Algebra Linear

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1 Exercício 1

Considere as bases do Respaço vetorial R3, A = $\{(4,2,0),(1,1,1),(5,3,3)\}$ e B = $\{(1,2,1),(1,5,2),(1,\ 0,1)\}$. Exiba as matrizes de mudança de base MB $\rightarrow$ A e MA $\rightarrow$ B. Escreva também os vetores abaixo nas bases indicadas:

- v = $(0,1,2)$A em B
- v = $(1,3,1)$B em A

Mudança B $\rightarrow A(b1)$
x. $a_1 + y \cdot a_2 + z \cdot a_3 = b_1$
x. $(4,2,0)$+ y. $(1,-1,1)$+ z. $(5,3,3) = (1,-2,1)$
$4x + y + 5z = 1$
$2x - y + 3z = -2$
$y + 3z = 1$
$4x + y + 5z = 1$
$2x - y + 3z = -2$
$6x + 8z = -1$
$2x - y + 3z = -2$
$y + 3z = 1$
$2x + 6z = -1$
$6x + 8z = -1$
$2x + 6z = -1(-3)$
$6x + 8z = -1$
$-6x - 18z = 3$
$-10z = 2$
$z = -2/10$
$z = -1/5$
$2x + 6z = -1$
$2x + 6 \cdot (-1/5) = -1$
x = $1/10$
$y + 3z = 1$
$y + 3(-1/5) = 1$

$-1 + 3/5$
$y = 8/5$
Mudança B $\rightarrow A(b2)$

$$4x + y + 5z = 1$$
$$2x = y + 3z = 5$$
$$y + 3z = 2$$
$$4x + y + 5z = 1$$
$$2x - y + 3z = 5$$
$$2x - y + 3z = 5$$
$$\mathbf{y + 3 = 2}$$
$$\mathbf{6x + 8}x = \mathbf{6}$$
$$\mathbf{2x + 6}z = 7(-3)$$
$$-\mathbf{10}z = -15$$
$$\mathbf{z = 3/2}$$
$$\mathbf{2x + 6} \cdot (3/2) = 7$$
$$\mathbf{x = -1}$$
$$y + 3 \cdot (3/2) = 2$$
$$\mathbf{y = -5/2}$$

Mudança $\mathbf{B} \rightarrow A(b3)$

$4x + y + 5z = 1$

$2x - y + 3z = 0$

$y + 3z = 1$

$4x + y = 5z = 1$

$2x - y + 3z = 0$

$2x - y + 3z = 0$

$y + 3z = 1$

$6x + 8z = 1$

$2x + 6z = 1$

x $= -1/10$

$z = 1/5$

$y + 3(1/5) = 1$

$y + 3/5 = 1$

$\mathbf{y} = 2/5$

Mudança $\mathbf{A} \rightarrow B(a1)$

x. $\mathbf{b}_1 + y \cdot b_2 + z \cdot b_3 = a_1$

x $\cdot (1, -2, 1) +$ y $\cdot (1, 5, 2) +$ z $\cdot (1, 0, 1) = (4, 2, 0)$

$x + y + z = 4$

$-2x + 5y = 2$

$x + 2y + z = 0$

$x + y + z = 4(.2\}$

$-2x + 5y = 2$

$x + y + z = 4$

$x + 2y + 2 = 0(-1)$
$-2x + 2y + 2z = 8$
$-2x + 5y = 2$
$7y + 2z = 10$
$x + y + z = 4$
$-x - 2y - z = 0$
$-y = 4$
$y = -4$
$-2x + 5(-4) = 2$
-20
$-2x = 2$
$-2x = 22$
$\mathbf{x} = -11$
$7y + 2z = 10$
7. $(-4) + 2z = 10$
z $= 19$
Mudança $\mathbf{A} \to B(a2)$
x + y + z $= 1$
$-2x + 5y = -1$
$x + 2y + z = 1$
$\mathbf{x + y + z = 1} \cdot (2)$
$-2x + 5y = -1$
$x + y + z = 1$
$z = 2y + z = 1$
$2x + 2y = 2z = 2$
$-2x + 5y = -1$
$7y + 2z = 1$
$\mathbf{y = 0}$
$7.0 + 2z = 1$
$z = 1/2$
$-2x + 0 = -1$
$-2x + 0 = -1$
$-2x = -1$
x $= 1/2$

Mudança $\mathbf{A} \to B(a3)$

$$\text{x} + \text{y} + \text{z} = 5$$
$$-2x + 5y = 3$$
$$x + 2y + z = 3$$
$$x + y + z = 5$$
$$-2x + 5y = 3$$
$$x + y + z = 5$$
$$x + 2y + z = 3$$

