PBH Lensing

Summer 2019

Abstract

Abstract

1 Introduction

Blah Blah Dark Matter exists [1] [2].

Over the past few decades, Primordial black holes (PBHs) have become an ever more compelling DM candidate. In addition to being well motivated by various inflationary models [3], PBHs are the only DM prospect that don't require beyond standard model physics, relying only on general relativity. The parameter space where PBHs can make up 100% of DM has been squeezed in by Hawking radiation evaporation [4], CMB anisotropies [5], and gravitational lensing [6]. However, there still exists a window in parameter space where PBHs could make up all of the DM. The microlensing constraints near this critical window, have repeatedly been subject to revision [7], [8]. Our goal is to understand this region in parameter space by examining how the finite size and wave diffraction effects shape these constraints [9] [10].

2 Microlensing Formalism

We define the magnification factor $A = \frac{\phi}{\phi_0}$ where ϕ_0 is the size of the image in the absence of lensing. If we ignore the effects of wave optics (the geometric optics approximation) then the magnification for a point source can be shown to be

$$A_{geo} = \frac{u^2+2}{u\sqrt{u^2+4}}, \tag{1}$$

where u is the dimensionless impact parameter $u = \boldsymbol{\theta}/\theta_E$ which gives the distance between the PBH lens and the source star in units of the Einstein radius [9].

Defining the dimensionless quantities

$$\mathbf{x} = \boldsymbol{\theta}_l/\theta_*, \quad \mathbf{y} = \boldsymbol{\theta}/\theta_*, \quad w = \frac{r_l r_0}{r_l o}\theta_*^2 \omega, \tag{2}$$

it can be shown that for the case of a point mass lens (such as a PBH), the amplification factor becomes

$$|A|^2 = \frac{\pi w}{1-e^{-\pi w}} \left| {}_1F_1\left(\frac{1}{2}iw, 1; \frac{1}{2}iwy^2\right) \right|^2, \tag{3}$$

where ${}_1F_1$ is the confluent hypergeometric function [9].

[11]

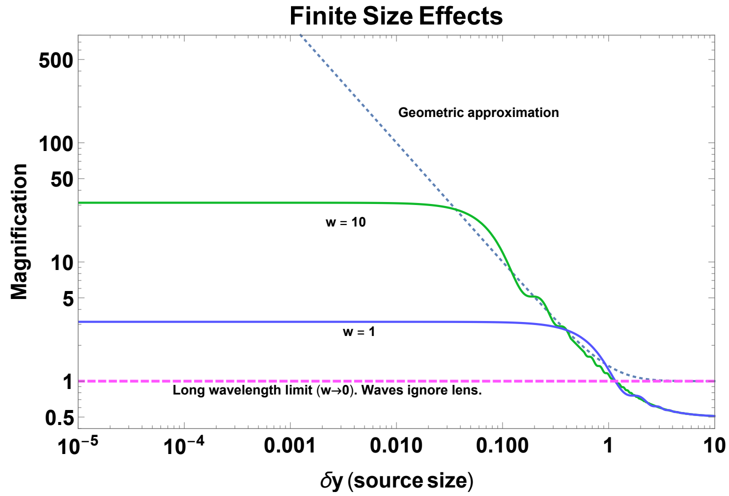

Figure 1: The magnification of point sources and finite sources for different values of the dimensionless frequency w.

3 Finite Size Effects

The magnification of a source by a PBH depends on the size of the source. In general, the peak magnification for an extended source can be either smaller or larger than the peak magnification for a point source. The size of the source, along with the magnitude of the impact parameter, determine whether the peak magnification is enhanced or diminished [12]. When the impact parameter becomes small ($u << r_{source}$), however, the peak magnification for a point source diverges while the peak magnification for an extended source remains finite. This means that for PBHs that cross very near the center of our line-of-sight to an extended source, the peak magnification will be significantly lower than in the point source case.

