GoSam 2.0.2: $u\bar{u} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$

scyboz

2016-08-23 (10:33:49)

Abstract

This process consists of 14 tree-level diagrams and 334 NLO diagrams. GoSam has identified 13 groups of NLO diagrams by analyzing their one-loop integrals.

Contents

1 Helicities

Index	1	2	3	4	5	6	7	8
0	+	−	+	−	−	+	+	−
1	−	+	+	−	−	+	+	−
2	+	−	+	−	−	+	−	+
3	−	+	+	−	−	+	−	+

2 Wave Functions

In this section, we use $l_i = k_i$ for massless particles; in spinors $|i\rangle$ (resp. $|i]$) denote $|l_i\rangle$ (resp. $|l_i]$).

All helicity amplitudes are defined in terms of the following wave functions:

- $u(k_1)$

$$u_+(k_1) = |1\rangle \tag{1}$$
$$u_-(k_1) = |1] \tag{2}$$

- $\bar{u}(k_2)$

$$\bar{v}_+(k_2) = \langle 2| \tag{3}$$
$$\bar{v}_-(k_2) = [2| \tag{4}$$

- $e^+(k_3)$

$$v_+(k_3) = |3] \tag{5}$$
$$v_-(k_3) = |3\rangle \tag{6}$$

- $\nu_e(k_4)$

$$\bar{u}_+(k_4) = [4| \tag{7}$$
$$\bar{u}_-(k_4) = \langle 4| \tag{8}$$

- $\mu^-(k_5)$

$$\bar{u}_+(k_5) = [5| \tag{9}$$
$$\bar{u}_-(k_5) = \langle 5| \tag{10}$$

- $\bar{\nu}_\mu(k_6)$

$$v_+(k_6) = |6] \tag{11}$$
$$v_-(k_6) = |6\rangle \tag{12}$$

- $b(k_7)$

$$\bar{u}_+(k_7) = [7| \tag{13}$$
$$\bar{u}_-(k_7) = \langle 7| \tag{14}$$

- $\bar{b}(k_8)$

$$v_+(k_8) = |8] \tag{15}$$
$$v_-(k_8) = |8\rangle \tag{16}$$

3 Colour Basis

$$|c_1\rangle = 1q_{i_1}^{(1)}\bar{q}_{i_1}^{(1)}\bar{q}_{i_8}^{(8)}q_{i_8}^{(8)} \tag{17}$$

$$|c_2\rangle = 1q_{i_1}^{(1)}\bar{q}_{i_8}^{(8)}\bar{q}_{i_1}^{(1)}q_{i_8}^{(8)} \tag{18}$$

4 Tree Diagrams

QGraf Setup

```
--------------------------------------------------------------------------------
                                  qgraf-3.1.4
--------------------------------------------------------------------------------

output = 'diagrams-0.hh';
style = 'form.sty';
model = 'model';
in = U[k1], Ubar[k2];
out = ep[k3], ne[k4], mum[k5], nmubar[k6], B[k7], Bbar[k8];
loops=0;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[phim,php,chi,H,ep,em,ne,nebar,mup,mum,nmu,nmubar,0,0];
true=vsum[QCD,2,2];
true=vsum[QED,4,4];

--------------------------------------------------------------------------------

  24P  ---  7+  17-  ---  5N+  2C+  17C-

  145V  ---  3^118  4^27

--------------------------------------------------------------------------------

   -    4^3  ---  0 diagrams
 3^2    4^2  ---  0 diagrams
 3^4    4^1  ---  0 diagrams
 3^6     -   ---  14 diagrams

  total =  14 diagrams
```

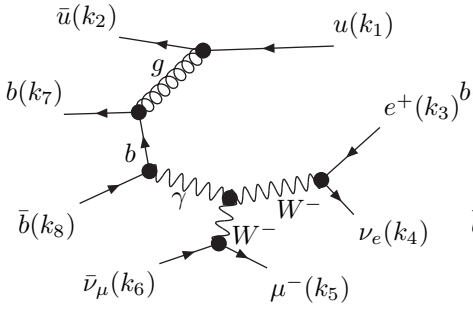


Diagram 1

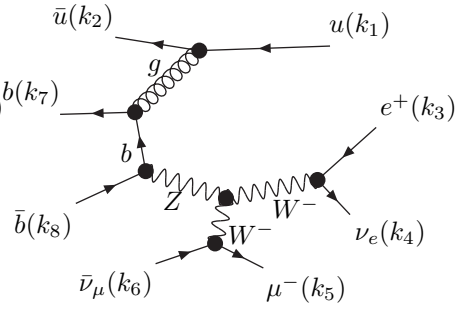


Diagram 2

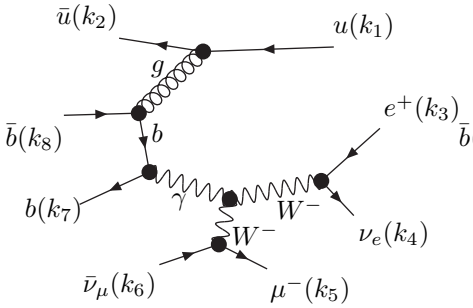


Diagram 3

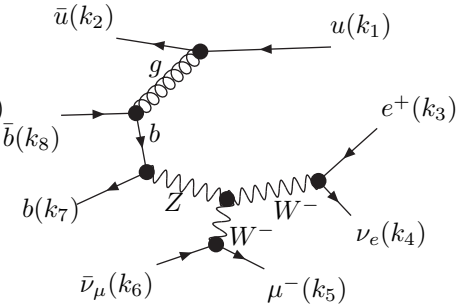


Diagram 4

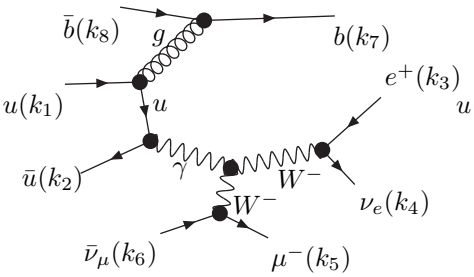


Diagram 5

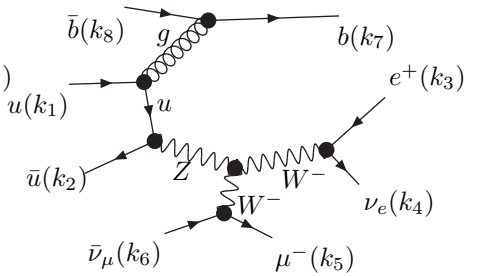


Diagram 6

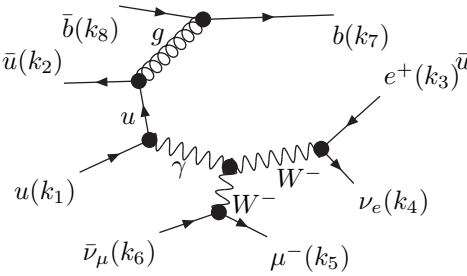


Diagram 7

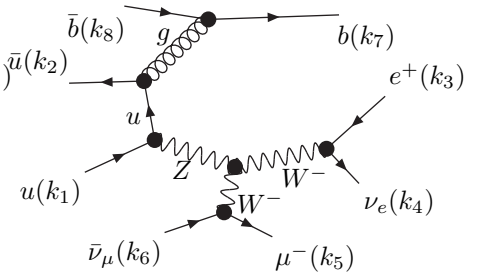


Diagram 8

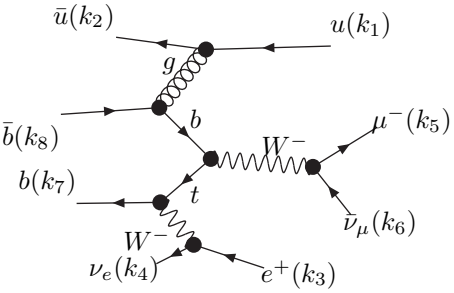


Diagram 9

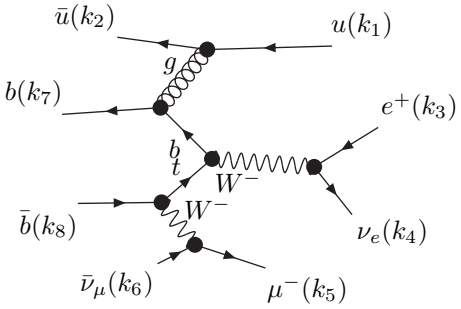


Diagram 10

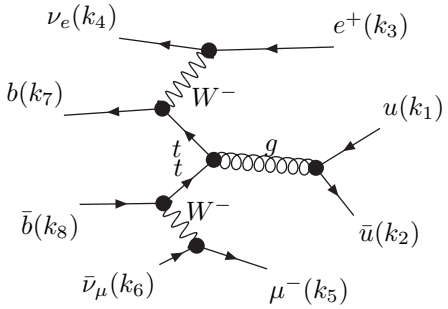


Diagram 11

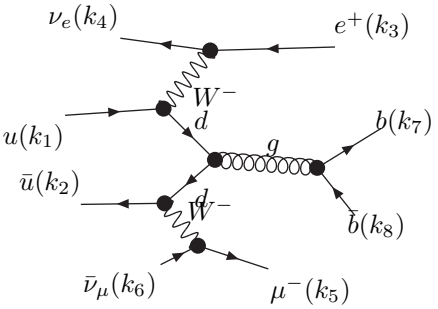


Diagram 12

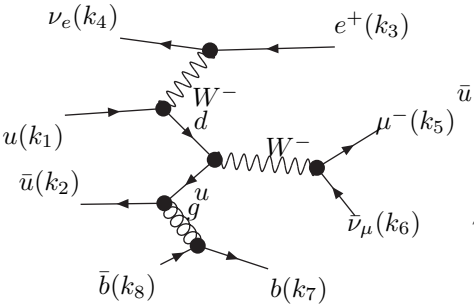


Diagram 13

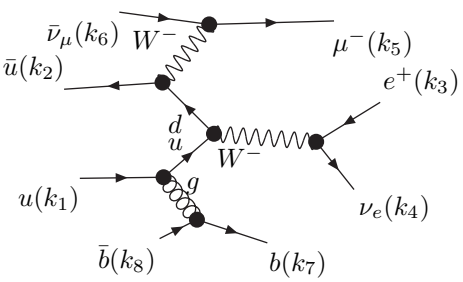


Diagram 14

5 One-Loop Diagrams

General Information

QGraf Setup

```
qgraf-3.1.4

output = 'diagrams-1.hh';
style = 'form.sty';
model = 'model';
```

```
in = U[k1], Ubar[k2];
out = ep[k3], ne[k4], mum[k5], nmubar[k6], B[k7], Bbar[k8];
loops=1;
loop_momentum=p;
options=onshell, notadpole, nosnail;
true=iprop[phim,phip,chi,H,ep,em,ne,nebar,mup,mum,nmu,nmubar,0,0];
true=vsum[QCD,4,4];
true=vsum[QED,4,4];

------------------------------------------------------------------------

  24P  ---  7+  17-  ---  5N+  2C+  17C-

  145V  ---  3^118  4^27

------------------------------------------------------------------------

   -   4^4  ---  0 diagrams
 3^2   4^3  ---  0 diagrams
 3^4   4^2  ---  0 diagrams
 3^6   4^1  ---  0 diagrams
 3^8    -   ---  424 diagrams

 total =  424 diagrams
```

Loop diagrams are grouped into sets of diagrams which share loop-propagators. A loop integral can be written as

$$\int \frac{\mathrm{d}^n k}{i\pi^{\frac{n}{2}}} \frac{\mathcal{N}(q)}{\prod_{j=1} N\left[(k+r_j)^2 - (m_j^2 - i m_j \Gamma_j) + i\delta\right]}. \tag{19}$$

For each group we list r_j, m_j and Γ_j. For m_j and Γ_j only non-vanishing symbols are listed. Furthermore, we give the matrix S which is defined as

$$S_{\alpha\beta} = (r_\alpha - r_\beta)^2 - (m_\alpha^2 - i m_\alpha \Gamma_\alpha) - (m_\beta^2 - i m_\beta \Gamma_\beta). \tag{20}$$

For each diagram we denote how the matrix S' for the specific diagram is obtained from the original S. The notation

$$S' = S_{Q\to q'}^{\{l_1,l_2,\ldots\}} \tag{21}$$

means, that the rows and columns labeled by $l_1, l_2, \ldots$ should be removed from S (likewise $r_{l_1}, r_{l_2}, \ldots$ are removed from the list of propagators) and $\mathcal{N}(q)$ has to be replaced by $\mathcal{N}(q')$. The maximum effective rank of a group is the rank that has to be passed to SAMURAI if the whole group is reduced at once; this number is calculated as

$$\max_{\text{diagrams}} \left\{(\text{rank of diagram}) + (\text{number of pinches})\right\}. \tag{22}$$

Diagrams with massless closed quark lines are multiplied by a factor `Nfrat` = `Nf`/`Nfgen`. This multiplication is indicated by the symbol N_f following the rank. By default `Nfrat` evaluates to one but can be changed by modifying `Nf` or `Nfgen` in the model file.

5.1 Group 0 (4-Point)

General Information

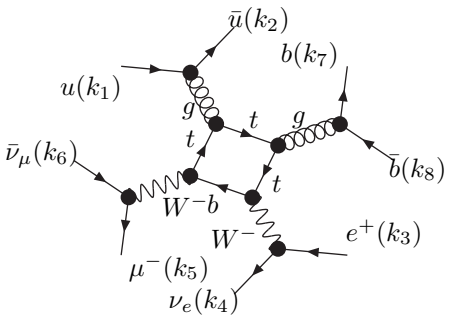
The maximum effective rank in this group is 4.

$$r_1 = -k_3 - k_7 - k_8 - k_4, \quad m_1 = m_t, \quad \Gamma_1 = \Gamma_t \tag{23a}$$
$$r_2 = -k_3 - k_4, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{23b}$$
$$r_3 = 0 \tag{23c}$$
$$r_4 = k_6 + k_5, \quad m_4 = m_t, \quad \Gamma_4 = \Gamma_t \tag{23d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{24}$$

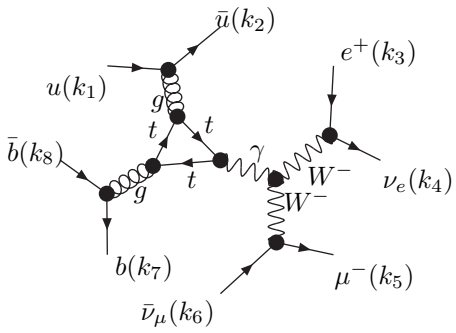
$$S_{1,1} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{25a}$$
$$S_{1,2} = -2m_t^2 + s_{78} + 2i \cdot m_t \cdot \Gamma_t \tag{25b}$$
$$S_{1,3} = -m_t^2 - s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} + i \cdot m_t \cdot \Gamma_t \tag{25c}$$
$$S_{1,4} = s_{12} - 2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{25d}$$
$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{25e}$$
$$S_{2,3} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{25f}$$
$$S_{2,4} = -2m_t^2 + s_{3456} + 2i \cdot m_t \cdot \Gamma_t \tag{25g}$$
$$S_{3,4} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{25h}$$
$$S_{4,4} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{25i}$$

5.1.1 Diagrams (12)



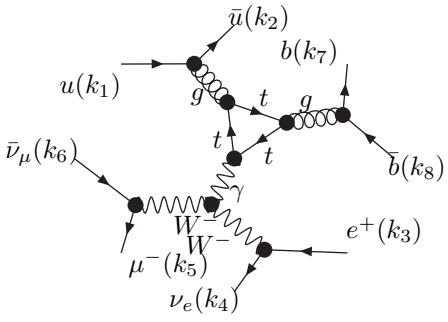
-Diagram 17

$S' = S_{Q\to q-(-k6-k5)}$, rk = 4



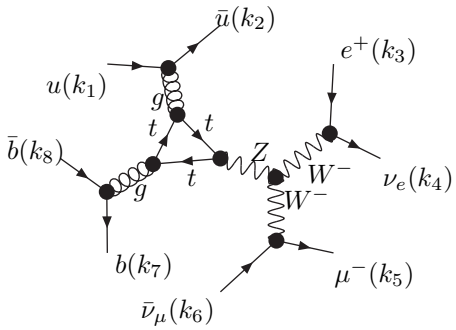
-Diagram 33

$S' = S^{\{3\}}_{Q\to -q-(-k3-k8-k7-k4)}$, rk = 3



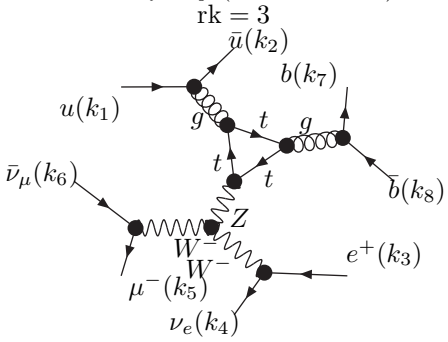
-Diagram 34

$S' = S^{\{3\}}_{Q\to -q-(-k3-k8-k7-k4)}$, rk $= 3$



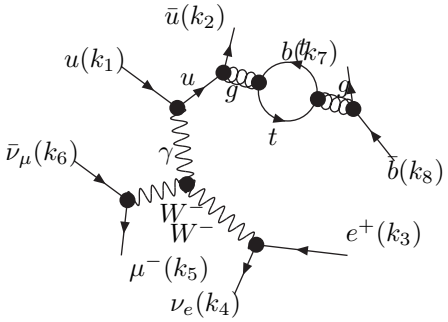
-Diagram 45

$S' = S^{\{3\}}_{Q\to -q-(-k3-k8-k7-k4)}$, rk $= 3$



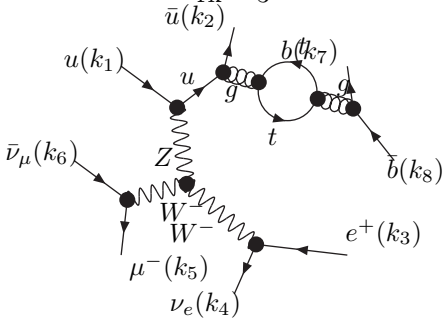
-Diagram 46

$S' = S^{\{3\}}_{Q\to -q-(-k3-k8-k7-k4)}$, rk $= 3$



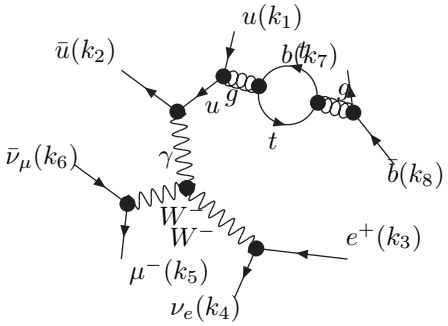
-Diagram 133

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



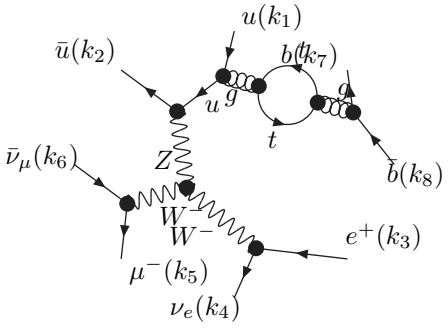
-Diagram 141

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



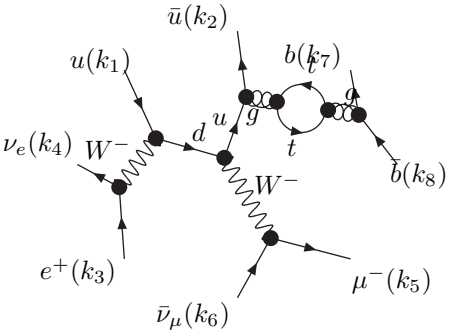
-Diagram 149

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



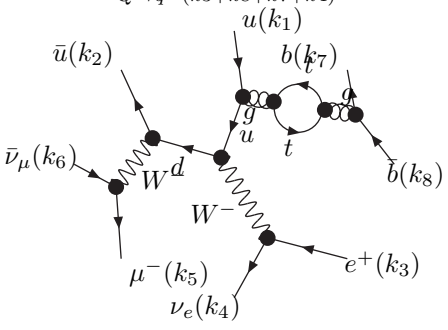
-Diagram 157

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



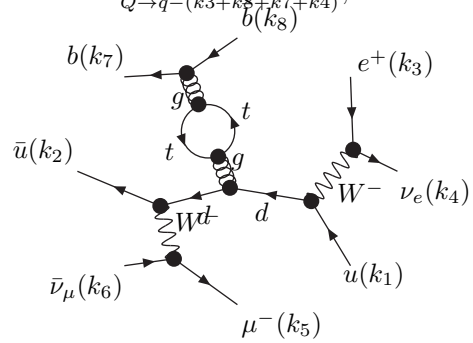
-Diagram 205

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



-Diagram 221

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$



-Diagram 333

$S' = S^{\{3,4\}}_{Q\to q-(k3+k8+k7+k4)}$, rk $= 2$

5.2 Group 1 (4-Point)

General Information

The maximum effective rank in this group is 4.

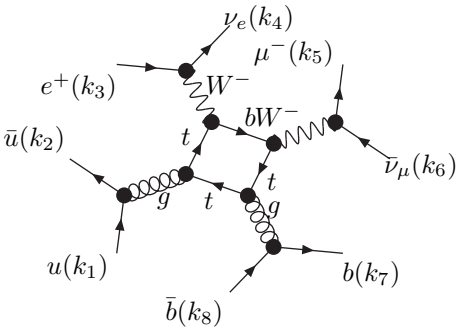
$$r_1 = k_7 + k_8 + k_5 + k_6, \quad m_1 = m_t, \quad \Gamma_1 = \Gamma_t \tag{26a}$$
$$r_2 = k_6 + k_5, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{26b}$$
$$r_3 = 0 \tag{26c}$$
$$r_4 = -k_3 - k_4, \quad m_4 = m_t, \quad \Gamma_4 = \Gamma_t \tag{26d}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & 0 & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} \end{pmatrix} \tag{27}$$

$$S_{1,1} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{28a}$$
$$S_{1,2} = -2m_t^2 + s_{78} + 2i \cdot m_t \cdot \Gamma_t \tag{28b}$$
$$S_{1,3} = -m_t^2 + s_{1234} + i \cdot m_t \cdot \Gamma_t \tag{28c}$$
$$S_{1,4} = s_{12} - 2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{28d}$$

$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{28e}$$
$$S_{2,3} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{28f}$$
$$S_{2,4} = -2m_t^2 + s_{3456} + 2i \cdot m_t \cdot \Gamma_t \tag{28g}$$
$$S_{3,4} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{28h}$$
$$S_{4,4} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{28i}$$

5.2.1 Diagrams (1)



-Diagram 11

$S' = S_{Q\to -q-(-k3-k4)}$, rk $= 4$

5.3 Group 2 (4-Point)

General Information

The maximum effective rank in this group is 4.

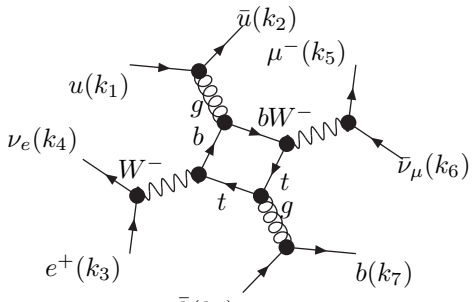
$$r_1 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{29a}$$
$$r_2 = -k_3 - k_7 - k_8 - k_4, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{29b}$$
$$r_3 = -k_3 - k_4, \quad m_3 = m_t, \quad \Gamma_3 = \Gamma_t \tag{29c}$$
$$r_4 = 0 \tag{29d}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} \\ S_{4,1} & S_{4,2} & S_{4,3} & 0 \end{pmatrix} \tag{30}$$

$$S_{1,2} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{31a}$$
$$S_{1,3} = -m_t^2 + s_{1234} + i \cdot m_t \cdot \Gamma_t \tag{31b}$$
$$S_{1,4} = s_{12} \tag{31c}$$
$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{31d}$$
$$S_{2,3} = -2m_t^2 + s_{78} + 2i \cdot m_t \cdot \Gamma_t \tag{31e}$$
$$S_{2,4} = -m_t^2 - s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} + i \cdot m_t \cdot \Gamma_t \tag{31f}$$

$$S_{3,3} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{31g}$$
$$S_{3,4} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{31h}$$

5.3.1 Diagrams (1)



-Diagram 6
$S' = S$, rk $= 4$

5.4 Group 3 (5-Point)

General Information

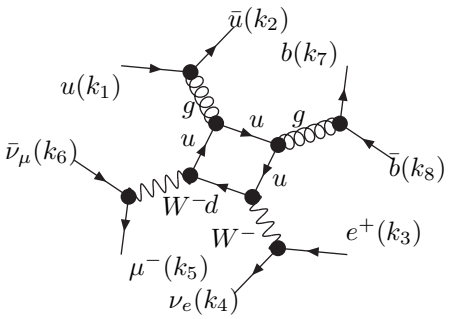
The maximum effective rank in this group is 5.

$$r_1 = -k_6 - k_5 \tag{32a}$$
$$r_2 = -k_3 - k_6 - k_5 - k_4 \tag{32b}$$
$$r_3 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{32c}$$
$$r_4 = -k_2 \tag{32d}$$
$$r_5 = 0 \tag{32e}$$

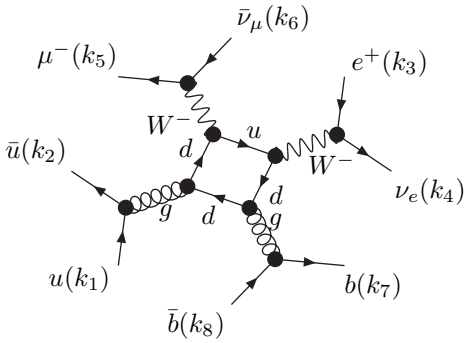
$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{33}$$

$$S_{1,2} = s_{34} \tag{34a}$$
$$S_{1,3} = -s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} \tag{34b}$$
$$S_{1,4} = s_{56} + s_{781} - s_{3456} - s_{234} + s_{34} \tag{34c}$$
$$S_{1,5} = s_{56} \tag{34d}$$
$$S_{2,3} = s_{78} \tag{34e}$$
$$S_{2,4} = s_{781} \tag{34f}$$
$$S_{2,5} = s_{3456} \tag{34g}$$
$$S_{3,5} = s_{12} \tag{34h}$$

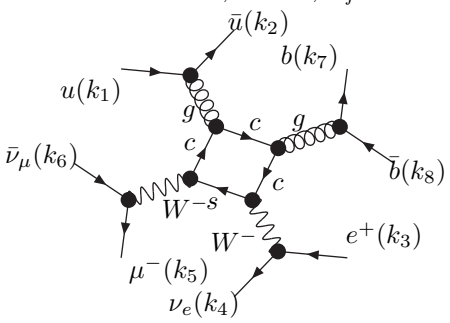
5.4.1 Diagrams (28)



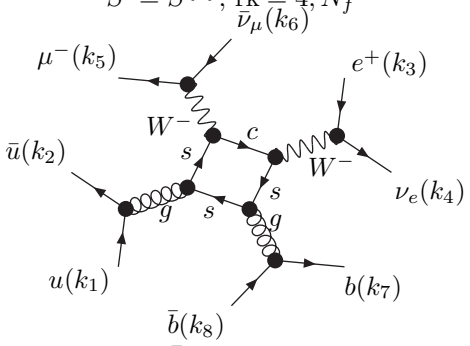
-Diagram 13
$S' = S^{\{4\}}$, $\mathrm{rk} = 4, N_f$