$7y + 2z = 13$

$y = -2$

$7y + 2z = 13$

$y = -2$

$7 \cdot (-2) = 2\text{z} = 13$

$\text{z} = 27/2$

$-2\text{x} + 5(-2) = 3$

$-2\text{x} - 10 = 13$

$-2\text{x} = 13$

$\text{x} = -13/2$

$$\mathbf{M_B} \to_{\mathbf{A}}: \begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5} \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \to \begin{bmatrix} \frac{1}{10}{}_*1 & -1_*3 & \frac{-1}{10}{}_*(-1) \\ \frac{8}{5}{}_*1 & \frac{-5}{2}{}_*3 & \frac{2}{5}{}_*(-1) \\ \frac{-1}{5}{}_*! & \frac{3}{2}{}_*3 & \frac{1}{5}{}_*(-1) \end{bmatrix} = \begin{bmatrix} \frac{-25}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$$

$$\mathbf{M}_A \to_B: \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \to \begin{bmatrix} -11_*0 & \frac{1}{2}{}_*1 & \frac{-13}{2}{}_*2 \\ 4_*0 & 0_*1 & -2_*2 \\ 19_*0 & \frac{1}{2}{}_*1 & \frac{27}{2}{}_*2 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{bmatrix}$$

2 Exercício 2

Considere o conjunto $S = \{(1, 1, 1, 1, 1), (2, 0, 1, 1, 3), (3, 1, 0, 2, 4), (2, 2, 5, 8, 1), (0, 1, 0, 2, 3)\}$

- S é LI ou LD?

$$\text{S} = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] l2 - 1 * l1 \to l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$l3 - 1 * l1 \to l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] l4 - 1 * l1 \to l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$
l5 - 1 * l1 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] -\tfrac{1}{2} * l2 \rightarrow l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]
$$

$$
-\tfrac{1}{3} * l3 \rightarrow l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] -1 * l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]
$$

$$
l3 - 1 * l2 \rightarrow l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] l4 - 1 * l2 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & -\frac{3}{2} & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]
$$

$$
l5 - 1 * l2 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] -\tfrac{1}{6} * l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right]
$$

$$
-\frac{1}{3} * l5 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & -\frac{7}{6} & 0 \end{array}\right] l4 - 1 * l3 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \\ 0 & 0 & 0 & 1 & -\frac{7}{6} & 0 \end{array}\right]
$$

$$
l5 - 1 * l3 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 0 \end{array}\right] \frac{4}{3} * l4 \rightarrow l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & 0 \end{array}\right]
$$

$$
-\frac{3}{2} * l5 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l5 - 1 * l4 \rightarrow l5 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]
$$

R : O conjunto S é LD (Linearmente Dependente).

b) Forma base do R-espaço vetorial R5?

R.: O conjunto S não forma base, pois se trata de um conjunto LD (Linearmente Dependente).

3 Exercício 3

Considere o conjunto $\mathrm{W} = \{(\mathrm{x,y,z,w,t,u}) \mid x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.

Mostre que conjunto W é um subespaço do R-espaço vetorial R^6.

$\mathrm{t - x = 0}$

$\mathrm{t = x}$

$\mathrm{y - w - z = 0}$

$\mathrm{y = w + z}$

$\mathrm{x + y + w + z + t + u = 0} \rightarrow x + w + z + w + z + x + u = 0$

$\mathrm{u = -x - y - w - z - t} \rightarrow u = -2x - 2w - 2z$

$\mathrm{W = \{(x, w + z, z, w, x, -x - w - z - w - z - x)\}} \rightarrow$

$\mathrm{W = \{(x, w + z, z, w, x, -2x - 2w - 2z)} \mid x, z, w \in R\}$

I) $0 \in W$ para$\mathrm{x}\ = 0 z = 0 w = 0$

(w, w, w, w, w, -w) $\rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$

$= (0, 0, 0, 0, 0, -0)$

$= 0$

Logo, $0 \in W$

II) $\mathrm{u, v} \in W \rightarrow u + v \in W$, sendo :

$\mathrm{u = (u1, u2, u3, u4, u5, -u6)} \rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$

$\mathrm{v = (v1, v2, v3, v4, v5, -v6)} \rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2 - 2w_2 - 2z_2)$

u+v $= (\mathrm{x}_1 + x_2, (w_1 + z_1) + (w_2 + z2)\,, z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2$

u+v $= (\mathrm{x}_1 x_2, w_1 z_1 w_2 z_2, z_1, z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2, -2w_1 - 2w_2, -2z_1 - 2z_2)$

Logo, $\mathrm{u + v} \in W$

III)$\mathrm{a} \in R, v \in W \rightarrow av \in W,$ sendo :

$\mathrm{v = (v1, v2, v3, v4, v5, -v6)} \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$

$\mathrm{av = a} \cdot (\mathrm{x}_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$

$\mathrm{av} = (\mathrm{a.x}_1, a.w_1 + z_1, a.z_1, a.w_1, a.x_1, a. - 2x_1 - 2w_1 - 2z_1)$

$\mathrm{av} = (\mathrm{ax}_1, aw_1 w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1)$

Logo, $\mathrm{av} \in W$

Logo W é subespaço vetorial de R6.

