GoSam 2.0.4: $gg \to hh$

scyboz

2017-05-19 (11:53:02)

Abstract

This process consists of 4 tree-level diagrams and 195 NLO diagrams. GoSam has identified 12 groups of NLO diagrams by analyzing their one-loop integrals.

Contents

1 Helicities

Index	1	2	3	4
0	−	−	0	0
$1 \to 0$	+	−	0	0
$2 \to 0$	−	+	0	0
$3 \to 0$	+	+	0	0

2 Wave Functions

In this section, we use $l_i = k_i$ for massless particles; in spinors $|i\rangle$ (resp. $|i]$) denote $|l_i\rangle$ (resp. $|l_i]$). For the massive particles we have:

$$l_3 = k_3 - \frac{mdlMh^2}{2k_3 \cdot k_2} k_2 \tag{1}$$

$$l_4 = k_4 - \frac{mdlMh^2}{2k_4 \cdot k_2} k_2 \tag{2}$$

All helicity amplitudes are defined in terms of the following wave functions:

- $g(k_1)$

$$\varepsilon_+^\mu(k_1) = \frac{\langle 2|\gamma^\mu|1]}{\sqrt{2}\langle 2|1\rangle} \tag{3}$$

$$\varepsilon_-^\mu(k_1) = \frac{[2|\gamma^\mu|1\rangle}{\sqrt{2}[1|2]} \tag{4}$$

- $g(k_2)$

$$\varepsilon_+^\mu(k_2) = \frac{\langle 1|\gamma^\mu|2]}{\sqrt{2}\langle 1|2\rangle} \tag{5}$$

$$\varepsilon_-^\mu(k_2) = \frac{[1|\gamma^\mu|2\rangle}{\sqrt{2}[2|1]} \tag{6}$$

- $h(k_3)$

$$\epsilon(k_3) = 1 \tag{7}$$

- $h(k_4)$

$$\epsilon(k_4) = 1 \tag{8}$$

3 Colour Basis

$$|c_1\rangle = g_{(1)}^{A_1} g_{(2)}^{A_2} \mathrm{tr}\left\{T^{A_2} T^{A_1}\right\} \tag{9}$$

4 Tree Diagrams

QGraf Setup

```
-------------------------------------------------------------------------------
                                qgraf-3.1.4
-------------------------------------------------------------------------------

output = 'diagrams-0.hh';
style = 'form.sty';
model = 'model';
in = part21[k1], part21[k2];
out = part5000000[k3], part5000000[k4];
loops=0;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[part1,part2,part3,part4,part5,0,0];
true=vsum[QED,2,2];

===============================================================================
 warning: repeated vertices found
===============================================================================


===============================================================================
 warning: model splits into disjoint components
===============================================================================


===============================================================================
 warning: model contains at least one non-interacting
          field
===============================================================================


-------------------------------------------------------------------------------

 24P  ---  7+  17-  ---  5N+  2C+  17C-

 76V  ---  3^59  4^11  5^4  6^2

-------------------------------------------------------------------------------

  -   4^1   -    -   ---  1 diagram
3^2   -    -    -   ---  3 diagrams

 total =  4 diagrams
```

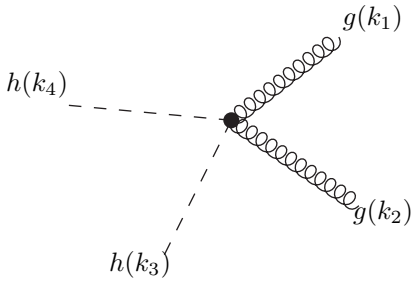


Diagram 1

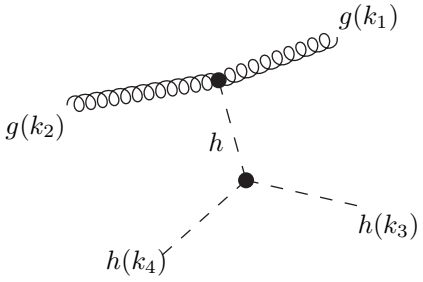


Diagram 2

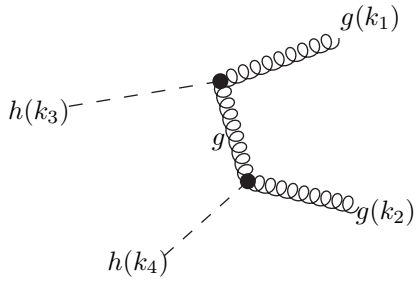


Diagram 3

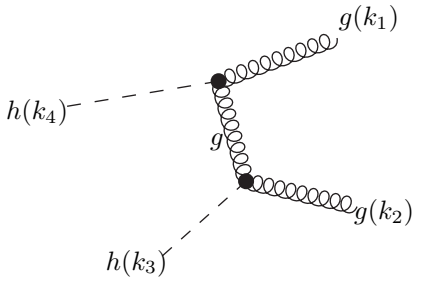


Diagram 4

5 One-Loop Diagrams

General Information

QGraf Setup

```
_____________________________________________________________

                        qgraf-3.1.4
_____________________________________________________________

output = 'diagrams-1.hh';
style = 'form.sty';
model = 'model';
in = part21[k1], part21[k2];
out = part5000000[k3], part5000000[k4];
loops=1;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[part1,part2,part3,part4,part5,0,0];
true=vsum[QED,2,2];

=============================================================
 warning: repeated vertices found
=============================================================
```

```
================================================================
warning: model splits into disjoint components
================================================================


================================================================
warning: model contains at least one non-interacting
         field
================================================================


----------------------------------------------------------------

24P  ---  7+  17-  ---  5N+  2C+  17C-

76V  ---  3^59  4^11  5^4  6^2

----------------------------------------------------------------

 -    -    -   6^1  ---  0 diagrams
3^1   -   5^1   -   ---  4 diagrams
 -   4^2   -    -   ---  8 diagrams
3^2  4^1   -    -   ---  75 diagrams
3^4   -    -    -   ---  116 diagrams

total =  203 diagrams
```

Loop diagrams are grouped into sets of diagrams which share loop-propagators. A loop integral can be written as

$$\int \frac{\mathrm{d}^n k}{i\pi^{\frac{n}{2}}} \frac{\mathcal{N}(q)}{\prod_{j=1} N\left[(k+r_j)^2 - (m_j^2 - im_j\Gamma_j) + i\delta\right]}. \tag{10}$$

For each group we list r_j, m_j and Γ_j. For m_j and Γ_j only non-vanishing symbols are listed. Furthermore, we give the matrix S which is defined as

$$S_{\alpha\beta} = (r_\alpha - r_\beta)^2 - (m_\alpha^2 - im_\alpha\Gamma_\alpha) - (m_\beta^2 - im_\beta\Gamma_\beta). \tag{11}$$

For each diagram we denote how the matrix S' for the specific diagram is obtained from the original S. The notation

$$S' = S_{Q\to q'}^{\{l_1,l_2,\ldots\}} \tag{12}$$

means, that the rows and columns labeled by $l_1, l_2, \ldots$ should be removed from S (likewise $r_{l_1}, r_{l_2}, \ldots$ are removed from the list of propagators) and $\mathcal{N}(q)$ has to be replaced by $\mathcal{N}(q')$. The maximum effective rank of a group is the rank that has to be passed to SAMURAI if the whole group is reduced at once; this number is calculated as

$$\max_{\text{diagrams}} \left\{(\text{rank of diagram}) + (\text{number of pinches})\right\}. \tag{13}$$

Diagrams with massless closed quark lines are multiplied by a factor `Nfrat` = `Nf`/`Nfgen`. This multiplication is indicated by the symbol N_f following the rank. By default `Nfrat` evaluates to one but can be changed by modifying `Nf` or `Nfgen` in the model file.

5.1 Group 0 (4-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_2 + k_4 \tag{14a}$$
$$r_2 = -k_2 \tag{14b}$$
$$r_3 = 0 \tag{14c}$$
$$r_4 = -k_3 \tag{14d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & 0 \\ S_{2,1} & 0 & 0 & S_{2,4} \\ S_{3,1} & 0 & 0 & S_{3,4} \\ 0 & S_{4,2} & S_{4,3} & 0 \end{pmatrix} \tag{15}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{16a}$$
$$S_{1,3} = 2\mathrm{Mh}^2 - s_{23} - s_{12} \tag{16b}$$
$$S_{2,4} = s_{23} \tag{16c}$$
$$S_{3,4} = \mathrm{Mh}^2 \tag{16d}$$

5.1.1 Diagrams (1)

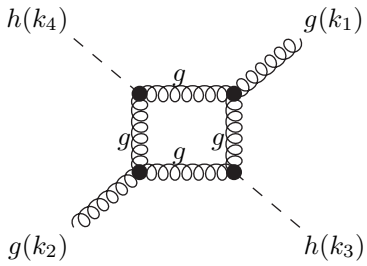


Diagram 195
$S' = S_{Q\to -q-(-k3)}$, rk = 6

5.2 Group 1 (4-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_3 - k_4 \tag{17a}$$
$$r_2 = -k_3 \tag{17b}$$
$$r_3 = 0 \tag{17c}$$
$$r_4 = -k_2 \tag{17d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & 0 \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & 0 \\ 0 & S_{4,2} & 0 & 0 \end{pmatrix} \tag{18}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{19a}$$
$$S_{1,3} = s_{12} \tag{19b}$$
$$S_{2,3} = \mathrm{Mh}^2 \tag{19c}$$
$$S_{2,4} = s_{23} \tag{19d}$$

