Modified Method of the Kinematic Analysis of Planar Linkage Mechanism for Non-stationary Motion Modes

J. Drewniak, P. Garlicka, J. Kopeć and S. Zawiślak

Abstract The aim of the paper is presentation of the modified method of utilization of the contour graphs for an analysis of kinematics of the closed crane mechanisms. An introductory phase of calculations is necessary for performance of further dynamical analysis of the mechanisms because—in the considered duty cycle—the linkages of the mechanisms are subjected to a non-stationary motion. The non-stationary motion mode is characteristic for a startup and a braking as well as during an unstable motion of the system. The results of calculations: velocities and accelerations for particular linkages of the mechanism are shown in various figures mainly as a function of the rotational angle of the drive linkage.

Keywords Linkage mechanism · Contour graph method · Analysis of kinematics

1 Introduction

The basic, classical method of kinematical analysis of mechanisms consists in differentiation (with respect to time) of the radius-vectors of positions of the mechanism nodes [1]. However, the method of an analysis of kinematics of the mechanisms based upon utilization of contour graphs allows for generation of algebraic equations. The idea of usage of graphs, block-schemes, dyads and other

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B. Corves et al. (eds.), *New Advances in Mechanisms, Mechanical Transmissions and Robotics*, Mechanisms and Machine Science 46,
DOI 10.1007/978-3-319-45450-4_2

algebraic tools or structures for solution of some problems of mechanisms' modeling, analysis and simulation is presented in papers [1–6]. The graph-based approach allows e.g. for systematic creation of atlases or families of mechanisms layouts or for their algorithmic synthesis [2]. The basic ideas of contour graphs are described in the Marghitu's monograph [7]. Usage of contour graphs for planetary gears modeling is presented in [1, 7]. The method could be utilized for plane as well as for spatial mechanisms, for which we consider closed kinematical chains as their calculation model. It has been assumed that the closed kinematical chain can model a crane mechanism. The mechanism consists of n moving linkages. The linkages are numbered in a consecutive way: from 0 to n. The 0 linkage is fixed to the ground (or reference system) and it is considered as the base. The absolute coordinate system *OXY* is attached to it. The *i*-th linkage is connected with the linkage $i-1$ in point A_i, whereas with linkage $i+1$ in point A_{i+1}. Therefore every considered point belongs to two neighbor linkages i and $i+1$. Aiming for distinguishing of belongings of point A_i e.g. to the linkage $i-1$ ($A_i \in i-1$)—we can write the description of this point as one having two indexes $A_{i,i-1}$. Similarly, point $A_i \in i$ linkage is described as $A_{i,i}$.

The analysis of velocities of consecutive linkages of a mechanism (treated as a kinematic chain) is based on two fundamental kinematic relationships describing the complex motion [7]:

$$\sum_i \boldsymbol{\omega}_{i,i-1} = \mathbf{0}. \tag{1}$$

$$\sum_i \mathbf{r}_{Ai} \times \boldsymbol{\omega}_{i,i-1} + \sum_i \mathbf{v}^r_{Ai,i-1} = \mathbf{0}, \tag{2}$$

where $\boldsymbol{\omega}_{i,i-1}$ is a relative velocity of i linkage in relations to $i-1$ linkage, $\mathbf{r}_{Ai} = \mathbf{r}_{OAi}$, $\mathbf{r}_{Ai-1} = \mathbf{r}_{OAi-1}$ $\boldsymbol{\omega}_{i,i-1}$ and $\mathbf{v}^r_{Ai,i-1}$ is relative velocity in relation to $A_{i,i-1}$ point.

