

Liner shipping hub location-routing problem under demand uncertainty

Cihan Butun ^{*1}, Sanja Petrovic², and Luc Muyldermans³

¹Loughborough Business School, Loughborough University, Loughborough, LE11 3TU, United Kingdom. Email: c.butun@lboro.ac.uk

²Nottingham University Business School, Jubilee Campus, Nottingham NG81BB, United Kingdom, Email: sanja.petrovic@nottingham.ac.uk

³Nottingham University Business School, Jubilee Campus, Nottingham NG81BB, United Kingdom, Email: luc.muyldermans@nottingham.ac.uk

Algorithm pseudo-codes

1. Tabu Search

Table 1: Tabu Search parameters

Parameter	Explanation
\mathcal{L}^o	Set of open hubs in the initial solution
\mathcal{L}^c	Set of closed hubs in the initial solution
\mathcal{U}	Set of unallocated non-hub nodes in the initial solution
tf	Total flows routed in the network, $tf = \sum_{k=1}^H (1 - \beta)(o_{k(3)} + d_{k(3)}) + (\beta - \lambda)(o_{k(4)} + d_{k(4)})$
$tcap$	Total cargo handling capacity of hub candidates, $tcap = \sum_{k=1}^H cap_k$
I_k	Hub attractiveness score of k , $k \in \mathcal{H}$
sd_{ki}	The sea distance between non-hub terminal i and hub candidate terminal k in nautical miles, $k \in \mathcal{H}$, $i \in \mathcal{N}$
$volumeFactor$	The multiplier that indicates the minimum total capacity that the located hubs must have in the initial solution, $1 \leq volumeFactor \leq \frac{tcap}{tf}$
$iter$	Current number of consecutive iterations without improvement
$maxIter$	The Tabu Search algorithm terminates when $iter = maxIter$
$openFreq$	The list of hub location frequencies of currently open hubs
$maxFreq$	The maximum value of hub opening frequencies in $openFreq$
$limFreq$	Diversification is triggered when $maxFreq \geq limFreq$
S	A feasible solution
S_0	The initial feasible solution
S_{best}	The best solution obtained in the Tabu Search algorithm
tnc_s	The network cost of solution S

^{*}Corresponding author

Algorithm 1 Initial solution procedure

```
1  $S_0 \leftarrow \emptyset$ ,  $tnc_{S_0} \leftarrow \infty$ ,  $volumeFactor \leftarrow 1$ 
2 for all  $k \in \mathcal{H}$  do
3   Calculate  $I_k$ 
4 end for
5 while  $volumeFactor \leq tcap/tf$  do
6    $\mathcal{U} \leftarrow \mathcal{N}$ ,  $\mathcal{L}^o \leftarrow \emptyset$ ,  $\mathcal{L}^c \leftarrow \mathcal{H}$ 
7   Rank  $k \in \mathcal{L}^c$  in non-increasing order of  $I_k$ 
8   while  $\sum_{k=1}^{|\mathcal{L}^o|} cap_k < volumeFactor \cdot tf$  do
9     Open a new hub at the top element  $k \in \mathcal{L}^c$ 
10    Remove  $k$  from  $\mathcal{L}^c$  and  $\mathcal{U}$ 
11    Add  $k$  to  $\mathcal{L}^o$ 
12  end while
13  while  $\mathcal{U} \neq \emptyset$  do
14    for all  $k \in \mathcal{L}^o$  and  $i \in \mathcal{U}$  do
15      if  $k$  has enough capacity to accommodate  $i$  then
16        Allocate  $i$  to  $k$  (subject to sufficient capacity) |  $sd_{ki} = \min_{\forall k \in \mathcal{L}^o} sd_{ki}$ 
17        Remove  $i$  from  $\mathcal{U}$ 
18      end if
19      if  $i$  cannot be allocated to any  $k \in \mathcal{L}^o$  then
20        Discard the solution
21        break
22      end if
23    end for
24  end while
25  if  $\mathcal{U} = \emptyset$  then
26    Update  $S$  with the hub locations and node allocations
27    Apply Nearest-Neighbor heuristic to obtain the hub-level network for  $S$ 
28    if  $tnc_S < tnc_{S_0}$  then
29       $S_0 \leftarrow S$ ,  $tnc_{S_0} \leftarrow tnc_S$ 
30    end if
31  end if
32   $volumeFactor \leftarrow volumeFactor + 0.05$ 
33 end while
34 return  $S_0$ 
```

