Valuation of Express Certificate on Royal Dutch Shell Company

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Product Characteristics

The retail certificate used in this paper is the express certificate on Royal Dutch Shell PLC issued by Deutsche Bank on 17 July 2018. The certificate is traded in Börse Frankfurt and the historic price records are accessed via Finanzen.net.

The product described has two possible termination scenarios: early termination or termination on the maturity date. Early termination occurs if, on any autocall observation date, the reference price reaches or exceeds the autocall barrier price. In this case, the investor receives a cash payment on the following autocall payment date, equal to the applicable autocall payment.

If the product does not terminate early, it will reach its maturity date. At maturity, the investor will receive different cash payments based on the final reference price:

Observ. Dates	Barrier Prices	Payment Dates	Payments
19 July 2019	EUR 29.44	24 July 2019	EUR 108.10
17 July 2020	EUR 27.97	22 July 2020	EUR 116.20
16 July 2021	EUR 26.50	21 July 2021	EUR 124.30
15 July 2022	EUR 25.02	20 July 2022	EUR 132.40

Table 1. Early Termination Dates and Payments

If the final reference price is at or above EUR 23.55, the cash payment will be EUR 140.50. If the final reference price is at or above EUR 19.14 and below EUR 23.55, the cash payment will be EUR 100. If the final reference price is below EUR 19.14, the cash payment will be calculated by multiplying the product's notional amount by the ratio of the final reference price to EUR 29.44.

The payoff profile at the end of the certificate life is shown in Figure 1. When compared with the underlying payoff, it can be seen that the certificate provides the same return as the underlying until the price exceeds €19.14. Until the underlying price reaches €14.50, the certificate generates a greater return, but otherwise, it is the opposite. An important consideration is that early termination can cause the product to have a different payoff.

Nelson-Siegel-Svennson Method

Using Nelson-Siegel-Svennson method, the risk free rate is estimated 5.1%

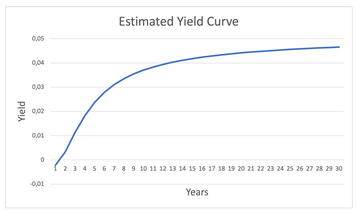

Figure 1. Estimated Yield Curve

Monte Carlo Simulation & Valuation

Geometric Brownian motion is used in Monte Carlo simulation with 65,536 observations to simulate the probable pricing paths of the underlying. The valuation is performed using the available information at the issue date of the certificate. The estimated price of the certificate is €112.49.

Figure 2. Payoff Profile

Figure 3. Certificate Price Estimation

Figure 3 presents the estimated prices of the certificate with respect to the underlying prices. The payoff profile reflects the payoff information, as shown in Figure 3. We observe a similar linear increase in the certificate price in line with the underlying price since there is a payment of the reference price multiplied by 100/29,44 if the reference price is less than €19.14. The peak in the graph is produced by the early redemption that provides greater returns than the underlying. Finally, the curve converges as it becomes highly probable that the certificate will be redeemed earlier.

Figure 4. The Underlying Stock Price History

Figure 5. Actual & Estimated Price of Certificate

In Figure 4, the daily closing prices of the underlying stock is plotted. The similarity between the this graph and the certificate's historic prices is apparent. An important observation in the graph is the drop during the COVID-19 period, which affected negatively both the certificate and the underlying.

Figure 5 plots the historic prices of the certificate and the estimated prices obtained by the valuation. The blue curve is the estimated price sequence and the red curve is the actual price sequence. The difference between the actual and predicted prices is due to uncertain factors. In the following table, error measures are shown.

MSE	MSE	RMSE
9.75884	196.4103	14.0146

Table 2. Performance Measure

Sensitivity Analysis

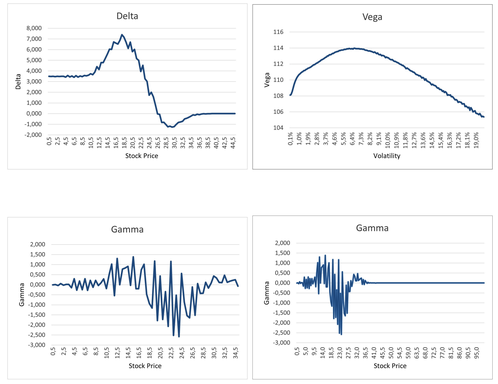

Figure 6. Sensitivity Analysis

Delta

The delta sensitivity analysis is in line with the certificate price estimate and the payoff profile. It shows the price movement of the underlying and the certificate. It has a shape similar to a sinusoidal wave as its characteristic. It starts with a positive value and stays stable until the price is around €10, indicating a linear increase in the certificate as the price of the underlying increases. At the end, it converges to zero as the payoff of the underlying becomes certain at €140.50. In between, there is a wave shape which results from the autocall payments.

