Bias and Variance Behavior in Model Fit Types

💡 Key Concepts

Term	Meaning
Bias	Error from overly simple assumptions in the model.
Variance	Error from excessive sensitivity to training data variations.
Underfitting	Model is too simple to learn the pattern — high bias.
Overfitting	Model is too complex and fits noise — high variance.

1️⃣ Underfitting

Aspect	Behavior
Bias	🔺 High – model makes strong, incorrect assumptions.
Variance	🔻 Low – too rigid to adapt to changes in data.
Error	🔺 High – on both training and test sets.
Why	Model is **too simple** to capture patterns.
Example	Linear model on sine wave.

2 Good Fit

Aspect	Behavior
Bias	🔻 Low – captures general patterns.
Variance	🔻 Low – stable across training subsets.
Error	🔻 Low – on both training and test sets.
Why	Just the **right complexity** for the data.
Example	Cubic polynomial on sine wave with minimal noise.

3 Best Fit (Sweet Spot)

Aspect	Behavior
Bias	🔻 Slightly low – minimal assumption error.
Variance	🔻 Slightly low – stable, but responsive to change.
Error	🔻 Minimal – generalizes well.
Why	Perfect **bias-variance tradeoff**.
Example	Polynomial degree that matches function shape and avoids noise.

4 Overfitting

Aspect	Behavior
Bias	🔻 Very low – even noise is captured.
Variance	🔺 High – predictions change drastically across training sets.
Error	🔻 Low on training, 🔺 High on test.

Aspect	Behavior
Why	Model is **too complex**, memorizing data including noise.
Example	15-degree polynomial matching all training noise.

📊 Summary Table

Fit Type	Bias	Variance	Training Error	Test Error	Generalization
Underfitting	High 🔺	Low 🔻	High 🔺	High 🔺	Poor
Good Fit	Low 🔻	Low 🔻	Low 🔻	Low 🔻	Good
Best Fit	Low 🔻	Low 🔻	Very Low 🔻	Very Low 🔻	Excellent
Overfitting	Low 🔻	High 🔺	Very Low 🔻	High 🔺	Poor

🧠 Final Insight

> 🔄 **Bias and variance behave inversely**:
>
> - Increasing model complexity ↓ bias but ↑ variance.
> - Simpler models have ↑ bias but ↓ variance.

The **ideal model** is not the most complex, but the one that **balances both** to generalize well to new data.