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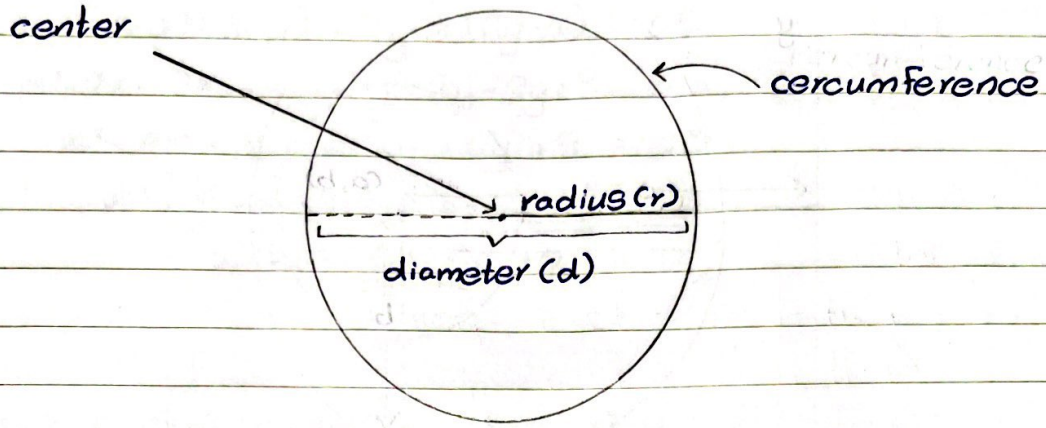
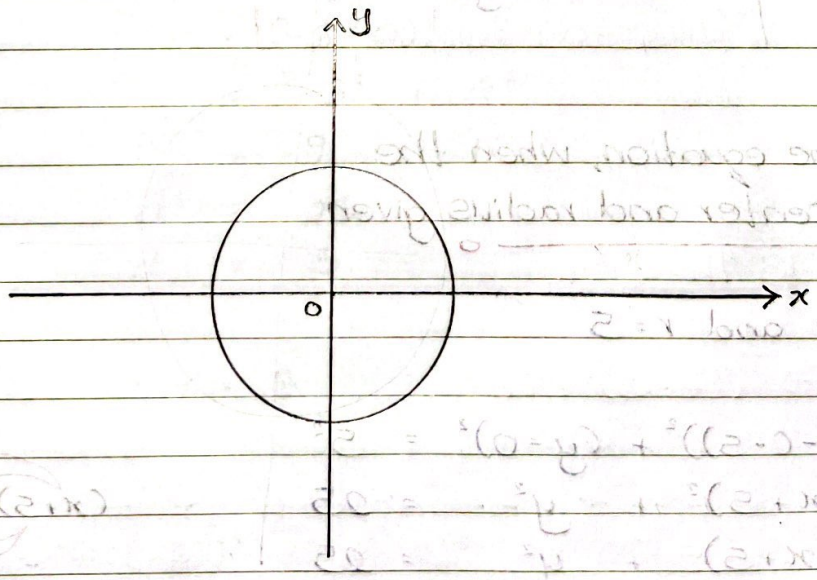
Circles



Standard Equation of a Circle.

a) When the center point is at Origin.