Gamma

Despite high noise in the graph resulting from not having enough samples, the trend movement of the graph is visible. The gamma starts stable and creates a bump in the positive region and then in the negative region, finally converging to zero.

Vega

The graph has a concave shape, which shows that there is a volatility that maximizes the expected return of the certificate, and higher and lower volatility does not provide higher expected returns. Generally, option price increases with volatility increase, but this opposite result comes from the payoff structure of the certificate. There is the same rate of return below €19.14 with the underlying and at the high underlying prices the certificate under-performs compared to the underlying. Thus, the only excess return provided by the certificate is between €19.25 and €41.50. As a result high volatility causes stock price to deviate from this range.

Portfolio Insurance

A fictitious portfolio is constructed using the underlying stock. Starting from 19 July 2018, using Brownian motion the stock price movement is simulated 10,000 times with 2 month average returns and volatility. The distribution is log-normal since it is not possible to observe negative stock prices.

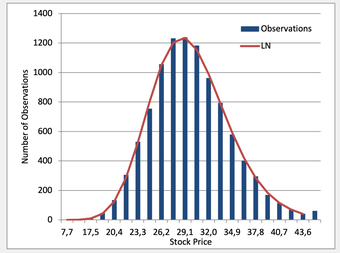

Figure 7. Estimated 1-Year Stock Price Distribution

Figure 7 shows the distribution of the stock price for a 1-year forecast. The expected stock price is €29.85 with a variance of 23.33. Thus, the confidence interval of the future stock price is [13.863, 32.797] with a 95% significance level. The red curve represents the expected distribution of Brownian motion equation, obtained using Equation **??**,

Risk Management

To manage tail risk put options can be included in the portfolio. Portfolios with 0%, 2%, 4%, 6%, 8% and 10% put options are generated for a 1-year time horizon using Monte Carlo simulation with 65,457 instances.

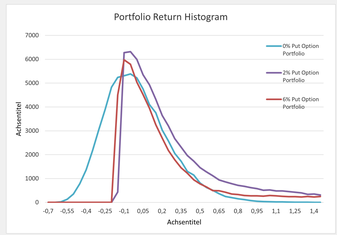

Figure 8. Portfolios with Put Options

	0%	2%	4%	6%	8%
No Stress	3.7%	34.1%	65.0%	95.4%	125.9%
Stress	-17.4%	80.7%	179.2%	277.4%	375.6%

Table 3. Average Estimated Returns Portfolios with Option Fractions

Stress Scenario Analysis

To measure the effectiveness of the put option fractions in the portfolios, 20% sudden drop is applied to the underlying prices at the half of the investment horizon. The same portfolios with put option shares are used in the analysis. The value of the put options are calculated using Black-Scholes-Merton model.

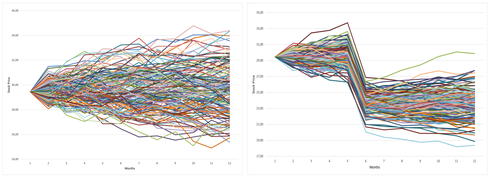

Figure 9. Stock Price Simulation (No Drop [1] & Sudden Drop [2])

The figure 8 is estimated evolution of the underlying stock prices using Brownian Motion and Monte Carlo simulation. It exhibits an increasing volatility

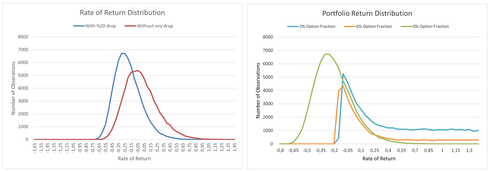

Figure 10. Expected Return Distribution (Stress No Stress)

Figure 11. Return Distribution Under Stress

In Figure 9, the expected return distribution of the underlying at the end of 1 year that has an expected rate of return 0.036 stock price of €29.53. When %20 sudden drop applied to the underlying stock prices, the return distribution changes by shifting the mean to the left. Also, the distribution shrinks since there is less time left.

In Figure 10, the rate of return distribution after the 20% drop is shown. The portfolio without a put option exhibits a normal distribution. However, as the put option fraction increases in the portfolio, a sudden cut-off appears, indicating protection against extreme drops. Compared to 8,

References

[1] John Hull.
Options, Futures, and Other Derivatives, Global Edition .
Pearson Deutschland, 10th edition, 2018.