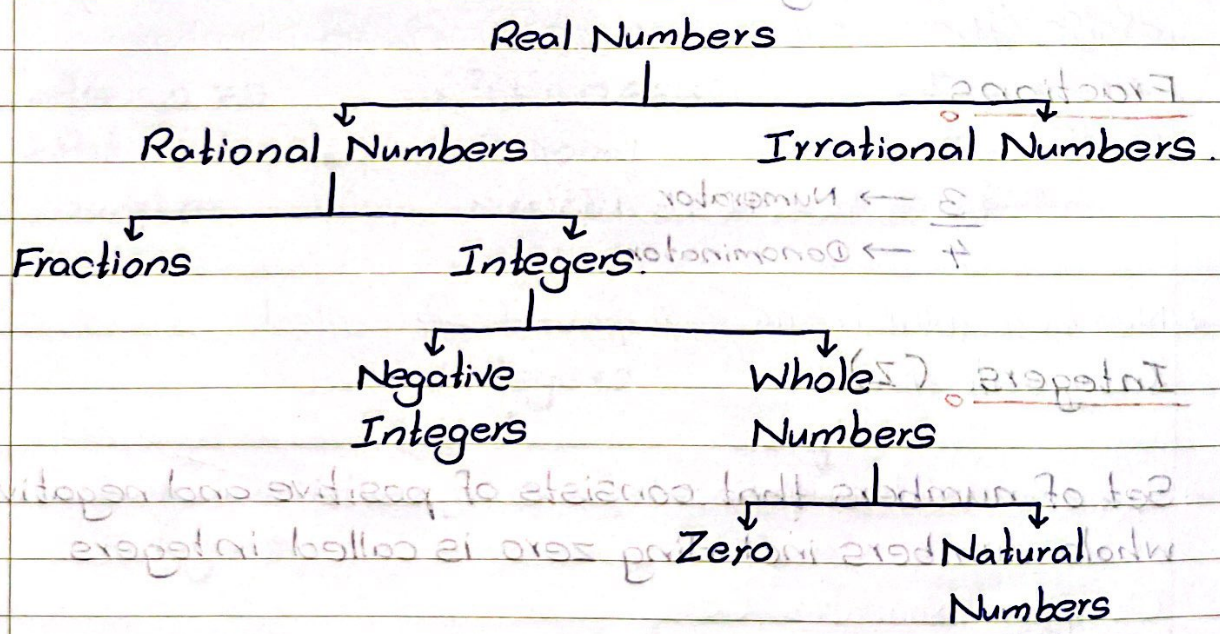
01. Types of Numbers

Real Numbers. (R)

Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers.

Types of Real Numbers.

01. Natural Numbers.
02. Whole Numbers.
03. Integers.
04. Fractions.
05. Rational Numbers.
06. Irrational Numbers.



There're 2 kinds of Real Numbers.

01.) Rational Numbers.

02.) Irrational Numbers.

Rational Numbers.

In mathematics, a rational number is any number that can be expressed as the quotient or fraction p/q of 2 integers, p and q, with the denominator q not equal to zero.

Ex:

$2/3, -1/3, 1/2, 5$

* 5 is also a rational number since $5 = 1/5$.

There're 2 kinds of Rational Numbers.

01.) Fractions.

02.) Integers.

Fractions.

$\frac{3}{4}$ — 3 ⟶ Numerator, 4 ⟶ Denominator

Integers. (Z)

Set of numbers that consists of positive and negative whole numbers including zero is called integers.

Ex:

$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

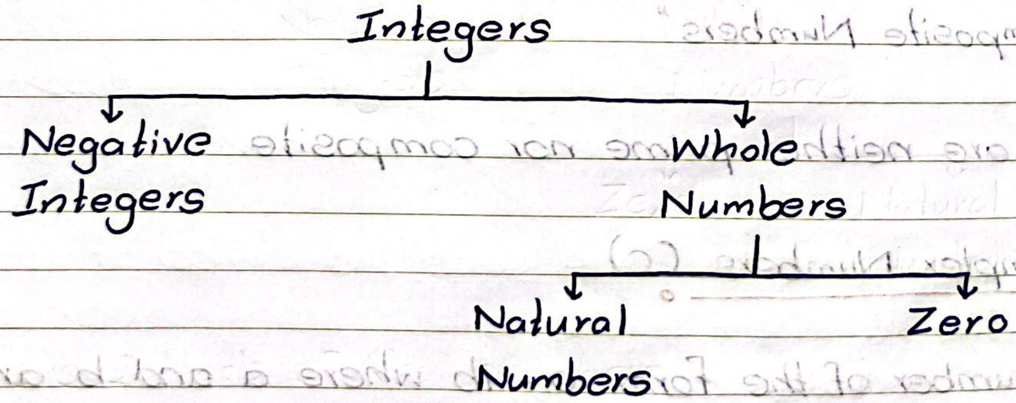
Irrational Numbers.

If a number cannot be expressed as the quotient or Fraction p/q of two integers, p and q, with the denominator q not equal to zero, then, the number is called an irrational number.

Ex:
$\sqrt{2}$, π, $\sqrt{3}/3$

Different Types of Decimal Numbers.

Terminating Decimals	Non-terminating Recurring Decimals.	Non-terminating Non-recurring Decimals.
* $3/4 = 0.75$	* $2/3 = 0.666\ldots$	* $\sqrt{2} = 1.414\ldots$
* Rational number	* Rational number.	* Irrational number.



Date No

Natural Numbers and Whole Numbers.

* Positive integers excluding zero are called "Natural Numbers."

$\{1, 2, 3, 4 \ldots\} \rightarrow$ counting numbers.

* Positive integers including zero are called "Whole Numbers."

$\{0, 1, 2, 3, 4 \ldots\} \rightarrow$ numbers.

Prime Numbers.

* If a natural number r including 1 is divisible by r alone, then r is known as a "Prime Number".

Ex:

$\{2, 3, 5, 7, 11, 13, 17 \ldots\}$

* Numbers which are not prime are called "Composite Numbers."

* 0, 1 are neither prime nor composite.

Complex Numbers (C)

* A number of the form $a + ib$ where a and b are real numbers.

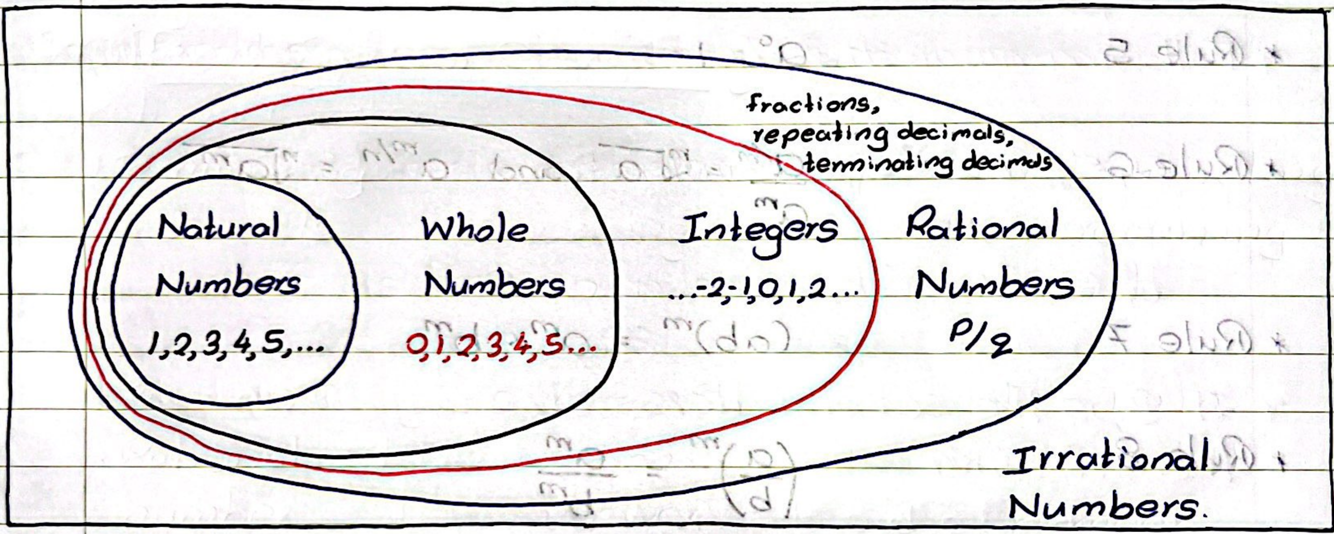
Date No

* $i^2 = -1$ where $i = \sqrt{-1}$

* A complex number has 2 parts:
  - Real part: a
  - Imaginary part: b

Ex:

$2+3i$, $i-3i$...



Real Numbers

02. Rules of Indices

Date No

* Rule 1 $a^m \times a^n = a^{(m+n)}$

  $2^3 \times 2^2 = 2^{(3+2)} = 2^5 = 32$

* Rule 2 $(a^m)^n = a^{m \times n}$

  $(2^3)^2 = 2^6 = 64$

* Rule 3 $a^m \div a^n = a^{(m-n)}$

  $2^3 \div 2^2 = 2^{(3-2)} = 2^1 = 2$

* Rule 4 $a^m = \frac{1}{a^{-m}}$ and $a^{-m} = \frac{1}{a^m}$

  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $2^3 = \frac{1}{2^{-3}}$

* Rule 5 $a^0 = 1$

  $2^0 = 1$

* Rule 6 $a^{1/m} = \sqrt[m]{a}$ and $a^{m/n} = \sqrt[n]{a^m}$

  $2^{1/3} = \sqrt[3]{2}$ and $2^{2/3} = \sqrt[3]{2^2} = \sqrt[3]{4} = {}^1/4^3$

* Rule 7 $(ab)^m = a^m \times b^m$

  $(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36$

* Rule 8 $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

  $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

Date No

03. Set Theory

Date No

* A set is any well defind collection of "objects".

Ex:

$A = \{a_1, a_2, a_3 \dots, a_n\}$ "A contains..."

* Usually we denote sets with upper-case letters, elements with lower-case letters.

