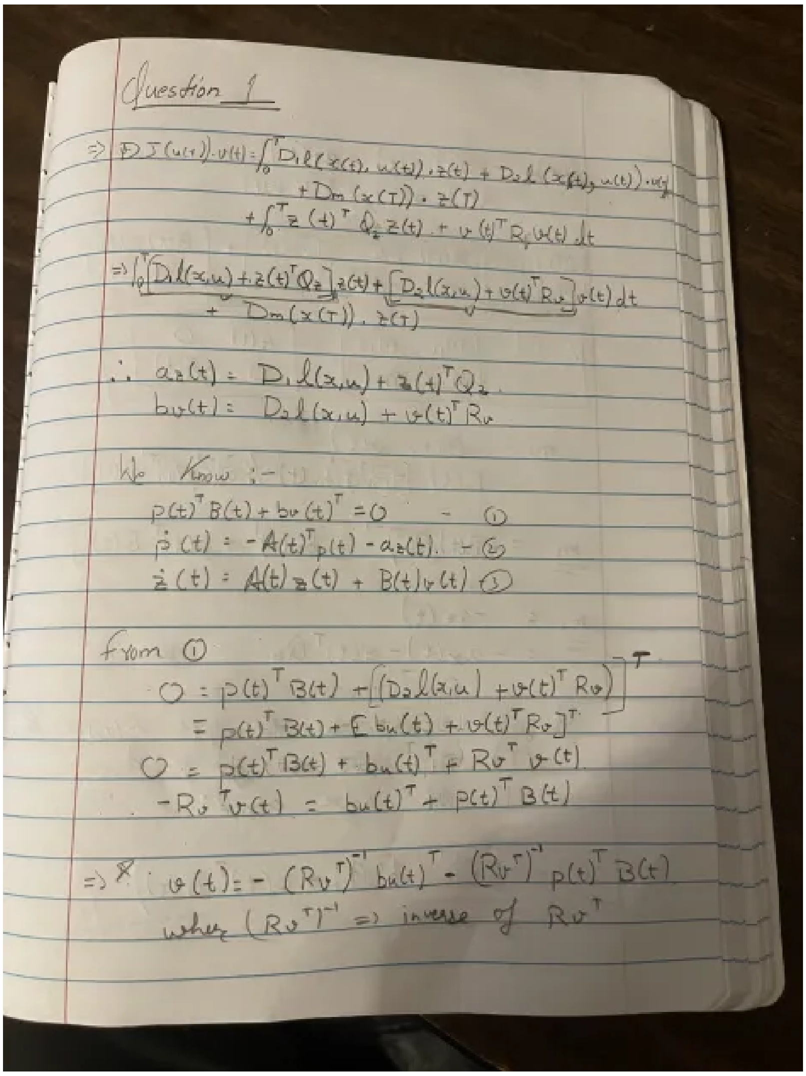
# HOMEWORK-4

## Question 1



Question 1

$$\Rightarrow DJ(u(t))\cdot v(t) = \int_0^T D_1 l(x(t), u(t))\cdot z(t) + D_2 l(x(t), u(t))\cdot v(t)$$
$$+ Dm(x(T))\cdot z(T)$$
$$+ \int_0^T z(t)^T Q_z z(t) + v(t)^T R_v v(t)\, dt$$

$$\Rightarrow \int_0^T \underbrace{[D_1 l(x,u) + z(t)^T Q_z]}\, z(t) + \underbrace{[D_2 l(x,u) + v(t)^T R_v]}\, v(t)\, dt$$
$$+ Dm(x(T))\cdot z(T)$$

$$\therefore\ a_z(t) = D_1 l(x,u) + z(t)^T Q_z$$
$$b_v(t) = D_2 l(x,u) + v(t)^T R_v$$

We know :-

$$p(t)^T B(t) + b_v(t)^T = 0 \quad - (1)$$
$$\dot{p}(t) = -A(t)^T p(t) - a_z(t) \quad - (2)$$
$$\dot{z}(t) = A(t) z(t) + B(t) v(t) \quad (3)$$

from (1)

$$0 = p(t)^T B(t) + [(D_2 l(x,u) + v(t)^T R_v)]^T$$
$$= p(t)^T B(t) + [b_u(t) + v(t)^T R_v]^T$$
$$0 = p(t)^T B(t) + b_u(t)^T + R_v^T v(t)$$
$$-R_v^T v(t) = b_u(t)^T + p(t)^T B(t)$$

$$\Rightarrow v(t) = -(R_v^T)^{-1} b_u(t)^T - (R_v^T)^{-1} p(t)^T B(t)$$

where $(R_v^T)^{-1} \Rightarrow$ inverse of $R_v^T$

$$\dot{p}(t) = -A(t)^{T} p(t) - a_z(t)$$

$$\dot{z}(t) = A(t)\, z(t) + B(t)\, v(t)$$

$$\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t) \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B(t)\, v(t) \\ -a_z(t) \end{bmatrix}$$

$$\boxed{\therefore M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t) \end{bmatrix}}$$

$$m_1 = B(t)\, v(t)$$

$$= B(t) \left[ -(R_v^T)^{-1} b_u(t)^{T} - (R_v^T)^{-1} p(t)^T B(t) \right]$$

$$\underline{\underline{m_1}} = -B(t) (R_v^T)^{-1} \left[ b_u(t)^{T} + p(t)^T B(t) \right]$$

