

Heap Sort

A = [7 3 5 1 9 6 7 1]

Sol 1 → 1) Create min-heap/max-heap.

2) getMin()/element N times & store in arr.
getMax()

$$TC = O(N + N \log(N)) = \underline{O(N \log(N))}$$

$$SC = \underline{O(N)}$$

O(1) Inplace Heap Build

i/p A[] → Heap → Sorted A[]

Min/Max

A = [7 3 5 1 9 6 7 1]

lc = A[1]

lc = A[5]

rc = A[2]

lc = A[3]

rc = A[6]

lc = A[7]

rc = A[4]

rc = A[8] x

structure → Complete Binary Tree

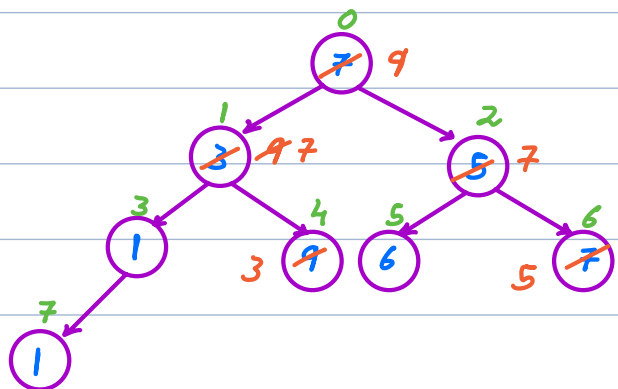
N nodes → # leaves = (N+1)/2

8 → # leaves = 4

i → lc = 2i+1

rc = 2i+2

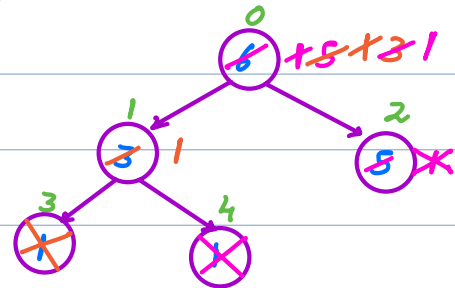
parent = (i-1)/2



TC = O(N)

Max Heap

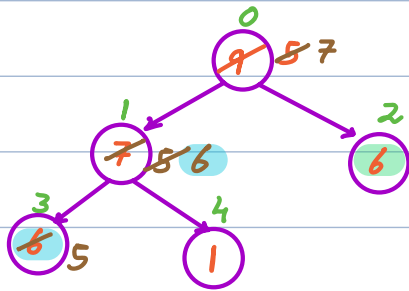
A = [7 3 5 1 9 6 7 1]

$$A = \begin{bmatrix} \cancel{8} & \cancel{3} & \cancel{5} & \cancel{1} & \cancel{1} & 7 & 7 & 9 \\ + & + & + & 5 & 6 & & & \\ \cancel{8} & & & & & & & \\ + & & & & & & & \\ \cancel{8} & & & & & & & \\ * & & & & & & & \\ + & & & & & & & \end{bmatrix}$$


