Computations

$$\bar{h} = 1$$

1 S_2 , char 2, τ trivial

1.1 Hilbert polynomial

$$t^4 + 2t^3 + 2t^2 + 2t + 1 = (t+1)^2(t^2+1).$$

1.2 Generators

$$x_0^2 + x_1^2$$

 x_0^4 .

2 S_3 , char 3, τ trivial

2.1 Hilbert polynomial

$$t^{12} + 3t^{11} + 6t^{10} + 8t^9 + 9t^8 + 9t^7 + 9t^6 + 9t^5 + 9t^4 + 8t^3 + 6t^2 + 3t + 1 = (t^2 + t + 1)^3(t^6 + t^3 + 1).$$

2.2 Generators

$$x_0^3 + x_1^3 + x_2^3$$

$$((2c+2)/c)x_0^3 + ((c+1)/c)x_1^3 - x_0^2x_1 - x_1^2x_2 - x_0x_2^2 + x_0x_1^2 + x_0^2x_2 + x_1x_2^2$$

 x_0^9 .

3 S_4 , char 2, τ trivial

3.1 Hilbert polynomial

$$t^6 + 4t^5 + 7t^4 + 8t^3 + 7t^2 + 4t + 1 = (t+1)^4(t^2+1).$$

3.2 Generators

$$\begin{split} x_0^2 + x_1^2 + x_2^2 + x_3^2 \\ &((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3 \\ &((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3 \\ &x_0^4. \end{split}$$

4 S_5 , char 5, τ trivial

4.1 Hilbert polynomial

PARTIAL

$$1 + 5t + 15t^2 + 35t^3 + 70t^4 + 122t^5 + 190t^6 + 270t^7 + \dots$$

4.2 Generators

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x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5
 ((4c+4)/c)x_0^5
 +((c+1)/c)x_3^5
+x_0^4x_1+x_0^4x_2+x_1^4x_3+x_2^4x_3+x_0^4x_4+x_3x_4^4
   +2x_0^4x_3
   +3x_0x_3^4
 -x_0x_1^4 - x_0x_2^4 - x_1x_3^4 - x_2x_3^4 - x_3^4x_4 - x_0x_4^4
 +\left((c+4)/(c+2)\right)x_0^2x_1^3+\left((c+4)/(c+2)\right)x_0^2x_2^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_2^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^3+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_3^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)x_1^2x_2^2+\left((c+4)/(c+2)\right)
2))x_3^3 x_4^2 + ((c+4)/(c+2))x_0^2 x_4^3
 +((2c+3)/(c+2))x_0^2x_3^3
+((3c+2)/(c+2))x_0^3x_3^2\\+((4c+1)/(c+2))x_0^3x_1^2+((4c+1)/(c+2))x_0^3x_2^2+((4c+1)/(c+2))x_1^3x_3^2+((4c+1)/(c+2))x_2^3x_3^2+((4c+1)/(c+2))x_1^3x_3^2+((4c+1)/(c+2))x_1^3x_3^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_3^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_2^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^3x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1^2+((4c+1)/(c+2))x_1
 1)/(c+2)x_0^3x_4^2 + ((4c+1)/(c+2))x_3^2x_4^3
 +(c/(c+2))x_0x_1^3x_2+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1^3x_4+(c/(c+2))x_0x_2^3x_4+(c/(c+2))x_0x_1x_4^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^3+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_0x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))x_1x_2^2+(c/(c+2))
 (2))x_0x_2x_4^3
 + \left(2c/(c+2)\right)x_0^3x_1x_3 + \left(2c/(c+2)\right)x_0^3x_2x_3 + \left(2c/(c+2)\right)x_1x_2x_3^3 + \left(2c/(c+2)\right)x_0^3x_3x_4 + \left(2c/(c+2)\right)x_1x_3^3x_4 + \left(2c/(c+2)\right)x_1x_3^2x_4 + \left(2c/(c+2)\right)x_1x_3^2x_4 + \left(2c/(c+2)\right)x_1x_3^2x_4 + \left(2c/(c+2)\right)x_1x_3^2x_4 + \left(2c/(c+2)\right)x_1x_3^2x_4 + \left(2c/(c+2)\right)x_
 (2c/(c+2))x_2x_3^3x_4
 + \left(3c/(c+2)\right)x_0^3x_1x_2 + \left(3c/(c+2)\right)x_0x_1x_3^3 + \left(3c/(c+2)\right)x_0x_2x_3^3 + \left(3c/(c+2)\right)x_0^3x_1x_4 + \left(3c/(c+2)\right)x_0^3x_2x_4 + \left(3c/(c+2)\right)x_0^3x_1x_2 + \left(3c/(c+2)\right)x_1^3x_1x_2 + \left(3c/(c+2)\right)x_1^3x_1 + \left(3c/(c+2)\right)x_1^3x_1 + \left(3c/(c+2)\right)x_1^3x_1 + \left(3c/(c+2)\right)x_1^3x_1 + \left(3c/(c+2)\right)x_1^3x_1 + \left(3
 (3c/(c+2))x_0x_3^3x_4
 (4c/(c+2))x_2x_3x_4^3
 1)x_1^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_2^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_0x_3^2x_4^2
+((2c^2+3c)/(c^2+1))x_1^2x_2x_3^2+((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_2^2+((2c^2+3c)/(c^2+1)x_1^2x_1^2+((2c^2+3c)/(c^2+1)x_1^2x_1^2+((2c^2+3c)/(c^2+1)x_1^2+((2c^2+3c)/(c^2+1)x_1^2+((2c^2+3c
1))x_2^2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_1x_3^2x_4^2 + ((2c^2+3c)/(c^2+1))x_2x_3^2x_4^2
```

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\begin{aligned} &1))x_0^2x_2^2x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_2x_4^2 \\ &+ ((4c^2+c)/(c^2+1))x_0x_1^2x_2^2 + ((4c^2+c)/(c^2+1))x_0^2x_1^2x_3 + ((4c^2+c)/(c^2+1))x_0^2x_2^2x_3 + ((4c^2+c)/(c^2+1))x_0^2x_2^2x_3^2 + ((4c^2+c)/(c^2+1))x_0^2x_2x_3^2 + ((4c^2+c)/(c^2+1))x_0^2x_3x_4^2 \\ &+ (c^2/(c^2+1))x_0^2x_1x_2x_4 \\ &+ (2c^2/(c^2+1))x_0x_1x_2x_3^2 + (2c^2/(c^2+1))x_1^2x_2x_3x_4 + (2c^2/(c^2+1))x_1x_2^2x_3x_4 + (2c^2/(c^2+1))x_0x_1x_3^2x_4 + (2c^2/(c^2+1))x_0x_2x_3^2x_4 + (2c^2/(c^2+1))x_0x_2x_3^2x_4 + (2c^2/(c^2+1))x_0x_1x_2x_3x_4^2 \\ &+ (3c^2/(c^2+1))x_0^2x_1x_2x_3 + (3c^2/(c^2+1))x_0x_1^2x_2x_4 + (3c^2/(c^2+1))x_0x_1x_2^2x_4 + (3c^2/(c^2+1))x_0x_1x_2x_3^2x_4 + (4c^2/(c^2+1))x_0x_1x_2x_3^2x_4 + (4c^2/(c^2+1))x_0x_1x_2x_3^2x_4
```

 $((4c+4)/c)x_0^5 + x_0^4x_1 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 - x_0x_1^4 + 2x_0^4x_2 + (2c/(c+2))x_0^3x_1x_2 + ((4c^2+1)/(c+2))x_0^2x_1^3 + ((4c+1)/(c+2))x_0^3x_1^2 + ((4c+1)/(c+2))x_0^3x_1^3 + ((4c+1)/(c+2))x_0^3 + ((4c+1)/($ $(c)/(c^2+1))x_0^2x_1^2x_2 + x_1^4x_2 + ((3c+2)/(c+2))x_0^3x_2^2 + ((c^2+4c)/(c^2+1))x_0x_1^2x_2^2 + ((4c+1)/(c+2))x_1^3x_2^2 + ((2c+1)/(c+2))x_1^3x_2^2 + ((2c+1)/(c+2))x_1^3x_1^2 + ((2c+1)/(c+2))x_1^3 + ((2c+1)/(c+2))x_1^3 + ((2c+1)/(c+2))x_1^3 + (2c+1)/(c+2)x_1^2 + (2c+1)/(c+2)x_1^2$ $3)/(c+2))x_0^2x_2^3 + (3c/(c+2))x_0x_1x_2^3 + ((c+4)/(c+2))x_1^2x_2^3 + 3x_0x_2^4 - x_1x_2^4 + ((c+1)/c)x_2^5 + x_0^4x_3 + (3c/(c+2))x_0x_1x_2^3 + (3c/(c+2))x_1x_2^3 + (3c/(c+2))x_1x_2^2 + (3c/(c+2))x_1x_2^2 + (3c/(c+2))x_1x_2^2 + (3c/(c+2))x_1x_2^2 + (3c/(c+2))x_1x_2^2 + ($ $2))x_0^3x_1x_3 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1^2x_3 + (c/(c+2))x_0x_1^3x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_3 + (3c^2/(c^2 + 1)$ $(4c/(c+2))x_1^3x_2x_3 + (2c^2/(c^2+1))x_0x_1x_2^2x_3 + ((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3 + (3c/(c+2))x_0x_2^3x_3 + (2c/(c+2))x_0x_2^3x_3 + (2c/(c+2))x_0x_2^2x_3 + (2c/(c+2))x_0x_3^2x_3 + (2c/(c+2))x_3^2x_3^2x_3^2 + (2c/(c+2))x_3^2x_3^2 + (2c/(c+2))x_3^2x_3^2 + (2c/(c+2$ $(4c+1)/(c+2))x_1^3x_2^3 + (4c+1)/(c+2))x_0^3x_3^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_3^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_3^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_1^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_1^2 + ((4c^2+c)/(c^2+1))x_0x_1^2 + ((4c^2+c)/(c^2+1))x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+1)x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+1)x_1^2 + ((4c^2+c)/(c^2+$ $c)/(c^2+1))x_0^2x_2x_3^2+((c^2+4c)/(c^2+1))x_1^2x_2x_3^2+((c^2+4c)/(c^2+1))x_0x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2+((2c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^2+3c)/(c^2+3c)/(c^2+3c)((2c^2+3c)/(c^$ $((c+4)/(c+2))x_2^3x_3^2 + ((c+4)/(c+2))x_0^2x_3^3 + (c/(c+2))x_0x_1x_3^3 + (4c/(c+2))x_1x_2x_3^3 + ((4c+1)/(c+2))x_2^2x_3^3 + ((4c+1)/(c+2))x_2^2x_3^2 + ((4c+1)/(c+2))x_3^2x_3^2 + ((4c+1)/(c+2))x_3^2 + ((4c+1)/(c+2))$ $2))x_0x_2^3x_4 + (2c/(c+2))x_1x_2^3x_4 - x_2^4x_4 + (3c/(c+2))x_0^3x_3x_4 + (c^2/(c^2+1))x_0^2x_1x_3x_4 + (3c^2/(c^2+1))x_0x_1^2x_3x_4 + (3c/(c+2))x_0x_1^2x_3x_4 + (3c/(c+2))x_1^2x_3x_4 + (3$ $(3c^2/(c^2+1))x_0^2x_2x_3x_4 + (2c^2/(c^2+1))x_1^2x_2x_3x_4 + (2c^2/(c^2+1))x_0x_2^2x_3x_4 + (4c^2/(c^2+1))x_1x_2^2x_3x_4 + (2c/(c^2+1))x_1x_2^2x_3x_4 + (2c/(c^2+1))x_1x_2^2x_3x_3x_4 + (2c$ $(3c^2 + 2c)/(c^2 + 1))x_0^2x_3^2x_4 + (3c^2/(c^2 + 1))x_0x_1x_3^2x_4 + (2c^2/(c^2 + 1))x_1x_2x_3^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1x_3^2x_3^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1x_3^2x_3^2x_3 + ((2c^2 + 3c)/(c^2 + 1))x_1x_3^2x_3^2x_3 + ((2c^2 + 3c)/($ $1))x_{2}^{2}x_{3}^{2}x_{4} + (c/(c+2))x_{0}x_{3}^{3}x_{4} + (4c/(c+2))x_{2}x_{3}^{3}x_{4} + ((4c+1)/(c+2))x_{0}^{3}x_{4}^{2} + ((3c^{2}+2c)/(c^{2}+1))x_{0}^{2}x_{1}x_{4}^{2} + ((4c+1)/(c+2))x_{0}^{2}x_{1}^{2}x_{2}^{2} + ((4c+1)/(c+2))x_{0}^$ $((4c^2+c)/(c^2+1))x_0x_1^2x_4^2 + ((4c^2+c)/(c^2+1))x_0^2x_2x_4^2 + ((c^2+4c)/(c^2+1))x_1^2x_2x_4^2 + ((c^2+4c)/(c^2+1))x_1^2x_2x_2^2 + ((c^2+4c)/(c^2+1))x_1^2x_2^2 + ((c^2+4c)/(c^2+1))x_1^2 + ((c^2+4c)/(c^2+1)x_1^2 + ((c^2+4c)/(c^2+1)x_1^2 + ((c^2+4c)/(c^2+1))x_1^2 + ((c^2+4$ $1))x_0x_2^2x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1x_2^2x_4^2 + ((c+4)/(c+2))x_2^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3x_4^2 + (3c^2/(c^2 + 1))x_1x_2^2x_4^2 + ((c+4)/(c+2))x_2^3x_4^2 + ((c+4)/(c+2))x_2^3x_2^2 + ((c+4)/(c+2))x_2^2 + ((c+4)/(c+$ $1))x_0x_1x_3x_4^2 + (2c^2/(c^2+1))x_1x_2x_3x_4^2 + ((2c^2+3c)/(c^2+1))x_2^2x_3x_4^2 + ((4c^2+c)/(c^2+1))x_0x_3^2x_4^2 + ((c^2+4c)/(c^2+1))x_0x_3^2x_4^2 + ((c^2+4c)/(c^2+1))x_0x_1x_3^2x_4^2 + ((c^2+4c)/(c^2+1))x_0x_1x_3^2x_4^2 + ((c^2+4c)/(c^2+1))x_0x_1x_2x_3^2 + ((c^2+4c)/(c^2+1))x_0x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1)x_1x_2x_3x_3^2 + ((c^2+4c)/(c^2+1)x_3x_3x_3^2 + ((c^2+4c)/(c^2+1))x_1x_3x_3x_3^2 + ((c^2+4$ $(c/(c+2))x_0x_3x_4^3 + (4c/(c+2))x_2x_3x_4^3 - x_0x_4^4 + x_2x_4^4$

