Tackling Deep Inelastic Scattering (DIS) Using Spectral Function Formalism

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Neutrinos are highly pertinent particles in the search for potential new physics beyond the Standard Model. Quite uniquely, these leptons come in three different "flavour" states: the electron-, muon-, and tau-neutrino. Neutrinos are able to oscillate from one flavour state to another given some mathematically described probability which is proportional to (disregarding the mixing parameters involved) the distance traveled and neutrino energy; importance being, these oscillation parameters are extremely valuable in determining the structure and function of these particles and subsequently answering many open questions in physics and cosmology. One can reduce the expected neutrino interaction rate to a relationship that can be predicted, in part, given a proper handle on the lepton-nucleus cross-section. This cross-section varies greatly in contribution due to reaction mechanisms that dominate in certain energy regimes. Specifically, we are interested in Deep Inelastic Scattering (DIS) contributions to the lepton-nucleus cross-section at high neutrino energies, relevant in the interpretation of signals for ongoing and future experiments as we probe higher and higher GeV scales. A Spectral Function formalism recently applied to all other regimes of this cross-section has yet to encompass DIS effects: using this mathematical framework, we wish to utilize different sets of parton distribution functions (PDFs) in conjunction with the LHAPDF library to better describe each elementary structure function, soon joining them together to simulate deuteron scattering and, finally, neutrino scattering.

I. INTRODUCTION

A. Mysteries of Neutrino Physics

Neutrinos are extremely elusive, the lightest and most puzzling of the known Standard Model (SM) elementary particles. Understanding their properties will give us crucial information which have subsequent implications on the universe and its formation; furthermore, they could potentially clarify important open questions in physics such as baryon asymmetry. There is still much to learn and lots to discover with these mysterious entities: are they Majorana or Dirac particles [12], what are their absolute masses and precise mixing angles [4], is there CP-violation [3], is our current picture of the SM correct, do sterile neutrinos exist [11]?

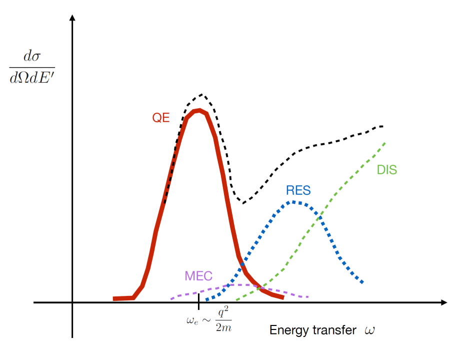

Figure 1. Different reaction mechanisms contributing to lepton-nucleus cross section.

Where do neutrino masses come from, why is there leptonic mixing, what was the neutrino's role in the creation and evolution of our known Universe?

1. Neutrino Oscillations

Neutrinos come in three different flavour states corresponding to each of the charged leptons: each of these is comprised of a unique superposition of neutrino mass eigenstates. When a neutrino comes into existence and propagates through matter for some distance L, it has a certain probability that we will observe it to change flavour states (similar to other observed quantomechanical effects such as spin precession)! Anytime one produces a state which is *not* an eigenstate of the Hamiltonian, this inevitably leads to these types of oscillatory behaviors. For example, a muon neutrino can be expected to shift into an electron neutrino with the following likelihood:

$$P_{\nu_\mu \to \nu_e}(E, L) \sim \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \to \frac{\Phi_e(E, L)}{\Phi_\nu(E, 0)}. \tag{1}$$

Above, θ represents the mixing angle, Δm^2 is the mass squared difference ($m^2_{\nu_e} - m^2_{\nu_\mu}$), and $\Phi(E, L)$ depicts the neutrino flux as a function of beam energy and distance propagated.

However, measuring the flux experimentally is entirely

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non-trivial: instead, are able to reconstruct the flux by, first, getting a proper handle on the interaction event rate. Subsequently, using physical detectors, we are able to indirectly observe neutrino scattering events with some regularity, dependent in theory on a cross-sectional term and smearing matrix. In essence, to get accurate results on neutrino fluxes and important oscillation parameters, one needs to precisely encapsulate the neutrino-nucleon cross-section σ [6].

Due to the fundamental mathematical structure behind quantum effects, neutrino mixing of the mass and flavour fields implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates. The existence and observational confirmation of neutrino oscillations cements the fact that neutrinos have an intrinsic mass and mix (as the various components of the initial state propagate with differing phases). Furthermore, this points to a currently gaping hole in the SM, a gap in which our understanding of particle physics comes to a halt in continuity. The Standard Model of Particle Physics would lead us to believe that neutrinos are massless objects, but we have proof that this is not the case.

