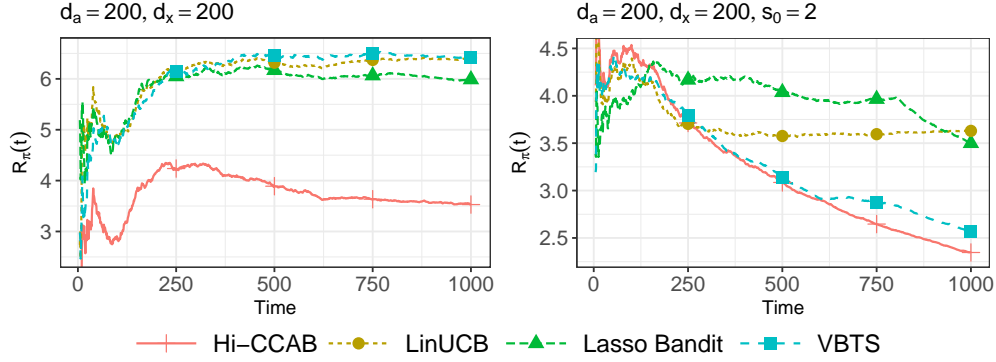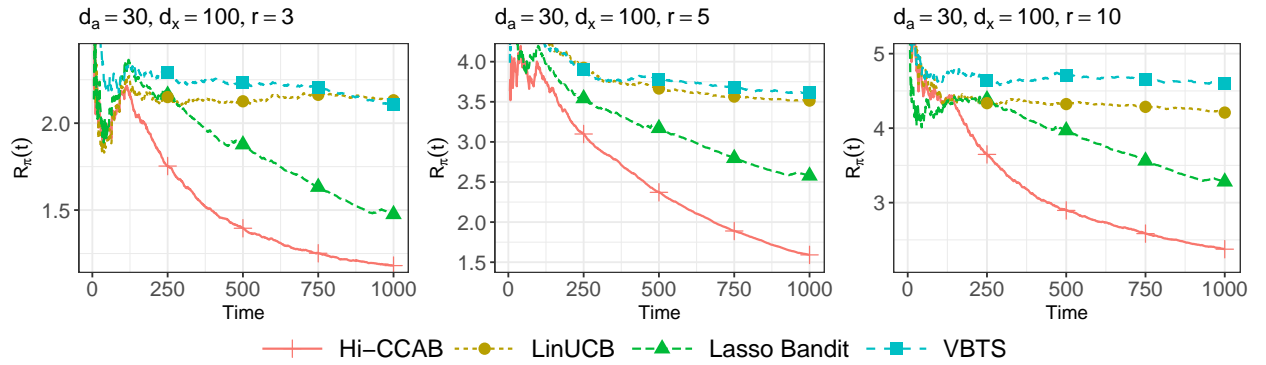**Figure A1:** Replicate Figure 1(a) with VBTS by Chakraborty et al.(2023).

Figure A2: Non-sparse (left) and sparse (right) settings with $d_a = d_x = 200$.

Figure A3: Sensitivity analysis of the rank $r = 3, 5, 10$ with $d_a = 30$ and $d_x = 100$.