 $((4c+4)/c)x_0^5 + 2x_0^4x_1 + ((3c+2)/(c+2))x_0^3x_1^2 + ((2c+3)/(c+2))x_0^2x_1^3 + 3x_0x_1^4 + ((c+1)/c)x_1^5 + x_0^4x_2 + (2c/(c+3)/(c+2))x_0^2x_1^3 + 3x_0x_1^4 + ((c+1)/c)x_1^5 + x_0^4x_2 + (2c/(c+3)/(c+2))x_1^3 + (2c/(c+3)/(c+3)/(c+2))x_1^3 + (2c/(c+3)/(c+3)/(c+2))x_1^3 + (2c/(c+3)/(c+3)/(c+3)/(c+3))x_1^3 + (2c/(c+3)/(c+3)/(c+3)/(c+3)/(c+3)/(c+3)$ $2))x_0^3x_1x_2 + (3c/(c+2))x_0x_1^3x_2 - x_1^4x_2 + ((4c+1)/(c+2))x_0^3x_2^2 + ((4c^2+c)/(c^2+1))x_0^2x_1x_2^2 + ((c^2+4c)/(c^2+1))x_0^2x_1x_2^2 + ((c^2+4c)/(c^2+1))x_1^2 + ((c^2+4c)/(c^2+1)x_1^2 + ((c^2+4c)/(c^2+1)x_1^2 + ((c^2+4c)/(c^2+1)/(c^2+1)x_1^2 + ((c^2+4c)/(c^2+1)/(c$ $1))x_0x_1^2x_2^2 + ((c+4)/(c+2))x_1^3x_2^2 + ((c+4)/(c+2))x_0^2x_2^3 + ((4c+1)/(c+2))x_1^2x_2^3 - x_0x_2^4 + x_1x_2^4 + x_0^4x_3 + (2c/(c+2))x_1^2x_2^2 + ((4c+1)/(c+2))x_1^2x_2^2 + ((4c+1)/(c+2))x_1^2 + ((4c+1)/(c+2))x_1^2 + ((4c+1)/(c+2))x_1^2 + ((4c+1)/(c+2))x_1^2 + ((4c+1)/(c+2))x_1^2 + ((4c+1)/(c+2))x_1^2 + ((4$ $2))x_0^3x_1x_3 + (3c/(c+2))x_0x_1^3x_3 - x_1^4x_3 + (3c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2+1))x_0^2x_1x_2x_3 + (2c^2/(c^2+1))x_0x_1^2x_2x_3 + (3c/(c+2))x_0x_1^2x_2x_3 + (3c/(c+2))x_1^2x_2x_3 + (3c/$ $(4c+1)/(c+2))x_0^3x_3^2 + ((4c^2+c)/(c^2+1))x_0^2x_1x_3^2 + ((c^2+4c)/(c^2+1))x_0x_1^2x_3^2 + ((c+4)/(c+1))x_0x_1^2x_3^2 + ((c+4)/(c+1))x_0x_1^2 + ((c+4)/(c+1))x_0x_1^2 + ((c+4)/(c+1))x_0x_1^2 + ((c+4)/(c+1))x_0x_1^2 + ((c+4)/(c+1))x_1^2 + ((c+4)/(c+1))x_1^2 + ((c+4)/(c+1)/(c+1)x_1^2 + ((c+4)/(c+1)/(c+1)x_1^2 + ((c+4)/(c+1)/(c+1)x_1^2 + ((c+4)/(c+1)/(c+1)x_1^2$ $2))x_1^3x_3^{\frac{7}{2}} + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_3^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_3^2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_3^2 + ((c^2 + 4c)/(c^2 + 1)x_3$ $1))x_1x_2^2x_3^2 + ((c+4)/(c+2))x_0^2x_3^3 + ((4c+1)/(c+2))x_1^2x_3^3 + (c/(c+2))x_0x_2x_3^3 + (4c/(c+2))x_1x_2x_3^3 - x_0x_3^4 + x_1x_3^4 +$ $1))x_0x_1^2x_2x_4 + (2c/(c+2))x_1^3x_2x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_2^2x_4 + ((2c^2+3c)/(c^2+1))x_1^2x_2^2x_4 + (c/(c+2))x_0x_2^3x_4 + (c/(c+2))x_0x_2^2x_4 + (c/(c+2))x_2^2x_4 + (c/(c+2))$ $(4c/(c+2))x_1x_2^3x_4 + (3c/(c+2))x_0^3x_3x_4 + (3c^2/(c^2+1))x_0^2x_1x_3x_4 + (2c^2/(c^2+1))x_0x_1^2x_3x_4 + (2c/(c+2))x_1^3x_3x_4 + (3c/(c+2))x_1^3x_3x_4 + (3c/(c+2))x_1^$ $(c^2/(c^2+1))x_0^2x_2x_3x_4 + (4c^2/(c^2+1))x_1^2x_2x_3x_4 + (3c^2/(c^2+1))x_0x_2^2x_3x_4 + (2c^2/(c^2+1))x_1x_2^2x_3x_4 + ((3c^2+1))x_1x_2^2x_3x_4 + (3c^2/(c^2+1))x_1x_2^2x_3x_4 + (3c$ $\frac{2c}{(c^2+1)}x_0^2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4 + (3c^2/(c^2+1))x_0x_2x_3^2x_4 + (2c^2/(c^2+1))x_1x_2x_3^2x_4 + (c/(c+1))x_1x_2x_3^2x_4 + (c/(c+1))x_1x_3x_3^2x_4 + (c/(c+1))x_1x_3x_3^2x_4 + (c/(c+1))x_1x_3x_3x_4 + (c/(c+1))x_3x_3x_4 + ($ $2))x_0x_3^3x_4 + (4c/(c+2))x_1x_3^3x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((4c^2+c)/(c^2+1))x_0^2x_1x_4^2 + ((c^2+4c)/(c^2+1))x_0x_1^2x_4^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_4^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_1^2 + ((4c^2+c)/(c^2+1))x_0x_1^2 + ((4c^2+c)/(c^2+1))x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+c)x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+c)x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+c)x_1^2 + ((4c^2+$ $((c+4)/(c+2))x_1^3x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_2x_4^2 + ((2c^2+3c)/(c^2+1))x_1^2x_2x_4^2 + ((4c^2+c)/(c^2+1))x_0x_2^2x_4^2 + ((4c^2+c)/(c^2+1))x_1^2x_2x_4^2 + ((4c^2+c)/(c^2+1))x_1^2x_2x_2^2 + ((4c^2+c)/(c^2+c)/(c^2+c)x_1^2 + ((4c^2+c)/(c^2+c)/(c^2+c)/(c^2+c)x_1^2 + ((4c^2+c)/(c^$ $((c^2+4c)/(c^2+1))x_1x_2^2x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_3x_4^2 + ((2c^2+3c)/(c^2+1))x_1^2x_3x_4^2 + (3c^2/(c^2+1))x_0x_2x_3x_4^2 + (3c^2+3c)/(c^2+1)x_1x_2^2x_3x_4^2 + (3c^2+3c)/(c^2+3c)/$ $(2c^2/(c^2+1))x_1x_2x_3x_4^{\tilde{2}} + ((4c^2+c)/(c^2+1))x_0x_3^2x_4^2 + ((c^2+4c)/(c^2+1))x_1x_3^2x_4^2 + ((c+4)/(c+2))x_0^2x_4^3 + ((4c+4)/(c+2))x_0x_3^2x_4^2 + ((4c+4)/(c+2))x_3^2x_4^2 + ((4c+4)/(c+2))x_3^2 + ((4c+4)/(c+2)/(c+2))x_3^2 + ((4$ $1)/(c+2))x_1^2x_4^3 + (c/(c+2))x_0x_2x_4^3 + (4c/(c+2))x_1x_2x_4^3 + (c/(c+2))x_0x_3x_4^3 + (4c/(c+2))x_1x_3x_4^3 - x_0x_4^4 + x_1x_4^4 + x_1$

conjecture: x_0^{25}

5 S_6 , char 3, τ trivial

5.1 Hilbert polynomial

PARTIAL

```
1 + 6t + 21t^2 + 51t^3 + 96t^4 + 147t^5 + 192t^6 + 222t^7 + \dots
```

