CSC 665: Artificial Intelligence

Games: Uncertainty

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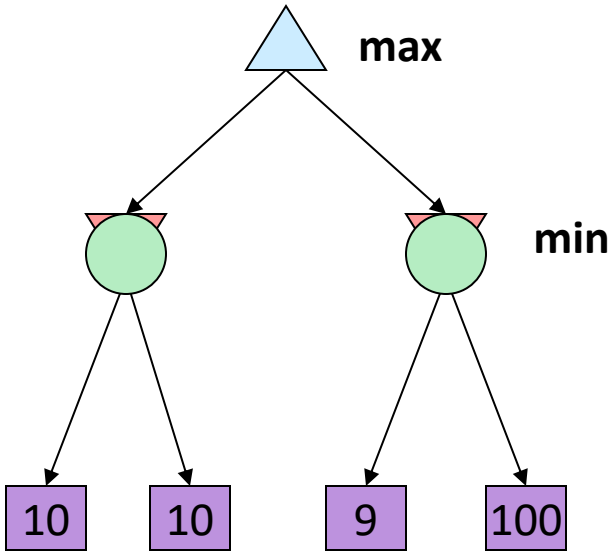
Uncertain Outcomes

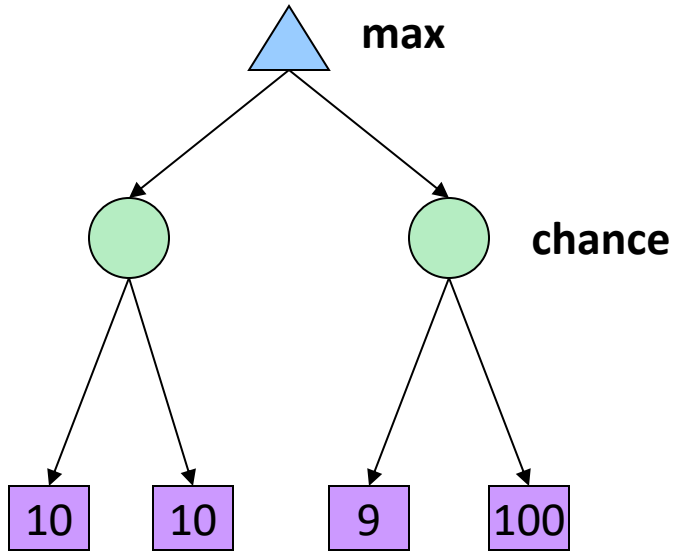
Worst-Case vs. Average Case



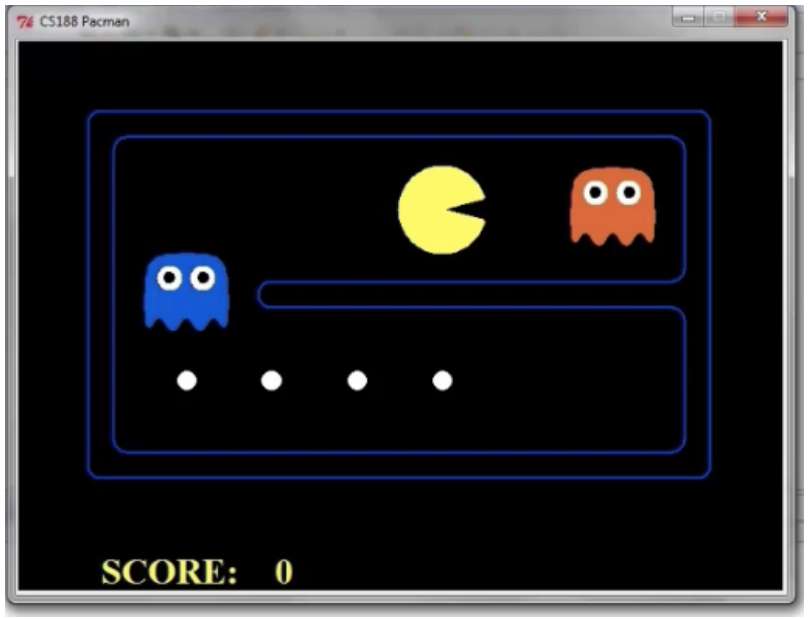
Idea: Uncertain outcomes controlled by chance, not an adversary!

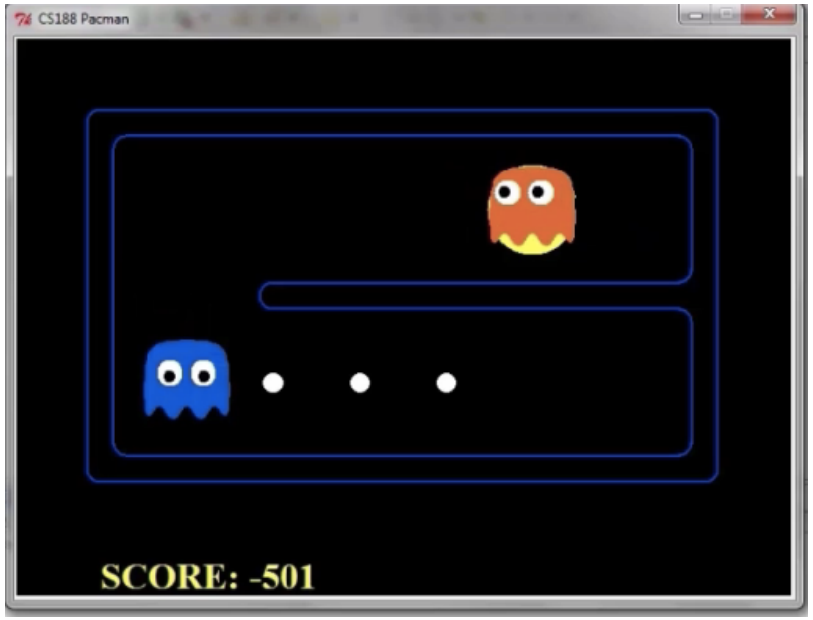
Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes: the outcome is uncertain (opponent or environment)
  - Calculate their expected utilities: take weighted average (expectation) of children

