Addition & Multiplication

Q → 10 Girls 7 Boys

How many different pairs can be formed (B–G).

G_1 B_1

G_2 B_2

$\vdots$ $\vdots$

G_{10} B_7

$10 * 7 = \underline{70}$ (Ans)

For every possibility of one we can select any possibility of second then use $*$

Q →

Find #ways to travel from Pune to Agra via Delhi.

a) Travel from Pune to Delhi → 3

AND

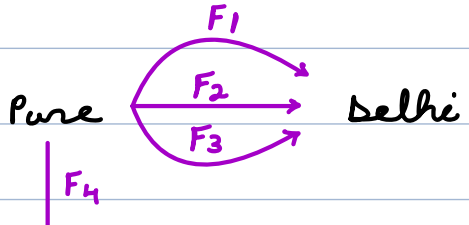
b) Travel from Delhi to Agra → 2

$F_1\ F_4$ $F_2\ F_4$ $F_3\ F_4$

$F_1\ F_5$ $F_2\ F_5$ $F_3\ F_5$

$\underline{6} \leftarrow 3 * 2$

AND → $*$



BLR

Find # ways to travel from Pune to Delhi or BLR.

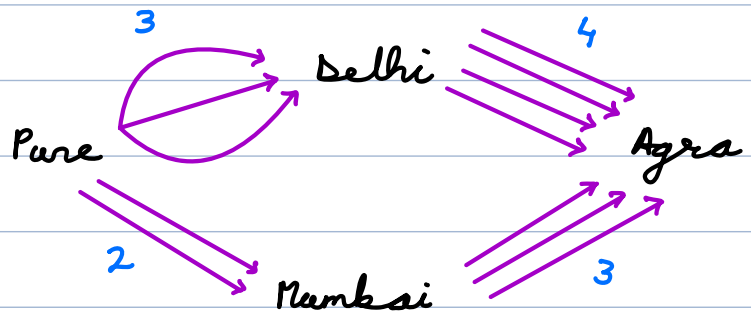
a) Travel from Pune to Delhi → 3

OR

b) Travel from Pune to BLR → 1

} 3 + 1 = 4

OR → +



ways Pune to Agra → Pune → Delhi → Agra → 3 * 4 = 12

OR

Pune → Mumbai → Agra → 2 * 3 = 6

Ans = 12 + 6 = 18

Permutation Basics

Arrangement of objects

⇒ order matters $(x, y \neq y, x)$

Q → Find # ways to arrange 3 distinct objects.

abc abc bac cab Ans = 6

acb bca cba

3 (AND) 2 (AND) 1 → #ways = 3 * 2 * 1 = 6 (3!)

#ways to arrange N distinct characters → N!

Q → Find #ways to arrange R out of N distinct objects.

d a t e 4 → 3 → 4 * 3 = 12

N = 4 R = 2

$$\frac{N * (N-1) * (N-2) * \ldots (N-(R-1)) * (N-R) * (N-(R+1)) * \ldots 2 * 1}{(N-R) * (N-(R+1)) * \ldots 2 * 1}$$

$$= \boxed{\frac{N!}{(N-R)!} = {}^{N}P_{R}}$$

Combinations Basics

Selection of objects
⇒ order do not matter (x, y = y, x)

Q → Find #ways to select 2 out of 4 distinct objects.

a b c d a b a c a d b c b d c d → 6

Arrangement ab bc ac bd ad cd } 12
ba cb ca db da dc

(#ways to select R out of N distinct objects) → ${}^{N}C_{R}$
* (#ways to arrange R objects) → R!
= (#ways to arrange R out of N distinct objects) → ${}^{N}P_{R}$

N = 5
R = 4

${}^{5}P_{4} = \frac{5!}{(5-4)!} = 120$

$4! = 24$

(#select) * 24 = 120

⇒ #select = $\frac{120}{24} = 5$

a b c d e ⇒ a b c d
a b c e
a b d e
a c d e
b c d e

${}^{N}C_{R} * R! = {}^{N}P_{R}$

⇒ $\boxed{{}^{N}C_{R} = {}^{N}P_{R} / R! = \frac{N!}{(N-R)! * R!}}$

Properties of Combinations

1) #ways to select 0 out of N items

${}^{N}C_{0} = \frac{N!}{(N-0)! * 0!} = 1$ [0! = 1]