5.2 Generators

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 \\ ((2c+2)/c)x_0^3 \\ + ((c+1)/c)x_4^3 \\ + x_0^2x_1 + x_0^2x_2 + x_0^2x_3 + x_0^2x_4 + x_1x_2^2 - x_2x_1^2 - x_3x_2^2 + x_1^2x_5 \\ - x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_0^2x_4 + x_1x_2^2 - x_2x_1^2 - x_3x_2^2 - x_1^2x_5 \\ - (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 \\ + (c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_3x_4x_5 \\ ((2c+2)/c)x_0^3 \\ + ((c+1)/c)x_0^3 \\ + x_0^2x_1 + x_0^2x_2 + x_1^2x_3 + x_2^2x_3 + x_0x_2^2 + x_1^2x_4 + x_3x_2^2 + x_2^2x_5 + x_3x_2^2 \\ - x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_1x_3^2 - x_2x_3^2 - x_2^2x_4 + x_0x_3^2 + x_2^2x_5 + x_0x_3^2 \\ + (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 \\ + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 + (2c/(c+2))x_3x_4x_5 \\ ((2c+2)/c)x_0^3 \\ + ((c+1)/c)x_0^3 \\ + ((c+1)/c)x_0^3 \\ + (c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_3x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_2x_3x_5 + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_1x_2x_5 +$$

conjecture: x_0^9 .

5.3 n-1 case

6 S_6 , char 2, τ trivial

6.1 Hilbert polynomial

$$t^8 + 6t^7 + 16t^6 + 26t^5 + 30t^4 + 26t^3 + 16t^2 + 6t + 1 = (t+1)^6(t^2+1)$$

6.2 Generators

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_0x_3 + x_1x_4 + x_2x_4 + x_3x_4 + ((c+1)/c)x_4^2 + x_0x_5 + x_4x_5$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_1x_3 + x_2x_3 + ((c+1)/c)x_3^2 + x_0x_4 + x_3x_4 + x_0x_5 + x_3x_5$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3 + x_0x_4 + x_2x_4 + x_0x_5 + x_2x_5$$

$$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3 + x_0x_4 + x_1x_4 + x_0x_5 + x_1x_5$$

$$x_0^4.$$

7 S_9 , char 3, τ trivial

7.1 Hilbert polynomial

PARTIAL

$$\cdots + 157t^3 + 45t^2 + 9t + 1$$

7.2 Generators

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + x_8^3 \\ ((2c+2)/c)x_0^3 \\ + ((c+1)/c)x_7^3 \\ + x_0^2x_1 + x_0^2x_2 + x_0^2x_3 + x_0^2x_4 + x_0^2x_5 + x_0^2x_6 + x_1^2x_7 + x_2^2x_7 + x_3^2x_7 + x_4^2x_7 + x_5^2x_7 + x_6^2x_7 + x_0x_7^2 + x_0^2x_8 + x_7x_8^2 \\ - x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_0x_4^2 - x_0x_5^2 - x_0x_6^2 - x_0^2x_7 - x_1x_7^2 - x_2x_7^2 - x_3x_7^2 - x_4x_7^2 - x_5x_7^2 - x_6x_7^2 - x_7^2x_8 - x_0x_8^2 \\ + (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/$$

```
2))x_0x_3x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (c/(c+2))x_0x_5x_6 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_1x_2x_7 + (2c/(c+2))x_1x_3x_7 + (2c/(c+2))x_2x_3x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 + (2c/(c+2))x_1x_7x_8 + (2c/(c+2))x_2x_7x_8 + (2c/(c+2))x_3x_7x_8 + (2c/(c+2))x_4x_7x_8 + (2c/(c+2))x_5x_7x_8 + (2c/(c+2))x_6x_7x_8
```

 $((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 - x_0x_5^2 - x_0^2x_6 + x_1^2x_6 + (2c/(c+2))x_1x_2x_6 + x_2^2x_6 + (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 + x_3^2x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 + x_2^2x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + (2c/(c+2))x_4x_5x_6 + x_5^2x_6 + x_0x_6^2 - x_1x_6^2 - x_2x_6^2 - x_3x_6^2 - x_4x_6^2 - x_5x_6^2 + ((c+1)/c)x_6^3 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_6x_8 + (2c/(c+2))x_2x_6x_8 + (2c/(c+2))x_2x_6$

 $((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 - x_0^2x_5 + x_1^2x_5 + (2c/(c+2))x_1x_2x_5 + x_2^2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 + x_3^2x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_2x_3x_5 + x_3^2x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_2x_3x_5 + x_3^2x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_0x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 - x_5^2x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_0x_4x_7 + (2c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_5x_8 + (2c/(c+2))x_2x_5x_8 + (2c/(c+2))x_3x_5x_8 + (2c/(c+2))x_3x_5x_8 + (2c/(c+2))x_3x_5x_8 + (2c/(c+2))x_5x_6x_8 + (2c/(c+2))x_5x_6x_8 + (2c/(c+2))x_5x_7x_8 - x_0x_8^2 + x_5x_8^2 + x_0x_8^2 + x_0x_8$

 $((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - x_0x_3^2 - x_0^2x_4 + x_1^2x_4 + (2c/(c+2))x_1x_2x_4 + x_2^2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + x_3^2x_4 + x_0x_4^2 - x_1x_4^2 - x_2x_4^2 - x_3x_4^2 + ((c+1)/c)x_4^3 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_4^2x_5 - x_0x_5^2 + x_4x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 - x_4^2x_6 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 - x_4^2x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_4x_5x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 - x_4^2x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_0x_5x_7 + (2c/(c+2))x_4x_5x_7 + (2c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 - x_4^2x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_0x_5x_7 + (2c/(c+2))x_0x_5x_8 + (2c/(c+2)$

 $((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 - x_0^2x_3 + x_1^2x_3 + (2c/(c+2))x_1x_2x_3 + x_2^2x_3 + x_0x_3^2 - x_1x_3^2 - x_2x_3^2 + ((c+1)/c)x_3^3 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 - x_3^2x_4 - x_0x_4^2 + x_3x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 - x_3^2x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_0x_5^2 + x_3x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 - x_3^2x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_3x_4x_6 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_5x_7 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_5x_8 + (c/(c+2))x_0x_5x$

 $(2))x_1x_2x_3 - x_2^2x_3 - x_0x_3^2 + x_2x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (2c/(c+2))x_1x_2x_4 - x_2^2x_4 + (c/(c+2))x_0x_3x_4 + (c/(c+2))x_0x_4 + (c/(c+2))x_0x_5 + (c/(c+2))x_0x_5 + (c/(c+2))x_0x_5 + (c/(c+2))x_0x_5 + (c/(c+2))x_0x_5 + (c/(c+2))x_0x_5 + (c/(c+2))x$ $(2c/(c+2))x_2x_3x_4 - x_0x_4^2 + x_2x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (2c/(c+2))x_1x_2x_5 - x_2^2x_5 + (c/(c+2))x_0x_3x_5 + x_0^2x_5 +$ $(2c/(c+2))x_2x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_2x_4x_5 - x_0x_5^2 + x_2x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (2c/(c+2))x_0x_1x_6 + (2c/(c+2))x_1x_6 + (2c/(c+2))x_1x_6 + (2c/(c+2))x_1x_6 + (2c/(c+2))x_1x_6 + (2c/(c+2))x_1x_6 + (2c/(c+2))x_6 + (2c/(c+2))$ $2))x_{1}x_{2}x_{6}-x_{2}^{2}x_{6}+(c/(c+2))x_{0}x_{3}x_{6}+(2c/(c+2))x_{2}x_{3}x_{6}+(c/(c+2))x_{0}x_{4}x_{6}+(2c/(c+2))x_{2}x_{4}x_{6}+(c/(c+2))x_{2}x_{3}x_{6}+(c/(c+2)$ $2))x_0x_5x_6 + (2c/(c+2))x_2x_5x_6 - x_0x_6^2 + x_2x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (2c/(c+2))x_1x_2x_7 - x_2^2x_7 + (c/(c+2))x_1x_2x_7 - x_2^2x_7 + x$ $2))x_0x_3x_7 + (2c/(c+2))x_2x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_2x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_5x_5x_7 + (2c/(c+2))x_5x_7 + (2c/(c+2))x_7 + (2c/(c$ $(c/(c+2))x_0x_6x_7 + (2c/(c+2))x_2x_6x_7 - x_0x_7^2 + x_2x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (2c/(c+2))x_1x_2x_8 - x_2^2x_8 + x_2^2x_8 - x_2^2x_8 -$ $(c/(c+2))x_0x_3x_8 + (2c/(c+2))x_2x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_2x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_0x_5x_8 + ($ $(2))x_2x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_2x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_2x_7x_8 - x_0x_8^2 + x_2x_8^2 + x_2x_8$ $((2c+2)/c)x_0^3 - x_0^2x_1 + x_0x_1^2 + ((c+1)/c)x_1^3 + x_0^2x_2 - x_1^2x_2 - x_0x_2^2 + x_1x_2^2 + x_0^2x_3 - x_1^2x_3 + (c/(c+2))x_0x_2x_3 + x_0x_1^2 + x_0x_1^2$ $(2c/(c+2))x_1x_2x_3 - x_0x_3^2 + x_1x_3^2 + x_0^2x_4 - x_1^2x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_1x_2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_1x_2x_4 - x_0x_4^2 + x_1x_4^2 + x_0^2x_5 - x_1^2x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_1x_5 + (2c/(c+2))x_1x_5 + (2c/(c+2))x_1x_5 + (2c/(c+2))x_1$ $(2c/(c+2))x_1x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_1x_4x_5 - x_0x_5^2 + x_1x_5^2 + x_0^2x_6 - x_1^2x_6 + (c/(c+2))x_0x_2x_6 + x_0x_5^2 + x_1x_5^2 + x_0x_5^2 +$ $(2c/(c+2))x_1x_2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_1x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_4x_6 + (c/(c+2))x_1x_2x_6 + (c/($ $(2))x_0x_5x_6 + (2c/(c+2))x_1x_5x_6 - x_0x_6^2 + x_1x_6^2 + x_0^2x_7 - x_1^2x_7 + (c/(c+2))x_0x_2x_7 + (2c/(c+2))x_1x_2x_7 + (c/(c+2))x_1x_2x_7 + (c/(c+2))x_1x_7 + (c/(c+2))x_1x_7 + (c/(c+2))x_1x_$ $2))x_0x_3x_7 + (2c/(c+2))x_1x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_1x_7 + (2c/(c+2))$ $(c/(c+2))x_0x_6x_7 + (2c/(c+2))x_1x_6x_7 - x_0x_7^2 + x_1x_7^2 + x_0^2x_8 - x_1^2x_8 + (c/(c+2))x_0x_2x_8 + (2c/(c+2))x_1x_2x_8 + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_3x_3 +$ $(c/(c+2))x_0x_3x_8 + (2c/(c+2))x_1x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_0x_5x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_0x_8x_8 + (2c/(c+2))x_8x_8 + (2c/(c+2))x_8x_8 + (2c/(c+2))x_8x_8 + (2c/(c+2))x_8x_8 + (2c/(c+2))x_8$ $2))x_1x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_1x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_1x_7x_8 - x_0x_8^2 + x_1x_8^2 + x_1x_8^$