B. Deep Inelastic Scattering (DIS)

Physical detector experiments measure neutrino interaction rates: a quantitative knowledge of the cross section and smearing matrix is essential for us to reconstruct the event count, leading to a precise extraction of neutrino oscillation parameters. Let's rewind for a minute: a neutrino cross section (in our case) is used to characterize the probability that a neutrino scattering event will occur. Typically, this can be thought of as a "characteristic area" in which a larger area implies a higher probability of interaction. Similarly, the smearing matrix here would be a means of simulating the smearing that occurs from the detection of particles. Since detectors are physical and have limits to their perfection, there is always some smearing that occurs which modifies what is measured from what was actually there (usually done with a simple Gaussian with some width).

Different reaction mechanisms contribute to the lepton-nucleus cross section for a fixed value of the beam energy (monochromatic). In legitimate neutrino experiments, these contributions are not nicely separated (rather, they are represented by the convolution of regimes depicted by the black, dashed line in Fig. 1). Realistically speaking, since neutrinos are neutral in charge, we have no way to create a perfectly monochromatic beam which adds additional complications. Again, for lepton-nucleus interactions, we are able to better control the energy of the beam but still have to calculate all contributing channels in our energy transfer region.

Relevant to our investigation, the lepton-nucleus kinematic region with $Q^2 \gtrapprox 1$ GeV^2 is defined as the region of Deep Inelastic Scattering (DIS). This reaction mechanism is used to probe inside of hadrons and provided the first convincing evidence for quarks (previously thought to be purely mathematical).

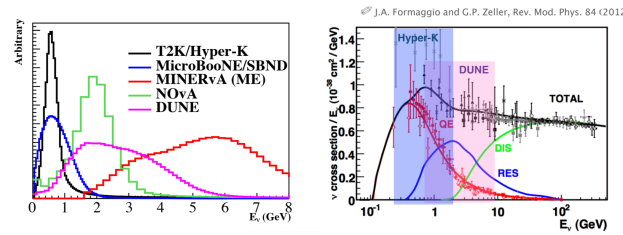

Figure 2. Addressing future precision experiments. The dominant reaction mechanism changes dramatically over the region of interest to oscillation experiment.

DIS processes induced by charged leptons and $\nu/\bar{\nu}$ on nucleons and nuclear targets are important tools to study the quark-parton structure of free nucleons and nucleons in a bound nucleus. We would argue that DIS processes for accelerator based neutrino experiments, such as DUNE, MINERνA, NOVA, and T2K will not really help us better understand the partonic structure of nucleons. Rather, we need to use our understanding of DIS from projects such as HERA, the Tevatron, and the LHC to improve our predictions for the neutrino experiments.

Interpretation of event signals relevant to ongoing and future experiments at higher neutrino energies require accurate predictions of nuclear cross sections in these *inelastic channels*. This is clearly confirmed by comparing the contributions to ν cross section to experimental flux for a given E_ν (Fig. 2).

C. Spectral Function Formalism

The intrinsic properties of the nucleus are described by the so-called **Spectral Function** [8]. Generically, our spectral function simply tells us the probability that a particle with a certain momentum k has some specific energy ω. The mean field approximation underlying our independent particle model of the nucleus leaves the sum over the states of the residual (A-1)-nucleon system restricted to bound one-hole states [8]. The corresponding spectral function can be expressed as:

$$P_{MF}(\mathbf{k}, E) = \sum_{\alpha \in |F|} |\phi_\alpha(\mathbf{k})|^2 \delta(E - \epsilon_\alpha), \tag{2}$$

where this sum includes all single particle states associated with the Fermi sea F (denoted by index α), with $\phi_\alpha(\mathbf{k})$ and ϵ_α showing the momentum-space wave function and energy, respectively [8].

There are still some issues which can be simplified vastly under certain assumptions. Neutrino-nucleus cross-sections involve three main elements: the nuclear weak current as well as the target final and initial states. At non-relativistic energies, the initial state can be safely modeled independent of kinematics; however, this quickly breaks down as we attempt to describe either the nuclear final state or nuclear current operator, which explicitly depends on the momentum transfer [13].

