AST5220 – lecture 2

An introduction to the CMB power spectrum

Hans Kristian Eriksen

January 20th, 2011

Cosmology in ~five slides

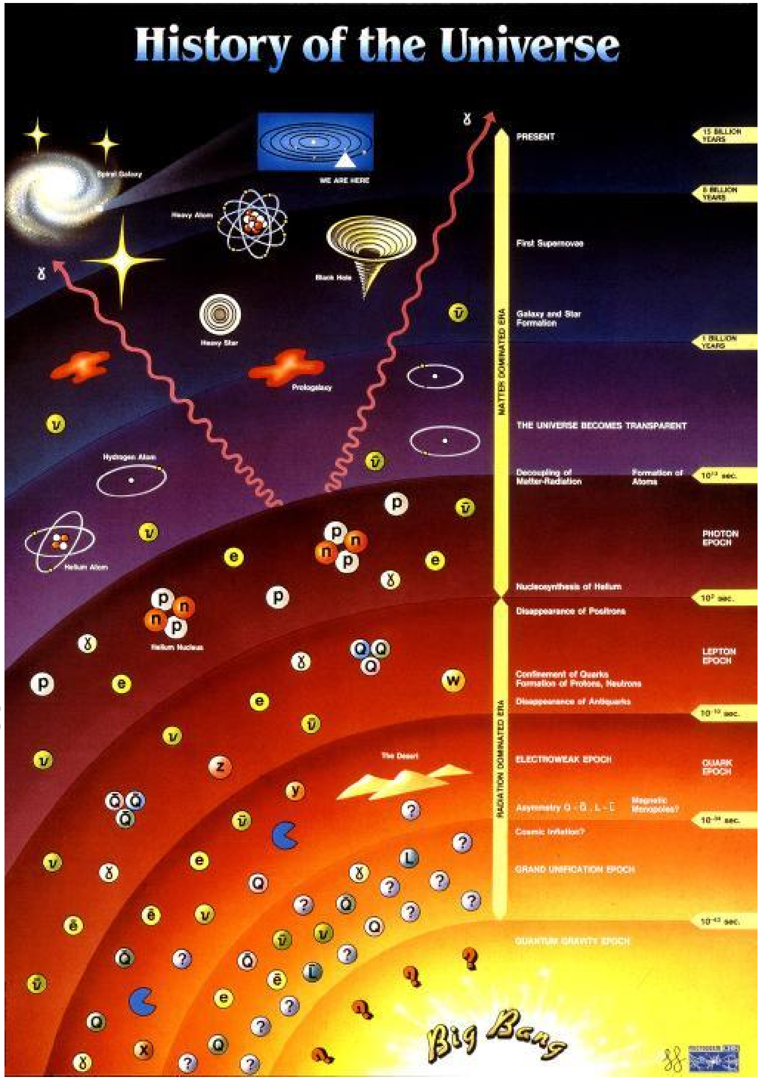
1) The Big Bang model

The basic ideas of Big Bang:

- The universe expands today
  - Therefore it must have previously been smaller
  - Very early it must have been very small
- When a gas is compressed, it heats up
  - The early universe must have been *very* hot
- High-energy photons destroys particles
  - Only elementray particles may have existed very early
  - More complex particles was formed as the temperature fell

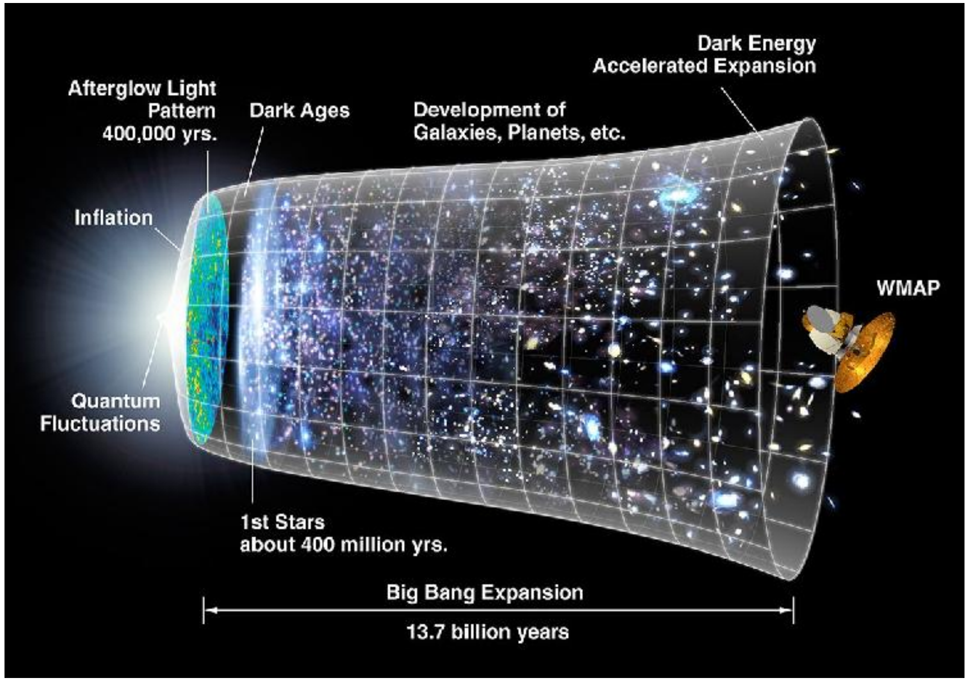
Important epochs in the CMB history of the universe:

- Creation (!) – about 14 billions years ago
- Inflation – fast expansion about 10^{-35} s after Big Bang; structures forms
- Recombination – the temperature falls below 3000K about 380,000 years after Big Bang; hydrogen is formed



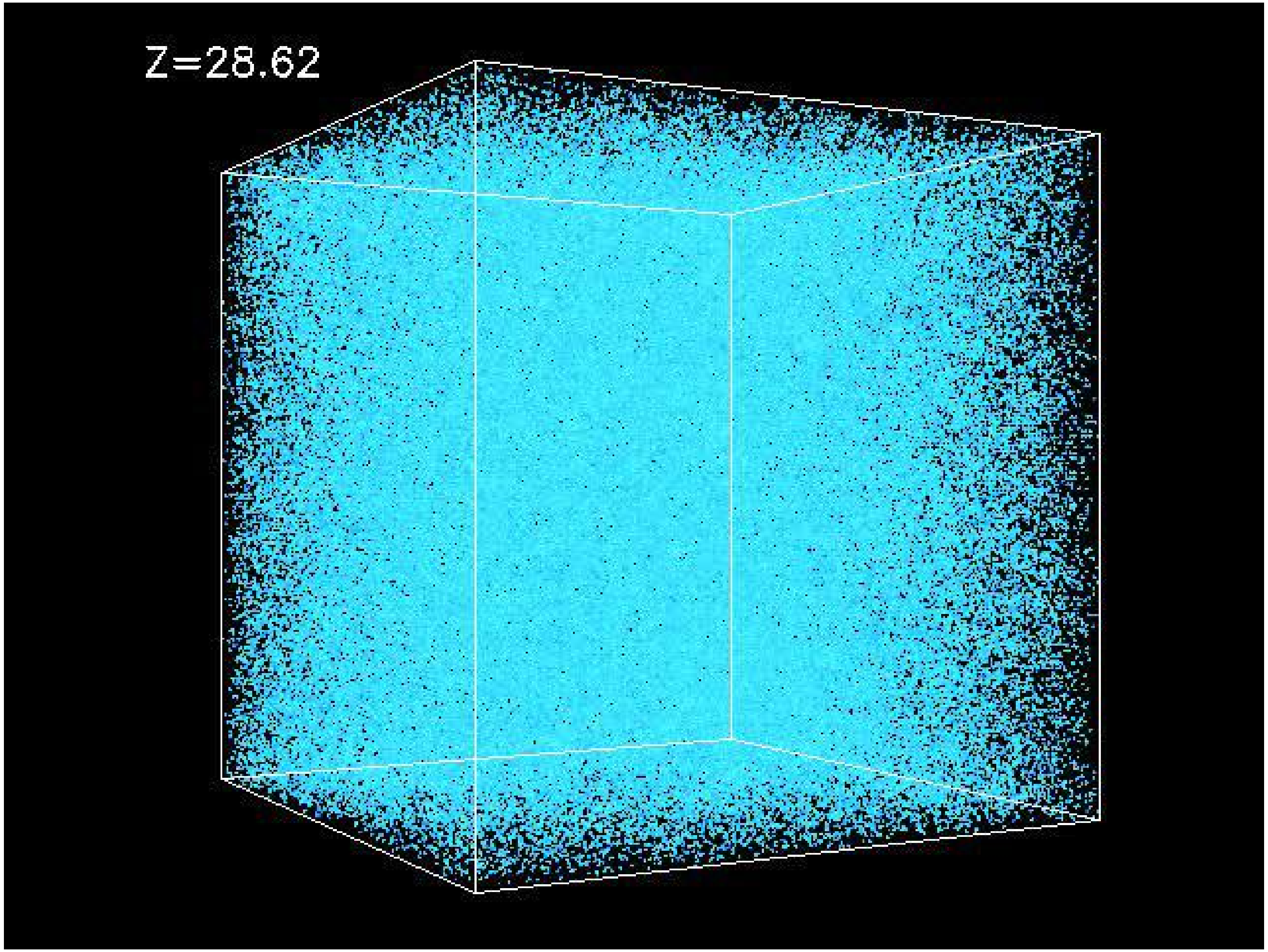
2) Inflation and initial conditions

- We observe that the universe is
  - very close to flat (euclidean)
  - isotropic (looks the same in all directions)
- Why? Best current idea: Inflation!
  - Short period with exponential expansion
  - The size of the universe increases by a factor of 10^{23} during 10^{-34} seconds!
- Implications:
  - The geometry is driven towards flat
  - All pre-inflation structure is washed (streched) out

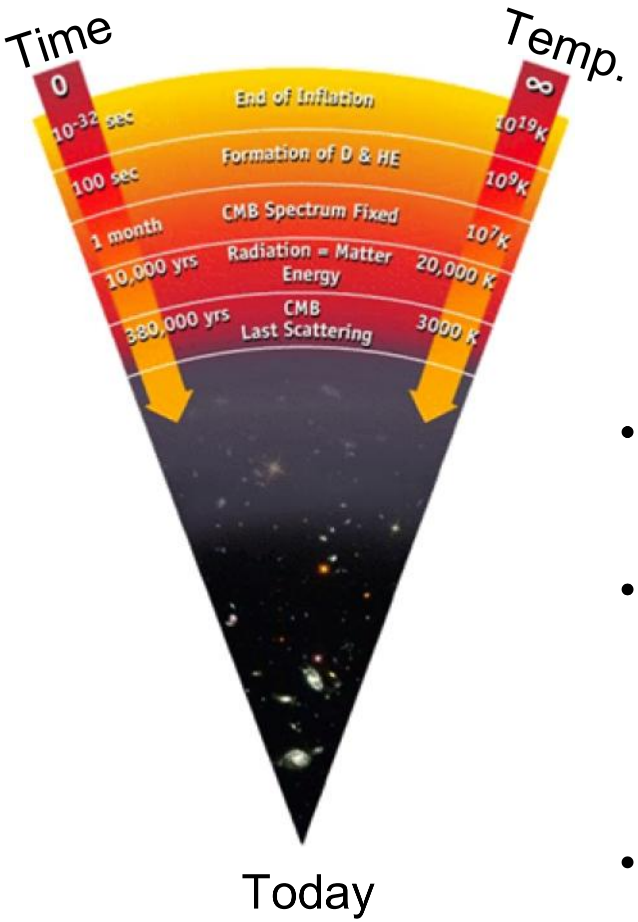


- But most importantly: The universe is filled with a plasma consisting of high-energy photons and elementary particles
- Quantum fluctuations created small variations in the plasma density

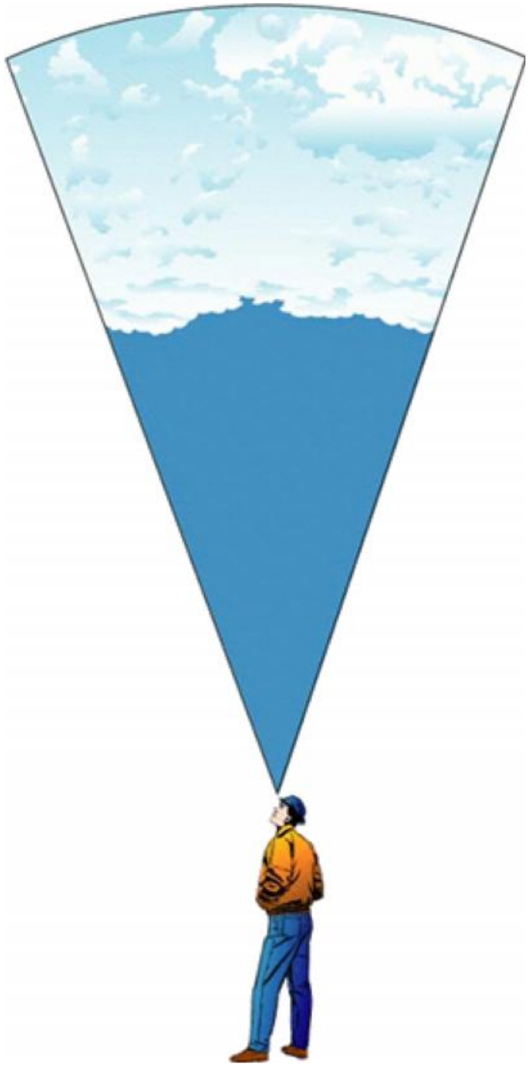
3) Gravitational structure formation



4) Cosmic background radiation



- The universe started as a hot gas of photons and free electrons
  - Frequent collisions implied thermodynamic equilibrium
  - Photons could only move a few meters before scattering on an electron
- The gass expanded quickly, and therefore cooled off
- Once the temperature fell below 3000°K, electrons and protons formed neutral hydrogen
- With no free electrons in the universe, photons could move freely through the universe!



