Causal Inference

Mixtape Session

MIXTAPE SESSIONS

Roadmap

Definition 7: Simple difference in mean outcomes (SDO)

A simple difference in mean outcomes (SDO) can be approximated by the sample averages:

$$
\begin{aligned}
SDO &= E[Y^1|D=1] - E[Y^0|D=0] \\
&= E_N[Y|D=1] - E_N[Y|D=0]
\end{aligned}
$$

in large samples. I'll usually use expectation operators but we use samples for estimation.

SDO vs. ATE

Notice the subtle difference between the SDO and ATE notation:

$$E[Y|D=1] - E[Y|D=0] \quad \lesseqgtr \quad E[Y^1] - E[Y^0]$$

- The SDO is an *estimate*, whereas ATE is a *parameter*
- SDO is a crank that turns data into numbers
- ATE is a parameter that is unknowable because of the fundamental problem of causal inference
- SDO might line up with the ATE but also might not
- Under what situations is SDO a biased estimate of the ATE?

Potentially biased comparisons

Decomposition of the SDO

The SDO can be decomposed into the sum of three parts:

$$\begin{aligned} E[Y^1|D=1] - E[Y^0|D=0] &= ATE \\ &+E[Y^0|D=1] - E[Y^0|D=0] \\ &+(1-\pi)(ATT - ATU) \end{aligned}$$

Seeing is believing so let's work through this identity!

Decomposition of SDO

Use LIE to decompose ATE into the sum of four conditional average expectations

$$
\begin{aligned}
\text{ATE} &= E[Y^1] - E[Y^0] \\
&= \{\pi E[Y^1|D=1] + (1-\pi)E[Y^1|D=0]\} \\
&\quad -\{\pi E[Y^0|D=1] + (1-\pi)E[Y^0|D=0]\}
\end{aligned}
$$

Substitute letters for expectations

$$
\begin{aligned}
E[Y^1|D=1] &= a \\
E[Y^1|D=0] &= b \\
E[Y^0|D=1] &= c \\
E[Y^0|D=0] &= d \\
\text{ATE} &= e
\end{aligned}
$$

Rewrite ATE

$$
e = \{\pi a + (1-\pi)b\} - \{\pi c + (1-\pi)d\}
$$

Move SDO terms to LHS

$$\begin{aligned}
e &= \{\pi a + (1-\pi)b\} - \{\pi c + (1-\pi)d\} \\
e &= \pi a + b - \pi b - \pi c - d + \pi d \\
e &= \pi a + b - \pi b - \pi c - d + \pi d + (\textbf{a} - \textbf{a}) + (\textbf{c} - \textbf{c}) + (\textbf{d} - \textbf{d}) \\
0 &= e - \pi a - b + \pi b + \pi c + d - \pi d - \textbf{a} + \textbf{a} - \textbf{c} + \textbf{c} - \textbf{d} + \textbf{d} \\
\textbf{a} - \textbf{d} &= e - \pi a - b + \pi b + \pi c + d - \pi d + \textbf{a} - \textbf{c} + \textbf{c} - \textbf{d} \\
\textbf{a} - \textbf{d} &= e + (\textbf{c} - \textbf{d}) + \textbf{a} - \pi a - b + \pi b - \textbf{c} + \pi c + d - \pi d \\
\textbf{a} - \textbf{d} &= e + (\textbf{c} - \textbf{d}) + (1-\pi)a - (1-\pi)b + (1-\pi)d - (1-\pi)c \\
\textbf{a} - \textbf{d} &= e + (\textbf{c} - \textbf{d}) + (1-\pi)(a-c) - (1-\pi)(b-d)
\end{aligned}$$

Substitute conditional means

$$\begin{aligned}
E[Y^1|D=1] - E[Y^0|D=0] &= \text{ATE} \\
&\quad +(E[Y^0|D=1] - E[Y^0|D=0]) \\
&\quad +(1-\pi)(\{E[Y^1|D=1] - E[Y^0|D=1]\} \\
&\quad -(1-\pi)\{E[Y^1|D=0] - E[Y^0|D=0]\}) \\
E[Y^1|D=1] - E[Y^0|D=0] &= ATE \\
&\quad +(E[Y^0|D=1] - E[Y^0|D=0]) \\
&\quad +(1-\pi)(ATT - ATU)
\end{aligned}$$

Decomposition of difference in means

$$
\begin{aligned}
\underbrace{E_N[y_i|d_i=1] - E_N[y_i|d_i=0]}_{\text{SDO}} &= \underbrace{E[Y^1] - E[Y^0]}_{\text{Average Treatment Effect}} \\
&+ \underbrace{E[Y^0|D=1] - E[Y^0|D=0]}_{\text{Selection bias}} \\
&+ \underbrace{(1-\pi)(ATT - ATU)}_{\text{Heterogenous treatment effect bias}}
\end{aligned}
$$

where $E_N[Y|D=1] \to E[Y^1|D=1]$, $E_N[Y|D=0] \to E[Y^0|D=0]$ and $(1-\pi)$ is the share of the population in the control group.