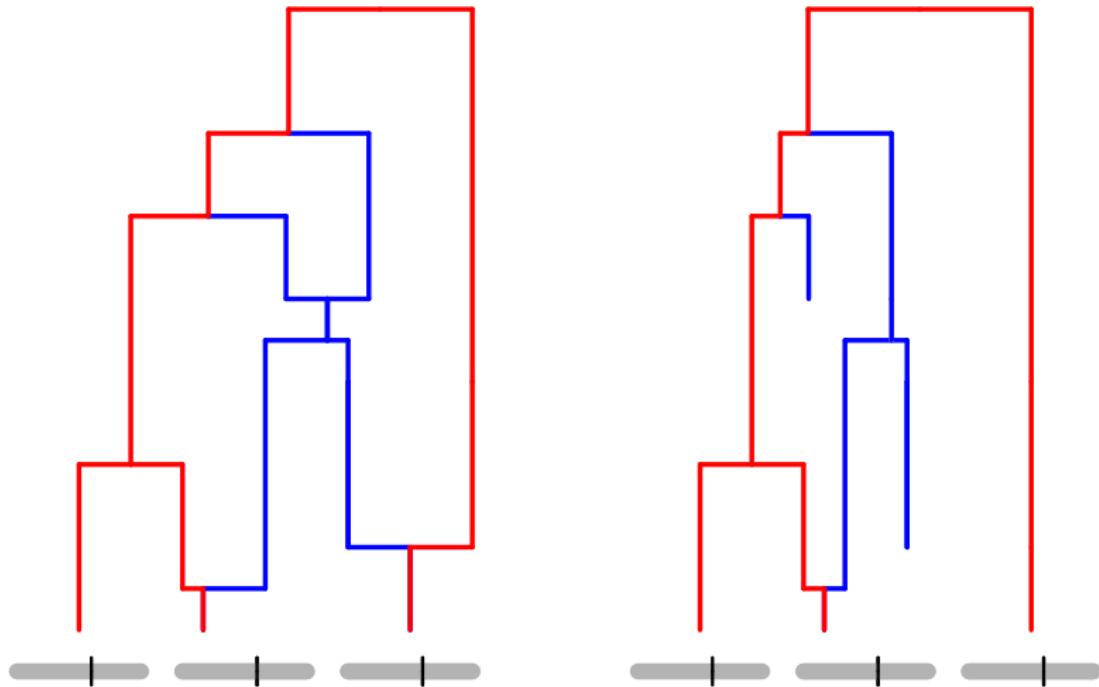
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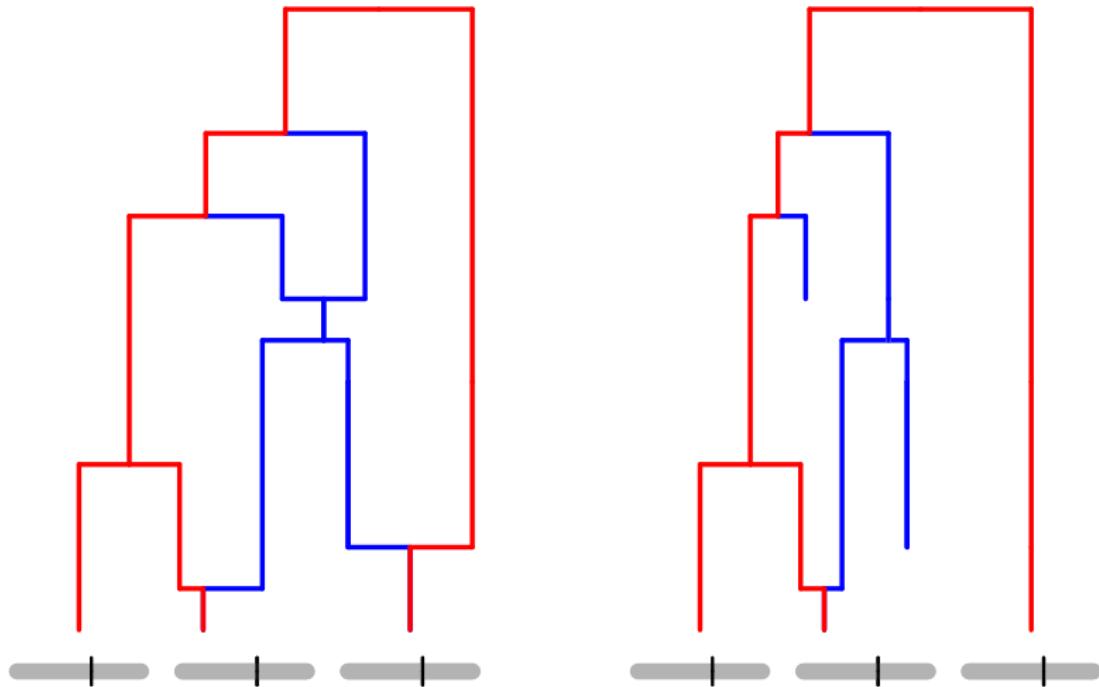
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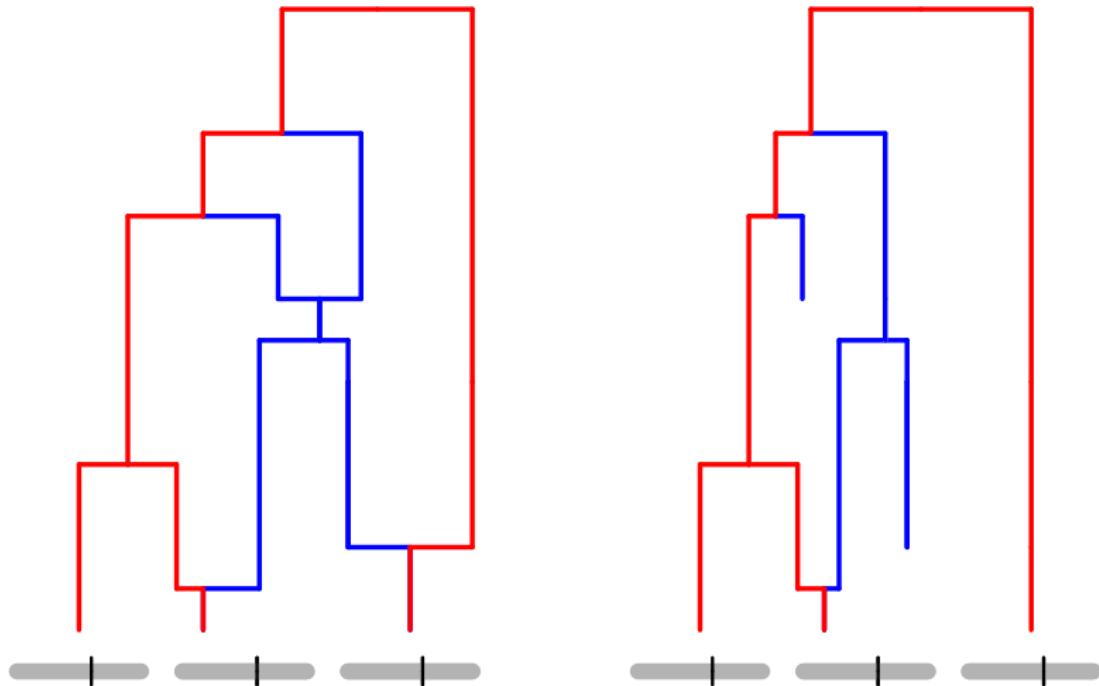
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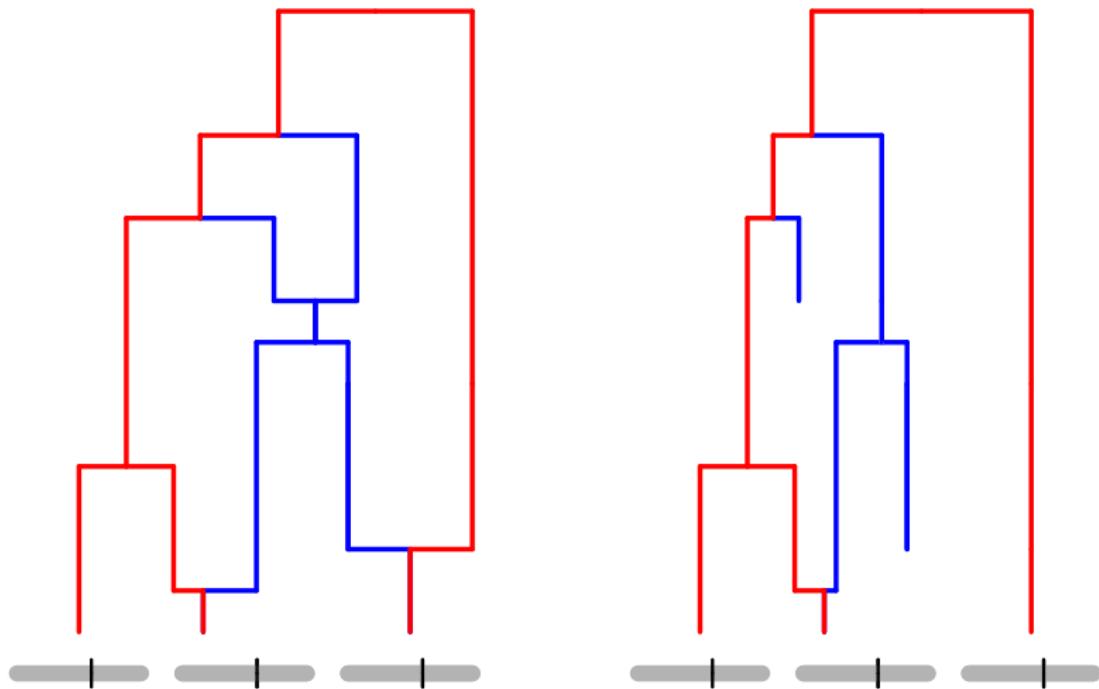
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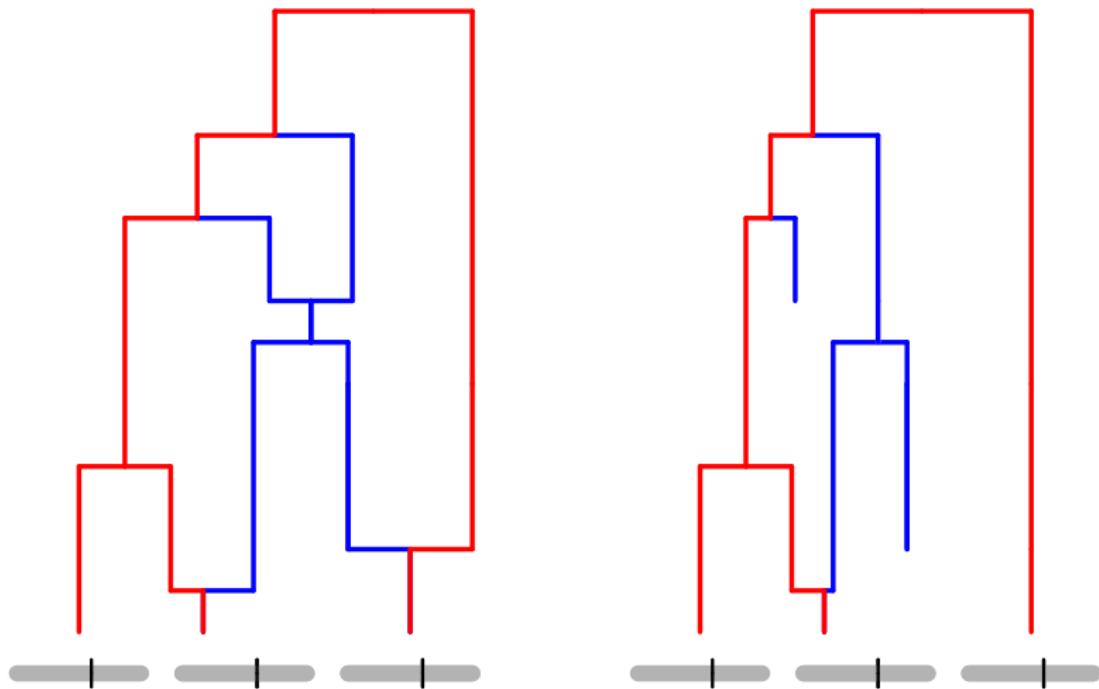
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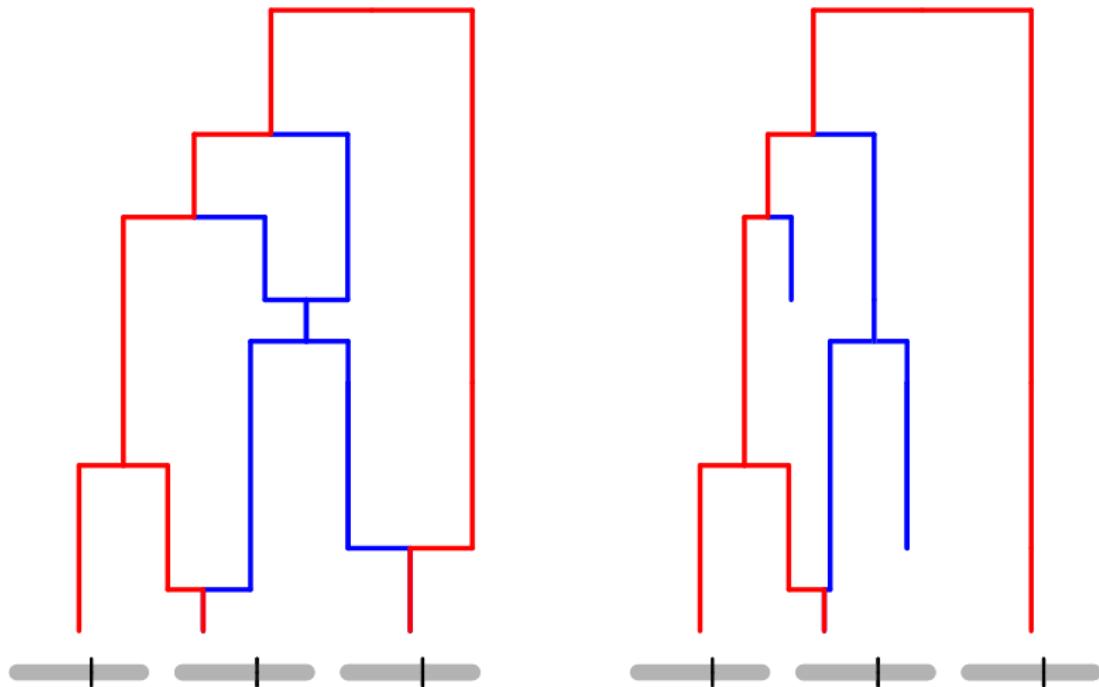
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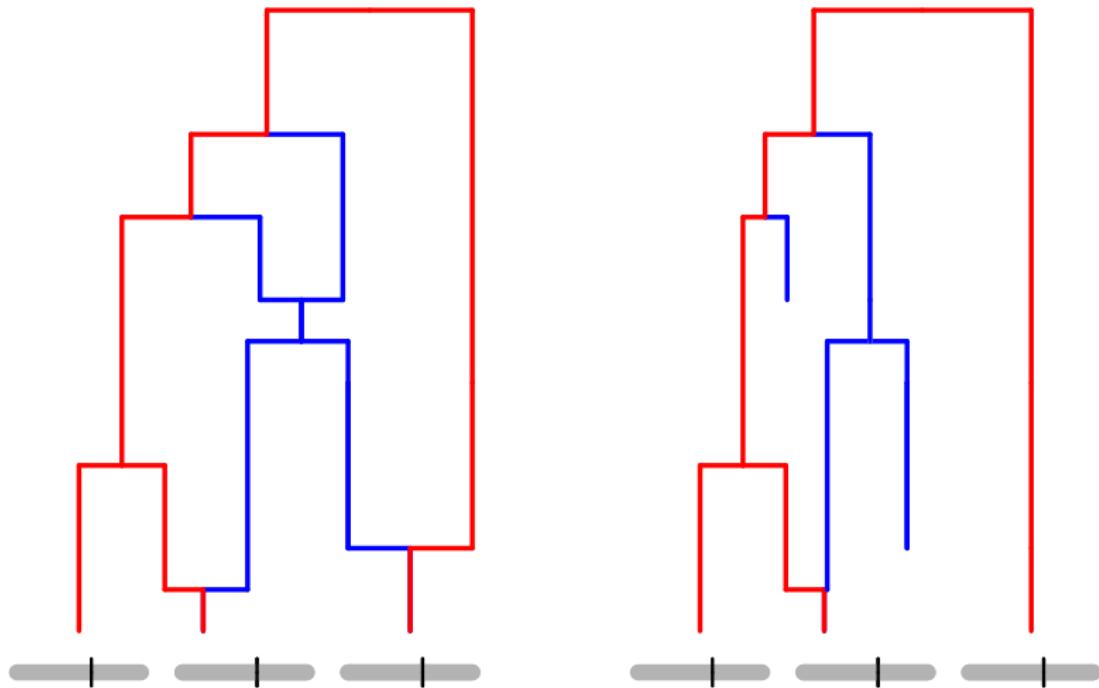
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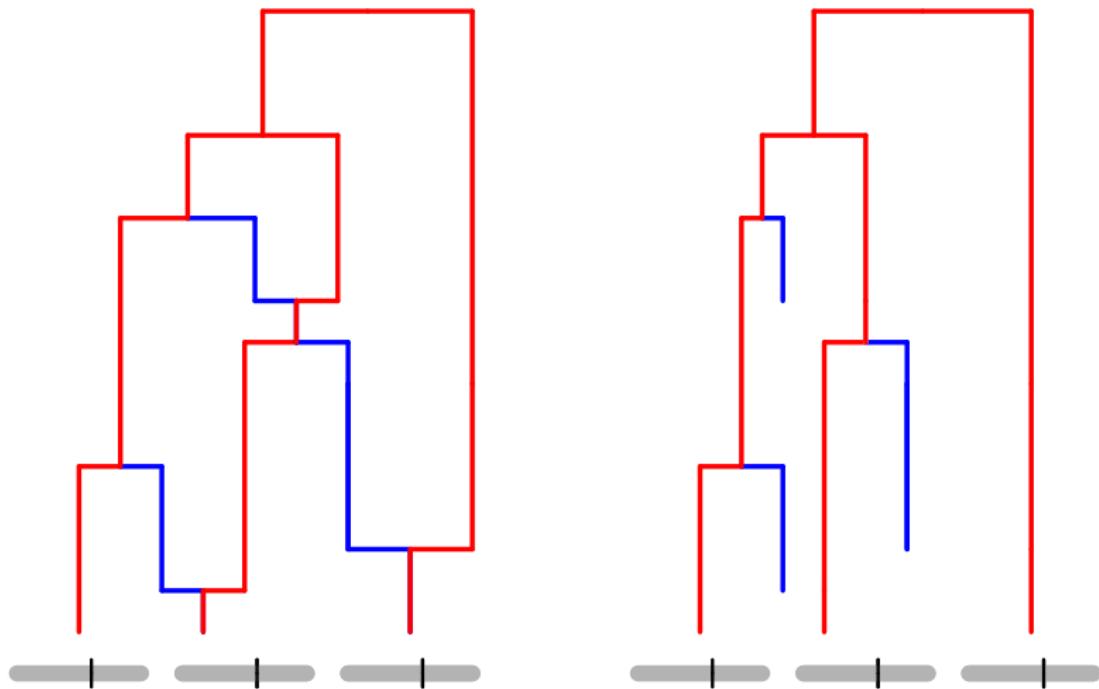
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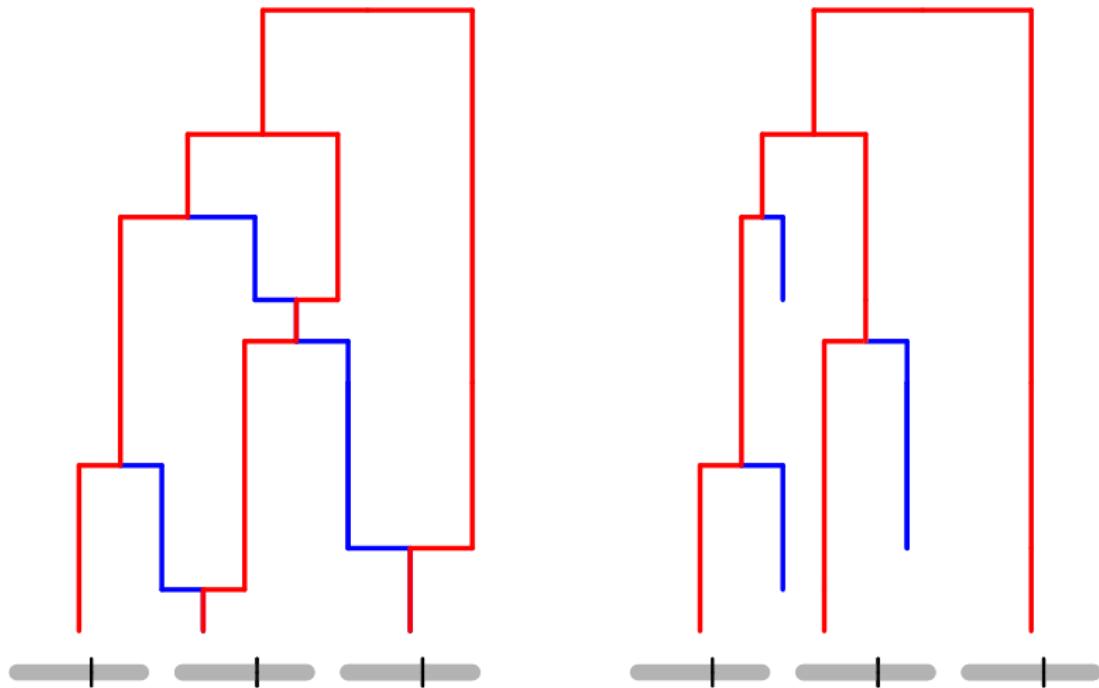
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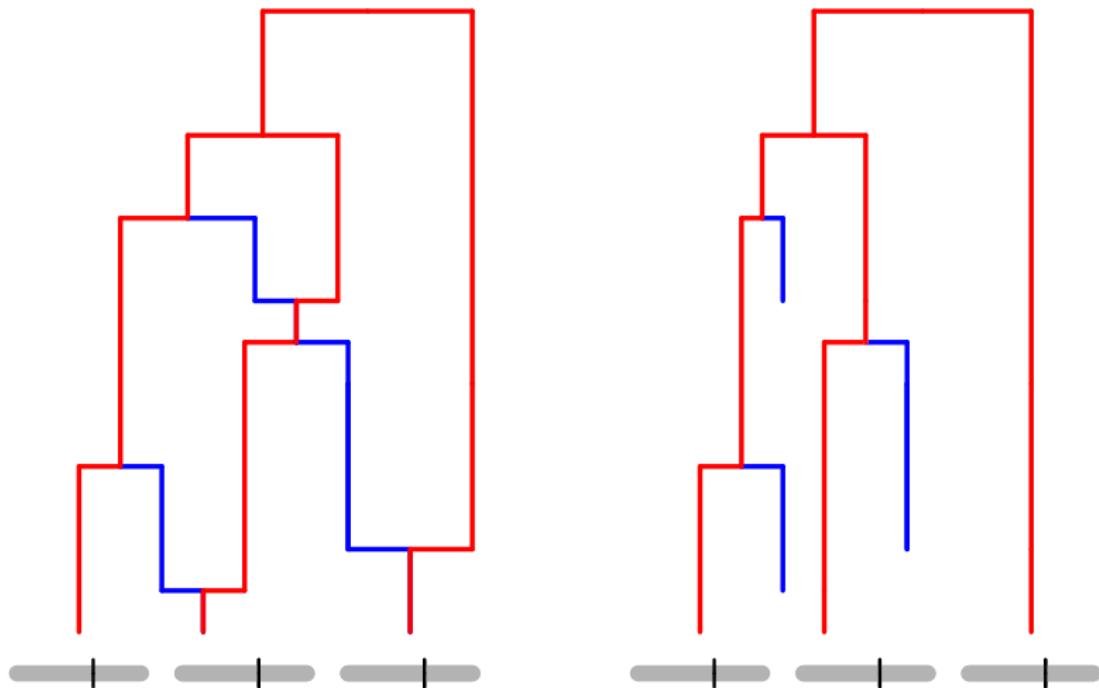
