Python Questions:

Question 1 – Algorithm Performance

In this question we compare the performance of the Franke-Wolfe Algorithm (FW) and Method of Successive Averages (MSA) for finding the user equilibrium link flow solution on a network. We use the Sioux Fall Transportation Network for this study. The network structure and demand files for this network were obtained from the Transportation Network Test Problems (TNTP) database available on the following website:

https://github.com/bstabler/TransportationNetworks

The convergence measure used is Average Excess Cost (AEC) with a target value of 10^{-6}. We test the performance of the two algorithms at different values of their respective parameters like step size for MSA and precision for FW. Apart from this we also test the performance of two different methods of finding step size for FW, that is, Newton's Method and Bisection method. We note the AEC values when running the two algorithms (with each of these variations) at different number of iterations and different amounts of times of execution of the algorithm.

Time is measured using the python function time.time() which provides the wall clock time for running the algorithms.

SPECIFICATIONS TESTED:

The following specifications were tested:

A) **MSA step size formula:**

1. Constant value 0.5

2. Constant value 0.1

3. Variable step size 1 / (1 + i); where i is the number of iteration

4. Variable step size $1 / i^{2/3}$

B) **FW Step Size calculation Method:**

1. Newton's method

2. Bisection method

C) **FW precision for step size calculation**

1. 10^{-1}

2. 10^{-4}

3. 10^{-6}

COMPARISONS WITHIN ALGORITHMS

(A) COMPARISON BETWEEN DIFFERENT STEP SIZE FORMULAE FOR MSA

AEC vs Iterations (MSA)

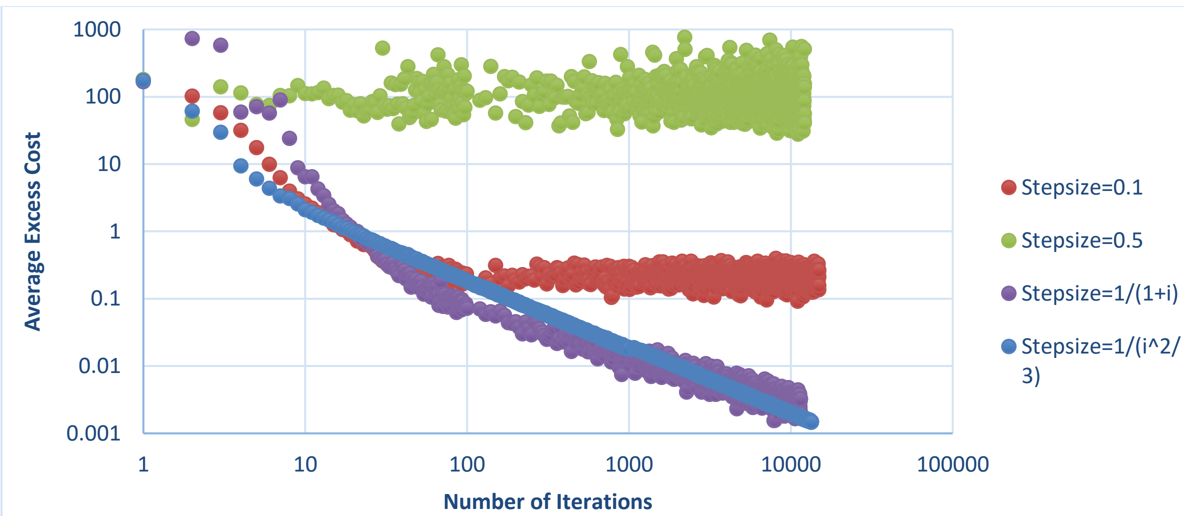

Figure 1 AEC vs Iterations (MSA)

AEC vs Computational Time (MSA)

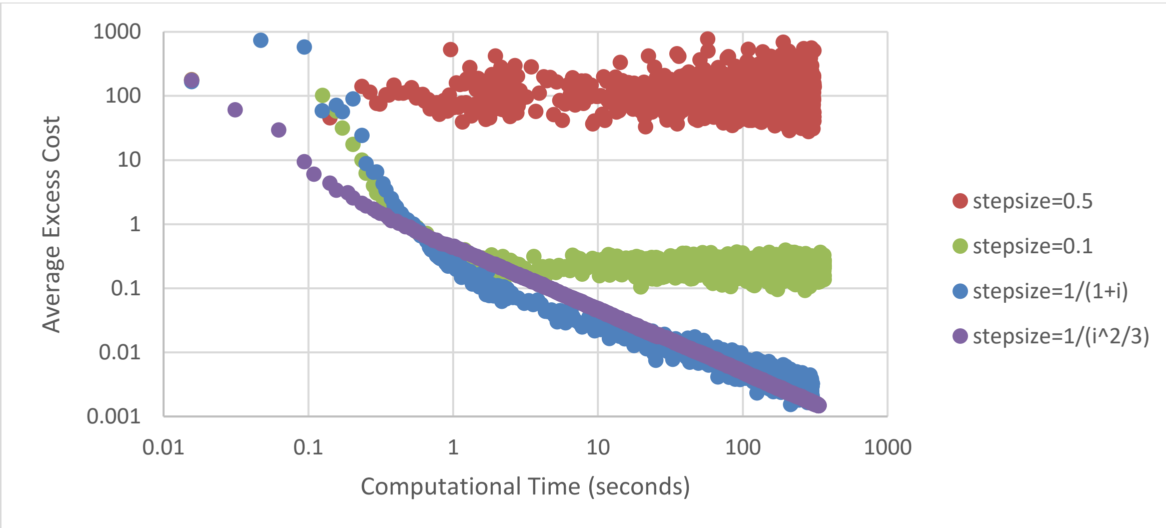

Figure 2 AEC vs Computational Time (MSA)

COMPARISON BETWEEN DIFFERENT PRECISION VALUES IN FW USING BISECTION METHOD:

AEC vs Iterations (FW-Bisection)