Similarly, the analysis of accelerations figures on two basic equations of the contour graphs method which initially consists in determination of relative angular accelerations $\boldsymbol{\varepsilon}_{i,i-1}$ of i linkage in relation to $i-1$ linkage, relative linear accelerations $\mathbf{a}^r_{Ai,i-1}$ and Coriolis's accelerations $\mathbf{a}^c_{Ai,i-1}$ [7]:

$$\sum_i \boldsymbol{\varepsilon}_{i,i-1} + \sum_i \boldsymbol{\omega}_i \times \boldsymbol{\omega}_{i,i-1} = \mathbf{0}, \tag{3}$$

and

$$\begin{aligned}\sum_i \mathbf{a}^r_{Ai,i-1} + \sum_i \mathbf{a}^c_{Ai,i-1} + \sum_i \mathbf{r}_{Ai} \times (\boldsymbol{\varepsilon}_{i,i-1} \times \boldsymbol{\omega}_i - \boldsymbol{\omega}_{i,i-1}) \\ + \sum_i \boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \mathbf{r}_{AiAi+1}) = \mathbf{0}.\end{aligned} \tag{4}$$

where $\mathbf{r}_{AiAi+1} = \mathbf{r}_{Ai} - \mathbf{r}_{Ai-1}$, $\mathbf{a}^r_{Ai,i-1} = \mathbf{a}^r_{Ai,iAi,i-1}$, $\mathbf{a}_{Ai,i} = \mathbf{a}_{Ai,i-1} + \mathbf{a}^r_{Ai,i-1} + \mathbf{a}^c_{Ai,i-1}$ and $\mathbf{a}^c_{Ai,i-1} = 2 \cdot \boldsymbol{\omega}_{i-1} \times \mathbf{v}^r_{Ai,i-1}$.

An algorithm of generation of the algebraic equations for velocities and angular accelerations describing the behavior of linear linkages of a particular mechanism based on the contour graph method can be formulated in the following steps:

(i) drawing the scheme of the analyzed mechanism, assuming of the coordinate system X0Y and determination of geometrical layout of particular nodes of the mechanism and determination of the radius-vectors of mechanism nodes,
(ii) calculation of the mobility of the mechanism $W = 3 \cdot n - 2 \cdot p_5 - p_4$, where n—number of movable linkages, p_5—number of kinematics pairs of 5-th class, p_4—number of kinematic pairs of 4-th class and checking the condition of problem solvability—i.e. the number of known velocities has to be equal to W,
(iii) drawing the contour graph (representing kinematics) of the mechanism having N independent contours:

$$N = c - n = c - p + 1, \tag{5}$$

where: $c = p_5 + p_4$—number of nodes (joints) of the mechanism, n—number of movable linkages, $p = n + 1$—total number of linkages,
(iv) decomposition of the mechanism and generating of the system of vector equations for velocities or/and accelerations for every contour (closed loop),
(v) finding a solution of the system of equations after transformation of the vector-type equations into the scalar ones (via projection of the vectors onto the axes x and y of the assumed coordinate system in case of planar case) and final solving of the system of algebraic equations.

2 Analysis of Velocities of the Mechanism Linkages

The analyzed mechanism is presented in Fig. 1a. The linkages 1, 3 and 5 are arms (swing-arms), whereas arm 1 is the driving linkage and arm 5—the driven one. The geometric and layout data for mechanism linkages are as follows: |AB| = 80, |BC| = 80, |BD| = 90, |CD| = 80, |ED| = 100, |CG| = 150, A(0; 0), E(60; 0),

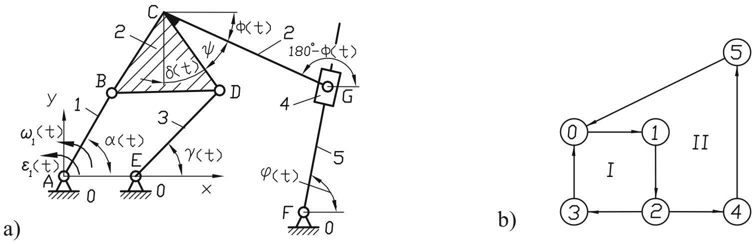

Fig. 1 Analyzed crane mechanism (**a**) and two-contour graph of the mechanism (**b**)

