ECE598 Dynamical Systems & Neural Networks: Project Proposals

proposed by Rainer Engelken engelken@illinois.edu, September 2025

Contents

A.) Lyapunov Spectra of Training Dynamics

Goal

Imagine training itself as a dynamical system: the weight updates of a neural network define trajectories in weight space. By calculating the Lyapunov spectrum of this process, you can quantify how nearby weight trajectories diverge or converge under (stochastic) gradient descent. The project explores how spectra depend on network architecture, initialization, optimizer choice, or the task being learned.

Project Rationale

Most analyses of deep learning optimization focus on the static geometry of the loss landscape (e.g., its Hessian). This project adopts a complementary, dynamical perspective, focusing on the stability of the training trajectory itself. The central question is what new insights this dynamical view offers. For instance, do stable, non-chaotic training dynamics correlate with better generalization? This is an opportunity to apply a classic tool from physics to a core problem in modern machine learning.

Optional extensions

- If you are more interested in analytical work, you could study an idealized toy-problem fully analytically.
- If you are more interested in numerical work, you could go to more modern and complicated tasks, architectures, and optimizers.

Key outcomes

- Quantify the largest Lyapunov exponent of (S)GD and compare it with brute-force numerical simulations in a simple scenario (e.g., ridge regression).
- **(Stretch):** Understand it analytically or study it numerically in more sophisticated tasks, architectures, or optimizers.

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★★
Comp Neuro:	-
ML:	★★★
Starter toolkit:	Reference example implementation exists (Julia and Python)

Literature

Core References

- Chemnitz, D., & Engel, M. (2024). *Characterizing Dynamical Stability of Stochastic Gradient Descent in Overparameterized Learning.* arXiv:2407.20209. https://doi.org/10.48550/arXiv.2407.20209
- Sharafi, N., Martin, C., & Hallerberg, S. (2024). *Weight dynamics of learning networks.* Physical Review E, 111(5), 054208. https://doi.org/10.1103/PhysRevE.111.054208
- Engelken R., Wolf F., Abbott L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks.* Phys. Rev. Research 5:043044. www.doi.org/10.1103/PhysRevResearch.5.043044

B.) Spiking Chaos in Space

Goal

This project aims to characterize how distance-dependent wiring in large-scale spiking networks shape their collective dynamics. The core task is to simulate networks with distance-dependent coupling, identify different dynamical regimes (e.g., asynchronous chaos, traveling waves, Turing instability), and then quantify the stability and chaoticity of each regime by computing their full Lyapunov spectra.

Project Rationale

While models with spatial structure are known to generate complex patterns, a rigorous analysis of their collective dynamics is largely missing. This project directly connects the well-established field of pattern formation with the quantitative framework of dynamical systems theory. The work involves applying a sophisticated analysis tool (Lyapunov spectra) to a biologically motivated problem, providing a deeper understanding of how network structure constrains network dynamics.

Optional Extensions

- **Analytical:** Refine the analytical understanding of the phase diagram.
- **Dynamical Systems:** Link the collective activity modes to covariant Lyapunov vectors (the directions of largest instability along a trajectory).
- **Biological Applications:** Link these spatiotemporal dynamics to activity patterns in healthy brains (e.g., traveling waves during sleep, grid cell formation) or pathological states (e.g., epilepsy).

Key outcomes

- Reimplement the simplest nontrivial scenario of spatiotemporal activity patterns you can find (e.g., Pyle & Rosenbaum, 2019) in event-based simulations and carefully test it.
- **(Stretch):** Quantify, analyze, and visualize the chaos in this spatiotemporal dynamics and interpret the results.

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★★
Comp Neuro:	★★
ML:	-
Starter toolkit:	Reference example implementation exists (Julia)

Literature

Core References

- Pyle, R., & Rosenbaum, R. (2019). *Spatiotemporal Dynamics of Balanced Networks with Spatial Connectivity*. Physical Review X, 9(2), 021051. https://doi.org/10.1103/PhysRevX.9.021051
- Engelken, R. (2023). *SparseProp: Efficient event-based simulation and training of sparse recurrent spiking neural networks*. Advances in Neural Information Processing Systems, 36. https://doi.org/10.48550/arXiv.2312.17216

Additional Literature

- Huang, C., et al. (2019). *Circuit Models of Low-Dimensional Shared Variability in Cortical Networks*. Neuron, 101(2), 337-348.e4. https://doi.org/10.1016/j.neuron.2018.11.034
- Ginelli, F., et al. (2007). *Characterizing Dynamics with Covariant Lyapunov Vectors*. Physical Review Letters, 99(13), 130601. https://doi.org/10.1103/PhysRevLett.99.130601

C.) Adversarial Examples and Dynamical Instability

Goal

Adversarial examples occur when tiny changes in input (e.g., pixels of an image) cause a large change in output (e.g., flipping a classifier's label). This project reframes adversarial robustness through a dynamical systems lens: under what conditions do small perturbations grow into large deviations? You will study how adversarial vulnerability relates to the end-to-end Jacobian spectrum of a network and to its finite-time Lyapunov exponents. The project also explores how basins of attraction shape robustness: how small must a perturbation be to push an input across the decision boundary into a different attractor basin?