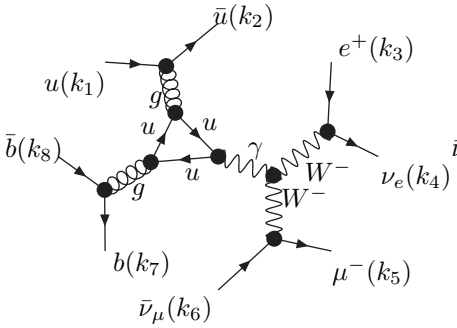
-Diagram 14
$S' = S^{\{4\}}$, $\mathrm{rk} = 4, N_f$



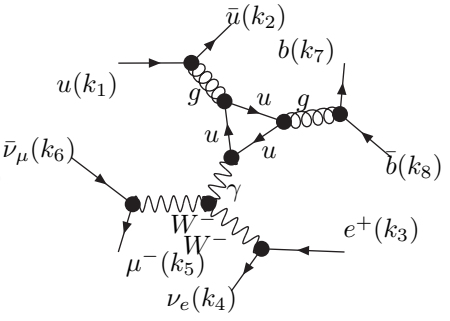
-Diagram 15
$S' = S^{\{4\}}$, $\mathrm{rk} = 4, N_f$



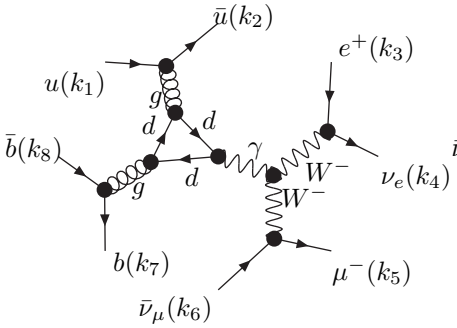
-Diagram 16
$S' = S^{\{4\}}$, $\mathrm{rk} = 4, N_f$



-Diagram 25
$S' =$
$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



-Diagram 26
$S' =$
$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$

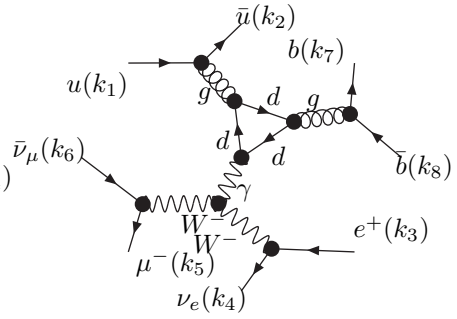


-Diagram 27

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

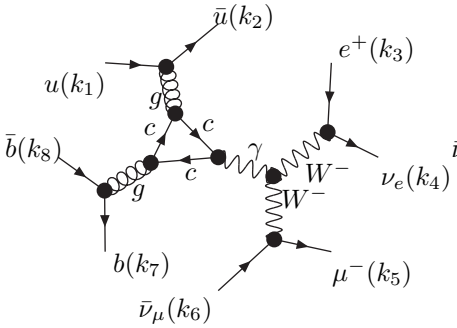


-Diagram 28

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

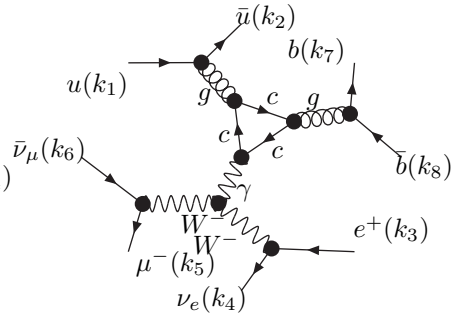


-Diagram 29

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

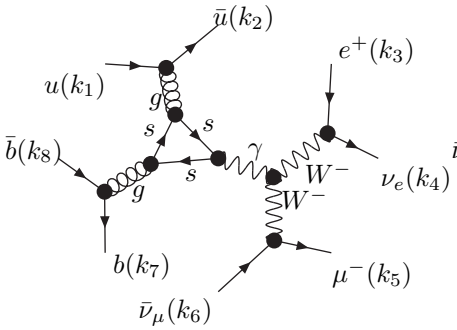


-Diagram 30

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

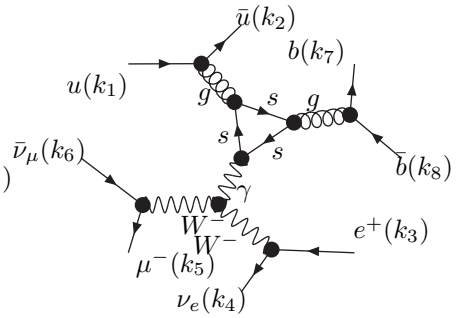


-Diagram 31

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

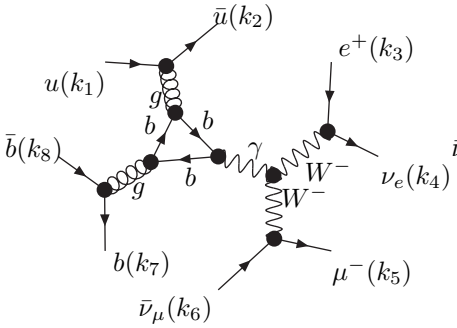


-Diagram 32

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

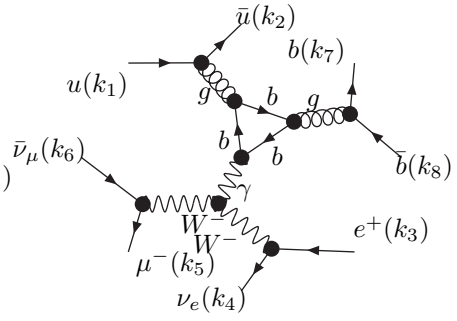


-Diagram 35

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

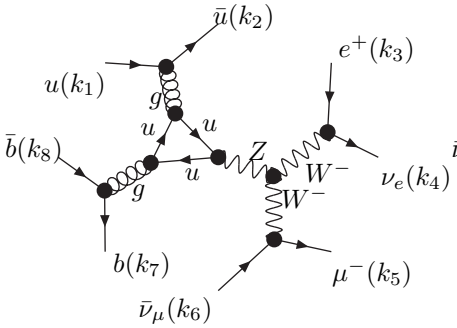


-Diagram 36

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

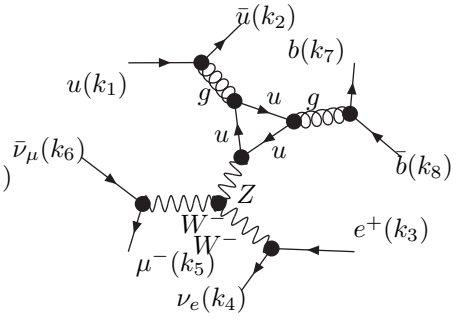


-Diagram 37

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

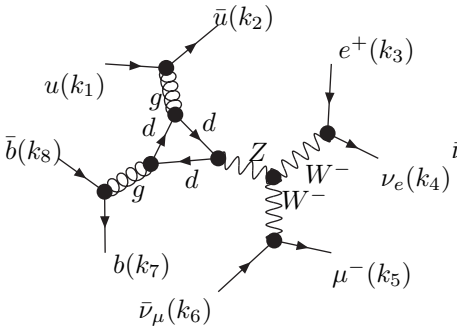


-Diagram 38

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

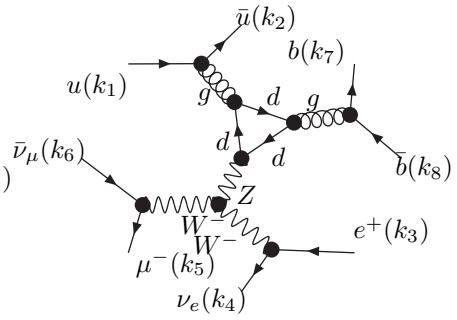


-Diagram 39

$S' =$

$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$

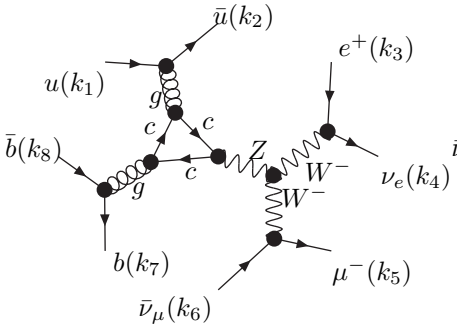


-Diagram 40

$S' =$

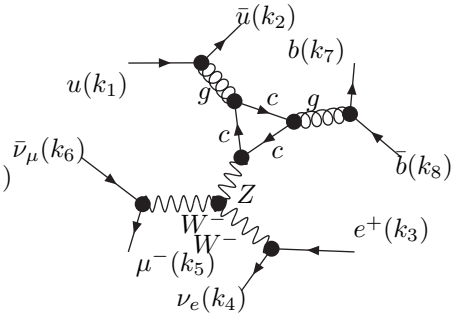
$S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 3, N_f$



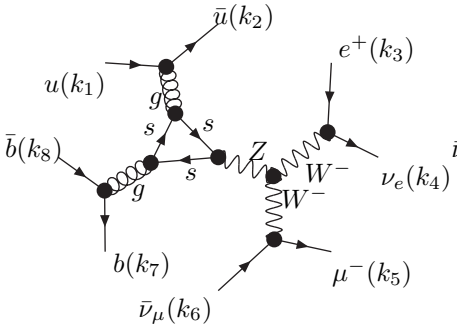
-Diagram 41

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



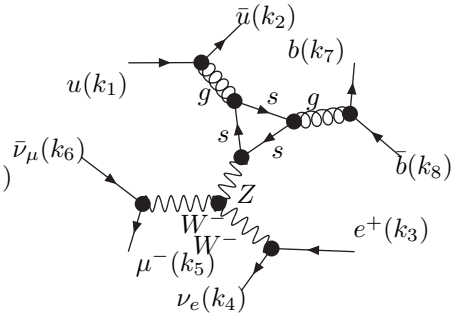
-Diagram 42

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



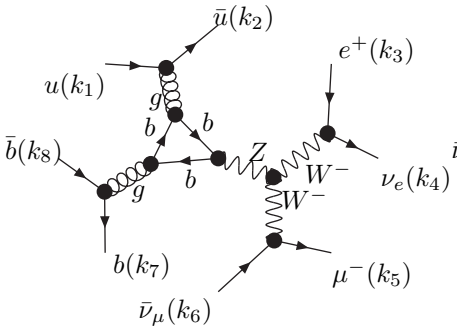
-Diagram 43

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



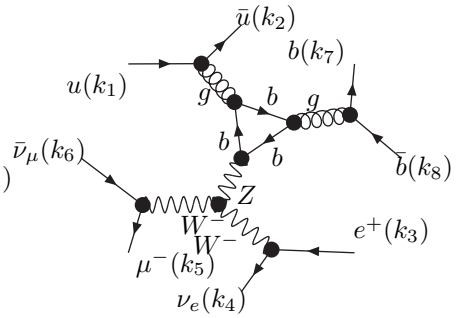
-Diagram 44

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



-Diagram 47

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$



-Diagram 48

$S' = S^{\{1,4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
$\mathrm{rk} = 3, N_f$

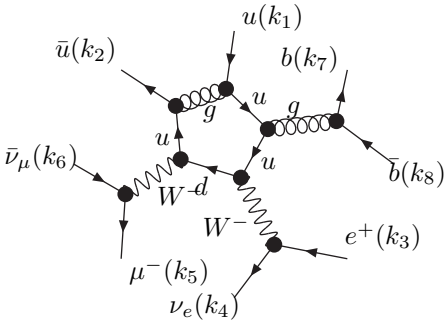


Diagram 68
$S' = S_{Q\to -q-(-k2)}$, rk $= 4$

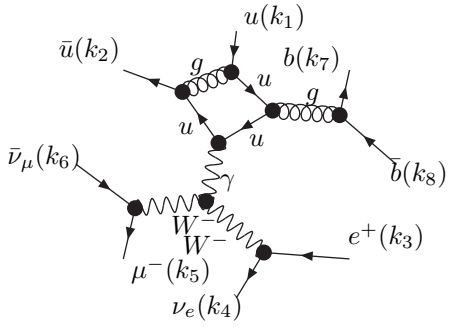


Diagram 93
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk $= 3$

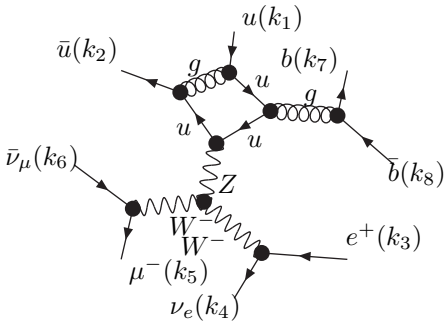


Diagram 94
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk $= 3$

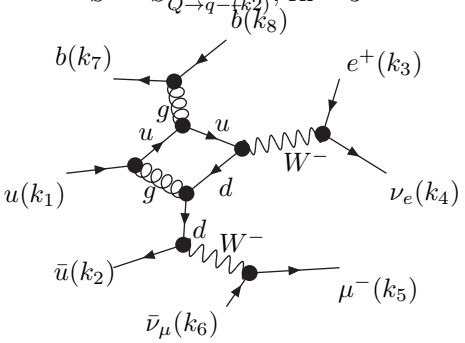


Diagram 170
$S' = S^{\{5\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 3$

5.5 Group 4 (5-Point)

General Information

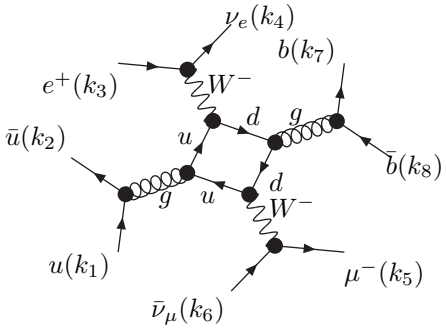
The maximum effective rank in this group is 5.

$$r_1 = -k_7 - k_8 - k_5 - k_6 \tag{35a}$$
$$r_2 = -k_6 - k_5 \tag{35b}$$
$$r_3 = 0 \tag{35c}$$
$$r_4 = -k_2 \tag{35d}$$
$$r_5 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{35e}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{36}$$

$$S_{1,2} = s_{78} \tag{37a}$$
$$S_{1,3} = s_{1234} \tag{37b}$$
$$S_{1,4} = s_{1234} + s_{34} - s_{234} - s_{12} \tag{37c}$$
$$S_{1,5} = s_{34} \tag{37d}$$
$$S_{2,3} = s_{56} \tag{37e}$$
$$S_{2,4} = s_{56} + s_{781} - s_{3456} - s_{234} + s_{34} \tag{37f}$$
$$S_{2,5} = -s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} \tag{37g}$$
$$S_{3,5} = s_{12} \tag{37h}$$

5.5.1 Diagrams (7)

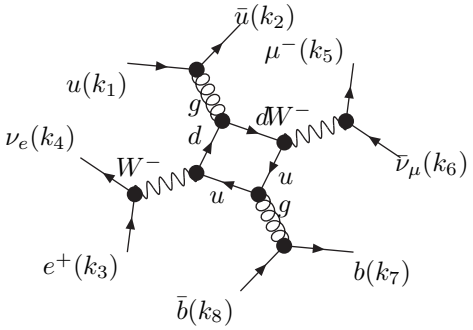


-Diagram 1

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

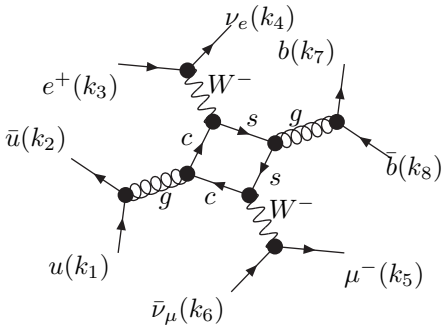


-Diagram 2

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

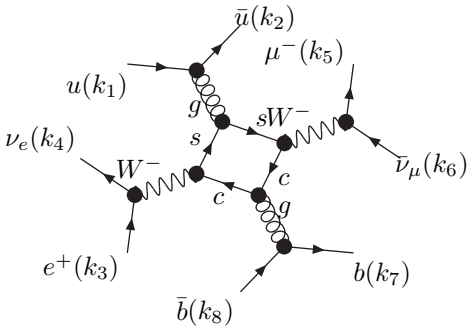


-Diagram 3

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$



-Diagram 4

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

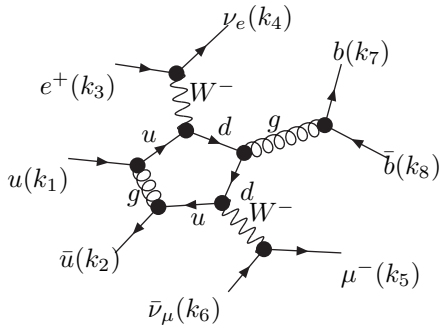


Diagram 66
$S' = S_{Q\to q-(k2)}$, rk = 4

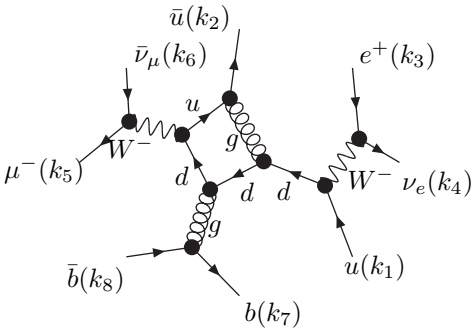


Diagram 165
$S' = S^{\{5\}}$, rk = 3

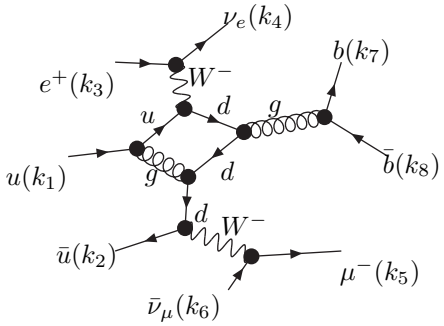


Diagram 169
$S' = S^{\{3\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk = 3

5.6 Group 5 (5-Point)

General Information

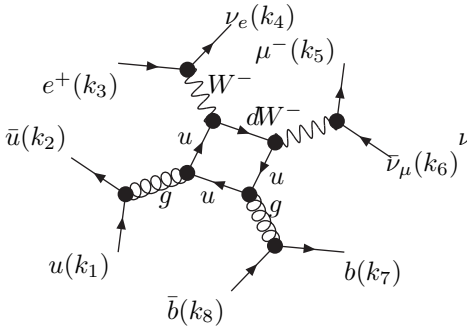
The maximum effective rank in this group is 5.

$$r_1 = -k_7 - k_8 - k_5 - k_6 \tag{38a}$$
$$r_2 = -k_7 - k_8 \tag{38b}$$
$$r_3 = 0 \tag{38c}$$
$$r_4 = -k_2 \tag{38d}$$
$$r_5 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{38e}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & 0 & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{39}$$

$$S_{1,2} = s_{56} \tag{40a}$$
$$S_{1,3} = s_{1234} \tag{40b}$$
$$S_{1,4} = s_{1234} + s_{34} - s_{234} - s_{12} \tag{40c}$$
$$S_{1,5} = s_{34} \tag{40d}$$
$$S_{2,3} = s_{78} \tag{40e}$$
$$S_{2,4} = s_{78} - s_{781} + s_{3456} - s_{12} \tag{40f}$$
$$S_{2,5} = s_{3456} \tag{40g}$$
$$S_{3,5} = s_{12} \tag{40h}$$

5.6.1 Diagrams (8)

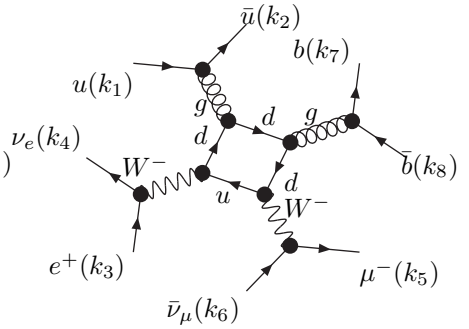


-Diagram 7

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

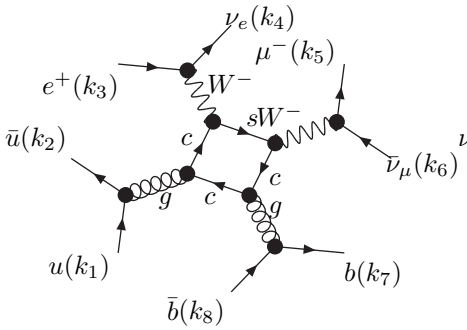


-Diagram 8

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

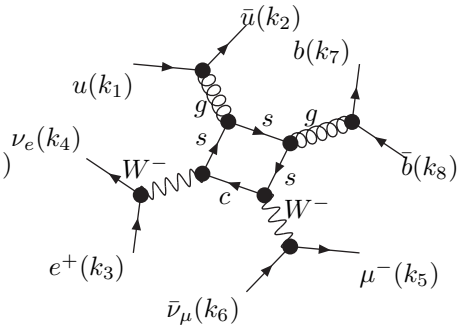


-Diagram 9

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$



-Diagram 10

$S' =$

$S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 4, N_f$

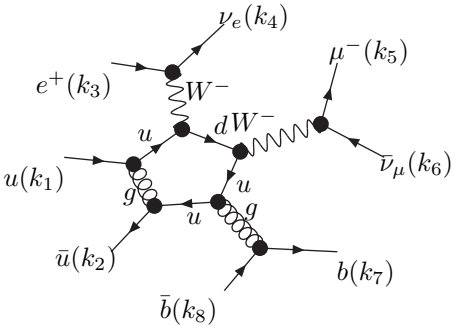


Diagram 67
$S' = S_{Q\to q-(k2)}$, rk $= 4$

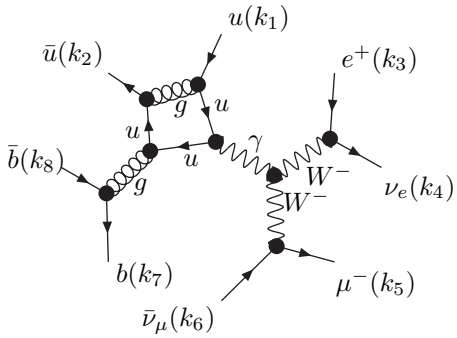


Diagram 95
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 3$

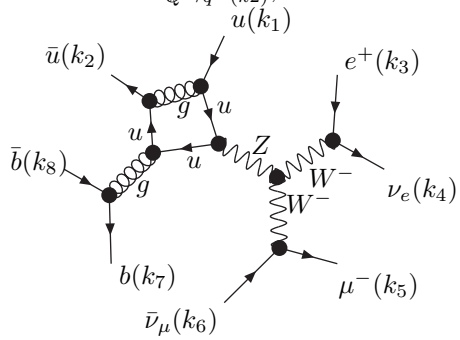


Diagram 96
$S' = S^{\{1\}}_{Q\to -q-(-k2)}$, rk $= 3$

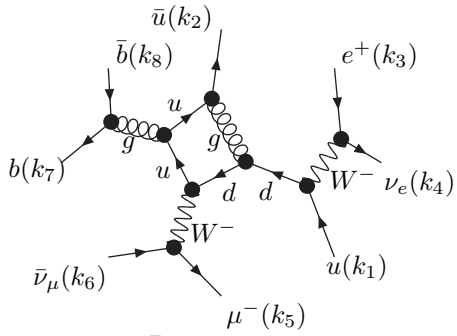


Diagram 166
$S' = S^{\{5\}}$, rk $= 3$

5.7 Group 6 (5-Point)

General Information

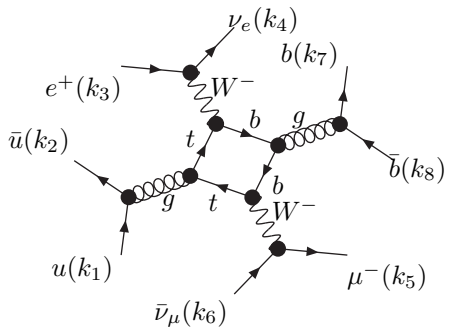
The maximum effective rank in this group is 5.

$$r_1 = -k_3 - k_7 - k_4, \quad m_1 = m_t, \quad \Gamma_1 = \Gamma_t \tag{41a}$$
$$r_2 = k_8 + k_5 + k_6, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{41b}$$
$$r_3 = k_8 \tag{41c}$$
$$r_4 = 0 \tag{41d}$$
$$r_5 = -k_7 \tag{41e}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{42}$$

$$S_{1,1} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{43a}$$
$$S_{1,2} = s_{12} - 2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{43b}$$
$$S_{1,3} = -m_t^2 - s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} + i \cdot m_t \cdot \Gamma_t \tag{43c}$$
$$S_{1,4} = -m_t^2 + s_{56} - s_{567} + s_{812} - s_{3456} + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{43d}$$
$$S_{1,5} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{43e}$$
$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{43f}$$
$$S_{2,3} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{43g}$$
$$S_{2,4} = -m_t^2 + s_{1234} + s_{56} - s_{567} - s_{78} + i \cdot m_t \cdot \Gamma_t \tag{43h}$$
$$S_{2,5} = -m_t^2 + s_{1234} + i \cdot m_t \cdot \Gamma_t \tag{43i}$$
$$S_{3,5} = s_{78} \tag{43j}$$

5.7.1 Diagrams (12)



-Diagram 5

$S' = S^{\{4\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 4$

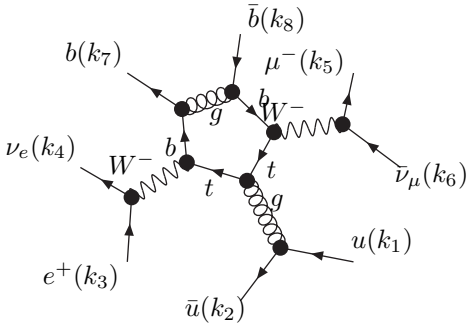
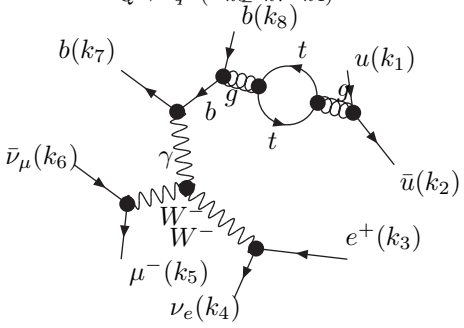


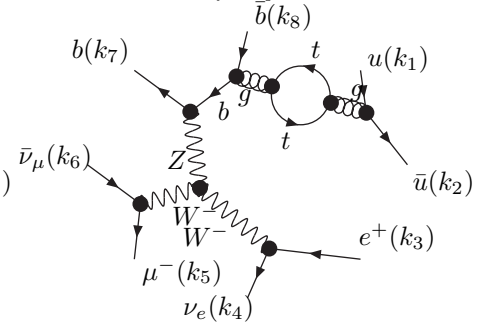
Diagram 65

$S' = S_{Q\to -g}$, rk $= 4$



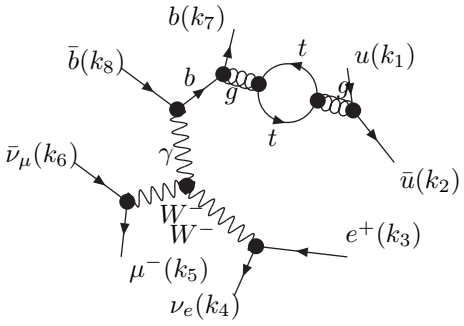
-Diagram 101

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$



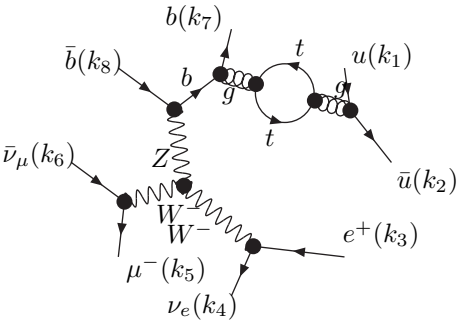
-Diagram 109

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$



-Diagram 117

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$



-Diagram 125

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$

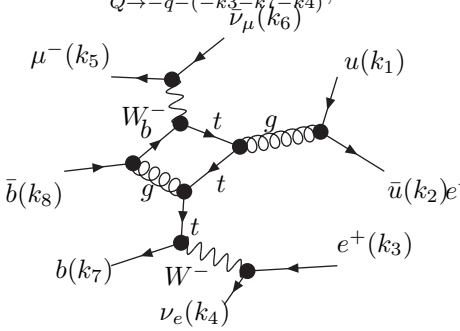


Diagram 164

$S' = S^{\{5\}}_{Q\to -q-(k8)}$, rk $= 3$

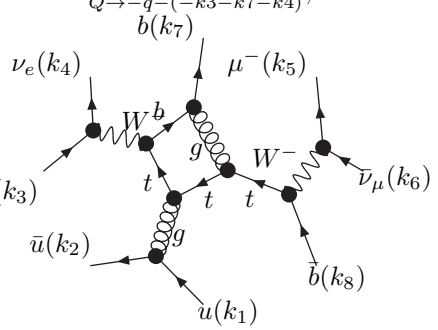
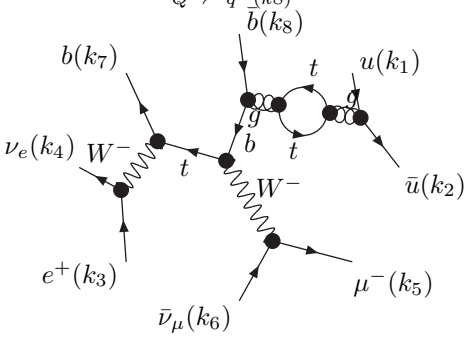


