

Boolean Algebra

Algebra associated with Binary Numbers is called Boolean Algebra.

Variables used in Boolean Algebra are called Boolean Variables.

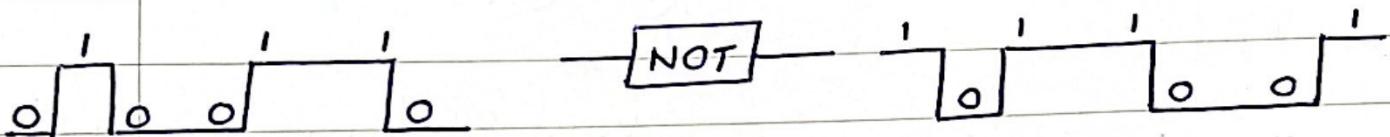
The two entities 0 and 1 together with the three operations; AND, OR, NOT is called Boolean Algebra.

Normal Variable \rightarrow any value.

Boolean Variable \rightarrow 1 or 0

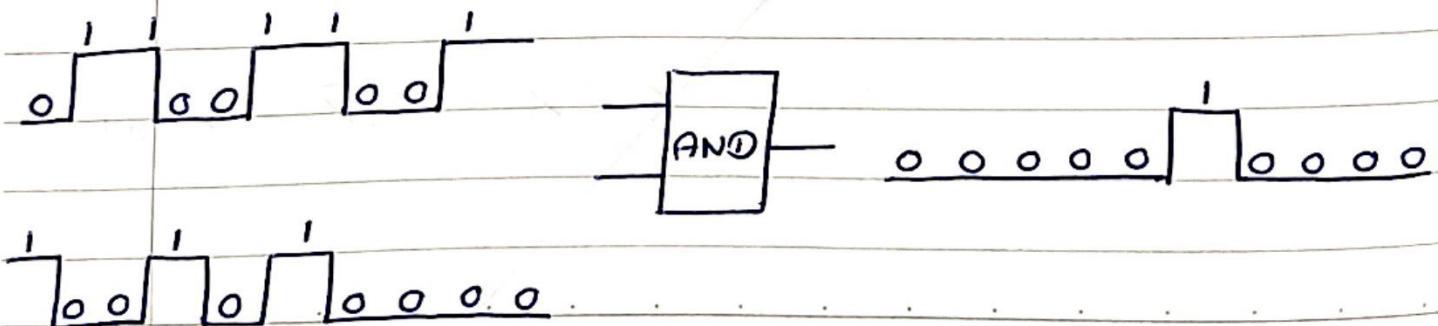
NOT Operation (Complementation)

X	$\text{NOT}(X) / \bar{X}$
0	1
1	0



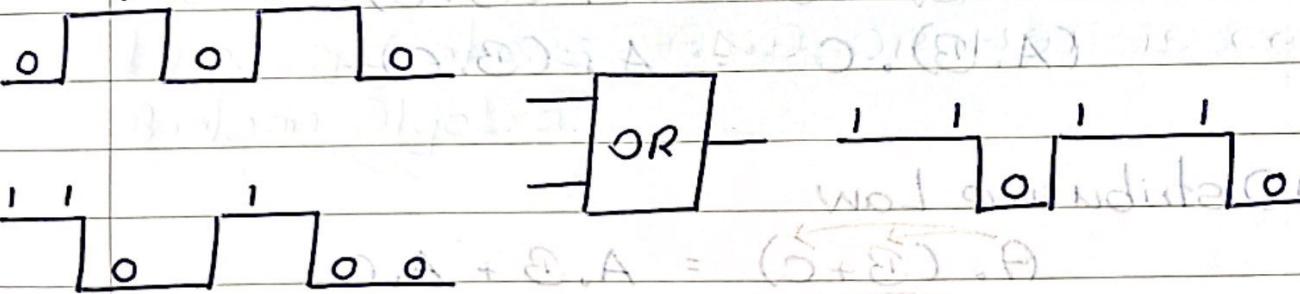
AND Operation (Logical Multiplication)

X	Y	$X \text{ AND } Y / (X \cdot Y)$
0	0	0
0	1	0
1	0	0
1	1	1



OR Operation (Logical Addition)

x	y	$x \text{ OR } y / (x + y)$
0	0	0
0	1	1
1	0	1
1	1	1



Boolean Algebra Rules

- 01) $X + 0 = X$
- 02) $\Theta + 1 = 1$
- 03) $X \cdot 0 = 0$
- 04) $X \cdot 1 = X$
- 05) $X + X = X$
- 06) $X \cdot X = X$

07) $X + \bar{X} = 1$

08) $X \cdot \bar{X} = 0$

09) $(\bar{X}) = X$

10.) Commutative Law

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

11.) Associative Law

$$(A + B) + C = A + (B + C)$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

* 12.) Distributive Law

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A + (B \cdot C) = (A + B)(A + C)$$

13.) Redundance Law

$$A + A \cdot B = A$$

$$A(A + B) = A$$

$$X = 0 + X \quad (1)$$

$$X = 1 + X \quad (2)$$

$$0 = 0 \cdot X \quad (3)$$

$$X = 1 \cdot X \quad (4)$$

14.) $X + \bar{X}Y = X + Y$

$$X = X + X \quad (5)$$

$$X = X \cdot X \quad (6)$$

Prove, $A + A \cdot B = A$

$$A = (0+1)A \text{ and}$$

Prove, $A(A+B) = A$ $A = A \cdot A + A \cdot B$

Perfect Induction

Perfect induction employs a truth table, which describes the validity of the Boolean entity for all the possible value combinations of the Boolean Variables.

Prove the following using Perfect Induction.

$$\text{Q1) } \underbrace{X + XY}_{\text{L.H.S}} = \underbrace{X}_{\text{R.H.S}}$$

X	Y	XY	X+XY
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

*no of variables = 2^n

↑
R.H.S

↑
L.H.S
 $(X+1)+X =$

$$\therefore \text{R.H.S} = \text{L.H.S} //$$

Date _____ No. _____

Q2) $\underbrace{X(X+Y)}_{L.H.S} = \underbrace{X}_{R.H.S}$

X	Y	$X+Y$	$X(X+Y)$
0	0	0	0
0	1	1	0
1	1	1	1

\downarrow
R.H.S

\downarrow
L.H.S

$X = XY + X(10)$

$\therefore R.H.S = L.H.S$

Prove the above using Boolean Variable.

Q1) $X + XY = X$

$$\begin{aligned}
 L.H.S &= X + XY \\
 &= X \underbrace{(1+Y)}_1
 \end{aligned}$$

$$= X$$

$$= R.H.S$$

$$02) X(X + Y) = X$$

$$\text{L.H.S} = X(X + Y)$$

$$= X \cdot X + X \cdot Y$$

$$= X + X \cdot Y$$

$$= X + \underbrace{X}_{1} (1 + Y)$$

$$= X + 1$$

$$= X + 1 = \text{R.H.S}$$

Using Boolean Algebra, simplify the following expressions.

$$01) A = \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XYZ$$

$$= \bar{X}Y \underbrace{(\bar{Z} + Z)}_1 + XZ \underbrace{(Y + Y)}_{1}$$

$$= \bar{X}Y + XZ$$

De Morgan's Theorem.

$$\overline{X} = (\overline{X} + \overline{Y}) \cdot (\overline{X} + \overline{Z})$$

$$(\overline{X} + \overline{Y}) \cdot \overline{X} = 2H1$$

$$I.) \quad \overline{X + Y} = \overline{X} \cdot \overline{Y}$$

$$II.) \quad \overline{X \cdot Y} = \overline{X} + \overline{Y}$$

Apply De Morgan's Theorem to the following expressions.

$$01) Z = \overline{A + \overline{B}}$$

$$= \overline{A} \cdot \overline{\overline{B}} = A \cdot B$$

$$02) Z = \overline{AB + CD}$$

$$= \overline{AB} \cdot \overline{CD} = \overline{A} \cdot \overline{B} + \overline{C} \cdot \overline{D}$$

$$03) Z = \overline{\overline{AB} \cdot \overline{CD}}$$

$$= \overline{\overline{AB}} + \overline{\overline{CD}} = \overline{A} \cdot \overline{B} + \overline{C} \cdot \overline{D}$$

$$= AB + CD$$

Basic Logic Gates.

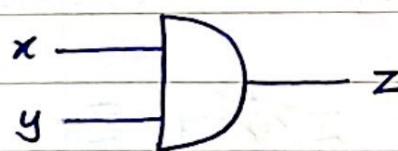
(01) NOT Gate (Inverter)



x	y
0	1
1	0

Σ	Σ	Σ
0	0	0
1	1	0
1	0	1
0	1	1

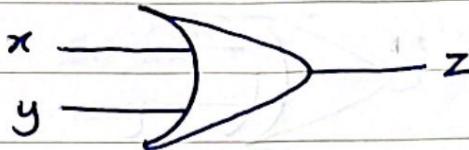
(02) AND Gate (Logical Multiplication)



x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

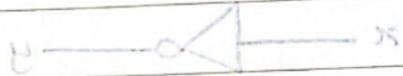
Σ	Σ	Σ
0	0	0
1	1	0
1	0	1
0	1	1

(03.) OR Gate (Addition)



(Invert) also TO 1 (0)

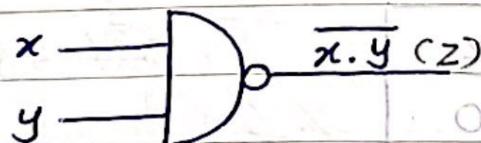
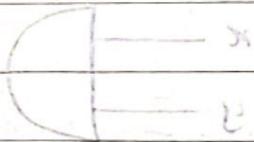
x	y	z
0	0	0
0	1	1
1	0	1
1	1	1



x	y
1	0
0	1
1	1

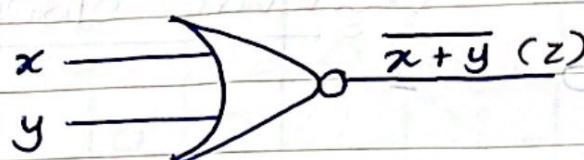
Other Logic Gates

(04.) NAND Gate (Not AND)



