

Q → Given a rotated sorted array of distinct elements find index of element K.

$A = [8, 12, 15, 19, 2, 5, 6]$ $K = 12$

Ans = 1

Bruteforce → for $i \rightarrow 0$ to $N-1$ {
if ($A[i] == K$) return i
}
TC = $O(N)$ SC = $O(1)$

$A = [8, 12, 15, 19, 2, 5, 6]$

Part 1 | Part 2

Elements in Part 2 < Elements in part 1.

check if an element 'x' is in part 1 or 2?

if ($x < A[0]$) ⇒ 'x' is in Part 2
else 'x' is in part 1

// Search Space

$l = 0$ $r = N-1$

while ($l \leq r$) {

$mid = (l+r)/2$

// check mid

if ($A[mid] == K$) return mid

// Decide left/right

if ($K < A[0]$) { // K is in part 2

if ($A[mid] < A[0]$) { // mid is in Part 2

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        if (K < A[mid]) r = mid - 1
        else l = mid + 1
    } else { // mid is in Part 1
        l = mid + 1
    }
} else { // K is in part 1
    if (A[mid] >= A[0]) { // mid is in Part 1
        if (K < A[mid]) r = mid - 1
        else l = mid + 1
    } else { // mid is in Part 2
        r = mid - 1
    }
}
}
} return -1

```

$TC = O(\log_2(N))$ $SC = O(1)$

Q → Find square root of N ($N \rightarrow$ perfect sq.)

$N = 49$

Ans = 7

$N = 25$ Ans = 5

Bruteforce → Range of sqrt of $N \rightarrow [1 \quad N]$

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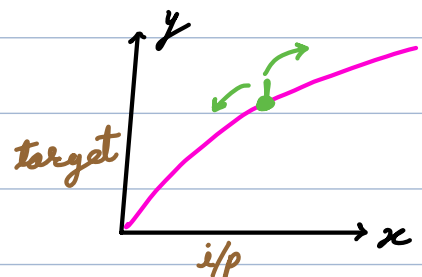
for i → 1 to N {
    if (i * i == N) return i
}

```

$TC = O(\sqrt{N})$

Search Space → Integers 1 to N

Target → \sqrt{N}



$$N = 45$$

$$x = 5$$

$$5 * 5 < N$$

Binary Search on Answer

$$l = 1 \quad r = N$$

while ($l \leq r$) {

$$\text{mid} = l + (r - l) / 2$$

if ($\text{mid} * \text{mid} == N$) return mid

if ($\text{mid} * \text{mid} < N$) $l = \text{mid} + 1$

else $r = \text{mid} - 1$

}

$$TC = O(\log_2(N))$$

$$SC = O(1)$$

Q → Find N^{th} magical number.

Magical No. → Divisible by either x or y or both.

$$x = 2$$

} 2 4 5 6 8 10 12 ...

$$y = 5$$

$$N = 3 \rightarrow \text{Ans} = \underline{5}$$

$$N = 6 \rightarrow \text{Ans} = \underline{10}$$

$$N = 4$$

$$x = 3$$

}

$$3$$

$$5$$

$$6$$

$$\underline{9}$$

$$y = 5$$

Bruteforce →

$$i = 1 \quad \text{cnt} = 0$$

while ($\text{cnt} < N$) {

if ($i \% x == 0 \parallel i \% y == 0$)

cnt++

i++

} return (i-1)

Smallest $\rightarrow \underline{\text{Min}(x, y)}$

Largest $\rightarrow \underline{N * \text{Min}(x, y)}$

of multiples of $x \leq K \Rightarrow \underline{K/x}$

$$\begin{array}{l} x = 6 \\ K = 50 \end{array} \} 50/6 = \underline{8}$$

of multiples of x or y or both $\leq K \rightarrow$

$x = 3 \rightarrow 3 \quad 6 \quad 9 \quad 12 \quad 15 \quad 18 \quad 21 \quad 24 \quad 27 \quad \underline{30}$
 $33 \quad 36 \quad 39$

$y = 10 \rightarrow 10 \quad 20 \quad \underline{30} \quad 40$

$K = 40$

$$\begin{array}{r} \frac{K}{x} + \frac{K}{y} - \frac{K}{\cancel{x*y}} \\ 13 + 4 - 1 = \underline{16} \end{array} \quad \text{Ans} = \underline{16}$$

$x = 6 \rightarrow 6 \quad 12 \quad 18 \quad 24 \quad \underline{30} \quad 36$

$y = 10 \rightarrow 10 \quad 20 \quad \underline{30} \quad 40$

$K = 41$

$$\frac{K}{x} = 6 \quad \frac{K}{y} = 4 \quad \frac{K}{\text{lcm}(x, y)} = 1 \quad \text{Ans} = 6 + 4 - 1 = \underline{9}$$

of multiples of x or y or both $\leq K \rightarrow \frac{K}{x} + \frac{K}{y} - \frac{K}{\text{lcm}(x, y)}$

multiples $\propto K$

$$\frac{x * y}{\text{gcd}(x, y)} = \text{lcm}(x, y)$$

$l = \min(X, Y)$

$r = N * \min(X, Y)$

while ($l \leq r$) {

mid = $l + (r - l) / 2$

// check mid

cnt = mid / x + mid / y - mid / lcm(x, y)

if (cnt == N && (mid % x == 0 || mid % y == 0))

return mid

// Decide left / right

if (cnt < N) $l = \text{mid} + 1$

else $r = \text{mid} - 1$

}

TC = $O(\log(N * \min(X, Y)))$

SC = $O(\log(\max(X, Y))) \rightarrow \text{gcd}()$

Median of Array \rightarrow Middle element in sorted order.

[2 8 1 10 5] \rightarrow 1 2 5 8 10

2 4 5 7 8 12

average $\rightarrow \frac{5+7}{2} = 6$

Q \rightarrow Find median of array.

Sol \rightarrow 1) Sort the array } TC = $O(N \log(N))$
2) Ans = $A[N/2]$ } SC = $O(1)$

Q \rightarrow Given two sorted arrays, find median of combined data.

$$A = [6 \quad 8 \quad 11]$$

$$B = [1 \quad 7]$$

$$\text{Ans} = \underline{7}$$

$$A = [1 \quad 3 \quad 5 \quad 6]$$

$$B = [2 \quad 8 \quad 10]$$

$$\text{Ans} = \underline{5}$$

Sol 1 → 1) Merge two sorted arrays.

2) Ans = middle element.

$$TC = O(N+M)$$

$$SC = O(N+M)$$

$$O(1)$$

Apply merge 2 sorted array without storing the result, just counting the selection.

Sol 2 →

$$A = [1 \quad 3 \quad 5 \quad 6 \quad 7 \quad 9]$$

$$B = [2 \quad 4 \quad 7 \quad 10 \quad 12]$$

↑
mid

cnt # element
 $< A[mid] = x$

$$(x + mid) + 1$$

$$len = N+M$$

$$\text{median position} = (N+M)/2$$

$$TC = O(\log(N))$$

$$(\# \text{element} < K) \propto (\text{position of } K \text{ in sorted order})$$

1) Pick any array & apply BS to find ans by counting $\# \text{elements} < A[i]$ in both arrays.

2) If ans not found, try with another array.

Pseudo-code → HW