4) Cosmic background radiation

- The universe started as a hot gas of photons and free

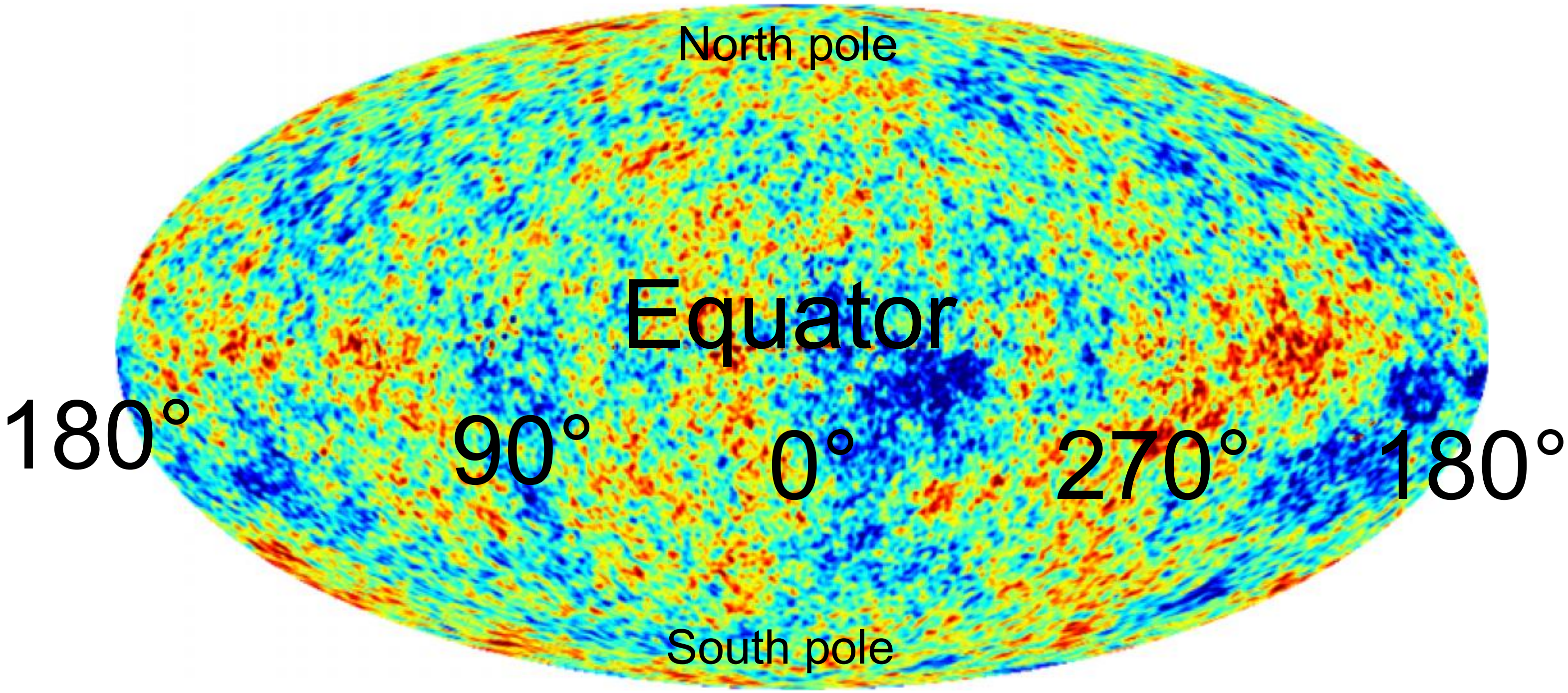
With no free electrons in the universe, photons could move freely through the universe!

The CMB is our oldest and cleanest source of information in the early universe!

Mathematical description of CMB fluctuations

CMB observations and maps

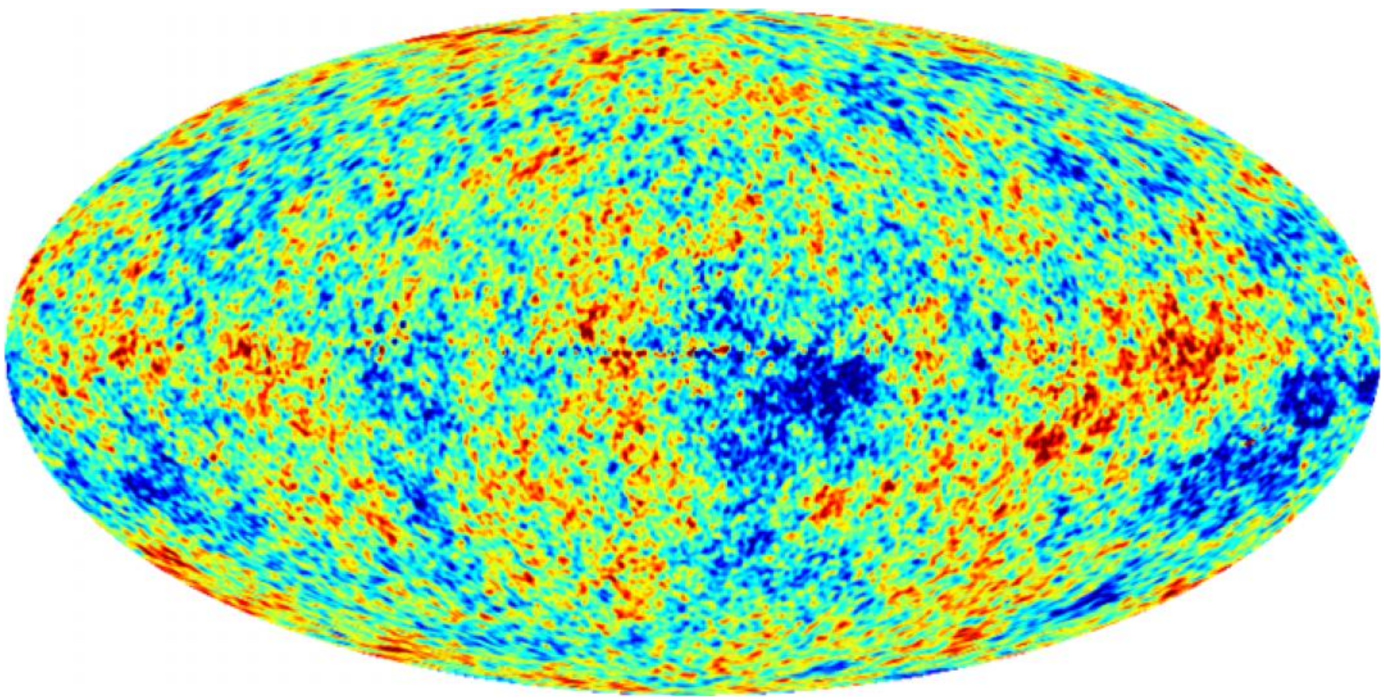
- A CMB telescope is really just an expensive TV antenna
- You direct the antenna in some direction, and measure a voltage
  - The higher the voltage, the stronger the incident radiation
  - The stronger the incident radiation, the hotter the CMB temperature
- You scan the sky with the antenna, and produce a map of the CMB temperature
- Often displayed in the Mollweide projection:



CMB observations and maps

- We are more interested in physics than in the details of a CMB map
- Different physical effects affect different physical *scales*
  - Inflation works on all scales, from very small to very large
  - Radiation diffusion only works on small scales

$\Rightarrow$ Useful to split the map into well-defined scales



Fourier transforms

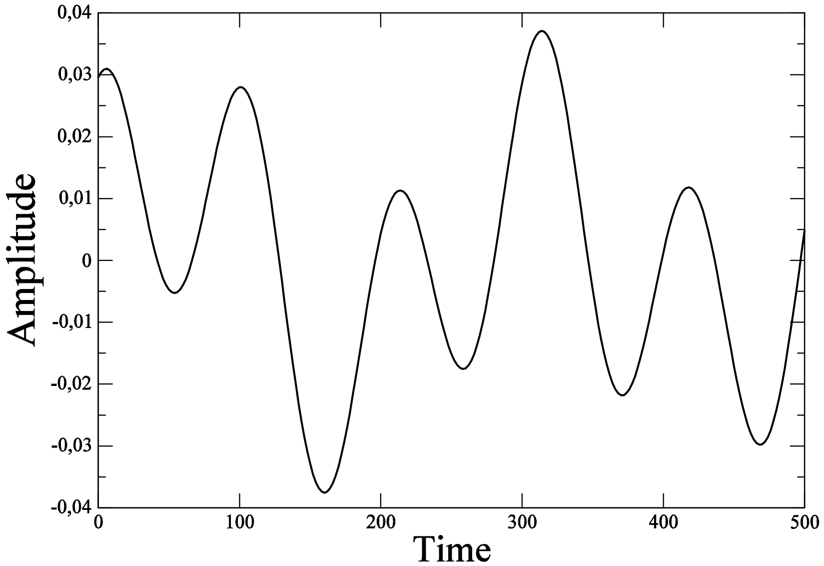
- "Theorem": Any function may be expanded into wave functions
- In flat space, this is called the Fourier transform:

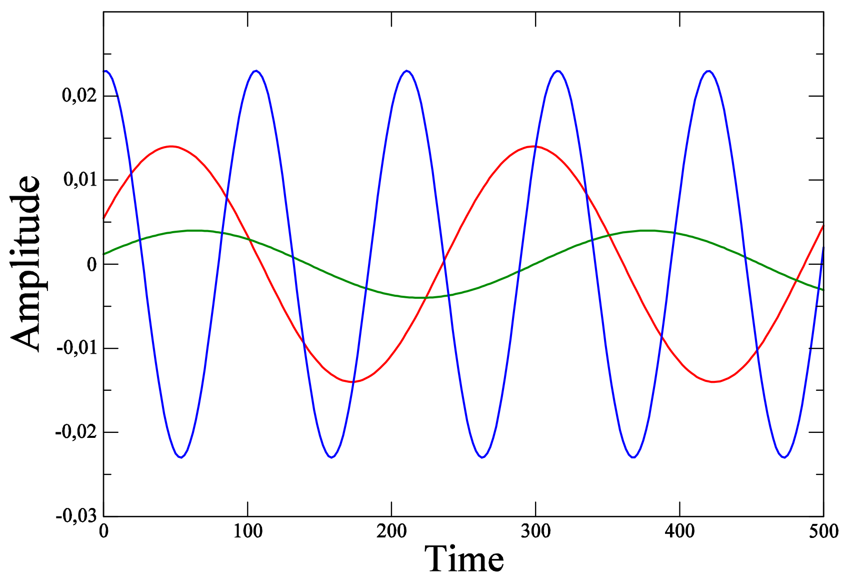
$$f(x) = \sum_k \left[b_k \cos(kx) + c_k \sin(kx)\right] \;= \sum_k a_k e^{ikx}$$

  - $|a_k|$ describes the amplitude of the mode (ie., wave)
  - The phase of a_k determines the position of the wave along the x axis

- The Fourier coefficents are given by

$$a_k = \int f(x) e^{-ikx} dx$$

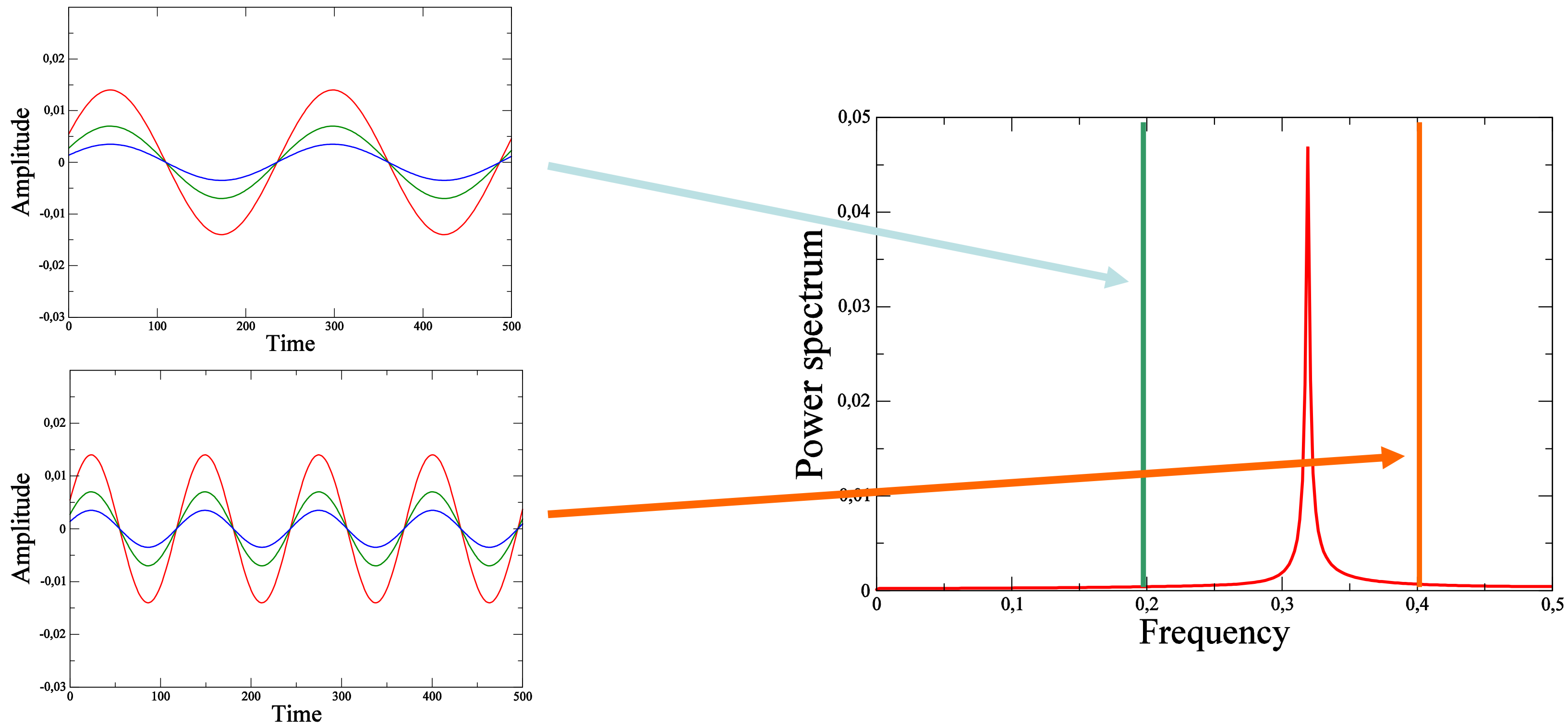




The power spectrum

- For ”noise-like” phenomena, we are only interested the *amplitude* of the fluctuations as a function of *scale*
  - Remember that the CMB is just noise from the Big Bang!
  - The specific position of a given maximum or minimum is irrelevant irrelevant

