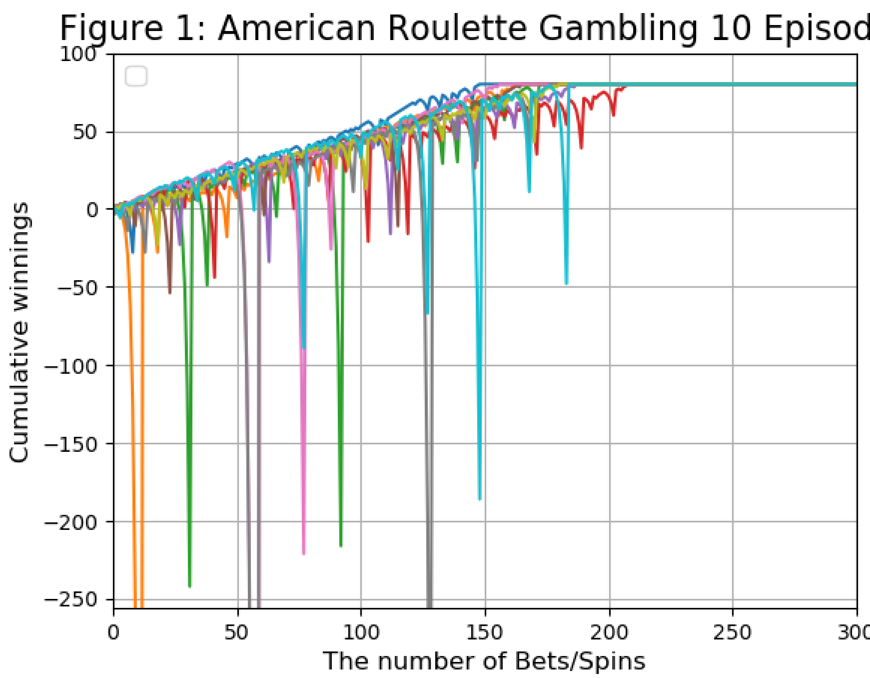
American Roulette: A Statistical Study in Risk, Probability and Observation

Shichao Zhou
szhou401@gatech.edu

1 EXPERIMENT 1

First and foremost, it is essential to clarify that in the simulated Experiment 1 strategy, each episode involves 1000 bet rotations. It is presumed that the target for winnings is set at $80, and there is no upper limit to the bankroll, meaning the betting funds are continually replenished. Based on this foundation, an initial simulation of 10 episodes was conducted, followed by an extended simulation of 1000 episodes. The results are illustrated in the subsequent figures and a detailed analysis is provided below.



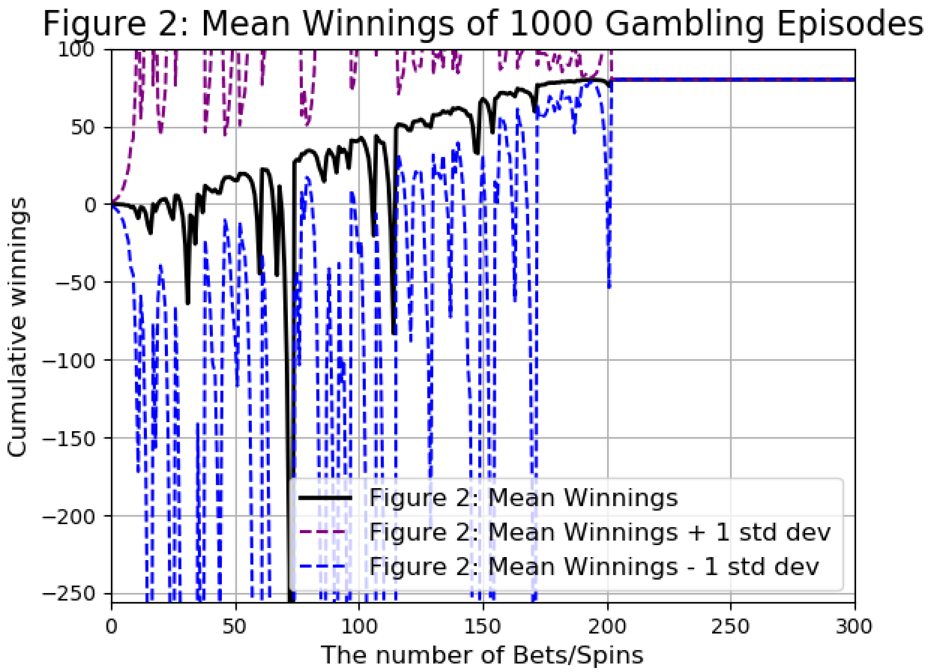


Figure 2: Mean Winnings of 1000 Gambling Episodes
Cumulative winnings
The number of Bets/Spins
Figure 2: Mean Winnings
Figure 2: Mean Winnings + 1 std dev
Figure 2: Mean Winnings - 1 std dev