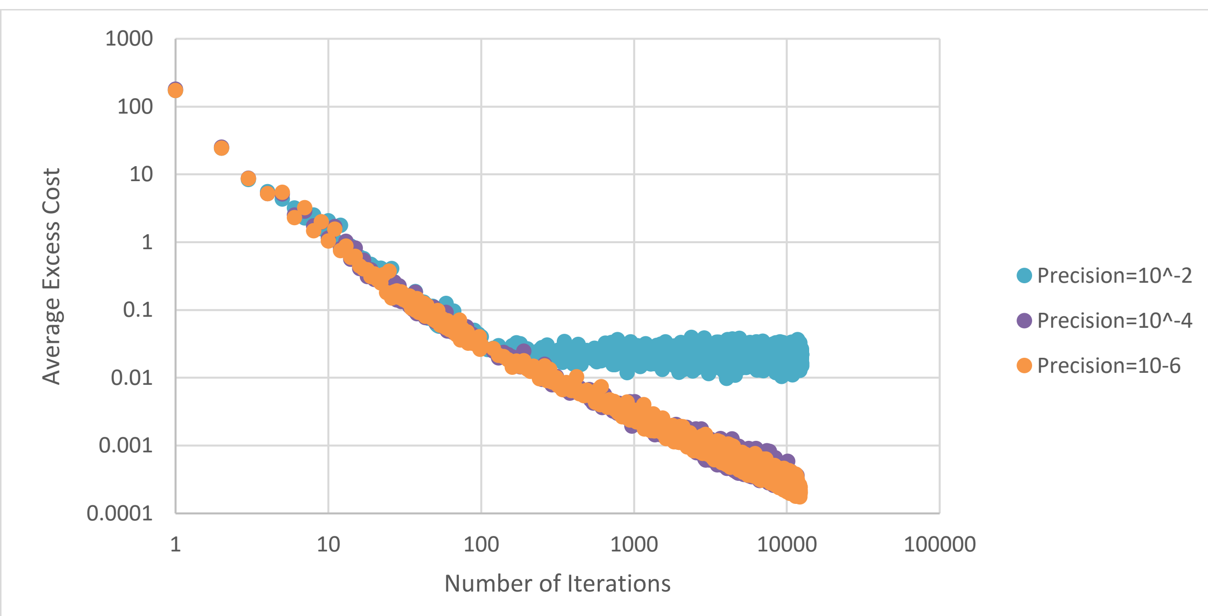

Figure 3 AEC vs Iterations (FW-Bisection)

AEC vs Computational Time (FW-Bisection)

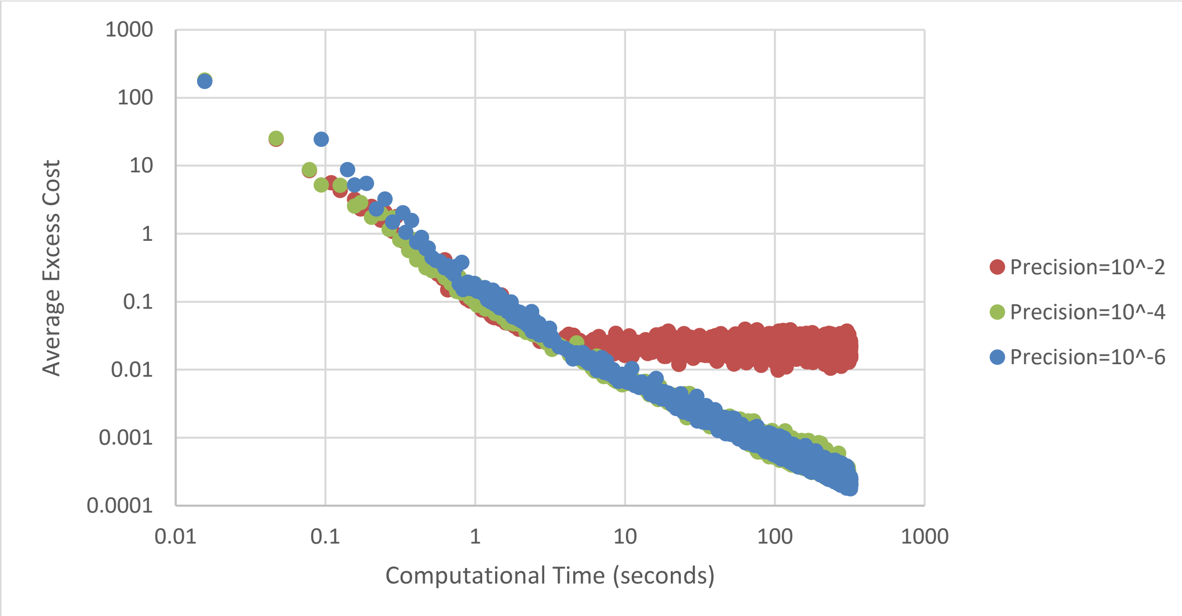

Figure 4 AEC vs Computational Time (FW-Bisection)

COMPARISON BETWEEN DIFFERENT PRECISION VALUES IN FW FOR NEWTON'S METHOD:

AEC vs Iterations (FW-Newton's)

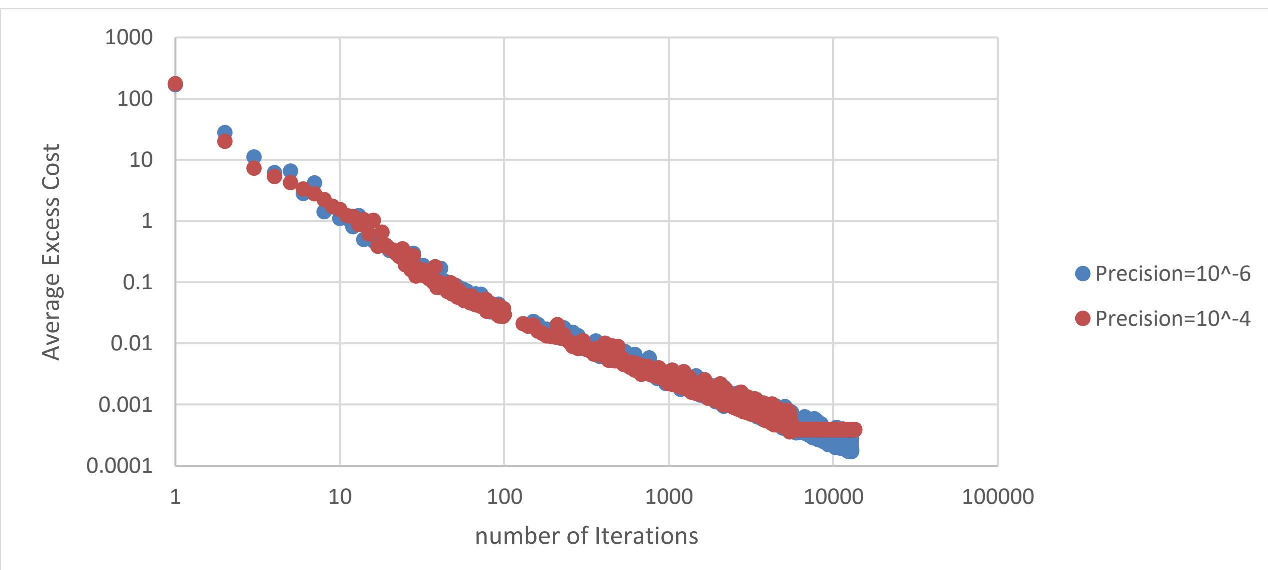

Figure 5 AEC vs Iterations (FW-Newton's)

AEC vs Computational Time (FW-Newton's)