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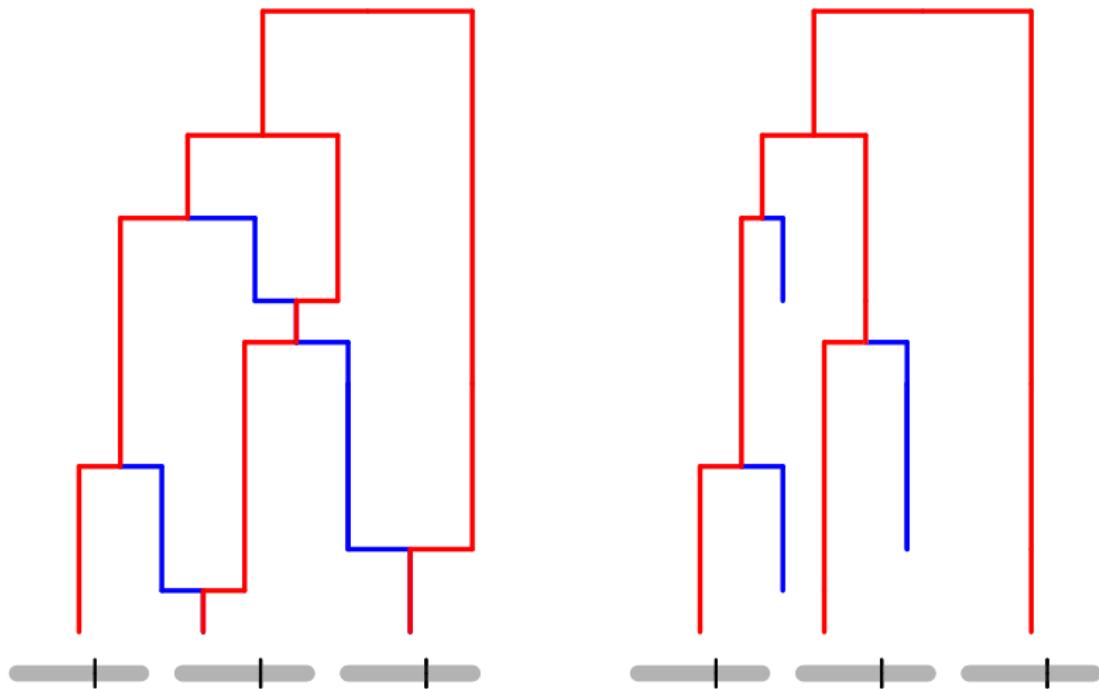
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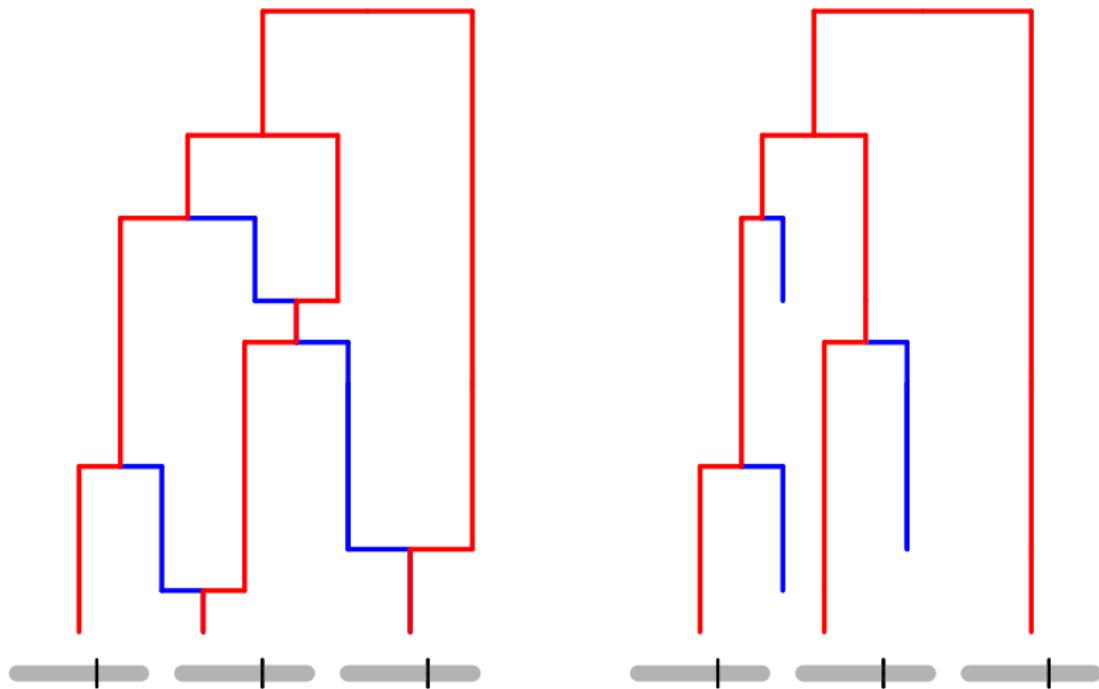
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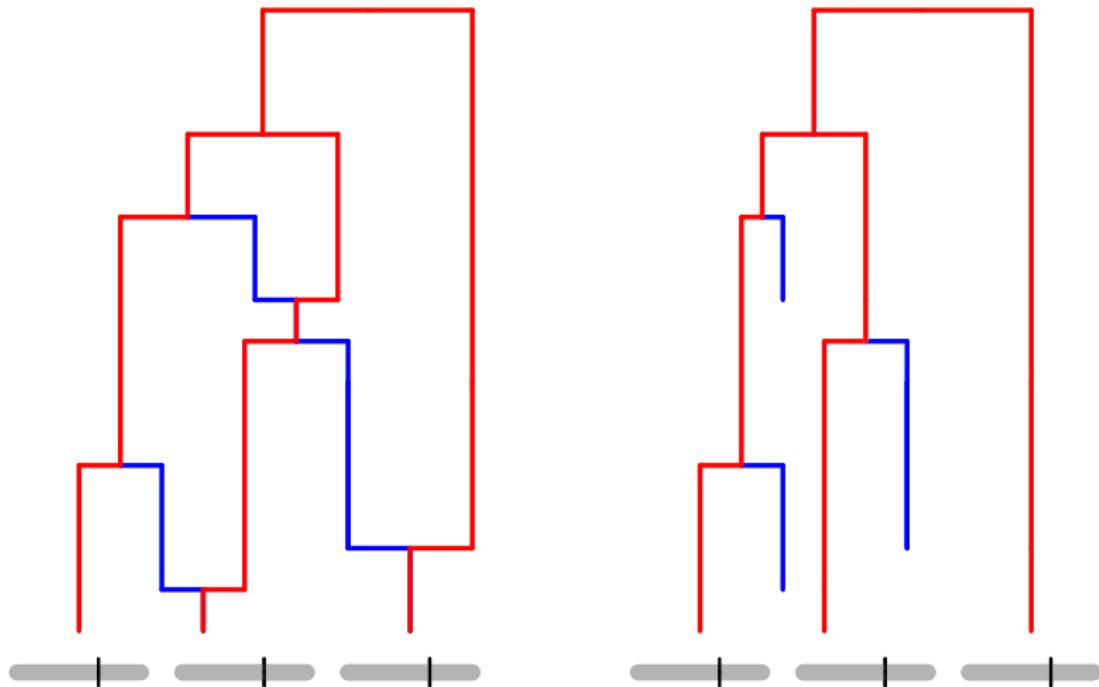
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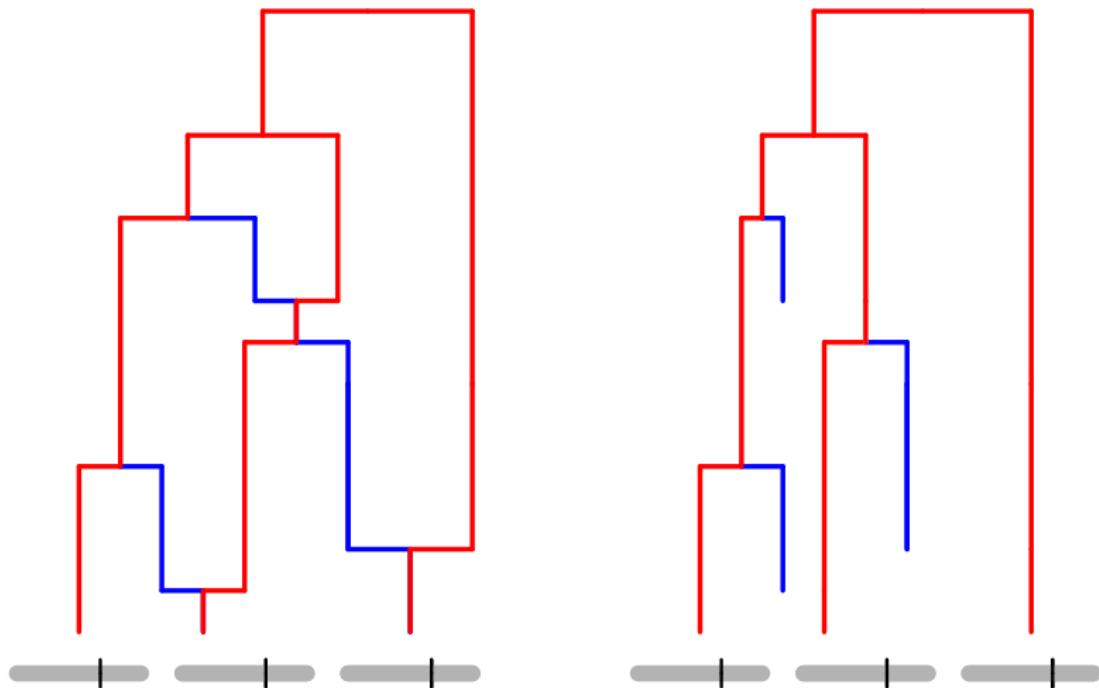
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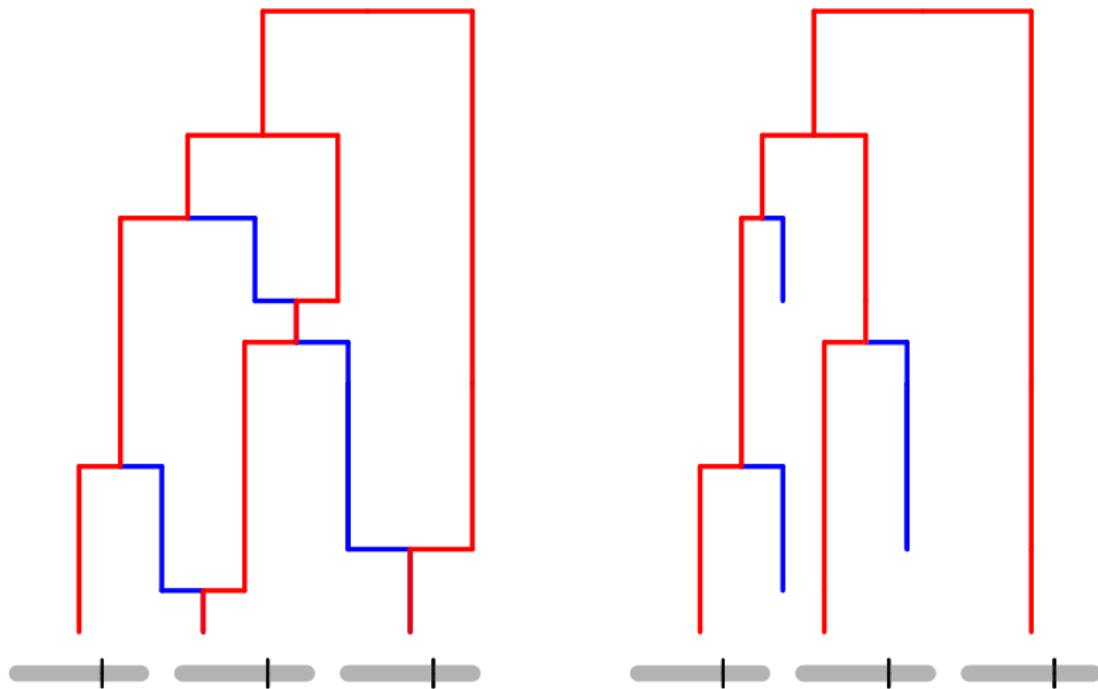
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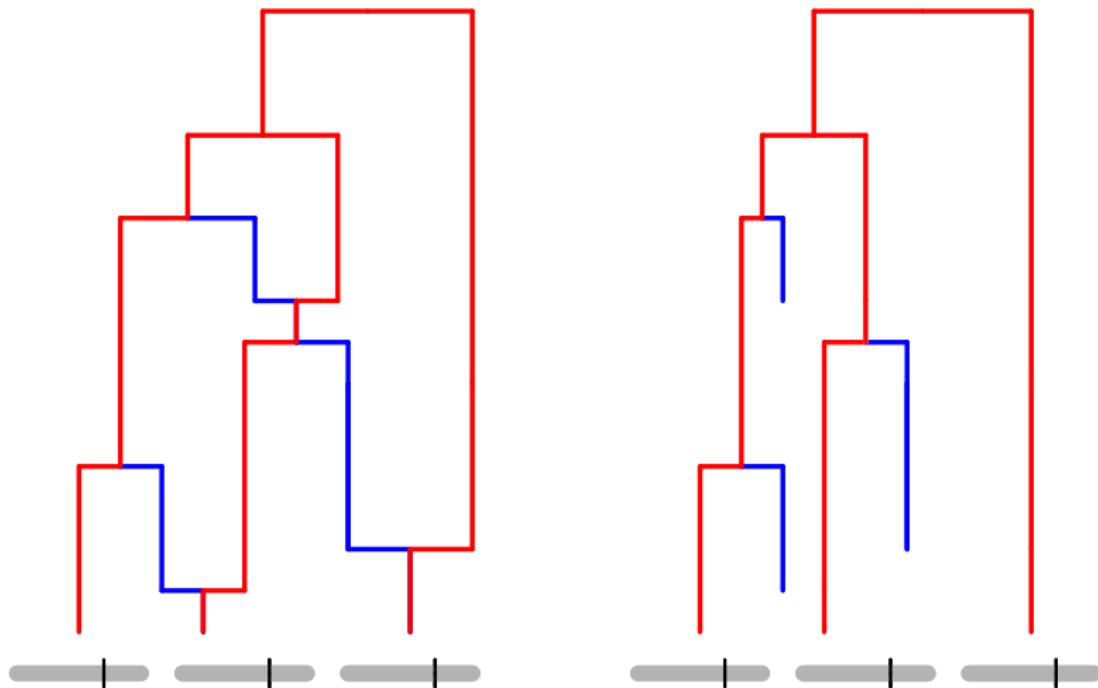
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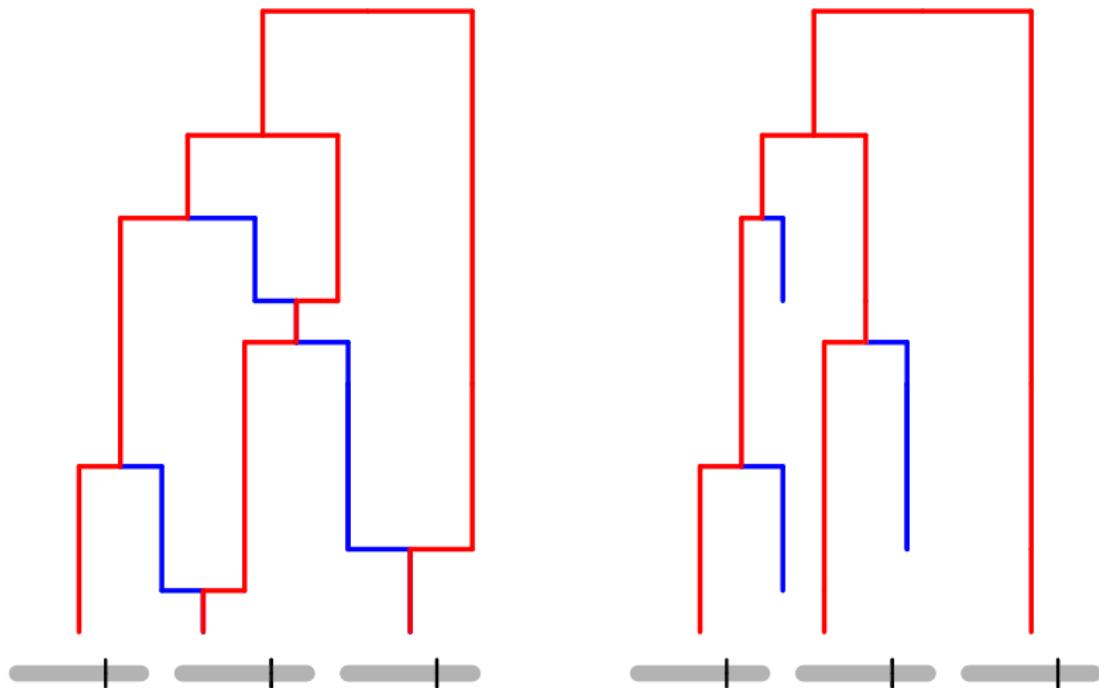
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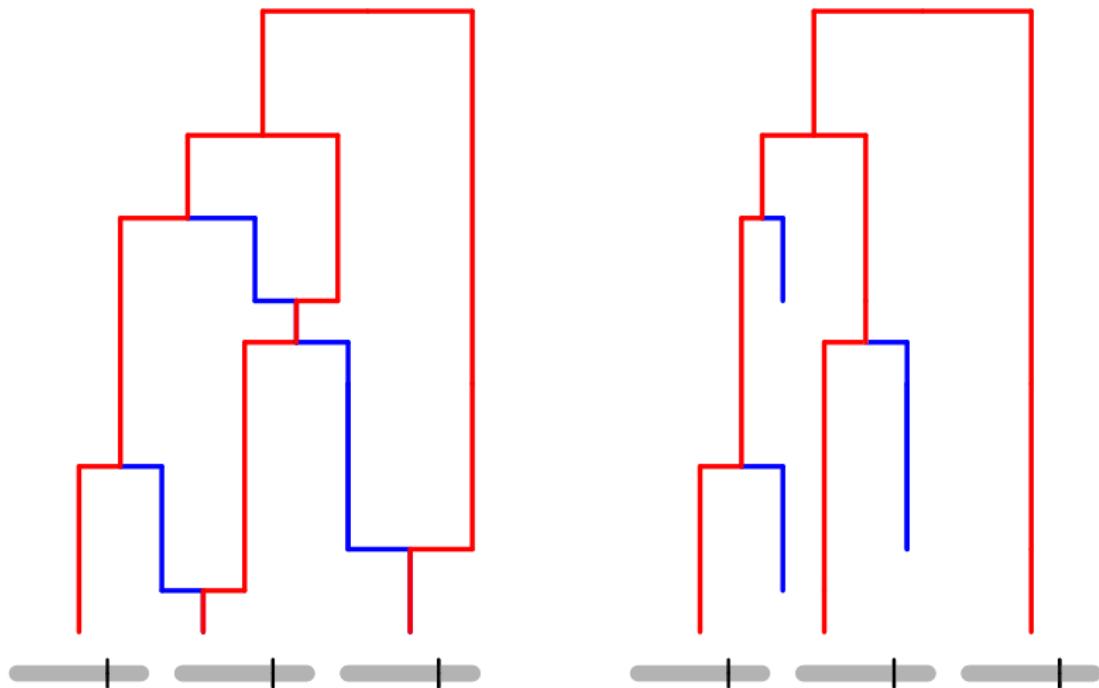
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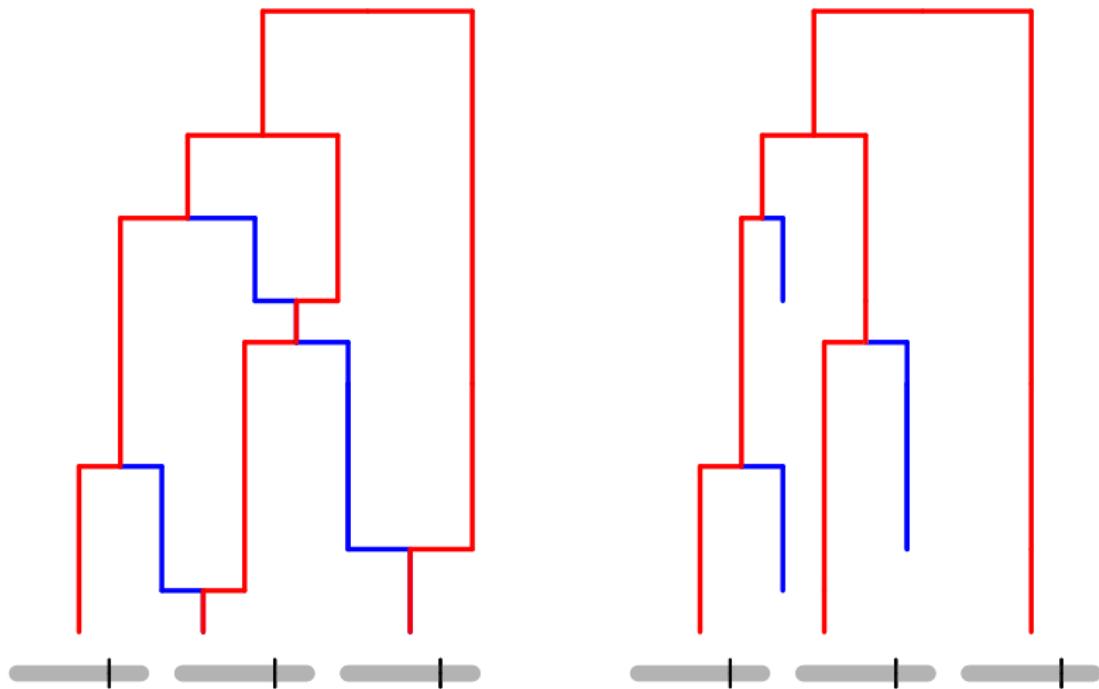
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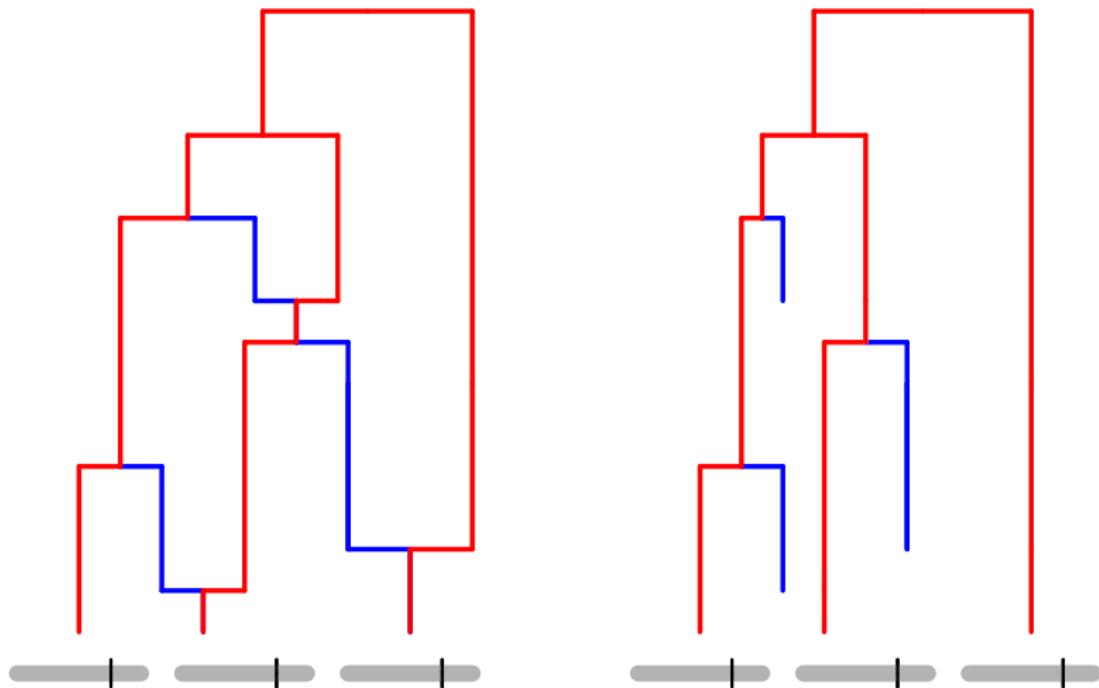