- This is quantified by the power spectrum, $P(k) = |a_k|^2$
  - Power of a given scale = the square of the Fourier amplitude



Laplace' equation on the sphere

- The Fourier transform is only defined in flat space
- The required basis wave functions in a given space is found by solving Laplace' equation:

$$\nabla^2 \psi = 0$$

- Since the CMB field is defined on a sphere, on has to solve the following equation in spherical coordinates (where $\psi(\theta,\phi) = \Theta(\theta)\Phi(\phi)$):

$$\frac{\Phi(\phi)}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta(\theta)}{d\theta}\right) + \frac{\Theta(\theta)}{\sin^2\theta}\frac{d^2\Phi(\phi)}{d\phi^2} + \ell(\ell+1)\Theta(\theta)\Phi(\phi) = 0$$

- This is (fortunately!) done in other courses, and the answer is

$$\psi = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}P_{\ell m}(\cos\theta)e^{im\phi} \equiv Y_{\ell m}(\theta,\phi)$$

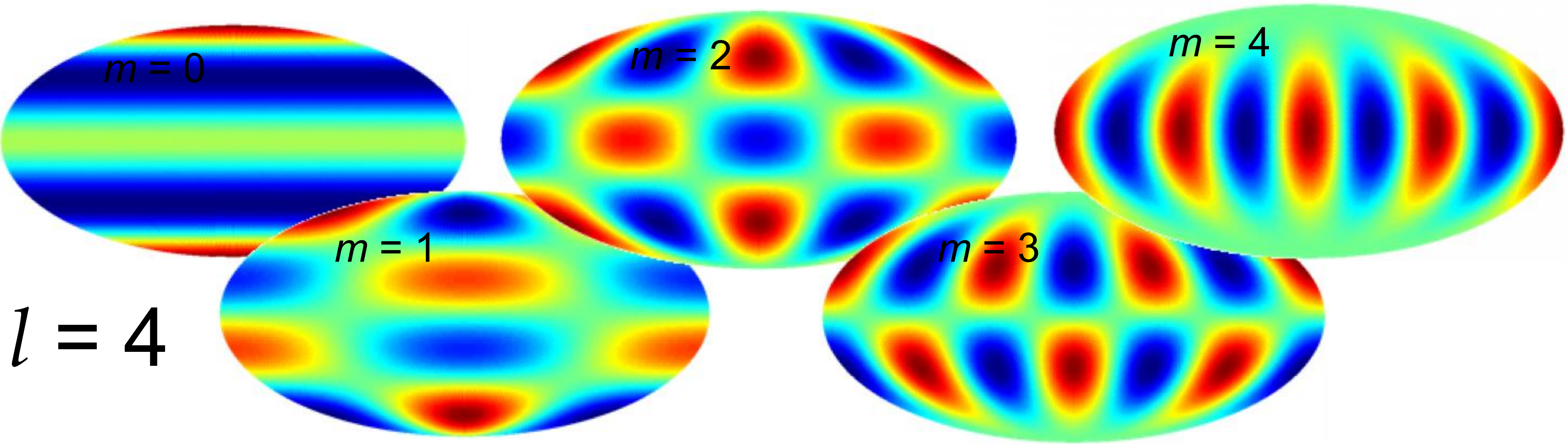
for $l \geq 0$ and m = - l, ..., l

Spherical harmonics

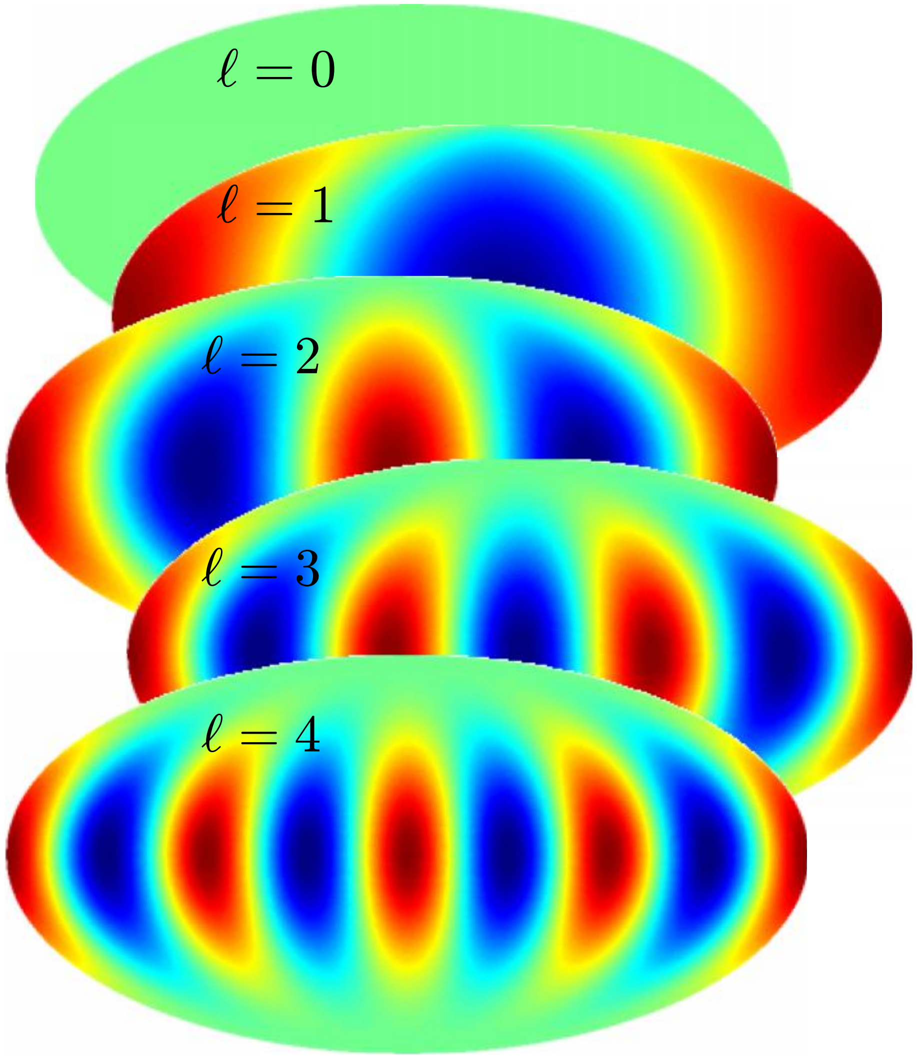
- The spherical harmonics are wave functions on the sphere
  - Completely analogous to the complex exponential in flat space

$$e^{ikx} \leftrightarrow Y_{\ell m}(\theta, \phi)$$

- Instead of wave number k, these are described by l and *m*
  - l determines ”the wave length” of the mode
    - l is the number of waves along a meridian
  - *m* determines the ”shape” of the mode
    - m is the number of modes along equator



Relationship between l and scale

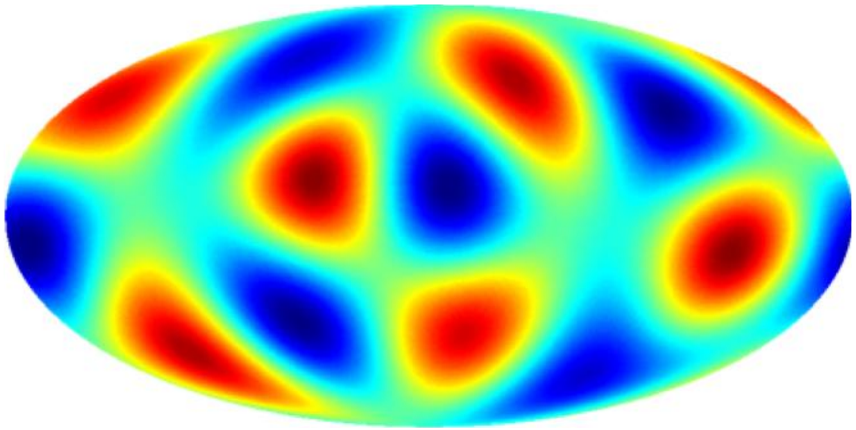


- If l increases by one, the number of waves between 0 og 2π increases by one
- The wave length is therefore

$$\lambda = \frac{2\pi}{\ell}$$

- This only holds along equator
- For a general mode (summed over m) we say more generally that the typical ”size ”of a spot is

$$\lambda \sim \frac{180^\circ}{\ell}$$



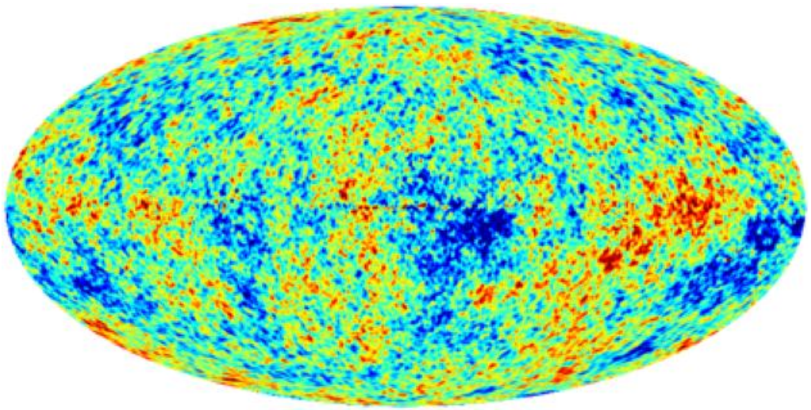
Spherical harmonics transforms

”Theorem”: Any function defined on the sphere may be expanded into spherical harmonics:

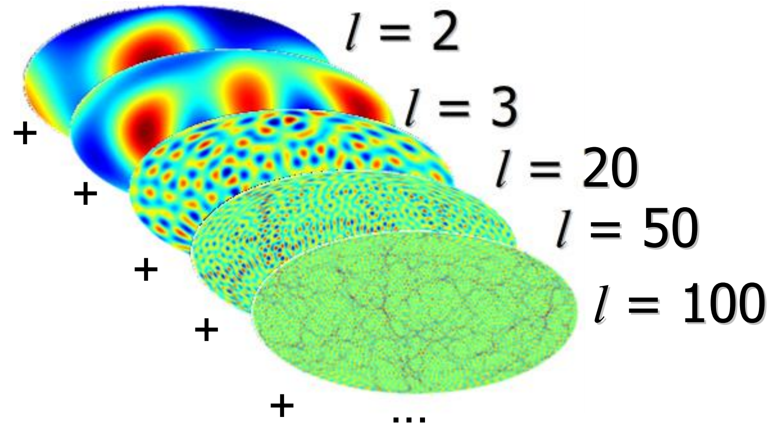
$$T(\hat{n}) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n})$$