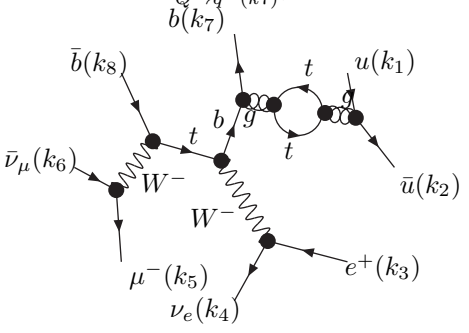
Diagram 168

$S' = S^{\{3\}}_{Q\to q-(k7)}$, rk $= 3$



-Diagram 197

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$



-Diagram 213

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk $= 2$

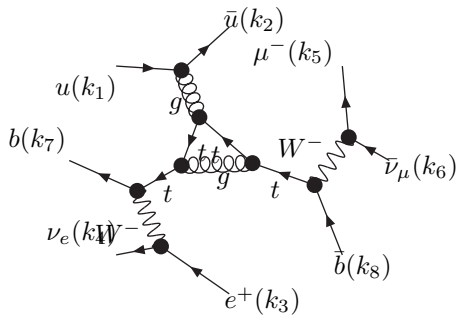
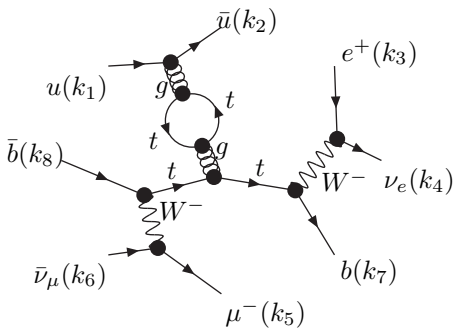


Diagram 315

$S' = S^{\{3,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk = 2



-Diagram 325

$S' = S^{\{3,4,5\}}_{Q\to -q-(-k3-k7-k4)}$, rk = 2

5.8 Group 7 (5-Point)

General Information

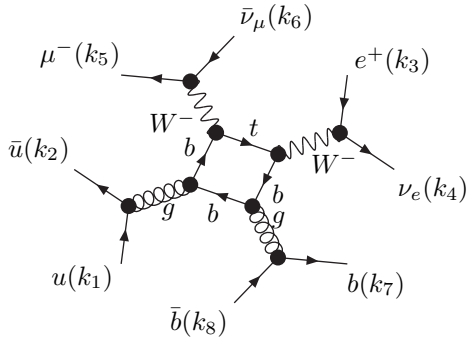
The maximum effective rank in this group is 5.

$$r_1 = -k_3 - k_7 - k_6 - k_5 - k_4 \tag{44a}$$
$$r_2 = -k_3 - k_7 - k_4, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{44b}$$
$$r_3 = -k_7 \tag{44c}$$
$$r_4 = 0 \tag{44d}$$
$$r_5 = k_8 \tag{44e}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{45}$$

$$S_{1,2} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{46a}$$
$$S_{1,3} = s_{3456} \tag{46b}$$
$$S_{1,4} = s_{812} \tag{46c}$$
$$S_{1,5} = s_{12} \tag{46d}$$
$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{46e}$$
$$S_{2,3} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{46f}$$
$$S_{2,4} = -m_t^2 + s_{56} - s_{567} + s_{812} - s_{3456} + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{46g}$$
$$S_{2,5} = -m_t^2 - s_{1234} + s_{78} + s_{56} + s_{34} - s_{3456} + s_{12} + i \cdot m_t \cdot \Gamma_t \tag{46h}$$
$$S_{3,5} = s_{78} \tag{46i}$$

5.8.1 Diagrams (5)



-Diagram 18

$S' = S^{\{4\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$, rk = 4

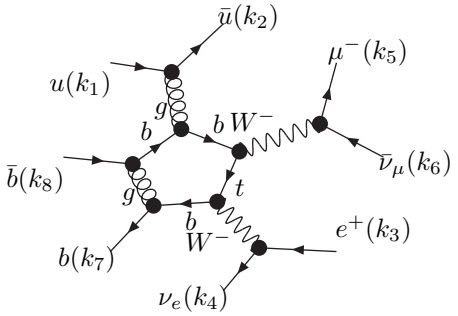


Diagram 63

$S' = S$, rk = 4

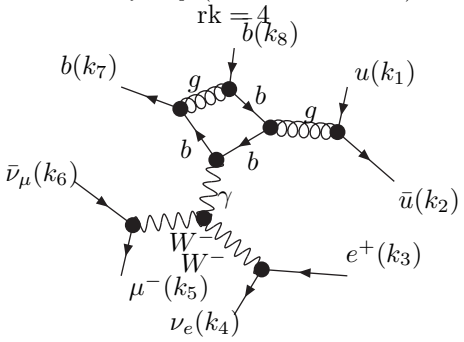


Diagram 91

$S' = S^{\{2\}}$, rk = 3

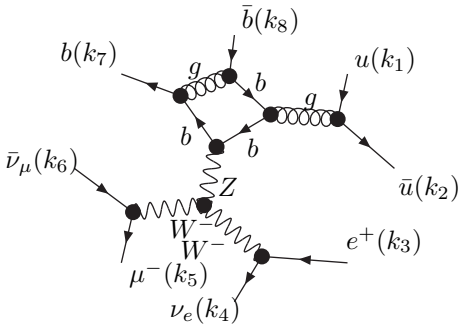


Diagram 92

$S' = S^{\{2\}}$, rk = 3

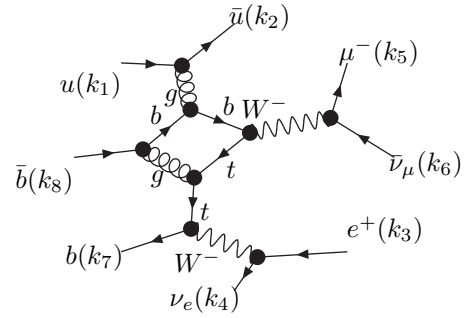


Diagram 163

$S' = S^{\{3\}}_{Q\to -q-(k8)}$, rk = 3

5.9 Group 8 (5-Point)

General Information

The maximum effective rank in this group is 5.

$$r_1 = k_3 + k_6 + k_5 + k_4 + k_8 \tag{47a}$$

$$r_2 = k_8 + k_5 + k_6, \quad m_2 = m_t, \quad \Gamma_2 = \Gamma_t \tag{47b}$$

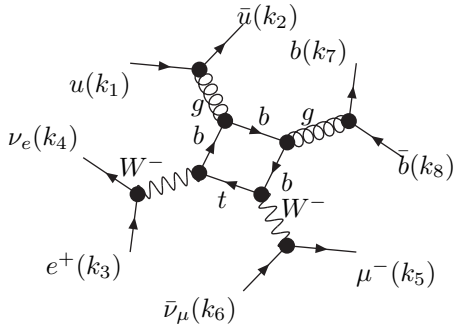
$$r_3 = k_8 \tag{47c}$$

$$r_4 = 0 \tag{47d}$$
$$r_5 = -k_7 \tag{47e}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} & S_{2,5} \\ S_{3,1} & S_{3,2} & 0 & 0 & S_{3,5} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 \end{pmatrix} \tag{48}$$

$$S_{1,2} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{49a}$$
$$S_{1,3} = s_{3456} \tag{49b}$$
$$S_{1,4} = -s_{78} - s_{812} + s_{3456} + s_{12} \tag{49c}$$
$$S_{1,5} = s_{12} \tag{49d}$$
$$S_{2,2} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{49e}$$
$$S_{2,3} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{49f}$$
$$S_{2,4} = -m_t^2 + s_{1234} + s_{56} - s_{567} - s_{78} + i \cdot m_t \cdot \Gamma_t \tag{49g}$$
$$S_{2,5} = -m_t^2 + s_{1234} + i \cdot m_t \cdot \Gamma_t \tag{49h}$$
$$S_{3,5} = s_{78} \tag{49i}$$

5.9.1 Diagrams (5)



-Diagram 12

$S' = S^{\{4\}}_{Q\to q-(-k3-k8-k6-k5-k4)}$, rk = 4

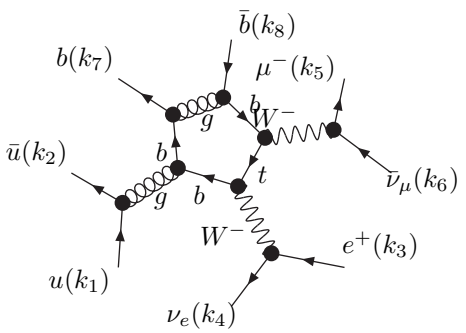


Diagram 64

$S' = S_{Q\to -q}$, rk = 4

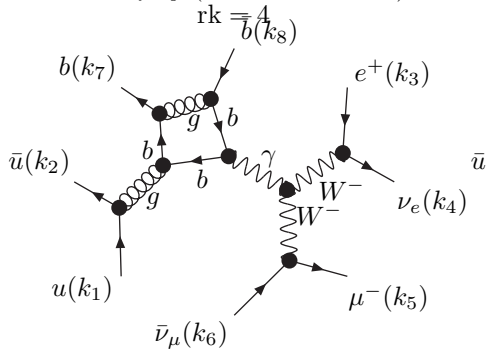


Diagram 89

$S' = S^{\{2\}}_{Q\to -q}$, rk = 3

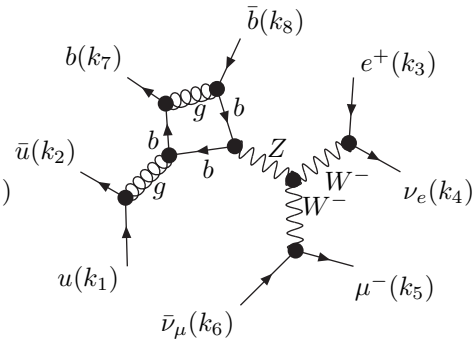


Diagram 90

$S' = S^{\{2\}}_{Q\to -q}$, rk = 3

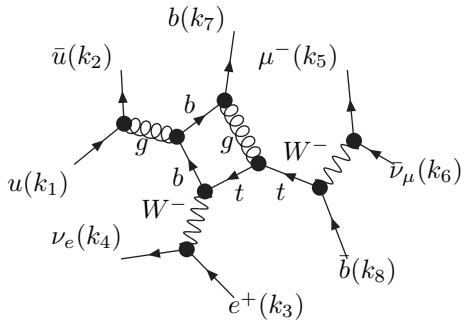


Diagram 167
$S' = S^{\{3\}}_{Q\to q-(k7)}$, rk $= 3$

5.10 Group 9 (6-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_2 + k_6 + k_5 \tag{50a}$$
$$r_2 = -k_2 \tag{50b}$$
$$r_3 = 0 \tag{50c}$$
$$r_4 = -k_7 \tag{50d}$$
$$r_5 = -k_7 - k_8 \tag{50e}$$
$$r_6 = k_3 - k_2 + k_6 + k_5 + k_4 \tag{50f}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & 0 & 0 & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & 0 & 0 & 0 & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 & 0 \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & 0 & 0 \end{pmatrix} \tag{51}$$

$$S_{1,2} = s_{56} \tag{52a}$$
$$S_{1,3} = s_{56} + s_{781} - s_{3456} - s_{234} + s_{34} \tag{52b}$$
$$S_{1,4} = s_{567} + s_{81} - s_{812} - s_{234} + s_{34} \tag{52c}$$
$$S_{1,5} = s_{1234} + s_{34} - s_{234} - s_{12} \tag{52d}$$
$$S_{1,6} = s_{34} \tag{52e}$$
$$S_{2,4} = s_{3456} + s_{81} - s_{812} - s_{781} \tag{52f}$$
$$S_{2,5} = s_{78} - s_{781} + s_{3456} - s_{12} \tag{52g}$$
$$S_{2,6} = s_{3456} \tag{52h}$$
$$S_{3,5} = s_{78} \tag{52i}$$
$$S_{3,6} = s_{781} \tag{52j}$$
$$S_{4,6} = s_{81} \tag{52k}$$

5.10.1 Diagrams (116)

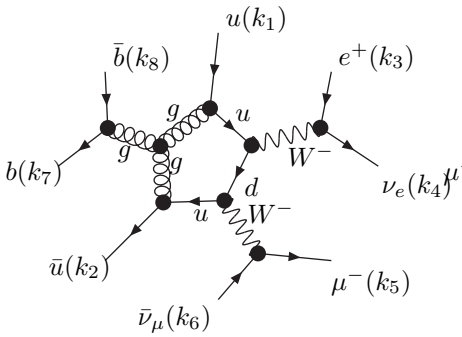


Diagram 62

$S' = S^{\{4\}}_{Q\to -q-(-k8-k7)}$, rk $= 4$

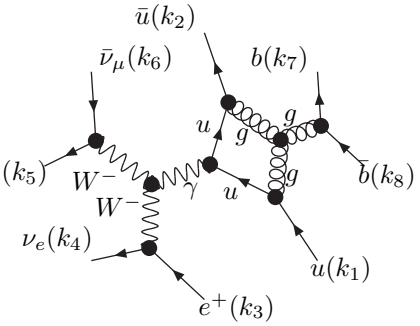


Diagram 71

$S' = S^{\{1,4\}}_{Q\to -q-(-k8-k7)}$, rk $= 3$

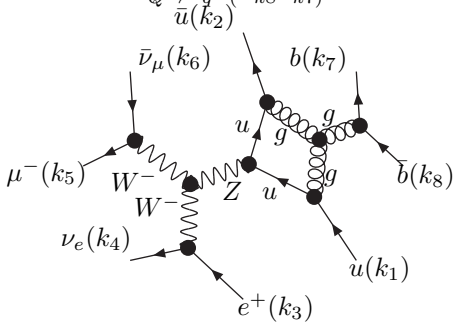


Diagram 72

$S' = S^{\{1,4\}}_{Q\to -q-(-k8-k7)}$, rk $= 3$

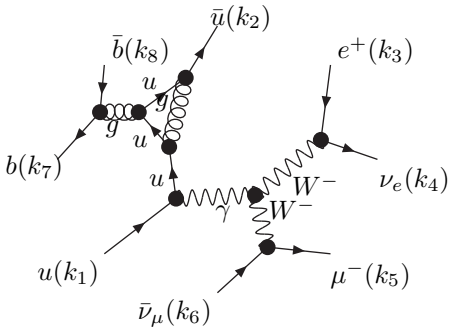


Diagram 81

$S' = S^{\{1,4,6\}}$, rk $= 2$

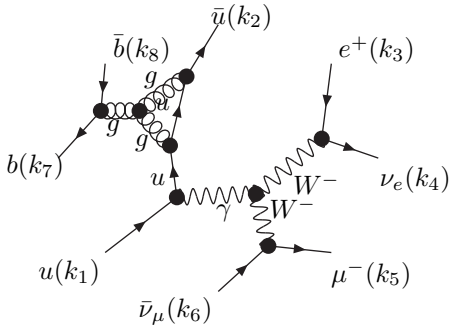


Diagram 82

$S' = S^{\{1,4,6\}}$, rk $= 2$

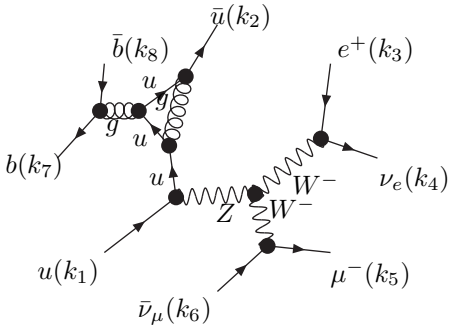


Diagram 83

$S' = S^{\{1,4,6\}}$, rk $= 2$

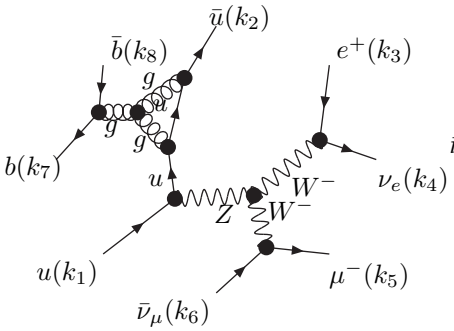


Diagram 84

$S' = S^{\{1,4,6\}}$, rk = 2

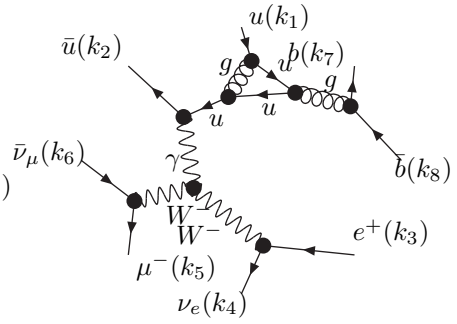


Diagram 85

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

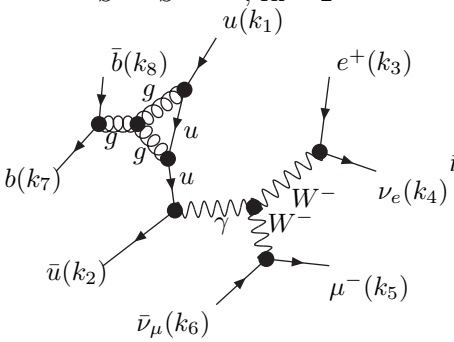


Diagram 86

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

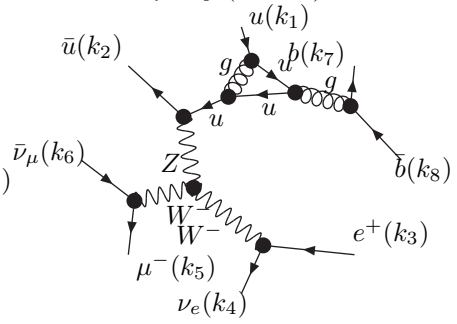


Diagram 87

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

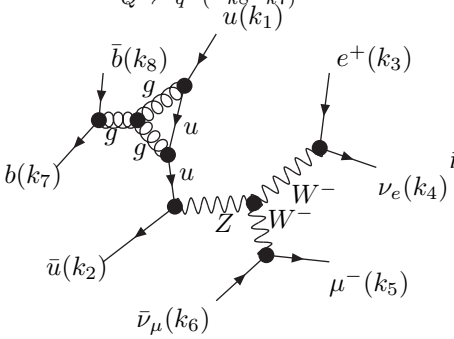
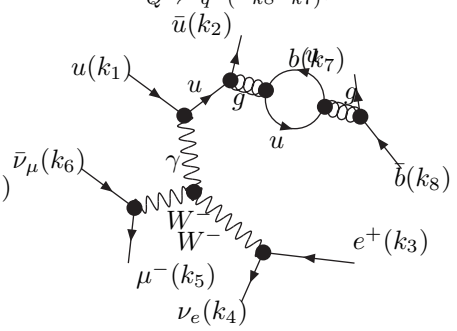


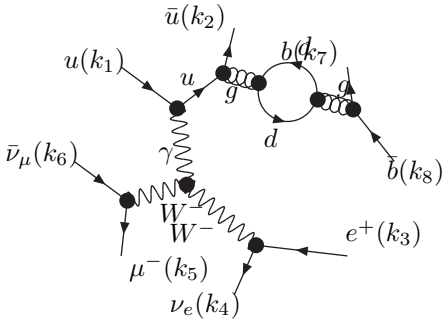
Diagram 88

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

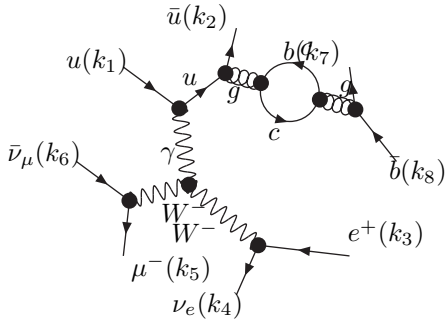


-Diagram 129

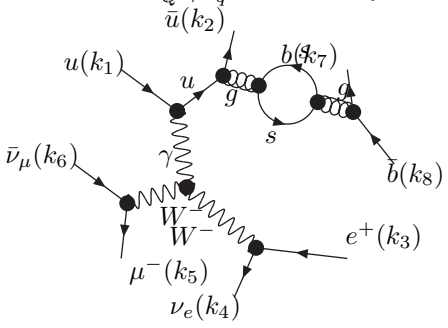
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



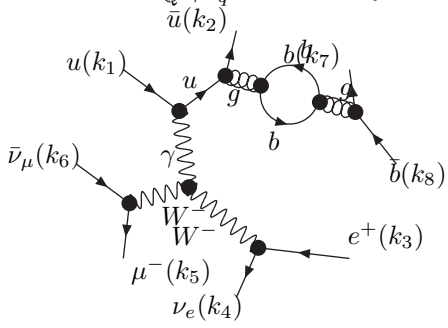
-Diagram 130
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



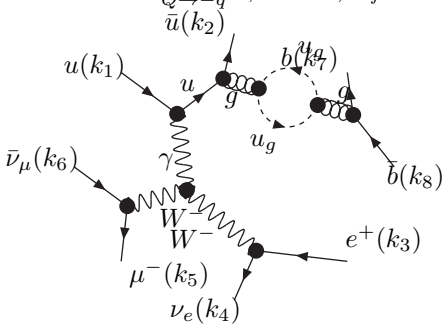
-Diagram 131
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 132
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 134
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 135
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

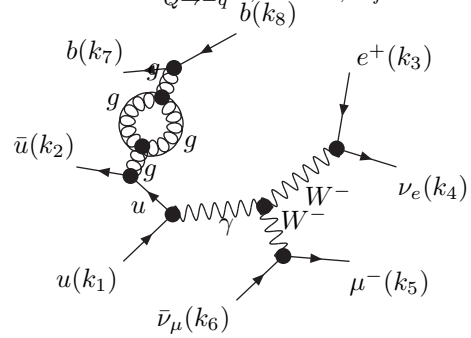
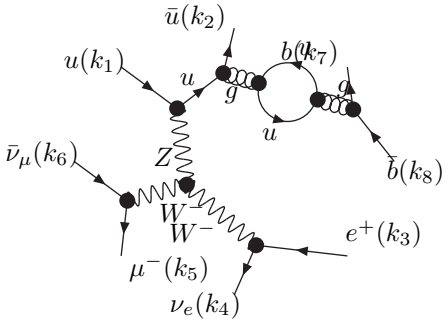
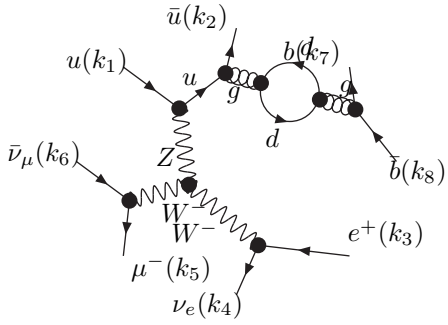


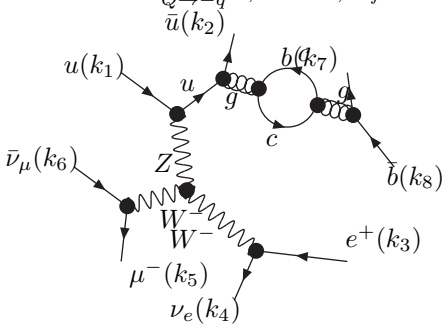
Diagram 136
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$



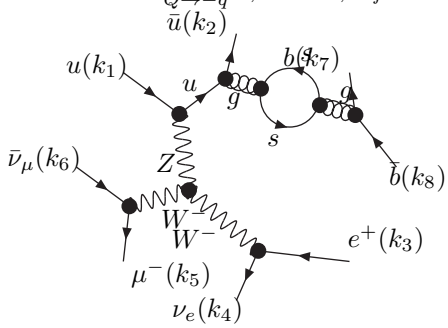
-Diagram 137
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



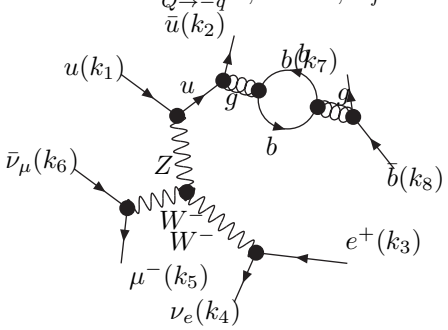
-Diagram 138
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



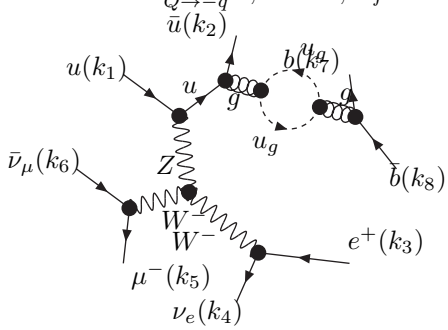
-Diagram 139
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 140
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 142
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 143
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

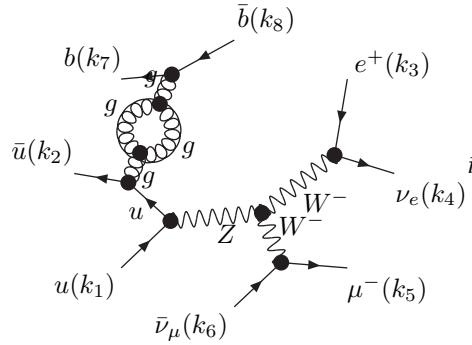
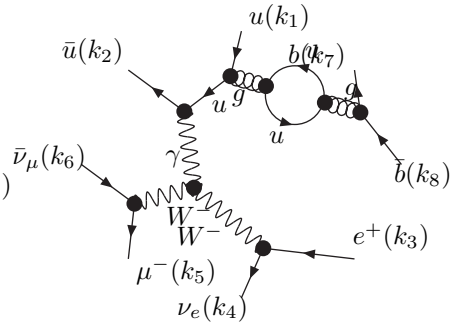
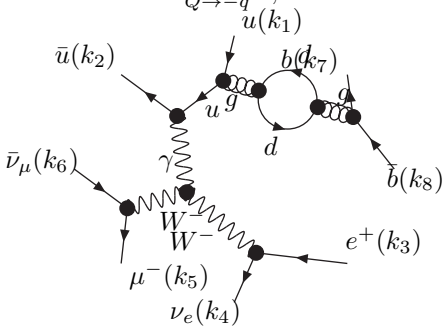