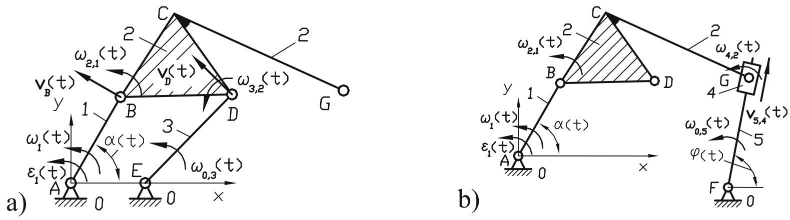

Fig. 2 First and second component of mechanism (after decomposition)

F(200; −30), $\psi = 60°$, $30° \leq \alpha \leq 210°$, where $30° \leq \alpha \leq 45°$—startup, $45° \leq \alpha \leq 195°$—stable motion, $195° \leq \alpha \leq 210°$—braking.

In Fig. 1b, N independent contour graphs of the mechanism are presented, whereas: $N = c - n = 7 - 5 = 2$ based on Eq. 5. The nodes—shown as circles—represent the particular linkages of the mechanism corresponding to the same descriptions, whereas the edges connecting the graph nodes represent the mechanism joints. The schemas are utilized to drawing the so called expanded contour graphs (Fig. 3) corresponding to particular subsystem obtained upon the decomposition of the mechanism (Fig. 2). The considered mechanism (Fig. 1a) is a complex mechanical system which can be decomposed into two simple subsystems (Fig. 2). It allows for essential simplification of generation and solution of the system of equations corresponding to linkages velocities. Further simplification consists in creation of the extended contour graphs for particular decomposed subsystems (Fig. 2).

The system of vector equations of velocities for the first contour (Figs. 1b and 3) can be written in the following form:

$$\boldsymbol{\omega}_{1.0} + \boldsymbol{\omega}_{2.1} + \boldsymbol{\omega}_{3.2} + \boldsymbol{\omega}_{0.3} = \mathbf{0}, \tag{6a}$$

$$\mathbf{r}_{AB} \times \boldsymbol{\omega}_{2.1} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{3.2} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{0.3} = \mathbf{0}, \tag{6b}$$

The angular velocity $\boldsymbol{\omega}_{1.0}(\alpha) = \omega_1(\alpha)$ of arm 1 is presented in Fig. 2. We are looking for the relative angular velocities $\boldsymbol{\omega}_{2.1} = \boldsymbol{\omega}_{2.1} \cdot \mathbf{k}$, $\boldsymbol{\omega}_{3.2} = \boldsymbol{\omega}_{3.2} \cdot \mathbf{k}$ and $\boldsymbol{\omega}_{0.3} = \boldsymbol{\omega}_{0.3} \cdot \mathbf{k}$. After adequate rewriting, the system of algebraic equations is obtained, via projections on z, x, y axes respectively (Fig. 4):

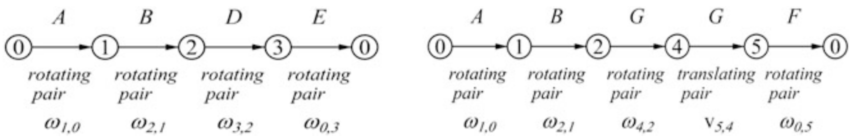

Fig. 3 Extended contour graphs corresponding to first and second component of mechanism

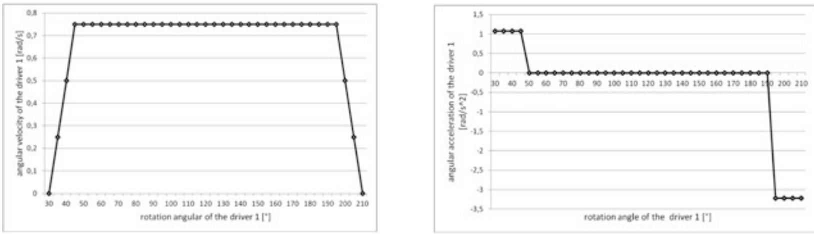

Fig. 4 Set angular velocities $\omega_1(\alpha)$ and accelerations $\varepsilon_1(\alpha)$ of driving arm 1