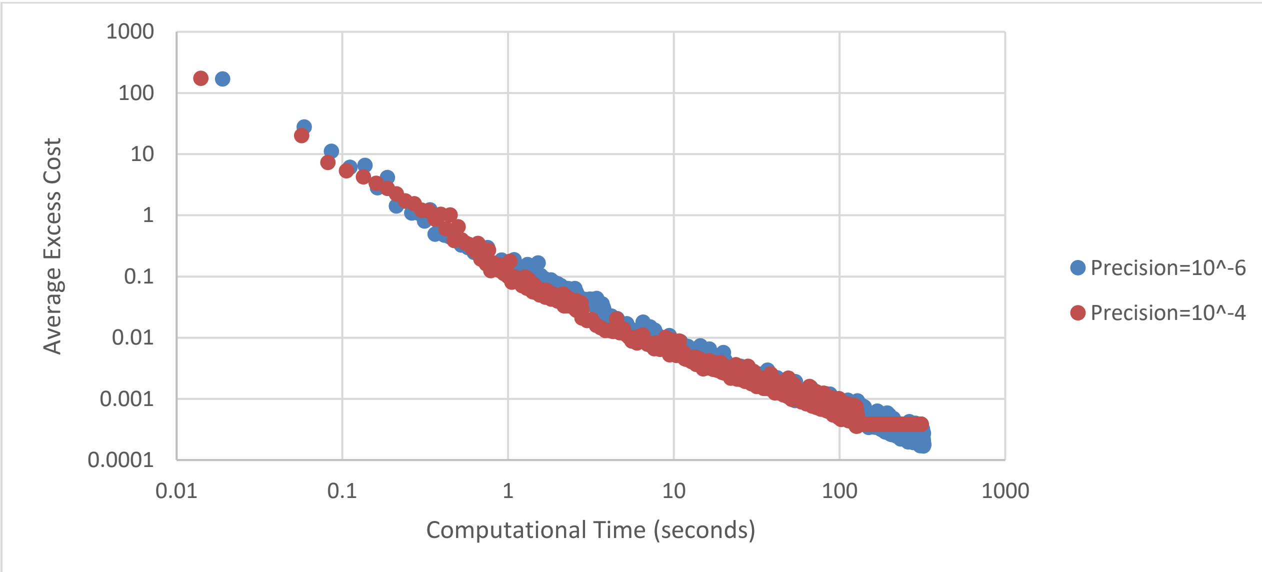

Figure 6 AEC vs Computational Time (FW-Newton's)

COMPARISON BETWEEN FW USING NEWTON'S METHOD AND BISECTION METHOD FOR A PRECISION VALUE OF 10^{-6}

AEC vs Iterations (FW)

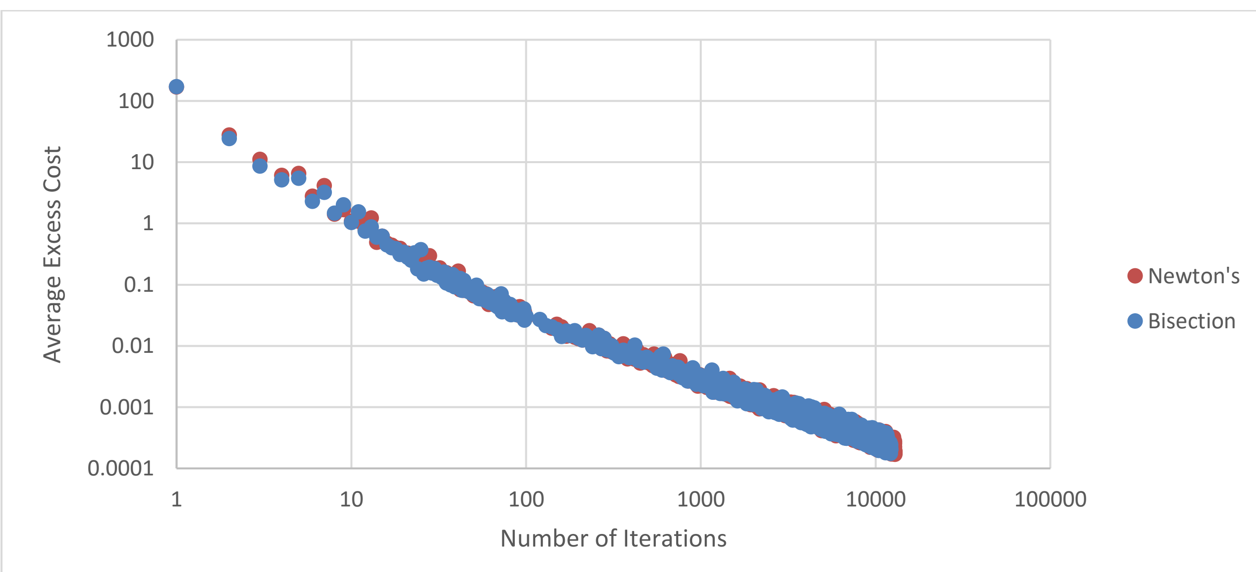

Figure 7 AEC vs Iterations (FW-Newton and Bisection)

OBSERVATIONS

- In MSA, the step size of 1/(i^2/3) is the best over large number of iterations and large computation time.

- In FW, precision value of 10^-6 is better than its competitors.

- The performance of Newton and Bisection Methods in FW appear to be almost exactly the same for the given Sioux Fall network.

COMPARISON BETWEEN MSA & FWA

Based on the above observations we will finally compare MSA with step size 1/(i^2/3) with FW with precision 10^-6 and both Newton's Method and Bisection Method.

AEC vs Iterations (FW and MSA)

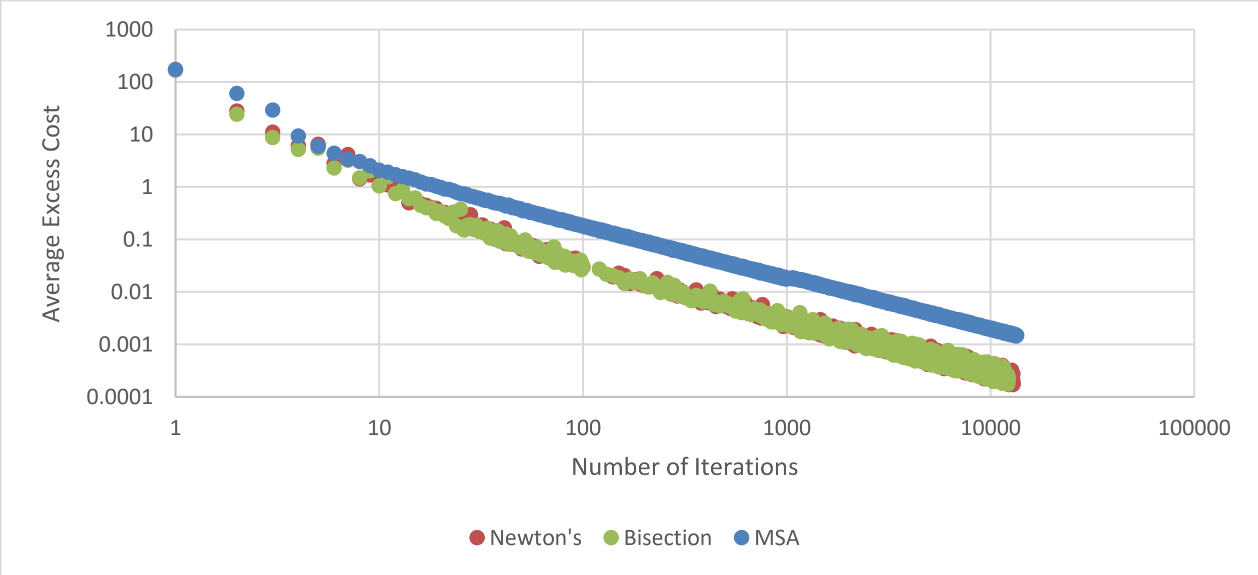

Figure 8 AEC vs Iterations (FW and MSA)

