

# Calculations

Calculations of motors:

There are two possible approaches for designing of an UAV. First approach is design by using available standard components in the market. This approach is preferred when the designer has standardly available components required and he/she designs the frame of UAV according to it. Second approach is complete opposite, where the designer designs the frame of UAV and selects components according to the design. Both approaches have their own pros and cons but after considering them we decided to settle for the first approach. The main reason we opted for first approach is availability of components.

## A2212/13T TECHNICAL DATA



No. of Cells:	2 - 3 Li-Poly 6 - 10 NiCd/NiMH
Kv:	1000 RPM/V
Max Efficiency:	80%
Max Efficiency Current:	4 - 10A (>75%)
No Load Current:	0.5A @10V
Resistance:	0.090 ohms
Max Current:	13A for 60S
Max Watts:	150W
Weight:	52.7 g / 1.86 oz
Size:	28 mm dia x 28 mm bell length

We selected a BLDC 1000 KV motor for our UAV.

KV rating indicates how fast the motor will spin for 1V of applied voltage.

According to electromechanical relation, the torque constant can be calculated by formula [1];

$$K_t = (0.01794) * K_v$$

where;

$$K_t = \text{torque constant [Nm/A]}$$

$$KV = \text{RPM per volt}$$

$$= 0.01794 * 1000$$

$$= 17.94 \text{ Nm /A}$$

The torque constant is 17.94 Nm/A.

Torque of motor:

Torque can be calculated as [1];

$$T = I * K_t$$

where;

$$T = \text{Motor Torque [Nm]}$$

$$I = \text{Motor Current [A]}$$

$$K_t = \text{Torque Constant [Nm/A]}$$

$$T = I * K_t$$

$$= 10 * 17.94$$

$$= 179.4 \text{ Nm}$$

Calculate speed of motor [1]:

$$N = k_v * \text{Voltage Input}$$

where;

$$N = \text{Angular velocity of motor}$$

For maximum voltage input (V = 12V)

$$N = 1000 * 12$$

$$= 12000 \text{ rpm}$$

For minimum voltage input ( $V = 7V$ )

$$\begin{aligned} N &= 1000 * 7 \\ &= 7000 \text{ rpm} \end{aligned}$$

The range of angular velocity of motor is 7000 *rpm* to 12000 *rpm*.

ESC (Electronic Speed Control) Calculations [2]:

For selection of ESC, we design for 20-50% extra amp current than actually required as it is considered a good rule to ensure that your ESC does not burn out.

If you remember our motor draws maximum current of 13 Amps and according to motor data sheet maximum power motor can draw is 150W. So watt value for 3s and 4s will be as follows;

$$3s \text{ battery} = 11.1 * 13 = 144.3 \text{ watts}$$

$$4s \text{ battery} = 14.8 * 13 = 192.4 \text{ watts}$$

Since our motor and ESC are not much efficient and not capable of drawing power output by 4s batteries we opted to use 3s batteries.

Since our motor draws maximum current of 13 Amps, we can select our ESC using following formula [1][2];

$$\begin{aligned} ESC(A) &= (1.2 \text{ to } 1.5) * \text{Maximum Amps of motor} \\ &= 1.5 * 13 \\ &= 19.5 \text{ Amps} \end{aligned}$$

Considering the above results we opted the ESC of 30A.

Battery Calculations:

Before we calculate the current being drawn by the motor through battery, we have to first calculate the current discharge the battery is capable of

$$\begin{aligned} \text{Max Source Current} &= \text{Discharge rate} * \text{Capacity} \\ &= (35 * 3300) / 1000 \text{ mApms} \\ &= 115.5 \text{ Amps} \end{aligned}$$

Force analysis for propeller:

Calculation of centrifugal force on the propeller can be calculated using following formula [1];

$$F_c = m * R * \omega^2$$

where;

$F_c$  = Centrifugal force ( $N$ )

$R$  = Radius of propeller ( $m$ )

$N$  = Speed of propeller ( $rpm$ )

$m$  = Mass of propeller ( $Kg$ )

$$\begin{aligned} F_c &= 0.014 * 0.127 * w^2 \\ &= 0.014 * 0.127 * \left(\frac{2\pi * N}{60}\right) \\ &= 0.001778 * \left(\frac{2\pi * 12000}{60}\right) \\ &= 0.001778 * (1256.64)^2 \\ &= 0.001778 * 1579136.70 \\ &= 2807.70 \text{ } N \end{aligned}$$

$$F_c = 2.807 \text{ } kN$$

The centrifugal force on propeller is 2.807  $kN$ .

Calculate Moment:

$$\begin{aligned} \text{Moment} &= F_c * (\text{Perpendicular distance of propeller center w.r.t ground surface}) \\ &= 2.8007 * (25 + 37.5 + 110) * 10^{-3} \\ &= 2.807 * 0.1725 \\ M &= 0.48420 \text{ } kNm \\ M &= 484.20 \text{ } Nm \end{aligned}$$

Calculate power of propeller [1]:

$$\text{Power} = K_p * D^4 * p * N^3$$

where;

$K_p$  = propeller constant (1.11 for APC controller)

$p$  = pitch of propeller (According to data sheet)

$$\begin{aligned} \text{Power} &= 1.11 * (0.254)^4 * (4.5 * 10^{-3}) * (12000)^3 \\ &= 1.11 * (0.254)^4 * 0.0045 * (12000)^3 \\ &= 35926.432 \text{ } kwatt \end{aligned}$$

Calculate thrust of motor [2]:

$$T = [(Eta * P)^2 * 2\Pi * r^2 * air\ density]^{1/3}$$

where;

