HW5 wCOS510

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April 2014

1. A1: What kinds of *DML* programs fail to type check? Explain in a single sentence

Solution 1: Those programs with free variables. A variable is well-formed only if it is in the context and that only happens if it is assigned by a function call, so free variables are not well formed.

2. A2: There is either a missing typing rule. What rule is missing? Please write out a rule with all the necessary conditions that will complete the semantics properly. Which theorem(s) (Progress and/or Preservation) fail due to the absence of this rule? Explain exactly which specific case of the proof of either Progress or Preservation can't be proven without the rule and why. (If multiple cases of the proof can't be proven, you only need to refer to one case of the proof in your explanation.)

Solution 2: The following rule completes the semantics:

$$\frac{}{\textbf{error}}\ \text{T10}$$

Without this rule preservation fails. Every case where the programe takes a step to **error** will fail to type check. For example, in the case of rule $O16$: since $o(v_1, v_2) \longrightarrow$ **error** , it's impossible to prove well-formedness of **error** without rule $T10$.

3. A3: Give the proof for the cases of the Progress Lemma that involve the typing rule T6. Do not show the proofs of the other cases. If you require an additional rule you specified in question A2, then use it. If you require auxiliary lemmas such as Exchange, Weakening, Canonical Forms, Inversion, Substitution or other similar lemmas then carefully write down the statement of the lemma that you require, but do not bother to write out the proof for it.

Solution 3: In the proof of progress we do an induction in the derivation of $\vdash_d e$ wf, here we show the case of rule $T6$.

$$\frac{(1)\ \vdash_d e_1 \text{ wf} \qquad (2)\ \vdash_d e_2 \text{ wf}}{\vdash_d e_1(e_2) \text{ wf}}\ \text{T6}$$

The induction hypothesis says: *If* $\vdash_d e' wf$ *then* e' *is not stuck*. So, applying the IH in (1) we get e_1 is not stuck, thus one of the following cases holds:

1. e_1 is an expression that can take an execution step $e_1 \longrightarrow e_1'$. Then

$$\frac{e_1 \longrightarrow e_1'}{e_1(e_2) \longrightarrow e_1'(e_2)} \text{ O12}$$

2. e_1 is **error** . Then

$$\frac{}{\textbf{error } (e_2) \longrightarrow \textbf{ error}} \text{ O24}$$

3. e_1 is a value v_1. Then, we apply the IH to (2) above, to get the following cases:

   (a) e_2 is an expression that can take an execution step $e_2 \longrightarrow e_2'$. Then

   $$\frac{e_2 \longrightarrow e_2'}{v_1(e_2) \longrightarrow v_1(e_2')} \text{ O13}$$

   (b) e_2 is **error** . Then

   $$\frac{}{v_1(\textbf{ error }) \longrightarrow \textbf{ error}} \text{ O25}$$

   (c) e_2 is a value v_2. Finally we use induction on v_1 to get the final two cases:

      i. $v_1 = \text{fun } f(x) = e \text{ end}$, so:

      $$\frac{(v_1 = \text{fun } f(x) = e \text{ end })}{v_1(v_2) \longrightarrow e[v_1, v_2/f, x]}$$

      ii. Otherwise, v_1 isnt_fun holds, so

      $$\frac{v_1 \text{ isnt\_fun}}{v_1(v_2) \longrightarrow \textbf{ error}} \text{ O18}$$

In all the cases, the expression is not stuck, which is what we wanted to prove.

4. A4: Give the proof for the cases of the Preservation Lemma that involve operational rules O5, O7, and O11. Do not show the proofs of the other cases. Be clear about what you must prove in each case. If you require the additional rule you specified in question A2, then use it. If you require auxiliary lemmas such as Exchange, Weakening, Canonical Forms, Inversion, Substitution or other similar lemmas then carefully write down the statement of the lemma that you require, but do not bother to write out the proof for it.

Solution 4: For this proof, we will need the following lemmas, which we state here without proof:

Lemma 1 ***Substitution***: *If* $\Delta, x \text{ } val \vdash_d e$ *and* $\Delta \vdash_d v$ *then* $\Delta \vdash_d e[v/x]$

In the proof of Preservation we do induction on the derivation of $e \longrightarrow e'$, here we show the case of rules $O5$, $O7$ and $O11$.

1. Case $O5$:

$$\frac{(v_1 = \ \textbf{fun}\ f(x) = e\ \textbf{end}\ )}{v_1(v_2) \longrightarrow e[v_1, v_2/f, x]}\ \text{O5}$$

The hypothesis is $\vdash_d v_1(v_2)$ wf and we want to prove $\vdash_d e[v_1, v_2/f, x]$ wf. By inspecting the well-formed rules, for the hypothesis the case must be:

$$\frac{(1) \vdash_d v_1 \text{ wf} \qquad (2) \vdash_d v_2 \text{ wf}}{\vdash_d v_1(v_2) \text{ wf}}\ \text{T6}$$

Since, $v_1 = \ \textbf{fun}\ f(x) = e\ \textbf{end}$ then (1) means $\vdash_d\ \textbf{fun}\ f(x) = e\ \textbf{end}$ wf. Again, by inspecting the well-formed rules the case must be:

$$\frac{(3) f \text{ val}, x \text{ val} \vdash_d e \text{ wf}}{\vdash_d\ \textbf{fun}\ f(x) = e\ \textbf{end}\ \text{ wf}}\ \text{T7}$$

Then, by the substitution lemma on (3) and (1) we get

$$x \text{ val} \vdash_d e[v_1/f] \text{ wf} \tag{4}$$

Applying again the substitution lemma on (4) and (2) we get

$$\vdash_d e[v_1, v_2/f, x] \text{ wf} \tag{5}$$

Which is what we wanted.

2. Case $O7$:

$$\frac{}{\textbf{check}\ (n, \text{Int}) \longrightarrow n}\ \text{O7}$$

The hypothesis is $\vdash_d\ \textbf{check}\ (n, \text{Int})$ wf and we want to prove $\vdash_d n$ wf. By inspecting the well-formed rules, for the hypothesis the case must be:

$$\frac{\vdash_d n \text{ wf}}{\vdash_d\ \textbf{check}\ (n, \text{Int}) \text{ wf}}$$

Which gives $\vdash_d n$ wf as we wanted.

3. Case $O11$

$$\frac{e_2 \longrightarrow e_2'}{v_1(e_2) \longrightarrow v_1(e_2')}\ \text{O11}$$

The hypothesis (H) is $\vdash_d\ v_1(e_2)$ wf and the induction hypothesis (IH) is that *if* $\vdash_d\ e_2$ *wf, since* $e_2 \longrightarrow e_2'$*, then* $\vdash_d\ e_2'$ *wf.* We want to prove $\vdash_d v_1(e_2')$ wf. By inspecting the well-formed rules, for the hypothesis (H) the case must be:

$$\frac{(1) \vdash_d v_1 \text{ wf} \qquad (2) \vdash_d e_2 \text{ wf}}{\vdash_d v_1(e_2) \text{ wf}}$$

By the induction hypothesis and (2) we learn (3) $\vdash_d e_2'$ wf. Then:

$$\frac{(1) \vdash_d v_1 \text{ wf} \qquad (3) \vdash_d e_2' \text{ wf}}{\vdash_d v_1(e_2') \text{ wf}}$$

Which is what we wanted.

That concludes the three cases of the proof of Preservation that we needed to show.