$A = [1 \ 1 \ 3 \ 5 \ 6 \ 7 \ 7 \ 9]$ $SC = O(1)$

$$TC = \underline{O(N \log(N))}$$

Unstable Sorting



$A = [8 \quad 5 \quad 2 \quad 4 \quad 1 \quad 7]$ $K = 3$ $Ans = \underline{5}$

$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$ $K = 5$ Ans = 1

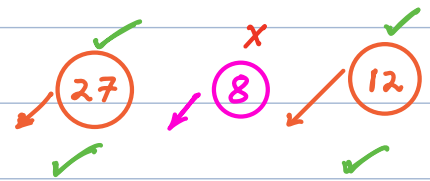
Ans = $A[N-K]$

$$TC = \underline{O(N \log(N))}$$
$$SL = O(1)$$

K	Ans
1	$A[N-1]$
2	$A[N-2]$
\vdots	\vdots

Team of 5 players

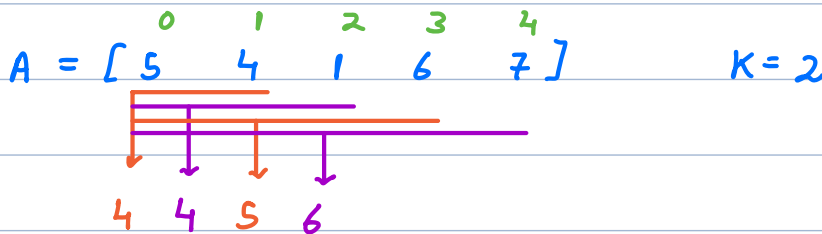
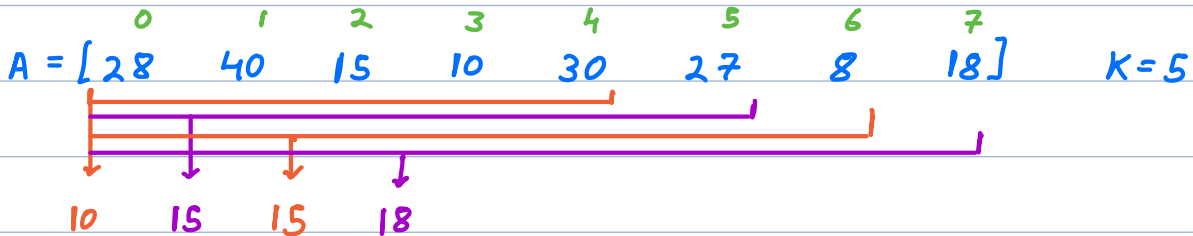
28 ✓ 40 ✓ 15 10 30 ✓



$A = [28 \quad 40 \quad 15 \quad 10 \quad 30 \quad 27 \quad 8 \quad 12]$

10_x

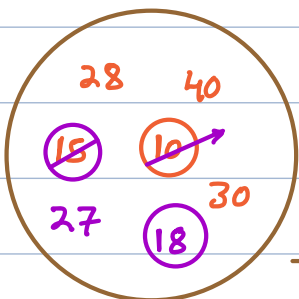
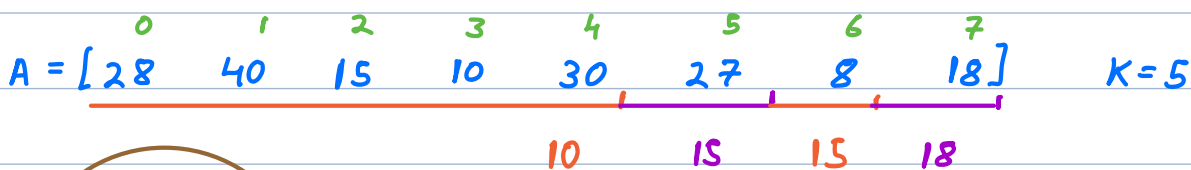
\Rightarrow Find K^{th} largest element \forall subarray from 0 to i where $i \geq (K-1)$.



K^{th} largest \rightarrow Min Heap

of size K

to store K largest elements.



Min Heap

```

// h → minHeap
for i → 0 to (K-1) {
    h.insert(A[i])
}

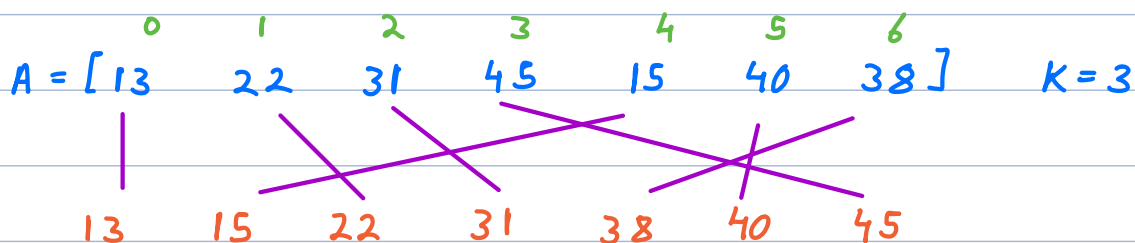
print(h.root)           // h[0]

for i → K to (N-1) {
    if (h.root < A[i]) {
        h.getMin()      // delete root
        h.insert(A[i])
    }
    print(h.root)
}

SC =  $O(K)$ 
TC =  $O(N \log(K))$ 

```

Q → Sort the given array where every element is $\leq K$ distance away from its position in sorted order. ($N \gg K$)



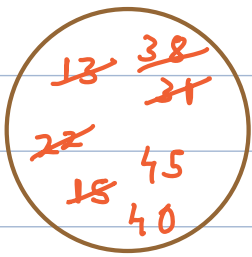
Smallest → (0 — K)

use minHeap
of size (K+1)

Array A: [13, 22, 31, 45, 15, 40, 38] with indices 0 to 6. $K=3$

Elements in the min-heap (underlined): 13, 15, 22, 31, 38, 40, 45.

