Project 2 PHY 494

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1 Background

In this problem, we are modeling the trajectory of a soccer ball using Newton's laws of motion to find a set of parameters that lead to a goal in the free kick and a set of parameters that lead to a goal in the corner kick. We use an integration algorithm (RK4) to model the trajectories of the soccer ball while taking into consideration the force due to gravity, the Magnus force, and the drag force. Both the free kick and corner kick are modeled separately using real-life constants and dimensions. We also give examples of trajectories with parameters that lead to missed and blocked goals.

1.1 Soccer physics

A projectile hurling through the air is governed by the Newtonian equations of motion and experiences forces due to gravity, drag, and lift.

$$\mathbf{F}_D = -\frac{1}{2}C_D(v)\rho A v^2 \times \frac{\mathbf{v}}{v}$$

The force due to gravity is a familiar one which causes our ball to descend back to Earth in what is defined in the $-\hat{y}$ direction.

$$\mathbf{F}_g = -mg\hat{\mathbf{y}}$$

where g is the acceleration due to gravity and m is the given mass of the soccer ball.
$\mathbf{F_M}$ or the Magnus force, accounts for the changing airflow surrounding a spinning object. It is a function of the ball's angular velocity ω and the translations velocity v.

$$\mathbf{F}_M = \frac{1}{2}C_L \rho A \frac{v}{\omega}\boldsymbol{\omega} \times \mathbf{v}$$

The Magnus force is also dependent on the air density ρ, cross-sectional area of the soccer ball, and a lift coefficient C_L.

$$C_L = \frac{1}{2} \times S^{0.4}$$

$$S = \frac{r\omega}{v}$$

where S, or the spin parameter, is a ratio of rotational and translational speeds. These parameters were modeled specifically for soccer physics and were given.
Quadratic drag force was used due to the high Reynold's number corresponding to turbulent air flow surrounding the ball. The equation is given as

$$\mathbf{F}_D = -\frac{1}{2}C_D(v)\rho A v^2 \times \frac{\mathbf{v}}{v}$$

where once again ρ is the air density and A is the cross-sectional area.
The drag coefficient C_D was calculated according to the provided equation

$$C_D(v,S) = \begin{cases} a + \frac{b}{1+\exp[(v-v_c)/v_s]} & \text{if } v < v_c \text{ or } S < 0.05 \\ cS^d & \text{if } v \geq v_c \text{ and } S \geq 0.05 \end{cases} \quad v_c = 12.19\text{ms}^{-1}, \quad v_s = 1.309\text{ms}^{-1}, \quad , a = 0.155,$$

Upon reaching a critical velocity v_c and minimum spin parameter S, the ball will suddenly experience drastically lower drag. See the results for an analysis of this drop in air resistance.
The lift coefficient C_L was dependent on the spin parameter and was calculated according to the equation

$$C_L(S) = \frac{1}{2}S^{0.4}$$

This coefficient increases nonlinearly as S increased for low values of S, but then leveled out and increases at a relatively linear rate for larger values of S. See results for more discussion on the lift coefficient as well.

1.2 Code

An RK4 algorithm, courtesy of Oliver Beckstein, was applied to calculate the trajectories of the ball's motion. The Chodes created a function that, given the initial parameters $\vec{v_0}$, $\vec{\omega}$, and the initial position – which was set at the origin for both the free and corner kick – would store all initial position and velocity components in a y-vector and feed the y-vector into a hungry little force function. This function would then unpack the velocity components, use them to calculate the accelerations due to force of gravity, Magnus force, drag force, and team spirit, and vomit back up the velocities and accelerations in an f-vector. This function was utilized in a while loop that repeatedly used the force function to find the accelerations and then utilized the RK4 algorithm to calculate the new positions and velocities. The old positions were stored in a list, the y-vector was updated with the new quantities, and the loop would repeat, until one of the boundaries was encountered. The drag coefficient C_D and the lift coefficient C_L were calculated using functions that were called upon by the main function.