Within this formalism, nucleon structure functions can easily be written in terms of parton distribution functions if we choose to take the limit of $Q^2 \to \infty$ and $\omega \to \infty$. This so-called "Impulse Approximation" (IA) allows us to reduce the nuclear interactions to the incoherent sum of elementary processes involving individual nucleons at sufficiently large momentum transfers. Moreover, critical values associated with electron-deuteron scattering have been precisely measured in previous experiments, allowing for a simple cross-check as we work through each scattering case.

D. Parton Distribution Functions (PDFs)

Factorization theorems in quantum chromodynamics (QCD) show that structure functions can be expressed in terms of universal parton distribution functions (PDFs); that is, the cross-sections can be factorized into process-dependent perturbative pieces governed by Wilson coefficients (pQCD) and the non-perturbative universal PDFs.

We utilize various sets of PDFs using the LHAPDF [1] library from CERN and compare with nuclear nCTEQ PDFs, allowing for an accurate prediction of DIS values needed to extract further oscillation parameters. Specifically, we use the LHAPDF library to facilitate our own calculations, but actually use PDFs such as MMHT2014lo68cl [5] for comparison.

II. NUCLEON SCATTERING CONTRIBUTIONS

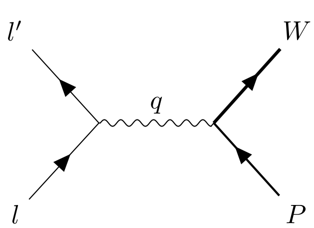

Fig. 2 Kinematics of deeply inelastic scattering.

If we take some hadron structure and examine it on a minuscule distance or time scale (as in large-momentum-transfer collisions), it will behave as if it were composed of non-interacting point-like structures named *partons* by Richard Feynman.

In this conceptual framework, a nucleus can be thought of as a collection of structure-less and non-interacting nucleons: however, how can partons be in this quasi-free state within hadrons if they interact so strongly that they cannot separate [10]? Quantum chromodynamics resolves this issue: we are able to formulate the language of observables and kinematics conducive to lepton-hadron scattering processes.

We begin our investigation with a reaction of the generic form:

$$l^-(l) + N(p) \to l^-(l') + X(W)$$
$$\begin{cases} l = (E, \vec{k}), & \text{4-momenta} \\ l' = (E', \vec{k'}), & \text{definitions} \\ P = (M_p, \vec{0}). & \end{cases} \tag{3}$$

On shell conditions posit that $p^2 = M_p^2 \to$ mass of the proton is 938 MeV. $l^2 = l'^2 = m_l^2 \approx 0$.

There are only two independent variables needed to describe the kinematics of inclusive DIS: we may choose to describe our system in terms of the out-going electron energy and scattering angle (E', θ) or (x, Q^2) or even (x, y).

We define the quad-momentum transfer Q^2 as

$$Q^2 = S \cdot x \cdot y = 2M_N \cdot E \cdot x \cdot y. \tag{4}$$

The invariant mass (Mandelstam variable) goes as

$$\begin{aligned} S &= (l + P)^2 \\ &= l^2 + P^2 + 2l \cdot P = 0 + M_N^2 + 2E \cdot M_N, \end{aligned} \tag{5}$$

where M_N represents the mass of a nucleon (either proton or neutron, depending on the scattering case).

In practice, we start with some given initial beam energy ($E = 30$ GeV, for example). It is important to mention that there are only two structure functions due to the allowed Lorentz structures in this calculation. We wish to compute the following cross-sectional term:

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 \cdot S}{Q^4} \left[F_2(x) \cdot (1 - y) + F_1(x) \cdot y^2 \cdot x \right], \tag{6}$$

where the fine structure constant $\alpha \approx \frac{1}{137}$.

The energy transfer term y is defined as

$$y = \frac{\nu}{E} = \frac{E - E'}{E}, \tag{7}$$

which varies in the bound range of $0 \le y \le 1$.

To obtain sensible results and for ease of computation, we rewrite Eq. (6) in terms of Eq's (4) and (5); subsequently, we evaluate Eq. (6) for a fixed value of x ($x = 0.5$, for example), and, again, some $0 \le y \le 1$.

The incoherence assumption, or impulse approximation (IA), means that our structure function for e-p scattering reduces to the sum over the contributions of individual partons:

$$W_2(Q^2, \nu) = \sum_i \int_0^1 dx_i f_i(x_i) W_2^{(i)}(Q^2, \nu; x_i), \tag{8}$$

where $f_i(x_i)$ yields the probability of finding the ith parton with momentum fraction x_i.

