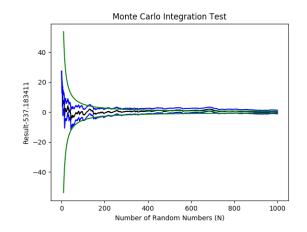
Computing Tasks

May 13, 2018

Exercise 1

Integration Practice



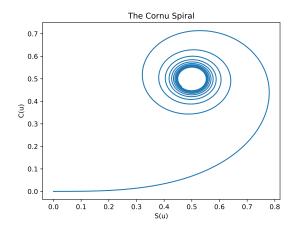


Figure 1

0.0.1 IntegrateMK2.py Code

NError = [-x for x in Error]

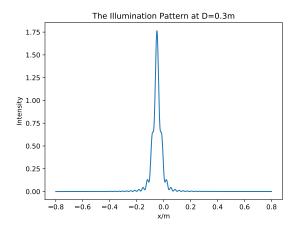
```
import numpy as np
import operator
import matplotlib.pyplot as plt
def function (coordinate):
        return np. sin (coordinate)
def Monte_Gradual_integrate (Lower_Limit=0, Upper_Limit=np.pi/8, Dimensions=8, N=10):
        count_in=0; square_count_in=0; Results = []; Error = []
        Volume = (Upper_Limit-Lower_Limit) ** Dimensions
        for i in range (1, int(N)+1):
                 coordinate = 0
                 for j in range(int(Dimensions)):
                         coordinate += np.random.rand()*(Upper_Limit-Lower_Limit) + Lower_Limit
                 count_in+=float(function(coordinate)); square_count_in += float((function(coordinate
                 \#print(count_in, square_count_in)
                 Results.append(Volume*count_in/i)
                 Error.append(Volume*(np.power(((-np.power(((count_in)/i),2)+(square_count_in/i))/i),
        return [x*10**6 for x in Results], [x*(10**6) for x in Error]
N = 1000
Results, Error = Monte_Gradual_integrate(N=N)
plt.plot(range(1,len(Results)+1),Results,color='black')
plt.plot(range(1,len(Results)+1),list(map(operator.sub, Results, Error)),color='blue')
NError = [-x \text{ for } x \text{ in } Error]
plt.plot(range(1,len(Results)+1), list(map(operator.sub,Results,NError)), color='blue')
plt.xlabel('Number_of_Random_Numbers_(N)')
plt.title('Monte_Carlo_Integration_Test')
plt.savefig('New_Plot.pdf')
plt.clf()
```

plt.plot(range(1,len(Results)+1),[x-537.1873411 for x in list(map(operator.sub, Results, Error))], co

plt.plot(range(1,len(Results)+1),[x-537.1873411 for x in Results],color='black')

```
plt.plot(range(1,len(Results)+1),[x-537.1873411 for x in list(map(operator.sub,Results,NError))], column columns (sub, negative sub), and the substitution of the su
plt.xlabel('Number_of_Random_Numbers_(N)')
plt.ylabel('Result -537.183411')
plt.title('Monte_Carlo_Integration_Test')
plt.savefig('Comparative_Plot.pdf')
u = [537.183411/x \text{ for } x \text{ in } range(10,1000)]
1 = [-537.183411/x \text{ for } x \text{ in range}(10,1000)]
plt.plot(range(10,1000),u,color='green')
plt.plot(range(10,1000),l,color='green')
plt.savefig('Compartative_Plot_Add_Lines.png')
0.0.2 CornuIntegrate.py Code
import scipy
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import numpy.random as rand
from scipy.integrate import quad
def cosser(x):
                        return np. \cos(\text{np.pi}*(x**2)/2)
def sinner(x):
                        return np. \sin(\text{np.pi}*(x**2)/2)
def Integrator(u):
                       C = quad(cosser, 0, u)[0]
                        S = quad(sinner, 0, u)[0]
                        return C,S
x = []; y = []
for i in np. linspace(0,2*np.pi,num=1000):
                        C,S = Integrator(i)
                        y.append(S); x.append(C)
plt.plot(x,y)
plt.xlabel('S(u)')
plt.ylabel('C(u)')
plt.title('The_Cornu_Spiral')
plt.savefig('Cornu_Spiral.pdf')
```

Apertures



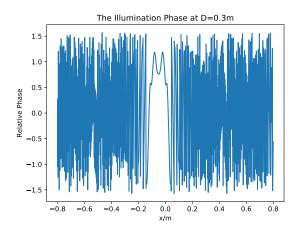
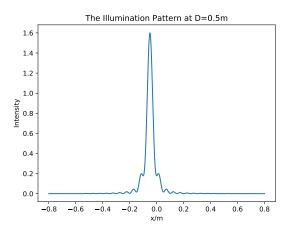