Project Rationale

By connecting adversarial robustness to dynamical instability, this project offers a new way to frame one of ML's central challenges. You can test hypotheses such as whether keeping finite-time Lyapunov exponents near zero (e.g., via gradient flossing) enhances robustness, or how finite-size perturbations interact with basins of attraction. This is an open-ended and exploratory project with many possible paths, an excellent opportunity for ambitious students to push into new territory.

Key Outcomes

- Reimplement a method to compute the finite-time Lyapunov exponents for a standard image classifier (Storm 2024)
- **(Stretch):** Investigate the relationship between these exponents and the network's vulnerability to a standard adversarial attack.
- **(Stretch):** Relate the directions of greatest instability to the most vulnerable adversarial attack vectors.

Project profile

Analytical Intensity:	★ – ★★ (pick your level)
Coding Intensity:	★★ – ★★★
Comp Neuro:	★★
ML:	★★★
Starter toolkit:	No example implementation exists

Core References

- Storm, L., et al. (2024). *Finite-time Lyapunov exponents of deep neural networks*. Physical Review Letters 132.5: 057301.
- Engelken R. (2023). *Gradient Flossing: Improving Gradient Descent through Dynamic Control of Jacobians*. NeurIPS. https://doi.org/10.48550/arXiv.2312.17306

Additional References

- Szegedy, C., et al. (2013). *Intriguing properties of neural networks.* arXiv preprint arXiv:1312.6199.

- Goodfellow, I. J., Shlens, J., & Szegedy, C. (2014). *Explaining and harnessing adversarial examples.* arXiv preprint arXiv:1412.6572.

- Ginelli, F., et al. (2007). *Characterizing Dynamics with Covariant Lyapunov Vectors.* Physical Review Letters, 99(13), 130601. https://doi.org/10.1103/PhysRevLett.99.130601

D.) Rate Variability and Chaos

Goal

Investigate how chaos in spiking networks influences population rate variability. In chaotic regimes, two nearby spike-train trajectories diverge, which leads to different measurable macroscopic rate flucutations that should be measurable (e.g., trial-to-trial variability). In contrast, non-chaotic networks are expected to show reproducible rate dynamics across trials. This project bridges the microscopic spiking level with macroscopic rate variability, revealing how chaos leaves signatures across scales and can potentially make experimentally testable predictions.

Project Rationale

This project links fundamental dynamical instability with experimentally measurable quantities like rate variability. It highlights how chaos shapes observable neural dynamics, offering potentially a new conceptual bridge between theory and experiment.

Key Outcomes

- Implement measures of firing rate variability and synchrony (Fano factor, population synchrony) in a spiking network with strong population fluctuations. Test and benchmark them carefully.

- Implement and test a power-law recurrent network architecture or another architecture that gives rise to macroscopic rate fluctuations.

- Compare microscopic measures of dynamic instability to macroscopic measures of firing rate variability.

- **(Stretch):** Corroborate your findings in an input-driven scenario where the network receives time-varying external input and relate your findings to the existing experimental literature.

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	Reference example implementation exists (Julia)

Literature

Core References

- Farkhooi, F. *Weak-correlation universality and macroscopic fluctuations in power-law recurrent networks.* (PRL 2025). https://doi.org/10.1103/l6pc-y1x6
- Engelken R., Monteforte M., Wolf F. (2024). *Sparse chaos in neural circuits.* arXiv 2412.21188. www.doi.org/10.48550/arXiv.2412.21188

Additional Literature

- Huang, C., et al. (2019). *Circuit Models of Low-Dimensional Shared Variability in Cortical Networks.* Neuron, 101(2), 337-348.e4. https://doi.org/10.1016/j.neuron.2018.11.034

E.) Gradient Flossing in Diverse Network Architectures

Goal

Extend the idea of gradient flossing, nudging Lyapunov exponents of the end-to-end Jacobian toward zero, beyond simple recurrent networks. The project explores how flossing affects trainability across different architectures, including convolutional networks, very deep feedforward nets, transformers (standard and looped/latent reasoning models), and modern gated architectures such as MesaNet, DeltaNet xLSTM, or Mamba2. Each student can focus on one architecture, or multiple students can pursue this project in parallel on different systems.

