Meeting Some Probability Distributions

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Discrete Probability Distributions

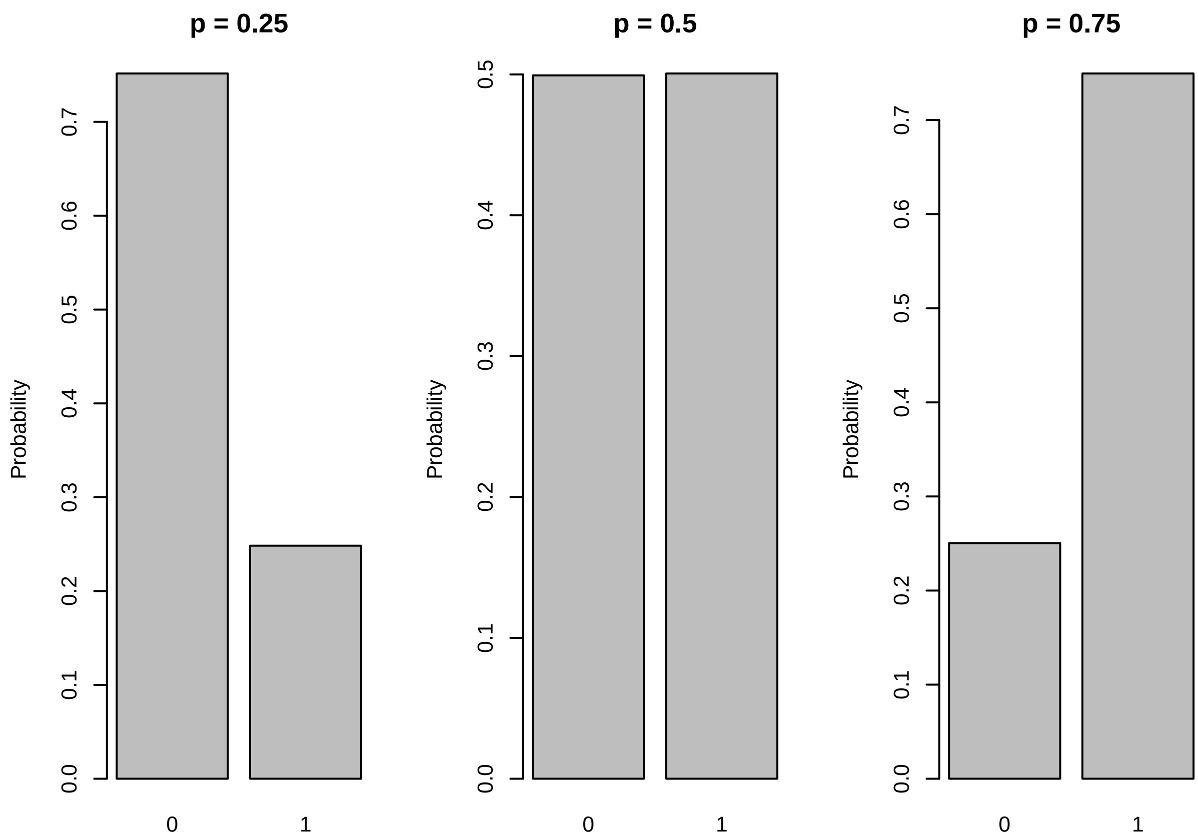
Bernoulli

Support: $y \in 0, 1$ (which says: "y in the set of values containing only 0 and 1")
Parameters: p, the probability of success, and q, the probability of failure ($q = 1 - p$)
First Moment (Population Mean): p
Second Moment (Population Variance): $p * q$

Playing Around with the Shape of the Bernoulli

A probability distribution (governed by specific parameter values) is most commonly visualized using a plot composed of supported values on the x-axis, and the associated probability of observing those supported values on the y-axis.