AEC vs Time (MSA vs FW)

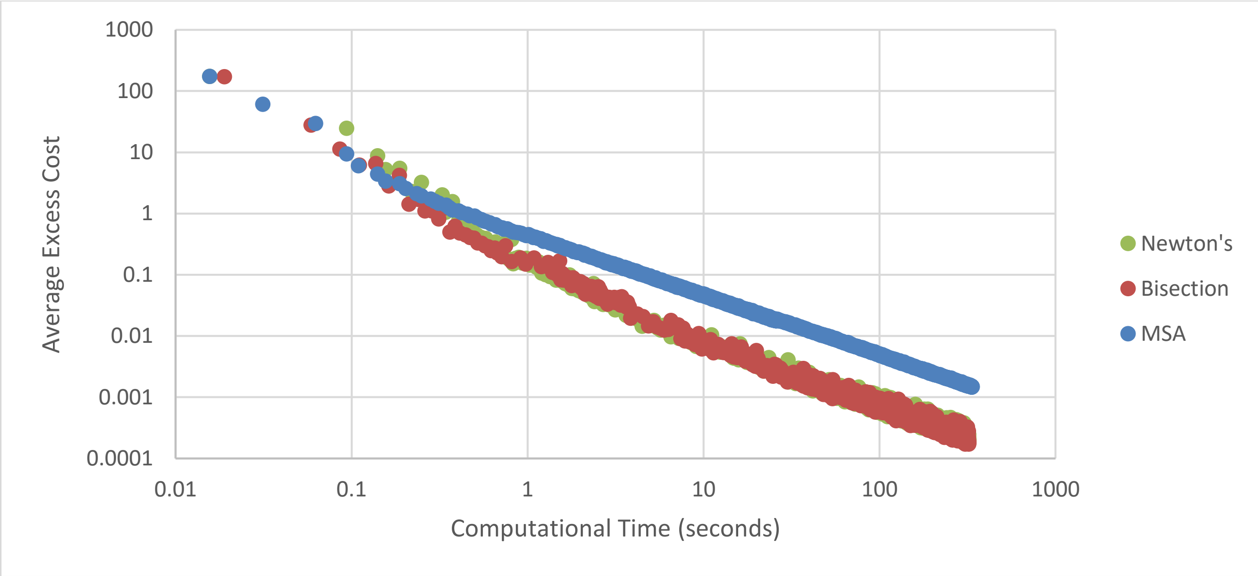

Figure 9 AEC vs Time (MSA vs FW)

From the above two graphs we can say that FW (with Newton or Bisection method) works much better than MSA over large number of iterations and Computational Time. The difference is almost of one complete order over about 10,000 iterations and a computation time of 300 seconds.

<u>Question 2</u> – Effects of a network change

In this question we evaluate the changes in the flows and travel times in a network after the introduction of a major change in the link structure. A highway in Sioux Falls is to be modified such that 10 of its links now have twice the capacity and half the free flow time from before. The network structure in given in figure below. The red line indicates the proposed highway:

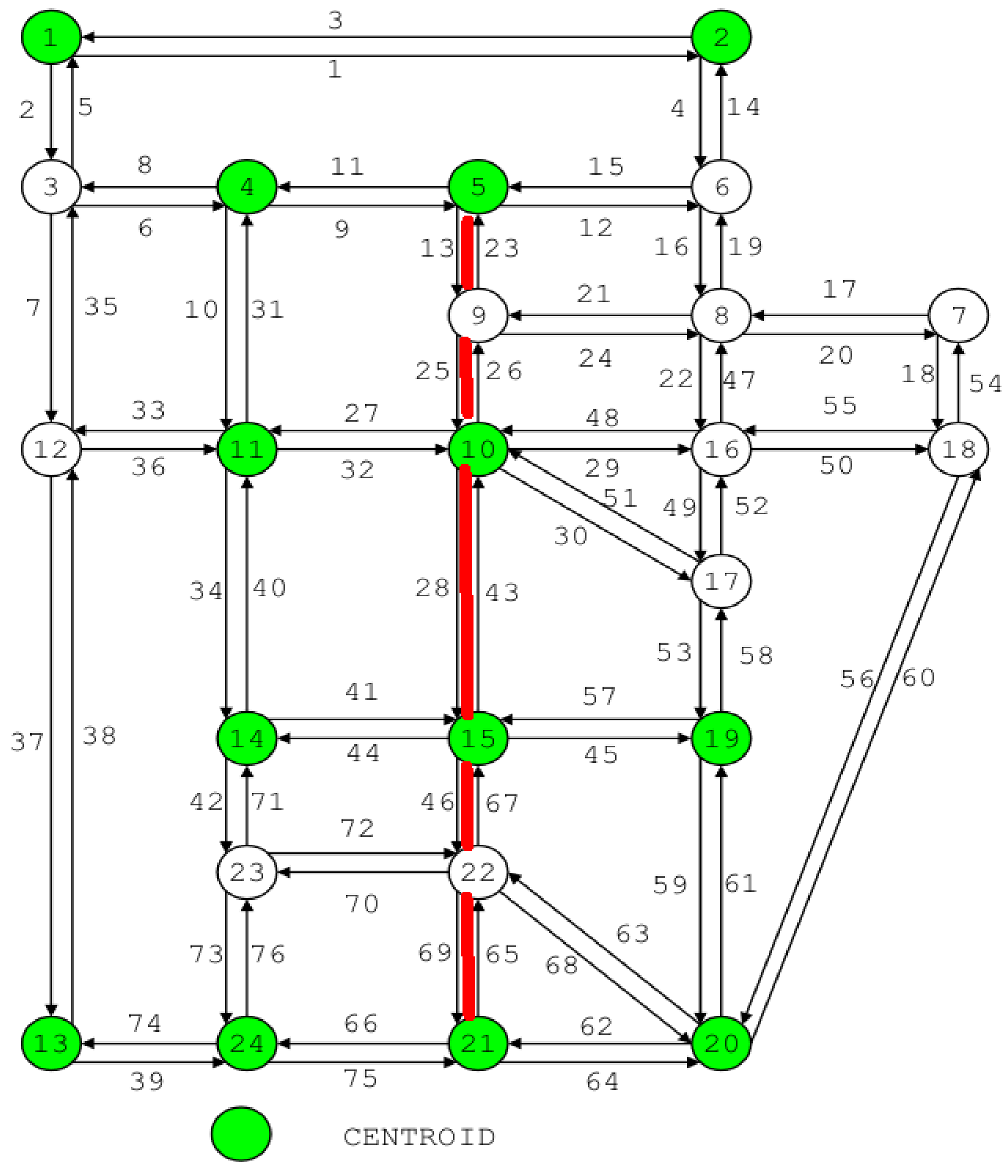

Figure 10 Sioux Falls Network

To make this evaluation, we first run a user equilibrium solving algorithm on the network before the change, and once after the change is incorporated. By taking the difference between the equilibrium flows and travel times in the before and after states of the network, we can understand how the change affects the users of the network.