Notations

* Following notation is used to show set membership

$a_1 \in A$ "a_1 is an element of A"

$b_1 \in A$ "b_1 is not an element of A"

* If $\mathcal{B}$ is the set of letters in the English Alphabet, then we have,

$\mathcal{B} = \{a, b, c, d, \dots, x, y, z\}$

* We can write $a \in \mathcal{B}$, $b \in \mathcal{B}$, $c \in \mathcal{B}, \dots,$ $z \in \mathcal{B}$

* Number of elements of a given set A is denoted by $n(A)$.

* In the above example since the number of letters in the Alphabet is 26,

$n(B) = 26.$

Date No

Ways of describing Sets.

* List the elements.
  $A = \{1, 2, 3, 4, 5, 6\}$

* Give a verbal description.
  "A is the set of all positive integers from 1 to 6, inclusive."

* Denote using the set builder notation /mathematical notation.
  $A = \{x \mid x \in Z^+, x < 7\}$

* Read as
  "A is the set of x such that x is a positive integer and less than 7.

Examples for Sets.

"Standard" Sets.

* Natural Numbers: $N = \{0, 1, 2, 3 \ldots\}$
* Integers: $Z = \{\ldots -2, -1, 0, 1, 2 \ldots\}$
* Positive Integers: $Z^+ = \{1, 2, 3, 4 \ldots\}$
* Real Numbers: $R = \{47.3, -12, \pi, \ldots\}$
* Rational Numbers: $Q = \{1.5, 2.6, -3.8, 15 \ldots\}$

ProMate

Date No

$\mathbb{Q}$ denotes the set of rational numbers, then

$$\mathbb{Q} = \{x \mid x = p/q \text{ where } p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$$

Set Equality.

* Sets A and B are equal if and only if they contain exactly the same elements.

Ex:

$A = \{9, 2, 7, -3\}$
$B = \{7, 9, -3, 2\}$

$A = B$

* Order of the elements doesn't matter.

Some Special Sets.

(01) Null Set or Empty Set $\{\}$

* This is a set with no elements.
* Often symbolized by '$\emptyset$'.

(02) Universal Set.

* This is the set of all elements currently under consideration.
* Often symbolized by 'Ω'.
* Also symbolized by 'U'.

Date No

Membership Relationships.

Subset

* "A is a subset of B"

  $x \in A \Rightarrow x \in B$ (all the members of A are also members of B.)

* The notation for subset means, in terms of the sets, "included in or equal to"

  $A \subseteq B$: "A is a subset of B"

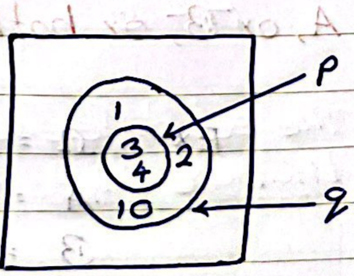
Ex:

$B = \{9, 2, 7, -3\}$

$\{9\}, \{9,2\}, \{2,7,-3\}$ are subsets of B and $\{9,2,7,-3\}$ is also a subset of B.

(01) Does the empty set is a subset of any set. Prove.

Let's take 2 non empty sets p, q.



Such that $p \subset q$

$x \in p \;,\; x \in q$

$p \cap q = p \,,\; p \cup q = q$

thus; $\emptyset \cap q = \emptyset \,,\; \emptyset \cup q = q$

$\emptyset \subset q$

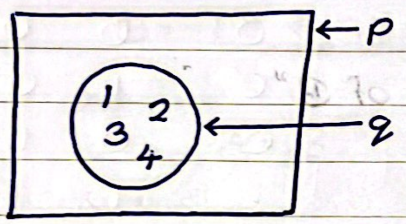
Date No

Proper Subset.

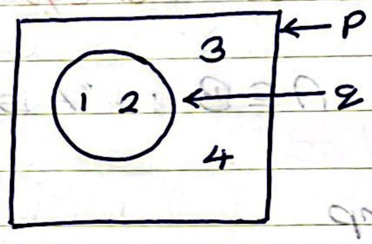
* To become a proper subset, that subset cannot be equal to the owning set.

* The notation for proper subset means, in terms of the sets, "included in but not equal to."

$A \subset B$: "A is a proper subset of B"



Not a proper subset.



A proper subset.

Combining Sets.

Set Union

$A \cup B$: A union B

* "A union B" is the set of all elements that are in A, or B, or both.

Ex: $\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3\}$

$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Set Intersection

$A \cap B$: A intersection B

* "A intersection B" is the set of all elements that is in both A and B.

Ex: $\Omega = \{1, 2, 3, 4, 5, 6\}$
$A = \{1, 2, 3\}$
$B = \{3, 4, 5, 6\}$
$A \cap B = \{3\}$

Set Complement

$\bar{A}$, A' : A complement

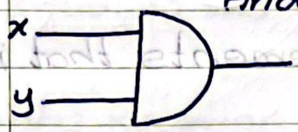
* "A complement" or "not A" is the set of all elements not in A.

Note: $\bar{\bar{A}} = A$

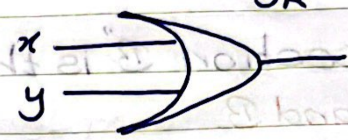
Ex:
$\Omega = \{1, 2, 3, 4, 5, 6\}$
$B = \{3, 4, 5, 6\}$
$\bar{B} = \{1, 2\}$

Relationship between Basic Logic Gates and Set Notations.

And



OR

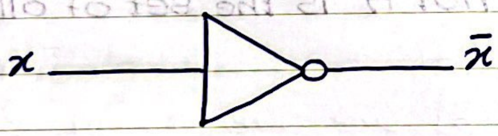


$\cap$ - Ientersection

x	y	out
0	0	0
0	1	0
1	0	0
1	1	1

$\cup$ - Union

x	y	out
0	0	0
0	1	1
1	0	1
1	1	1



complement

Set Difference..

$A - B$: A minus B

* The set difference "A minus B" is the set of elements that are in A, with those that are in B subtracted out.

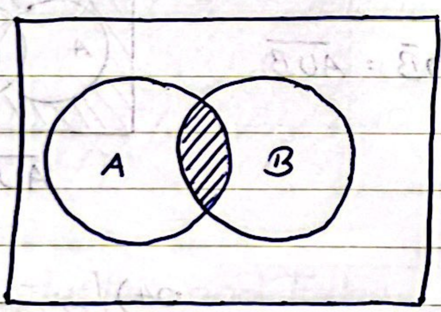
* Another way of putting it is, it is the set of elements that are in A, and not in B, so

$$A - B = A \cap \bar{B}$$

Ex: $\Omega = \{1, 2, 3, 4, 5, 6\}$
$A = \{3, 4, 5, 6\}$
$B = \{1, 2, 3\}$
$A - B = \{4, 5, 6\}$

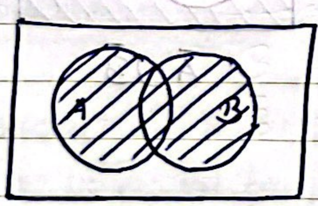
Venn Diagrams.

* Venn diagrams use topological areas to stand for sets.



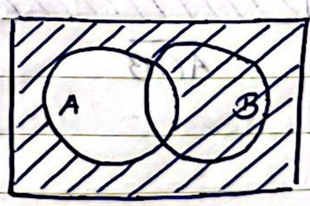
$A \cap B$

01.)



$A \cup B$

02.)

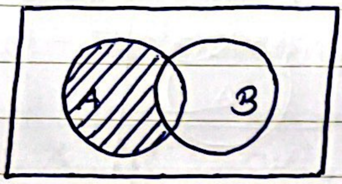


$\bar{A}$

Date No

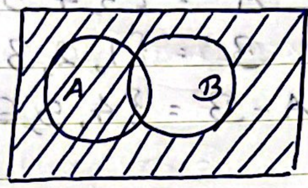
03.)

04.)



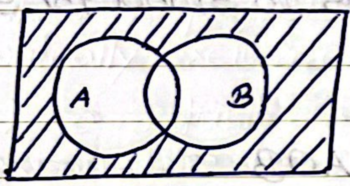
$A-B$

04.)



$\bar{B}$

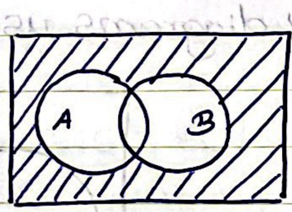
01.)



$\bar{A} \cap \bar{B}$

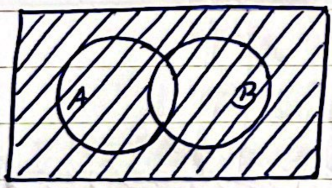
$\bar{A} \cap \bar{B} = \overline{A \cup B}$

02.)



$\overline{A \cup B}$

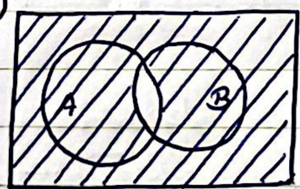
03.)



$\overline{A \cap B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

04.)



$\bar{A} \cup \bar{B}$

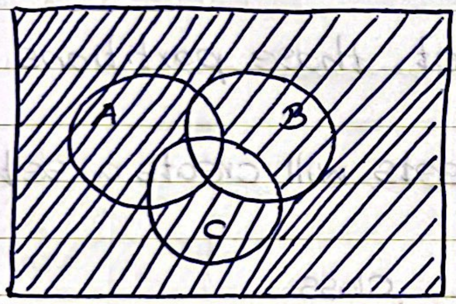
De Morgan's Law for Sets.

* $\overline{A \cup B} = \bar{A} \cap \bar{B}$

* $\overline{A \cap B} = \bar{A} \cup \bar{B}$

* $\overline{(A \cup B) \cap C} = (\bar{A} \cap \bar{B}) \cup \bar{C}$

$(A \cup B) \cap C = (\bar{A} \cap \bar{B}) \cup \bar{C}$



Mutually Exclusive and Exhaustive Sets.

Exhaustive Sets.

