

# Assignment 1

Computational Intelligence, SS2017

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# 1 Linear Regression

## 1.1 Derivation of Regularized Linear Regression

## 1.2 Linear Regression with polynomial features

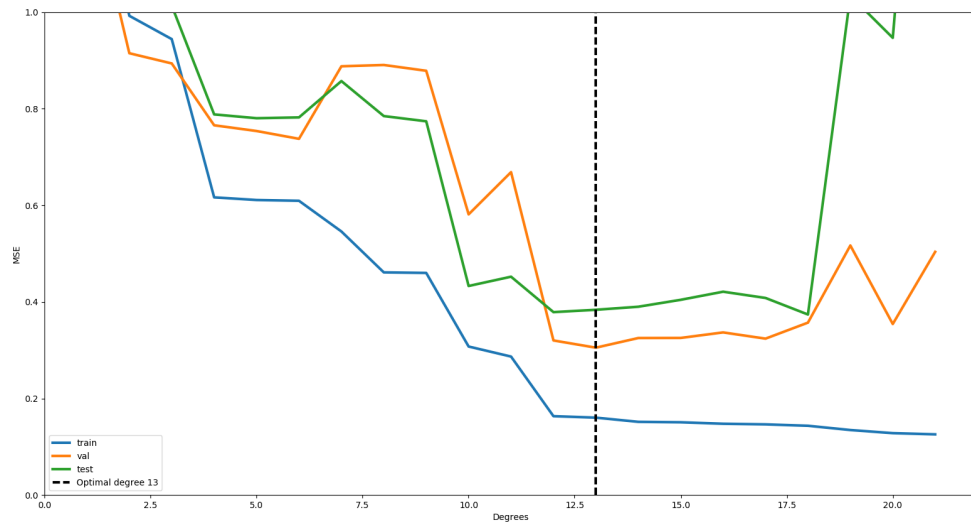


Figure 1: Training, validation and testing errors

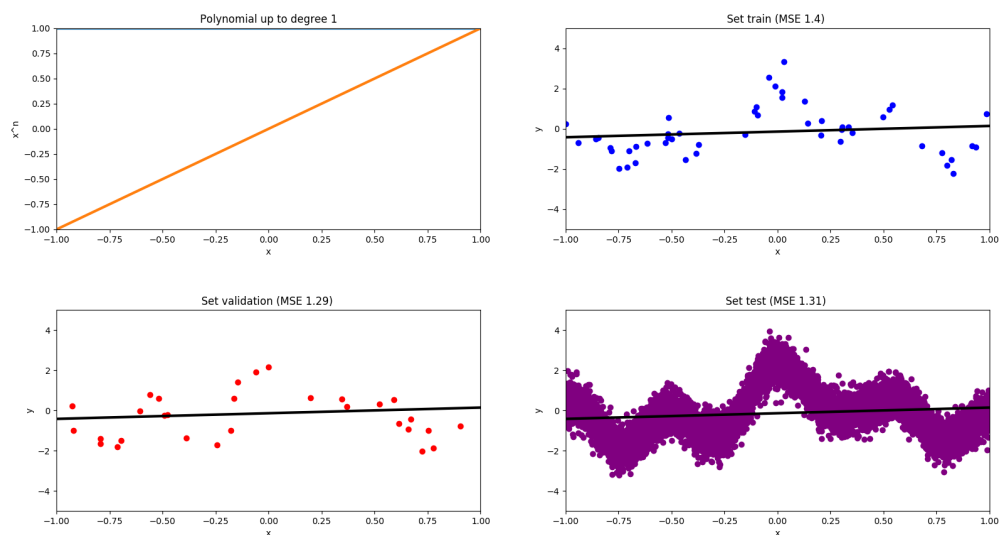


Figure 2: Linear Regression (Polynomial, Degree 1)

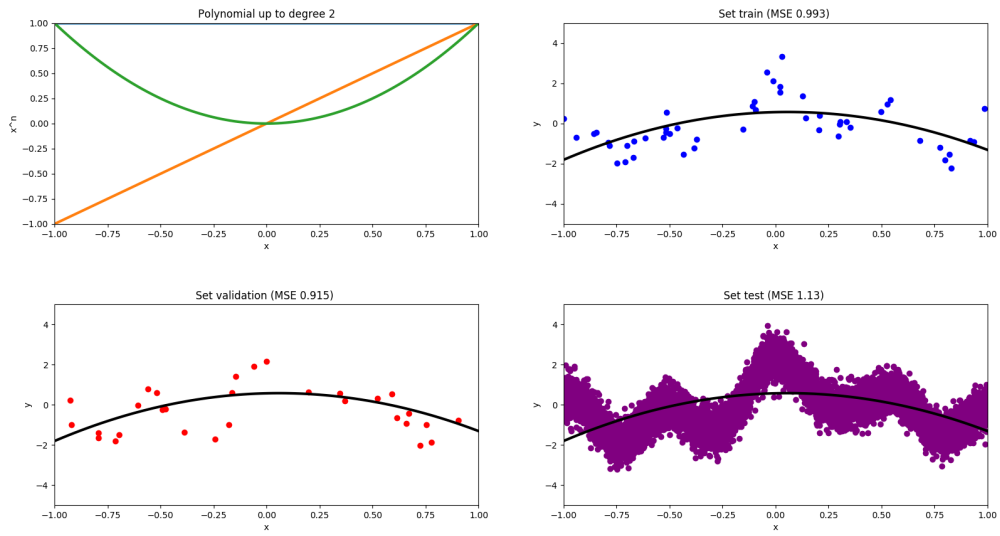


Figure 3: Linear Regression (Polynomial, Degree 2)

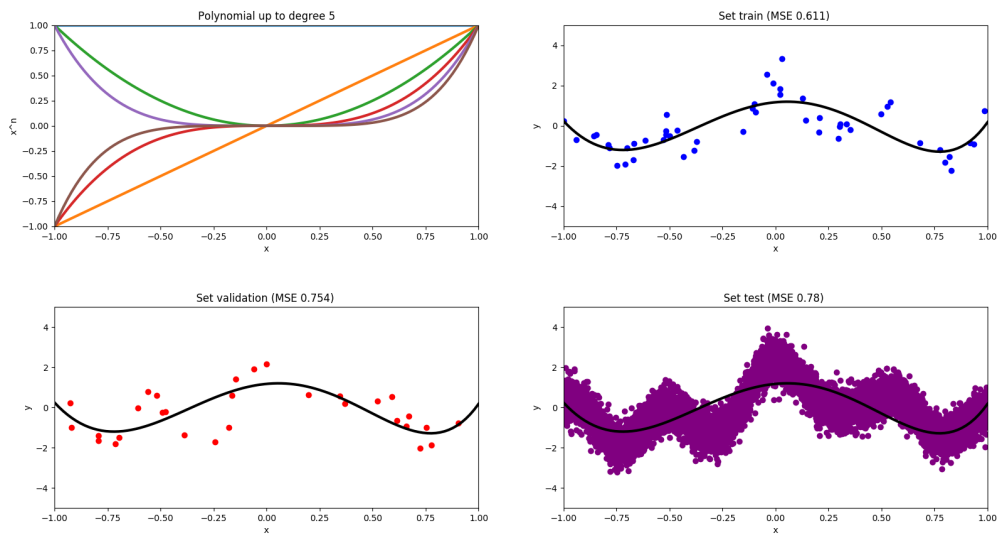


Figure 4: Linear Regression (Polynomial, Degree 5)