$$\omega_{2.1}(\alpha) + \omega_{3.2}(\alpha) + \omega_{0.3}(\alpha) = -\omega_{1.0}(\alpha) \tag{7a}$$

$$y_B(\alpha) \cdot \omega_{2.1}(\alpha) + y_D(\alpha) \cdot \omega_{3.2}(\alpha) + y_E(\alpha) \cdot \omega_{0.3}(\alpha) = 0 \tag{7b}$$

$$-x_B(\alpha) \cdot \omega_{2.1}(\alpha) - x_D(\alpha) \cdot \omega_{3.2}(\alpha) - x_E(\alpha) \cdot \omega_{0.3}(\alpha) = 0 \tag{7c}$$

Unknown velocities $\omega_{2.1}(\alpha)$, $\omega_{3.2}(\alpha)$ and $\omega_{0.3}(\alpha)$ are shown in Fig. 5, where $30° \leq \alpha \leq 210°$.

The system of vector equations for the second contour (Figs. 1b and 3):

$$\boldsymbol{\omega}_{1.0} + \boldsymbol{\omega}_{2.1} + \boldsymbol{\omega}_{4.2} + \boldsymbol{\omega}_{0.5} = \mathbf{0}, \tag{8a}$$

$$\mathbf{r}_{AB} \times \boldsymbol{\omega}_{2.1} + \mathbf{r}_{AG} \times \boldsymbol{\omega}_{4.2} + \mathbf{v}^{r}_{G5.4} + \mathbf{r}_{AF} \times \boldsymbol{\omega}_{0.5} = \mathbf{0}, \tag{8b}$$

After adequate rewriting, the system of vector equations (8a, 8b) can be presented in an algebraic form:

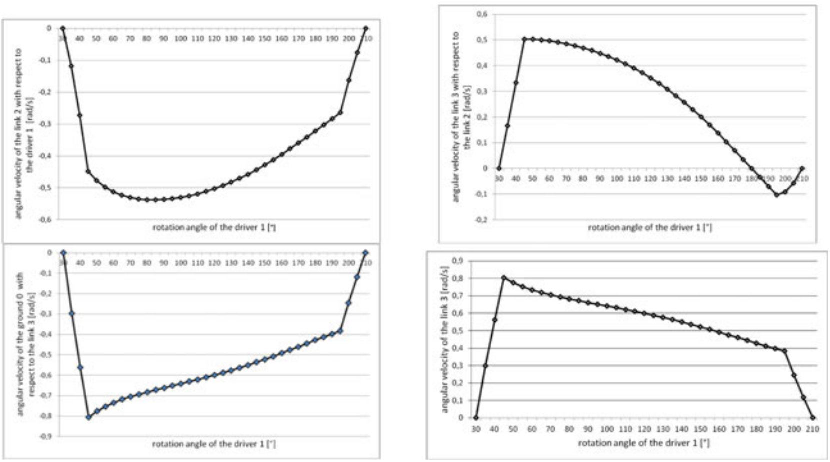

Fig. 5 Angular velocities as function of angular angle α of arm 1 (I contour)

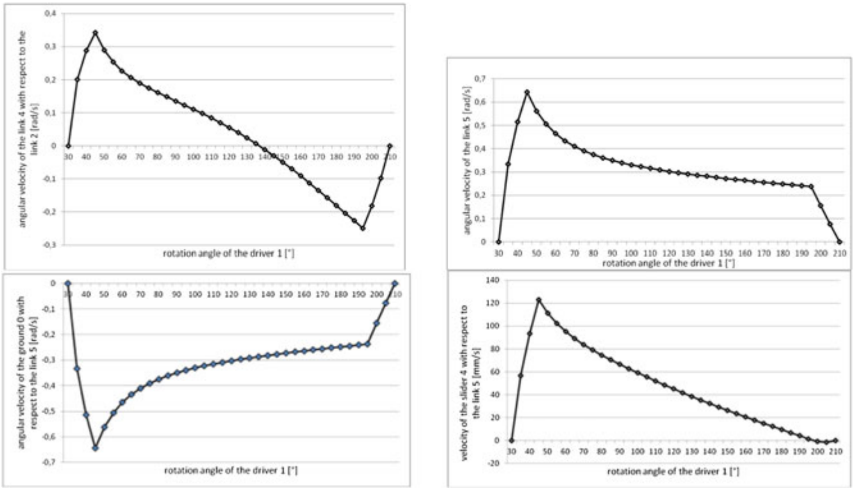

Fig. 6 Angular and linear velocities as functions of rotational angle of α arm 1 (II contour)

