TO TOLL OR NOT TO TOLL?

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CE392C Course Project, Fall 2018

WHAT WE WILL BE TALKING ABOUT…

- Background on Tolling
- Tolling strategies
- Formulating the problems
- Small network implementation
- Large network implementation
- Effect of demand variation on tolling
- Inferences

BACKGROUND

Recap from the course:

- The motivation behind tolling.
- MSCP (Marginal Social Cost Pricing) tolls: $\beta_{MSCP} = t'(x)x$
- Mathematical Equivalence of UE and SO optimization problems

- Disadvantage of MSCP tolling:

  Tolls on almost every link in the network! $\Rightarrow$ (1) large infrastructure cost involved in setting up of tollbooths, and (2) large revenue in tolls collected from travelers on network

TOLLING STRATEGIES

Different types of tolls based on priorities

- **MINSYS** – Minimizing the total tolls collected
- **MINTB** – Minimizing the number of toll booths
- **MINMAX** – Minimizing the maximum toll on any link
- **ROBINHOOD** – Constraining net tolls collected to be zero (by subsidizing other users)

Ref: "Solving Congestion Toll Pricing Models" – Hearn and Ramana (1998)

FEASIBLE TOLL SET

By writing the LP duality of the UE optimization problem and the KKT conditions for the SO optimization problem, the following feasibility conditions can be obtained:

$$Z^T (\ t(x^*) + \beta\ ) \geq A^T \rho_{rs}$$

$$(x^*)^T (\ t(x^*) + \beta\ ) = d_{rs}^T \rho_{rs}$$

Z - arc-path incidence matrix

A - OD-path incidence matrix

ρ_{rs} *- unknown constant specific to each OD pair*

Ref: "Congestion Toll Pricing of Traffic Networks" – Bergendorff et al. (1997)

OPTIMIZATION FORMULATION

- MINSYS

$$\min_{(\beta,\rho)} \quad \beta^T x^*$$

$$\begin{aligned} s.t. \quad & Z^T(\, t(x^*) + \beta\,) \geq A^T \rho \\ & (x^*)^T(\, t(x^*) + \beta\,) = d^T \rho \\ & \beta \geq 0 \end{aligned}$$

- MINTB

$$\min_{(y,\beta,\rho)} \quad \sum y_{ij}$$

$$\begin{aligned} s.t. \quad & Z^T(\, t(x^*) + \beta\,) \geq A^T \rho \\ & (x^*)^T(\, t(x^*) + \beta\,) = d^T \rho \\ & \beta \leq M y_a \\ & \beta \geq 0 \\ & y_{ij} \in \{0,1\} \end{aligned}$$

METHODOLOGY ADOPTED

Solve for SO flows using MSCP

↓

Enumerate all paths for each OD pair

↓

Find arc-path incidence matrix and OD pair-path incidence matrix

↓

Set up equality and inequality constraints and variable bounds for the linear/integer program

↓

Run the solver

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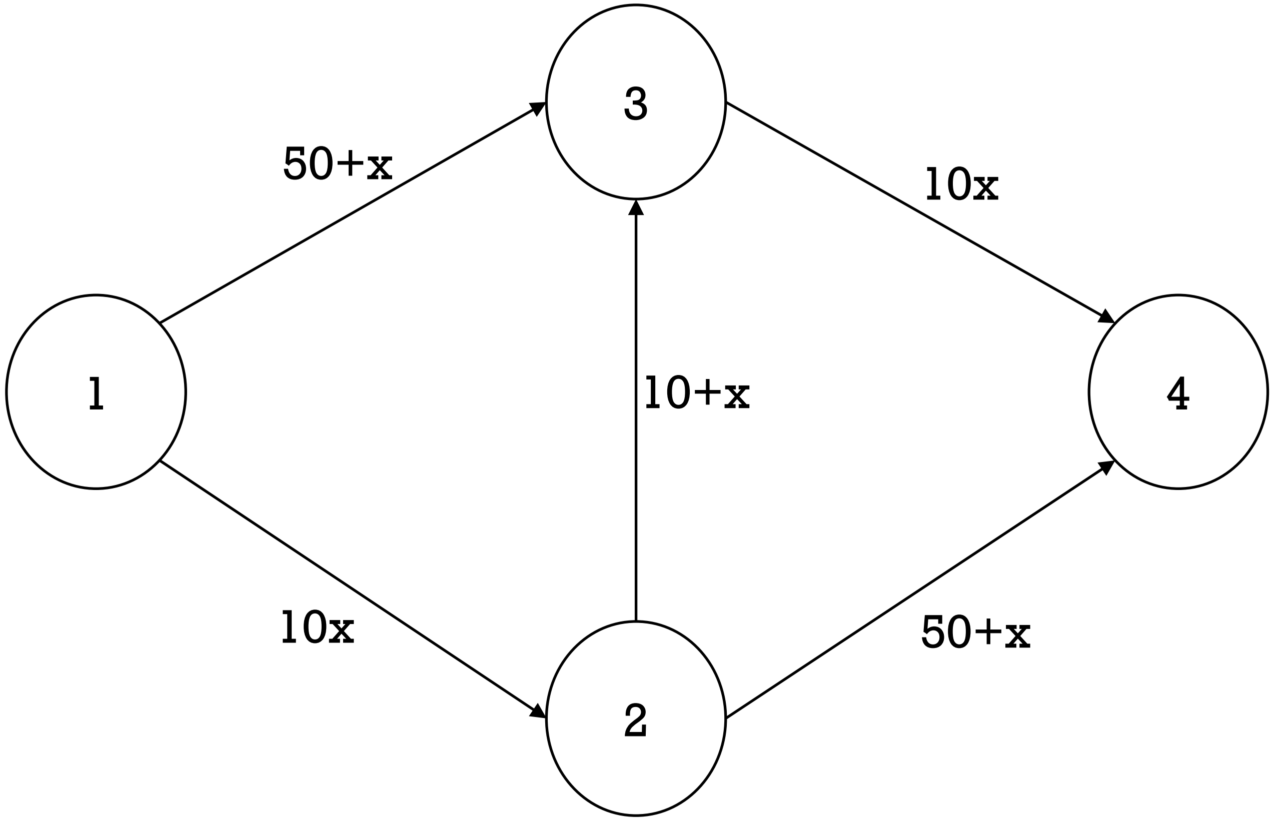
Run the solver