Project Rationale

This project applies the link between trainability (in backprop) and stability (in dynamical systems) that was previously explored in recurrent neural networks to other network architectures. It is open-ended, technically challenging, and potentially highly rewarding, making it an excellent project for self-driven students eager to push into new research territory.

Key Outcomes

- Implement gradient flossing for a chosen modern architecture (e.g., a Transformer).
- **(Stretch):** Systematically evaluate its effect on training stability, speed, and final performance compared to standard training methods.
- **(Stretch):** Experiment with cheaper proxies of flossing, e.g. only regularize Jacobians or randomized linear algebra approaches.

Project profile

Analytical Intensity:	★
Coding Intensity:	★★★
Comp Neuro:	-
ML:	★★★
Starter toolkit:	Reference example implementation exists (Julia and Python)

Literature

Core References

- Engelken R. (2023). *Gradient Flossing: Improving Gradient Descent through Dynamic Control of Jacobians.* NeurIPS. https://doi.org/10.48550/arXiv.2312.17306
- Doshi, D., He, T., & Gromov, A. (2023). *Critical initialization of wide and deep neural networks using partial jacobians.* Advances in Neural Information Processing Systems, 36, 40054-40095.

Optional Literature

- Cowsik, A., et al. (2024). *Geometric dynamics of signal propagation predict trainability of transformers.* arXiv preprint arXiv:2403.02579.
- Gonzalez, X., et al. (2025). *Predictability Enables Parallelization of Nonlinear State Space Models.* arXiv preprint arXiv:2508.16817.
- Vock, S., & Meisel, C. (2025). *Critical dynamics governs deep learning.* arXiv preprint arXiv:2507.08527.
- Vogt, R., et al. (2022). *On lyapunov exponents for rnns: Understanding information propagation using dynamical systems tools.* Frontiers in Applied Mathematics and Statistics, 8, 818799.
- Zheng, C., & Shlizerman, E. (2025). *Hyperpruning: Efficient Search through Pruned Variants of Recurrent Neural Networks Leveraging Lyapunov Spectrum.* arXiv preprint arXiv:2506.07975.
- Hazelden, J., et al. (2025). *KPFlow: An Operator Perspective on Dynamic Collapse Under Gradient Descent Training of Recurrent Networks.* arXiv preprint arXiv:2507.06381.
- Bordelon, B., et al. (2025). *Dynamically learning to integrate in recurrent neural networks.* arXiv preprint arXiv:2503.18754.

F.) How Network Structure Shapes Dynamics & Chaos (Firing-Rate Networks)

Goal

How does network structure shape the chaos and complexity of the dynamics? This project studies how different connectivity patterns (for example random, small-world, (anti-)symmetry, or second-order motifs) reshape the Lyapunov spectrum and topological or metric invariants like the Kaplan–Yorke dimension and Kolmogorov–Sinai entropy.

Project Rationale

Even small changes in network features can transform dynamics in surprising ways. By mapping how the Lyapunov spectrum and related invariants depend on specific features, you may uncover new links between wiring statistics and collective dynamics.

Optional extensions

- If you want to push the theoretical frontier, you could try to study this using dynamic mean-field theory
- If you are interested more in biological questions, you could also ask how experimentally observed network structures in biological data shape network dynamics.
- If you are more interested in neural computation, you could also investigate the computational properties of different network structures.

Key Outcomes

- Implement and test a firing-rate network with a specific, non-random structural feature.
- Quantify and analyze the full Lyapunov spectrum for this network and compare it to a random network, interpreting the differences in the context of the structural feature.
- **(Stretch):** link the dynamical properties to computational properties, e.g. trainability

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★
Comp Neuro:	★★
ML:	★ – ★★★ (pick your level)
Starter toolkit:	Reference example implementation exists (Julia and Python)

Literature

Core References

- Sompolinsky, H., Crisanti, A., & Sommers, H. J. (1988). *Chaos in random neural networks.* Physical Review Letters, 61(3), 259–262. https://doi.org/10.1103/PhysRevLett.61.259
- Engelken, R., Wolf, F., & Abbott, L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks.* Physical Review Research, 5(4), 043044. https://doi.org/10.1103/PhysRevResearch.5.04304

Additional literature (optional)

- Rajan, K., & Abbott, L. F. (2006). *Eigenvalue spectra of random matrices for neural networks.* Physical Review Letters, 97(18), 188104.
- Ahmadian, Y., Fumarola, F., & Miller, K. D. (2015). *Properties of networks with partially structured and partially random connectivity.* Physical Review E, 91(1), 012820.
- Watts & Strogatz, 1998; Barabási & Albert, 1999; Zhao et al., 2011 (SONET); Wainrib & Touboul, 2013; Hu & Sompolinsky, 2022; Ganguli et al., on embedded feedforward structure.

