Polynomial representations of rational Cherednik algebras of type A in positive characteristic $p \mid n$

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Abstract

We study the polynomial representation of the ratioal Cherednik algebra of type A_{n-1} with generic parameter in characteristic p for $p \mid n$. We give explicit formulas for generators for the maximal ideal, show that they cut out a complete intersection, and thus compute the Hilbert series of the irreducible quotient. Our methods are motivated by generalizing from characteristic 0 to characteristic p.

1 Introduction

In this paper we study lowest-weight representations of rational Cherednik algebras associated to the symmetric group Σ_n in characteristic p dividing n. This problem is an interesting test case for moving results from characteristic 0 to positive characteristic. Cherednik algebras, originally double affine Hecke algebras, were introduced by Cherednik in 1993; for an introduction to these algebras we refer to [4]. The representation theory of these algebras has been studied more extensively in characteristic 0 than in positive characteristic, including the case of Cherednik algebras of type A_{n-1} (associated to the symmetric group Σ_n). In general new techniques are required to study positive characteristc as opposed to characteristic 0. In this paper we directly connect the characteristc 0 case to the positive characteristic case; instead of taking complex residues, we consider the coefficients of formal power series in positive characteristic. While we only consider the case of the symmetric group Σ_n in this paper, this technique could also be used for the general case of complex reflection groups.

1.1 Definitions

Given the symmetric group Σ_n, we let $\mathcal{S}$ be the set of reflections in Σ_n. Let $\mathfrak{h}$ be the irreducible $(n-1)$-dimensional representation of Σ_n, realized as the submodule of k^n where the sum of the coordinates is 0. For each $s \in \mathcal{S}$ we assign a vector $\alpha_s \in \mathfrak{h}^*$ spanning the image of $1-s$, and choose $\alpha_s^\vee \in \mathfrak{h}$ so that $(1-s)x = \langle \alpha_s^\vee, x\rangle \alpha_s$ for all $x \in \mathfrak{h}^*$, where $\langle \cdot, \cdot \rangle$ indicates the pairing between $\mathfrak{h}$ and $\mathfrak{h}^*$.

Let $T(\mathfrak{h} \oplus \mathfrak{h}^*)$ be the tensor algebra of $\mathfrak{h} \oplus \mathfrak{h}^*$. We choose $\hbar, c \in k$. Then we define the *rational Cherednik algebra of type A* $\mathcal{H}_{\hbar,c}(\mathfrak{h})$ as the quotient of $k[\Sigma_n] \ltimes T(\mathfrak{h} \oplus \mathfrak{h}^*)$ by the relations

$$[x, x'] = 0, \quad [y, y' = 0], \quad [y, x] = \hbar\langle y, x\rangle - \sum_{s \in \mathcal{S}} c\langle y, \alpha_s\rangle\langle \alpha_s^\vee, x\rangle s$$

for all $x, x' \in \mathfrak{h}^*, y, y' \in \mathfrak{h}$. We can give $\mathcal{H}_{\hbar,c}(\mathfrak{h})$ a $\mathbb{Z}$-grading by setting $\deg x = 1$ for $x \in \mathfrak{h}^*$, $\deg y = -1$ for $y \in \mathfrak{h}$, and $\deg g = 0$ for $g \in k[G]$. We get the PBW-type decomposition $\mathcal{H}_{\hbar,c}(\mathfrak{h}) = \mathrm{Sym}(\mathfrak{h}) \otimes_k k[\Sigma_n] \otimes_k \mathrm{Sym}(\mathfrak{h}^*)$ ([4], section 3.2).

In general, for any $\alpha \neq 0$, $\mathcal{H}_{\hbar,c}(\mathfrak{h}) \simeq \mathcal{H}_{\alpha\hbar,\alpha c}(\mathfrak{h})$, so we restrict to $\hbar = 0$ or $\hbar = 1$.

1.2 Representations of rational Cherednik algebras

We are concerned with the *polynomial representation* $\mathrm{Sym}(\mathfrak{h}^*)$ of $\mathcal{H}_{\hbar,c}$, which is a polynomial ring. We refer to the polynomial representation as A; we can give this a $\mathbb{Z}$-grading in an obvious way.