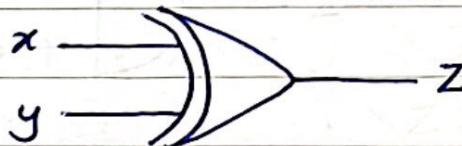
x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(02) NOR Gate (Not OR)



x	y	z
0	0	1
0	1	0
1	0	0
1	1	0

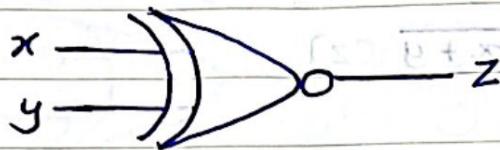
Gate (03) XOR (exclusive OR gate) ⊕



$$\begin{aligned} z &= \bar{x}y + x\bar{y} \\ &= x \oplus y \end{aligned}$$

x	y	z
0	0	0
0	1	1
1	0	1
1	1	0

(04) XNOR Gate (exclusive NOR gate)

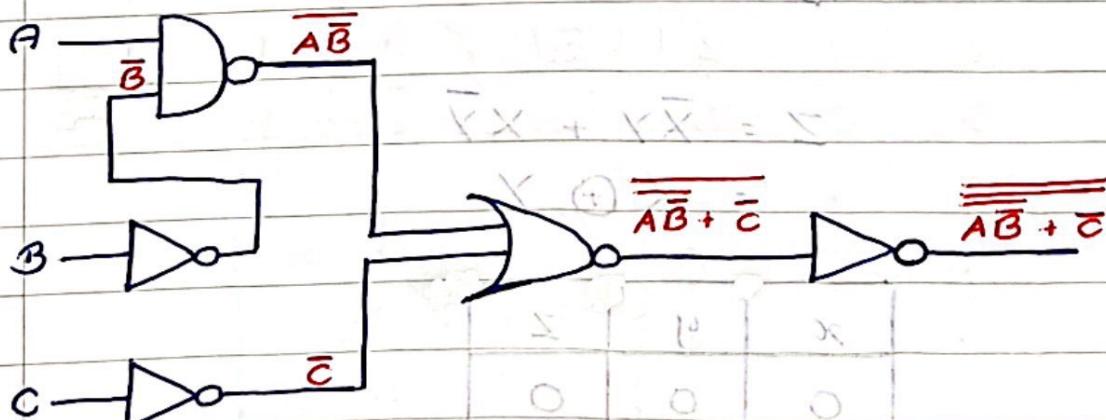


$$Z = \overline{X} \overline{Y} + XY$$

$$= \overline{X \oplus Y}$$

x	y	z
0	0	1
0	1	0
1	0	0
1	1	1

Example:

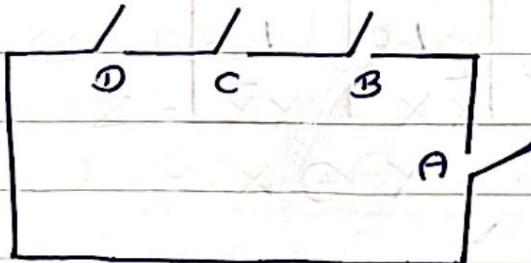


Σ	0	1	0
0	0	0	0
1	1	0	0
0	1	1	1

A	B	C	\bar{B}	\bar{C}	$\bar{A}\bar{B}$	$\bar{A}\bar{B} + \bar{C}$	$\bar{A}\bar{B} + \bar{C}$
0	0	0	1	1	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	1	0	0	1
0	1	1	0	0	1	0	1
1	0	0	1	1	0	0	0
1	0	1	1	0	0	1	0
1	1	0	0	1	1	0	1
1	1	1	0	0	1	0	1

Logic Circuit Design

Example:



- If any 2 doors are open, bulb must turn ON.
- If door A is open, without considering other doors, bulb must turn ON.

DOOR OPEN = 1

DOOR CLOSE = 0

ProMate

A	B	C	D	Z					
0	0	0	0	10	1010000000	1010000000	1010000000	1010000000	1010000000
0	0	0	1	10	0110000000	0110000000	0110000000	0110000000	0110000000
0	0	10	0	10	1000000000	1000000000	1000000000	1000000000	1000000000
0	0	10	1	11	0011000000	0011000000	0011000000	0011000000	0011000000
0	1	0	0	00	1100000000	1100000000	1100000000	1100000000	1100000000
0	1	0	1	01	0111000000	0111000000	0111000000	0111000000	0111000000
0	1	1	0	11	1000000000	1000000000	1000000000	1000000000	1000000000
0	1	1	1	11	0010000000	0010000000	0010000000	0010000000	0010000000
1	0	0	0	1	1100000000	1100000000	1100000000	1100000000	1100000000
1	0	0	1	1	0111000000	0111000000	0111000000	0111000000	0111000000
1	0	1	0	1	1010000000	1010000000	1010000000	1010000000	1010000000
1	0	1	1	0	0100000000	0100000000	0100000000	0100000000	0100000000
1	1	0	0	11	1000000000	1000000000	1000000000	1000000000	1000000000
1	1	0	1	11	0111000000	0111000000	0111000000	0111000000	0111000000
1	1	1	0	1	1010000000	1010000000	1010000000	1010000000	1010000000
1	1	1	1	11	0010000000	0010000000	0010000000	0010000000	0010000000

No multi-layered steps are needed as per TE.

~~que se producen en el tránsito entre los países~~

~~not want him off his work~~

BOOK OPEN = 1
BOOK CLOSE = 0

ProMate

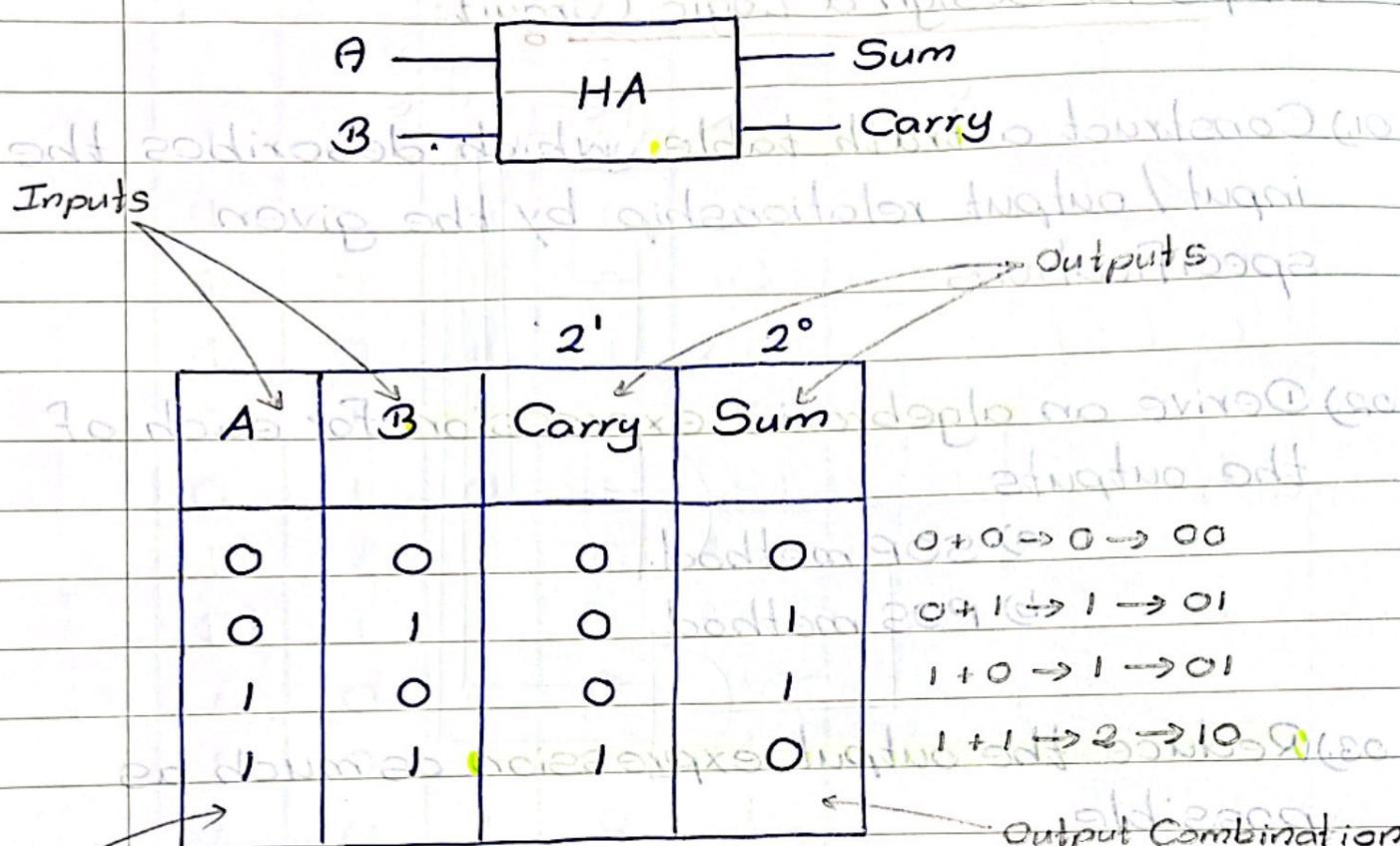
Steps for Design a Logic Circuit.

- 01.) Construct a **truth table**, which describes the input / output relationship by the given specifications.
- 02.) Derive an **algebraic expression** for each of the outputs.
 - a) SOP method.
 - b) POS method.
- 03.) Reduce the output expression as much as possible.
 - a) Algebraically.
 - b) Using K Maps.
 - c) Using Tabular Method.
- 04.) Implement the circuit using gates.

Binary Adders.

Binary Half Adder.