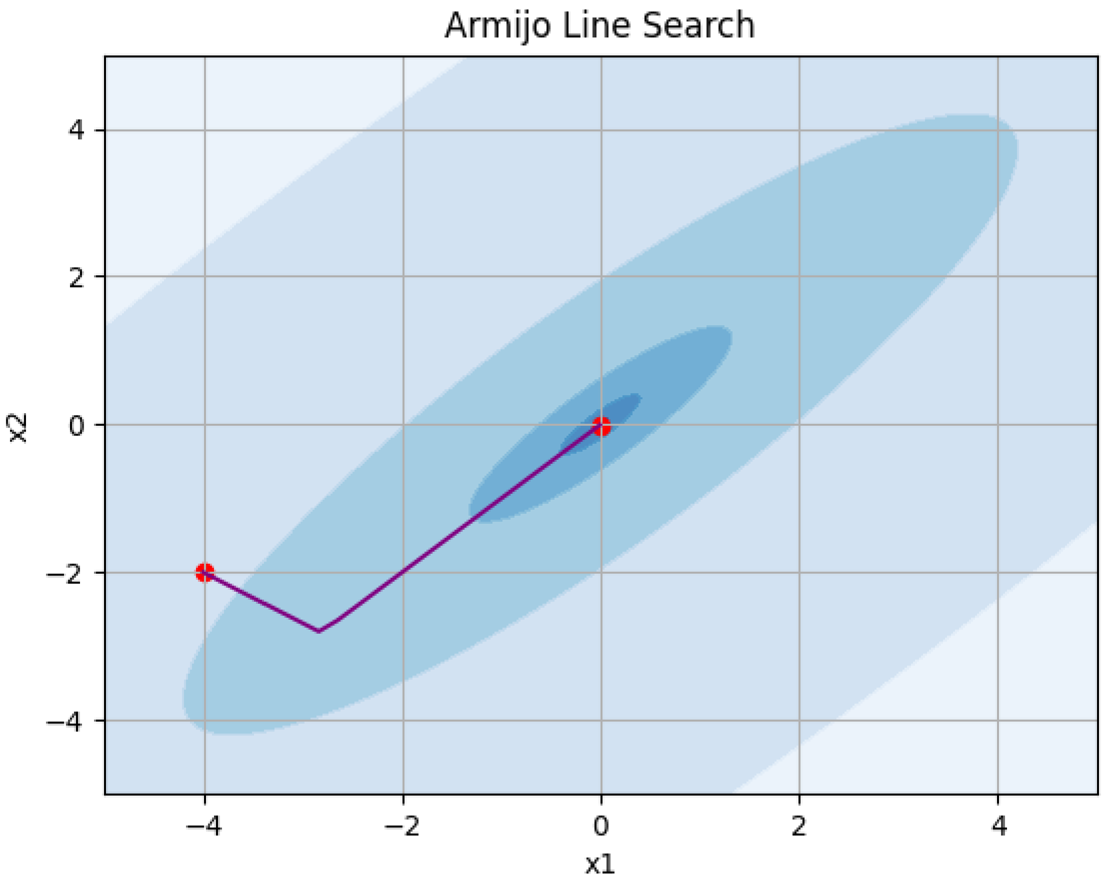
$$\underline{\underline{m_2}} = -a_z(t)$$

$$= -a_x(t) - z(t)^T Q_z$$

Using solve_bvp, with condits $z(0) = 0$ & $p(T) = p_1$, we can get the desired $v(t)$ as solve above :-

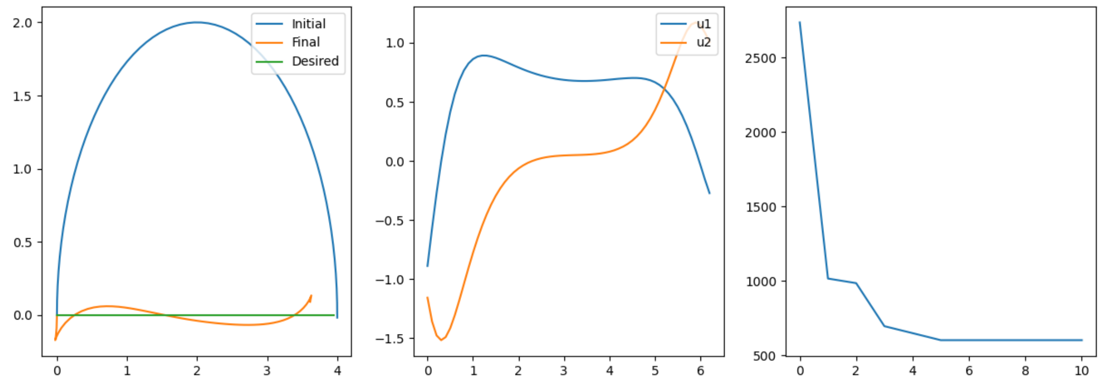
$$\boxed{v(t) = -(R_v^T)^{-1} \left( b_u(t) + p(t)^T B(t) \right)}$$