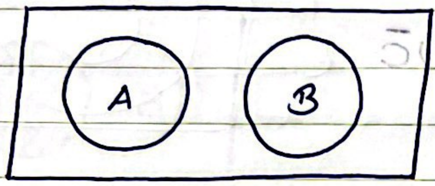
* A group of sets is 'Exhaustive' of another set if their union is equal to that set.

Ex:

If $A \cup B = C$, A and B are exhaustive with respect to C.

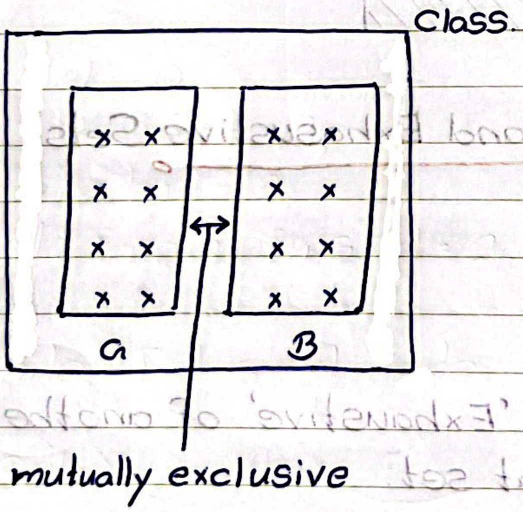
Mutually Exclusive Sets

* Two sets A and B are 'mutually exclusive' if $A \cap B = \emptyset$, that is, the sets have no elements in common.



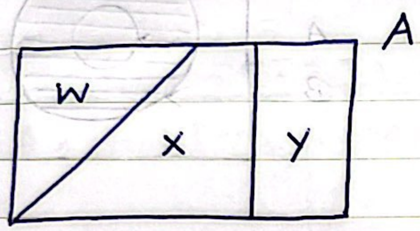
Set Partition.

* When we partitioning a set, those partitions will be mutually exclusive.
* These mutually exclusive sets will creats a set that is exhaustive.

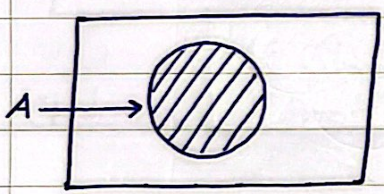


* A group of sets partitions another set if they are mutually exclusive and exhaustive with respect to that set.

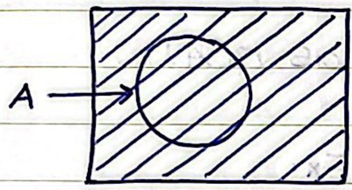
* When we "partition a set," we break it down into mutually exclusive and exhaustive regions. (i.e., regions with no overlap.)
* The below diagram, the set A (the rectangle) is partitioned into sets W, X and Y.



(01.) $A \cup \emptyset = A$



(02.) $A \cup \bar{A} = \Omega$



(03.) $A \cap \bar{A} = \emptyset$

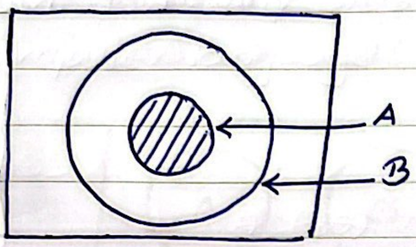
(04.) $A \cup \Omega = \Omega$

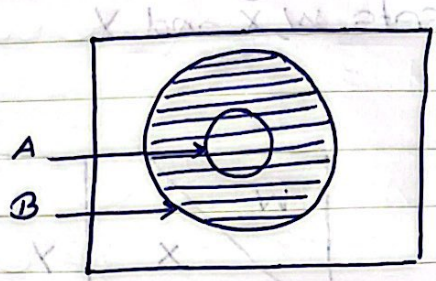
(05.) $A \cap \Omega = A$

(06.) If $A \subset B$, then,

I. $A \cap B = A$

II. $A \cup B = B$





Cardinality and Finiteness.

* $|A|$ (read "the cardinality of set A") is a measure of how many different elements A has.

* Same as $n(A)$.

Ex:

$|\emptyset| = 0,$

$|\{1,2,3\}| = 3,$

$|\{a,b\}| = 2,$

$|\{\{1,2,3\},\{4,5\}\}| = 2$

* A is infinite, if it is not finite.

* Infinite Sets

N, Z, R

The Power Set Operation.

* The power set $P(A)$ of set A is the set of all subsets of A.

$$P(A) = \{x \mid x \subseteq A\}$$

Ex:

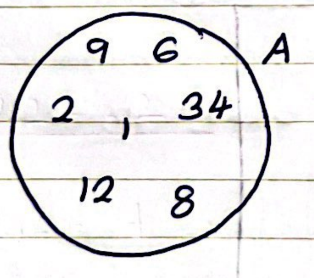
if $A = \{a, b\}$ then $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$.

Cardinality of power set $P(A)$ is given by:

$$|P(A)| = 2^{|A|} \quad \text{(For finite A)}$$

* It turns out that $|P(A)| > |A|$.

(01.) $|P(A)| = 2^7 = 128$

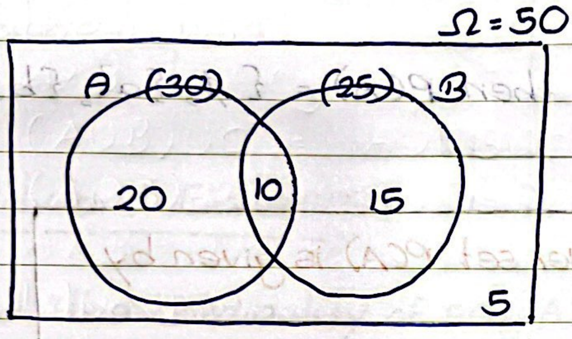


$|A| = 7$

Cardinality of A

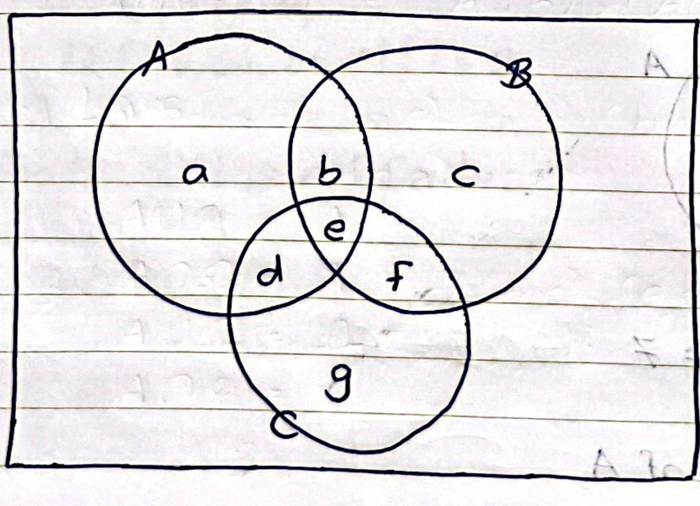
Counting and Venn Diagrams

01.) In a class of 50 college freshmen, 30 are studying BASIC, 25 are studying PASCAL and 10 are studying both. How many freshmen are studying either computer language?



$$\boxed{* |A \cup B| = |A| + |B| - |A \cap B|}$$

02.)



- $|A| = a + b + d + e$
- $|B| = b + c + e + f$
- $|C| = d + e + f + g$

- $|A \cap B| = b + e$
- $|B \cap C| = e + f$
- $|A \cap C| = d + e$

- $|A \cap B \cap C| = e$

$$|A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = a + b + c + d + e + f + g$$

$$= |A \cup B \cup C|$$

$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|}$$

Date No

Laws of the Algebra of Sets.

* Idempotent Laws

$$A \cup A = A$$
$$A \cap A = A$$

* Associative Laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

* Commutative Laws

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

* Distributive Laws.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

* Identity Laws.

$$A \cup \emptyset = A$$
$$A \cap \Omega = A$$
$$A \cup \Omega = \Omega$$
$$A \cap \emptyset = \emptyset$$

* **Involution Law**

$$\bar{\bar{A}} = A \quad \text{or} \quad A'' = A$$

* **Complement Laws**

$$A \cup A' = \Omega$$
$$A \cap A' = \emptyset$$
$$\Omega' = \emptyset$$
$$\emptyset' = \Omega$$

* **De Morgan's Laws**

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

Relations & Functions.

Cartesian Product

* Given two non-empty sets A and B, the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$ is called Cartesian Product of A and B.

$$\boxed{A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}}$$

$$\boxed{A \times B = \{(a, b) \mid a \in A \wedge b \in B\}}$$

* The Cartesian product deals with ordered pairs, so the order in which the sets are considered is important.

Ex:

$A = \{a, b, c\}$
$B = \{d, e, f\}$

$A \times B = \{(a,d), (a,e), (a,f), (b,d), (b,e), (b,f), (c,d), (c,e), (c,f)\}$

$B \times A = \{(d,a), (d,b), (d,c), (e,a), (e,b), (e,c), (f,a), (f,b), (f,c)\}$

* For finite sets A, B

$$n(A \times B) = n(A) . n(B)$$

Relations

* A 'relation' is just a relationship between sets of information.

Ex:

All of the students in a class and their heights. The pairing of names and heights is a relation.

* In relations, the pairs of names and heights are "ordered", which means one comes first and other comes second.

Relations - Definition

* A relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product set $A \times B$.

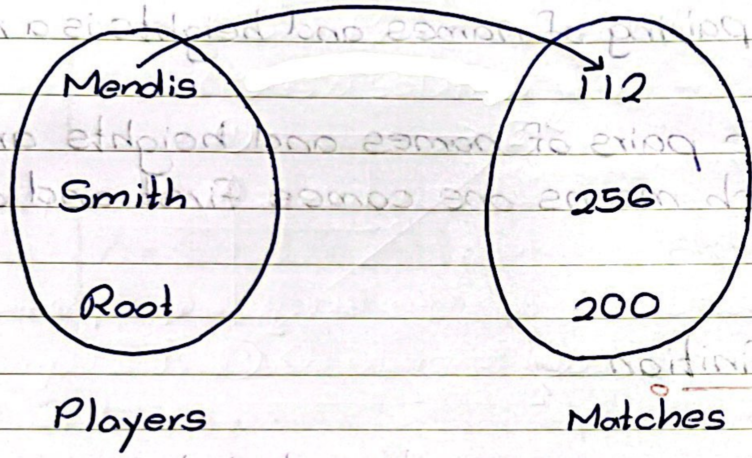
* The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$.