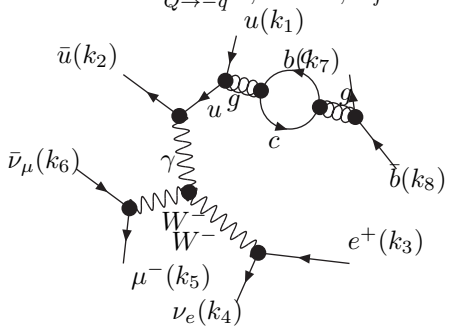
Diagram 144
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$



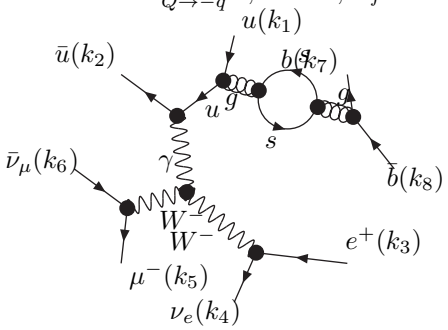
-Diagram 145
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



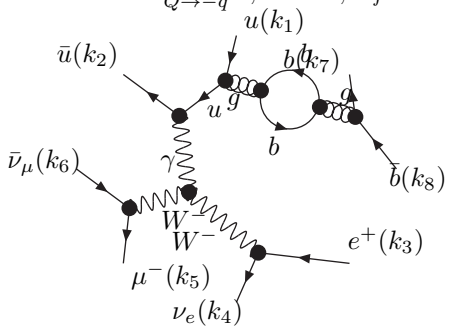
-Diagram 146
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



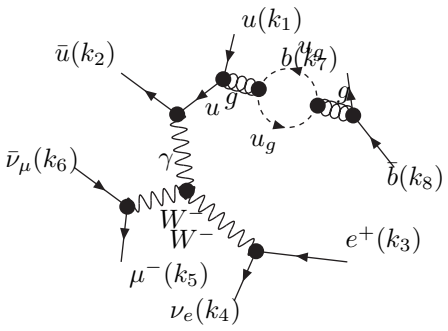
-Diagram 147
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 148
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 150
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 151
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

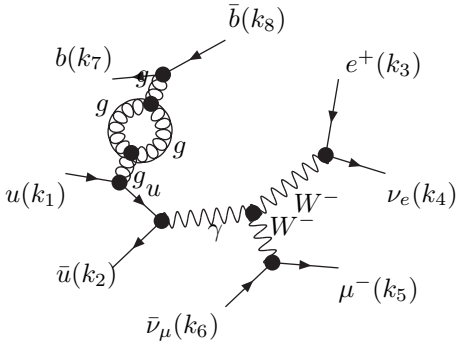
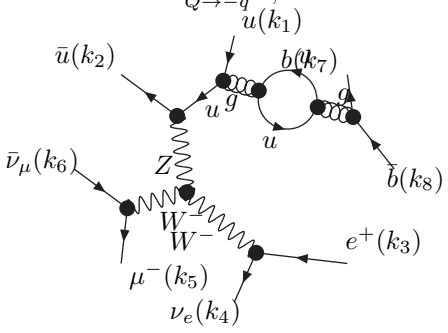
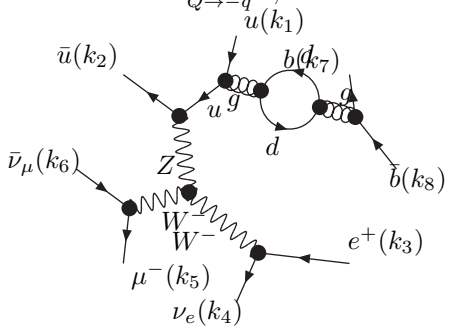


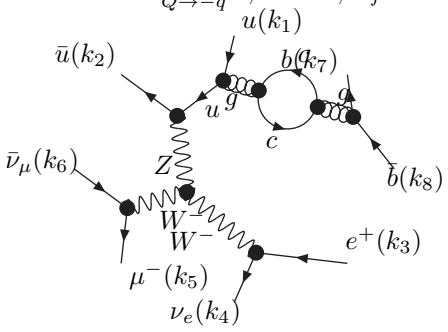
Diagram 152
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$



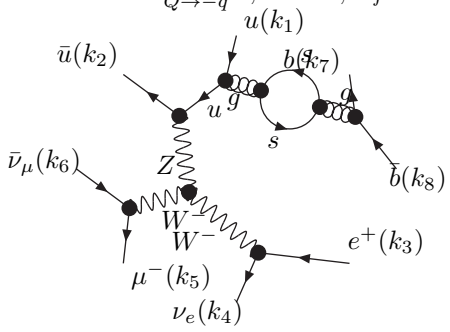
-Diagram 153
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



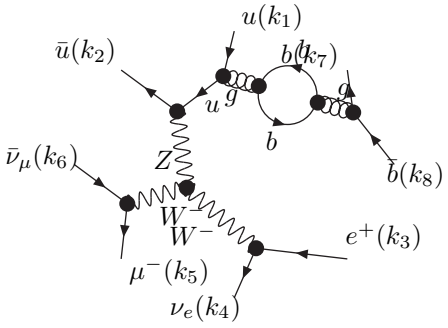
-Diagram 154
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 155
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$

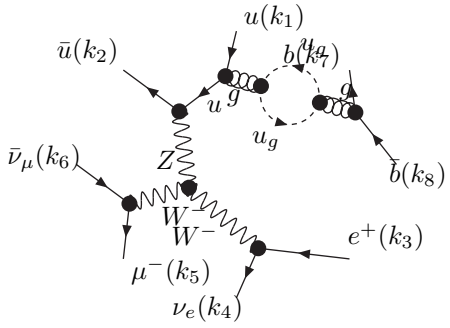


-Diagram 156
$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 158

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 159

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

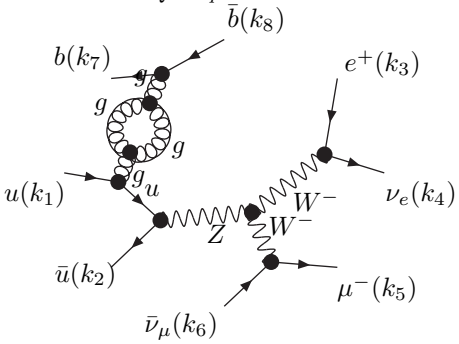


Diagram 160

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

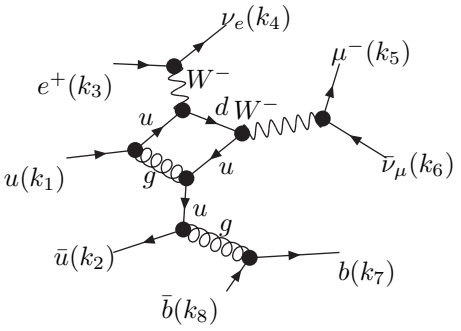


Diagram 171

$S' = S^{\{3,4\}}_{Q\to q-(-k3+k2-k6-k5-k4)}$, rk $= 3$

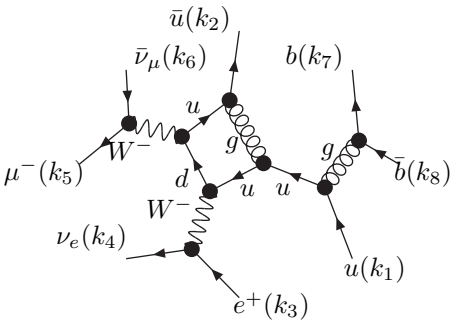


Diagram 172

$S' = S^{\{4,5\}}_{Q\to -q-(-k2)}$, rk $= 3$

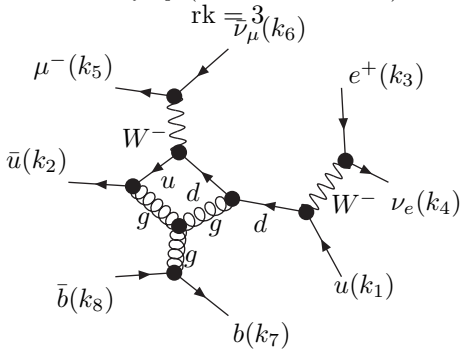


Diagram 174

$S' = S^{\{4,6\}}_{Q\to -q-(-k2)}$, rk $= 3$

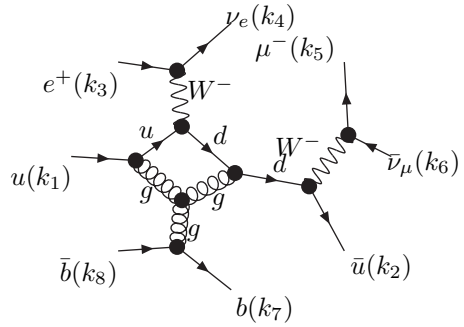


Diagram 176

$S' = S^{\{2,4\}}_{Q\to q-(-k3+k2-k6-k5-k4)}$, rk = 3

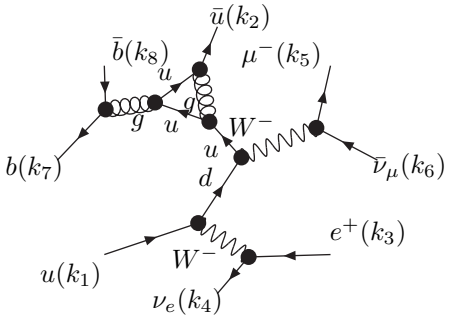


Diagram 182

$S' = S^{\{1,4,6\}}$, rk = 2

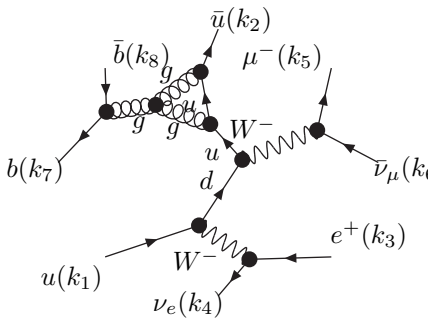


Diagram 183

$S' = S^{\{1,4,6\}}$, rk = 2

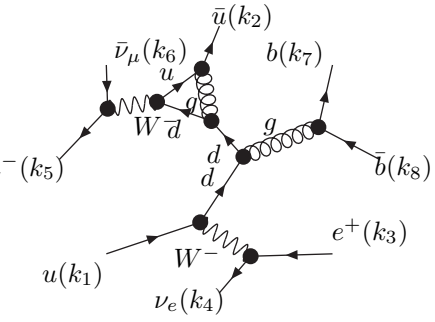


Diagram 184

$S' = S^{\{4,5,6\}}_{Q\to -q-(-k2)}$, rk = 2

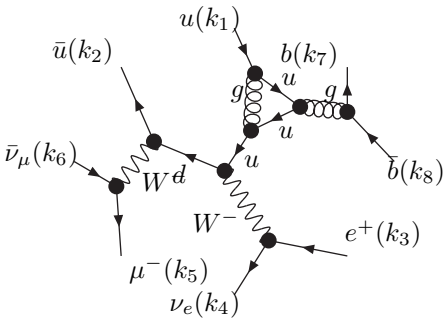


Diagram 188

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

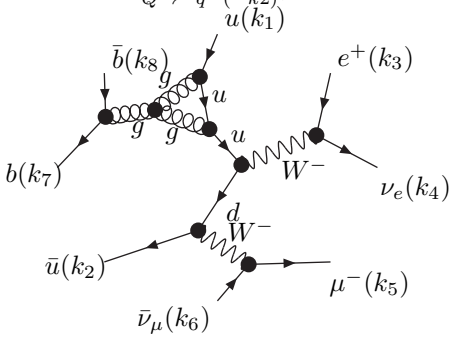


Diagram 189

$S' = S^{\{1,2,4\}}_{Q\to -q-(-k8-k7)}$, rk = 2

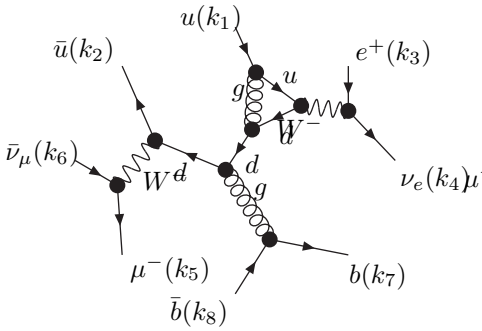


Diagram 190

$S' = S^{\{2,3,4\}}_{Q\to q-(-k3+k2-k6-k5-k4)}$, rk $= 2$

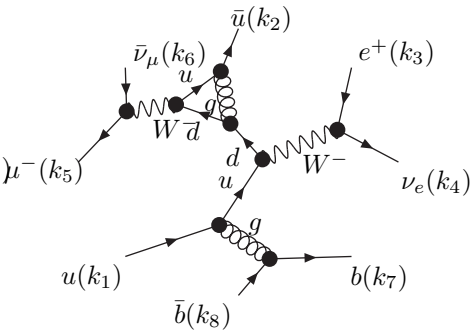


Diagram 191

$S' = S^{\{4,5,6\}}_{Q\to -q-(-k2)}$, rk $= 2$

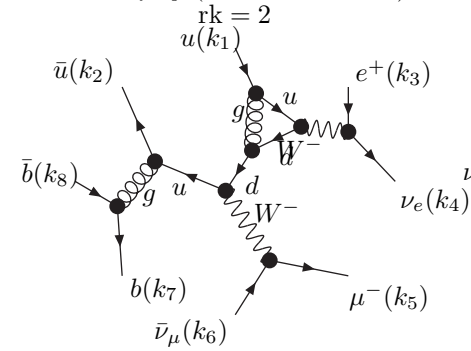
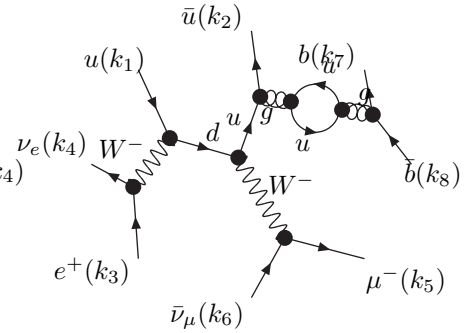


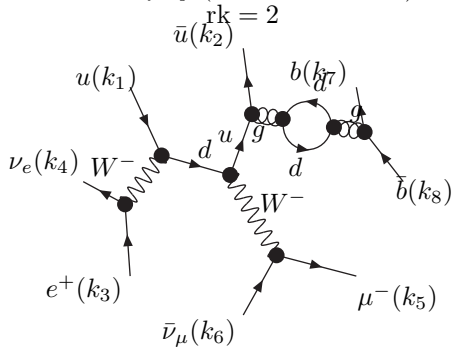
Diagram 192

$S' = S^{\{2,3,4\}}_{Q\to q-(-k3+k2-k6-k5-k4)}$, rk $= 2$



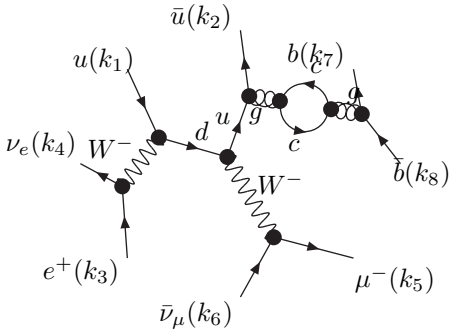
-Diagram 201

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



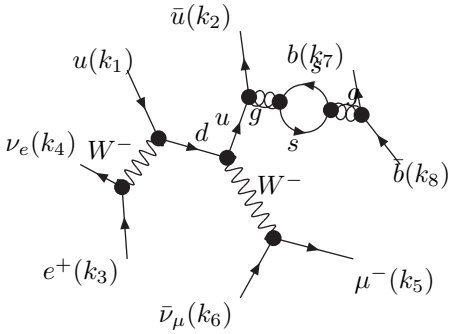
-Diagram 202

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



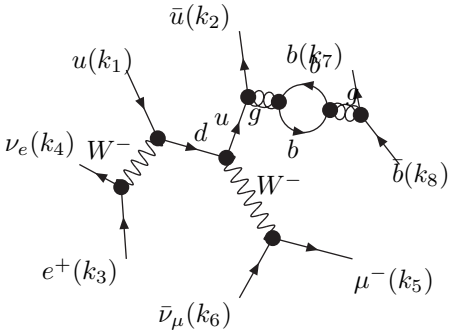
-Diagram 203

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



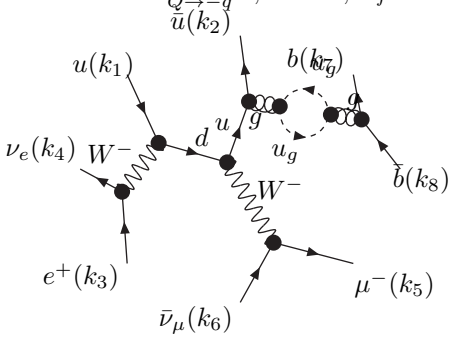
-Diagram 204

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 206

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



-Diagram 207

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$

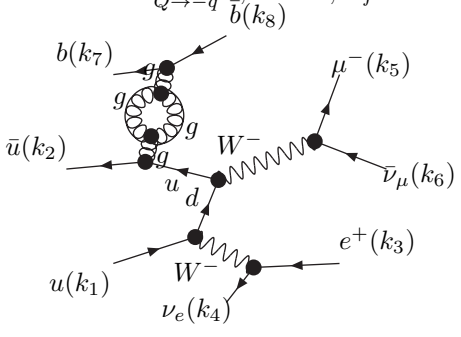
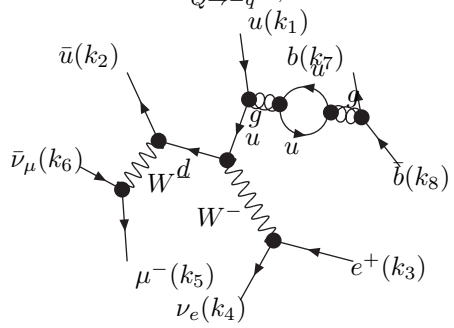


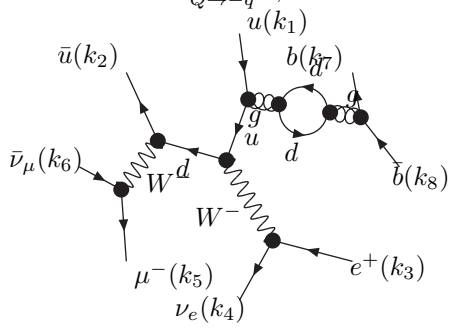
Diagram 208

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2$



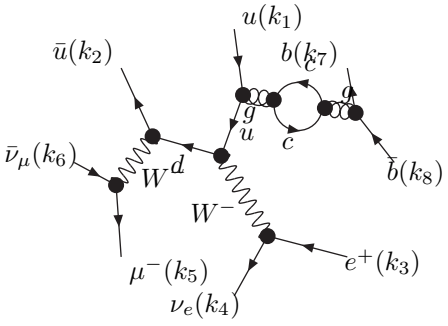
-Diagram 217

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



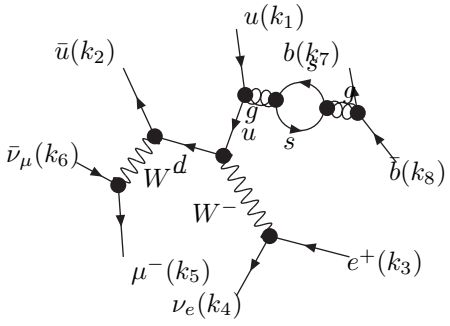
-Diagram 218

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



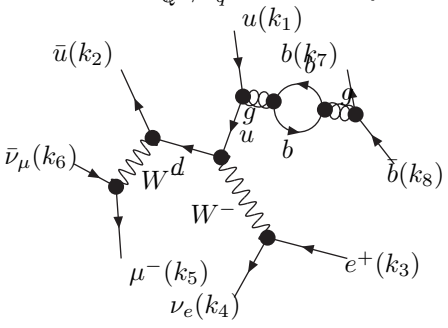
-Diagram 219

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



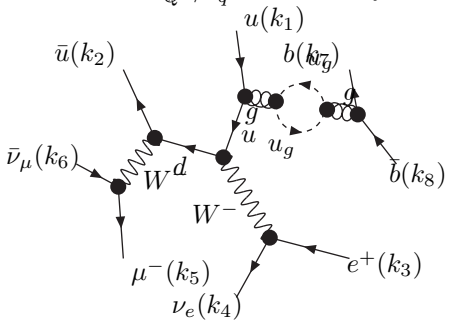
-Diagram 220

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



-Diagram 222

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



-Diagram 223

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2$

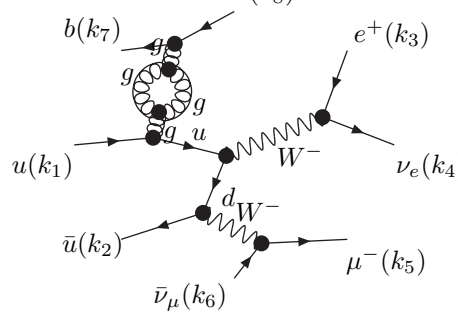


Diagram 224

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2$

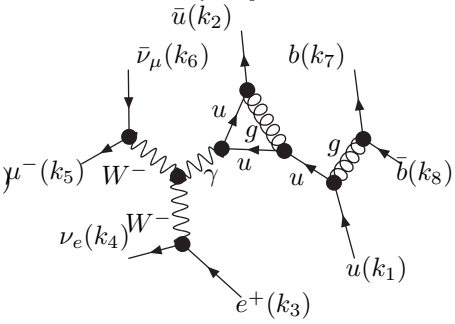


Diagram 253

$S' = S^{\{1,4,5\}}$, $\mathrm{rk} = 2$

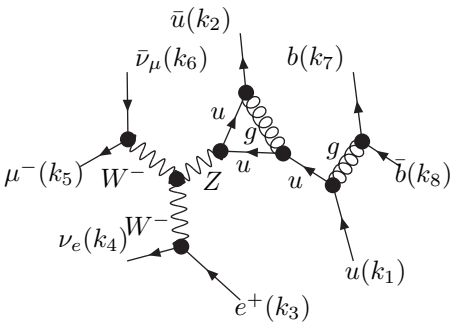


Diagram 254

$S' = S^{\{1,4,5\}}$, rk $= 2$

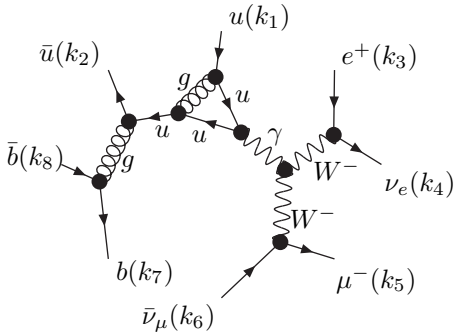


Diagram 255

$S' = S^{\{1,3,4\}}_{Q\to -q-(-k8-k7)}$, rk $= 2$

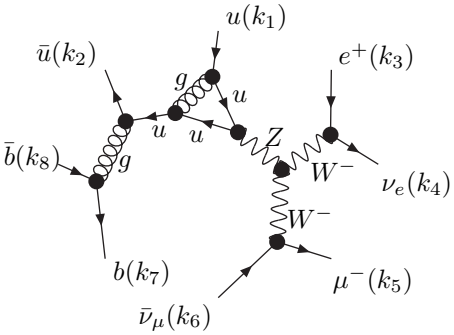


Diagram 256

$S' = S^{\{1,3,4\}}_{Q\to -q-(-k8-k7)}$, rk $= 2$

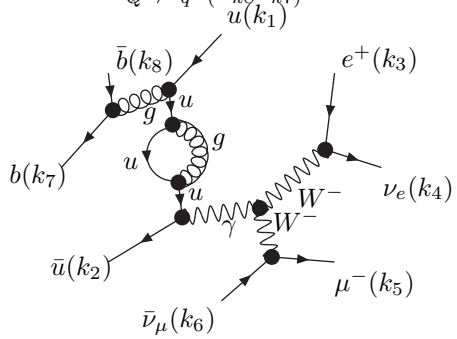


Diagram 261

$S' = S^{\{1,2,4,5\}}_{Q\to -q}$, rk $= 1$

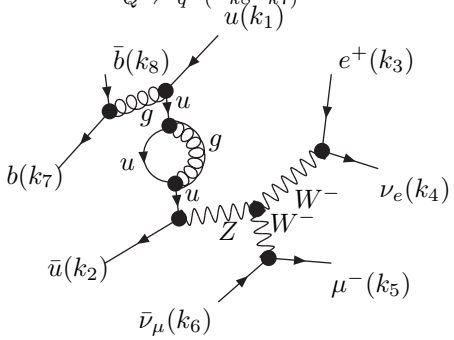


Diagram 262

$S' = S^{\{1,2,4,5\}}_{Q\to -q}$, rk $= 1$

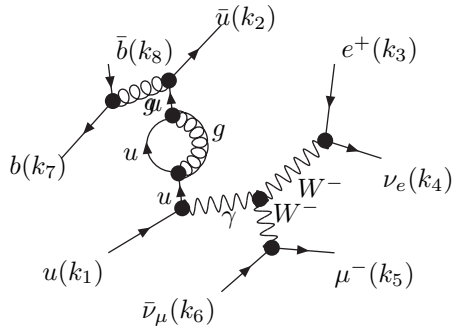


Diagram 263

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k2)}$, rk $= 1$

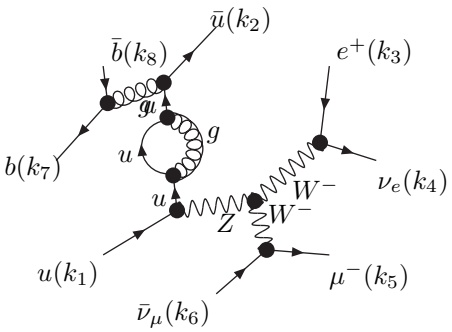


Diagram 264

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k2)}$, rk $= 1$

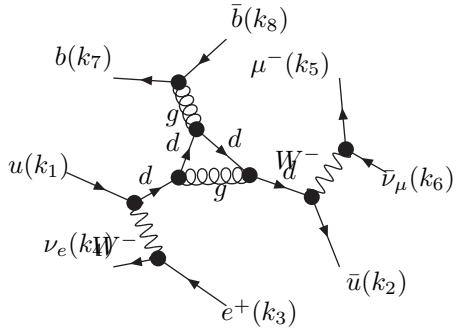


Diagram 317

$S' = S^{\{2,4,6\}}_{Q\to q-(k8+k7)}$, rk $= 2$

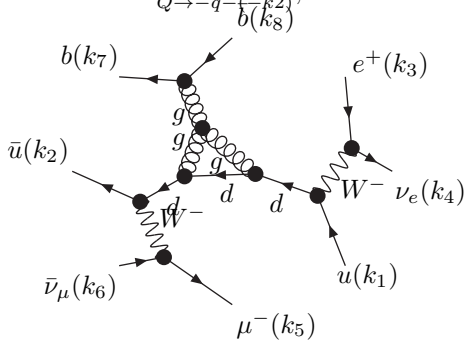


Diagram 318

$S' = S^{\{2,4,6\}}_{Q\to q-(k8+k7)}$, rk $= 2$

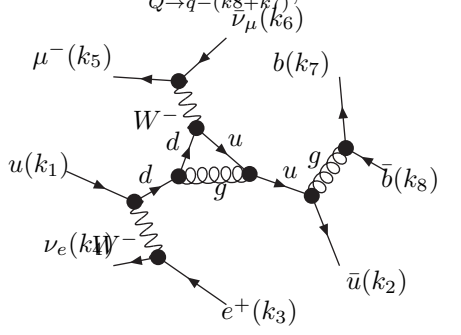


Diagram 319

$S' = S^{\{3,4,6\}}_{Q\to -q-(-k2+k6+k5)}$, rk $= 2$

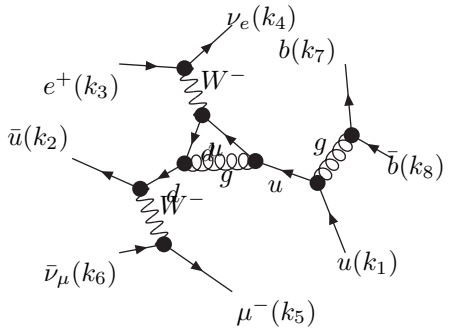
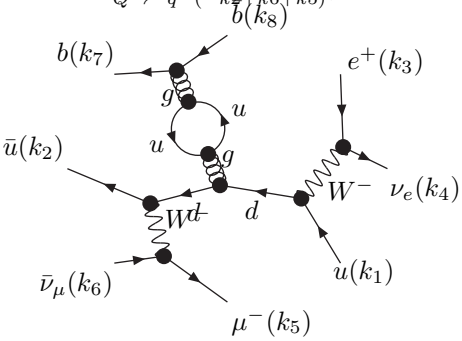