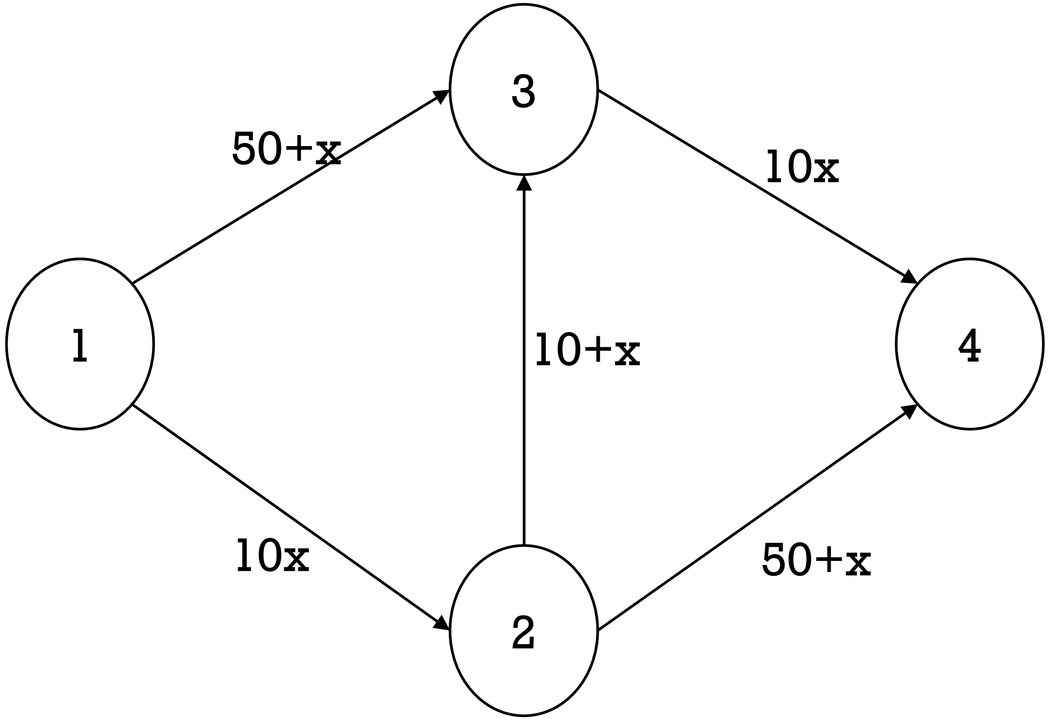
TOLLS FOUND!!!

BRAESS NETWORK !



BRAESS NETWORK !

User Equilibrium

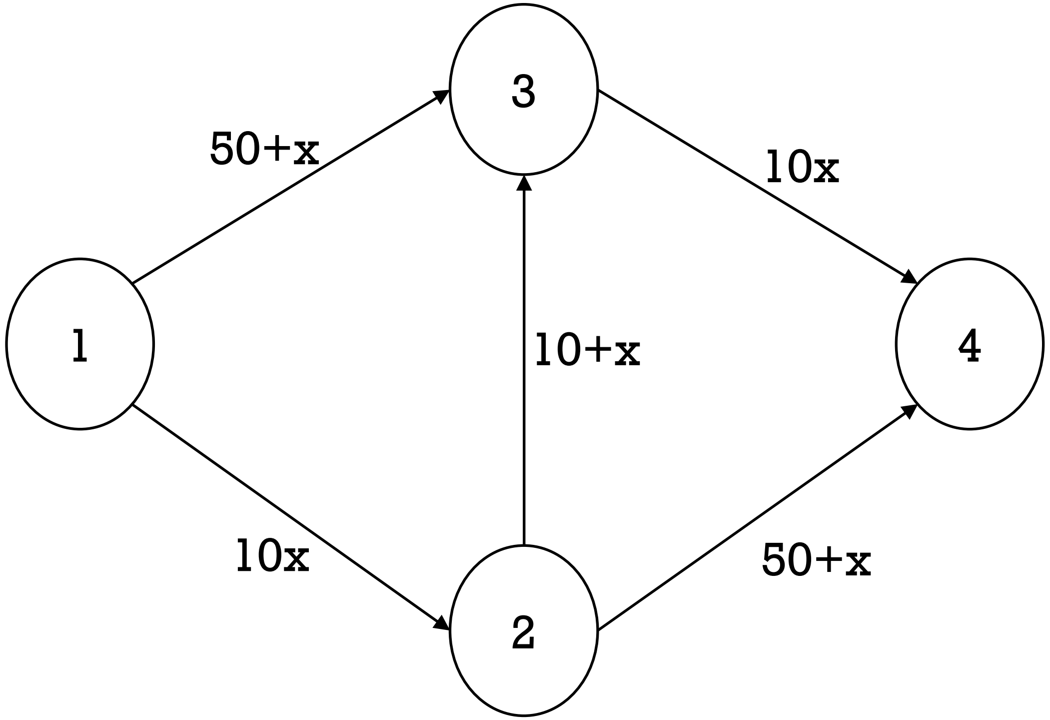


Path cost = 92

BRAESS NETWORK !