5.2.1 Diagrams (20)

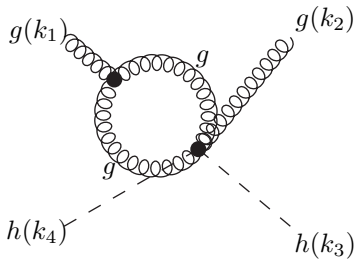


Diagram 1
$S' = S^{\{2,3\}}_{Q\to -q-(-k3-k4)}$, rk = 2

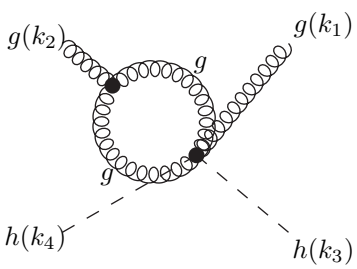


Diagram 2
$S' = S^{\{1,2\}}$, rk = 2

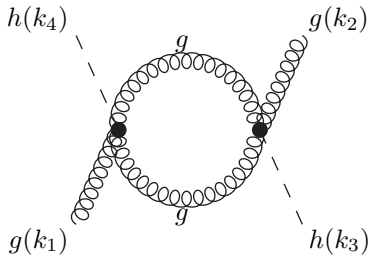


Diagram 10
$S' = S^{\{1,3\}}_{Q\to -q-(-k3)}$, rk = 2

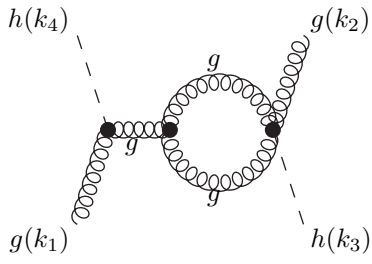


Diagram 21
$S' = S^{\{1,3\}}_{Q\to -q-(-k3)}$, rk = 2

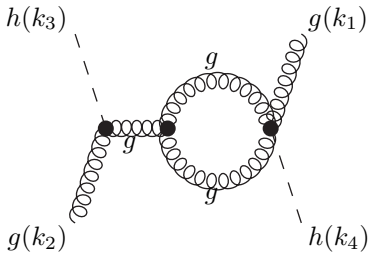


Diagram 24
$S' = S^{\{1,3\}}_{Q\to q-(k3)}$, rk $= 2$

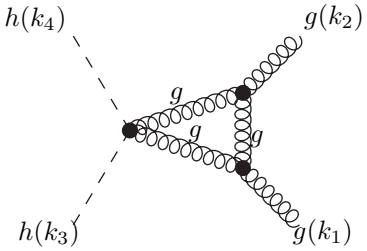


Diagram 33
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 4$

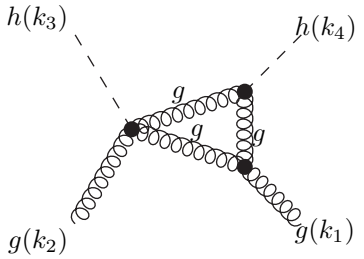


Diagram 40
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

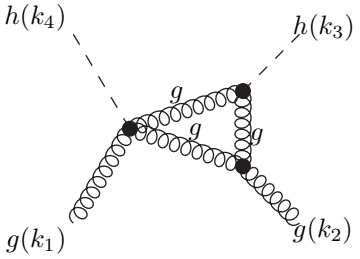


Diagram 44
$S' = S^{\{1\}}$, rk $= 4$

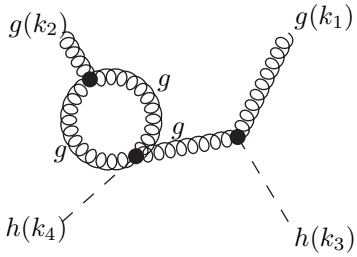


Diagram 64
$S' = S^{\{1,2\}}$, rk $= 2$

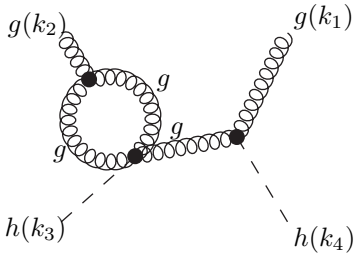


Diagram 69
$S' = S^{\{1,2\}}$, rk $= 2$

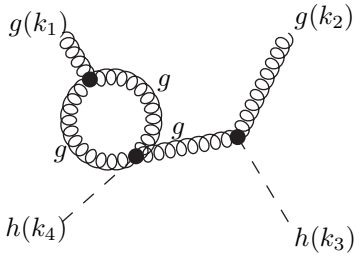


Diagram 74
$S' = S^{\{2,3\}}_{Q\to -q-(-k3-k4)}$, rk $= 2$

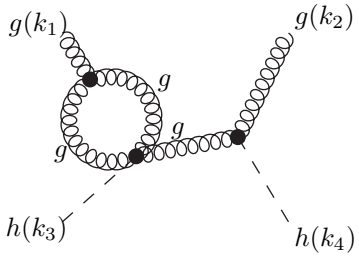


Diagram 79
$S' = S^{\{2,3\}}_{Q\to -q-(-k3-k4)}$, rk $= 2$

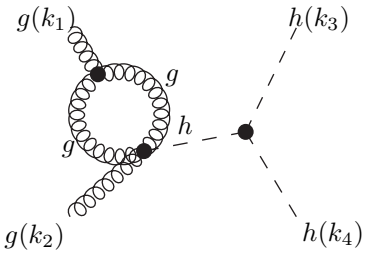


Diagram 84
$S' = S^{\{2,3\}}_{Q\to -q-(-k3-k4)}$, rk $= 2$

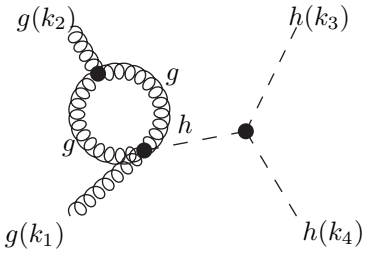


Diagram 86
$S' = S^{\{1,2\}}$, rk $= 2$

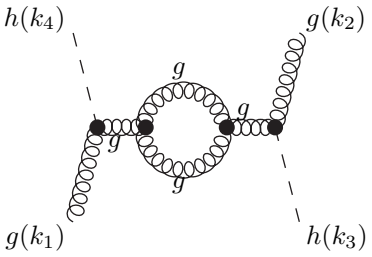
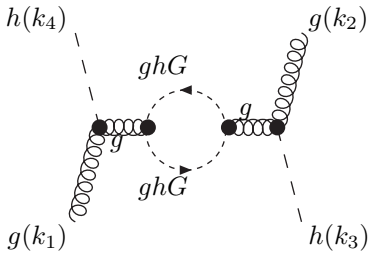


Diagram 102
$S' = S^{\{1,3\}}_{Q\to -q-(-k3)}$, rk $= 2$



-Diagram 104
$S' = S^{\{1,3\}}_{Q\to -q-(-k3)}$, rk $= 2$

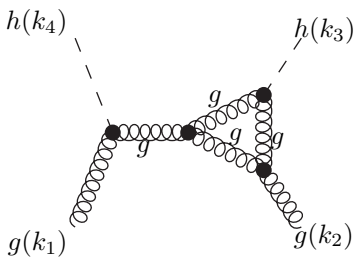


Diagram 132
$S' = S^{\{1\}}$, rk $= 4$

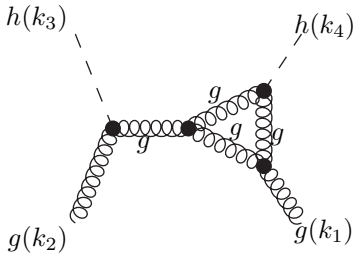


Diagram 144
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

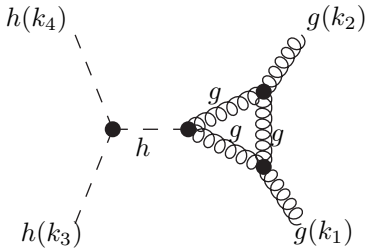


Diagram 168
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 4$

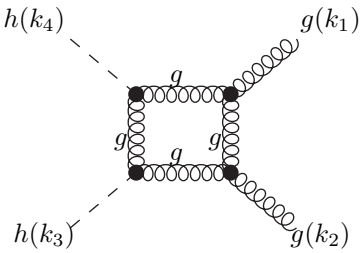


Diagram 185
$S' = S_{Q\to q-(k2)}$, rk $= 6$

5.3 Group 2 (4-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_3 - k_4 \tag{20a}$$
$$r_2 = -k_4 \tag{20b}$$
$$r_3 = 0 \tag{20c}$$
$$r_4 = -k_2 \tag{20d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & 0 \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & 0 \\ 0 & S_{4,2} & 0 & 0 \end{pmatrix} \tag{21}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{22a}$$
$$S_{1,3} = s_{12} \tag{22b}$$
$$S_{2,3} = \mathrm{Mh}^2 \tag{22c}$$
$$S_{2,4} = 2\mathrm{Mh}^2 - s_{23} - s_{12} \tag{22d}$$

5.3.1 Diagrams (10)

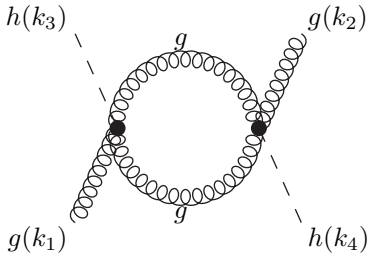


Diagram 7
$S' = S^{\{1,3\}}_{Q\to -q-(-k4)}$, rk $= 2$

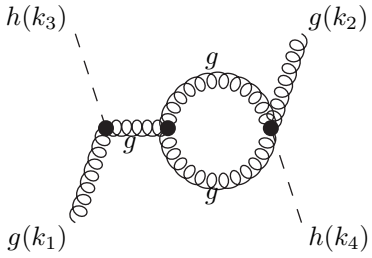


Diagram 18
$S' = S^{\{1,3\}}_{Q\to -q-(-k4)}$, rk $= 2$

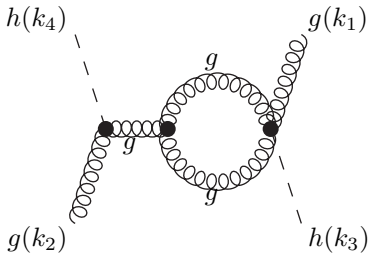


Diagram 27
$S' = S^{\{1,3\}}_{Q\to q-(k4)}$, rk $= 2$

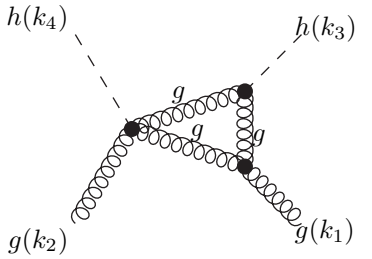


Diagram 36
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

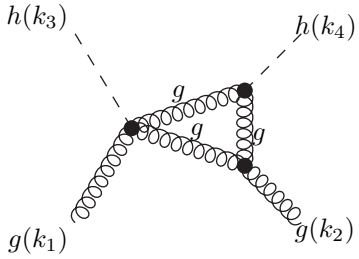


Diagram 48
$S' = S^{\{1\}}$, rk $= 4$

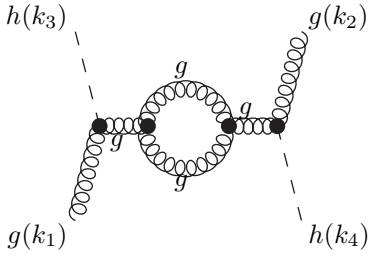
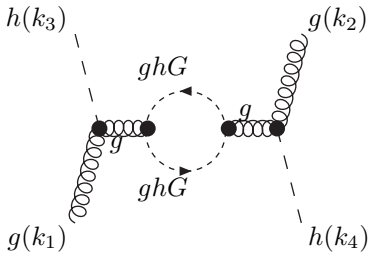


Diagram 93
$S' = S^{\{1,3\}}_{Q\to -q-(-k4)}$, rk $= 2$



-Diagram 95
$S' = S^{\{1,3\}}_{Q\to -q-(-k4)}$, rk = 2

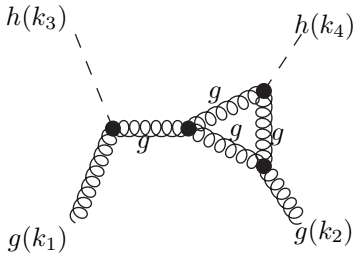


Diagram 120
$S' = S^{\{1\}}$, rk = 4

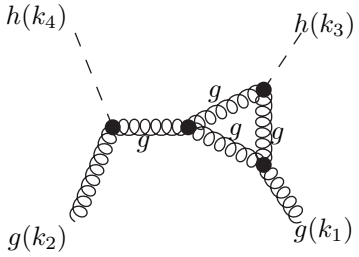


Diagram 156
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 4

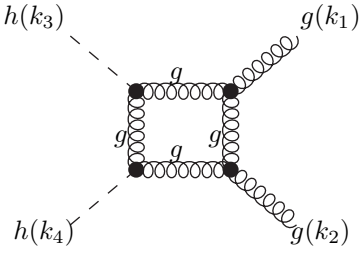


Diagram 175
$S' = S_{Q\to q-(k2)}$, rk = 6

5.4 Group 3 (4-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_2 + k_4, \quad m_1 = \mathrm{Mh}, \quad \Gamma_1 = \mathrm{Wh} \tag{23a}$$
$$r_2 = -k_2, \quad m_2 = \mathrm{Mh}, \quad \Gamma_2 = \mathrm{Wh} \tag{23b}$$
$$r_3 = 0 \tag{23c}$$
$$r_4 = -k_3 \tag{23d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & 0 \end{pmatrix} \tag{24}$$

$$S_{1,1} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25a}$$
$$S_{1,2} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25b}$$
$$S_{1,3} = \mathrm{Mh}^2 - s_{23} - s_{12} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25c}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25d}$$

$$S_{2,2} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25e}$$
$$S_{2,3} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25f}$$
$$S_{2,4} = -\mathrm{Mh}^2 + s_{23} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{25g}$$
$$S_{3,4} = \mathrm{Mh}^2 \tag{25h}$$