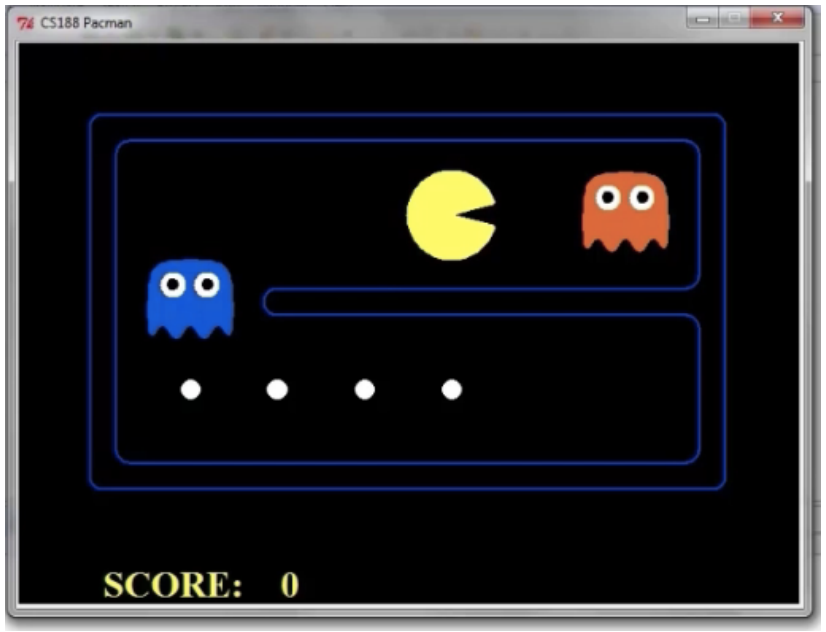


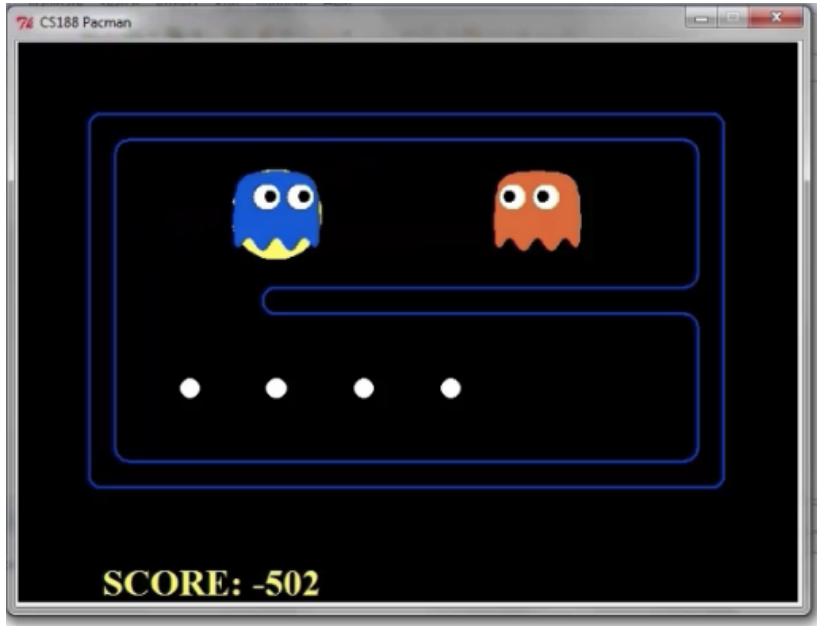
Video of Demo Minimax vs Expectimax (Min)

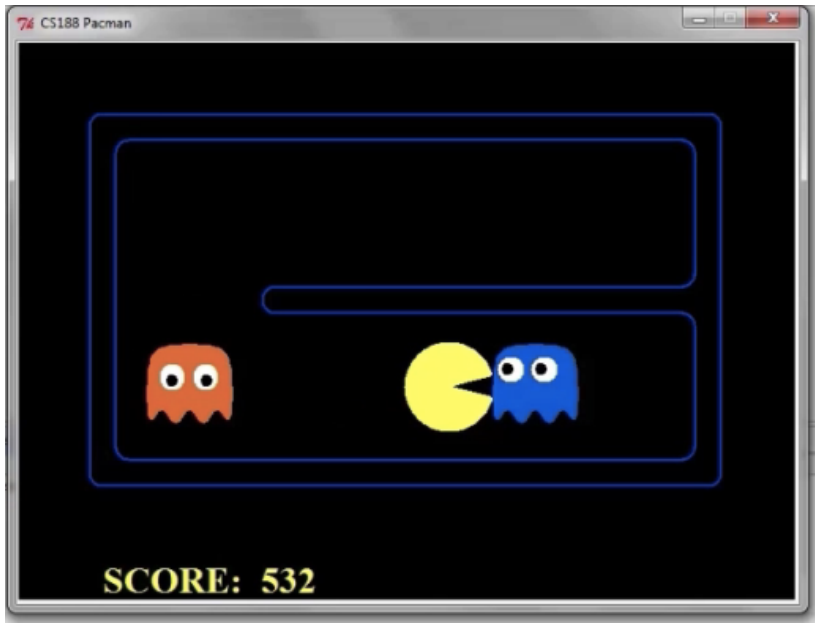




Video of Demo Minimax vs Expectimax (Exp)