Eta = Propeller hover efficiency = 0.7

P = Shaft Power = *Voltage x Current x motor efficiency*

r = Radius of propeller

Air Density =  $\rho = 1.22\text{ Kg/m}^3$

Voltage = 10 *V*

Current = 13 *A*

Motor efficiency = 80% = 0.8

$$\begin{aligned} T &= [(0.7 * 10 * 13 * 0.8)^2 * 2\Pi * (0.127)^2 * 1.22]^{1/3} \\ &= [(72.8)^2 * 2\Pi * (0.127)^2 * 1.22]^{0.33} \\ &= (655.254)^{0.33} \\ &= 8.499\text{ N} \approx 8.5\text{ N} \end{aligned}$$

Therefore, calculated thrust is [2],

$$\begin{aligned} T &= 8.49 * 0.101\text{ kg} \\ &= 858.5\text{ gm} \end{aligned}$$

Therefore, calculated total thrust is;

$$\begin{aligned} T_{Total} &= 858.5 * 4 \\ &= 3434\text{ gm} \\ &= 3.434\text{ kg} \end{aligned}$$

Now, let us consider this 80% efficiency of motor to be maximum efficiency at which it can operate (theoretical) i.e consider now at its 100% efficiency motor generates thrust of 3.434 Kg and not at 80% (Actual). There is a thumb rule that while designing the maximum weight of UAV should be kept below 50% efficiency of motor. Therefor now thrust will be;

$$\begin{aligned} T_{actual} &= 3.434 * \frac{50}{100} \\ T_{actual} &= 1.717\text{ kg} \end{aligned}$$

The reason behind keeping weight below 50% efficiency of motor thrust can be explained by ship in water analogy. Consider a ship floating on water having a weight of say 5 Kg. At equilibrium condition, the buoyant force applied by water is equal to the weight of ship i.e 5 Kg. Now at equilibrium if we apply any external force on the ship in downward direction, due to buoyant force

being less than weight, the ship will sink. To avoid this from happening, the weight of ship is kept below 50% of buoyant force for safety of ship i.e 2.5 Kg. If we use this analogy on UAV, here buoyant force is replaced by pressure gradient generated above and below the propellers. So to be able to lift and fly the UAV safely the weight of UAV should be kept below 1.717 Kg or 1717 gms. This is done so that UAV could fly stable even if there is excess air drag or air conditions keep changing.

So, the minimum amount of thrust required by a single motor is;

$$T_{each} = \frac{1717}{4}$$

$$T_{each} = 429.25 \text{ gms}$$

Since the minimum required thrust of each motor is 429.25 gms and since each motor can generate 858.5 gms of thrust, the system will fly safely.

Calculate total weight of UAV:

$$\begin{aligned} W &= \frac{\text{Thrust}}{\text{Factor of safety}} \\ &= \frac{3.434}{2} \\ &= 1.717 \text{ kg} = 1717 \text{ gms} \end{aligned}$$

Calculate lift of UAV [1]:

$$\text{Lift} = \frac{W * D^4 * N^2 * \left( \frac{\rho * 24}{C_f * 29.9} \right)}{2.2}$$

Where;

W = Weight of UAV (Kg)

D = Diameter of propeller (m)

N = Speed of motor (rpm)

$\rho$  = Air density = 1.22 Kg/m<sup>3</sup>

C<sub>f</sub> = Lift Coefficient = 0.8 to 1.4

$$\begin{aligned} \text{Lift} &= \frac{1.717 * (0.254)^4 * (12000)^2 * \left( \frac{1.22 * 24}{1.4 * 29.9} \right)}{2.2} \\ &= 3.27 \text{ kg} \end{aligned}$$

The lift of UAV is 3.27 Kg.

Calculate flight time [1]:

$$\begin{aligned} \text{Duration (flight time)} &= \frac{\text{Capacity of battery in Ah} * \text{battery discharge}}{\text{Maximum current drawn by motor}} \\ &= \frac{3.3 * 35}{10} \\ t_{\text{flight}} &= 11.55 \text{ min} \end{aligned}$$

Calculation of Center of Gravity (C.G) for 'I' shaped frame:

(Diagram here)

Dimension of frame;

$$\text{Length} = 328 \text{ mm}$$

$$\text{Width} = 363 \text{ mm}$$

$$\text{Diagonal distance} = 450 \text{ mm}$$

For rod 1;

$$A_1 = 25 * 363 = 9075 \text{ mm}^2$$

$$y_1 = 181.5 \text{ mm} \quad x_1 = 12.5 \text{ mm}$$

For rod 2;

$$A_2 = 25 * 278 = 6950 \text{ mm}^2$$

$$y_2 = 181.5 \text{ mm} \quad x_2 = 164 \text{ mm}$$

For rod 3;

$$A_3 = 25 * 363 = 9075 \text{ mm}^2$$

$$\begin{aligned} y_3 &= 181.5 \text{ mm} \quad x_3 = 25 + 278 + 12.5 \\ &= 315.5 \text{ mm} \end{aligned}$$

Find C.G position of 'I' shape frame:

For x direction;

$$\begin{aligned} x &= \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} \\ &= \frac{(9075 * 12.5) + (6950 * 164) + (9075 * 315.5)}{9075 + 6950 + 9075} \end{aligned}$$

$$= \frac{113437.5 + 1139800 + 2863162.5}{25100}$$

$$= \frac{4116400}{25200}$$

$$x = 164 \text{ mm}$$

For y direction;

$$y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(9075 * 181.5) + (6950 * 181.5) + (9075 * 181.5)}{25100}$$

$$= \frac{1647112.5 + 1261425 + 1647112.5}{25100}$$

$$y = 181.5 \text{ mm}$$

The centre of gravity (C.G) of 'I' shape frame is (164, 181.5).