

# Computations

$$\bar{h} = 1$$

## 1 $S_2$ , char 2, $\tau$ trivial

### 1.1 Hilbert polynomial

$$t^4 + 2t^3 + 2t^2 + 2t + 1 = (t + 1)^2(t^2 + 1).$$

### 1.2 Generators

$$x_0^2 + x_1^2$$

$$x_0^4.$$

## 2 $S_3$ , char 3, $\tau$ trivial

### 2.1 Hilbert polynomial

$$t^{12} + 3t^{11} + 6t^{10} + 8t^9 + 9t^8 + 9t^7 + 9t^6 + 9t^5 + 9t^4 + 8t^3 + 6t^2 + 3t + 1 = (t^2 + t + 1)^3(t^6 + t^3 + 1).$$

### 2.2 Generators

$$x_0^3 + x_1^3 + x_2^3$$

$$\begin{aligned} & ((2c + 2)/c)x_0^3 \\ & + ((c + 1)/c)x_1^3 \\ & - x_0^2x_1 - x_1^2x_2 - x_0x_2^2 \\ & + x_0x_1^2 + x_0^2x_2 + x_1x_2^2 \end{aligned}$$

$$x_0^9.$$

## 3 $S_4$ , char 2, $\tau$ trivial

### 3.1 Hilbert polynomial

$$t^6 + 4t^5 + 7t^4 + 8t^3 + 7t^2 + 4t + 1 = (t + 1)^4(t^2 + 1).$$

### 3.2 Generators

$$x_0^2 + x_1^2 + x_2^2 + x_3^2$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3$$

$$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3$$

$$x_0^4.$$

## 4 $S_5$ , char 5, $\tau$ trivial

### 4.1 Hilbert polynomial

PARTIAL

$$1 + 5t + 15t^2 + 35t^3 + 70t^4 + 122t^5 + 190t^6 + 270t^7 + \dots$$

### 4.2 Generators

PARTIAL

$$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5$$

$$\begin{aligned} & ((4c+4)/c)x_0^5 \\ & + ((c+1)/c)x_3^5 \\ & + x_0^4x_1 + x_0^4x_2 + x_1^4x_3 + x_2^4x_3 + x_0^4x_4 + x_3x_4^4 \\ & + 2x_0^4x_3 \\ & + 3x_0x_3^4 \\ & - x_0x_1^4 - x_0x_2^4 - x_1x_3^4 - x_2x_3^4 - x_3x_4^4 - x_0x_4^4 \\ & + ((c+4)/(c+2))x_0^2x_1^3 + ((c+4)/(c+2))x_0^2x_2^3 + ((c+4)/(c+2))x_1^2x_3^3 + ((c+4)/(c+2))x_2^2x_3^3 + ((c+4)/(c+2))x_3^3x_4^2 + ((c+4)/(c+2))x_0^2x_4^3 \\ & + ((2c+3)/(c+2))x_0^2x_3^3 \\ & + ((3c+2)/(c+2))x_0^3x_2^2 \\ & + ((4c+1)/(c+2))x_0^3x_1^2 + ((4c+1)/(c+2))x_0^3x_2^2 + ((4c+1)/(c+2))x_1^3x_3^2 + ((4c+1)/(c+2))x_2^3x_3^2 + ((4c+1)/(c+2))x_3^3x_4^2 + ((4c+1)/(c+2))x_0^2x_4^3 \\ & + (c/(c+2))x_0x_1^3x_2 + (c/(c+2))x_0x_1x_2^3 + (c/(c+2))x_0x_1^3x_4 + (c/(c+2))x_0x_2^3x_4 + (c/(c+2))x_0x_1x_4^3 + (c/(c+2))x_0x_2x_4^3 \\ & + (2c/(c+2))x_0^3x_1x_3 + (2c/(c+2))x_0^3x_2x_3 + (2c/(c+2))x_1x_2x_3^3 + (2c/(c+2))x_0^3x_3x_4 + (2c/(c+2))x_1x_3^3x_4 + (2c/(c+2))x_2x_3^3x_4 \\ & + (3c/(c+2))x_0^3x_1x_2 + (3c/(c+2))x_0x_1x_3^3 + (3c/(c+2))x_0x_2x_3^3 + (3c/(c+2))x_0^3x_1x_4 + (3c/(c+2))x_0^3x_2x_4 + (3c/(c+2))x_0x_3^3x_4 \\ & + (4c/(c+2))x_1^3x_2x_3 + (4c/(c+2))x_1x_2^3x_3 + (4c/(c+2))x_1^3x_3x_4 + (4c/(c+2))x_2^3x_3x_4 + (4c/(c+2))x_1x_3^3x_4 + (4c/(c+2))x_2x_3^3x_4 \\ & + ((c^2+4c)/(c^2+1))x_1^2x_2^2x_3 + ((c^2+4c)/(c^2+1))x_0x_1^2x_3^2 + ((c^2+4c)/(c^2+1))x_0x_2^2x_3^2 + ((c^2+4c)/(c^2+1))x_1^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_2^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_0x_3^2x_4^2 \\ & + ((2c^2+3c)/(c^2+1))x_1^2x_2x_3^2 + ((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2 + ((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_2^2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_1x_2^2x_4^2 + ((2c^2+3c)/(c^2+1))x_2x_2^2x_4^2 \\ & + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_2^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_2^2x_4 + ((3c^2+2c)/(c^2+1))x_1^2x_2^2x_4 + ((3c^2+2c)/(c^2+1))x_2^2x_2^2x_4 \end{aligned}$$

$$\begin{aligned}
& 1))x_0^2x_2^2x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_4^2 \\
& + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_2^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1^2x_3 + ((4c^2 + c)/(c^2 + 1))x_0^2x_2^2x_3 + ((4c^2 + c)/(c^2 + 1))x_0x_1^2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_3x_4^2 \\
& + (c^2/(c^2 + 1))x_0^2x_1x_2x_4 \\
& + (2c^2/(c^2 + 1))x_0x_1x_2x_3^2 + (2c^2/(c^2 + 1))x_1^2x_2x_3x_4 + (2c^2/(c^2 + 1))x_1x_2^2x_3x_4 + (2c^2/(c^2 + 1))x_0x_1x_3^2x_4 + (2c^2/(c^2 + 1))x_0x_2x_3^2x_4 + (2c^2/(c^2 + 1))x_1x_2x_3x_4^2 \\
& + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (3c^2/(c^2 + 1))x_0x_1^2x_2x_4 + (3c^2/(c^2 + 1))x_0x_1x_2^2x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_3x_4 + (3c^2/(c^2 + 1))x_0^2x_2x_3x_4 + (3c^2/(c^2 + 1))x_0x_1x_2x_4^2 \\
& + (4c^2/(c^2 + 1))x_1x_2x_3^2x_4
\end{aligned}$$

$$\begin{aligned}
& ((4c+4)/c)x_0^5 + x_0^4x_1 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 - x_0x_1^4 + 2x_0^4x_2 + (2c/(c+2))x_0^3x_1x_2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1^2x_2 + x_1^4x_2 + ((3c+2)/(c+2))x_0^3x_2^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_2^2 + ((4c+1)/(c+2))x_1^3x_2^2 + ((2c+3)/(c+2))x_0^2x_3^2 + (3c/(c+2))x_0x_1x_3^2 + ((c+4)/(c+2))x_1^2x_3^2 + 3x_0x_2^4 - x_1x_2^4 + ((c+1)/c)x_0^5 + x_0^4x_3 + (3c/(c+2))x_0^3x_1x_3 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1^2x_3 + (c/(c+2))x_0x_1^3x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (4c/(c+2))x_1^3x_2x_3 + (2c^2/(c^2 + 1))x_0x_1x_2^2x_3 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_3 + (3c/(c+2))x_0x_3^2x_3 + (2c/(c+2))x_1x_3^2x_3 - x_1^4x_3 + ((4c+1)/(c+2))x_0^3x_3^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0x_1^2x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_1^2x_2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_2^2x_3^2 + ((2c^2 + 3c)/(c^2 + 1))x_1x_2^2x_3^2 + ((c+4)/(c+2))x_2^3x_3^2 + ((c+4)/(c+2))x_0^2x_3^3 + (c/(c+2))x_0x_1x_3^3 + (4c/(c+2))x_1x_2x_3^3 + ((4c+1)/(c+2))x_2^2x_3^3 - x_0x_3^4 + x_2x_3^4 + x_1^4x_4 + (3c/(c+2))x_0^3x_1x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1^2x_4 + (c/(c+2))x_0x_1^3x_4 + (2c/(c+2))x_0^3x_2x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_4 + (4c/(c+2))x_1^3x_2x_4 + (2c^2/(c^2 + 1))x_0x_1x_2^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_4 + (3c/(c+2))x_0x_3^2x_4 + (2c/(c+2))x_1x_3^2x_4 - x_1^4x_4 + (3c/(c+2))x_0^3x_3x_4 + (c^2/(c^2 + 1))x_0^2x_1x_3x_4 + (3c^2/(c^2 + 1))x_0x_1^2x_3x_4 + (3c^2/(c^2 + 1))x_0^2x_2x_3x_4 + (2c^2/(c^2 + 1))x_1^2x_2x_3x_4 + (2c^2/(c^2 + 1))x_0x_2^2x_3x_4 + (4c^2/(c^2 + 1))x_1x_2^2x_3x_4 + (2c/(c+2))x_2^3x_3x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3^2x_4 + (3c^2/(c^2 + 1))x_0x_1x_2^2x_4 + (2c^2/(c^2 + 1))x_1x_2x_3^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_2^2x_3^2x_4 + (c/(c+2))x_0x_3^3x_4 + (4c/(c+2))x_2x_3^3x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_1x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_1^2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1^2x_2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_2^2x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1x_2^2x_4^2 + ((c+4)/(c+2))x_2^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3x_4^2 + (3c^2/(c^2 + 1))x_0x_1x_3x_4^2 + (2c^2/(c^2 + 1))x_1x_2x_3x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_2^2x_3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_3^2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_2x_3^2x_4^2 + ((c+4)/(c+2))x_0^2x_4^3 + (c/(c+2))x_0x_1x_4^3 + (4c/(c+2))x_1x_2x_4^3 + ((4c+1)/(c+2))x_2^2x_4^3 + (c/(c+2))x_0x_3x_4^3 + (4c/(c+2))x_2x_3x_4^3 - x_0x_4^4 + x_2x_4^4
\end{aligned}$$

$$\begin{aligned}
& ((4c+4)/c)x_0^5 + 2x_0^4x_1 + ((3c+2)/(c+2))x_0^3x_1^2 + ((2c+3)/(c+2))x_0^2x_1^3 + 3x_0x_1^4 + ((c+1)/c)x_0^5 + x_0^4x_2 + (2c/(c+2))x_0^3x_1x_2 + (3c/(c+2))x_0x_1^2x_2 - x_1^4x_2 + ((4c+1)/(c+2))x_0^3x_2^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_2^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_2^2 + ((c+4)/(c+2))x_1^3x_2^2 + ((c+4)/(c+2))x_0^2x_3^2 + ((4c+1)/(c+2))x_1^2x_3^2 - x_0x_2^4 + x_1x_2^4 + x_0^4x_3 + (2c/(c+2))x_0^3x_1x_3 + (3c/(c+2))x_0x_1^2x_3 - x_1^4x_3 + (3c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (2c^2/(c^2 + 1))x_0x_1^2x_2x_3 + (2c/(c+2))x_1^3x_2x_3 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2^2x_3 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_3 + (c/(c+2))x_0x_2^2x_3 + (4c/(c+2))x_1x_3^2x_3 + ((4c+1)/(c+2))x_0^3x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_3^2 + ((c+4)/(c+2))x_1^3x_3^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_3^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_2^2x_3^2 + ((c+4)/(c+2))x_0^2x_3^3 + ((4c+1)/(c+2))x_1^2x_3^3 + (c/(c+2))x_0x_2x_3^3 + (4c/(c+2))x_1x_2x_3^3 - x_0x_3^4 + x_1x_3^4 + x_0^4x_4 + (2c/(c+2))x_0^3x_1x_4 + (3c/(c+2))x_0x_1^2x_4 - x_1^4x_4 + (3c/(c+2))x_0^3x_2x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_4 + (2c^2/(c^2 + 1))x_0x_1^2x_2x_4 + (2c/(c+2))x_1^3x_2x_4 + (2c/(c+2))x_1^3x_2x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_4 + (c/(c+2))x_0x_3^2x_4 + (4c/(c+2))x_1x_3^2x_4 + (3c/(c+2))x_0^3x_3x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_3x_4 + (2c^2/(c^2 + 1))x_0x_1^2x_3x_4 + (2c/(c+2))x_1^3x_3x_4 + (c^2/(c^2 + 1))x_0^2x_2x_3x_4 + (4c^2/(c^2 + 1))x_1^2x_2x_3x_4 + (3c^2/(c^2 + 1))x_0x_2^2x_3x_4 + (2c^2/(c^2 + 1))x_1x_2^2x_3x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_3^2x_4 + (3c^2/(c^2 + 1))x_0x_2x_3^2x_4 + (2c^2/(c^2 + 1))x_1x_2x_3^2x_4 + (c/(c+2))x_0x_3^3x_4 + (4c/(c+2))x_2x_3^3x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_4^2 + ((c+4)/(c+2))x_1^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_2^2x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_3x_4^2 + (3c^2/(c^2 + 1))x_0x_2x_3x_4^2 + (2c^2/(c^2 + 1))x_1x_2x_3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_3^2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_3^2x_4^2 + ((c+4)/(c+2))x_0^2x_4^3 + ((4c+1)/(c+2))x_1^2x_4^3 + (c/(c+2))x_0x_2x_4^3 + (4c/(c+2))x_1x_2x_4^3 + (c/(c+2))x_0x_3x_4^3 + (4c/(c+2))x_1x_3x_4^3 - x_0x_4^4 + x_1x_4^4
\end{aligned}$$