ALGORITHM SPECIFICATIONS:

For this analysis we use Frank Wolfe algorithm with precision value 10^-9, and Newton's Method to calculate the step size at every iteration. The convergence criteria is taken as Average Excess cost of which the final value obtained in both the runs (before and after) of the algorithm is around 4 x 10^-5.

RESULTS:

The following Table contains the link flows before and after the change: (Red fill indicates decrease in flow, no fill indicates increase in flow, and green fills indicate the links belonging to the highway)

Table 1 Link Flows

Link ID	Link Flow before highway	Link Flow after highway	Change in Flows
(1,2)	4494.670151	4265.954278	-228.716
(1,3)	8119.091851	7873.505861	-245.586
(2,1)	4519.092784	4273.498398	-245.594
(2,6)	5967.344656	6065.953241	98.60858
(3,1)	8094.669219	7865.961742	-228.707
(3,4)	14006.41489	17304.82762	3298.413
(3,12)	10022.3741	13073.42504	3051.051
(4,3)	14030.60131	17338.46031	3307.859
(4,5)	18006.41387	22370.47694	4364.063
(4,11)	5200.04135	5134.362626	-65.6787
(5,4)	18030.60425	22534.29022	4503.686
(5,6)	8798.262074	8534.050648	-264.211
(5,9)	15780.80603	22136.43214	6355.626
(6,2)	5991.767289	6073.49736	81.73007
(6,5)	8806.479888	8526.509632	-279.97
(6,8)	12492.92959	10600.00448	-1892.93
(7,8)	12101.53717	10664.66674	-1436.87
(7,18)	15794.04647	10435.33763	-5358.71
(8,6)	12525.57004	10600.00758	-1925.56
(8,7)	12040.93198	10632.85317	-1408.08
(8,9)	6882.659538	7064.65793	181.9984
(8,16)	8388.691436	6100.006116	-2288.69
(9,5)	15796.7786	22307.78643	6511.008
(9,8)	6836.694428	7032.846395	196.152
(9,10)	21744.09343	34401.07566	12656.98
(10,9)	21814.1009	34640.61841	12826.52

(10,11)	17726.61967	15156.72562	-2569.89
(10,15)	23125.81732	32833.77264	9707.955
(10,16)	11047.09264	10501.60869	-545.484
(10,17)	8100.002369	7947.923343	-152.079
(11,4)	5300.037392	5104.182041	-195.855
(11,10)	17604.22811	15079.73477	-2524.49
(11,12)	8365.291464	5692.022197	-2673.27
(11,14)	9776.124292	6563.551262	-3212.57
(12,3)	9973.765044	13032.24823	3058.483
(12,11)	8404.936404	5708.924988	-2696.01
(12,13)	12287.66411	10803.94527	-1483.72
(13,12)	12378.69999	10879.67126	-1499.03
(13,24)	11121.3507	9020.296811	-2101.05
(14,11)	9814.083836	6539.477037	-3274.61
(14,15)	9036.33141	8499.992369	-536.339
(14,23)	8400.426751	4860.52774	-3539.9
(15,10)	23192.29732	33014.81809	9822.521
(15,14)	9079.826681	8499.992742	-579.834
(15,19)	19083.30065	19370.22295	286.9223
(15,22)	18409.90554	26582.95695	8173.051
(16,8)	8406.691805	6100.007181	-2306.68
(16,10)	11073.00996	10522.64144	-550.369
(16,17)	11694.96989	10867.97286	-826.997
(16,18)	15278.38031	11633.61988	-3644.76
(17,10)	8100.004073	7962.378746	-137.625
(17,16)	11683.8035	10843.90017	-839.903
(17,19)	9953.01447	9905.6016	-47.4129
(18,7)	15854.65166	10467.15121	-5387.5
(18,16)	15333.46439	11678.72639	-3654.74
(18,20)	18976.85719	15644.71028	-3332.15
(19,15)	19116.72947	19417.1113	300.3818
(19,17)	9941.849782	9895.984311	-45.8655
(19,20)	8688.358723	7874.245873	-814.113
(20,18)	18992.54646	15621.63037	-3370.92
(20,19)	8710.622851	7911.516932	-799.106
(20,21)	6302.032445	6541.933861	239.9014
(20,22)	7000.011961	8091.272892	1091.261
(21,20)	6239.993183	6489.585382	249.5922
(21,22)	8619.526026	12725.85745	4106.331
(21,24)	10309.38381	9735.496137	-573.888
(22,15)	18386.45199	26617.11443	8230.662
(22,20)	7000.00462	8057.812512	1057.808
(22,21)	8607.369647	12725.15701	4117.787

(22,23)	9661.824178	8800.000318	-861.824
(23,14)	8394.891024	4836.453142	-3558.44
(23,22)	9626.206907	8799.996966	-826.21
(23,24)	7902.995343	4360.543245	-3542.45
(24,13)	11112.38658	8996.022792	-2116.36
(24,21)	10259.50092	9683.848105	-575.653
(24,23)	7861.842344	4336.465296	-3525.38

We can see from the table that the links which are a part of the highway show a significant increase in the flow values while most of the other links in the network face decreased flow values. This can be attributed to the speculation that people traversing the network would like to take the highway to go through the network as it now offers a reduced free flow time and increased capacity, and it also goes from one end of the network to the other, rather than taking other links around it.

