Causal Inference

Mixtape Session

MIXTAPE SESSIONS

Roadmap

Counterfactuals and causality
- Causality and models
- Potential outcomes
- Randomization and selection bias
- Randomization inference

Directed Acyclic Graphs
- Graph notation
- Backdoor criterion
- Collider bias
- Front door criterion
- Concluding remarks

Potential outcomes notation

- Let the treatment be a binary variable:

$$D_{i,t} = \begin{cases} 1 \text{ if hospitalized at time } t \\ 0 \text{ if not hospitalized at time } t \end{cases}$$

  where i indexes an individual observation, such as a person
- Potential outcomes:

$$Y^j_{i,t} = \begin{cases} 1 \text{ health if hospitalized at time } t \\ 0 \text{ health if not hospitalized at time } t \end{cases}$$

  where j indexes a counterfactual state of the world

Moving between worlds

- I'll drop t subscript, but note – these are potential outcomes for the same person at the exact same moment in time
- A potential outcome Y^1 is not the historical outcome Y either conceptually or notationally
- Potential outcomes are hypothetical states of the world but historical outcomes are ex post realizations
- Major philosophical move here: go from the potential worlds to the actual (historical) world based on your treatment assignment

Important definitions

Definition 1: Individual treatment effect

The individual treatment effect, δ_i, equals $Y_i^1 - Y_i^0$

Definition 2: Average treatment effect (ATE)

The average treatment effect is the population average of all i individual treatment effects

$$\begin{aligned} E[\delta_i] &= E[Y_i^1 - Y_i^0] \\ &= E[Y_i^1] - E[Y_i^0] \end{aligned}$$

Definition 3: Switching equation

An individual's observed health outcomes, Y, is determined by treatment assignment, D_i, and corresponding potential outcomes:

$$\begin{aligned} Y_i &= D_i Y_i^1 + (1 - D_i) Y_i^0 \\ Y_i &= \begin{cases} Y_i^1 \text{ if } D_i = 1 \\ Y_i^0 \text{ if } D_i = 0 \end{cases} \end{aligned}$$

So what's the problem?

Definition 4: Fundamental problem of causal inference

If you need both potential outcomes to know causality with certainty, then since it is impossible to observe both Y_i^1 and Y_i^0 for the same individual, δ_i, is *unknowable*.

Conditional Average Treatment Effects

Definition 5: Average Treatment Effect on the Treated (ATT)

The average treatment effect on the treatment group is equal to the average treatment effect conditional on being a treatment group member:

$$\begin{aligned} E[\delta|D=1] &= E[Y^1 - Y^0|D=1] \\ &= E[Y^1|D=1] - E[Y^0|D=1] \end{aligned}$$

Definition 6: Average Treatment Effect on the Untreated (ATU)

The average treatment effect on the untreated group is equal to the average treatment effect conditional on being untreated:

$$\begin{aligned} E[\delta|D=0] &= E[Y^1 - Y^0|D=0] \\ &= E[Y^1|D=0] - E[Y^0|D=0] \end{aligned}$$

Causality and comparisons

- Does the ventilator make someone have severe COVID symptoms? Or are they sick with COVID symptoms, and that's why they are on a ventilator?
- Why can't I just compare symptoms for those on vents versus those who aren't? After all, there's a control group
- What are we actually measuring if we compare average health outcomes for those on vents versus those who aren't?
- Let's look at our first estimator and see if we can't better understand what comparisons are and are not saying.