1.3 Definitions

1.3.1 Various constants and dimensions

We have the following constants which were used in our modeling of the trajectories:

- air density $\rho = 1.2\frac{kg}{m^3}$
- acceleration due to gravity $g = 9.81\frac{m}{s^2}$
- mass of the ball $m = 0.436kg$
- radius of the ball $r = 0.109m$
- initial velocity (chosen in range) $4.5\frac{m}{s} \leq v_0 \leq 31\frac{m}{s}$
- angular velocity $0 \leq \omega_0 \leq 75.4\frac{rad}{s}$

The dimensions of the soccer field in this problem are $105m$ in length, $68.52m$ in width (goal line), and a goal width and height of $7.32m$ and $2.44m$ respectively. The dimensions that are most important to this problem are discussed in the following "free kick" and "corner kick" sections.

1.3.2 Free kick

For the free kick we used the following definitions and distances. We defined the origin of our axes to be at the starting point of the ball; the positive x-axis, positive y-axis, and positive z-axis were defined as follows in Figure 1. The azimuthal angle (sweeping the xz-plane) is defined to be ϕ, and the polar angle (sweeping the yx-plane) is defined to be θ.

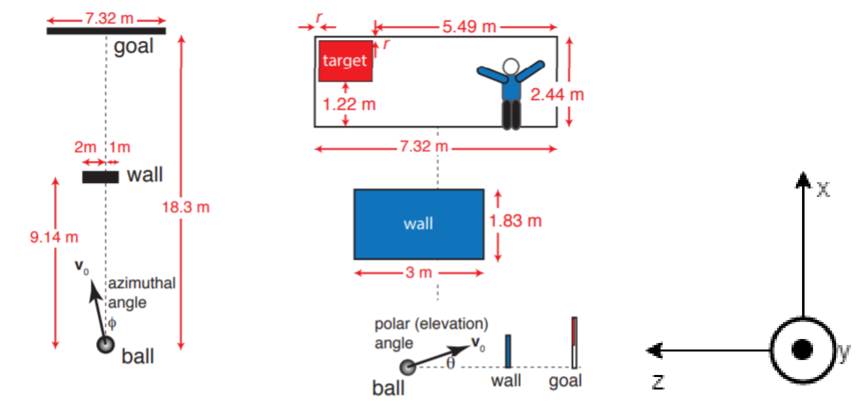

Figure 1: Distances and Axes defined free kick *(project2.pdf Beckstein)*

A successful kick, i.e. a "goal" is a kick where the ball lands in the target area (red box in goal, Figure 1). The kicker must clear the wall and put the ball in the target area in order to score. The letter r in the figure is the ball's radius which was also taken into consideration to test if the ball entered the target area.

1.3.3 Corner kick

Our definitions for the corner kick were very similar to the free kick in that the kick has to strike the target area in order to score. The origin is at the starting place for the ball again but is now starting in the corner of the field rather than directly in front of the goal. The axes were defined in the same way as the free kick, except now the goal is parallel with the positive x-axis since the origin has now moved. The azimuthal (xz-plane) and polar (xy-plane) angles are defined again as ϕ and θ respectively.