$$x^2 + y^2 = r^2$$

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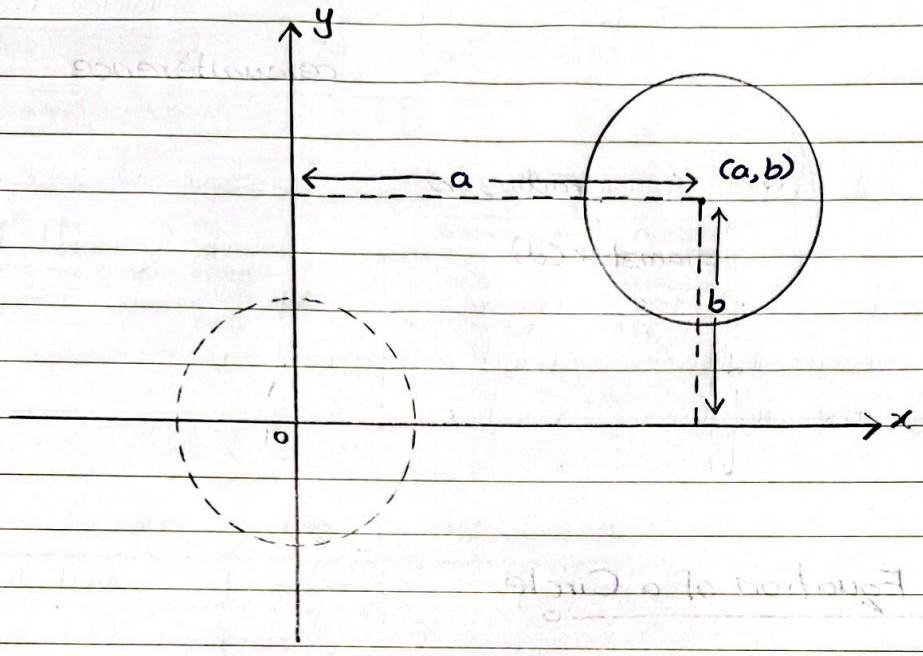
b) When the circles not at the Origin.



$$(x-a)^2 + (y-b)^2 = r^2$$

Writing the equation, when the center and radius given.

(01.) C = (-5, 0) and r = 5

$$(x-(-5))^2 + (y-0)^2 = 5^2$$
$$(x+5)^2 + y^2 = 25$$
$$(x+5)(x+5) + y^2 = 25$$
$$x^2 + x + 5x + 25 + y^2 = 25$$
$$x^2 + 10x + 25 + y^2 = 25$$
$$x^2 + y^2 + 10x = 0 //$$

$(x+5)(x+5)$

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(02.) C = (2, 3) and r = 5

$$(x-2)^2 + (y-3)^2 = 5^2$$
$$(x-2)(x-2) + (y-3)(y-3) = 25$$
$$x^2-2x-2x+4 + y^2-3y-3y+9 = 25$$
$$x^2-4x+4 + y^2-6y+9 = 25$$
$$x^2+y^2-4x-6y+13 = 25$$
$$x^2+y^2-4x-6y-12 = 0 \;//$$

Writing the equation, when the center and a point on the circle given.

(01.) C = (-3, 0) and point A (0, -4).

distance between two points,

$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$= (-3-0)^2 + (0-(-4))^2$$
$$= -3^2 + 4^2$$
$$= 9 + 16$$
$$d^2 = 25$$
$$d = 5$$

$\therefore r = 5$

$$(x-(-3))^2 + (y-0)^2 = 5^2$$
$$(x+3)^2 + y^2 = 25$$
$$(x+3)(x+3) + y^2 = 25$$
$$x^2+3x+3x+9 + y^2 = 25$$
$$x^2+6x+9 + y^2 = 25$$
$$x^2+y^2+6x-16 = 0 \;//$$

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Checking whether a given point is Out, In or On the Circle.

Once we substitute the given point in the left hand side of the equation,

* if it is = to r^2, point is on the circle.
* if it is < to r^2, point is inside the circle.
* if it is > to r^2, point is outside the circle.

(01.) $(x-13)^2 + (y-6)^2 = 5^2$

$(x-13)^2 + (y-6)^2 = 25$

a) $(5,6) \rightarrow = (5-13)^2 + (6-6)^2$

$= 8^2 + 0^2$

$= 64$

$64 > 25$

$\therefore$ Outside the circle. //

b) $(14,8) \rightarrow = (14-13)^2 + (8-6)^2$

$= 1^2 + 2^2$

$= 1 + 4$

$= 5$

$5 < 25$

$\therefore$ inside the circle. //

c) $(16,2) \rightarrow = (16-13)^2 + (2-6)^2$

$= 3^2 + (-4)^2$

$= 9 + 16$

$= 25$

$25 = 25$

$\therefore$ On the circle

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General Equation of a Circle.