Expectimax Pseudocode

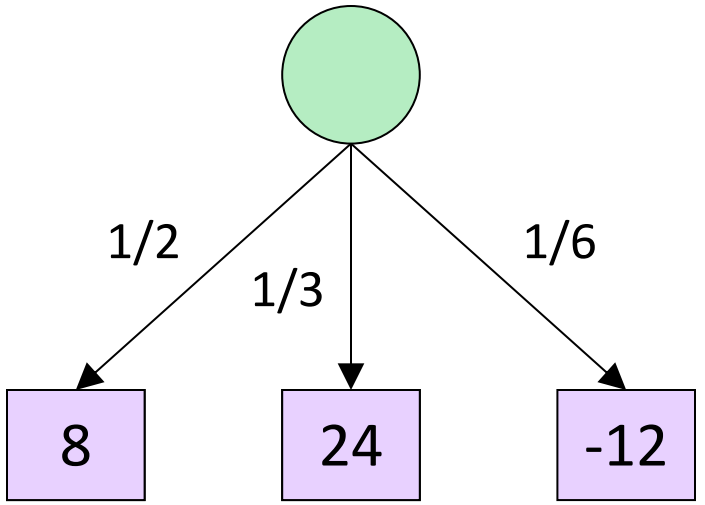
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is CHANCE: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

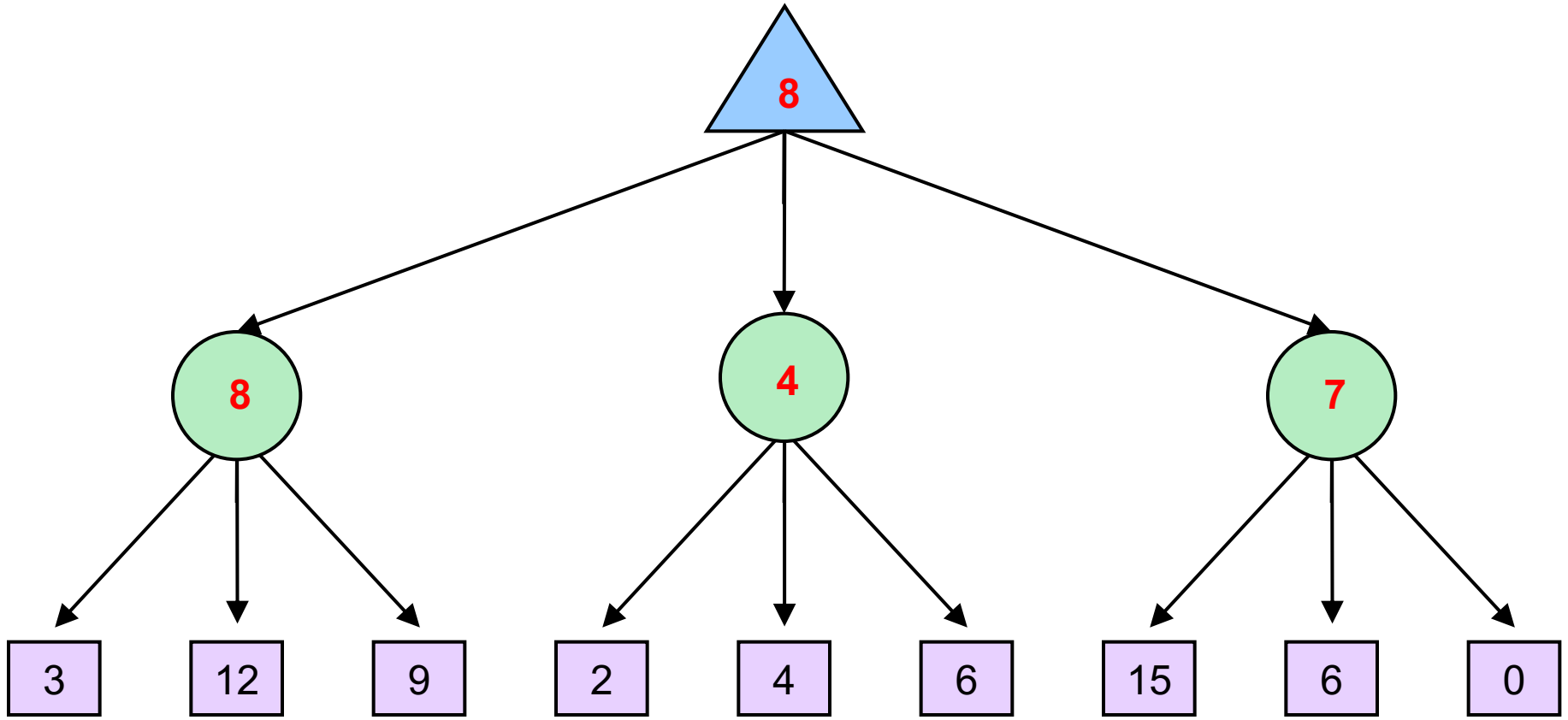
Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