$$\omega_{4.2}(\alpha) + \omega_{0.5}(\alpha) = -\,\omega_{1.0}(\alpha) - \omega_{2.1}(\alpha), \tag{9a}$$

$$\begin{aligned} &y_G(\alpha)\cdot\omega_{4.2}(\alpha) + y_F(\alpha)\cdot\omega_{0.5}(\alpha) + v^r_{G5.4}(\alpha)\cdot\ \cos(\alpha) \\ &\quad = -\,y_B(\alpha)\cdot\omega_{2.1}(\alpha), \end{aligned} \tag{9b}$$

$$\begin{aligned} &-x_G(\alpha)\cdot\omega_{4.2}(\alpha) - x_F(\alpha)\cdot\omega_{0.5}(\alpha) + v^r_{G5.4}(\alpha)\cdot\ \sin\varphi \\ &\quad = x_B(\alpha)\cdot\omega_{2.1}(\alpha). \end{aligned} \tag{9c}$$

The solutions of the system of Eqs. (9a–9c) are shown in Fig. 6 in case of angles $30° \le \alpha \le 210°$.

3 Analysis of Accelerations of Mechanism Linkages

Calculations of accelerations by means of the contour graph method is performed according to the algorithm described in Chap. 1. Angular acceleration $\varepsilon_1 = \varepsilon_{1.0}$ of the driving arm 1 for a startup state is presented in Fig. 7.

Based on Figs. 7a and 8a, the following system of vector equations of accelerations can be derived:

$$\boldsymbol{\varepsilon}_{1.0} + \boldsymbol{\varepsilon}_{2.1} + \boldsymbol{\varepsilon}_{3.2} + \boldsymbol{\varepsilon}_{0.3} = \mathbf{0}, \tag{10a}$$

$$\mathbf{r}_{AB}\times\boldsymbol{\varepsilon}_{2.1} + \mathbf{r}_{AD}\times\boldsymbol{\varepsilon}_{3.2} + \mathbf{r}_{AE}\times\boldsymbol{\varepsilon}_{0.3} - \omega^2_{1.0}\cdot\mathbf{r}_{AB} - \omega^2_{0.3}\cdot\mathbf{r}_{DE} = \mathbf{0}, \tag{10b}$$

where: $\varepsilon_{1.0}(\alpha) = \dot{\omega}_{1.0}(\alpha)\cdot\mathbf{k}$—set angular acceleration (Fig. 8).

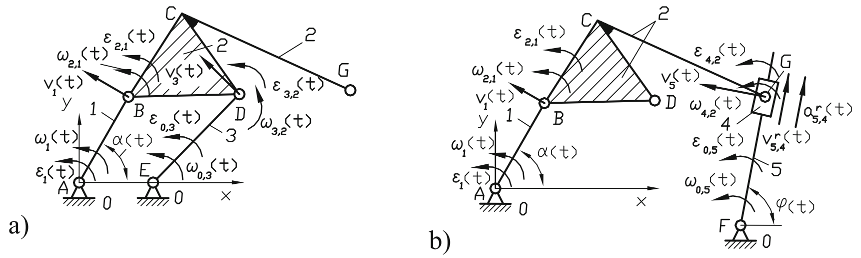

Fig. 7 First and second component of the mechanism (after decomposition)

