Chapter 5: Mining Frequent Patterns, Association and Correlations

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Frequent Pattern Mining

Frequently occuring items in a DB

- □ Frequent pattern: a pattern (a set of co-purchased items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed in 1993 in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent patterns in data
 - What products were often purchased together? (this will be our main example) Beer and diapers
 - What are the subsequent purchases after buying a digital camera?

SD memory

- What kinds of DNA are sensitive to this new drug?
- Applications
 - Basket data analysis, DNA sequence analysis



Basic Concepts

- **□** Itemset $X = \{x_1, ..., x_k\}$
 - □ Frequent pattern is defined on an itemset
- \square Association rules $X \rightarrow Y$ If someone purchases X, then there's a good chance he'll buy Y
 - It is defined on two itemsets X and Y, where X U Y must be a frequent pattern. Minimum confidence 를 넘어야 association rule이라고 부를 수 있음
- □ Support and Confidence: Computed on each item

Transaction -id	Items bought
10	A, B, D purchase together
20	A , C, D
30	A , D , E
40	B, E, F
50	B, C, D, E, F

6 items -> 2^6 - 1 possible combinations

- **Support**, s, is probability (or, frequency) that a transaction contains X. $E^{x} \times E^{x} \times E^{x} = \{A, D\} x = \{A, D\}$
 - **Minimum support:** a threshold that decides whether X is a frequent pattern or not, based on its support
- □ Confidence, c, conditional probability that a transaction having X also contains Y
 - **Minimum confidence:** it is also a threshold Ex) $X = \{A, D\} Y = \{B\} - 1/3$

Let $sup_{min} = 50\%$, $conf_{min} = 50\%$, then:

- Q: Find all freqent patterns. A: {A:3, B:3, D:4, E:3, AD:3} size 2 item sets

- Q: Find all association rules. A:

> 1) defined on two item sets A and D 2) A U D is frequent

$$A \rightarrow D$$
 (60%, 100%)
 $D \rightarrow A$ (60%, 75%)

size 1 item sets



Closed Patterns and Max-Patterns

- A long pattern contains too many number of sub-patterns, e.g., $\{a_1, ..., a_{100}\}$ contains $2^{100} - 1$ sub-patterns! Too many patterns!!
- □ Solution: Mine closed patterns and max-patterns instead, which can be representatives of those sub-patterns
- An itemset X is closed if X is frequent and there exists no superpattern $Y \supset X$, with **the same support** as X
- □ An itemset X is a max-pattern if X is frequent and there exists no **frequent** super-pattern $Y \supset X$
- Closed pattern is a lossless compression of freq. patterns
 - Reducing the # of redundant patterns and rules

support

 $X = \{a.b.c\}: 10$ $Y = \{a.b.c.d\}: 10$

Y is more informative (includes X) -> Y is closed.

 $X = \{a,b,c\}$: $Y = \{a.b.c.d\}: 10$

 $Z = \{a.b.c.d.e\} : 8$

Y is still closed because support of Z is 8.

Sup-min: 8 Y is not a max-pattern Y is a max-pattern because Z is frequent.

Sup-min: 9 because Z is not frequent.

Closed Patterns and Max-Patterns

- Exercise. DB = $\{\langle a_1, ..., a_{100} \rangle, \langle a_1, ..., a_{50} \rangle\}$ including only two transactions and 100 items
 - □ Let Min sup = 1.

```
X = {a1 ··· a49} : 2

Y = {a1 ··· a50} : 2

Z = {a1 ··· a51} : 1

Q = {a1 ··· a100}

Y is still closed.

-> Y and Q is closed
```

Questions:

- How many frequent patterns are there?
 - $2^{100} 1$
- □ What is the set of closed patterns? (write each one's support as well)
 - <a₁, ..., a₁₀₀>: 1
 - < a₁, ..., a₅₀>: 2
- □ What is the set of max-patterns? (write each one's support as well)
 - <a₁, ..., a₁₀₀>: 1
 X = {a1 ··· a50} : 2
 Y = {a1 ··· a100} : 1
 Y is a max-pattern.
 (Y is frequent and is a superset of X)

Scalable Methods for Mining Frequent Patterns

- The downward closure property of frequent patterns
 - □ Any subset of a frequent itemset must be frequent
 - □ If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
 - □ i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}

If {abc} is not frequent, {abcd} cannot be frequent.

- Scalable mining methods: Three major approaches
 - □ **Apriori** (Agrawal & Srikant@VLDB′ 94)
 - □ Freq. pattern growth (**FP-growth**, @SIGMOD′ 00)
 - □ Vertical data format approach (Charm, @SDM′ 02)



Apriori: A Candidate Generation-and-Test Approach

□Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested!

■ Method:

- □ Initially, scan DB once to get frequent 1-itemset
- □ Repeat with index [k]:
 - **Generate candidate** itemsets of **Size** (k+1) from frequent itemsets of length k
 - Test the candidates against DB
 - Terminate when no frequent or candidate set can be generated



The Apriori Algorithm—An Example



Tid	Items
10	A, C, D
20	В, С, Е
30	A, B, C, E
40	B, E

 $Sup_{min} = 2$

1 st scan	

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

	Itemset	sup
L_1	{A}	2
	{B}	3
	{C}	3
	{E}	3

			_
L_2	Itemset	sup	
_	{A, C}	2	
	{B, C}	2	←
	{B, E}	3	
	{C, E}	2	
			_

2	Itemset	sup
	{A, B}	1
	{A, C}	2
	{A, E}	1
ſ	{B, C}	2
Γ	{B, E}	3
	{C, E}	2

C_2	Itemset
^d scan	{A, B}
	{A, C}
	{A, E}
	{B, C}
	{B, E}

{C, E}

C_3	Itemset	
J	{B, C, E}	

3 rd scan	L_3

Itemset	sup
{B, C, E}	2

The Apriori Algorithm: Pseudo-code

■Pseudo-code:

```
C<sub>k</sub>: Candidate itemset of size k
L_k: frequent itemset of size k
L_1 = \{ \text{frequent items} \};
for (k = 1; L_k! = \emptyset; k++) do begin
    C_{k+1} = candidates generated from L_k
   for each transaction t in database do
       increment the count of all candidates in C_{k+1} that are contained in t
    L_{k+1} = candidates in C_{k+1} with min_support
    end
return \cup_k L_k
```

Important Details of Apriori

- □ How to generate candidates, from L_k to C_{k+1} ?
 - \square Step 1: self-joining L_k
 - □ Step 2: pruning
- Example of candidate generation via self-joining and pruning
 - $\square L_3 = \{abc, abd, acd, ace, bcd\}$
 - □ Self-joining: L_3*L_3
 - abcd can be a candidate from abc, abd, bcd, all of which are frequent
 - Pruning:
 - *acde* cannot be included because *ade* is not in L_3 , i.e., not frequent!
 - \Box C_{4} ={abcd}



Challenges of Frequent Pattern Mining

Challenges

- Multiple scans of a transaction database (about k times)
- Huge number of candidates
- □ Tedious workload of support counting for candidates

General ideas of improving efficiency of frequent pattern mining

- Reduce the number of transaction database scans
- □ Reduce the number of candidates
- □ Facilitate support counting of candidates

Thank You