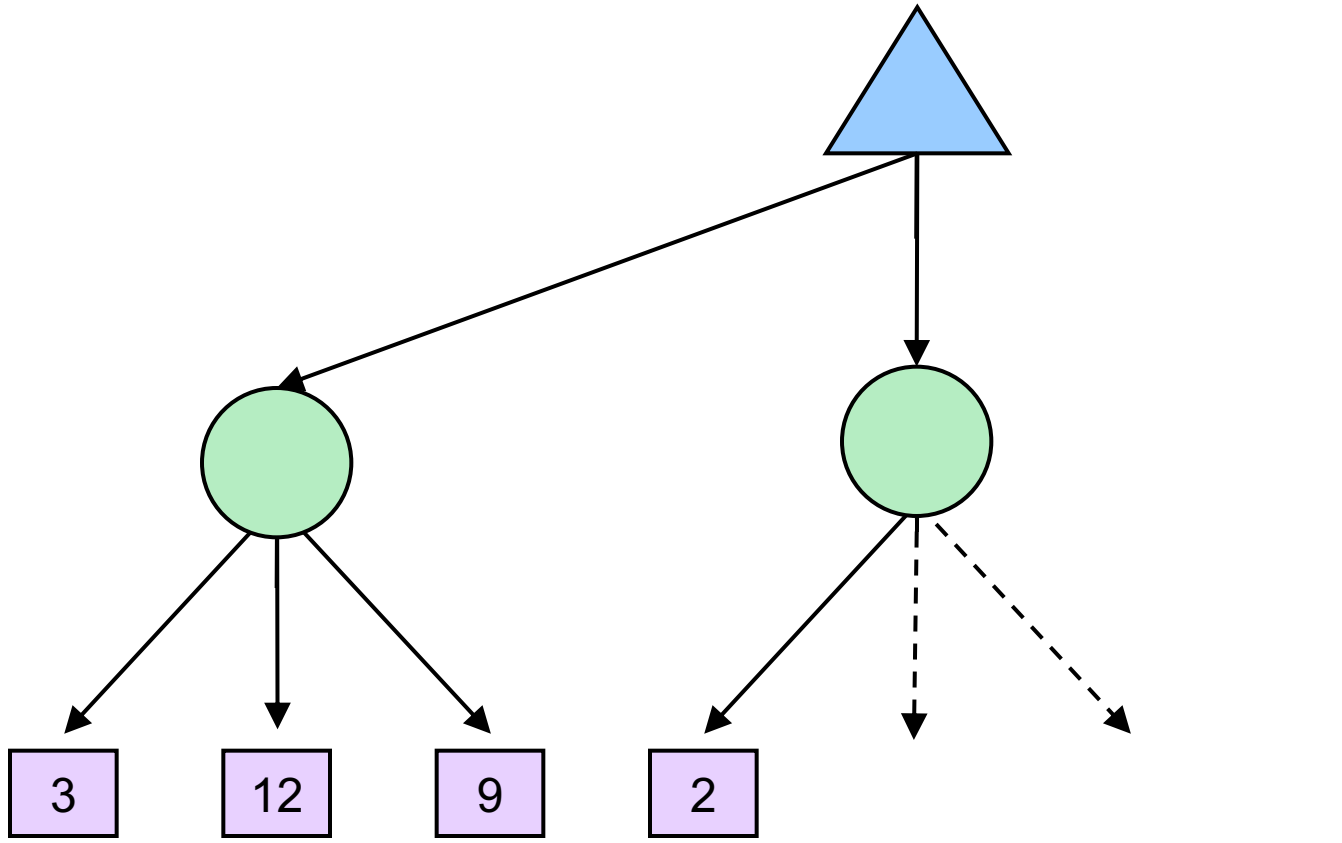


$$v = (1/2)\,(8) + (1/3)\,(24) + (1/6)\,(-12) = 10$$

Expectimax Example

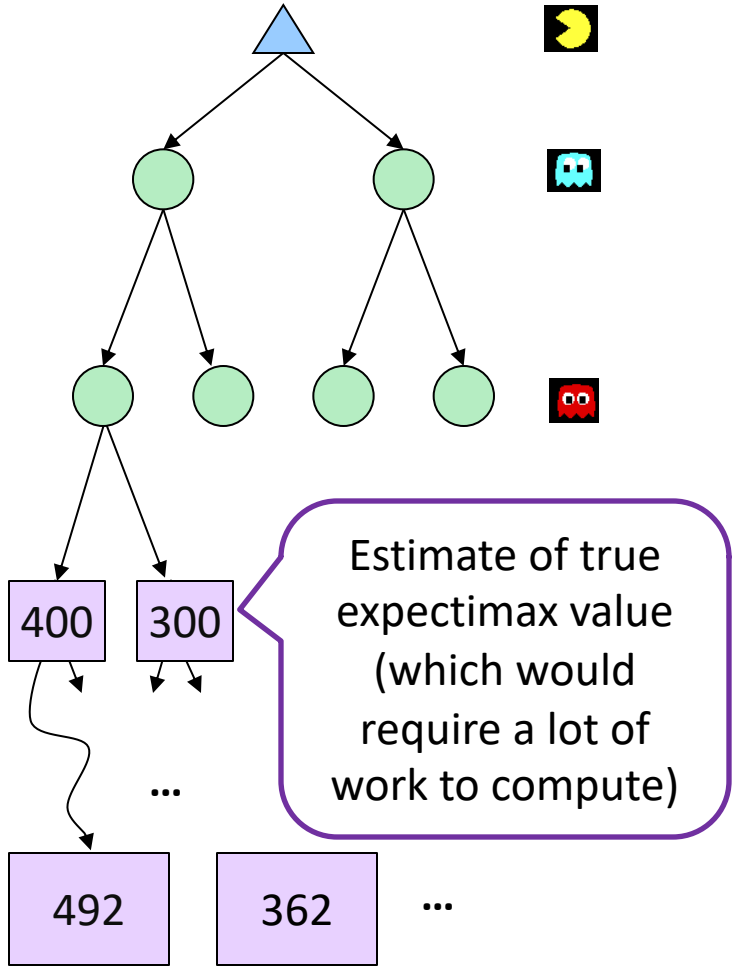


Expectimax Pruning?

