

Q → Find the smallest number that can be formed by re-arranging the digits of the given number in an array.

$[\# \text{ digits} \leq 10^5]$ $[0 \leq A[i] \leq 9]$

$A = [\overset{0}{5} \ \overset{1}{6} \ \overset{2}{3} \ \overset{3}{5} \ \overset{4}{4} \ \overset{5}{2} \ \overset{6}{0} \ \overset{7}{8} \ \overset{8}{5}]$
 $\hookrightarrow \quad 0 \quad 2 \quad 3 \quad 4 \quad 5 \quad 5 \quad 5 \quad 6 \quad 8$

$A = [5 \quad 3 \quad 3 \quad 2 \quad 4 \quad 2]$
 $\hookrightarrow \quad 2 \quad 2 \quad 3 \quad 3 \quad 4 \quad 5$

sort in asc. order

Sorting Algo.

$TC = O(N \log(N))$

$SC = O(1)$

$A = [\overset{0}{5} \ \overset{1}{6} \ \overset{2}{3} \ \overset{3}{5} \ \overset{4}{4} \ \overset{5}{2} \ \overset{6}{0} \ \overset{7}{8} \ \overset{8}{5} \ \overset{9}{6} \ \overset{10}{3} \ \overset{11}{4}]$
 $\rightarrow \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark \ \checkmark$

Freq. array $F[i] = \text{frequency of } i$
 size = 10

$F = [\overset{0}{1} \ \overset{1}{0} \ \overset{2}{1} \ \overset{3}{2} \ \overset{4}{2} \ \overset{5}{3} \ \overset{6}{2} \ \overset{7}{0} \ \overset{8}{1} \ \overset{9}{0}]$