We can recalculate the magnification, taking into account the finite size effects following the work of [12] [7]. The relevant scale here is size of the source in the plane of the lensing PBH. So it is convenient to define the parameter

$$U \equiv \frac{\theta_S}{\theta_E} = \frac{R_S/d_S}{R_E/d_L}. \tag{4}$$

The finite size effects are most prominent in the regime where $U >> 1$, but are not negligible even when $U < 1$.

The magnification is given by integrating Eq. (1) over the source star in the plane of the lensing PBH

$$A_{finite}(u, U) \equiv \frac{1}{\pi U^2} \int_{|\mathbf{y}| \leq U} d^2\mathbf{y} A_{geo}(|\mathbf{u} - \mathbf{y}|). \tag{5}$$

The HSC is sensitive to a microlensing event when the magnification is $A \geq 1.34$. Under the geometric approximation Eq. (1), this corresponds to a threshold impact parameter value of $u = 1$. When taking into account the finite size effects, the threshold impact parameter will be different from unity in general. Following the prescription of [7], we can calculate the value of the impact parameter that corresponds to a detectable event by setting $A_{finite} = 1.34$ for a particular set of parameters d_L, M_{PBH}, r_{source}, and solving for u_{thresh} in Eq. (5). The value of u_{thresh} for the case of a point source, extended source, and extended source accounting for wave effects is shown in Fig. (2).

The threshold impact parameter depends both on d_L and r_{source}. In Fig. (3), this dependence is explicitly shown for different values of M_{PBH} and r_{source}.

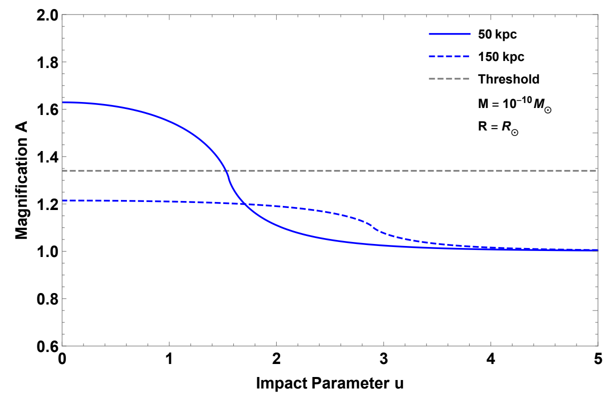

Figure 2: The magnification of point sources and finite sources for different values of the dimensionless frequency w.

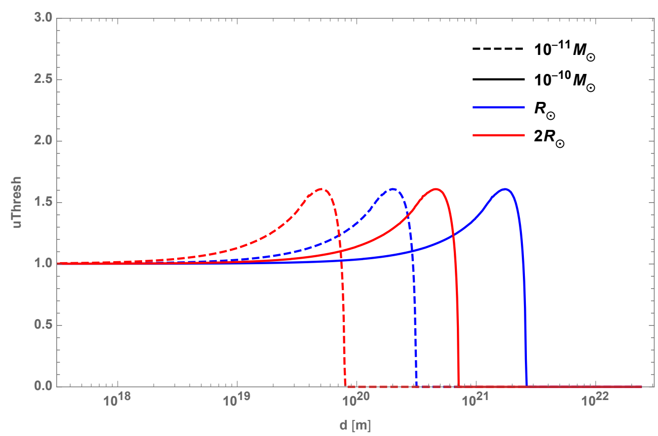

Figure 3: The dependence of the threshold impact parameter for different values of M_{PBH} and r_{source}

3.1 Size of Source Stars in M31

The HSC constraints assume source size of 1 solar radius for simplicity. But considering the sensitivity of the HSC (30-20% for $m_r = 25 - 26$ mag, and 70-60% for $m_r = 23 - 24$ mag), a back-of-the-envelope calculation shows that the Sun has an apparent magnitude of ≈ 29 mag in the r-band if it were located in the center of M31. In the most optimistic case, a main sequence star located in M31 would need to have an absolute magnitude of ≈ 1.57 in the r-band. This corresponds to a radius of at least 2 solar radii.