As described in Section 2.5 of [1], $A = \mathrm{Sym}(\mathfrak{h}^*)$ has a unique maximal graded proper submodule J which can be realized as the kernel of the contravariant form $\beta_c : \mathrm{Sym}(\mathfrak{h}^*) \times \mathrm{Sym}(\mathfrak{h}) \to k$; β_c can be characterized by the property that for all $x \in \mathfrak{h}^*, y \in \mathfrak{h}, f \in \mathrm{Sym}(\mathfrak{h}^*), g \in \mathrm{Sym}(\mathfrak{h})$:

$$\beta_c(xf, g) = \beta_c(f, xg), \quad \beta_c(f, yg) = \beta_c(yf, g), \quad \beta_c(1,1) = 1.$$

The quotient A/J is a finite-dimensional irreducible $\mathbb{Z}$-graded representation of $\mathcal{H}_{\hbar,c}(\mathfrak{h})$. The *Hilbert series* $h_{A/J}(t)$ of A/J is $\sum_{j=0}^{\infty}(\dim_k L_j)t^k$ where L_j is the j-graded factor of A/J.

1.3 Main results

We call c *generic* if we do not specify its value. We will be concerned with the case $\hbar = 1$, c generic, and the characteristic p of k dividing n. We realize $\mathfrak{h}$ as the subspace of k^n with the sum of the coordinates 0, which is an irreducible representation of Σ_n of dimension $n-1$. We state our result for this case below:

Theorem 3.4. *The irreducible representation A/J of $\mathcal{H}_{\hbar,c}(\mathfrak{h})$ is a complete intersection with Hilbert series $\left(\frac{1-t^p}{1-t}\right)^{n-1}$. The maximal ideal J is generated by the coefficients of z^p in the formal power series*

$$F_a = \frac{1}{1-x_a z}\prod_{j=0}^{n-1}\left(\sum_{k=0}^{p-1}\binom{c}{k}(-1+\prod_{j=0}^{n-1}(1-x_j z))^k\right)$$

for $a = 0, \dots, n-2$.

We assume $p > 2$ since the case $p = 2$ is fully characterized in [5].

1.4 Connection to characteristic 0 results

In this paper we consider the case where the characteristic of K divides n. This is related to the case in characteristic 0 where c takes the specific value p/n, as described in [2]. In that case the generators of the ideal J were the residues at infinity of $\frac{1}{z-x_a}\prod_{i=0}^{n-1}(z-x_i)^c$ for each a. To get a similar result in positive characteristic, we must consider formal power series in z with coefficients from A. The formal power series would be $\frac{1}{z^{p+1}}\frac{1}{1-x_a z}\prod_{j=0}^{n-1}(1-x_j z)^c$, with the corresponding generator as the coefficient of $1/z$. We simplify and truncate the formal power series so we can define it in positive characteristic.

1.5 Relation to previous work

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2 The rational Cherednik algebra

2.1 Dunkl operators

To understand the action of $\mathcal{H}_{\hbar,c}(\mathfrak{h})$ on A, we can use the PBW decompositions. The action of $\mathrm{Sym}(\mathfrak{h}^*)$ on A is by left multiplication; $k[\Sigma_n]$ acts by the diagonal action, and $\mathrm{Sym}(\mathfrak{h})$ acts via *Dunkl operators*. For $y \in \mathfrak{h}$, the Dunkl operator D_y acts on A by:

$$D_y(f) = \hbar\partial_y f - \sum_{s\in\mathcal{S}} c\frac{(y,\alpha_s)}{\alpha_s}(1-s).f.$$

We choose bases for $\mathfrak{h}, \mathfrak{h}^*$ in the following way: let V be the vector space spanned by $y_0, \ldots, y_{n-1}$ and let $\mathfrak{h}$ be the subspace spanned by $y_i - y_j$ for $i \neq j$; Σ_n acts by permuting indices. Then if $x_0, \ldots, x_{n-1}$ is the dual basis for V^*, we see that $\mathfrak{h}^*$ is the span of $x_0, \ldots, x_{n-1}$ under the relation $x_0 + \cdots + x_{n-1} = 0$; alternatively we can consider $\mathfrak{h}^*$ as the span of $x_0, \ldots, x_{n-2}$ with x_{n-1} defined as $-x_0 - \cdots - x_{n-2}$. If D_i is the Dunkl operator corresponding to y_i, the Dunkl operators for the elements of $\mathfrak{h}$ are spanned by $D_i - D_j$ with $i \neq j$.