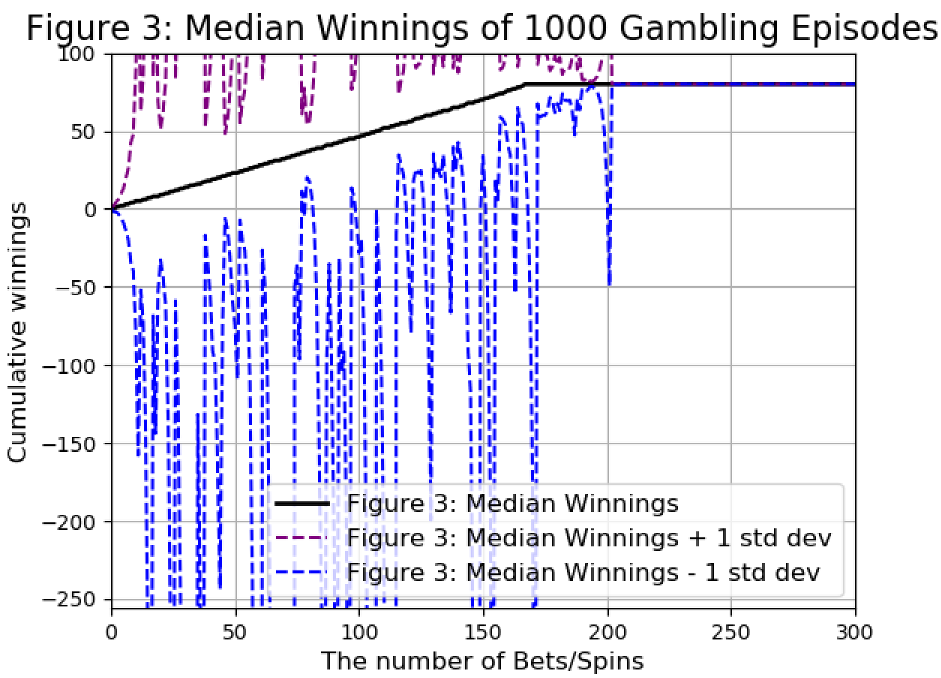


Figure 3: Median Winnings of 1000 Gambling Episodes
Cumulative winnings
The number of Bets/Spins
Figure 3: Median Winnings
Figure 3: Median Winnings + 1 std dev
Figure 3: Median Winnings - 1 std dev

1.1 Answer to Q1

Based on the analysis of the hypotheses and results derived from Experiment one, it is evident that the proposed ideal strategy assures consistent performance for gamblers. Regardless of 10 episodes, or 1000 episodes, the gambler can achieve winnings of $80 in each episode. When the number of spins falls between 139 and 207, the winnings will approach or precisely meet the target of $80. Thus, it implies that the gambler is guaranteed a 100% success rate in obtaining the $80 winning, that is, the probability is 1.

1.2 Answer to Q2

The answer is $80. For an American wheel, the expected probability of winning in a spin is 0.474 if the betting is on black. The goal for a gambler is to win $80 within 1000 spins. One significant premise is the absence of a bankroll constraint for the gambler. And the gambler needs to secure one win in every 12.5 spin (1000/80=12.5). The probability of not winning in a single spin is (1-0.474) and the probability of not winning for 12.5 bets is (1-0.474) ^12.5 = 0.0003254. Rounding off, the gambler is almost certain not to lose. So, the estimated expected value of winnings after 1000 sequential bets is $80.

1.3 Answer to Q3

Yes. Prior to achieving the $80 winning target, the standard deviation line shows significant fluctuations. The reason for this is that as the number of spins increases, there is a risk for the gambler to incur substantial losses. However, once the gambler reaches a profit of $ 80, the gambling is stopped. Therefore, around the 207th spin, the standard deviation starts to stabilize and converge.

2 EXPERIMENT 2

2.1 Answer to Q4

Compared to experiment 1, experiment 2 has the condition about the gambler with only $256 bankrolls. Within 1000 spins and under the condition of only $256 bankrolls, the gambler can only get 636 successes. So, the probability of winning $80 is reduced to 0.636 (636/1000 = 0.636).