$$Path_{134} = 83$$
$$Path_{124} = 82$$
$$Path_{1234} = 70!$$

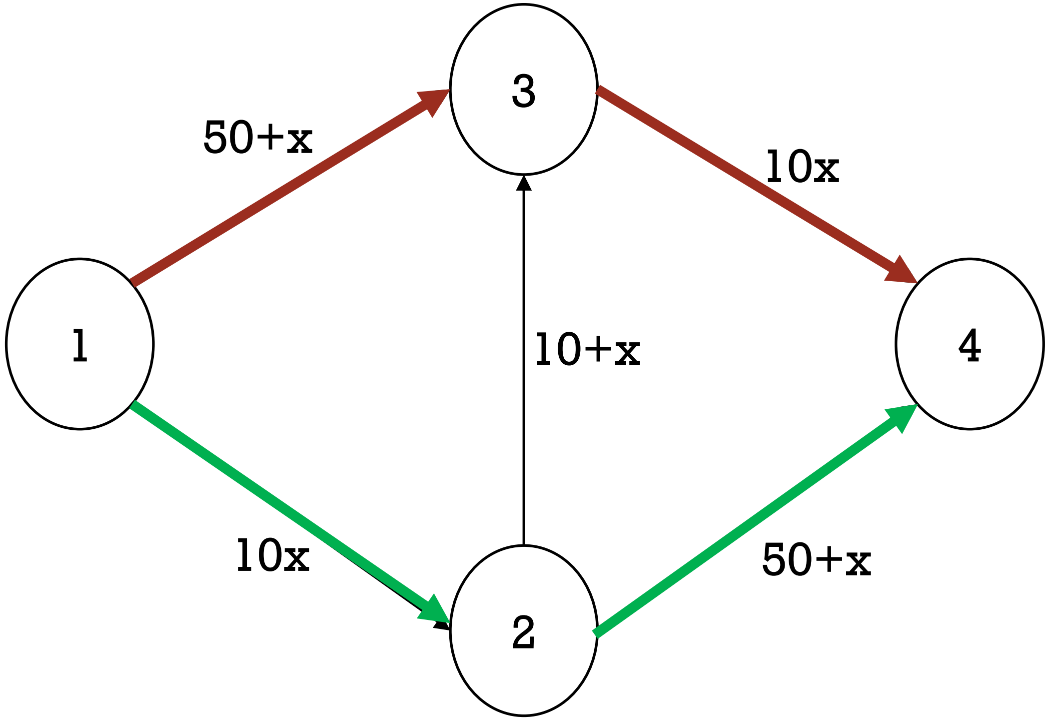
User Equilibrium



Path cost = 92

System Optimal

Flow=3, Path cost =83



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BRAESS NETWORK !

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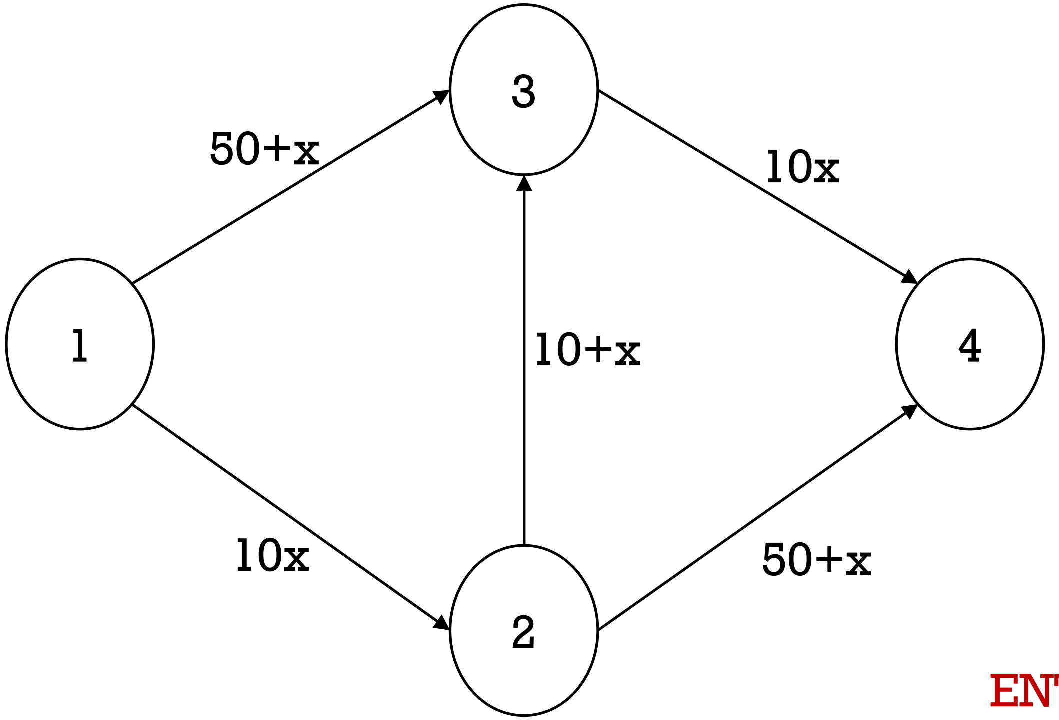

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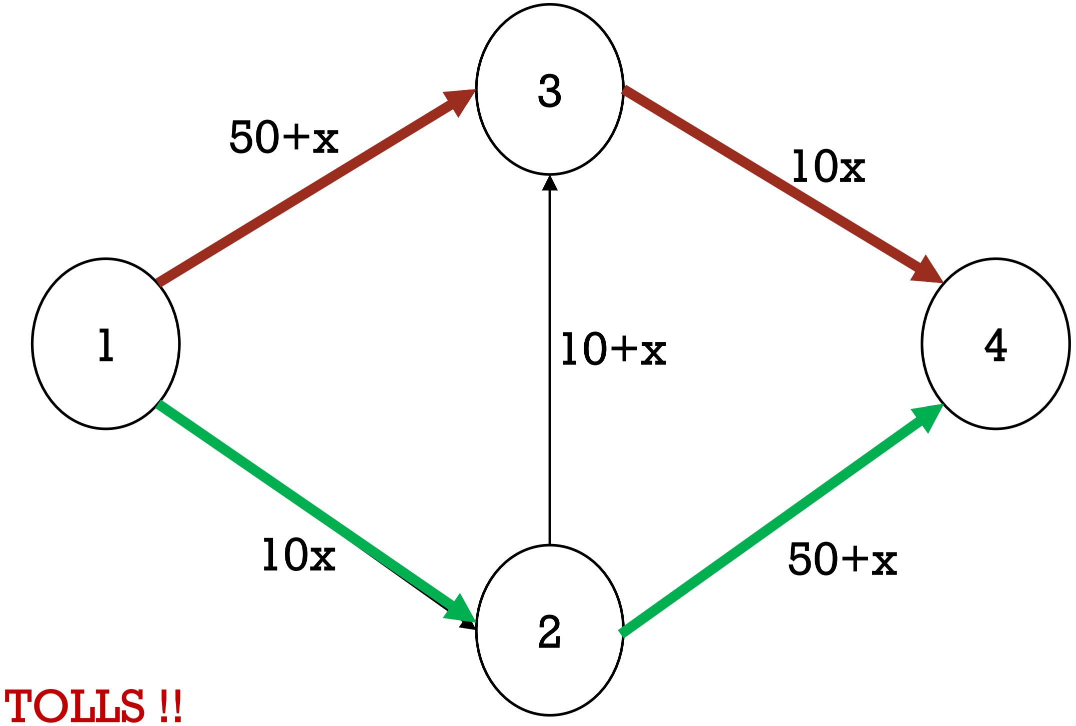
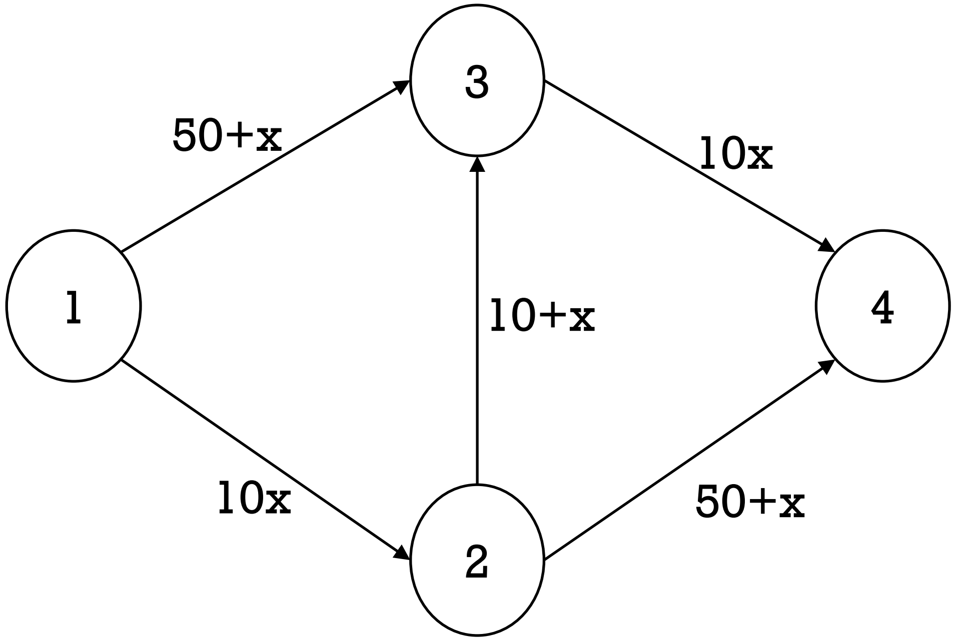