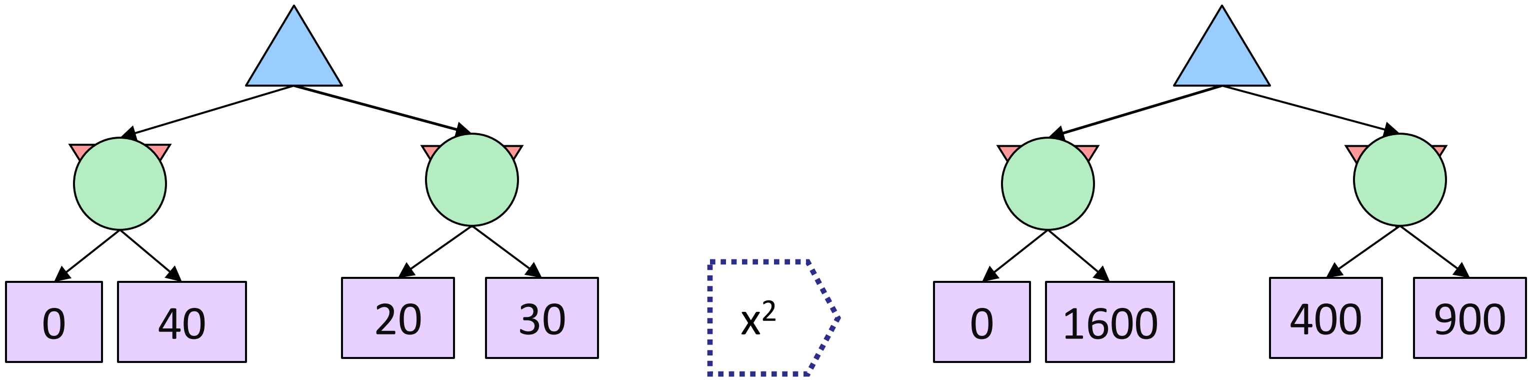


No Pruning!

Depth-Limited Expectimax



What Values to Use?



x>y => f(x)>f(y)

f(x) = Ax+B where A>0

- For worst-case minimax reasoning, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - Minimax decisions are ***invariant with respect to monotonic transformations on values***
- Expectimax decisions are ***invariant with respect to positive affine transformations***

Probabilities

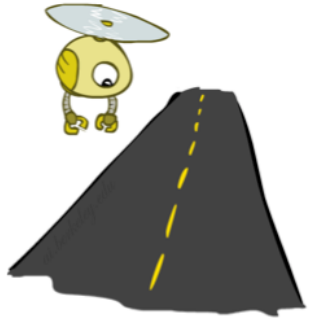
Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

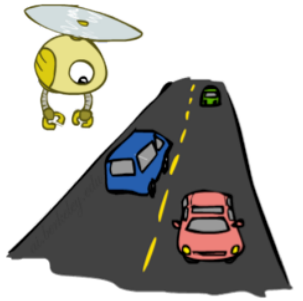
- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later



0.25



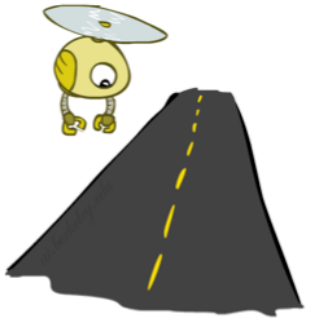
0.50

0.25

Reminder: Expectations

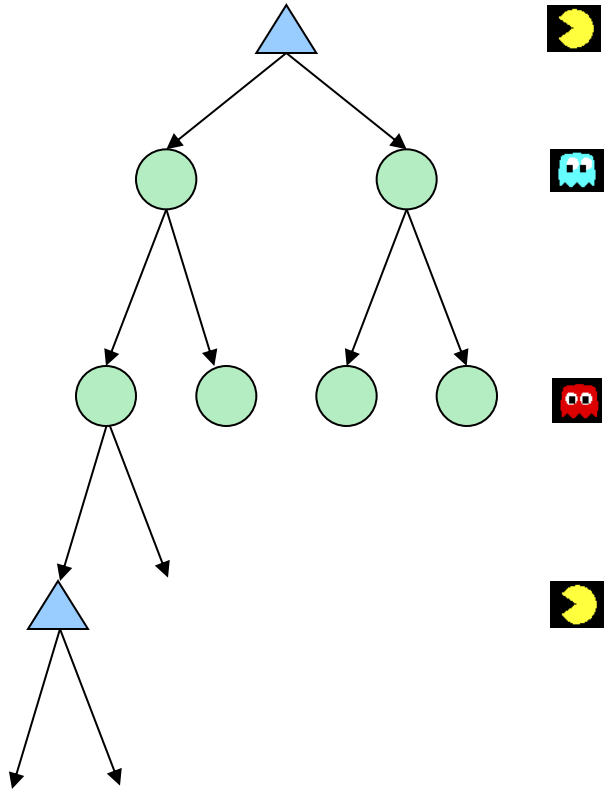
- The expected value of a random variable is the average, weighted by the probability distribution over outcomes
- Example: How long to get to the airport?