The expansion coeffients are given by

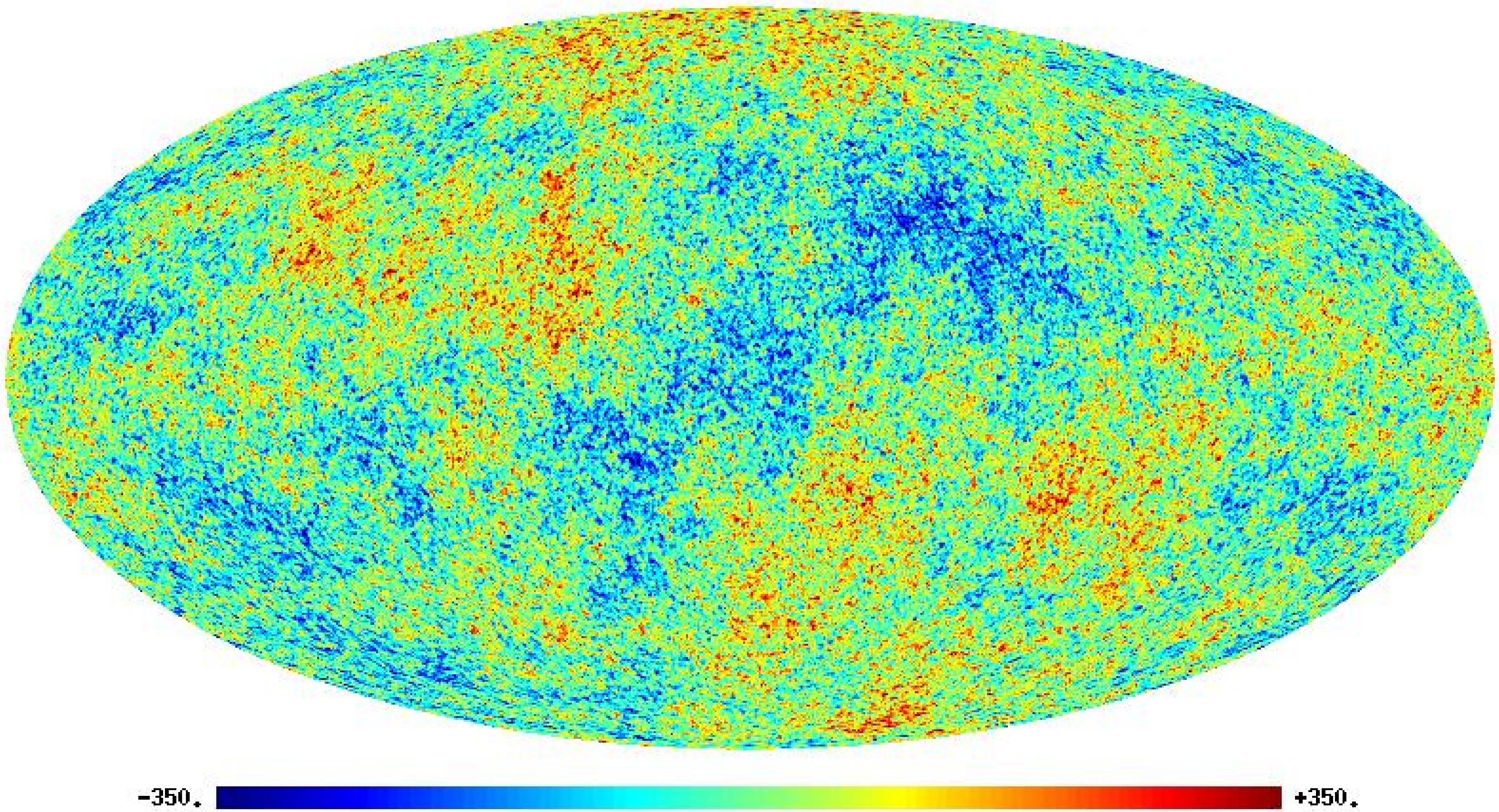
$$a_{\ell m} = \int_{4\pi} T(\hat{n}) Y^*_{\ell m}(\hat{n}) d\Omega$$



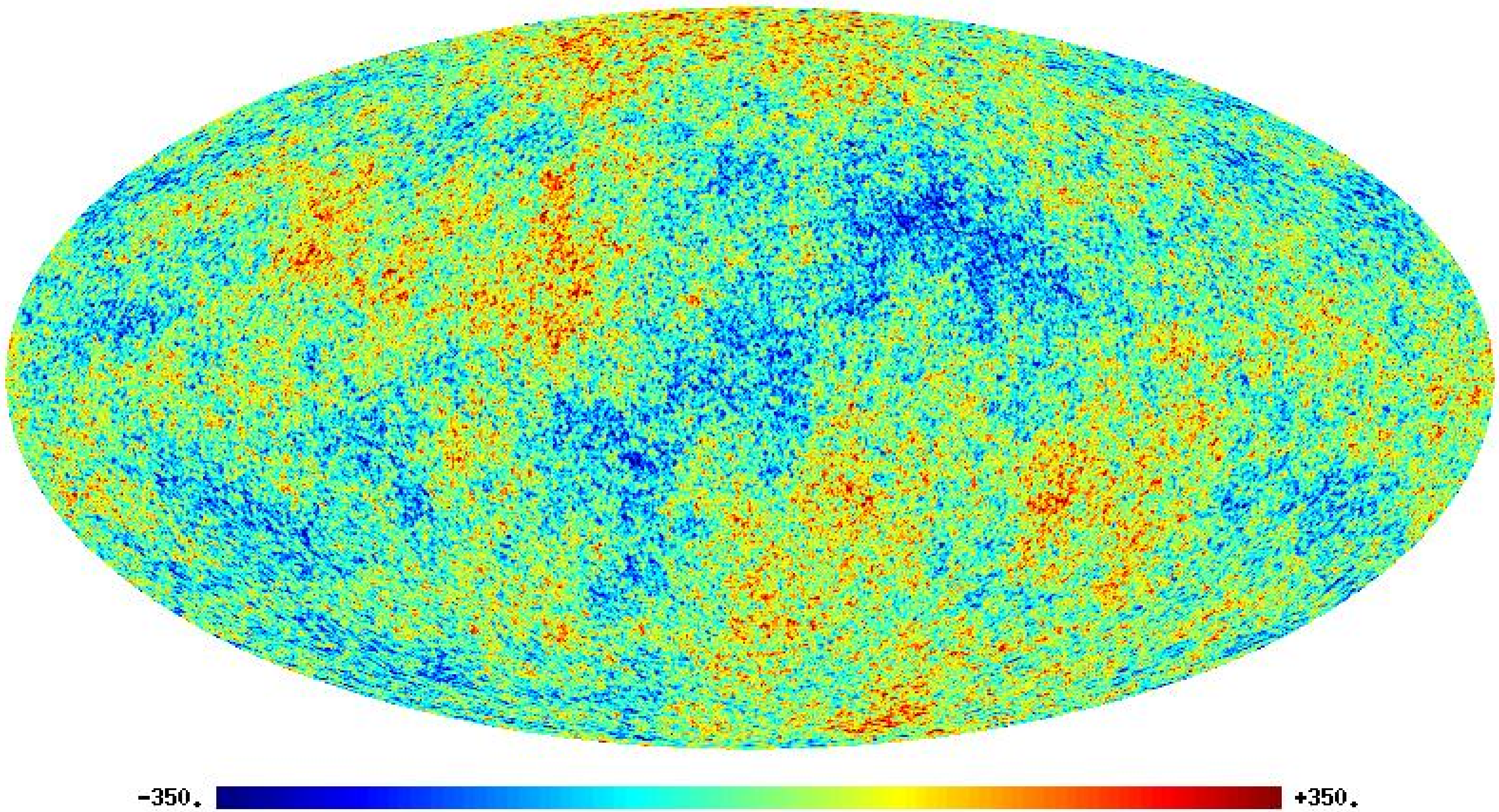
=



Spherical harmonics transforms



Spherical harmonics transforms

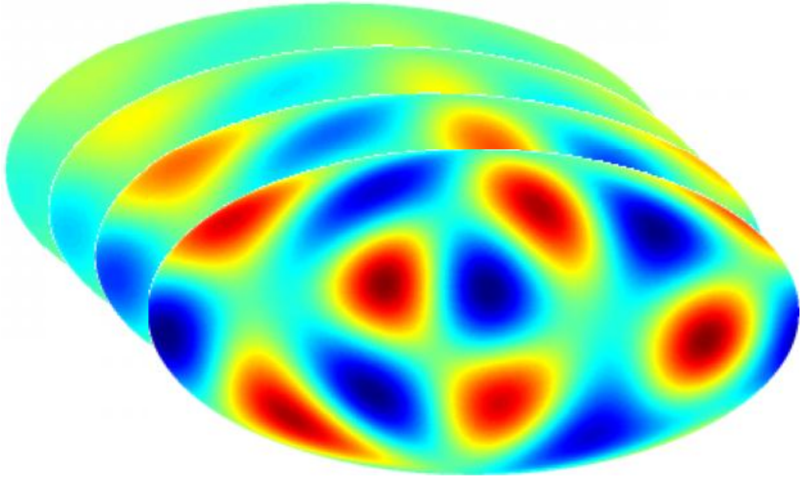


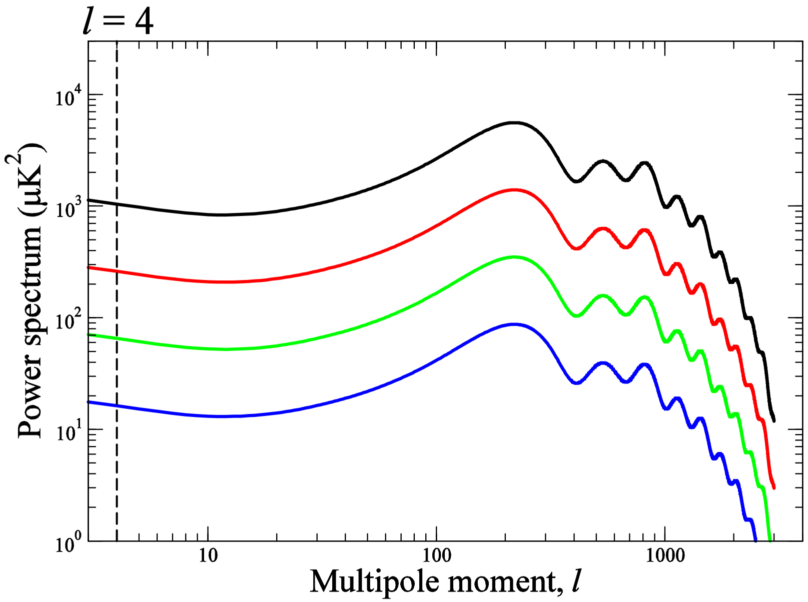
The angular power spectrum

- The angular power spectrum measures *amplitude* as a function of *wavelength*

- Defined as an average over over *m* for every *l*:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$





Theoretical and observed spectrum

- There are two types of power spectra:
  1. Given a specific map, compute

$$\hat{C}_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}|a_{\ell m}|^2$$

     This is the *observed* spectrum of a given *realization*

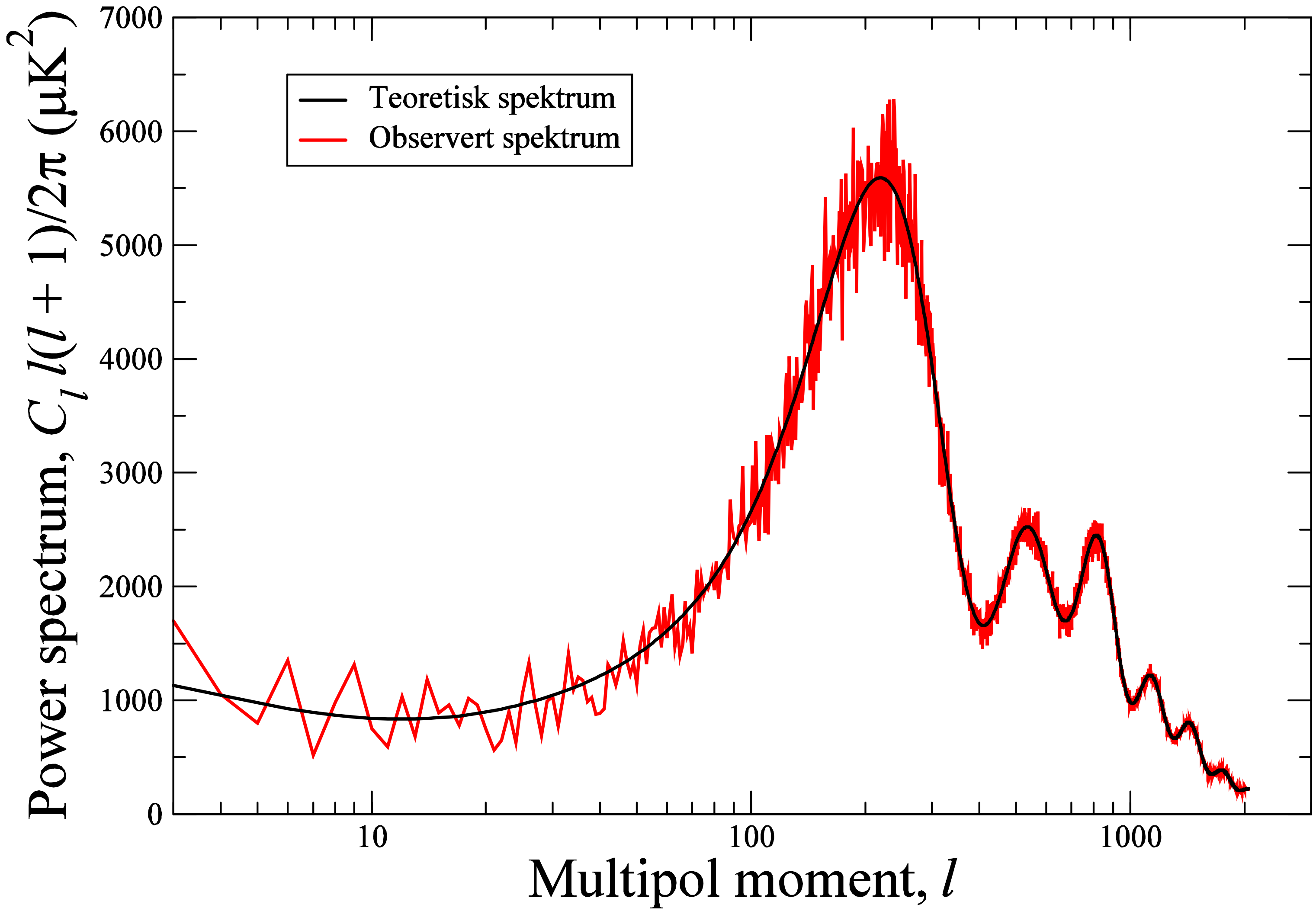
  2. Given an ensemble of maps (think thousands of independent realizations), compute

$$C_\ell = \left\langle \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}|a_{\ell m}|^2 \right\rangle_{\mathrm{ensemble}}$$