* The set of all first elements in a relation R, is called the domain of the relation R, and the set of all second elements called images, is called the range of R.

* Let A and B be sets. A binary relation or simply a relation from A to B is a subset of $A \times B$.

* Suppose R is a relation from A to B. Then R is the set of ordered pairs where each first element comes from A and each second element comes from B.

Ex:



(Mendis, 112) ✓

(Mendis, 256) ✗

(Smith, 256) ✗

Ordered Pairs.

Relations will be defined in terms of ordered pairs (a, b) of elements, where a is designated as the first element and b as the second element.

$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

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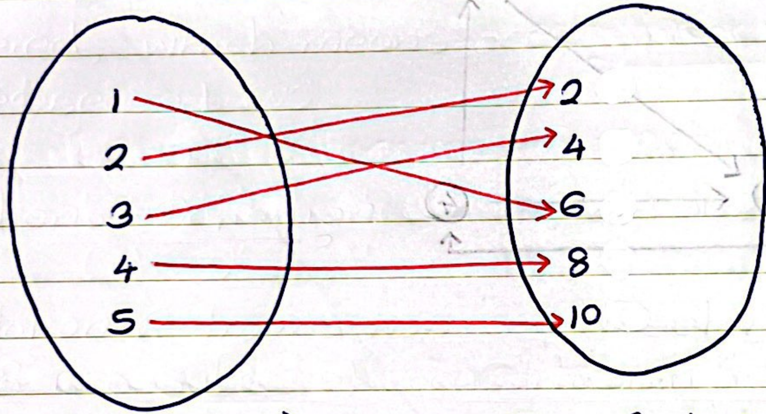
Pictorial Representations of Relations.

(01.) Arrow Diagram.

* Suppose A and B are two finite sets.
* Write down the elements of A and the elements of B in two disjoint disks.
* Then draw an arrow from $a \in A$ to $b \in B$ whenever a is related to b

Ex:



Domain (set of all x's) Range (set of all y's)

Relation: $\{(1,6), (2,2), (3,4), (4,8), (5,10)\}$

(02.) Directed Graphs.

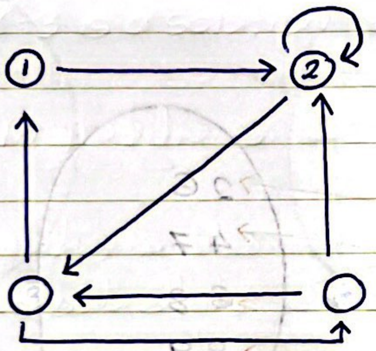
* These graphs are used to picture a relation R, which is a relation from a finite set to itself.

Ex:

① Draw the directed graph of the relation R on the set.

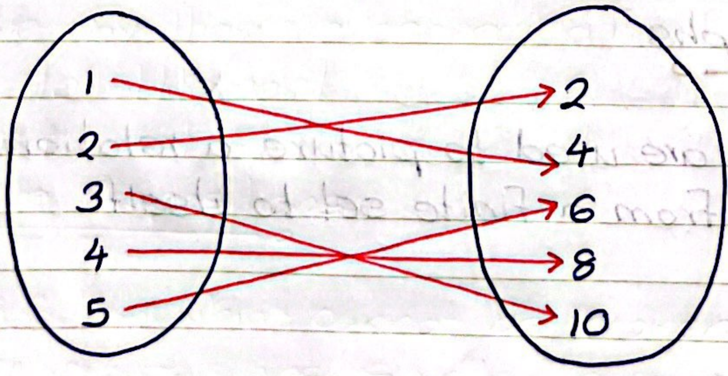
$A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,2), (2,4), (3,2), (3,4), (4,1), (4,3)\}$



Functions.

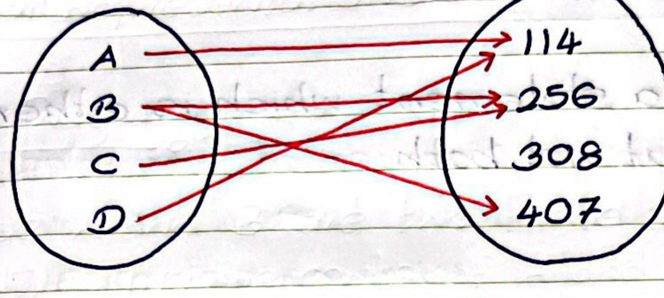
A function f is a relation in which each element of the domain is paired with excatly one element of the range.



Set A is the domain.

Set B is the range.

Ex:



B has 2 elements in range
So, not a function.

04. Propositional Logic

Propositions

* A proposition is a statement which is either TRUE or FALSE but not both.

Ex:

I. Colombo is in Sri Lanka. → T
II. 17 is a Prime number. → T
III. $x = 3$ is a solution of $x^2 = 4$. → F

Composite Propositions.

* Some propositions are composite, that is they are composed of sub statements.

Ex:

17 is a prime number and 10 is an even number.

* A fundamental property of a composite statement is that its truth value is completely determined by the truth value of each of its sub-statements and the way they are connected to form the composite statement.

Primitive Propositions.

* A proposition which cannot be broken down into simpler propositions is called a primitive proposition.
* Each primitive proposition can be represented by a name.

Ex:

I. 3+8 = 11

II. Elephants can fly.

* A primitive statement has a truth value either TRUE or FALSE.

Truth Tables.

* A truth table is a computational device by which we can determine the truth value of a proposition once we know the truth value of each of its components.

* We can combine primitive propositions using the basic logical operations.

Constructing Truth Tables.

* We are using truth table to prove a combinational, logical relation between variables or the validity of a statement.

* Truth tables depends on variables.

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Designing the Truth Tables.

* Number of columns = Number of Variables
  +
  Working columns
  +
  Output

* Number of rows = $2^{\text{number of variables}}$.

* Since Variables are for propositions, they can have only two values, TRUE or FALSE.

Ex:

Single Variable Truth Table.

2°

X	Output
F	
T	

* Write 'F' First.
* Use 'F,T', Not '0,1'.

Ex:

Two variable truth table.

2^1	2^0	
X	Y	Output
F	F	
F	T	
T	F	
T	T	

$* 2^0 = 1 =$ F T

$* 2^1 = 2 =$ F F T T

$* 2^2 = 4 =$ F F F F T T T T

Ex:

Three variable truth table.

2^2	2^1	2^0	
X	Y	Z	Output
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

Negation (NOT)

* When any proposition is given, another proposition called Negation of p can be formed by inserting the word "not" before p.

* Negation of p is denoted by '$\sim p$' or '$\neg p$' or 'not p'.

p	$\sim p$
F	T
T	F

Conjunction (AND)

* Any two given propositions can be combined with the word "and" to form a compound proposition called the conjunction of the two given propositions.

* If p and q are propositions then, "$p \wedge q$" (read as 'p and q') denotes the conjunction of the propositions p, q.

* Consider the two propositions p, q. If p is TRUE, then $p \wedge q$ is TRUE. Otherwise, $p \wedge q$ is FALSE.

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction (OR)

* Any two given propositions can be combined with the word "or" to form a compound proposition called the disjunction of the two given propositions.

* If p and q are propositions then, $p \vee q$ (read as 'p or q') denotes the disjunction of the propositions p, q.

* Consider the two propositions p, q. If p is TRUE or q is TRUE, then $p \vee q$ is TRUE. Otherwise $p \vee q$ is False.

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Implication

* An implication is of the form "if p is TRUE, then q follows." or "if p, then q".

* An implication is denoted by $p \Rightarrow q$.

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

* If p is FALSE it doesn't matter what will be the truth value of q, $p \Rightarrow q$ is always TRUE.

Equivalence (XNOR)

* An equivalence is denoted by $<=>$.

* An equivalence $p <=> q$ is TRUE if and only if p and q have the same truth value.

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p	q	p <=> q
F	F	T
F	T	F
T	F	F
T	T	T

Precedence Rules.

* Propositional logic uses precedence rules.

$\sim$	Highest
$\wedge$	
$\vee$	
$\Rightarrow$	
$\Leftrightarrow$	Lowest

Brackets override
B, N, C, D, I, E

Exclusive OR

* Let p and q be two propositions. The exclusive OR of p and q (denoted by $p \oplus q$) is a proposition that simply means excatly one of p and q will be true but both cannot be true.

Ex:

When you buy a car from XYZ company, you get either Rs. 50,000 cashback or accessories worth Rs. 50,000.

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* If the inputs are different, outcome will be true.

Tautology.

* Some compound propositions contain only "T" in the last column of their truth tables.

* A compound proposition which is true under all possible assignments of truth values to its prime propositions is called a tautology or a valid proposition.

Consider the statements.

- The new born baby is either male or female.
- The train will either arrive in time or will not arrive in time.

* The above propositions are tautologies because they're always TRUE.

* The propositions $p \vee \sim p$ is a tautology.

p	$\sim p$	$p \vee \sim p$
F	T	T
T	F	T

ProMate

Contradiction

* Similarly, some compound propositions contain only "F" in the last column of their truth tables.

* A compound proposition which is FALSE under all possible assignments of truth values to its prime propositions is called a contradiction or an inconsistent proposition.

* The propositions $p \wedge \sim p$ is a contradiction.

p	$\sim p$	$p \wedge \sim p$
F	T	F
T	F	F

Contingent Proposition

* A compound proposition which is neither a tautology nor a contradiction is called a contingent proposition.

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Ex:

$p \Rightarrow \sim p$

p	$\sim p$	$p \Rightarrow \sim p$
F	T	F
T	F	T

Laws in Propositional Logic.

