Assignment 4

Computational Intelligence, SS2017

Team Members		
Last name	First name	Matriculation Number
Reeh	Lucas	00630128

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1 Maximum Likelihood Estimation

1.2 Maximum Likelihood Estimation of Model Parameters

1. Scenario 2 distribution

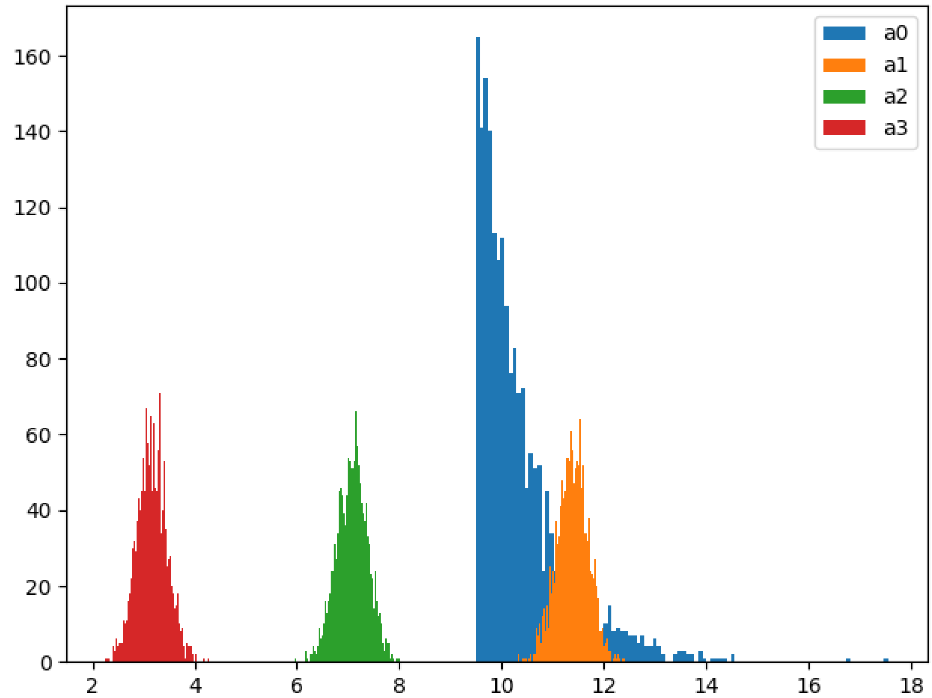

Figure 1: Histograms for Scenario 2

Discussion

Data from anchor 0 “seems” to be **exponentially** distributed whereas the others are look like a normal distribution.

2. Maximum Likelihood for exponential distribution

$$\text{Case II Exponential} : p(r_i|\mathbf{p}) = \begin{cases} \lambda_i e^{-\lambda_i(r_i - d_i(\mathbf{p}))}, & r_i \geq d_i(\mathbf{p}) \\ 0 & \text{else} \end{cases}$$

and

$$\hat{\mathbf{p}}_{ML} = \arg\max_{\mathbf{p}} p(\mathbf{r}|\mathbf{p}) = \arg\max_{\mathbf{p}} \prod_{i=1}^{N_A} p(r_i|\mathbf{p})$$

We are using log-Likelihood for easier derivation. This is possible since we only want the max for the arguments and not any probability values themselfs [1, p. 9].

$$
\begin{aligned}
\lambda_{ML} &= \arg\max_{\lambda} L(r|\lambda) \\
L(R|\lambda) &= \sum_{i=1}^{N} \ln(Exp(r_i|\lambda)) \quad \text{ln helps with } e \text{ in } Exp \\
\frac{\partial L(r|\lambda)}{\partial \lambda} &\overset{!}{=} 0 \\
&= \frac{\partial}{\partial \lambda} \sum_{i=1}^{N} \ln(\lambda_i e^{-\lambda(r_i - d_i(\mathbf{p}))}) \\
&= \frac{N}{\lambda} - \left(\sum_{i=1}^{N} (r_i - d_i(\mathbf{p})) \right) \\
\implies \lambda &= \frac{N}{\sum_{i=1}^{N} (r_i - d_i(\mathbf{p}))}
\end{aligned}
$$