For a transposition $s_{ij} \in \Sigma_n$ with $i < j$, we let the corresponding vector $\alpha_{s_{ij}} \in \mathfrak{h}^*$ be $x_i - x_j$.

3 Proof of the main result

We let $g = \prod_{j=0}^{n-1}(1 - x_j z)$. Let F_a for $a = 0, \ldots, n-1$ be the formal power series in z defined by $F_a = \frac{1}{1-x_a z}\left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right)$ where $\binom{c}{k} = \frac{c(c-1)\ldots(c-k+1)}{k!}$.

Because of the definition of the contravariant form β_c, showing that an element f of A is in the kernel of β_c is equivalent to showing that the Dunkl operators corresponding to the basis elements of $\mathfrak{h}$ annihilate f.

Proposition 3.1. *Let f_a be the coefficient of z^p in the power series F_a. Then f_a for $a = 0, \ldots, n-1$ are annihilated by the Dunkl operators.*

Proof. Since $x_{n-1} = -x_0 - \cdots - x_{n-2}$, we see that $\frac{\partial g}{\partial x_i} = -\frac{zg}{1-x_i z} + \frac{zg}{1-x_{n-1}z}$ for all $0 \leq i < n-1$. Let $F = \left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right)$. Note that F is symmetric and $F_a = \frac{F}{1-x_a z}$ for all a. Then we see that for all $0 \leq i < n-1$:

$$
\begin{aligned}
\frac{\partial F}{\partial x_i} &= \frac{\partial}{\partial x_i}\left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right) \\
&= \left(\sum_{k=1}^{p-1} k\binom{c}{k}(g-1)^{k-1}\right)\frac{\partial g}{\partial x_i} \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(\sum_{k=1}^{p-1} k\binom{c}{k}(g-1)^{k-1}\right) g \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(\sum_{k=0}^{p-2} (k+1)\binom{c}{k+1}(g-1)^{k}\right) g \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(\sum_{k=0}^{p-2} c\binom{c-1}{k}(g-1)^{k}\right) (g-1+1) \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(\left(\sum_{k=0}^{p-2} c\binom{c-1}{k}(g-1)^{k}\right)+\left(\sum_{k=0}^{p-2} c\binom{c-1}{k}(g-1)^{k+1}\right)\right) \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(\left(\sum_{k=0}^{p-2} c\binom{c-1}{k}(g-1)^{k}\right)+\left(\sum_{k=1}^{p-1} c\binom{c-1}{k-1}(g-1)^{k}\right)\right) \\
&= \left(-\frac{z}{1-x_i z}+\frac{z}{1-x_{n-1}z}\right)\left(-c\binom{c-1}{p-1}(g-1)^{p-1}+\sum_{k=0}^{p-1} c\binom{c}{k}(g-1)^{k}\right) \\
&= \left(-\frac{zc}{1-x_i z}+\frac{zc}{1-x_{n-1}z}\right)\left(-\binom{c-1}{p-1}(g-1)^{p-1}+F\right)
\end{aligned}
$$

Let $G_i = \left(-\frac{zc}{1-x_i z}+\frac{zc}{1-x_{n-1}z}\right)\left(-\binom{c-1}{p-1}(g-1)^{p-1}\right)$. Then $\frac{\partial F}{\partial x_i} = G_i + \left(-\frac{zc}{1-x_i z}+\frac{zc}{1-x_{n-1}z}\right)F$.

We also see that:

$$
\begin{aligned}
\frac{\partial F_0}{\partial x_1} &= \frac{\partial}{\partial x_1}\left(\frac{1}{1-x_0 z}\left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right)\right) \\
&= \frac{1}{1-x_0 z}\frac{\partial F}{\partial x_1} \\
&= \frac{1}{1-x_0 z}\left(-\frac{zc}{1-x_1 z}+\frac{zc}{1-x_{n-1}z}\right)F+\frac{G_1}{1-x_0 z} \\
&= \left(-\frac{zc}{1-x_1 z}+\frac{zc}{1-x_{n-1}z}\right)F_0+\frac{G_1}{1-x_0 z}
\end{aligned}
$$

and that:

$$
\begin{aligned}
\frac{\partial F_0}{\partial x_0} &= \frac{\partial}{\partial x_1}\left(\frac{1}{1-x_0 z}\left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right)\right)\\
&= \frac{z}{(1-x_0 z)^2}F + \frac{1}{1-x_0 z}\frac{\partial F}{\partial x_0}\\
&= \frac{z}{1-x_0 z}F_0 + \frac{1}{1-x_0 z}\left(-\frac{zc}{1-x_0 z} + \frac{zc}{1-x_{n-1}z}\right)F + \frac{G_0}{1-x_0 z}\\
&= \frac{z}{1-x_0 z}F_0 + \left(-\frac{zc}{1-x_0 z} + \frac{zc}{1-x_{n-1}z}\right)F_0 + \frac{G_0}{1-x_0 z}\\
&= \left(\frac{z(1-c)}{1-x_0 z} + \frac{zc}{1-x_{n-1}z}\right)F_0 + \frac{G_0}{1-x_0 z}
\end{aligned}
$$

We note that F_0 is invariant under s_{ij} where $0 < i, j$. Therefore for transpositions we need only consider transpositions of the form s_{0i} for $0 < i \leq n-1$.

$$
\begin{aligned}
\frac{1-s_{0i}}{x_0 - x_i}(F_0) &= \frac{1}{x_0 - x_i}\left(\frac{F}{1-x_0 z} - \frac{F}{1-x_i z}\right)\\
&= \frac{1}{x_0 - x_i}\left(\frac{1}{1-x_0 z} - \frac{1}{1-x_i z}\right)F\\
&= \frac{1}{x_0 - x_i}\left(\frac{(1-x_i z)-(1-x_0 z)}{(1-x_0 z)(1-x_i z)}\right)F\\
&= \frac{x_0 z - x_i z}{(1-x_0 z)(1-x_i z)(x_0 - x_i)}F\\
&= \frac{z}{(1-x_i z)(1-x_0 z)}F\\
&= \frac{z}{1-x_i z}F_0
\end{aligned}
$$

We recall that we need only consider the action of $D_1 - D_0$ on F_0. We consider D_0F_0, D_1F_0 separately first. We see that $D_0 = \left(\frac{\partial}{\partial x_0} - c\sum_{j>0}\frac{1-s_{0j}}{x_0 - x_j}\right), D_1 = \left(\frac{\partial}{\partial x_1} - c\frac{1-s_{01}}{x_1 - x_0}\right)$ since F_0 is invariant under s_{ij} where $0 < i, j$.

$$
\begin{aligned}
D_0 F_0 &= \left(\frac{\partial}{\partial x_0} - c\sum_{j>0}\frac{1-s_{0j}}{x_0 - x_j}\right)(F_0)\\
&= \frac{\partial F_0}{\partial x_0} - c\sum_{j>0}\frac{1-s_{0j}}{x_0 - x_j}(F_0)\\
&= \frac{G_0}{1-x_0 z} + \left(\frac{z(1-c)}{1-x_0 z} + \frac{zc}{1-x_{n-1}z}\right)F_0 - \sum_{j>0}\frac{zc}{1-x_j z}F_0
\end{aligned}
$$

$$
\begin{aligned}
D_1F_0 &= \left(\frac{\partial}{\partial x_1} - c\frac{1-s_{01}}{x_1-x_0}\right)(F_0) \\
&= \frac{\partial F_0}{\partial x_1} + c\frac{1-s_{01}}{x_0-x_1}(F_0) \\
&= \frac{G_1}{1-x_0z} + \left(-\frac{zc}{1-x_1z} + \frac{zc}{1-x_{n-1}z}\right)F_0 + \frac{zc}{1-x_1z}F_0 \\
&= \frac{G_1}{1-x_0z} + \frac{zc}{1-x_{n-1}z}F_0
\end{aligned}
$$

It is then easy to see that $(D_1 - D_0)(F_0) = \frac{G_1-G_0}{1-x_0z} + \frac{z(c-1)}{1-x_0z}F_0 + \sum_{j>0}\frac{zc}{1-x_jz}F_0$.

In order to show that the pth coefficient in this formal power series is 0, we must consider $\frac{\partial F_0}{\partial z}$.