Time:	20 min		30 min		60 min		
	x	+	x	+	x	→	35 min
Probability:	0.25		0.50		0.25		



What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



Modeling Assumptions

The Dangers of Optimism and Pessimism

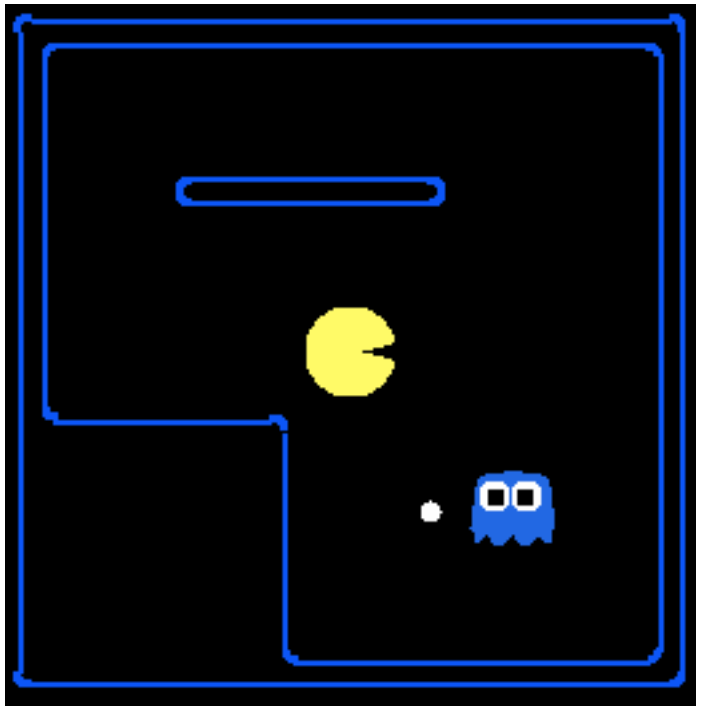
Dangerous Optimism

Assuming chance when the world is adversarial

Dangerous Pessimism

Assuming the worst case when it’s not likely

Assumptions vs. Reality

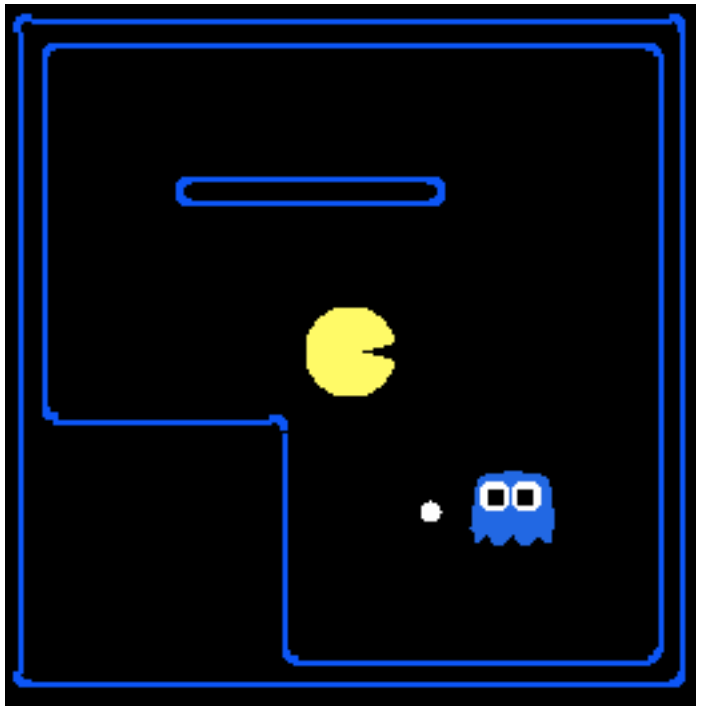


Reality

Assumption	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

Assumptions vs. Reality



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Results from playing 5 games

Video of Demo World Assumptions
Random Ghost – Expectimax Pacman

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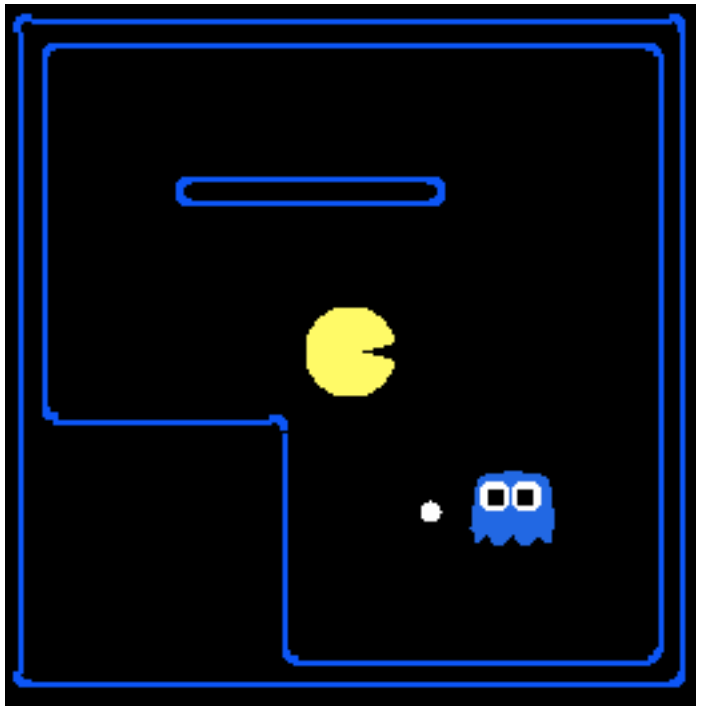