When g, f, c are constants, the general equation of a circle is given by,

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Center = $(-g, -f)$

Radius = $\sqrt{g^2 + f^2 - c}$

(01.) $x^2 + y^2 + \underbrace{2gx}_{\downarrow} + \underbrace{2fy}_{\downarrow} + c = 0$

$x^2 + y^2 - 2x - 4y + 1 = 0$

$x^2 + y^2 + 2\underbrace{(-1)}_{g}x + 2\underbrace{(-2)}_{f}y + 1 = 0$

Center = $(-g, -f)$

$= (1, 2)$

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + (-2)^2 - 1}$

$= \sqrt{1 + 4 - 1}$

$= \sqrt{4}$

$= 2$ //

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Finding the equation of a circle given 3 points on the circle.

(01.) $x^2 + y^2 + 2gx + 2fy + c = 0$

$(0,0) \rightarrow 0 + 0 + 0 + 0 + c = 0 \rightarrow c = 0$

$(c=0)$

$(1,0) \rightarrow 1 + 0 + 2g + 0 + 0 = 0$

$2g = -1$

$g = -1/2$

$c=0$ ලබාගැනීම

$(c=0)$

$(0,1) \rightarrow 0 + 1 + 0 + 2f + 0 = 0$

$2f = -1$

$f = -1/2$

$x^2 + y^2 + 2x - \frac{1}{2}x + 2x - \frac{1}{2}y = 0$

$x^2 + y^2 - x - y = 0$ //

(02.) $x^2 + y^2 + 2gx + 2fy + c = 0$

$(4,2) \rightarrow 16 + 4 + (2 \times 4x) + (2 \times 2y) + c = 0$

$20 + 8x + 4y + c = 0$

$8g + 4f + c = -20$ —— (1)

$(2,0) \rightarrow 4 + 0 + 4g + 0 + c = 0$

$4g + c = -4$ —— (2)

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$(0,2) \rightarrow 0 + 4 + 0 + 4f + c = 0$