5.4.1 Diagrams (7)

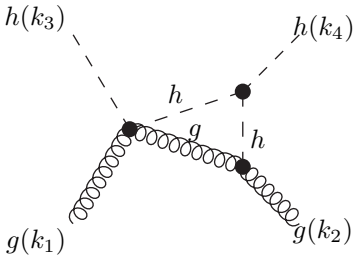


Diagram 49
$S' = S^{\{4\}}_{Q\to -q-(-k2)}$, rk $= 4$

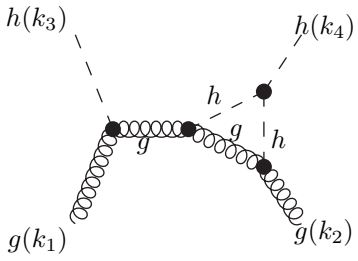


Diagram 126
$S' = S^{\{4\}}_{Q\to -q-(-k2)}$, rk $= 4$

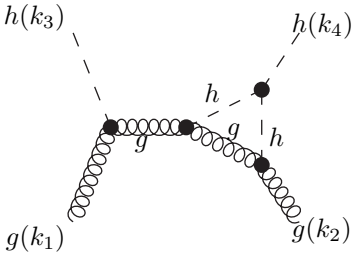


Diagram 130
$S' = S^{\{4\}}_{Q\to -q-(-k2)}$, rk $= 4$

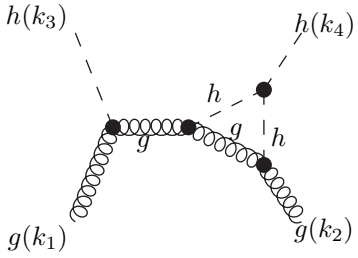


Diagram 131
$S' = S^{\{4\}}_{Q\to -q-(-k2)}$, rk $= 4$

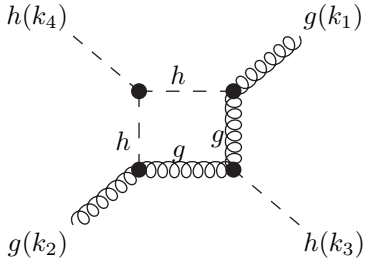


Diagram 196
$S' = S_{Q\to -q-(-k3)}$, rk $= 6$

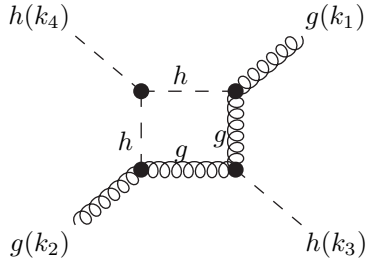


Diagram 197
$S' = S_{Q\to -q-(-k3)}$, rk $= 6$

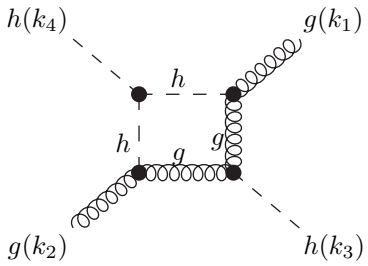


Diagram 198
$S' = S_{Q\to -q-(-k3)}$, rk = 6

5.5 Group 4 (4-Point)

General Information

The maximum effective rank in this group is 8.

$$r_1 = -k_3 - k_4 \tag{26a}$$
$$r_2 = -k_3 \tag{26b}$$
$$r_3 = 0 \tag{26c}$$
$$r_4 = -k_2, \quad m_4 = \mathrm{Mh}, \quad \Gamma_4 = \mathrm{Wh} \tag{26d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{27}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{28a}$$
$$S_{1,3} = s_{12} \tag{28b}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{28c}$$
$$S_{2,3} = \mathrm{Mh}^2 \tag{28d}$$
$$S_{2,4} = -\mathrm{Mh}^2 + s_{23} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{28e}$$
$$S_{3,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{28f}$$
$$S_{4,4} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{28g}$$