     This is the *ensemble averaged* power spectrum

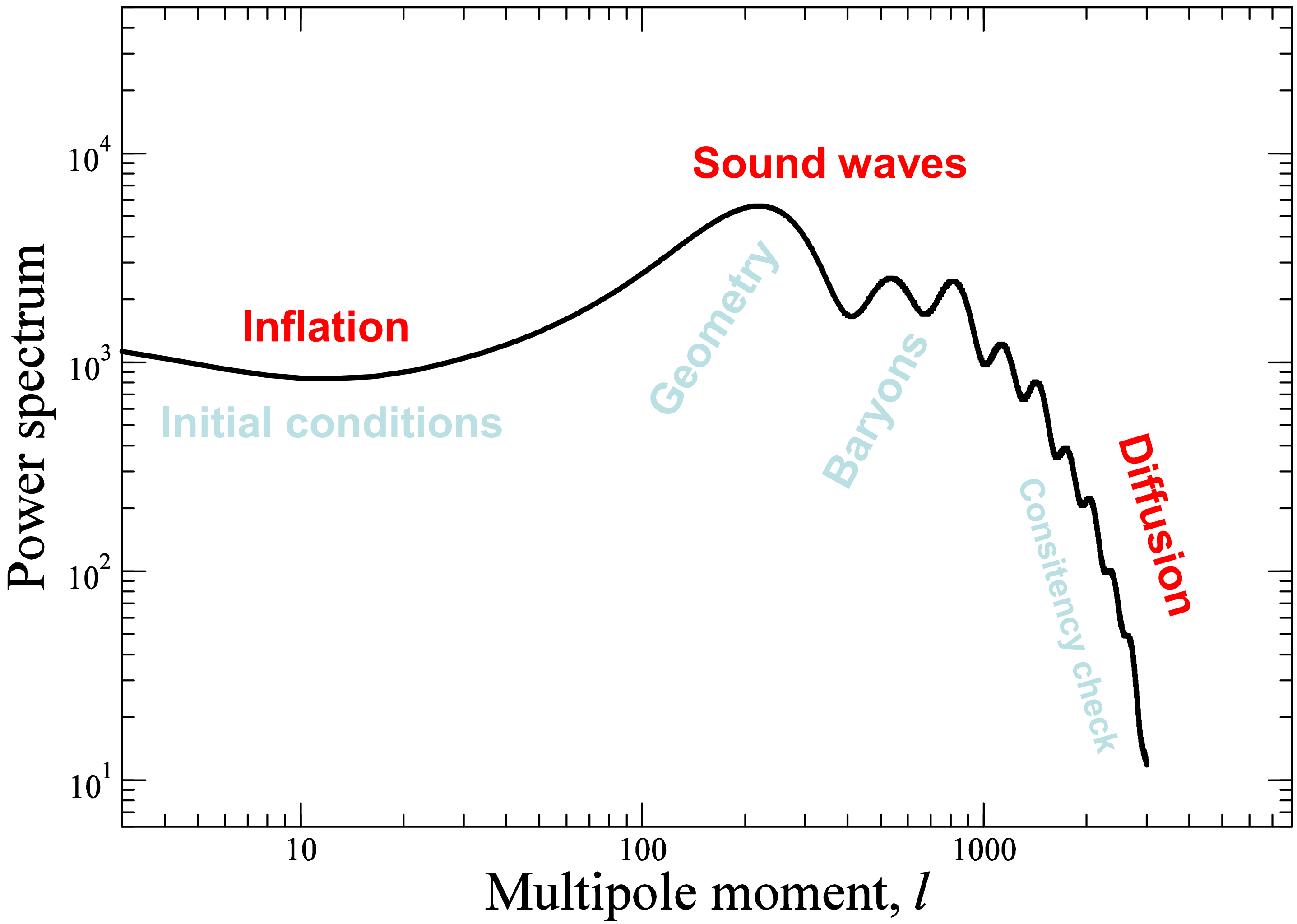
- The physics is given by C_ℓ , while we only observe $\hat{C}_\ell$
  – All CMB measurements are connected with an uncertainty called *cosmic variance*
  – The cosmic variance is given by $\Delta C_\ell = \sqrt{\frac{2}{2\ell+1}}C_\ell$

Theoretical and observed spectrum

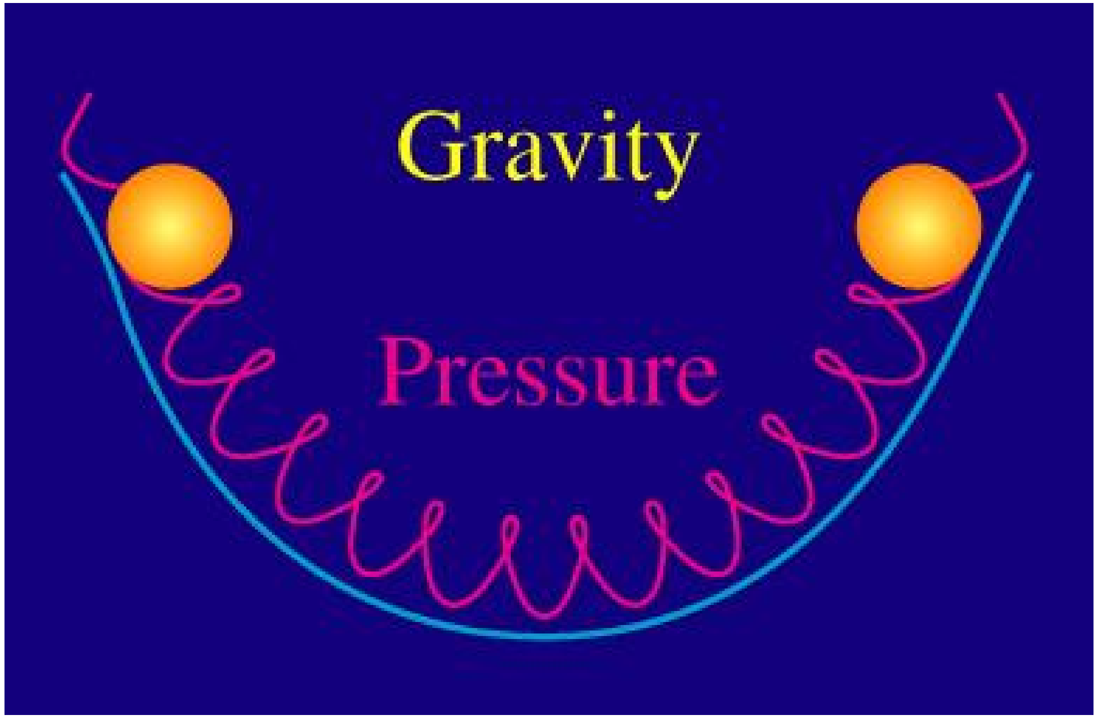


Physics and the
CMB power spectrum

Overview of the CMB spectrum

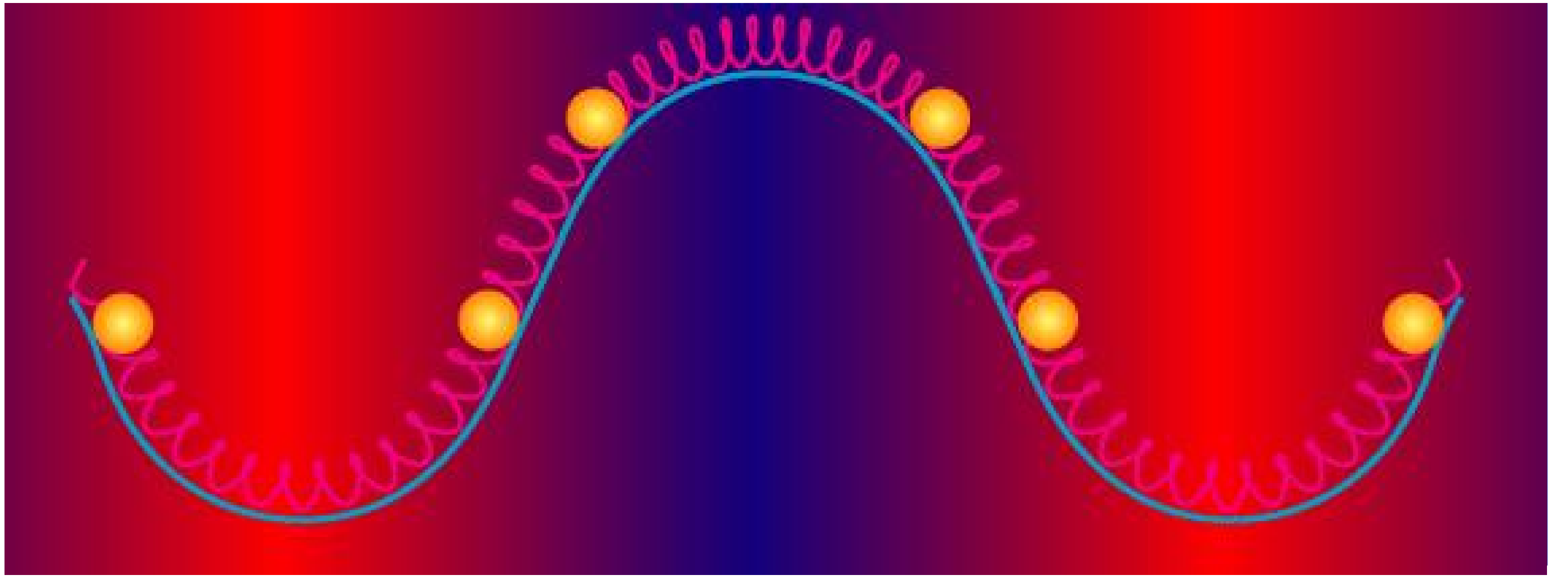


Main idea 1: Graviation vs. pressure



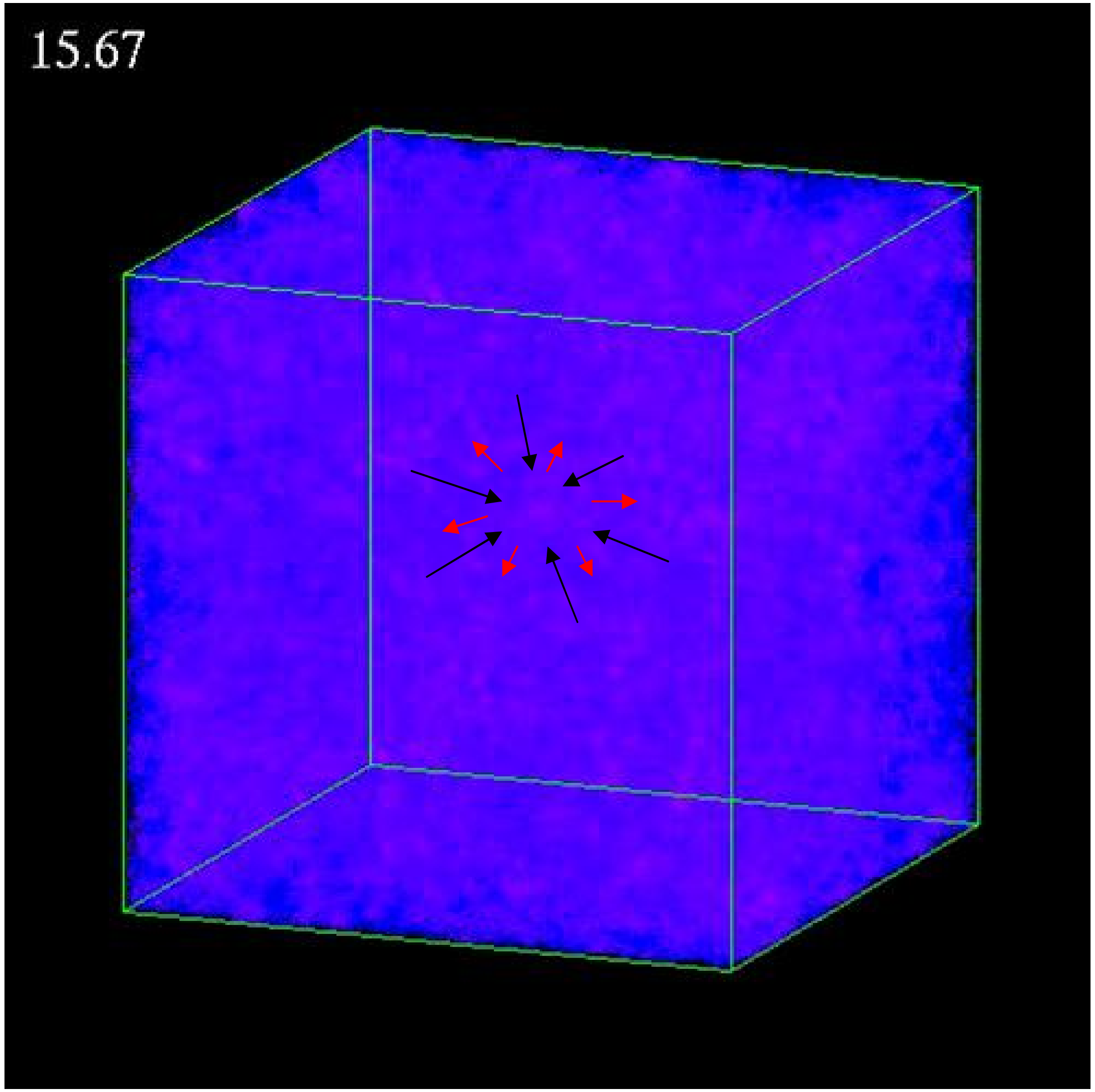
- The early universe was filled with baryonic matter (yellow balls) and photons (red spring), interacting in a graviational potential set up by dark matter (blue line)
  - Matter concentrations attract each other because of gravity
  - But when the density increases, the pressure also increases $\Rightarrow$ repulsion