Figure 2



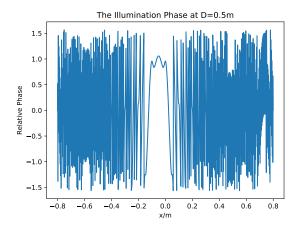
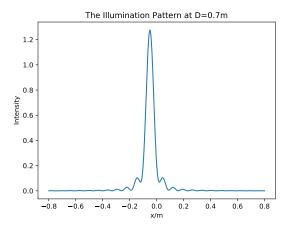


Figure 3



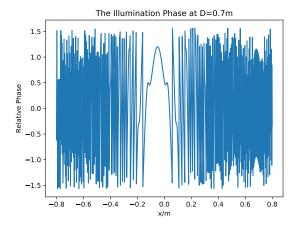


Figure 4

Aperture Code

```
import scipy
import numpy as np
import matplotlib
\mathbf{import} \hspace{0.2cm} \mathtt{matplotlib.pyplot} \hspace{0.2cm} \mathtt{as} \hspace{0.2cm} \mathtt{plt}
import numpy.random as rand
from scipy.integrate import quad
def cosser(x):
         return np. cos (np. pi *(x**2)/2)
def sinner(x):
         return np. \sin(\text{np.pi}*(x**2)/2)
def Integrator(u):
         C = quad(cosser, 0, u)[0]
         S = quad(sinner, 0, u)[0]
         return C,S
x = []; y = []
def Two_Ended(x0, x1, lam = 0.01, D = 0.3):
         Scaling = (2/(lam*D))**0.5
         u2 = x1*Scaling; u1 = x0*Scaling
         imag1, real1 = Integrator(u1)
         imag2, real2 = Integrator (u2)
         imag = -imag1 + imag2
         real = -real1 + real2
         mag = (imag**2+real**2)*0.5
         arg = np. arctan(imag/real)
         return mag, arg
def Plot_Mag_Arg(d=0.1, lam = 0.01, D=0.3):
         mag\_list = []; arg\_list = []
         x = np. linspace(-0.8, 0.8, num=1000)
         for i in x:
                  mag, arg = Two\_Ended(i, i+d, D=D, lam=lam)
                  mag_list.append(mag); arg_list.append(arg)
         plt.plot(x, mag_list)
         plt.xlabel('x/m')
plt.ylabel('Intensity')
         plt.title('The_Illumination_Pattern_at_D='+str(D)+'m')
         plt.savefig('Apperture_Pattern_'+str(D)+'_-'+'.pdf')
         plt.clf()
         plt.plot(x, arg_list)
         plt.xlabel('x/m')
         plt.ylabel('Relative_Phase')
         plt.title('The_Illumination_Phase_at_D='+str(D)+'m')
         plt.savefig('Phase_Pattern_'+str(D)+'_.pdf')
         plt.clf()
Plot_Mag_Arg()
Plot_Mag_Arg(D=0.5)
Plot_Mag_Arg(D=0.7)
```

Exercise 2

The effect of order

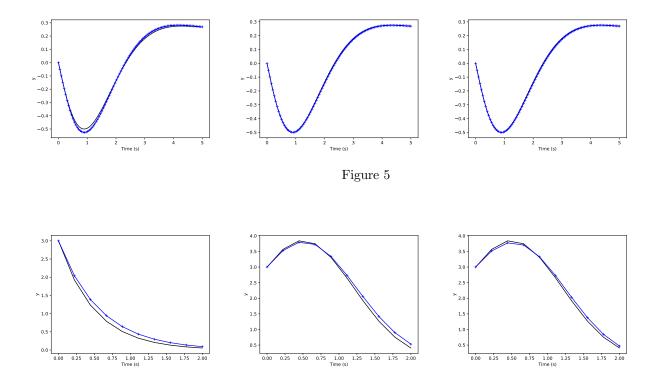
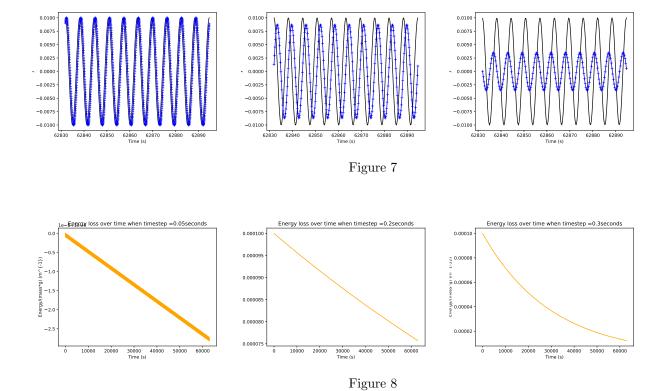
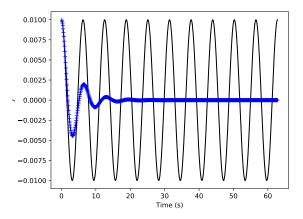