conjecture: x_0^9

8 S_4 , char 3, τ trivial

8.1 Polynomial

$$(t+1)^2(t^4+t^2+1)^2(t^2+t+1)^2(t^6+t^3+1)$$

8.2 Generators

$$x_0^3 + x_1^3 + x_2^3 + x_2^3$$

$$((2c+2)/c)x_0^6 - x_0^5x_1 + x_0^4x_1^2 - x_0^2x_1^4 + x_0x_1^5 + ((c+1)/c)x_1^6 + x_0^5x_2 + x_0^3x_1^2x_2 - x_0^2x_1^3x_2 - x_1^5x_2 - x_0^4x_2^2 - x_0^3x_1x_2^2 + x_0x_1^3x_2^2 + x_1^4x_2^2 + ((c+1)/c)x_0^3x_2^3 + x_0^2x_1x_2^3 - x_0x_1^2x_2^3 + ((2c+2)/c)x_1^3x_2^3 + x_0^2x_2^4 - x_1^2x_2^4 - x_0x_2^5 + x_1x_2^5 + x_0^3x_1^2x_3 - x_0^2x_1^3x_3 - x_0^3x_2^2x_3 + x_1^3x_2^2x_3 + x_0^3x_2^2x_3 - x_1^3x_2^2x_3 - x_0^3x_2^2x_3 + x_1^3x_2^2x_3 - x_0^2x_1^2x_3 - x_0^3x_1x_3^2 + x_0x_1^3x_3^2 + x_0x_1^3x_2^2 - x_0x_1^3x_2^$$

$$x_0^6 + x_0^3 x_1^3 + x_1^6 + x_0^3 x_2^3 + x_1^3 x_2^3 + x_2^6$$

$$x_0^9 + x_0^6 x_1^3 + x_0^3 x_1^6 + x_1^9$$

9 S_4 , char 3, τ trivial, n-1 representation

9.1 Polynomial

$$(t+1)^2(t^4+t^2+1)^2(t^2+t+1)(t^6+t^3+1)$$

9.2 Generators

$$x_0^3x_1^3 + x_0^3x_2^3 + x_1^3x_2^3 \\ ((2c+2)/c)x_0^3x_1^3 - x_0^3x_1^2x_2 + x_0^2x_1^3x_2 + x_0^3x_1x_2^2 - x_0x_1^3x_2^2 + ((c+1)/c)x_0^3x_2^3 - x_0^2x_1x_2^3 + x_0x_1^2x_2^3 \\ x_0^9 + (c^2+2)x_0^6x_1^3 + cx_0^5x_1^4 + (2c^2+2c)x_0^4x_1^5 + x_1^9 + (2c^2+c)x_0^6x_1^2x_2 + (2c^2+2c)x_0^5x_1^3x_2 + c^2x_0^4x_1^4x_2 + c^2x_0^3x_1^5x_2 + (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9 \\ (c^2+2c)x_0^2x_1^2x_2^2 + cx_0^2x_1^2x_2^2 + cx_0^2x_1^2x_1^2 + cx_0^2x_1^2x_1$$

10 S_7 , char 3, τ trivial

10.1 Polynomial

PARTIAL

$$434t^5 + 203t^4 + 83t^3 + 28t^2 + 7t + 1$$

10.2 Generators

PARTIAL

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3$$

11 S_7 , char 3, τ trivial, n-1 representation

11.1 Generators

$$x_0^3x_1^3 + x_0^3x_2^3 + x_1^3x_2^3 + x_0^3x_3^3 + x_1^3x_3^3 + x_2^3x_3^3 + x_0^3x_4^3 + x_1^3x_4^3 + x_2^3x_4^3 + x_0^3x_5^3 + x_1^3x_5^3 + x_2^3x_5^3 + x_3^3x_5^3 + x_4^3x_5^3 + x_4^3x_5^3 + x_2^3x_5^3 + x_3^3x_5^3 + x_4^3x_5^3 + x_4^3x_5^3 + x_2^3x_1^3 + x_0^3x_1^2x_2 + x_0^2x_1^3x_2 - x_0^3x_1x_2^2 + (c/(c+2))x_0^2x_1^2x_2^2 - x_0x_1^3x_2^2 + (2c/(c+2))x_0^3x_1^2x_3 + (2c/(c+2))x_0^3x_1^2x_3 + (c/(c^2+c+1))x_0x_1^3x_2x_3 + ((c+1)/(c+2))x_0^3x_2^2x_3 + ((c^2+c)/(c^3+2))x_0^2x_1^2x_2x_3 + (c/(c^2+c+1))x_0x_1^3x_2x_3 + ((c+1)/(c+2))x_1^3x_2^2x_3 + ((c+1)/(c+2))x_0^3x_2^2x_3 + ((c+1)/(c+2))x_0^3x_1x_3^2 + (2c^2+c)/(c^3+2))x_0x_1x_2^3x_3 + ((c+1)/(c+2))x_0^3x_1x_3^2 + (2c^2/(c^2+c+1))x_0x_1x_2^3x_3 + ((c+1)/(c+2))x_0^3x_1x_3^2 + (2c^2/(c^2+c+1))x_0x_1x_2^2x_3^2 + ((c+2)/(c+2))x_0x_1x_2^2x_3^2 + ((c+2)/(c+2))x_0x_1x_2^2x_3^2 + ((c+2)/(c^2+c+1))x_0^2x_1x_2x_3^2 + ((c+2)/(c+2))x_0x_2x_3^2 + ((c^2+c)/(c^2+c+1))x_0^2x_1x_2x_3^2 + ((c^2+c)/(c^2+c+1))x_0^2x_1x_2x_3^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2^2x_3^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2^2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2x_2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_1x_2 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x$$

