Monoids

aka Semigroups with a sense of self

David Pullara, January 2021

Semigroup

(*S*, *op*)

a set *S* with a closed, associative binary operation *op*

Semigroup

(*S*, *op*)

```
S: Set[A]
op: A => A => A
```

Semigroup

Axioms

$$(ab)c = a(bc)$$

Semigroup

Examples

```
(Set[Int], +)
(Set[Int], *)
(Set[String], ++)
(Set[RelativeFilePaths], ++)
```

Monoid

(*S*, *op*, *e*)

a set *S* with a closed, associative binary operation *op* and an identity element *e*

Monoid

(*S*, *op*, *e*)

```
S: Set[A]
op: A => A => A
S contains e
```

Monoid

Axioms

$$(ab)c = a(bc)$$

$$ea = ae = a$$

Monoid

Examples

```
(Set[Int], +, 0)
(Set[Int], *, 1)
(Set[String], ++, “”)
(Set[RelativeFilePaths], ++, “”)
```

Scala

```scala
trait Semigroup[A] {
  def combine(x: A, y: A): A
}
trait Monoid[A] extends Semigroup[A] {
  def empty: A
}
```

Scala

If only we had an example to play with...

More to Explore

- Monoid (Wikipedia)
- Monoids: what they are, why they are useful, and what they teach us about software (blog)
- What are monoids and how to use them (Jakub Kozłowski, youtube)
- Practical Category Theory: Monoids (David Koontz, youtube)
- Monoids, Monoids, Monoids (Luka Jacobowitz, youtube)
- Chain – Replacing the List Monoid (blog)
- Monoid Morphisms, Products, and Coproducts (blog)
- A great series of posts on the power of monoids (blog)