Causal Inference

Mixtape Session

MIXTAPE SESSIONS

Roadmap

Counterfactuals and causality
- Causality and models
- Potential outcomes
- Randomization and selection bias
- Randomization inference

Directed Acyclic Graphs
- Graph notation
- Backdoor criterion
- Collider bias
- Front door criterion
- Concluding remarks

Independence assumption

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Treatment is independent of potential outcomes

$$(Y^0, Y^1) \perp\!\!\!\perp D$$

In plain language: Random assignment means the treatment has been assigned to units without regard to their potential outcomes. This ensures that mean potential outcomes for the treatment group and control group are the same. Also ensures other variables are distributed the same for a large sample.

$$\begin{aligned} E[Y^0|D=1] &= E[Y^0|D=0] \\ E[Y^1|D=1] &= E[Y^1|D=0] \end{aligned}$$

Random Assignment Solves the Selection Problem

$$\begin{aligned}\underbrace{E_N[y_i|d_i=1]-E_N[y_i|d_i=0]}_{\text{SDO}} &= \underbrace{E[Y^1]-E[Y^0]}_{\text{Average Treatment Effect}}\\ &+\underbrace{E[Y^0|D=1]-E[Y^0|D=0]}_{\text{Selection bias}}\\ &+\underbrace{(1-\pi)(ATT-ATU)}_{\text{Heterogenous treatment effect bias}}\end{aligned}$$

- If treatment is independent of potential outcomes, then swap out equations and **selection bias** zeroes out:

$$E[Y^0|D=1]-E[Y^0|D=0] = 0$$

Random Assignment Solves the Heterogenous Treatment Effects

- How does randomization affect heterogeneity treatment effects bias from the third line? Rewrite definitions for ATT and ATU:

$$\text{ATT} = E[Y^1|D=1] - E[Y^0|D=1]$$
$$\text{ATU} = E[Y^1|D=0] - E[Y^0|D=0]$$

- Rewrite the third row bias after $1-\pi$:

$$\begin{aligned} ATT - ATU &= \mathbf{E[Y^1 \mid D{=}1]} - E[Y^0|D=1] \\ &\quad -\mathbf{E[Y^1 \mid D{=}0]} + E[Y^0|D=0] \\ &= 0 \end{aligned}$$

- If treatment is independent of potential outcomes, then:

$$\begin{aligned} E_N[y_i|d_i=1] - E_N[y_i|d_i=0] &= E[Y^1] - E[Y^0] \\ SDO &= ATE \end{aligned}$$