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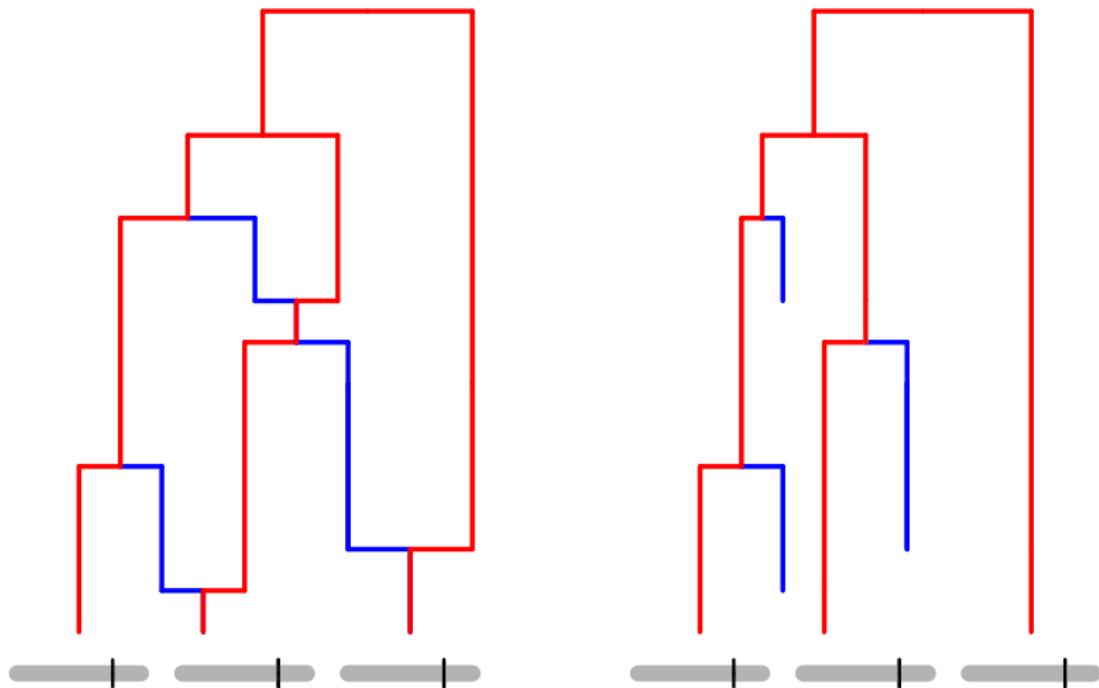
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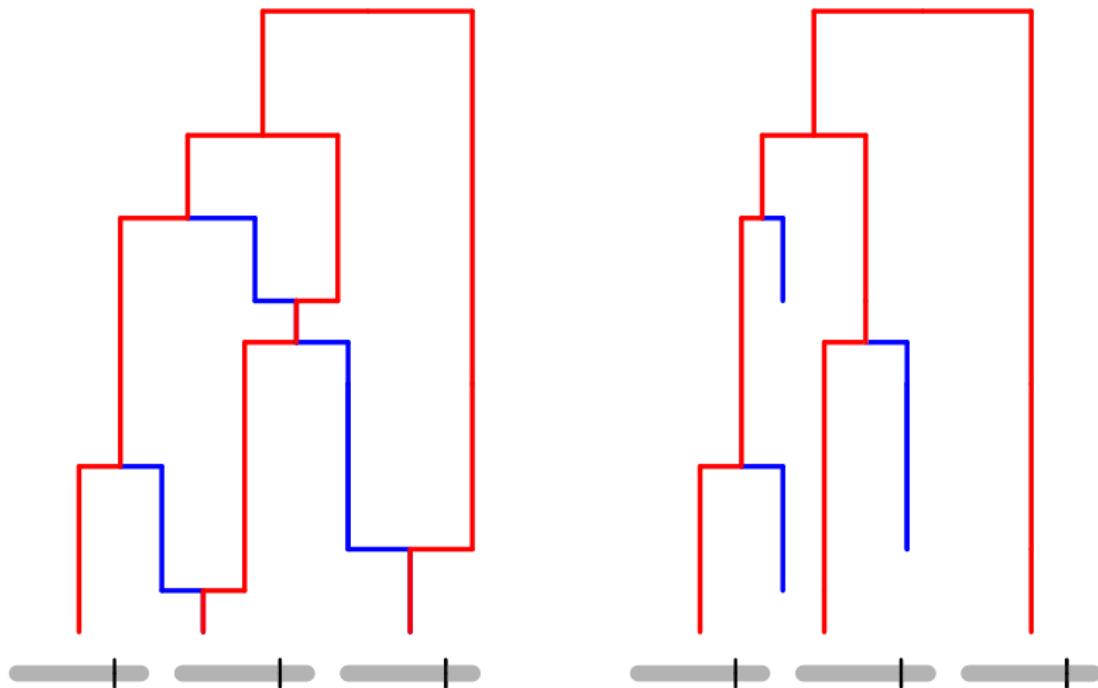
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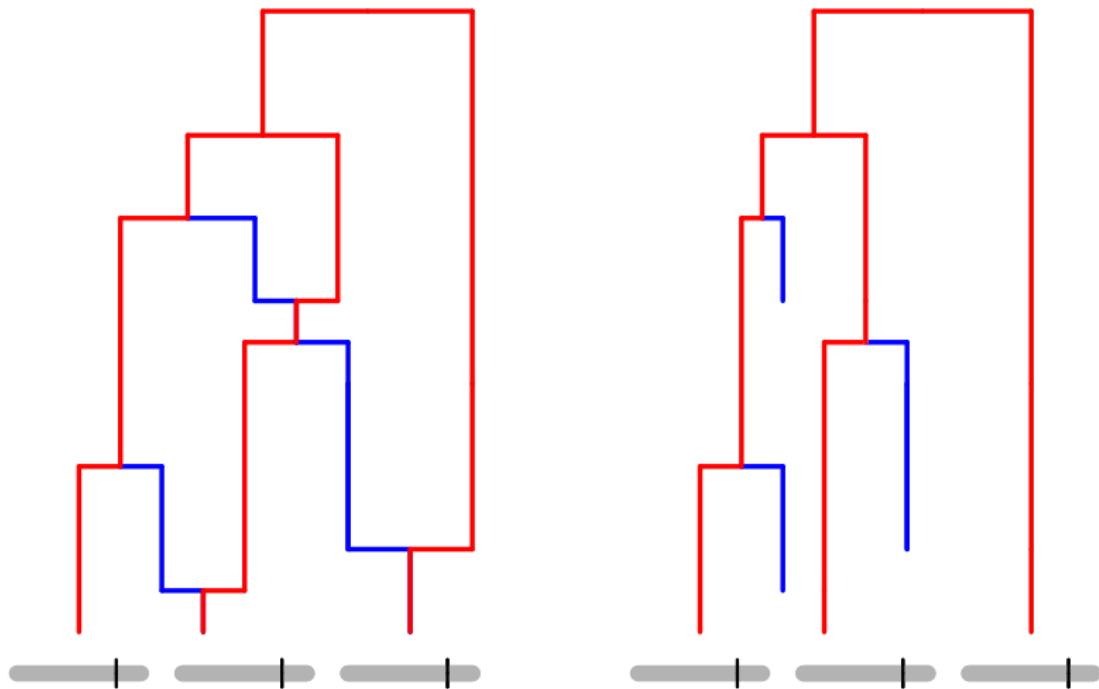
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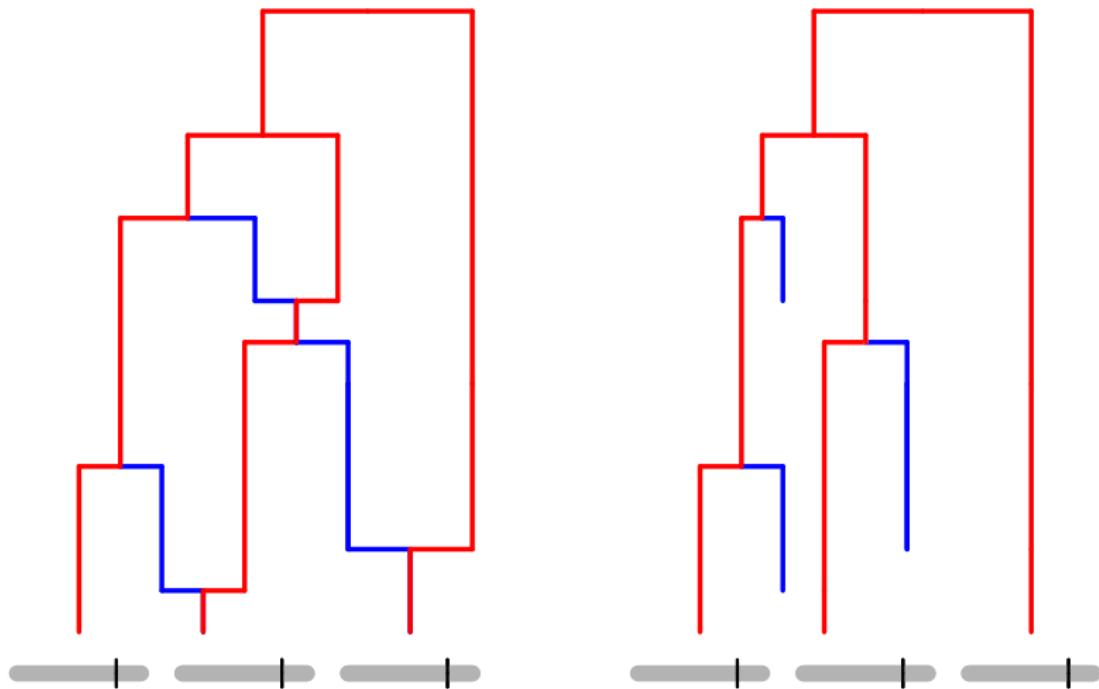
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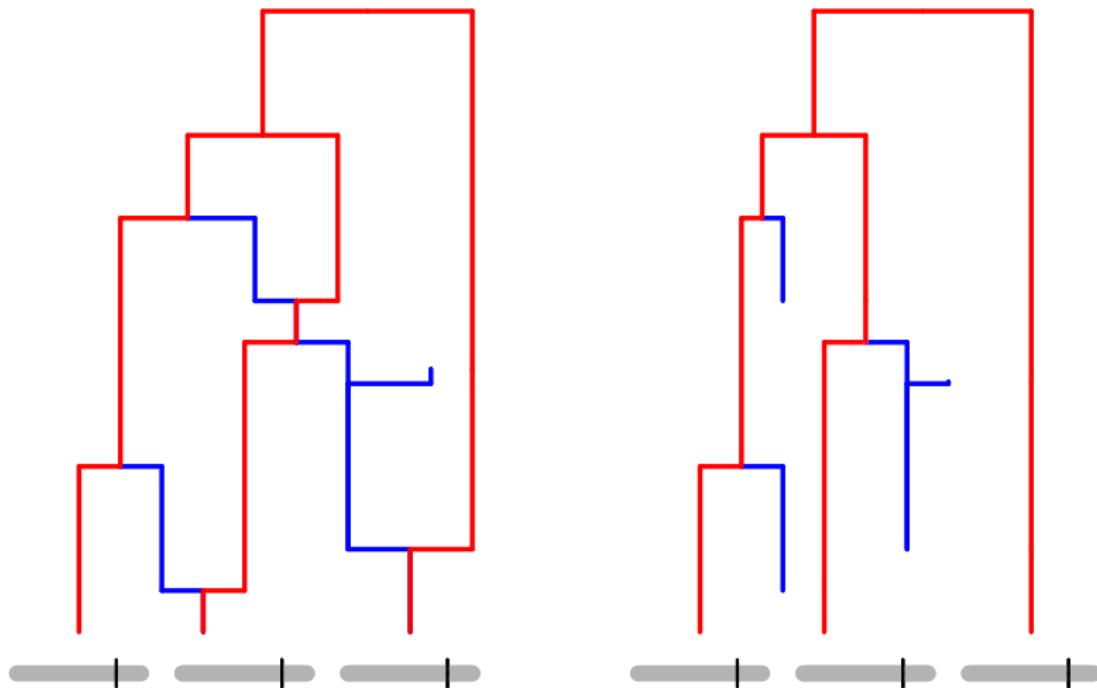
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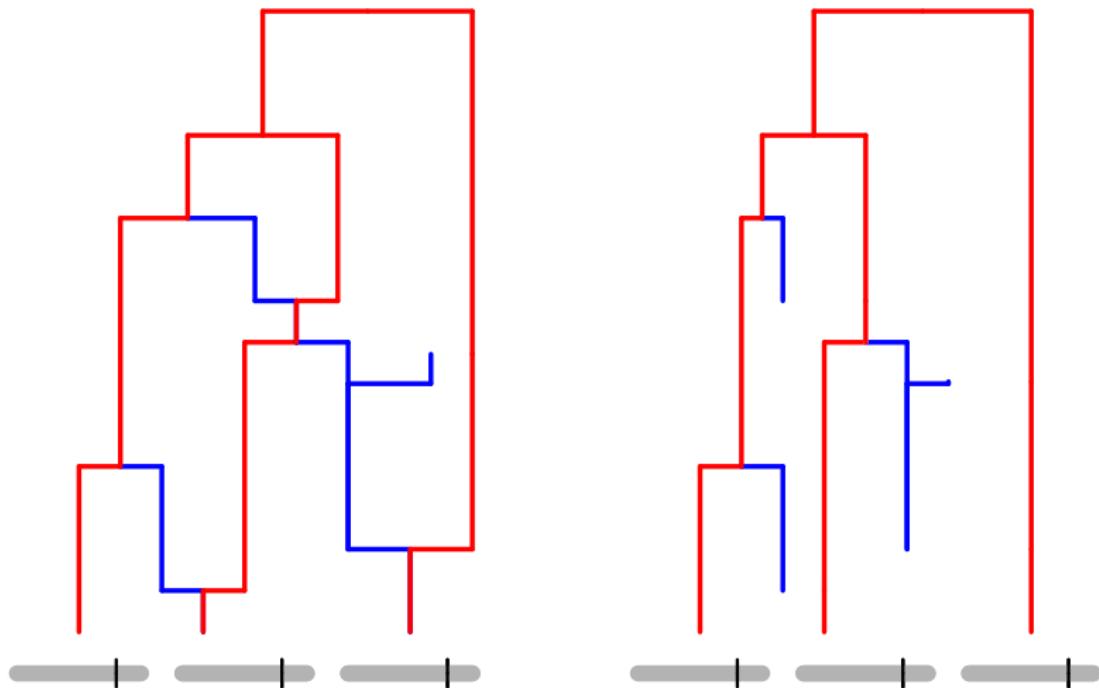
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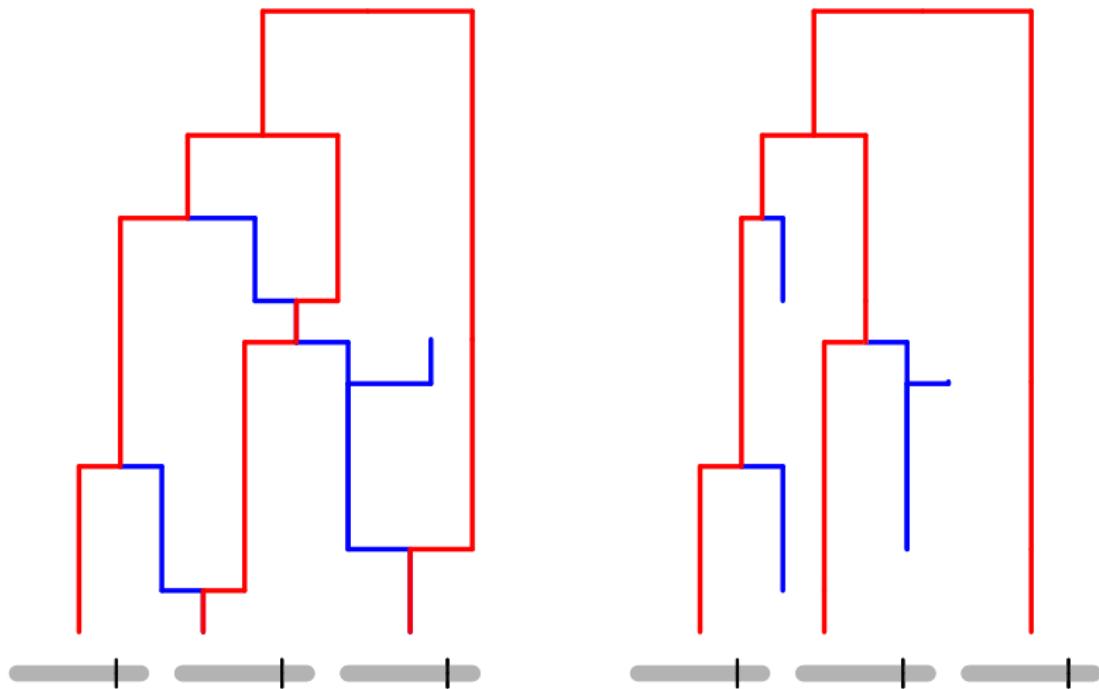
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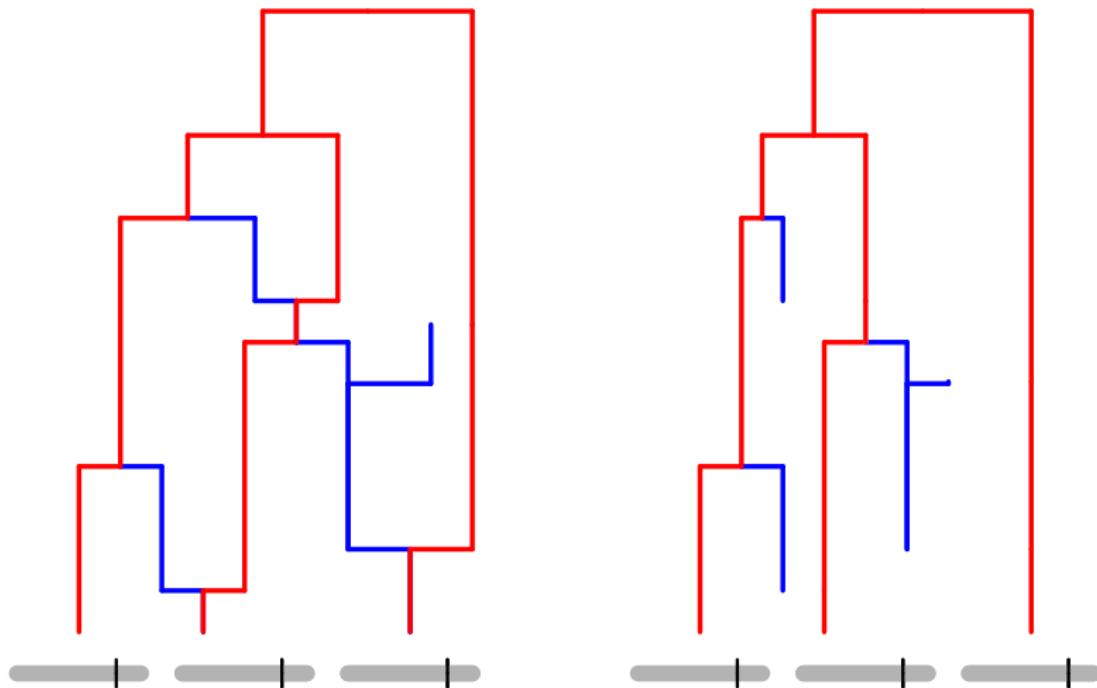
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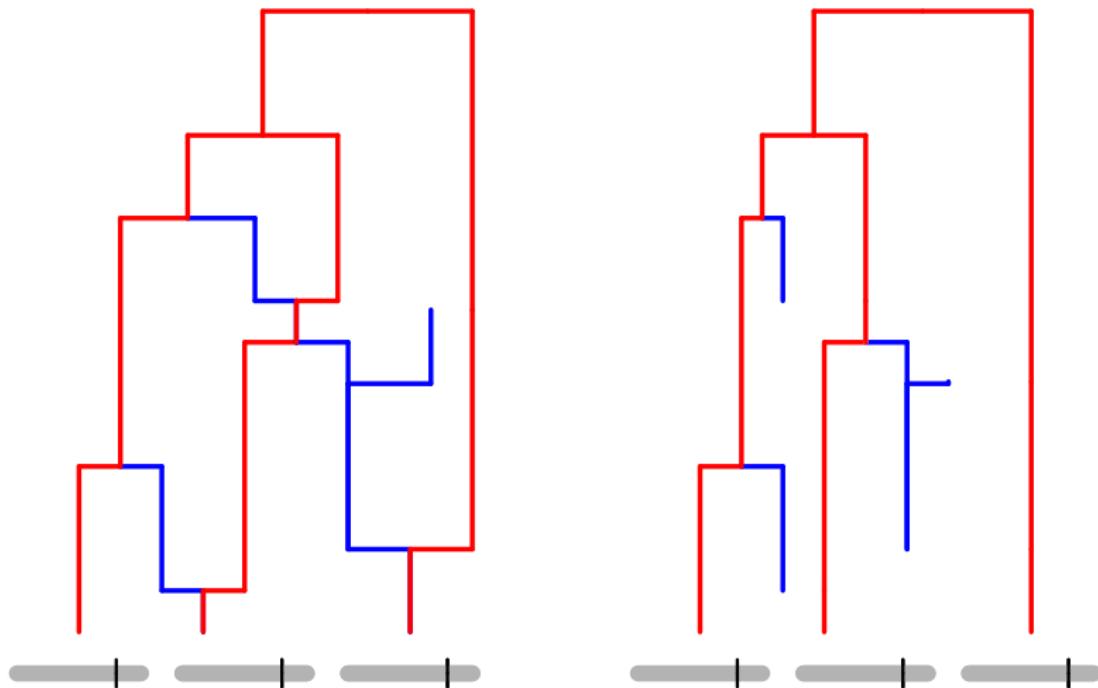
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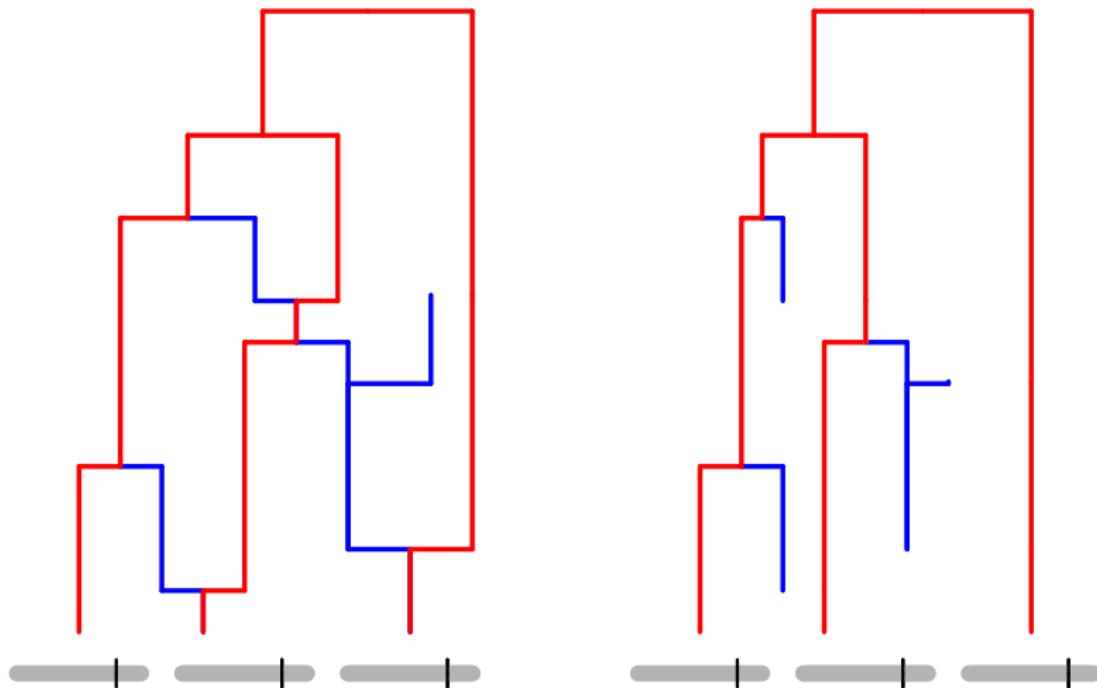