3. Parameter Estimation

Scenario 1

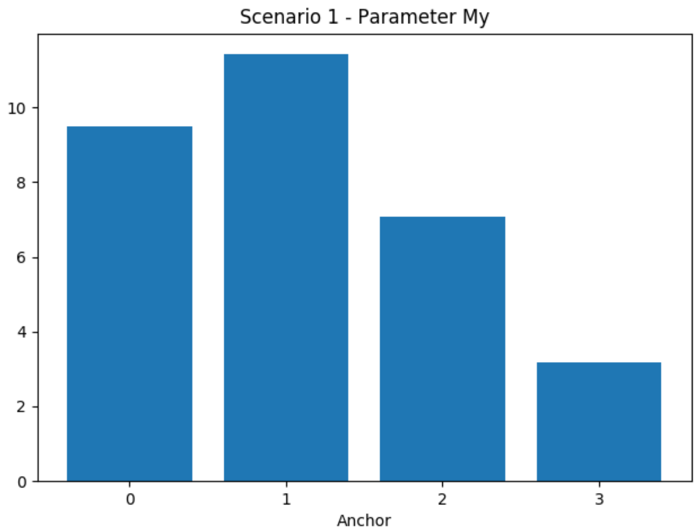

Figure 2: Parameter Estimation Scenario 1 - Parameter μ

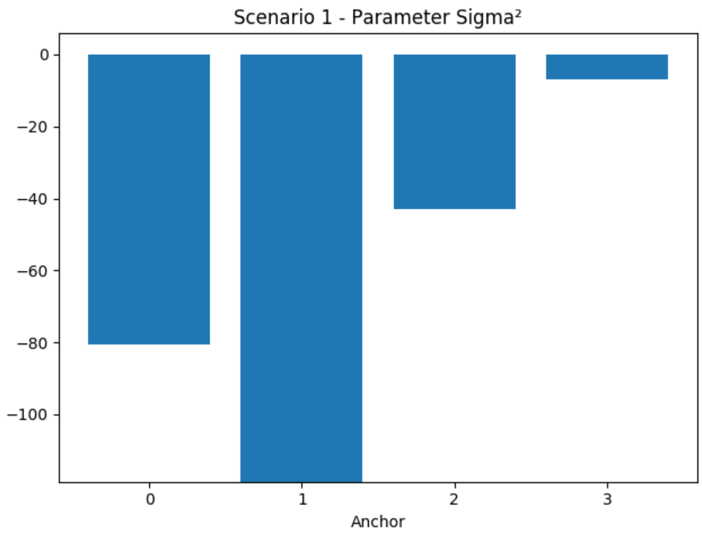

Figure 3: Parameter Estimation Scenario 1 - Parameter σ^2

Parameter	Anchor 1	Anchor 2	Anchor 3	Anchor 4
μ	9.48795736	11.40808013	7.08071941	3.16140767
σ^2	-80.53337746	-118.73621222	-43.05586795	-6.83309077

Scenario 2

Parameter	Anchor 1	Anchor 2	Anchor 3	Anchor 4
μ	–	11.39544817	7.0798979	3.16310352
σ^2	–	-118.46079088	-43.04505635	-6.84212036
λ	$-3.82138765e^{-16}$	–	–	–

Scenario 3

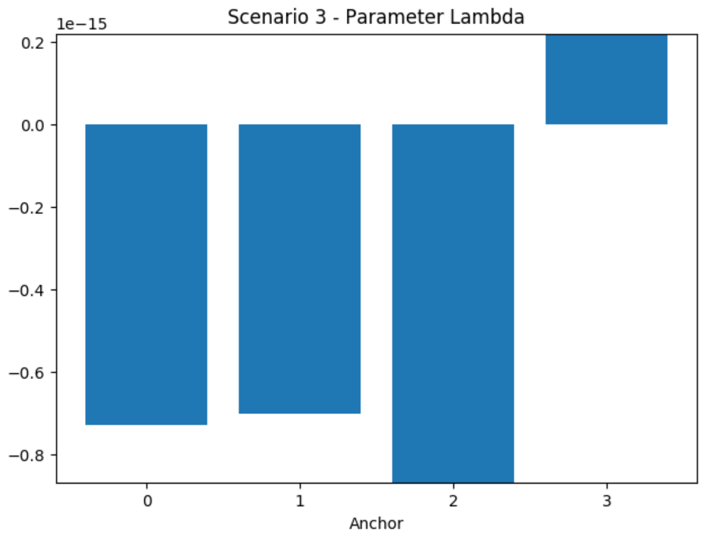

Figure 4: Parameter Estimation Scenario 3 - Parameter λ

Parameter	Anchor 1	Anchor 2	Anchor 3	Anchor 4
λ	$-7.27418126e^{-16}$	$-7.00772773e^{-16}$	$-8.68638494e^{-16}$	$2.21156427e^{-16}$

1.3 Least-Squares Estimation of the Position

1. $\hat{\mathbf{p}}_{ML} \stackrel{?}{=} \hat{\mathbf{p}}_{LS}$

$$\arg\max_{\mathbf{p}} \prod_{i=1}^{N_A} p(r_i|\mathbf{p}) \stackrel{?}{=} \arg\min_{\mathbf{p}} \sum_{i=1}^{N_A} (r_i - d_i(\mathbf{p}))^2$$

   Again using log-Likelihood will help. Product will become sum[1, p. 9] ($ln[\prod(x)] \rightarrow \sum[ln(x)]$).

2. code is still bugy.

1.4 Numerical Maximum-Likelihood Estimation of the Position

???

References

[1] Franz Pernkopf. *Computational Intelligence: Teil 2 (Vorlesungsmitschrift)*. [Online; Stand 29. Juni 2016]. 2016.