ECEN 2350 Spring 2017 — Homework 2

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1. *Book Problems: 2.12, 2.13 and 2.14*

(2.12) *Find the minimum SOP expression for* $f = x_1x_3 + x_1\overline{x_2} + \overline{x_1}x_2x_3 + \overline{x_1}\overline{x_2}\overline{x_3}$

$$
\begin{aligned}
& f = x_1x_3 + x_1\overline{x_2} + \overline{x_1}x_2x_3 + \overline{x_1}\overline{x_2}\overline{x_3} && \\
& f = x_3(x_1 + \overline{x_1}x_2) + \overline{x_2}(x_1 + \overline{x_1}\overline{x_3}) && \textit{Distributive} \\
& f = x_3(x_1 + x_2) + x_2(x_1 + \overline{x_3}) && \textit{DeMorgan's} \\
& f = x_3x_1 + x_3x_2 + x_2x_1 + x_2\overline{x_3} && \textit{Distributive} \\
\textit{For Consensus,}\quad & \textit{Note: } ab + bc + \bar{a}c = ab + \bar{a}c && \\
\textit{For Consensus,}\quad & \textit{Let: } a = x_3,\ b = x_1,\ \textit{and } c = x_2 && \\
\textit{For Consensus,}\quad & \textit{Substitute } x_3x_1 + x_1x_2 + x_2\overline{x_3} \textit{ for } x_3x_1 + x_2\overline{x_3} && \\
& f = x_3x_1 + x_3x_2 + x_2\overline{x_3} && \textit{Consensus} \\
& f = \boxed{\mathbf{x_3x_1 + x_3x_2 + x_2\overline{x_3}}} &&
\end{aligned}
$$

(2.13) *Find the minimum SOP expression for* $f = x_1\overline{x_2}\overline{x_3} + x_1x_2x_4 + x_1\overline{x_2}x_3\overline{x_4}$

$$
\begin{aligned}
& f = x_1\overline{x_2}\overline{x_3} + x_1x_2x_4 + x_1\overline{x_2}x_3\overline{x_4} && \\
& f = x_1(\overline{x_2}\overline{x_3} + x_2x_4 + \overline{x_2}x_3\overline{x_4}) && \textit{Distributive} \\
& f = x_1(\overline{x_2}(\overline{x_3} + x_3\overline{x_4}) + x_2x_4) && \textit{Distributive} \\
& f = x_1(\overline{x_2}(\overline{x_3} + \overline{x_4}) + x_2x_4) && \textit{DeMorgan's} \\
& f = x_1(\overline{x_2}\overline{x_3} + \overline{x_2}\overline{x_4} + x_2x_4) && \textit{Distributive} \\
& f = x_1\overline{x_2}\overline{x_3} + x_1\overline{x_2}\overline{x_4} + x_1x_2x_4 && \textit{Distributive} \\
& f = \boxed{\mathbf{x_1\overline{x_2}\overline{x_3} + x_1\overline{x_2}\overline{x_4} + x_1x_2x_4}} &&
\end{aligned}
$$

(2.14) *Find the minimum POS expression for* $f = (x_1 + x_3 + x_4)(x_1 + \overline{x_2} + x_3)(x_1 + \overline{x_2} + \overline{x_3} + x_4)$