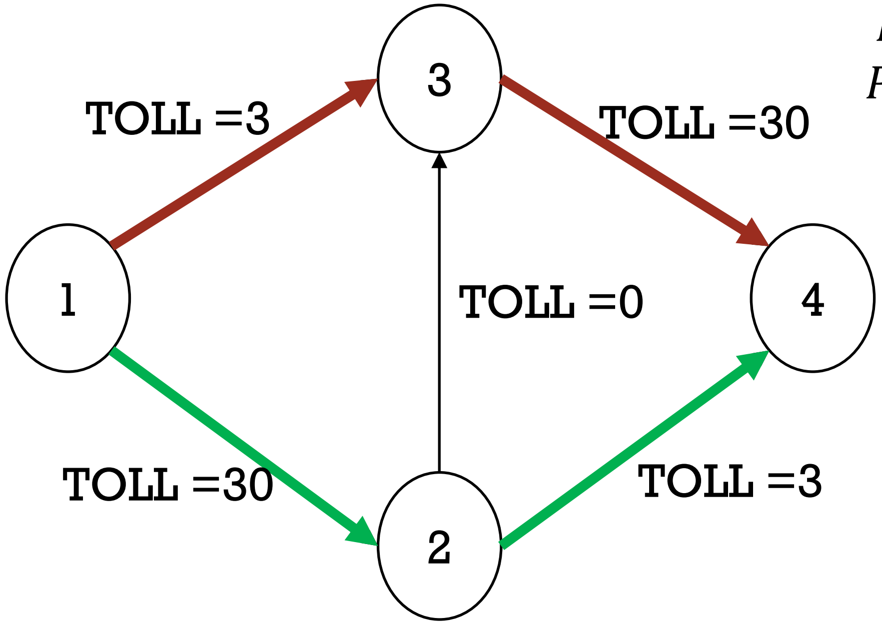
ENTER TOLLS !!

Flow=3, Path cost =83

BRAESS NETWORK !