 $1))x_0x_1x_3^3x_4 + ((2c+2)/(c+2))x_1^2x_3^3x_4 + ((c^2+c)/(c^2+c+1))x_0x_2x_3^3x_4 + ((c^2+c)/(c^2+c+1))x_1x_2x_3^3x_4 + ((2c+2)/(c^2+c+1))x_1x_2x_3^3x_4 + ((2c+2)/(c^2+c+1)x_2x_3^3x_4 + ((2c+2)/(c^2+c+1)x_3^2x_4 + ((2c+2)/(c^2+c+1)x_3^2x_4 + ((2c+2)/(c^2+c+1)x_3^2x_4 + ((2c+2)/(c^2+c+1)x_4 + ((2c+2)/(c^2+c+1)x_3^2x_4 + ((2c+2$ $2))x_{1}^{2}x_{2}x_{3}x_{4}^{2} + ((c^{2}+c)/(c^{3}+2))x_{0}x_{2}^{2}x_{3}x_{4}^{2} + ((c^{2}+c)/(c^{3}+2))x_{1}x_{2}^{2}x_{3}x_{4}^{2} + ((c+1)/(c+2))x_{2}^{3}x_{3}x_{4}^{2} + ((c^{2}+c)/(c^{3}+2))x_{1}x_{2}^{2}x_{3}x_{4}^{2} + ((c+1)/(c+2))x_{2}^{3}x_{3}x_{4}^{2} + ((c^{2}+c)/(c^{3}+2))x_{1}x_{2}^{2}x_{3}x_{4}^{2} + ((c+1)/(c+2))x_{2}^{3}x_{3}x_{4}^{2} + ((c^{2}+c)/(c^{3}+2))x_{1}x_{2}^{2}x_{3}x_{4}^{2} + ((c+1)/(c+2))x_{2}^{3}x_{3}x_{4}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{4}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2} + ((c+1)/(c+2))x_{3}^{3}x_{3}^{2$ $(c+1))x_0^2x_3^2x_4^2 + (c^2/(c^3+2))x_0x_1x_3^2x_4^2 + ((2c^2+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c^2+c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c^2+c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c^2+c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c)/(c^3+2))x_0x_2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_4^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2x_3^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2 + ((c^3+2c)/(c^2+c+1))x_1^2x_3^2 + ((c^3+2c)/(c^2+c+1)x$ $2c^2+c)/(c^3+2))x_1x_2x_3^2x_4^2+((2c^2+2c)/(c^2+c+1))x_2^2x_3^2x_4^2+((c+1)/(c+2))x_0x_3^3x_4^2+((c+1)/(c+2))x_1x_3^3x_4^2+((c+1)/(c+2))x_1x_3^2x_3^2+((c+1)/(c+2))x_3^2x_3^2+((c+1)/(c+2))x_3^2x_3^2+((c+1)/(c+2))x_3^2+((c+1)/(c+2))x_3^2+((c+1)/(c+2))x_3^2+((c+1)/(c+2))x_3^2+((c+1)/(c+2))x$ $((c^2+c)/(c^2+c+1))x_0^2x_1x_2^2x_5 + ((c^2+c)/(c^2+c+1))x_0x_1^2x_2^2x_5 + (c/(c^2+c+1))x_0^3x_1x_3x_5 + ((2c^3+c^2+c)/(c^3+c+1))x_0x_1^2x_1x_2^2x_5 + (c/(c^2+c+1))x_0x_1^2x_1x_2^2x_5 + (c/(c^2+c+1))x_0x_1^2x_1x_2^2x_3 + (c/(c^2+c+1))x_1^2x_1x_2^2x_3 + (c/(c^2+c+1))x_1^2x_1x_2^2x_2^2x_3 + (c/(c^2+c+1))x_1^2x_1x_2^2x_3 + (c/(c^2+c+1))x_1^2x_1x_2^$ $2))x_0^2x_1^2x_3x_5 + (c/(c^2+c+1))x_0x_1^3x_3x_5 + ((2c^2+2c)/(c^2+c+1))x_0^3x_2x_3x_5 + (2c^2/(c^3+2))x_0^2x_1x_2x_3x_5 + (2c^2/(c^3+2))x_0^2x_1x_3x_3x_5 + (2c^2/(c^2+2))x_0^2x_1x_3x_3x_5 + (2c^2/(c^2+2))x_0^2x_1x_3x_3x_5 + (2c^2/(c^2+2))x_0^2x_1x_3x_3x_5 + (2c^2/(c^2+2))x_0^2x_1x_3x_3x_5 + (2c^2/(c^2+2))x_0^2x_3x_3x_5 + (2c^2/(c^2/($ $(2c^2/(c^3+2))x_0x_1^2x_2x_3x_5 + ((2c^2+2c)/(c^2+c+1))x_1^3x_2x_3x_5 + ((2c^3+2c)/(c^3+2))x_0^2x_2^2x_3x_5 + ((c^3+c^2)/(c^3+2))x_1^2x_2x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_2x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^3+c^2))x_1^2x_3x_3x_5 + ((c^3+c^2)/(c^2+c^2))x_1^2x_3x_3x_5 + ((c^3+c^$ $(2c^{2}+2c)/(c^{2}+2c)/(c^{3}+2c)/(c^{3}+2c)/(c^{3}+2c)/(c^{3}+2c)/(c^{2}+2c)/(c^{2}+2c)/(c^{2}+c+1))x_{0}x_{2}^{3}x_{3}x_{5}+((2c^{2}+2c)/(c^{2}+c+1))x_{0}x_{0}^{3}x_{3}x_{5}+((2c^{2}+2c)/(c^{2}+c+1))x_{0}^{3}x_{0}+((2c^{2}+2c)/(c^{2}+c+1)x_{0}^{3}x_{0}+((2c^{2}+2c)/(c^{2}+c+1))x_{0}^{3}x_{0}+((2c^{2}+2c)/(c^{2}+c+1))x_{0}^{3}x_{0}+((2c^{2}+2c)/(c^{2}+c+1))x_{0}^{3}x_{0}+((2c^{2}+2c)/(c^{2}$ $((2c+2)/(c+2))x_1^3x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_0^2x_2x_3^2x_5 + (c^2/(c^2+c+1))x_0x_1x_2x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_0x_1x_2x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_1x_2x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_3^2x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_3x_5 + ((c^2+c)/(c^2+c+1))x_1x_5 + ((c^2+c)/(c^2+c+1)x_5 + ((c^2+c)/(c^2+c+1))x_5 + (($ $2))x_{0}^{2}x_{3}^{3}x_{5} + (2c/(c^{2}+c+1))x_{0}x_{1}x_{3}^{3}x_{5} + (1/(c+2))x_{1}^{2}x_{3}^{3}x_{5} + (2c/(c^{2}+c+1))x_{0}x_{2}x_{3}^{3}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}x_{3}^{3}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{3}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{3}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{3}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{3}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{3}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{2}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}x_{2}^{2}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}^{2}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}^{2}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{1}^{2}x_{3}^{2}x_{5} + (2c/(c^{2}+c+1))x_{2$ $2c)/(c^2+c+1))x_1^3x_3x_4x_5+(c^3/(c^3+2))x_0^2x_2x_3x_4x_5+(c^3/(c^3+2))x_1^2x_2x_3x_4x_5+(c^3/(c^3+2))x_0x_2^2x_3x_4x_5+(c^3/(c^3+2))x_1^2x_2x_3x_3x_4x_5+(c^3/(c^3+2))x_1^2x_2x_3x_3x$ $(c^3/(c^3+2))x_1x_2^2x_3x_4x_5 + ((2c^2+2c)/(c^2+c+1))x_2^3x_3x_4x_5 + ((2c^3+2c^2)/(c^3+2))x_0^2x_3^2x_4x_5 + (c^2/(c^2+c+1))x_2^3x_3x_4x_5 + ((2c^3+2c)/(c^3+2))x_1x_2^2x_3x_4x_5 + ((2c^3+2c)/(c^2+c+1))x_2^3x_3x_4x_5 + ((2c^3+2c)/(c^3+2))x_1x_2^2x_3x_4x_5 + ((2c^3+2c)/(c^2+c+1))x_2^3x_3x_4x_5 + ((2c^3+2c)/(c^3+2))x_2^2x_3x_4x_5 + ((2c^3+2c)/(c^3+2))x_3^2x_4x_5 + ((2c^3+2c)/(c^3+2))x_3^2x_4x_5 + ((2c^3+2c)/(c^3+2))x_3^2x_4x_5 + ((2c^3+2c)/(c^3+2))x_3^2x_5x_5 + ((2c^3+2c)/(c^3+2))x_5^2x_5 + ((2c^3+2c)/(c^3+2c)/(c^3+2c))x_5^2x_5 + ((2c^3+2c)/(c^3+2c)/(c^3+2c))x_5^2x_5 + ((2c^3+2c)/(c^3+2c)/(c^3+2c)/(c^3+2c))x_5^2x_5 + ((2c^3+2c)/(c^3+2c)/(c^3+2c)/(c^3+2c))x_5^2x_5 + ((2c^3+2c)/($ $1))x_0x_1x_3^2x_4x_5 + ((2c^3 + 2c^2)/(c^3 + 2))x_1^2x_3^2x_4x_5 + ((2c^3 + 2c^2)/(c^3 + 2))x_2^2x_3^2x_4x_5 + (c^2/(c^2 + c + 1))x_0x_3^3x_4x_5 + (c^2/(c^2 + c + 1))x_0x_3^2x_4x_5 + (c^2/(c^2 + c + 1))x_0x_3^2x_5 + (c^2/(c^2 + c + 1))x_0x_3^2x_5 + (c^2/(c^2 + c + 1))x_0x_5^2x_5 + (c^2/(c^2 + c + 1))x_5^2x_5 + (c^2/(c^2 + c + 1))x_5^2x_5$ $(c^{2})/(c^{3}+2))x_{0}x_{3}^{2}x_{4}^{2}x_{5} + ((c^{3}+c^{2})/(c^{3}+2))x_{1}x_{3}^{2}x_{4}^{2}x_{5} + ((c^{3}+c^{2})/(c^{3}+2))x_{2}x_{3}^{2}x_{4}^{2}x_{5} + x_{3}^{3}x_{4}^{2}x_{5} - x_{0}^{3}x_{1}x_{5}^{2} + (c/(c+c^{3}+c^{2})/(c^{3}+2))x_{2}x_{3}^{2}x_{4}^{2}x_{5} + x_{3}^{3}x_{4}^{2}x_{5} + x_{$ $((c+1)/(c+2))x_0^3x_3x_5^2 + ((c^2+c)/(c^3+2))x_0^2x_1x_3x_5^2 + ((c^2+c)/(c^3+2))x_0x_1^2x_3x_5^2 + ((c+1)/(c+2))x_1^3x_3x_5^2 + ((c+1)/(c+2))x_1^3x_5^2 + ((c+1)/(c+2))x_1^3x_5^2 + ((c+1)/(c+2))x_1^2 + ((c+1)/(c+2))x_1^2 + ((c+1)/(c+2))x_1^2 + ($ $((2c^3+c^2+2c)/(c^3+2))x_0^2x_2x_3x_5^2+(c^2/(c^2+c+1))x_0x_1x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^3+2))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_5^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_2x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^3+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c))x_1^2x_3x_3x_3x_3^2+((2c^2+c^2+2c)/(c^2+2c)x_3x_3x_3x_3x_3x_3x_3x_3x_3x_3x_3x_3$ $(c^2 + 2c)/(c^3 + 2))x_0x_2^2x_3x_5^2 + ((2c^3 + c^2 + 2c)/(c^3 + 2))x_1x_2^2x_3x_5^2 + ((c+1)/(c+2))x_2^3x_3x_5^2 + (c^2/(c^2 + c + c^2))x_1x_2^2x_3x_5^2 + (c^2/(c^2 + c + c^2))x_1x_2^2x_3x_3^2 + (c^2/(c^2 + c + c^2))x_1x_2^2x_3x_3^2 + (c^2/(c^2 + c + c^2))x_1x_2^2x_3x_3^2 + (c^2/(c^2 + c + c^2))x_1x_3^2x_3^2 + (c^2/(c^2 + c^2))x_3^2x_3^2 + (c^2/(c^2 + c^2))x_3^2x_3^2 + (c^2/(c^2 + c^2))x_3^2x_3^2 + (c^2/(c^2 + c^2))x_3^2x_3^2 + (c^2/(c^2 +$ $\frac{2c)/(c^3+2))x_1x_2x_3^2x_5^2+(c^2/(c^2+c+1))x_2^2x_3^2x_5^2+(2/(c+2))x_0x_3^3x_5^2+(2/(c+2))x_1x_3^3x_5^2+(2/(c+2))x_2x_3^3x_5^2+(2/(c+2))x_2x_3^3x_5^2+(2/(c+2))x_2x_3^3x_5^2+(2/(c+2))x_2x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3x_3^2x_5^2+(2/(c+2))x_3^2x_5^2+(2/(c+2))x_3^2x_5^2+(2/(c+2))x_3^2x_5^2+(2/(c+2))x_3^$ $2))x_0^2x_3x_4x_5^2 + (2c^3/(c^3+2))x_0x_1x_3x_4x_5^2 + ((c^3+c^2)/(c^3+2))x_1^2x_3x_4x_5^2 + ((c^3+c^2)/(c^3+2))x_0x_2x_3x_4x_5^2 + ((c^3+c^2)/(c^3+2))x_0x_3x_4x_5^2 + ((c^3+c^2)/(c^3+c^2))x_0x_5^2 + ((c^3+c^2)/(c^3+c^2))x_0x_5^2 + ((c^3+c^2)/(c^3+c^2))x_0x_5^2 + ((c^3+c^2)/(c^3+c^2))x_0x_5^2 + ((c^3+c^2)/(c^3+c^2))x_0x_5^2 + ((c^3+c^2)/(c^3+c^2))x_5^2 + ((c^3+c^2)/(c^3+c^2)x_5^2 + ((c^3+c^2)/(c^3+c^2)x_5^2 + ((c^3+c^2)/(c^3+c^2))x_5^2 + ((c^3+c^2)/($ $(c^{3})/(c^{3}+2))x_{1}x_{2}x_{3}x_{4}x_{5}^{2} + ((c^{3}+c^{2})/(c^{3}+2))x_{2}^{2}x_{3}x_{4}x_{5}^{2} + (2c^{3}/(c^{3}+2))x_{0}x_{3}^{2}x_{4}x_{5}^{2} + (2c^{3}/(c^{3}+2))x_{1}x_{3}^{2}x_{4}x_{5}^{2} + (2c^{3}/(c^{3}+2))x_{1}x_{2}^{2}x_{3}x_{4}x_{5}^{2} + (2c^{3}/(c^{3}+2))x_{1}^{2}x_{3}x_{4}x_{5}^{2} + (2c^{3}/(c^{3}+2))x_{1}^{2}x_{3}^{2}x_{5}^{2} + ($ $(2c^3/(c^3+2))x_2x_3^2x_4x_5^2 - x_3^3x_4x_5^2 + (2c/(c+2))x_0x_1x_4^2x_5^2 + (2c/(c+2))x_3^2x_4^2x_5^2 + x_0^2x_3x_5^3 + (2c/(c+2))x_0x_1x_3x_5^3 + (2c/(c+2))x_0x_1x_3x_5^3 + (2c/(c+2))x_0x_1x_3x_5^3 + (2c/(c+2))x_0x_1x_3x_5^3 + (2c/(c+2))x_0x_1x_3x_5^3 + (2c/(c+2))x_0x_1x_5^2 + (2c/(c+2))x_0x_1x_5 + (2c/(c+2))x_1x_5 + (2c/(c+2))x$ $x_1^2 x_3 x_5^3 + (2c/(c+2)) x_0 x_2 x_3 x_5^3 + (2c/(c+2)) x_1 x_2 x_3 x_5^3 + x_2^2 x_3 x_5^3 - x_0 x_3^2 x_5^3 - x_1 x_3^2 x_5^3 - x_2 x_3^2 x_5^3 + ((c+1)/c) x_3^3 x_5^3 + ((c+1)/c) x_5^3 x_5^3 x_5^3 + ((c+1)/c) x_5^3 x_5^3 + ((c+1)/c) x_5^3 x_5^3 x_5^3 + ((c+1)/c) x_5^3 x_5^$ $(2c/(c+2))x_0x_3x_4x_5^3 + (2c/(c+2))x_1x_3x_4x_5^3 + (2c/(c+2))x_2x_3x_4x_5^3 - x_3^2x_4x_5^3 + x_3x_4^2x_5^3$