Figure 6

The effect of time-step



Light Damping



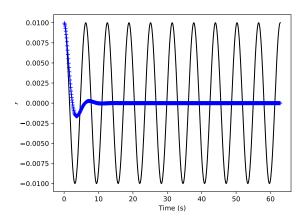
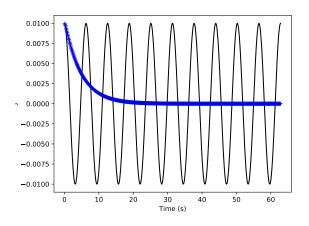


Figure 9

Overdamping



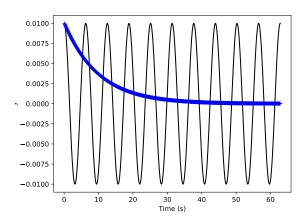
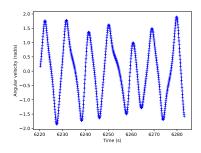
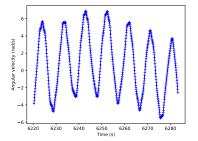


Figure 10

Forced





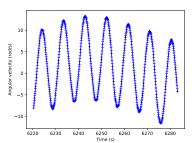


Figure 11: Apparent period of oscillation (when Forcing=0.5) (from sign changes of ang velocity): 9.42477796077 Apparent period of oscillation (when Forcing=1.2) (from sign changes of ang velocity): 11.0879740715 Apparent period of oscillation (when Forcing=1.44) (from sign changes of ang velocity): 9.42477796077 Apparent period of oscillation (when Forcing=1.465) (from sign changes of ang velocity): 9.42477796077

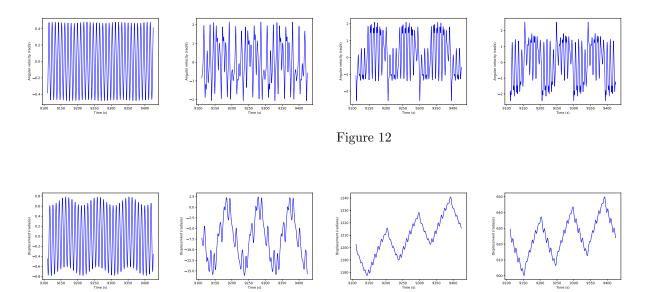


Figure 13

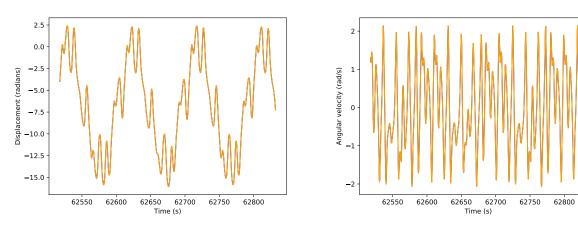
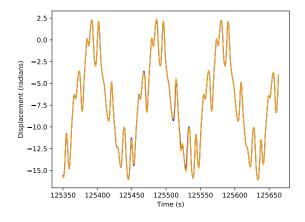


Figure 14



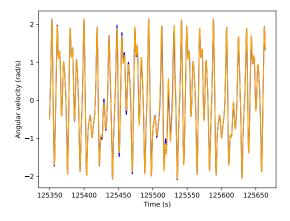
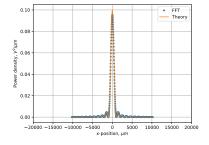
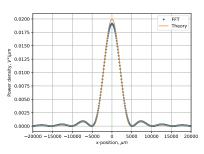


Figure 15

Exercise 3

Task 1





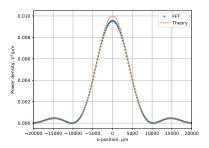


Figure 16

Task 2

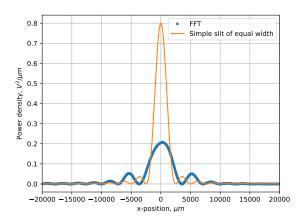
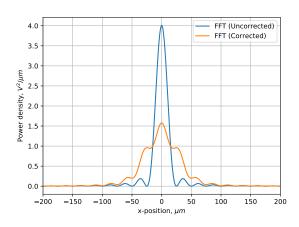