$$
\begin{aligned}
f &= (x_1 + x_3 + x_4)(x_1 + \overline{x_2} + x_3)(x_1 + \overline{x_2} + \overline{x_3} + x_4) && \\
\overline{f} &= \overline{(x_1 + x_3 + x_4)(x_1 + \overline{x_2} + x_3)(x_1 + \overline{x_2} + \overline{x_3} + x_4)} && \\
\overline{f} &= \overline{(x_1 + x_3 + x_4)} + \overline{(x_1 + \overline{x_2} + x_3)} + \overline{(x_1 + \overline{x_2} + \overline{x_3} + x_4)} && \textit{DeMorgan's} \\
\overline{f} &= (\overline{x_1}\,\overline{x_3}\,\overline{x_4}) + (\overline{x_1}\,\overline{\overline{x_2}}\,\overline{x_3}) + (\overline{x_1}\,\overline{\overline{x_2}}\,\overline{\overline{x_3}}\,\overline{x_4}) && \textit{DeMorgan's} \\
\overline{f} &= (\overline{x_1}\,\overline{x_3}\,\overline{x_4}) + (\overline{x_1}x_2\overline{x_3}) + (\overline{x_1}x_2x_3\overline{x_4}) && \textit{Single Variable Thrm} \\
\overline{f} &= (\overline{x_1}\,\overline{x_3}\,\overline{x_4}) + (\overline{x_1}x_2\overline{x_3}) + (\overline{x_1}x_2x_3\overline{x_4}) + (\overline{x_1}x_2x_3\overline{x_4}) && \textit{Repeating Term} \\
\overline{f} &= \overline{x_1}(\overline{x_3}\,\overline{x_4} + x_2\overline{x_3} + x_2x_3\overline{x_4} + x_2x_3\overline{x_4}) && \textit{Distributive} \\
\overline{f} &= \overline{x_1}(\overline{x_4}(\overline{x_3} + x_2x_3) + x_2(\overline{x_3} + x_3\overline{x_4})) && \textit{Distributive} \\
\overline{f} &= \overline{x_1}(\overline{x_4}(\overline{x_3} + x_2) + x_2(\overline{x_3} + \overline{x_4})) && \textit{DeMorgan's} \\
\overline{f} &= \overline{x_1}(\overline{x_4}\,\overline{x_3} + \overline{x_4}x_2 + x_2\overline{x_3} + x_2\overline{x_4}) && \textit{Distributive} \\
\overline{f} &= \overline{x_1}(\overline{x_4}\,\overline{x_3} + \overline{x_4}x_2 + x_2\overline{x_3}) && \textit{Distributive} \\
\overline{f} &= \overline{x_1}\,\overline{x_4}\,\overline{x_3} + \overline{x_1}\,\overline{x_4}x_2 + \overline{x_1}x_2\overline{x_3} && \textit{Distributive} \\
\overline{\overline{f}} &= \overline{\overline{x_1}\,\overline{x_4}\,\overline{x_3} + \overline{x_1}\,\overline{x_4}x_2 + \overline{x_1}x_2\overline{x_3}} && \\
f &= (\overline{\overline{x_1}\,\overline{x_4}\,\overline{x_3}})(\overline{\overline{x_1}\,\overline{x_4}x_2})(\overline{\overline{x_1}x_2\overline{x_3}}) && \textit{DeMorgan's} \\
f &= \boxed{(\mathbf{x_1} + \mathbf{x_4} + \mathbf{x_3})(\mathbf{x_1} + \mathbf{x_4} + \overline{\mathbf{x_2}})(\mathbf{x_1} + \overline{\mathbf{x_2}} + \mathbf{x_3})} && \textit{DeMorgan's}
\end{aligned}
$$

2. *Given the truth table below, answer the following:*

a	b	c	f	minterm	maxterm
0	0	0	0	$\bar{a}\bar{b}\bar{c}$	a+b+c
0	0	1	1	$\bar{a}\bar{b}c$	$a + b + \bar{c}$
0	1	0	0	$\bar{a}b\bar{c}$	$a + \bar{b} + c$
0	1	1	1	$\bar{a}bc$	$a + \bar{b} + \bar{c}$
1	0	0	1	$a\bar{b}\bar{c}$	$\bar{a} + b + c$
1	0	1	0	$a\bar{b}c$	$\bar{a} + b + \bar{c}$
1	1	0	1	$ab\bar{c}$	$\bar{a} + \bar{c} + c$
1	1	1	1	abc	$\bar{a} + \bar{b} + \bar{c}$