for $i \rightarrow 0$ to $(N-1)$ {

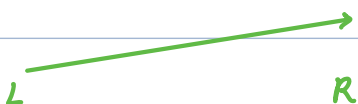
$F[A[i]]++$

$TC = O(N)$

$SC = O(A[i])$

}

$= O(10) = O(1)$


 L R

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for d → 0 to 9 {
  for i → 1 to F[d] {    TC = O(N)
    print(d)
  }
}

```

Count Sort TC = O(N) SC = O(A[i]) → O(1)

If $A[i] \leq 10^9$, can we use count sort?

No, \because SC will be high \Rightarrow MLE error.

usual array max length \rightarrow 10^6 to 10^7

If $-5 \leq A[i] \leq 5$, can we use count sort?

Yes

$A = [\overset{0}{2} \quad \overset{1}{-3} \quad \overset{2}{0} \quad \overset{3}{2} \quad \overset{4}{-1} \quad \overset{5}{-5} \quad \overset{6}{-3} \quad \overset{7}{2}]$
 \rightarrow ✓ ✓ ✓ ✓ ✓ ✓ ✓

Range = [min max] \rightarrow (max - min + 1)

$5 - (-5) + 1 = \underline{11}$

$F = [\overset{0}{1} \quad \overset{1}{0} \quad \overset{2}{2} \quad \overset{3}{0} \quad \overset{4}{1} \quad \overset{5}{1} \quad \overset{6}{0} \quad \overset{7}{3} \quad \overset{8}{0} \quad \overset{9}{0} \quad \overset{10}{0}]$

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for i → 0 to (N-1) {

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  F[A[i] - minA] ++

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}

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$A[i] = -3 \rightarrow$ index = 2

smallest $\rightarrow 0$

$A[i] - \text{smallest} \rightarrow$ index in F[]
-5

$A[i] - (-5) = \underline{A[i] + 5}$

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for x → minA to maxA {
  for i → 1 to F[x - minA] {