5.5.1 Diagrams (73)

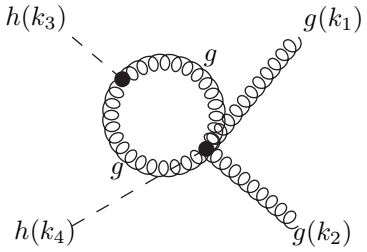


Diagram 3
$S' = S^{\{1,4\}}_{Q\to q-(k3)}$, rk $= 2$

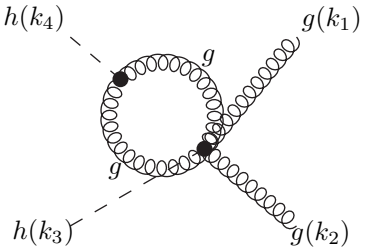


Diagram 4
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 2$

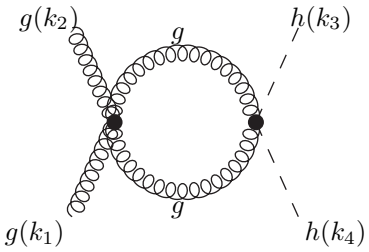


Diagram 5
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 2$

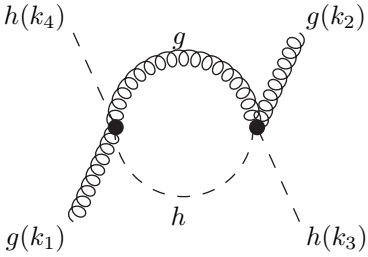


Diagram 11
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

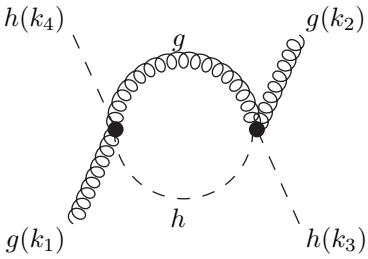


Diagram 12
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

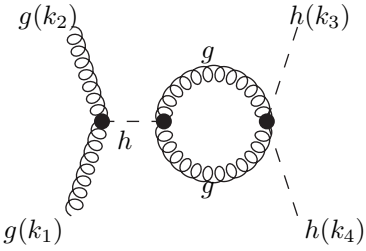


Diagram 14
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

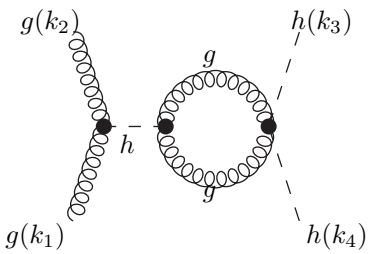


Diagram 15
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

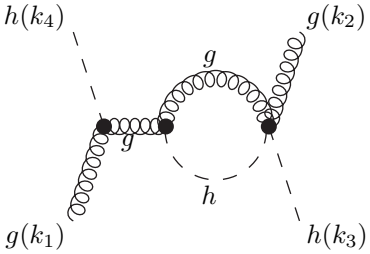


Diagram 22
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

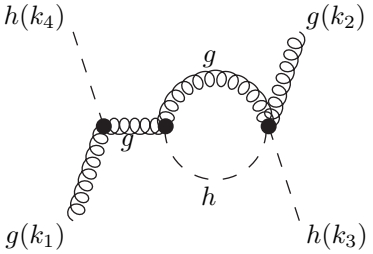


Diagram 23
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

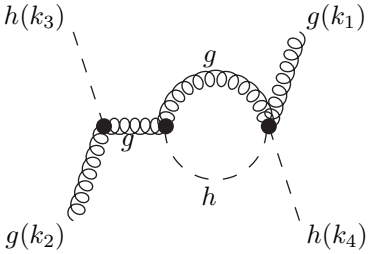


Diagram 25
$S' = S^{\{1,3\}}_{Q\to -q-(-k2)}$, rk $= 4$

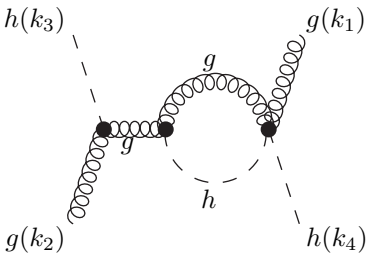


Diagram 26
$S' = S^{\{1,3\}}_{Q\to -q-(-k2)}$, rk $= 4$

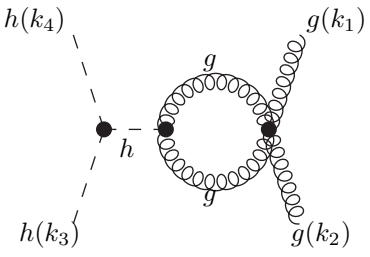


Diagram 30
$S' = S^{\{2,4\}}_{Q\to q-(k3+k4)}$, rk $= 2$

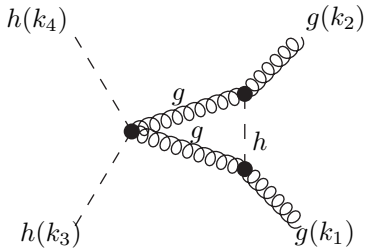


Diagram 32
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk = 6

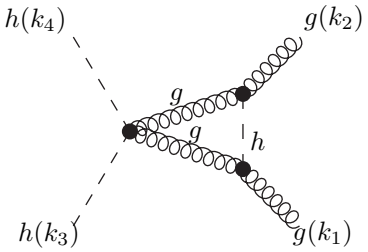


Diagram 34
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk = 6

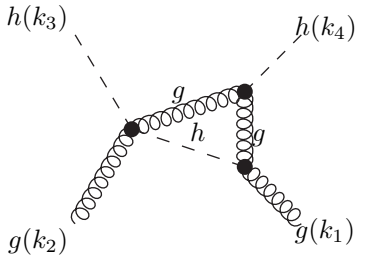


Diagram 42
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

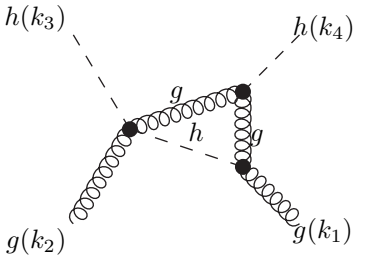


Diagram 43
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

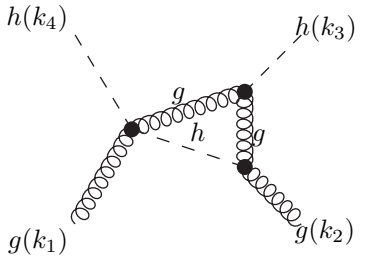


Diagram 46
$S' = S^{\{1\}}$, rk = 6

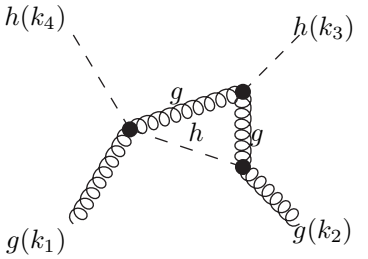


Diagram 47
$S' = S^{\{1\}}$, rk = 6

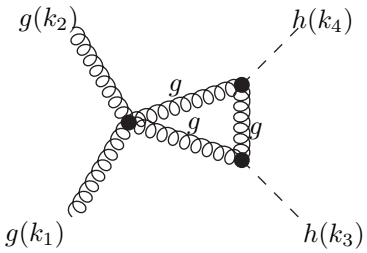


Diagram 52
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 4$

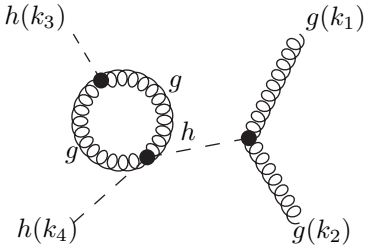


Diagram 55
$S' = S^{\{1,4\}}_{Q\to q-(k3)}$, rk $= 4$

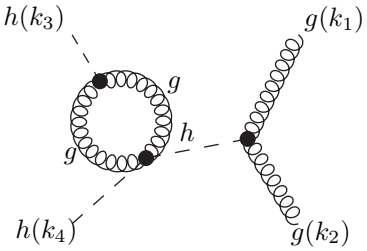


Diagram 56
$S' = S^{\{1,4\}}_{Q\to q-(k3)}$, rk $= 4$

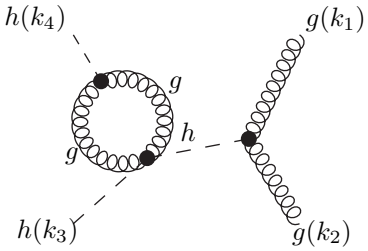


Diagram 60
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 4$

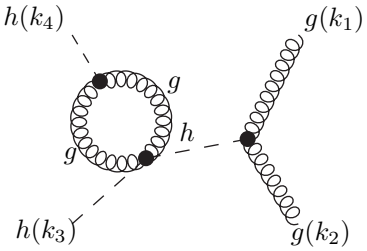


Diagram 61
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 4$

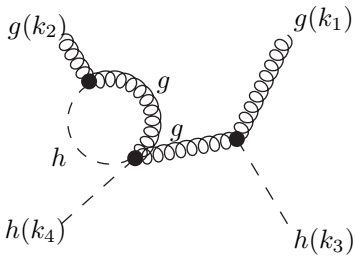


Diagram 65
$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 4$

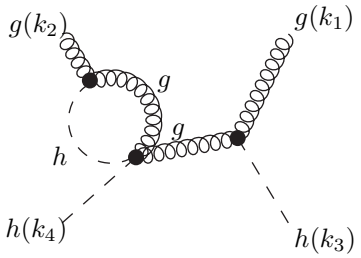


Diagram 66
$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 4$

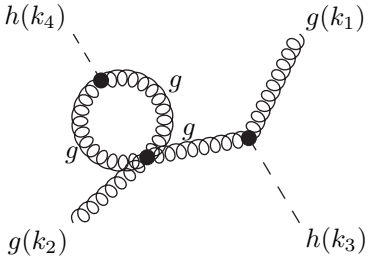


Diagram 67
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 2$

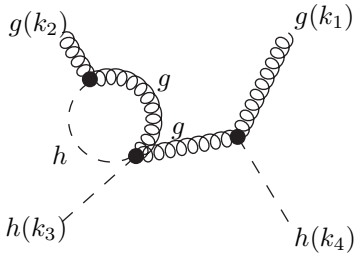


Diagram 70
$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 4$

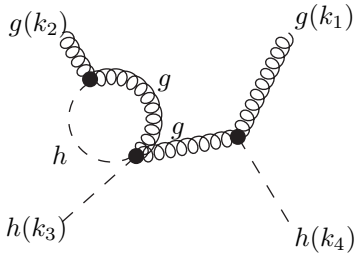


Diagram 71
$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 4$

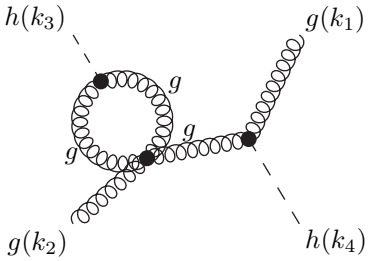


Diagram 72
$S' = S^{\{1,4\}}_{Q\to q-(k3)}$, rk $= 2$

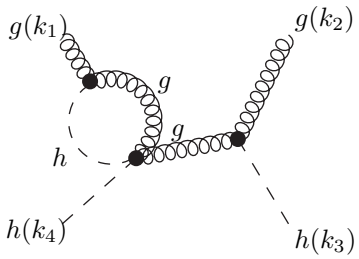


Diagram 75
$S' = S^{\{2,3\}}_{Q\to q-(k2)}$, rk $= 4$

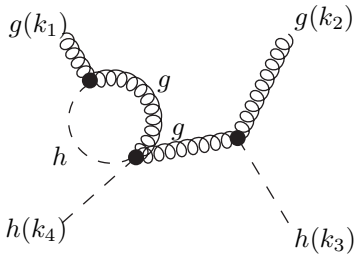


Diagram 76
$S' = S^{\{2,3\}}_{Q\to q-(k2)}$, rk $= 4$

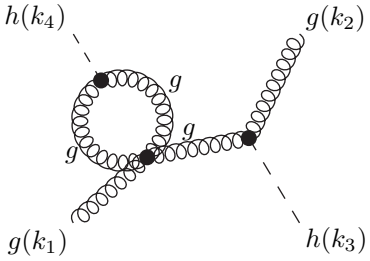


Diagram 77
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 2$

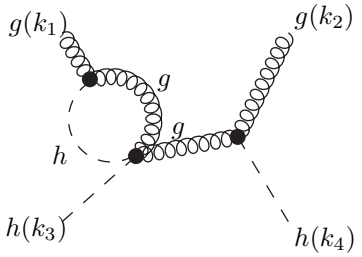


Diagram 80
$S' = S^{\{2,3\}}_{Q\to q-(k2)}$, rk $= 4$

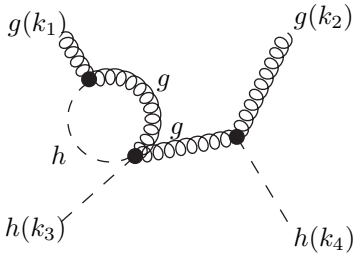


Diagram 81
$S' = S^{\{2,3\}}_{Q\to q-(k2)}$, rk $= 4$

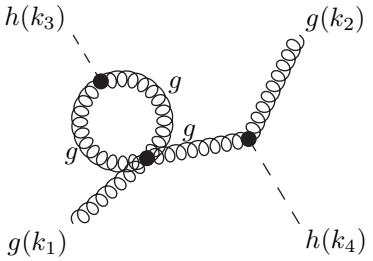


Diagram 82
$S' = S^{\{1,4\}}_{Q\to q-(k3)}$, rk $= 2$

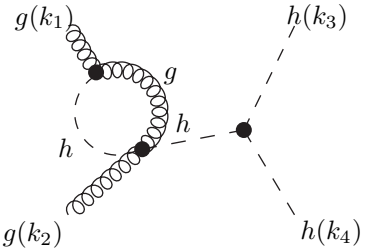


Diagram 85
$S' = S^{\{2,3\}}_{Q\to q-(k2)}$, rk $= 4$

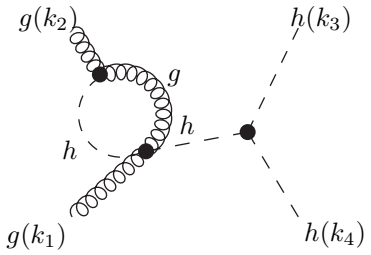


Diagram 87
$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 4$

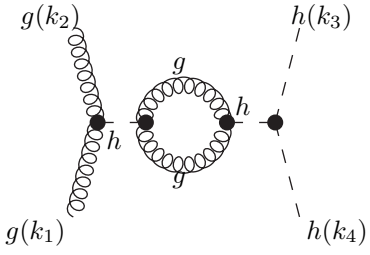


Diagram 90
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

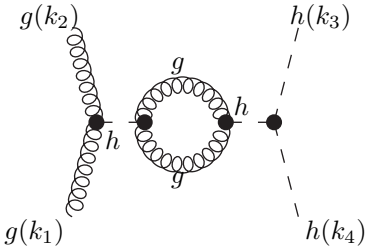


Diagram 91
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

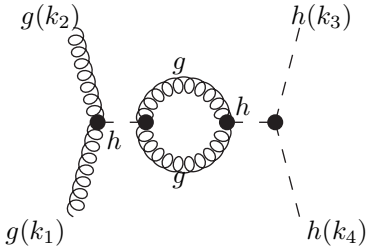


Diagram 92
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 4$

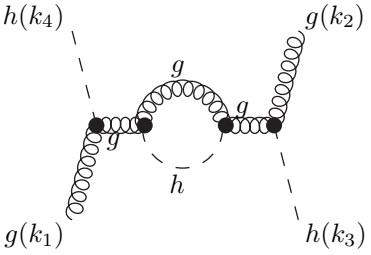


Diagram 103
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

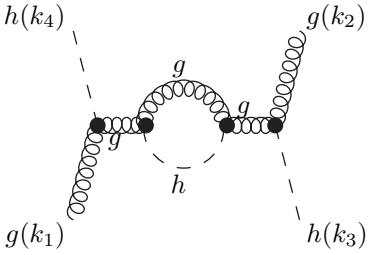


Diagram 106
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

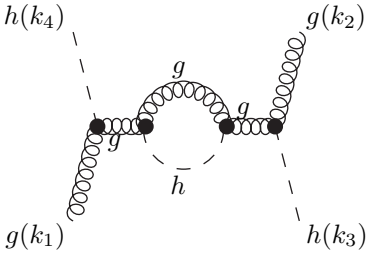


Diagram 107
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

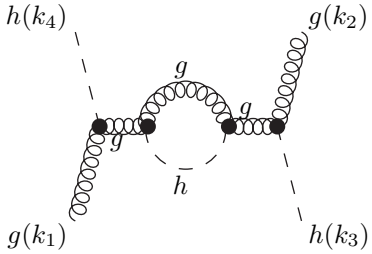


Diagram 108
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

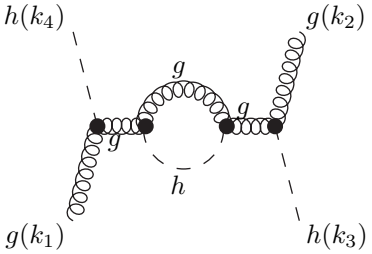


Diagram 109
