this is a better fitting line
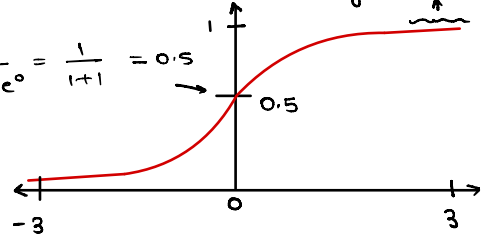
malignant

1

0.7 threshold

0

tumour size x

this line is a sigmoid function

never crosses 1 no matter how big z is.

$g(0) = \dfrac{1}{1+e^0} = \dfrac{1}{1+1} = 0.5$

1

0.5

-3   0   3

SIGMOID FUNCTION

Formula :- $g(z) = \dfrac{1}{1+e^{-z}}$

$\{ 0 < g(z) < 1 \}$

e is a constant ≈ 2.7

also called as logistics function

outputs between 0 and 1

$g(z)$ never crosses 1 because if z gets very large it will equal to 1.

$\Rightarrow \displaystyle\lim_{z \to \infty} \dfrac{1}{1+e^{-z}} = \lim_{z \to \infty} \dfrac{1}{1+\frac{1}{e^z}} = \dfrac{1}{1+\frac{1}{e^\infty}} = \dfrac{1}{1+\frac{1}{\infty}} = \dfrac{1}{1+0} = 1$

$g(z)$ is not less than 0,

$\Rightarrow \displaystyle\lim_{z \to -\infty} \dfrac{1}{1+e^{-z}} = \dfrac{1}{1+e^{-(-\infty)}} = \dfrac{1}{1+e^\infty} = \dfrac{1}{1+\infty} = \dfrac{1}{\infty} = 0$

Defining logistic regression function

Let $z = \vec{w} \cdot \vec{x} + b$ (straight line function)

$$g(z) = \frac{1}{1 + e^{-z}} \quad \Rightarrow \quad f_{w,b}(\vec{x}) = g(\vec{w} \cdot \vec{x} + b) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

Interpretation of output from logistic regression

Suppose $f(x) = 0.7$ then that means probability that tumour is malignant is 70% and not malignant is $1 - P(M) = 1 - 0.7 = 0.3$ (30%).