It is a logic circuit for the addition of **two, 1-bit binary numbers.**

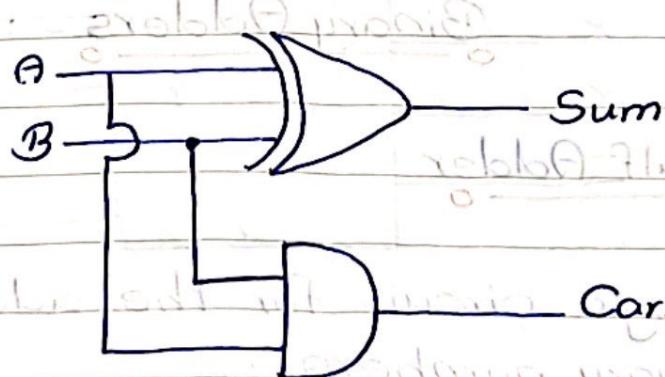


Input Combinations

$$\text{Sum} = \bar{A}\bar{B} + A\bar{B}$$

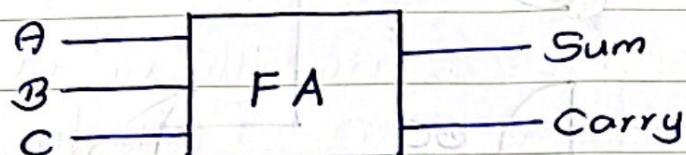
$$= A \oplus B$$

$$\text{Carry} = AB$$



Binary Full Adder

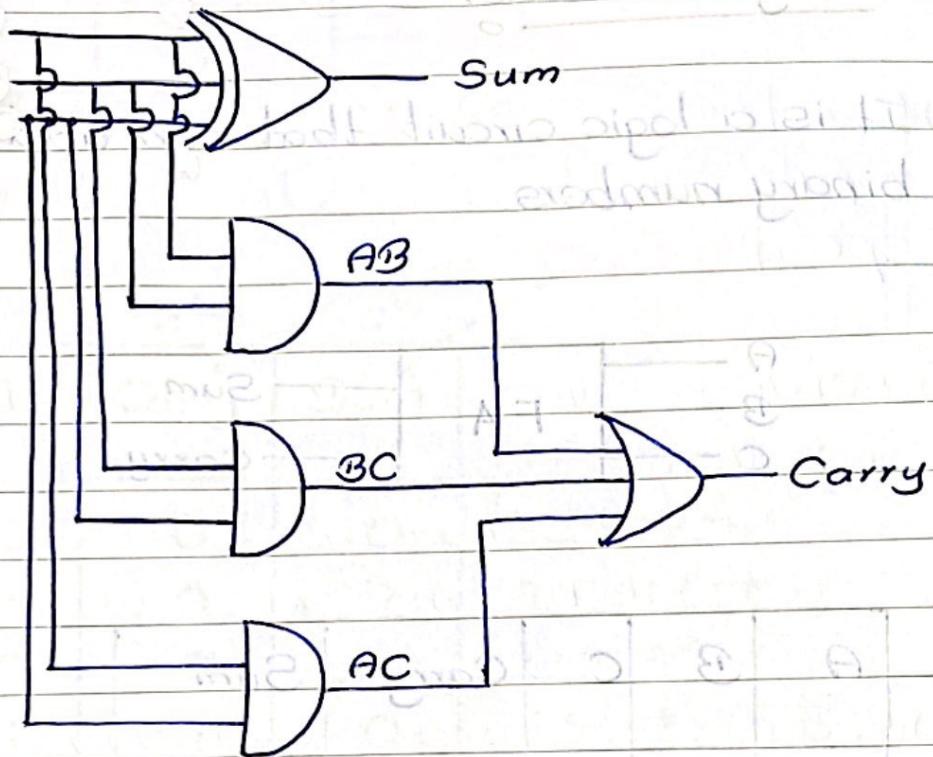
It is a logic circuit that can add three, 1-bit binary numbers.



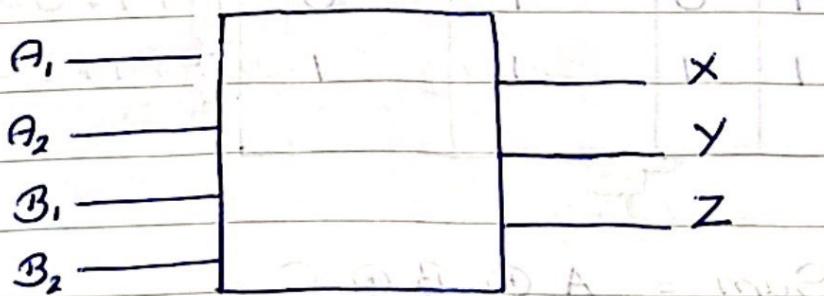
A	B	C	Carry	Sum	
0	0	0	0	0	$0+0+0 \rightarrow 0 \rightarrow 00$
0	0	1	0	1	$0+0+1 \rightarrow 1 \rightarrow 01$
0	1	0	0	1	$0+1+0 \rightarrow 1 \rightarrow 01$
0	1	1	1	0	$0+1+1 \rightarrow 2 \rightarrow 10$
1	0	0	0	1	$1+0+0 \rightarrow 1 \rightarrow 01$
1	0	1	1	0	$1+0+1 \rightarrow 2 \rightarrow 10$
1	1	0	1	0	$1+1+0 \rightarrow 2 \rightarrow 10$
1	1	1	1	1	$1+1+1 \rightarrow 3 \rightarrow 11$

$$\text{Sum} = A \oplus B \oplus C$$

$$\text{Carry} = AB + BC + AC$$



Design a logic circuit to add two, 2-bit binary numbers. Implement the circuit using logic gates.



$$DA + DC + BA = \text{Sum}$$

A_1	A_2	B_1	B_2	X	Y	Z
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

S X X E E A A

O O O O

I O O O

O I O O

I I O O

O I I O

Draw the

Chair

I I I O

O O O I

I O O I

O I O I

I I O I

O O I I

I I O I

O I I I

I I I I

(Q1) In a digital system, an error comparator is required which will compare two, 2-bit binary numbers, A, B and gives separate outputs for the conditions $A = B$, $A > B$ & $A < B$. Design a logic circuit to perform the above functions.

A_1	A_2	B_1	B_2	$A = B$	$A > B$	$A < B$
0	0	0	0	1	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	1	0	0

Isomerism is a phenomenon where one molecule has two or more different forms due to the arrangement of atoms in different ways. These different forms are called isomers. Isomers have the same chemical formula but different structural arrangements.

$E > A$ $E \wedge A$ $A \cdot A$ $A \wedge A$ A A A

O O O O O O O O

Draw the
Cramis

O I O O O X I O

O O I I O X I O

I I O O O X I O

O I I O O X O I

O I I O O X O I

O I I O O X O I

I O O I I X O I

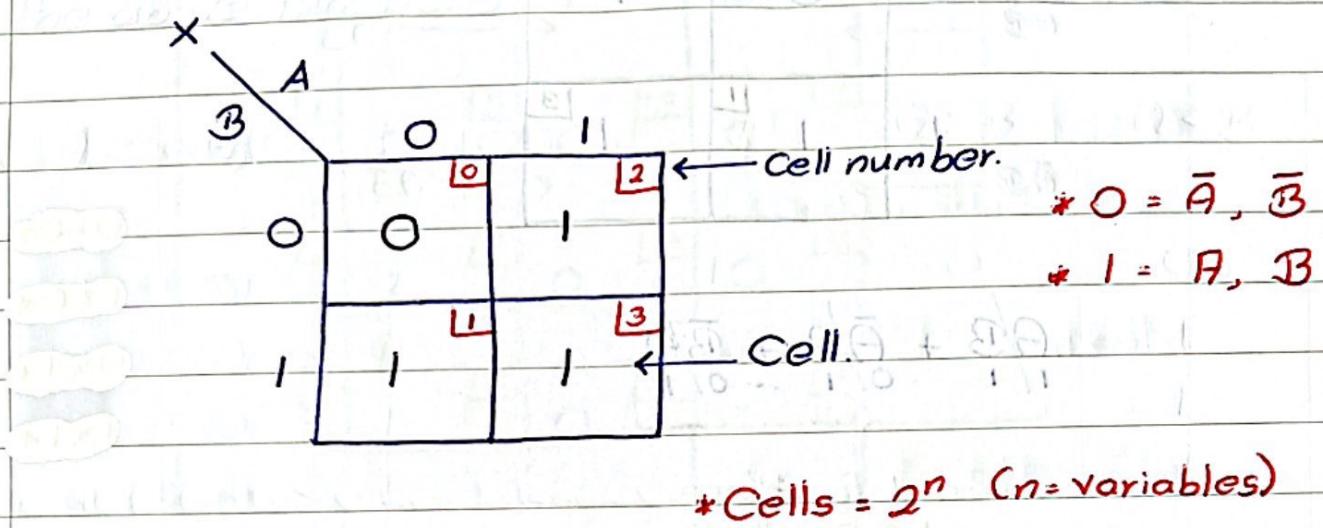
O I I O O X I I

O I I O O X I I

O I I O O X I I

Karnaugh Maps (K-Maps)

A boolean expression can be represented graphically using a K-Map



How to Find the Value.

Fill

How to Find the Cell Number.

$$\begin{array}{c}
 \begin{array}{cc} A & B \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{array} \\
 0 + 0 \rightarrow 00 \rightarrow 0 \\
 0 + 1 \rightarrow 01 \rightarrow 1 \\
 1 + 0 \rightarrow 10 \rightarrow 2 \\
 1 + 1 \rightarrow 11 \rightarrow 3
 \end{array}$$

* Only one variable changing at a time. ProMate

2 Variables.

	\bar{A}	A	
\bar{B}	0	0	1
B	1	1	1
	$\bar{A}\bar{B}$	$\bar{A}B$	AB

$$\bar{A}\bar{B} + \bar{A}B + AB$$

Put 1 for these
and 0 for all others.

3 Variables.

	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	
\bar{C}	000	001	011	110	
C	111	010	001	101	

* 11 comes
before 10.

$$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C$$

4 Variables

X AB CD	00	01	11	10	
00	0	0	1	12	18
01	11	0	15	0	9
11	0	1	17	0	19
10	0	2	6	14	10

$$\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

due to minimum 2 inputs to reduce number of terms

Ex: $\bar{A}C + A\bar{B}$ variable cost

C AB	00	01	11	10
0	0	0	0	1
1	1	1	0	1

$A \rightarrow 0 / C \rightarrow 1$
000 000

&
 $A \rightarrow 1 / B \rightarrow 0$
000 000

$$\bar{A}C + A\bar{B}$$

variable cost
costs zero value
no cost.

With up times effect operating time

ProMate

Simplification of Boolean Expressions

Using K-Map Method.

Sub Cube.

A set of 2^n adjacent cells is referred to as a sub cube.

Maximal Sub Cube.

It is the largest possible sub cube for a cell.

Minimal Map.

Map with minimum number of maximal sub cubes that cover all the minterms of the given expression.

2^n	1	2	4	8	16	32
2^0	↑	↑	↑	↑	↑	↑
	2^1	2^2	2^3	2^4	2^5	

* Go for the maximum.