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

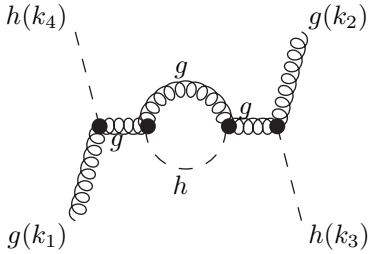


Diagram 110
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

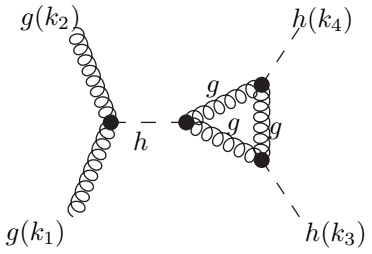


Diagram 114
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

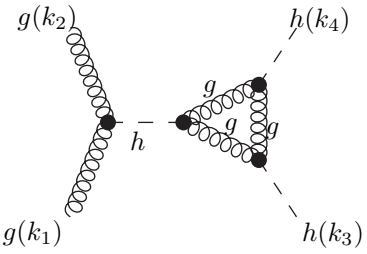


Diagram 115
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

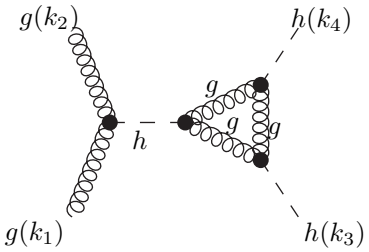


Diagram 116
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

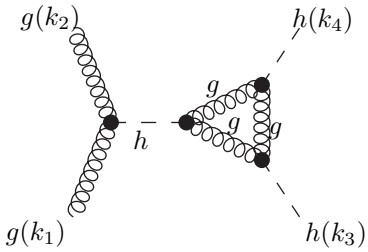


Diagram 117
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

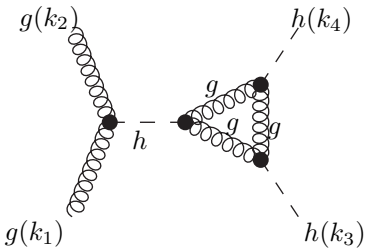


Diagram 118
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

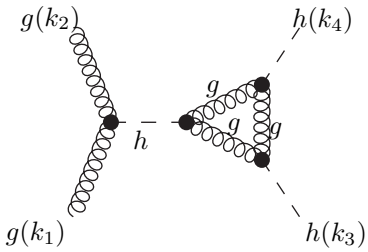


Diagram 119
$S' = S^{\{4\}}_{Q\to q-(k3)}$, rk $= 6$

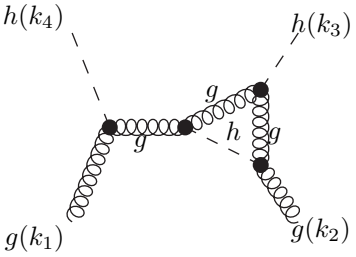


Diagram 133
$S' = S^{\{1\}}$, rk $= 6$

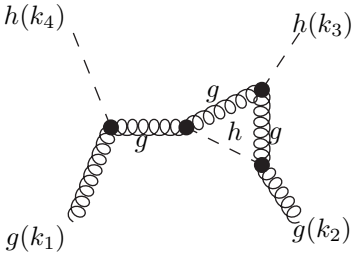


Diagram 134
$S' = S^{\{1\}}$, rk $= 6$

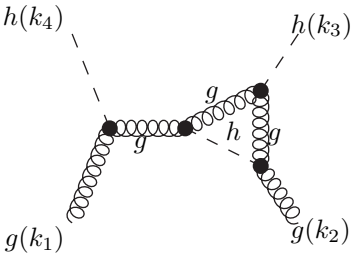


Diagram 135
$S' = S^{\{1\}}$, rk $= 6$

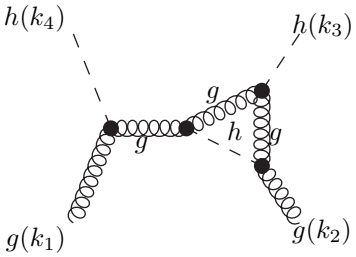


Diagram 139
$S' = S^{\{1\}}$, rk $= 6$

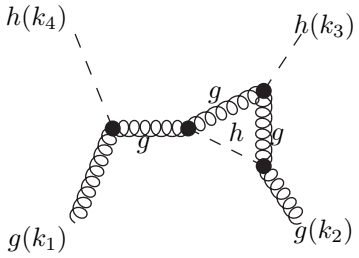


Diagram 140
$S' = S^{\{1\}}$, rk $= 6$

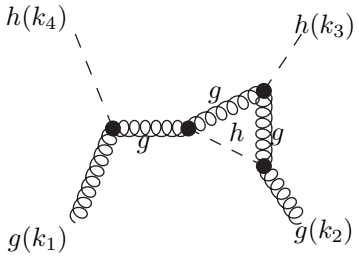


Diagram 141
$S' = S^{\{1\}}$, rk $= 6$

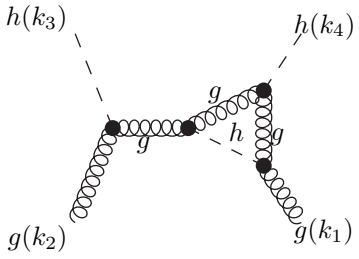


Diagram 145
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

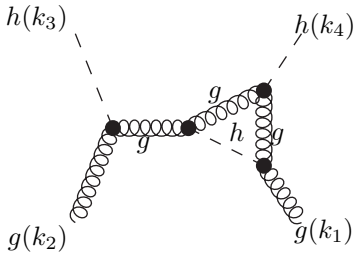


Diagram 146
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

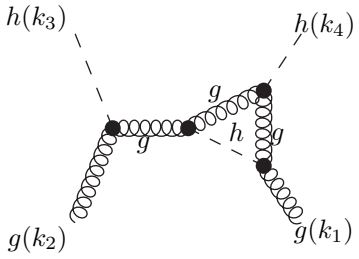


Diagram 147
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

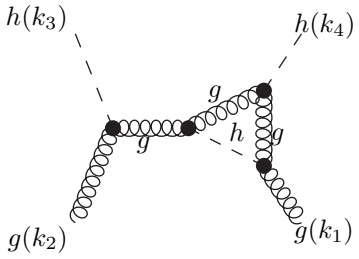


Diagram 151
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

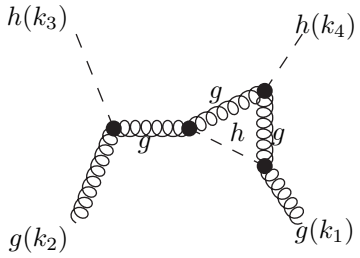


Diagram 152
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

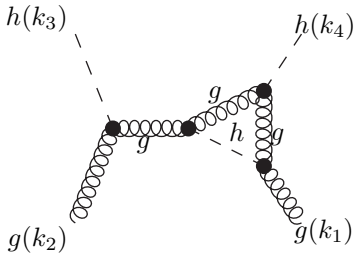


Diagram 153
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

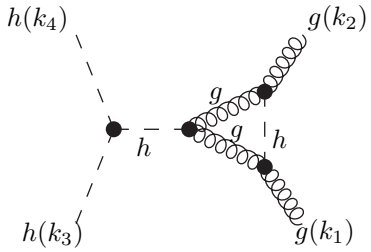


Diagram 172
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 6$

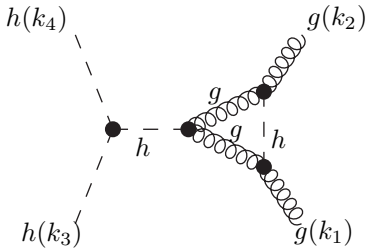


Diagram 173
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 6$

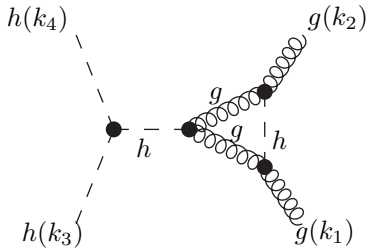


Diagram 174
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 6$

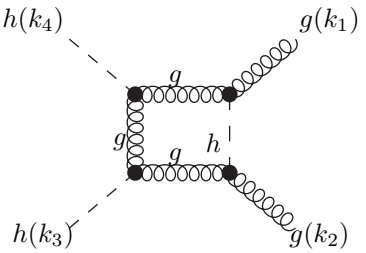


Diagram 189
$S' = S_{Q\to q-(k2)}$, rk $= 8$

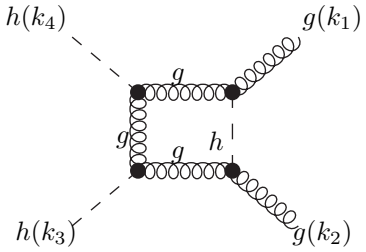


Diagram 190
$S' = S_{Q\to q-(k2)}$, rk $= 8$

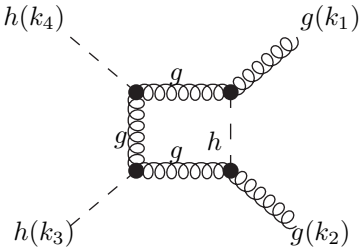


Diagram 191
$S' = S_{Q\to q-(k2)}$, rk $= 8$

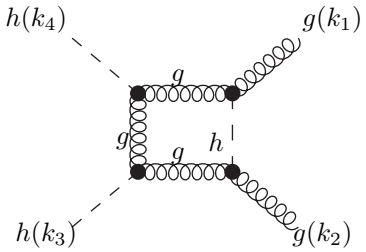


Diagram 192
$S' = S_{Q\to q-(k2)}$, rk $= 8$

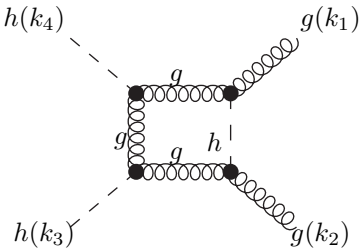


Diagram 193
$S' = S_{Q\to q-(k2)}$, rk $= 8$

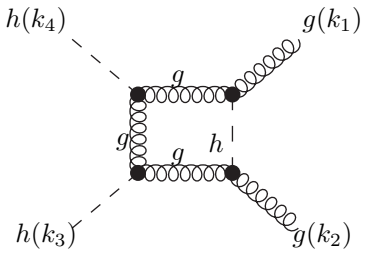


Diagram 194
$S' = S_{Q\to q-(k2)}$, rk $= 8$

5.6 Group 5 (4-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_4 \tag{29a}$$
$$r_2 = 0 \tag{29b}$$
$$r_3 = -k_2, \quad m_3 = \mathrm{Mh}, \quad \Gamma_3 = \mathrm{Wh} \tag{29c}$$
$$r_4 = k_3 - k_2, \quad m_4 = \mathrm{Mh}, \quad \Gamma_4 = \mathrm{Wh} \tag{29d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{30}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{31a}$$
$$S_{1,3} = \mathrm{Mh}^2 - s_{23} - s_{12} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31b}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31c}$$
$$S_{2,3} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31d}$$
$$S_{2,4} = -\mathrm{Mh}^2 + s_{23} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31e}$$
$$S_{3,3} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31f}$$
$$S_{3,4} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31g}$$
$$S_{4,4} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{31h}$$

5.6.1 Diagrams (3)

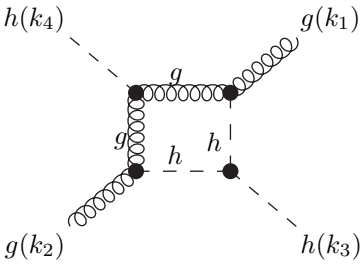


Diagram 201
$S' = S_{Q \to q-(-k3+k2)}$, rk $= 6$

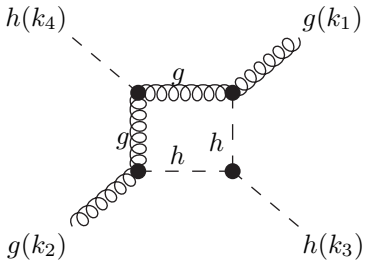


Diagram 202
$S' = S_{Q \to q-(-k3+k2)}$, rk $= 6$

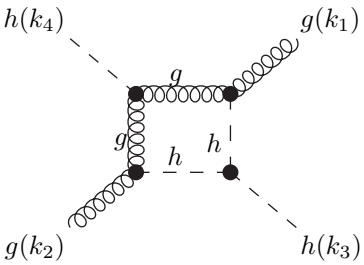


Diagram 203
$S' = S_{Q \to q-(-k3+k2)}$, rk $= 6$

5.7 Group 6 (4-Point)

General Information

The maximum effective rank in this group is 8.

$$r_1 = -k_3 - k_4 \tag{32a}$$
$$r_2 = -k_4 \tag{32b}$$
$$r_3 = 0 \tag{32c}$$
$$r_4 = -k_2, \quad m_4 = \mathrm{Mh}, \quad \Gamma_4 = \mathrm{Wh} \tag{32d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{33}$$

$$S_{1,2} = \mathrm{Mh}^2 \tag{34a}$$
$$S_{1,3} = s_{12} \tag{34b}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{34c}$$
$$S_{2,3} = \mathrm{Mh}^2 \tag{34d}$$
$$S_{2,4} = \mathrm{Mh}^2 - s_{23} - s_{12} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{34e}$$
$$S_{3,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{34f}$$
$$S_{4,4} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{34g}$$