If the source star were off-main sequence, which is quite possible considering the over-density of RGB and AGB stars in the disk of M31 [13], a conservative estimate for the source radius could be much larger, ≥ 10 solar radii. In this case, the finite size effects become much more significant. The constraints are significantly weakened in the lower mass range of the parameter space as shown in Figure (5).

The faintest main sequence stars that are detectable are found in Brick 21 of the PHAT survey [13]. Using the luminosity, temperature, and density of sources for stars in this region [14], the most generous estimate for the typical size of a detectable main sequence star is ≈ 2 solar radii.

4 Constraints on PBH as DM

Following the work of Niikura et all [6], the differential event rate for microlensing of a single star by a PBH is given by

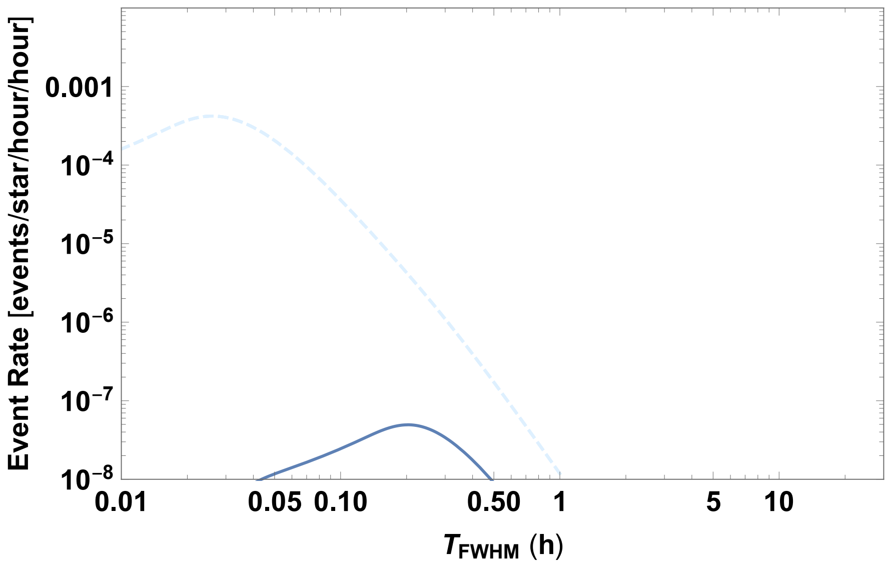

Figure 4: The rate of lensing events detected per hour per hour of observation for a single star

$$\frac{d\Gamma_{PBH}}{d\hat{t}} = 2\frac{\Omega_{PBH}}{\Omega_{DM}} \int_0^{d_s} dd_L \int_0^{U_T} du_{min} \frac{1}{\sqrt{u_T^2 - u_{min}^2}} \frac{\rho_{DM}(d_L)}{M_{PBH} v_c^2(d_L)} v^4 \exp\Big[- \frac{v^2}{v_c^2(d_L)} \Big], \tag{6}$$

where $v = 2R_E\sqrt{u_T^2 - u_{min}^2}/\hat{t}$.
where d_L is the distance to the lensing PBH and d_S is the distance to the source star.
The relevant scale for lensing by a PBH is the Einstein radius R_E, which is defined as

$$R_E = \sqrt{\frac{4GM_{PBH}d_L(1 - d_L/d_S)}{c^2}}. \tag{7}$$

The angular size of the Einstein radius, therefore, is given by $\theta_E \equiv \frac{R_E}{d_L}$.

The duration of observation was 7 hours, of which the greatest sensitivity to detection of events was from 0.07 hours to 3 hours. Integrating over this time, we find the total expected rate of events. Assuming a Poisson distribution for events, we can compare the actual dectections (1 perhaps) to the predicted number of detections. This leads to the constraints shown in figure (5).

If the source radius were to double, in order to keep the same parameter U, the lens would need to be brought closer to the Earth according to:

$$\frac{\theta_S}{\theta_E} = \frac{R_S d_L}{R_E d_S} = R_S \Big(\frac{x}{1-x}\Big)^{1/2} \tag{8}$$

where $x = d_L/d_S$.