Figure 17

Task 3



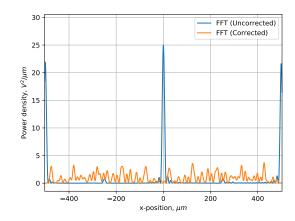


Figure 18

Appendix

Task 1 Code

```
#More complicated pendulum for task1
import numpy as np
import matplotlib.pyplot as plt
                             # Time step
z = 0.0 \ \# \ z is damping factor gamma – q was already in use
b=1 ## b = g/l ## = w0**2 in small angle limit --> w0 = 1 rad/s --> f = 1/(2pi) Hz --> T = 2pi secon
F=0 \# no \ forcing \ in \ this \ task
omega_d=1 \# not used in this task
initial_displacement = 0.01
oscillations=170
A = np. array([[0,1],[0,-z]])
B =np.array([[0],[1]])
mint = (oscillations -10)*2*np.pi
\max t = \operatorname{oscillations} *2*\operatorname{np.pi} \# \operatorname{given} that b = 1
t = np. linspace(0, maxt, num=int(maxt/h))
def y_maker(time):
         return initial_displacement*np.cos(time) # the theoretical value in the small angle limit
def k_maker(q_value, time):
         return A. dot(q_value) + B*(-np. sin(q_value[0][0]) * b+F*np. sin(omega_d*time))
displacement = []
acceleration = []
pend_time = []
energy = [] # Energy by weight
def Pendulum(init=initial_displacement):
         k1 = np.zeros((2,1))
         q1 = np.zeros((2,1))
         q0 = np.zeros((2,1))
         counter = 0
         clicker = 0
         for i in t:
                  if i < 0.95*h:
                           q0[0][0] = init; q0[1][0] = 0
                  else:
                           k1 = k_{-}maker(q0, i*h) \# Approx for y gives approx for deriv
                           q1 = (q0 + k1*h/2)
                           k2 = k_maker(q1, i*h +h/2)
                           q2 = (q0 + k2*h/2)
                           k3 = k_maker(q2, i*h+h/2)
                           q3 = (q0 + k3*h)
                           k4 = k_maker(q3, (i+1)*h)
                           q1=q0+(k1+2*k2+2*k3+k4)*(h/6)
                                                                   # Intermediate value
                           if q1[0][0]*q0[0][0] < 0: # should click every time passes through origin
                                    clicker +=1
                           q0=q1
                  if counter\%20 = 0 and False:
                           energy.append\,(\,q0\,[\,0\,]\,[\,0\,]\,*\,*\,2\ +q0\,[\,1\,]\,[\,0\,]\,*\,*\,2\,\,)\,;\ pend\_time.append\,(\,i\,)
                  counter +=1
                  if i > mint and False:
                           displacement.append(q0[0][0])
                           acceleration.append(q0[1][0])
                           pend_time.append(i)
        return (2*i)/clicker
theta = np.linspace(0.001, np.pi, num = 50)
period = []
print(Pendulum(init=np.pi/2))
if True:
```

```
for init_d in theta:
                  period.append(Pendulum(init=init_d))
        plt.plot(theta, period, color='blue', marker='+')
        plt.xlabel('Initial_Displacement_(radians)')
        plt.ylabel('Period_(seconds)')
        plt.title('Period_vs_Initial_Displacement_for_a_Pendulum_(T(pi/2)='+" {:.2 f}".format(Pendulum
        plt.savefig('Period_Displacement.pdf')
        plt.clf()
if False:
        a = Pendulum()
if False:
        yexact = [y_maker(x) for x in pend_time]
        plt.plot(pend_time, yexact, color='black')
        plt.plot(pend_time, displacement, color='blue', marker='+')
        plt.xlabel('Time_(s)')
plt.ylabel('y')
        plt.savefig('Task1Pendulum.pdf')
if False:
        plt.plot(pend_time, energy, color='orange')
        plt.xlabel('Time_(s)')
        plt.ylabel('Energy/(mass*g)\lfloor (m^{-1})')
        plt.title('Energy_loss_over_time_when_timestep =='+str(h)+'seconds')
        plt.savefig('Energy_Loss_h'+str(h)+'_.pdf')
        plt.clf()
#legend('Exact', 'Approximate');
Task 2 Code
#Pendulum for task2 by sdat2
\#Usage: 2\_Task2Pendulum.py \ [damping \ coeff] \ [forcing \ coeff] \ [timestep] \ [oscillations \ modelled]
\#Example: 2\_Task2Pendulum.py 0.5 0.5 0.1 10
import numpy as np
import matplotlib.pyplot as plt
import sys
### Global Variable defaults ###
                           # Time step
h = 0.05;
z = 0.0 ## z is damping factor gamma - q was already in use
b=1 \# b = g/l
F=0 # Forcing
omega_d=2/3 # the freq of driving
initial_displacement = 0.01
oscillations=170
graph_os = 10
sample_os = 20
### Command Line Inputs ###
if len(sys.argv) > 1: z = float(sys.argv[1])
if len(sys.argv) > 2: F = float(sys.argv[2])
if len(sys.argv) > 3: h = float(sys.argv[3])
if len(sys.argv) >4: oscillations = int(sys.argv[4])
if len(sys.argv) >5: graph_os = int(sys.argv[5])
if len(sys.argv) >6: sample_os = int(sys.argv[6])
#Matrices from solving 2nd order ODE
A = np. array([[0,1],[0,-z]])
B = np. array([[0], [1]])
#it is only ever useful to plot about 10 oscillations worth
mint = (oscillations-graph_os)*2*np.pi
```

```
mint2=(oscillations -sample_os)*2*np.pi
\max t = \operatorname{oscillations} *2*\operatorname{np.pi} \# \operatorname{given} \ \operatorname{that} \ b = 1
t = np. linspace(0, maxt, num=int(maxt/h))
#sometimes it is useful to compare to perfect SHM
def y_maker(time):
         return initial_displacement*np.cos(time)
\mathbf{def} \ \mathbf{k}_{-}\mathbf{maker}(\mathbf{q}_{-}\mathbf{value}, \mathbf{time}) \colon \#(Otherwise \ known \ as \ the \ derivative!)
         \textbf{return } A. \det (q\_value) + B*(-np. \sin (q\_value [0][0]) * b + F*np. \sin (omega\_d*time))
displacement = []
velocity = []
pend_time = []
def Pendulum(init=initial_displacement):
         k1 = np.zeros((2,1))
         q1 = np.zeros((2,1))
         q0 = np.zeros((2,1))
         counter = 0
         \mathrm{clicker} \, = \, 0
         for i in t:
                   if i < 0.95*h:
                            q0[0][0] = init; q0[1][0] = 0
                   else:
                            k1 = k_{-}maker(q0,i) # Approx for y gives approx for deriv
                            q1 = (q0 + k1*h/2)
                            k2 = k_{\text{-}maker}(q1, i + h/2)
                            q2 = (q0 + k2*h/2)
                            k3 = k_maker(q2, i+h/2)
                            q3 = (q0 + k3*h)
                            k4 = k_{-}maker(q3, i*h)
                                                                     # Intermediate value
                            q1=q0+(k1+2*k2+2*k3+k4)*(h/6)
                            if q1[1][0]*q0[1][0]<0 and i>mint2: \# should click every time passes throug
                                     clicker +=1
                            q0=q1
                   counter +=1
                   i\,f\ i\ >\ \min t:
                            displacement.append(q0[0][0])
                            velocity.append(q0[1][0])
                            pend_time.append(i)
         return (2*(i-mint2))/clicker
if True:
         apparent_period = Pendulum(init=initial_displacement)
if True:
         \#y exact = [y_-maker(x) for x in pend_time]
         \#plt. plot(pend\_time, yexact, color = 'black')
         plt.plot(pend_time, displacement, color='blue')
         plt.xlabel('Time_(s)')
         plt.ylabel('Displacement_(radians)')
         plt.savefig('T2Displacement_'+str(F)+'_Forcing.pdf')
         plt.clf()
if True:
         plt.plot(pend_time, velocity, color='blue')
         plt.xlabel('Time_(s)')
         plt.ylabel ('Angular_velocity_(rad/s)')
         plt.savefig('T2Velocity\_'+str(F)+'\_Forcing.pdf')
print('Apparent_period_of_oscillation_(when_Forcing='+str(F)+')_(from_sign_changes_of_ang_velocity):
Task 3 Code
```

```
#Pendulum for task2 by sdat2
\#Usage: \ 2\_Task2Pendulum.py \ [damping \ coeff] \ [forcing \ coeff] \ [timestep] \ [oscillations \ modelled]
#Example: 2_Task2Pendulum.py 0.5 0.5 0.1 10
import numpy as np
import matplotlib.pyplot as plt
import sys
### Global Variable defaults ####
h = 0.05;
                             # Time step
z\!=\!0.0~\#\!\#~z~is~damping~factor~gamma-q~was~already~in~use
b=1 \# b = g/l
F=0 \# Forcing
omega_d=2/3 # the freq of driving
initial_displacement = 0.01
oscillations=170
graph_os = oscillations
sample_os = 20
### Command Line Inputs ###
if len(sys.argv) > 1: z = float(sys.argv[1])
if len(sys.argv) > 2: F = float(sys.argv[2])
if len(sys.argv) > 3: h = float(sys.argv[3])
if len(sys.argv) >4: oscillations = int(sys.argv[4])
if len(sys.argv) >5: graph_os = int(sys.argv[5])
if len(sys.argv) > 6: sample_os = int(sys.argv[6])
\#Matrices\ from\ solving\ 2nd\ order\ ODE
A = np. array([[0,1],[0,-z]])
B =np.array([[0],[1]])
\#it is only ever useful to plot about 10 oscillations worth
mint \, = \, \big(\, o\, s\, cill\, a\, t\, i\, o\, n\, s\, -g\, r\, a\, p\, h\, \_o\, s\, \big) *2 * np\, .\,\, p\, i
mint2=(oscillations -sample_os)*2*np.pi
\max t = \operatorname{oscillations} *2*\operatorname{np.pi} \# \operatorname{given} that b = 1
t = np. linspace(0, maxt, num=int(maxt/h))
#sometimes it is useful to compare to perfect SHM
def y_maker(time):
         return initial_displacement*np.cos(time)
def k_maker(q_value, time, z=z, F=F): #(Otherwise known as the derivative!)
         A = np. array([[0,1],[0,-z]])
         B = np. array([[0], [1]])
         return A. dot(q_value) + B*(-np. sin(q_value[0][0]) * b+F*np. sin(omega_d*time))
def Pendulum(init=initial_displacement, z=z,F=F):
         displacement = []
         velocity = []
         pend_time = []
         k1 = np.zeros((2,1))
         q1 = np.zeros((2,1))
         q0 = np.zeros((2,1))
         counter = 0
         clicker = 0
         for i in t:
                  if i < 0.95*h:
                           q0[0][0] = init; q0[1][0] = 0
                  else:
                           k1 = k_maker(q0, i, z=z, F=F)
                           q1 = (q0 + k1*h/2)
                           k2 = k_maker(q1, i +h/2, z=z, F=F)
                           q2 = (q0 + k2*h/2)
                           k3 = k_maker(q2, i+h/2, z=z, F=F)
```

```
q3 = (q0 + k3*h)
                             k4 = k_maker(q3, i*h, z=z, F=F)
                                                                         \# Intermediate value
                             q1=q0+(k1+2*k2+2*k3+k4)*(h/6)
                             if q1[1][0]*q0[1][0]<0 and i>mint2: \# should \ click \ every \ time \ passes \ throug
                                        clicker +=1
                             q0=q1
                    counter +=1
                    if i > mint:
                             displacement.append(q0[0][0])
                              velocity.append(q0[1][0])
                             pend_time.append(i)
         \textbf{return} \hspace{0.2cm} (2*(i-mint2))/\hspace{0.1cm} clicker \hspace{0.1cm}, \hspace{0.1cm} displacement \hspace{0.1cm}, \hspace{0.1cm} velocity \hspace{0.1cm}, \hspace{0.1cm} pend\_time
if True:
          apparent_period, DispA, VelA, TA = Pendulum(init=0.20)
          apparent_period, DispB, VelB, TB = Pendulum(init = 0.20001)
if True:
          plt.plot(TA, DispA, color='blue')
          plt.plot(TB, DispB, color='orange')
          plt.xlabel('Time_(s)')
          plt.ylabel('Displacement_(radians)')
          plt.savefig('T3_Displacement.pdf')
         plt.clf()
if True:
          plt.plot(TA, VelA, color='blue')
          plt.plot(TB, VelB, color='orange')
          plt.xlabel('Time_(s)')
          plt.ylabel('Angular_velocity_(rad/s)')
          plt.savefig('T3_Velocity.pdf')
\mathbf{print}(\ '\mathrm{Apparent\_period\_of\_oscillation\_(when\_Forcing='+\mathbf{str}(F)+')\_(from\_sign\_changes\_of\_ang\_velocity):
```