5.7.1 Diagrams (34)

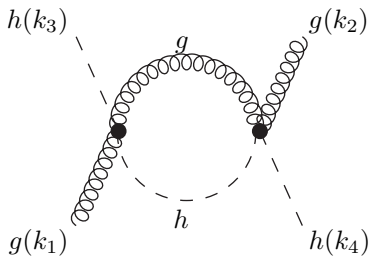


Diagram 8
$S' = S^{\{1,3\}}_{Q \to q-(k2)}$, rk $= 4$

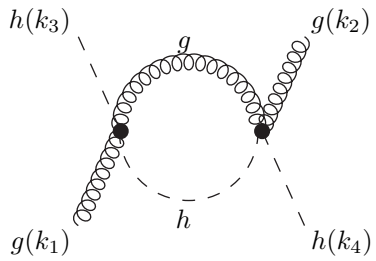


Diagram 9
$S' = S^{\{1,3\}}_{Q \to q-(k2)}$, rk $= 4$

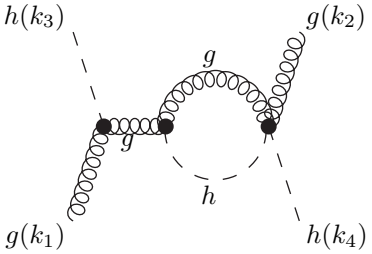


Diagram 19
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

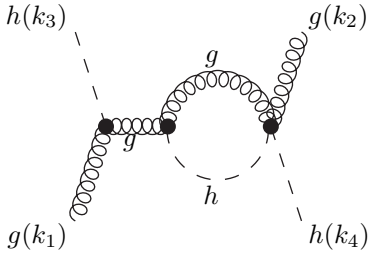


Diagram 20
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

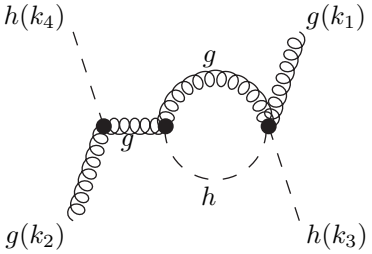


Diagram 28
$S' = S^{\{1,3\}}_{Q\to -q-(-k2)}$, rk $= 4$

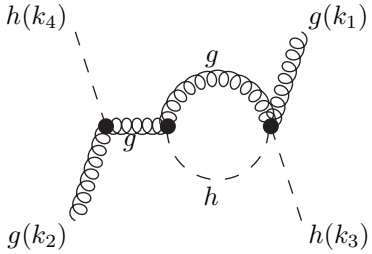


Diagram 29
$S' = S^{\{1,3\}}_{Q\to -q-(-k2)}$, rk $= 4$

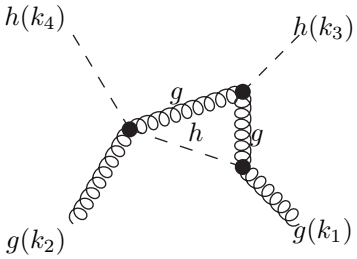


Diagram 38
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

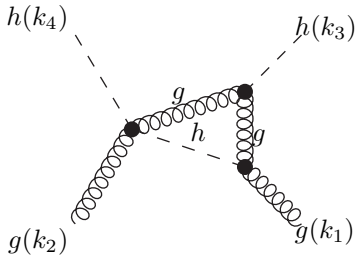


Diagram 39
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 6$

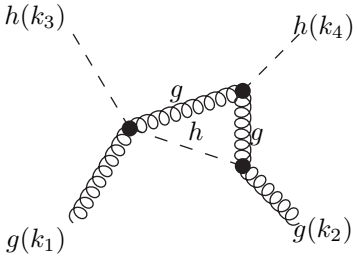


Diagram 50
$S' = S^{\{1\}}$, rk $= 6$

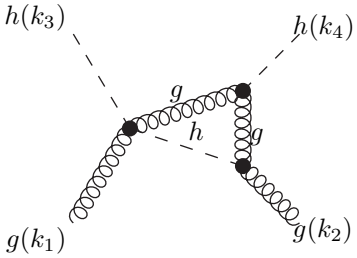


Diagram 51
$S' = S^{\{1\}}$, rk $= 6$

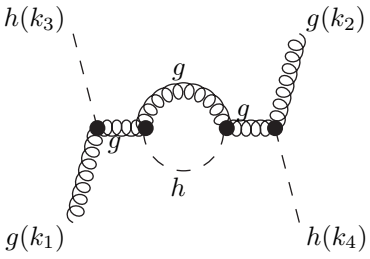


Diagram 94
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

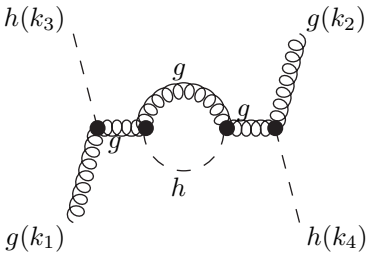


Diagram 97
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

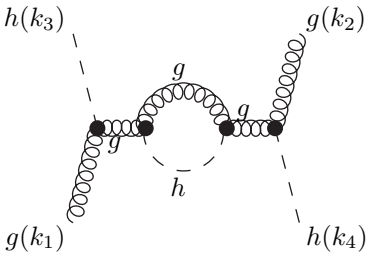


Diagram 98
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

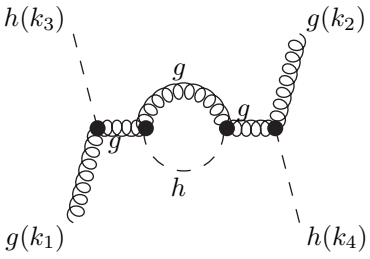


Diagram 99
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

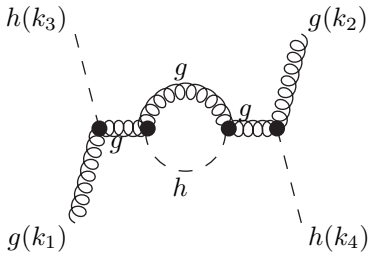


Diagram 100
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

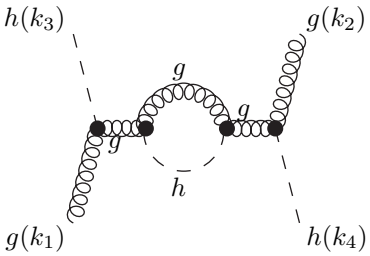


Diagram 101
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 4$

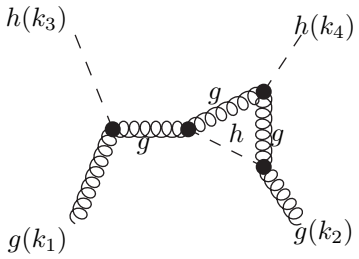


Diagram 121
$S' = S^{\{1\}}$, rk $= 6$

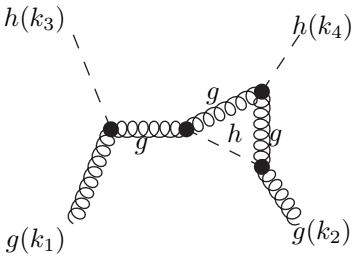


Diagram 122
$S' = S^{\{1\}}$, rk $= 6$

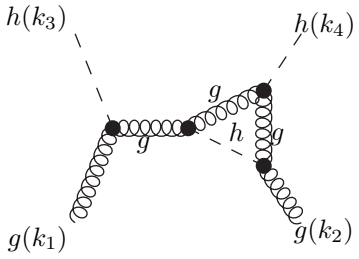


Diagram 123
$S' = S^{\{1\}}$, rk $= 6$

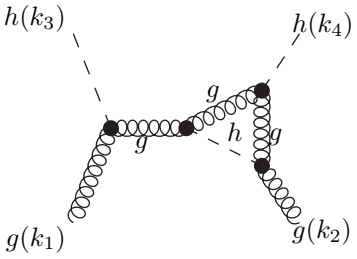


Diagram 127
$S' = S^{\{1\}}$, rk $= 6$

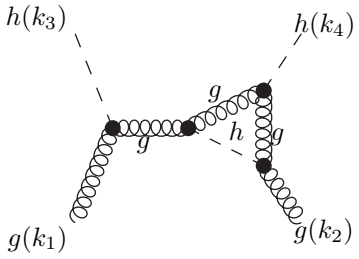


Diagram 128
$S' = S^{\{1\}}$, rk $= 6$

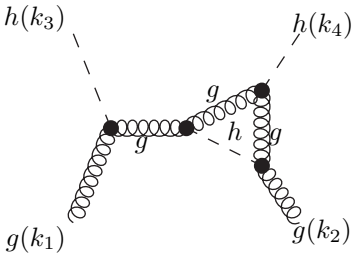


Diagram 129
$S' = S^{\{1\}}$, rk $= 6$

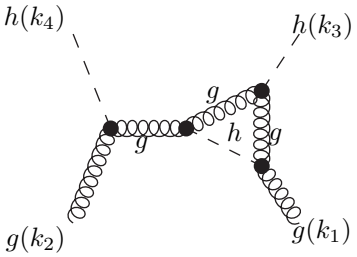


Diagram 157
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

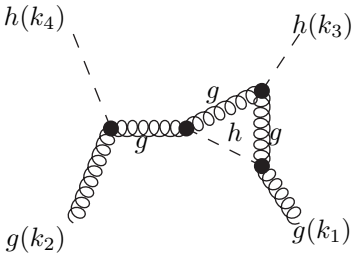


Diagram 158
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

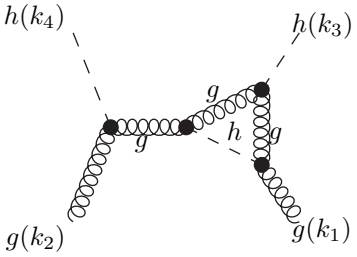


Diagram 159
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

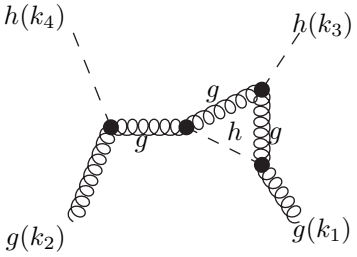


Diagram 163
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

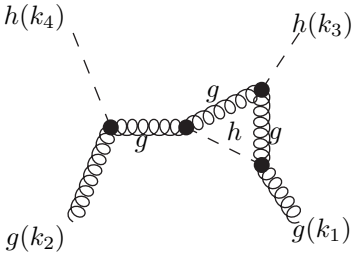


Diagram 164
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

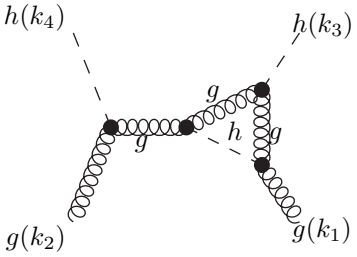


Diagram 165
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk = 6

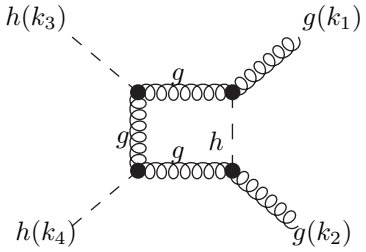


Diagram 179
$S' = S_{Q\to q-(k2)}$, rk = 8

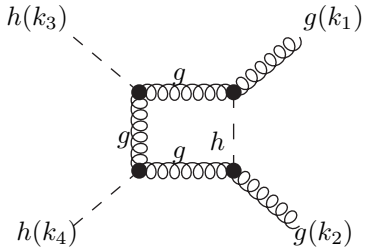


Diagram 180
$S' = S_{Q\to q-(k2)}$, rk = 8

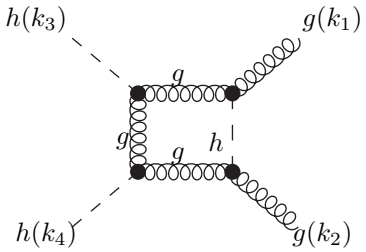


Diagram 181
$S' = S_{Q\to q-(k2)}$, rk = 8

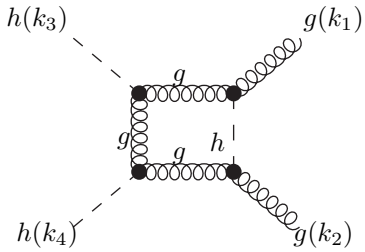


Diagram 182
$S' = S_{Q\to q-(k2)}$, rk = 8

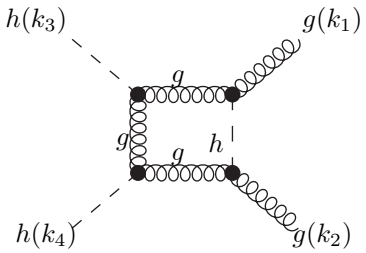


Diagram 183
$S' = S_{Q\to q-(k2)}$, rk = 8

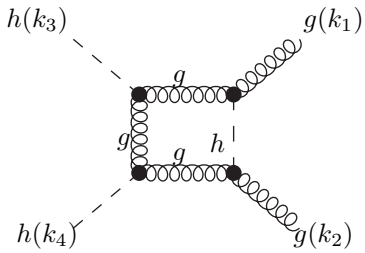


Diagram 184
$S' = S_{Q\to q-(k2)}$, rk = 8

5.8 Group 7 (4-Point)

General Information

The maximum effective rank in this group is 5.

$$r_1 = k_3 - k_2 + k_4, \quad m_1 = \mathrm{Mh}, \quad \Gamma_1 = \mathrm{Wh} \tag{35a}$$
$$r_2 = -k_2 + k_4, \quad m_2 = \mathrm{Mh}, \quad \Gamma_2 = \mathrm{Wh} \tag{35b}$$
$$r_3 = -k_2, \quad m_3 = \mathrm{Mh}, \quad \Gamma_3 = \mathrm{Wh} \tag{35c}$$
$$r_4 = 0 \tag{35d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & 0 \end{pmatrix} \tag{36}$$

$$S_{1,1} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37a}$$
$$S_{1,2} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37b}$$
$$S_{1,3} = -2\mathrm{Mh}^2 + s_{12} + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37c}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37d}$$
$$S_{2,2} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37e}$$
$$S_{2,3} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37f}$$
$$S_{2,4} = \mathrm{Mh}^2 - s_{23} - s_{12} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37g}$$
$$S_{3,3} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37h}$$
$$S_{3,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{37i}$$

5.8.1 Diagrams (5)

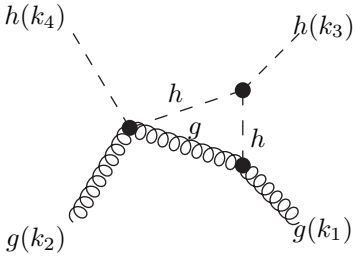


Diagram 37
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

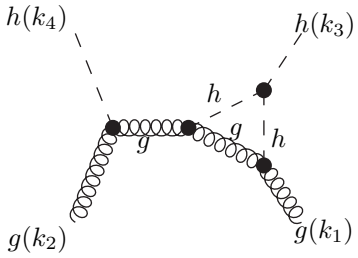


Diagram 162
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

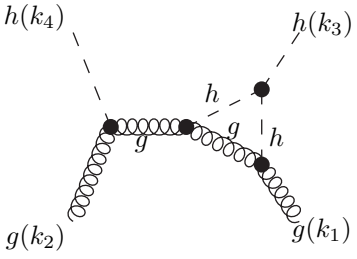


Diagram 166
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

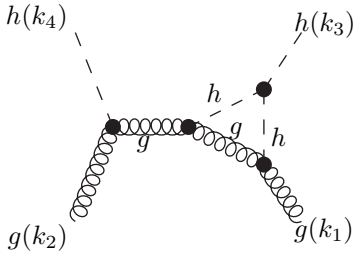


Diagram 167
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

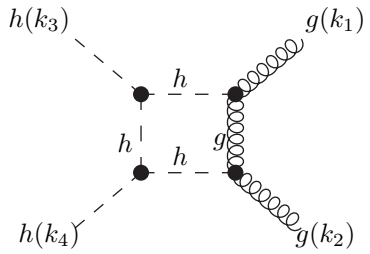


Diagram 176
$S' = S_{Q\to -q}$, rk $= 4$

5.9 Group 8 (4-Point)

General Information

The maximum effective rank in this group is 5.