(If you can go for 4, can't go for 2)

Ex:

$$X = \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$$

X	$\bar{A}B$	00	01	11	10	
C	0	0	0	0	1	$100 \rightarrow \bar{A}\bar{B}C$
A	1	0	1	1	1	$101 \rightarrow A\bar{B}C$
B		011	111			
	$\bar{A}BC$	$A\bar{B}C$				
	$\underbrace{\bar{A}BC \quad A\bar{B}C}_{BC (000S)}$					

Therefore,

$$X = A\bar{B} + BC //$$

Ex:

Y

	AB 00	01	10
C	0	0	0
	1	0	1

$$\begin{array}{ccc}
 011 & 111 & 101 \\
 \downarrow & \downarrow & \downarrow \\
 \bar{A}BC & ABC & A\bar{B}C \\
 \overbrace{\quad\quad\quad}^{\text{BC}} & \overbrace{\quad\quad\quad}^{\text{AC}} & \overbrace{\quad\quad\quad}^{\text{AC}}
 \end{array}$$

Therefore,

$$X = BC + AC //$$

01) Y

$\bar{A}B\bar{C}$	010				
C	AB	00	01	11	10
0	0	1	0	0	
1	0	1	1	0	

$\bar{A}B + BC //$

02) Y

$\bar{A}B\bar{C}$	011				
C	AB	00	01	11	10
0	0	1	0	0	
1	0	0	1	1	

$\bar{A}B\bar{C} + AC //$

03.) Y

$\bar{A}B\bar{C}$	000				
C	AB	00	01	11	10
0	1	0	0	0	1
1	0	0	0	0	1

$\bar{B}\bar{C} + A\bar{B} //$

04.) Y

$\bar{A}B\bar{C}$	000	001	010	110	100	011	101	111	000
C	AB	00	01	11	10	01	11	10	00
0	1	1	1	1	1	1	1	1	0
1	0	0	0	0	1	0	0	0	0

$\bar{C} + A\bar{B} //$

05.)

$$\begin{array}{cc} 000 & 010 \\ \bar{A}\bar{B}\bar{C} & \bar{A}\bar{B}\bar{C} \end{array}$$

	$\bar{A}B$	C	00	01	11	10
0	1	Y1	1	0	0	0
1	1	1	1	1	0	0
	$\bar{A}\bar{B}C$		001	$\bar{A}\bar{B}C$		

$$\bar{A} + BC //$$

06.)

$$\begin{array}{cc} \bar{A}\bar{B}C & \bar{A}BC \\ 011 & 111 \end{array}$$

	$\bar{A}B$	CD	00	01	011	110
00	0	0	0	0	0	0
01	0	0	0	1	1	0
	$\bar{A}\bar{B}CD$		0011	$\bar{A}\bar{B}CD$		
11	1	0	0	0	1	0
	$\bar{A}\bar{B}C\bar{D}$		10	1	0	0
0010	1	0	0	0	1	0

$$\begin{array}{l} \bar{A}BCD \\ 1101 \end{array}$$

$$\begin{array}{l} \bar{A}\bar{B}CD \\ 1001 \end{array}$$

$$\begin{array}{l} \bar{A}\bar{B}CD \\ 1011 \end{array}$$

$$\begin{array}{l} \bar{A}\bar{B}CD \\ 1010 \end{array}$$

$$A\bar{C}D + \bar{B}C //$$

07.)

	$\bar{A}B$	CD	00	01	11	10
00	0	0	0	0	0	0
01	0	0	1	1	1	1
11	1	1	1	1	1	1
10	1	1	1	1	1	1

$$\begin{array}{l} \bar{A}\bar{B}CD \\ 1101 \end{array}$$

$$\begin{array}{l} A\bar{B}CD \\ 1001 \end{array}$$

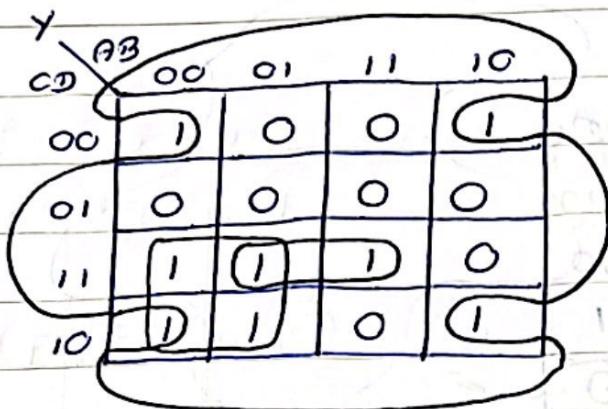
$$\begin{array}{l} A\bar{B}CD \\ 1111 \end{array}$$

$$\begin{array}{l} \bar{A}\bar{B}CD \\ 1011 \end{array}$$

$$C + AD //$$

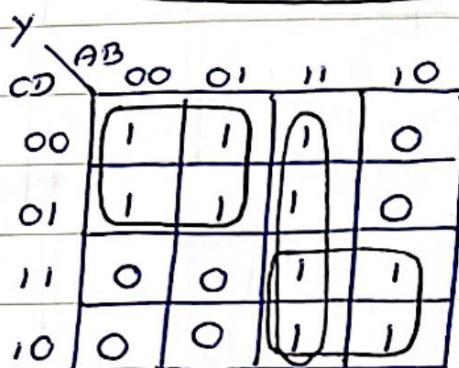
$$\begin{array}{cccc} \bar{A}\bar{B}CD & \bar{A}BC\bar{D} & A\bar{B}CD & \bar{A}\bar{B}CD \\ 0010 & 0110 & 1110 & 1010 \end{array}$$

08)



$$\bar{A}C + \bar{B}CD + \bar{B}\bar{D}$$

09)



$$\bar{A}\bar{C} + A\bar{B} + AC$$

Getting the Simplified POS expression.

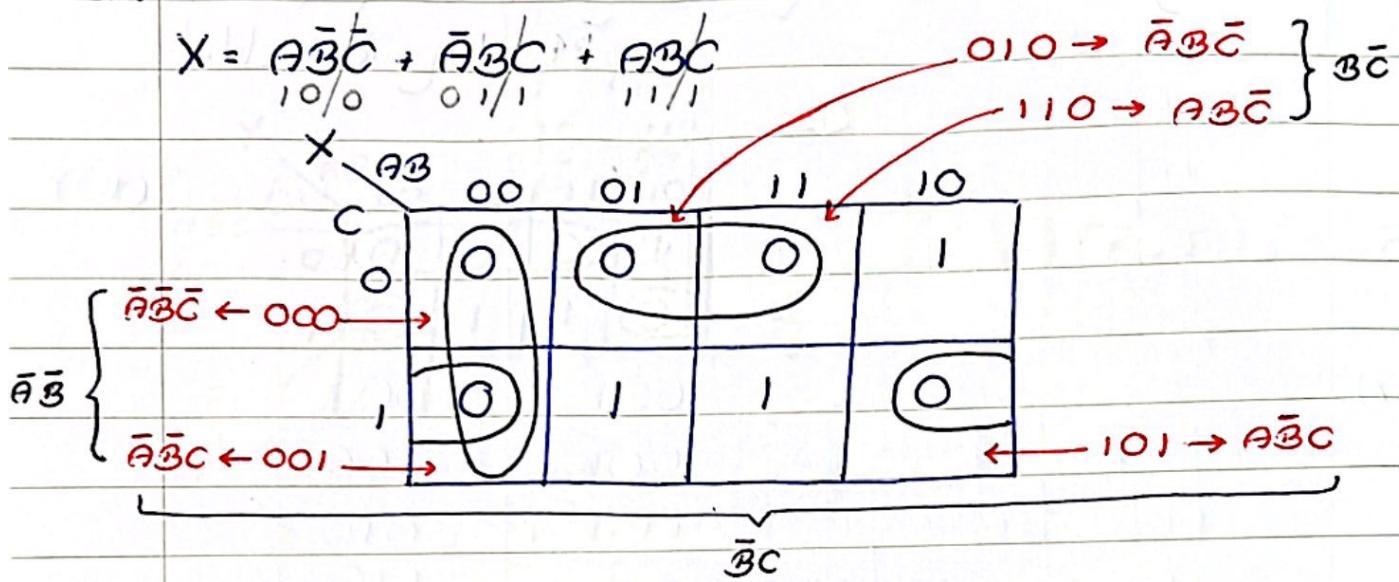
01) Draw the K-Map for X.

02) Get the minimal map for \bar{X} . (Cover all 0's)

03) Get \bar{X} in SOP form.

04) Complement both sides.

Ex:



SOP expression;

$$\bar{X} = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$$

$$\bar{\bar{X}} = \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}}$$

$$X = \overline{\bar{A}\bar{B}} \cdot \overline{\bar{B}\bar{C}} \cdot \overline{\bar{A}\bar{C}}$$

POS expression;

$$X = (A+B)(C\bar{B}+C)(B+\bar{C})$$

(01)

	AB	00	01	11	10
C	0	1	1	0	0
	1	1	1	1	0

 $110 \rightarrow A\bar{B}\bar{C}$ $100 \rightarrow A\bar{B}C$ $101 \rightarrow A\bar{B}C$ $A\bar{C}$ $A\bar{B}$

SOP expression:

$$\bar{X} = A\bar{C} + A\bar{B}$$

001 000

001 100

110 010

110 110

100 001

111 101

011 011

101 111

POS expression:



(02)

	AB	00	01	11	10
C	00	0	0	0	0
	000	01	0	1	0
0000 - 01000	000	01	0	1	0
0100 - 00100	000	01	0	1	0
1100 - 00010	000	01	0	1	0
1000 - 00001	000	01	0	1	0
1001 - 01000	000	01	0	1	0
1101 - 00010	000	01	0	1	0
1111 - 00001	000	01	0	1	0

0100 - 00100

1000 - 00010

1001 - 01000

1101 - 00010

SOP expression



POS expression

01	11	10	00
01	11	10	00
10	11	10	00
10	11	10	00

ProMate

Simplify X, Y, Z given by the following truth table using K-Maps.

(Q1.)

ABC	XYZ
000	100
001	100
010	011
011	011
100	001
101	111
110	110
111	101

01	01	10	00	1
0	1	1	1	1

noice2019x0902

$$\bar{A}A + \bar{B}B = \bar{X}$$

noice2019x0209

* X

AB		00	01	11	10
C	0	0	0	1	0
	1	1	0	1	1

01	01	10	00	1
0	1	1	1	0

(Q2)

* Y

AB		00	01	11	10
C	0	0	1	1	0
	1	0	1	0	1

noice2019x0902

ProMate

* Z

	AB	00	01	11	10
C	0	0	0	0	0
	1	0	1	1	1

01 11 10 00 / 03

00
10
11
01

(02.)

