HW3 - Data Science

S Deepak Narayanan
16110142

4)

a) No. This is because it could so happen that we have a sample like $\boxed{1}$ $\boxed{1}$ $\boxed{1}$ in the first case, whereas that won't occur in case two (second case).

In second case, we will not have reservoirs holding same values multiple times.

In case 1, it is ~~always~~ could be the case ~~that~~ we have same items sampled multiple times.

We could have all possible samples that we have when we have a reservoir of size ~~of~~ k ~~and~~ in the multiple reservoir case but the vice versa is not true.

k size storage $\Rightarrow$ k 1 size storage

k 1 size storage $\not\Rightarrow$ k size storage

Eg: $\boxed{1}$ $\boxed{1}$ $\boxed{1}$ $\longleftrightarrow$ $\boxed{1 \mid 2 \mid 3}$,

Never in case 2

b) By induction hypothesis, assume that we have i+1 elements in stream and we have reservoir of size k.

Suppose first i elements follow the fact that $Pr[\text{element in reservoir}] = \frac{k}{i}$

For the $(i+1)^{th}$ item,

Probability of it getting added is $\frac{k}{i+1}\left(\frac{1}{k}+-\frac{1}{k}\right)$

$= \frac{k}{i+1}$ (Since we choose up $\frac{k}{i+1}$) $\boxed{\frac{k}{k}=1}$

($\frac{1}{k} \rightarrow$ Pr of replacing an element) and dont choose up $1-\frac{k}{i+1}$) $\rightarrow$ for n element

~~Probability of an existing element staying at time i a ~~new element getting chosen~~ and that it it staying in reservoir at time i is not replaced at time i+1~~

$$\cancel{\Rightarrow \frac{k}{i}\left(1-\left(\frac{k}{i+1}\right)\cdot\left(\frac{1}{k}\right)\right) = \frac{k}{i+1}}$$

(b) Probability an old element stayed in stream at time i, and the probability it wasn't kicked out at time i+1.

$$\Rightarrow \frac{k}{i}\left(1-\left(\frac{k}{i+1}\right)\left(\frac{1}{k}\right)\right) = \frac{k}{i+1}$$

(c) No. This is not independent sampling. Our sample space reduces if we want to have the followings.

Suppose element x_1 is present in reservoir and we want to compute probability that x_2 is also added to reservoir.

without kicking out x_1. ~~This is~~ Assume at time $(t+1)$ x_2 comes. If we add x_2 to reservoir we need to ensure that x_1 is not kicked out,

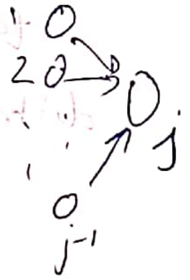
ie, Pr of [x_2 along with x_1] $= \left(\frac{k}{t+1}\right)\left(\frac{k-1}{k}\right)$

$\Rightarrow$ Not independent, since this varies ~~as~~ number of samples varies.

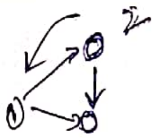
---

3) $p \to$ Probability Vector

$$T \quad \begin{array}{c|c} & \quad\quad j \\ i & \begin{bmatrix} \cancel{p_2} & p_j & p_n \end{bmatrix} \end{array} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$



$p_{1j} \quad p_{2j} - p_{j-1\,j}$



$$\begin{bmatrix} p_2 & p_3 \\ p_1 & p_3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \quad p = Tp \quad \text{for stationarity}$$

$$\delta_{ij} = \begin{cases} 1, & \text{edge from } i \to j \\ 0, & \text{else} \end{cases}$$

$p_1p_2 + p_3p_3$
$p_1p_1 + p_3p_3$
0

$$\begin{pmatrix} p_2^2 + p_3^2 \\ p_1^2 + p_3^2 \\ 0 \end{pmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Added to make stationarity

$$\boxed{\delta_{ij} = \begin{cases} 1 & \text{edge from } i \to j \\ 0 & \text{else} \end{cases}}$$

$$p_i = \sum_{j=1}^{n} p_j^2 \delta_{ij} + x$$

~~xo~~ We can edit only the p_ith element if we have to add

Pro self loops. So, $x = p_i x'$

(

$$p_i = \sum_{j=1}^{n} p_j^{2} \delta_{ij} + p_i x'$$

$$x' = \frac{1 - \sum_{j=1}^{n} p_j^{2} \delta_{ij}}{p_i}$$

We need to add self loops summing upto the above value to make p, a stationary distribution.