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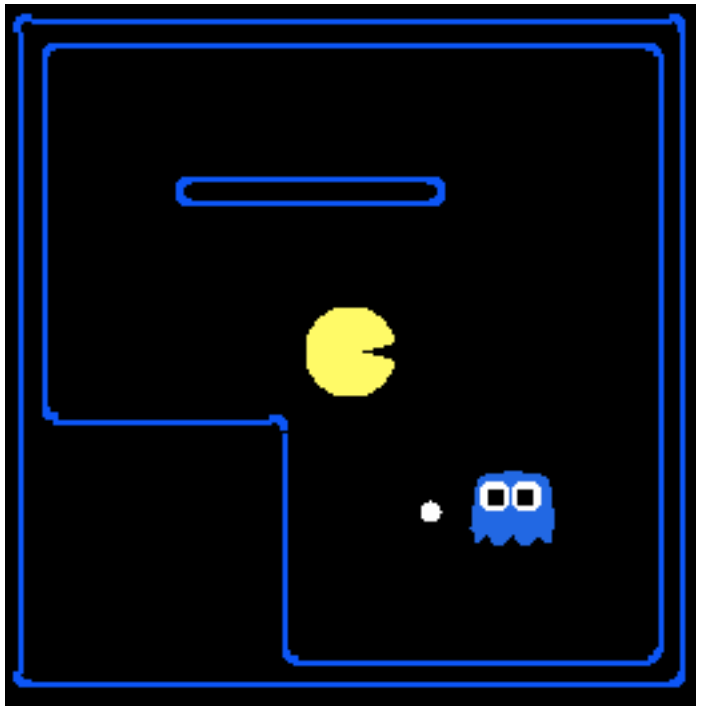

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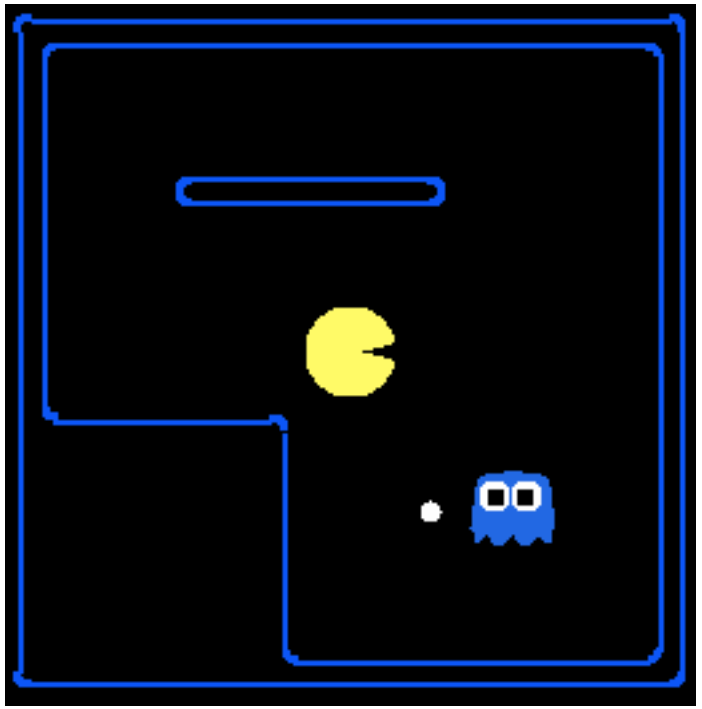

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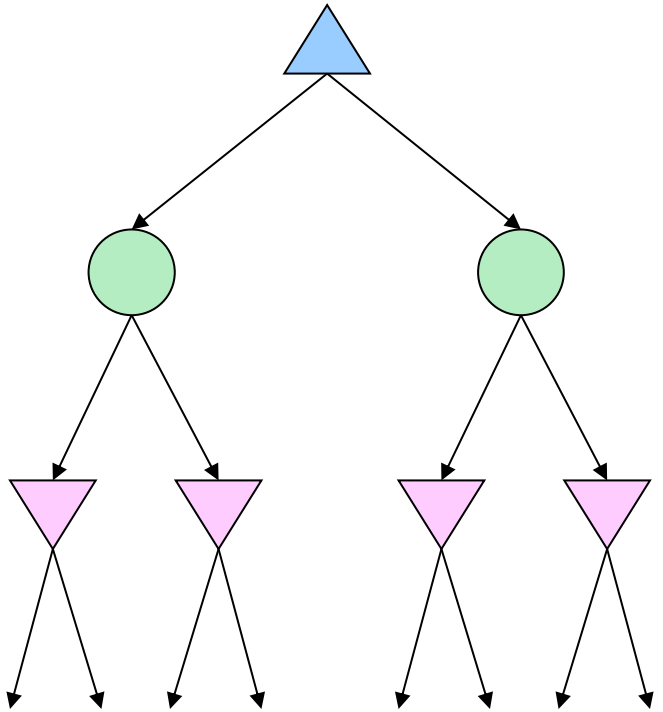
Other Game Types