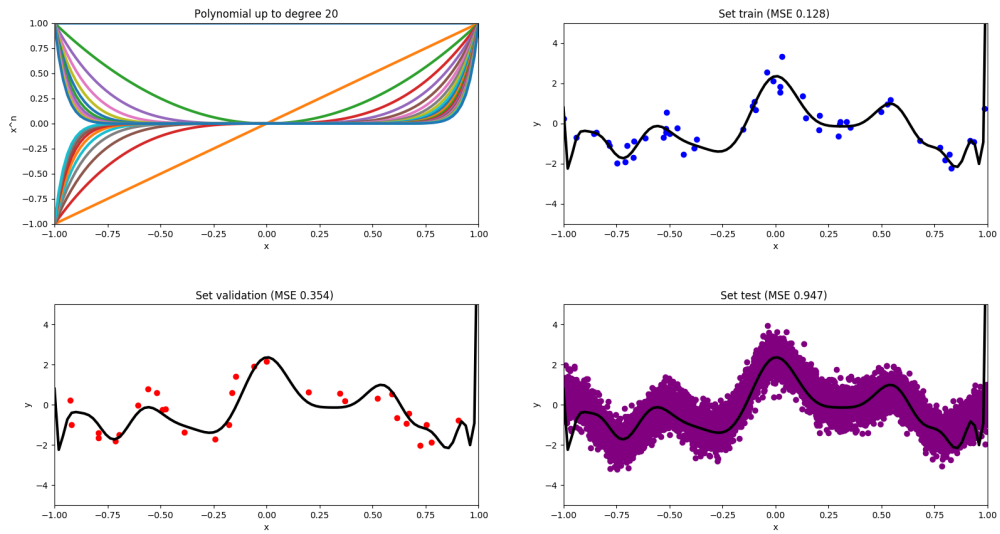


Figure 5: Linear Regression (Polynomial, Degree 20)

- Lowest training error when using degree 21

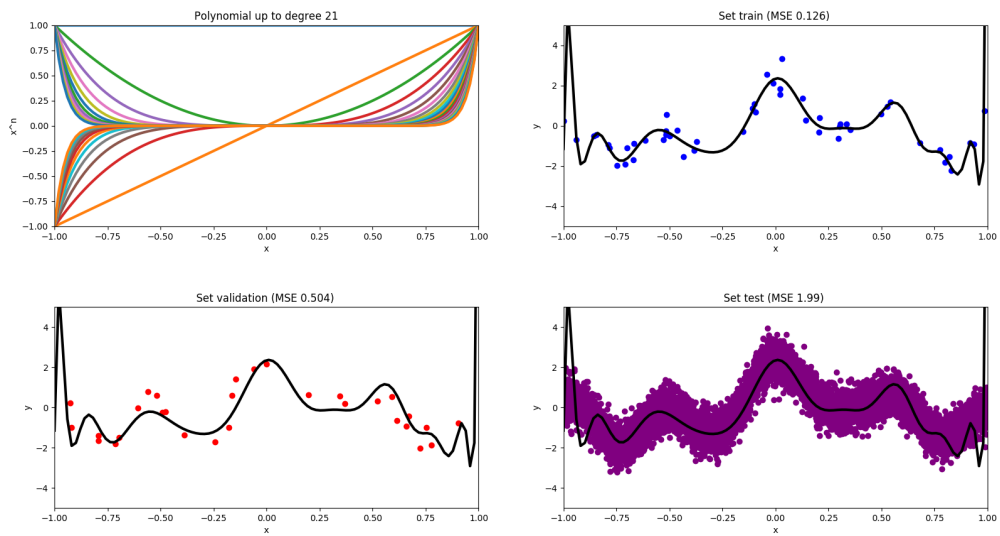


Figure 6: Linear Regression (Polynomial, Degree 21)

- Lowest validation error occurs when using degree 13

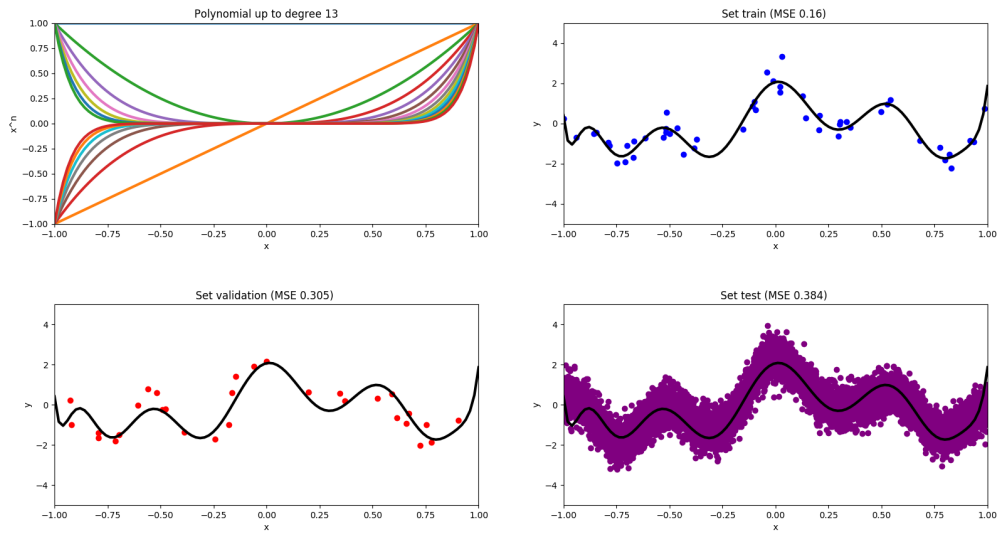


Figure 7: Linear Regression (Polynomial, Degree 13)

### • Discussion

Validation sets help to estimate performance of algorithms used for predictions and also to select a hypothesis (lowes error on set data). According to the error in the test set no over-fitting occured up to a degree of 13 (but would on higher degrees as can clearly be seen in Figure for degree 21, outliers and lesser data).

## 1.3 Linear Regression with radial basis functions

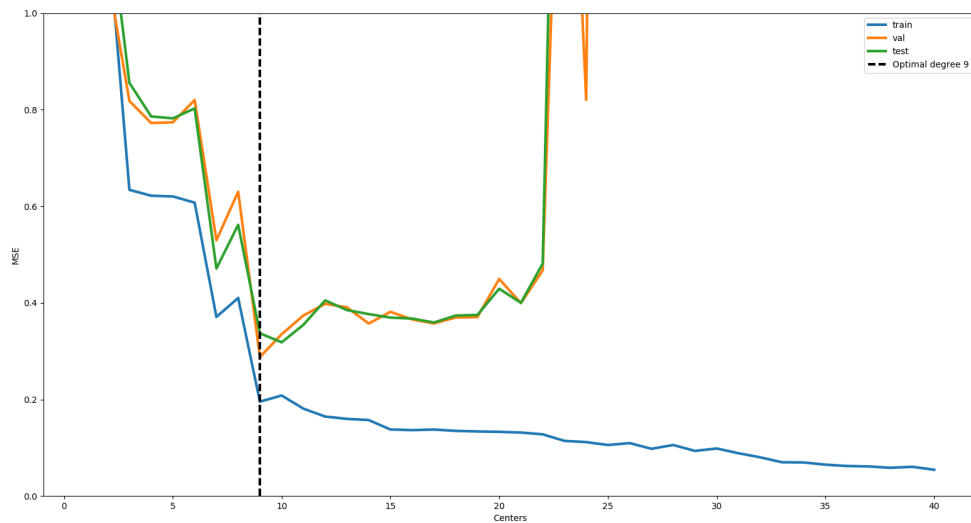


Figure 8: Training, validation and testing errors

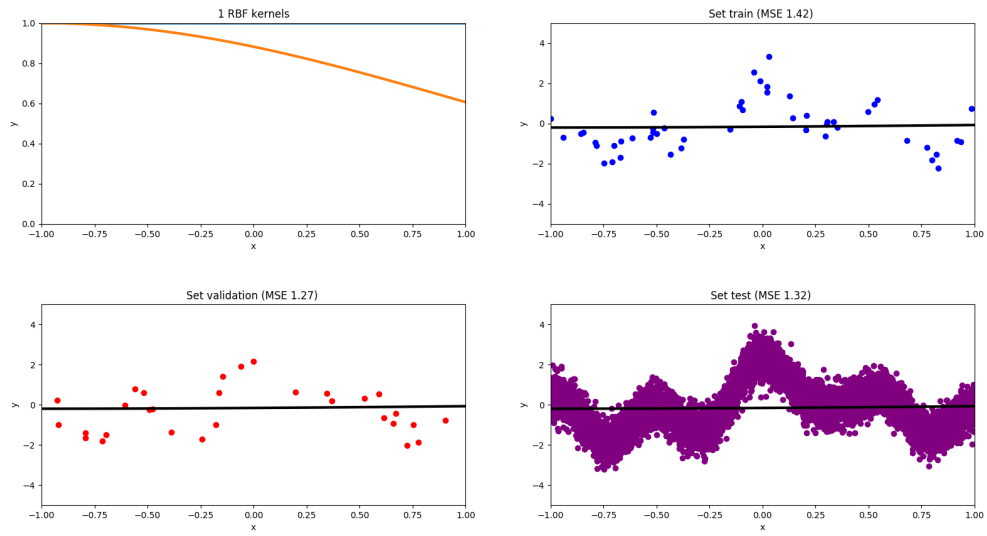


Figure 9: Linear Regression (Bias, Center 1)