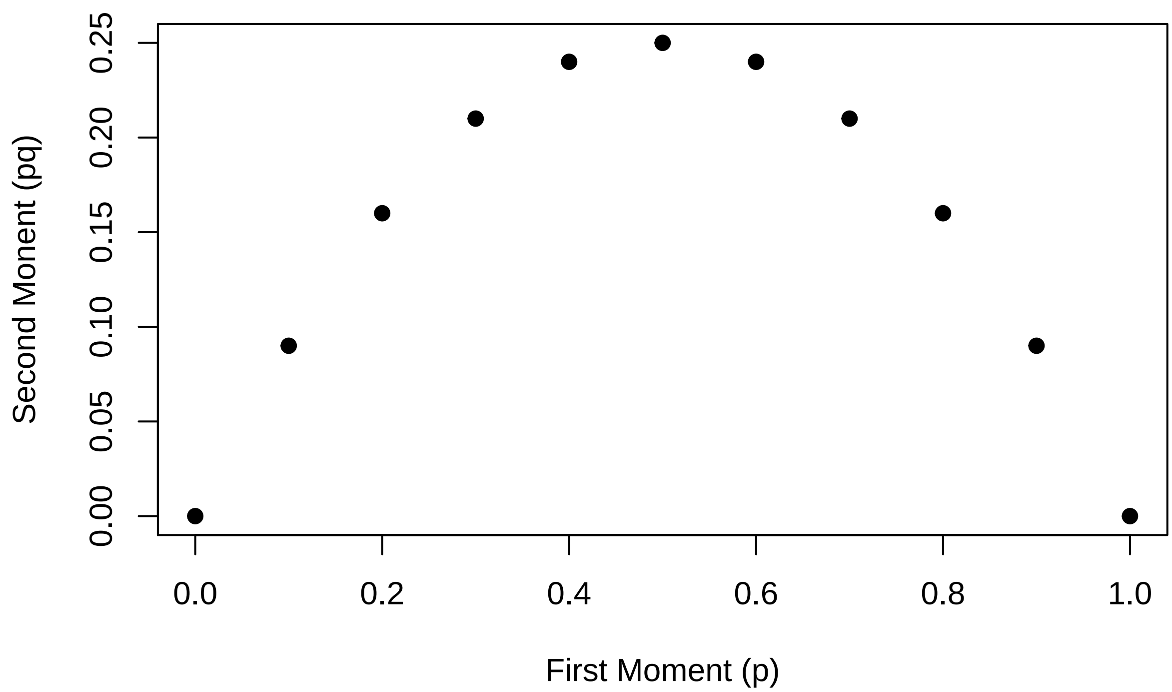
```r
par(mfrow = c(1, 3), mar = c(2, 4, 4, 2))
barplot(table(rbinom(100000, 1, 0.25))/100000, ylab = "Probability", main = "p = 0.25")
barplot(table(rbinom(100000, 1, 0.5))/100000, ylab = "Probability", main = "p = 0.5")
barplot(table(rbinom(100000, 1, 0.75))/100000, ylab = "Probability", main = "p = 0.75")
```



The Relationship Between the Two Moments of the Bernoulli

The spread of a distribution is described by its second moment, the population variance. Distributions described by smaller variances have most of their values close to the population mean, while distribution described by larger variances are more "spread out", with more values falling farther from the mean. Let's take a look at the relationship between the first two moments of the Bernoulli distribution.

```
m1 <- seq(0, 1, 0.1)
q <- 1-m1
m2 <- m1*q
plot(m1, m2, pch=19, xlab = 'First Moment (p)', ylab = 'Second Monent (pq)')
```



Does this make sense? The variance of the Bernoulli distribution is largest when the probability of success is 0.5. Such a distribution would have a mean of 0.5, resulting in all draws (0 or 1), being 0.5 from the mean. When the probability of success is 0.75, most draws are 1, and many fewer draws are 0, meaning most deviations from the mean are only 0.25, resulting in a smaller variance. Neat!

Working with the Bernoulli Distribution in R

we've so far only used probability distributions in R to produce random data. Remembering that the Bernoulli distribution is a special case of the binomial distribution when `size = 1`, we can generate a sample of data from a specific Bernoulli distribution using the `rbinom()` function. R also allows us to call on the probability mass function (PMF) of the Bernoulli distribution (probability *mass* because data described by a Bernoulli is discrete). We will rarely worry about the exact form of a distribution's PMF (or probability density function (PDF)), but the Bernoulli PMF is very straight forward, so I show it here.

$$PMF = f(y) = P(Y = y) = \begin{cases} q = 1 - p & if \;\; y = 0 \\ p & if \;\; y = 1 \end{cases}$$

As we've discussed, PMFs (and PDFs) compute the probability of the random variables taking on some value. So here we're saying, "the probability of Y taking on the value of 0 (y = 0) is $q = 1 - p$ and the probability of Y taking on the value 1 (y = 1) is p." Straight forward indeed!

```
# What is the probability of drawing a value of 0 from a Bernoulli distribution
# with a probability of success equal to 0.68?
dbinom(0, 1, 0.68)
```

```
## [1] 0.32
```