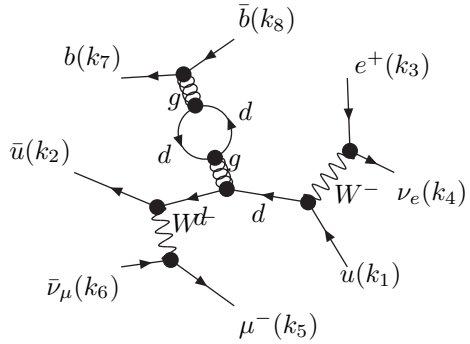
Diagram 320

$S' = S^{\{2,4,5\}}_{Q\to q-(k2-k6-k5)}$, rk $= 2$



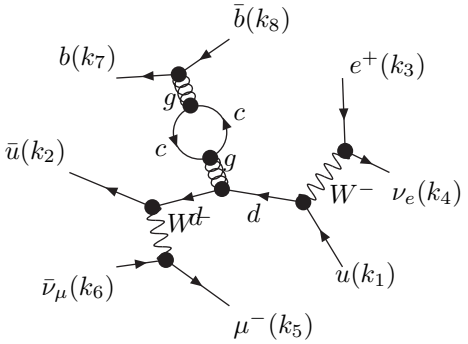
-Diagram 329

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, rk $= 2, N_f$



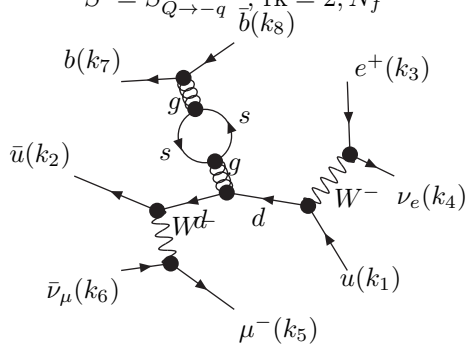
-Diagram 330

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



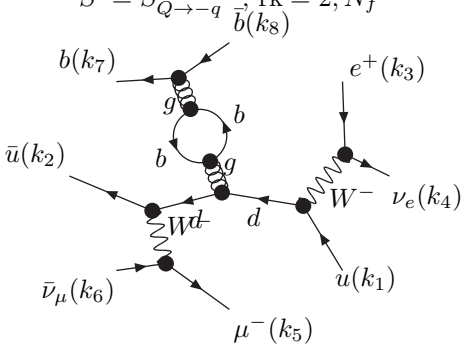
-Diagram 331

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



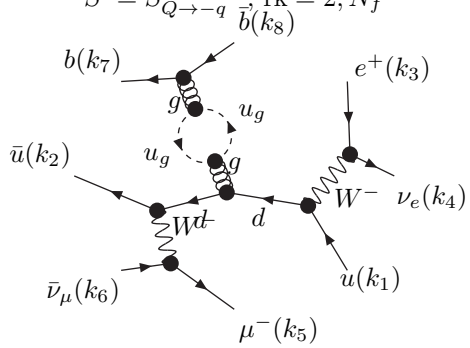
-Diagram 332

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



-Diagram 334

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2, N_f$



-Diagram 335

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2$

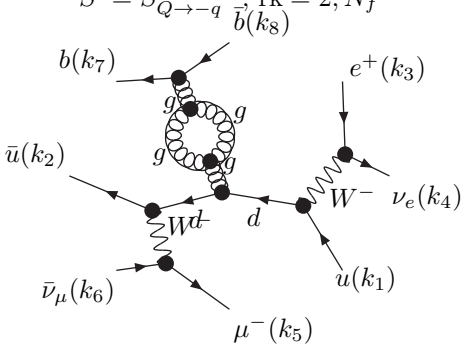


Diagram 336

$S' = S^{\{1,2,4,6\}}_{Q\to -q}$, $\mathrm{rk} = 2$

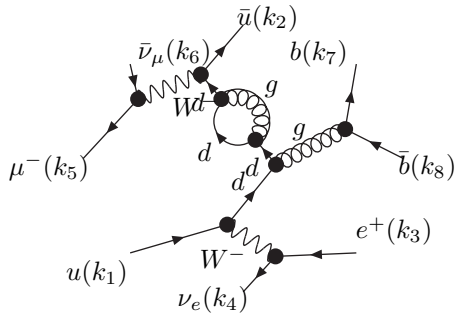


Diagram 341

$S' = S^{\{2,4,5,6\}}_{Q\to -q-(-k2+k6+k5)}$, rk = 1

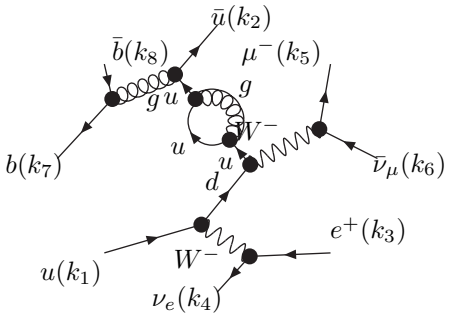


Diagram 342

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k2)}$, rk = 1

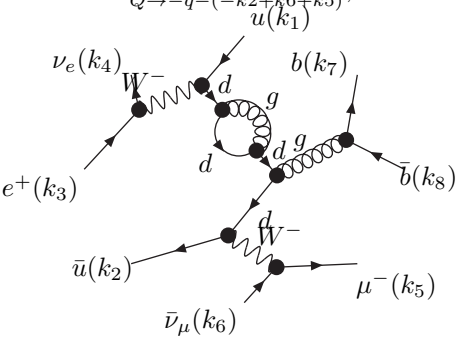


Diagram 345

$S' = S^{\{2,3,4,6\}}_{Q\to q-(k2-k6-k5)}$, rk = 1

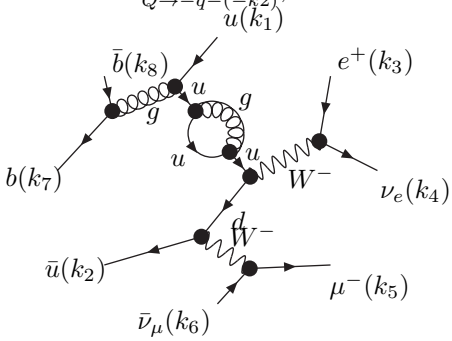


Diagram 346

$S' = S^{\{1,2,4,5\}}_{Q\to -q}$, rk = 1

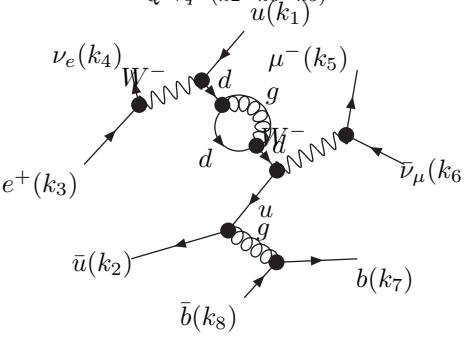


Diagram 347

$S' = S^{\{2,3,4,6\}}_{Q\to q-(k2-k6-k5)}$, rk = 1

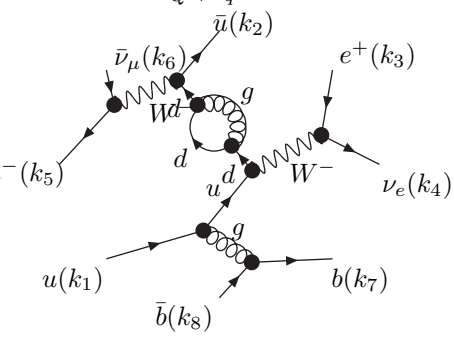


Diagram 348

$S' = S^{\{2,4,5,6\}}_{Q\to -q-(-k2+k6+k5)}$, rk = 1

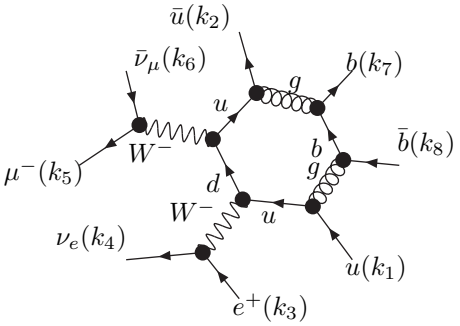


Diagram 352

$S' = S_{Q\to -q-(-k7)}$, rk $= 4$

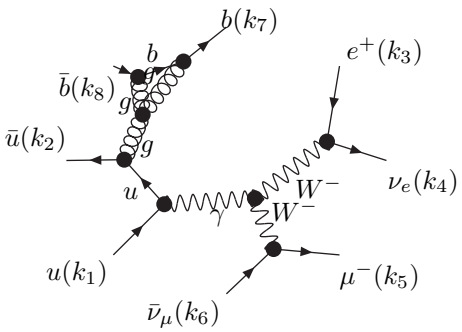


Diagram 353

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

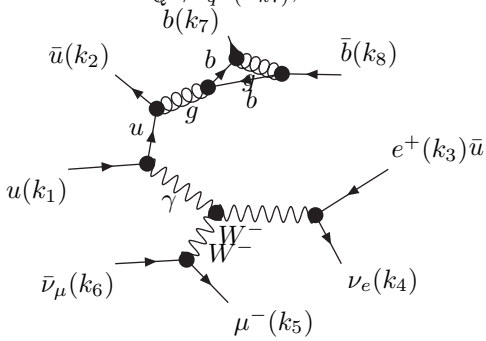


Diagram 354

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

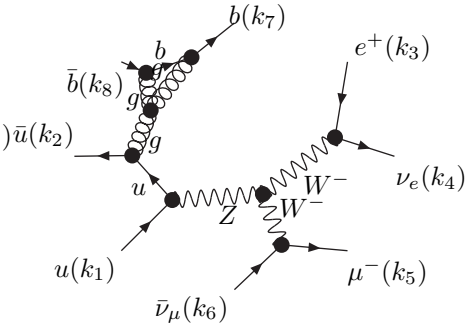


Diagram 355

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

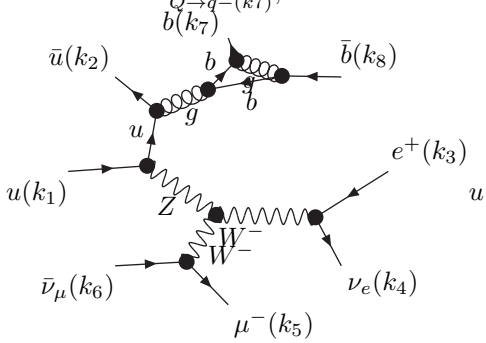


Diagram 356

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

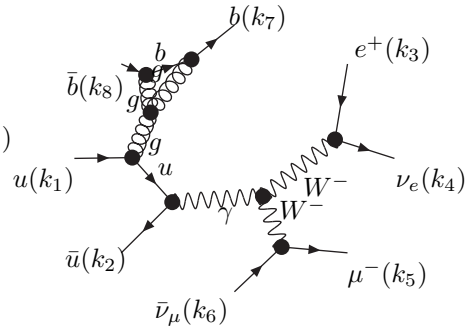


Diagram 357

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

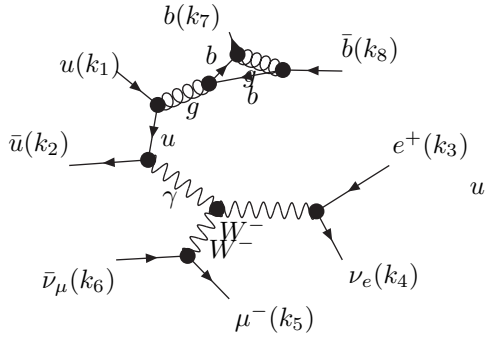


Diagram 358

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

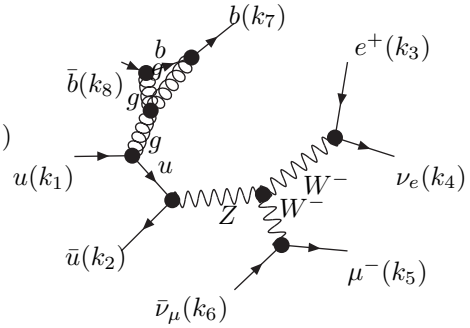


Diagram 359

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

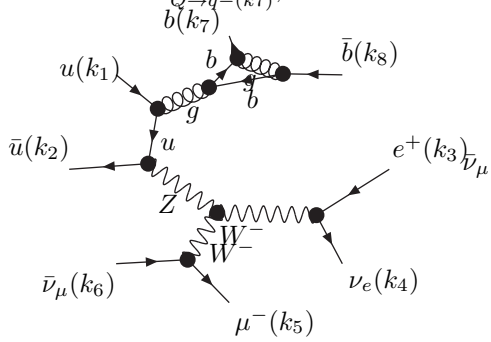


Diagram 360

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

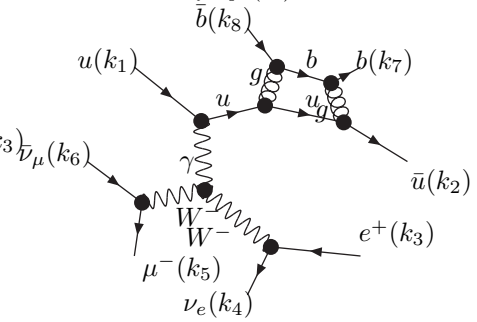


Diagram 379

$S' = S^{\{1,6\}}_{Q\to -q}$, rk $= 2$

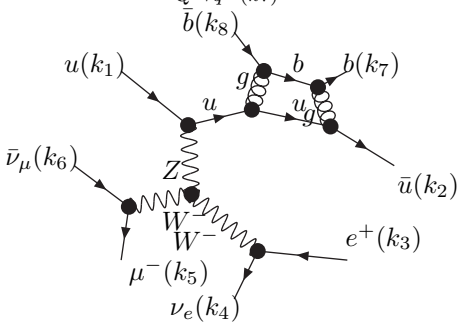


Diagram 380

$S' = S^{\{1,6\}}_{Q\to -q}$, rk $= 2$

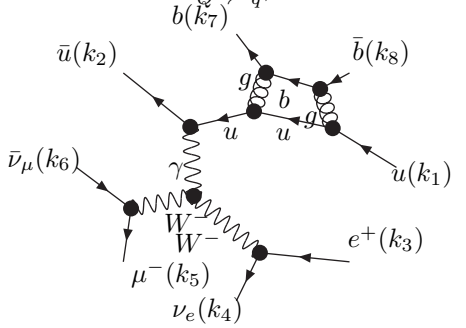


Diagram 381

$S' = S^{\{1,2\}}_{Q\to q-(k8+k7)}$, rk $= 2$

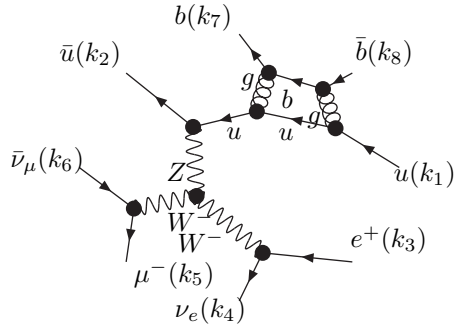


Diagram 382
$S' = S^{\{1,2\}}_{Q\to g-(k8+k7)}$, rk = 2

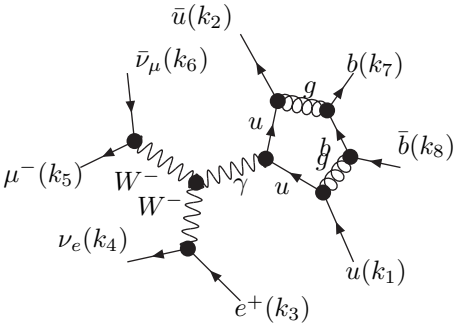


Diagram 391
$S' = S^{\{1\}}_{Q\to q-(k7)}$, rk = 3

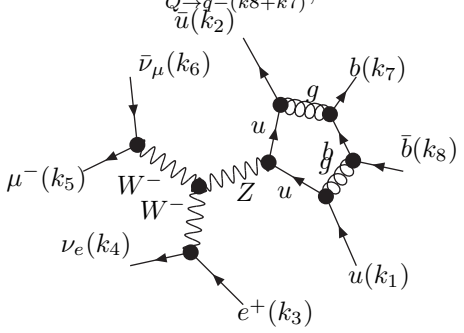


Diagram 392
$S' = S^{\{1\}}_{Q\to q-(k7)}$, rk = 3

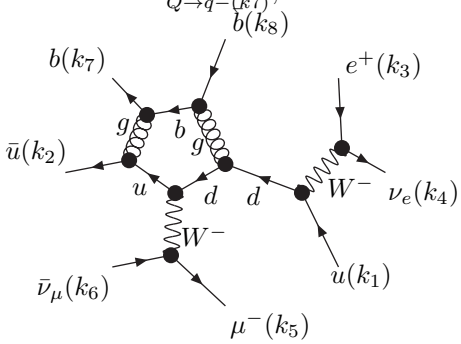


Diagram 393
$S' = S^{\{6\}}_{Q\to -q}$, rk = 3

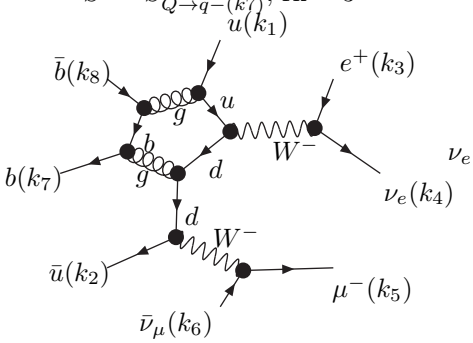


Diagram 398
$S' = S^{\{2\}}_{Q\to q-(k8+k7)}$, rk = 3

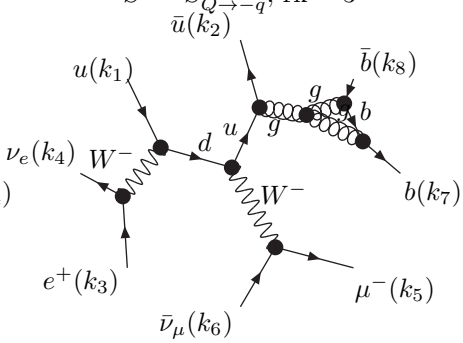


Diagram 401
$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk = 2

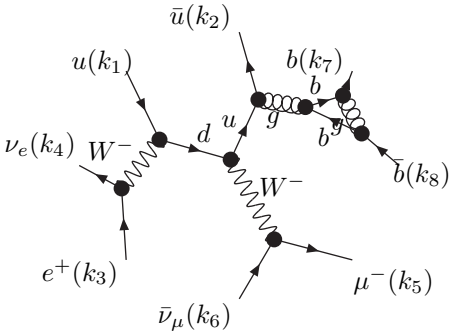


Diagram 402

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

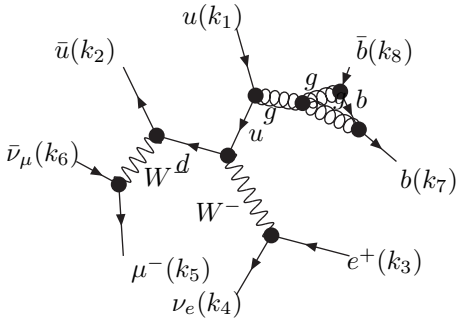


Diagram 405

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

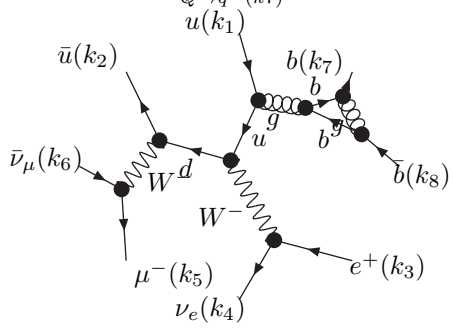


Diagram 406

$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk $= 2$

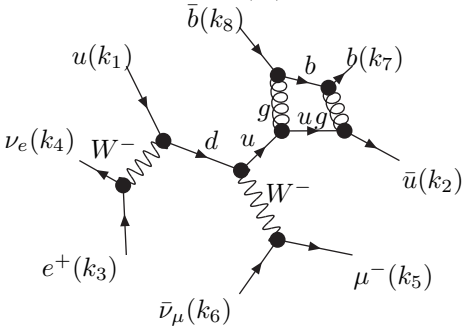


Diagram 409

$S' = S^{\{1,6\}}_{Q\to -q}$, rk $= 2$

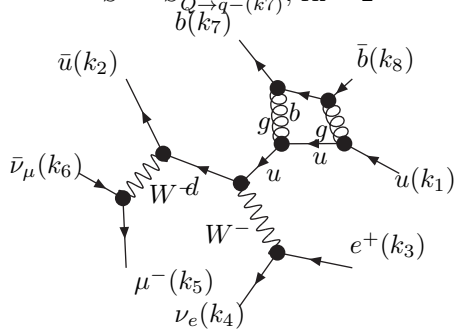


Diagram 414

$S' = S^{\{1,2\}}_{Q\to q-(k8+k7)}$, rk $= 2$

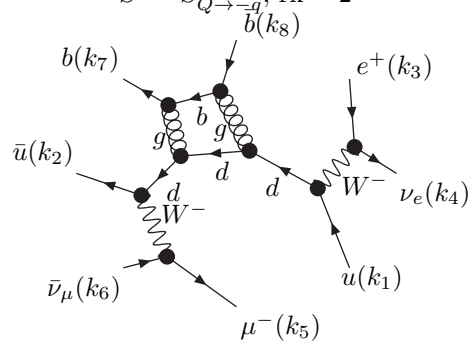


Diagram 418

$S' = S^{\{2,6\}}_{Q\to -q-(-k7)}$, rk $= 2$

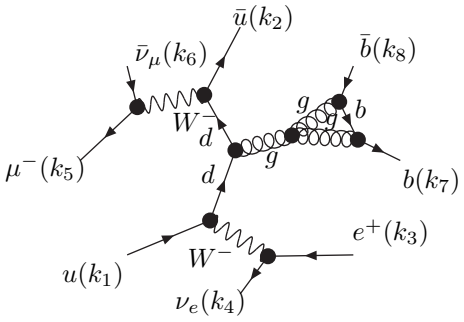


Diagram 421
$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk = 2

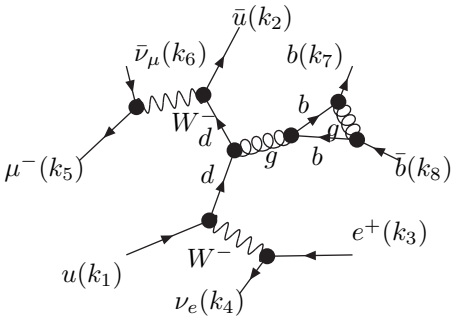


Diagram 422
$S' = S^{\{1,2,6\}}_{Q\to q-(k7)}$, rk = 2

5.11 Group 10 (6-Point)

General Information

The maximum effective rank in this group is 6.

$$r_1 = -k_3 - k_7 - k_4, \quad m_1 = m_t, \quad \Gamma_1 = \Gamma_t \tag{53a}$$
$$r_2 = -k_3 - k_7 - k_6 - k_5 - k_4 \tag{53b}$$
$$r_3 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{53c}$$
$$r_4 = -k_2 \tag{53d}$$
$$r_5 = 0 \tag{53e}$$
$$r_6 = -k_7 \tag{53f}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & 0 & 0 & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & 0 & 0 & 0 & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 & 0 \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & 0 & 0 \end{pmatrix} \tag{54}$$

$$S_{1,1} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{55a}$$
$$S_{1,2} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{55b}$$
$$S_{1,3} = -m_t^2 + s_{1234} + s_{56} - s_{567} - s_{78} + i \cdot m_t \cdot \Gamma_t \tag{55c}$$
$$S_{1,4} = -m_t^2 + s_{56} - s_{567} + s_{81} - s_{781} + s_{234} + i \cdot m_t \cdot \Gamma_t \tag{55d}$$
$$S_{1,5} = -m_t^2 + s_{56} - s_{567} + s_{812} - s_{3456} + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{55e}$$
$$S_{1,6} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{55f}$$
$$S_{2,4} = s_{81} \tag{55g}$$
$$S_{2,5} = s_{812} \tag{55h}$$
$$S_{2,6} = s_{3456} \tag{55i}$$
$$S_{3,5} = s_{12} \tag{55j}$$

$$S_{3,6} = -s_{78} - s_{812} + s_{3456} + s_{12} \tag{55k}$$
$$S_{4,6} = s_{3456} + s_{81} - s_{812} - s_{781} \tag{55l}$$