conjecture:  $x_0^{25}$

## 5 $S_6$ , char 3, $\tau$ trivial

### 5.1 Hilbert polynomial

PARTIAL

$$1 + 6t + 21t^2 + 51t^3 + 96t^4 + 147t^5 + 192t^6 + 222t^7 + \dots$$

### 5.2 Generators

PARTIAL

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 \\ & + ((c+1)/c)x_4^3 \\ & + x_0^2x_1 + x_0^2x_2 + x_0^2x_3 + x_1^2x_4 + x_2^2x_4 + x_3^2x_4 + x_0x_4^2 + x_0^2x_5 + x_4x_5^2 \\ & - x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_0^2x_4 - x_1x_4^2 - x_2x_4^2 - x_3x_4^2 - x_4^2x_5 - x_0x_5^2 \\ & + (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 \\ & + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + \\ & (2c/(c+2))x_3x_4x_5 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 \\ & + ((c+1)/c)x_3^3 \\ & + x_0^2x_1 + x_0^2x_2 + x_1^2x_3 + x_2^2x_3 + x_0x_3^2 + x_0^2x_4 + x_3x_4^2 + x_0^2x_5 + x_3x_5^2 \\ & - x_0x_1^2 - x_0x_2^2 - x_0^2x_3 - x_1x_3^2 - x_2x_3^2 - x_3^2x_4 - x_0x_4^2 - x_3^2x_5 - x_0x_5^2 \\ & + (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_4x_5 \\ & + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 + \\ & (2c/(c+2))x_3x_4x_5 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 \\ & + ((c+1)/c)x_2^3 \\ & + x_0^2x_1 + x_1^2x_2 + x_0x_2^2 + x_0^2x_3 + x_2x_3^2 + x_0^2x_4 + x_2x_4^2 + x_0^2x_5 + x_2x_5^2 \\ & - x_0x_1^2 - x_0^2x_2 - x_1x_2^2 - x_2^2x_3 - x_0x_3^2 - x_2^2x_4 - x_0x_4^2 - x_2^2x_5 - x_0x_5^2 \\ & + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_3x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 \\ & + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_2x_3x_4 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_2x_3x_5 + \\ & (2c/(c+2))x_2x_4x_5 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 \\ & + ((c+1)/c)x_1^3 \\ & - x_0^2x_1 - x_1^2x_2 - x_0x_2^2 - x_2^2x_3 - x_0x_3^2 - x_1^2x_4 - x_0x_4^2 - x_1^2x_5 - x_0x_5^2 \\ & + x_0x_1^2 + x_0^2x_2 + x_1x_2^2 + x_0^2x_3 + x_1x_3^2 + x_0^2x_4 + x_1x_4^2 + x_0^2x_5 + x_1x_5^2 \\ & + (c/(c+2))x_0x_2x_3 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 \\ & + (2c/(c+2))x_1x_2x_3 + (2c/(c+2))x_1x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_1x_2x_5 + (2c/(c+2))x_1x_3x_5 + \\ & (2c/(c+2))x_1x_4x_5 \end{aligned}$$

conjecture:  $x_0^9$ .

### 5.3 $n - 1$ case

## 6 $S_6$ , char 2, $\tau$ trivial

### 6.1 Hilbert polynomial

$$t^8 + 6t^7 + 16t^6 + 26t^5 + 30t^4 + 26t^3 + 16t^2 + 6t + 1 = (t + 1)^6(t^2 + 1)$$

### 6.2 Generators

$$\begin{aligned} & x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \\ & ((c + 1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_0x_3 + x_1x_4 + x_2x_4 + x_3x_4 + ((c + 1)/c)x_4^2 + x_0x_5 + x_4x_5 \\ & ((c + 1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_1x_3 + x_2x_3 + ((c + 1)/c)x_3^2 + x_0x_4 + x_3x_4 + x_0x_5 + x_3x_5 \\ & ((c + 1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c + 1)/c)x_2^2 + x_0x_3 + x_2x_3 + x_0x_4 + x_2x_4 + x_0x_5 + x_2x_5 \\ & ((c + 1)/c)x_0^2 + ((c + 1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3 + x_0x_4 + x_1x_4 + x_0x_5 + x_1x_5 \\ & x_0^4. \end{aligned}$$

## 7 $S_9$ , char 3, $\tau$ trivial

### 7.1 Hilbert polynomial

PARTIAL

$$\dots + 157t^3 + 45t^2 + 9t + 1$$

### 7.2 Generators

PARTIAL

$$\begin{aligned} & x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + x_8^3 \\ & ((2c + 2)/c)x_0^3 \\ & + ((c + 1)/c)x_7^3 \\ & + x_0^2x_1 + x_0^2x_2 + x_0^2x_3 + x_0^2x_4 + x_0^2x_5 + x_0^2x_6 + x_1^2x_7 + x_2^2x_7 + x_3^2x_7 + x_4^2x_7 + x_5^2x_7 + x_6^2x_7 + x_0x_7^2 + x_0^2x_8 + x_7x_8^2 \\ & - x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_0x_4^2 - x_0x_5^2 - x_0x_6^2 - x_1x_7^2 - x_2x_7^2 - x_3x_7^2 - x_4x_7^2 - x_5x_7^2 - x_6x_7^2 - x_7x_8^2 - x_0x_8^2 \\ & + (c/(c + 2))x_0x_1x_2 + (c/(c + 2))x_0x_1x_3 + (c/(c + 2))x_0x_2x_3 + (c/(c + 2))x_0x_1x_4 + (c/(c + 2))x_0x_2x_4 + (c/(c + 2))x_0x_3x_4 + (c/(c + 2))x_0x_1x_5 + (c/(c + 2))x_0x_2x_5 + (c/(c + 2))x_0x_3x_5 + (c/(c + 2))x_0x_4x_5 + (c/(c + 2))x_0x_5^2 + (c/(c + 2))x_1x_7x_8 + (c/(c + 2))x_2x_7x_8 + (c/(c + 2))x_3x_7x_8 + (c/(c + 2))x_4x_7x_8 + (c/(c + 2))x_5x_7x_8 + (c/(c + 2))x_6x_7x_8 + (c/(c + 2))x_7x_8^2 + (c/(c + 2))x_0x_8^2 + (c/(c + 2))x_1x_8^2 + (c/(c + 2))x_2x_8^2 + (c/(c + 2))x_3x_8^2 + (c/(c + 2))x_4x_8^2 + (c/(c + 2))x_5x_8^2 + (c/(c + 2))x_6x_8^2 + (c/(c + 2))x_7x_8^3 + (c/(c + 2))x_8^3. \end{aligned}$$

$$\begin{aligned}
& 2))x_0x_3x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 + (c/(c+2))x_0x_1x_6 + \\
& (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (c/(c+2))x_0x_5x_6 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + \\
& (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (c/(c+2))x_0x_6x_8 \\
& + (2c/(c+2))x_1x_2x_7 + (2c/(c+2))x_1x_3x_7 + (2c/(c+2))x_2x_3x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + \\
& (2c/(c+2))x_3x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 + \\
& (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 + \\
& (2c/(c+2))x_1x_7x_8 + (2c/(c+2))x_2x_7x_8 + (2c/(c+2))x_3x_7x_8 + (2c/(c+2))x_4x_7x_8 + (2c/(c+2))x_5x_7x_8 + \\
& (2c/(c+2))x_6x_7x_8
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\
& x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + \\
& (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 - x_0x_5^2 - x_0^2x_6 + x_1^2x_6 + (2c/(c+2))x_1x_2x_6 + x_2^2x_6 + \\
& (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 + x_3^2x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 + \\
& x_4^2x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + (2c/(c+2))x_4x_5x_6 + x_5^2x_6 + x_0x_6^2 - x_1x_6^2 - \\
& x_2x_6^2 - x_3x_6^2 - x_4x_6^2 - x_5x_6^2 + ((c+1)/c)x_0^3 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + \\
& (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + \\
& (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 - x_6^2x_7 - x_0x_7^2 + x_6x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + \\
& (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_6x_8 + (2c/(c+2))x_2x_6x_8 + (2c/(c+2))x_3x_6x_8 + \\
& (2c/(c+2))x_4x_6x_8 + (2c/(c+2))x_5x_6x_8 - x_6^2x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_6x_7x_8 - x_0x_8^2 + x_6x_8^2
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\
& x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 - x_0^2x_5 + x_1^2x_5 + (2c/(c+2))x_1x_2x_5 + \\
& x_2^2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 + x_3^2x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + \\
& (2c/(c+2))x_3x_4x_5 + x_4^2x_5 + x_0x_5^2 - x_1x_5^2 - x_2x_5^2 - x_3x_5^2 - x_4x_5^2 + ((c+1)/c)x_0^3 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + \\
& (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + \\
& (2c/(c+2))x_4x_5x_6 - x_5^2x_6 - x_0x_6^2 + x_5x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + \\
& (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 - x_5^2x_7 + \\
& (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_5x_6x_7 - x_0x_7^2 + x_5x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + \\
& (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_5x_8 + (2c/(c+2))x_2x_5x_8 + (2c/(c+2))x_3x_5x_8 + (2c/(c+2))x_4x_5x_8 - x_5^2x_8 + (c/(c+2))x_0x_6x_8 + \\
& (2c/(c+2))x_5x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_5x_7x_8 - x_0x_8^2 + x_5x_8^2
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\
& x_0x_3^2 - x_0^2x_4 + x_1^2x_4 + (2c/(c+2))x_1x_2x_4 + x_2^2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + x_3^2x_4 + x_0x_4^2 - \\
& x_1x_4^2 - x_2x_4^2 - x_3x_4^2 + ((c+1)/c)x_0^3 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_4x_5 + \\
& (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_2^2x_5 - x_0x_5^2 + x_4x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + \\
& (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 - x_2^2x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_4x_5x_6 - x_0x_6^2 + x_4x_6^2 + \\
& x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 - \\
& x_4^2x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_4x_5x_7 + (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_4x_6x_7 - x_0x_7^2 + x_4x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + \\
& (c/(c+2))x_0x_3x_8 + (2c/(c+2))x_1x_4x_8 + (2c/(c+2))x_2x_4x_8 + (2c/(c+2))x_3x_4x_8 - x_4^2x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_4x_5x_8 + \\
& (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_4x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_4x_7x_8 - x_0x_8^2 + x_4x_8^2
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 - x_0^2x_3 + x_1^2x_3 + (2c/(c+2))x_1x_2x_3 + x_2^2x_3 + \\
& x_0x_3^2 - x_1x_3^2 - x_2x_3^2 + ((c+1)/c)x_0^3 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 - \\
& x_3^2x_4 - x_0x_4^2 + x_3x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 - x_3^2x_5 + \\
& (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_0x_5^2 + x_3x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (2c/(c+2))x_1x_3x_6 + \\
& (2c/(c+2))x_2x_3x_6 - x_3^2x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_3x_4x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_3x_5x_6 - x_0x_6^2 + x_3x_6^2 + \\
& x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (2c/(c+2))x_1x_3x_7 + (2c/(c+2))x_2x_3x_7 - x_3^2x_7 + (c/(c+2))x_0x_4x_7 + \\
& (2c/(c+2))x_3x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_3x_5x_7 + (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_3x_6x_7 - x_0x_7^2 + x_3x_7^2 + x_0^2x_8 + \\
& (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (2c/(c+2))x_1x_3x_8 + (2c/(c+2))x_2x_3x_8 - x_3^2x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_3x_4x_8 + \\
& (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_3x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_3x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_3x_7x_8 - x_0x_8^2 + x_3x_8^2
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 - x_0^2x_2 + x_1^2x_2 + x_0x_2^2 - x_1x_2^2 + ((c+1)/c)x_2^3 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (2c/(c+2))x_1x_2x_3 - x_2^2x_3 - x_0x_3^2 + x_2x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (2c/(c+2))x_1x_2x_4 - x_2^2x_4 + (c/(c+2))x_0x_3x_4 + \\
& (2c/(c+2))x_2x_3x_4 - x_0x_4^2 + x_2x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (2c/(c+2))x_1x_2x_5 - x_2^2x_5 + (c/(c+2))x_0x_3x_5 + \\
& (2c/(c+2))x_2x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_2x_4x_5 - x_0x_5^2 + x_2x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (2c/(c+2))x_1x_2x_6 - x_2^2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_2x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_2x_4x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_2x_5x_6 - x_0x_6^2 + x_2x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (2c/(c+2))x_1x_2x_7 - x_2^2x_7 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_2x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_2x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_2x_5x_7 + \\
& (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_2x_6x_7 - x_0x_7^2 + x_2x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (2c/(c+2))x_1x_2x_8 - x_2^2x_8 + (c/(c+2))x_0x_3x_8 + (2c/(c+2))x_2x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_2x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_2x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_2x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_2x_7x_8 - x_0x_8^2 + x_2x_8^2
\end{aligned}$$