ABCD	XZY
0000	100
0001	001
0010	001
0011	001
0100	010
0101	100
0110	001
0111	001
1000	010
1001	010
1010	100
1011	001
1100	010
1101	010
1110	010
1111	100

01 11 10 00 / 03

00
10
11
01

* X

		AB	00	01	11	10
		00				
		01				
		11				
		10				

01	11	10	00	11
00	10	01	11	00
11	01	10	00	11
10	00	01	11	00
00	11	10	01	11

* Y

		AB	00	01	11	10
		00				
		01				
		11				
		10				

SKX 50000

001	0000
100	1000
100	0100
100	1100

* Z

		AB	00	01	11	10
		00				
		01				
		11				
		10				

010	0010
001	1010
100	0110
100	1110
010	0001
010	1001

001 1010

100 1101

010 0011

010 1011

010 0111

001 0111

Incompletely Specified Boolean Functions. (Don't Cares) $\rightarrow X$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	X
1	1	0	0
1	1	1	1

} Don't Cares

The value of the function at "don't cares" conditions, is either not significant or those A, B, C values will never occur. Therefore, don't cares can be either included or excluded in a maximal subcube so as to get the minimal map.

included in a maximal subcube (to make $2^n = 2^2 = 4$)

	Y	C	AB	00	01	11	10
0				1	X	X	0
1				1	1	0	0

excluded from a maximal subcube.

Ex:

Z position of output based on input assignment

		AB		CD		Z			
		00	01	11	10	00	01	11	10
00		1	0	0	X				
01		1	X	0	0				
11		X	0	1	X				
10		1	0	0	1				

$$Z = \bar{A}\bar{B} + \bar{B}C + ACD$$

position of output

don't care

recall POS - ②

How to find SOP

Ex:

 Z

		AB		CD		Z			
		00	01	11	10	00	01	11	10
00		1	0	0	X				
01		1	X	0	0				
11		X	0	1	X				
10		1	0	0	1				

$$(A+\bar{C}).(\bar{A}+B).(\bar{B}+C+\bar{D})$$

all together

gray term in

(01)

		AB		CD		Z			
		00	01	11	10	00	01	11	10
00		0	1	X	0				
01		0	1	X	0				
11		0	0	X	0				
10		1	1	X	1				

$$X \bar{Z} = X\bar{B}\bar{C} + CD$$

$$X Z = \bar{B}\bar{C} + CD$$

$$= \bar{B}\bar{C} + CD$$

$$= (B+C).(\bar{C}+\bar{D})$$

Miniterms (Σ) & Maxterms (Π)

Ex:

$$01) Y = (A+B+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+\bar{C})$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

		Y	AB	C	00	01	11	10
					0	1	1	1
					1	1	0	1
0	0				0	1	1	1
1	1				1	0	0	0

$$02) Y = (A+B+C) \cdot (\bar{A}+\bar{B}+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$

$$Y = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$

		Y	AB	C	00	01	11	10
					0	1	0	1
					1	1	1	0
0	0				0	1	0	1
1	1				1	1	1	0

Ex: $Z = f(A, B, C, D)$

$Z = 1$ for the minterms. (0, 2, 5, 7, 8, 13)

$Z = \text{Don't care}$ for the minterms. (10, 15)

$Z = 0$ for the remaining minterms.

a) Draw the truth table.

b) Simplify Z using K-Map method.

c) Draw the circuit diagram.

a)	A	B	C	D	Z
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	X
11	1	0	1	1	0
12	1	1	0	0	0

13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	X

b)

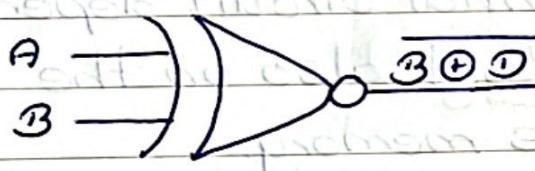
 Z

CD	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	X	0
10	1	0	0	X

$$Z = \overline{B}D + \overline{B}\overline{D}$$

$$= \overline{B} + D$$

c)

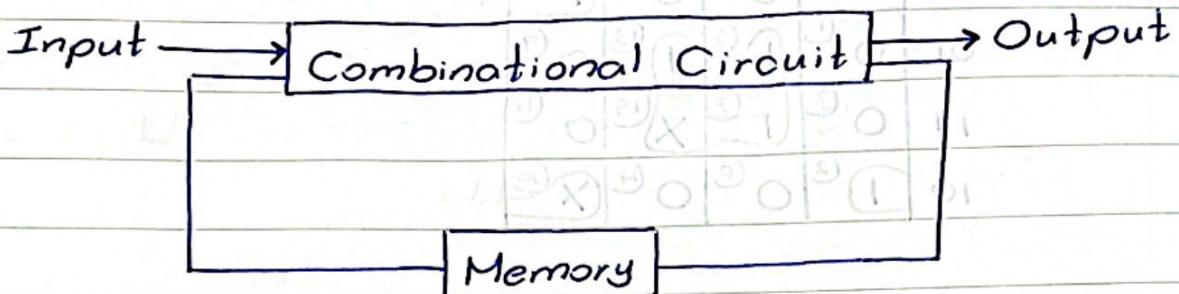


Combinational Circuits

Input \rightarrow Combinational Circuit \rightarrow Output

A combinational circuit generally consists of several input signals, several output signals and an interconnection of gates. When applying the same inputs, a combinational circuit **always produces the same outputs**. i.e every input has a single, unique output.

Sequential Logic Circuits



Sequential circuits are built out of **memory elements** and **combinational logic devices**.

The outputs of a sequential circuit depends on not only the inputs, but also on the current contents of its memory.

Therefore, these circuits produce different outputs for the same set of inputs depending on what is stored in memory.

Sequential Logic Circuits

Synchronous Asynchronous

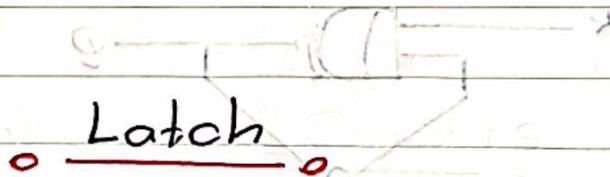
01.) Synchronous Sequential Logic Circuits.

Various blocks of the logic circuit are triggered by the same clock signal, called the master clock. Therefore, the output of these blocks (devices) change at the same time.

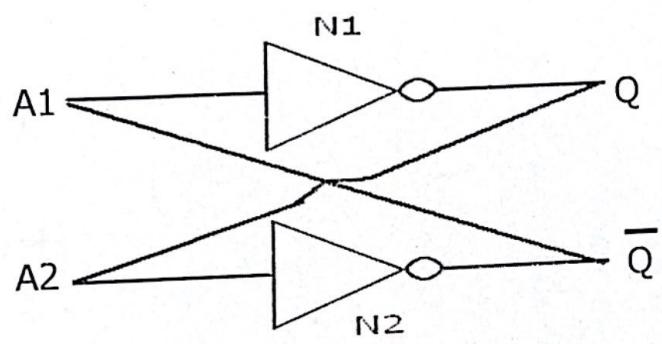
02.) Asynchronous Sequential Logic Circuits.

In this types of circuits, different parts of the circuit are triggered by different signals.

Therefore, the outputs of different blocks of the circuit can change at different instances.

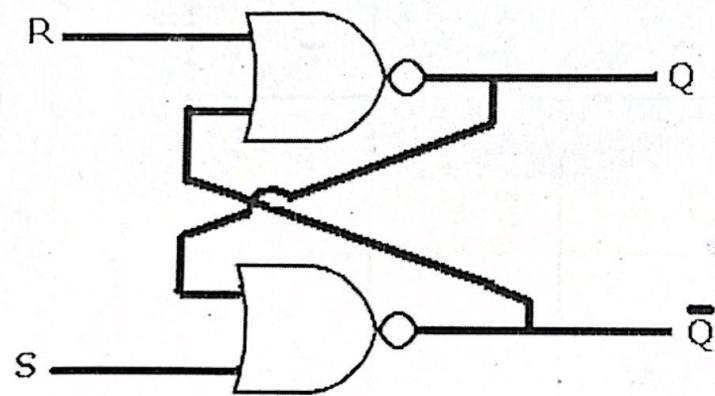


- * A latch is a data storage (memory) device that can store one bit of information.
- * The basic digital memory circuit is obtained by cross-coupling two NOT gates.



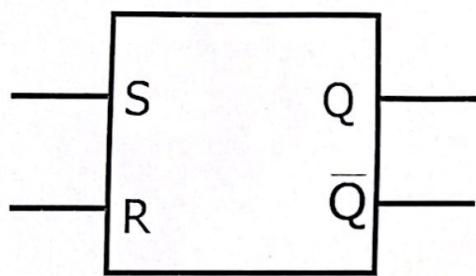
The output of each gate is connected to the input of the other and this feedback combination is called a latch.