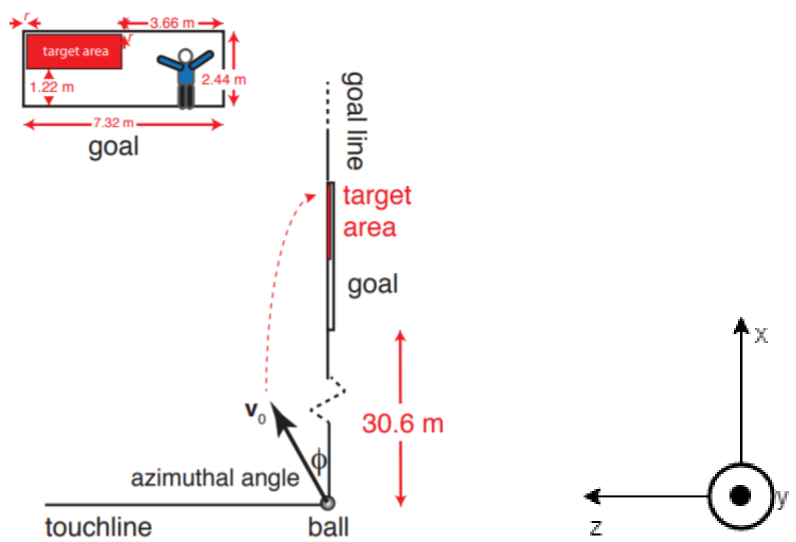

Figure 2: Distances and Axes defined corner kick *(project2.pdf Beckstein)*

2 Results

2.1 Parameters used for free kick goal

$$\theta = \frac{\pi}{8} \quad radians$$

$$\phi = \frac{\pi}{18} \quad radians$$

$$v_0 = 27 * \left[cos(\frac{\pi}{8})cos(\frac{\pi}{18})\hat{\mathbf{x}} + sin(\frac{\pi}{8})\hat{\mathbf{y}} + cos(\frac{\pi}{8})sin(\frac{\pi}{18})\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = \left[-60\hat{\mathbf{z}} \right] \frac{rad}{s}$$

2.2 Parameters used for corner kick goal

$$\theta = \frac{\pi}{5} \quad radians$$

$$\phi = \frac{\pi}{10} \quad radians$$

$$v_0 = 27 * \left[cos(\frac{\pi}{5})cos(\frac{\pi}{10})\hat{\mathbf{x}} + sin(\frac{\pi}{5})\hat{\mathbf{y}} + cos(\frac{\pi}{5})sin(\frac{\pi}{10})\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = \left[63\hat{\mathbf{y}} - 15\hat{\mathbf{z}} \right] \frac{rad}{s}$$

2.3 Figures for drag and lift coefficients; corner and free kick goals side/top views

2.3.1 Notes about graphs:

The red lines on all of the two dimensional plots represent the target area defined in the introduction. The large red box on the three dimensional plots represent the wall (for the free kicks); the other boxes in 3D plots represent the goal box and target area – purple and red respectively.