The following table displays the cost of traversing the links: (Red indicates reduced cost, green indicates highway links and no fill indicates increase in cost)

Table 2 Costs per vehicle

Link ID	Cost (before)	Cost (After)	Change in cost
(1,2)	6.000816246	6.000662361	-0.00015
(1,3)	4.008690801	4.00768604	-0.001
(2,1)	6.000834132	6.000667059	-0.00017
(2,6)	6.573606968	6.680227214	0.10662
(3,1)	4.008586703	4.007656624	-0.00093
(3,4)	4.269405207	4.62772171	0.358317
(3,12)	4.020179594	4.058423493	0.038244
(4,3)	4.27127088	4.632615974	0.361345
(4,5)	2.315377108	2.751315292	0.435938
(4,11)	7.133336528	7.077154133	-0.05618
(5,4)	2.317075277	2.773564957	0.45649
(5,6)	9.998209826	9.30951609	-0.68869
(5,9)	9.651338966	3.062784053	-6.58855
(6,2)	6.599526841	6.688601496	0.089075
(6,5)	10.0206512	9.290774179	-0.72988
(6,8)	14.6909722	8.577516075	-6.11346
(7,8)	5.552167167	4.539345536	-1.01282
(7,18)	2.062226252	2.011858398	-0.05037
(8,6)	14.8241244	8.577523779	-6.2466
(8,7)	5.501424353	4.521059587	-0.98036
(8,9)	15.17469135	15.74412465	0.569433
(8,16)	10.72941444	6.601970227	-4.12744

(9,5)	9.67019902	3.080413088	-6.58979
(9,8)	15.03783547	15.64136021	0.603525
(9,10)	5.682541615	2.025193518	-3.65735
(10,9)	5.717255747	2.0399752	-3.67728
(10,11)	12.40568049	8.958064057	-3.44762
(10,15)	13.72239704	3.980611386	-9.74179
(10,16)	20.08480274	17.1355143	-2.94929
(10,17)	16.30802687	15.70144091	-0.60659
(11,4)	7.22305907	7.05204985	-0.17101
(11,10)	12.20326203	8.878252452	-3.32501
(11,12)	13.59024872	7.627039282	-5.96321
(11,14)	13.69130456	5.969114227	-7.72219
(12,3)	4.019790945	4.057690907	0.0379
(12,11)	13.7351622	7.64645189	-6.08871
(12,13)	3.02279698	3.01362478	-0.00917
(13,12)	3.023480111	3.014010805	-0.00947
(13,24)	17.66097207	9.912004711	-7.74897
(14,11)	13.84270431	5.940382998	-7.90232
(14,15)	12.2343304	10.66374748	-1.57058
(14,23)	9.079319933	4.569290421	-4.51003
(15,10)	13.81157926	4.002419301	-9.80916
(15,14)	12.37462553	10.66374847	-1.71088
(15,19)	4.326215105	4.407791951	0.081577
(15,22)	9.088104669	2.327063859	-6.76104
(16,8)	10.77874943	6.601971346	-4.17678
(16,10)	20.23627945	17.24106274	-2.99522
(16,17)	9.501373761	7.594207282	-1.90717
(16,18)	3.163467161	3.054951649	-0.10852
(17,10)	16.30803386	15.75762239	-0.55041
(17,16)	9.472765428	7.544806769	-1.92796
(17,19)	7.436611589	7.333756584	-0.10286
(18,7)	2.063186867	2.012003668	-0.05118
(18,16)	3.165837374	3.055808865	-0.11003
(18,20)	4.259374426	4.119811939	-0.13956
(19,15)	4.335532242	4.421472558	0.08594
(19,17)	7.412258757	7.313072682	-0.09919
(19,20)	9.459042603	7.682997254	-1.77605
(20,18)	4.26023325	4.11910649	-0.14113
(20,19)	9.515213765	7.753224621	-1.76199
(20,21)	8.165673968	8.514751835	0.349078
(20,22)	7.713148544	9.843349826	2.130201
(21,20)	8.081646584	8.435220776	0.353574
(21,22)	4.213491782	1.328656792	-2.88483

(21,24)	11.92396635	10.09674902	-1.82722
(22,15)	9.057139806	2.331322965	-6.72582
(22,20)	7.713137162	9.763729313	2.050592
(22,21)	4.201031155	1.328584439	-2.87245
(22,23)	12.36580531	9.757076288	-2.60873
(23,14)	9.065944448	4.558094972	-4.50785
(23,22)	12.24312721	9.757067517	-2.48606
(23,24)	3.759314354	2.163056807	-1.59626
(24,13)	17.61698079	9.848623261	-7.76836
(24,21)	11.75249822	9.9473466	-1.80515
(24,23)	3.72295472	2.159485073	-1.56347

Again, we can see that the costs of the highway links have decreased substantially. Most of the links showing a decrease in flow show a decrease in cost as well, while those with increased flow show increased cost. Exception are the highway links that show a substantial decrease in cost despite increase in flow which is beneficial for every traversing it.

The table below presents the total costs on the links (flow*cost) and change in the same. (Red indicates reduced cost, green indicates highway links and no fill indicates increase in cost):

Table 3 Total cost on the link

Link ID	Total Link cost before	Total Link cost after	Change in total link cost
(1,2)	26971.68967	25598.55	-1373.14
(1,3)	32546.92882	31554.54	-992.389
(2,1)	27118.32622	25643.84	-1474.49
(2,6)	39226.97842	40521.95	1294.968
(3,1)	32448.18339	31524.07	-924.11
(3,4)	59799.06065	80081.93	20282.87
(3,12)	40291.74386	53057.5	12765.75
(4,3)	59928.49882	80322.43	20393.93
(4,5)	41691.63847	61548.24	19856.6
(4,11)	37093.64491	36336.68	-756.969
(5,4)	41778.26734	62500.32	20722.05
(5,6)	87966.87032	79447.88	-8518.99
(5,9)	152305.9081	67799.11	-84506.8
(6,2)	39542.82905	40623.2	1080.374
(6,5)	88246.66327	79217.88	-9028.79
(6,8)	183533.2814	90921.71	-92611.6
(7,8)	67189.75733	48410.61	-18779.1
(7,18)	32570.89727	20994.42	-11576.5
(8,6)	185680.6085	90921.82	-94758.8
(8,7)	66242.27642	48071.76	-18170.5

(8,9)	104442.2341	111226.9	6784.621
(8,16)	90005.74703	40272.06	-49733.7
(9,5)	152757.9929	68717.2	-84040.8
(9,8)	102809.086	110003.3	7194.198
(9,10)	123561.7158	69668.84	-53892.9
(10,9)	124716.7937	70666	-54050.8
(10,11)	219910.7797	135774.9	-84135.9
(10,15)	317341.6472	130698.5	-186643
(10,16)	221878.6765	179950.5	-41928.2
(10,17)	132095.0563	124793.8	-7301.21
(11,4)	38282.48316	35994.95	-2287.54
(11,10)	214829.0084	133881.7	-80947.3
(11,12)	113686.3916	43413.28	-70273.1
(11,14)	133847.8951	39178.59	-94669.3
(12,3)	40092.45041	52880.84	12788.38
(12,11)	115443.1648	43653.02	-71790.1
(12,13)	37143.11395	32559.04	-4584.08
(13,12)	37426.75321	32791.45	-4635.31
(13,24)	196413.8642	89409.22	-107005
(14,11)	135853.4606	38847	-97006.5
(14,15)	110553.4641	90641.77	-19911.7
(14,23)	76270.16205	22209.16	-54061
(15,10)	320322.2526	132139.1	-188183
(15,14)	112359.4551	90641.78	-21717.7
(15,19)	82558.46353	85379.91	2821.449
(15,22)	167311.1485	61860.24	-105451
(16,8)	90613.62448	40272.07	-50341.6
(16,10)	224076.5239	181421.5	-42655
(16,17)	111118.2801	82533.64	-28584.6
(16,18)	48332.65438	35540.15	-12792.5
(17,10)	132095.1407	125468.2	-6626.98
(17,16)	110677.9299	81815.13	-28862.8
(17,19)	74016.70275	72645.27	-1371.43
(18,7)	32711.10908	21059.95	-11651.2
(18,16)	48543.25464	35687.96	-12855.3
(18,20)	80829.54021	64453.26	-16376.3
(19,15)	82881.19696	85852.22	2971.028
(19,17)	73691.56311	72370.05	-1321.51
(19,20)	82183.55531	60497.81	-21685.7
(20,18)	80912.67793	64347.16	-16565.5
(20,19)	82883.43845	61339.77	-21543.7
(20,21)	51460.34228	55702.94	4242.601
(20,22)	53992.13206	79645.23	25653.1