Main idea 1: Gravitation vs. pressure

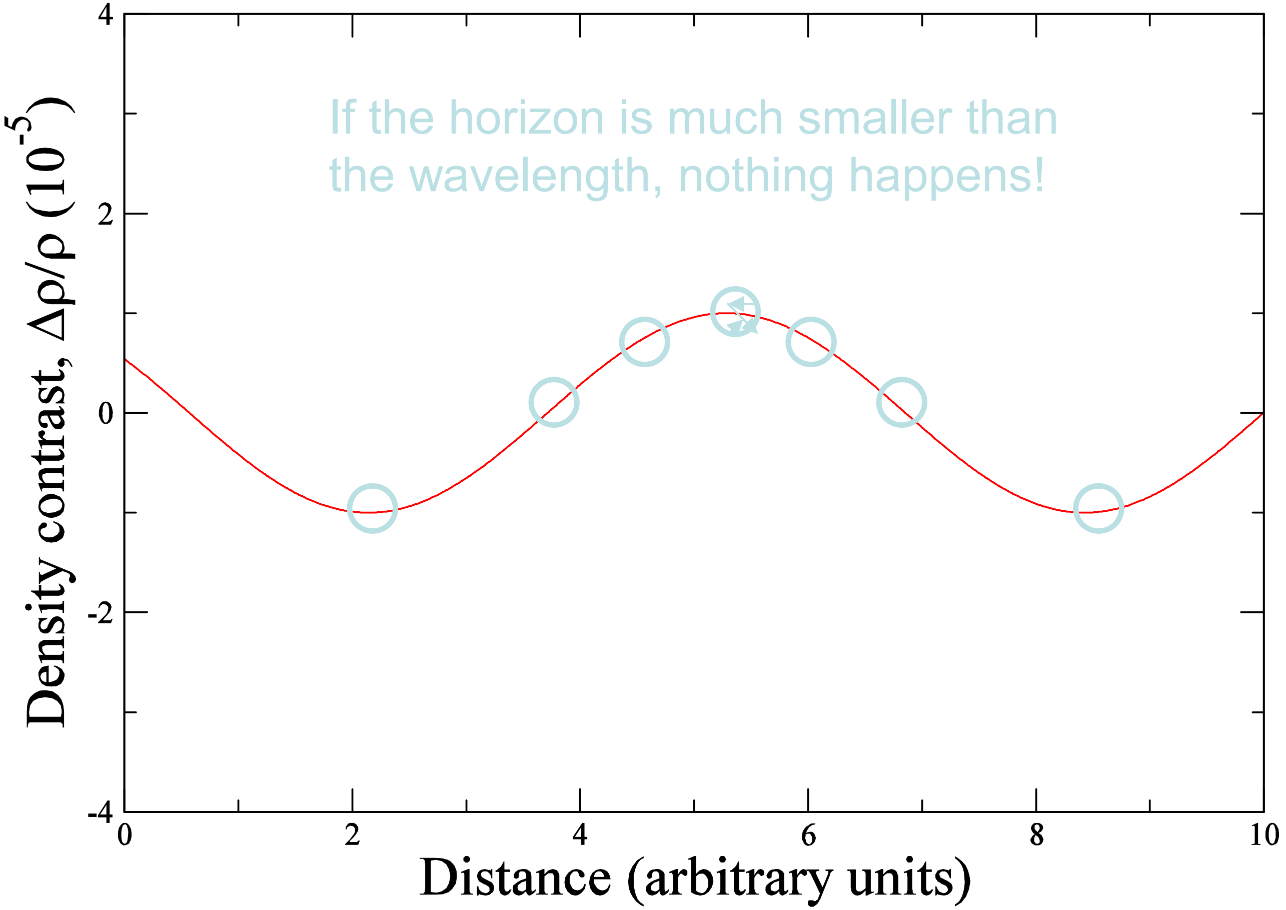


- The universe consists of an entire landscape of potential wells and peaks
- The baryon-photon plasma oscillates in this potential landscape
  - Sound waves propagate through the universe
- The baryon density corresponds directly to the CMB temperature
  - Areas with high density become cold spots in a CMB map
  - Areas with low density become warm spots in a CMB map

Main idea 2: Acoustics and the horizon

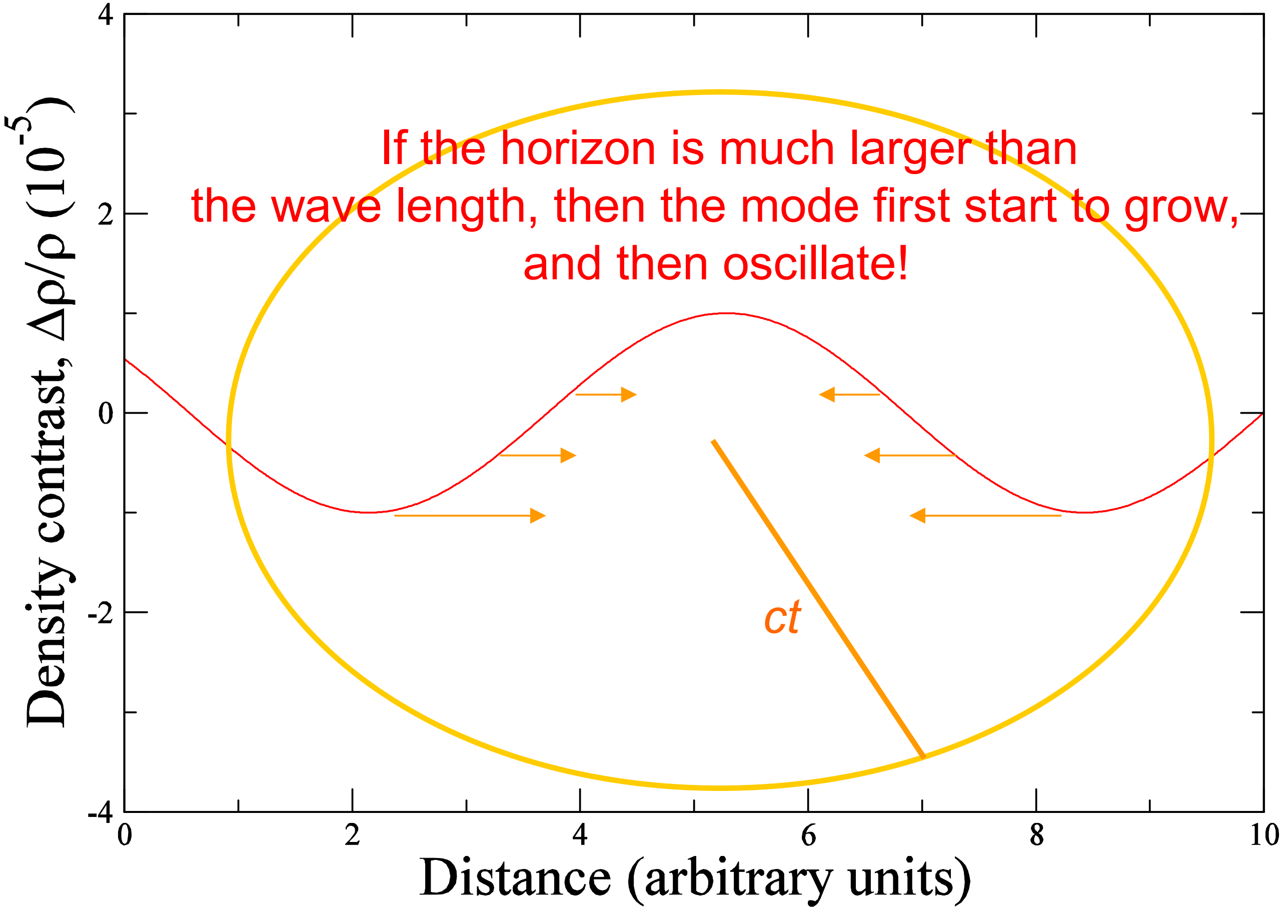


Main idea 2: Acoustics and the horizon



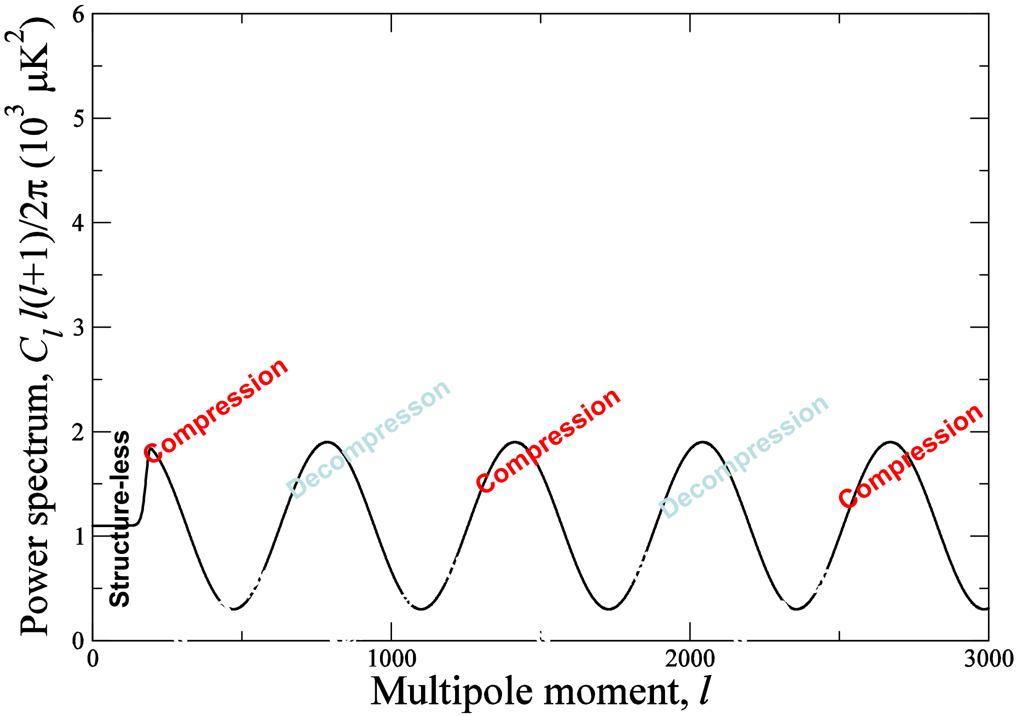
- Fourier decompose the density field, and look at one single mode enkelt mode
- Remember from AST4220: The horizon is how far light has travelled since the Big Bang
  - Gravity can only act within a radius of ~*ct*

Main idea 2: Acoustics and the horizon



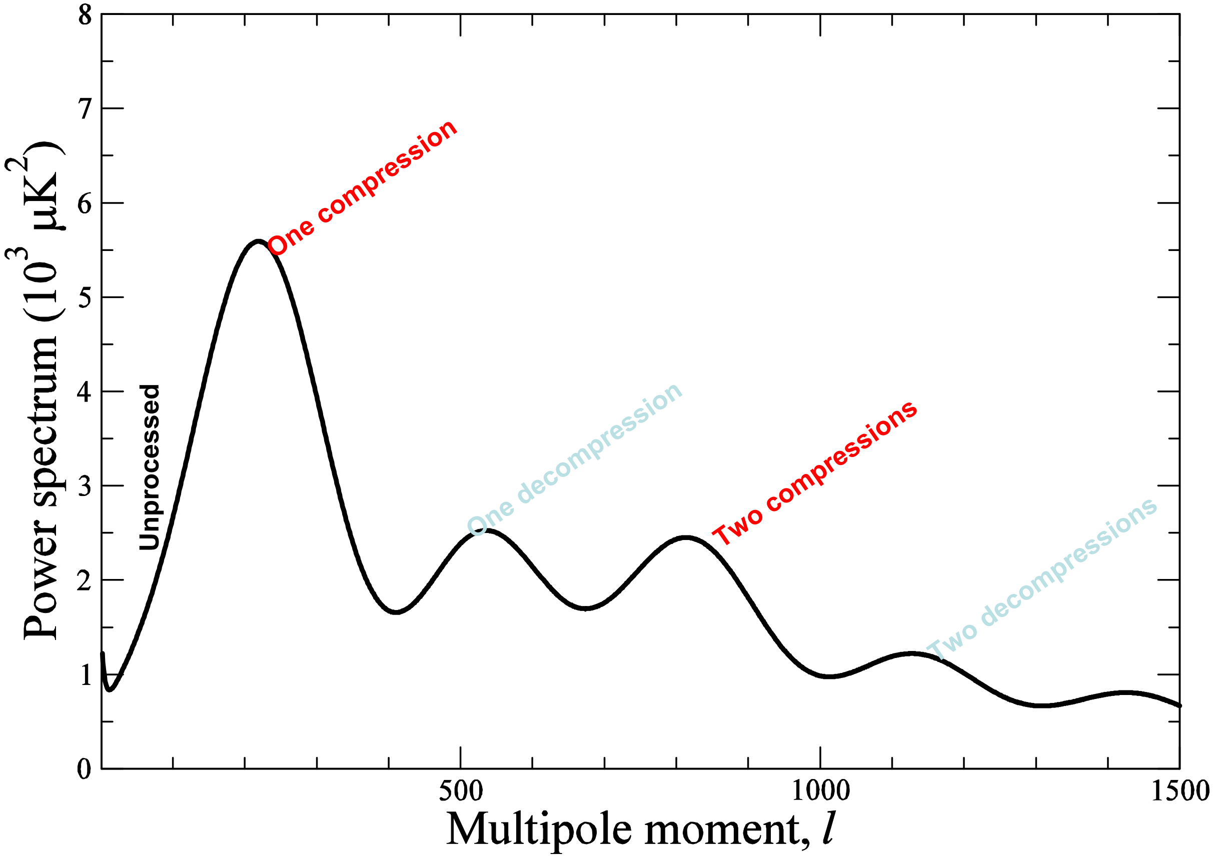
- Fourier decompose the density field, and look at one single mode enkelt mode
- Remember from AST4220: The horizon is how far light has travelled since the Big Bang
  - Gravity can only act within a radius of ~*ct*

Main idea 2: Acoustics and the horizon



- Inflation sets up a flat spectrum of fluctuations
- Shortly after inflation, the horizon is small
  - Only small scales are processed by gravity and pressure
- As time goes by, larger and larger scales start to oscillate
- Then, one day, recombination happens, and the CMB is ”frozen”

Main idea 2: Acoustics and the horizon



- Question: What can these fluctuations tell us about the processes that acts in the universe?