SR Latch (Set Reset Latch)



SR Latch (Set Reset Latch)

Symbol:



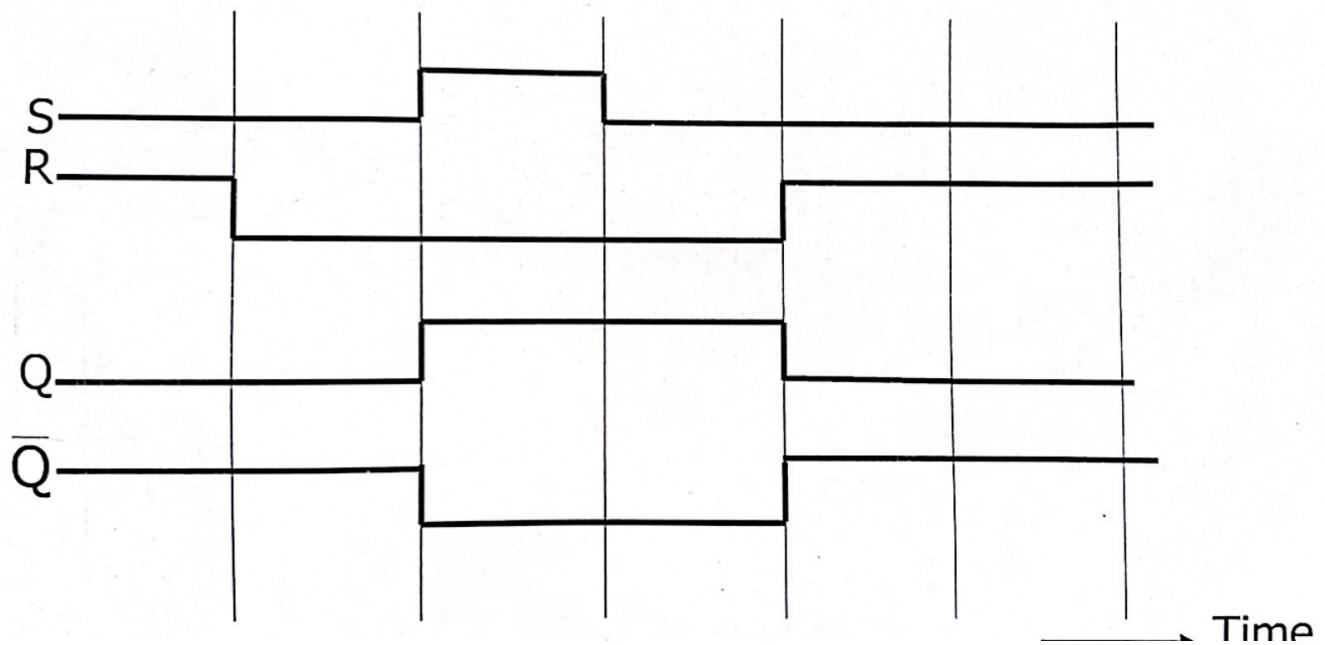
Truth table:

S	R	Q_n	\bar{Q}_n	Q_{n+1}	\bar{Q}_{n+1}
0	0	0	1	0	1
0	1	1	0	1	0
1	0	0	1	0	1
1	1	0	1	1	0
		1	0	Not allowed	

Reduced Truth Table:

	S	R	Q_{n+1}
Hold state	0	0	Q_n
Reset state	0	1	0
Set state	1	0	1
Not allowed state	1	1	NA

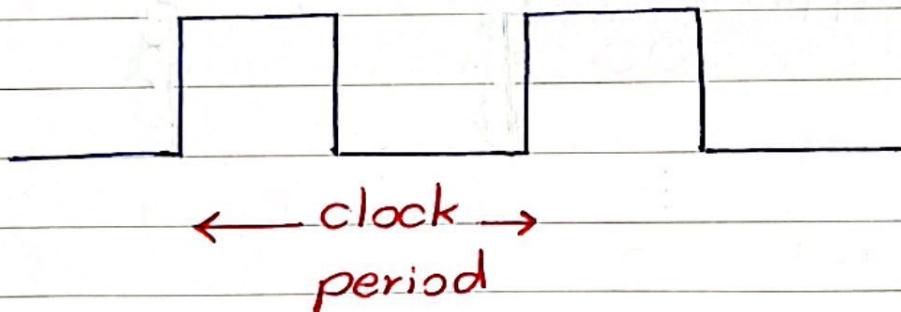
Draw the timing diagram showing the operation of a SR Latch:



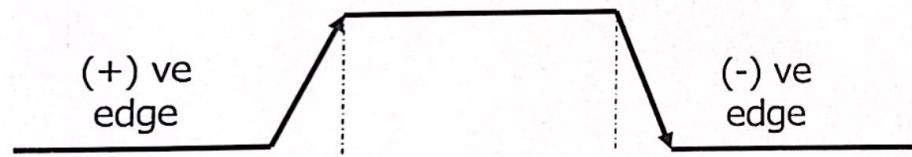
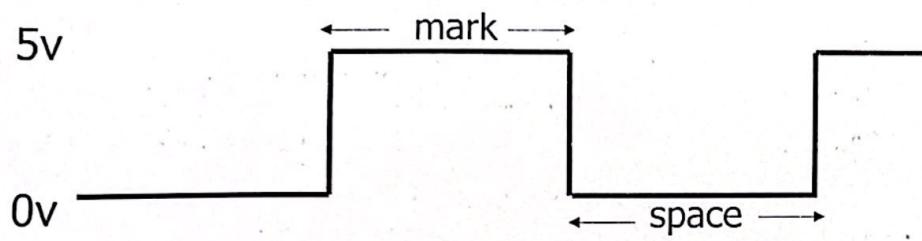
Flip Flops (Triggered Latch)

Clock

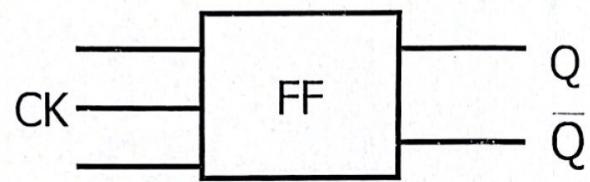
A clock is a special device that continuously outputs 0's and 1's. The time it takes for the clock to change from 1 to 0 and back to 1 is called a **clock period** or **clock cycle**.



Types of triggering

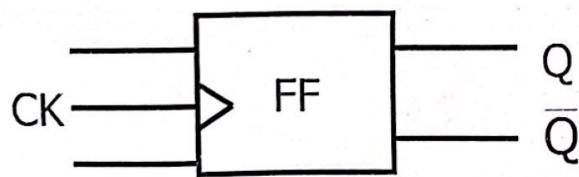


Level triggering:

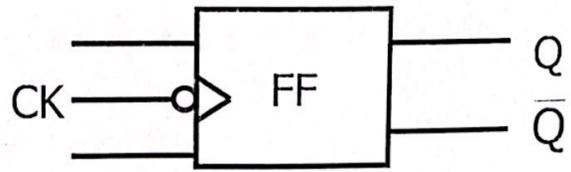


Activated by the
mark

Edge triggering:



Activated by the
(+)ve edge



Activated by the
(-)ve edge

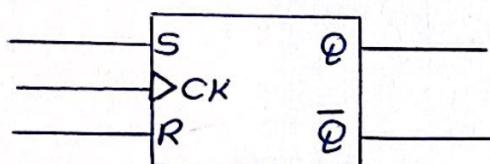
SR (Set Reset) Flip-Flop

Truth table:

CK	S	R	Q_{n+1}
0	0	0	Q_n
0	0	1	Q_n
0	1	0	Q_n
0	1	1	Q_n
1	0	0	Q_n
1	0	1	0
1	1	0	1
1	1	1	NA

When CK is low $Q_{n+1} = Q_n$

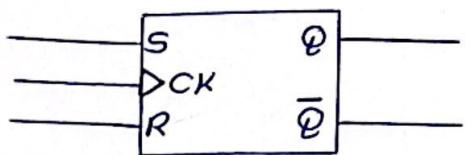
*Symbol:



0	1	0	Q_n
0	1	1	Q_n
1	0	0	Q_n
1	0	1	0
1	1	0	1
1	1	1	NA

low $Q_{n+1} = Q_n$

*Symbol:

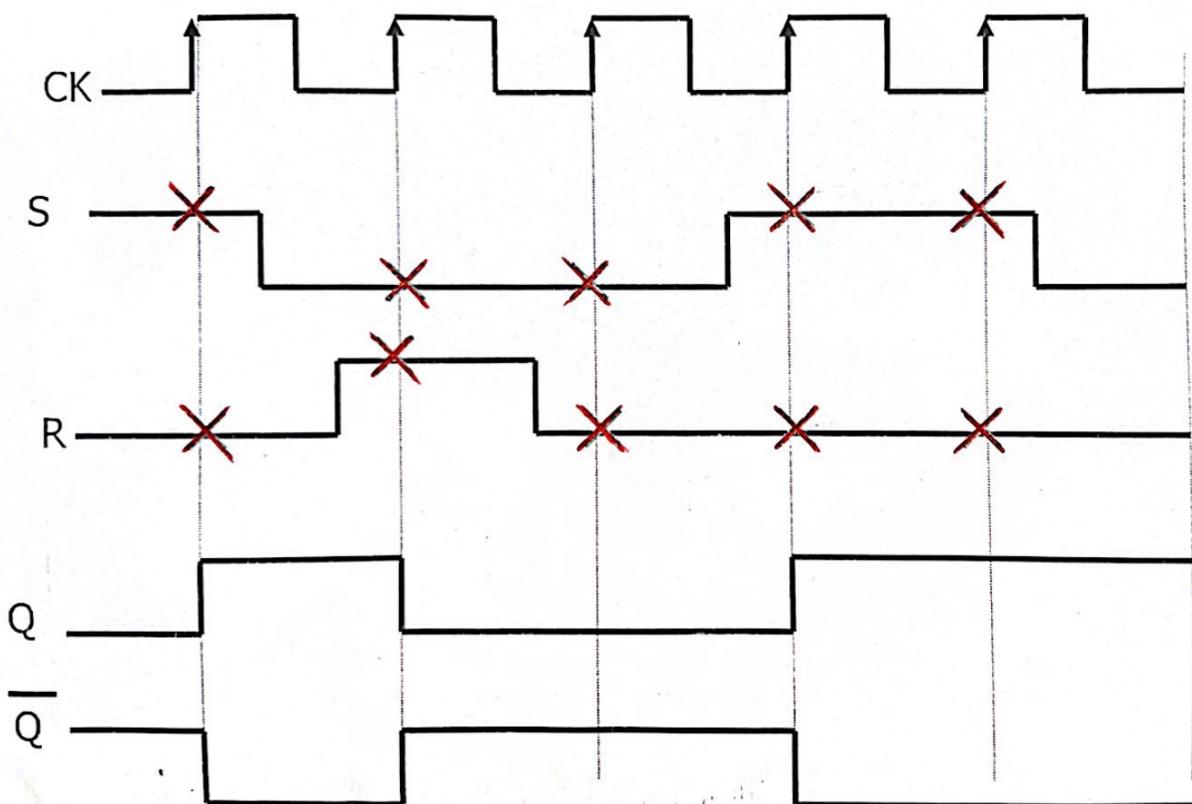


Complete Truth Table of a SR FF

When CK = 1

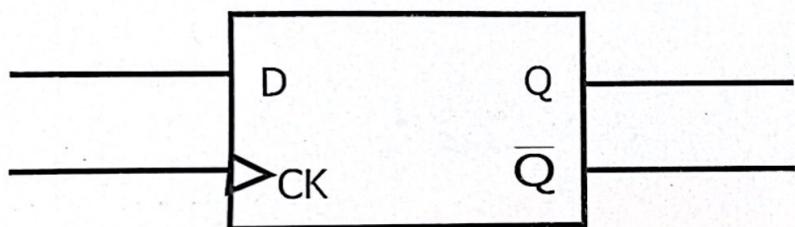
S	R	Q_n	Q_{n+1}
0	0	0	0
0	1	1	1
0	1	0	0
1	0	1	1
1	1	0	NA

Draw the output waveform for the (+)ve edge triggered SR FF.