G.) How Network Structure Shapes Dynamics & Chaos (Spiking Networks)

Goal

Same as Project F, but in spiking networks. This project investigates how different network connectivity patterns and structural motifs shape the dynamics in large-scale spiking networks.

Project Rationale

While the dynamics of random spiking networks are increasingly well understood, the impact of specific, non-random structures remains an open area of research. This project applies the rigorous framework of dynamical systems analysis to understand the structure-dynamics relationship in more biologically detailed models.

Optional extensions

- If you are interested more in biological questions, how do structural motifs found in real neural circuits (e.g., from connectomics data) influence the stability of spiking dynamics?
- If you are more interested in neural computation, do certain network structures promote computationally relevant dynamics, such as stable representations or flexible responses to input?

Key Outcomes

- Implement a spiking network with a specific structural feature (e.g., embedded feedforward chains).
- Quantify the Lyapunov spectrum and relate its features to the underlying network topology, comparing the results to a randomly connected control network.

Project profile

Analytical Intensity:	-
Coding Intensity:	★
Comp Neuro:	★★★
ML:	★
Starter toolkit:	Reference example implementation exists (Julia)

Literature

Core References

- Monteforte, M., & Wolf, F. (2010). *Dynamical entropy production in spiking neuron networks in the balanced state.* Physical Review Letters, 105(26), 268104. https://doi.org/10.1103/PhysRevLett.105.268104
- My tutorial on quantifying chaos (week 3)
- Engelken, R., Wolf, F., & Abbott, L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks.* Physical Review Research, 5(4), 043044. https://doi.org/10.1103/PhysRevResearch.5.043044
- Sonnet paper; AB paper; feedforward paper embedded paper (Ganguli)

H.) Improving Spiking Network Training using Dynamical Systems

Goal

The goal is to overcome a key obstacle in training brain-inspired Spiking Neural Networks (SNNs). While SNNs are promising, their all-or-nothing "spikes" are non-differentiable, clashing with standard training algorithms. The common workaround, "surrogate gradients," often fails on temporal tasks due to unstable error signals that either explode or vanish. This project reframes the problem using dynamical systems theory. You will evaluate a novel technique called surrogate gradient flossing, which measures the stability of the training process by calculating "surrogate Lyapunov exponents." By actively tuning the network to control these exponents, you will stabilize learning and enable SNNs to tackle challenging temporal credit assignment problems.

Project Rationale

The "surrogate gradient flossing" idea has been implemented and showed initial promise, but it needs more rigorous evaluation and a deeper understanding of its mechanics. The central question is whether explicitly controlling the stability of the gradient flow is a viable and robust strategy for improving learning in SNNs. This project offers a chance to explore the fundamental link between a system's dynamics and its capacity to learn.

Key Outcomes

- Train simple SNN with surrogate gradient flossing and analysis of gradients.
- **(Stretch):** Compare their performance on a standard benchmark tasks (spiking spoken digits classification) and analyze the resulting network dynamics.
- **(Stretch):** Implement a multi-layer spiking network and quantify gradients and performance with and without surrogate gradient flossing.
- **(Stretch):** Optimize the gradient flossing schedule.

Project profile

Analytical Intensity:	-
Coding Intensity:	★★★
Comp Neuro:	★★★
ML:	★★★
Starter toolkit:	Reference example implementation exists (Julia and Python)

Literature

- Neftci, E. O., Mostafa, H., & Zenke, F. (2019). *Surrogate gradient learning in spiking neural networks.* IEEE Signal Processing Magazine, 36(6), 51-63.
- Engelken R. (2023). *Gradient Flossing: Improving Gradient Descent through Dynamic Control of Jacobians.* NeurIPS.
  https://doi.org/10.48550/arXiv.2312.17306
- Gygax, J., & Zenke, F. (2025). *Elucidating the theoretical underpinnings of surrogate gradient learning in spiking neural networks.* Neural Computation, 37(5), 886-925.
- Rossbroich, J., Gygax, J., & Zenke, F. (2022). *Fluctuation-driven initialization for spiking neural network training.* Neuromorphic Computing and Engineering, 2(4), 044016.
- Yik, J., et al. (2025). *The neurobench framework for benchmarking neuromorphic computing algorithms and systems.* Nature communications, 16(1), 1545.