* Commutative Laws.

$$p \wedge q = q \wedge p$$
$$p \vee q = q \vee p$$

* Associative Laws.

$$p \wedge (q \wedge r) = (p \wedge q) \wedge r$$
$$p \vee (q \vee r) = (p \vee q) \vee r$$

* ① Distributive Laws.

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

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* ① De Morgan's Laws.

$$\sim(p \wedge q) = \sim p \vee \sim q$$
$$\sim(p \vee q) = \sim p \wedge \sim q$$

* Law of Negation.

$$\sim(\sim p) = p$$

* Law of Excluded Middle

$$p \vee \sim p = TRUE$$

* Law of Contradiction.

$$p \wedge \sim p = FALSE$$

* Law of Implication.

$$p \Rightarrow q = \sim p \vee q$$

* Contrapositive Law.

$$p \Rightarrow q = (\sim q \Rightarrow \sim p)$$

* Law of Equivalence.

$$p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$$

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* Idempotence.

$$p \vee p = p$$
$$p \wedge p = p$$

* Laws of Simplification - 1

$$p \wedge true = p$$
$$p \vee true = true$$
$$p \wedge false = false$$
$$p \vee false = p$$

* Laws of Simplification - 2

$$p \vee (p \wedge q) = p$$
$$p \wedge (p \vee q) = p$$

* Distributive Laws.

$$a \vee (b \wedge c \wedge d) = (a \vee b) \wedge (a \vee c) \wedge (a \vee d)$$
$$a \wedge (b \vee c \vee d) = (a \wedge b) \vee (a \wedge c) \vee (a \wedge d)$$

07. Matrices.

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In computing matrices are advanced data structures used to store data and perform execution.

What is a Matrix?

A matrix is a rectangular array of numbers.

The numbers in the array are called the entries of the matrix.

$$\begin{bmatrix} 4 & 8 & 9 & 10 \\ 3 & 5 & -4 & 1 \\ 6 & 8 & 12 & 0 \end{bmatrix} \quad \overbrace{\underbrace{3}_{\text{rows}} \times \underbrace{4}_{\text{columns}}}^{\text{dimension.}}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Usually, we specify a position in the matrix as row, column.

The size of a matrix is written in terms of;

the number of it's rows $\times$ the number of its columns

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a_{ij} is called ij-entry or ij element appears in row i and column j.

We also denote the above Matrix as;

$$A = [a_{ij}]_{m \times n}$$

Square Matrix

A square matrix is a matrix with same number of rows as columns.

An $n \times n$ square matrix is said to be of order n and is sometimes called an "n-square matrix".

Square matrices eligable for, to conduct number of arithmatic operations. (The operations of addition, multiplication, scalar multiplication and transpose can be performed on any $n \times n$ matrices would result in an $n \times n$ matrix.)

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 3 \\ 1 & 4 & 3 & -1 \end{bmatrix}$$

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Identity Matrix

If a square matrix contain only 1's in diagonal and all the other positions with 0's; is an identity matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{n \times n}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{n \times n}$$

For any square matrix A,

$$AI = IA = A$$

Matrix Operations.

Matrix Addition & Subtraction.

First you have to check whether dimensions are same.

If that so, you can do either addition or subtraction to respective position value.

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Matrix Addition.

$$\begin{bmatrix} 6 & -9 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} -12 & -11 \\ 15 & -6 \end{bmatrix} = \begin{bmatrix} 6+(-12) & (-9)+(-11) \\ 0+15 & (-7)+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -20 \\ 15 & -13 \end{bmatrix} //$$

Matrix Subtraction.

$$\begin{bmatrix} 6 & -9 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} -12 & -11 \\ 15 & -6 \end{bmatrix} = \begin{bmatrix} 6-(-12) & (-9) \\ 0-15 & (-7)-(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 2 \\ -15 & -1 \end{bmatrix} //$$

Scalar Multiplication.

The product of the matrix A by a scalar k, written kA is the matrix obtained by multiplying each element of A by k.

That is,

$$kA = \begin{bmatrix} ka_{11} & ka_{12} & \dots & ka_{1n} \\ ka_{21} & ka_{22} & \dots & ka_{2n} \\ \dots & \dots & \dots & \dots \\ ka_{m1} & ka_{m2} & \dots & ka_{mn} \end{bmatrix}_{m \times n}$$

$$4\begin{bmatrix} 6 & 15 \\ 3 & -8 \end{bmatrix} = \begin{bmatrix} 24 & 60 \\ 12 & -32 \end{bmatrix}$$

(01.) If... $2A - kB = \begin{bmatrix} 46 & -13 & 65 \\ -12 & 3 & -1 \end{bmatrix}$, find k?

$$A = \begin{bmatrix} 5 & -11 & 7 \\ -6 & 3 & -8 \end{bmatrix} \qquad B = \begin{bmatrix} -12 & -3 & -17 \\ 0 & 1 & -5 \end{bmatrix}$$

$$2\begin{bmatrix} 5 & -11 & 7 \\ -6 & 3 & -8 \end{bmatrix} - k\begin{bmatrix} -12 & -3 & -17 \\ 0 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 46 & -13 & 65 \\ -12 & 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -22 & 14 \\ -12 & 6 & 16 \end{bmatrix} - k\begin{bmatrix} -12 & -3 & -17 \\ 0 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 46 & -13 & 65 \\ -12 & 3 & -1 \end{bmatrix}$$

$$k\begin{bmatrix} -12 & -3 & -17 \\ 0 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 46 & -13 & 65 \\ -12 & 3 & -1 \end{bmatrix} - \begin{bmatrix} 10 & -22 & 14 \\ -12 & 6 & -16 \end{bmatrix}$$

$$k\begin{bmatrix} -12 & -3 & -17 \\ 0 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 36 & 9 & 51 \\ 0 & -3 & 15 \end{bmatrix}$$

$$k = 3 \;//$$

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Multiplying Two Matrices.

To multiply two matrices, the number of columns in the first matrix must be equal to number of rows in the second matrix.

If, $A \times B = C$,

01. Check whether the two matrices are compatible for multiplication.

2×2 (answer)

A is a 2×3 matrix. B is a 3×2 matrix.

(No. of columns of A = No. of rows of B.)

02. Identify the dimensions of the resultant matrix. (2×2)

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}_{2\times 2}$$

Ex:

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \times \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}_{3\times 2}$$

shape

$$= \begin{bmatrix} (1\times 3)+(2\times 2)+(3\times 1) & (1\times -1)+(2\times 0)+(3\times 1) \\ (4\times 3)+(5\times 2)+(6\times 1) & (4\times -1)+(5\times 0)+(6\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 2 \\ 28 & 2 \end{bmatrix}_{2\times 2}$$

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① Determinant of a 2×2 matrix.

If;

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \quad (a\times d) - (b\times c)$$

the determinant $|A|$ is given by;

$$|A| = ad - bc$$

Inverse of a 2×2 matrix.

A square matrix A is said to be "invertible" or "nonsingular" if the following relationship holds;

$$AA^{-1} = A^{-1}A = I$$

If;

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

swap

Multiply by (-)

$$A^{-1} = \frac{1}{\text{det or } |A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

ad-bc

where $|A| = ad - bc$
and $|A| \neq 0$

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Solving simultaneous equations

$2x + 4y = 14$
$4x - 4y = 4$

$$\underbrace{\begin{bmatrix} 2 & 4 \\ 4 & -4 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 14 \\ 4 \end{bmatrix}}_{C}$$

$$aX \xrightarrow{a^{-1}} = C$$

$$\boxed{X = a^{-1} . C}$$

Find the determinant of a;

$$\begin{bmatrix} 2 & 4 \\ 4 & -4 \end{bmatrix} \longrightarrow (2 \times -4) - (4 \times 4) = -8 - 16 = -24$$

$\therefore |a| = -24$

Find the inverse of a;

$$a^{-1} = \frac{1}{-24} \begin{bmatrix} -4 & -4 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{vmatrix} 1/6 & 1/6 \\ 1/6 & -1/12 \end{vmatrix}$$

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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/6 & 1/6 \\ 1/6 & -1/12 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} (14/6 + 14/6) \\ (14/6 - 4/12) \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\left.\begin{matrix} x = 3 \\ y = 2 \end{matrix}\right\} //$$

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(01.) $\begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

a X c

$\overset{\frown}{aX} = c$

$X = a^{-1}.c$

Find the determinant of a;

$|a| = 2-2$
$= 0$

Because $|a| = 0$ can't go further.

(02.) $\begin{bmatrix} 3 & -4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ 24 \end{bmatrix}$

a X c

$\overset{\frown}{aX} = c$

$X = a^{-1}.c$

Find the determinant of a;

$|a| = 9-16$
$= -7$

$X = \frac{1}{-7} \begin{vmatrix} 3 & -4 \\ -4 & -3 \end{vmatrix} \begin{vmatrix} 25 \\ 24 \end{vmatrix}$

$= \begin{vmatrix} -3/7 & 4/7 \\ 4/7 & -3/7 \end{vmatrix} \begin{vmatrix} 25 \\ 24 \end{vmatrix}$

$= \begin{vmatrix} \frac{(-75+96)}{7} \\ \frac{(100-72)}{7} \end{vmatrix}$

$= \begin{vmatrix} 3 \\ 4 \end{vmatrix}$ //

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Finding the Inverse of 3×3 square matrix

There're 4 steps we need to complete.

01. Minors.
02. Co-factor Matrix.
03. Transpose of a Matrix.
04. Determinent of 3×3 Matrix.

Finding Minors.