Mixed Layer Types

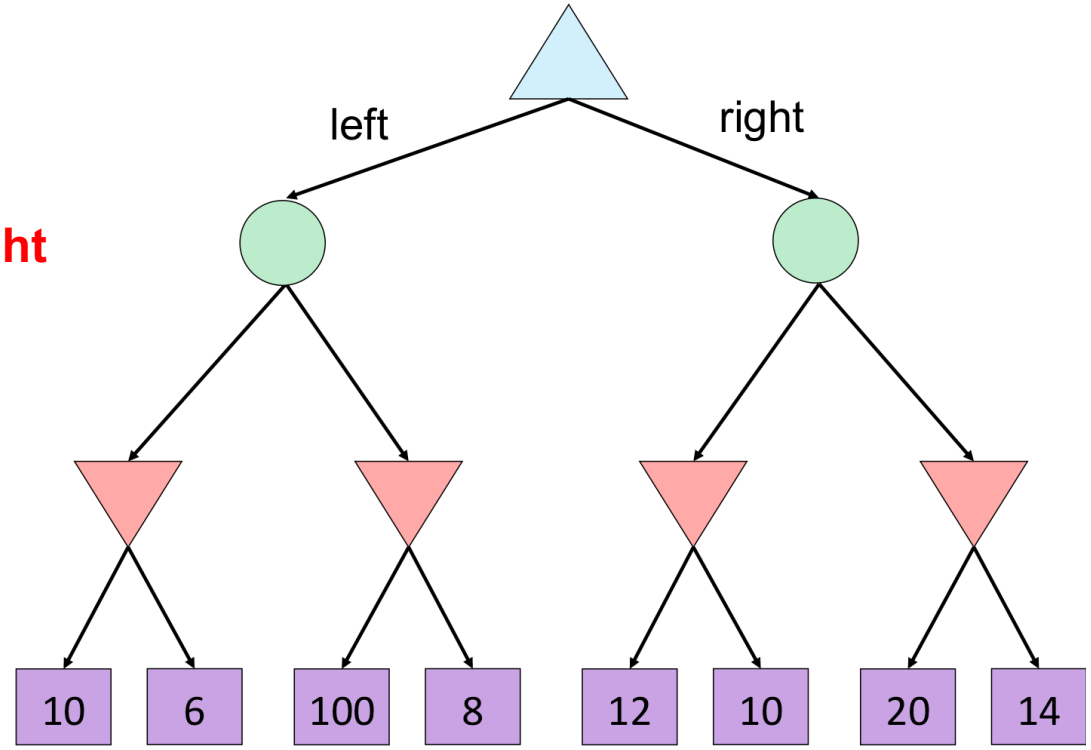
- E.g. Backgammon
- Expectiminimax
  - Rolling dice is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



Quiz: Mixed Layers Game

Consider the mixed layers game below, which has a maximizer playing first, followed by a chance node (assume each outcome at a chance node is equally likely), followed by a minimizer.

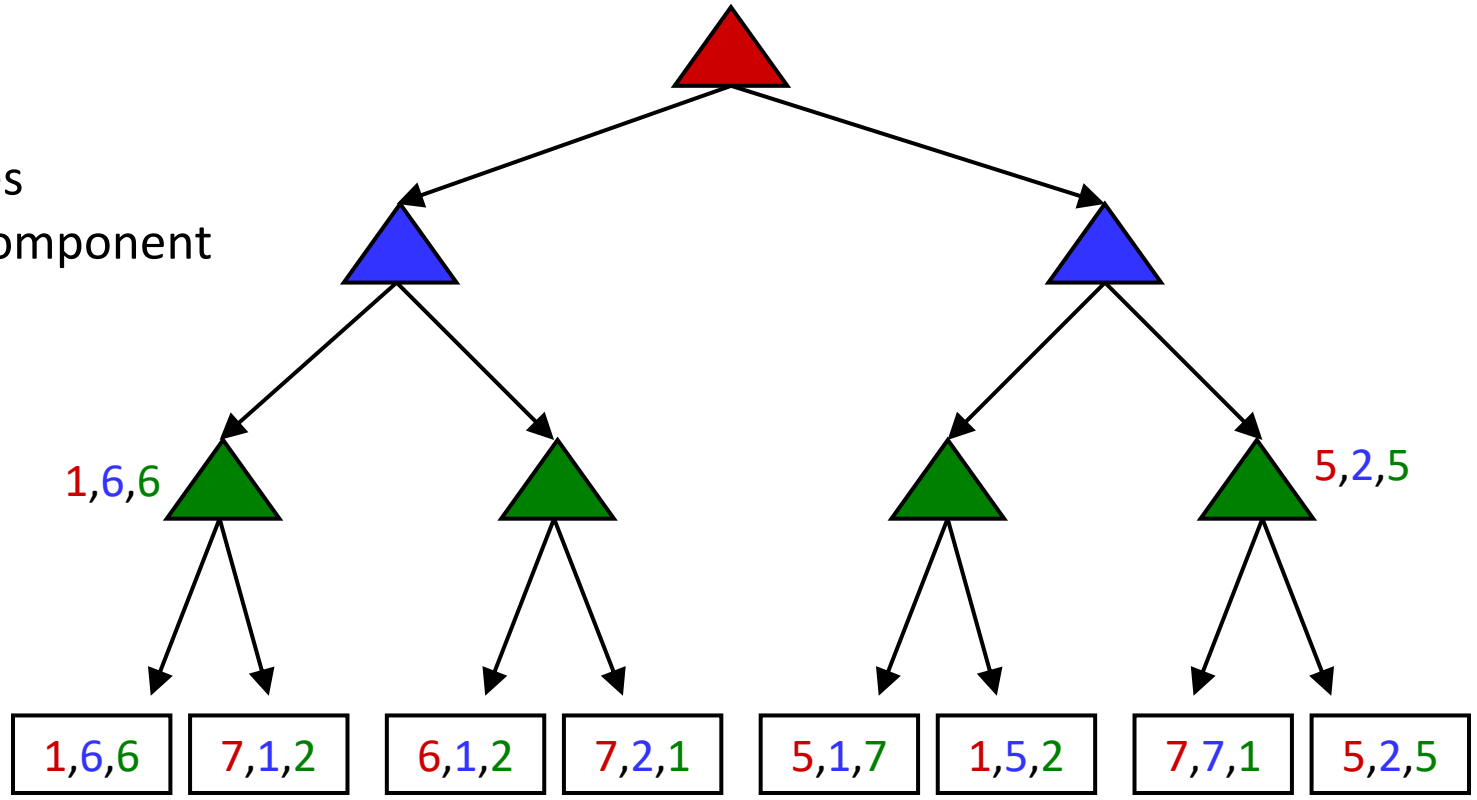
- What is the value of the game? **12**
- Which action is optimal for the maximizer? **Right**



Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



Reading

- Chapter 5.1-5.3, 5.5 in the AIMA textbook