1 Pumping lemma

Let L be a regular language. Then there exists some positive integer n such that for every w in L with length at least N we can decompose w into 3 strings, w = xyz, such that :

- $y \neq \lambda$
- $|xy| \leq n$
- for every $k \geq 0$ the string $xy^k z$ is also in L

1.1 Example

Consider : $L_{eq} = w \epsilon 0, 1* \mid w\ has\ an\ equal\ number\ of\ 0s\ and\ 1s$
Claim : L_{eq} is not regular
Proof : The proof is by contradiction. Assume that L_{eq} is regular and let n be the positive integer that exists according to the pumping lemma.
We pick a string w in L_{eq}, namely w = $0^n 1^n$. Clearly $|w| \geq n$ so by the pumping lemma w=xyz such that $|xy| \leq n$ and $y \neq \lambda$
We can deduce from this that xy is made up entirely of 0s, and hence y consists of one or more 0s.
By the pumping lemma xy^2 must be in the language but this equals $0^p 1^n$, where $p > n$ so there are more 0s than 1s and therefore it isnt in the language and our assumption that L_{eq} is regular must be incorrect.

1.2 Boilerplate

Proof: The proof is by contradiction. Assume that L_{ww} is regular and let n be the positive integer that exists according to the pumping lemma.
We pick a string w in L_{ww}, namely w = . Clearly $|w| \geq n$ so by the pumping lemma w = xyz, such that $|xy| \leq n$ and $y \neq \lambda$
We can deduce from this that xy is , and hence that y consists of .
By the pumping lemma, xyz must be in the language but this quals and therefore isnt in the language.
Hence we have a contradiction and our assumption that L_{ww} is regular must be wrong.