Task 1 Code

```
#program for task 1 by sdat2 drawing on
\#\ http://kmdouglass.github.io/posts/approximating-diffraction-patterns-of-rectangular-apertures-with
import numpy as np
import matplotlib.pyplot as plt
from scipy.fftpack import fft
from scipy.fftpack import fftshift, ifftshift
import sys
\mathbf{def} \operatorname{sinc}(\mathbf{x}):
    if (x != 0):
        \# Prevent divide-by-zero
        return \operatorname{np.sin}(\operatorname{np.pi} * x) / (\operatorname{np.pi} * x)
    else:
        return 1
sinc = np.vectorize(sinc)
\#globals
                      # Volt / sqrt(micron)
amplitude
             = 1
                        \# microns
slitWidth
             = 100
wavelength = 0.5 \# microns
propDistance = 10**6 \# microns (= 1 mm)
bins = 2048
AppertureWidth = 5000
#redefine by Command Line
if len(sys.argv)>1: bins = int(sys.argv[1])
if len(sys.argv)>2: wavelength = float(sys.argv[2])
if len(sys.argv)>3: slitwidth = float(sys.argv[3])
if len(sys.argv)>4: AppertureWidth = int(sys.argv[4])
if len(sys.argv)>5: propagation_distance = float(sys.argv[5])
      = np.linspace(-AppertureWidth/2, AppertureWidth/2, num = bins)
field = np.zeros(x.size, dtype='complex128') # Field complex
field [np.logical\_and (x > -slitWidth / 2, x <= slitWidth / 2)] = amplitude + 0j
#creates an apperture of the correct width and amp of 1
dx \,=\, x\,[\,1\,] \,\,-\,\, x\,[\,0\,] \,\,\#\,\, \textit{Spatial sampling period}\,\,,\,\,\, \textit{microns}
fS = 1 / dx
                  \#\ Spatial\ sampling\ frequency\,,\ units\ are\ inverse\ microns
f = (fS / x.size) * np.arange(0, x.size, step = 1) # a frequency type vector
diffracted Field = dx * np. fft. fft (np. fft. fftshift (field)) # take the FFT of the apperture function
# fftshift uses the symmetry of the appeture to get rid of negative part
        = \text{np.hstack}((f[-int((f.size/2)):] - fS, f[0:int(f.size/2)])) * wavelength * propDistance
IntensTheory = amplitude / (wavelength * propDistance) * \
    (slitWidth * sinc(xPrime * slitWidth / wavelength / propDistance))**2
IntensFFT
             = np.fft.fftshift(diffractedField * np.conj(diffractedField)) / wavelength / propDistan
plt.plot(xPrime, np.abs(IntensFFT), '.', label = 'FFT')
plt.plot(xPrime, IntensTheory, label = 'Theory')
plt.xlim((-20000, 20000))
plt.xlabel(r'x-position, _$\mu_m$')
plt.ylabel(r'Power_density, _$V^2_/_\mu_m$')
plt.grid(True)
plt.legend()
plt.savefig('Task1_lam='+str(wavelength)+'_d='+str(slitWidth)+'_D='+str(AppertureWidth)+'_microns.pd
Task 2 Code
#program for task 2 by sdat2 drawing on
\#\ http://kmdouglass.github.io/posts/approximating-diffraction-patterns-of-rectangular-apertures-with
import numpy as np
```

