Descriptive Statistics

Gov 51: Data Analysis and Politics

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Week 2, Thursday
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Today's Roadmap

1. **The Question: What Does "Typical" Mean?**
   - Summarizing presidential approval across states
2. **Measures of Center**
   - Mean, median, and when they differ
3. **Measures of Spread**
   - Range, percentiles, variance, standard deviation
4. **Weighted Statistics**
   - Why some observations "count" more than others
5. **Visualizing Distributions**
   - Histograms and what they tell us

The Question: What Does “Typical” Mean?

A Simple Question

What does the typical American think about the president?

Gallup and other pollsters survey Americans constantly. But:

- Different states have different opinions
- Different states have different populations
- How do we summarize all this into one number?

Today we'll use state-level presidential approval data to learn how to summarize distributions.

Loading Our Data

```
# Load data
approval <- read.csv("state_approval.csv")

# What do we have?
dim(approval)
## [1] 50  5

head(approval, 4)
##      state abbrev approval population  region
## 1  Alabama     AL       38    5024279   South
## 2   Alaska     AK       41     733391    West
## 3  Arizona     AZ       44    7151502    West
## 4 Arkansas     AR       36    3011524   South
```

50 states, with approval rating (%) and population.

Note: The R script on the course website has the full URL to load this data.

First Look: The Raw Numbers

Here are approval ratings for a few states:

State	Approval (%)	Population (millions)
California	52	39.5
Texas	41	29.1
Wyoming	32	0.6
Vermont	57	0.6
Ohio	42	11.8

What's the "typical" approval rating?

Should Wyoming (0.6 million people) count the same as California (39.5 million)?

Measures of Center

Let's Start Simple

Before we use R, let's calculate by hand.

Here are approval ratings for 5 states:

38, 41, 44, 36, 52

Calculate the mean:

1. Add them up: $38 + 41 + 44 + 36 + 52 = 211$
2. Divide by the count: $211 \div 5 = 42.2$

The mean approval rating is 42.2%.

That's all the mean is: sum divided by count.

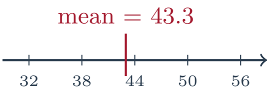
The Mean: What R Does

The `mean()` function does exactly what we just did:

```r
mean(approval$approval)
## [1] 43.26
```

For all 50 states, the mean approval is **43.3%**.

Simple, right? But there's a catch...



The Median: The Middle Value

The **median** is the middle value when you sort the data.

```
# Sort the values
sort(approval$approval)
## [1] 32 34 34 35 35 36 36 37 37 38 38 38 39 39 39 39 40
## [18] 40 41 41 42 42 43 43 44 44 45 45 45 46 46 47 47 47
## [35] 48 48 49 49 50 50 50 51 52 52 53 56 57 58

# The median (middle value)
median(approval$approval)
## [1] 43.5
```

With 50 values, the median is the average of the 25th and 26th values.

Median = 43.5%

Mean vs. Median: Why Both?

In our data: Mean = 43.3%, Median = 43.5%

They're almost the same! But that's not always true.

The Mean

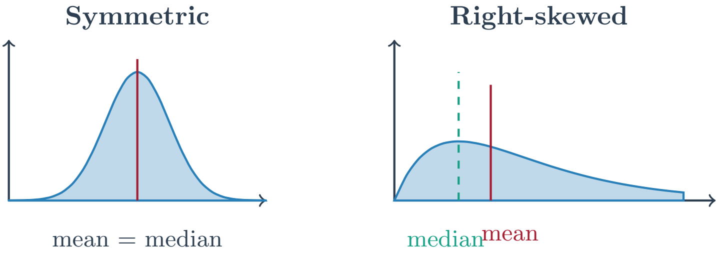
- Uses every value
- Sensitive to outliers
- Gets "pulled" by extreme values

The Median

- Only uses the middle
- Robust to outliers
- Ignores extreme values

When mean > median: distribution is right-skewed (pulled up by high values)
When mean < median: distribution is left-skewed (pulled down by low values)

Visualizing the Difference



Income is famously right-skewed: a few billionaires pull the mean way above the median. The median household income tells you more about the "typical" American than the mean does.