I.) A Better Dynamical Systems Theory of Exploding and Vanishing Gradients

Goal

Gradient flossing exploits a link between Lyapunov exponents and exploding/vanishing gradients to understand and mitigate these problems. However, the theoretical understanding of when/how it works and when/how it fails is at best rudimentary. Your task will be to work towards a deeper theoretical understanding. By starting in an analytically tractable scenario (e.g., deep linear networks), you will explore both with pen and paper and in very controlled simulations the mathematical basis of gradient flossing.

Project Rationale

Developing a rigorous theory of gradient stability is a fundamental challenge in deep learning. This project aims to move beyond empirical observations by building a solid mathematical foundation in simplified settings. Success in this area could lead to more principled and efficient methods for training deep networks.

Key Outcomes

- Re-derive the connection between Jacobians and gradients in a deep linear network.
- **(Stretch):** Provide a new theoretical insight into why controlling the Lyapunov spectrum (via gradient flossing) mitigates the vanishing/exploding gradient problem in this context, and verify it numerically.

Project profile

Analytical Intensity:	★★ – ★★★
Coding Intensity:	★ – ★★
Comp Neuro:	★
ML:	★★★
Starter toolkit:	Reference implementation exists (Julia and Python)

Literature

Core References

- Engelken, R. (2023). *Gradient Flossing: Improving Gradient Descent through Dynamic Control of Jacobians*. Advances in Neural Information Processing Systems, 36. https://doi.org/10.48550/arXiv.2312.17306
- Saxe, A. M., McClelland, J. L., & Ganguli, S. (2013). *Exact solutions to the nonlinear dynamics of learning in deep linear neural networks*. arXiv preprint arXiv:1312.6120. https://doi.org/10.48550/arXiv.1312.6120

Additional Literature

- Engelken, R., Wolf, F., & Abbott, L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks*. Physical Review Research, 5, 043044. https://doi.org/10.1103/PhysRevResearch.5.043044

J.) Graph Rules for Spiking Networks

Goal

The dynamics of a neural circuit are fundamentally shaped by its wiring diagram. Spiking networks, for instance, can settle into stable limit cycles or generate chaotic activity, with each regime having unique computational properties. This project aims to move from empirical observation to a predictive understanding by finding the network structures that maximize or minimize chaos. Using efficient event-based simulations to quantify chaos, your challenge will be to discover these structures, first through exhaustive search in very small networks, and then using optimization algorithms for intermediate-sized ones. The final step is to analyze the resulting topologies and identify the properties that promote or suppress chaotic dynamics.

Project Rationale

While it is widely accepted that network structure governs dynamics, a theoretical framework linking the two is largely missing, especially for spiking networks. This project tackles that gap directly. By systematically searching for network architectures that extremize a key dynamical property (chaoticity), the work aims to uncover the underlying principles of this relationship. The overarching goal is to formulate a set of "graph rules" that can predict the dynamical behavior of a recurrent spiking network from its connectivity alone, providing a foundational link between the static structure and the emergent function of neural circuits.

Key Outcomes

- Simulate comprehensively all small spiking network structures (directed digraphs) and find systematic relationships between network structure and network dynamics.
- Visualize network states with Poincaré sections.
- Characterize the dynamical regime (e.g., synchronous, chaotic) for each motif and attempt to formulate a "graph rule" that predicts the outcome.
- **(Stretch 1):** try to tackle the structure → dynamics problem analytically.
- **(Stretch 2):** run large-scale well-controlled simulations of small-scale networks and find graph rules using graph neural networks.

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★ – ★★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	Example implementation exists and can be shared

Literature

Core References

- Curto, C., & Morrison, K. (2023). *Graph rules for recurrent neural network dynamics: extended version.* ArXiv: arXiv-2301.
- Monteforte, M., & Wolf, F. (2010). *Dynamical entropy production in spiking neuron networks in the balanced state.* Physical Review Letters, 105(26), 268104. https://doi.org/10.1103/PhysRevLett.105.268104

Optional Literature

- Engelken R., Monteforte M., Wolf F. (2024). *Sparse chaos in neural circuits.* arXiv 2412.21188. www.doi.org/10.48550/arXiv.2412.21188
- For analytical work: Memmesheimer & Timme (2006); Börner et al. (2023).
- (review) Smeal, R. M., Ermentrout, G. B., & White, J. A. (2010). *Phase-response curves and synchronized neural networks.* Philosophical Transactions of the Royal Society B, 365, 2407–2422.
- Goel, P. & Ermentrout, B. (2002). *Synchrony, stability, and firing patterns in pulse-coupled oscillators.* Physica D, 163, 191–216.
- Canavier, C. C. & Achuthan, S. (2010). *Pulse coupled oscillators and the phase resetting curve.* Mathematical Biosciences, 226, 77–96.
- For numerical work: check literature on Graph Neural Networks.