5.11.1 Diagrams (115)

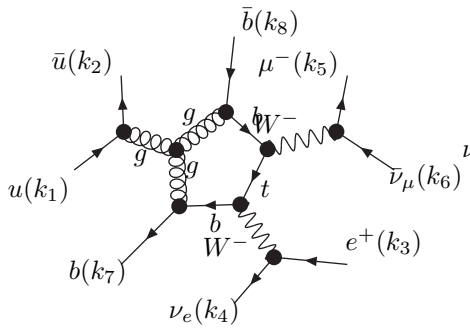


Diagram 61
$S' = S^{\{4\}}_{Q\to -q}$, rk = 4

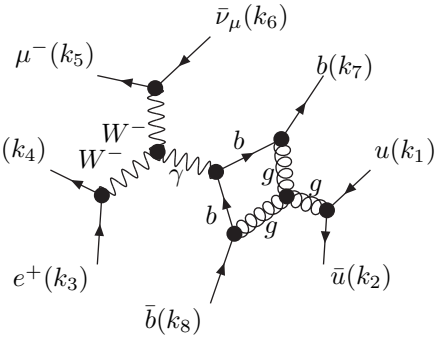


Diagram 69
$S' = S^{\{1,4\}}_{Q\to -q}$, rk = 3

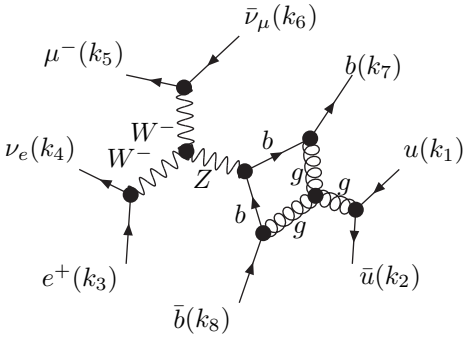


Diagram 70
$S' = S^{\{1,4\}}_{Q\to -q}$, rk = 3

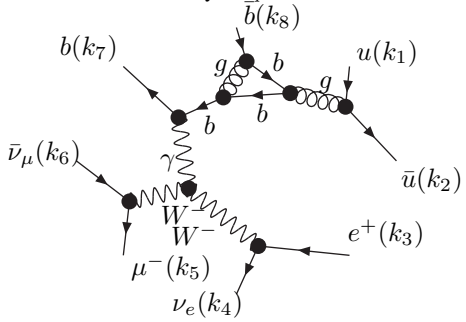


Diagram 73
$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

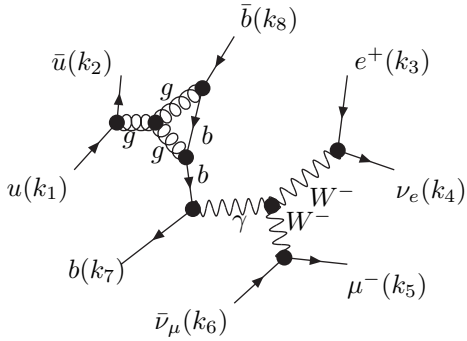


Diagram 74
$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

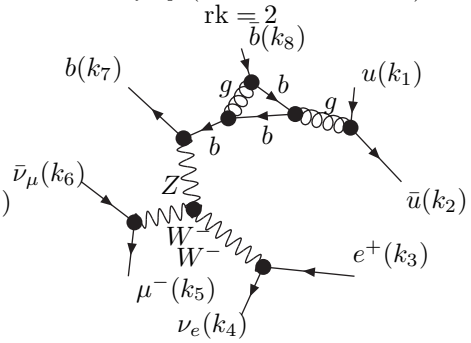


Diagram 75
$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

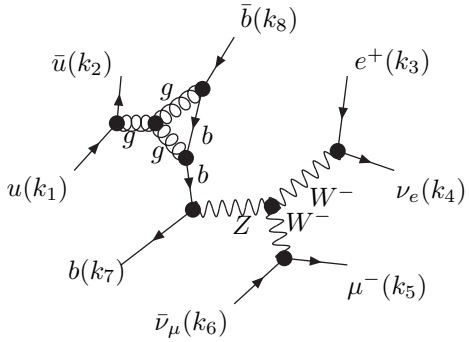


Diagram 76

$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

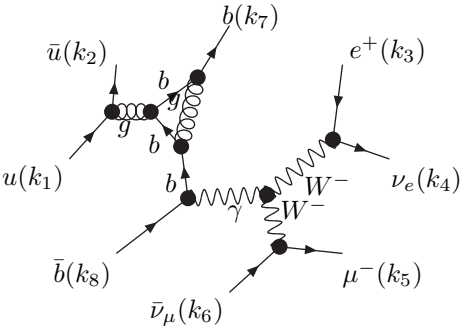


Diagram 77

$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2

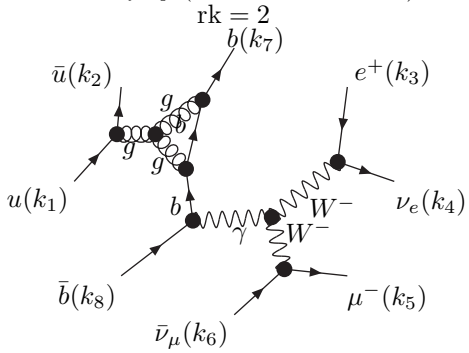


Diagram 78

$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2

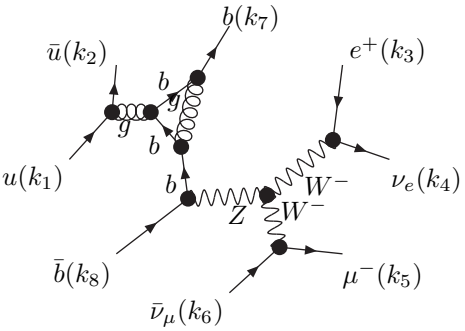


Diagram 79

$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2

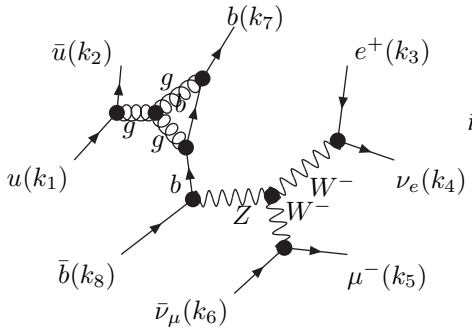
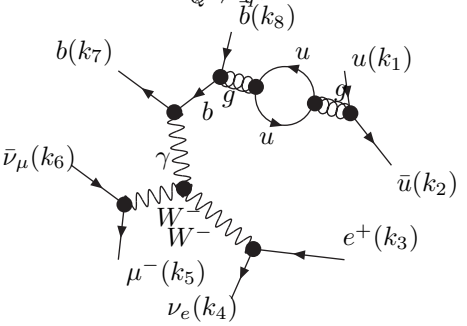


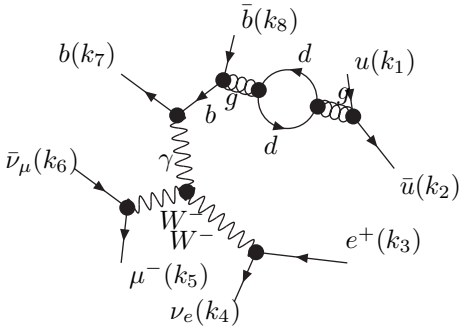
Diagram 80

$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2



-Diagram 97

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = 2, N_f

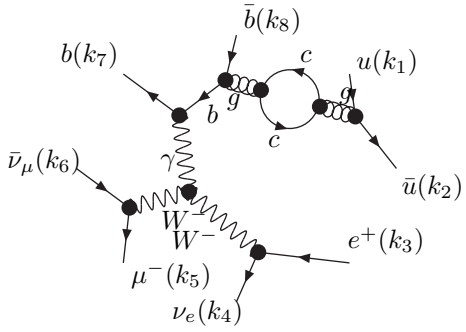


-Diagram 98

$S' =$

$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2, N_f$

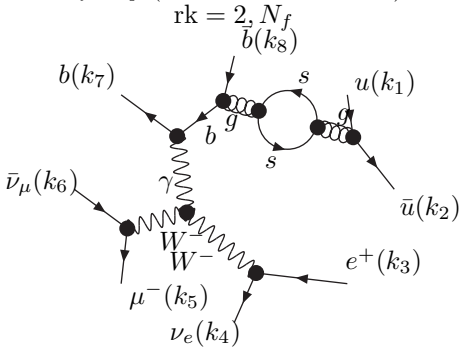


-Diagram 99

$S' =$

$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2, N_f$

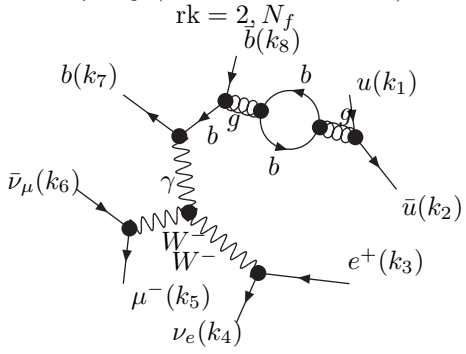


-Diagram 100

$S' =$

$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2, N_f$

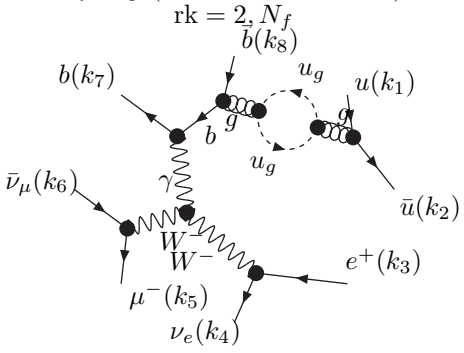


-Diagram 102

$S' =$

$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2, N_f$



-Diagram 103

$S' =$

$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2$

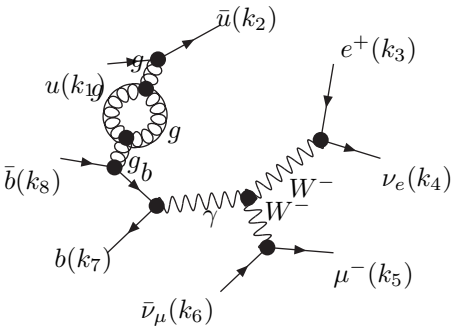
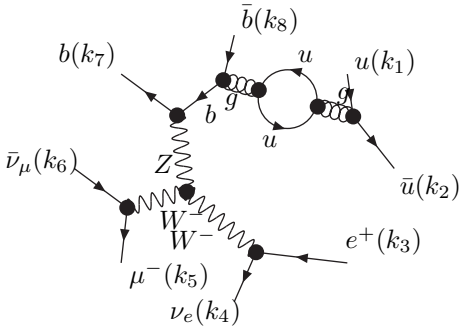


Diagram 104

$S' =$

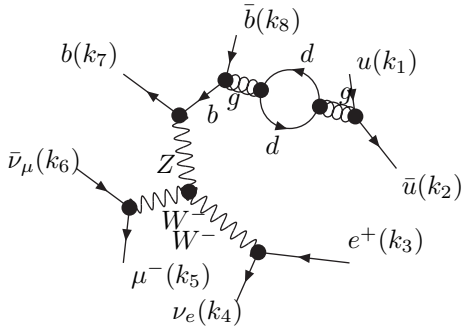
$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,

$\mathrm{rk} = 2$



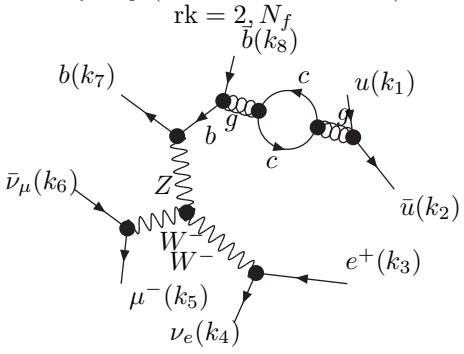
-Diagram 105

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



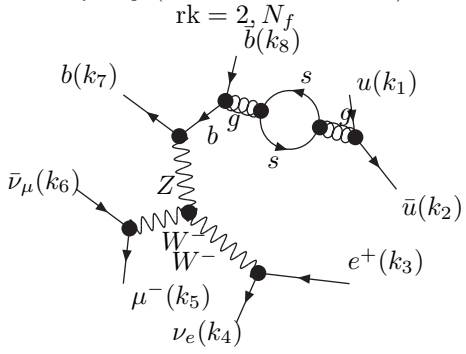
-Diagram 106

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



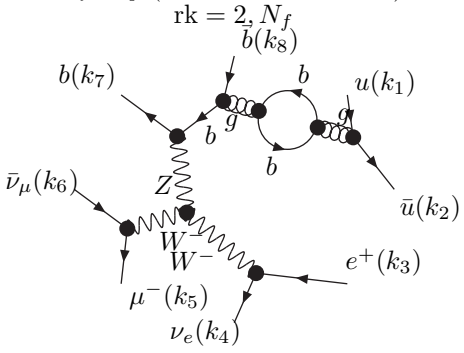
-Diagram 107

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



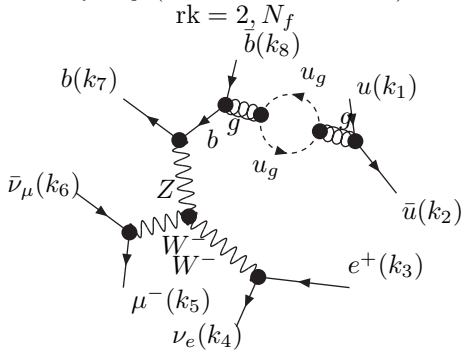
-Diagram 108

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



-Diagram 110

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



-Diagram 111

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2$

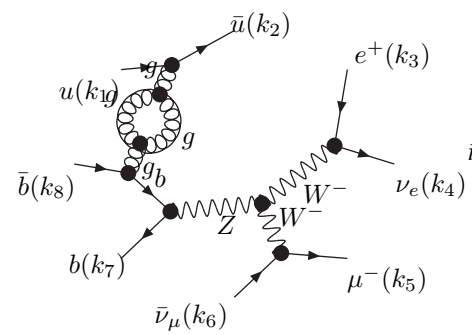
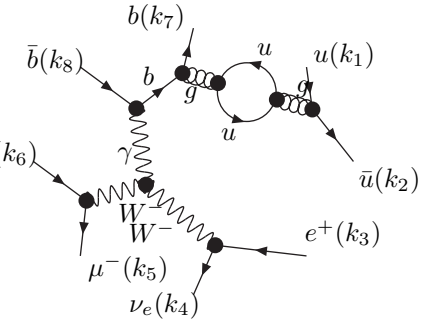


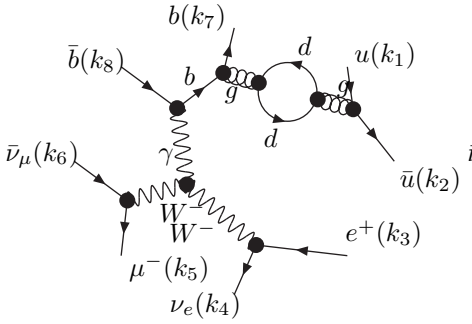
Diagram 112

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2$



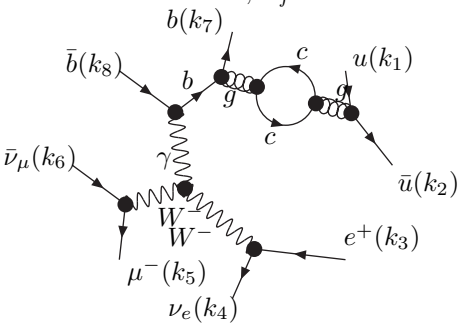
-Diagram 113

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



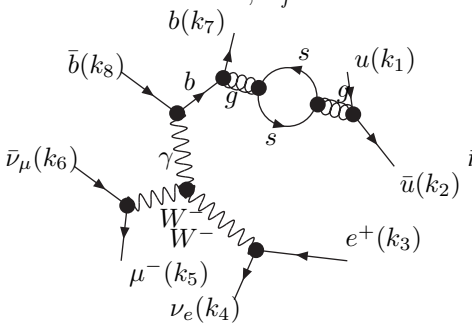
-Diagram 114

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



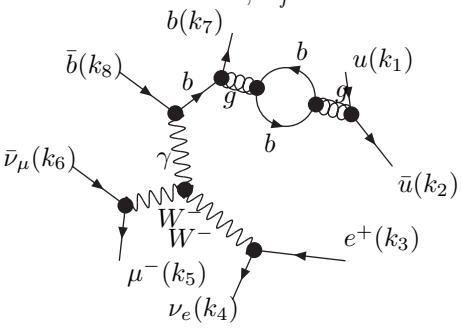
-Diagram 115

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



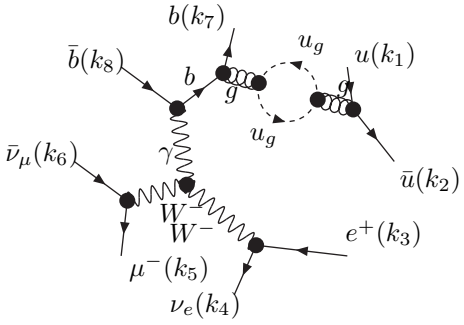
-Diagram 116

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



-Diagram 118

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



-Diagram 119

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2$

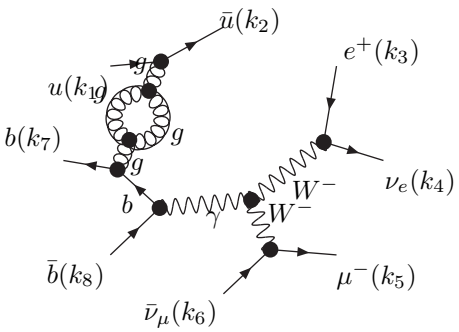
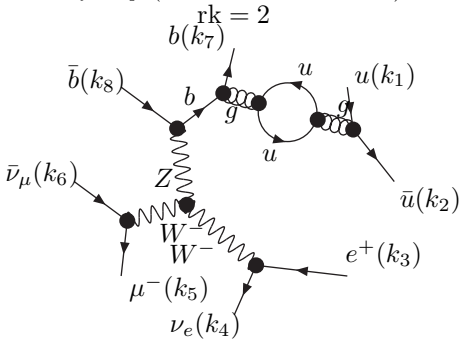


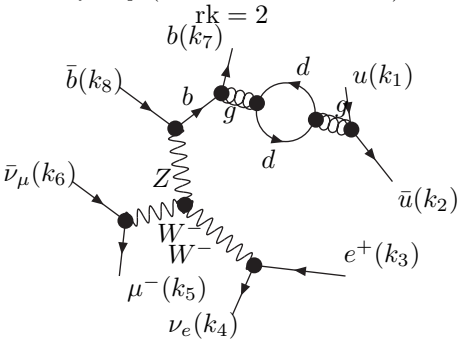
Diagram 120

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2$



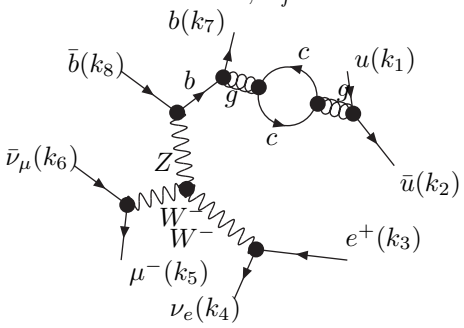
-Diagram 121

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



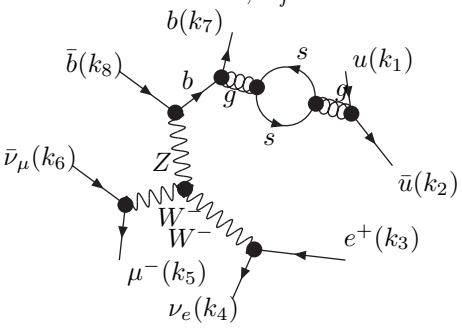
-Diagram 122

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



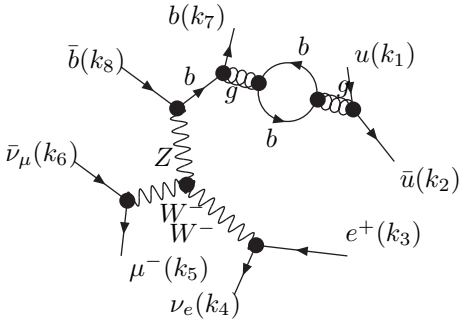
-Diagram 123

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



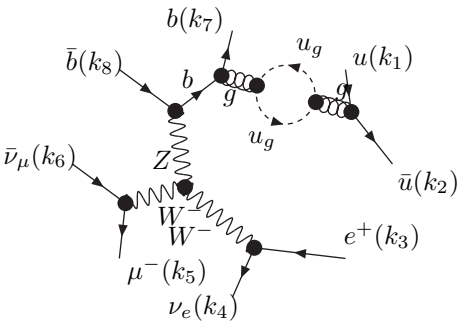
-Diagram 124

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



-Diagram 126

$S' =$
$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2, N_f$



-Diagram 127

$S' =$
$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2$

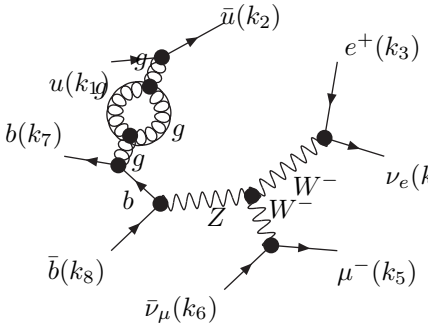


Diagram 128

$S' =$
$S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2$

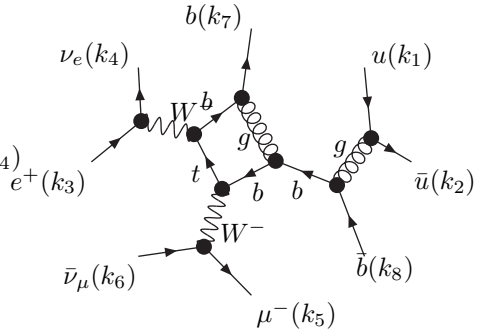


Diagram 161

$S' = S^{\{3,4\}}_{Q\to q-(k7)}$, rk $= 3$

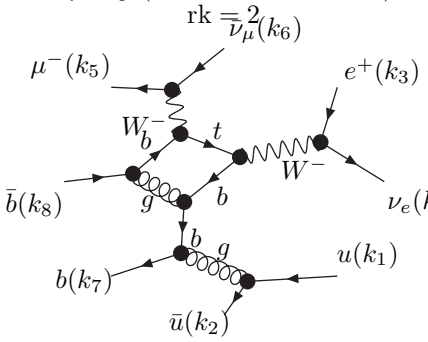


Diagram 162

$S' = S^{\{4,5\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$,
rk $= 3$

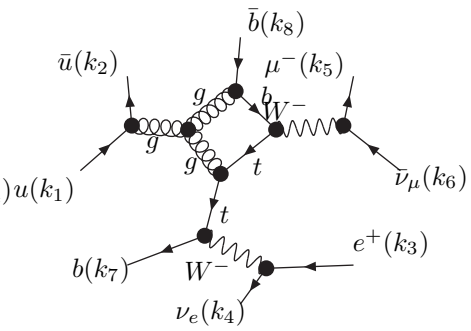


Diagram 173

$S' = S^{\{4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$,
rk $= 3$

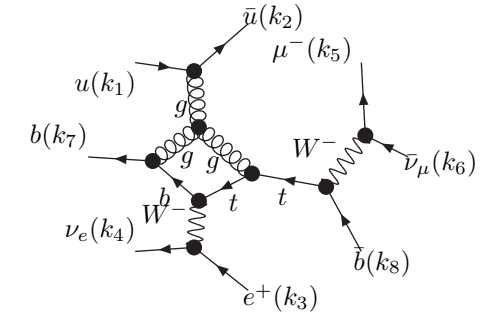


Diagram 175
$S' = S^{\{2,4\}}_{Q\to -q}$, rk = 3

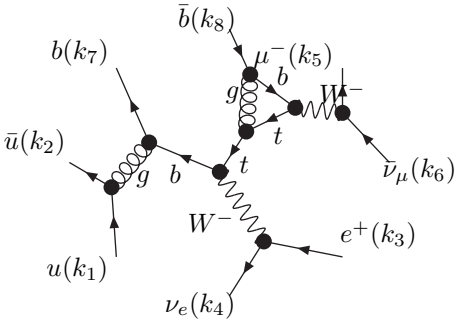


Diagram 177
$S' = S^{\{4,5,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$, rk = 2

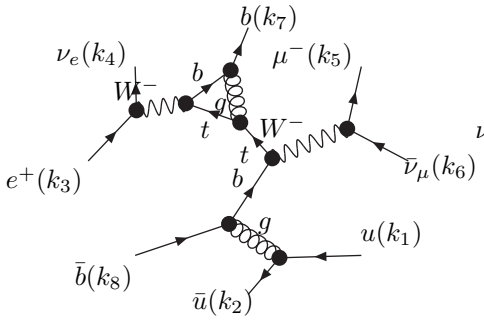


Diagram 178
$S' = S^{\{2,3,4\}}_{Q\to q-(k7)}$, rk = 2

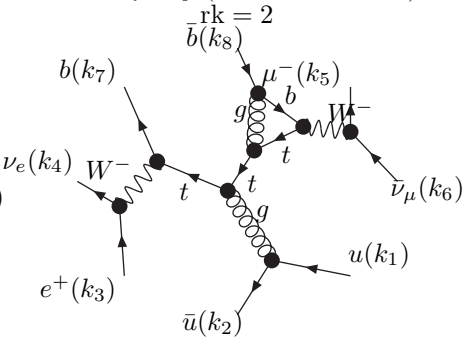


Diagram 179
$S' = S^{\{4,5,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$, rk = 2

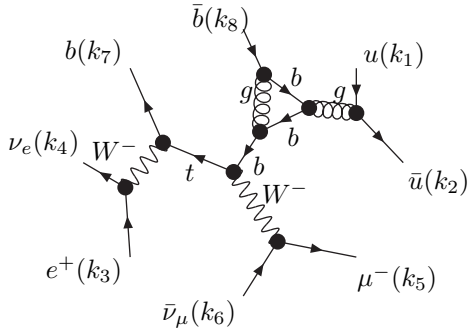


Diagram 180
$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

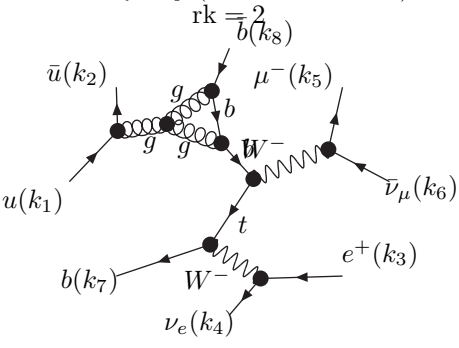


Diagram 181
$S' = S^{\{1,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

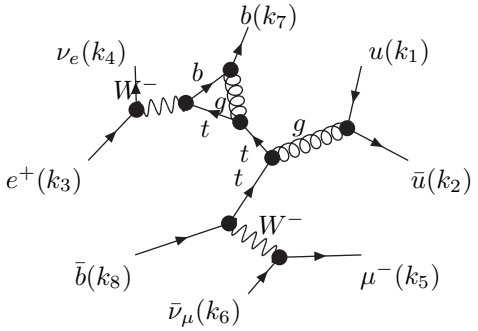


Diagram 185

$S' = S^{\{2,3,4\}}_{Q\to q-(k7)}$, rk = 2

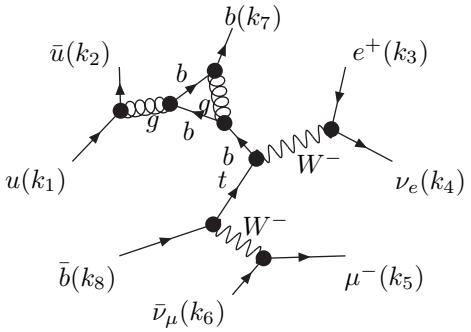


Diagram 186

$S' = S^{\{1,2,4\}}_{Q\to -g}$, rk = 2

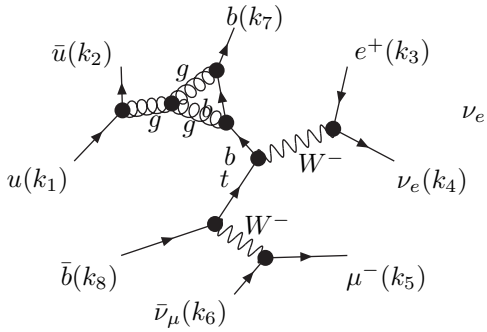
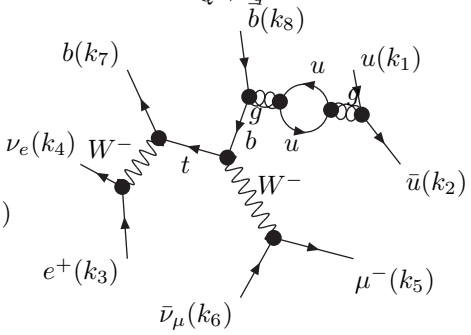