Connection to Problem Set 1

In PS1, you'll calculate mean and median commute times.

The question we'll ask:

- Is the mean larger or smaller than the median?
- What does that tell you about the shape of the distribution?
- Does it match what you see in the histogram?

This is how you interpret data, not just calculate it.

Now the Math

We've seen what the mean does. Here's the formula:

$$\text{Sample Mean: } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

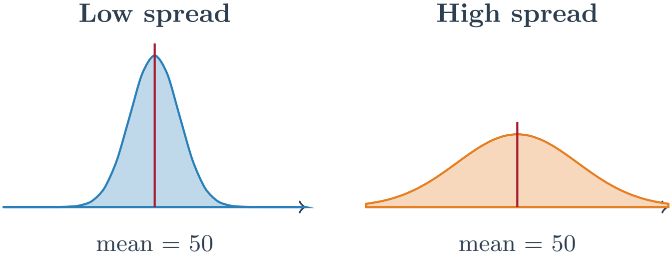
- x_i = each individual value
- n = number of values
- $\sum$ = "add them all up"
- $\bar{x}$ = the mean (pronounced "x-bar")

The formula just says: add up all the values, divide by how many there are.

Measures of Spread

Center Isn't Everything

Two datasets can have the same mean but look completely different:



We need measures of spread: How much do the values vary around the center?

The Range: Simplest Measure

The **range** is just max minus min:

```
min(approval$approval)
## [1] 32

max(approval$approval)
## [1] 58

range(approval$approval)
## [1] 32 58
```

Range $= 58 - 32 = 26$ percentage points

Problem: The range only uses two values. One outlier can make it huge.

Percentiles: More Robust

Percentiles tell you where values fall in the distribution:

```
quantile(approval$approval)
##    0%   25%   50%   75%  100%
##    32    39    43    49    58
```

- **0th percentile** (min): 32%
- **25th percentile** (Q1): 39%
- **50th percentile** (median): 43.5%
- **75th percentile** (Q3): 49%
- **100th percentile** (max): 58%

The middle 50% of states have approval between 39% and 49%.

Getting Specific Percentiles

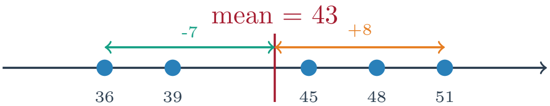
You can ask for any percentile:

```
# The 90th percentile
quantile(approval$approval, 0.90)
## 90%
##  52

# Multiple percentiles at once
quantile(approval$approval, c(0.10, 0.50, 0.90))
## 10% 50% 90%
##  36  43  52
```

In PS1, you'll calculate the 90th percentile of commute times and write:
"90% of commuters travel ___ minutes or less."

Variance: Average Squared Distance from Mean

The **variance** measures how far values typically are from the mean.



Steps:

1. Calculate each deviation: $(x_i - \bar{x})$

2. Square them: $(x_i - \bar{x})^2$

3. Average the squared deviations

Why Square the Deviations?

Problem: Deviations add to zero!

- Some values are above the mean (positive deviation)
- Some values are below the mean (negative deviation)
- They cancel out: $\sum(x_i - \bar{x}) = 0$

Solution: Square them first

- Squaring makes everything positive
- Bigger deviations get emphasized (squared distance)

There are other solutions (like absolute value), but squaring has nice mathematical properties.

Variance in R

```r
var(approval$approval)
## [1] 42.28
```

The variance is 42.28... but 42.28 *what*?

Units are "percent squared"—not very interpretable!

Solution: Take the square root to get back to original units.

Variance Is Always Non-Negative

Look at the formula:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Each term $(x_i - \bar{x})^2$ is a **squared number**.

- Squaring any real number gives something ≥ 0
- Sum of non-negative terms is non-negative
- Dividing by $(n-1) > 0$ keeps it non-negative

Result: Variance ≥ 0, always.

Variance $= 0$ only when every value equals the mean (no spread at all).