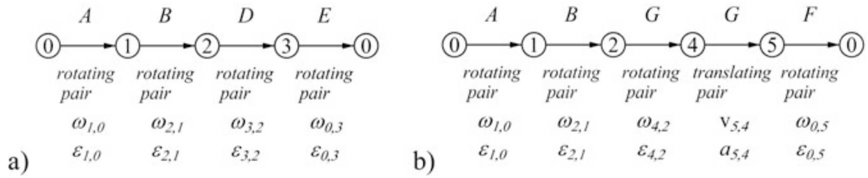

Fig. 8 Expanded contour graphs corresponding to first and second component of the mechanism

Rewritten of Eqs. (10a, 10b) give the algebraic form of the considered equation system:

$$\varepsilon_{2.1}(\alpha)+\varepsilon_{3.2}(\alpha)+\varepsilon_{0.3}(\alpha)=-\varepsilon_{1.0}(\alpha), \tag{11a}$$

$$y_B\cdot\varepsilon_{2.1}(\alpha)+y_D\cdot\varepsilon_{3.2}(\alpha)+y_E\cdot\varepsilon_{0.3}(\alpha)=\omega_{1.0}^2(\alpha)\cdot x_B+\omega_{0.3}^2(\alpha)\cdot(x_E-x_D), \tag{11b}$$

$$-x_B\cdot\varepsilon_{2.1}(\alpha)-x_D\cdot\varepsilon_{3.2}(\alpha)-x_E\cdot\varepsilon_{0.3}(\alpha)=\omega_{1.0}^2(\alpha)\cdot y_B-\omega_{0.3}^2(\alpha)\cdot y_D. \tag{11c}$$

The solutions of the system Eqs. (11a–11c) are presented in Fig. 9.

Based on Fig. 7b—where the second component of the mechanism is shown—and based on Fig. 8b—where the expended contour graph of this component, it is possible to generate the following system of vector equations for accelerations:

$$\boldsymbol{\varepsilon}_{1.0}+\boldsymbol{\varepsilon}_{2.1}+\boldsymbol{\varepsilon}_{4.2}+\boldsymbol{\varepsilon}_{0.5}=\mathbf{0}, \tag{12a}$$

$$\mathbf{r}_{AB}\times\boldsymbol{\varepsilon}_{2.1}+\mathbf{r}_{AG}\times\boldsymbol{\varepsilon}_{4.2}+\mathbf{r}_{AF}\times\boldsymbol{\varepsilon}_{0.5}+\mathbf{a}_{G5.4}^{r}+\mathbf{a}_{G5.4}^{c}-\boldsymbol{\omega}_{1.0}^{2}\cdot\mathbf{r}_{AB}-\boldsymbol{\omega}_{0.5}^{2}\cdot\mathbf{r}_{GF}=\mathbf{0}, \tag{12b}$$

After rewriting, Eqs. (12a, 12b) can be presented in an algebraic form:

$$\varepsilon_{2,1}(\alpha)+\varepsilon_{4,2}(\alpha)+\varepsilon_{0,5}(\alpha)=-\varepsilon_{1,0}(\alpha), \tag{13a}$$

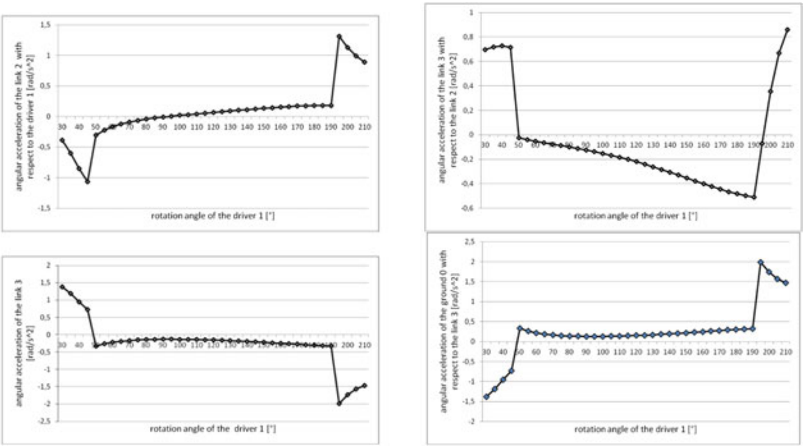

Fig. 9 Angular accelerations as functions of rotational angle α of arm 1 (I contour)

