# Undecidability and Forcing

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**Abstract**

In this talk, I will give a brief introduction (or review) to *undecidability*, one of the most important notion in mathematical logic. The idea will be explained in three different forms: *Gödel's Incompleteness Theorem*, *Universal Turing Machine and the Halting Problem*, *Computable Functions*. But we will find out they are essentially the same.

As the most famous example of undecidability, I will explain in detail the independence of the *Continuum Hypothesis*(CH) in ZFC axiomatic set theory. The proof of its independence consists of two parts. The first part is to show that CH is *consistent* with ZFC (or informally, CH can hold in ZFC). This part of proof was finished by Kurt Gödel in 1938 (along with the consistency of the Axiom of Choice(AC)). The other side of the proof was done by Paul Cohen in 1966. He devised a extremely genius and influential technique, *forcing*, to show that it does no harm even if CH (or AC) fails within ZFC. I will sketch the the proof and introduce the inspiring idea of forcing. If we have enough time, I will show its origine and the connection between set theory, topology and measure theory.

No prerequisite in logic is needed, but the audience are assumed to have some knowledge in axiomatic set theory (our set theory seminar is enough). Those who are not familiar with basic concepts in set theory may refer to the first chapter of [3] to catch some definitions and basic results in ordinals, cardinals, transfinite induction and the equivalent forms of AC. I will make the talk accessible to all audience. But in case of getting lost in the middle of the talk, there is an introductory paper([6]) that I strongly recommend you to read (as much as you can, with or without understanding) before the cocktail.

## References

[1] Herbert B. Enderton, A Mathematical Introduction to Logic(2nd Ed.), Elsevier, 2001.

[2] Thomas Jech, Set Theory (3rd Ed.), Springer, 1995.

[3] Kenneth Kunen, Set Theory: An Introduction to Independence Proofs, North Holland Publishing Company, 1980.

[4] Harry R. Lewis, Elements of the Theory of Computation(2nd Ed.), Prentice-Hall, 1997.

[5] Robert I. Soare, Recursively Enumerable Sets and Degrees, Springer, 1987.

[6] Timothy Y. Chow, A Beginner's Guide to Forcing, http://arxiv.org/abs/0712.1320, 2008.