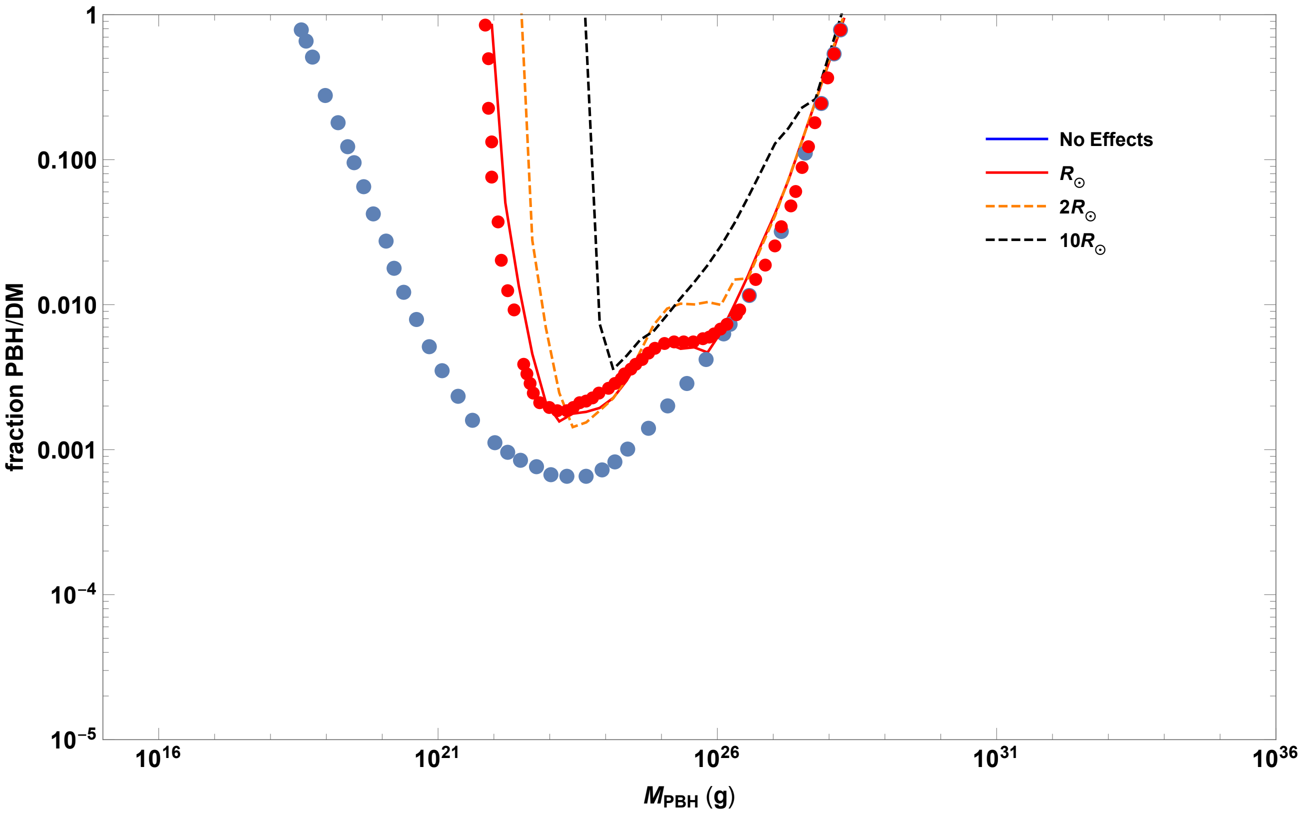

Figure 5: Here are the constraints for various stellar radii

References

[1] F. Zwicky. On the Masses of Nebulae and of Clusters of Nebulae. *The Astrophysical Journal*, 86:217, October 1937.

[2] Vera C. Rubin and W. Kent Ford. Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions. *The Astrophysical Journal*, 159:379, February 1970.

[3] Keisuke Inomata, Masahiro Kawasaki, Kyohei Mukaida, and Tsutomu T. Yanagida. Double inflation as a single origin of primordial black holes for all dark matter and LIGO observations. *Physical Review D*, 97(4):043514, February 2018. arXiv: 1711.06129.