Standard Deviation: Variance in Original Units

The **standard deviation** is the square root of variance:

```
sd(approval$approval)
## [1] 6.50
```

The standard deviation is 6.5 percentage points.

Interpretation: On average, states are about 6.5 percentage points away from the mean approval rating.

This is the measure of spread you'll report in your summary statistics tables.

The Formulas

Sample Variance: $$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Sample Standard Deviation: $$s = \sqrt{s^2}$$

Wait... why $n - 1$ instead of n?

The $n - 1$ Question

Quick intuition (don't worry about the proof):

When we calculate variance, we first calculated the mean. That "used up" one piece of information.

- We have n data points
- But only $n - 1$ independent pieces of information left
- This is called **degrees of freedom**

Dividing by $n - 1$ instead of n corrects for this, giving us an unbiased estimate of the population variance.

R's `var()` and `sd()` functions use $n - 1$ by default. That's what you want.

Why Does Estimation "Use Up" Information?

The problem: We want to measure spread around the *true* mean μ.
But we don't know μ—we estimated it with $\bar{x}$.

The catch: $\bar{x}$ is the point that *minimizes* squared deviations.

- Deviations from $\bar{x}$ are artificially small
- Dividing by n would systematically underestimate variance
- This is called **bias**

The constraint: Once you know $\bar{x}$, the deviations must sum to zero.

- If you know $n - 1$ deviations, you can calculate the last one
- Only $n - 1$ deviations are "free to vary"

Dividing by $n - 1$ corrects for the bias. The estimate becomes **unbiased**.

The summary() Shortcut

R's `summary()` function gives you many statistics at once:

```
summary(approval$approval)
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   32.00   39.00   43.50   43.26   49.00   58.00
```

This gives you:

- Min and Max (range)
- 1st and 3rd Quartiles (25th and 75th percentiles)
- Median (50th percentile)
- Mean

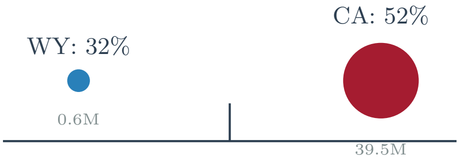
Note: `summary()` doesn't give you standard deviation—use `sd()` for that.

Weighted Statistics

Not All States Are Equal

Our mean approval (43.3%) treated every state equally.

But should Wyoming (576,000 people) count the same as California (39.5 million)?



If we want to know what the typical *American* thinks (not the typical *state*), California should count more.

Weighted Mean

The **weighted mean** lets each observation count according to its weight:

```
# Unweighted mean (each state counts equally)
mean(approval$approval)
## [1] 43.26

# Weighted mean (weight by population)
weighted.mean(approval$approval, approval$population)
## [1] 44.52
```

- Unweighted: 43.3% (average across states)
- Weighted: 44.5% (average across people)

The weighted mean is higher because large, high-approval states (CA, NY) pull it up.

The Weighted Mean Formula

$$\textbf{Weighted Mean: } \bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Where w_i is the weight for observation i.

Intuition: Instead of each value counting once, it counts w_i times.

When all weights are equal, this reduces to the regular mean.

Connection to Problem Set 1

In PS1, you'll use American Community Survey (ACS) data.

The ACS has a variable called `PERWT` (person weight):

- Each person in the survey represents many people in the US
- `PERWT` tells you how many

Two calculations:

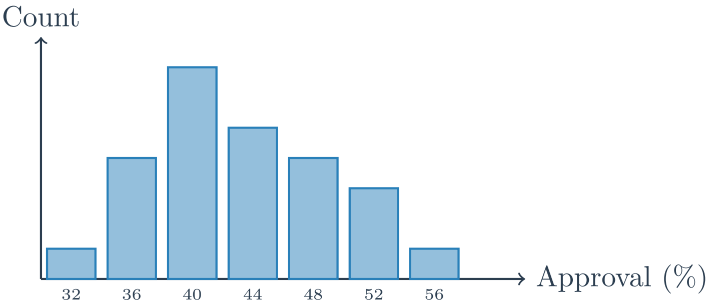
```
# Unweighted (average in the sample)
mean(commuters$TRANTIME)

# Weighted (average in the US population)
weighted.mean(commuters$TRANTIME, commuters$PERWT)
```

You'll compare these and explain what each number represents.

