ECEN 2350 Spring 2017 Homework 3
Samuel Cuthbertson

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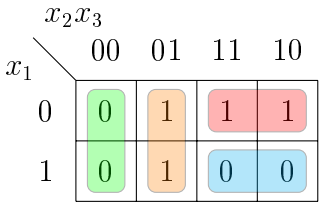
1. *Book Problems: 2.37, 2.39 and 2.46*

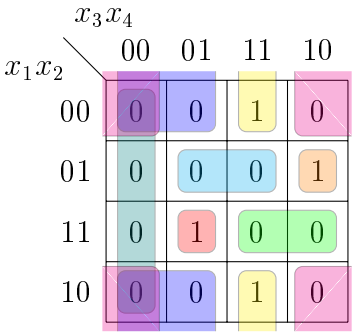
(2.37) *Find the minimum cost SOP and POS forms for* $f(x_1, x_2, x_3) = \sum m(1, 2, 3, 5)$



Therefore, we have the simplified (minimum cost) SOP and POS equations:

$$SOP = \overline{x_1}x_2 + \overline{x_2}x_3$$
$$POS = (\overline{x_1} + \overline{x_2})(x_2 + x_3)$$

(2.39) *Find the minimum cost SOP and POS forms for* $f(x_1, x_2, x_3, x_4) =$
$= \prod M(0, 1, 2, 4, 5, 7, 8, 9, 10, 12, 14, 15)$



Therefore, we have the simplified (minimum cost) SOP and POS equations:

$$SOP = x_1x_2\overline{x_3}x_4 + \overline{x_2}x_3x_4 + \overline{x_1}x_2x_3\overline{x_4}$$
$$POS = (x_2 + x_3)(x_1 + \overline{x_2} + \overline{x_4})(\overline{x_1} + \overline{x_2} + \overline{x_3})(x_3 + x_4)(x_2 + x_4)$$

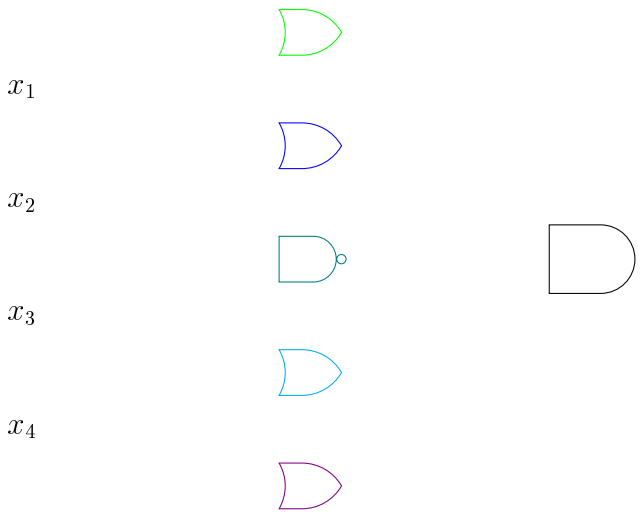
(2.46) *Derive a minimum cost realization of the four-variable function that is equal to 1 if exactly two or exactly three of its variables are equal to 1; otherwise it is equal to 0.*

x_1x_2 \ x_3x_4	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	0	1
10	0	1	1	1

As there are fewer 0's than 1's ($5 < 10$), the lowest cost formula will be the POS using maxterms, as shown here:

$$POS = (x_1 + x_2 + x_3)(x_1 + x_3 + x_4)(x_1 + x_2 + x_4)(\overline{x_1} + \overline{x_2} + \overline{x_3} + \overline{x_4})(x_2 + x_3 + x_4)$$

Also, note that $(\overline{x_1} + \overline{x_2} + \overline{x_3} + \overline{x_4}) = \overline{(x_1x_2x_3x_4)}$