To find minor at the each position we need to exclude (cut off) the row and the column containing that position value. And we have to write the remain as a 2×2 matrix.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} e & f \\ h & i \end{bmatrix} \quad M_{12} = \begin{bmatrix} d & f \\ g & i \end{bmatrix} \quad M_{13} = \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} b & c \\ h & i \end{bmatrix} \quad M_{22} = \begin{bmatrix} a & c \\ g & i \end{bmatrix} \quad M_{23} = \begin{bmatrix} a & b \\ g & h \end{bmatrix}$$

$$M_{31} = \begin{bmatrix} b & c \\ e & f \end{bmatrix} \quad M_{32} = \begin{bmatrix} a & c \\ d & f \end{bmatrix} \quad M_{33} = \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

Creating the Co-factor Matrix

Co-factors are obtained as follows:

$$\boxed{C_{ij} = (-1)^{i+j} \cdot M_{ij}}$$

C_{ij} is the elements of the co-factor matrix

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 0 & 8 \\ 4 & -5 & 1 \end{bmatrix}$$

$$C_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & 8 \\ -5 & 1 \end{vmatrix}$$

$$= -1^{2} \cdot \begin{vmatrix} 0 & 8 \\ -5 & 1 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 0 & 8 \\ -5 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 8 \\ -5 & 1 \end{vmatrix}$$

$$C_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix}$$

$$= -1^{3} \cdot \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix}$$

$$= -1 \cdot \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix}$$

(01.) $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 0 & 8 \\ 4 & -5 & 1 \end{bmatrix}$

$$\begin{vmatrix} \begin{vmatrix} 0 & 8 \\ -5 & 1 \end{vmatrix} & -\begin{vmatrix} 6 & 8 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 6 & 0 \\ 4 & -5 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 4 & -5 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 0 & 8 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 6 & 8 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 6 & 0 \end{vmatrix} \end{vmatrix}$$

$$\begin{vmatrix} 0-(-40) & -(6-32) & -30-0 \\ -(1-(-10)) & 2-8 & -(-10-4) \\ 8-0 & -(16-12) & 0-6 \end{vmatrix}$$

$$CA = \begin{vmatrix} 40 & 26 & -30 \\ -11 & -6 & 14 \\ 8 & -4 & -6 \end{vmatrix} //$$

Transpose of a Matrix

* While keeping the diagonal values constant, swap the opposite values. (Or write the rows as columns)

$$\begin{bmatrix} -4 & 6 & -3 \\ 6 & -9 & 4 \\ 1 & -2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -4 & 6 & 1 \\ 6 & -9 & -2 \\ -3 & 4 & 1 \end{bmatrix}$$

Determinent of a $n \times n$ matrix.

The determinent $|A|$ is given by,

$$\boxed{|A| = \sum_{j=1}^{n} a_{ij} \; (-1)^{i+j} \; M_{ij}}$$

$$A = \begin{vmatrix} 2 & -1 & 3 \\ -0 & -2 & -4 \\ 5 & -6 & 1 \end{vmatrix}$$

$$|A| = \left(2 \times 1. \begin{vmatrix} -2 & 4 \\ 6 & 1 \end{vmatrix}\right) + \left(1 \times -1. \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix}\right) + \left(3 \times 1. \begin{vmatrix} 0 & -2 \\ 5 & 6 \end{vmatrix}\right)$$

$$= 2. \begin{vmatrix} -2 & 4 \\ 6 & 1 \end{vmatrix} - 1. \begin{vmatrix} 0 & 4 \\ 5 & 1 \end{vmatrix} + 3. \begin{vmatrix} 0 & -2 \\ 5 & 6 \end{vmatrix}$$

$$= 2.(-2-24) - 1.(0-20) + 3.(0-(-10))$$
$$= 2\times(-26) - 1\times(-20) + 3\times(10)$$
$$= -52 + 20 + 30$$
$$= -2$$

Inverse of 3×3 matrix.

$$\boxed{A^{-1} = \frac{(\text{Cofactor matrix})^T}{|A|}}$$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 8 & 0 & 6 \\ 5 & 1 & -2 \end{bmatrix}$$

Find the Co-factor Matrix

$$= \begin{vmatrix} \begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 8 & 6 \\ 5 & -2 \end{vmatrix} & \begin{vmatrix} 8 & 0 \\ 5 & 1 \end{vmatrix} \\ -\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 0 & 6 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 8 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 8 & 0 \end{vmatrix} \end{vmatrix}$$

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$$= \begin{vmatrix} 0-6 & -(-16-30) & 8-0 \\ -(-6-(-1)) & -4-(-5) & -(-2-15) \\ 18-0 & -(12-(-8)) & 0-24 \end{vmatrix}$$

$$CA = \begin{vmatrix} -6 & 46 & 8 \\ 5 & 1 & 13 \\ 18 & -20 & -24 \end{vmatrix}$$

Transpose of Matrix

$$(C.M)^T = \begin{vmatrix} -6 & 5 & 18 \\ 46 & 1 & -20 \\ 8 & 13 & -24 \end{vmatrix}$$

Find the Determinent

$$|A| = \left(2 \times 1 \times \begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix}\right) + \left(3 \times -1 \times \begin{vmatrix} 8 & 6 \\ 5 & -2 \end{vmatrix}\right) + \left(-1 \times 1 \begin{vmatrix} 8 & 0 \\ 5 & 1 \end{vmatrix}\right)$$

$$= 2\begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix} - 3\begin{vmatrix} 8 & 6 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 8 & 0 \\ 5 & 1 \end{vmatrix}$$

$$= 2(0-6) - 3(-16-30) - 1(8-0)$$

$$= 2 \times (-6) - 3 \times (-46) - 1(8)$$

$$= -12 + 138 - 8$$

$$= 118$$

Date No

$$A^{-1} = \frac{1}{118}\begin{vmatrix} -6 & 5 & 13 \\ 46 & 1 & -20 \\ 8 & 13 & -24 \end{vmatrix}$$

$$= \begin{vmatrix} -0.0048 & 0.04 & 0.104 \\ 0.368 & 0.008 & -0.16 \\ 0.064 & 0.104 & -0.192 \end{vmatrix}$$

System of Linear Equations.

$x + y + z = 6$
$x + 2y - z = 2$
$2x + y - 2z = 10$

$$\underset{a}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 2 \end{bmatrix}} \times \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{c}{\begin{bmatrix} 6 \\ 2 \\ 10 \end{bmatrix}}$$

$$aX = c$$

$$\boxed{X = a^{-1} \cdot c}$$

cf

$$\frac{(cf)^T}{|A|} \rightarrow \frac{(cf)^T}{|A|}$$

ProMate

Date No

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 2 \end{bmatrix} \rightarrow \begin{vmatrix} \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{vmatrix}$$

$$= \begin{vmatrix} 4-(-1) & -(2-(-2)) & 1-4 \\ -(2-1) & 2-2 & -(1-2) \\ -1-2 & -(-1-1) & 2-1 \end{vmatrix}$$

$$C.M = \begin{vmatrix} 5 & -4 & -3 \\ -1 & 0 & 1 \\ -3 & 2 & 1 \end{vmatrix} \rightarrow (C.M)^T = \begin{vmatrix} 5 & -1 & -3 \\ -4 & 0 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

$$|A| = \left(1 \times 1 \times \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}\right) + \left(1 \times -1 \times \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix}\right) + \left(1 \times 1 \times \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}\right)$$

$$= 1 \times \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= 1 \times (4-(-1)) - 1 \times (2-(-2)) + 1 \times (1-4)$$

$$= 5 - 4 - 3$$

$$= -2$$

ProMate

Date No

$$x = \frac{1}{-2} \begin{vmatrix} 5 & -1 & -3 \\ -4 & 0 & 2 \\ -3 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 6 \\ 2 \\ 10 \end{vmatrix}$$

$$= -\frac{1}{2} \begin{vmatrix} 30 + (-2) + (-30) \\ (-24) + 0 + 20 \\ (-18) + 2 + 10 \end{vmatrix}$$

$$= -\frac{1}{2} \begin{vmatrix} -2 \\ -4 \\ -6 \end{vmatrix}$$

$$= \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

$$\left.\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}\right\} //$$

ProMate

(01.)

RAM = x
VGA = y
SSD = z

$$2x + 3y + 4z = 38\cancel{000}$$
$$4x + 2y + 3z = 32\cancel{000}$$ *add the zeros last.
$$3x + 4y + 2z = 38\cancel{000}$$

$$\underbrace{\begin{vmatrix} 2 & 3 & 4 \\ 4 & 2 & 3 \\ 3 & 4 & 2 \end{vmatrix}}_{a} \times \underbrace{\begin{vmatrix} x \\ y \\ z \end{vmatrix}}_{X} = \underbrace{\begin{vmatrix} 38 \\ 32 \\ 38 \end{vmatrix}}_{c}$$

$$aX = c$$
$$X = a^{-1}.c$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 4 & 2 & 3 \\ 3 & 4 & 2 \end{vmatrix} \rightarrow \begin{vmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} \\ -\begin{vmatrix} 3 & 4 \\ 4 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \\ \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} \end{vmatrix}$$

$$= \begin{vmatrix} 4-12 & -(8-9) & 16-6 \\ -(6-16) & 4-12 & -(8-9) \\ 9-8 & -(6-6) & 4-12 \end{vmatrix}$$

$$C.M = \begin{vmatrix} -8 & 1 & 10 \\ 10 & -8 & 1 \\ 1 & 10 & -8 \end{vmatrix}$$

$$(C.M)^T = \begin{vmatrix} -8 & 10 & 1 \\ 1 & -8 & 10 \\ 10 & 1 & -8 \end{vmatrix}$$

$$|A| = \left(2\times 1\times \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix}\right) + \left(3\times -1\times \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix}\right) + \left(4\times 1\times \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix}\right)$$

$$= 2\times \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} + 4\begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix}$$

$$= 2(4-12) - 3(8-9) + 4(16-6)$$

$$= (2\times -8) - (3\times -1) + 4(10)$$

$$= -16 + 3 + 40$$

$$= 27$$

$$x = \frac{1}{27} \begin{vmatrix} -8 & 10 & 1 \\ 1 & -8 & 10 \\ 10 & 1 & -8 \end{vmatrix} \times \begin{vmatrix} 38 \\ 32 \\ 38 \end{vmatrix}$$

$$= \frac{1}{27} \begin{vmatrix} (-304 + 320 + 38) \\ (38 + (-256) + 380) \\ (380 + 32 + (-304)) \end{vmatrix}$$

$$= \frac{1}{27} \begin{vmatrix} 34 \\ 162 \\ 108 \end{vmatrix}$$

$$= \begin{vmatrix} 2 \\ 6 \\ 4 \end{vmatrix}$$

$$\left.\begin{aligned} x &= 2\,\underline{000} \\ y &= 6\,\underline{000} \\ z &= 4\,\underline{000} \end{aligned}\right\} \text{* adding the zeros.}$$

//

Coordinate Geometry

Date No

Outline

01. Equation of a straight line.
02. Parallel lines.
03. Perpendicular lines.
04. Distance between two points.
05. Mid point between two points.
06. Perpendicular Distance from a point to a line.