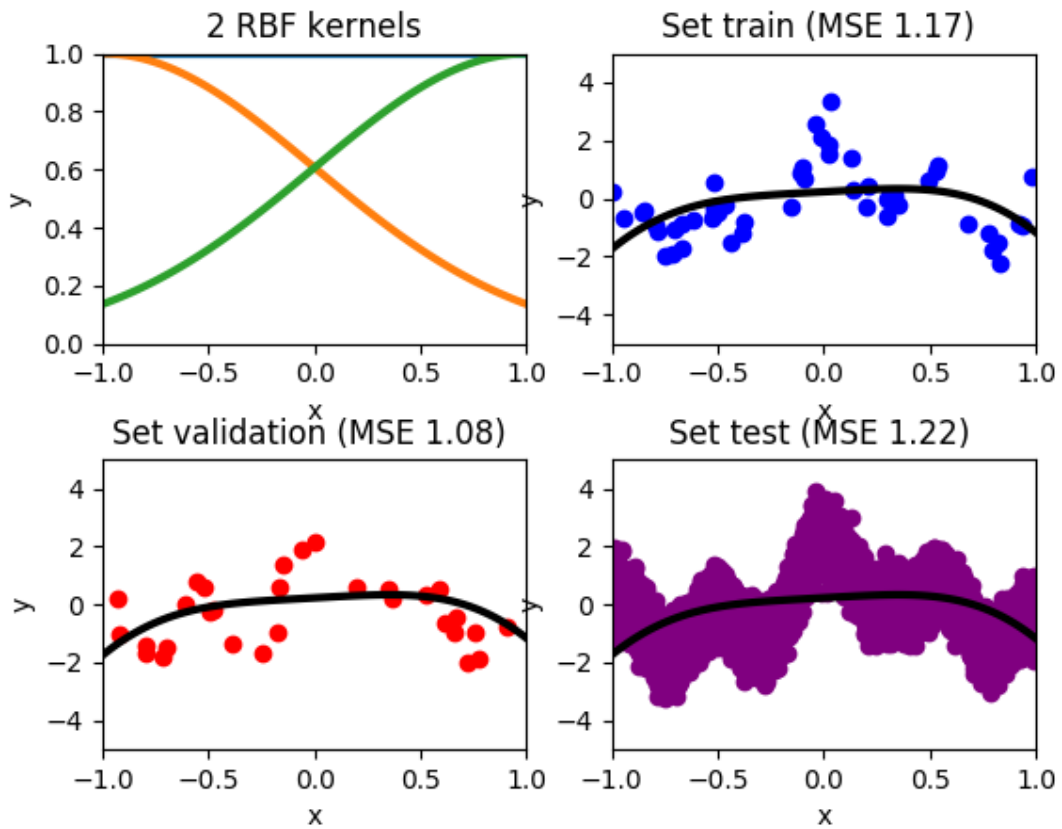


Figure 10: Linear Regression (Bias, Center 2)



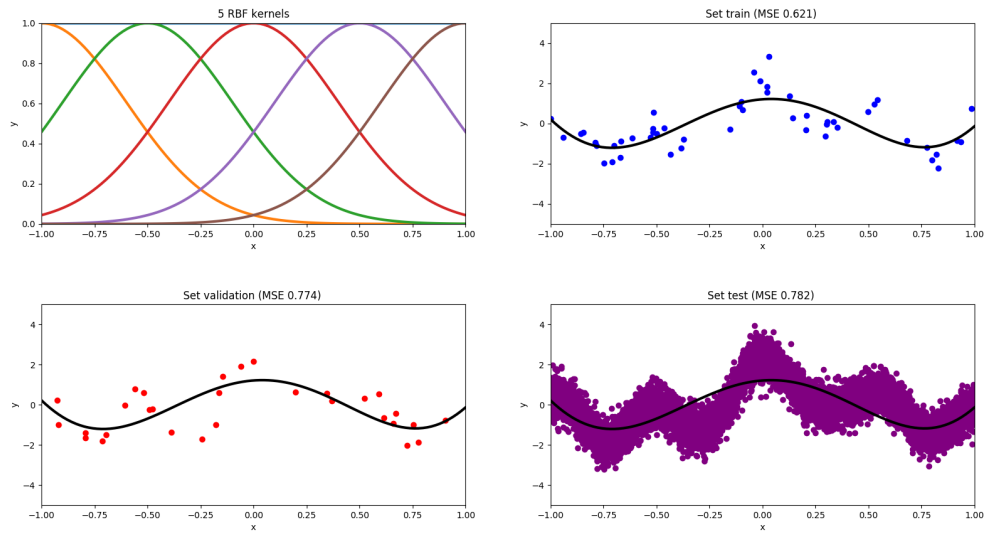


Figure 11: Linear Regression (Bias, Center 5)

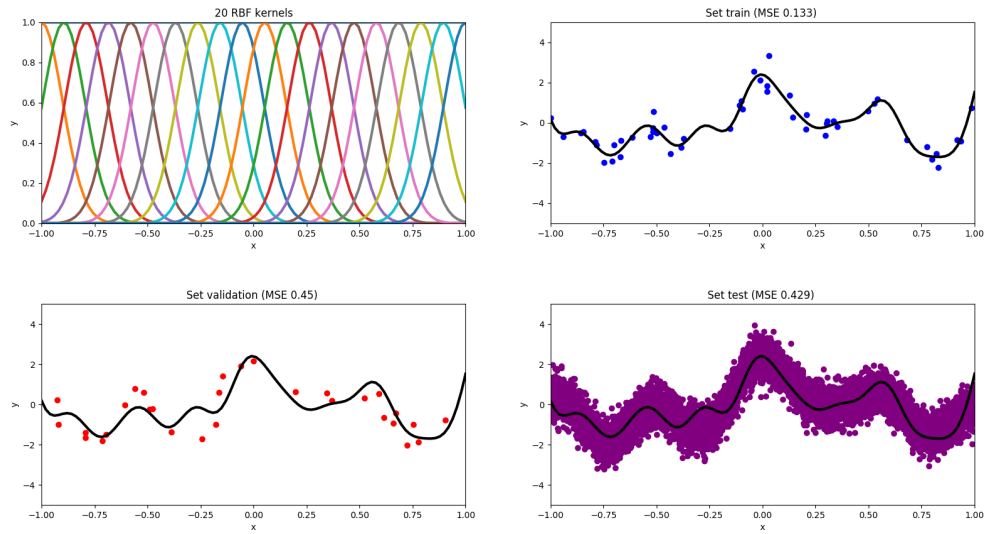


Figure 12: Linear Regression (Bias, Center 20)

- Lowest training error when using center 40

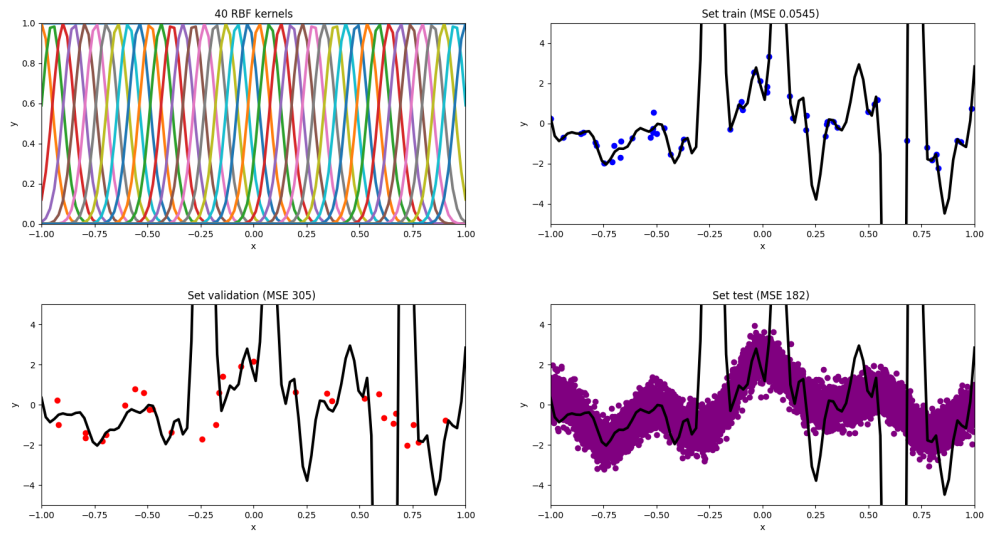


Figure 13: Linear Regression (Bias, Center 40)