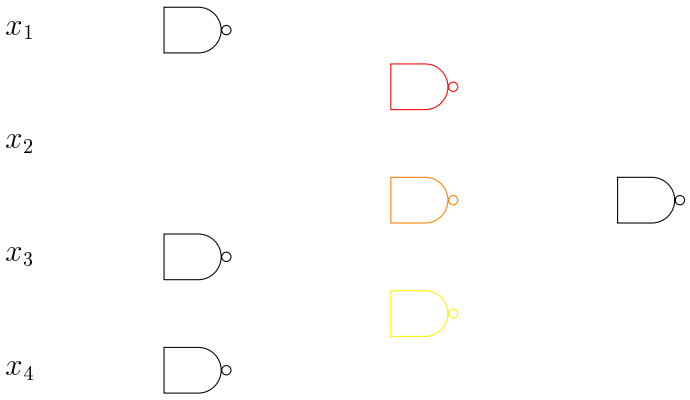


2. *Find the logic equation for the lowest cost circuit for the implementation below in SOP, and then synthesize the circuit with only NAND gates.*

$$f(x_1, x_2, x_3, x_4) = \sum m(0, 4, 5, 6, 7, 14)$$

x_1x_2 \ x_3x_4	00	01	11	10
00	1	0	0	0
01	1	1	1	1
11	0	0	0	1
10	0	0	0	0

$$SOP = \overline{x_1}\,\overline{x_3}\,\overline{x_4} + \overline{x_1}x_2 + x_2x_3\overline{x_4}$$

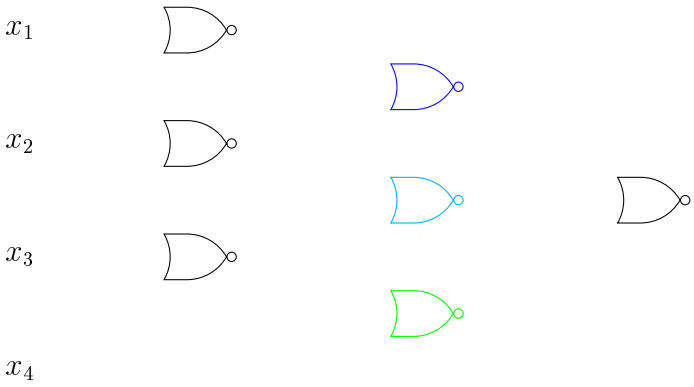


3. *Find the logic equation for the lowest cost circuit for the implementation below in POS, and then synthesize the circuit with only NOR gates.*

$$f(x_1, x_2, x_3, x_4) = \prod M(0, 2, 6, 7, 8, 10, 12, 13, 14, 15)$$

x_1x_2 \ x_3x_4	00	01	11	10
00	0	1	1	0
01	1	1	0	0
11	0	0	0	0
10	0	1	1	0

$$POS = (x_2 + x_4)(\overline{x_2} + \overline{x_3})(\overline{x_1} + \overline{x_2})$$

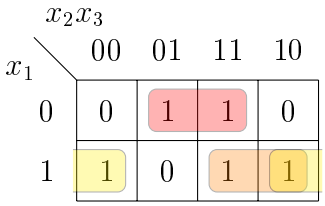


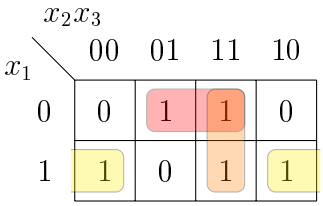
4. *Given the following truth table (from HW1,HW2) use a Karnaugh map to:*

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

x_1 \ x_2x_3	00	01	11	10
0	0	1	1	0
1	1	0	1	1

(a) *Show that there are two minimized solutions for SOP form.*
Both minimized forms are shown below, as there are two ways to select all the one's from the K-Map with a minimum number of selections.

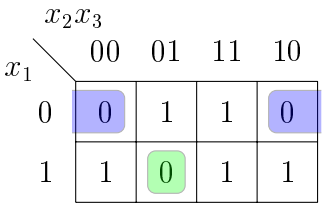




$f(x_1, x_2, x_3) = \overline{x_1}x_3 + x_1x_2 + x_1\overline{x_3}$

$f(x_1, x_2, x_3) = \overline{x_1}x_3 + x_2x_3 + x_1\overline{x_3}$

(b) *Show the minimized POS form.*



$f(x_1, x_2, x_3) = (x_1 + x_3)(\overline{x_1} + x_2 + \overline{x_3})$