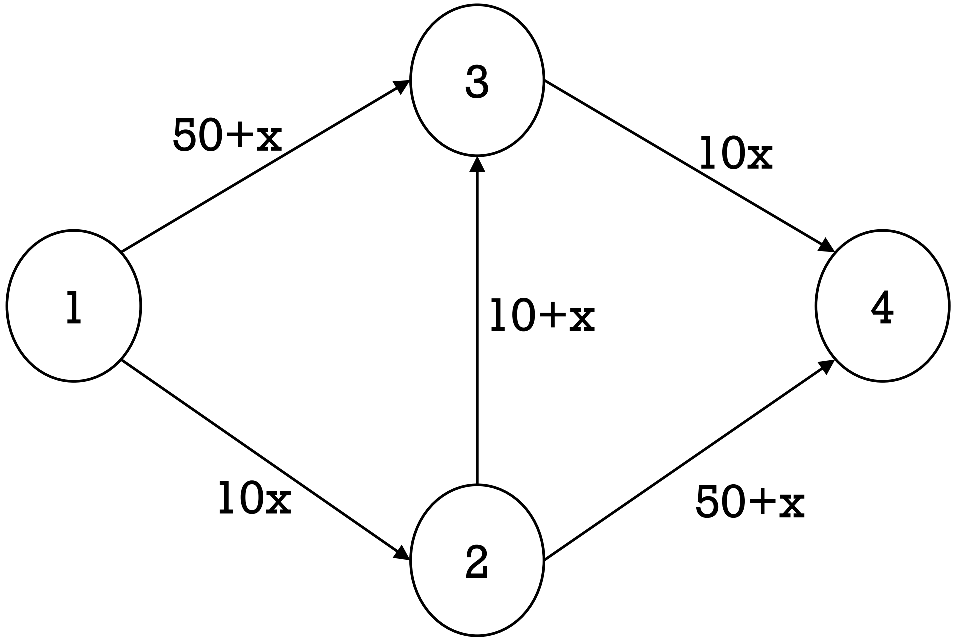
MSCP



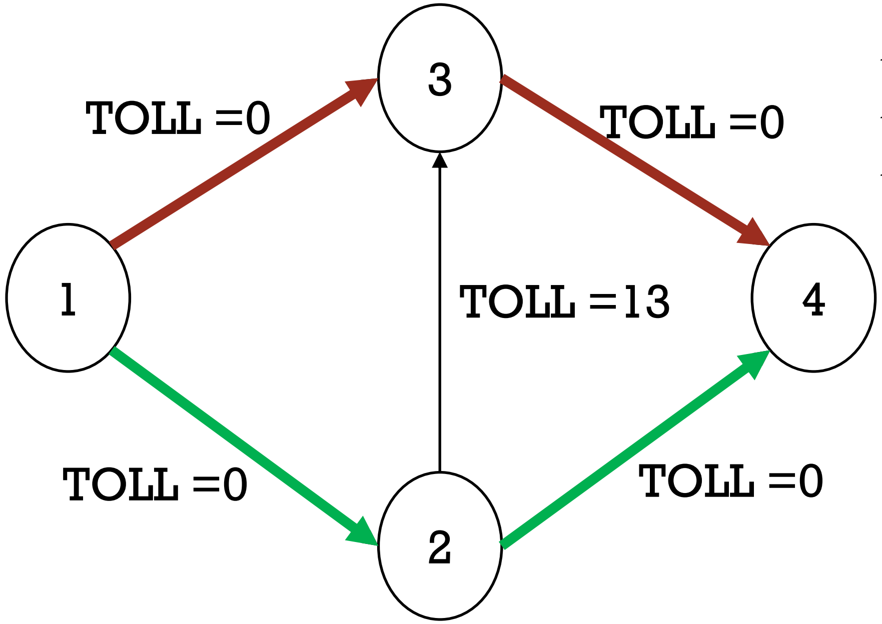
$Path_{134} = 116$
$Path_{124} = 116$
$Path_{1234} = 130$

	MSCP		
Total revenue ($\beta^T x^*$) in minutes	198		
Average toll per user in minutes	33		
Number of toll booths	4		

BRAESS NETWORK !



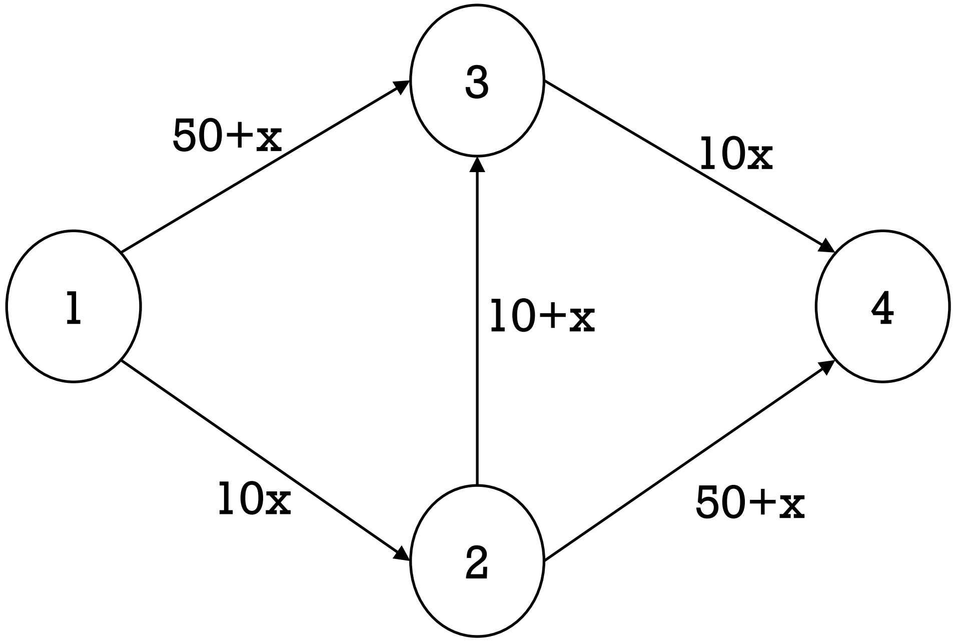
MINSYS



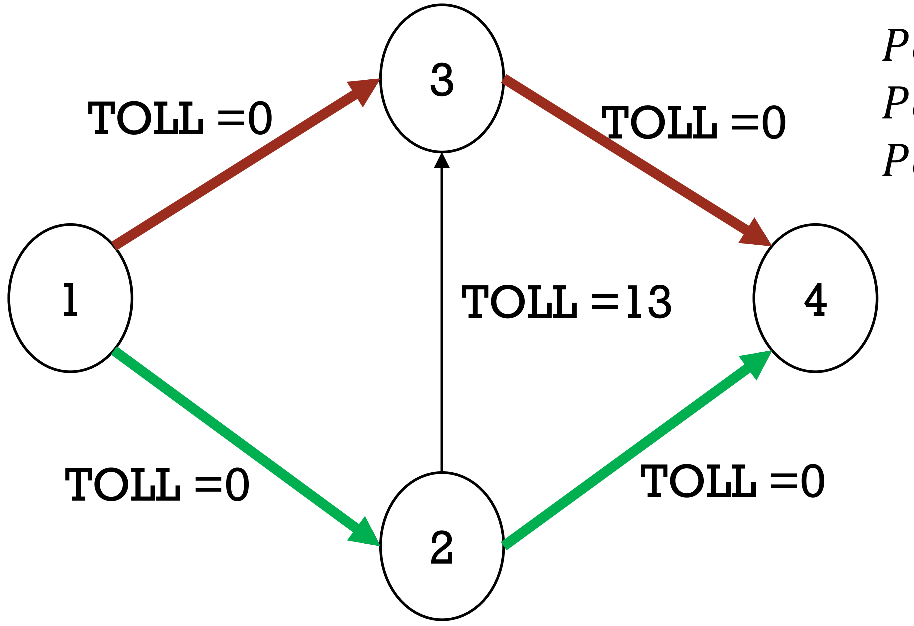
$Path_{134} = 83$
$Path_{124} = 83$
$Path_{1234} = 83$

	MSCP	MINSYS	
Total revenue $(\beta^T x^*)$ in minutes	198	0	
Average toll per user in minutes	33	0	
Number of toll booths	4	1	

BRAESS NETWORK !