- Lowest validation error occurs when using center 9

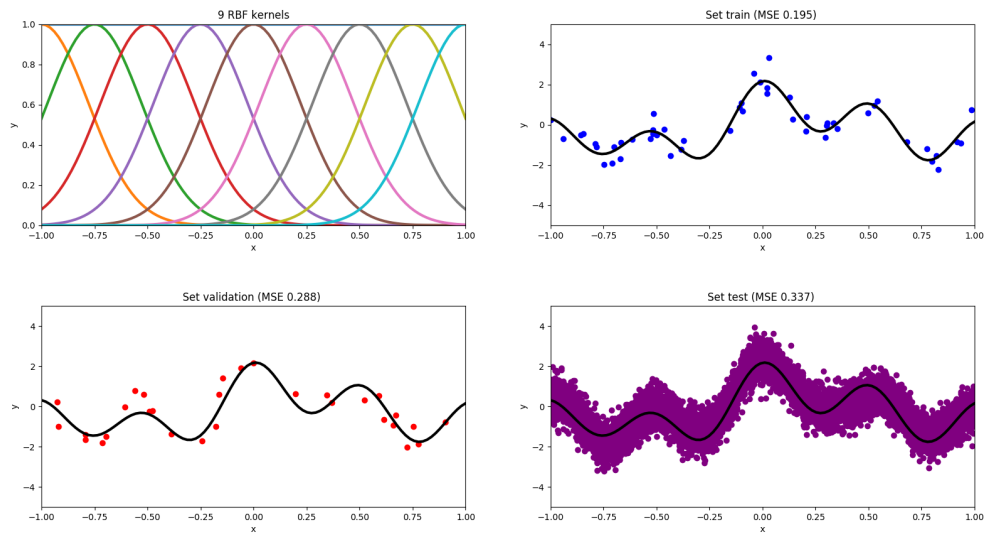


Figure 14: Linear Regression (Polynomial, Degree 9)

## • Discussion

Bias function is better because it fits natural phenomenon better. Overfitting occurs very early on parameter center 10.

## 2 Logistic Regression

### 2.1 Derivation of Gradient

### 2.2 Logistic Regression training with gradient descent and `scipy.optimize`

#### 2.2.1 Gradient descent

##### 1. `check_gradient` explanation

The function check whether the regression functions are really converging at a certain rate. To avoid divergence ;)

##### 2. `gradient descent` degree $l = 1$ , 20 and 2000 iterations, learning rate $\eta = 1$

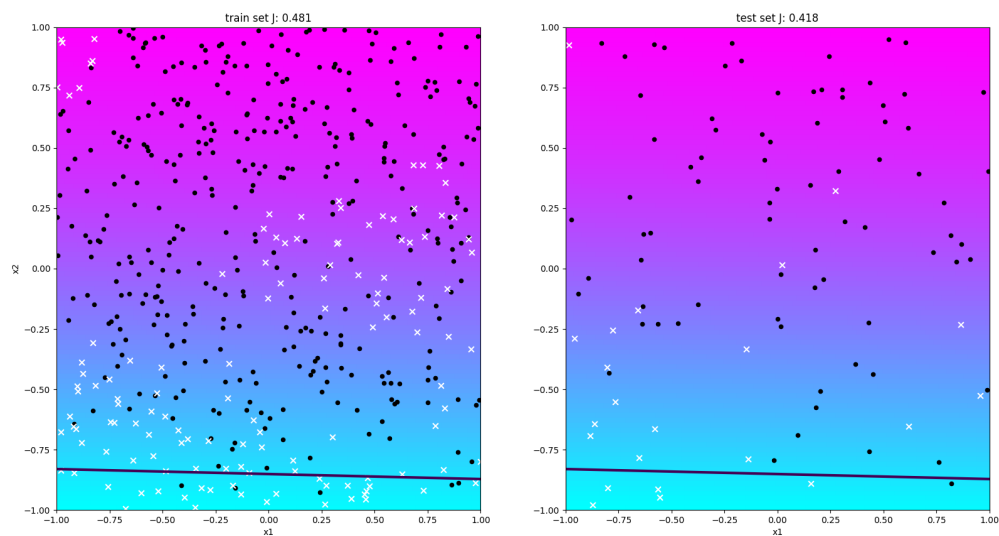


Figure 15: Logistic Regression ( $\eta = 1$ ,  $l = 1$ , 20 iterations)

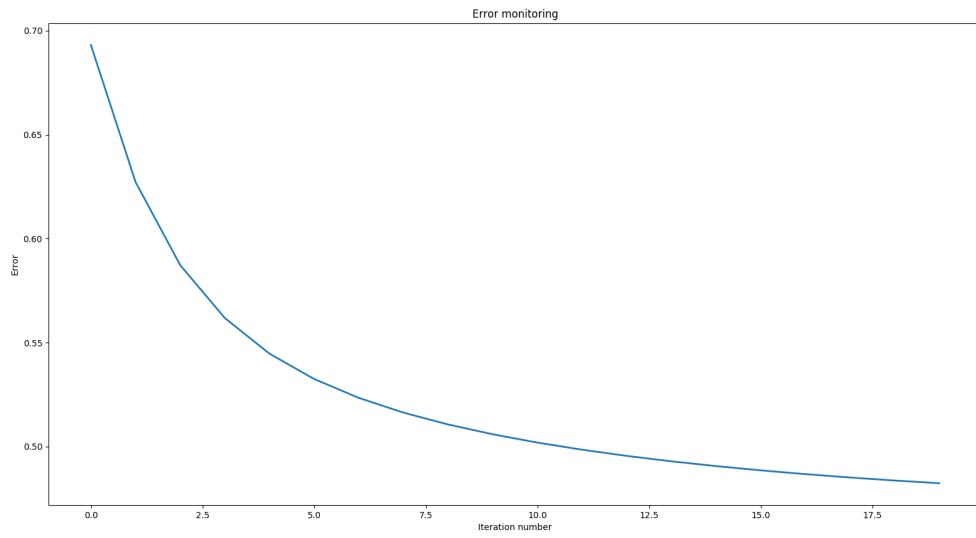


Figure 16: Logistic Regression Errors ( $\eta = 1$ ,  $l = 1$ , 20 iterations)

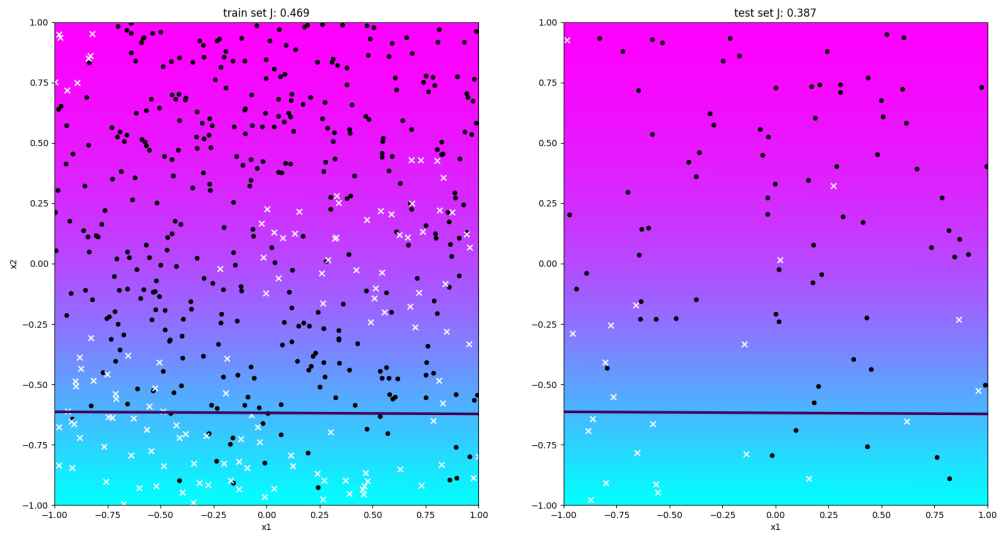


Figure 17: Logistic Regression ( $\eta = 1$ ,  $l = 1$ , 2000 iterations)