(21,20)	50429.41959	54741.09	4311.666
(21,22)	36318.30208	16908.3	-19410
(21,24)	122928.7456	98296.86	-24631.9
(22,15)	166528.6662	62053.09	-104476
(22,20)	53991.99577	78674.3	24682.3
(22,21)	36159.82805	16906.45	-19253.4
(22,23)	119476.2367	85862.27	-33614
(23,14)	76107.61567	22045.01	-54062.6
(23,22)	117854.8757	85862.16	-31992.7
(23,24)	29709.84383	9432.103	-20277.7
(24,13)	195766.701	88598.44	-107168
(24,21)	120574.7663	96328.59	-24246.2
(24,23)	29269.28306	9364.532	-19904.8

The Total System Travel Time of the network decreases by 2,372,146.

REMARKS:

Out of the 76 links in the network, only 16 links are such that they show an increase in the total cost of travel after the introduction of the highway. All the rest of the network greatly benefits from this proposal. A notable observation here is that out of the 16 links with increased cost, 10 are those which allow people to get onto and off the new highway. These are (4,5), (5,4), (8,9), (9,8), (15,19), (19,15), (20,21), (21,20), (20,22), and (22,20). A reason for this is that since there is a considerable benefit in terms of cost on travelling the highway, most people would want to travel on it, and this will result in increased flows on the links incoming and outgoing from the highway and result in larger costs on the same.

The losers in the scenario of the highway being built will be the people who travel on these 16 links which will have increased cost. Those people will have to face a larger cost for using the same links as before. The greatest problem here would be faced by those people whose workplaces or homes or any other places of interest lie by these links as that would mean that they will have to use the path even after the highway is built and will have to necessarily pay the higher cost (in terms of time or money).

The winners on the other hand will be all the other users of the network which form most of the demand on the network and seem worth implementing the proposal for. Even the TSTT is decreasing which proves that it is better for the overall system, at least cost wise.

Question 3 – Speeding up traffic assignment (in group with Aupal Mondal)

To improve the speed of Traffic Assignment we have implemented the Conjugate Frank-Wolfe (CFW) algorithm. The original code consisted of the implementation of normal Frank-Wolfe (FW) algorithm and MSA. How Conjugate Frank-Wolfe algorithm is different from the Frank-Wolfe algorithm is that in CFW the All-Or-Nothing (AON) assignment is no longer the target flow. So, the shift no longer talks place between the AON and the current flows; instead, CFW calculates a new target flow solution for each iteration (which in turn uses the AON assignment and target flow from the previous iteration) and shifts flows between this new target flow and the current flow solution. This way it somewhat (though not fully) overcomes the limitation of FW of only proceeding towards one of the corner points in the solution set. This makes it converge quicker in comparison to the original FW algorithm. The rest of the algorithm is same as FW, in that, it continues to shift flows using either the Bisection or the Newton's Method and calculated flow solutions and travel times just like before.

IMPLEMENTATION

To implement the CFW algorithm, we have defined an additional function **alphacj( ),** which calculates the factor alpha which is used to calculate the new target flow vector.

The alphacj( ) function takes as parameters the old target flow vector, the current AON vector, and an epsilon value used to avoid equating alpha to 1 (which could be disastrous for the algorithm). Certain specific conditions had to be coded, for instance a condition of the denominator in the expression of alpha becoming 0, and alpha going out of its acceptable range [0, 1-epsilon], in which case it is projected back to the set.

A few changes are also made to the userEquilibrium( ) function which now calculates the target flows as per the CFW rule by calling alphacj( ). The first parameter of userEqulibrium( ) (earlier FW or MSA) now should only be entered as "FW" for which it executes the Conjugate FW algorithm by default. Anything else will raise a Bad Step Size rule Exception.

RESULTS

The algorithm is tested on 4 different networks from the TNTP dataset namely- Sioux Falls, Anaheim, Berlin-Mitte-Center, and Berlin-Mitte-Prenzlauerberg-Friedrichshain-Center.

The characteristics of the networks are presented in Table 4.

Table 4 Networks used for testing the code

Network	Nodes	Links	Zones
Sioux Falls	24	76	24
Berlin-Mitte-Center	398	871	36
Anaheim	416	914	38
Berlin-M-P-F-C	975	2184	98

The two algorithms being compared are FW with Newton's Method and precision 10^-6, and Conjugate Frank-Wolfe with Newton's Method and precision 10^-6.

On the first three networks, the algorithms are run for 300 seconds, while on the 4th one they are run for 600 seconds. We compare the AEC values attained by both the algorithms over different values of run time. The results are presented in Figures 2, 3, 4 and 5. It can be seen that CFW converges much faster than normal FW for all the tested networks irrespective of network size.