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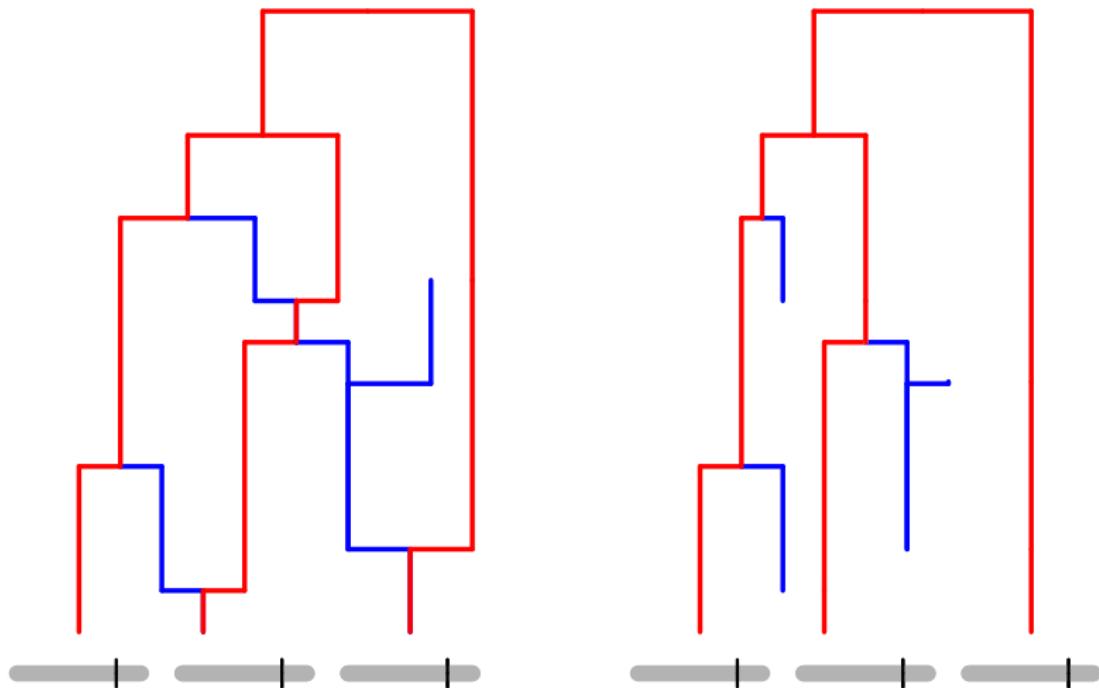
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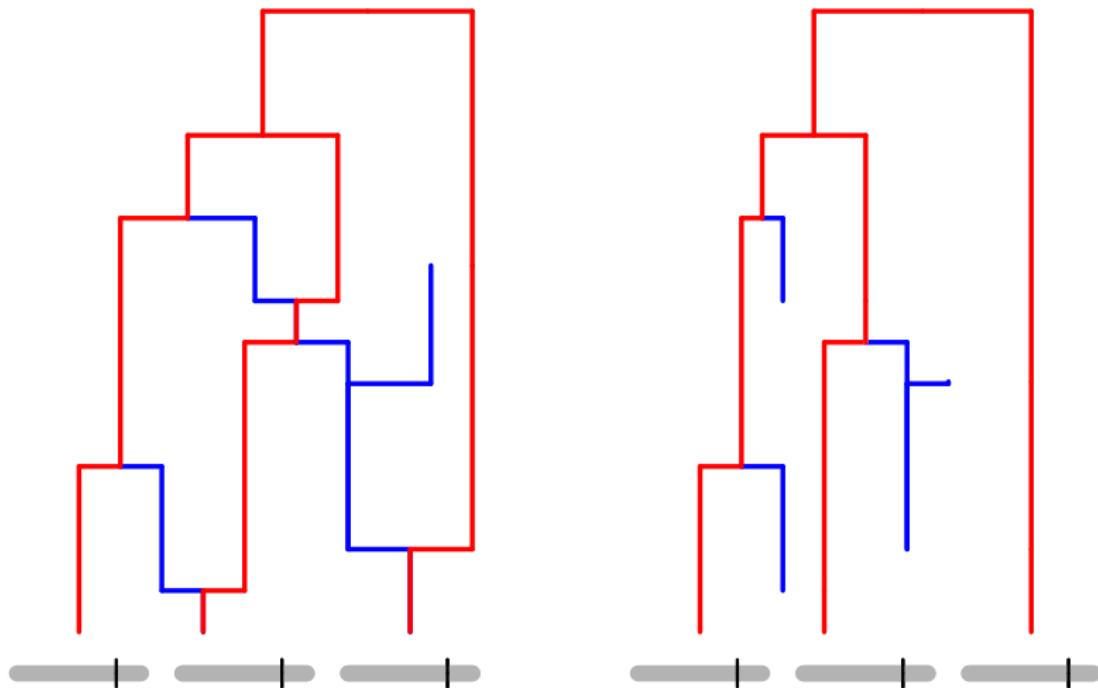
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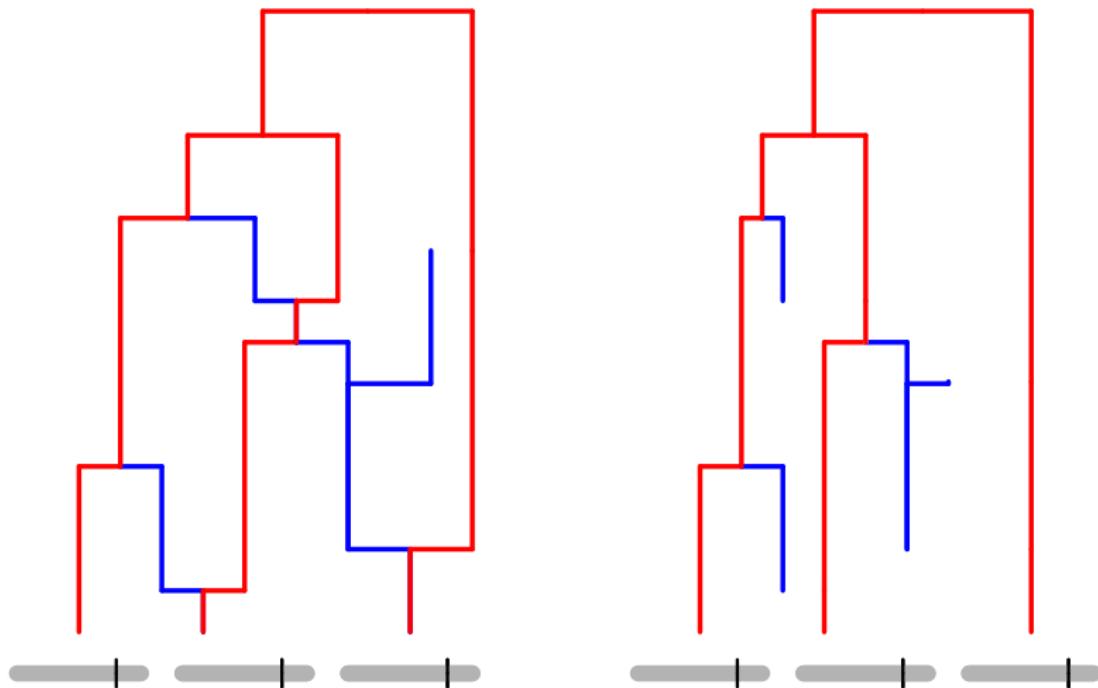
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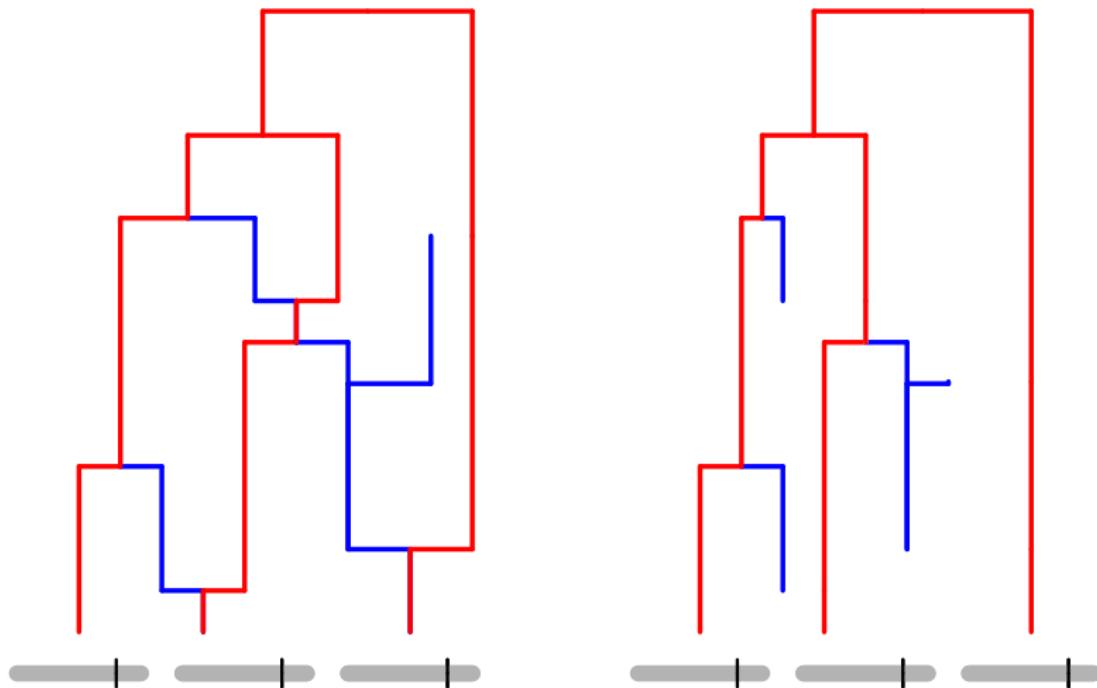
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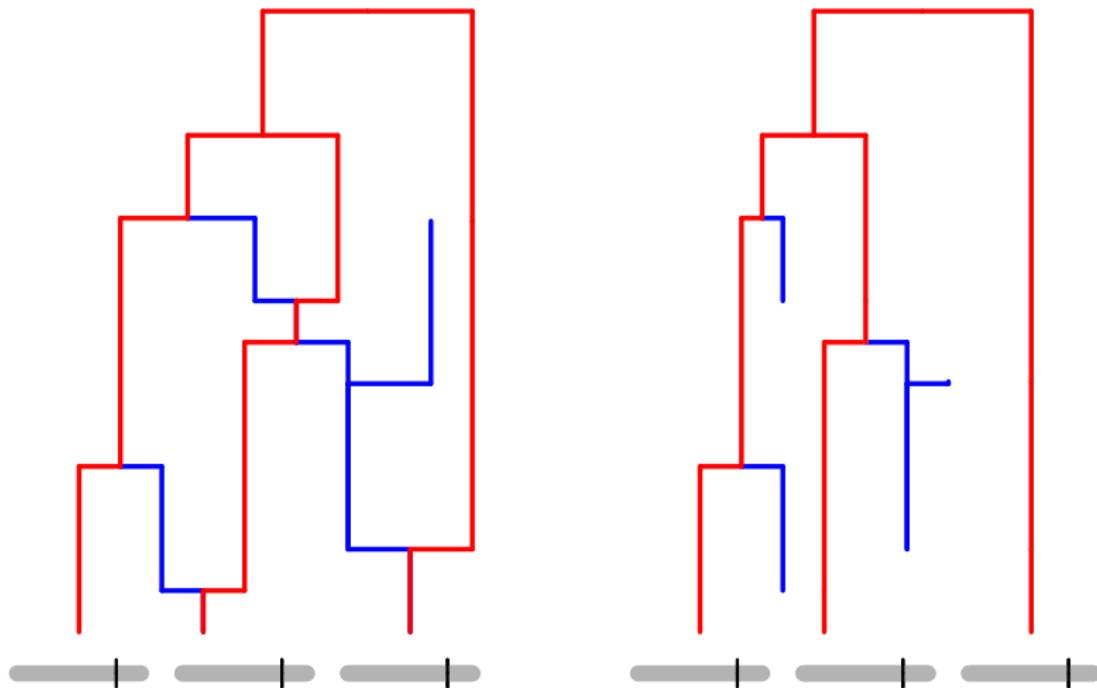
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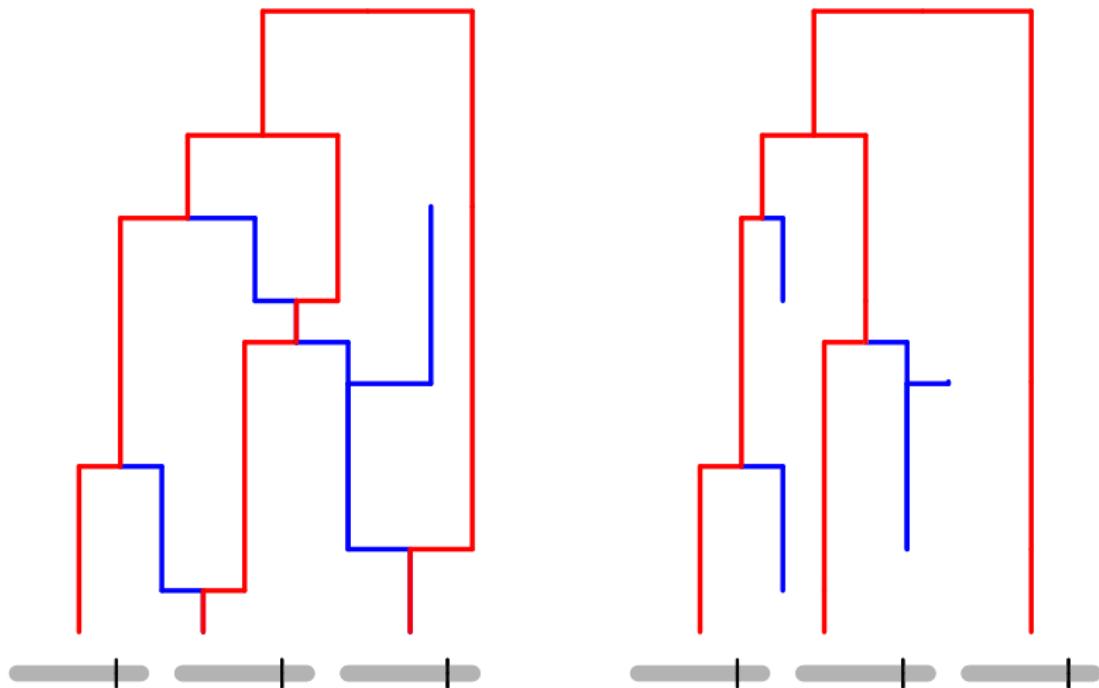
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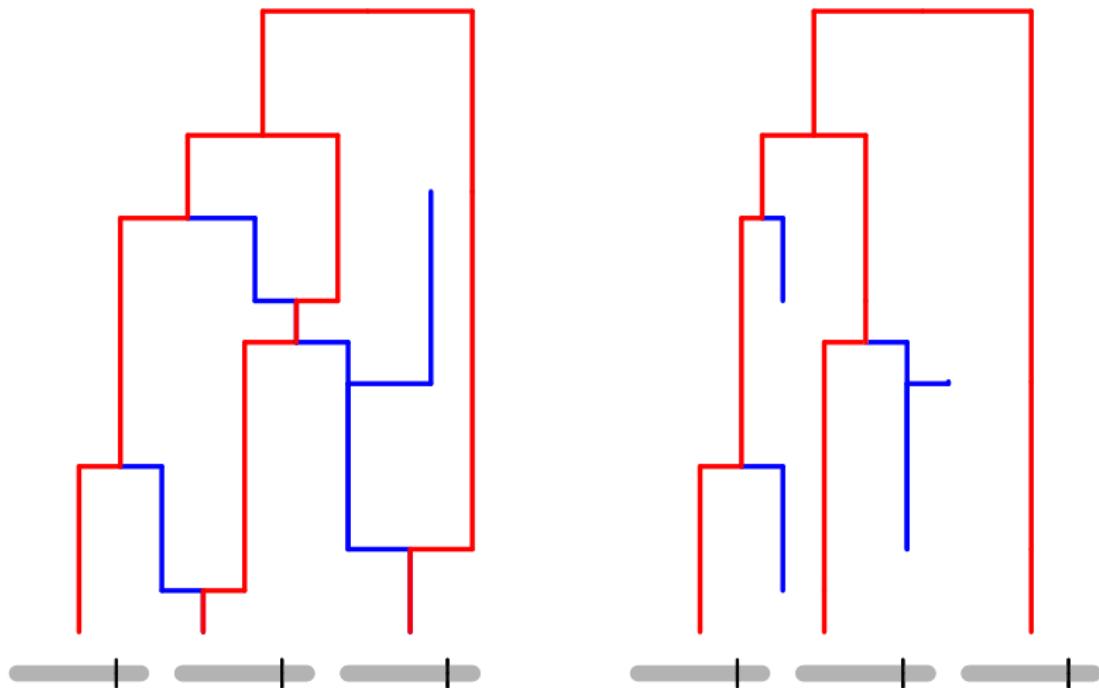
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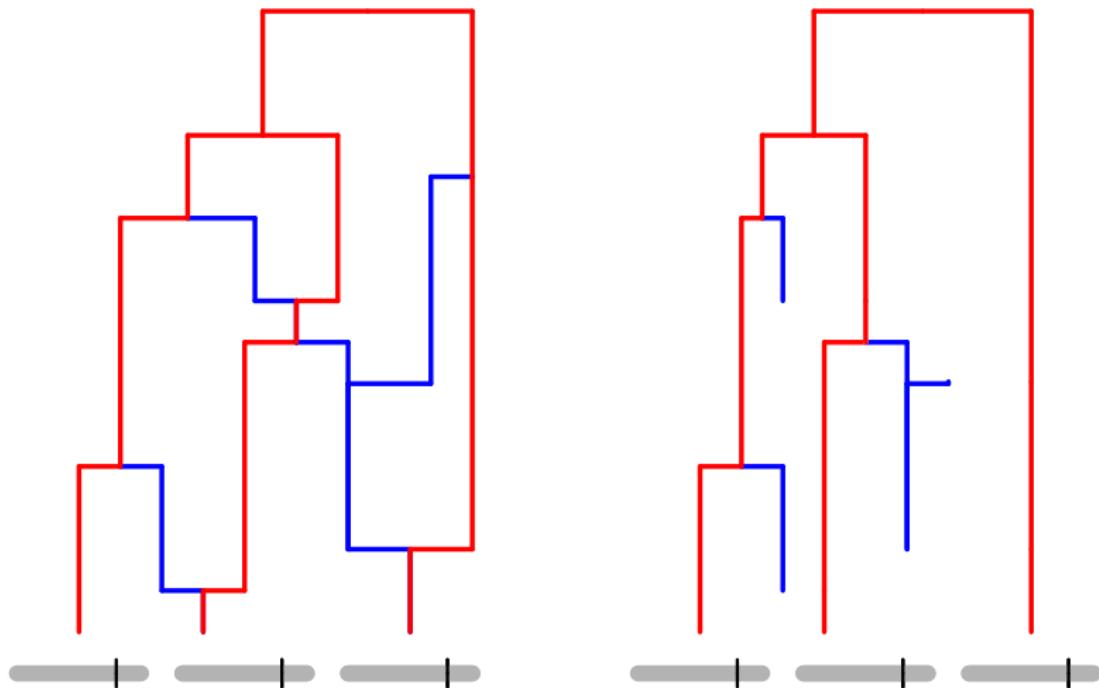
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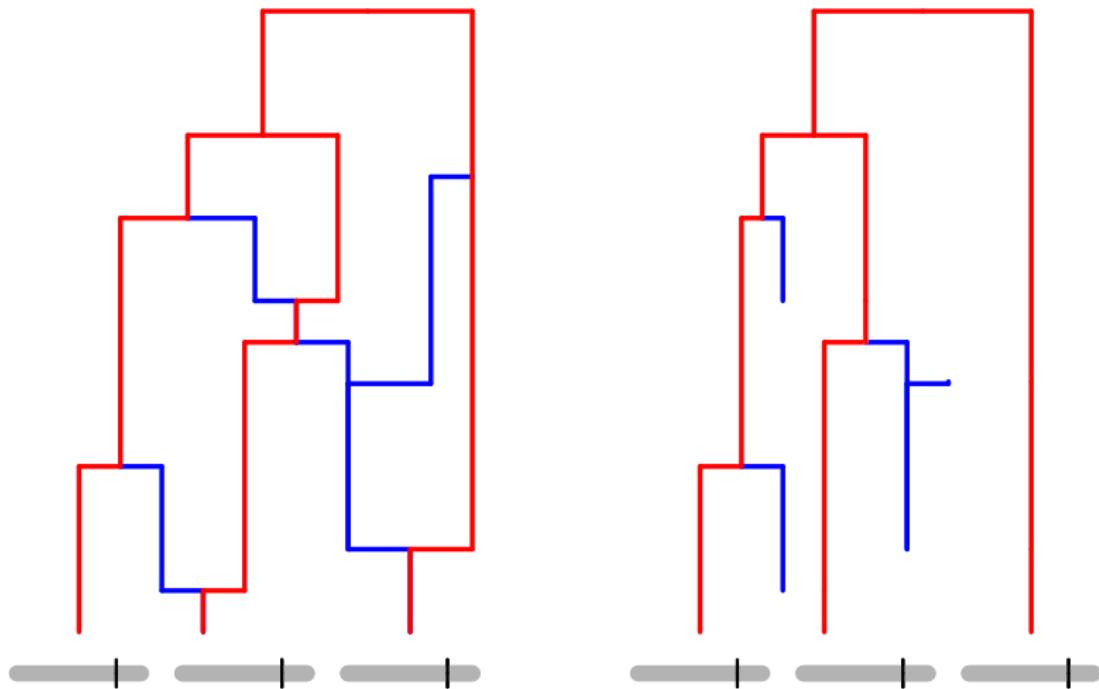
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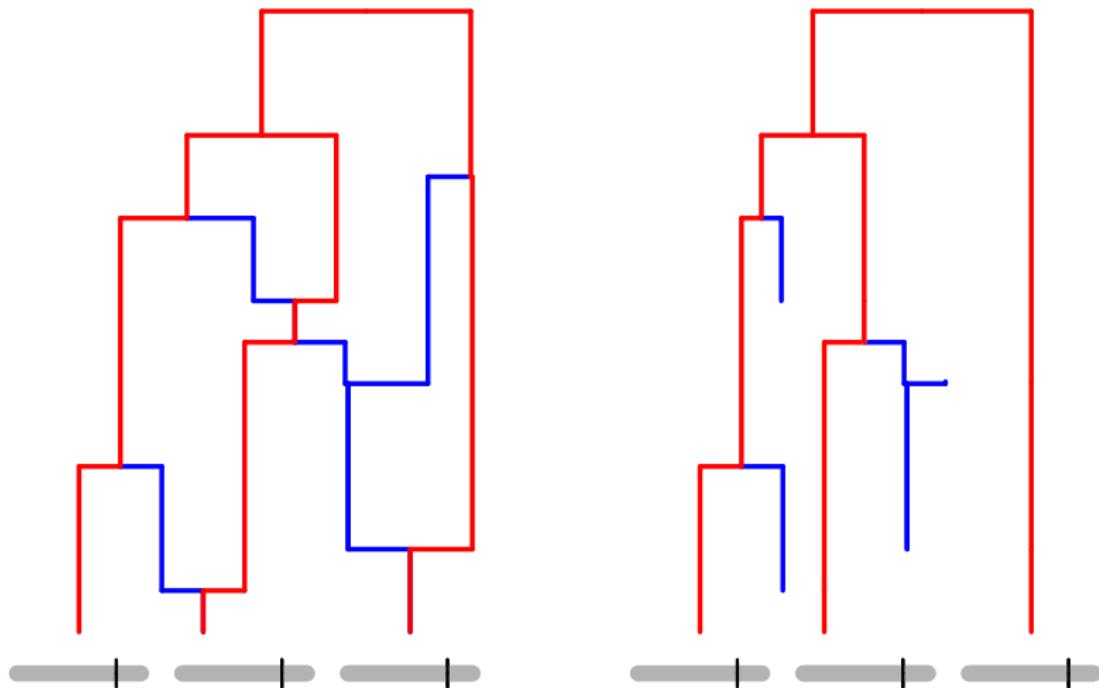