$$\begin{aligned}
& ((2c+2)/c)x_0^3 - x_0^2x_1 + x_0x_1^2 + ((c+1)/c)x_1^3 + x_0^2x_2 - x_1^2x_2 - x_0x_2^2 + x_1x_2^2 + x_0^2x_3 - x_1^2x_3 + (c/(c+2))x_0x_2x_3 + \\
& (2c/(c+2))x_1x_2x_3 - x_0x_3^2 + x_1x_3^2 + x_0^2x_4 - x_1^2x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_2x_4 + (c/(c+2))x_0x_3x_4 + \\
& (2c/(c+2))x_1x_3x_4 - x_0x_4^2 + x_1x_4^2 + x_0^2x_5 - x_1^2x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_0x_3x_5 + \\
& (2c/(c+2))x_1x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_1x_4x_5 - x_0x_5^2 + x_1x_5^2 + x_0^2x_6 - x_1^2x_6 + (c/(c+2))x_0x_2x_6 + \\
& (2c/(c+2))x_1x_2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_1x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_4x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_1x_5x_6 - x_0x_6^2 + x_1x_6^2 + x_0^2x_7 - x_1^2x_7 + (c/(c+2))x_0x_2x_7 + (2c/(c+2))x_1x_2x_7 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_1x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_5x_7 + \\
& (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_1x_6x_7 - x_0x_7^2 + x_1x_7^2 + x_0^2x_8 - x_1^2x_8 + (c/(c+2))x_0x_2x_8 + (2c/(c+2))x_1x_2x_8 + (c/(c+2))x_0x_3x_8 + (2c/(c+2))x_1x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_1x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_1x_7x_8 - x_0x_8^2 + x_1x_8^2
\end{aligned}$$

conjecture:  $x_0^9$

## 8 $S_4$ , char 3, $\tau$ trivial

### 8.1 Polynomial

$$(t+1)^2(t^4+t^2+1)^2(t^2+t+1)^2(t^6+t^3+1)$$

### 8.2 Generators

$$x_0^3 + x_1^3 + x_2^3 + x_3^3$$

$$\begin{aligned}
& ((2c+2)/c)x_0^6 - x_0^5x_1 + x_0^4x_1^2 - x_0^2x_1^4 + x_0x_1^5 + ((c+1)/c)x_1^6 + x_0^5x_2 + x_0^3x_1^2x_2 - x_0^2x_1^3x_2 - x_1^5x_2 - x_0^4x_2^2 - x_0^3x_1x_2^2 + \\
& x_0x_1^3x_2^2 + x_1^4x_2^2 + ((c+1)/c)x_0^3x_2^3 + x_0^2x_1x_2^3 - x_0x_1^2x_2^3 + ((2c+2)/c)x_1^3x_2^3 + x_0^2x_2^4 - x_1^2x_2^4 - x_0x_2^5 + x_1x_2^5 + x_0^3x_1^2x_3 - \\
& x_0^2x_1^3x_3 - x_0^3x_2^2x_3 + x_1^3x_2^2x_3 + x_0^2x_2^3x_3 - x_1^2x_2^3x_3 - x_0^3x_1x_2^3 + x_0x_1^3x_2^3 + x_0^3x_2x_2^3 - x_1^3x_2x_2^3 - x_0x_2^3x_2^3 + x_1x_2^3x_2^3
\end{aligned}$$

$$x_0^6 + x_0^3x_1^3 + x_1^6 + x_0^3x_2^3 + x_1^3x_2^3 + x_2^6$$

$$x_0^9 + x_0^6x_1^3 + x_0^3x_1^6 + x_1^9$$

## 9 $S_4$ , char 3, $\tau$ trivial, $n-1$ representation

### 9.1 Polynomial

$$(t+1)^2(t^4+t^2+1)^2(t^2+t+1)(t^6+t^3+1)$$

## 9.2 Generators

$$x_0^3 x_1^3 + x_0^3 x_2^3 + x_1^3 x_2^3$$

$$((2c+2)/c)x_0^3x_1^3 - x_0^3x_1^2x_2 + x_0^2x_1^3x_2 + x_0^3x_1x_2^2 - x_0x_1^3x_2^2 + ((c+1)/c)x_0^3x_2^3 - x_0^2x_1x_2^3 + x_0x_1^2x_2^3$$

$$x_0^9 + (c^2 + 2)x_0^6x_1^3 + cx_0^5x_1^4 + (2c^2 + 2c)x_0^4x_1^5 + x_1^9 + (2c^2 + c)x_0^6x_2^2x_1 + (2c^2 + 2c)x_0^5x_1^3x_2 + c^2x_0^4x_1^4x_2 + c^2x_0^3x_1^5x_2 + (c^2 + 2c)x_0^6x_1x_2^2 + 2c^2x_0^5x_1^2x_2^2 + cx_0^4x_1^3x_2^2 + c^2x_0^3x_1^4x_2^2 + 2c^2x_0^2x_1^5x_2^2 + x_2^9$$

## 10 $S_7$ , char 3, $\tau$ trivial

## 10.1 Polynomial

PARTIAL

$$434t^5 + 203t^4 + 83t^3 + 28t^2 + 7t + 1$$

## 10.2 Generators

PARTIAL

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3$$

**11**  $S_7$ , char 3,  $\tau$  trivial,  $n - 1$  representation

## 11.1 Generators

$$x_0^3x_1^3+x_0^3x_2^3+x_1^3x_2^3+x_0^3x_3^3+x_1^3x_3^3+x_2^3x_3^3+x_0^3x_4^3+x_1^3x_4^3+x_2^3x_4^3+x_3^3x_4^3+x_0^3x_5^3+x_1^3x_5^3+x_2^3x_5^3+x_3^3x_5^3+x_4^3x_5^3$$

[illegible]







$$\begin{aligned}
& 1/c)x_0^3x_1x_3^2 + x_0^2x_1^2x_3^2 + ((2c+1)/c)x_0x_1^3x_3^2 + ((c+2)/c)x_0^3x_2x_3^2 + (2c/(c+2))x_0x_1^2x_2x_3^2 - x_0^2x_2^2x_3^2 + (c/(c+2))x_0x_1x_2^2x_3^2 + ((c+2)/c)x_0x_2^3x_3^2 + ((c+2)/c)x_0^3x_1^2x_4 + ((c+2)/c)x_0^2x_1^3x_4 + (2c/(c+2))x_0^2x_1^2x_2x_4 + x_0x_1^3x_2x_4 + ((2c+1)/c)x_0^3x_2^2x_4 + (c/(c+2))x_0^2x_1x_2^2x_4 + ((2c+1)/c)x_0^2x_2^3x_4 - x_0x_1x_2^3x_4 + x_0^3x_1x_3x_4 + (1/(c+2))x_0^2x_1^2x_3x_4 + x_0x_1^3x_3x_4 - x_0^3x_2x_3x_4 + (2c/(c+2))x_0x_1^2x_2x_3x_4 + (2/(c+2))x_0^2x_2^2x_3x_4 + (c/(c+2))x_0x_1x_2^2x_3x_4 - x_0x_2^3x_3x_4 + ((c+1)/(c+2))x_0^2x_1x_3^2x_4 + ((c+1)/(c+2))x_0x_1^2x_3^2x_4 + ((2c+2)/(c+2))x_0^2x_2x_3^2x_4 + ((2c+2)/(c+2))x_0x_2^2x_3^2x_4 + ((2c+1)/c)x_0^3x_1x_4^2 + x_0^2x_1^2x_4^2 + ((2c+1)/c)x_0x_1^3x_4^2 + ((c+2)/c)x_0^3x_2x_4^2 + (2c/(c+2))x_0x_1^2x_2x_4^2 - x_0^2x_2^2x_4^2 + (c/(c+2))x_0x_1x_2^2x_4^2 + ((c+2)/c)x_0x_2^3x_4^2 + ((c+1)/(c+2))x_0^2x_1x_3x_4^2 + ((c+1)/(c+2))x_0x_1^2x_3x_4^2 + ((2c+2)/(c+2))x_0^2x_2x_3x_4^2 + ((2c+2)/(c+2))x_0x_2^2x_3x_4^2 - x_0x_1x_2^3x_4^2 + x_0x_2x_2^3x_4^2 + ((c+2)/c)x_0^3x_1^2x_5 + ((c+2)/c)x_0^2x_1^3x_5 + (2c/(c+2))x_0^2x_2^2x_5 + x_0x_1^3x_2x_5 + ((2c+1)/c)x_0^3x_2^2x_5 + (c/(c+2))x_0^2x_1x_2^2x_5 + ((2c+1)/c)x_0^2x_2^3x_5 - x_0x_1x_2^3x_5 + x_0^3x_1x_3x_5 + (1/(c+2))x_0^2x_1^2x_3x_5 + x_0x_1^3x_3x_5 - x_0^3x_2x_3x_5 + (2c/(c+2))x_0x_1^2x_2x_3x_5 + (2/(c+2))x_0^2x_2^2x_3x_5 + (c/(c+2))x_0x_1x_2^2x_3x_5 - x_0x_2^3x_3x_5 + ((c+1)/(c+2))x_0^2x_1x_3^2x_5 + ((c+1)/(c+2))x_0x_1^2x_3^2x_5 + ((2c+2)/(c+2))x_0^2x_2x_3^2x_5 + ((2c+2)/(c+2))x_0x_2^2x_3^2x_5 + x_0^3x_1x_4x_5 + (1/(c+2))x_0^2x_1^2x_4x_5 + x_0x_1^3x_4x_5 - x_0^3x_2x_4x_5 + (2c/(c+2))x_0x_1x_2^2x_4x_5 + (2/(c+2))x_0^2x_2^2x_4x_5 + (c/(c+2))x_0x_1x_2^2x_4x_5 - x_0x_2^3x_4x_5 + (c/(c+2))x_0x_1x_2^3x_4x_5 + ((c+1)/(c+2))x_0^2x_1x_4^2x_5 + ((c+1)/(c+2))x_0x_1^2x_4^2x_5 + ((2c+2)/(c+2))x_0^2x_2x_4^2x_5 + ((2c+2)/(c+2))x_0x_2^2x_4^2x_5 + (c/(c+2))x_0x_1x_3x_4^2x_5 + (2c/(c+2))x_0x_2x_3x_4^2x_5 + ((2c+1)/c)x_0^3x_1x_5^2 + x_0^2x_1^2x_5^2 + ((2c+1)/c)x_0x_1x_2^2x_5^2 + ((c+2)/c)x_0x_2^3x_5^2 + ((c+1)/(c+2))x_0^2x_1x_3x_5^2 + ((c+1)/(c+2))x_0x_1^2x_3x_5^2 + ((2c+2)/(c+2))x_0^2x_2x_3x_5^2 + ((2c+2)/(c+2))x_0x_2^2x_3x_5^2 - x_0x_1x_2^3x_5^2 + x_0x_2x_2^3x_5^2 + ((c+1)/(c+2))x_0^2x_1x_4x_5^2 + ((c+1)/(c+2))x_0x_1^2x_4x_5^2 + ((2c+2)/(c+2))x_0^2x_2x_4x_5^2 + ((2c+2)/(c+2))x_0x_2^2x_4x_5^2 + (c/(c+2))x_0x_1x_3x_4x_5^2 + (2c/(c+2))x_0x_2x_3x_4x_5^2 - x_0x_1x_4^2x_5^2 + x_0x_2x_4^2x_5^2
\end{aligned}$$

## 12 Conjecture for $p = 3, 3 \mid n$

Variables are  $x_0, \dots, x_{n-1}$ .