 $((2c^2+1)/c^2)x_0^3x_1^3 + ((c+2)/c)x_0^3x_1^2x_2 + ((c+2)/c)x_0^2x_1^3x_2 + ((2c+1)/c)x_0^3x_1x_2^2 + x_0^2x_1^2x_2^2 + ((2c+1)/c)x_0x_1^3x_2^2 + ((2c+1)/c)x_0x_1^3x_1^2 + ((2c+1)/c)x_0x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^$ $2)/c)x_1^3x_2^2x_3 + ((2c+1)/c)x_0^3x_1x_3^2 + ((c+2)/c)x_0x_1^3x_3^2 + (c/(c+2))x_0x_1^2x_2x_3^2 + ((2c+1)/c)x_1^3x_2x_3^2 + (c/(c+2)/c)x_1^3x_2x_3^2 + (c/(c+2)/c)x_1^3x_2^2 + (c/(c+2)/c)x_1^3x_1^2 + (c/(c+2)/c)x_1^2 + ($ $2))x_0x_1x_2^2x_3^2 - x_1^2x_2^2x_3^2 + ((c+2)/c)x_0^2x_1x_3^3 + ((2c+1)/c)x_0x_1^2x_3^3 + ((c^2+2)/c^2)x_1^3x_3^3 - x_0x_1x_2x_3^3 + ((2c+1)/c)x_0x_1^2x_3^3 + ((2c+1)/c)x_0x_1^2x_3^2 + ((2c+1)/c)x_0x_1^2x_1^2x_3^2 + ((2c+1)/c)x_1^2x_1^2x_1^2 + ((2c+1)/c)x_1^2x_1^2x_1^2 + ((2c+1)/c)x_1^2x_1^2 + ((2c+1)/c)x_1^2x_1^2 + ((2c+1)/c)x_1^2x_1^2 + ((2c+1)/c)x_1^2x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^2$ $((2c+1)/c)x_0^3x_1x_4^2 + x_0^2x_1^2x_4^2 + ((2c+1)/c)x_0x_1^3x_4^2 + ((c+1)/(c+2))x_0^2x_1x_2x_4^2 + ((c+1)/(c+2))x_0x_1^2x_2x_4^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2^2 + ((c+1)/(c+2))x_0x_1^2 + ((c+1)/(c+2))x_1^2 + ((c+1)/(c+2))$ $x_0x_1x_2^2x_4^2 + (2c/(c+2))x_0^2x_1x_3x_4^2 + ((c+2)/c)x_1^3x_3x_4^2 + ((2c+2)/(c+2))x_1^2x_2x_3x_4^2 + x_1x_2^2x_3x_4^2 + (c/(c+2)/c)x_1^2x_2x_3x_4^2 + (c/(c+2)/c)x_1^2x_3x_4^2 + (c/(c+2)/c)x_1^2x_3x_3^2 + (c/(c+2)/c)x_1^2x_3^2 + (c/(c+2)/c)x_1^2x_3^2 + (c/(c+2)/c)x_1^2x_3^2 + (c/(c+2)/c)x_1^2x_3^2 + (c/(c+2)/c)$ $(2c/(c+2))x_0^2x_1^2x_3x_5 + (2c/(c+2))x_0^2x_1x_2x_3x_5 - x_1^3x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2^2x_3x_5 + (c/(c+2))x_0x_1^2x_3^2x_5 + (c/(c+2))x_0x_1^2x_3x_5 + (c/(c+2))x_0x_1^2x_3x_5 + (c/(c+2))x_0x_1x_2x_3x_5 + (c/(c+2))x_1x_2x_3x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_5 + (c/(c+2))x_1x_5 + (c/(c+2))x_1x_5 + (c/(c+2))x_5 + (c/(c+2))x$ $((2c+1)/c)x_1^3x_3^2x_5 + (c/(c+2))x_0x_1x_2x_3^2x_5 + (2/(c+2))x_1^2x_2x_3^2x_5 + ((2c+2)/(c+2))x_1x_2^2x_3^2x_5 - x_0x_1x_3^3x_5 + (2/(c+2))x_1x_2x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3^2x_5 + (2/(c+2))x_1x_3x_3x_5 + (2/(c+2))x_1x_3x_5 + (2/(c+2))x_1x_3x_5 + (2/(c+2))x_1x_3x_5 + (2/(c+2))x_1x_3x_5 + (2/(c+2))x_1x_3x_5 + (2/(c+2))x_1x_5 + (2/(c+2))x_1x_5 + (2/(c+2))x_5 +$ $(2c/(c+2))x_0^2x_1x_3x_4x_5 - x_1^3x_3x_4x_5 + (2c/(c+2))x_1x_2^2x_3x_4x_5 + (c/(c+2))x_0x_1x_3^2x_4x_5 + (2/(c+2))x_1^2x_3^2x_4x_5 - (c/(c+2))x_1x_2^2x_3x_4x_5 + (c/(c+2))x_1x_2^2x_3x_5 + (c/(c+2))x_1x_2^2x_5 + (c/(c+2))x_1x_2^2x_5 + (c/(c+2))x_$ $x_1x_3^3x_4x_5 + ((c+1)/(c+2))x_0^2x_1x_4^2x_5 + ((c+1)/(c+2))x_0x_1^2x_4^2x_5 + (c/(c+2))x_0x_1x_2x_4^2x_5 + ((2c+2)/(c+2))x_1x_2x_3x_4^2x_5 + ((2c+2)/(c+2))x_1x_3^2x_4^2x_5 + ((2c+1)/c)x_0^3x_1x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1^3x_5^2 + ((c+1)/(c+2))x_0^2x_1x_2x_5^2 + ((c+1)/(c+2))x_0x_1^2x_2x_5^2 - x_0x_1x_2^2x_5^2 + (2c/(c+2))x_0^2x_1x_3x_5^2 + ((c+2)/c)x_1^3x_3x_5^2 + ((c+2)/c)x_1^2x_2x_3x_5^2 + (c/(c+2))x_0^2x_1x_3x_5^2 + ((c+1)/(c+2))x_0x_1x_3x_5^2 + (c/(c+2)/c)x_1x_3x_5^2 + ((c+2)/c)x_1x_3x_5^2 + ((c+1)/(c+2))x_0x_1x_4x_5^2 + ((c+1)/(c+2))x_0x_1x_4x_5^2 + ((c+2)/(c+2))x_0x_1x_2x_4x_5^2 + ((c+2)/(c+2))x_1x_2x_3x_4x_5^2 + ((c+2)/(c+2)/(c+2))x_1x_2x_3x_4x_5^2 + ((c+2)/(c+2)/(c+2))x_1x_2x_3x_4x_5^2 + ((c+2)/(c+2)/(c+2))x_1x_2x_3x_4x_5^2 + ((c+2)/(c+2)/(c+2)/(c+2))x_1x_2x_3x_4x_5^2 + ((c+2)/($