D Flip-Flop(Data FF)

Symbol:



Reduced TT:

CK	D	Q_{n+1}
0	0	Q_n
0	1	Q_n
1	0	0
1	1	1

When CK is low
 $Q_{n+1} = Q_n$

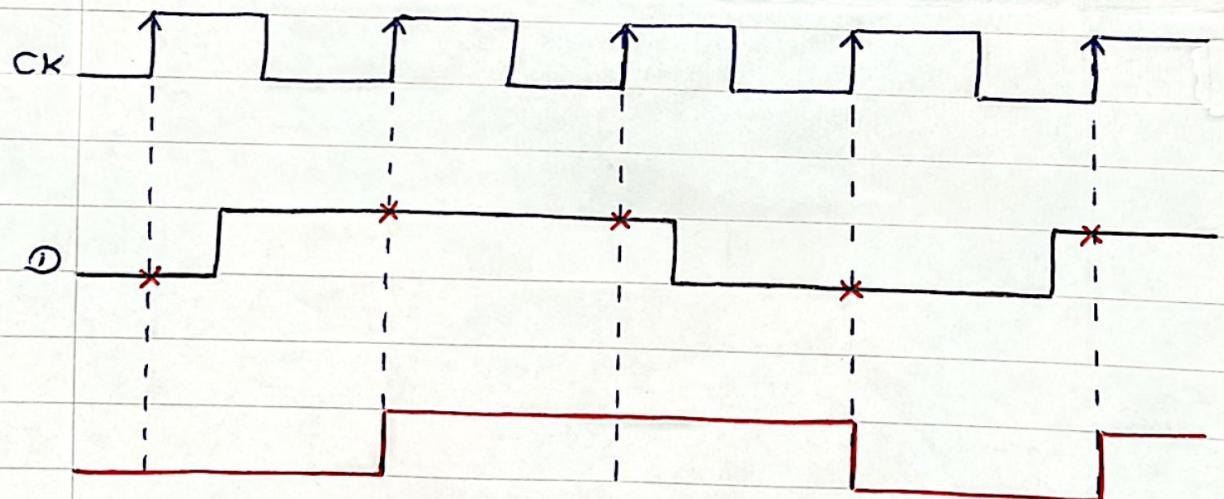
Complete Truth Table of a D FF

When CK = 1

D	Q_n	Q_{n+1}
0	0	0
	1	0
1	0	1
	1	1

Date

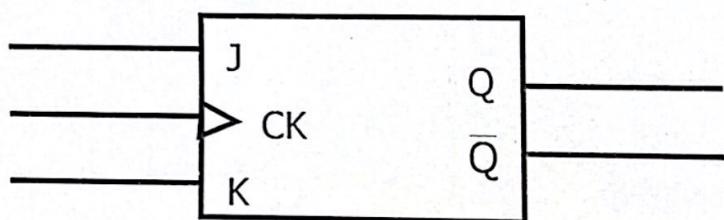
No



PROMATE™
World Stationer

JK flip-Flop

Symbol:



Reduced TT:

When CK=1

J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	\bar{Q}_n

When CK = 0
 $Q_{n+1} = Q_n$

Complete Truth Table of a JK FF

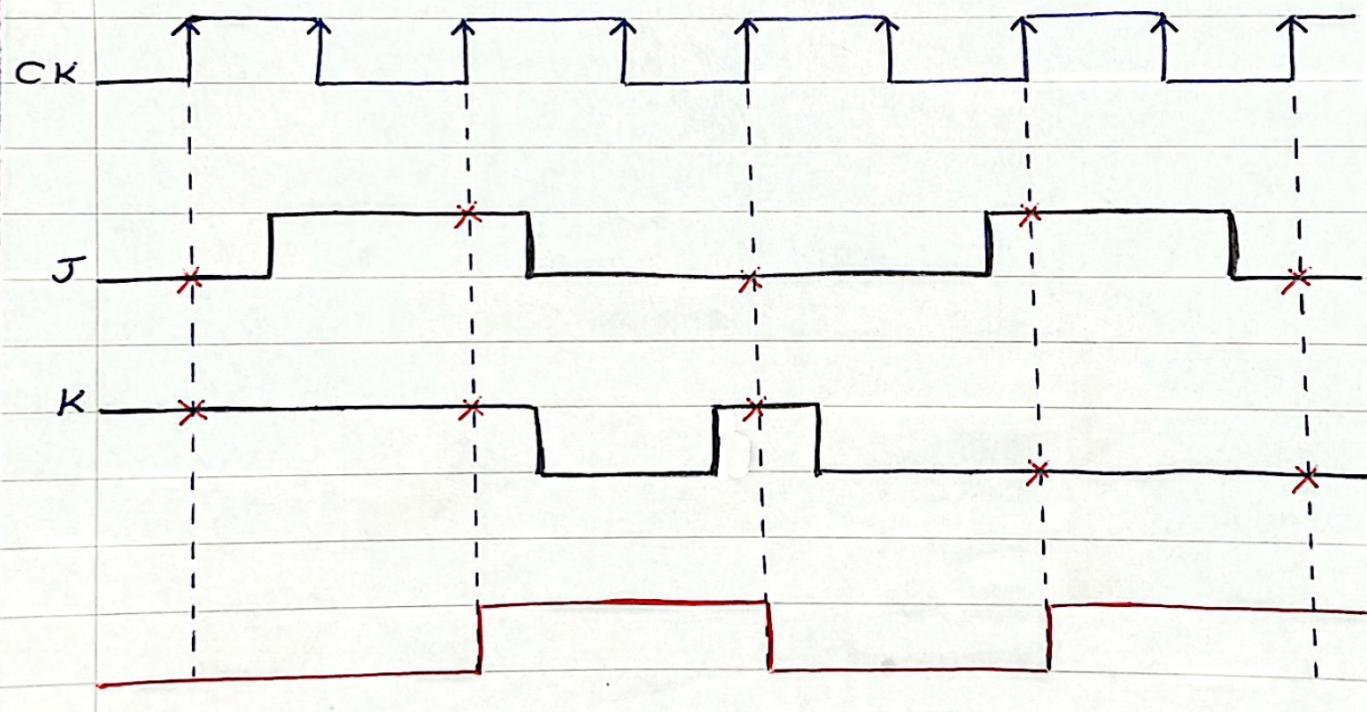
When CK =1

J	K	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Toggle Flip-flop(T-FF)

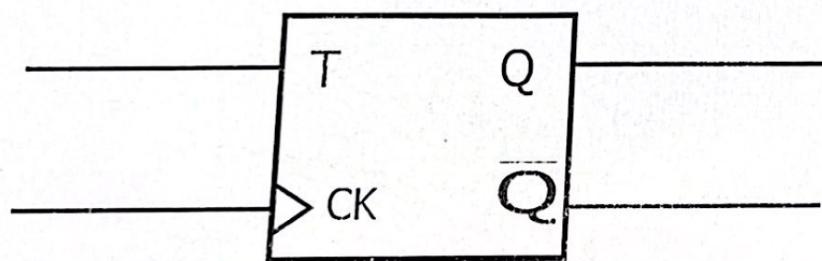
Symbol:

ProMate™
world stationer



Toggle Flip-flop(T-FF)

Symbol:



Reduced TT

When CK=1 {

T	Q_{n+1}
0	Q_n
1	\bar{Q}_n

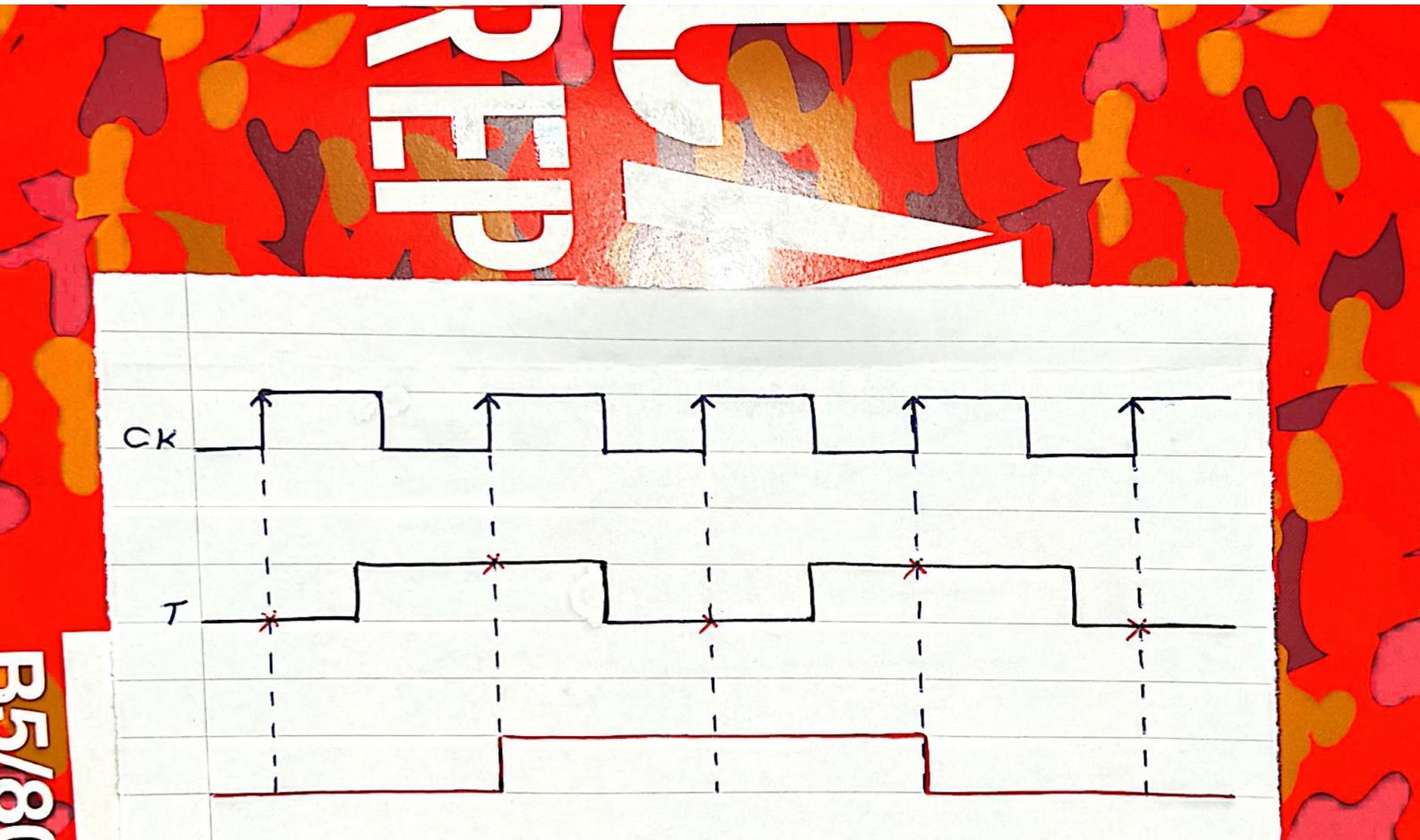
When CK = 0
 $Q_{n+1} = Q_n$

Complete Truth Table of a T FF

When CK = 1

T	Q_n	Q_{n+1}
0	0	0
	1	1
1	0	1
	1	0

R5/8



SR Flip-flop

Truth table:

S	R	Q _n	Q _{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	NA
1	1	1	NA

Excitation Table:

Q _n	Q _{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

JK Flip Flop

Truth Table:

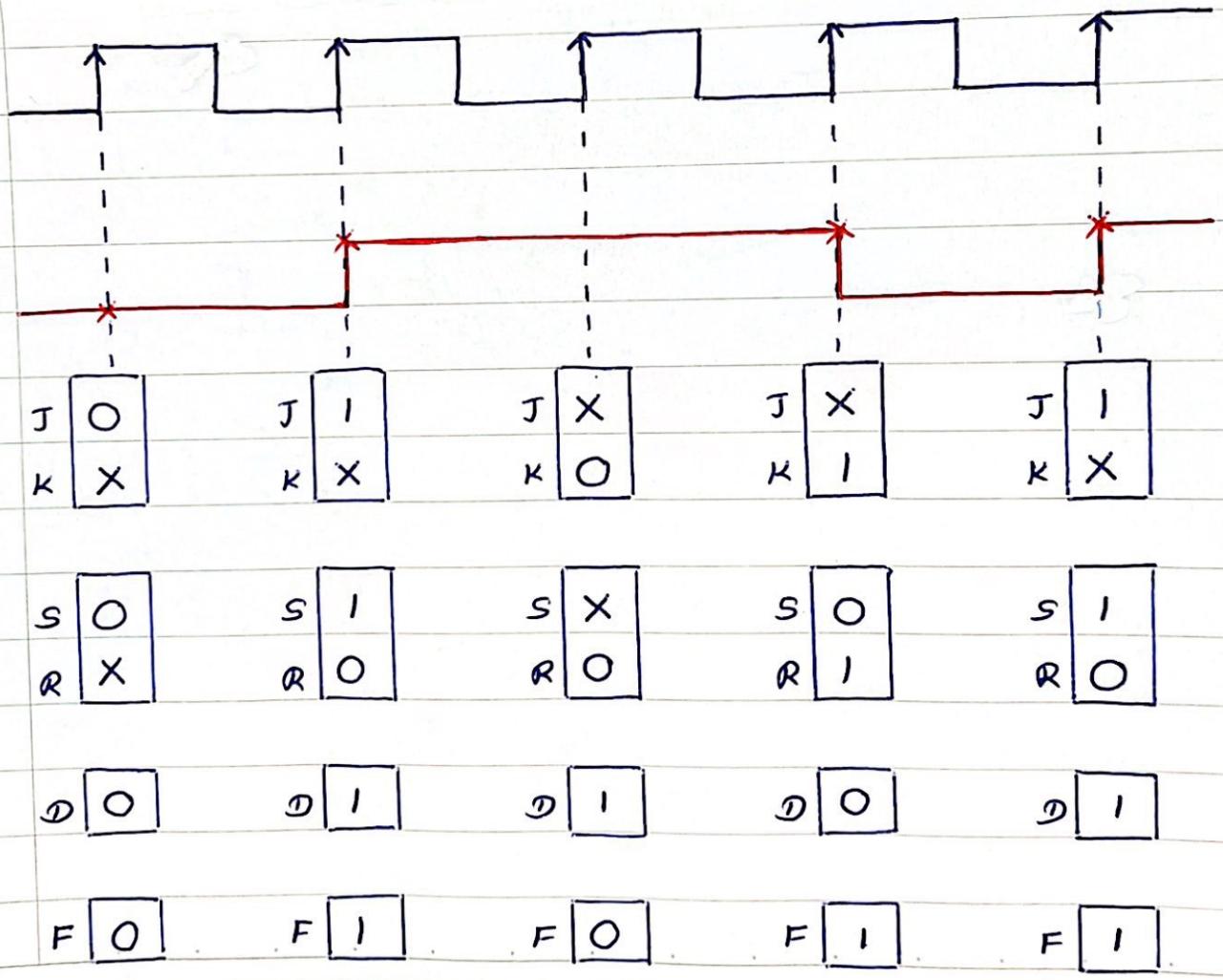
J	K	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Excitation Table:

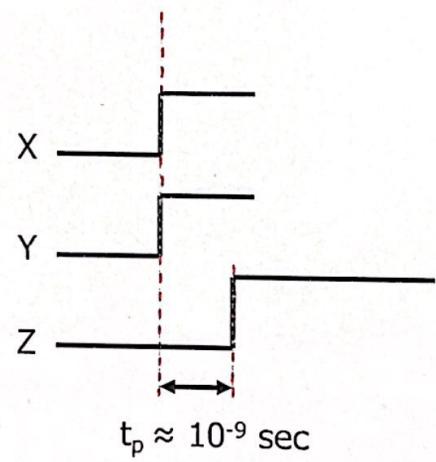
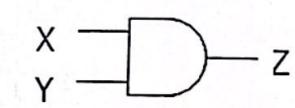
Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation tables:

Q_n	Q_{n+1}	SR	D	JK	T
0	0	0X	0	0X	0
0	1	10	1	1X	1
1	0	01	0	X1	1
1	1	X0	1	X0	0



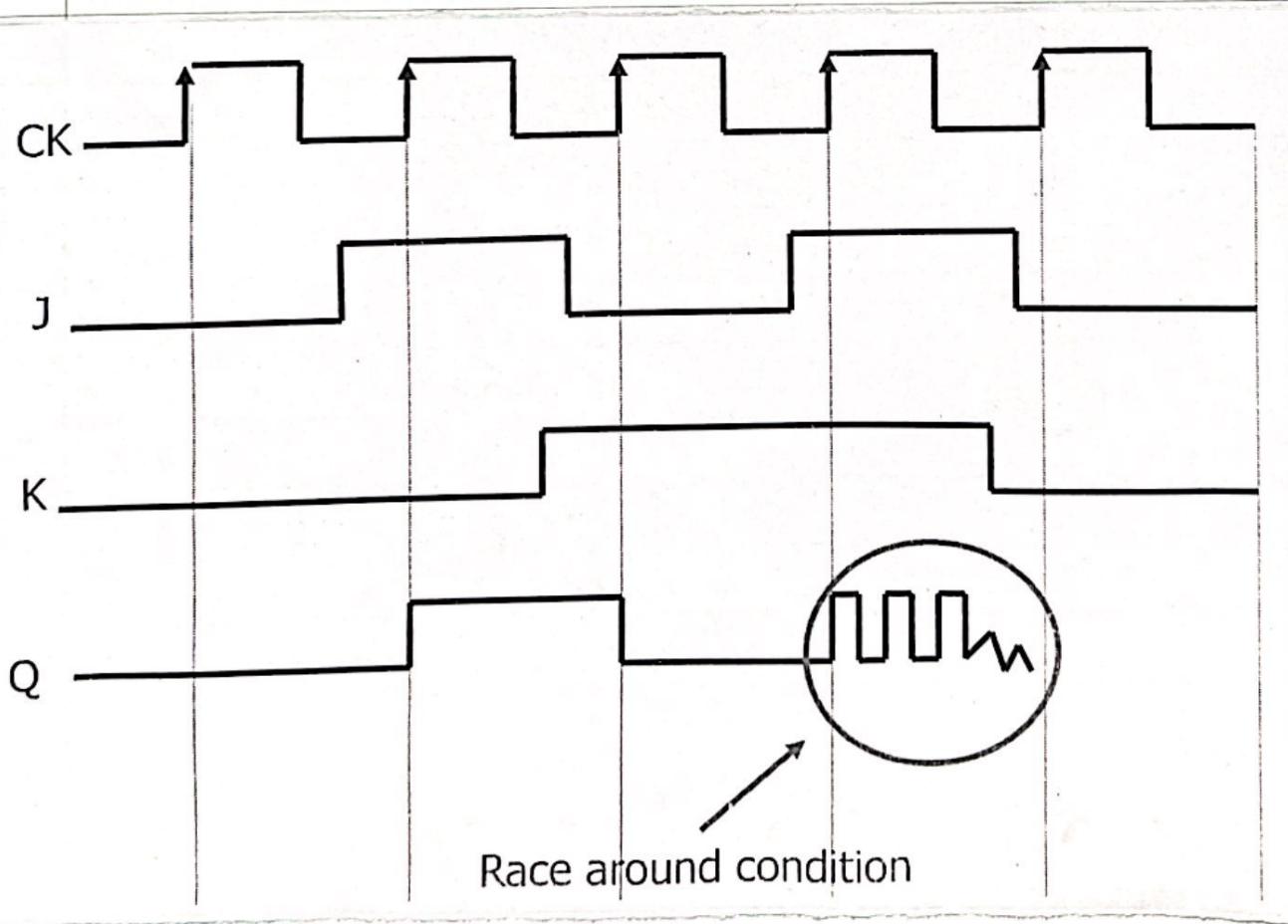
Propagation Delay



Race Around Condition

When, $J = K = 1$ and the clock is enabled, the output Q_{n+1} of a JK FF is \bar{Q}_n .

Therefore, the output oscillates at a frequency determined by the propagation delay, till the clock is disabled. Therefore, at the end of the clock pulse, the value of the output cannot be determined. The output can be either a 0 or a 1, depending on the **clock duration** and the **frequency of oscillation**.



Draw the output waveform for a:

- (a) (+)ve edge triggered T FF
- (b) (-)ve edge triggered T-FF
- (c) C(+) & C(-) ve edge triggered T-FF