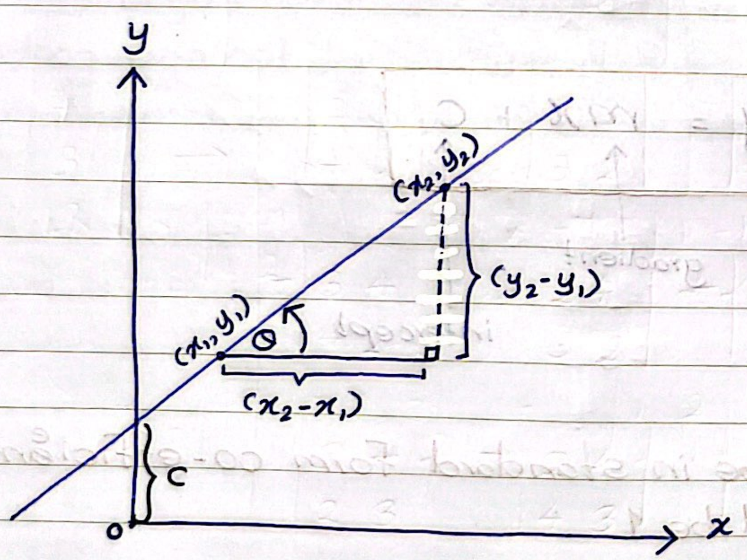
} Straight line

07. Equation of a circle.
08. Polar coordinates.
09. Tagent to a circle.

} circle.

Straight Line.

In computing straight line properties are used to model a relationship between 2 parameters.



* The angle θ is measured counter clockwise from the positive x-axis.

*The domain and range of a linear function are all real numbers.

ProMate

Gradient / Slope (m) (අනුක්‍රමණය)

* The change of the function is refers to the gradient.
* The angle, function creates with X axis can use to measure the gradient by taking its tan value.

$$m = \tan\theta$$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$\left\{ \frac{\text{ස.ව}}{\text{ඔ.ව}} \right.$

Intercept (c) (අන්තඃඛණ්ඩය)

It refers to the constance of the function. (Where it cuts the y axis)

Standard Equation of a Straight Line.

$$\boxed{y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{\text{intercept}}}$$

* Note that to be in standard form co-efficient of the y should be 1.

Ex:

I. $2y = 8x - 12$

$$\frac{2y}{2} = \frac{8x}{2} - \frac{12}{2}$$

$$y = 4x - 6$$

$\left.\begin{array}{l} m = 4 \\ C = 6 \end{array}\right\}$ //

II. $\frac{x+3}{2y-5} = -4$

$$x + 3 = -4(2y - 5)$$

$$x + 3 = -8y + 20$$

$$\frac{8y}{8} = -\frac{x}{8} + \frac{17}{8}$$

$$y = -\frac{1}{8}x + \frac{17}{8}$$

$\left.\begin{array}{l} m = -1/8 \\ C = 17/8 \end{array}\right\}$ //

(01.) Explain why straight line properties are important for computing?

Let's take the example of automatic mobile display brightness adjusment. In here brightness is responding against light. This can be represented using mathematical model which is linear regration.

To represent the interception we can use by default minimum brightness in any mobile.

Plotting a graph from the line equation.

a) Plot the line by identifying x and y intercepts.

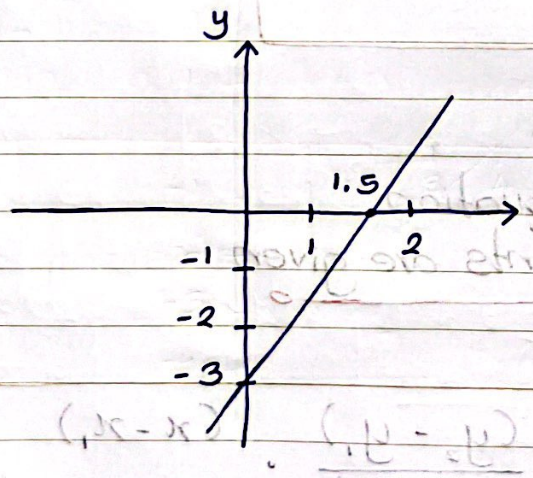
$y = 2x - 3$

Finding the x interception

$$y = 0$$
$$0 = 2x - 3$$
$$2x = 3$$
$$x = 1.5$$

Finding the y interception

$$y = (2 \times 0) - 3$$
$$= -3$$

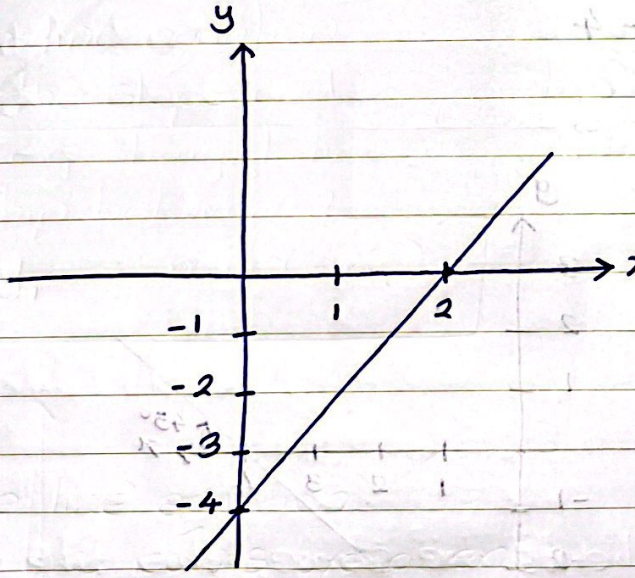


Date No

b) Plotting the line using two points.

$(0, -4)\ (2, 0)$
$x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - (-4)}{2 - 0}$$
$$= \frac{4}{2}$$
$$= 2$$



(01) Interprit differences and similarities of above 2 graphs.

Both has same gradient.
Therefore graphs are parellel.

Both have negative interception with different values.

Date No

Plotting the line by identifying the slope and the intercept.

$y = x - 4$

Finding the slope

$m = 1$

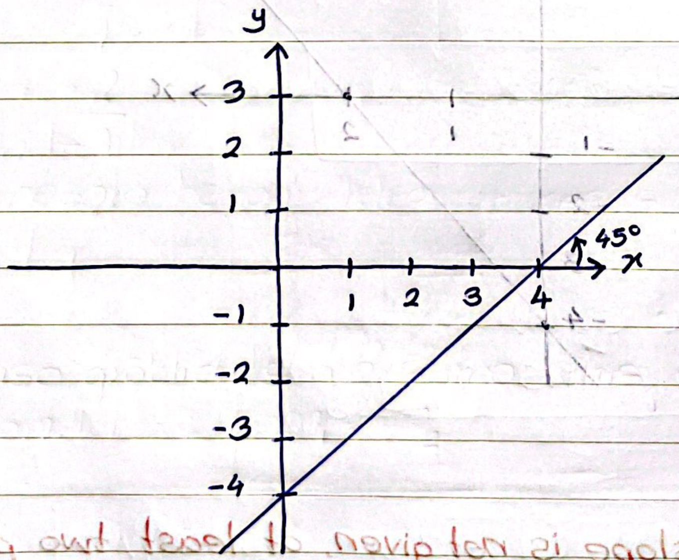
$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$= 45^\circ$

Finding the intercept

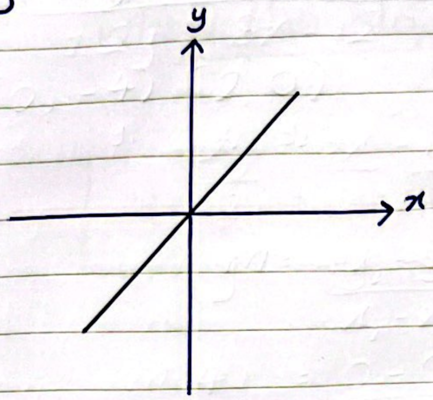
$C = -4$



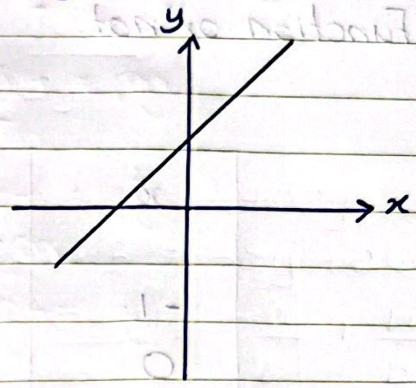
* Note that if the slope is not given, at least two points are required to plot a line. (previous graph)