K.) SparseProp with Delays

Goal

Extend the SparseProp simulation framework by incorporating synaptic delays. Start with homogeneous delays (all neurons share the same delay) and then experiment with heterogeneous delays across neurons or synapses. The main task is to implement and validate delayed communication within SparseProp simulations.

Project Rationale

Adding delays is a natural but challenging step toward more biologically realistic spiking networks. Once implemented, students can choose to explore how delays shape network activity, for example, traveling waves or novel spatio-temporal patterns, but the core achievement is extending SparseProp itself.

Key Outcomes

- Implement homogeneous delays in SparseProp, test it carefully, and benchmark it.
- **(Stretch):** Implement heterogeneous delays (different outgoing delays for each neuron or even different delays for every synapse).
- **(Stretch):** Study traveling waves (e.g., the scenario in Davis et al., 2021).

Project profile

Analytical Intensity:	-
Coding Intensity:	★★★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	Reference example implementation exists (Julia)

Literature

- Engelken, R. (2023). *SparseProp: Efficient event-based simulation and training of sparse recurrent spiking neural networks.* Advances in Neural Information Processing Systems, 36. https://doi.org/10.48550/arXiv.2312.17216
- Davis, Z. W., et al. (2021). *Spontaneous traveling waves naturally emerge from horizontal fiber time delays and travel through locally asynchronous-irregular states.* Nature Communications, 12, 6057. https://doi.org/10.1038/s41467-021-26175-1

L.) Lyapunov Spectra of EIF Spiking Networks

Goal

Study how chaos emerges in networks of exponential integrate-and-fire (EIF) neurons by calculating their Lyapunov spectra. The project focuses on how intrinsic neuron parameters shape stability and chaos in large recurrent EIF networks, and how these results compare to other spiking models.

Project Rationale

This project is a low-hanging fruit: it extends methods for chaos analysis to a widely used spiking neuron model and allows direct comparisons with other neuron types. Results can validate or challenge existing insights about chaos in recurrent circuits and connect to broader questions about how biophysical properties at the single-cell level shape collective network dynamics.

Key Outcomes

- Quantify λ_1, KS entropy rate, and Kaplan-Yorke attractor dimension as a function of neuron parameters of a balanced EIF network.

Project profile

Analytical Intensity:	-
Coding Intensity:	★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	Reference example implementation exists (Julia)

Literature

- Engelken R., Monteforte M., Wolf F. (2024). *Sparse chaos in neural circuits.* arXiv 2412.21188. www.doi.org/10.48550/arXiv.2412.21188
- Fourcaud-Trocmé, N., et al. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating inputs.* Journal of neuroscience, 23(37), 11628-11640.
- Badel, L., et al. (2008). *Dynamic IV curves are reliable predictors of naturalistic pyramidal-neuron voltage traces.* Journal of Neurophysiology, 99(2), 656-666.
- Badel, L., et al. (2008). *Extracting non-linear integrate-and-fire models from experimental data using dynamic I–V curves.* Biological cybernetics, 99(4), 361-370.
- Brunel, N. (2000). *Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons.* Journal of computational neuroscience, 8(3), 183-208.
- Monteforte, M., & Wolf, F. (2010). *Dynamical entropy production in spiking neuron networks in the balanced state.* Physical review letters, 105(26), 268104.

M.) Covariance Spectrum of Input-driven Chaotic Rate Networks

Goal

The Sompolinsky–Crisanti–Sommers (SCS) model is a classic framework for high-dimensional chaotic networks. Recent theoretical advances have computed its covariance spectrum and dimensionality using advanced tools. This project asks: how does the covariance spectrum change when the network is driven by external input? You will extend existing theoretical approaches to this input-driven case from a random dynamical systems perspective and verify the predictions numerically.

Project Rationale

Extending the SCS covariance spectrum to include input signals is a natural but unexplored step. It connects directly to questions about the dimensionality of cortical activity under sensory drive and the representational capacity of recurrent networks in machine learning. The project combines analytical thinking with straightforward numerical checks, making it both accessible and impactful.

