Recap

"Logistics regression is done in two ways"

→ $z = \vec{\omega} \cdot \vec{x} + b$

→ $f(z) = \dfrac{1}{1+e^{-z}}$

$$f_{w,b}(\vec{x}) = g(\vec{\omega} \cdot \vec{x} + b) = \frac{1}{1+e^{-(\vec{\omega} \cdot \vec{x} + b)}}$$

$$= P(y = 1 \mid x; \vec{\omega}, b)$$

A threshold makes it easier to make decisions because we only have to predict categories.

- Is $f_{\vec{\omega},b}(\vec{x}) \geq 0.5$ → Yes: $\hat{y} = 1$ No: $\hat{y} = 0$

$\Rightarrow$ $g(z) \geq 0.5$

$\Rightarrow$ $\vec{\omega} \cdot \vec{x} + b \geq 0$ (refer from graph in prev. notes; $g(0) = 0.5$)

$\Rightarrow$ $\hat{y} = 1$

- Is $f_{\vec{\omega},b}(\vec{x}) < 0.5$ → Yes: $\hat{y} = 0$ No: $\hat{y} = 1$

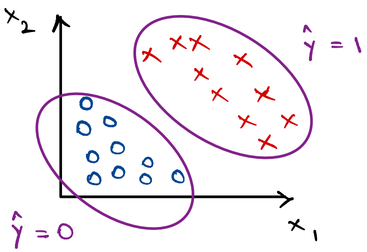
$\Rightarrow$ $g(z) < 0.5$

$\Rightarrow$ $\vec{\omega} \cdot \vec{x} + b < 0.5$

$\Rightarrow$ $\hat{y} = 0$

This threshold is graphically shown using a decision boundary.

Decision Boundary



$$f_{w,b}(\vec{x}) = g(z) = g(w_1 x_1 + w_2 x_2 + b)$$

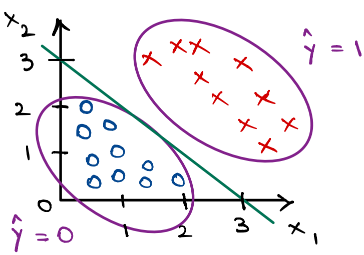
two variables

Let $w_1 = 1$, $w_2 = 1$ and $b = -3$

decision boundary equation

$$z = \vec{w} \cdot \vec{x} + b = 0, \quad z = x_1 + x_2 - 3 = 0$$

0 because it is the deciding factor (less than 0 = $\hat{y} = 0$ and greater than or equal to 0 = $\hat{y} = 1$)



$$x_1 + x_2 - 3 = 0$$

$\Rightarrow x_1 + x_2 = 3$ is the decision boundary line

Non-linear decision boundaries

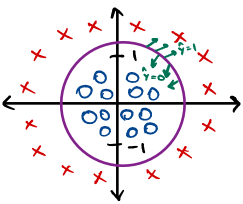
Decision boundaries are not necessarily lines, it can be curves

for eg. let's take a polynomial expression

$$f_{\vec{w},b}(\vec{x}) = g(z) = g(w_1 x_1^2 + w_2 x_2^2 + b)$$

Let $w_1 = 1$, $w_2 = 1$ and $b = -1$

$$z = x_1^2 + x_2^2 - 1 = 0 \quad ; \quad x_1 + x_2 = 1$$



eg. of more complex decision boundaries

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{w_1 x + w_2 x_2 + w_3 x_1^2 + w_4 x_1 x_2 + w_5 x_2^2}_{\text{formed using feature engineering and polynomials of higher order}})$$