- O conjunto $\mathrm{W} = \{(\mathrm{x,y,z}) \mid x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$

é um subsespaço vetorial de R3? Esboce graficamente W.

$\mathrm{x - z = 1}$ à $\mathrm{x = 1 + z}$

$\mathrm{y + x = 0}$ à $\mathrm{y + 1 + z = 0}$ à $\mathrm{y = -1 - z}$.

$\mathrm{W = \{(1 + z, -1 - z, z)\}}$

I) $0 \in W$, para$\mathrm{z} = 0$

$\mathrm{(1 + z, -1 - z, z)}$ à $(1 + 0, -1 - 0, 0) = (1, -1, 0)$.

Logo 0NAAO pertence a W para z = 0. Portanto, W NÃO é subespaço vetorial.

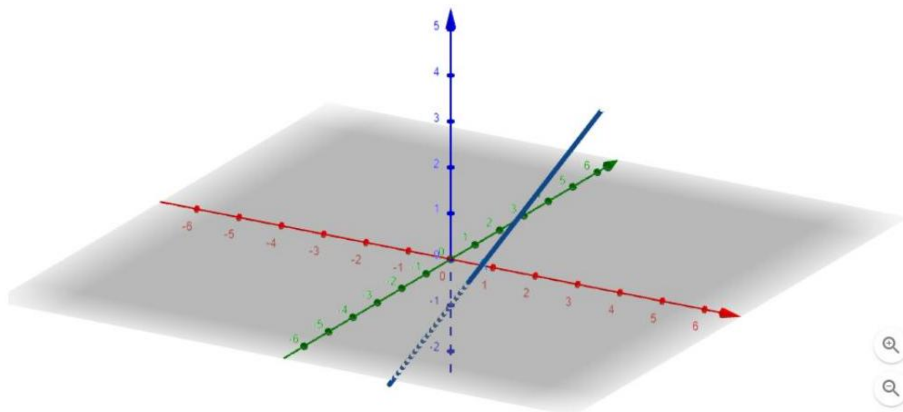

Figure 1: Representação gráfica.

- Invente seu subespaço vetorial em qualquer R n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

$\mathrm{Z} = \{(\mathrm{x}, \mathrm{y}, \mathrm{z}) \mid 2y + z = 0 \wedge x + y = 0\}$

$\mathrm{2y + z = 0}$

$\mathrm{x + y = 0}$

$z = -2y$

$\mathrm{x = -y}$

$\mathrm{Z = (-y, y, -2y)}$

$\mathrm{Z} = \{(\mathrm{-z, z, -z}) \mid Z \in R\}$

I) $0 \in Z, \mathrm{para z = 0}$

$\mathrm{(z, z, z)}$ à $\mathrm{(-y, y, -2y)}$

y à 0

$\mathrm{(-y, y, -2y) = (-0, 0, -0) = (0, 0, 0)}$

Logo, $0 \in Z$ II) $\mathrm{u, v} \in Z \rightarrow u + v \in Z$, sendo :

$\mathrm{u = (u1, u2, u3)}$

$\mathrm{v = (v1, v2, v3)}$ à $\mathrm{(-y, y, -2y)}$

$\mathrm{u + v = (u1, u2, u3) + (-y, y, -2y)}$

$\mathrm{u + v = (u1 - y, u2 + y, -u3 - 2y)}$

Logo, $u + v \in Z$

III) $\mathrm{a} \in \mathrm{R}, \mathrm{v} \in Z \rightarrow av \in Z$. Sendo :

$\mathrm{v = (v1, v2, v3)}$ à(-y, y, -2y)

$\mathrm{a.v} = a \cdot (-y, y, -2y)$

$\mathrm{a.v = (a \cdot (-y), a \cdot y, a \cdot (-2y))}$

$\mathrm{a.v} = (-ay,$ ay, $-$ a $2y)$

Logo, av $\in Z$

Logo Z é subespaço vetorial de $R3$

4 Exercício 4

Mostre que o conjunto $\{(1,1,1,1,0,1,1),(1,0,1,1,1,1,0),(2,2,1,1,1,1,1),(1,0,0,1,2,1,1),(2,0,2,0,2,0,2),(1,1,1,1,1,1,1),(3,0,2,0,2,1,2)\}$ forma uma base para o Respaço vetorial R7. Escreva o vetor $(0,1,1,1,1,0,1)$ nesta base.