MINTB



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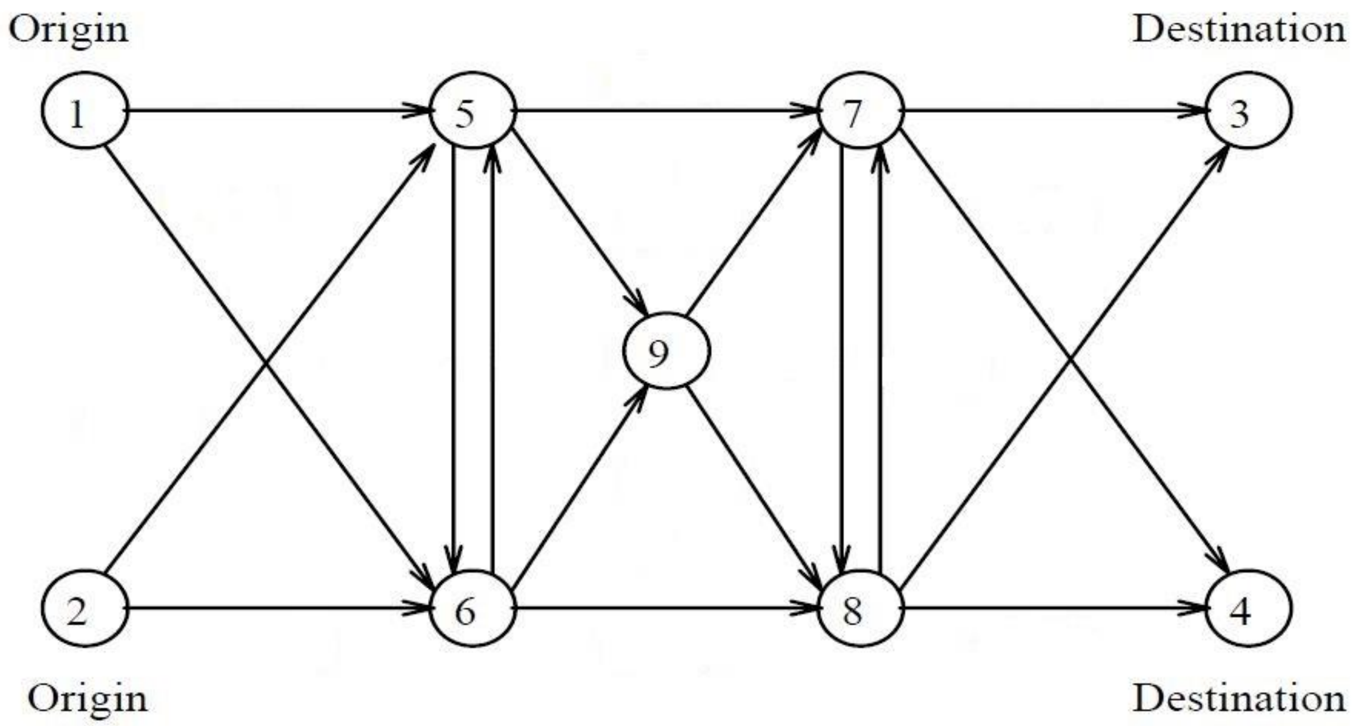
	MSCP	MINSYS	MINTB
Total revenue ($\beta^T x^*$) in minutes	198	0	0
Average toll per user in minutes	33	0	0
Number of toll booths	4	1	1

THE NINE NODE NETWORK

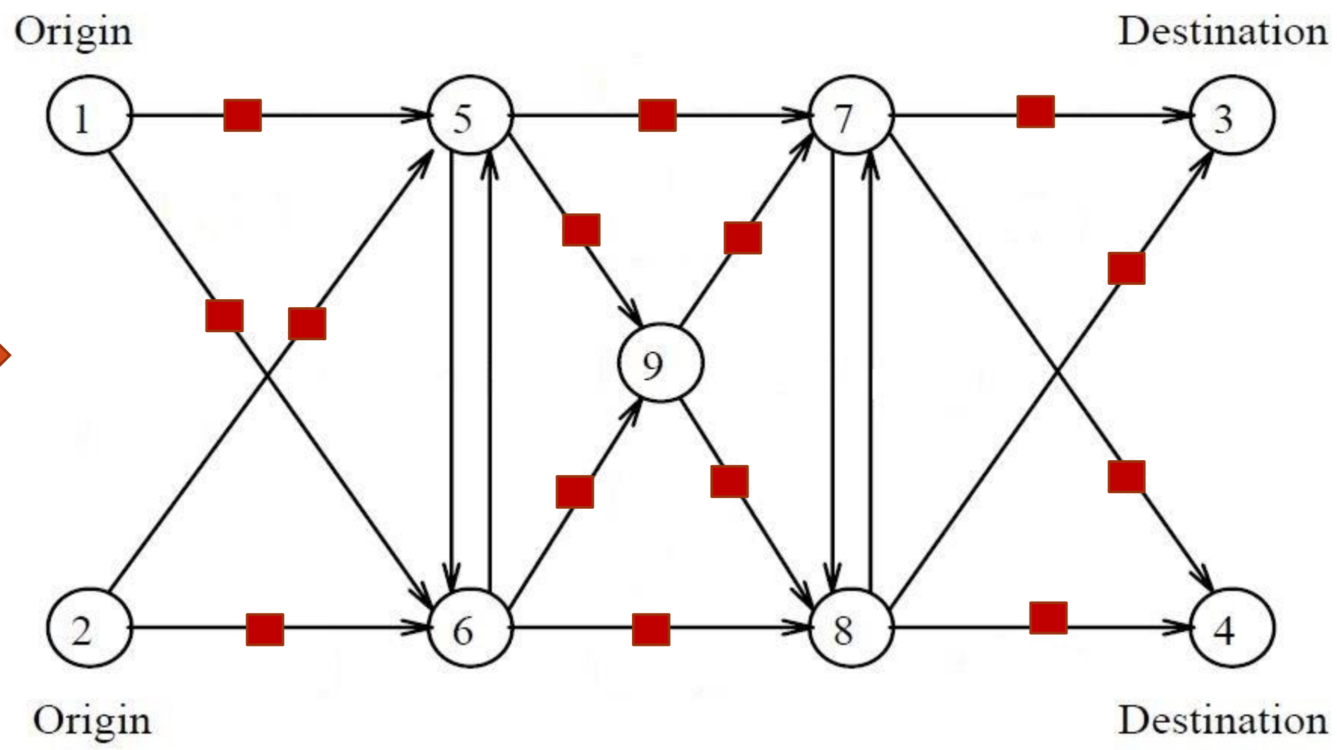


OD pair	Demand
1-3	10
1-4	20
2-3	30
2-4	40

Link performance functions ~ $t_{ij} = t_{ij}^{0}\left[1 + 0.15\left(\frac{x}{c}\right)^{4}\right]$