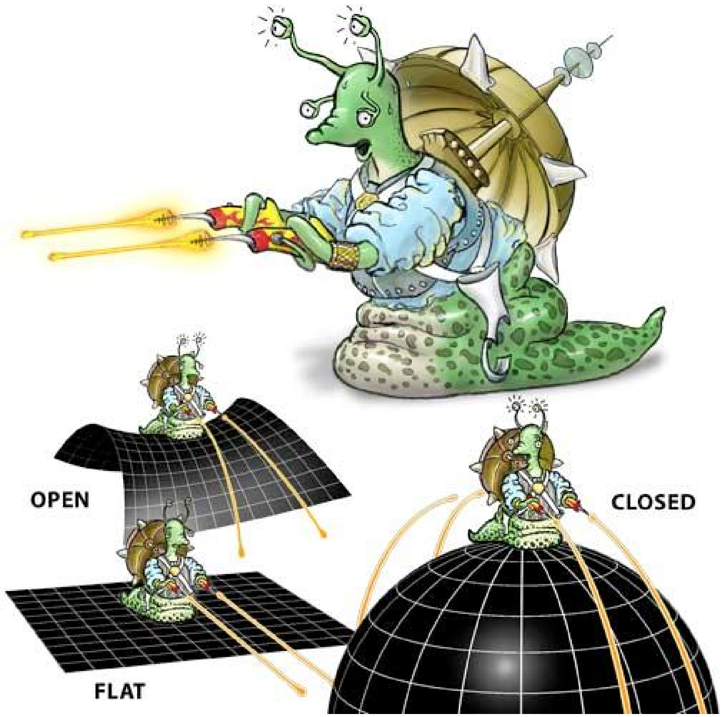
Inflation from low l's

- The size of the horizon at recombination is today ~1° on the sky
  - This corresponds to multipoles l ~ 180° / 1° ~ 200
  - Scales larger than this are only weakly processed by gravity and pressure
- The CMB field at l < 50 is a ”direct” picture of the fluctuations generated by inflation!
- Some predictions from inflation:
  - The fluctuations are Gaussian and isotropic
  - The spectrum is nearly scale invariant [$P(k) = A\ k^n$, n ~ 1]
    - There is no characteristic scale
    - The fluctuations are equally strong on all scales (ie., flat spectrum)
    - The relevant parameters for initial conditions from inflation are
      - an amplitude *A*
      - a tilt parameter n_s, that should be close to 1

$$C_l = A\left(\frac{\ell}{\ell_0}\right)^{n_s - 1}$$

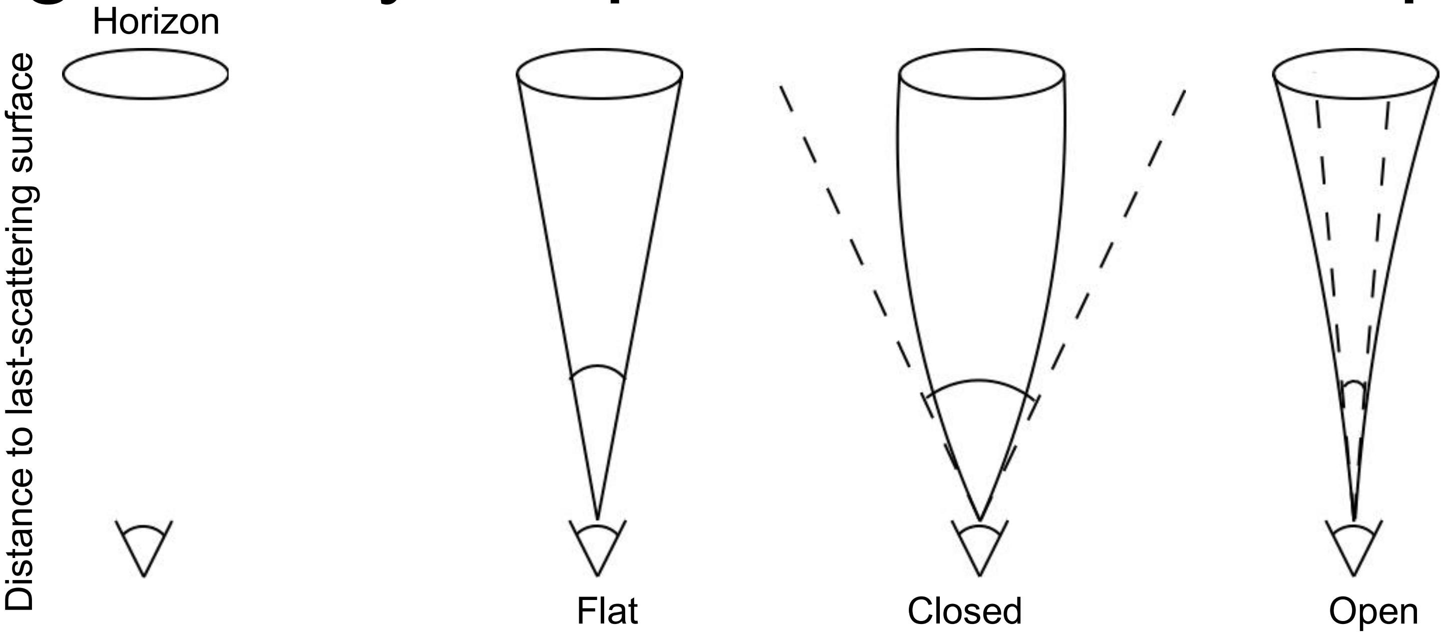
- By fitting this function to real data at l < 50, we get a direct estimate of *A* and n_s!

The geometry of space from the first peak



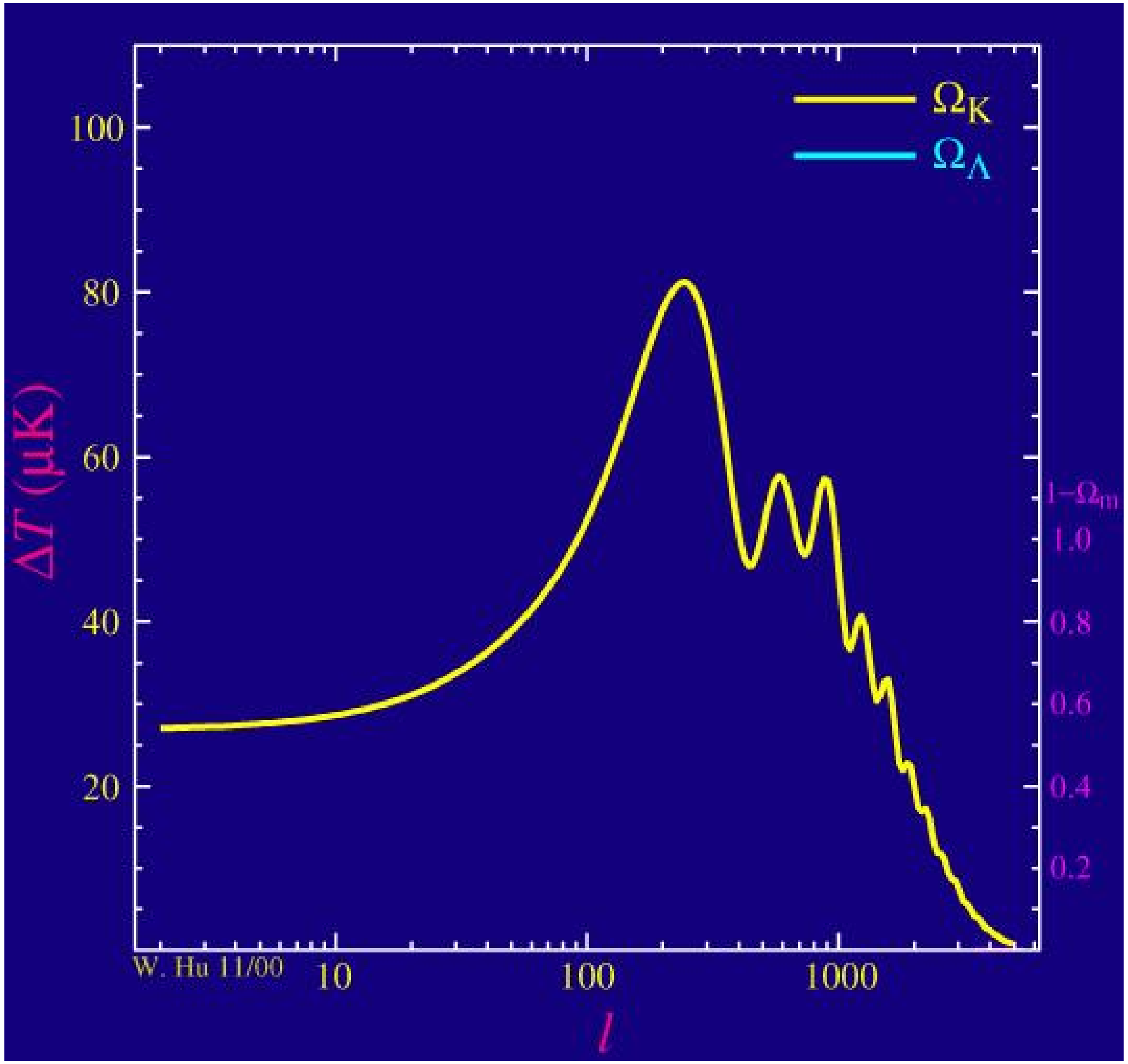
- According to GR, light propagates along geodesics in space
  - In flat space, these are straight lines
  - In open spaces, the geodesics diverge
  - In closed spaces, the geodesics converge

The geometry of space from the first peak



- Assume that we know:
  - the size of the horizon at recombination
    - Given by the properties of the plasma (pressure, density etc.)
  - The distance to the last scattering surface
    - Given by the expansion history of the universe
- The geometry of the universe is given by the angular size of the horizon
- The first acoustic peak is a standard ruler for the horizon size
  - If the first peak is at l ~220, then the universe is flat
  - If the first peak is at l > 220, then the universe is open
  - If the first peak is at l < 220, then the universe is closed

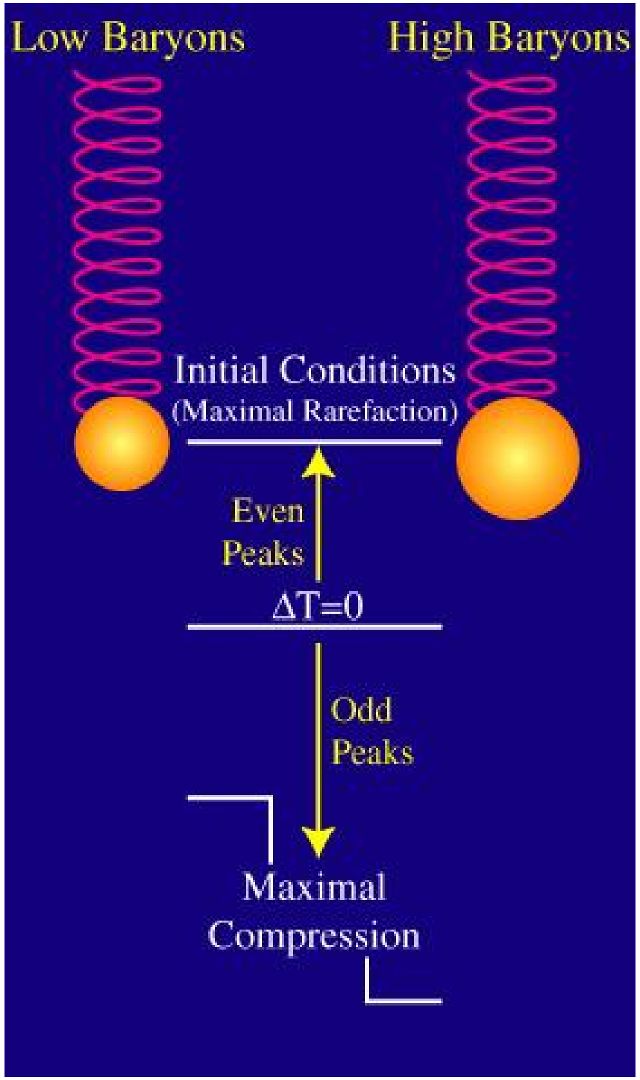
The geometry of space from the first peak