Visualizing Distributions

The Histogram

A **histogram** shows how values are distributed:



- X-axis: the variable (approval rating)
- Y-axis: how many observations fall in each bin
- **Shape** tells us about the distribution

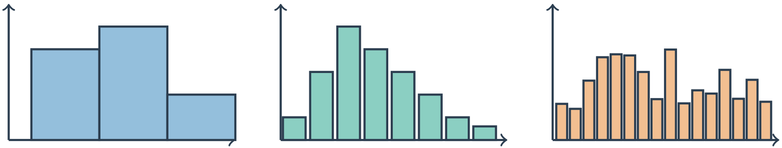
Creating a Histogram in R

```
library(ggplot2)

ggplot(approval, aes(x = approval)) +
  geom_histogram(binwidth = 4,
                 fill = "steelblue",
                 color = "white") +
  labs(x = "Approval Rating (%)",
       y = "Number of States",
       title = "Distribution of Presidential Approval")
```

Key choices:

- `binwidth`: How wide is each bar? (Experiment!)
- `fill`: Color of the bars
- `color`: Color of the bar borders

Bin Width Matters



Too few bins
Hides patterns

Good
Shows shape

Too many bins
Too noisy

There's no perfect answer—try different values and see what tells the clearest story.

Describing Shape

When you look at a histogram, describe:

Shape

- Symmetric?
- Right-skewed?
- Left-skewed?
- Bimodal?

Center

- Where's the "middle"?
- Mean and median

Spread

- How wide?
- Are values clustered or dispersed?

Also note:

- Outliers (unusual values)
- Notable features (gaps, spikes)

Example Description

Looking at our approval rating histogram:

"The distribution of presidential approval across states is roughly symmetric, centered around 43%. Most states fall between 36% and 52% approval. There are no obvious outliers, though Vermont (57%) and Hawaii (58%) are notably high, while Wyoming (32%) and West Virginia (34%) are notably low."

In PS1, you'll write a similar description for commute times.

Putting It Together

The Summary Statistics Table

In PS1, you'll create a table like this:

Variable	N	Mean	Std. Dev.	Min	Max
Approval (%)	50	43.3	6.5	32	58
Population (millions)	50	6.6	7.2	0.6	39.5

This table should be:

- Generated by code (not typed manually)
- Readable on its own (clear variable names, units)
- Rendered cleanly in your PDF

Building the Table in R

```r
# Calculate statistics
stats <- data.frame(
  Variable = c("Approval (%)", "Population (millions)"),
  N = c(length(approval$approval),
        length(approval$population)),
  Mean = c(mean(approval$approval),
           mean(approval$population)/1e6),
  SD = c(sd(approval$approval),
         sd(approval$population)/1e6),
  Min = c(min(approval$approval),
          min(approval$population)/1e6),
  Max = c(max(approval$approval),
          max(approval$population)/1e6)
)

# Render as a table
knitr::kable(stats, digits = 1)
```

What You've Learned

Concepts:

- Mean vs. median
- Skewness and shape
- Variance and standard deviation
- Degrees of freedom $(n - 1)$
- Weighted statistics

R Functions:

- `mean()`, `median()`
- `min()`, `max()`, `range()`
- `var()`, `sd()`
- `quantile()`
- `weighted.mean()`
- `summary()`
- `ggplot() + geom_histogram()`

Numbers summarize. Visuals reveal. Use both.

Looking Ahead

Problem Set 1:

- Due Wednesday, February 11 at 11:59pm
- You now have all the statistics you need!
- Don't forget: GitHub URL in your document

Next Week: Probability

- Foundation for everything that comes after
- Why we need it for inference

Section this week: Help with PS1, IPUMS setup

Questions?

Scott: Tue/Thu 3–5pm | George: Thu 2–3pm, K455 | CA: Harrison Huang