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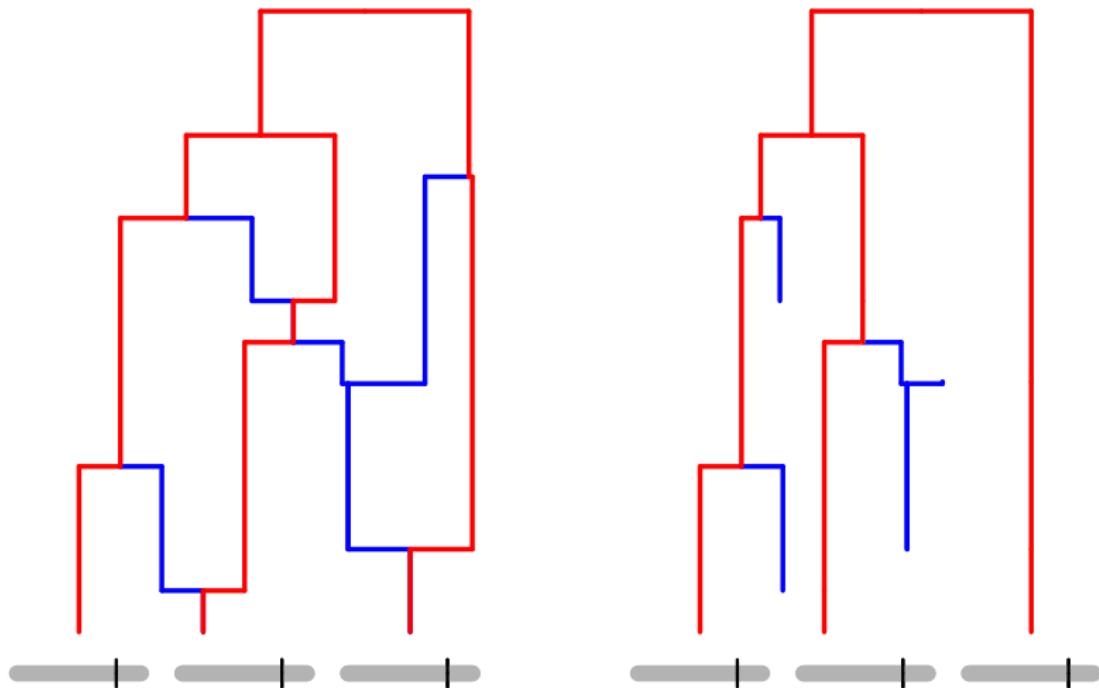
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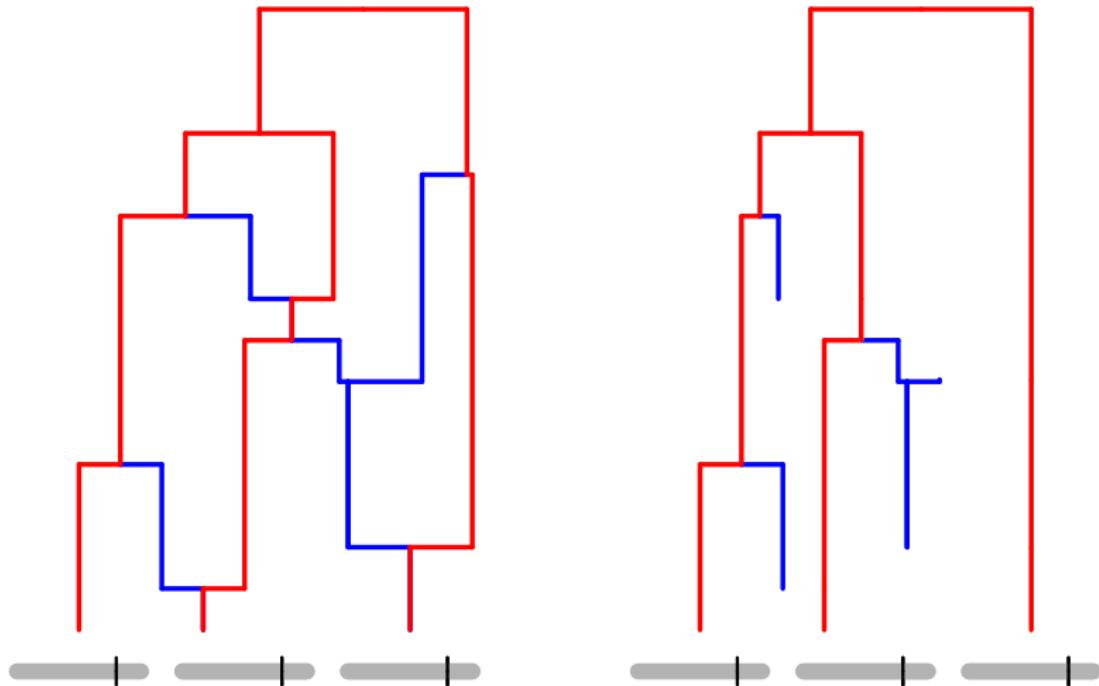
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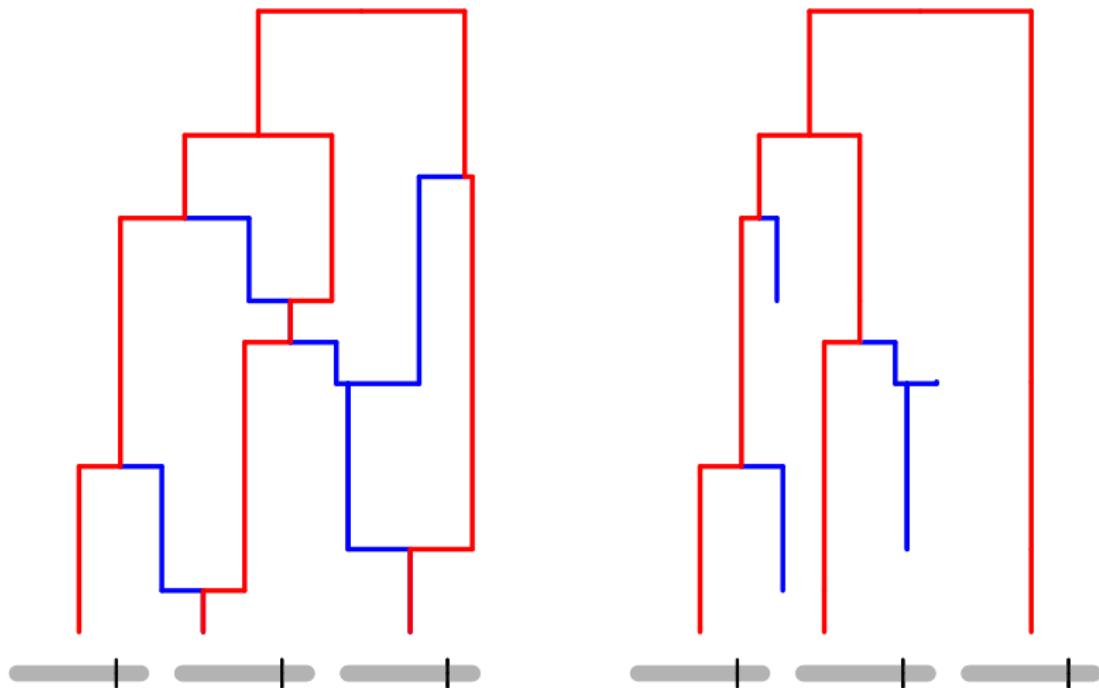
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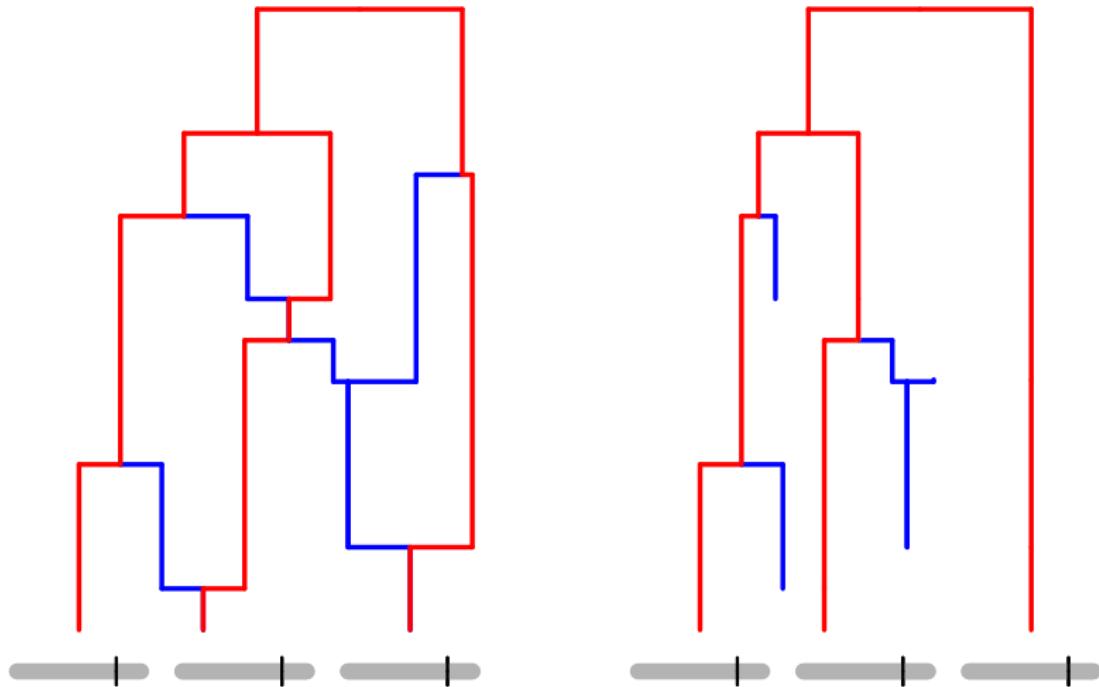
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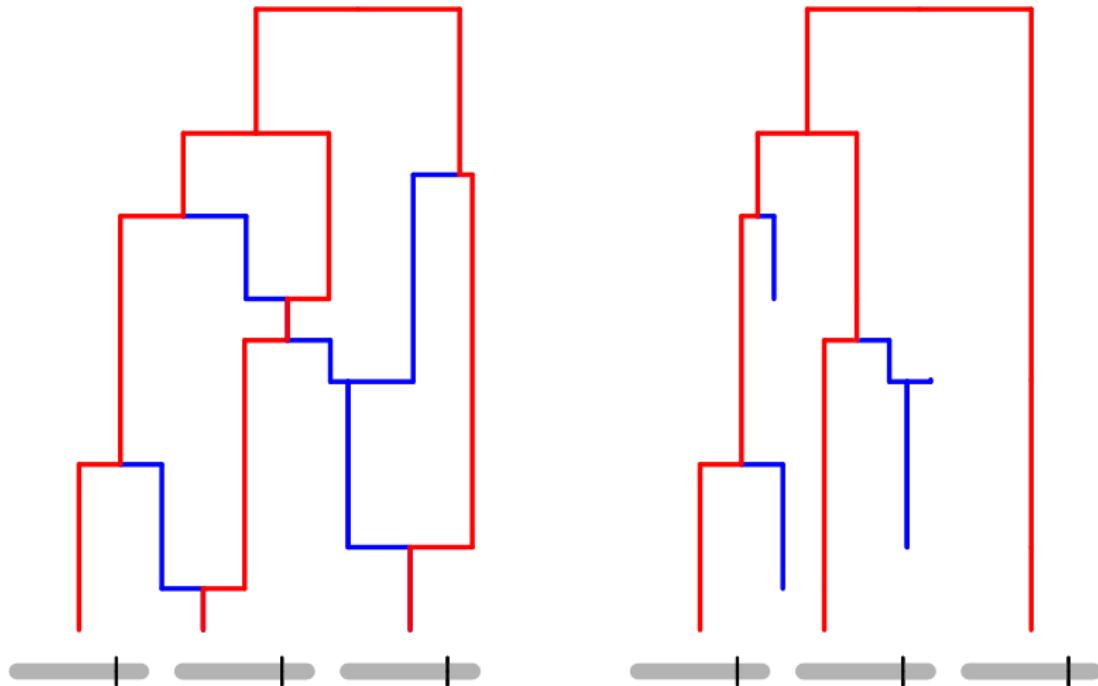
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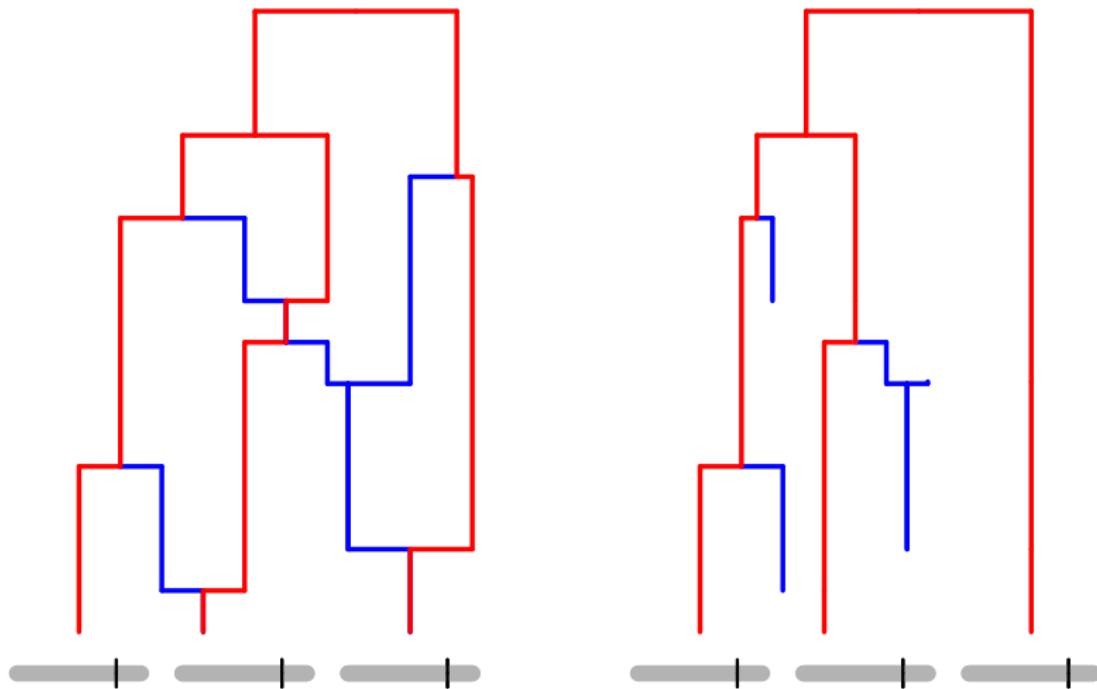
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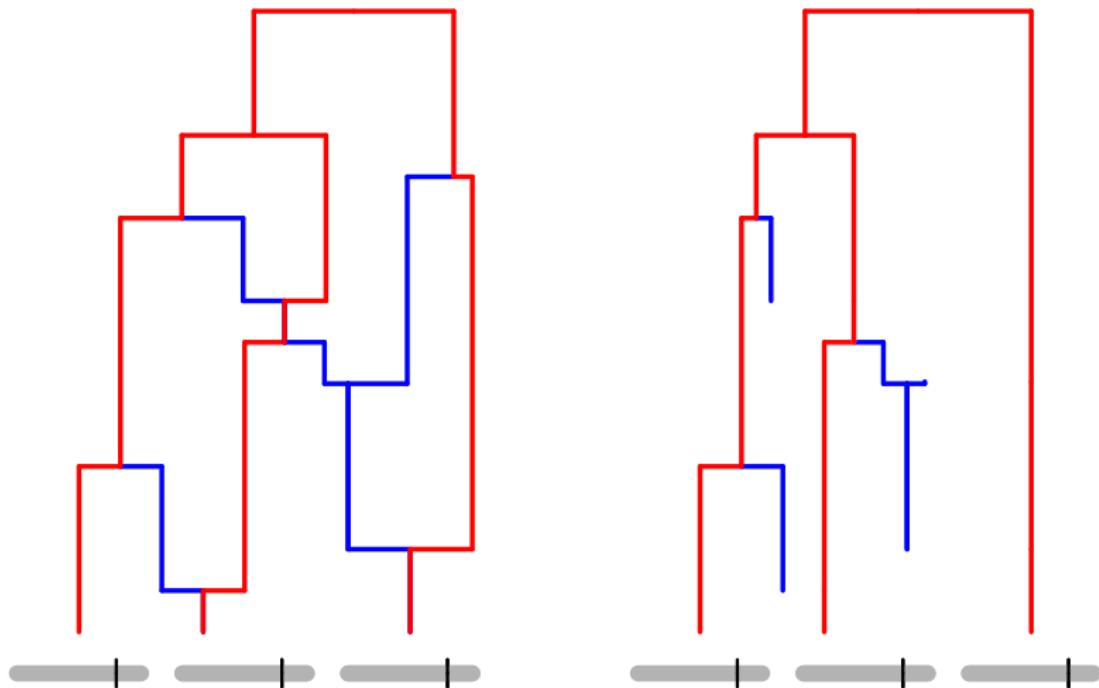
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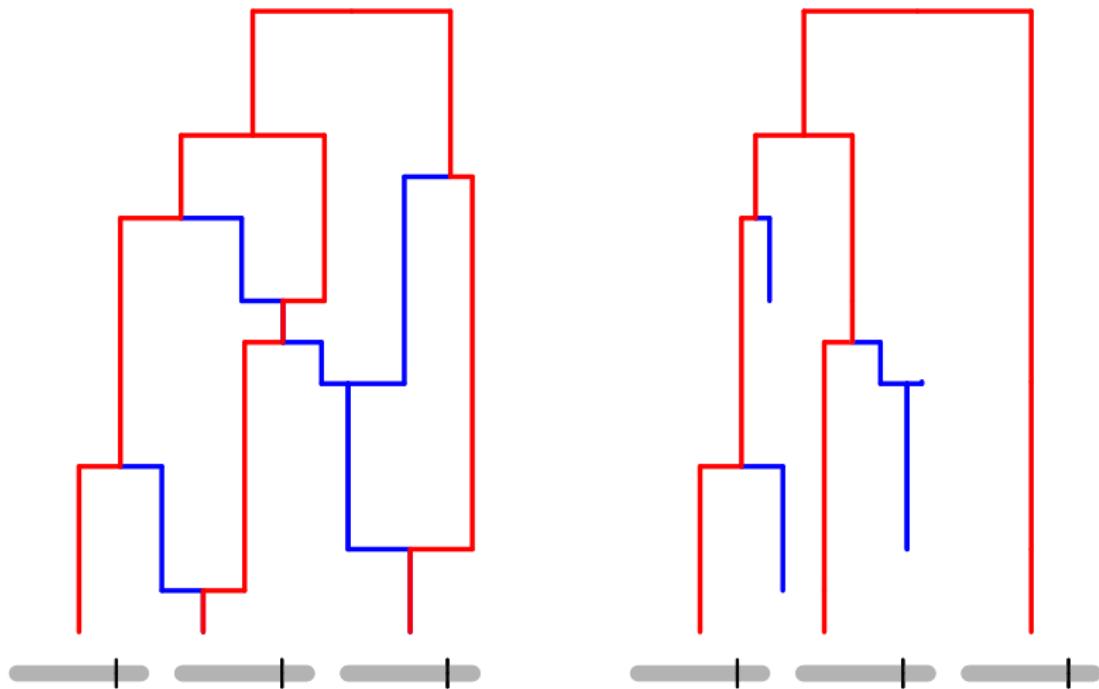
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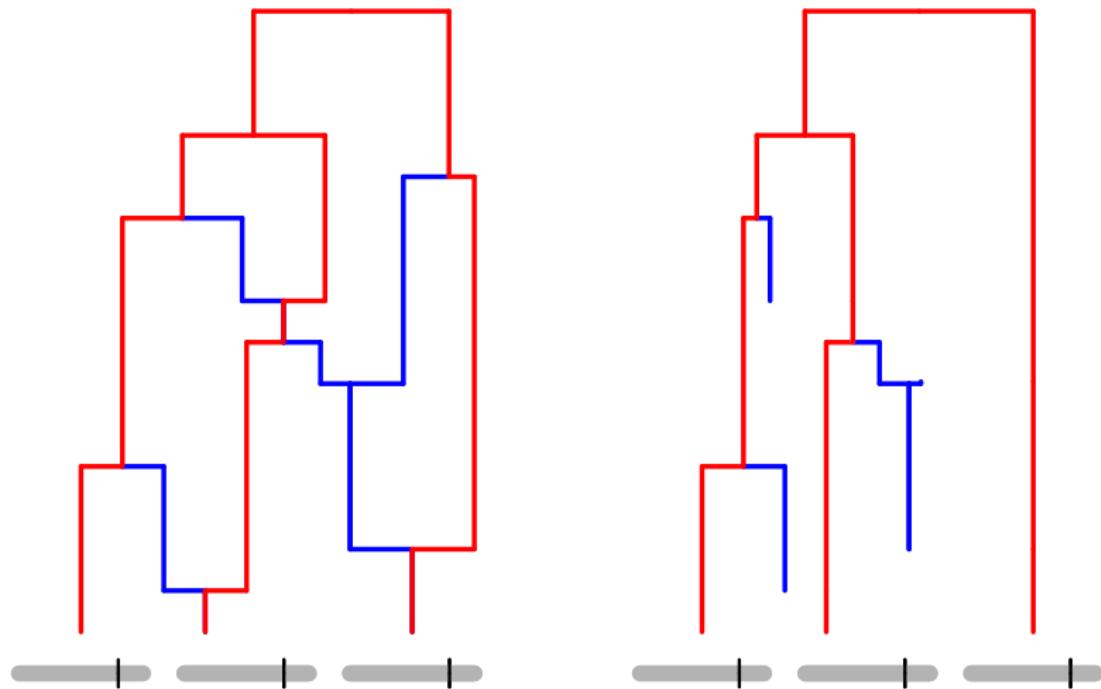
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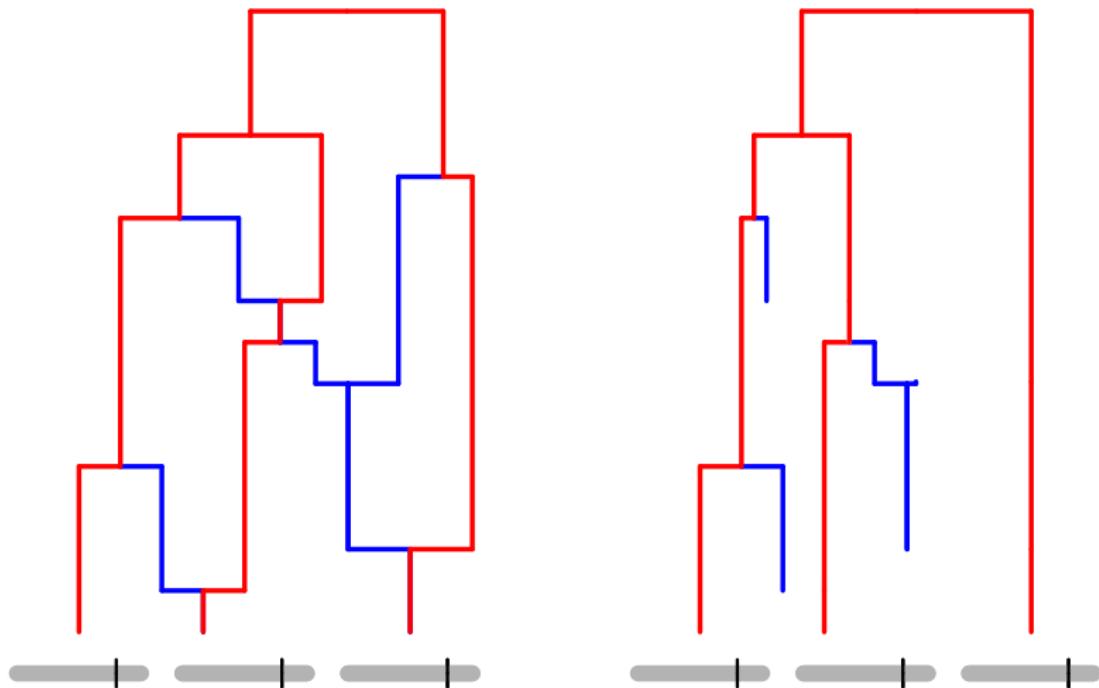