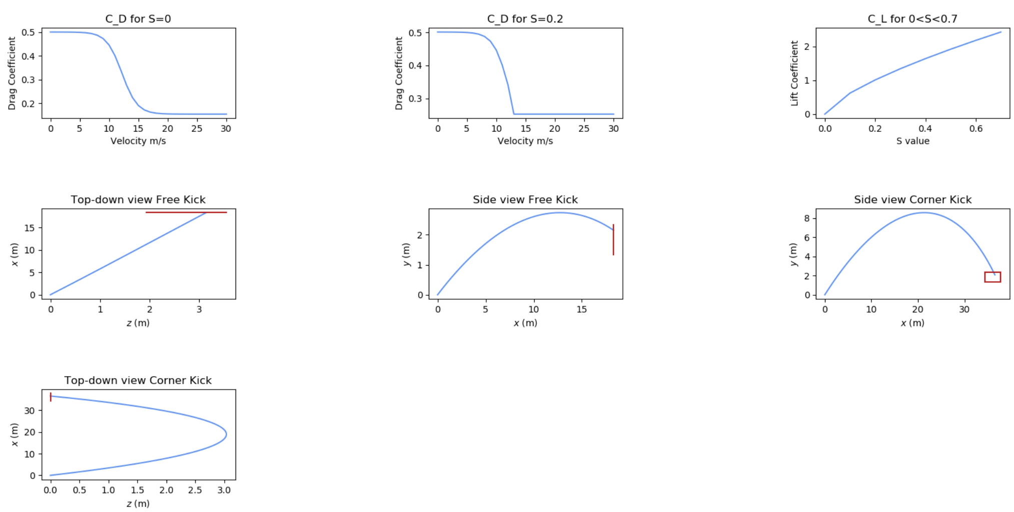

Figure 3: Top and side views of free and corner kicks

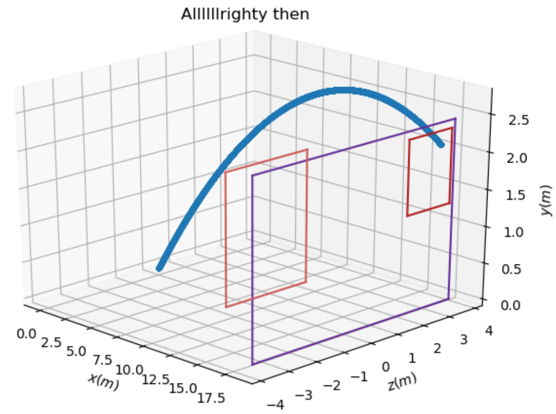

Figure 4: Free kick

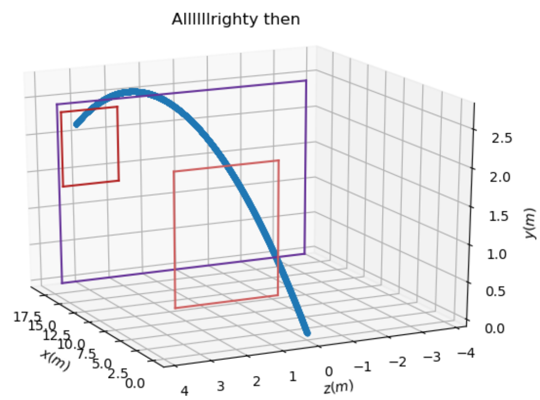

Figure 5: Free kick No. 2

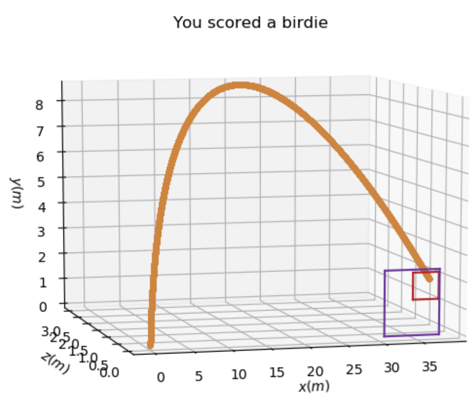

Figure 6: Corner kick

2.4 Goal parameters used but with no spin

$$\theta = \frac{\pi}{5} \quad radians$$

$$\phi = \frac{\pi}{10} \quad radians$$

$$v_0 = 27 * \left[cos(\frac{\pi}{5})cos(\frac{\pi}{10})\hat{\mathbf{x}} + sin(\frac{\pi}{5})\hat{\mathbf{y}} + cos(\frac{\pi}{5})sin(\frac{\pi}{10})\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = 0$$