import matplotlib.pyplot as plt
from scipy.fftpack import fft

```
from scipy.fftpack import fftshift, ifftshift
import sys
\mathbf{def} \operatorname{sinc}(\mathbf{x}):
        if (x != 0):
               \# Prevent divide-by-zero
               return np. \sin (\text{np. pi} * x) / (\text{np. pi} * x)
        else:
               return 1
sinc = np. vectorize(sinc)
\#globals
                                        # Volt / sqrt(micron)
amplitude
                        = 1
slitWidth
                         = 2000
                                              # microns
wavelength = 0.5 \# microns
propDistance = 10*10**6 \# microns (= 10 m)
bins = 2048
AppertureWidth = 100000
m=8
s = 100
#redefine by Command Line
if len(sys.argv)>1: bins = int(sys.argv[1])
if len(sys.argv)>2: wavelength = float(sys.argv[2])
if len(sys.argv)>3: slitwidth = float(sys.argv[3])
if len(sys.argv)>4: AppertureWidth = int(sys.argv[4])
if len(sys.argv) > 5: propogation_distance = float(sys.argv[5])
           = np. linspace(-AppertureWidth/2, AppertureWidth/2, num = bins)
field = np.zeros(x.size, dtype='complex128') # Field complex
\mathbf{def} \ \mathrm{phase}(\mathrm{x}):
       return m/2*np. sin(2*np. pi*x/s)
for i in range(len(x)):
        if x[i] > -slitWidth / 2 and x[i] \le slitWidth / 2:
                field [i] = amplitude*(np.cos(phase(x[i])) + np.sin(phase(x[i]))*1j)
\#creates an apperture of the correct width and amp of 1
dx = x[1] - x[0] \# Spatial sampling period, microns
                                \# Spatial sampling frequency, units are inverse microns
fS = 1 / dx
f = (fS \ / \ x.\,size) \ * \ np.\,arange(0\,, \ x.\,size \,, \ step = 1) \ \# \ a \ frequency \ type \ vector
diffracted Field = dx * np. fft. fft (np. fft. fftshift (field)) # take the FFT of the apperture function
# fftshift uses the symmetry of the appeture to get rid of negative part
              = np. hstack ((f[-int((f.size/2)):] - fS, f[0:int(f.size/2)])) * wavelength * propDistance
IntensTheory = amplitude / (wavelength * propDistance) * \
        (slitWidth * sinc(xPrime * slitWidth / wavelength / propDistance))**2
                        = np.\,fft.\,fftshift\,(\,diffractedField\,\,*\,\,np.\,conj\,(\,diffractedField\,))\,\,\,/\,\,\,wavelength\,\,/\,\,propDistanter
IntensFFT
plt.plot(xPrime, np.abs(IntensFFT), '.', label = 'FFT')
plt.plot(xPrime, IntensTheory, label = `Simple\_slit\_of\_equal\_width')
plt.xlim((-20000, 20000))
plt.xlabel(r'x-position, _$\mu_m$')
plt.ylabel(r'Power_density, \_$V^2_/_\mu_m$')
plt.grid(True)
plt.savefig('Task2\_lam='+str(wavelength)+'\_d='+str(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_D='+str(AppertureWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'\_microns.pdf(slitWidth)+'
Task 3 Code
#program for task 2 by sdat2 drawing on
\#\ http://kmdouglass.github.io/posts/approximating-diffraction-patterns-of-rectangular-apertures-with
```