THE NINE NODE NETWORK

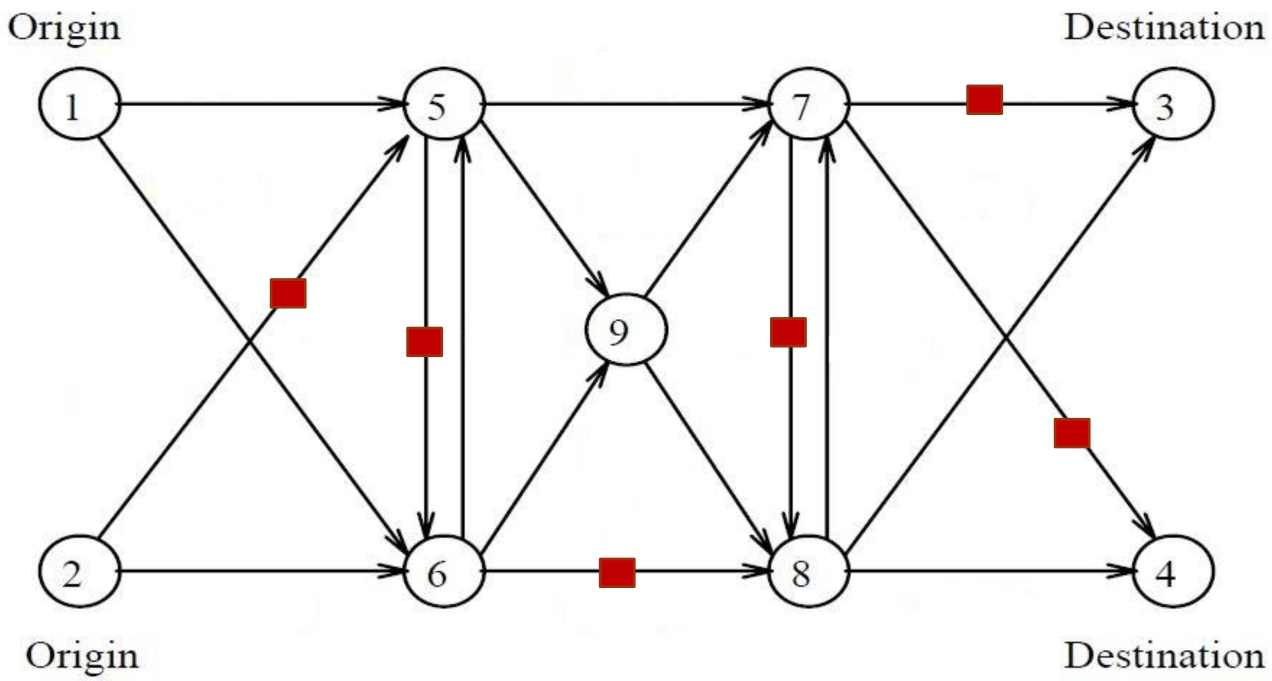
MSCP TOLLING



	MSCP		
Total revenue ($\beta^T x^*$) in minutes	1493.478		
Average toll per user in minutes	14.93		
Number of toll booths	14		

THE NINE NODE NETWORK

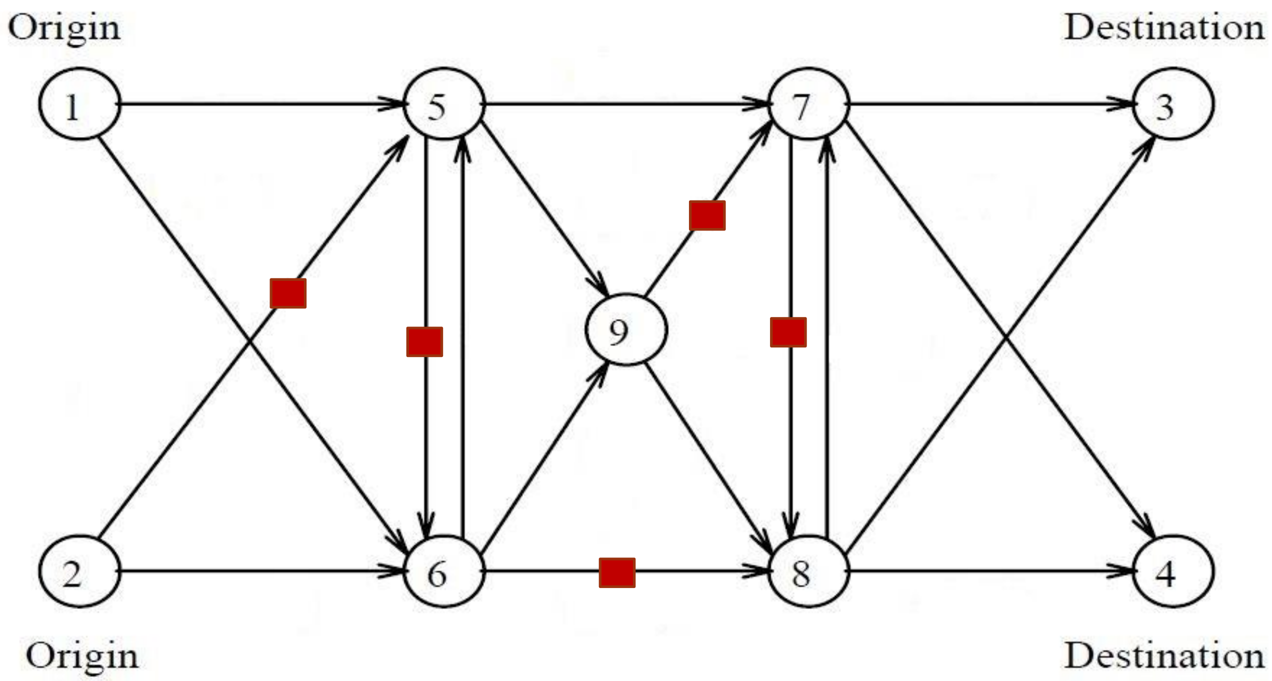
MINSYS TOLLING →



	MSCP	MINSYS	
Total revenue ($\beta^T x^*$) in minutes	1493.478	887.56	
Average toll per user in minutes	14.93	8.88	
Number of toll booths	14	6	

THE NINE NODE NETWORK

MINTB TOLLING



	MSCP	MINSYS	MINTB
Total revenue ($\beta^T x^*$) in minutes	1493.478	887.56	887.57
Average toll per user in minutes	14.93	8.88	8.88
Number of toll booths	14	6	5

BEWARE OF CONVERGENCE!

RELATIVE GAP	E-3	E-4	E-6
TSTT	2254.033	2253.918	2253.916
Total revenue ($\beta^T x^*$) in minutes	1281.92	887.57	887.56
Average toll per user in minutes	128.19	8.88	8.88
Number of toll booths	**8**	**5**	**6**

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IMPLEMENTATION ON LARGER NETWORKS

Obtain the SO flows using the general MSCP formulation

Enumerate all possible paths using a recursive function (1.71 million paths for SF!)