Project profile

Analytical Intensity:	★ – ★★★ (pick your level)
Coding Intensity:	★ – ★★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	Example implementation (Julia/Python) exists and can be shared

Literature

Core References

- Shen, Xuanyu, and Yu Hu. "Covariance spectrum in nonlinear recurrent neural networks." arXiv preprint arXiv:2508.05288 (2025).
- Clark, David G., L. F. Abbott, and Ashok Litwin-Kumar. "Dimension of activity in random neural networks." Physical Review Letters 131.11 (2023): 118401.
- Engelken, R., Wolf, F., & Abbott, L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks.* Physical Review Research, 5(4), 043044. https://doi.org/10.1103/PhysRevResearch.5.043044

Additional Literature

- Rajan, K., Abbott, L. F., & Sompolinsky, H. (2010). *Stimulus-dependent suppression of chaos in recurrent neural networks.* Physical Review E, 82(1), 011903. https://doi.org/10.1103/PhysRevE.82.011903

N.) Limit Cycles in Transformers: A Dynamical Systems Stability Analysis

Goal

When run deterministically, language models often fall into repetitive loops which is a critical failure mode. This project treats this pathology as a problem in nonlinear dynamics. Your goal is to move beyond simply observing these loops and instead analyze their stability. By treating the transformer as a high-dimensional discrete-time map, you will use tools such as Jacobian analysis and Floquet multipliers to build a mechanistic understanding of why these repetitive sequences are stable attractors. This project directly applies the analytical toolkit of theoretical neuroscience to a central problem in modern AI, offering a path from deep analysis to real-world impact on model reliability.

Project Rationale

Understanding when and why transformers fall into limit cycles connects generative model pathologies to nonlinear dynamics. The project combines hands-on exploration of language models with theoretical analysis and could lead to practical insights for mitigating repetition failures in modern AI.

Key Outcomes

- Reproduce a repetitive loop in a 'small LLM' that you can study locally.
- **(Stretch):** Implement a method to analyze the stability of this loop by computing the eigenvalues of the relevant Jacobian, providing a mechanistic explanation for the repetition.

Project profile

Analytical Intensity:	★ – ★★ (pick your level)
Coding Intensity:	★★ – ★★★
Comp Neuro:	-
ML:	★★★
Starter toolkit:	No example implementation exists

Literature

- *A Theoretical Analysis of the Repetition Problem in Text Generation.* https://doi.org/10.48550/arXiv.2012.14660
- Engelken, Wolf, Abbott, 2023 (PRR)
- Holtzman et al. (2019). *The Curious Case of Neural Text Degeneration.* https://arxiv.org/abs/1904.09751
- Xu, J., et al. (2022). *Learning to break the loop: Analyzing and mitigating repetitions for neural text generation.* https://arxiv.org/abs/2203.08398
- Li, H., et al. (2023a). *Repetition in repetition out: Towards understanding neural text degeneration from the data perspective.* In NeurIPS. https://arxiv.org/abs/2305.13241
- *Unveiling Attractor Cycles in Large Language Models: A Dynamical Systems View of Successive Paraphrasing.*
- Mahaut, M., & Franzon, F. *Repetitions are not all alike: distinct mechanisms sustain repetition in language models.* https://doi.org/10.48550/arXiv.2504.01100

O.) Benchmarking SparseProp

Goal

SparseProp is an ultra-efficient event-based algorithm for simulating spiking networks, but it has not yet been systematically benchmarked against other simulators. This project will perform a direct performance comparison of SparseProp with standard frameworks (e.g., Brian2, NEST). Benchmarks will start with simple cases (such as current-based leaky integrate-and-fire models without exponential synapses) and then extend to more complex settings.

Project Rationale

Benchmarking different simulators is a straightforward but important step to understand how SparseProp compares across network sizes, firing rates, and neuron models. This is a practical project that fills a clear gap, making it a useful contribution for anyone interested in efficient simulation of large spiking networks.

Project profile

Analytical Intensity:	-
Coding Intensity:	★★ – ★★★
Comp Neuro:	★★★
ML:	★
Starter toolkit:	Reference implementation exists (Julia)

Literature

Core References

- Engelken, R. (2023). *SparseProp: Efficient event-based simulation and training of sparse recurrent spiking neural networks.* Advances in Neural Information Processing Systems, 36. https://doi.org/10.48550/arXiv.2312.17216

- Zenke, F. & Gerstner, W. (2014). *Limits to high-speed simulations of spiking neural networks using general-purpose computers.* Front Neuroinform 8, 76. doi: 10.3389/fninf.2014.00076

- NEST/Brian2 papers

Additional Literature (Simulator Papers)

- Gewaltig, M. O., & Diesmann, M. (2007). *NEST (NEural Simulation Tool).* Scholarpedia, 2(4), 1430. https://doi.org/10.4249/scholarpedia.1430

- Stimberg, M., Brette, R., & Goodman, D. F. (2019). *Brian 2, an intuitive and efficient neural simulator.* eLife, 8, e47314. https://doi.org/10.7554/eLife.47314

P.) Conductance-based Rapid Theta Model

Goal

The Theta model is a phase description of the quadratic integrate-and-fire (QIF) neuron, which can often be solved analytically in networks with exponentially decaying synapses. This project extends the model by adding conductances together with the rapidness variable, leading to a piecewise quadratic form that requires more advanced analytic solutions (involving Whittaker functions). The task is to work out this extended formulation and test how networks of adaptive rapid Theta neurons behave.