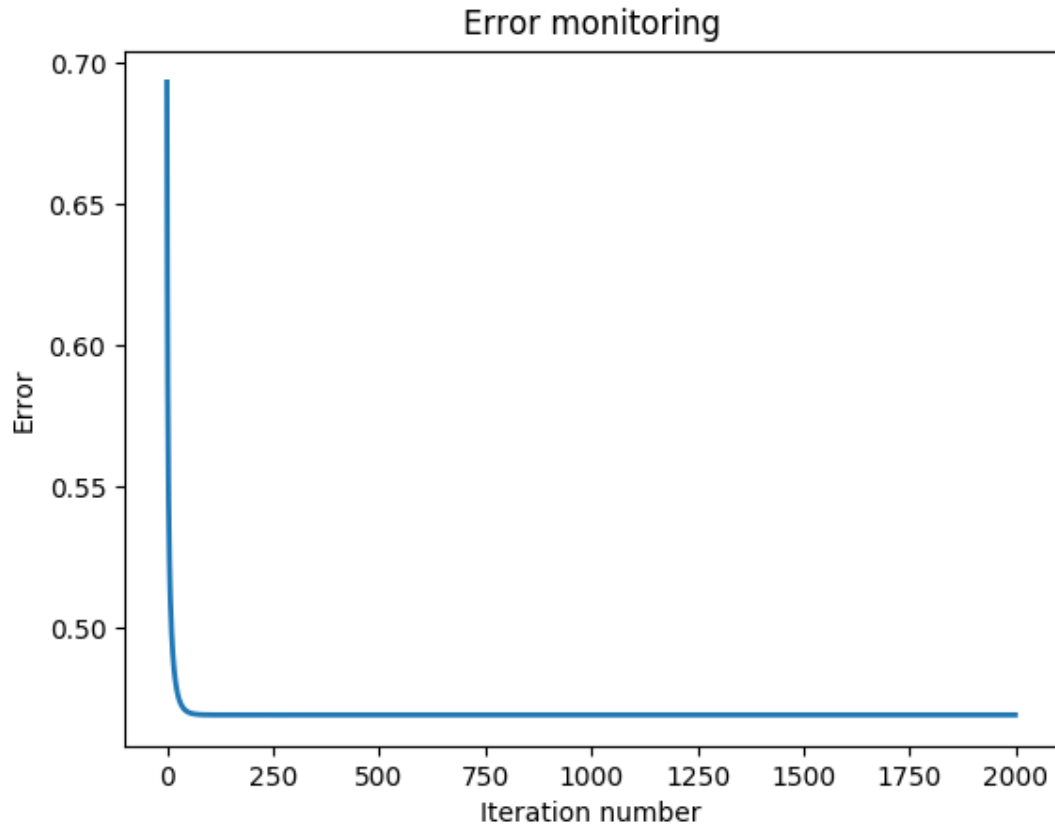


Figure 18: Logistic Regression Errors ( $\eta = 1$ ,  $l = 1$ , 2000 iterations)

3. **gradient descent** degree  $l = 2$ , 200 iterations, learning rate  $\eta = 0.15, 1.5, 15$

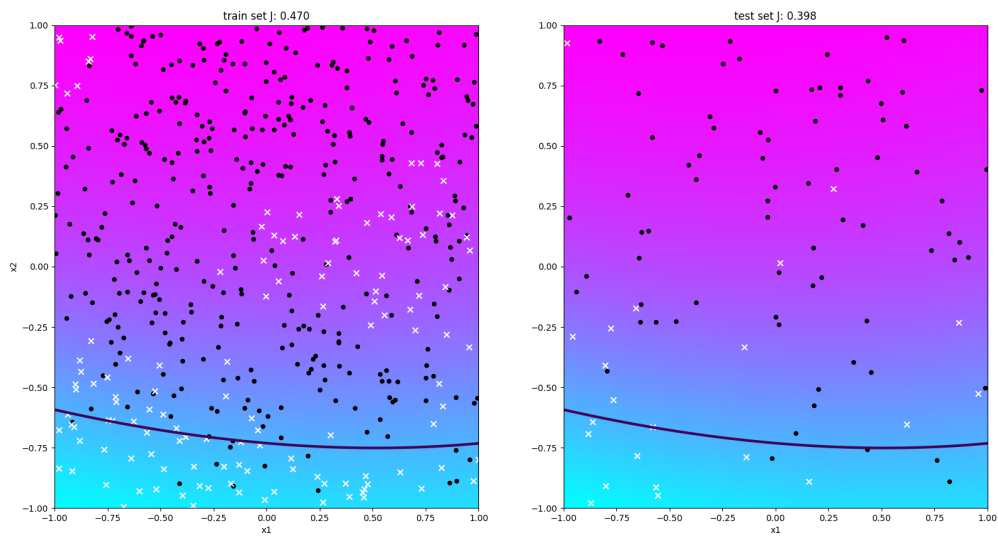


Figure 19: Logistic Regression ( $\eta = 0.15$ ,  $l = 1$ , 200 iterations)

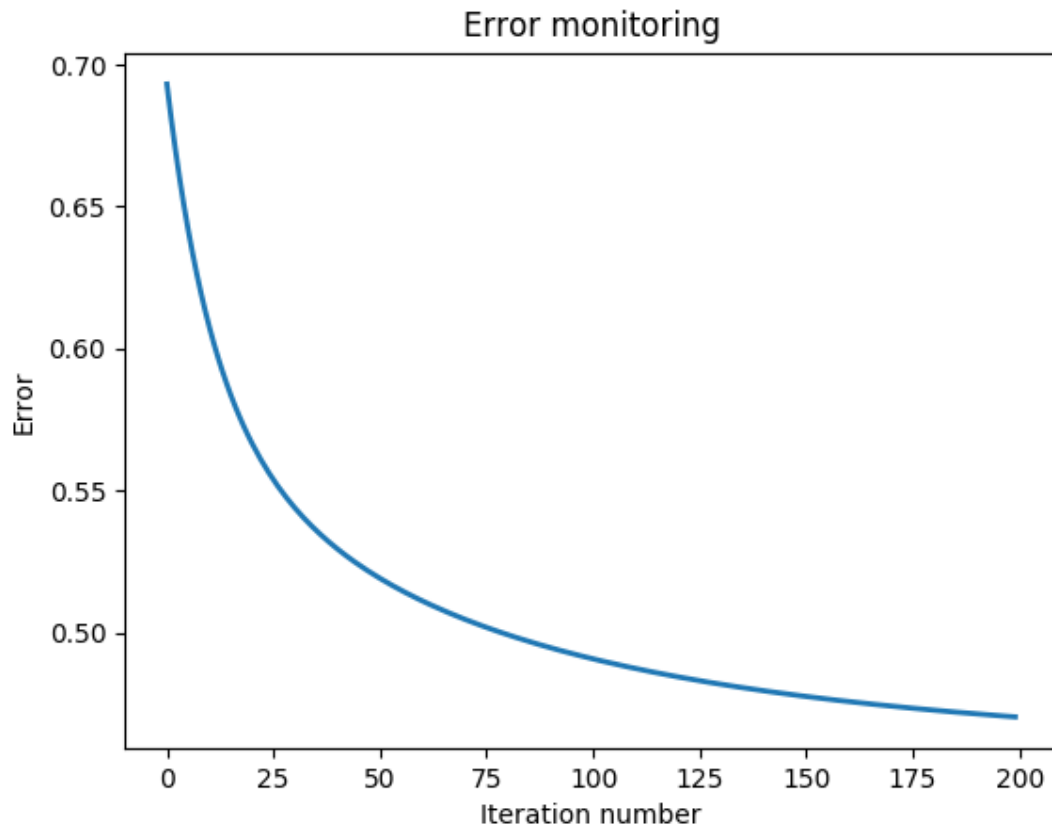


Figure 20: Logistic Regression Errors ( $\eta = 0.15$ ,  $l = 1$ , 200 iterations)