ProMate

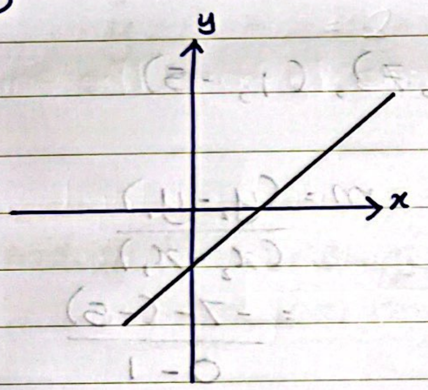
$y = mx$



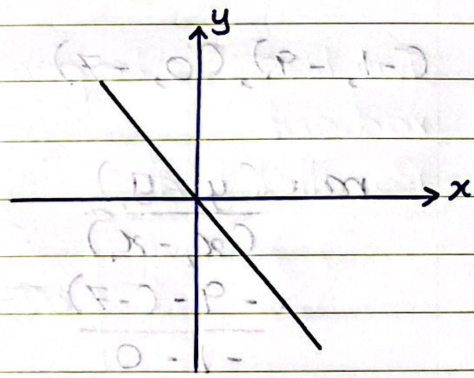
$y = mx + c$



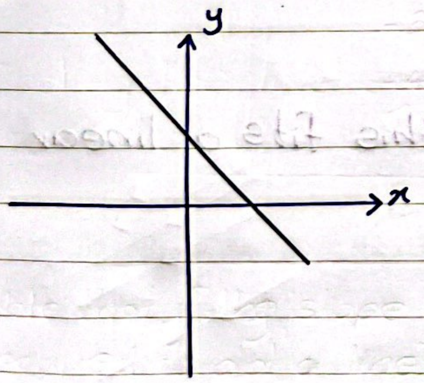
$y = mx - c$



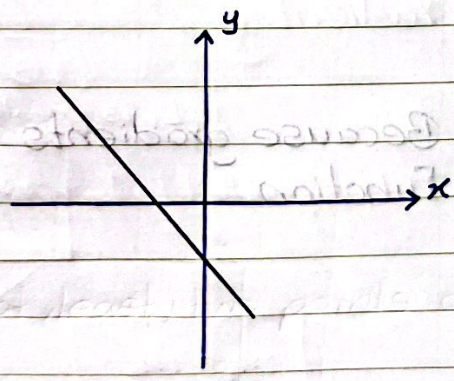
$y = -mx$



$y = -mx + c$



$y = -mx - c$



$m: + \rightarrow$ /

$m: - \rightarrow$ \

Date No

(01.) Identify whether the following data fits a linear Function or not.

x	y
-1	-9
0	-7
1	-5

$(-1,-9), (0,-7)$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-9-(-7)}{-1-0} = \frac{-2}{-1} = 2$$

$(0,7), (1,-5)$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-7-(-5)}{0-1} = \frac{-2}{-1} = 2$$

Because gradients are same, this fits a linear Function.

Date No

Finding the line equation
when the slope and a point is given.

$$\boxed{y - y_1 = m(x - x_1)}$$

(01.) $m = -2$, $(3, 4)$

$$y - 4 = -2(x - 3)$$
$$y - 4 = -2x + 6$$
$$y = -2x + 10$$

Finding the line equation
when two points are given.

$$\boxed{y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1)}$$

(01.) $(3, 4)$, $(-1, 6)$

$$y - 4 = \frac{(6 - 4)}{(-1 - 3)} \cdot (x - 3)$$
$$y - 4 = \frac{2}{-4} \cdot (x - 3)$$
$$y - 4 = -\frac{1}{2} \cdot (x - 3)$$

$$y-4 = -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2} + 4$$

$$y = -\frac{1}{2}x + \frac{3}{2} + \frac{8}{2}$$

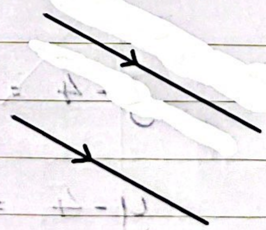
$$y = -0.5x + \frac{11}{2} \;//$$

Parallel lines and Perpendicular lines.-

Parallel Lines.

Two lines to be paralleled, it needs to have the same radient.

If L_1 is parallel to L_2,



$$\boxed{m_1 = m_2}$$

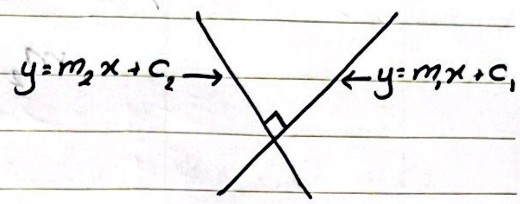
Date No

Perpendicular Lines

Two lines to be perpendicular, their gradient multiplication must be negative one. (-1).

If L_1 is perpendicular to L_2,

$$\boxed{m_1 \times m_2 = -1}$$



Perpendicular lines have the opposite reciprocal slopes.

(01.) Find the equation of a line (L_1) through points $(3,4)$ and $(-4,-6)$. Now write the equation of the line perpendicular to L_1 containing point $(2,3)$.

$$(y-4) = \frac{(-6-4)}{(-4-3)} \cdot (x-3)$$

$$y-4 = \frac{10}{7} \cdot (x-3)$$

$$y-4 = \frac{10x}{7} - \frac{30}{7}$$

$$y = \frac{10x}{7} - \frac{30}{7} + 4$$

$$= \frac{10x}{7} - \frac{30}{7} + \frac{28}{7}$$

$$y = \frac{10x}{7} - \frac{2}{7}$$

Date No

if $L_1 \perp$ (perpendicular) L_2,

$$m_1 \times m_2 = -1$$

$$\frac{10}{7} \times m_2 = -1$$

$$m_2 = \frac{-7}{10}$$

$$(y-3) = \frac{-7}{10}(x-2)$$

$$y-3 = \frac{-7x}{10} + \frac{14}{10}$$

$$y = \frac{-7x}{10} + \frac{14}{10} + 3$$

$$= \frac{-7x}{10} + \frac{14}{10} + \frac{30}{10}$$

$$y = \frac{-7x}{10} + \frac{44}{10}$$

Date No

(Q2) Identify the following pairs of lines are parallel, perpendicular or not.

$-3x + 4y + 1 = 0$ Not Parallel.//

$4x - 3y - 6 = 0$

$-3x + 4y + 1 = 0$

$4y = 3x - 1$

$y = \frac{3}{4}x - \frac{1}{4}$

$4x - 3y - 6 = 0$

$-3y = -4x + 6$

$-y = \frac{-4x}{3} + \frac{6}{3}$

$y = \frac{4x}{3} - \frac{6}{3}$

$y = \frac{4}{3}x - 2$

Check $\perp \rightarrow \cancel{\frac{3}{4}} \times \cancel{\frac{4}{3}} = +1$

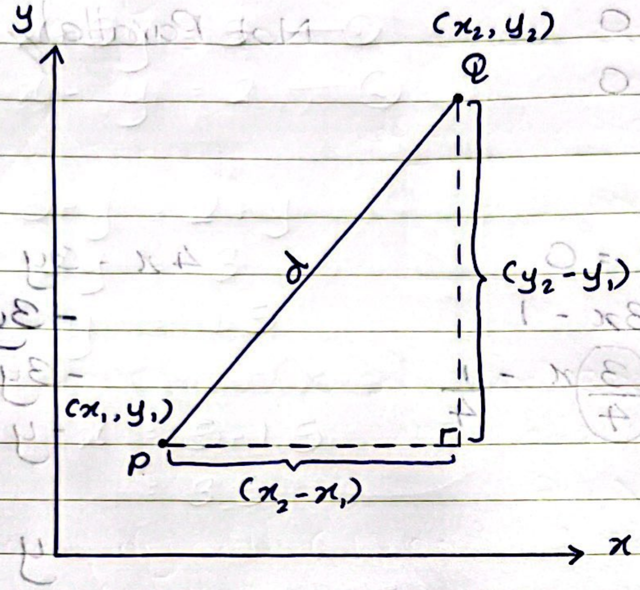
Not Perpendicular.//

ProMate

Scanned with CamScanner

Date No

Distance Between Two Points.



$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(01.) (-1, -2), (3, 4)

$$d = \sqrt{(3-(-1))^2 + (4-(-2))^2}$$

$$= \sqrt{4^2 + 6^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$= 7.21$$

Mid Point Between Two Points.

$$\boxed{M = \underbrace{\left(\frac{x_1 + x_2}{2}\right)}_{a} , \underbrace{\left(\frac{y_1 + y_2}{2}\right)}_{b}}$$

(01.) $(-1, -2), (3, 4)$

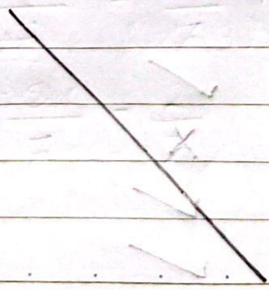
$$M = \left(\frac{-1+3}{2}\right), \left(\frac{-2+4}{2}\right)$$

$$= \frac{2}{2}, \frac{2}{2}$$

$$= (1, 1)$$ //

Identifying the Intersection Point of Two Intersecting Lines.

To get intersection at a point, both lines must satisfy that point. Because of that we can find that line by solving the 2 equations simulta neously.



Date No

(01.) $x + y = 5$
$x - y = 2$

$x + y = 5$ — ①
$x - y = 2$ — ②

$5 - y = 2 + y$
$2y = 3$
$y = 1.5$

$x = 5 - 1.5$
$= 3.5$

$\left.\begin{array}{l} x = 3.5 \\ y = 1.5 \end{array}\right\}$ //

Identifying whether a given point is on a defined line.

A point to be on line, it should satisfy the equation of the line.

$$\boxed{y_1 = mx_1 + c}$$

(01.) $x + y = 5$

A) (2, 3) ✓
B) (-2, -3) X
C) (1, 4) ✓
D) (3, 2) ✓

Perpendicular distance from a point to a line.

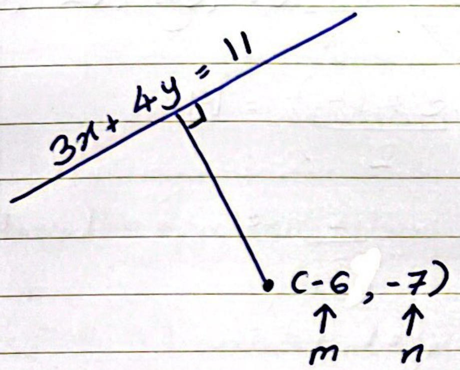
(-) ගොරක් දාන්නත් බෑ.

$$\boxed{PQ = \frac{|am + bn + c|}{\sqrt{a^2 + b^2}}}$$

$|-7| = 7$

$|7| = 7$

(01.)



$$\underset{a}{3x} + \underset{b}{4y} - \underset{c}{11} = 0$$

$$d = \frac{|(3 \times -6) + (4 \times -7) - 11|}{\sqrt{9+16}}$$

$$= \frac{18 + 28 + 11}{\sqrt{25}}$$

$$= \frac{57}{5}$$

$$= 11.4$$