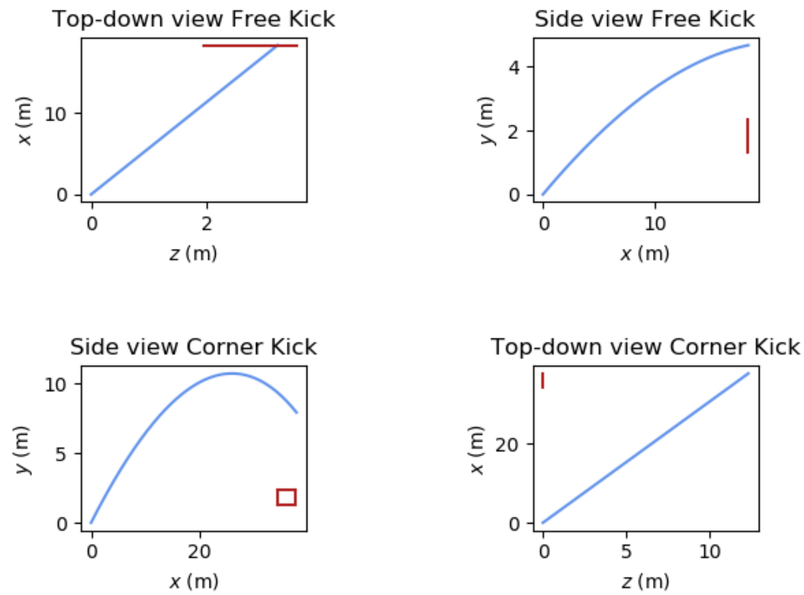

Figure 7: Top and side views of free and corner kicks

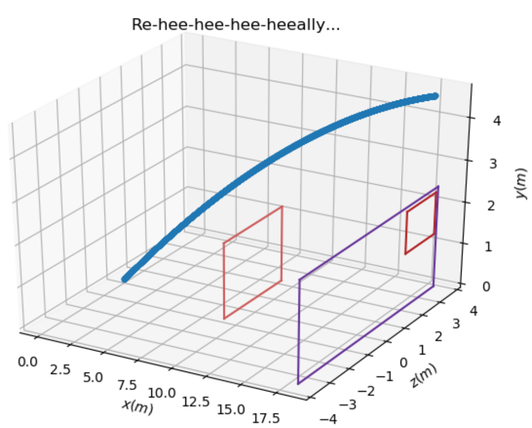

Figure 8: Free kick

Figure 9: Corner kick

2.5 Parameters used for missed goal No. 1: Free Kick (hits wall)

$$\theta = \frac{\pi}{15} \quad radians$$

$$\phi = 0 \quad radians$$

$$v_0 = 30 * \left[cos(\frac{\pi}{15})cos(0)\hat{\mathbf{x}} + sin(\frac{\pi}{15})\hat{\mathbf{y}} + cos(\frac{\pi}{15})sin(0)\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = \left[-1\hat{\mathbf{z}} \right] \frac{rad}{s}$$

2.6 Parameters used for missed goal No. 1: Corner Kick (lands short)

$$\theta = \frac{\pi}{20} \quad radians$$

$$\phi = 0 \quad radians$$

$$v_0 = 30 * \left[cos(\frac{\pi}{20})cos(0)\hat{\mathbf{x}} + sin(\frac{\pi}{20})\hat{\mathbf{y}} + cos(\frac{\pi}{20})sin(0)\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = \left[-1\hat{\mathbf{z}} \right] \frac{rad}{s}$$

2.7 Figures corner and free kick missed No. 1

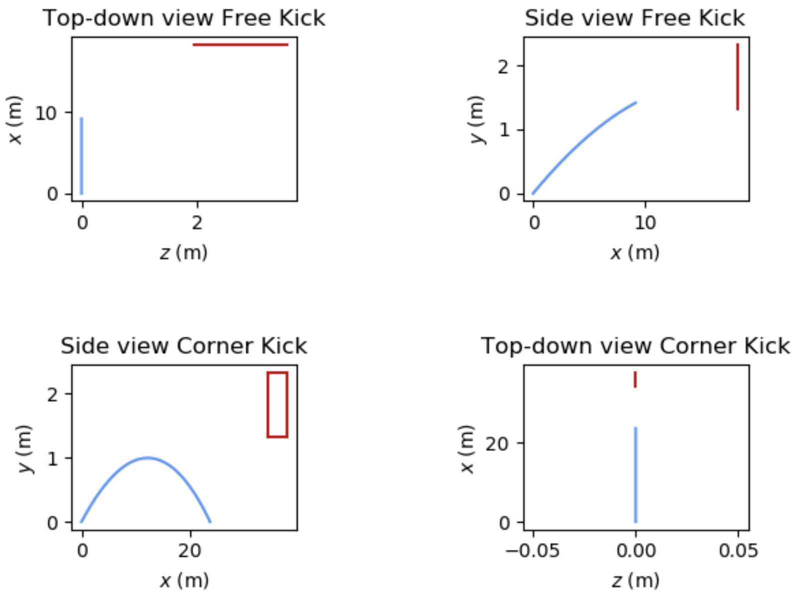

Figure 10: Top and side views of free and corner kicks misses No.1

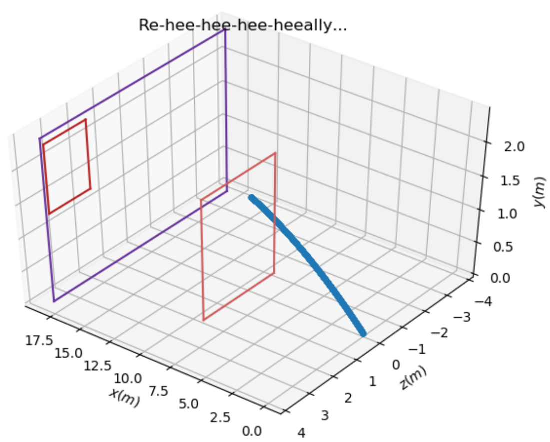

Figure 11: Free kick miss No.1

Figure 12: Corner kick miss No.1

2.8 Parameters used for missed goal No. 2 (both free and corner kick) fly-over and complete miss

$$\theta = \frac{\pi}{3} \quad radians$$

$$\phi = \frac{\pi}{6} \quad radians$$

$$v_0 = 30 * \left[cos(\frac{\pi}{3})cos(\frac{\pi}{6})\hat{\mathbf{x}} + sin(\frac{\pi}{3})\hat{\mathbf{y}} + cos(\frac{\pi}{3})sin(\frac{\pi}{6})\hat{\mathbf{z}} \right] \frac{m}{s}$$