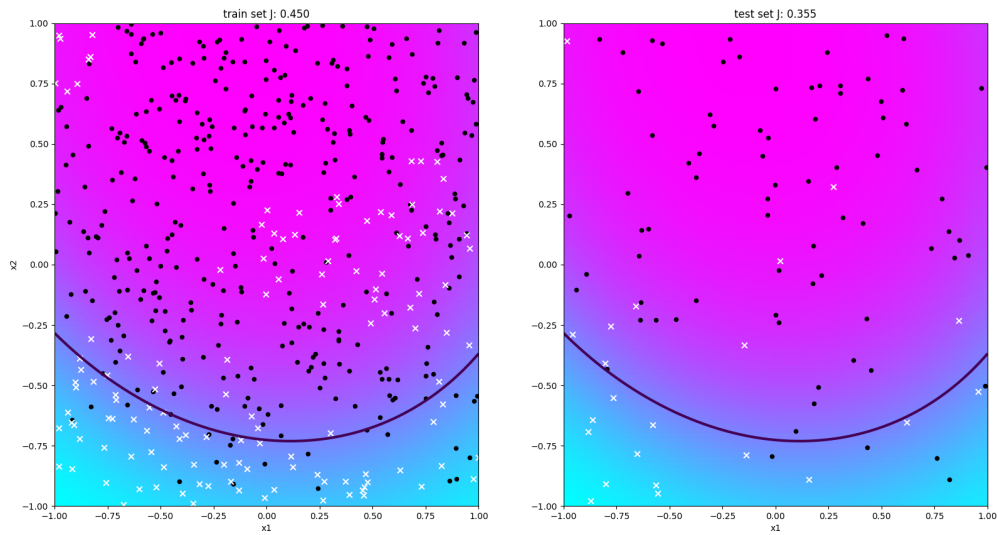


Figure 21: Logistic Regression ( $\eta = 1.5$ ,  $l = 1$ , 200 iterations)

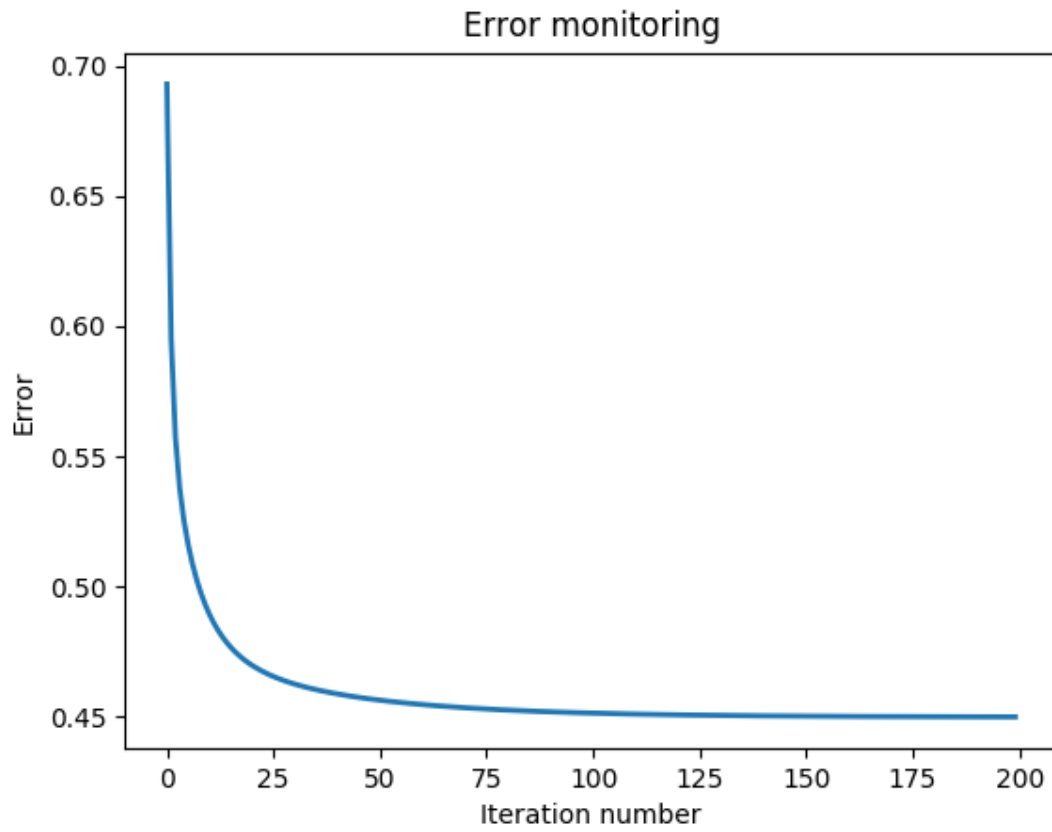


Figure 22: Logistic Regression Errors ( $\eta = 1.5$ ,  $l = 1$ , 200 iterations)

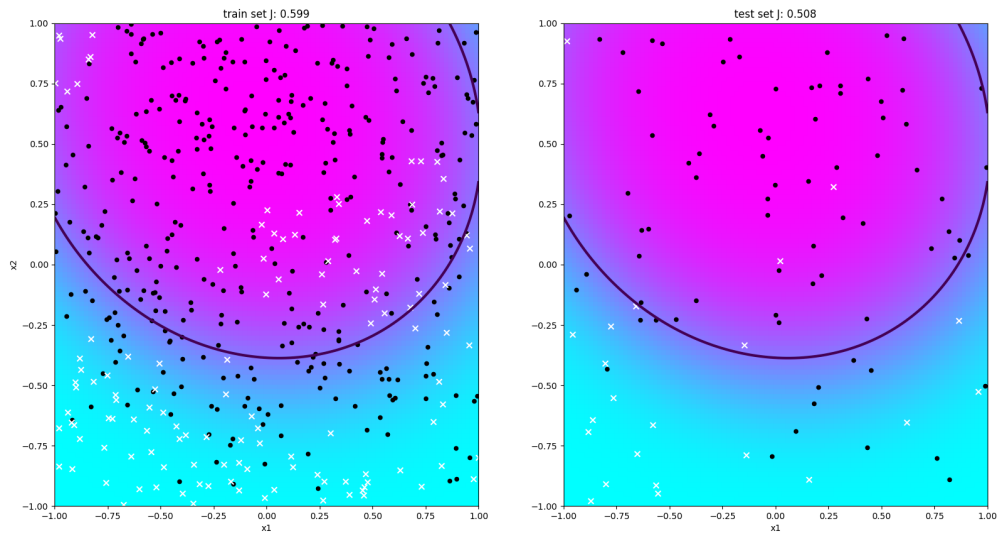


Figure 23: Logistic Regression ( $\eta = 15$ ,  $l = 1$ , 200 iterations)

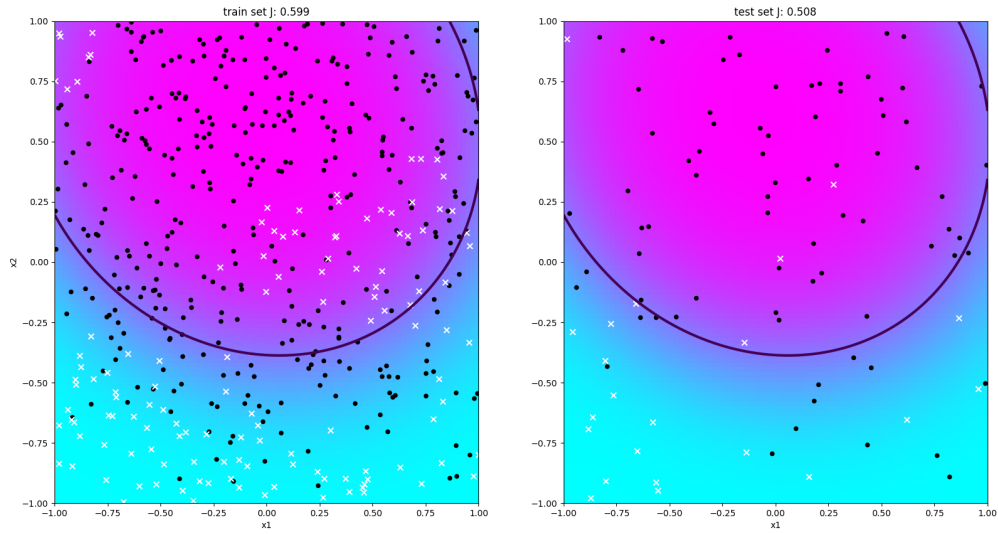


Figure 24: Logistic Regression Errors ( $\eta = 15$ ,  $l = 1$ , 200 iterations)

Discussion: Too low or too high learning rates lead to divergence or spinning between lower and high cost (oscillates).

