ECEN 2350 Spring 2017 Homework 4
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1. *Book Problems: 3.2, 3.3, 3.5 and 3.5*

(3.2) *Determine the decimal values of the following decimal values of the following one's complement numbers.*

(a) *0111011110*
First, note the that given number is positive (MSB is 0) and thus it's value is simply given by the below formula:

$$\begin{aligned} V(b) &= b_0 * 2^0 + b_1 * 2^1 ... b_i * 2^i \\ &= 0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 1 * 2^6 + 1 * 2^7 + 1 * 2^8 \\ &= \boxed{478} \end{aligned}$$

(b) *1011100111*
Now, note that the value is negative (MSB is 1) and thus the magnitude will be given by examining it's complement (0100011000) with the formula mentioned in the previous problem.

$$\begin{aligned} V(b) &= 0 * 2^0 + 0 * 2^1 + 0 * 2^2 + 1 * 2^3 + 1 * 2^4 + 0 * 2^5 + 0 * 2^6 + 0 * 2^7 + 1 * 2^8 \\ &= 280 \end{aligned}$$

Thus, our value is $\boxed{-280}$

(c) *1111111110*
Again, note the the value is negative and that the magnitude is given by it's complement (0000000001).

$$\begin{aligned} V(b) &= 1 * 2^0 \\ &= 1 \end{aligned}$$

Thus, our value is $\boxed{-1}$

(3.3) *Determine the values of the following 2's complement numbers.*
First, recall the below equations for the value of 2's complements numbers.

$$V(b) = b_0 * 2^0 + b_1 * 2^1 + ... + b_{i-1} * 2^{i-1} - b_i * 2^i$$

(a) *0111011110*

$$\begin{aligned} V(b) &= 1 * 2^1 + 1 * 2^2 + 1 * 2^3 + 1 * 2^4 + 1 * 2^6 + 1 * 2^7 + 1 * 2^8 \\ V(b) &= \boxed{478} \end{aligned}$$

(b) *1011100111*

$$V(b) = 1*2^0 + 1*2^1 + 1*2^2 + 1*2^5 + 1*2^6 + 1*2^7 - 1*2^9$$
$$V(b) = \boxed{-281}$$

(c) *1111111110*

$$V(b) = 1*2^1 + 1*2^2 + 1*2^3 + 1*2^4 + 1*2^5 + 1*2^6 + 1*2^7 + 1*2^8 - 1*2^9$$
$$V(b) = \boxed{-2}$$

(3.4) *Convert 73, 1906, -95 and -1630 into signed 12-bit numbers in the following representations.*

(a) *Sign and Magnitude*

$$73 = 64 + 8 + 1 = \boxed{(000001001001)_2}$$
$$1906 = 1024 + 512 + 256 + 64 + 32 + 16 + 2 = \boxed{(011101110010)_2}$$
$$\text{Note:} \quad 95 = 64 + 16 + 8 + 4 + 2 + 1 = (000001011111)_2$$
$$\Longrightarrow \ -95 = \boxed{(100001011111)_2}$$
$$\text{Note:} \quad 1630 = 1024 + 512 + 64 + 16 + 8 + 4 + 2 = (011001011110)_2$$
$$\Longrightarrow \ -1630 = \boxed{(111001011110)_2}$$

(b) *1's Complement*
Take the magnitudes from above, and for the negatives bitwise complement them.

$$73 = \boxed{(000001001001)_2}$$
$$1906 = \boxed{(011101110010)_2}$$
$$-95 = \boxed{(111110100000)_2}$$
$$-1630 = \boxed{(100110100001)_2}$$

(c) *2's Complement*
Take the answers from immediately above, and for the negatives add one.

$$73 = \boxed{(000001001001)_2}$$
$$1906 = \boxed{(011101110010)_2}$$
$$-95 = \boxed{(111110100001)_2}$$
$$-1630 = \boxed{(100110100010)_2}$$

(3.5) *Perform the following operations using 8-bit 2's complement numbers and indicate whether arithmetic overflow occurs.*

	0	0	0	0	1	0	0		Carry
	0	0	1	1	0	1	1	0	x
+	0	1	0	0	0	1	0	1	y
	0	1	1	1	1	0	1	1	x+y

No Arithmetic Overflow, $54 + 69 = 123$.

1	1	1	1	1	1	0	0		Carry
	0	1	1	1	0	1	0	1	x
+	1	1	0	1	1	1	1	0	y
	0	1	0	1	0	0	1	1	x+y

No Arithmetic Overflow, $117 - 34 = 83$.