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.fftpack import fft
from scipy.fftpack import fftshift, ifftshift
import sys
\mathbf{def} \operatorname{sinc}(\mathbf{x}):
     if (x != 0):
         \# Prevent divide-by-zero
         return np. sin (np. pi * x) / (np. pi * x)
     else:
         return 1
sinc = np. vectorize(sinc)
\#globals
                         # Volt / sqrt(micron)
amplitude
               = 1
slitWidth
               = 2000
                            # microns
wavelength = 0.5 \# microns
propDistance = 10*10**6 \# microns (= 10 m)
bins = 2048
AppertureWidth = 100000
m=8
s = 100
sinusoidal=False
plt_theory=False
x_{lim} = 20000
#redefine by Command Line
if len(sys.argv)>1: bins = int(sys.argv[1])
if len(sys.argv)>2: wavelength = float(sys.argv[2])
if len(sys.argv)>3: slitWidth = float(sys.argv[3])
if len(sys.argv)>4: AppertureWidth = int(sys.argv[4])
if len(sys.argv)>5: propDistance = float(sys.argv[5])
if len(sys.argv)>6: x_lim = float(sys.argv[6])
       = np.linspace(-AppertureWidth/2, AppertureWidth/2, num = bins)
field = np.zeros(x.size, dtype='complex128') # Field complex
newfield = np.zeros(x.size,dtype='complex128')
\mathbf{def} \ \mathrm{phase}(\mathrm{x}):
    return m/2*np.sin(2*np.pi*x/s)
for i in range (len(x)):
     \label{eq:if_x[i]} \textbf{if} \ x\hspace{.1cm}[\hspace{.1cm} i\hspace{.1cm}] \ > -slitWidth \hspace{.1cm} / \hspace{.1cm} 2 \hspace{.1cm} \textbf{and} \hspace{.1cm} x\hspace{.1cm}[\hspace{.1cm} i\hspace{.1cm}] \ <= \hspace{.1cm} slitWidth \hspace{.1cm} / \hspace{.1cm} 2 \colon
         if sinusoidal:
              field \, [\,i\,] \, = \, amplitude \, *(np.\cos(\,phase\,(x\,[\,i\,]\,)) \, + \, np.\sin(\,phase\,(x\,[\,i\,]\,)) \, *1\,j\,)
              field [i] = amplitude + 0j
         newfield[i] = field[i]*np.exp(1j*np.pi*(x[i]**2)/(wavelength*propDistance))
#creates an apperture of the correct width and amp of 1
dx = x[1] - x[0] \# Spatial sampling period, microns
                    \# Spatial sampling frequency, units are inverse microns
fS = 1 / dx
  = (fS / x.size) * np.arange(0, x.size, step = 1) # a frequency type vector
diffracted Field = dx * np. fft. fft (field) # take the FFT of the apperture function
newdiffractedField = dx * np.fft.fft(newfield)
# fftshift uses the symmetry of the appeture to get rid of negative part
          = np.\,hstack\,((\,f[-\textbf{int}\,((\,f.\,size\,/\,2)):]\,\,-\,\,fS\,,\ f\,[\,0:\textbf{int}\,(\,f.\,size\,/\,2)]))\ *\ wavelength\ *\ propDistance]
IntensTheory = amplitude / (wavelength * propDistance) * \
     (slitWidth * sinc(xPrime * slitWidth / wavelength / propDistance))**2
             = np.fft.fftshift(diffractedField * np.conj(diffractedField)) / wavelength / propDistan
newIntensFFT =np.fft.fftshift(newdiffractedField * np.conj(newdiffractedField)) / wavelength / propI
```

```
plt.plot(xPrime, np.abs(IntensFFT), label = 'FFT_(Uncorrected)')
plt.plot(xPrime, np.abs(newIntensFFT), label = 'FFT_(Corrected)')
if plt_theory: plt.plot(xPrime, IntensTheory, label = 'Simple_slit_of_equal_width')
plt.xlim((-x_lim , x_lim))
plt.xlabel(r'x-position, _$\mu_m$')
plt.ylabel(r'Power_density, _$V^2_/_\mu_m$')
plt.grid(True)
plt.legend()
plt.savefig('Task3_lam='+str(wavelength)+'_d='+str(slitWidth)+'_D='+str(propDistance)+'_microns.pdf')
```