$$\omega = \left[-70\hat{\mathbf{z}} \right] \frac{rad}{s}$$

2.9 Figures for missed corner and free kicks No. 2

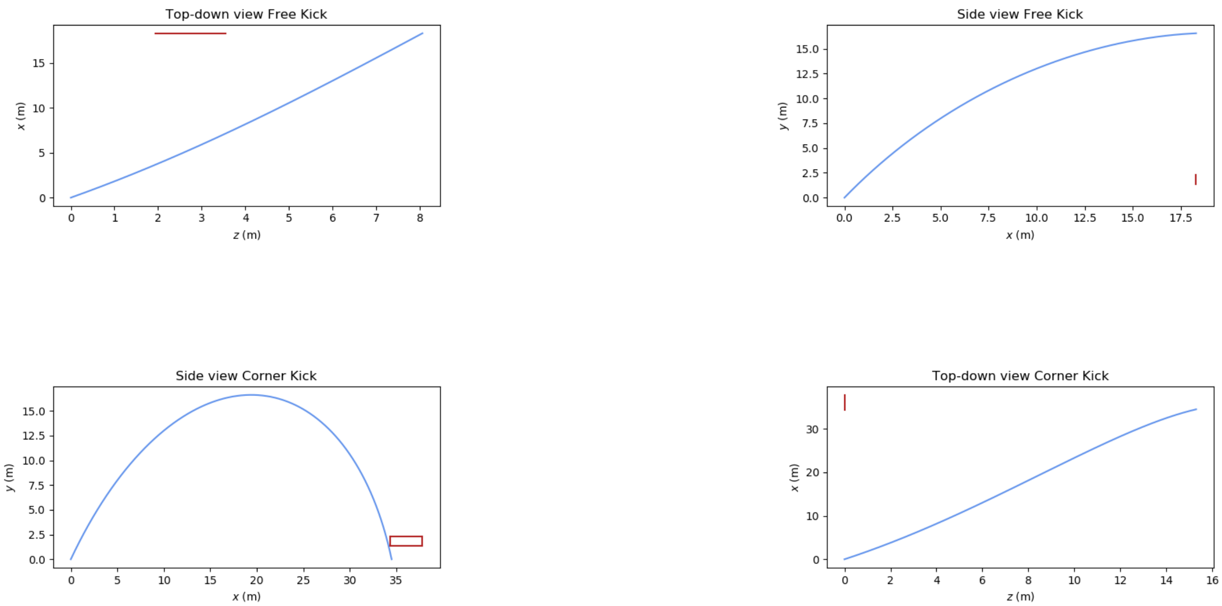

Figure 13: Top and side views of free and corner kicks misses No.2

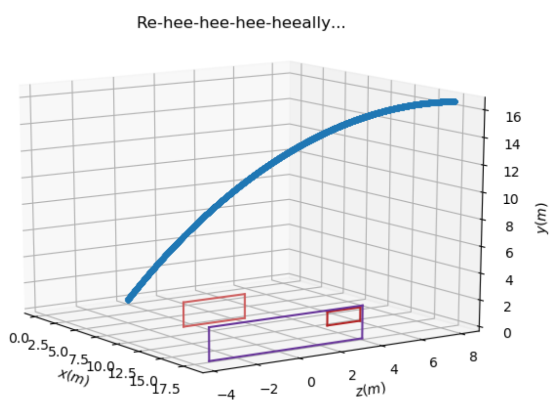

Figure 14: Free kick miss No.2

Figure 15: Corner kick miss No.2

3 Discussion

For the free kick, by setting a shallow angle ϕ between the trajectory and the x-axis and a larger angle θ between the trajectory and the x-z plane, the Code Chodes managed to "kick" the ball over the "human" wall of opposing teammates and into the target area. However, in order to set θ to a larger magnitude without causing the ball to soar over the goal, the Code Chodes had to add a spin component to the ball. Due to the cross product nature of the Magnus force, in order to cause the ball to experience a downward acceleration and arc down towards the goal after passing over the wall, the Code Chodes added a negative z component to the spin of the ball, ω. No other components were necessary for this course of action, but if the Chodes had decided to arc the ball around the side of the wall instead of over it, depending on the choice of around the right or left sides of the wall, a positive or negative y component would have been added to ω instead.

