Prob. Distr. Exercise Solutions

1. a) $f(0) = 0.25 - 0 = 0.25$ (the vertical jump)

   $f(1) = 0.75 - .25 = 0.5$

   $f(2) = 1 - .75 = 0.25$

   b) $f(x) = F'(x) = 1, \quad 0 \le x \le 1$

   Uses Fundamental Thm. of Calculus: $f(x) = \frac{d}{dx} \int_a^x f(y) \, dy$

2. a) $S = \{0, 1, 2, 3, 4\}$

   b) $P(X \ge 3) = P(3) + P(4) = 0.3 + 0.3 = 0.6$

   c) $\mu = 0(.05) + 1(.1) + 2(.25) + 3(.3) + 4(.3) = 2.7$

3. a) $P(X \le 1) = \int_0^1 (1 - x/2) \, dx = x - \frac{x^2}{4} \Big|_0^1 = 1 - \frac{1}{4} - (0 - 0) = \frac{3}{4}$

   b) $E(X) = \int_0^2 x(1 - x/2) \, dx = \frac{x^2}{2} - \frac{x^3}{6} \Big|_0^2 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

   c) $F(x) = \int_0^x (1 - \frac{y}{2}) \, dy = y - \frac{y^2}{4} \Big|_0^x = x - \frac{x^2}{4}, \quad 0 \le x \le 2$

   $F(x) = 0, \quad x < 0$

   $F(x) = 1, \quad x > 2$

4. a) $S = (-\infty, \infty)$

   b) $S = (0, \infty)$, b/c it is the range of the function $g(x) = x^2$.

5. $S = \{0, 1, ..., 12\}$

6. a) $\sigma_Y = 1.2(50) = 60$ the spread is multiplied by 1.2

   b) $\sigma_T = 60$. adding a constant does not change the width, or σ

   c) $\sigma_{aX+b} = a\sigma_X \qquad \sigma^2_{aX+b} = a^2 \sigma^2_X$