$4f + c = -4$ ——— ③

③, ① හි ආදේශය,

$8g - 4 = -20$

$8g = -16$

$g = -2$

$g = -2$, ② හි ආදේශය,

$4 \times (-2) + c = -4$

$-8 + c = -4$

$c = 4$

$c = 4$, ③ හි ආදේශය,

$4f + 4 = -4$

$4f = -8$

$f = -2$

$$x^2 + y^2 - 4x - 4y + 4 = 0 //$$

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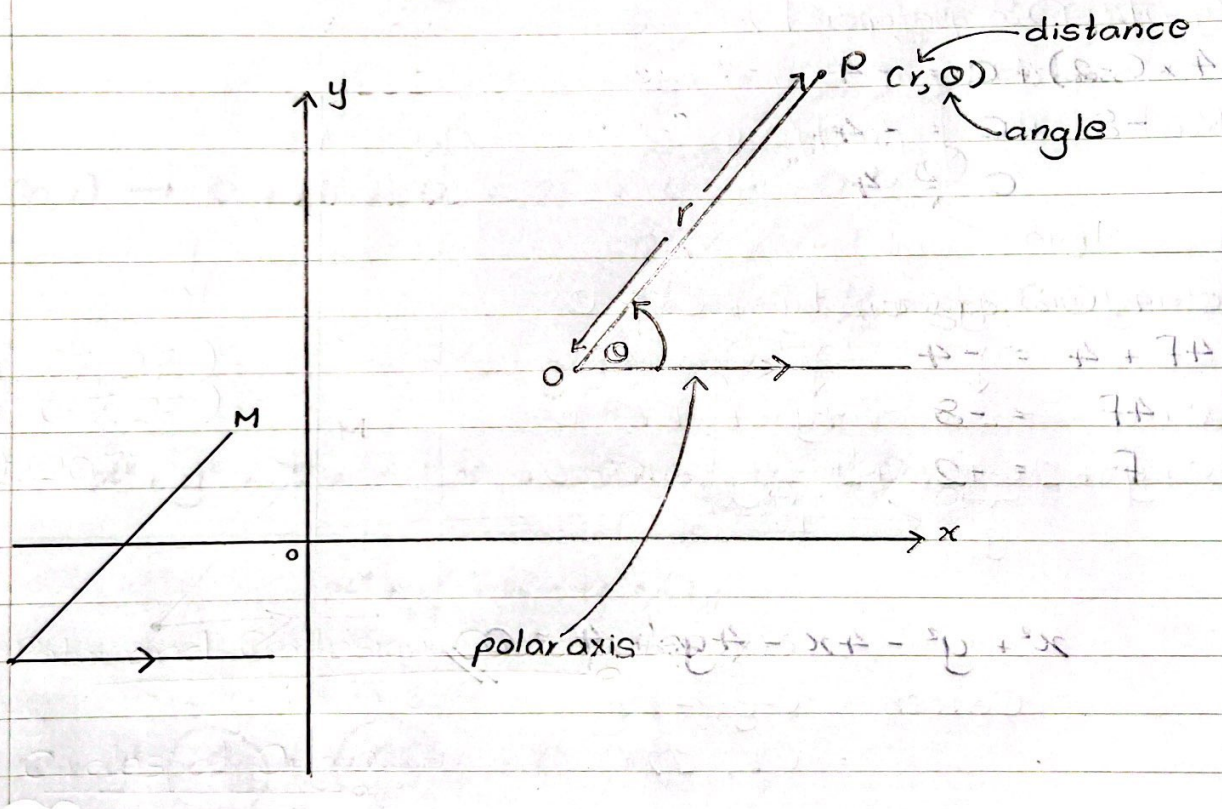
Polar Coordinates.

We use polar coordinates to represent a point on a circle using its radius and an angle.

If P is any other point in the plane, let;

* r be the distance from O to P.
* θ be the angle (usually measured in radians) between the polar axis and the line OP.



* P is represented by the ordered pair (r, θ).
* r, θ are called polar coordinates of P.

Measuring the Angle.

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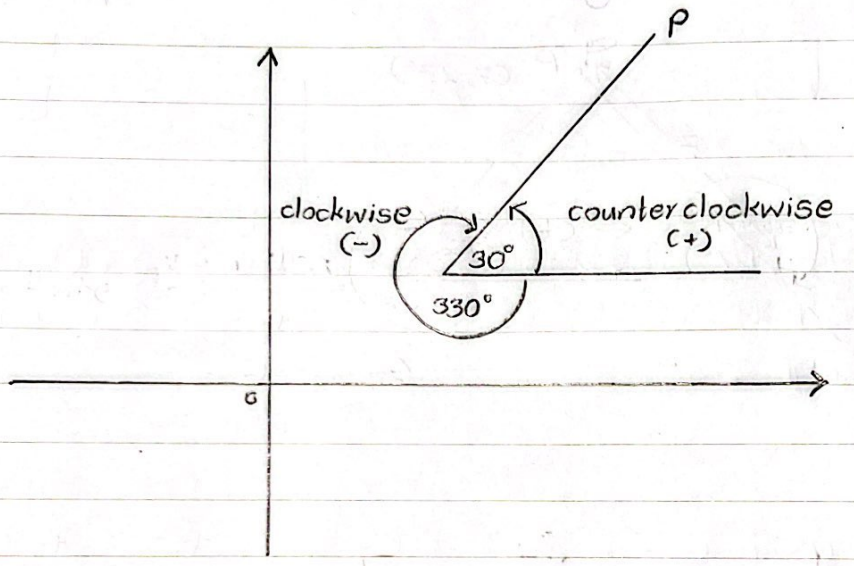
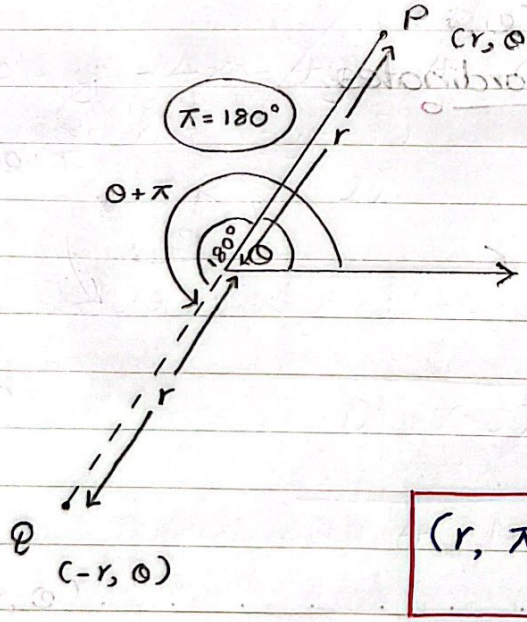
* We use the convention that an angle is;
  - Positive - if measured in the counterclockwise direction from the polar axis.
  - Negative - if measured in the clockwise direction from the polar axis.