Project Rationale

By extending the Theta framework to include adaptation, this project builds a new solvable model class that links biophysically realistic spiking behavior with analytical tractability. It offers the chance to derive exact solutions and explore how adding conductance changes collective network dynamics.

Key Outcomes

- implement the solution of a conductanced-based Theta neuron model.
- **(Stretch):** Find an analytical or semi-analytical solution for its dynamics and use it to predict the behavior of a small network.

Project profile

Analytical Intensity:	★★★
Coding Intensity:	★★ – ★★★
Comp Neuro:	★★★
ML:	-
Starter toolkit:	No reference implementation exists

Literature

Core References

- Tonnelier, A., Belmabrouk, H., & Martinez, D. (2007). *Event-driven simulations of nonlinear integrate-and-fire neurons.* Neural Computation, 19(12), 3226-3238.
- Monteforte, M., & Wolf, F. (2010). *Dynamical entropy production in spiking neuron networks in the balanced state.* Physical review letters, 105(26), 268104.
- Engelken R., Monteforte M., Wolf F. (2024). *Sparse chaos in neural circuits.* arXiv 2412.21188. www.doi.org/10.48550/arXiv.2412.21188

Optional Literature

- Engelken, R. (2023). *SparseProp: Efficient event-based simulation and training of sparse recurrent spiking neural networks.* Advances in Neural Information Processing Systems, 36. https://doi.org/10.48550/arXiv.2312.17216

Q.) Quantifying Dynamics of Ecological Networks

Goal

Quantify high-dimensional chaos in generalized Lotka–Volterra (gLV) systems with random interactions by computing the *full* Lyapunov spectrum, Kaplan–Yorke dimension, and Kolmogorov–Sinai entropy. Extend the analysis to input-driven or seasonally forced settings to assess how external drive shapes dimensionality and stability.

Project Rationale

Classic theoretical ecology often focused on conditions for stability, yet recent work demonstrates persistent fluctuations and chaos in high-diversity ecosystems. Prior studies typically report only the largest Lyapunov exponent; the full spectrum provides a richer picture of instability and effective dimensionality. This project imports the quantitative toolkit from neural-network chaos analysis to ecological community models, enabling direct comparisons across fields.

Key Outcomes

- implement a gLV system with random interactions (as in Roy et al., 2020) and validate dynamical regimes reported in the literature.
- compute the *full* Lyapunov spectrum numerically (e.g., QR/Benettin methods) and derive Kaplan–Yorke dimension and Kolmogorov–Sinai entropy; contrast with largest-exponent-only characterizations.
- map how the spectrum depends on interaction statistics (mean, variance, symmetry, sparsity), self-interaction strengths, and species richness N.
- **(Stretch):** compare numerical spectra and dimensionality with dynamical mean-field theory (DMFT) predictions (Roy et al., 2019); assess finite-size and sampling effects.
- **(Stretch):** add external drive or seasonal forcing and quantify how covariance structure and dimensionality change under input.

Project profile

Analytical Intensity:	★ – ★★★
Coding Intensity:	★
Comp Neuro:	★
ML:	-
Starter toolkit:	No example implementation exists.

Literature

Core References

- Roy, F., Biroli, G., Bunin, G., & Cammarota, C. (2020). *Complex interactions can create persistent fluctuations in high-diversity ecosystems.* PLoS Computational Biology, 16(5), e1007827. https://doi.org/10.1371/journal.pcbi.1007827
- Engelken, R., Wolf, F., & Abbott, L. F. (2023). *Lyapunov spectra of chaotic recurrent neural networks.* Physical Review Research, 5, 043044. https://doi.org/10.1103/PhysRevResearch.5.043044

Optional Literature

- Roy, F., Biroli, G., Bunin, G., & Cammarota, C. (2019). *Numerical implementation of dynamical mean field theory for disordered systems: Application to the Lotka–Volterra model of ecosystems.* Journal of Physics A: Mathematical and Theoretical, 52(48), 484001. https://doi.org/10.1088/1751-8121/ab4dfe

R.) Propose Your Own Project

Goal

If you are really excited about your own project ideas, I'd be keen to hear about them, but it is not my expectation that you bring your own project. Original, well-motivated ideas that are well-aligned with the course topic are always welcome. Please discuss your idea with me early to ensure it aligns with the course goals and is feasible within the semester.