    print(x)
  }
}

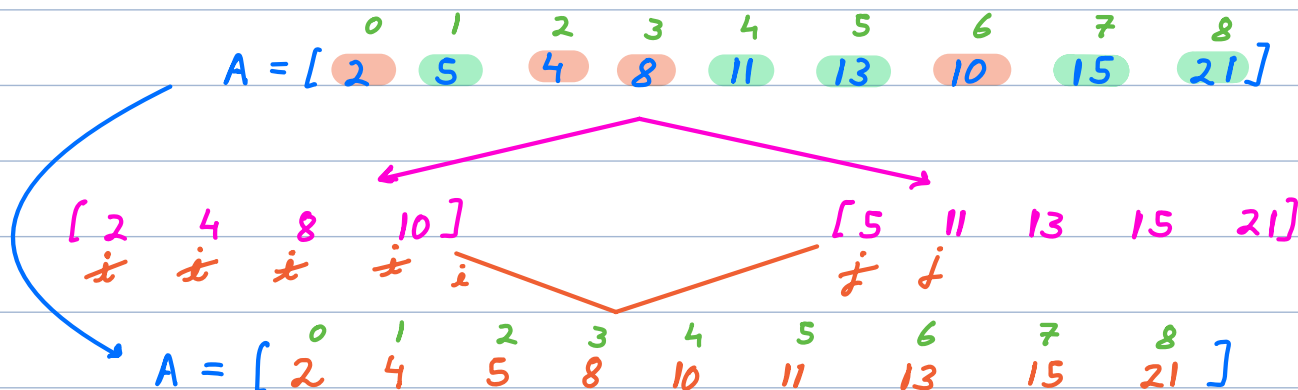
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$TC = O(N)$ $SC = O(A[i])$

Q → Given an integer array where all **odd elements** are sorted & all **even elements** are sorted. Sort the array.

$A = [\overset{0}{2}, \overset{1}{5}, \overset{2}{4}, \overset{3}{8}, \overset{4}{11}, \overset{5}{13}, \overset{6}{10}, \overset{7}{15}, \overset{8}{21}]$

Sorting Algo → $TC = O(N \log(N))$ → First sol. with $TC < O(N \log(N))$



Steps → 1) Divide the array into 2 parts,
one containing all even elements &
other containing all odd elements. → $TC = O(N)$

$SC = O(N)$

2) Merge two sorted arrays into one.

$A[N], B[M]$

$C[N+M]$

$i=0, j=0, k=0$

while ($i < N \ \&\& \ j < M$) {

if ($A[i] \leq B[j]$) {

$C[k] = A[i] \quad i++ \quad k++ \quad // \ C[k++] = A[i++]$

} else {

$C[k] = B[j] \quad j++ \quad k++$

}

}

while ($i < N$) {

$C[k] = A[i] \quad i++ \quad k++$

}

while ($j < M$) {

$C[k] = B[j] \quad j++ \quad k++$

}

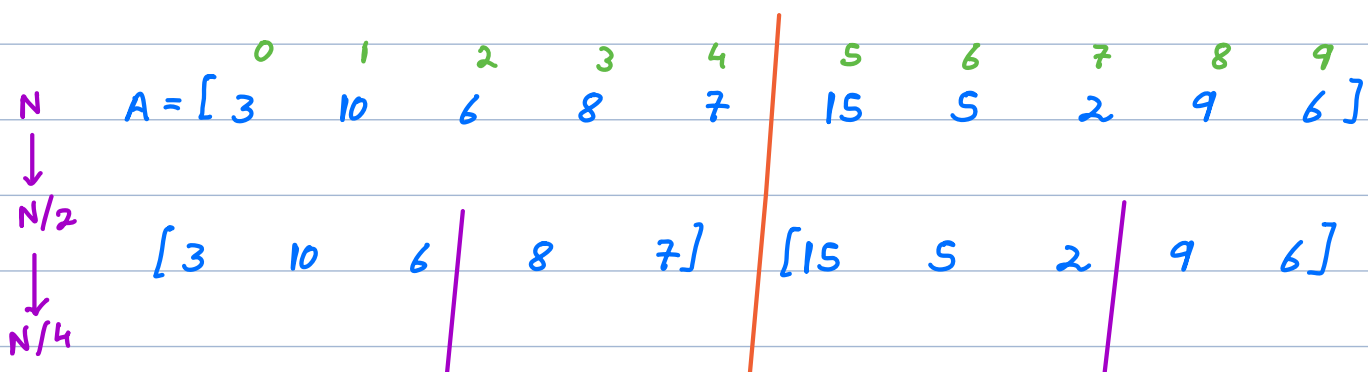
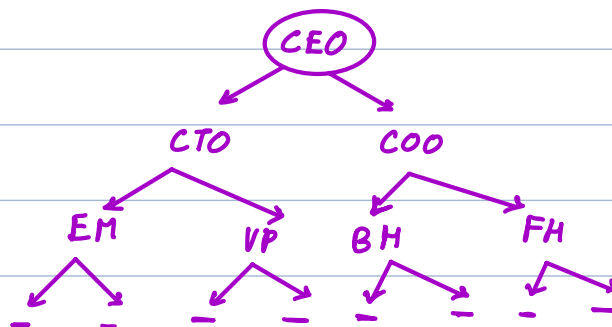
return C

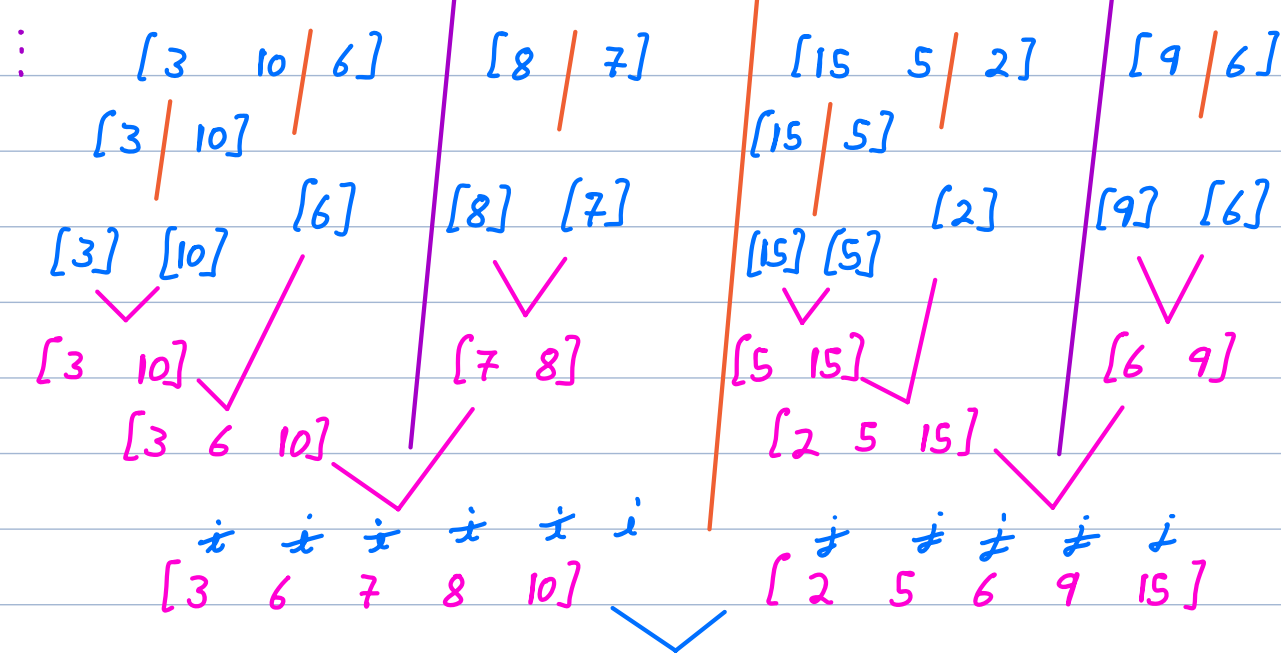
$TC = O(N+M)$

Overall $\rightarrow TC = \underline{O(N)} \quad SC = \underline{O(N)}$

Merge Sort

Divide & Conquer





$A = [2 \ 3 \ 5 \ 6 \ 6 \ 7 \ 8 \ 9 \ 10 \ 15]$

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void sort(A[], 0l, N-1r) {
    if (l >= r) return
    mid = (l+r)/2
    sort(A, l, mid)
    sort(A, mid+1, r)
    merge(A, l, mid, r) → TC = O(N) SC = O(N)
    l ← mid      (mid+1) ← r
}