Generators are:

$x_0^9$  in degree 9, and  $\sum x_i^3$  in degree 3. There are  $n-2$  remaining generators in degree 3, each with the following form:

$$\frac{c+1}{c}(x_1^3 - x_0^3) + (x_1 - x_0)(x_0x_1) + (x_1 - x_0) \left( \sum_{i \geq 2} x_i^2 - x_i(x_1 + x_0) \right) + \frac{2c}{c+2}(x_1 - x_0) \left( \sum_{i, j \geq 2; i < j} x_i x_j \right).$$

(The other generators are created from this one by switching  $x_1$  with  $x_k$  for some  $k \geq 2$ .)

We note that since  $3 \mid n$  that  $\sum_{i < j} (x_i - x_j)^2 = \sum_{i < j} x_i x_j + \sum_{i < j} x_i^2 + x_j^2 = -\sum_i x_i^2 + \sum_{i < j} x_i x_j$ .

We also note that  $\sum_i (x_i^2 - x_i x_1 - x_i x_0) = x_0 x_1 + \sum_{i \geq 2} (x_i^2 - x_i x_1 - x_i x_0)$ .

We also note that  $n x_1^2 = n x_0^2 = n x_0 x_1 = 0$ .

$$\begin{aligned}
& \frac{c+1}{c}(x_1^3 - x_0^3) + (x_1 - x_0)(x_0x_1) + (x_1 - x_0) \left( \sum_{i \geq 2} x_i^2 - x_i(x_1 + x_0) \right) + \frac{2c}{c+2}(x_1 - x_0) \left( \sum_{i,j \geq 2; i < j} x_ix_j \right) = \\
& = \frac{x_1 - x_0}{c(c+2)} \left( (c+1)(c+2)(x_0^2 + x_0x_1 + x_1^2) + c(c+2)(x_0x_1) + c(c+2) \left( \sum_{i \geq 2} x_i^2 - x_i(x_1 + x_0) \right) + 2c^2 \left( \sum_{i,j \geq 2; i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( (c^2 - 1)(x_0^2 + x_0x_1 + x_1^2) + (c^2 - c)(x_0x_1) + (c^2 - c) \left( \sum_{i \geq 2} x_i^2 - x_i(x_1 + x_0) \right) - c^2 \left( \sum_{i,j \geq 2; i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( (c^2 - 1)(x_0^2 + x_0x_1 + x_1^2) + (c^2 - c)(x_0x_1) + c^2 \left( \sum_{i \geq 2} x_i^2 \right) - c^2 \left( \sum_{i \geq 2} x_ix_1 + x_ix_0 \right) \right. \\
& \quad \left. - c \left( \sum_{i \geq 2} x_i^2 - x_ix_1 - x_ix_0 \right) - c^2 \left( \sum_{i,j \geq 2; i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( (c^2 - 1)(x_0^2 + x_0x_1 + x_1^2) + (c^2 - c)(x_0x_1) + c^2 \left( \sum_{i \geq 2} x_i^2 \right) + c^2 x_0x_1 - c \left( \sum_{i \geq 2} x_i^2 - x_ix_1 - x_ix_0 \right) \right. \\
& \quad \left. - c^2 \left( \sum_{i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( c^2 x_0^2 + c^2 x_1^2 - x_0^2 - x_1^2 - x_0x_1 - cx_0x_1 + c^2 \left( \sum_{i \geq 2} x_i^2 \right) - c \left( \sum_{i \geq 2} x_i^2 - x_ix_1 - x_ix_0 \right) - c^2 \left( \sum_{i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( -x_0^2 - x_1^2 - x_0x_1 + c^2 \left( \sum_i x_i^2 \right) - c \left( \sum_i x_i^2 - x_ix_1 - x_ix_0 \right) - c^2 \left( \sum_{i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( -(x_0 - x_1)^2 + c^2 \left( \sum_i x_i^2 \right) - c \left( \sum_i x_i^2 - x_ix_1 - x_ix_0 \right) - c^2 \left( \sum_{i < j} x_ix_j \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( -(x_0 - x_1)^2 - c \left( \sum_i x_i^2 - x_ix_1 - x_ix_0 + x_0x_1 \right) - c^2 \left( \sum_{i < j} (x_i - x_j)^2 \right) \right) \\
& = \frac{x_1 - x_0}{c(c+2)} \left( -(x_0 - x_1)^2 - c \left( \sum_i (x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{i < j} (x_i - x_j)^2 \right) \right) \\
& = \frac{1}{c(c+2)} \left( x_0^3 - x_1^3 - c \left( \sum_i (x_1 - x_0)(x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{i < j} (x_1 - x_0)(x_i - x_j)^2 \right) \right)
\end{aligned}$$

(Checked with Sage)

Let this generator equal  $g$ . We see that for  $\{i, j\} \cap \{0, 1\} = \emptyset$  that  $s_{ij}g = g$ . Therefore  $\frac{g-s_{ij}g}{x_i-x_j} = 0$ .

We see easily that  $s_{0k}g$  for  $k \neq 1$  is  $\frac{x_1-x_k}{c(c+2)} \left( -(x_k - x_1)^2 - c \sum_i (x_i - x_1)(x_i - x_k) - c^2 \sum_{i < j} (x_i - x_j)^2 \right)$ . From there further simple algebra tells us that  $(x_1 - x_0)(x_i - x_0)(x_i - x_1) - (x_1 - x_k)(x_i - x_1)(x_i - x_k) =$

$(x_1 - x_i)(x_0 - x_1 + x_k - x_i)(x_k - x_0)$  for all  $i$ . From this we see easily that

$$\frac{g-s_{0k}g}{x_k-x_0} = \frac{1}{c(c+2)} \left( -(x_k - x_0)^2 - c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) - c^2 \sum_{i < j} (x_i - x_j)^2 \right).$$

Similar algebra shows us that for  $k \neq 0, 1$ :

$$\frac{g-s_{1k}g}{x_k-x_1} = \frac{1}{c(c+2)} \left( (x_k - x_1)^2 + c \sum_i (x_0 - x_i)(x_1 - x_0 + x_k - x_i) + c^2 \sum_{i < j} (x_i - x_j)^2 \right).$$

The final case is  $s_{01}g$ . We see easily that all the terms inside the largest parentheses are left untouched by  $s_{01}$ . Therefore  $s_{01}g = -g$ , so  $\frac{g-s_{01}g}{x_1-x_0} = \frac{g-(-g)}{x_1-x_0} = \frac{2g}{x_1-x_0} = -\frac{g}{x_1-x_0}$ ; this is just

$$\frac{1}{c(c+2)} \left( (x_0 - x_1)^2 + c \left( \sum_i (x_i - x_1)(x_i - x_0) \right) + c^2 \left( \sum_{i < j} (x_i - x_j)^2 \right) \right).$$

We can use these to calculate the values for the Dunkl operators. We need only check  $D_0g, D_1g, D_2g$ , because the rest are essentially equivalent to  $D_2g$ .

## 12.1 $D_0g$

We start with  $D_0g$ . We see that  $D_0g = \partial_0g - c \sum_{i \geq 1} \frac{g-s_{0i}g}{x_0-x_i}$ .

We consider the partial derivative.

$$\begin{aligned} \partial_0g &= \partial_0 \left( \frac{1}{c(c+2)} \left( x_0^3 - x_1^3 - c \left( \sum_i (x_1 - x_0)(x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{i < j} (x_1 - x_0)(x_i - x_j)^2 \right) \right) \right) \\ &= \frac{1}{c(c+2)} \left( -c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i < j} \partial_0((x_1 - x_0)(x_i - x_j)^2) \right) \right) \\ &= \frac{1}{c(c+2)} \left( -c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i \geq 1} \partial_0((x_1 - x_0)(x_i - x_0)^2) \right) - c^2 \left( \sum_{0 < i < j} \partial_0((x_1 - x_0)(x_i - x_j)^2) \right) \right) \\ &= \frac{1}{c(c+2)} \left( -c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i \geq 1} \partial_0((x_1 - x_0)(x_i - x_0)^2) \right) + c^2 \left( \sum_{0 < i < j} (x_i - x_j)^2 \right) \right) \\ &= \frac{1}{c(c+2)} \left( -c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) + c^2 \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 - x_i) \right) + c^2 \left( \sum_{0 < i < j} (x_i - x_j)^2 \right) \right) \\ &= \frac{c}{c(c+2)} \left( - \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) + c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 - x_i) \right) + c \left( \sum_{0 < i < j} (x_i - x_j)^2 \right) \right) \end{aligned}$$

We note that in  $\sum_{0 < i < j} (x_i - x_j)^2$  that in each term at least one of the  $x_i, x_j$  has index  $\geq 2$ . Therefore  $\sum_{0 < i < j} (x_i - x_j)^2 = \sum_{0 < i < j} -2(x_i - x_j)^2 = -\sum_{0 < i, j; i \neq j} (x_i - x_j)^2 = -\left( \sum_{i \geq 2} \sum_{j \geq 1} (x_i - x_j)^2 \right) - \left( \sum_{i \geq 2} (x_i - x_1)^2 \right)$  because of the double-counting. Then since  $n - 1 = -1$ , we see that this is equal to  $-\left( \sum_{i \geq 2} \sum_{j \geq 1} (x_i - x_j)^2 - (x_i - x_1)^2 \right) = \sum_{i \geq 2} \sum_{j \geq 1} (x_i - x_1)^2 - (x_i - x_j)^2 = \sum_{i \geq 2} \sum_{j \geq 1} (x_1 - x_j)(x_1 +$

$$x_j + x_i).$$

Therefore we have (there is a switch of indices):

$$\begin{aligned}
\partial_0 g &= \frac{c}{c(c+2)} \left( - \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) + c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 - x_i) \right) + c \left( \sum_{i \geq 2} \sum_{j \geq 1} (x_1 - x_j)(x_1 + x_j + x_i) \right) \right) \\
&= \frac{c}{c(c+2)} \left( - \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 + x_1 + x_i) \right) + c \left( \sum_{i \geq 2} (x_1 - x_i)(x_0 - x_i) \right) + c \left( \sum_{i \geq 2} \sum_{j \geq 2} (x_1 - x_i)(x_1 + x_j + x_i) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} \left( -((x_1 - x_i)(x_0 + x_1 + x_i)) + c((x_1 - x_i)(x_0 - x_i)) + c \left( \sum_{j \geq 2} (x_1 - x_i)(x_1 + x_j + x_i) \right) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_i - x_1) \left( (x_0 + x_1 + x_i) - c(x_0 - x_i) - c \left( \sum_{j \geq 2} (x_1 + x_j + x_i) \right) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_i - x_1) \left( (x_0 + x_1 + x_i) - cx_0 + cx_i - cx_1 - cx_i - c \left( \sum_{j \geq 2} x_j \right) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_i - x_1) \left( (x_0 + x_1 + x_i) - c \left( \sum_j x_j \right) \right) \right)
\end{aligned}$$