1	1	1	1	1	0	0	0		Carry
	1	1	0	1	1	1	1	1	x
+	1	0	1	1	1	0	0	0	y
	1	0	0	1	0	1	1	1	x+y

No Arithmetic Overflow, $-33 - 72 = -105$.

0	0	0	0	1	0	1	1		Borrow
	0	0	1	1	0	1	1	0	x
-	0	0	1	0	1	0	1	1	y
	0	0	0	0	1	0	1	1	x-y

No Arithmetic Overflow, $54 - 43 = 11$.

1	0	0	0	0	1	1	0		Borrow
	0	1	1	1	0	1	0	1	x
-	1	1	0	1	0	1	1	0	y
	1	0	1	0	1	1	1	1	x-y

Arithmetic Overflow, $117 - -42 = 117 + 42 = 158 \neq -81$.

1	1	1	0	1	1	0	0		Borrow
	1	1	0	1	0	0	1	1	x
-	1	1	1	0	1	1	0	0	y
	1	1	1	0	0	1	1	1	x-y

No Arithmetic Overflow, $-45 - -20 = -45 + 20 = -25$.

2. *Multiply the following numbers like in the manner shown in Figure 3.36 of the textbook.*

(a) *20*15*

Note: $20 = 16 + 4 = (010100)_2$ and $15 = 8 + 4 + 2 + 1 = (001111)_2$

$$
\begin{array}{cccccccccccc|c}
 & & & & & & 0 & 1 & 0 & 1 & 0 & 0 & 20 \\
 & & & & & \times & 0 & 0 & 1 & 1 & 1 & 1 & 15 \\
\hline
 & & & 1 & 1 & 1 & 1 & & & & & & \text{Carry} \\
 & & & & & & 0 & 1 & 0 & 1 & 0 & 0 & P_0 \\
 & & & & & 0 & 1 & 0 & 1 & 0 & 0 & & P_1 \\
 & & & & 0 & 1 & 0 & 1 & 0 & 0 & & & P_2 \\
 & & & 0 & 1 & 0 & 1 & 0 & 0 & & & & P_3 \\
 & & 0 & 0 & 0 & 0 & 0 & 0 & & & & & P_4 \\
+ & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & P_5 \\
\hline
 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 20 * 15
\end{array}
$$

$$20 * 15 = 300$$

$$(010100)_2 * (001111)_2 = \boxed{(0100101100)_2}$$

(b) *-7*10*

Note: $-7 = -8 + 1 = (11001)_2$ and $10 = 8 + 2 = (01010)_2$

$$
\begin{array}{ccccccccccc|c}
 & & & & & & 1 & 1 & 0 & 0 & 1 & -7 \\
 & & & & & \times & 0 & 1 & 0 & 1 & 0 & 10 \\
\hline
 & 1 & 1 & 1 & & & & & & & & \text{Carry} \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & P_0 \\
 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & & P_1 \\
 & & & & 0 & 0 & 0 & 0 & 0 & & & P_2 \\
 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & & & & P_3 \\
+ & & 0 & 0 & 0 & 0 & 0 & & & & & P_4 \\
\hline
\ldots & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & -7 * 10
\end{array}
$$

$$-7 * 10 = -70$$

$$(11001)_2 * (01010)_2 = \boxed{(10111010)_2}$$

(c) *-3*-12*

Note that $-3 * -12 = 3 * 12$, and $3 = (00011)_2$ while $12 = (01100)_2$

$$
\begin{array}{cccccccccc|c}
 & & & & & 0 & 0 & 0 & 1 & 1 & 3 \\
 & & & & \times & 0 & 1 & 1 & 0 & 0 & 12 \\
\hline
 & & & & 1 & 1 & & & & & \text{Carry} \\
 & & & & & 0 & 0 & 0 & 0 & 0 & P_0 \\
 & & & & 0 & 0 & 0 & 0 & 0 & & P_1 \\
 & & & 0 & 0 & 0 & 1 & 1 & & & P_2 \\
 & & 0 & 0 & 0 & 1 & 1 & & & & P_3 \\
+ & 0 & 0 & 0 & 0 & 0 & & & & & P_4 \\
\hline
 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 3 * 12
\end{array}
$$

$$3 * 12 = 36$$

$$(00011)_2 * (01100)_2 = \boxed{(0100100)_2}$$