Opposite Polar Coordinates.



$$(r, \pi+\theta) \rightarrow (-r, \theta)$$

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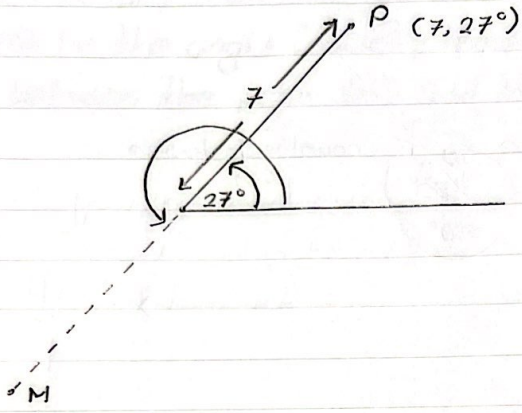
(01.) A circle with a radius of 7, having a polar coordinate with an angle of 27°.

I. If the point is P, write down its polar coordinates.

II. If the opposite polar coordinate is M, write down its polar coordinates.

III. Measure the angle creates with P's polar axis.



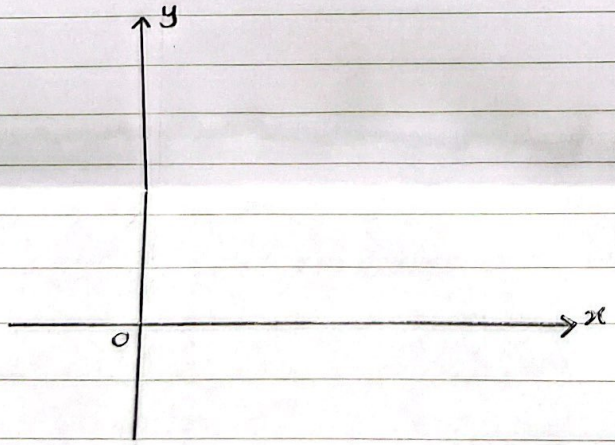
I. (7, 27°)

II. (-7, 27°)

III. 207°

Cartesian Coordinates.

* A coordinate system represents a point in the plane by an ordered pair of numbers called
* Usually, we use Cartesian coordinates, which are directed distances from two perpendicular axes.