Let  $G_1 = \frac{g - s_{01}g}{x_0 - x_1} = -\frac{1}{c(c+2)} \left( (x_0 - x_1)^2 + c(\sum_i (x_i - x_1)(x_i - x_0)) + c^2 \left( \sum_{i < j} (x_i - x_j)^2 \right) \right)$ . Let  $G_2 = \sum_{i \geq 2} \frac{g - s_{0i}g}{x_0 - x_i}$ .

$$\begin{aligned}
G_2 &= \sum_{i \geq 2} \frac{g - s_{0i}g}{x_0 - x_i} \\
&= \sum_{k \geq 2} \left( \frac{1}{c(c+2)} \left( (x_k - x_0)^2 + c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) + c^2 \sum_{i < j} (x_i - x_j)^2 \right) \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0)^2 + c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) \right) + (n-2)c^2 \sum_{i < j} (x_i - x_j)^2 \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0)^2 + c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) \right) + c^2 \sum_{i < j} (x_i - x_j)^2 \right) \\
&= \frac{1}{c(c+2)} \left( c \sum_{k \geq 2} \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) + c^2 \sum_{i < j} (x_i - x_j)^2 + \left( \sum_{k \geq 2} (x_k - x_0)^2 \right) \right)
\end{aligned}$$

We note that  $n - 2 - 1 = 0$  since  $3 \mid n$ . Therefore:

$$\begin{aligned}
G_2 + G_1 &= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0)^2 + c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) \right) + (n-2)c^2 \sum_{i < j} (x_i - x_j)^2 \right) \\
&\quad - \frac{1}{c(c+2)} \left( (x_0 - x_1)^2 + c \left( \sum_i (x_i - x_1)(x_i - x_0) \right) + c^2 \left( \sum_{i < j} (x_i - x_j)^2 \right) \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0)^2 + c \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) \right) \right. \\
&\quad \left. - (x_0 - x_1)^2 - c \left( \sum_i (x_i - x_1)(x_i - x_0) \right) \right)
\end{aligned}$$

We consider  $\sum_{k \geq 2} \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) - (\sum_i (x_i - x_1)(x_i - x_0))$ . We note that  $n - 2 = 1$ , so this is equal to  $\sum_{k \geq 2} \sum_i (x_1 - x_i)(x_0 - x_1 + x_k - x_i) - \left( \sum_{k \geq 2} \sum_i (x_i - x_1)(x_i - x_0) \right)$ ; then this is equal to  $\sum_{k \geq 2} \sum_i ((x_1 - x_i)(x_0 - x_1 + x_k - x_i) - (x_i - x_1)(x_i - x_0))$ . This simplifies to  $\sum_{k \geq 2} \sum_i (x_1 - x_i)(x_k - x_1)$ .

Therefore we see that (again using  $n - 2 = 1$ ):

$$\begin{aligned}
G_1 + G_2 &= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0)^2 - (x_1 - x_0)^2 + c \sum_i (x_1 - x_i)(x_k - x_1) \right) \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_0 + x_1 - x_0)(x_k - x_1) - c \sum_i x_i(x_k - x_1) \right) \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} (x_k - x_1) \left( x_k + x_1 + x_0 - c \sum_i x_i \right) \right)
\end{aligned}$$

Then we can relabel indices and see that  $c(G_1 + G_2) = \partial_0 g$ , so the Dunkl operator is 0 as desired.

## 12.2 $D_1 g$

We see that  $D_1 g = \partial_1 g - c \sum_{i \neq 1} \frac{g - s_{1i} g}{x_1 - x_i}$ .

We consider the partial derivative.

$$\begin{aligned}
\partial_1 g &= \partial_1 \left( \frac{1}{c(c+2)} \left( x_0^3 - x_1^3 - c \left( \sum_i (x_1 - x_0)(x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{i < j} (x_1 - x_0)(x_i - x_j)^2 \right) \right) \right) \\
&= \frac{1}{c(c+2)} \left( c \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i < j} \partial_1((x_1 - x_0)(x_i - x_j)^2) \right) \right) \\
&= \frac{1}{c(c+2)} \left( c \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i \neq 1} \partial_1((x_1 - x_0)(x_i - x_1)^2) \right) - c^2 \left( \sum_{1 \neq i < j \neq 1} \partial_1((x_1 - x_0)(x_i - x_j)^2) \right) \right) \\
&= \frac{1}{c(c+2)} \left( c \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i \geq 2} (x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{1 \neq i < j \neq 1} \partial_1((x_1 - x_0)(x_i - x_j)^2) \right) \right) \\
&= \frac{1}{c(c+2)} \left( c \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c^2 \left( \sum_{i \geq 2} (x_i - x_1)(x_i - x_0) \right) - c^2 \left( \sum_{1 \neq i < j \neq 1} (x_i - x_j)^2 \right) \right) \\
&= \frac{c}{c(c+2)} \left( \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c \left( \sum_{i \geq 2} (x_i - x_1)(x_i - x_0) \right) - c \left( \sum_{1 \neq i < j \neq 1} (x_i - x_j)^2 \right) \right)
\end{aligned}$$

We note that in  $\sum_{1 \neq i < j \neq 1} (x_i - x_j)^2$  that in each term at least one of the  $x_i, x_j$  has index  $\geq 2$ . Therefore  $\sum_{1 \neq i < j \neq 1} (x_i - x_j)^2 = \sum_{1 \neq i < j \neq 1} -2(x_i - x_j)^2 = -\sum_{1 \neq i, j; i \neq j} (x_i - x_j)^2 = -\left( \sum_{i \geq 2} \sum_{j \neq 1} (x_i - x_j)^2 \right) - \left( \sum_{i \geq 2} (x_i - x_0)^2 \right)$  because of the double-counting. Then since  $n - 1 = -1$ , we see that this is equal to  $-\left( \sum_{i \geq 2} \sum_{j \neq 1} (x_i - x_j)^2 - (x_i - x_0)^2 \right) = \sum_{i \geq 2} \sum_{j \neq 1} (x_i - x_0)^2 - (x_i - x_j)^2 = \sum_{i \geq 2} \sum_{j \neq 1} (x_0 - x_j)(x_0 + x_j + x_i) = \sum_{i \geq 2} \sum_{j \geq 2} (x_0 - x_j)(x_0 + x_j + x_i)$ .

Therefore we have (there is a switch of indices):

$$\begin{aligned}
\partial_1 g &= \frac{c}{c(c+2)} \left( \left( \sum_{i \geq 2} (x_0 - x_i)(x_0 + x_1 + x_i) \right) - c \left( \sum_{i \geq 2} (x_i - x_1)(x_i - x_0) \right) - c \left( \sum_{i \geq 2} \sum_{j \geq 2} (x_0 - x_i)(x_0 + x_j + x_i) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} ((x_0 - x_i)(x_0 + x_1 + x_i)) - c((x_i - x_1)(x_i - x_0)) - c \left( \sum_{j \geq 2} (x_0 - x_i)(x_0 + x_j + x_i) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_0 - x_i) \left( (x_0 + x_1 + x_i) - c(x_1 - x_i) - c \left( \sum_{j \geq 2} (x_0 + x_j + x_i) \right) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_0 - x_i) \left( (x_0 + x_1 + x_i) - cx_1 + cx_i - cx_0 - cx_i - c \left( \sum_{j \geq 2} x_j \right) \right) \right) \\
&= \frac{c}{c(c+2)} \left( \sum_{i \geq 2} (x_0 - x_i) \left( (x_0 + x_1 + x_i) - c \left( \sum_j x_j \right) \right) \right)
\end{aligned}$$



Let  $G_1 = \frac{g-s_{01}g}{x_1-x_0} = \frac{1}{c(c+2)} \left( (x_0-x_1)^2 + c \sum_i (x_i-x_1)(x_i-x_0) + c^2 \left( \sum_{i<j} (x_i-x_j)^2 \right) \right)$ . Let  $G_2 = \sum_{i \geq 2} \frac{g-s_{1i}g}{x_1-x_i}$ .

$$\begin{aligned}
G_2 &= \sum_{k \geq 2} \frac{g-s_{1k}g}{x_1-x_k} \\
&= - \sum_{k \geq 2} \left( \frac{1}{c(c+2)} \left( (x_k-x_1)^2 + c \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) + c^2 \sum_{i<j} (x_i-x_j)^2 \right) \right) \\
&= - \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k-x_1)^2 + c \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) \right) + (n-2)c^2 \sum_{i<j} (x_i-x_j)^2 \right) \\
&= - \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k-x_1)^2 + c \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) \right) + c^2 \sum_{i<j} (x_i-x_j)^2 \right)
\end{aligned}$$

We note that  $n-2-1=0$  since  $3 \mid n$ . Therefore:

$$\begin{aligned}
G_2 + G_1 &= - \frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k-x_1)^2 + c \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) \right) + c^2 \sum_{i<j} (x_i-x_j)^2 \right) \\
&\quad + \frac{1}{c(c+2)} \left( (x_0-x_1)^2 + c \left( \sum_i (x_i-x_1)(x_i-x_0) \right) + c^2 \left( \sum_{i<j} (x_i-x_j)^2 \right) \right) \\
&= \frac{1}{c(c+2)} \left( - \left( \sum_{k \geq 2} \left( (x_k-x_1)^2 + c \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) \right) \right) \right. \\
&\quad \left. + \left( (x_0-x_1)^2 + c \left( \sum_i (x_i-x_1)(x_i-x_0) \right) \right) \right)
\end{aligned}$$

We consider  $-\sum_{k \geq 2} \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) + (\sum_i (x_i-x_1)(x_i-x_0))$ . We note that  $n-2=1$ , so this is equal to  $-\sum_{k \geq 2} \sum_i (x_0-x_i)(x_1-x_0+x_k-x_i) + \left( \sum_{k \geq 2} \sum_i (x_1-x_i)(x_0-x_i) \right)$ ; then this is equal to  $\sum_{k \geq 2} \sum_i ((x_0-x_i)(x_0-x_k))$ . Therefore we see that (again using  $n-2=1$ ):

$$\begin{aligned}
G_1 + G_2 &= -\frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_1)^2 - (x_0 - x_1)^2 - c \sum_i (x_0 - x_i)(x_0 - x_k) \right) \right) \\
&= -\frac{1}{c(c+2)} \left( \sum_{k \geq 2} \left( (x_k - x_1 + x_0 - x_1)(x_k - x_0) - c \sum_i x_i(x_k - x_0) \right) \right) \\
&= -\frac{1}{c(c+2)} \left( \sum_{k \geq 2} (x_k - x_0) \left( x_k + x_1 + x_0 - c \sum_i x_i \right) \right) \\
&= \frac{1}{c(c+2)} \left( \sum_{k \geq 2} (x_0 - x_k) \left( x_k + x_1 + x_0 - c \sum_i x_i \right) \right)
\end{aligned}$$

Then we can relabel indices and see that  $c(G_1 + G_2) = \partial_1 g$ , so the Dunkl operator is 0 as desired.