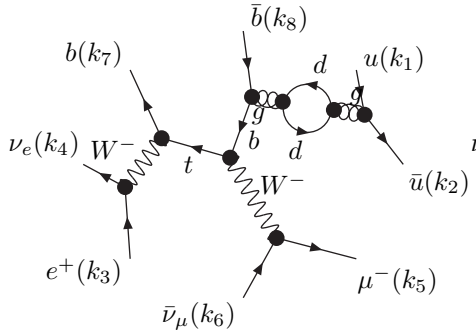
Diagram 187

$S' = S^{\{1,2,4\}}_{Q\to -q}$, rk = 2



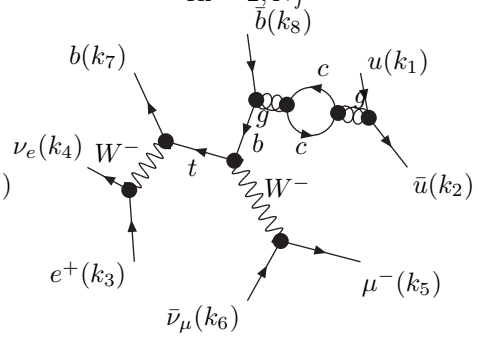
-Diagram 193

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = $2, N_f$



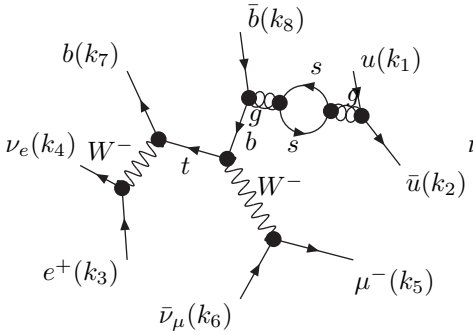
-Diagram 194

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = $2, N_f$



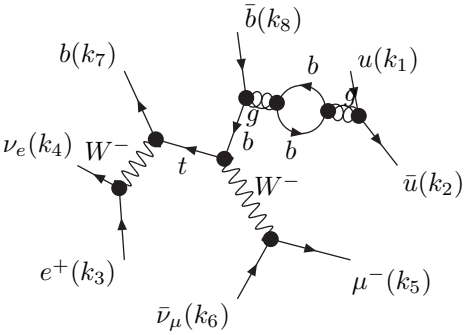
-Diagram 195

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = $2, N_f$



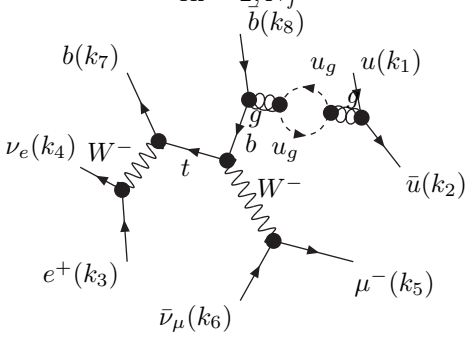
-Diagram 196

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



-Diagram 198

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



-Diagram 199

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2$

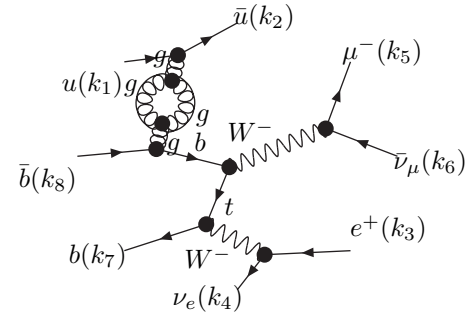
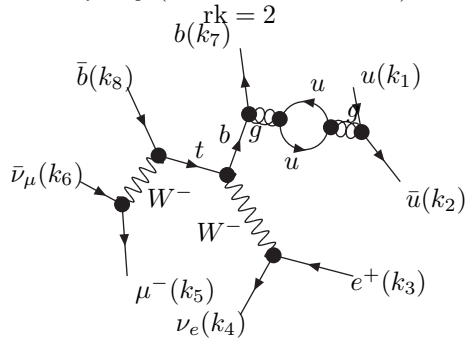


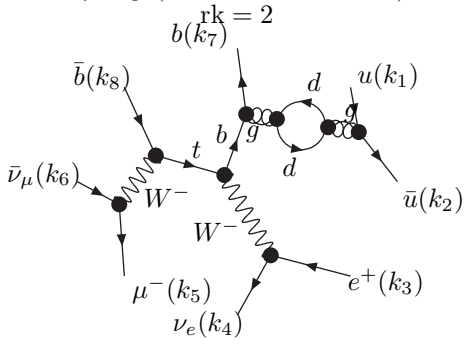
Diagram 200

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2$



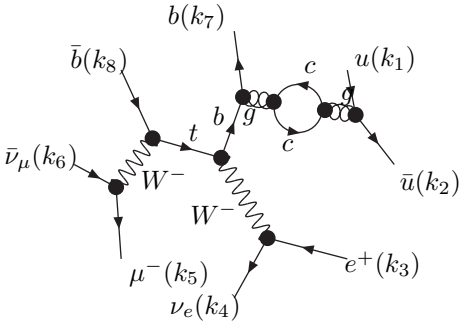
-Diagram 209

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



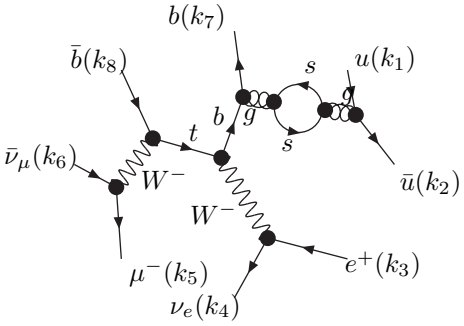
-Diagram 210

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, $\mathrm{rk} = 2, N_f$



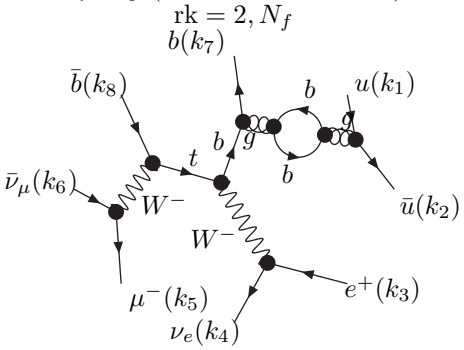
-Diagram 211

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2, N_f$



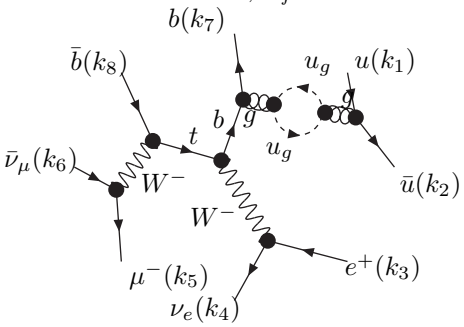
-Diagram 212

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2, N_f$



-Diagram 214

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2, N_f$



-Diagram 215

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2$

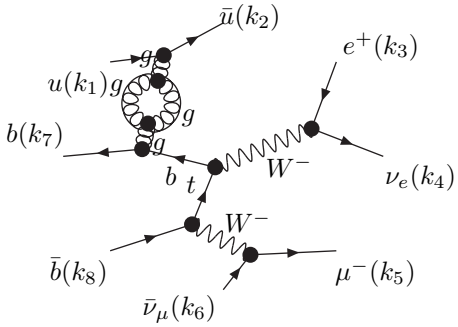


Diagram 216

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk $= 2$

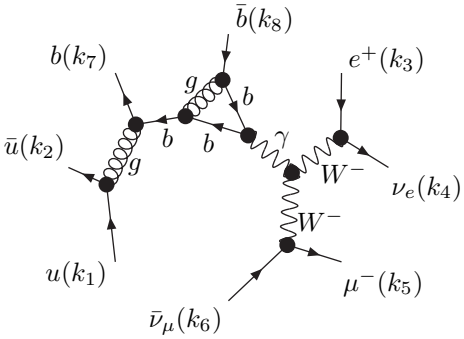


Diagram 249

$S' = S^{\{1,4,5\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$,
rk $= 2$

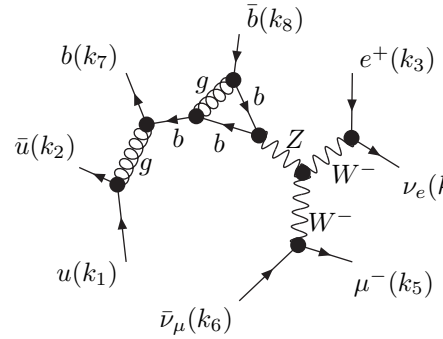


Diagram 250

$S' = S^{\{1,4,5\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$,
rk = 2

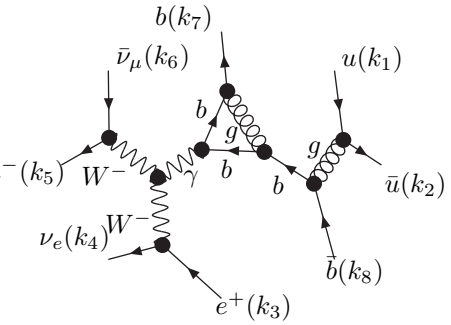


Diagram 251

$S' = S^{\{1,3,4\}}_{Q\to -q}$, rk = 2

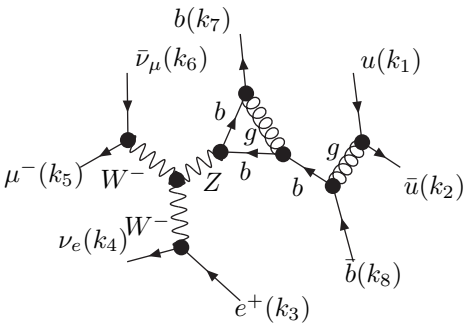


Diagram 252

$S' = S^{\{1,3,4\}}_{Q\to -q}$, rk = 2

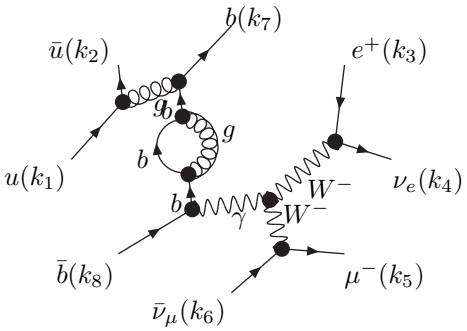


Diagram 257

$S' =$
$S^{\{1,2,4,5\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk = 1

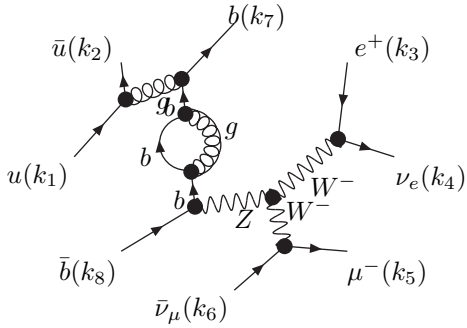


Diagram 258

$S' =$
$S^{\{1,2,4,5\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$,
rk = 1

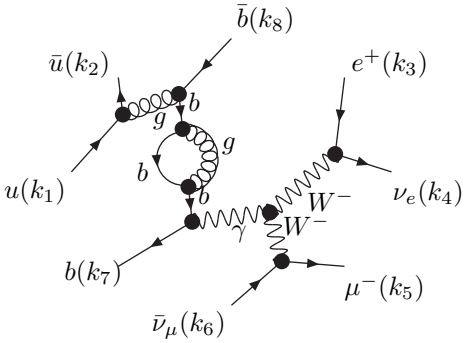


Diagram 259

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$,
rk = 1

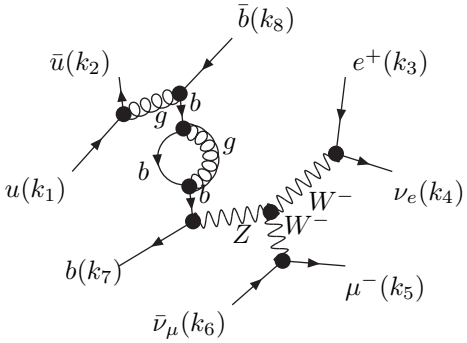


Diagram 260

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$, rk = 1

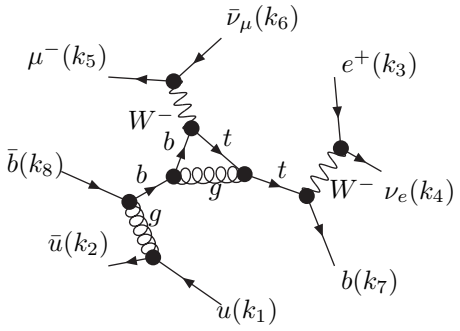


Diagram 313

$S' = S^{\{3,4,6\}}_{Q\to q-(k3+k7+k6+k5+k4)}$, rk = 2

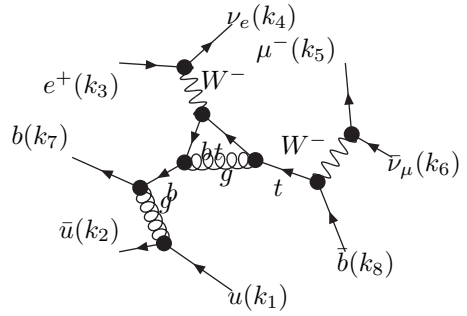


Diagram 314

$S' = S^{\{2,4,5\}}_{Q\to -q-(-k7)}$, rk = 2

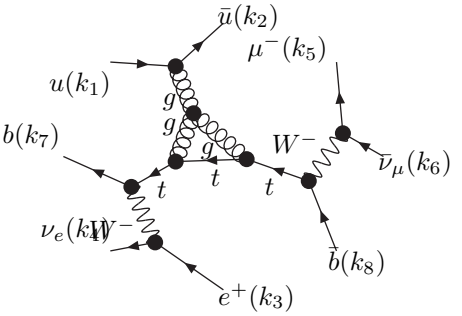
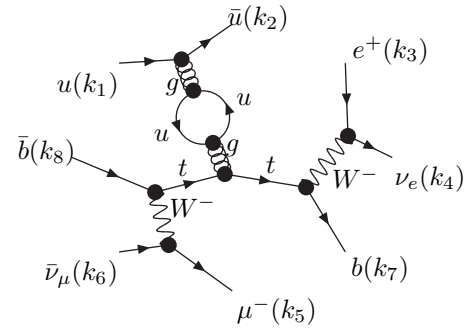


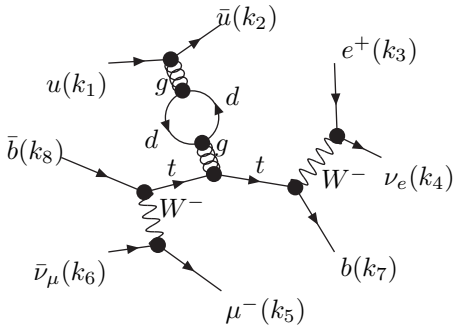
Diagram 316

$S' = S^{\{2,4,6\}}$, rk = 2



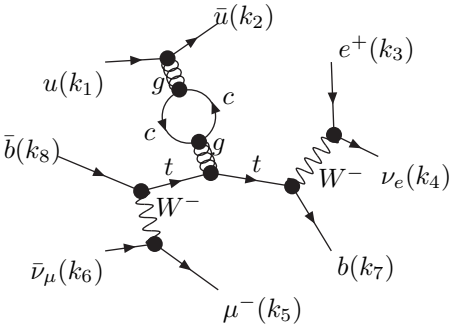
-Diagram 321

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = $2, N_f$



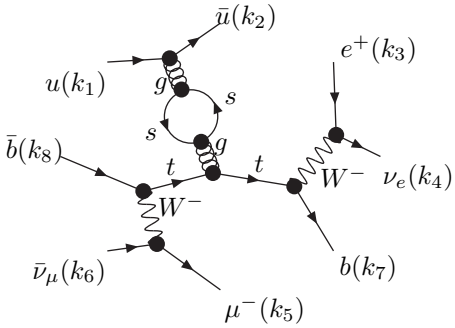
-Diagram 322

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = $2, N_f$



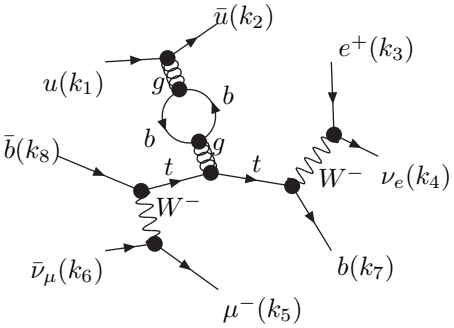
-Diagram 323

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



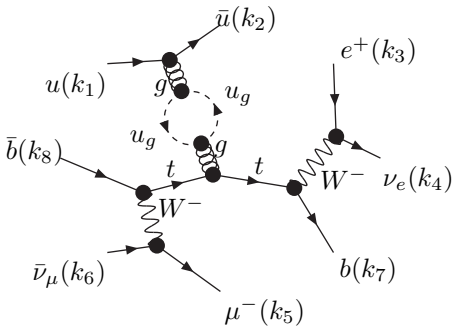
-Diagram 324

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



-Diagram 326

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2, N_f$



-Diagram 327

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2$

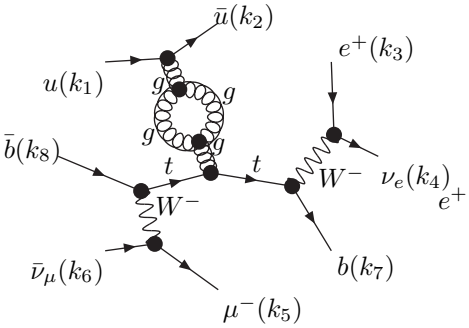


Diagram 328

$S' = S^{\{1,2,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 2$

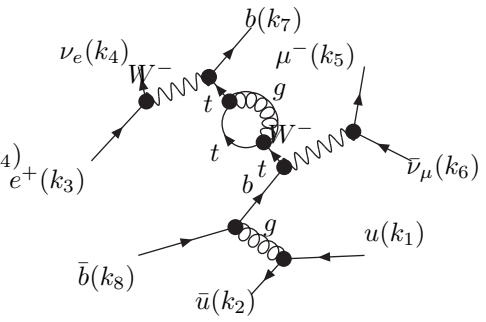


Diagram 337

$S' = S^{\{2,3,4,6\}}_{Q\to -q}$, rk $= 2$

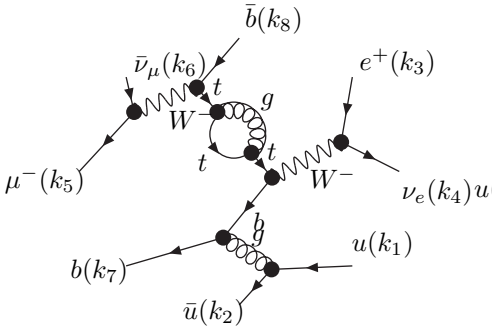


Diagram 338

$S' = S^{\{2,4,5,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

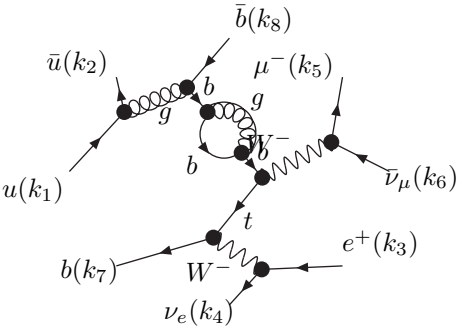


Diagram 339

$S' = S^{\{1,3,4,6\}}_{Q\to -q-(-k3-k7-k6-k5-k4)}$, rk = 1

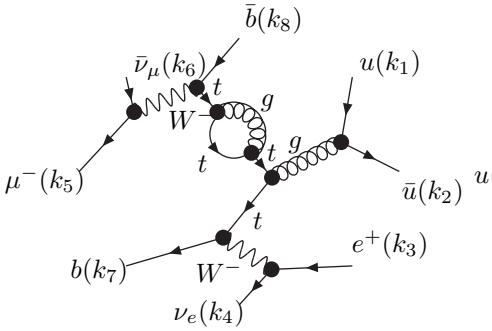


Diagram 340

$S' = S^{\{2,4,5,6\}}_{Q\to q-(k3+k7+k6+k5+k4+k8)}$, rk = 2

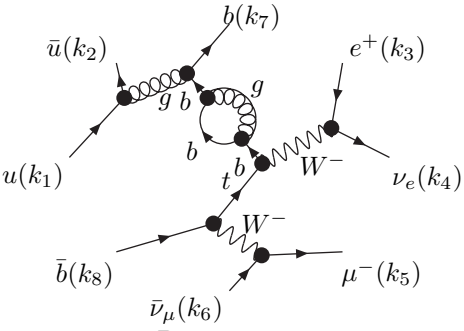


Diagram 343

$S' =$
$S^{\{1,2,4,5\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = 1

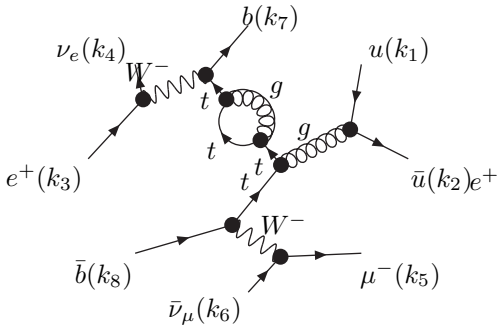


Diagram 344

$S' = S^{\{2,3,4,6\}}_{Q\to -q}$, rk = 2

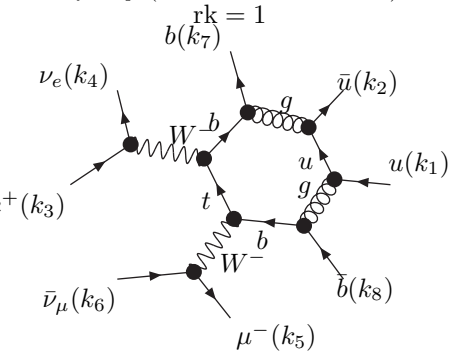


Diagram 350

$S' = S_{Q\to -q-(-k2)}$, rk = 4

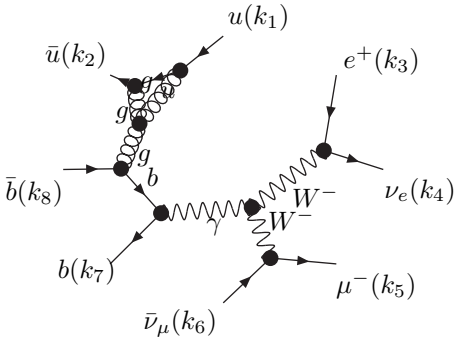


Diagram 361

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

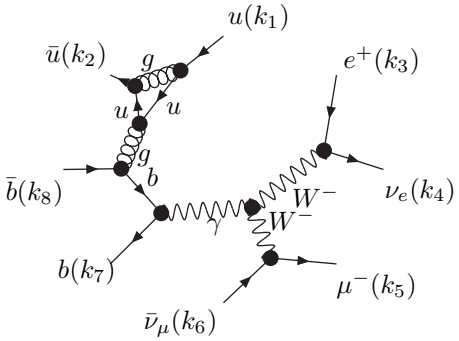


Diagram 362

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

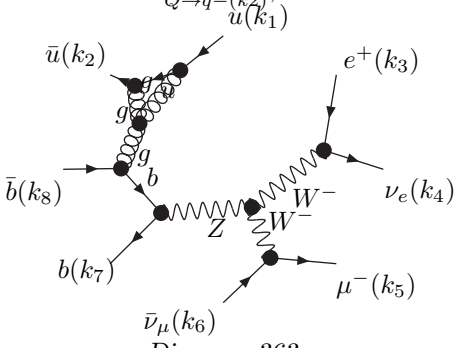


Diagram 363

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

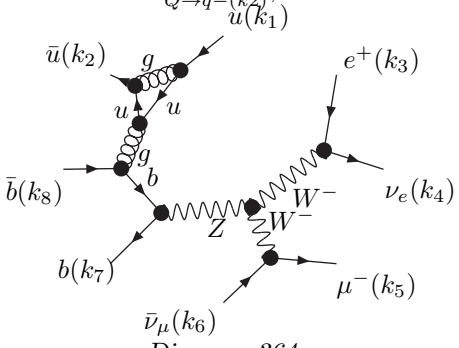


Diagram 364

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

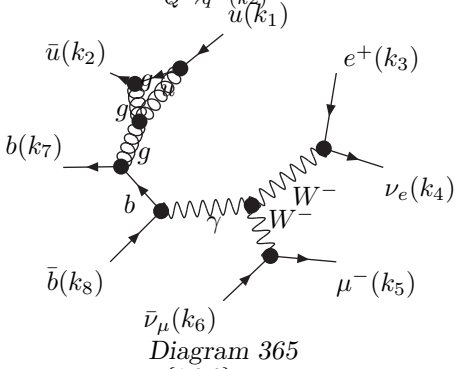


Diagram 365

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

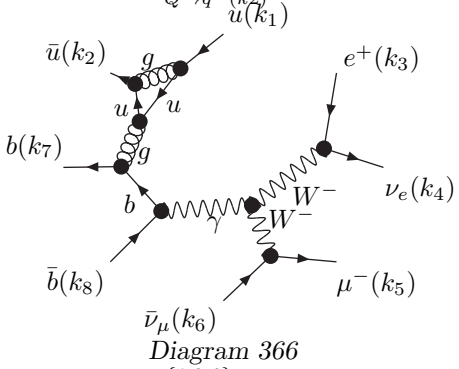


Diagram 366

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

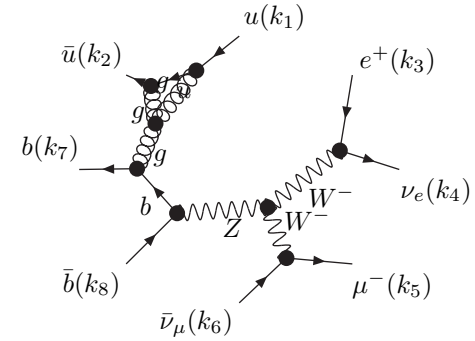


Diagram 367

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

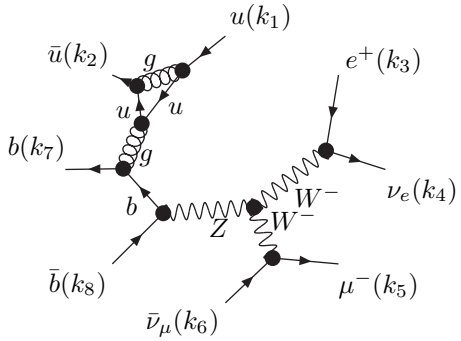


Diagram 368

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

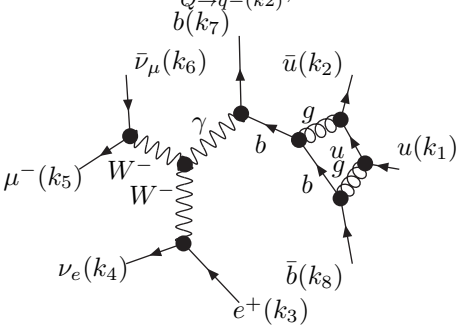


Diagram 371

$S' = S^{\{1,6\}}_{Q\to q-(k2)}$, rk $= 2$

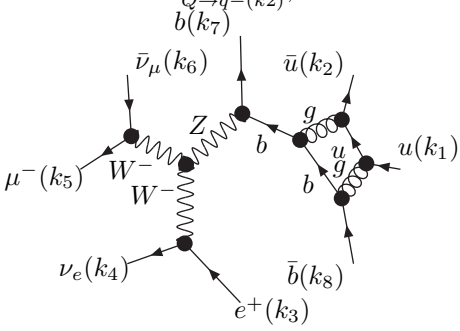


Diagram 372

$S' = S^{\{1,6\}}_{Q\to q-(k2)}$, rk $= 2$

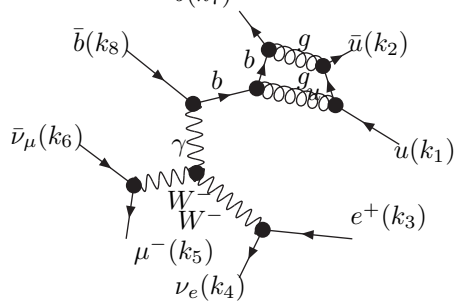


Diagram 373

$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 2$

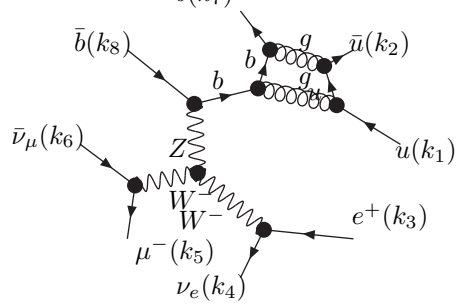


Diagram 374

$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 2$

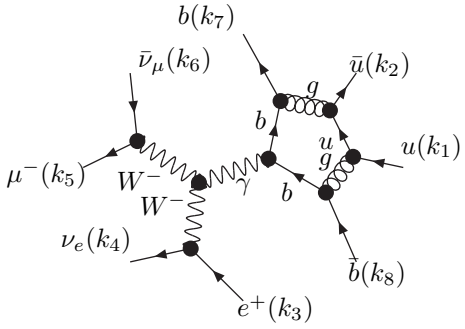


Diagram 387
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk = 3

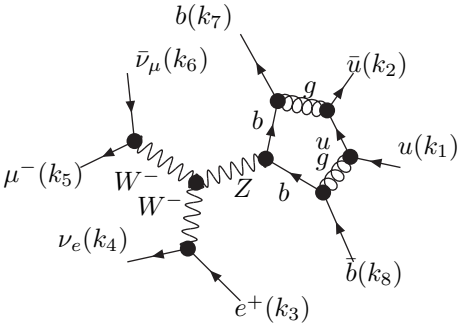


Diagram 388
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk = 3

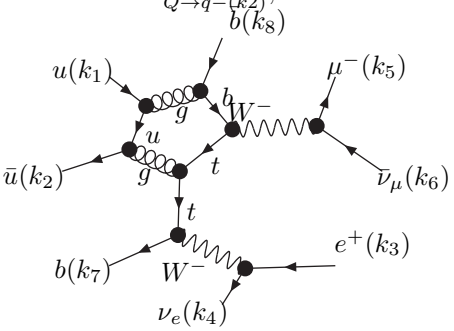


Diagram 395
$S' = S^{\{6\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk = 3