[4] B. J. Carr and S. W. Hawking. Black Holes in the Early Universe. *Monthly Notices of the Royal Astronomical Society*, 168(2):399–415, August 1974.

[5] Yacine Ali-Hamoud and Marc Kamionkowski. Cosmic microwave background limits on accreting primordial black holes. *Physical Review D*, 95(4):043534, February 2017.

[6] Hiroko Niikura, Masahiro Takada, Naoki Yasuda, Robert H. Lupton, Takahiro Sumi, Surhud More, Toshiki Kurita, Sunao Sugiyama, Anupreeta More, Masamune Oguri, and Masashi Chiba. Microlensing constraints on primordial black holes with the Subaru/HSC Andromeda observation. *Nature Astronomy*, 3(6):524–534, June 2019. arXiv: 1701.02151.

[7] Sunao Sugiyama, Toshiki Kurita, and Masahiro Takada. Revisiting the wave optics effect on primordial black hole constraints from optical microlensing search. *arXiv:1905.06066 [astro-ph]*, May 2019. arXiv: 1905.06066.

[8] Paulo Montero-Camacho, Xiao Fang, Gabriel Vasquez, Makana Silva, and Christopher M. Hirata. Revisiting constraints on asteroid-mass primordial black holes as dark matter candidates. *arXiv:1906.05950 [astro-ph]*, June 2019. arXiv: 1906.05950.

[9] Takahiro T. Nakamura and Shuji Deguchi. Wave Optics in Gravitational Lensing. *Progress of Theoretical Physics Supplement*, 133:137–153, January 1999.

[10] Norihito Matsunaga and Kazuhiro Yamamoto. The finite source size effect and wave optics in gravitational lensing. *Journal of Cosmology and Astroparticle Physics*, 2006(01):023–023, January 2006.

[11] Ryuichi Takahashi and Takashi Nakamura. Wave Effects in the Gravitational Lensing of Gravitational Waves from Chirping Binaries. *The Astrophysical Journal*, 595(2):1039–1051, October 2003.

[12] Hans J. Witt and Shude Mao. Can lensed stars be regarded as pointlike for microlensing by MACHOs? *The Astrophysical Journal*, 430:505–510, August 1994.

[13] Julianne J. Dalcanton, Benjamin F. Williams, Dustin Lang, Tod R. Lauer, Jason S. Kalirai, Anil C. Seth, Andrew Dolphin, Philip Rosenfield, Daniel R. Weisz, Eric F. Bell, Luciana C. Bianchi, Martha L. Boyer, Nelson Caldwell, Hui Dong, Claire E. Dorman, Karoline M. Gilbert, Lo Girardi, Stephanie M. Gogarten, Karl D. Gordon, Puragra Guhathakurta, Paul W. Hodge, Jon A. Holtzman, L. Clifton Johnson, Sren S. Larsen, Alexia Lewis, Jason L. Melbourne, Knut A. G. Olsen, Hans-Walter Rix, Keith Rosema, Abhijit Saha, Ata Sarajedini, Evan D. Skillman, and Krzysztof Z. Stanek. THE PANCHROMATIC HUBBLE ANDROMEDA TREASURY. *The Astrophysical Journal Supplement Series*, 200(2):18, May 2012.

[14] Karl D. Gordon, Morgan Fouesneau, Heddy Arab, Kirill Tchernyshyov, Daniel R. Weisz, Julianne J. Dalcanton, Benjamin F. Williams, Eric F. Bell, Luciana Bianchi, Martha Boyer, Yumi Choi, Andrew Dolphin, Leo Girardi, David W. Hogg, Jason S. Kalirai, Maria Kapala, Alexia R. Lewis, Hans-Walter Rix, Karin Sandstrom, and Evan D. Skillman. The Panchromatic Hubble Andromeda Treasury XV. The BEAST: Bayesian Extinction and Stellar Tool. *The Astrophysical Journal*, 826(2):104, July 2016. arXiv: 1606.06182.