2) # ways to select N out of N items

$$^{N}C_{N} = \frac{N!}{(N-N)! * N!} = 1$$

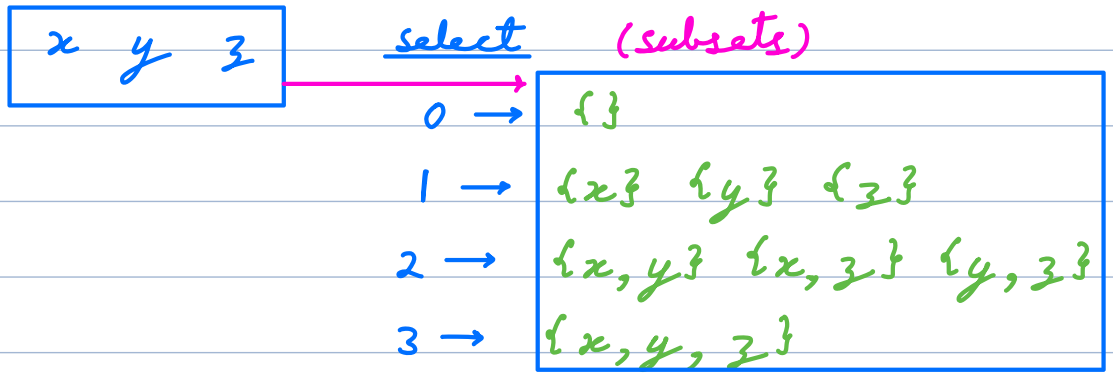
3) # ways to select (N-R) out of N objects

$$^{N}C_{N-R} = \frac{N!}{(N-(N-R))! * (N-R)!} = \frac{N!}{R! * (N-R)!} = {}^{N}C_{R}$$

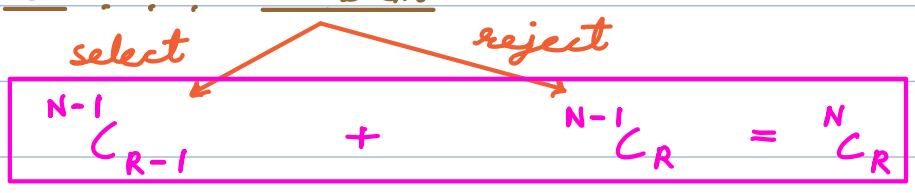
4) # ways to select <u>any no. of items</u> out of N items

$\{0, 1, 2 \ldots N\}$

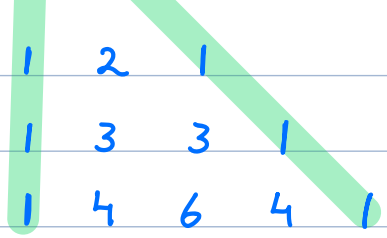
$$^{N}C_{0} + {}^{N}C_{1} + {}^{N}C_{2} \ldots {}^{N}C_{N} = 2^{N}$$



5) 1 2 3 . . . N^{th} item



Pascal Triangle



$^2C_0$ $^2C_1$ $^2C_2$

$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$

$^4C_0$ $^4C_1$ $^4C_2$ $^4C_3$ $^4C_4$

Q → Print pascal triangle for given value of N.

$$^{N-1}C_{R-1} + {}^{N-1}C_R = {}^NC_R$$

$$^NC_0 = 1 \qquad {}^NC_N = 1$$

```
C[N+1][N+1] = {0}
for i → 0 to N {
    C[i][0] = 1        C[i][i] = 1
    for j → 1 to (i-1) {
        C[i][j] = (C[i-1][j-1] + C[i-1][j]) % M
    }
}
```

TC = $O(N^2)$ SC = $O(N^2)$ / $O(1)$

Q → Given an integer N, find N^{th} column title.

N →	1	2	3	4	...	26	27	28	...	52	53	...
	A	B	C	D	...	Z	AA	AB	...	AZ	BA	...

N = 50 → AX

26	(50 − 1) = 49	23	X
26	(1 − 1) = 0	0	A
	0		

A	AA	BA
B	AB	BB
C	AC	BC
D	AD	BD
E	.	.
F	.	.
G	.	.
H	.	.
⋮	⋮	⋮
Z	AZ	BZ ...

A —— Z
1 —— 26
0 —— 25

Base → 26

N = 1000

ALL

26	(1000 - 1) = 999	11	L
26	(38 - 1) = 37	11	L
26	(1 - 1) = 0	0	A
	0		

26	(53 - 1) = 52	0	A
26	(2 - 1) = 1	1	B
	0		

53 → BA

```
ans = ""
while (N > 0) {
    r = (N-1) % 26
    ans = (char) (r + 'A') + ans      // append; 'A' = 65
    N = (N-1) / 26
}
return ans
```

TC = $O(\log_{26}(N))$ SC = $O(1)$