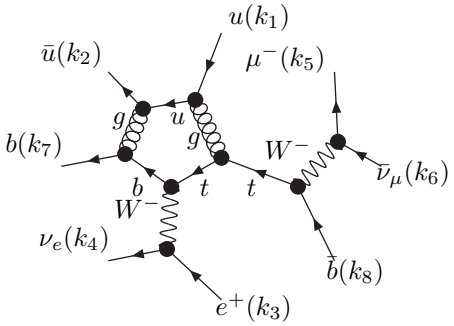


Diagram 400
$S' = S^{\{2\}}$, rk = 3

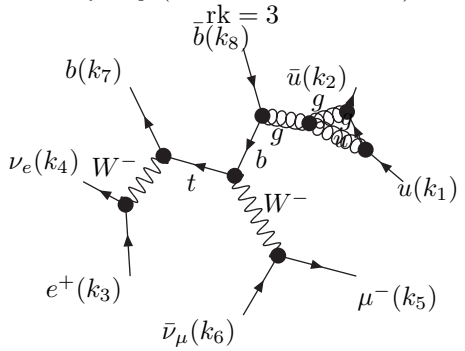


Diagram 403
$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk = 2

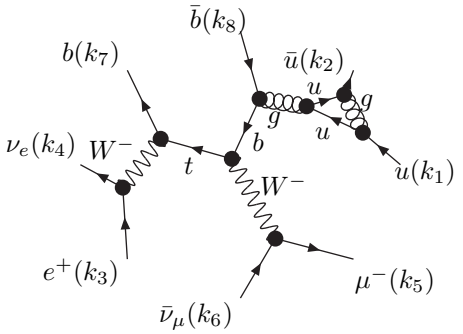


Diagram 404
$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk = 2

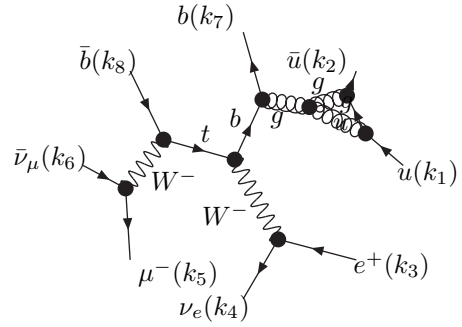


Diagram 407

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

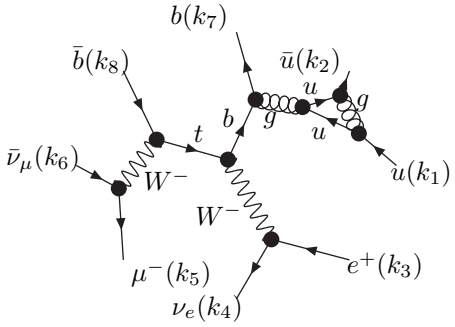


Diagram 408

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

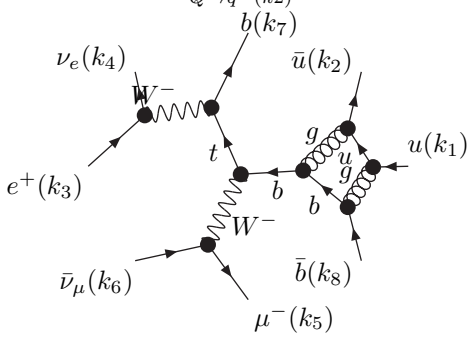


Diagram 411

$S' = S^{\{1,6\}}_{Q\to q-(k2)}$, rk $= 2$

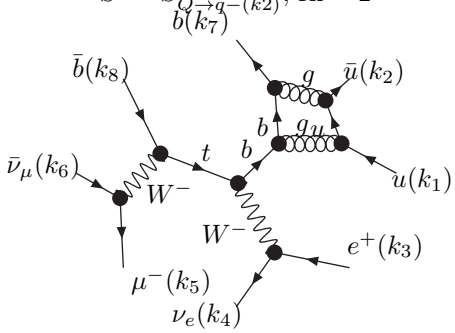


Diagram 416

$S' = S^{\{1,2\}}_{Q\to -q-(-k2)}$, rk $= 2$

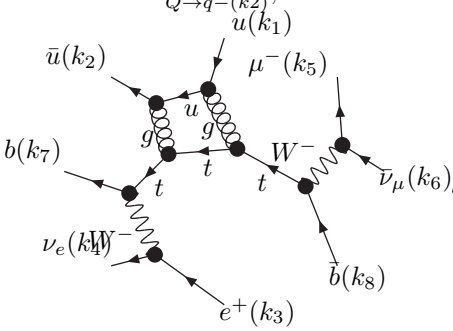


Diagram 420

$S' = S^{\{2,6\}}_{Q\to -q-(-k2)}$, rk $= 2$

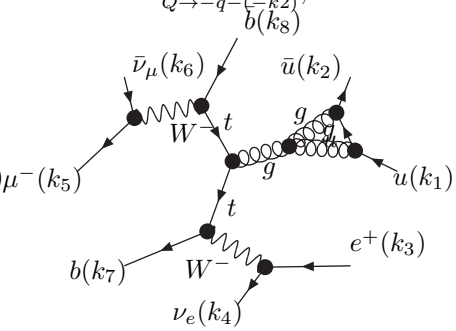


Diagram 423

$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk $= 2$

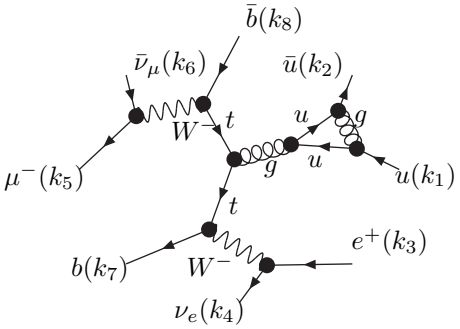


Diagram 424
$S' = S^{\{1,2,6\}}_{Q\to q-(k2)}$, rk = 2

5.12 Group 11 (6-Point)

General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_2 + k_6 + k_5 \tag{56a}$$
$$r_2 = -k_2 \tag{56b}$$
$$r_3 = 0 \tag{56c}$$
$$r_4 = -k_8 \tag{56d}$$
$$r_5 = -k_7 - k_8 \tag{56e}$$
$$r_6 = k_3 - k_2 + k_6 + k_5 + k_4 \tag{56f}$$

$$S = \begin{pmatrix} 0 & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & 0 & 0 & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & 0 & 0 & 0 & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 & 0 \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & 0 & 0 \end{pmatrix} \tag{57}$$

$$S_{1,2} = s_{56} \tag{58a}$$
$$S_{1,3} = s_{56} + s_{781} - s_{3456} - s_{234} + s_{34} \tag{58b}$$
$$S_{1,4} = -s_{567} + s_{56} + s_{1234} - s_{78} - s_{81} + s_{812} + s_{781} - s_{3456} + s_{34} - s_{234} - s_{12} \tag{58c}$$
$$S_{1,5} = s_{1234} + s_{34} - s_{234} - s_{12} \tag{58d}$$
$$S_{1,6} = s_{34} \tag{58e}$$
$$S_{2,4} = -s_{81} + s_{812} - s_{12} \tag{58f}$$
$$S_{2,5} = s_{78} - s_{781} + s_{3456} - s_{12} \tag{58g}$$
$$S_{2,6} = s_{3456} \tag{58h}$$
$$S_{3,5} = s_{78} \tag{58i}$$
$$S_{3,6} = s_{781} \tag{58j}$$

$$S_{4,6} = -s_{81} + s_{781} - s_{78} \tag{58k}$$

5.12.1 Diagrams (12)

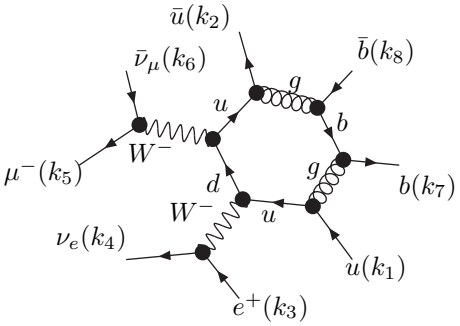


Diagram 351
$S' = S_{Q\to -q-(-k8)}$, rk = 4

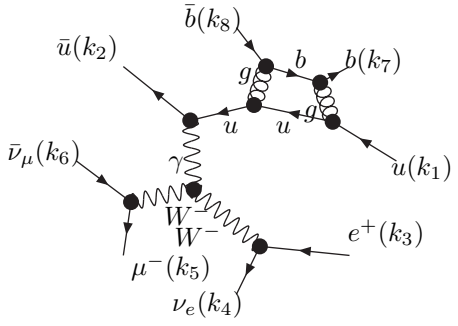


Diagram 377
$S' = S^{\{1,2\}}_{Q\to q-(k8+k7)}$, rk = 2

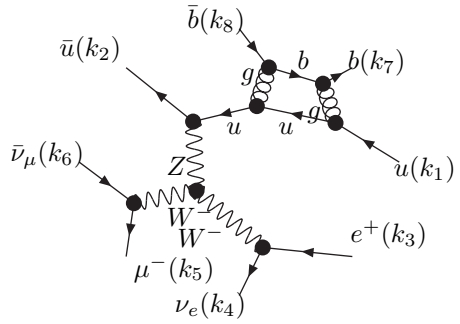


Diagram 378
$S' = S^{\{1,2\}}_{Q\to q-(k8+k7)}$, rk = 2

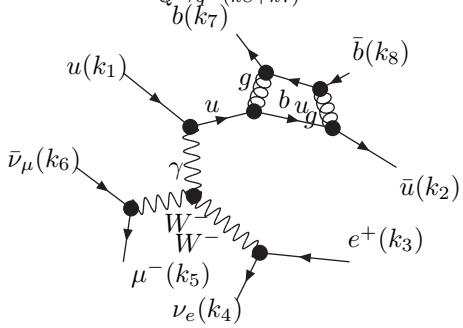


Diagram 383
$S' = S^{\{1,6\}}_{Q\to -q}$, rk = 2

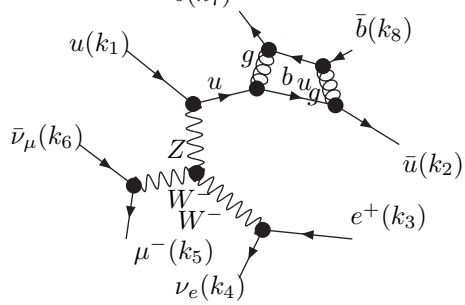


Diagram 384
$S' = S^{\{1,6\}}_{Q\to -q}$, rk = 2

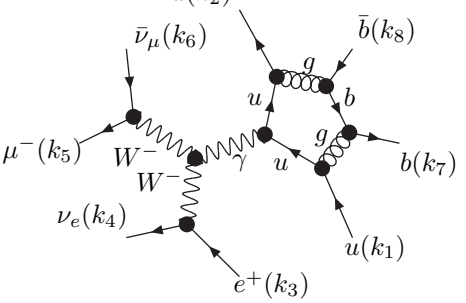


Diagram 389
$S' = S^{\{1\}}_{Q\to -q-(-k8)}$, rk = 3

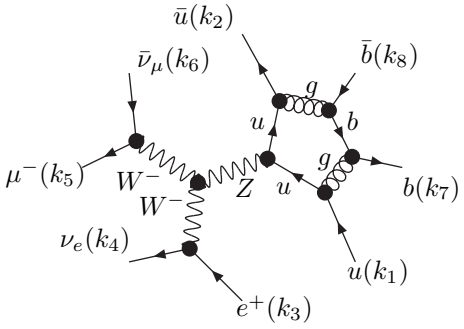


Diagram 390

$S' = S^{\{1\}}_{Q\to -q-(-k8)}$, rk $= 3$

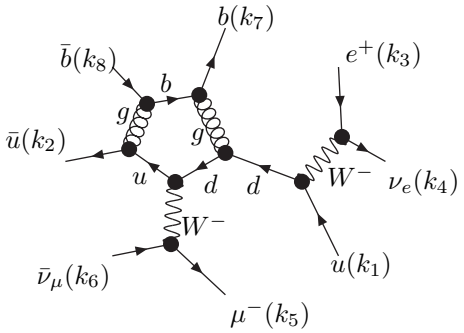


Diagram 394

$S' = S^{\{6\}}_{Q\to -q}$, rk $= 3$

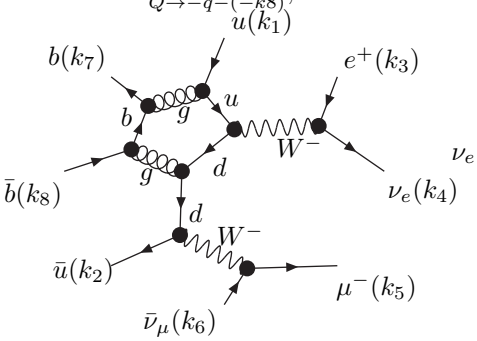


Diagram 397

$S' = S^{\{2\}}_{Q\to q-(k8+k7)}$, rk $= 3$

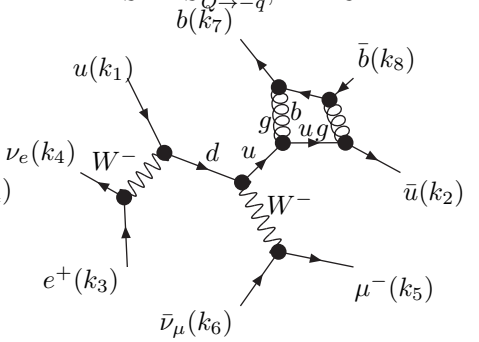


Diagram 410

$S' = S^{\{1,6\}}_{Q\to -q}$, rk $= 2$

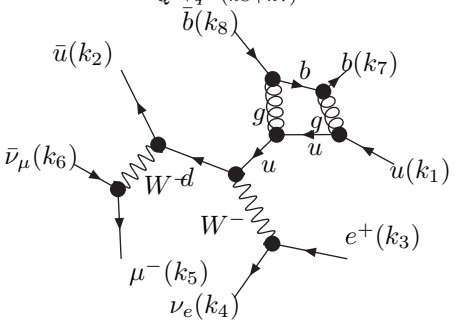


Diagram 413

$S' = S^{\{1,2\}}_{Q\to q-(k8+k7)}$, rk $= 2$

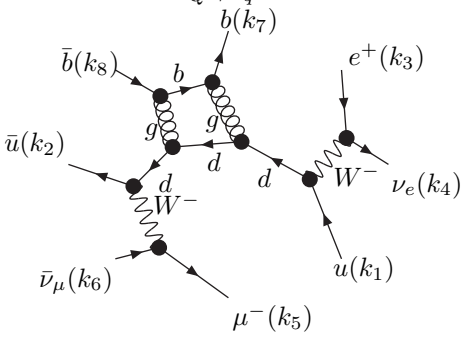


Diagram 417

$S' = S^{\{2,6\}}_{Q\to -q-(-k8)}$, rk $= 2$

5.13 Group 12 (6-Point)

General Information

The maximum effective rank in this group is 4.

$$r_1 = -k_8 - k_5 - k_6, \quad m_1 = m_t, \quad \Gamma_1 = \Gamma_t \tag{59a}$$

$$r_2 = -k_8 \tag{59b}$$

$$r_3 = 0 \tag{59c}$$

$$r_4 = -k_2 \tag{59d}$$

$$r_5 = -k_3 - k_7 - k_6 - k_5 - k_4 - k_8 \tag{59e}$$
$$r_6 = -k_3 - k_6 - k_5 - k_4 - k_8 \tag{59f}$$

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & 0 & 0 & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & 0 & 0 & 0 & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & 0 & 0 & 0 & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & 0 & 0 & 0 \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & 0 & 0 \end{pmatrix} \tag{60}$$

$$S_{1,1} = -2m_t^2 + 2i \cdot m_t \cdot \Gamma_t \tag{61a}$$
$$S_{1,2} = -m_t^2 + s_{56} + i \cdot m_t \cdot \Gamma_t \tag{61b}$$
$$S_{1,3} = -m_t^2 + s_{1234} + s_{56} - s_{567} - s_{78} + i \cdot m_t \cdot \Gamma_t \tag{61c}$$
$$S_{1,4} = -s_{567} - m_t^2 + s_{56} + s_{1234} - s_{78} - s_{81} + s_{812} + s_{781} - s_{3456} + s_{34} - s_{234} - s_{12} + i \cdot m_t \cdot \Gamma_t \tag{61d}$$
$$S_{1,5} = -m_t^2 + s_{56} - s_{567} + s_{812} - s_{3456} + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{61e}$$
$$S_{1,6} = -m_t^2 + s_{34} + i \cdot m_t \cdot \Gamma_t \tag{61f}$$
$$S_{2,4} = -s_{81} + s_{812} - s_{12} \tag{61g}$$
$$S_{2,5} = s_{812} \tag{61h}$$
$$S_{2,6} = s_{3456} \tag{61i}$$
$$S_{3,5} = s_{12} \tag{61j}$$
$$S_{3,6} = -s_{78} - s_{812} + s_{3456} + s_{12} \tag{61k}$$
$$S_{4,6} = -s_{81} + s_{781} - s_{78} \tag{61l}$$

5.13.1 Diagrams (12)

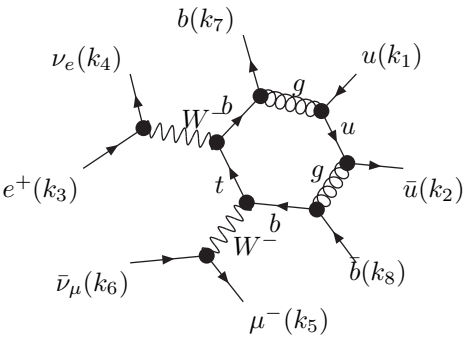


Diagram 349
$S' = S_{Q \to q-(k2)}$, rk = 4

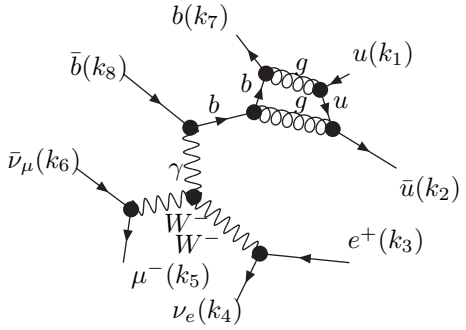


Diagram 369
$S' = S^{\{1,2\}}_{Q \to q-(k2)}$, rk = 2

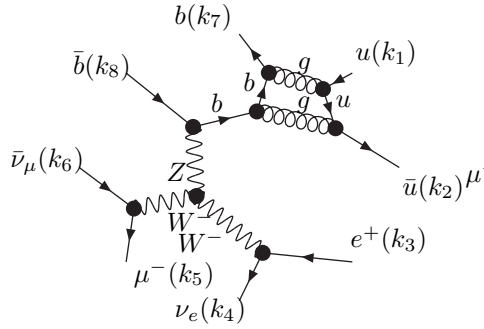


Diagram 370
$S' = S^{\{1,2\}}_{Q\to q-(k2)}$, rk $= 2$

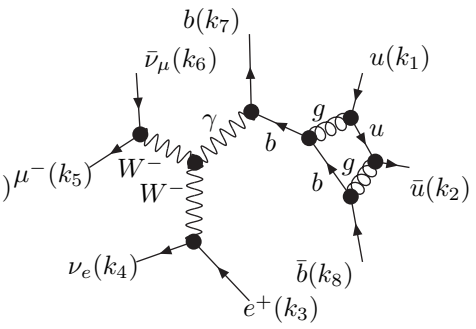


Diagram 375
$S' = S^{\{1,6\}}_{Q\to \bar{q}-q-(-k2)}$, rk $= 2$

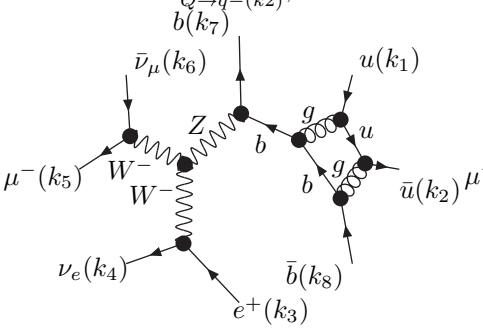


Diagram 376
$S' = S^{\{1,6\}}_{Q\to \bar{q}-q-(-k2)}$, rk $= 2$

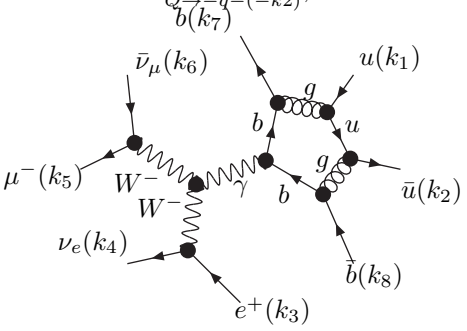


Diagram 385
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk $= 3$

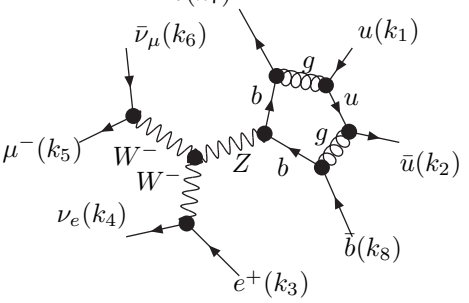


Diagram 386
$S' = S^{\{1\}}_{Q\to q-(k2)}$, rk $= 3$

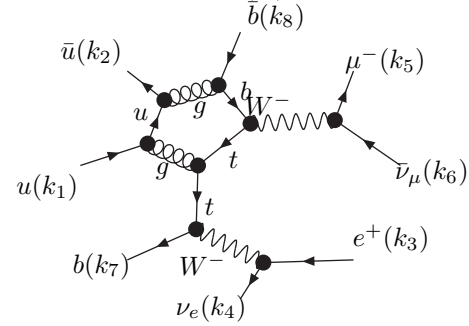


Diagram 396
$S' = S^{\{6\}}$, rk $= 3$

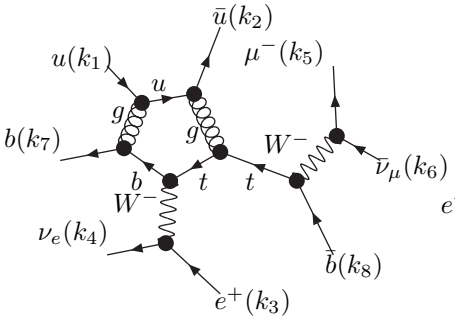


Diagram 399

$S' =$

$S^{\{2\}}_{Q\to -q-(-k3-k7-k6-k5-k4-k8)}$, rk $= 3$

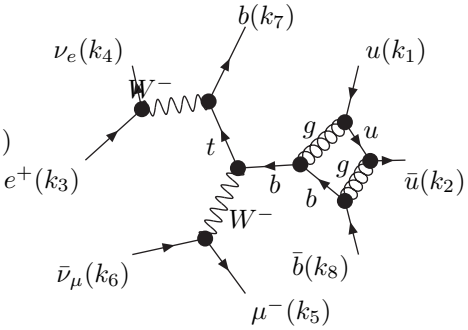


Diagram 412

$S' = S^{\{1,6\}}_{Q\to -q-(-k2)}$, rk $= 2$

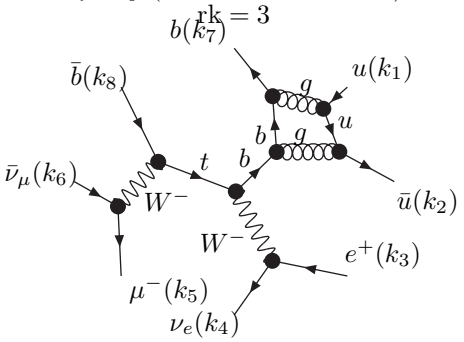


Diagram 415

$S' = S^{\{1,2\}}_{Q\to q-(k2)}$, rk $= 2$

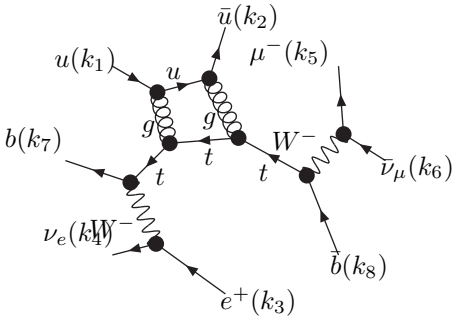


Diagram 419

$S' = S^{\{2,6\}}_{Q\to q-(k2)}$, rk $= 2$

Index of all Loop Diagrams

6 Related Work

If you publish results obtained by using this matrix element code please cite the appropriate papers in the bibliography of this document.

Scientific publications prepared using the present version of GoSam or any modified version of it or any code linking to GoSam or parts of it should make a clear reference to the publications [1, 2].

For graph generation we use QGraf [3]. The Feynman diagrams are further processed with the symbolic manipulation program FORM [4, 5] using the FORM library SPINNEY [6]. The Fortran 90 code is generated using FORM [4, 5] and HAGGIES [7]. For the reduction of the tensor integrals the code uses an implementation of the Laurent series expansion method [9] from the library Ninja [8]. For the reduction of the tensor integrals the code uses the implementation of the OPP method [10, 11] and extensions thereof from the package SAMURAI [12].For the reduction of the tensor integrals, the code uses the package GOLEM95 [13, 14, 15]. The tensor coefficients are obtained using tensorial reconstruction at the integrand level [20].

Please, make sure, you also give credit to the authors of the scalar loop libraries, if you configured the amplitude code such that it calls other libraries than the ones mentioned so far. Depending on your configuration you might use one or more of the following programs for the evaluation of the scalar integrals:

- OneLOop [16],
- QCDLoop [17], which uses FF [18],
- LoopTools [19], which uses FF [18].
- GOLEM95 [14, 13] which uses OneLOop [16] and may be configured such that it uses LoopTools [19, 18].

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