Bayesian Network

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Introduction

In this project, we use **Bayesian networks** to estimate the location of gems on a grid based on distance observations. We implement an **enumeration-based inference** method to update the beliefs after each observation by computing the probability of gem presence at every grid cell. We then analyze experimental results by examining belief updates on a grid across several iterations.

1 Methodology

1.1 Likelihood

We define the likelihood as a decreasing function of the distance between actual observations and hypothetical predictions. Let d be the Euclidean distance between the observation vector and the vector that would have been observed if gems were at the hypothetical positions. The likelihood is computed as:

$$\text{likelihood} = 2^{-d}.$$

This formulation favors hypothetical positions close to the real observations: the smaller the distance, the higher the likelihood, reflecting a greater probability that the hypothetical position is correct.

1.2 Enumeration Inference

To update beliefs after each observation, we use **enumeration-based inference**. We compute the probabilities for every possible gem configuration by iterating over all combinations. Each configuration yields a likelihood; the cell-wise beliefs over the grid are updated accordingly and then **normalized** so that total probability sums to 1.

2 Experimental Results

2.1 Belief Distribution

On a 10×10 grid, the belief distribution evolves over multiple update iterations. Beliefs progressively concentrate around the most probable gem locations as new observations are incorporated. Initially, probabilities are spread roughly uniformly across the grid; over time, they **converge** toward the true gem positions.

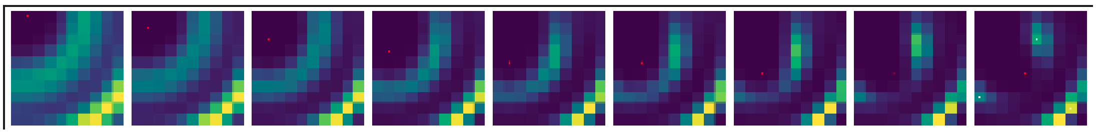

Figure 1: Evolution of the belief distribution over iterations

This update sequence highlights the ability of Bayesian inference to refine beliefs in response to observations. Initially, the probabilities are distributed uniformly, but they progressively converge toward the true gem positions over the iterations.

3 Conclusion

In conclusion, this project demonstrates how enumeration-based inference in a Bayesian network makes it possible to model and predict the positions of gems from distance observations. The likelihood formulation, which decreases exponentially with distance, favors positions close to the observations. The obtained results confirm the effectiveness of the method in improving belief accuracy after each observation. Possible improvements include using approximations to reduce the complexity of enumeration-based inference, as well as more advanced Bayesian methods such as particle filtering to handle larger grids.