We see easily that $\frac{\partial g}{\partial z} = g\sum_j \frac{-x_j}{1-x_jz}$. We now consider $\frac{\partial F}{\partial z}$:

$$
\begin{aligned}
\frac{\partial F}{\partial z} &= \frac{\partial}{\partial z}\left(\sum_{k=0}^{p-1}\binom{c}{k}(g-1)^k\right) \\
&= \left(\sum_{k=1}^{p-1}k\binom{c}{k}(g-1)^{k-1}\right)\frac{\partial g}{\partial z} \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(\sum_{k=1}^{p-1}k\binom{c}{k}(g-1)^{k-1}\right)g \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(\sum_{k=0}^{p-2}(k+1)\binom{c}{k+1}(g-1)^{k}\right)g \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(\sum_{k=0}^{p-2}c\binom{c-1}{k}(g-1)^{k}\right)(g-1+1) \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(\left(\sum_{k=0}^{p-2}c\binom{c-1}{k}(g-1)^{k}\right) + \left(\sum_{k=0}^{p-2}c\binom{c-1}{k}(g-1)^{k+1}\right)\right) \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(\left(\sum_{k=0}^{p-2}c\binom{c-1}{k}(g-1)^{k}\right) + \left(\sum_{k=1}^{p-1}c\binom{c-1}{k-1}(g-1)^{k}\right)\right) \\
&= \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(-c\binom{c-1}{p-1}(g-1)^{p-1} + \sum_{k=0}^{p-1}c\binom{c}{k}(g-1)^{k}\right) \\
&= \left(\sum_j \frac{-cx_j}{1-x_jz}\right)F + \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(-c\binom{c-1}{p-1}(g-1)^{p-1}\right)
\end{aligned}
$$

Let $V = \left(\sum_j \frac{-x_j}{1-x_jz}\right)\left(-c\binom{c-1}{p-1}(g-1)^{p-1}\right)$. Then $\frac{\partial F}{\partial z} = V + \left(\sum_j \frac{-cx_j}{1-x_jz}\right)F$.

From this it follows that:

$$
\begin{aligned}
\frac{\partial F_0}{\partial z} &= \frac{\partial}{\partial z}\left(\frac{F}{1-x_0z}\right) \\
&= \frac{1}{1-x_0z}\frac{\partial F}{\partial z} + \frac{x_0}{1-x_0z}F \\
&= \frac{V}{1-x_0z} + \frac{1}{1-x_0z}\left(\sum_j \frac{-cx_j}{1-x_jz}\right)F + \frac{x_0}{(1-x_0z)^2}F \\
&= \frac{V}{1-x_0z} + \left(\sum_j \frac{-cx_j}{1-x_jz}\right)F_0 + \frac{x_0}{1-x_0z}F_0
\end{aligned}
$$

We now again consider $(D_1 - D_0)(F_0)$. Recall that $n \equiv 0 \bmod p$, so in particular we can add n times any multiple of F_0 since that is 0 in characteristic p.

$$
\begin{aligned}
(D_1 - D_0)(F_0) &= \frac{G_1 - G_0}{1-x_0z} + \frac{z(c-1)}{1-x_0z}F_0 + \sum_{j>0}\frac{zc}{1-x_jz}F_0 \\
&= \frac{G_1 - G_0}{1-x_0z} - \frac{z}{1-x_0z}F_0 + \sum_{j}\frac{zc}{1-x_jz}F_0 \\
&= \frac{G_1 - G_0}{1-x_0z} - \frac{z}{1-x_0z}F_0 + \left(\sum_{j}\frac{zc}{1-x_jz}F_0\right) - nzcF_0 \\
&= \frac{G_1 - G_0}{1-x_0z} - zF_0 + zF_0 - \frac{z}{1-x_0z}F_0 + \left(\sum_{j} -zcF_0 + \frac{zc}{1-x_jz}F_0\right) \\
&= \frac{G_1 - G_0}{1-x_0z} - zF_0 + \frac{z - x_0z^2}{1-x_0z}F_0 - \frac{z}{1-x_0z}F_0 + \left(\sum_{j} \frac{-zc + x_jcz^2}{1-x_jz}F_0 + \frac{zc}{1-x_jz}F_0\right) \\
&= \frac{G_1 - G_0}{1-x_0z} - zF_0 + \frac{-x_0z^2}{1-x_0z}F_0 + \left(\sum_{j} \frac{x_jcz^2}{1-x_jz}F_0\right) \\
&= \frac{G_1 - G_0}{1-x_0z} - zF_0 - z^2\left(\frac{x_0}{1-x_0z}F_0 + \left(\sum_{j} -\frac{x_jc}{1-x_jz}F_0\right)\right) \\
&= \frac{G_1 - G_0}{1-x_0z} - zF_0 - z^2\frac{\partial F_0}{\partial z} + z^2\frac{V}{1-x_0z} \\
&= \frac{G_1 - G_0 + z^2V}{1-x_0z} - zF_0 - z^2\frac{\partial F_0}{\partial z}
\end{aligned}
$$