Algorithm 2 Tabu Search algorithm

```

35 Run the Initial solution procedure and obtain  $S_0$ 
36  $iter \leftarrow 1$ ,  $S_{iter}, S_{best} \leftarrow S_0$ ,  $tnc_{S_{best}} \leftarrow tnc_{S_0}$ 
37 while  $iter \neq maxIter$  do
38     Do a neighborhood search and update  $S_{iter}$ 
39     Run local improvement with 2-opt and update  $S_{iter}$ 
40     if  $tnc_{S_{iter}} < tnc_{S_{best}}$  then
41          $S_{best} \leftarrow S_{iter}$ ,  $tnc_{S_{best}} \leftarrow tnc_{S_{iter}}$ ,  $iter \leftarrow 0$ 
42     else
43          $iter \leftarrow iter + 1$ 
44     end if
45     Update  $openFreq$  and Tabu lists
46     if  $maxFreq \geq limFreq$  then
47         Run Diversification
48          $limFreq \leftarrow maxFreq + limFreq$ 
49         Update Tabu lists
50     end if
51 end while
52 return  $S_{best}$ 

```

2. Decomposition Heuristic

Table 2: Decomposition heuristic parameters

Parameter	Explanation
t	The current iteration number
tnc_{DH}	The lowest total network cost obtained by the decomposition heuristic
$SP1$	Subproblem 1, locates hubs and determines non-hub to hub allocations
$SP2$	Subproblem 2, designs the hub-level network and determines cargo routing between hubs.
$SP3$	Subproblem 3, re-optimizes non-hub to hub allocations terminals.
p_t	The number of hubs to be located at iteration t
p_{max}	The current maximum number of hubs that can be located
θ_t^{SP1}	The objective function value of SP1 calculated at iteration t with $p = p_t$
$\theta_{p_{max}}^{SP1}$	The objective function value of SP1 calculated with $p = p_{max}$
θ_t^{SP2}	The objective function value of SP2 calculated at iteration t
θ_t^{SP3}	The objective function value of SP3 calculated at iteration t

Algorithm 3 Decomposition heuristic

```
53  $t \leftarrow 1$ ,  $tnc_{DH} \leftarrow \infty$ ,  $p_{max} \leftarrow H$ ,  $p_t \leftarrow 2$ 
54 while  $p_t < p_{max}$  do
55     Solve SP1 with  $p = p_t$ , obtain  $\theta_t^{SP1}$ 
56     if  $t = 1$  or ( $t > 1$  and  $\theta_t^{SP1} < \theta_{t-1}^{SP1}$ ) then
57         Solve SP2, obtain  $\theta_t^{SP2}$ 
58         Solve SP3, obtain  $\theta_t^{SP3}$ 
59         if  $\theta_t^{SP3} < tnc_{DH}$  then
60              $tnc_{DH} \leftarrow \theta_t^{SP3}$ 
61             while true do
62                 Solve SP1 with  $p = p_{max}$ , obtain  $\theta_{p_{max}}^{SP1}$ 
63                 if  $\theta_{p_{max}}^{SP1} \geq tnc_{DH}$  then
64                      $p_{max} \leftarrow p_{max} - 1$ 
65                 else
66                     break
67                 end if
68             end while
69         end if
70     end if
71      $t \leftarrow t + 1$ ,  $p_t \leftarrow p_t + 1$ 
72 end while
73 return  $tnc_{DH}$ 
```