 $((2c^2+1)/c^2)x_0^3x_1^3 + ((c+2)/c)x_0^3x_1^2x_2 + ((c+2)/c)x_0^2x_1^3x_2 + ((2c+1)/c)x_0^3x_1x_2^2 + x_0^2x_1^2x_2^2 + ((2c+1)/c)x_0x_1^3x_2^2 + ((2c+1)/c)x_0x_1^3x_1^2 + ((2c+1)/c)x_0x_1^3x_1^2 + ((2c+1)/c)x_1^3x_1^2 + ((2c+1)/c)x_1^3 + (2c+1)/c + ((2c+1)/c)x_1^3 + ((2c+1)/c)x_1^3 + ((2c+1)/c)x_1^3 + ((2c+1)/c)x_1^3 + ((2c+1)/$ $(c/(c+2))x_0x_1x_2^2x_3^2 + ((c^2+2)/c^2)x_0^3x_3^3 + ((2c+1)/c)x_0^2x_1x_3^3 + ((c+2)/c)x_0x_1^2x_3^3 + ((2c+1)/c)x_0^2x_2x_3^3 - ((2c+1)/c)x_0x_1^2x_3^2 + ((2c+1)/c)x_0x_1^2 + ((2c+1)/c)x_1^2x_3^2 + ((2c+1)/c)x_1^2x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^2 + ((2c+1)/c)x_1^2 +$ $x_0x_1x_2x_3^3 + ((c+2)/c)x_0x_2^2x_3^3 + ((c+2)/c)x_0^3x_1^2x_4 + ((c+2)/c)x_0^2x_1^3x_4 + x_0^3x_1x_2x_4 + (1/(c+2))x_0^2x_1^2x_2x_4 + (1/(c+2)/c)x_0x_1^2x_2x_3 + ((c+2)/c)x_0x_1^2x_2x_3 + ((c+2)/c)x_0x_1^2x_2x_3 + ((c+2)/c)x_0x_1^2x_3 + ((c+2)/c)x_1^2x_3 + (($ $x_0x_1^3x_2x_4 + ((c+1)/(c+2))x_0^2x_1x_2^2x_4 + ((c+1)/(c+2))x_0x_1^2x_2^2x_4 + (2c/(c+2))x_0^2x_1^2x_3x_4 + x_0x_1^3x_3x_4 - x_0^3x_2x_3x_4 + x_0x_1^3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_4 + x_0x_1^3x_3x_3x_4 + x_0x_1^3x_3x_4 + x_0x_1^3x_3x$ $(2c/(c+2))x_0x_1^2x_2x_3x_4 + ((2c+2)/(c+2))x_0^2x_2^2x_3x_4 + ((2c+1)/c)x_0^3x_3^2x_4 + (c/(c+2))x_0^2x_1x_3^2x_4 + (2/(c+2))x_0^2x_1x_3^2x_4 + (2/(c+2))x_0x_1x_3^2x_4 + (2/(c+2))x_0x_1x_3x_4 + (2/(c+2))x_1x_3x_4 + (2/(c+2))x_1x_3x$ $2))x_0^2x_2x_3^2x_4 + (c/(c+2))x_0x_1x_2x_3^2x_4 + ((2c+2)/(c+2))x_0x_2^2x_3^2x_4 + ((2c+1)/c)x_0^2x_3^3x_4 - x_0x_1x_3^3x_4 - x_0x_2x_3^3x_4 + ((2c+2)/(c+2))x_0x_2^2x_3^2x_4 + ((2c+2)/(c+2))x_0x_3^2x_4 + ((2c+2)/(c+2)/(c+2))x_0x_3^2x_4 + ((2c+2)/(c+2)/(c+2)x_3^2x_4 + ((2c+2)/(c+2)/(c+2)x_3^2x_4 + ((2c+2)/(c+2)/(c+2)x_3^2x_4 + ((2c+2)/(c+2)/(c+2)x_3^2x_4 + (($ $((2c+1)/c)x_0^3x_1x_4^2 + x_0^2x_1^2x_4^2 + ((2c+1)/c)x_0x_1^3x_4^2 + ((c+1)/(c+2))x_0^2x_1x_2x_4^2 + ((c+1)/(c+2))x_0x_1^2x_2x_4^2 - ((c+1)/(c+2))x_0x_1^2x_2x_4^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2x_4^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2^2 + ((c+1)/(c+2))x_0x_1^2x_2x_2^2 + ((c+1)/(c+2))x_1^2x_2x_2^2 + ((c+1)/(c+2))x_1^2x_2x_2^2 + ((c+1)/(c+2))x_1^2x_2x_2^2 + ((c+1)/(c+2))x_1^2x_2x_2^2 + ((c+1)/(c+2))x_1^2x_2^2 + ((c+1)/(c+2))x_1^2 + ((c+1)/(c+2))x_1^2x_2^2 + ((c+1)/(c+2))x_1^2x_2^2 + ((c+1)/(c+2))x_1^2 + ((c+1$ $x_0x_1x_2^2x_4^2 + ((c+2)/c)x_0^3x_3x_4^2 + (2c/(c+2))x_0x_1^2x_3x_4^2 + ((2c+2)/(c+2))x_0^2x_2x_3x_4^2 + x_0x_2^2x_3x_4^2 + x_0x_2^2x_3x_4^2 + (c/(c+2)/(c+2))x_0x_1^2x_3x_4^2 + x_0x_2^2x_3x_4^2 + x_0x_2^2x_3x_2^2 + x_0x_2^2x_3x_2^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3x_3^2 + x_0x_2^2x_3^2x_3^2 + x_0x_2^2x_3^2x_3^2 + x_0x_2^2x_3^2x_3^2 + x_0x_3^2x_3^2x_3^2 + x_0x_3^2x_3^2 + x_0x_3^2x_3^2x_3^2 + x_0x_3^2x_3^2 + x_0x_3^2x_3^2x_3^2 + x_0x_3^2x_3^2 + x_0x_3^2x_3^2 + x_0x_3^2x_3^2 + x_0x_3^$ $2))x_0x_1x_3^2x_4^2 + ((2c+2)/(c+2))x_0x_2x_3^2x_4^2 + ((c+2)/c)x_0x_3^3x_4^2 + ((c+2)/c)x_0^3x_1^2x_5 + ((c+2)/c)x_0^2x_1^3x_5 + x_0^3x_1x_2x_5 + x_0^3x_1x_5 + x_0^3x_$ $x_0x_1^3x_3x_5 - x_0^3x_2x_3x_5 + (2c/(c+2))x_0x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_0^2x_2^2x_3x_5 + ((2c+1)/c)x_0^3x_3^2x_5 + (c/(c+2))x_0x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_0x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_2x_3x_5 + ((2c+2)/(c+2))x_1^2x_3x_5 + ((2c+2)/(c+2))x_1^2x_3x_5 + ((2c+2)/(c+2))x_1^2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1^2x_5 + ((2c+2)/(c+2)/(c+2))x_1^2x_5 + ((2c+2)/(c+2)/(c+2)x_5 + ($ $2))x_0^2x_1x_3^2x_5 + (2/(c+2))x_0^2x_2x_3^2x_5 + (c/(c+2))x_0x_1x_2x_3^2x_5 + ((2c+2)/(c+2))x_0x_2^2x_3^2x_5 + ((2c+1)/c)x_0^2x_3^2x_5 + ((2c+1)/c)x_0x_1x_2x_3^2x_5 + ((2c+2)/(c+2))x_0x_1x_2x_3^2x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_3x_5 + ((2c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2))x_1x_3x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_3x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_2x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_3x_5 + ((2c+2)/(c+2)/(c+2))x_1x_3x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)x_3x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)/(c+2)/(c+2)x_5 + ((2c+2)/(c+2)/(c+2)/(c+2)$ $x_0x_1x_3^3x_5 - x_0x_2x_3^3x_5 + x_0^3x_1x_4x_5 + (1/(c+2))x_0^2x_1^2x_4x_5 + x_0x_1^3x_4x_5 + (c/(c+2))x_0x_1x_2^2x_4x_5 - x_0^3x_3x_4x_5 + (c/(c+2))x_0x_1x_2^2x_4x_5 - x_0^3x_3x_4x_5 + (c/(c+2))x_0x_1x_2^2x_4x_5 + (c/(c+2))x_1x_2^2x_4x_5 + (c/(c+2))x_1x_2^2x_5 + (c/(c+2))x_1x_2^$ $(2c/(c+2))x_0x_1^2x_3x_4x_5 + (2c/(c+2))x_0x_2^2x_3x_4x_5 + (2/(c+2))x_0^2x_3^2x_4x_5 + (c/(c+2))x_0x_1x_3^2x_4x_5 - x_0x_3^3x_4x_5 + (2/(c+2))x_0x_1x_3^2x_4x_5 + (2/(c+2))x_1x_3^2x_4x_5 + (2/(c+2))x_1x_3^2x_5 + (2/(c+2))x_3^2x_5 + (2/(c+2))x_5^2x_5 + (2/(c$ $((2c+2)/(c+2))x_0^2x_2x_3x_5^2 + x_0x_2^2x_3x_5^2 - x_0^2x_3^2x_5^2 + (c/(c+2))x_0x_1x_3^2x_5^2 + ((2c+2)/(c+2))x_0x_2x_3^2x_5^2 + ((c+2)/(c+2))x_0x_2x_3x_5^2 + (c/(c+2)/(c+2))x_0x_2x_3x_5^2 + ((c+2)/(c+2))x_0x_2x_3x_5^2 + ((c+2)/(c+2))x_0x_3x_5^2 + ((c+2)/(c+2))x_0x_5^2 + ((c+2)/(c+2))x_5^2 + ((c$ $\frac{2)/c)x_0x_3^3x_5^2 + ((c+1)/(c+2))x_0^2x_1x_4x_5^2 + ((c+1)/(c+2))x_0x_1^2x_4x_5^2 + (c/(c+2))x_0x_1x_2x_4x_5^2 + ((c+2)/(c+2))x_0x_2x_3x_4x_5^2 + ((c+2)/(c+2))x_0x_3x_4x_5^2 + ((c+2)/(c+2))x_0x_5^2 + ((c+2)/(c+2$