### 12.3 $D_2 g$

We see that  $D_2 g = \partial_2 g - c \frac{g - s_{02}g}{x_2 - x_0} - c \frac{g - s_{12}g}{x_2 - x_1}$  because the rest cancel. Let  $G = \frac{g - s_{02}g}{x_2 - x_0} + \frac{g - s_{12}g}{x_2 - x_1}$ . From the above we see that:

$$\begin{aligned}
G &= \frac{g - s_{02}g}{x_2 - x_0} + \frac{g - s_{12}g}{x_2 - x_1} \\
&= \frac{1}{c(c+2)} \left( (x_2 - x_1)^2 - (x_2 - x_0)^2 + c \sum_i (x_0 - x_i)(x_1 - x_0 + x_2 - x_i) - (x_1 - x_i)(x_0 - x_1 + x_2 - x_i) \right) \\
&= \frac{1}{c(c+2)} \left( (x_2 - x_1 + x_2 - x_0)(x_0 - x_1) + c \sum_i (x_1 - x_0)(x_0 + x_1 - x_2 - x_i) \right) \\
&= \frac{1}{c(c+2)} \left( (x_2 - x_1 + x_2 - x_0)(x_0 - x_1) + c \sum_i x_i(x_0 - x_1) \right)
\end{aligned}$$

We consider the partial derivative.

$$\begin{aligned}
\partial_2 g &= \frac{x_1 - x_0}{c(c+2)} \left( -c(x_2 - x_1 + x_2 - x_0) - c^2 \sum_{i \neq 2} 2(x_2 - x_i) \right) \\
&= \frac{x_1 - x_0}{c(c+2)} \left( -c(x_2 - x_1 + x_2 - x_0) + c^2 \sum_{i \neq 2} x_2 - x_i \right) \\
&= \frac{x_1 - x_0}{c(c+2)} \left( -c(x_2 - x_1 + x_2 - x_0) + c^2 \sum_{i \neq 2} -x_i + c^2(n-1)x_2 \right) \\
&= \frac{x_1 - x_0}{c(c+2)} \left( -c(x_2 - x_1 + x_2 - x_0) - c^2 \sum_i x_i \right)
\end{aligned}$$

We see that  $\partial_2 g = cG$ , so  $D_2 g = 0$  as desired.

### 13 Conjecture for $p = 5, 5 \mid n$

Variables are  $x_0, \dots, x_{n-1}$ .

Generators are:

$x_0^{25}$  in degree 25, and  $\sum x_i^5$  in degree 5. There are  $n-2$  remaining generators in degree 3, each with the following form:

$$\begin{aligned}
&\frac{c+1}{c}(x_1^5 - x_0^5) + 2x_0^4 x_1 + 3x_0 x_1^4 + \left( \sum_{i \geq 2} x_0^4 x_i + x_1 x_i^4 - x_0 x_i^4 - x_1^4 x_i \right) + \frac{2c+3}{c+2} x_0^2 x_1^3 + \frac{3c+2}{c+2} x_0^3 x_1^2 + \\
&\frac{c+4}{c+2} \left( \sum_{i \geq 2} x_0^2 x_i^3 + x_1^3 x_i^2 - x_0^3 x_i^2 - x_1^2 x_i^3 \right) + \frac{c}{c+2} \left( \sum_{i,j \geq 2; i < j} x_0 x_i^3 x_j + x_0 x_i x_j^3 - x_1 x_i^3 x_j - x_1 x_i x_j^3 \right) + \\
&\frac{2c}{c+2} \left( \sum_{i \geq 2} x_0^3 x_1 x_i - x_0 x_1^3 x_i \right) + \frac{2c}{c+2} \left( \sum_{i,j \geq 2; i < j} x_i x_j (x_1^3 - x_0^3) \right) + \\
&\frac{c^2+4c}{c^2+1} \left( \sum_{i \geq 2} x_0 x_1^2 x_i^2 - x_0^2 x_1 x_i^2 \right) + \frac{c^2+4c}{c^2+1} \left( \sum_{i,j \geq 2; i < j} x_1 x_i^2 x_j^2 - x_0 x_i^2 x_j^2 \right) + \\
&\frac{2c^3+3c}{c^2+1} \left( \sum_{i,j \geq 2; i < j} x_1^2 x_i^2 x_j + x_1^2 x_i x_j^2 - x_0^2 x_i^2 x_j - x_0^2 x_i x_j^2 \right) + \frac{c^2}{c^2+1} \left( \sum_{i,j,k \geq 2; i < j < k} x_0^2 x_i x_j x_k - x_1^2 x_i x_j x_k \right) + \\
&\frac{2c^2}{c^2+1} \left( \sum_{i,j \geq 2; i < j} x_0 x_1^2 x_i x_j - x_0^2 x_1 x_i x_j \right) + \\
&\frac{2c^2}{c^2+1} \left( \sum_{i,j,k \geq 2; i < j < k} x_1 x_i^2 x_j x_k + x_1 x_i x_j^2 x_k + x_1 x_i x_j x_k^2 - x_0 x_i^2 x_j x_k - x_0 x_i x_j^2 x_k - x_0 x_i x_j x_k^2 \right)
\end{aligned}$$

It is definitely possible to factor out  $x_1 - x_0$  from this.

(The other generators are created from this one by switching  $x_1$  with  $x_k$  for some  $k \geq 2$ .)

#### 13.1 $x_0^9$

This is the final generator. We note that all partial derivatives are 0, so we disregard them. For Dunkl operators other than  $D_0$ , the only relevant term is  $\frac{x_0^9 - x_k^9}{x_k - x_0} = (x_k - x_0)^8$ . We will show this is in the ideal.

We note that then  $D_0(x_0^9) = -c \sum_{k \geq 1} \frac{x_0^9 - x_k^9}{x_0 - x_k}$  will also be in the ideal.

We therefore need only consider the case  $D_1$  since all others will be symmetric and follow.

$$\frac{x_0^9 - x_1^9}{x_1 - x_0} = -\sum_{i=0}^8 x_0^i x_1^{8-i}.$$

We note that  $D_0(x_0^9) = -c \sum_{k \geq 1} \frac{x_0^9 - x_k^9}{x_0 - x_k} = -c \sum_{i=0}^8 x_0^i (\sum_j x_j^{8-i})$ .

Let  $m_{i,j} = \sum_{k=0}^8 x_i^k x_j^{8-k}$  and  $g_{i,j}$  be the generators described above. Ex:  $g_{0,1}$  is the generator we were working with above.

We wish to write  $m_{0,1} = \sum_{i=1}^5 p_i g_{0,i}$  where  $p_i$  is a homogeneous degree 5 polynomial for all  $i$ .

We must consider the symmetries. We note that  $g_{i,j} = -g_{j,i}$ ,  $g_{i,j} - g_{k,j} = g_{i,k}$  and  $g_{i,j} - g_{i,k} = g_{k,j}$  for all  $i, j, k$ . We also note that  $g_{i,i} = 0$  for all  $i$ , and  $\sum_{j=0}^5 g_{i,j} = \frac{c+1}{c} \sum_{j=0}^5 x_j^3$  for all  $i$ .

Let  $s_{ij}$  be the transposition switching  $x_i$  and  $x_j$ . By letting these act on  $m_{0,1}$  we see that we must have for all  $i, j \geq 2$  that  $s_{ij}p_k = p_k$  for  $k \neq i, k \neq j$  and  $s_{ij}p_i = p_j$ ,  $s_{ij}p_j = p_i$ .

We also note that  $s_{01}m_{0,1} = m_{0,1}$  and  $s_{01}g_{0,1} = -g_{0,1}$ , while  $s_{0,1}g_{0,k}$  for  $k \geq 2$  is  $g_{1,k} = g_{0,k} - g_{0,1}$ .

Then  $m_{0,1} = -(s_{0,1} * p_1) * g_{0,1} + \sum_{i=2}^5 (s_{0,1}p_i) * (g_{0,i} - g_{0,1})$ .

From this we see that  $-\sum_{i=1}^5 s_{0,1}p_i = p_1$  and  $s_{0,1}p_i = p_i$  for  $i \geq 2$ . This means that  $p_1 + s_{0,1}p_1 + \sum_{i=2}^5 p_i = 0$ .

## 14 Conjecture for general $p \mid n$

Variables are  $x_0, \dots, x_{n-1}$ .

Generators are:

$x_0^{p^2}$  in degree  $p^2$ , and  $\sum x_i^p$  in degree  $p$ . There are  $n - 2$  remaining generators in degree  $p$ . It is clear that each such generator contains a term of the form  $\frac{c+1}{c}(x_k^p - x_0^p)$ . If we assume we are in the generator with  $k = 1$ , then the generator also contains a term  $\left(\sum_{i \geq 2} x_0^{p-1} x_i + x_1 x_i^{p-1} - x_0 x_i^{p-1} - x_1^{p-1} x_i\right)$

## 15 $p = 3, 3 \mid n, n - 1$ case, $n - 2$ case

In the case where we use the  $n - 1$  representation, the Hilbert polynomial appears to be  $\left(\frac{1-t^p}{1-t}\right)^{n-1}$ .

There are  $n - 1$  generators in degree 3. If we choose as basis  $e_i = x_i - x_0$  for  $i = 1, \dots, n - 1$  we will find that these generators are the same as the ones in the  $n$ -dimensional case, which we can express in terms of the  $e_i$ . Therefore we already know that they are killed.

We note that in this expression of the generators, the monomial  $e_i^3$  appears only in the  $i$ th generator with nonzero coefficient; therefore these generators must be linearly independent. Let  $I$  be the ideal generated by these generators.

Therefore to show that  $A/I$  is a complete intersection, we must show that it is finite dimensional. We will do this by showing that  $e_i^7$  is in the ideal for all  $i$ . TODO

Then we see that the Hilbert polynomial of  $A/I$  is  $h_{A/I}(t) = (t^2 + t + 1)^{n-1}$ . By Proposition 3.4 in <http://arxiv.org/abs/1107.0504>, we see that the Hilbert polynomial of  $A/J$  is  $(t^2 + t + 1)^{n-1} h(t^3)$  for some

polynomial  $h$  with nonnegative integer coefficients; since the Dunkl operators kill the generators of  $I$ ,  $I \subseteq J$ , so  $h_{A/I}(t) \geq h_{A/J}(t)$  coefficientwise; however by this restriction of the form of  $h_{A/J}(t)$ , we see that the only possible choice for  $h$  is  $h(t) = 1$ . Therefore  $h_{A/I}(t) = h_{A/J}(t)$ , so  $I = J$  and these  $n - 1$  generators generate the whole ideal.

In the case  $3 \mid n$  where we use the  $n - 2$  representation, the Hilbert polynomial appears to be  $\left(\frac{1-t^p}{1-t}\right)^{n-2}$ .

There are  $n - 3$  generators in degree 3. If we choose as basis  $e_i = x_i - x_0$  for  $i = 1, \dots, n - 2$  with  $e_{n-1} = -\sum_{i=1}^{n-2} e_i$ , we will find that these generators are the same as the ones in the  $n$ -dimensional case and the  $n - 1$  dimensional case, which we can express in terms of the  $e_i$  (the last generator is the negative of the sum of the others and therefore can be disregarded).

TODO:

$n = 3$ :

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{2c^2+2}{c^2+2}\right)x_0x_1^3 + \left(\frac{2c}{c+2}\right)x_1^4$$

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{c}{c+2}\right)x_0^2x_1^2 + \left(\frac{c^2}{c^2+2}\right)x_0x_1^3$$

$n = 4$ :

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{2c^2+2}{c^2+2}\right)x_0x_1^3 + \left(\frac{2c}{c+2}\right)x_1^4 + \left(\frac{2c^5+c^4+2c^3+c^2+2c+2}{c^3+2c^2+c+2}\right)x_0x_1^2x_2 + \left(\frac{c^6+c^5+c^4+c^3+2c^2+1}{c^4+2c^3+c^2+2c}\right)x_1^3x_2 + \left(\frac{2c^5+2c^3+c^2+2c+1}{c^4+2}\right)x_0x_1x_2^2 + \left(\frac{c^4+2c^3+2c^2}{c^2+2}\right)x_1^2x_2^2 + \left(\frac{2c^2+2}{c^2+2}\right)x_0x_2^3 + \left(\frac{2c^7+c^4+2c^3+2c^2+2c+2}{c^5+2c}\right)x_1x_2^3 + \left(\frac{2c}{c+2}\right)x_2^4$$

