Discrete Structures — Problem Set 2

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Problem 1

The “pile method” can be rewritten as $pile(n) = \sum_{i=1}^{n-1}(i)$ The proof of this is as follows:

$$Given\ pile(1) = 1,\ and\ pile(n) = \sum_{i=1}^{n-1}(i)\ for\ all\ n <= 10$$

$$Then\ pile(k+1) = \sum_{i=1}^{k}(i) = pile(k) + (k) = \sum_{i=1}^{k-1}(i) + k\ for\ all\ n \in R$$

Which is a valid strong induction proof, since $\sum_{i=1}^{k}(i) = \sum_{i=1}^{k-1}(i) + k$.

1 Problem 2

(a) Using the same method as described in the phrasing of question 2, we can obtain

$$J\ mod\ 3 = 2 \qquad J\ mod\ 5 = 0 \qquad J\ mod\ 7 = 3$$

$$3 * (-3) + 5 * (2) = 1 \implies 3 * (-3) * (0) + 5 * (2) * (2) = 20$$

And now solving for the final answer given that product and J mod 7 = 3

$$J\ mod\ 15 = 20 \qquad J\ mod\ 7 = 3$$

$$15 * (-\frac{4}{17}) + 5 * (\frac{11}{17}) = 1 \implies 15 * (-\frac{4}{17}) * (3) + 7 * (\frac{11}{17}) * (20) = 80$$

(b) And for the general, we can use the exact same method:

$$J\ mod\ 11 = 1 \qquad J\ mod\ 13 = 8 \qquad J\ mod\ 17 = 2$$

$$11 * (1) + 13 * (-\frac{10}{13}) = 1 \implies 11 * (1) * (8) + 13 * (\frac{-10}{13}) * (1) = 78$$

$$J\ mod\ 143 = 78 \qquad J\ mod\ 17 = 2$$

$$143 * (-\frac{347}{988}) + 17 * (\frac{229}{76}) = 1 \implies 143 * (-\frac{347}{988}) * 2 + 17 * (\frac{229}{76}) * (78) = 3895$$

2 Problem 3

(a) We can say that all the exponents must either be 3 or be 0, since any n^3 can be factored out into some number of primes $p_1^3 * p_2^3 * ... * p_i^3$.

(b) From staring at the list here (https://www.math.upenn.edu/ deturck/m170/wk2/divisors.html), the pattern that pops out is $\sum_{i=1}^{n}(2 * e_n)$

3 Problem 4

(a) This is the definition of Euler's Totient. In the example, 5 and 7 are distinct primes, so the totient is equal to (5-1)(7-1).

(b) If $m = p^k$, the numbers that have a common factor with m are all the multiples of p. There are p^{k-1} multiples, so the totient of p^k is $p^k - p^{k-1}$

4 Problem 5

Note that:
Regular 3-gons: 1
Regular 4-gons: 1
Regular 5-gons: 2
Regular 6-gons: 1
Regular 7-gons: 3
Regular 8-gons: 2
Regular 9-gons: 4
Regular 10-gons: 2
Regular 11-gons: 5

Thus, the number of distinct n-gons is $\frac{n-1}{2}$ if n is odd and $\frac{n-2}{2}$ if n is even, and n-gon(47) $= \frac{47-1}{2} = \frac{46}{2} = 23$.