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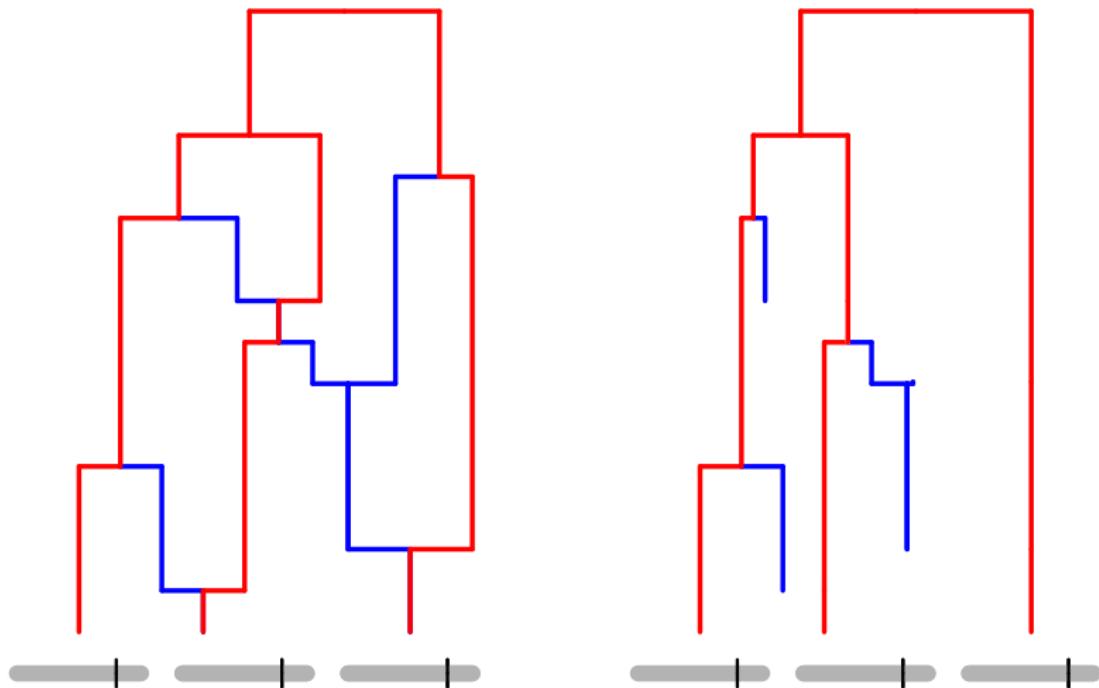
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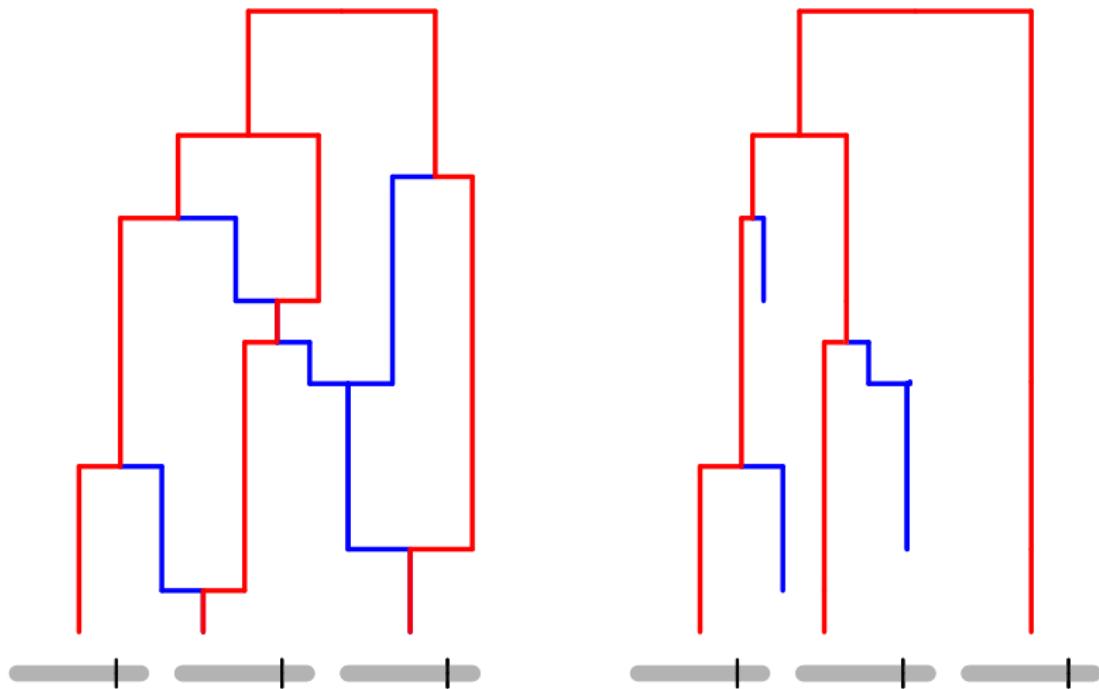
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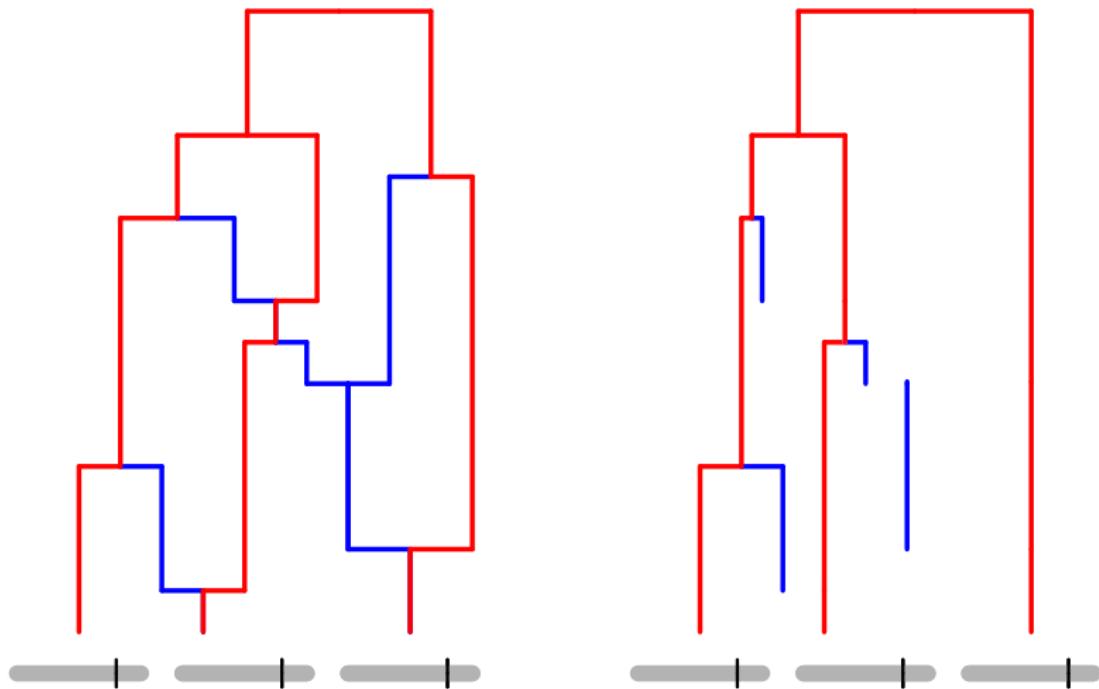
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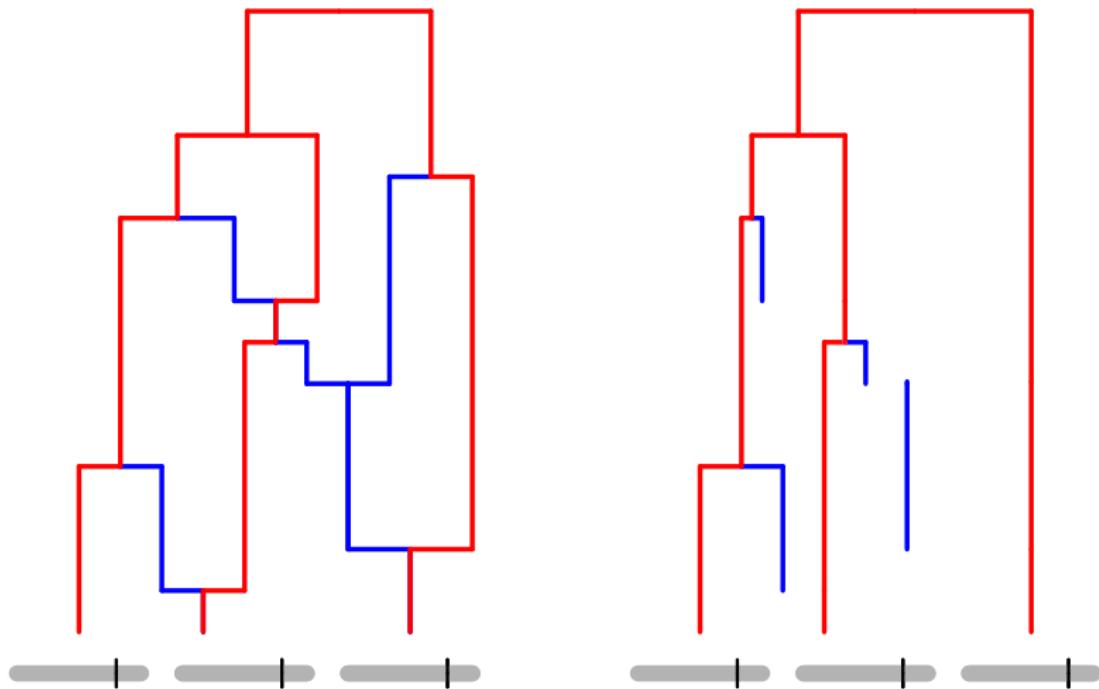
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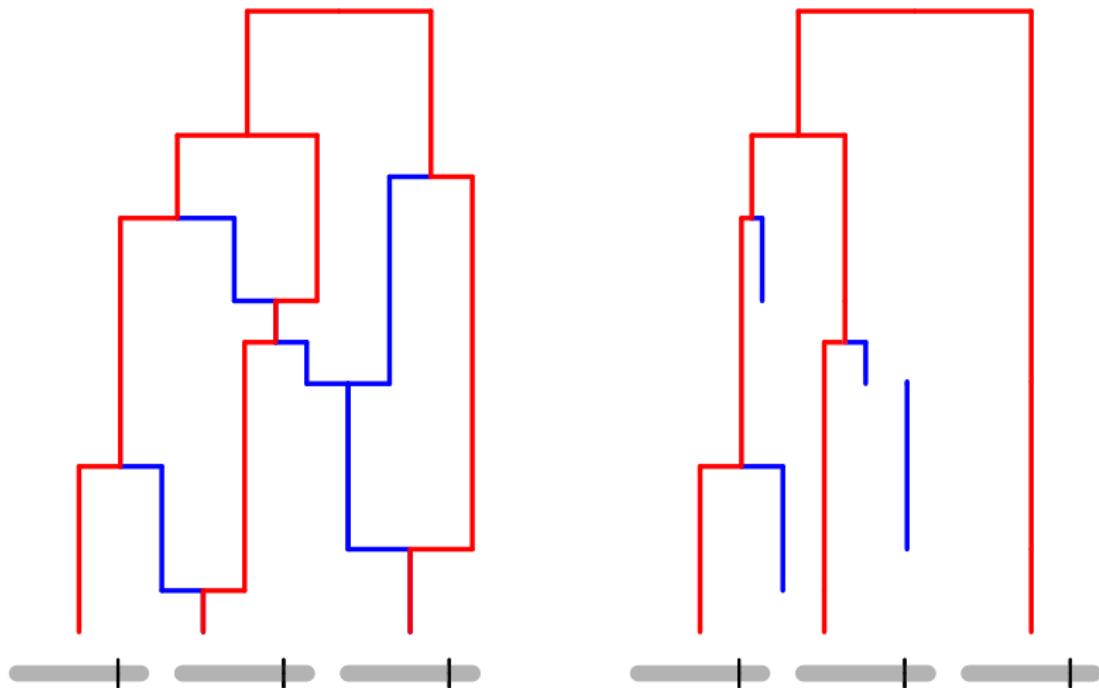
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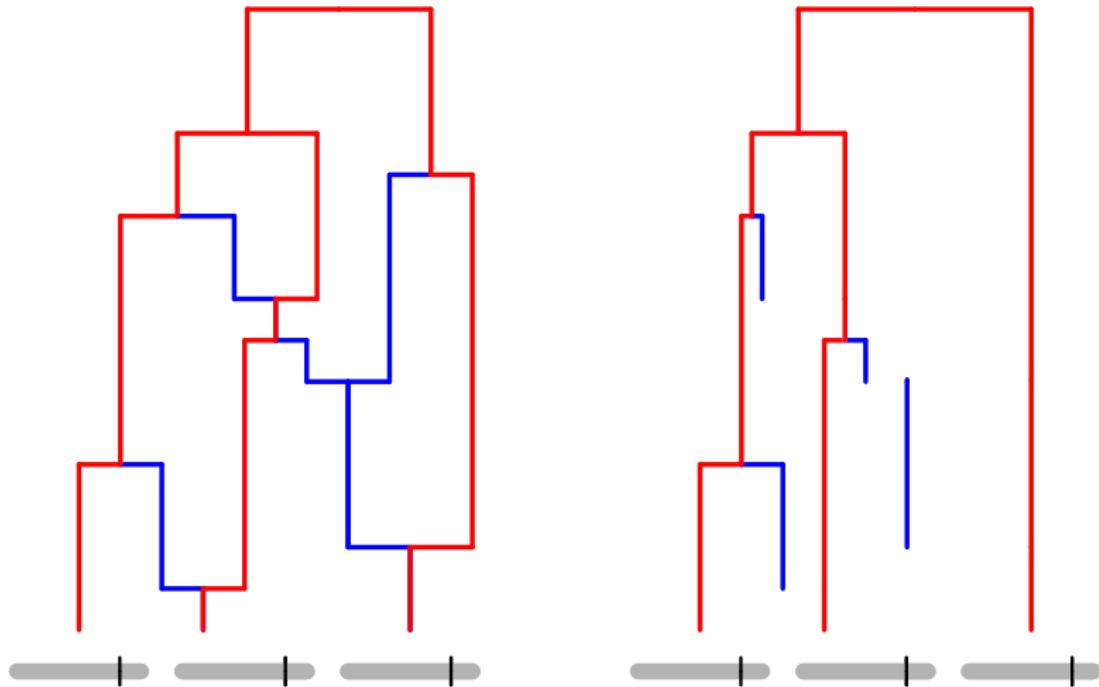
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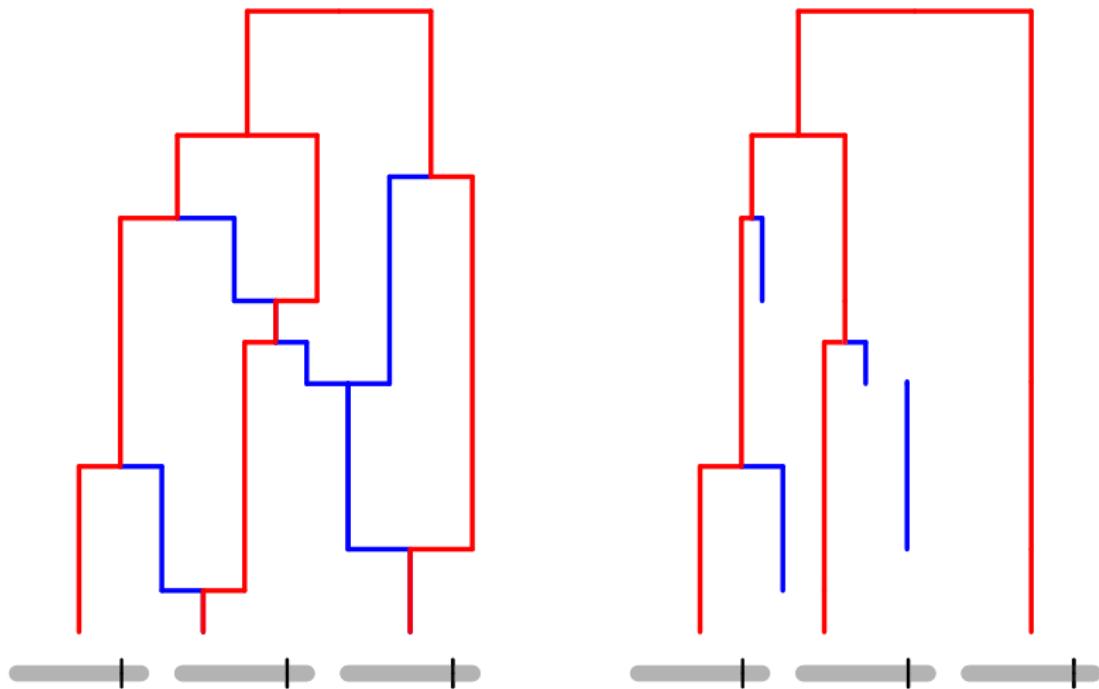
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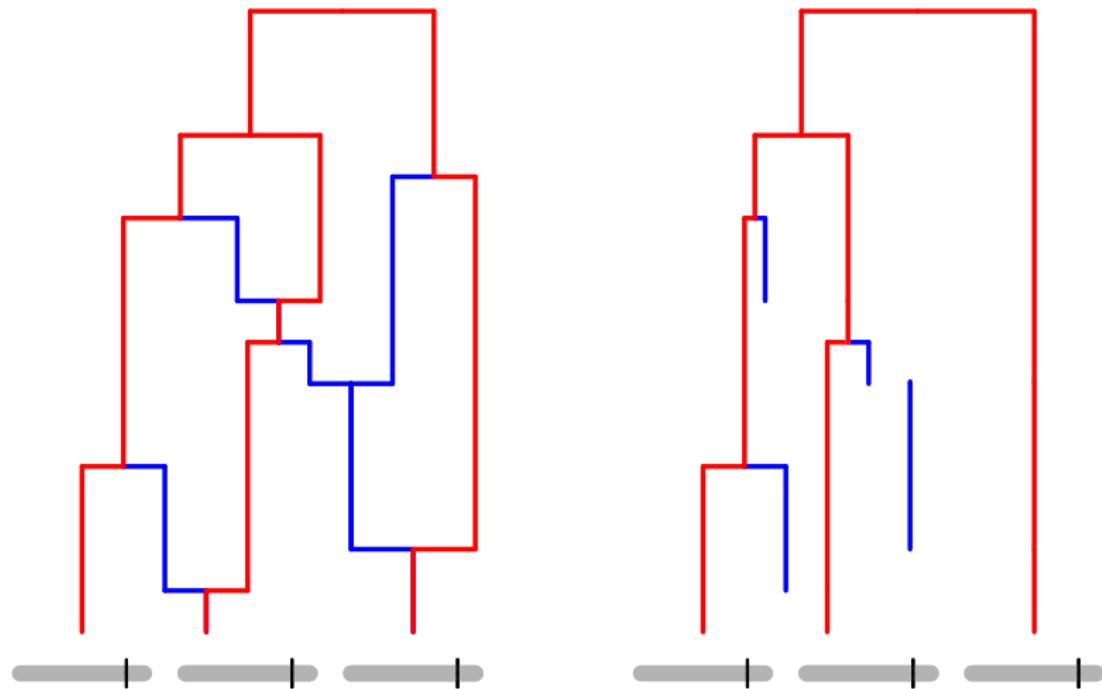
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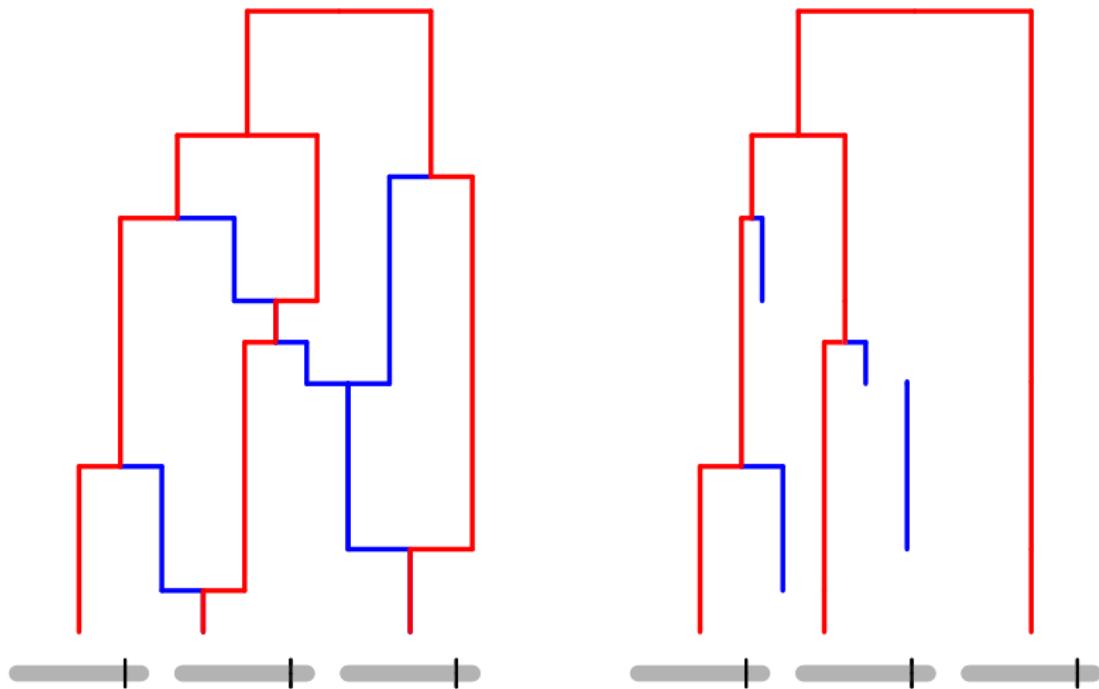
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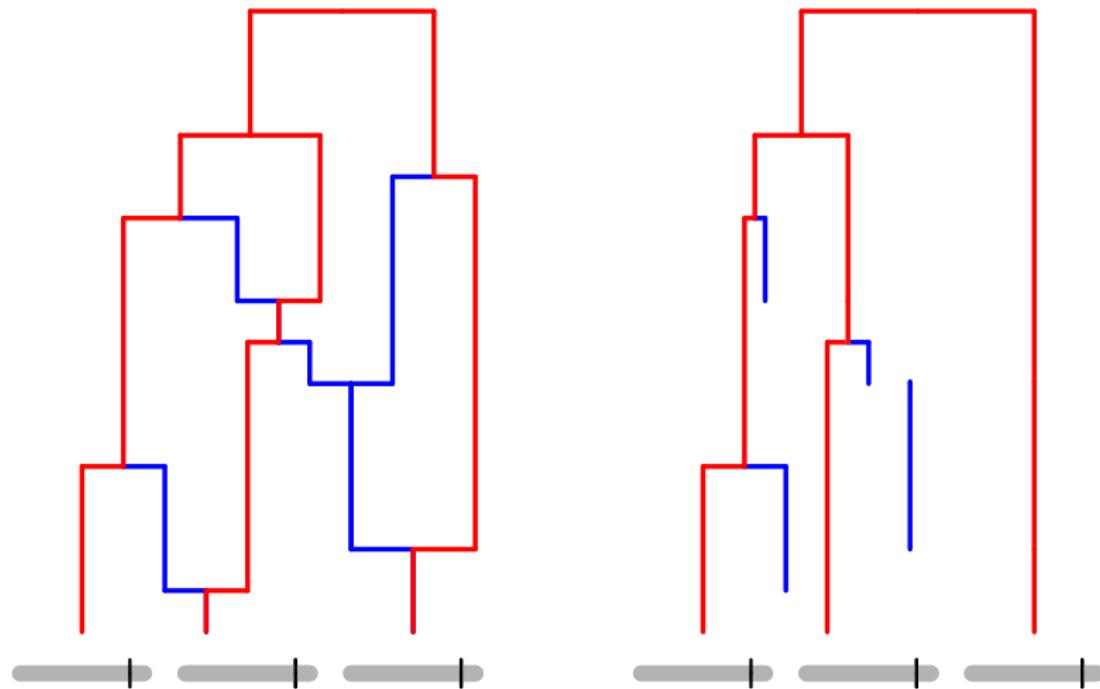