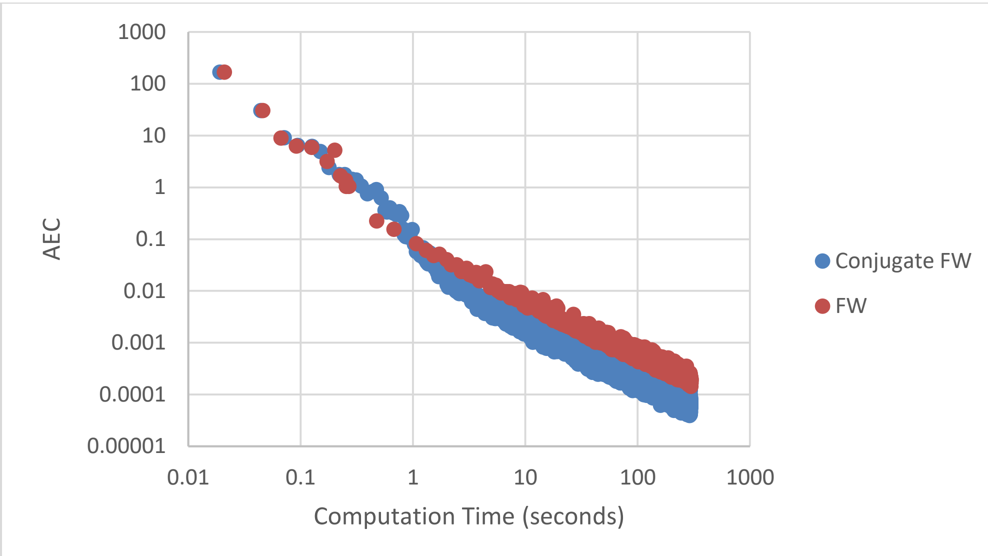

Figure 2 Performance comparison on Sioux Falls Network

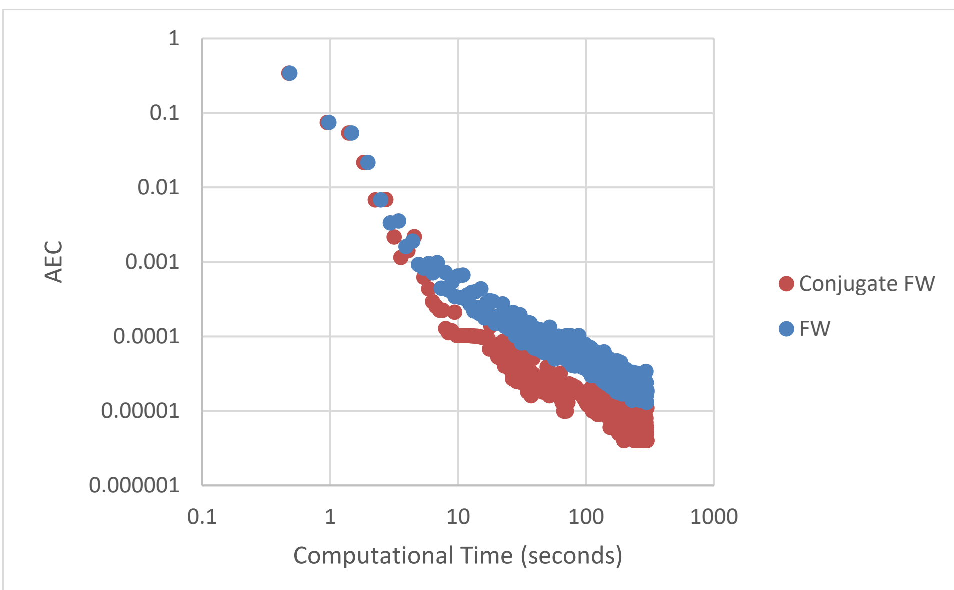

Figure 3 Performance comparison on Anaheim Network

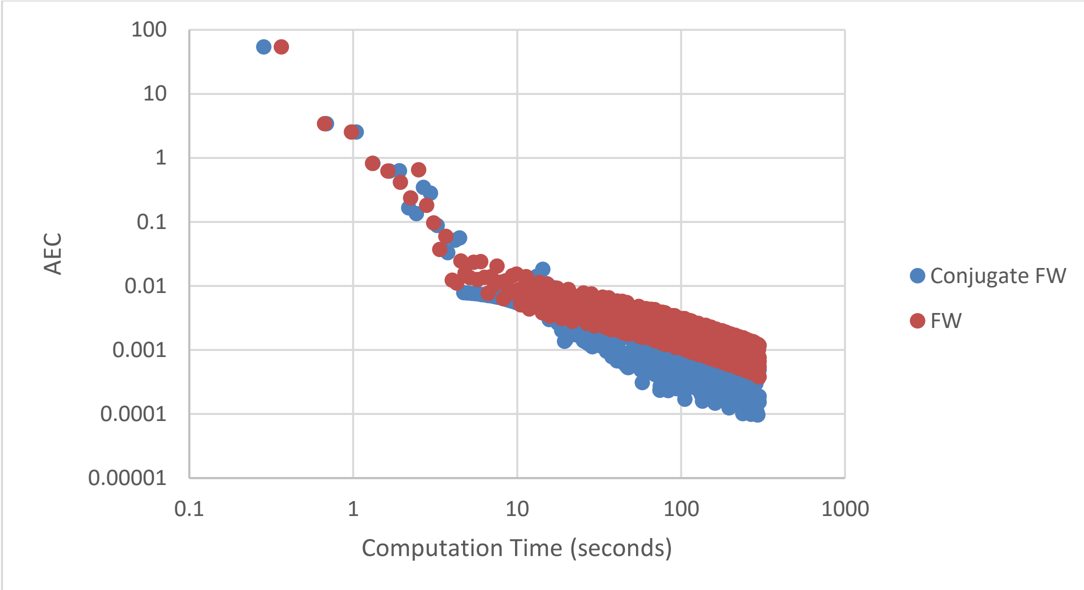

Figure 4 Performance comparison on Berlin-Mitte-Center Network

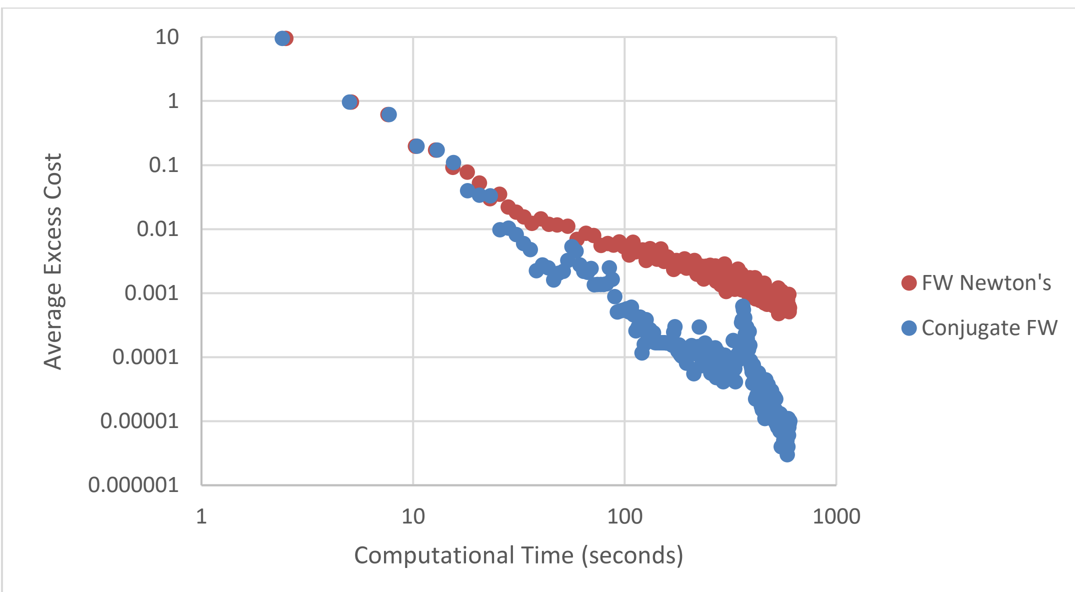

Figure 5 Performance comparison on Berlin-M-P-F-C Network