Use k-shortest paths to form a reasonable set of paths (here, k=100)

Tweak the optimization problem for implementing in large networks

Implement the MINSYS and MINTB strategies on the Sioux Falls network

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TWEAKING THE OPTIMIZATION PROBLEM FOR LARGE NETWORKS

- MINSYS

$$min_{(\beta,\rho)} \qquad \beta^T x^*$$

$$s.t. \qquad Z^T(\, t(x^*) + \beta\,) \geq A^T \rho$$

$$(x^*)^T(\, t(x^*) + \beta\,) = d^T \rho$$

$$\beta \geq 0$$

TWEAKING THE OPTIMIZATION PROBLEM FOR LARGE NETWORKS

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$$min_{(\beta,\rho)} \quad \beta^T x^*$$

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$$(x^*)^T(\, t(x^*) + \beta \,) = d^T\rho$$

$$\beta \geq 0$$

- MINSYS (for larger networks)

$$min_{(\beta,\rho)} \quad \beta^T x^* + \boldsymbol{\mu}[(\boldsymbol{x}^*)^{\boldsymbol{T}}(\, \boldsymbol{t}(\boldsymbol{x}^*) + \boldsymbol{\beta} \,) - \boldsymbol{d}^{\boldsymbol{T}}\boldsymbol{\rho}]$$

$$s.t. \quad Z^T(\, t(x^*) + \beta \,) \geq A^T\rho$$

$$\beta \geq 0$$

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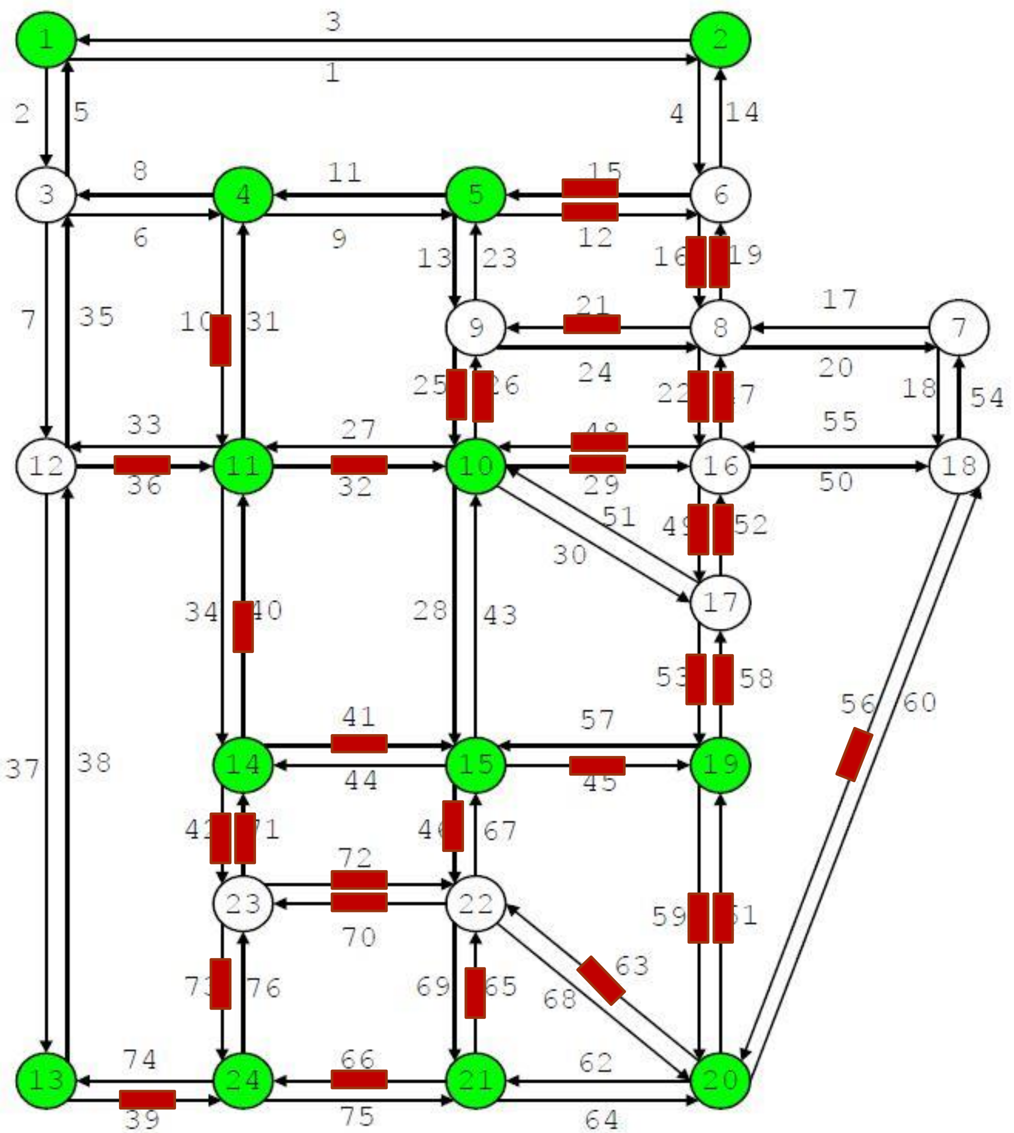
TOLLING THE SIOUX FALLS NETWORK

	MSCP	MINSYS	MINTB
Total revenue ($\beta^T x^*$) in minutes	14.93 E6	2.08 E6	2.23 E6
Toll revenue ($)	$2488K	$346K	$368K
Average toll per user	$6.9	$0.95	$1.0
Number of toll booths	**76**	**39**	**38**

Assumed VOTT = $10/hr



MINTB solution

DEMAND VARIATION ON NINE NODE NETWORK

	Base demand	50% increases	100% increased
Demand	100	150	200
Total revenue ($\beta^T x^*$) in minutes	887.56	1591.5	2060.9
Average toll per user in minutes	8.88	10.61	10.30
Number of toll booths	6	8	8

INFERENCES SO FAR…

- MINSYS and MINTB optimization problems provide much more realistic tolls than MSCP.
- MINSYS and MINTB strategies gave almost the same results for the studied networks. (Coincidence?!)
- Relative gap convergence to a "sufficient" level is crucial.
- Increase in the overall demand causes an increase in the total revenue collected in tolls but the average toll per user remains somewhat similar.

FUTURE WORK

- Analyse the impact of partial changes in the demands (some OD pairs at a time) on the tolls obtained for Sioux Falls network.

Thank you!
Questions?