Algebra Linear

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1 Exercício 1

Considere as bases do Respaço vetorial R3, A = $\{(4,2,0),(1,1,1),(5,3,3)\}$ e B = $\{(1,2,1),(1,5,2),(1,\ 0,1)\}$. Exiba as matrizes de mudança de base MB $\rightarrow$ A e MA $\rightarrow$ B. Escreva também os vetores abaixo nas bases indicadas:

- v = $(0,1,2)$A em B
- v = $(1,3,1)$B em A

Mudança B $\rightarrow A(b1)$

x. $a_1 + y \cdot a_2 + z \cdot a_3 = b_1$
x. $(4,2,0)$+ y. $(1,-1,1)$+ z. $(5,3,3) = (1,-2,1)$
$4x + y + 5z = 1$
$2x - y + 3z = -2$
$y + 3z = 1$
$4x + y + 5z = 1$
$2x - y + 3z = -2$
$6x + 8z = -1$
$2x - y + 3z = -2$
$y + 3z = 1$
$2x + 6z = -1$
$6x + 8z = -1$
$2x + 6z = -1(-3)$
$6x + 8z = -1$
$-6x - 18z = 3$
$-10z = 2$
$z = -2/10$
$z = -1/5$
$2x + 6z = -1$
$2x + 6 \cdot (-1/5) = -1$
x = $1/10$
$y + 3z = 1$
$y + 3(-1/5) = 1$

$-1 + 3/5$
$y = 8/5$
Mudança B $\rightarrow A(b2)$

$$4x + y + 5z = 1$$
$$2x = y + 3z = 5$$
$$y + 3z = 2$$
$$4x + y + 5z = 1$$
$$2x - y + 3z = 5$$
$$2x - y + 3z = 5$$
$$\mathbf{y + 3 = 2}$$
$$\mathbf{6x + 8}x = \mathbf{6}$$
$$\mathbf{2x + 6}z = 7(-3)$$
$$-\mathbf{10}z = -15$$
$$\mathbf{z = 3/2}$$
$$\mathbf{2x + 6} \cdot (3/2) = 7$$
$$\mathbf{x = -1}$$
$$y + 3 \cdot (3/2) = 2$$
$$\mathbf{y = -5/2}$$

Mudança $\mathbf{B} \rightarrow A(b3)$

$4x + y + 5z = 1$

$2x - y + 3z = 0$

$y + 3z = 1$

$4x + y = 5z = 1$

$2x - y + 3z = 0$

$2x - y + 3z = 0$

$y + 3z = 1$

$6x + 8z = 1$

$2x + 6z = 1$

$\mathrm{x} = -1/10$

$z = 1/5$

$y + 3(1/5) = 1$

$y + 3/5 = 1$

$\mathbf{y} = 2/5$

Mudança $\mathbf{A} \rightarrow B(a1)$

x. $\mathbf{b}_1 + y \cdot b_2 + z \cdot b_3 = a_1$

$\mathrm{x} \cdot (1, -2, 1) + \mathrm{y} \cdot (1, 5, 2) + \mathrm{z} \cdot (1, 0, 1) = (4, 2, 0)$

$x + y + z = 4$

$-2x + 5y = 2$

$x + 2y + z = 0$

$x + y + z = 4(.2\}$

$-2x + 5y = 2$

$x + y + z = 4$

$x + 2y + 2 = 0(-1)$
$-2x + 2y + 2z = 8$
$-2x + 5y = 2$
$7y + 2z = 10$
$x + y + z = 4$
$-x - 2y - z = 0$
$-y = 4$
$y = -4$
$-2x + 5(-4) = 2$
-20
$-2x = 2$
$-2x = 22$
$\mathbf{x} = -11$
$7y + 2z = 10$
7. $(-4) + 2z = 10$
z $= 19$
Mudança $\mathbf{A} \rightarrow B(a2)$
x + y + z $= 1$
$-2x + 5y = -1$
$x + 2y + z = 1$
$\mathbf{x + y + z = 1} \cdot (2)$
$-2x + 5y = -1$
$x + y + z = 1$
$z = 2y + z = 1$
$2x + 2y = 2z = 2$
$-2x + 5y = -1$
$7y + 2z = 1$
$\mathbf{y = 0}$
$7.0 + 2z = 1$
$z = 1/2$
$-2x + 0 = -1$
$-2x + 0 = -1$
$-2x = -1$
x $= 1/2$

Mudança $\mathbf{A} \rightarrow B(a3)$

$$\text{x} + \text{y} + \text{z} = 5$$
$$-2x + 5y = 3$$
$$x + 2y + z = 3$$
$$x + y + z = 5$$
$$-2x + 5y = 3$$
$$x + y + z = 5$$
$$x + 2y + z = 3$$

$7y + 2z = 13$

$y = -2$

$7y + 2z = 13$

$y = -2$

$7 \cdot (-2) = 2\text{z} = 13$

$\text{z} = 27/2$

$-2\text{x} + 5(-2) = 3$

$-2\text{x} - 10 = 13$

$-2\text{x} = 13$

$\text{x} = -13/2$

$$\mathbf{M}_A \rightarrow_B: \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} -11_* 0 & \frac{1}{2} {*} 1 & \frac{-13}{2} {*} 2 \\ 4_* 0 & 0_* 1 & -2_* 2 \\ 19_* 0 & \frac{1}{2} {*} 1 & \frac{27}{2} {*} 2 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{bmatrix}$$

2 Exercício 2

Considere o conjunto $S = \{(1,1,1,1,1), (2,0,1,1,3), (3,1,0,2,4), (2,2,5,8,1), (0,1,0,2,3)\}$

- S é LI ou LD?

$$\text{S} = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] l2 - 1 * l1 \rightarrow l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$l3 - 1 * l1 \rightarrow l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] l4 - 1 * l1 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$l5 - 1 * l1 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] -\frac{1}{2} * l2 \rightarrow l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]$$

$$-\tfrac{1}{3}*l3 \rightarrow l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] -1*l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]$$

$$l3-1*l2 \rightarrow l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] l4-1*l2 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & -\frac{3}{2} & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]$$

$$l5-1*l2 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] -\tfrac{1}{6}*l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right]$$

$$-\frac{1}{3}*l5 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & -\frac{7}{6} & 0 \end{array}\right] l4-1*l3 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \\ 0 & 0 & 0 & 1 & -\frac{7}{6} & 0 \end{array}\right]$$

$$l5-1*l3 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 0 \end{array}\right] \frac{4}{3}*l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 0 \end{array}\right]$$

$$-\frac{3}{2}*l5 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l5-1*l4 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

R : O conjunto S é LD (Linearmente Dependente).

b) Forma base do R-espaço vetorial R5?

R.: O conjunto S não forma base, pois se trata de um conjunto LD (Linearmente Dependente).

3 Exercício 3

Considere o conjunto W = $\{(\mathrm{x,y,z,w,t,u}) \mid x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.

Mostre que conjunto W é um subespaço do R-espaço vetorial R^6.

4 Exercício 4

Mostre que o conjunto $\{(1,1,1,1,0,1,1),(1,0,1,1,1,1,0),(2,2,1,1,1,1,1),(1,0,0,1,2,1,1),(2,0,2,0,2,0,2),(1,1,1,1,1,1,1),(3,0,2,0,2,1,2)\}$ forma uma base para o Respaço vetorial R7. Escreva o vetor $(0,1,1,1,1,0,1)$ nesta base.