4. **Adaptative gradient descent (GDad)** degree  $l = 1, 2, 5, 15$ , 1000 iterations, learning rate  $\eta = 1$

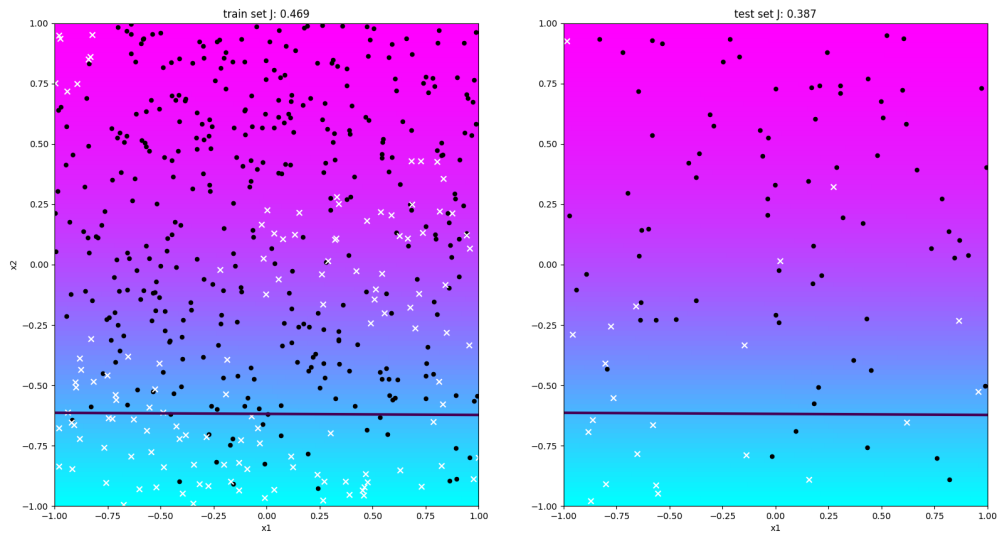


Figure 25: Logistic Regression (adaptive) ( $\eta = 1$ ,  $l = 1$ , 1000 iterations)



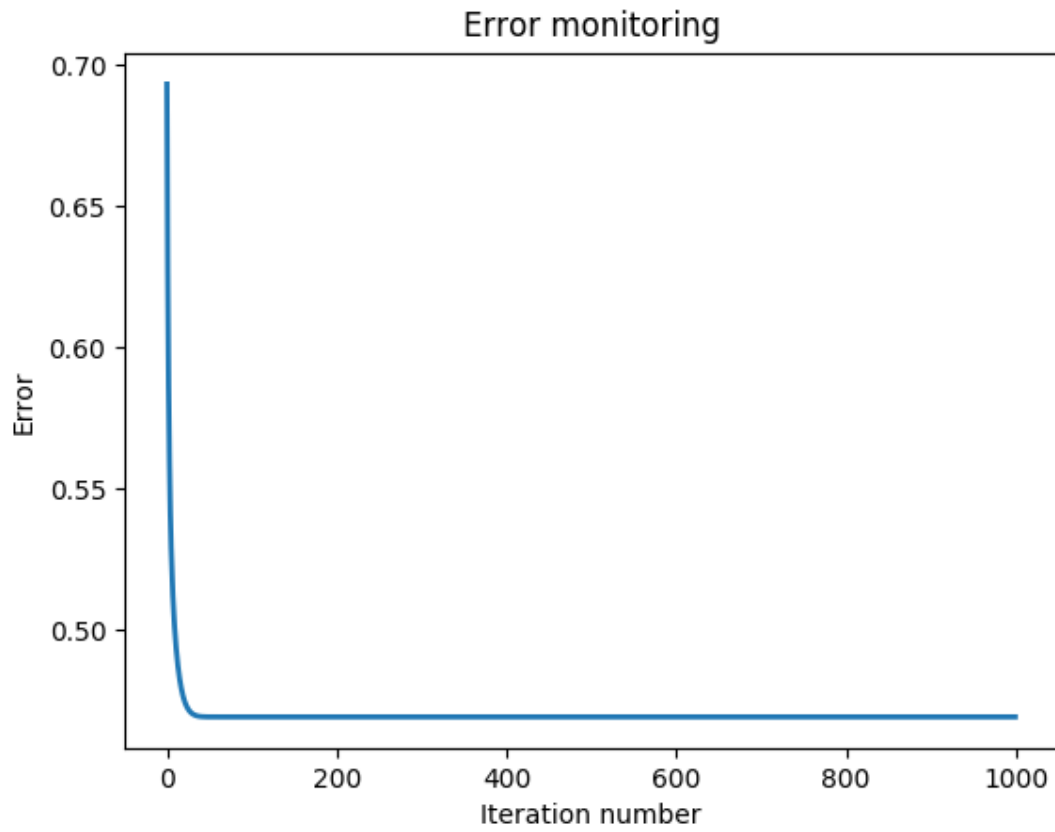


Figure 26: Logistic Regression (adaptive) Errors ( $\eta = 1$ ,  $l = 1$ , 1000 iterations)

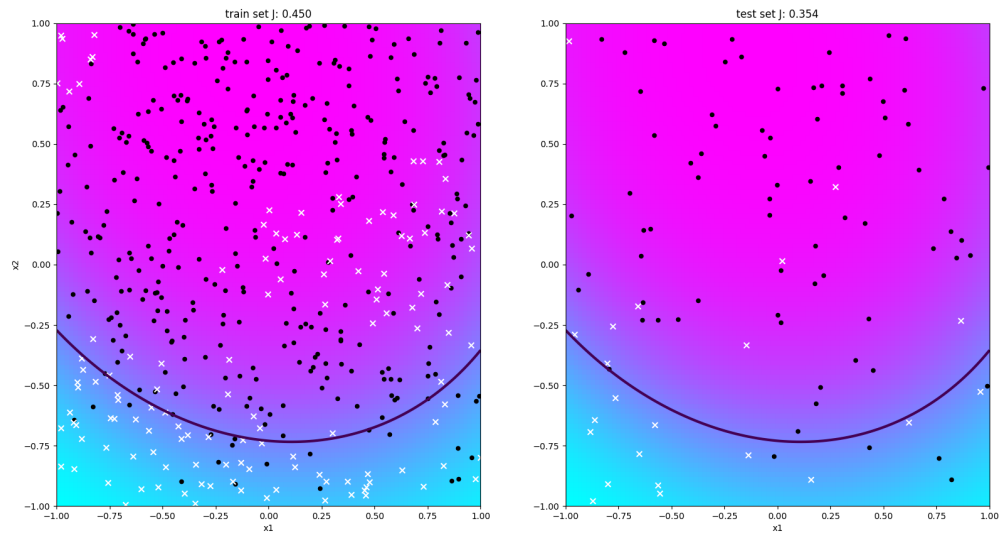


Figure 27: Logistic Regression (adaptive) ( $\eta = 1$ ,  $l = 2$ , 1000 iterations)

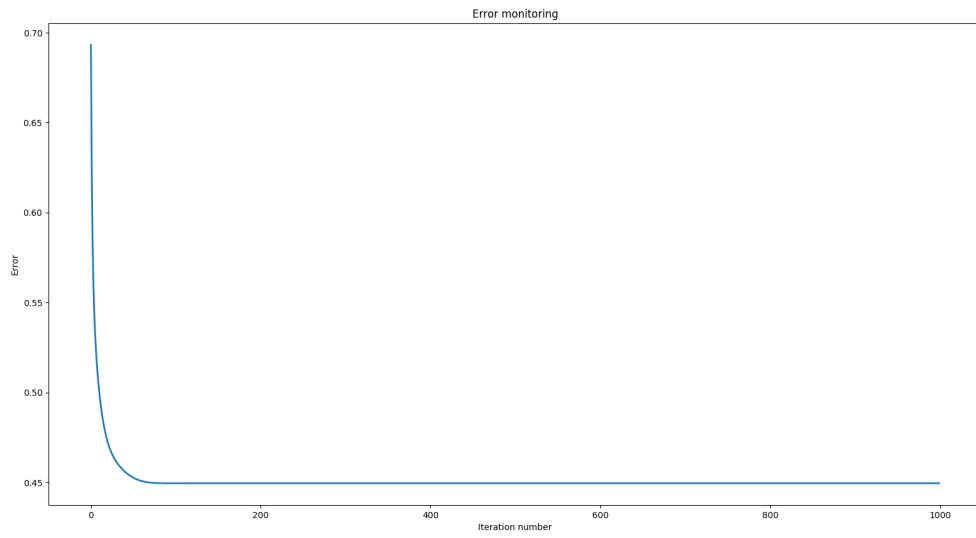


Figure 28: Logistic Regression (adaptive) Errors ( $\eta = 1$ ,  $l = 2$ , 1000 iterations)