General expression for the hadronic tensor:

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{m^2} \left(p^\mu - \frac{(p \cdot q)}{q^2} q^\nu \right) \left(p^\nu - \frac{(p \cdot q)}{q^2} q^\nu \right). \tag{9}$$

It is very convenient to define the scaling form as

$$F_2(x) \equiv \nu W_2(Q^2, \nu). \tag{10}$$

Again, it is typical and well-motivated to define yet another dimensionless form based on W_1, which will be written as

$$F_1(x) \equiv M W_1(Q^2, \nu), \tag{11}$$

otherwise seen as

$$F_1(x) = \begin{cases} 0, & \text{spin-zero partons,} \\ \left(\frac{1}{2x}\right) F_2(x), & \text{spin-}\frac{1}{2}\text{ partons.} \end{cases} \tag{12}$$

A. Electron-Proton Scattering

Each structure function (whether for the proton or neutron case) is directly related to its quark-parton distribution function. For ep scattering, we have that

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(s(x) + \bar{s}(x)) + ..., \tag{13}$$

with $u(x) = f_u(x)$, etc.

B. Electron-Neutron Scattering

Quite identically, the en scattering case can be solved by applying an isospin rotation to the proton structure functions, wherein we find

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(u(x) + \bar{u}(x)) + ..., \tag{14}$$

and note that $u(x)$, $d(x)$, ..., delineate the quark contents of the proton.

Each elementary case is calculated using LHAPDF's extensive collection of parton distribution functions applicable to our wanted configuration.

III. E-DEUTERON SCATTERING

The DIS electron-nucleus scattering cross section can be written as :

$$\frac{d^2\sigma}{dx_A dy_A} = \frac{4\pi\alpha^2 \cdot S_A}{Q^4} \times \left[F_2^A(x_A) \cdot (1 - y_A) + F_1^A(x_A) \cdot x_A \cdot y_A^2 \right], \tag{15}$$

with A number on nucleons (protons and neutrons) of the nucleus, for the deuteron case considered in this work $A = 2$. The following relations holds true also in the nuclear case

$$F_2^A(x_A) = \nu W_2^A, \tag{16}$$

$$F_1^A(x_A) = \nu W_1^A, \tag{17}$$

$$S = M_A^2 + 2E_x M_A, \tag{18}$$

$$x_A = \frac{Q^2}{2M_A \nu}, \tag{19}$$

$$y_A = \frac{\nu}{E_l}. \tag{20}$$

Note that, in this case, the proton (neutron) inside the deuteron is not at rest, its four-momentum reads

$$P_N^\mu = (\sqrt{p^2 + m_N^2}, P_x, P_y, P_z), \tag{21}$$

$$E_p = \sqrt{p^2 + m_N^2} \tag{22}$$

The four momentum transfer carried by the photon is give by $q^\mu = (\nu, 0, 0, |\vec{q}|)$. Using the Spectral Function formalism, we can rewrite the nuclear structure functions in terms of the proton and neutron ones as

$$F_2^A(x_A, Q^2) = \int \frac{dp \cdot d\cos\theta_p \cdot d\phi_p}{(2\pi)^3} p^2 \Bigg\{ n_p(p) \frac{m_N}{E(p)} \times \left[\frac{4M_A^2 x_A^2}{(Q^2 + 4M_A^2 x_A^2)} \frac{1}{2} p^2 \sin^2\theta_p \frac{F_2^N(x_N, Q^2)}{m_N} + \frac{Q^2 16 M_A^4 x_A^4}{(Q^2 + 4M_A^2 x_A^2)} \frac{F_2^N(x_N, Q^2)}{m_N^2} \cdot \left(E(p) + \frac{Q^2}{4M_A x_A x_N} \right) \right] + \text{same thing for neutron} \Bigg\} \tag{23}$$

Note that x_N is a function of p, $\cos\theta_p$, X_A, and Q^2:

$$x_N(\vec{p}, Q^2, x_A) = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2\left(E(p) \frac{Q^2}{2M_A x_A} - p \cdot \cos\theta_p \cdot \sqrt{\frac{Q^2(4M_A^2 x_A^2 + Q^2)}{4M_A^2 x_A^2}} \right)} \tag{24}$$

