# Why $r^2$ is meaningless

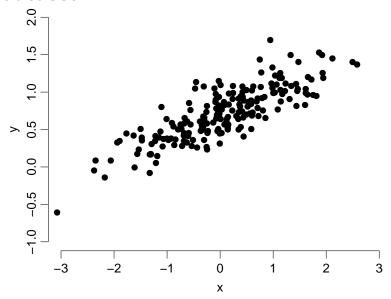
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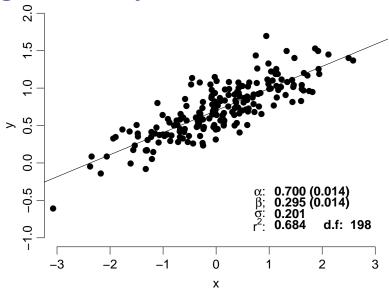
March 2015



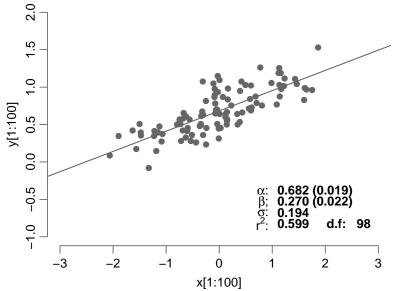
### A dataset



### Regression analysis

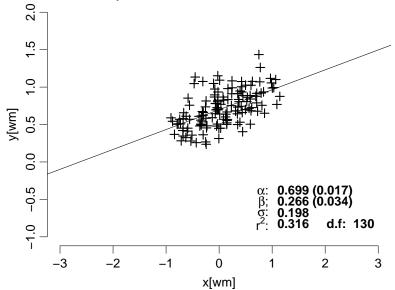


# A 50% random sample



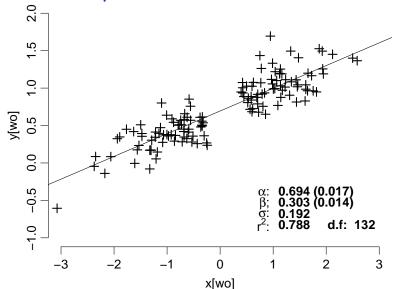
Same estimates of  $\alpha$ ,  $\beta$  and  $\sigma$  and same  $r^2$ 

# The middle 2/3 of the data



Same estimates of  $\alpha$ ,  $\beta$  and  $\sigma$  but **smaller**  $r^2$ 

# The outer 2/3 of the data



Same estimates of  $\alpha$ ,  $\beta$  and  $\sigma$  but larger  $r^2$ 

#### What $r^2$ is and is not

- ▶ r² is related to the **population** analysed: It is the fraction of the **population** variation in y which explained by x.
- ▶  $r^2$  does **not** convey **any** information on the **size** of the relationship. The relationship is judged from the estimates of  $\alpha$  and  $\beta$ : Is the effect clinically relevant?.
- ▶  $r^2$  does **not** convey any information on the **precision of predictions**. This is contained in the residual variation,  $\sigma$ . A 95% prediction interval for given  $x = x_0$  is:

$$\hat{y}_0 = \hat{\alpha} + \hat{\beta} \times x_0 \pm 1.96\hat{\sigma}$$

(disregarding the estimation error in  $\alpha$  and  $\beta$ ).

#### Moral:

- ▶ The clinically relevant parameters  $\alpha$ ,  $\beta$  and  $\sigma$  are the same no matter how the population is sampled.
- ightharpoonup They reflect the relationship between y and x.
- $ightharpoonup r^2$  involves the population distribution, which is alien to the relationship between y and x.
- lacktriangleright Hence,  $r^2$  is mathematical mumbo-jumbo where the link to subject matter relevance has been obscured by mixing in the distribution of y in the study population.