We consider V, G_1, G_0; since $z^{p+1} \mid z^{2p-2} \mid (g-1)^{p-1}$ and $(g-1)^{p-1}$ divides V, G_1, G_0, we see that for $\ell = 0, 1, \ldots, p$, the coefficient of z^ℓ in $G_1 - G_0 + z^2V$ is 0. Then in particular the coefficient of z^p in $\frac{G_1 - G_0 + z^2V}{1-x_0z}$ is 0, so the coefficient of z^p in $(D_1 - D_0)(F_0)$ is equal to the coefficient of z^p in $-zF_0 - z^2\frac{\partial F_0}{\partial z}$.

Let b be the coefficient of z^{p-1} in F_0. We see that the coefficient of z^p in $-zF_0$ is $-b$. Then the coefficient of z^{p-2} in $\frac{\partial F_0}{\partial z}$ is $(p-1)b = -b$, so the coefficient of z^p in $-z^2\frac{\partial F_0}{\partial z}$ is b. Therefore the coefficient of z^p in

$-zF_0 - z^2\frac{\partial F_0}{\partial z}$ is $-b+b=0$, so the coefficient of z^p in $(D_1 - D_0)(F_0)$ is 0. Then if f_0 is the coefficient of z^p in F_0, it is clear that $(D_1 - D_0)(f_0) = 0$ as desired. By the symmetry of the F_a, this is the only Dunkl operator we need consider; it is clear that all the f_a are killed by the Dunkl operators. □

Proposition 3.2. *The f_a for $a = 0, \ldots, n-2$ are linearly independent homogeneous polynomials of degree p.*

Proof. □

Proposition 3.3. *Let $I \subseteq A$ be the ideal generated by f_a for $a = 0, \ldots, n-2$. A/I is a complete intersection.*

Proof. □

Using these propositions, we are able to prove the main theorem.

Theorem 3.4. *The irreducible representation A/J of $\mathcal{H}_{\hbar,c}(\mathfrak{h})$ is a complete intersection with Hilbert series $\left(\frac{1-t^p}{1-t}\right)^{n-1}$. The maximal ideal J is generated by the coefficients of z^p in the formal power series*

$$F_a = \frac{1}{1-x_a z}\left(\sum_{k=0}^{p-1}\binom{c}{k}(-1+\prod_{j=0}^{n-1}(1-x_j z))^k\right)$$

for $a = 0, \ldots, n-2$.

Proof. It suffices to show that the f_a generate the ideal J and that A/J has Hilbert series $\left(\frac{1-t^p}{1-t}\right)^{n-1}$.

By Propositions 3.2, 3.3, A/I is a complete intersection with $n-1$ generators of degree p. It then must have Hilbert series $h_{A/I}(t) = \left(\frac{1-t^p}{1-t}\right)^{n-1}$. By Proposition 3.1, the generators of I are annihilated by the Dunkl operators, so $I \subseteq J$.

By Proposition 3.4 in [1], we see that the Hilbert series of A/J is $\left(\frac{1-t^p}{1-t}\right)^{n-1} h(t^p)$ for some polynomial h with nonnegative integer coefficients; since $I \subseteq J$, we see that $h_{A/I}(t) \geq h_{A/J}(t)$ coefficientwise; however by this restriction of the form of $h_{A/J}(t)$, we see that the only possible choice for h is $h(t) = 1$. Therefore $h_{A/I}(t) = h_{A/J}(t)$, so $I = J$ and these $n-1$ generators generate the whole ideal J.

□

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