Mainly, we're optimizing a cost function but instead of using gradient descent we will use this algorithm :-

repeat {

for $i = 1$ to m

$c^{(i)}$:- index of closest cluster centroid to the given point $x^{(i)}$

for $k = 1$ to K

μ_k = average of points assigned to cluster k.

}

Cost function for K-means which we will be minimizing using the above algorithm has some parameters :-

$c^{(i)}$ = index of cluster (1 to K) to which the cluster point $x^{(i)}$ is assigned to.

μ_K is the K^{th} cluster centroid

$\mu_{c^{(i)}}$ = cluster centroid to which cluster point $x^{(i)}$ is assigned. eg. $x^{(10)}$ $c^{(10)}$ $\mu_{c^{(10)}}$

cluster point

cluster centroid

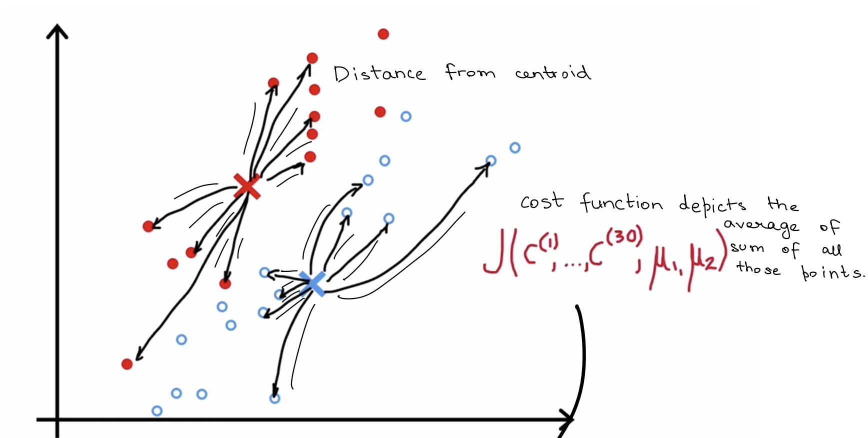
location of the cluster centroid

Using that, cost function $J(c^{(1)},\dots,c^{(m)},\mu_1,\dots,\mu_K)$

$$= \frac{1}{m}\sum_{i=1}^{m} \|x^{(i)} - \mu_{c^{(i)}}\|^2$$

cluster point

location of the training centroid which is close to $x^{(i)}$



$$J(c^{(1)},\dots,c^{(30)},\mu_1,\mu_2)$$

basically it will update $c^{(1)}, c^{(2)}, \dots, c^{(30)}$ or μ_1, μ_2 in order to keep on reducing the cost function J.

This cost function is called distortion function.

The K-means algorithm minimizes the cost function in two steps.

Repeat {

 # Assign points to cluster centroids

for i = 1 to m

$\underline{c^{(i)}}$:= index of cluster centroid closest to $x^{(i)}$ → first we only focus on the first step before moving to second step

 # Move cluster centroids

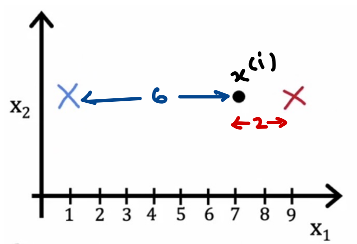
for k = 1 to K

μ_k := average of points in cluster k

}

Just choosing the closest cluster centroid will do half our job.

eg.

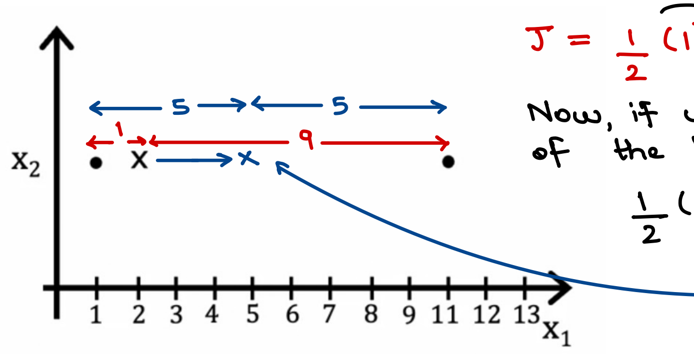


If we choose blue centroid our cost function will be high and choosing red centroid will make it much better.

∴ First it goes through all the training eg. and choose value for $c^{(m)}$ in order to minimize J without changing μ_k.

Secondly, we try to make μ as small as possible.

eg.



$J = \frac{1}{2}\overbrace{(1^2 + 9^2)}^{\text{distance}} = \frac{1}{2}(82) = 41$

Now, if we take average of the points.

$\frac{1}{2}(1 + 11) = 6$

becomes new index of centroid

New $J = \frac{1}{2}(5^2 + 5^2) = 25$ (< 41) which is better.

Since the k-means algorithm is optimizing the cost function J, it will always go down.

In case, it doesn't go down and goes up it is most likely due to a bug.

If cost function ever stops going down it means k-means has converged.