(a) *Provide the canonical sum of products logic equation for f(a,b,c)*

The canonical sum of products logic equation for f is given by taking the minterms from every line where $f(a, b, c) = 1$, and combining them into this:

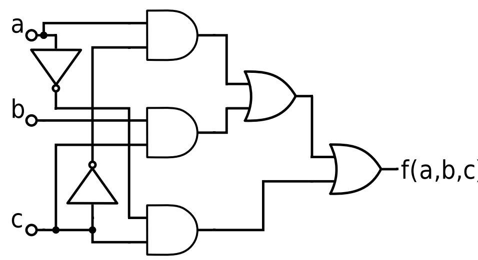
$$f(a,b,c) = \boxed{\mathbf{\bar{a}\bar{b}c + \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} + abc}}$$

(b) *Using Boolean algebra, derive the minimum cost sum-of-products logic equation for f. Specify which axiom, theorems, and properties were used for each step. Report the cost (number of gates and inputs).*

$$\begin{aligned}
f(a,b,c) &= \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} + abc && \\
f(a,b,c) &= \bar{a}c(\bar{b} + b) + a\bar{c}(\bar{b} + b) + abc && \textit{Distributive} \\
f(a,b,c) &= \bar{a}c(1) + a\bar{c}(1) + abc && \textit{Single Variable Thrm} \\
f(a,b,c) &= c(ba + \bar{a}) + a\bar{c} && \textit{Distributive} \\
f(a,b,c) &= c(b + \bar{a}) + a\bar{c} && \textit{DeMorgan's} \\
f(a,b,c) &= cb + c\bar{a} + a\bar{c} && \textit{Distributive} \\
f(a,b,c) &= \boxed{\mathbf{cb + c\bar{a} + a\bar{c}}} &&
\end{aligned}$$

As the answer has a, b and c feeding into three *and* gates which feed into an *or* gates, and only two of the outputs are inverted, the circuit has $3+2*1+3*2+1*3 = 14$ inputs and $3 + 1 + 2 = 6$ gates for a cost of $14 + 6 = \boxed{\mathbf{20}}$

(c) *Synthesize (draw the gates) the minimum cost sum-of-products circuit for the logic equation obtained above.*



3. *Given the Truth Table above, answer the following:*

(a) *Provide the canonical product of sums logic equation for f(a,b,c)*

The canonical product of sums logic equation for f is given by taking the maxterms from every line where $f(a,b,c) = 0$, and combining them into this:

$$f(a,b,c) = \boxed{(\mathbf{a}+\mathbf{b}+\mathbf{c})(\mathbf{a}+\bar{\mathbf{b}}+\mathbf{c})(\overline{\mathbf{a}}+\mathbf{b}+\overline{\mathbf{c}})}$$

(b) *Using Boolean algebra, derive the minimum cost product-of-sums logic equation for f. Specify which axiom, theorems, and properties were used for each step. Report the cost (number of gates and inputs).*

$$\begin{aligned}
f(a,b,c) &= (a+b+c)(a+\bar{b}+c)(\overline{a}+b+\overline{c}) && \\
f(a,b,c) &= (a+c)(a+c)(\overline{a}+b+\overline{c}) && \textit{Combining} \\
f(a,b,c) &= (a+c)(\overline{a}+b+\overline{c}) && \textit{Simplification} \\
f(a,b,c) &= \boxed{(\mathbf{a}+\mathbf{c})(\overline{\mathbf{a}}+\mathbf{b}+\overline{\mathbf{c}})} &&
\end{aligned}$$

As the answer has a, b and c feeding into two *or* gates which feed into one *and* gate and two of the inputs are inverted, the circuit has $3+2*1+2*2+3*1 = 12$ inputs and $2+2+2 = 6$ gates for a total cost of $12+6 = \boxed{\mathbf{18}}$

(c) *Synthesize (draw the gates) the minimum cost product-of-sums circuit for the logic equation obtained above.*