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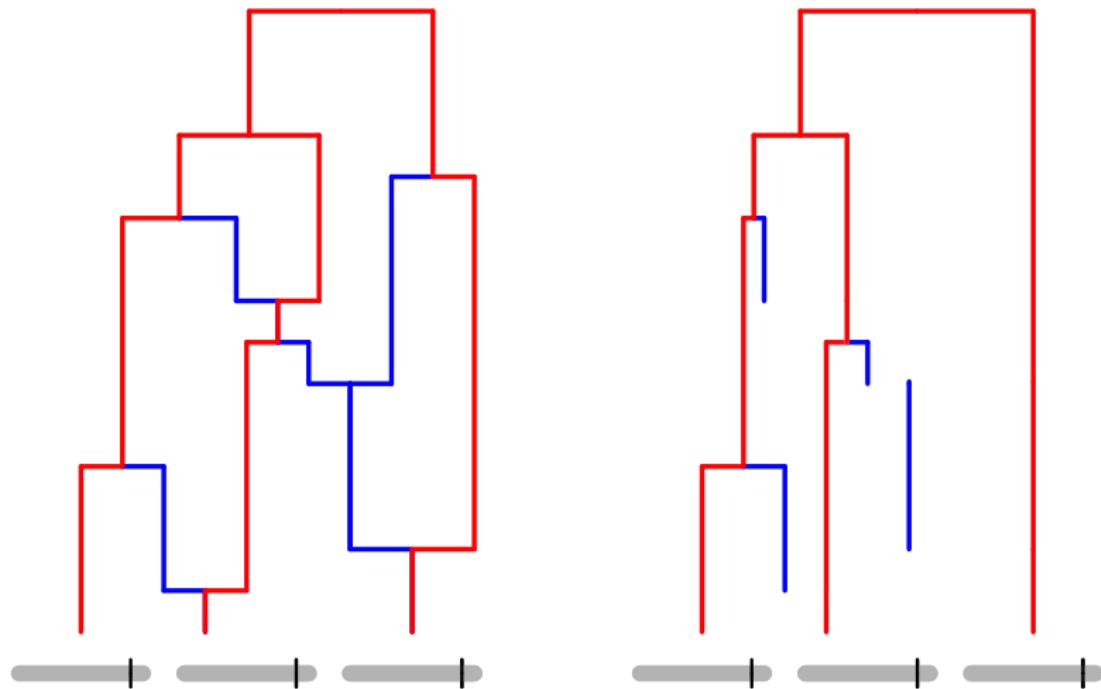
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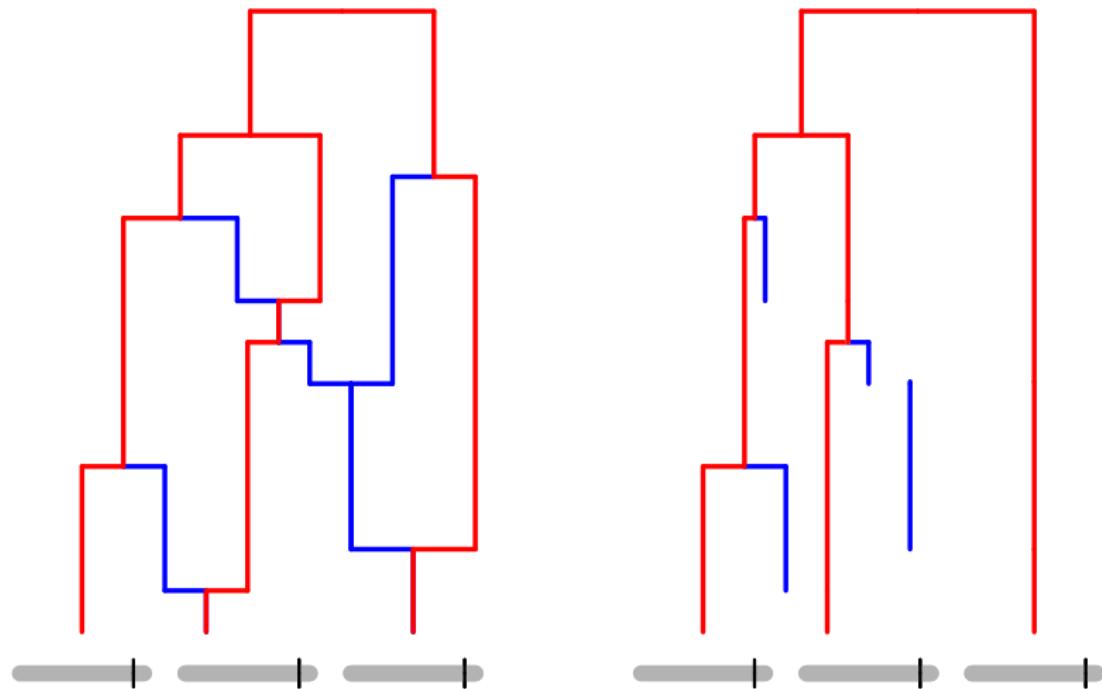
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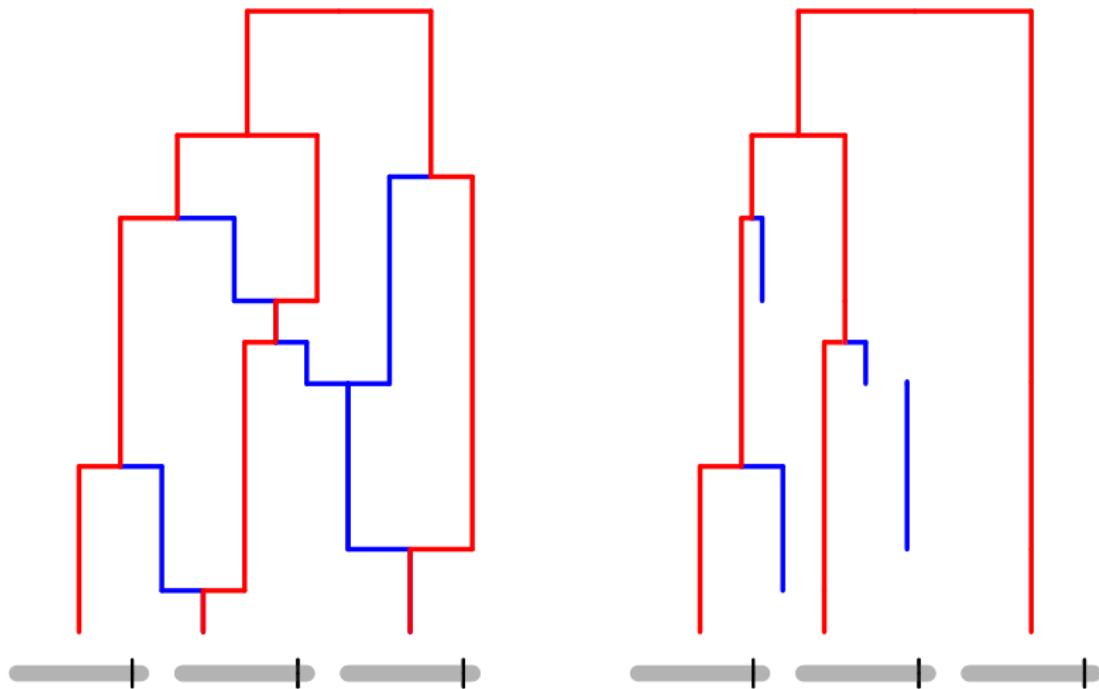
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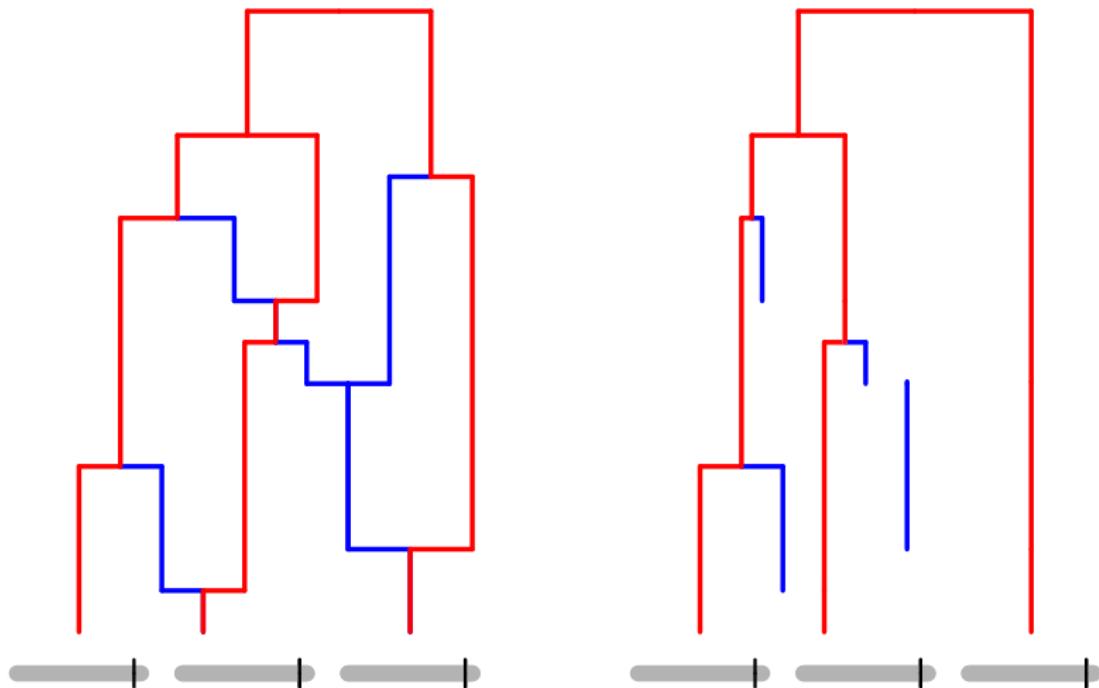
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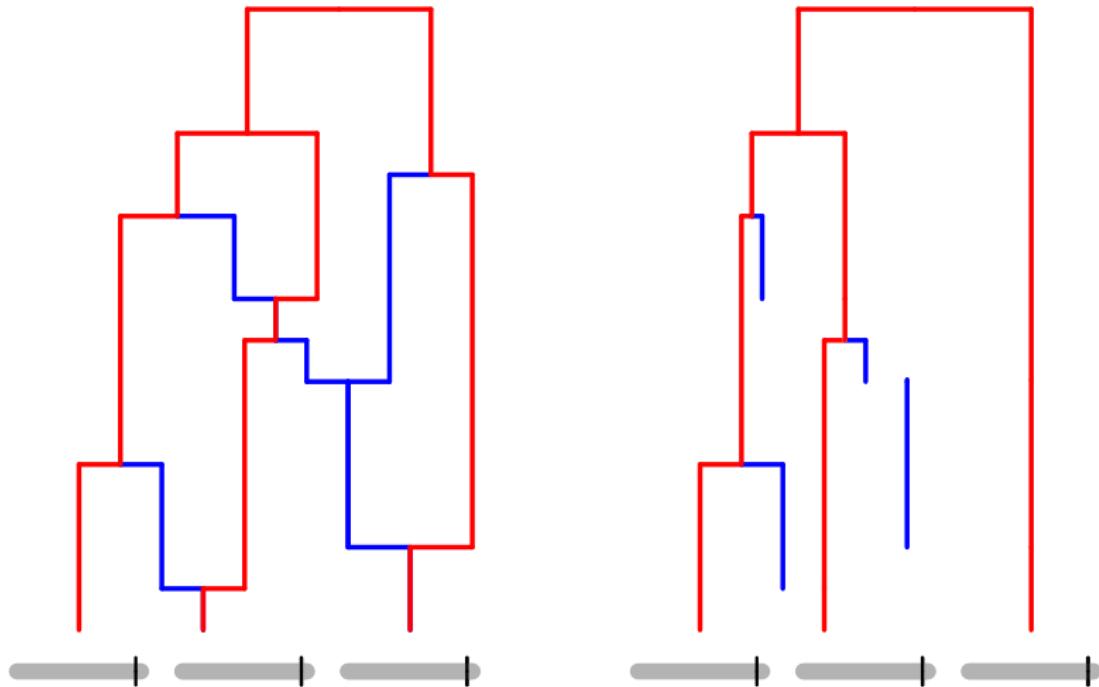
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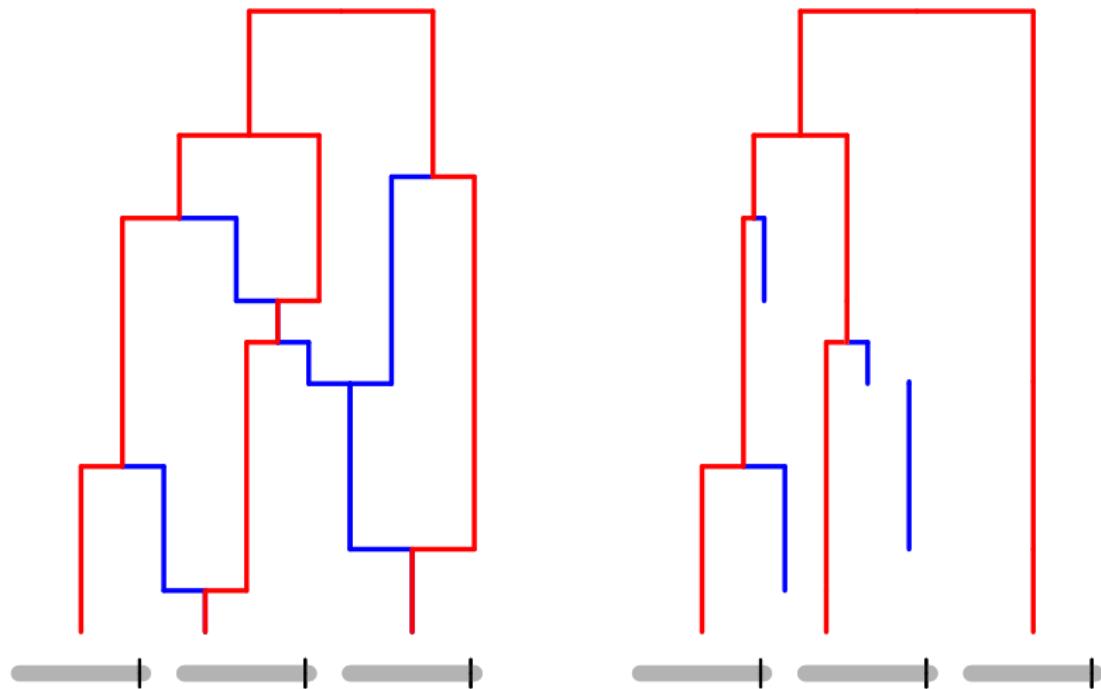
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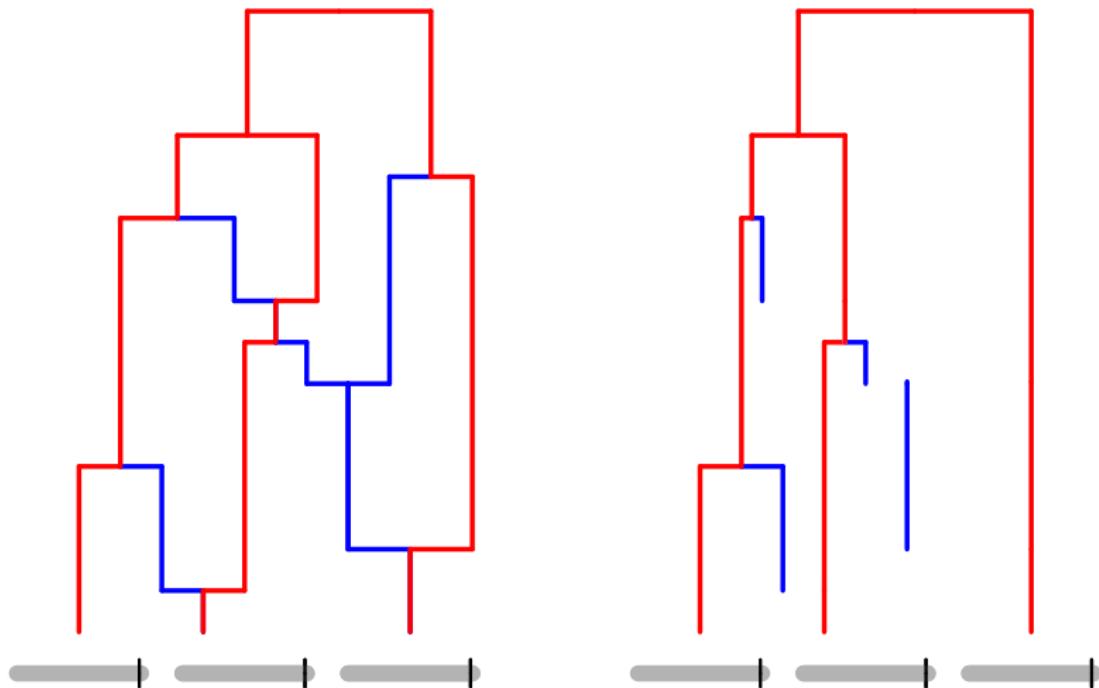
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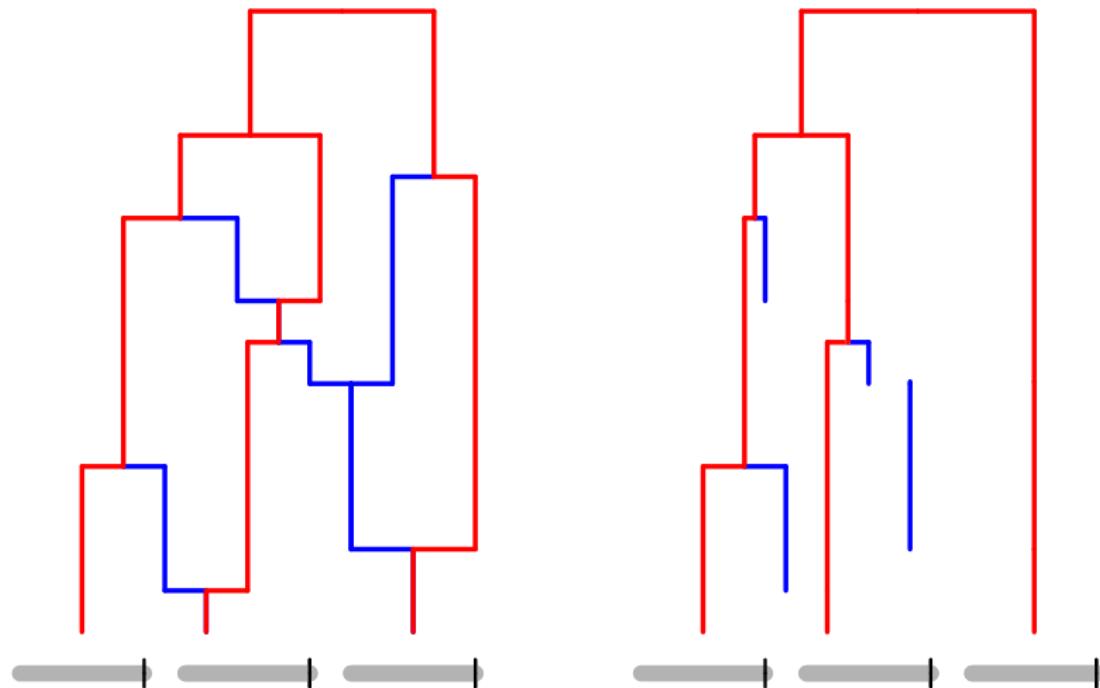
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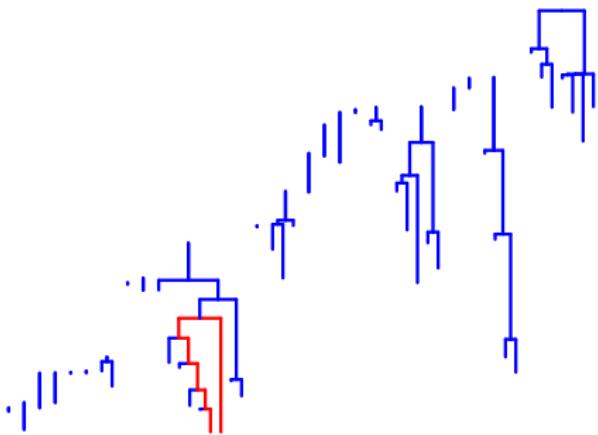
Both simplifications together