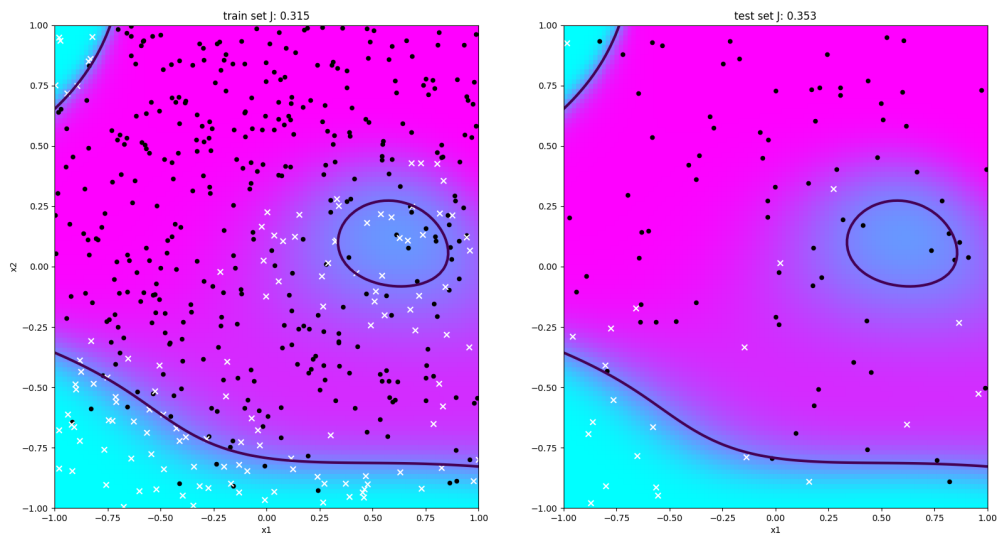


Figure 29: Logistic Regression (adaptive) ( $\eta = 1$ ,  $l = 5$ , 1000 iterations)

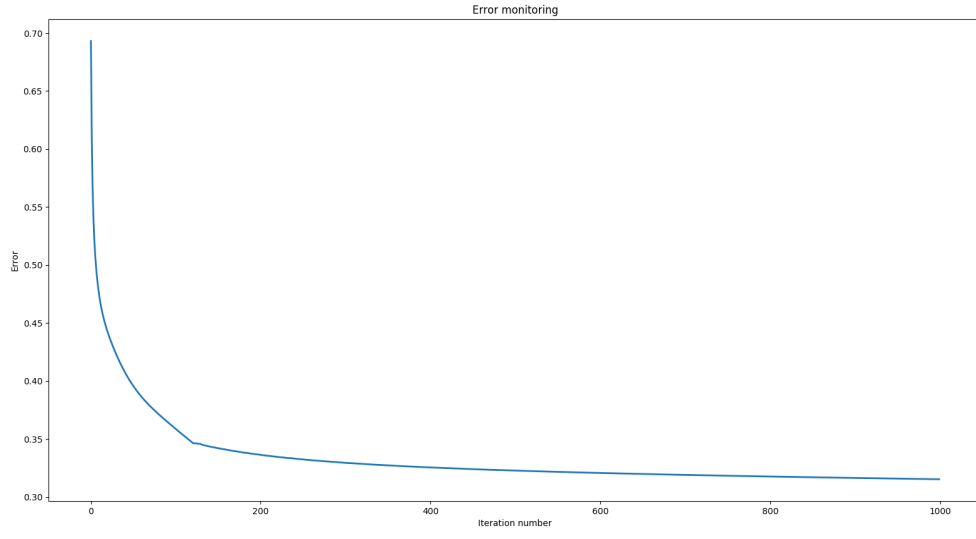


Figure 30: Logistic Regression (adaptive) Errors ( $\eta = 1$ ,  $l = 5$ , 1000 iterations)

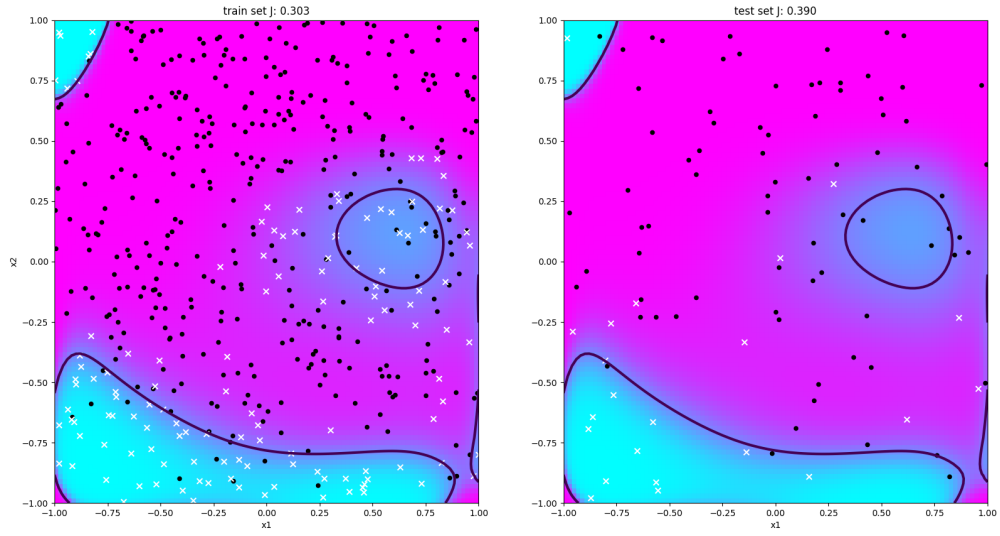


Figure 31: Logistic Regression (adaptive) ( $\eta = 1$ ,  $l = 15$ , 1000 iterations)

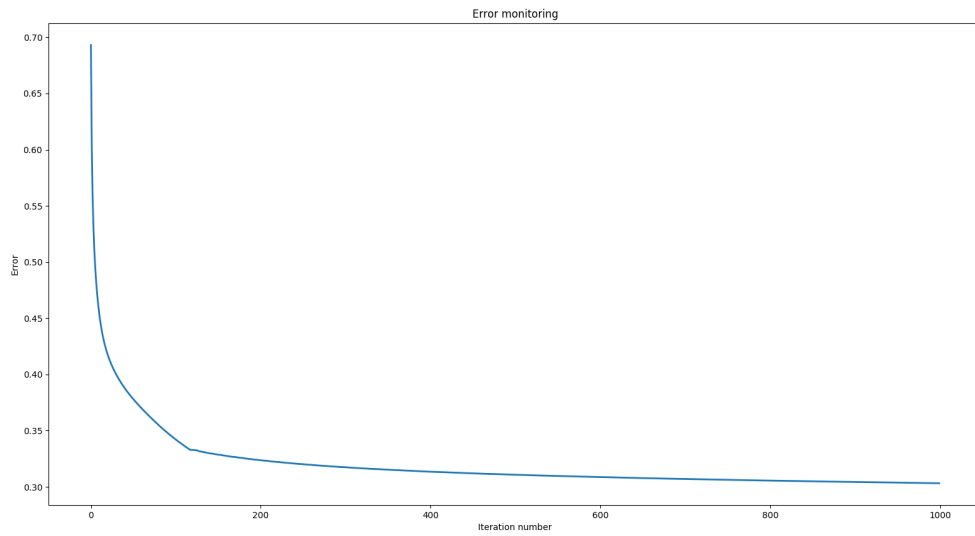


Figure 32: Logistic Regression (adaptive) Errors ( $\eta = 1$ ,  $l = 15$ , 1000 iterations)

5. Stopping When error between iteration becomes too low, threshold regression should be stopped.

### 2.2.2 Scipy optimizer