Ans → 13 15 22 31 38 40 45



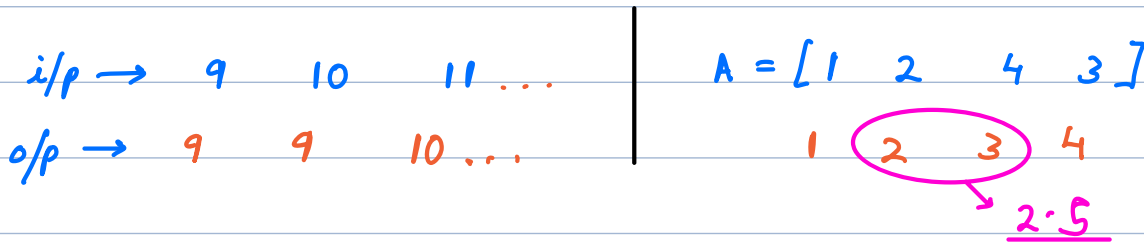
$$SC = O(K)$$

$$TC = O(N \log(K))$$

Q → Given a stream of integers.

Find median & integer intake.

→ middle element in sorted order.

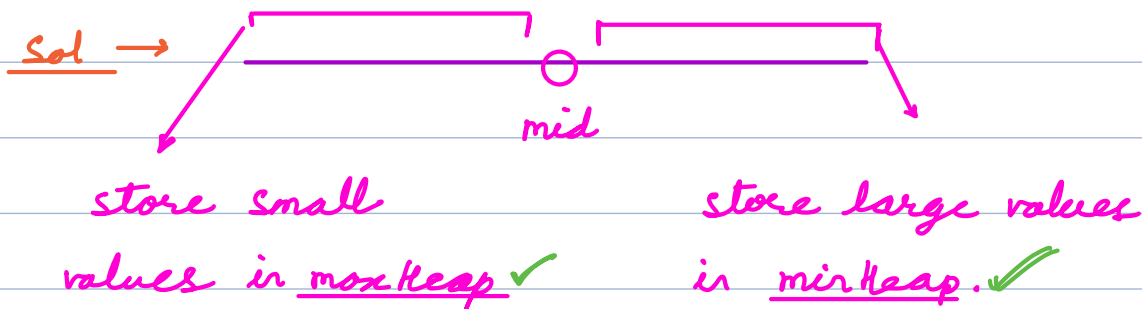


odd → middle is sorted

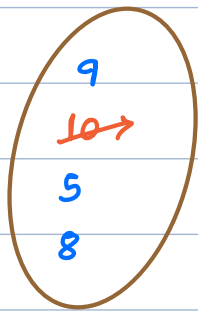
even → first middle is sorted. (2)

Brute force → & intake, sort & find middle.

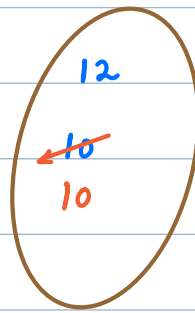
$$TC = O(N * N \log(N)) \quad SC = O(1)$$



Ans = maxHeap root node



Max Heap (small)



Min Heap (large)

i/p \rightarrow 9 12 10 5 8
 o/p \rightarrow 9 9 10 9 9

// i/p $\rightarrow x$

if ($x \leq \text{root of maxHeap}$) {
 insert in maxHeap

 if (size of maxHeap - size of minHeap > 1)
 move root of maxHeap to minHeap

} else {

 insert in minHeap

 if (size of minHeap - size of maxHeap > 0)
 move root of minHeap to maxHeap

}

$$TC = \underline{O(N \log(N))} \quad SC = \underline{O(N)}$$