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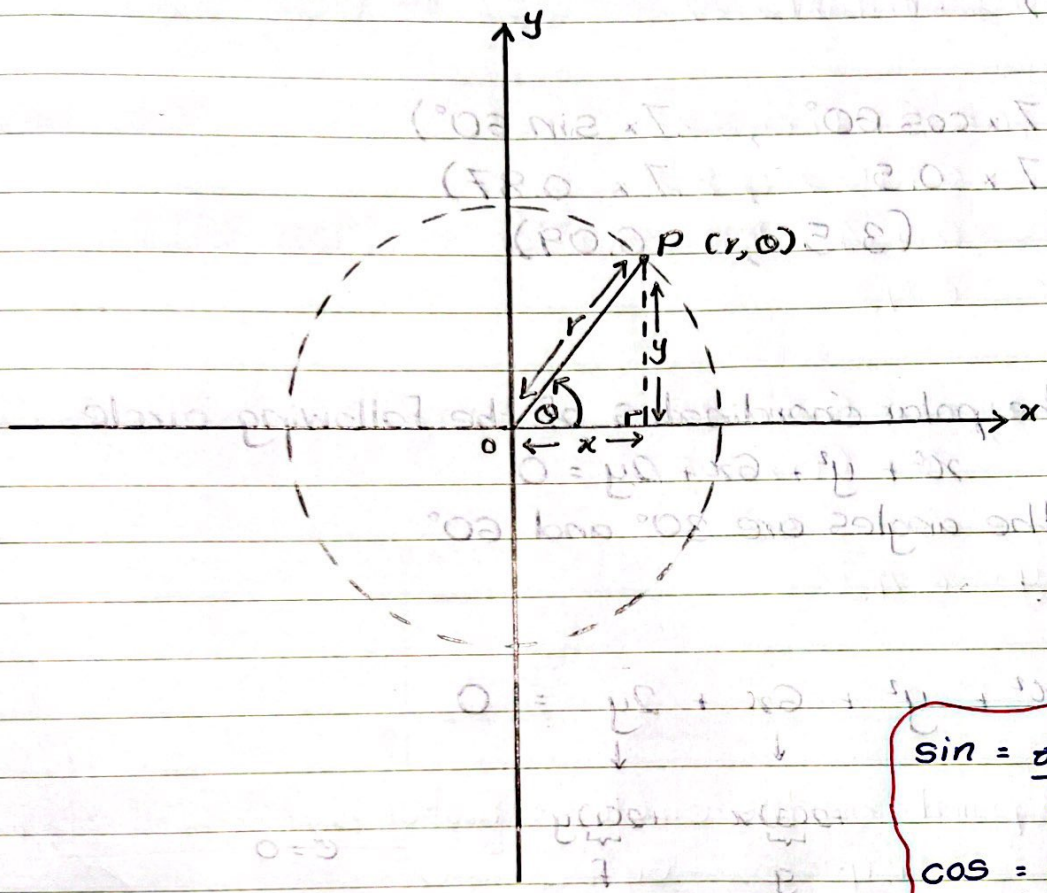
Polar and Cartesian Coordinates.

a) Circles at Origin.



sin = ඇ.ග / කර

cos = ඛ.ග / කර

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$

$x = r\cos\theta$ $y = r\sin\theta$

$P(r, \theta) \rightarrow (r\cos\theta, r\sin\theta)$

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(01) Find the cartesian coordinates of the following polar coordinates.

$(7, 60°)$

$(7 \times \cos 60° \;,\; 7 \times \sin 60°)$

$(7 \times 0.5 \;,\; 7 \times 0.87)$

$(3.5 \;,\; 6.09)$

(02) Find the polar coordinates of the following circle.

$$x^2 + y^2 + 6x + 2y = 0$$

When the angles are 30° and 60°.