Question 2

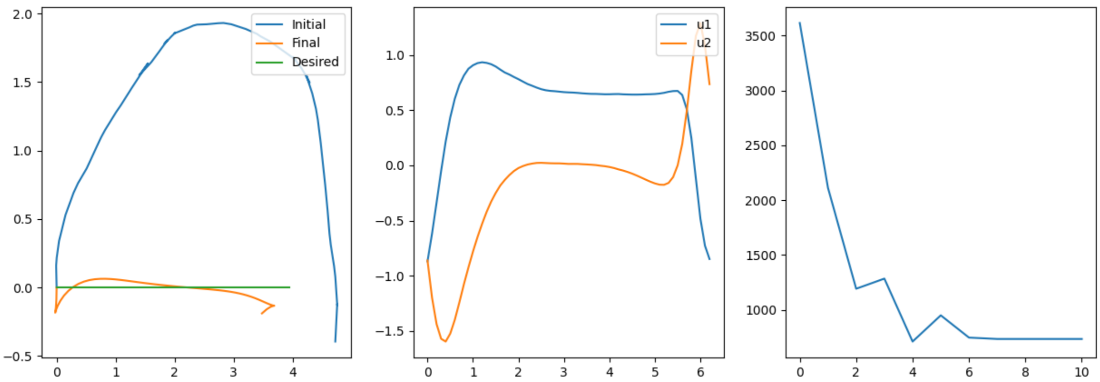


## Question 3

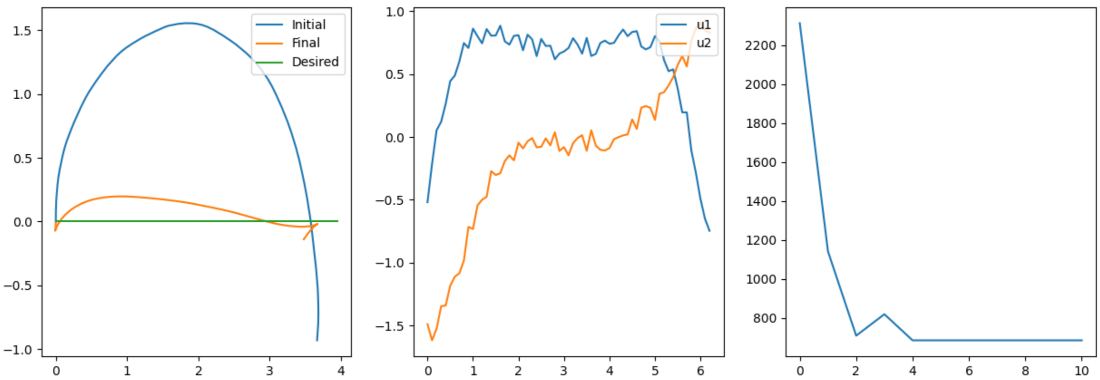
1. With Constant Control Signal



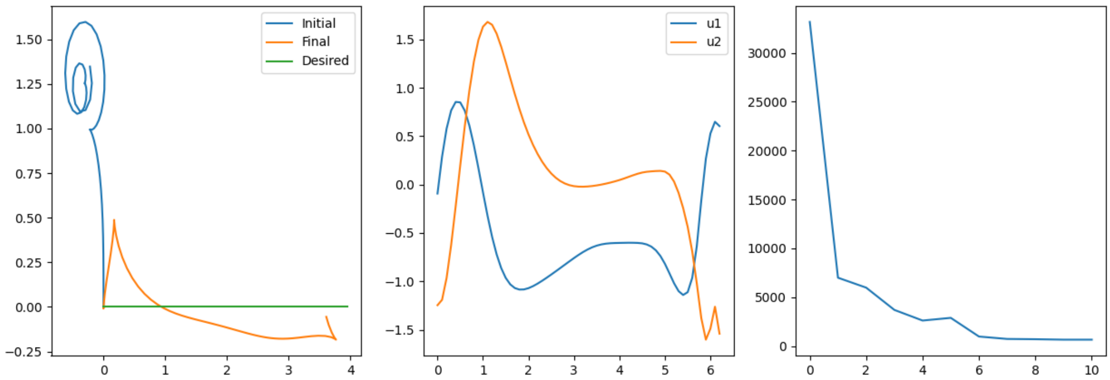

2. With Random Normal Distribution




3. With Random Uniform Distribution




3. With x as a cos distribition and y with input




4. With y as a cos distribition and x with input