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{c}{c+2}\right)x_0^2x_1^2 + \left(\frac{c^2}{c^2+2}\right)x_0x_1^3 + \left(\frac{2c^2+2c+1}{c^2+2}\right)x_0^2x_1x_2 + \left(\frac{c^7+2c^5+2c^4+2c^3+2c^2+2}{c^6+2c^5+2c^4+c}\right)x_0x_1^2x_2 + \left(\frac{c^6+2c^4+2c^3+c+2}{c^4+2}\right)x_1^3x_2 + \left(\frac{2c^2+2c+1}{c^2+2}\right)x_0^2x_2^2 + \left(\frac{c^6+2c^5+2c^4+2c^3+2c+1}{c^6+2c^5+2c^2+c}\right)x_0x_1x_2^2 + \left(\frac{2c^7+c^6+2c^5+c^4+c^3+1}{c^5+2c}\right)x_1^2x_2^2 + \left(\frac{2c^6+2c^4+2c^2+c+2}{c^4+c^2}\right)x_0x_2^3 + \left(\frac{c^6+c^5+2c^4+c^2+c+2}{c^4+c^3+c^2+c}\right)x_1x_2^3 + \left(\frac{c^6+c^5+c^3+c^2+2c+1}{c^5+2c^4+c^3+2c^2}\right)x_2^4$$

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{c^7+c^6+c^5+c^4+2c+1}{c^5+2c}\right)x_0^2x_1^2 + \left(\frac{2c^7+c^4+2c^3+c^2+c+1}{c^5+c^4+c^3+c^2}\right)x_0x_1^3 + \left(\frac{2c^6+c^4+c^3+2c+1}{c^4+2c^3+c^2+2c}\right)x_1^4 + \left(\frac{c^6+2c^5+2c^4+c^2+2c+2}{c^5+2c}\right)x_0^2x_1x_2 + \left(\frac{2c^7+c^5+c^3+1}{c^5+c^4+2c^3+c^2+c}\right)x_0x_1^2x_2 + \left(\frac{c^5+2c^3+c^2+c+1}{c^3+c^2}\right)x_1^3x_2 + \left(\frac{c}{c+2}\right)x_0^2x_2^2 + \left(\frac{2c^7+c^6+2c^5+c^4+c^3+1}{c^5+c^4+2c^3+c^2+c}\right)x_0x_1x_2^2 + \left(\frac{2c^7+c^6+2c^5+c^4+c^3+1}{c^5+2c}\right)x_1^2x_2^2 + \left(\frac{c^2}{c^2+2}\right)x_0x_2^3 + \left(\frac{2c^6+2c^5+2c^3+2c^2+c+2}{c^5+2c}\right)x_1x_2^3$$

$n = 5$ :

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{2c^2+2}{c^2+2}\right)x_0x_1^3 + \left(\frac{2c}{c+2}\right)x_1^4 + \left(\frac{2c^5+c^4+2c^3+c^2+2c+2}{c^3+2c^2+c+2}\right)x_0x_1^2x_2 + \left(\frac{c^6+c^5+c^4+c^3+2c^2+1}{c^4+2c^3+c^2+2c}\right)x_1^3x_2 + \left(\frac{2c^5+2c^3+c^2+2c+1}{c^4+2}\right)x_0x_1x_2^2 + \left(\frac{c^4+2c^3+2c^2}{c^2+2}\right)x_1^2x_2^2 + \left(\frac{2c^2+2}{c^2+2}\right)x_0x_2^3 + \left(\frac{2c^7+c^4+2c^3+2c^2+2c+2}{c^5+2c}\right)x_1x_2^3 + \left(\frac{2c}{c+2}\right)x_2^4 + \left(\frac{c^{18}+c^{17}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2}{c^{14}+2c^{11}+2c^{10}+c^9+2c^7+c^6+2c^4+2c^2}\right)x_0^2x_1x_3 + \left(\frac{2c^{16}+c^{13}+c^{11}+c^9+c^8+c^6+c^3+c^2+2c}{c^{12}+c^{11}+c^9+c^8+c^6+c^4+2c^3+2c^2+2c}\right)x_0x_1^2x_3 + \left(\frac{c^{19}+c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+2c^{13}+c^{11}+c^{10}+2c^7+2c^6+2c^5+2c^4+c^2+2}{c^{15}+2c^{13}+c^{12}+2c^{11}+2c^8+c^7+c^6+2c^5+2c^4+c^2}\right)x_1^3x_3 + \left(\frac{c^{19}+c^{18}+c^{15}+2c^{13}+c^{12}+c^{11}+c^{10}+c^9+c^8+c^4+c^3}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c^2+2}\right)x_0^2x_2x_3 + \left(\frac{c^{19}+c^{18}+c^{15}+2c^{13}+c^{12}+c^{11}+c^{10}+c^9+c^8+c^4+c^3}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c^2+2}\right)x_0x_1x_2x_3 + \left(\frac{c^{21}+c^{19}+2c^{18}+c^{16}+c^{14}+c^{18}+2c^{17}+2c^{16}}{c^{18}+2c^{17}+2c^{16}}\right)x_0^2x_1x_3 + \left(\frac{2c^{19}+c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+2c^{13}+c^{12}+c^{11}+2c^9+2c^8+c^6+2c^5+2c^4+2c^2+2}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c^2+2}\right)x_0x_2^2x_3 + \left(\frac{2c^{18}+c^{17}+c^{16}+c^{14}+2c^{12}+c^{10}+c^8+2c^4+2c^3+c^2+c+2}{c^{17}+2c^{16}+2c^{15}+2c^{11}+2c^6+c^5+2c^4+2c^3+c^2+c+2}\right)x_1x_2^2x_3 + \left(\frac{c^{20}+2c^{18}+2c^{17}+c^{16}+c^{15}+c^{12}+2c^{11}+2c^{10}+c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^2+1}\right)x_2^3x_3 + \left(\frac{c^{18}+c^{17}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^3+c^2}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c^2+2}\right)x_0^2x_3^2 + \left(\frac{2c^{20}+c^{18}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{10}+2c^8+c^7+c^6+c^5+c^4+2c^2+2c}{c^{16}+c^{15}+2c^{13}+c^{12}+c^{10}+2c^9+c^7+2c^6+2c^5+2c^2+2c}\right)x_0x_1x_2^3 + \left(\frac{2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{11}+2c^{10}+c^9+2c^8+c^7+2c^6+2c^5+c}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c^2+1}\right)x_0x_2^2x_3 + \left(\frac{c^{22}+2c^{20}+2c^{19}+c^{18}+2c^{17}+c^{14}+c^7+2c^6+c^5+2c^4+c^2+1}{c^{18}+2c^{17}+2c^{16}+2c^{12}+2c^7+c^6+2c^5+2c^4+c^3+c^2+2c}\right)x_1x_2x_2^3 + \left(\frac{2c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+2c^{12}+c^{11}+2c^{10}+2c^8+c^6+2c^5+2c^3}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^2+1}\right)x_2^2x_3^2 + \left(\frac{2c^{19}+c^{17}+2c^{16}+2c^{15}+2c^{13}+2c^{12}+c^{10}+2c^8+2c^7+2c^2+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^2+1}\right)x_0x_3^3 + \left(\frac{c^{20}+c^{19}+2c^{18}+c^{16}+c^{15}+2c^{10}+2c^9+2c^7+c^6+c^5+2c^4+2c^3+c^2+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^2+1}\right)x_1x_3^3 + \left(\frac{c^{20}+2c^{18}+c^{15}+2c^{14}+c^{12}+2c^{11}+2c^8+2c^7+c^6+c^5+2c^4+2}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^2+1}\right)x_2x_3^3 + \left(\frac{c^{18}+2c^{16}+2c^{15}+c^{14}+2c^{13}+2c^{10}+c^8+2c^7+2c^6+c^5+c^3}{c^{15}+2c^{14}+2c^{12}+2c^{10}+2c^9+2c^8+2c^7+2c^6+2c^5+c^4+2c+1}\right)x_3^4$$

$$\left(\frac{1}{c^2+2}\right)x_0^4 + \left(\frac{c}{c+2}\right)x_0^2x_1^2 + \left(\frac{c^2}{c^2+2}\right)x_0x_1^3 + \left(\frac{2c^2+2c+1}{c^2+2}\right)x_0^2x_1x_2 + \left(\frac{c^7+2c^5+2c^4+2c^3+2c^2+2}{c^6+2c^5+2c^4+c}\right)x_0x_1^2x_2 + \left(\frac{c^6+2c^4+2c^3+c+2}{c^4+2}\right)x_1^3x_2 + \left(\frac{2c^2+2c+1}{c^2+2}\right)x_0^2x_2^2 + \left(\frac{c^6+2c^5+2c^4+2c^3+2c+1}{c^6+2c^5+2c^2+c}\right)x_0x_1x_2^2 + \left(\frac{2c^7+c^6+2c^5+c^4+c^3+1}{c^5+2c}\right)x_1^2x_2^2 + \left(\frac{2c^6+2c^4+2c^2+c+2}{c^4+c^2}\right)x_0x_2^3 + \left(\frac{c^6+c^5+2c^4+c^2+c+2}{c^4+c^3+c^2+c}\right)x_1x_2^3 +$$

$$\begin{aligned} & \left( \frac{1}{c^{2+2}} x_0^4 + \left( \frac{c^{21}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{13}+c^{10}+c^9+c^8+2c^7+c^6+2c^5+c^4+2c^2+2c}{c^{17}+2c^{15}+c^{14}+2c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^6+c^5+2c^3+2c^2+c+2} \right) x_0^3 x_1 + \left( \frac{2c^{24}+c^{21}+2c^{20}+c^{19}+2c^{17}+2c^{16}+c^{10}+c^8+c^7+2c^6+c^5+c^4+2c^3+2c^2+2c}{c^{20}+2c^{19}+c^{17}+c^{15}+2c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+2c^5+c^4+2c^3+2c^2+c+2} \right) x_0^2 x_1^2 + \left( \frac{2c^{27}+2c^{26}+c^{25}+c^{24}+2c^{19}+2c^{15}+c^{13}+c^{10}+c^9+c^8+c^7+2c^6+2c^4+c^3+2c^2+c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_0 x_1^3 + \left( \frac{c^{22}+c^{20}+c^{19}+c^{17}+c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+2c^5+c^4+2c^3+2c^2+c+2}{c^{20}+2c^{19}+2c^{18}+2c^{14}+2c^9+c^8+2c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_0^3 x_2 + \left( \frac{2c^{27}+2c^{26}+2c^{25}+c^{24}+2c^{23}+2c^{22}+c^{20}+c^{19}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{10}+c^9+c^6+c^4+c^3+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_0 x_1^2 x_2 + \left( \frac{2c^{25}+2c^{24}+c^{23}+2c^{21}+c^{20}+c^{19}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{10}+c^9+c^6+c^4+c^3+1}{c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+c^{13}+c^{12}+c^{10}+c^9+2c^8+2c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_0^2 x_2^2 + \left( \frac{c^{17}+c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^7+2c^6+c^5+c^4+2c^3+c^2}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_1^2 x_2^2 + \left( \frac{c^{26}+c^{25}+c^{24}+2c^{22}+2c^{21}+2c^{19}+2c^{18}+2c^{16}+2c^{13}+2c^{12}+c^{11}+c^8+2c^7+c^6+c^5+2c^4+c^3+2c^2+c+1}{c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+2c^{17}+c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+c^{10}+2c^9+2c^8+2c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_1 x_2^3 + \left( \frac{2c^{17}+2c^{16}+2c^{13}+c^{11}+2c^{10}+2c^9+2c^8+2c^7+2c^6+2c^2+2c}{c^{20}+c^{19}+c^{17}+c^{16}+c^{15}+2c^{14}+c^{11}+c^9+2c^7+c^6+c^4+2c+1} \right) x_0^2 x_1 x_3 + \left( \frac{2c^{27}+2c^{26}+2c^{23}+c^{22}+2c^{21}+c^{20}+2c^{19}+2c^{18}+2c^{15}+2c^{14}+c^{12}+c^{11}+2c^8+2c^7+2c^6+2c^5+2c^4+1}{c^{25}+2c^{23}+c^{22}+2c^{21}+c^{19}+2c^{16}+c^{15}+2c^{13}+c^{12}+c^{11}+2c^{10}+2c^9+2c^8+2c^7+2c^6+2c^5+2c^4+2c^3+2c^2+c+1} \right) x_0 x_1^2 x_3 + \left( \frac{2c^{24}+c^{23}+c^{22}+2c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+c^{14}+c^{13}+c^{11}+2c^8+c^6+2c^5+2c^2+c+1}{c^{20}+2c^{19}+2c^{18}+2c^{14}+2c^9+c^8+2c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_0^2 x_2 x_3 + \left( \frac{2c^{27}+c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^9+c^8+2c^7+2c^6+c^4+2c^3+2c^2+2c+2}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_1^2 x_2 x_3 + \left( \frac{2c^{27}+2c^{24}+2c^{19}+c^{18}+c^{15}+2c^{12}+c^{11}+c^5+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_0 x_2^2 x_3 + \left( \frac{2c^{26}+c^{25}+c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c^2+c+2}{c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+2c^{16}+c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+2c^9+2c^8+2c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_0^2 x_1 x_2^3 + \left( \frac{c^{19}+2c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+2c^9+2c^8+2c^7+c^6+c^5+c^4}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+c^3+1} \right) x_0 x_1 x_2^3 + \left( \frac{c^{25}+2c^{24}+2c^{23}+2c^{20}+c^{19}+2c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+2c^9+c^8+c^7+2c^6+c^5+c^4+2c^3+2c^2+c+1}{c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{13}+c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+2c^3+2c^2+c+2} \right) x_0 x_2 x_2^3 + \left( \frac{2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{19}+2c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+c^8+2c^7+c^6+c^5+2c^4+2c^3+c^2+2c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_2^2 x_3^2 + \left( \frac{2c^{19}+c^{17}+c^{16}+2c^{15}+c^{14}+c^{11}+2c^9+c^8+c^7+2c^6+2c^4}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+1} \right) x_0 x_3^3 + \left( \frac{2c^{23}+2c^{22}+2c^{21}+c^{20}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+2c^9+c^8+c^7+2c^6+c^5+c^4+2c^3+2c^2+c+2}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^4+c^3+1} \right) x_2 x_3^3 \end{aligned}$$