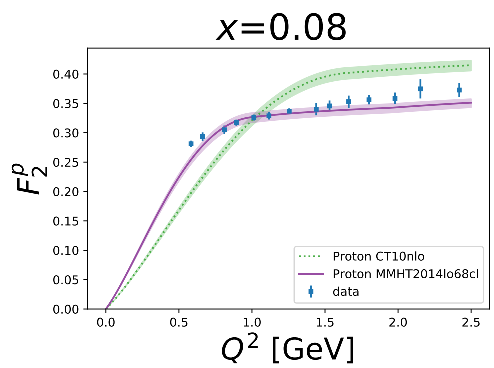

Figure 3. Simulated DIS electron-proton scattering comparison of CT10nlo and MMHT2014lo68cl PDFs against HEPData e-p experimental data [7]. Here, we plot the scaling factor F_2^p against Q^2 for $x = 0.08$ and find modest agreement.

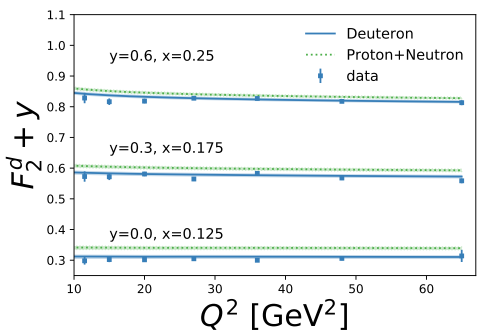

Figure 4. Simulated electron-deuteron scattering comparison against HEPData e-d experimental data [7]. Above, we plot the simple p+n case in dashed green, the deuteron case (with nuclear effects) in solid blue, and the experimental data's scaling factor $F_2^d + y$ versus Q^2 for various combinations of kinematic variables x, y.

The momentum integral goes as follows:

$$\begin{aligned}&\int dp \cdot p^2 \cdot d\cos\theta_p \cdot d\phi_p \\ &\to 2\pi \int_{-1}^{1} d\cos\theta_p \int_0^{p_{max}} dp \cdot p^2 ...\end{aligned} \tag{25}$$

since nothing here depends on ϕ_p, and p_{max} can simply be taken from momentum function tables for the proton and neutron, respectively.

IV. RESULTS

With all of these DIS kinematics in mind, we employed the use of several python packages to simulate results given various PDFs. Specifically, wrangled with the momentum integral's appearance in Eq. (25) as subsequently Eq. (23) by creating a uni-variate spline object for the nucleon momentum distribution, allowing for easier integration when using vegas [2] to take antiderivatives in conjunction with LHAPDF [1].

Python code allowed for the creation of class objects which could store each parton distribution function such that it only had to be initialized once: the DIS class object had self-calls which return the scaling functions F_1^N, F_2^N, for $N = p, n$. Then, using the long and arduous mathematical formalism described in III, we obtained our simulated data for electron-proton scattering as well as electron-deuteron scattering in the deep inelastic scattering momentum transfer regime.

We compare our results with observational data taken from a combined analysis of SLAC experiments on DIS e-p and e-d scattering [7]. Fig. 3, for instance, displays the observationally-pertinent scaling factor F_2^p in the case of electron-proton scattering. The blue data values are taken from previous DIS e-p experiments as a confirmation of our results; meanwhile, the green shows a CTEQ PDF, purple, MMHT. The discrepancy observed towards higher values of Q^2 may be explainable by the fit with these points. They use some order of a thousand points to fit the PDF, meaning that we would expect about 30% of them to fall outside the error bars. CTEQ's waning off could be due to the issue of using a next leading order PDF with our own leading order calculation.

Fig. 4 illustrates the alignment of experimental data with our calculation of the deuteron. Each dashed green line represents the naive exclusion of nuclear effects when joining together proton and neutron to form a deuteron. The gap seems to grow as we lower the value of our kinematic variable y, while only slightly varying x.

V. CONCLUSION

In short, we have gone through the exercise of bridging together this scattering spectral function formalism with the mathematics behind the reaction mechanism of deep inelastic scattering.

We have reached the end of the preliminary goals for simulating and confirming results on electron-proton and electron-deuteron scattering processes. At this point in time, we hope to continue our leading order approach and close the gap between electron and neutrino scattering: making this jump won't be non-trivial, but we've paved the way to get there. With our current Python framework, equations, and motivation from the modest agreement with experiment, this project will move forward and attempt to encompass DIS neutrino-deuteron scattering events in hopes of providing a more accurate handle on neutrino oscillation parameters.

ACKNOWLEDGMENTS

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