$$r_1 = k_3 - k_2 + k_4, \quad m_1 = \mathrm{Mh}, \quad \Gamma_1 = \mathrm{Wh} \tag{38a}$$
$$r_2 = k_3 - k_2, \quad m_2 = \mathrm{Mh}, \quad \Gamma_2 = \mathrm{Wh} \tag{38b}$$
$$r_3 = -k_2, \quad m_3 = \mathrm{Mh}, \quad \Gamma_3 = \mathrm{Wh} \tag{38c}$$
$$r_4 = 0 \tag{38d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & 0 \end{pmatrix} \tag{39}$$

$$S_{1,1} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40a}$$
$$S_{1,2} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40b}$$
$$S_{1,3} = -2\mathrm{Mh}^2 + s_{12} + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40c}$$
$$S_{1,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40d}$$
$$S_{2,2} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40e}$$
$$S_{2,3} = -\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40f}$$
$$S_{2,4} = -\mathrm{Mh}^2 + s_{23} + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40g}$$
$$S_{3,3} = -2\mathrm{Mh}^2 + 2i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40h}$$
$$S_{3,4} = -\mathrm{Mh}^2 + i \cdot \mathrm{Mh} \cdot \mathrm{Wh} \tag{40i}$$

5.9.1 Diagrams (21)

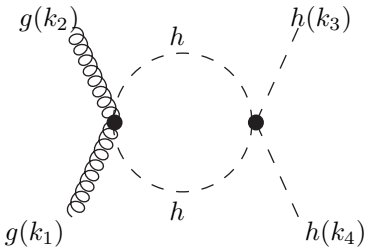


Diagram 6
$S' = S^{\{2,4\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 2$

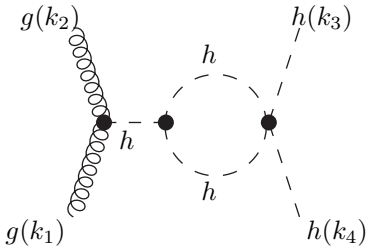


Diagram 17
$S' = S^{\{2,4\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 0$

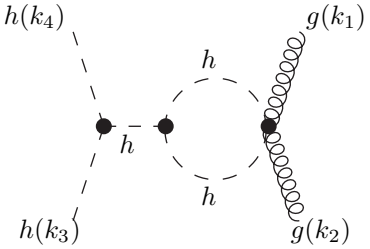


Diagram 31
$S' = S^{\{2,4\}}_{Q\to -q-(k3-k2+k4)}$, rk $= 2$

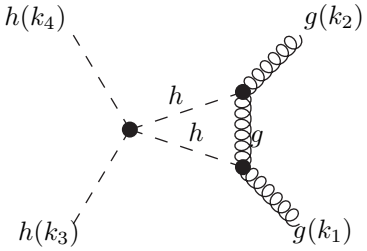


Diagram 35
$S' = S^{\{2\}}_{Q\to -q}$, rk $= 4$

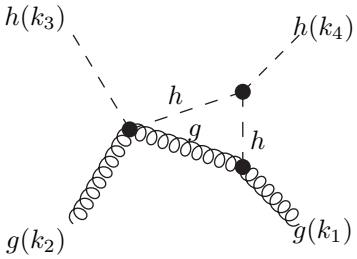


Diagram 41
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

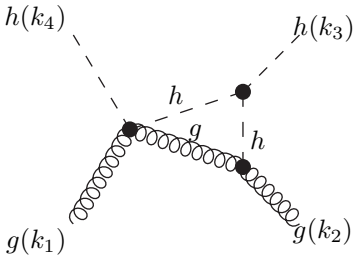


Diagram 45
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 4$

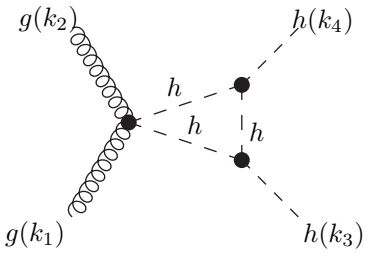


Diagram 53
$S' = S^{\{4\}}_{Q\to -q-(k3-k2)}$, rk $= 2$

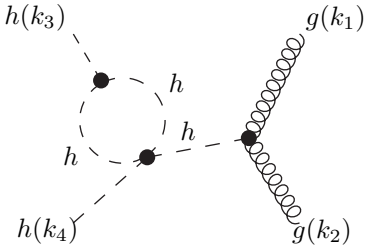


Diagram 58
$S' = S^{\{1,4\}}_{Q\to -q-(k3-k2)}$, rk $= 0$

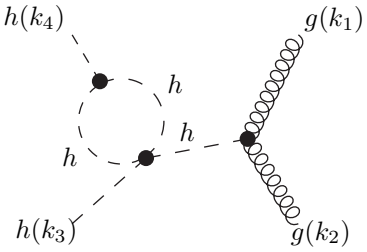


Diagram 63
$S' = S^{\{3,4\}}_{Q\to -q-(k3-k2+k4)}$, rk $= 0$

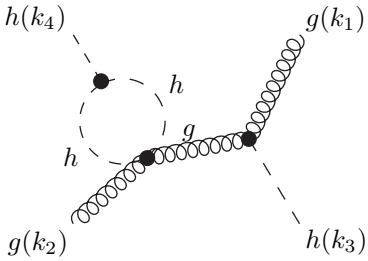


Diagram 68
$S' = S^{\{3,4\}}_{Q\to -q-(k3-k2+k4)}$, rk $= 2$

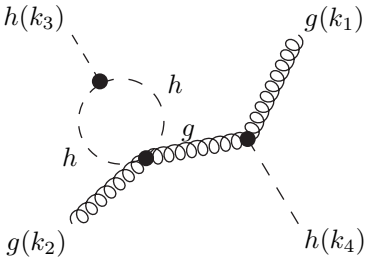


Diagram 73
$S' = S^{\{1,4\}}_{Q\to -q-(k3-k2)}$, rk $= 2$

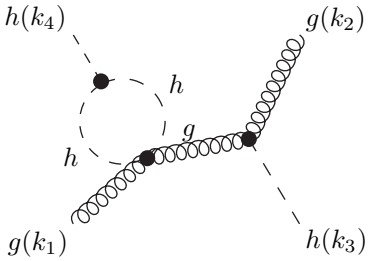


Diagram 78
$S' = S^{\{3,4\}}_{Q\to -q-(k3-k2+k4)}$, rk $= 2$

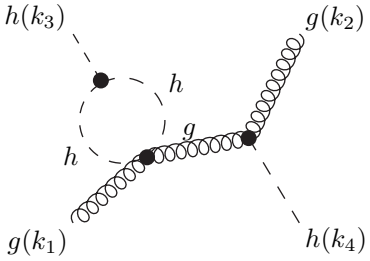


Diagram 83
$S' = S^{\{1,4\}}_{Q\to -q-(k3-k2)}$, rk $= 2$

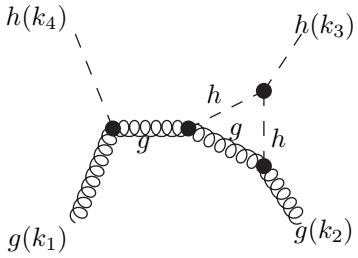


Diagram 138
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 4$

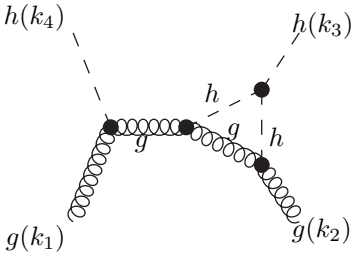


Diagram 142
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 4$

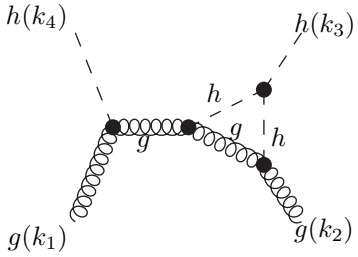


Diagram 143
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 4$

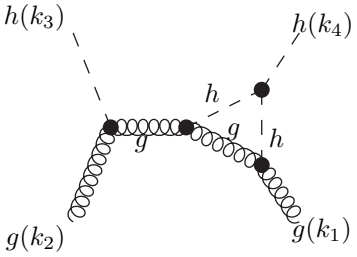


Diagram 150
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

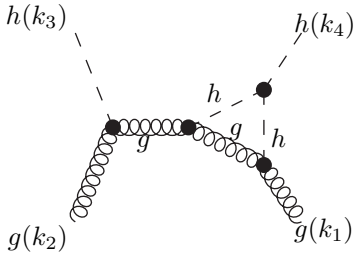


Diagram 154
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk $= 4$

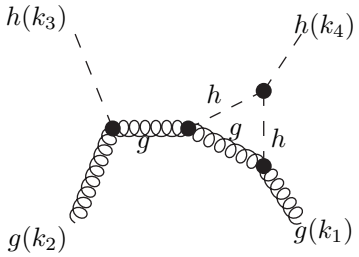


Diagram 155
$S' = S^{\{3\}}_{Q\to q-(-k3+k2-k4)}$, rk = 4

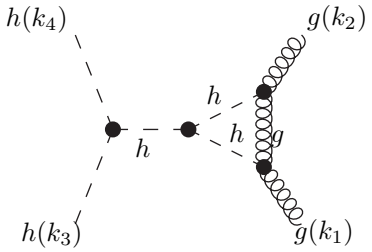


Diagram 169
$S' = S^{\{2\}}_{Q\to -q}$, rk = 4

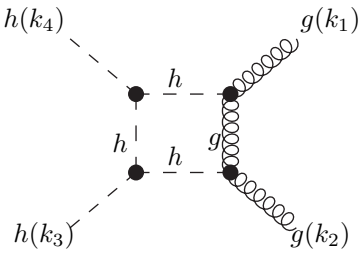


Diagram 186
$S' = S_{Q\to -q}$, rk = 4

5.10 Group 9 (4-Point)

General Information

The maximum effective rank in this group is 4.