 $((2c^2+1)/c^2)x_0^3x_1^3 + ((c+2)/c)x_0^3x_1^2x_2 + ((2c+1)/c)x_0^2x_1^3x_2 + ((2c+1)/c)x_0^3x_1x_2^2 + ((c+2)/c)x_0x_1^3x_2^2 + ((c+2)/c)x_0x_1^3 + ((c+2)/c)x_0x_1^3 + ((c+2)/c)x_0x_1^3 + ((c+2)/c)x_0x_1^3 + ((c+2)/c)x_0x_1^3 + ((c+2)/c)x_1^3 + (($ $2)/c)x_0^2x_1x_2^3 + ((2c+1)/c)x_0x_1^2x_2^3 + ((c^2+2)/c^2)x_1^3x_2^3 + ((c+2)/c)x_0^3x_1^2x_3 + ((c+2)/c)x_0^2x_1^3x_3 + x_0^3x_1x_2x_3 + ((c+2)/c)x_0^2x_1^2x_3 + ((c+2)/c)x_1^2x_3 + ((c+2)/c)x_1^2x_1^2x_2^2 + ((c+2)/c)x_1^2x_2^2 + ((c+2)/c)x_1^2x_1^2x_2^2 + ((c+2)/c)x_1^2x_2^2 + ((c+2)/c)x_1^2x_1^2x_2^2 + ((c+2)/c)x_1^2x_1^2x_2^2 + ((c+2)/c)x_1^2x_1^2x_1^2 + ((c+2)/c)x_1^2x_1^2 + ((c+2)/c)x_1^2x_1^2 + ((c+2)/c)x_1^2x_1^2 + ((c+2)/c)x_1^2x_1^2 + ((c+2)/c)x_1^2 + ((c+$ $1)/c)x_0^3x_1x_3^2 + x_0^2x_1^2x_3^2 + ((2c+1)/c)x_0x_1^3x_3^2 + (2c/(c+2))x_0^2x_1x_2x_3^2 + ((c+2)/c)x_1^3x_2x_3^2 + (c/(c+2))x_0x_1x_2^2x_3^2 - (c/(c+2))x_0x_1x_2^2x_3^2 + (c/(c+2)/c)x_1^2x_2x_3^2 + (c/(c+2)/c)x_1^2x_2^2x_3^2 + (c/(c+2)/c)x_1^2x_2^2x_3^2 + (c/(c+2)/c)x_1^2x_2^2x_3^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2x_2^2 + (c/(c+2)/c)x_1^2 + (c/(c+2)/c)x_1^2 + (c/(c+2)/c)x_1^2 + (c/$ $x_0x_1^3x_3x_4 + (2c/(c+2))x_0^2x_1x_2x_3x_4 - x_1^3x_2x_3x_4 + (c/(c+2))x_0x_1x_2^2x_3x_4 + (2/(c+2))x_1^2x_2^2x_3x_4 - x_1x_2^3x_3x_4 + ((c+2))x_1x_2^2x_3x_4 + (c/(c+2))x_1x_2^2x_3x_4 + (c/(c+2))x$ $1)/c)x_0^3x_1x_4^2 + x_0^2x_1^2x_4^2 + ((2c+1)/c)x_0x_1^3x_4^2 + (2c/(c+2))x_0^2x_1x_2x_4^2 + ((c+2)/c)x_1^3x_2x_4^2 + (c/(c+2))x_0x_1x_2^2x_4^2 - (c+2)x_0^2x_1x_2x_4^2 + (c+2)/c)x_1^2x_2x_4^2 + (c+2)/c(c+2)x_1^2x_2x_4^2 + (c+2)/c(c+2)x_1^2x_2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2x_2^2 + (c+2)/c(c+2)x_1^2 + (c+2)/c(c+2)x_1$ $((2c+2)/(c+2))x_1x_2^2x_3x_4^2 - x_0x_1x_3^2x_4^2 + x_1x_2x_3^2x_4^2 + ((c+2)/c)x_0^3x_1^2x_5 + ((c+2)/c)x_0^2x_1^3x_5 + x_0^3x_1x_2x_5 + (2c/(c+2)/c)x_0^2x_1^2x_5 + ((c+2)/c)x_0^2x_1^2x_5 + ((c+2)/c)x_1^2x_5 + ((c+2)/c$ $2))x_0^2x_1^2x_3x_5 + x_0x_1^3x_3x_5 + (2c/(c+2))x_0^2x_1x_2x_3x_5 - x_1^3x_2x_3x_5 + (c/(c+2))x_0x_1x_2^2x_3x_5 + (2/(c+2))x_1^2x_2^2x_3x_5 - x_1^3x_2x_3x_5 + (c/(c+2))x_0x_1x_2^2x_3x_5 + (c/(c+2))x_1x_2x_3x_5 - x_1^3x_2x_3x_5 + (c/(c+2))x_0x_1x_2x_3x_5 + (c/(c+2))x_1x_2x_3x_5 + (c/(c+2))x_1x_2x_$ $2))x_{1}x_{2}^{2}x_{3}^{2}x_{5} + x_{0}^{3}x_{1}x_{4}x_{5} + (1/(c+2))x_{0}^{2}x_{1}^{2}x_{4}x_{5} + x_{0}x_{1}^{3}x_{4}x_{5} + (2c/(c+2))x_{0}^{2}x_{1}x_{2}x_{4}x_{5} - x_{1}^{3}x_{2}x_{4}x_{5} + (c/(c+2))x_{0}^{2}x_{1}x_{2}x_{4}x_{5} + (c/(c+2))x_{0}^{2}x_{1}x_{2}x_{2}x_{2}x_{3}x_{3} + (c/(c+2))x_{0}^{2}x_{1}x_{2}x_{2}x_{3}x_{3} + (c/(c+2))x_{0}^{2}x_{1}x_{2}$ $2))x_0x_1x_2^2x_4x_5 + (2/(c+2))x_1^2x_2^2x_4x_5 - x_1x_2^3x_4x_5 + (c/(c+2))x_0x_1x_2^2x_4x_5 + (2c/(c+2))x_1x_2x_3^2x_4x_5 + ((c+2))x_1x_2x_3^2x_4x_5 + (c/(c+2))x_1x_2x_3^2x_4x_5 + (c/(c+2))x_1x_3x_3^2x_4x_5 + (c/(c+2))x_1x_3x_3^2x_4x_5 + (c/(c+2))x_1x_3x_3^2x_4x_5 + (c/(c+2))x_1x_3x_3x_4x_5 + (c/(c+2))x_1x_3x_4x_5 + (c/(c+2))x_1x_3x_5 + (c/(c+2))x_1x_5 + (c/(c+2))x_1x_5 + (c/(c+2))x_5 + (c/(c+2))x_5 + (c/(c+2))x_5 + (c/(c+2))x_5 + (c$ $\frac{1}{(c/(c+2))x_0x_1x_3x_4^2x_5 + (2c/(c+2))x_1x_2x_3x_4^2x_5 + ((2c+1)/c)x_0^3x_1x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1^3x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1^3x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1^2x_5^2 + x_0^2x_1^2x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1^2x_5^2 + x_0^2x_1^2x_5^2 + x_0^2x_1^2x_1^2x_1^2 + x_0^2x_1^2x_1^2x_1^2 + x_0^2x_1^2x_1^2x_1^2 + x_0^2x_1^2x_1^2x_1^2 + x_0^2x_1^2x_1^2x_1^2 + x_0^2x_1^2x_1^2 + x_0^2x_1^2x_1^2 + x_0^2x_1^2x_1^2 + x_0^2x_1^2 +$ $2))x_0^2x_1x_3x_5^2 + ((c+1)/(c+2))x_0x_1^2x_3x_5^2 + ((2c+2)/(c+2))x_1^2x_2x_3x_5^2 + ((2c+2)/(c+2))x_1x_2^2x_3x_5^2 - x_0x_1x_3^2x_5^2 + ((2c+2)/(c+2))x_1x_2x_3x_5^2 + ((2c+2)/(c+2)/(c+2)x_3x_5^2 + ((2c+2)/(c+2)/(c+2)x_5^2 + ((2c+2)/(c+2)/(c+2)x_5^2 + ((2c+2)/(c+2)/(c+2)x_5^2 + ((2c+2)/(c+2)/(c+2)x_5^2 + ((2c+2)/(c+2)/(c+2)x_5^2 + ((2c$ $x_1x_2x_3^2x_5^2 + ((c+1)/(c+2))x_0^2x_1x_4x_5^2 + ((c+1)/(c+2))x_0x_1^2x_4x_5^2 + ((2c+2)/(c+2))x_1^2x_2x_4x_5^2 + ((2c+2)/(c+2))x_1x_2^2x_4x_5^2 + (c/(c+2))x_0x_1x_3x_4x_5^2 + (2c/(c+2))x_1x_2x_3x_4x_5^2 - x_0x_1x_4^2x_5^2 + x_1x_2x_4^2x_5^2$

 $((2c^2+1)/c^2)x_0^3x_1^3 + ((2c+1)/c)x_0^3x_1^2x_2 + ((c+2)/c)x_0^2x_1^3x_2 + ((c+2)/c)x_0^3x_1x_2^2 + ((2c+1)/c)x_0x_1^3x_2^2 + ((2c+1)/c)x_0x_1^3x_1^2 + ((2c+1)/c)x_1^3x_1^2 + ((2c+1)/c)x_1^3 + ((2c+1)/c)x_1$

 $((c^2+2)/c^2)x_0^3x_2^3 + ((2c+1)/c)x_0^2x_1x_2^3 + ((c+2)/c)x_0x_1^2x_2^3 + ((c+2)/c)x_0^3x_1^2x_3 + ((c+2)/c)x_0^2x_1^2x_2x_3 + (c/c+2)/c)x_0^2x_1^2x_2x_3 + (c/c+1)/c)x_0^2x_2^2x_3 + (c/c+1)/c)x_0^2x_1^2x_2x_3 + (c/c+1)/c)x_0^2x_1^2x_2x_3 + (c/c+1)/c)x_0^2x_1^2x_2x_3 + (c/c+1)/c)x_0^2x_1^2x_2x_3 + (c/c+1)/c)x_0^2x_1^2x_2x_3^2 + (c/c+1)/c)x_0^2x_1^2x_2x_3^2 + (c/c+1)/c)x_0^2x_1^2x_2x_3^2 + (c/c+1)/c)x_0^2x_1^2x_2x_3^2 + (c/c+2)/c)x_0^2x_1^2x_2x_4 + (c/c+2)/c)x_0^2x_1^2x_2x_3x_4 + (c/c+2)/c)x_0^2x_1^2x_2x_3x_5 + (c/c+2)/c)x_0^2x_1^2x_2x_5 + (c/c+2)/c)x_0^2x_1^2x_2x_5 + (c/c+2)/c)x_0^2x_1^2x_2x_5 + (c/c$

12 Conjecture for $p = 3, 3 \mid n$

Variables are x_0, \ldots, x_{n-1} .

Generators are:

 x_0^9 in degree 9, and $\sum x_i^3$ in degree 3. There are n-2 remaining generators in degree 3, each with the following form:

$$\frac{c+1}{c}(x_1^3 - x_0^3) + (x_1 - x_0)(x_0x_1) + (x_1 - x_0)\left(\sum_{i \ge 2} x_i^2 - x_i(x_1 + x_0)\right) + \frac{2c}{c+2}(x_1 - x_0)\left(\sum_{i,j \ge 2; i < j} x_i x_j\right).$$

13 Conjecture for general $p \mid n$

Variables are x_0, \ldots, x_{n-1} .

Generators are:

 $x_0^{p^2}$ in degree p^2 , and $\sum x_i^p$ in degree p. There are n-2 remaining generators in degree p. It is clear that each such generator contains a term of the form $\frac{c+1}{c}(x_k^p-x_0^p)$. If we assume we are in the generator with k=1, then the generator also contains a term $\left(\sum_{i\geq 2}x_0^{p-1}x_i+x_1x_i^{p-1}-x_0x_i^{p-1}-x_1^{p-1}x_i\right)$

14 Conjectures, $n = 1 \mod 3$, p = 3

The degree 6 generators include ones of the form:

$$\frac{2c^2+1}{c^2}\left(x_0^3x_1^3-x_1^3x_2^3\right)+\frac{2c^2+1}{c^2}\left(x_0^3x_1^3-x_1^3x_2^3\right)+\frac{2c+2}{c}\left(\sum_{i>2}x_i(x_0^3x_1^2+x_0^2x_1^2+x_0^2x_1^2x_2+x_0^2x_1x_2^3-x_0^2x_1^3x_2-x_0^3x_1x_2^2-x_0x_1^2x_2^3\right)+\frac{2c+2}{c}\left(\sum_{i>2}x_i(x_0^3x_1^2+x_0^2x_1^3-x_2^2x_1^3-x_2^3x_1^2)+\sum_{i>2}x_i^2(x_1^3x_2+x_1x_2^3-x_0^3x_1-x_0x_1^3)\right)+\frac{2c}{c+2}\left(\sum_{i>2}x_ix_j(x_0^3x_1+x_0x_1^3-x_1^3x_2-x_1x_2^3)+\sum_{i>2}x_i^2(x_0^2x_1^2-x_1^2x_2^2)+\sum_{2< i< j}x_i^2x_j^2(x_1x_2-x_0x_1)+\frac{2c}{c+2}\left(\sum_{i>2}x_i(x_0^2x_1^2x_2-x_0x_1^2x_2^2)+\sum_{i>2}x_i^2(x_0^2x_1x_2-x_0x_1x_2^2)+\sum_{2< i< j}x_ix_j(x_0^2x_1x_2-x_0x_1x_2^2)\right)+\frac{2c}{c+2}\left(\sum_{2< i< j}x_ix_j(x_0^2x_1^2-x_1^2x_2^2)+\sum_{i>2}x_i^2(x_0^2x_1x_2-x_0x_1x_2^2)+\sum_{2< i< j}x_ix_j(x_0^2x_1x_2-x_0x_1x_2^2)\right)+\frac{1}{c+2}\left(\sum_{2< i< j}x_ix_j(x_0^2x_1^2-x_1^2x_2^2)+\sum_{2< i< j}x_ix_j(x_0^2x_1^2-x_1^2x_2^2)+\sum_{2< i< j}x_i^2x_j^2(x_1^2x_2-x_1x_2^2)\right)+\frac{1}{c+2}\left(\sum_{2< i< j}x_ix_j(x_0^2x_1^2-x_1^2x_2^2)+\sum_{2< i< j}x_i^2x_j^2(x_1^2x_2-x_1x_2^2)\right)$$

where x_0, x_1, x_2 can be swapped with any other three of the variables. TODO: figure out which set of these type of generators are linearly independent, simplify in some other fashion.

Another degree 6 generator is $\sum_{i < j} x_i^3 x_j^3$.

There is another type of degree 6 generator that I need to get a general form for.

Then there is a generator in degree 9 that I do not know the form of.