$$x^2 + y^2 + 6x + 2y = 0$$

$+2(3)x$ $\quad +2(1)y$ $\quad C = 0$

$g \qquad f$

$$r = \sqrt{g^2 + f^2 + c}$$

$$= \sqrt{3^2 + 1^2 + 0}$$

$$= \sqrt{9 + 1}$$

$$= \sqrt{10}$$

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When 30°,

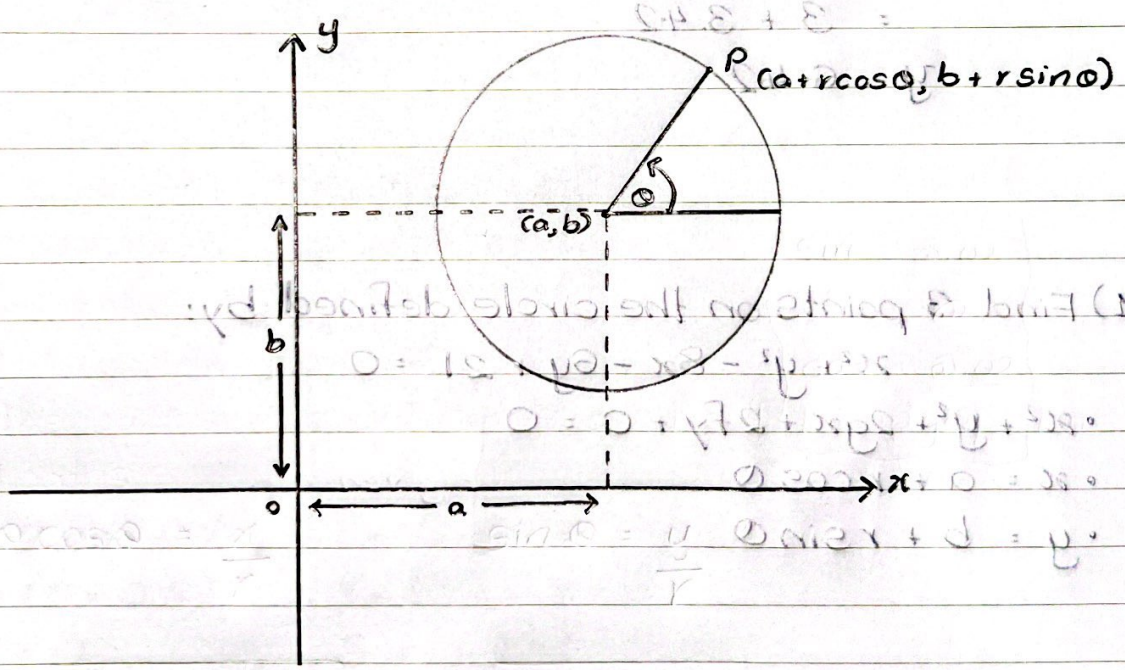
$$(\sqrt{10}, 30°) \rightarrow (\sqrt{10}\cos 30°, \sqrt{10}\sin 30°)$$

When 60°,

cartesian coordinates

$$(\sqrt{10}, 60°) \rightarrow (\sqrt{10}\cos 60°, \sqrt{10}\sin 60°)$$

b) Circles not at Origin.



$$x = (a + r\cos\theta)$$

$$y = (b + r\sin\theta)$$

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(01) Find the Cartesian points of following polar coordinates with center coordinates.

(6, 35°) ; center (2,3)

$x = 2 + 6\cos 35°$
$= 2 + (6 \times 0.82)$
$= 2 + 4.92$
$x = 6.92$

$y = 3 + 6\sin 35°$
$= 3 + (6 \times 0.57)$
$= 3 + 3.42$
$y = 6.42$

Q1) Find 3 points on the circle defined by:

$x^2 + y^2 - 8x - 6y + 21 = 0$

- $x^2 + y^2 + 2gx + 2fy + c = 0$
- $x = a + r\cos\theta$
- $y = b + r\sin\theta$

} given

$x^2 + y^2 - 8x - 6y + 21 = 0$

$+2(-4)x$ $+2(-3)y$ c

g f

$\therefore$ Center $\rightarrow$ (4, 3)

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$$\text{radius} = \sqrt{(-4)^2 + (-3)^2 - 21}$$

$$= \sqrt{16 + 9 - 21}$$

$$= \sqrt{4}$$

$$= 2$$

$(4 + 2\cos 30°, 3 + 2\sin 30°)$
$(4 + (2 \times 0.87), 3 + (2 \times 0.5))$
$(4 + 1.74, 3 + 1)$
$(5.74, 4)$ //

$(4 + 2\cos 45°, 3 + 2\sin 45°)$
$(4 + (2 \times 0.71), 3 + (2 \times 0.71))$
$(4 + 1.42, 3 + 1.42)$
$(5.42, 4.42)$ //

$(4 + 2\cos 60°, 3 + 2\sin 60°)$
$(4 + (2 \times 0.5), 3 + (2 \times 0.87))$
$(4 + 1, 3 + 1.74)$
$(5, 4.74)$ //