```

$$TC = \underline{O(N \log(N))}$$

$$SC = O(N + \log(N)) \rightarrow \underline{O(N)}$$

Q → Given 2 integer array A & B. Find the count of pairs (i, j) s.t $A[i] > B[j]$.

$A = [7 \ 3 \ 5]$

$B = [2 \ 0 \ 6]$

$(7, 2) \ (7, 0) \ (7, 6)$

$(3, 2) \ (3, 0)$

$(5, 2) \ (5, 0) \quad \text{Ans} = \underline{7}$

Brute force $\rightarrow \forall i, j$ check $A[i] > B[j]$. $TC = O(N * M)$

$SC = O(1)$

Sol \rightarrow Sort the array. ✓

$A = [3 \ 5 \ 7]$ $B = [0 \ 2 \ 6]$
 $i \ i \ i$ $i \ i \ i \ i$

$ans = 0 \ 3 \ 6 \ 7$ $[0 \ 2 \ 3 \ 5 \ 6 \ 7]$

\rightarrow select $B[j] \Rightarrow ans += \# \text{ remaining elements in } A$

$[i \ N-1] \rightarrow N-1-i+1 \Rightarrow \underline{N-i}$

$TC = O(N \log(N) + M \log(M) + N + M)$

$= \underline{O(N \log(N) + M \log(M))}$ $SC = \underline{O(N+M)}$

Q \rightarrow Find the # pairs (i, j) s.t $i < j$ & $A[i] > A[j]$.

$A = [10 \ 3 \ 8 \ 15 \ 6]$

$A = [5 \ 2 \ 6 \ 1]$

$Ans = \underline{4}$

i	j
0	1
0	3
1	3
2	3

i	j
0	1
0	2
0	4
2	4
3	4

$Ans = \underline{5}$

$A = [5 \ 3 \ 1 \ 4 \ 2]$

$Ans = \underline{7}$

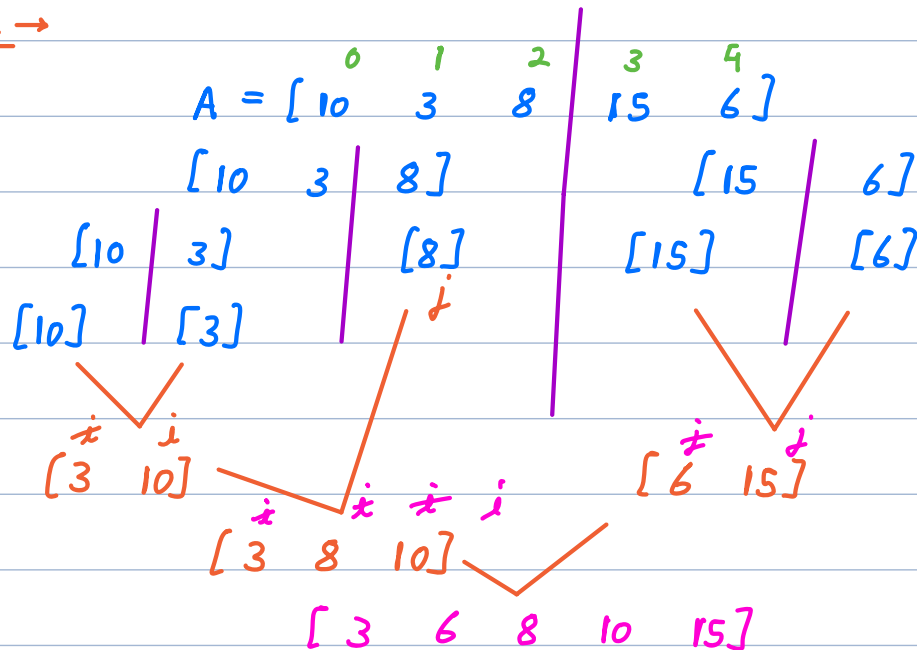
$i \rightarrow 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 3$
 $j \rightarrow 1 \ 2 \ 3 \ 4 \ 2 \ 4 \ 4$

ans = 0

Bruteforce → for $i \rightarrow 0$ to $(N-2)$ {
for $j \rightarrow (i+1)$ to $(N-1)$ {
if $(A[i] > A[j])$ ans ++
}
} return ans

TC = $O(N^2)$ SC = $O(1)$

Sol →



(10, 3) (8, 6)

(10, 8) (10, 6)

(15, 6)

TC = $O(N \log(N))$ SC = $O(N)$

select from right half → ans += # remaining elements in left

$l - mid$

$mid - i$

Stable Sort

while sorting relative order of equal elements should not change.

$A = [8 \quad 7_1 \quad 10 \quad 5 \quad 7_2 \quad 9]$
sort 5 $7_1 \quad 7_2$ 8 9 10

Relative order of equal data becomes critical if original data has a defined order.

Ensure stability in code \rightarrow For equal elements use index to compare.