For the corner kick, in order to keep the ball in the air long enough to enable it to travel far enough in the x direction to hit the target, θ was set fairly larger than that of the free kick. However, this once again required that some z-spin be added in order to force it to arc downward to hit at the correct height – less than that of the free kick, but not a negligible amount. The z axis was far trickier to deal with in this scenario. In order to get the ball to arc inwards into the target, the ball had to be given a not-insignificant component of z-velocity away from the goal, with a large amount of y-spin in order to cause it to curl inwards towards the goal in a parabolic trajectory as seen from the bird's eye view. Too much y-spin and the ball would hit the touchline before reaching the target, but not enough and it would pass the goal entirely. In addition, the curl in the y and the z jointly influenced the resulting trajectory of the ball – for example, even after finding a scoring trajectory that barely hit within the corner of the target, increasing just the y-spin would cause the ball to miss, while increasing both the y-spin and the z-spin would actually cause it to hit closer to the center of the target than the original values.

The effects of spin were paramount to the success of the shot. When the Chodes took the scoring trajectory parameters and removed the spin – setting it to

$$0\hat{x} + 0\hat{y} + 0.001\hat{z}$$

in order to avoid difficulties arising from the cross product operation used in the code – the ball completely missed the target, not even coming close to the goal itself. With the same initial velocity, the free kick passed directly over the target, since there was no acceleration along the z-axis due to the Magnus force, but without the Magnus force acting downward along the y-axis, the ball passed far and wide over the goal, not even hitting its flight apex by the time it passed over. The corner kick, with a z-component in its velocity and no Magnus force to spin it back toward the goal, went straight into the field. With a hefty upward component

of velocity and no Magnus force to spin back down, it traveled far into the defensive zone – although the Chodes admit that their collective knowledge of soccer/football/European football is admittedly lacking, so this term might be improperly applied.
For the free kick, when the ball was given a small θ and $\phi = 0$, the ball was obstructed by the wall, as could have been predicted. The Chodes pray for the health and safety of the players that made up the human wall. For the corner kick, when the ball was given similar angle measures, it hit the ground long before reaching the goal at all, resulting in a miss as well, and possibly great shame and embarrassment for the player in question. The Chodes can only hope that this terrible football player was ostracized forever by his friends and family for his great ineptitude.
Adjusting the parameters to give the ball more upward trajectory, less z trajectory, and a higher velocity, with only a negative z-spin but with a large magnitude, caused drastic effects. For the free kick, the ball presumably landed well outside of the stadium, while the corner kick's ball landed very solidly within the midfield.

4 Summary

Without the effects of spin and the Magnus force, there is a practically nonexistent chance of making either of these very common goal shots in soccer/football/European football. Perhaps with the aid of random weather events, a bird flying overhead, or an act of God, one of these shots could be made without the aid of spin, but otherwise, the average football player stands no chance of getting the ball into the goal without the aid of his or her teammates. Even with the aid of spin, however, the results of this analysis show that a very particular combination of kick strength, kick direction, and applied torque upon the ball is necessary to result in a successful goal, which is very difficult for the average *Homosapiens*. Perhaps this is the true lesson to be learned from this group project, that one can only truly hope to succeed by relying on one's teammates.

5 Contributions

Shoemaker and Farmer wrote code to implement free kick and corner kick with help from King. Geometry of kicks was also coded by Shoemaker. Farmer coded both goal success and failure parameters while King plotted all 2D figures. King also analyzed and plotted special $\omega = 0$ cases. All 3D graphs were produced by Shoemaker and Farmer. All contributed to the editing of the code and document with special emphasis by Farmer on Background, King on Definitions and Results, and Shoemaker on Discussion and Summary.

6 Acknowledgements

We would like to thank Professor Beckstein for his 3D plotting code that allowed us to make nice trajectory plots.

References

[1] baseball_solution.ipynb Retrieved April 8, 2018, from PHY 494 class resources repository for 3D plotting code.

[2] project_2.pdf Retrieved March 26, 2018 from PHY 494 class resources for kick figures.