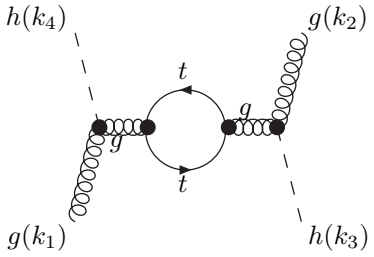
$$r_1 = -k_2 + k_4, \quad m_1 = \mathrm{MT} \tag{41a}$$
$$r_2 = -k_2, \quad m_2 = \mathrm{MT} \tag{41b}$$
$$r_3 = 0, \quad m_3 = \mathrm{MT} \tag{41c}$$
$$r_4 = -k_3, \quad m_4 = \mathrm{MT} \tag{41d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{42}$$

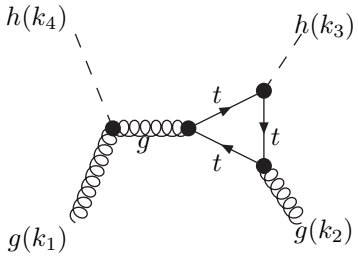
$$S_{1,1} = -2\mathrm{MT}^2 \tag{43a}$$
$$S_{1,2} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{43b}$$
$$S_{1,3} = 2\mathrm{Mh}^2 - s_{23} - 2\mathrm{MT}^2 - s_{12} \tag{43c}$$
$$S_{1,4} = -2\mathrm{MT}^2 \tag{43d}$$

$$S_{2,2} = -2\mathrm{MT}^2 \tag{43e}$$
$$S_{2,3} = -2\mathrm{MT}^2 \tag{43f}$$
$$S_{2,4} = s_{23} - 2\mathrm{MT}^2 \tag{43g}$$
$$S_{3,3} = -2\mathrm{MT}^2 \tag{43h}$$
$$S_{3,4} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{43i}$$
$$S_{4,4} = -2\mathrm{MT}^2 \tag{43j}$$

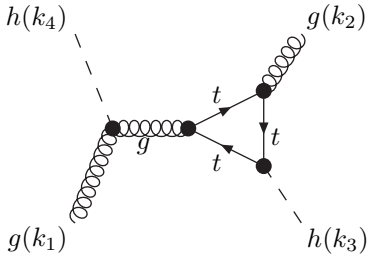
5.10.1 Diagrams (7)



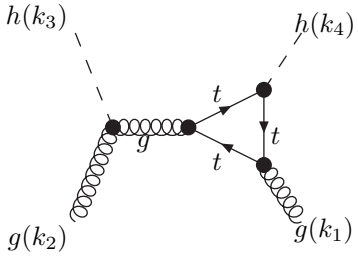
-Diagram 105
$S' = S^{\{1,3\}}_{Q\to q-(k2)}$, rk $= 2$



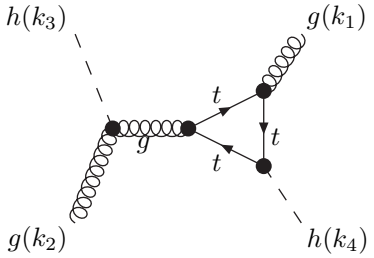
-Diagram 136
$S' = S^{\{1\}}$, rk $= 3$



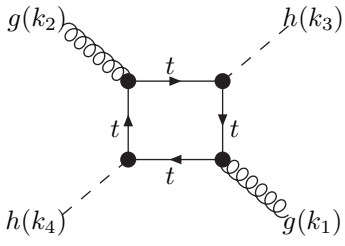
-Diagram 137
$S' = S^{\{1\}}$, rk $= 3$



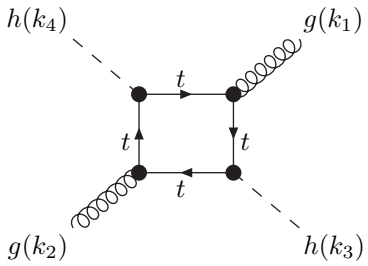
-Diagram 148
$S' = S^{\{3\}}_{Q\to q-(k2-k4)}$, rk $= 3$



-Diagram 149
$S' = S^{\{3\}}_{Q\to q-(k2-k4)}$, rk $= 3$



-Diagram 199
$S' = S_{Q\to -q-(-k3)}$, rk $= 4$



-Diagram 200
$S' = S_{Q\to -q-(-k3)}$, rk = 4

5.11 Group 10 (4-Point)

General Information

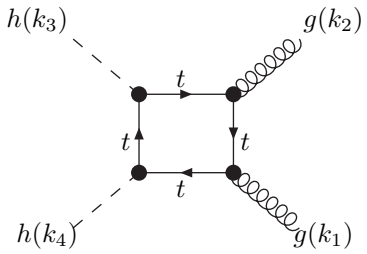
The maximum effective rank in this group is 4.

$$r_1 = -k_3 - k_4, \quad m_1 = \mathrm{MT} \tag{44a}$$
$$r_2 = -k_3, \quad m_2 = \mathrm{MT} \tag{44b}$$
$$r_3 = 0, \quad m_3 = \mathrm{MT} \tag{44c}$$
$$r_4 = -k_2, \quad m_4 = \mathrm{MT} \tag{44d}$$

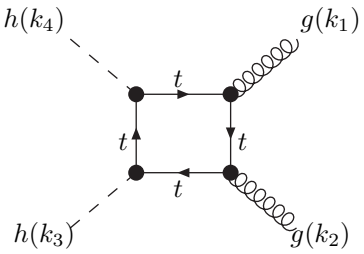
$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{45}$$

$$S_{1,1} = -2\mathrm{MT}^2 \tag{46a}$$
$$S_{1,2} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{46b}$$
$$S_{1,3} = -2\mathrm{MT}^2 + s_{12} \tag{46c}$$
$$S_{1,4} = -2\mathrm{MT}^2 \tag{46d}$$
$$S_{2,2} = -2\mathrm{MT}^2 \tag{46e}$$
$$S_{2,3} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{46f}$$
$$S_{2,4} = s_{23} - 2\mathrm{MT}^2 \tag{46g}$$
$$S_{3,3} = -2\mathrm{MT}^2 \tag{46h}$$
$$S_{3,4} = -2\mathrm{MT}^2 \tag{46i}$$
$$S_{4,4} = -2\mathrm{MT}^2 \tag{46j}$$

5.11.1 Diagrams (2)



-Diagram 187
$S' = S_{Q\to q-(k2)}$, rk = 4



-Diagram 188
$S' = S_{Q\to q-(k2)}$, rk = 4

5.12 Group 11 (4-Point)

General Information

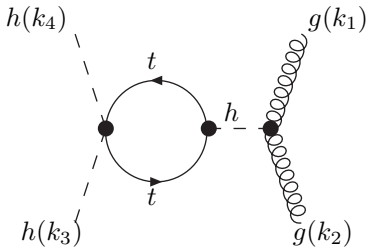
The maximum effective rank in this group is 4.

$$r_1 = -k_3 - k_4, \quad m_1 = \mathrm{MT} \tag{47a}$$
$$r_2 = -k_4, \quad m_2 = \mathrm{MT} \tag{47b}$$
$$r_3 = 0, \quad m_3 = \mathrm{MT} \tag{47c}$$
$$r_4 = -k_2, \quad m_4 = \mathrm{MT} \tag{47d}$$

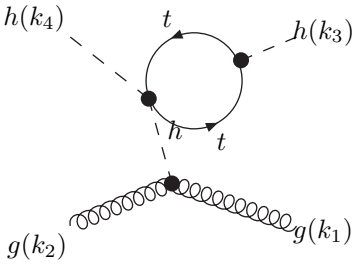
$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{48}$$

$$S_{1,1} = -2\mathrm{MT}^2 \tag{49a}$$
$$S_{1,2} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{49b}$$
$$S_{1,3} = -2\mathrm{MT}^2 + s_{12} \tag{49c}$$
$$S_{1,4} = -2\mathrm{MT}^2 \tag{49d}$$
$$S_{2,2} = -2\mathrm{MT}^2 \tag{49e}$$
$$S_{2,3} = \mathrm{Mh}^2 - 2\mathrm{MT}^2 \tag{49f}$$
$$S_{2,4} = 2\mathrm{Mh}^2 - s_{23} - 2\mathrm{MT}^2 - s_{12} \tag{49g}$$
$$S_{3,3} = -2\mathrm{MT}^2 \tag{49h}$$
$$S_{3,4} = -2\mathrm{MT}^2 \tag{49i}$$
$$S_{4,4} = -2\mathrm{MT}^2 \tag{49j}$$

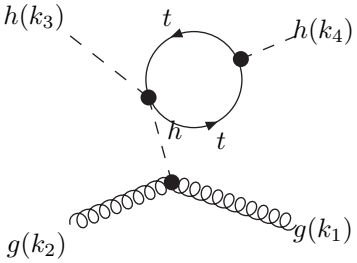
5.12.1 Diagrams (12)



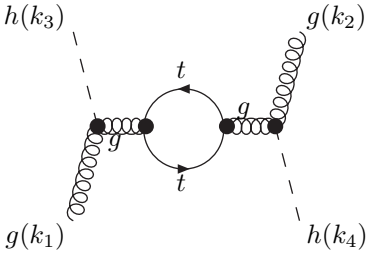
-Diagram 16
$S' = S^{\{2,4\}}_{Q\to -q-(-k3-k4)}$, rk $= 2$



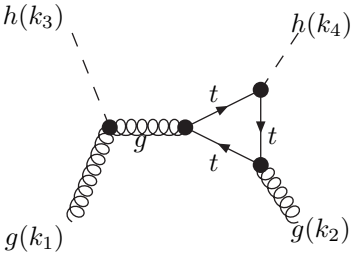
-Diagram 57
$S' = S^{\{3,4\}}_{Q\to q-(k3+k4)}$, rk $= 2$



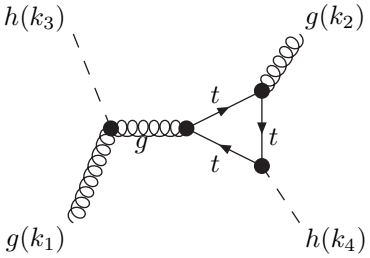
-Diagram 62
$S' = S^{\{1,4\}}_{Q\to q-(k4)}$, rk $= 2$



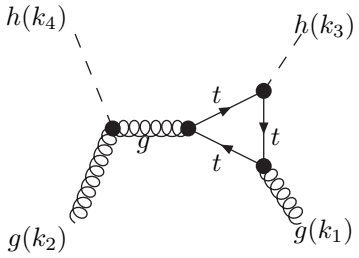
-Diagram 96
$S' = S^{\{1,3\}}_{Q\to -q-(-k4)}$, rk $= 2$



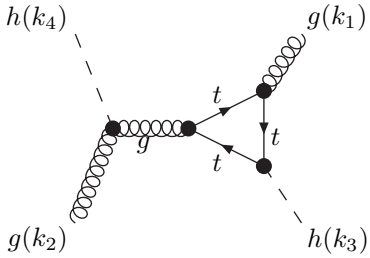
-Diagram 124
$S' = S^{\{1\}}$, rk $= 3$



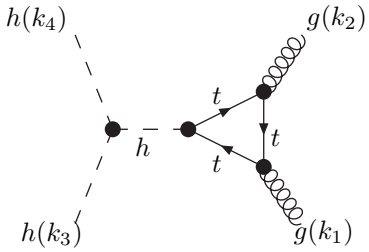
-Diagram 125
$S' = S^{\{1\}}$, rk $= 3$



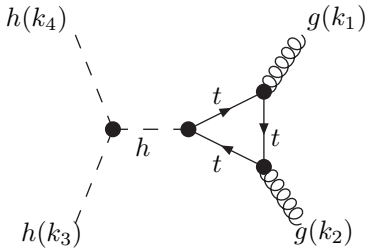
-Diagram 160
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 3$



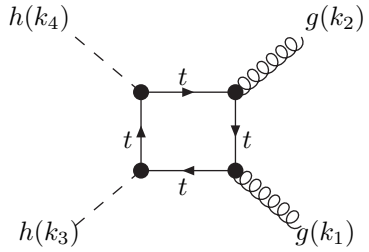
-Diagram 161
$S' = S^{\{3\}}_{Q\to -q-(-k3-k4)}$, rk $= 3$



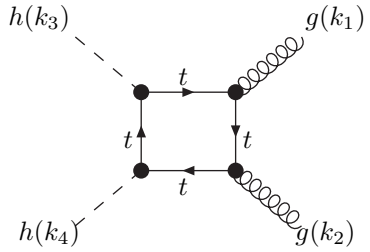
-Diagram 170
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 3$



-Diagram 171
$S' = S^{\{2\}}_{Q\to q-(k2)}$, rk $= 3$



-Diagram 177
$S' = S_{Q\to q-(k2)}$, rk $= 4$



-Diagram 178
$S' = S_{Q\to q-(k2)}$, rk $= 4$

Index of all Loop Diagrams

6 Related Work

If you publish results obtained by using this matrix element code please cite the appropriate papers in the bibliography of this document.

Scientific publications prepared using the present version of GoSam or any modified version of it or any code linking to GoSam or parts of it should make a clear reference to the publications [1, 2].

For graph generation we use QGraf [3]. The Feynman diagrams are further processed with the symbolic manipulation program FORM [4, 5] using the FORM library SPINNEY [6]. The Fortran 90 code is generated using FORM [4, 5]. For the reduction of the tensor integrals the code uses an implementation of the Laurent series expansion method [8] from the library Ninja [7].

Please, make sure, you also give credit to the authors of the scalar loop libraries, if you configured the amplitude code such that it calls other libraries than the ones mentioned so far. Depending on your configuration you might use one or more of the following programs for the evaluation of the scalar integrals:

- OneLOop [12],
- QCDLoop [13], which uses FF [14],
- LoopTools [15], which uses FF [14].
- GOLEM95 [10, 9] which uses OneLOop [12] and may be configured such that it uses LoopTools [15, 14].

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[2] G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, G. Ossola, T. Reiter and F. Tramontano, "Automated One-Loop Calculations with GoSam," Eur. Phys. J. C **72** (2012) 1889 [arXiv:1111.2034 [hep-ph]].

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