Low density

High density

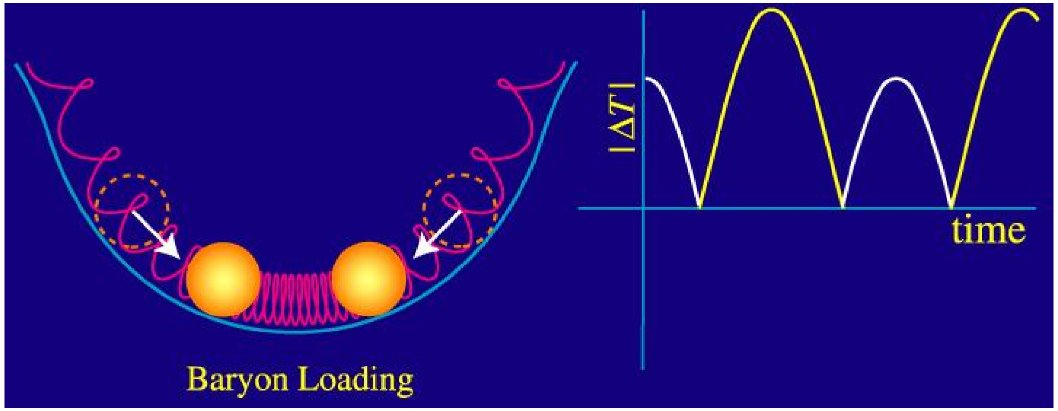
The baryon density from higher peaks



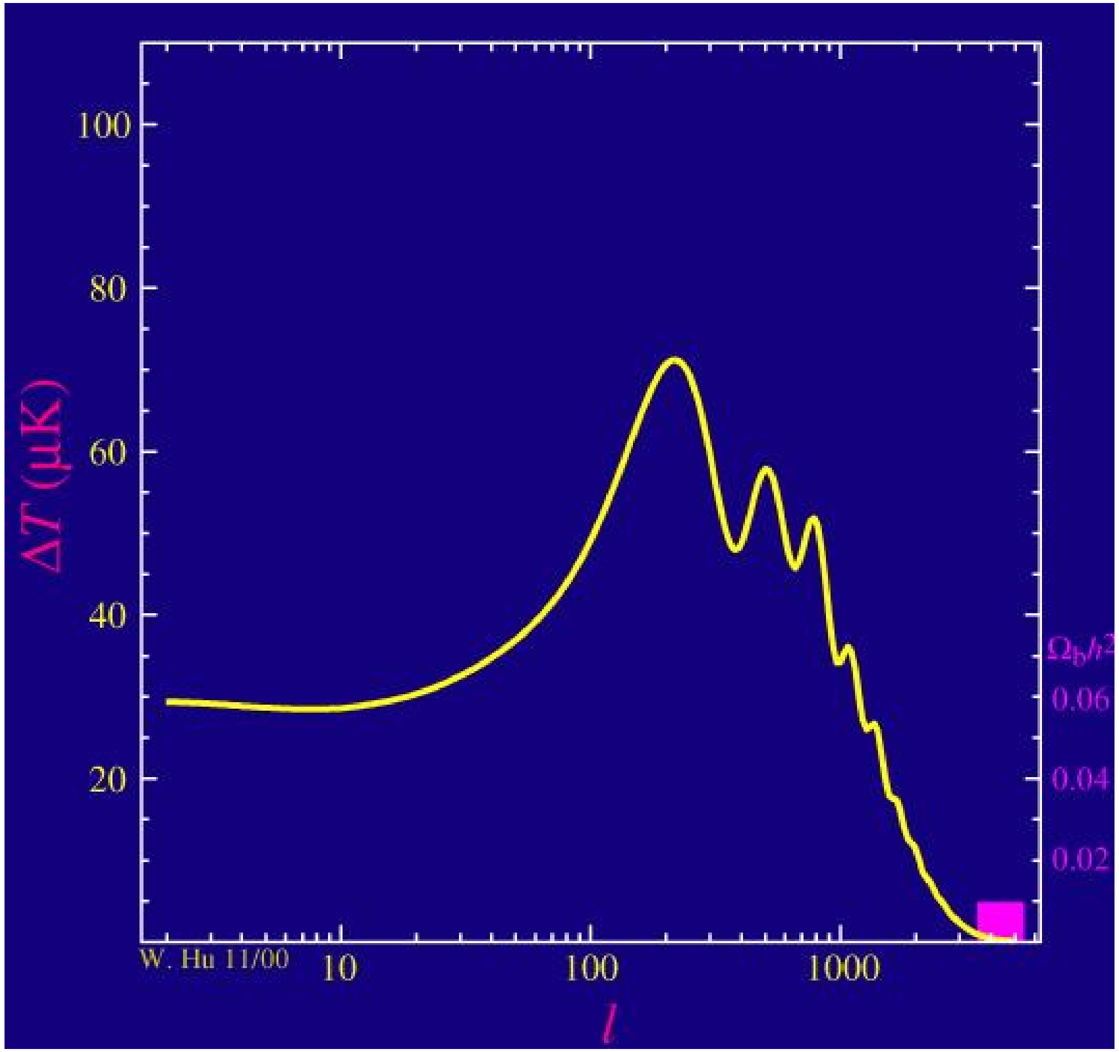
- The baryon density can be measured very accurately from the higher-ordered peaks
- Idea: More baryons means heavier load

1. The load falls deeper
2. If there are few baryons, these won't affect the graviational potential
   - ⇒ Symmetric oscillations around equlibrium
3. If there are many baryons, these add to the potential during *compressions*
   - Compressions are stronger than decompressions
   - But the power spectrum don't care about signs!

⇒ First and third peak are stronger than the second and fourth!



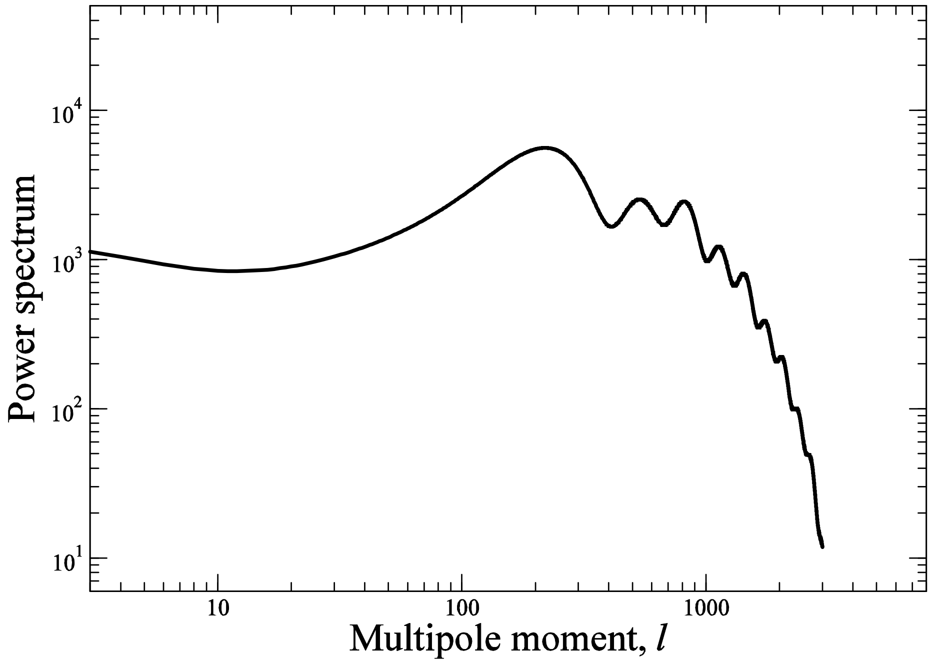
The baryon density from higher peaks



Much baryons

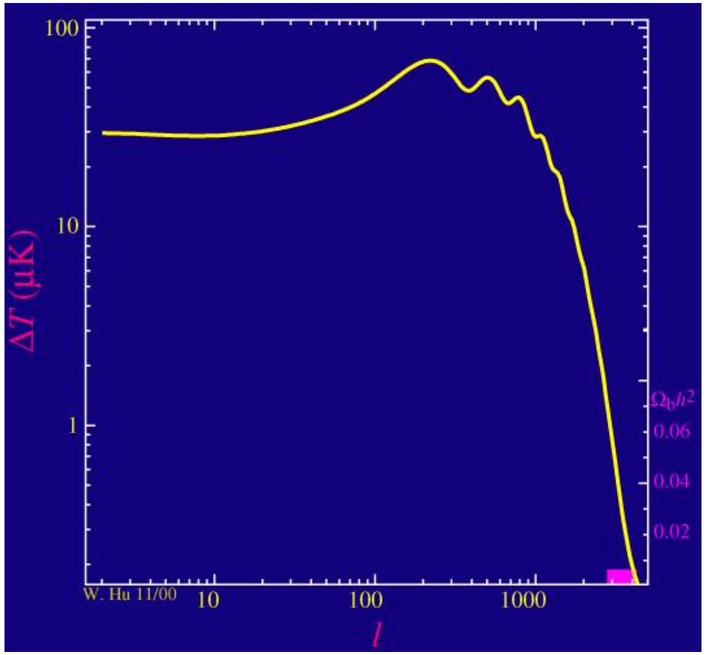
Little baryons

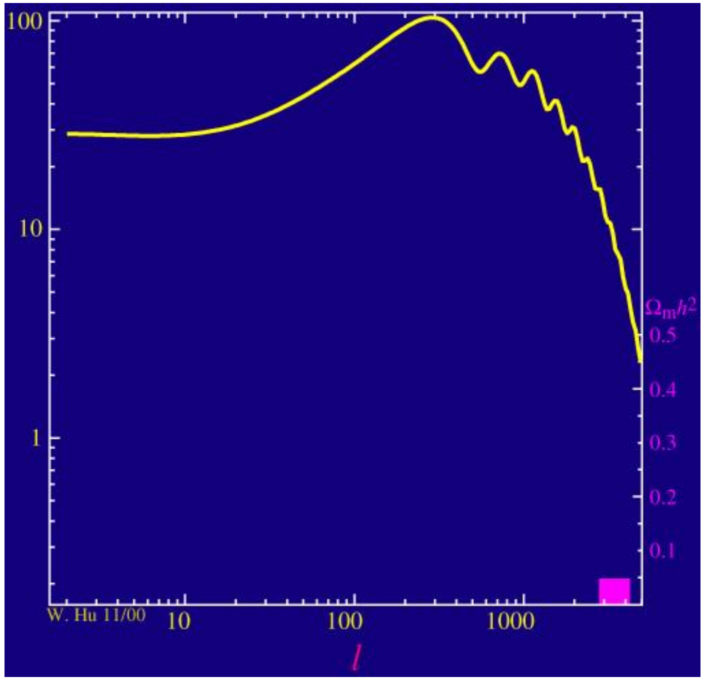
Exponential damping at high l's



- High l's correspond to "very small" physical scales
  - The initial fluctuations from inflation are washed out by photon diffusion
  - The power spectrum decays exponentially with l
- The precise damping rate depends on all cosmological parameters

Exponential damping at high l's





- High l's correspond to "very small" physical scales
  - The initial fluctuations from inflation are washed out by photon diffusion
  - The power spectrum decays exponentially with l
- The precise damping rate depends on all cosmological parameters
  - Example: High baryon density $\Rightarrow$ short free path for photons $\Rightarrow$ less diffusion
  - Example: High DM density $\Rightarrow$ old univers at recombination $\Rightarrow$ much diffusion
- High-l spectrum gives us a consistency check on other parameter estimates

Summary of main effects

- The cosmic background radiation was formed when the temperature in the universe fell below 3000° K, about 380,000 years after Big Bang
- The gas dynamics at the time determined the properties of the fluctuations in the CMB field
- Main effects that affect the CMB spectrum:
  - Inflation $\Rightarrow$ amplitude and tilt of primoridal structure
  - Gravitation vs. radiation pressure $\Rightarrow$ sound waves
    $\Rightarrow$ acoustic peaks
  - High baryon density $\Rightarrow$ heavy load in the waves
    $\Rightarrow$ strong compressions
    $\Rightarrow$ odd peaks stronger than even peaks
  - Photon diffusion on small scales $\Rightarrow$ exponential dampling at high l's
  - Lots of other effects too, but generally more compliated and less intuitive...

Summary

- Assumption: The very first structures were generated by inflation
  - These later grew by gravitational interaction, and formed the structures we see today
- Before recombination, the universe was opaque
  - Free electrons prevented light from travelling more than ~1 meter
- When electrons and protons formed neutral hydrogen, light could travel freely
  - At this time, the CMB radiation was formed
  - Happened ~380,000 years after Big Bang
- The CMB can be observed today, and we measure its power spectrum
  - The power spectrum is highly sensitive to small variations in many cosmological parameters
- Our goal: To quantitatively predict the CMB spectrum given cosmological parameters!