$$
\begin{aligned}
&y_B \cdot \varepsilon_{2.1}(\alpha) + y_G \cdot \varepsilon_{4.2}(\alpha) + y_F \cdot \varepsilon_{0.5}(\alpha) + a^r_{G5.4}(\alpha) \cdot \cos\varphi \\
&\quad = 2 \cdot \omega_{0.5}(\alpha) \cdot v^r_{G5.4}(\alpha) \cdot \sin\varphi + \omega^2_{1.0}(\alpha) \cdot x_B + \omega^2_{0.5}(\alpha) \cdot (x_F - x_G),
\end{aligned}
\tag{13b}
$$

$$
\begin{aligned}
&- x_B \cdot \varepsilon_{2.1}(\alpha) - x_G \cdot \varepsilon_{4.2}(\alpha) - x_F \cdot \varepsilon_{0.5}(\alpha) + a^r_{G5.4}(\alpha) \cdot \sin\varphi \\
&\quad = - 2 \cdot \omega_{0.5}(\alpha) \cdot v^r_{G5.4}(\alpha) \cdot \cos\varphi + \omega^2_{1.0}(\alpha) \cdot y_B + \omega^2_{0.5}(\alpha) \cdot (y_F - y_G).
\end{aligned}
\tag{13c}
$$

The solutions of the system of Eqs. (13a–13c) are presented in Fig. 10.

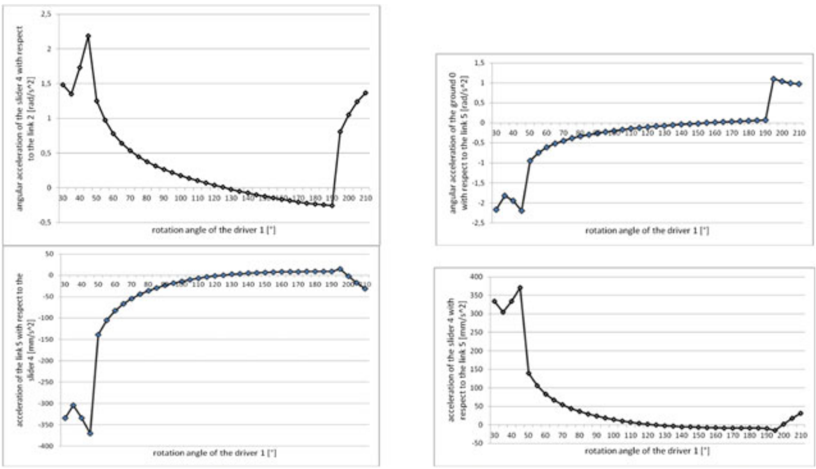

Fig. 10 Angular and linear acceleration as functions of rotational angle α of arm 1 (II contour)

4 Conclusions

It was shown, in the paper, that an application of the contour graphs method for an analysis of closed kinematical crane mechanisms essentially helps in generation and solving of the systems of equations describing the behavior of the mechanism. The main cause of this advantage is description of the motion by means of algebraic equations instead of differential ones like it is considered in classical approaches to solving similar problems. The benefits of the utilized methodology were especially evident in case of kinematical analysis of mechanisms which linkages are subjected to the non-stationary or unstable motions. It could be expected that these advantages would be also beneficiary in analysis of kinematics and dynamics of spatial mechanisms. The simulation results shown in figures confirm the design assumptions for the analyzed mechanism.

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http://www.springer.com/978-3-319-45449-8

New Advances in Mechanisms, Mechanical Transmissions and Robotics
Proceedings of The Joint International Conference of the XII International Conference on Mechanisms and Mechanical Transmissions (MTM) and the XXIII International Conference on Robotics (Robotics '16)
Corves, B.; Lovasz, E.-C.; Hüsing, M.; Maniu, I.; Gruescu, C. (Eds.)
2017, XVII, 497 p. 275 illus., Hardcover
ISBN: 978-3-319-45449-8