Both simplifications together



Example: forest of trees



Approximation: prune trees

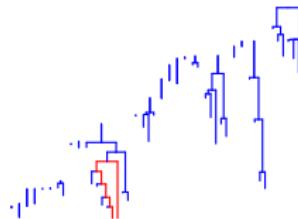
No pruning Exact

Partial pruning Nonancestral terminal branches $\leq T \times 2N_e$

Partial pruning ...

Full pruning SMC

Approximation: prune trees



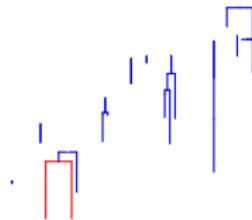
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Full pruning SMC

Computational complexity

model	n_{avg}	$n(\rho)$	runtime *
Wiuf & Hein	11700.5	super-exponential	3 days
Full	1502.1	super-exponential	1 hr
$T = 0.05$	378.8	constant	18'21"
$T = 0.1$	173.5	constant	9'47"
$T = 0.2$	66.2	constant	6'00"
$T = 0.4$	24.1	constant	3'23"
SMC	3	constant	1'04"

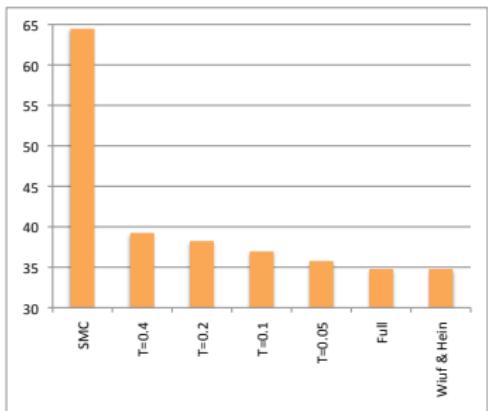
n_{avg} : average number of graph nodes

* 1000 iterations; Python implementation

Sequence length $\rho = 60$; two tips

Divergence times

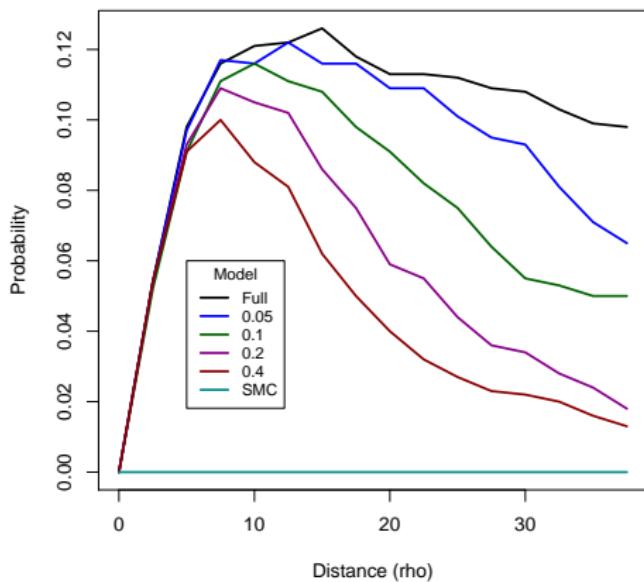
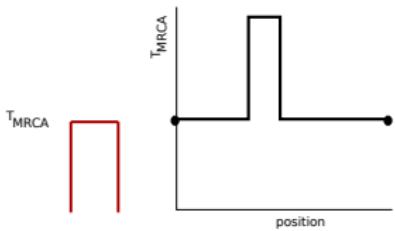
Average number of *distinct* divergence times between two samples:



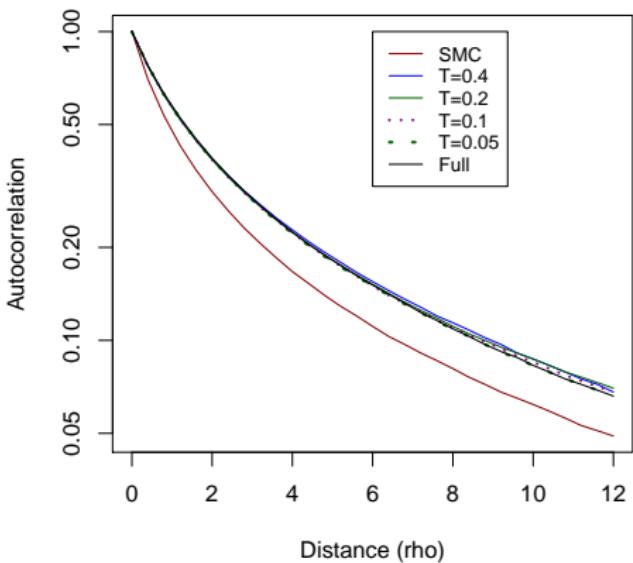
model	average
SMC	64.47
$T = 0.4$	39.23
$T = 0.2$	38.23
$T = 0.1$	36.95
$T = 0.05$	35.76
Full	34.78
Wiuf & Hein	34.78

Sequence length: $\rho = 60$

Probability of excursions



Autocorrelation of T_{MRCA}



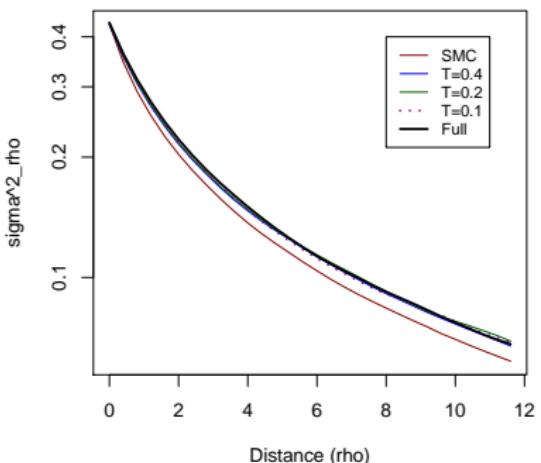
Estimates of LD under the model

$$E[r^2] = E \left[\frac{D}{p_1 q_1 p_2 q_2} \right] = ?$$

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$$E[r^2] = E \left[\frac{D}{p_1 q_1 p_2 q_2} \right] = ?$$

$$\sigma_{\rho}^2 := \frac{E[D]}{E[p_1 q_1 p_2 q_2]} = \frac{C_{ab,ab}^{\rho} - 2C_{ab,ac}^{\rho} + C_{ab,cd}^{\rho}}{E[t]^2 + C_{ab,cd}^{\rho}}$$



Summary

- Novel sequential algorithm for simulating under the ARG
 - State contains only *local* information, instead of (nonlocal) ARG
 - Simpler and faster than Wiuf & Hein's algorithm
 - Markovian
 - Can be adapted to complex demographic scenarios
- Tree pruning yields tunable algorithm
 - Good approximation to ARG
 - Time complexity of approximate algorithm $O(L)$
 - Scalable to full genomes
- Open questions:
 - Optimal pruning strategy?
 - Inference?

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Interested?

Postdoc position available soon!