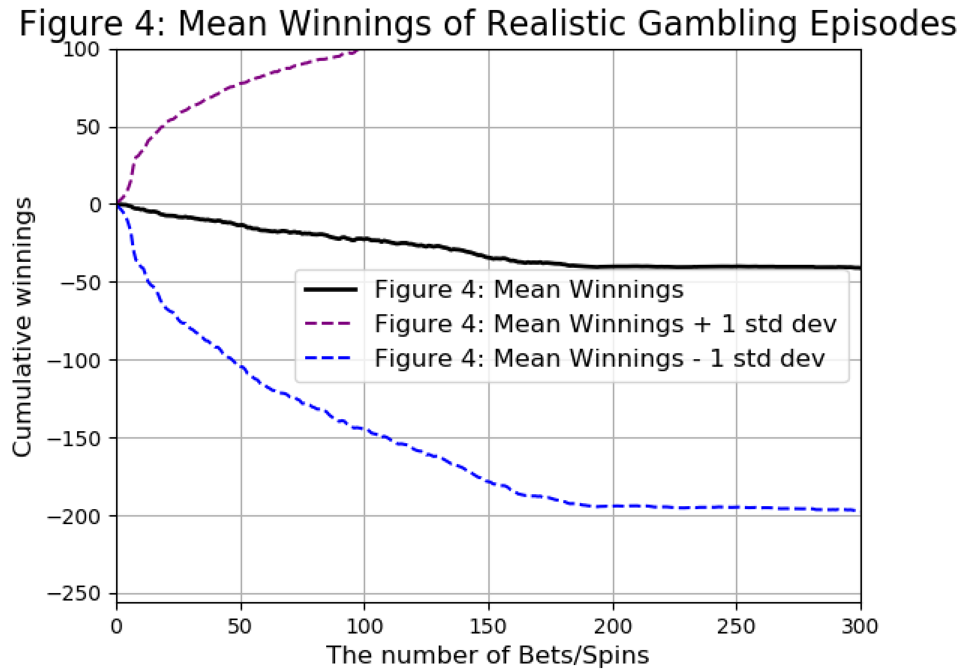


Figure 4: Mean Winnings of Realistic Gambling Episodes
100
50
0
−50
−100
−150
−200
−250
Cumulative winnings
0
50
100
150
200
250
300
The number of Bets/Spins
Figure 4: Mean Winnings
Figure 4: Mean Winnings + 1 std dev
Figure 4: Mean Winnings - 1 std dev

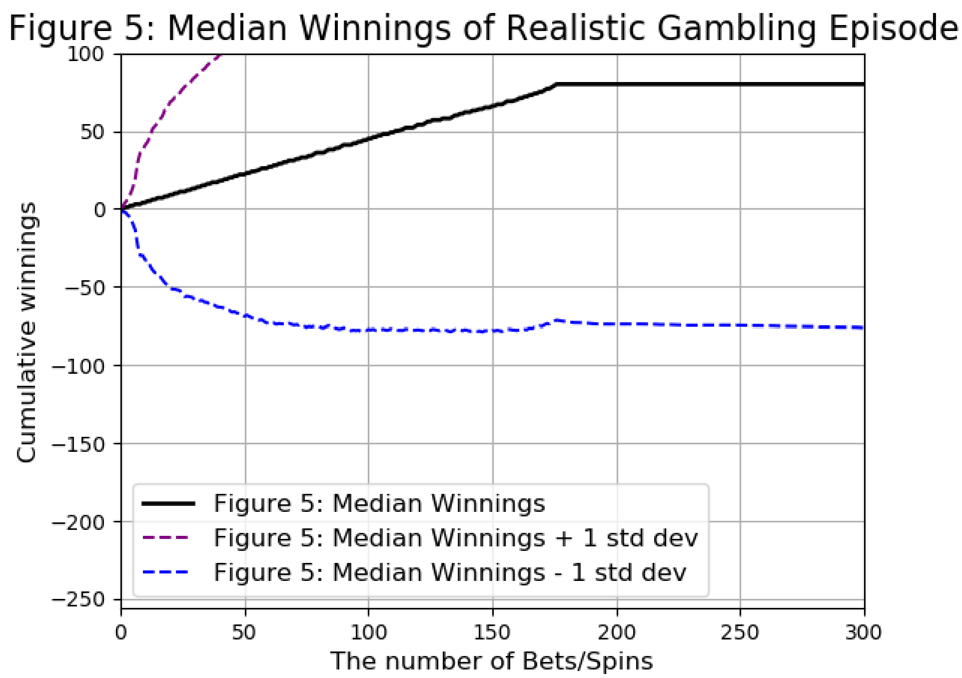


Figure 5: Median Winnings of Realistic Gambling Episode
100
50
0
−50
−100
−150
−200
−250
Cumulative winnings
0
50
100
150
200
250
300
The number of Bets/Spins
Figure 5: Median Winnings
Figure 5: Median Winnings + 1 std dev
Figure 5: Median Winnings - 1 std dev

2.2 Answer to Q5

Based on the condition mentioned above, the probability of winning $80 in 1000 sequential bets is 0.636. And the probability of losing $256 is 0.364. Therefore, the expected value of winning after 1000 sequential bets is -$42.30(0.636 * 80 -0.364 * 256 = - 42.30).

2.3 Answer to Q6

Yes. The divergence arises from 36.4% of simulations converging to -$256, while 63.6% gravitate towards $80. So, the upper standard deviation reaches a maximum and the lower standard deviation reaches a minimum and then stabilizes after 170 spins.

3 BENEFITS OF USING EXPECTED VALUES

3.1 Question 7

Simply using the result of one specific random episode is not informative because Expected value can help to address uncertainty on various outcomes by factoring in their respective probabilities, offering a comprehensive understanding of uncertain situations.

4 REFERENCES

1. Wikimedia Foundation. (2023, July 2). Expected value. Wikipedia. https://en.wikipedia.org/wiki/Expected_value
2. Expected values (EV). Default. (n.d.). https://kfknowledgebank.kaplan.co.uk/expected-values-(ev)-