$$\begin{aligned} & \left( \frac{1}{c^2+2} \right) x_0^4 + \left( \frac{2c^2+2}{c^2+2} \right) x_0 x_1^3 + \left( \frac{2c}{c+2} \right) x_1^4 + \left( \frac{2c^5+c^4+2c^3+c^2+2c+2}{c^3+2c^2+c+2} \right) x_0 x_1^2 x_2 + \left( \frac{c^6+c^5+c^4+c^3+2c^2+1}{c^4+2c^3+c^2+2c} \right) x_1^3 x_2 + \left( \frac{2c^5+2c^3+c^2+2c+1}{c^4+2c} \right) x_0 x_1 x_2^2 + \\ & \left( \frac{c^4+2c^3+c^2}{c^2+2} \right) x_1^2 x_2^2 + \left( \frac{2c^2+2}{c^2+2} \right) x_0 x_2^3 + \left( \frac{2c^7+c^4+2c^3+2c^2+2c+2}{c^5+2c} \right) x_1 x_2^2 + \left( \frac{2c}{c+2} \right) x_2^4 + \left( \frac{c^{18}+c^{17}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4}{c^{14}+2c^{11}+2c^{10}+c^9+2c^7+c^6+2c^4+2} \right) x_0^2 x_1 x_3 + \\ & \left( \frac{2c^{16}+c^{13}+c^{11}+c^9+c^8+c^6+c^3+c+2}{c^{12}+c^{11}+c^9+c^8+c^4+2c^3+2c^2+2c} \right) x_0 x_1^2 x_3 + \left( \frac{c^{19}+c^{18}+c^{15}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^4+c^3}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2} \right) x_0^2 x_2 x_3 + \left( \frac{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2} \right) x_0 x_1 x_2 x_3 + \left( \frac{c^{21}+c^{19}+2c^{18}+c^{16}+c^{14}}{c^{18}+2c^{17}+2c^{16}} \right) x_1^2 x_2^2 + \\ & \left( \frac{2c^{19}+c^{18}+2c^{17}+c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+c^{11}+2c^9+2c^4+c^3+2c^2+c}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2} \right) x_0 x_2^2 x_3 + \left( \frac{2c^{18}+c^{17}+c^{16}+c^{14}+2c^{12}+c^{10}+c^8+2c^4+2c^3+c^2+2c+1}{c^{17}+2c^{16}+2c^{15}+2c^{11}+2c^6+c^5+2c^4+2c^3+c^2+c+2} \right) x_1 x_2^2 x_3 + \\ & \left( \frac{c^{20}+2c^{18}+2c^{17}+c^{16}+c^{15}+c^{12}+2c^{11}+2c^{10}+c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0^2 x_3^2 + \left( \frac{c^{18}+c^{17}+c^{16}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2}{c^{15}+c^{14}+2c^{12}+c^{11}+c^9+2c^8+c^6+2c^5+2c^4+2c+2} \right) x_0^2 x_3^2 + \\ & \left( \frac{2c^{20}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+c^{11}+c^{10}+c^9+2c^8+2c^6+c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0 x_1 x_2^3 + \left( \frac{2c^{20}+2c^{18}+2c^{17}+c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+c^{11}+2c^{10}+2c^8+c^7+c^6+2c^5+2c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0 x_1 x_2^3 + \\ & \left( \frac{2c^{20}+2c^{19}+c^{18}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+c^7+2c^6+2c^5+c}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0 x_2^2 x_3^2 + \left( \frac{c^{22}+2c^{20}+2c^{19}+c^{18}+2c^{17}+c^{14}+c^7+2c^6+c^5+2c^4+c^2+1}{c^{18}+2c^{17}+2c^{16}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_1 x_2^2 x_3^2 + \\ & \left( \frac{2c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+c^{12}+c^{11}+c^{10}+2c^8+c^6+c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0^2 x_2^3 + \left( \frac{2c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+c^{12}+c^{11}+c^{10}+2c^8+c^6+c^4+2c^3+c^2+2c+1}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^4+2c^3+c^2+2c+1} \right) x_0^2 x_2^3 + \\ & \left( \frac{c^{18}+2c^{16}+2c^{15}+c^{14}+2c^{13}+c^{10}+c^8+2c^6+2c^5+c^4}{c^{15}+2c^{14}+2c^{12}+2c^{10}+2c^9+2c^8+2c^7+2c^6+2c^5+c^4+2c+1} \right) x_1^4 + \left( \frac{2c^{28}+c^{27}+c^{26}+2c^{25}+c^{22}+c^{21}+2c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4}{c^{23}+c^{22}+c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+2c^{12}+2c^{11}+2c^9+2c^8+2c^7+c^6+2c^5} \right) x_0^4 + \\ & \left( \frac{c^{32}+2c^{30}+c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+c^{24}+2c^{23}+c^{21}+2c^{20}+2c^{19}+c^{17}+2c^{14}+c^{13}+2c^{12}+2c^{10}+2c^8+2c^6+c^5+c^4+2c+2}{c^{27}+2c^{25}+2c^{23}+c^{22}+2c^{21}+c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+c^7+c^6+2c^5+2c^4} \right) x_0^2 x_1^4 + \left( \frac{2c^{26}+c^{25}+c^{24}+2c^{22}+2c^{21}+c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+c^{12}+c^{11}+2c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4}{c^{23}+c^{22}+c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+2c^{12}+2c^{11}+2c^9+2c^8+2c^7+c^6+2c^5} \right) x_0^2 x_2^4 + \left( \frac{2c^{28}+c^{27}+c^{26}+2c^{25}+c^{22}+c^{21}+2c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4}{c^{23}+c^{22}+c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+2c^{12}+2c^{11}+2c^9+2c^8+2c^7+c^6+2c^5} \right) x_0^2 x_2^4 + \\ & \left( \frac{2c^{20}+2c^{19}+2c^{18}+2c^{17}+c^{16}+c^{14}+2c^{13}+2c^{12}+c^{11}+c^{10}+c^9+c^8+2c^7+c^6+2c^3+c^2+2}{c^{17}+2c^{16}+c^{15}+2c^{13}+c^{11}+c^{10}+c^9+2c^8+2c^6+c^5+c^4+c^3} \right) x_1^2 x_2 x_4 + \left( \frac{c^{33}+c^{31}+c^{30}+2c^{29}+2c^{26}+c^{25}+c^{24}+c^{23}+2c^{22}+2c^{20}}{c^{28}+c^{27}+2c^{26}+2c^{25}+2c^{24}+c^{21}+2c^{19}+2c^{18}} \right) x_1^2 x_2^2 x_4 + \left( \frac{c^{33}+2c^{31}+2c^{29}+2c^{26}+2c^{25}+c^{24}+2c^{22}+2c^{21}+c^{17}+2c^{13}+2c^{11}+c^9+c^7+2c^6+2c^5+2c^3+1}{c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+c^{22}+2c^{20}+c^{19}+c^{18}+2c^{17}+c^{16}+2c^{15}$$

$$\begin{aligned} & \left( \frac{1}{c^2+2} \right) x_0^4 + \left( \frac{c}{c+2} \right) x_0^2 x_1^2 + \left( \frac{c^2}{c^2+2} \right) x_0 x_1^3 + \left( \frac{2c^2+2c+1}{c^2+2} \right) x_0^2 x_1 x_2 + \left( \frac{c^7+2c^5+2c^4+2c^3+2c^2+2}{c^6+2c^5+2c^2+c} \right) x_0 x_1^2 x_2 + \left( \frac{c^6+2c^4+2c^3+c+2}{c^4+2} \right) x_1^3 x_2 + \\ & \left( \frac{2c^2+2c+1}{c^2+2} \right) x_0^2 x_2^2 + \left( \frac{c^6+2c^5+2c^4+2c^3+2c+1}{c^6+2c^5+2c^2+c} \right) x_0 x_1 x_2^2 + \left( \frac{2c^7+c^6+2c^5+c^4+c^3+1}{c^5+2c} \right) x_1^2 x_2^2 + \left( \frac{2c^6+2c^4+2c^2+c+2}{c^4+c^2} \right) x_0 x_3^2 + \left( \frac{c^6+c^5+2c^4+c^2+c+2}{c^4+c^3+c+c} \right) x_1 x_3^2 + \\ & \left( \frac{c^6+c^5+c^3+c^2+c+1}{c^5+2c^4+c^3+2c+2} \right) x_4^2 + \left( \frac{c^{23}+c^{22}+c^{20}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{13}+2c^{11}+2c^9+c^8+2c^7+c^6+2c^5}{c^{19}+c^{16}+c^{15}+2c^{14}+c^{13}+c^{12}+c^{10}+c^9+c^7+2c^6+2c^4+c^2+c} \right) x_0^3 x_3 + \left( \frac{2c^{25}+c^{22}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{21}+c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+c^{13}+c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0 x_1^2 x_3 + \left( \frac{2c^{22}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{20}+c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0^2 x_3^2 + \\ & \left( \frac{c^{24}+c^{23}+2c^{22}+2c^{21}+c^{20}+c^{18}+c^{17}+c^{16}+c^{14}+c^{13}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+2c^2+c}{c^{21}+2c^{20}+c^{18}+c^{16}+2c^{15}+2c^{13}+c^{12}+2c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0^2 x_2 x_3 + \left( \frac{c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{22}+c^{21}+c^{20}+c^{19}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1^2 x_2 x_3 + \left( \frac{2c^{22}+2c^{21}+c^{20}+2c^{19}+2c^{17}+c^{16}+c^{15}+2c^{14}+c^{13}+c^{12}+c^{11}+c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{18}+c^{16}+2c^{13}+2c^{11}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1 x_2^2 x_3 + \left( \frac{c^{24}+c^{23}+2c^{22}+c^{21}+c^{20}+2c^{19}+2c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{20}+c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0^2 x_3^2 + \left( \frac{2c^{25}+2c^{24}+c^{21}+2c^{20}+c^{19}+c^{16}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{21}+2c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{12}+c^{11}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1^2 x_3^2 + \left( \frac{c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{19}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{22}+c^{20}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1 x_3^3 + \left( \frac{2c^{27}+c^{26}+c^{24}+c^{23}+2c^{22}+c^{21}+c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{23}+2c^{22}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0 x_3^3 + \left( \frac{2c^{27}+c^{26}+c^{24}+c^{23}+2c^{22}+c^{21}+c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{23}+2c^{22}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0 x_3^3 + \left( \frac{2c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{20}+2c^{19}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{22}+2c^{21}+c^{19}+c^{18}+2c^{16}+2c^{15}+2c^{14}+c^{13}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0^2 x_3^2 + \left( \frac{2c^{25}+2c^{24}+c^{21}+2c^{20}+c^{19}+c^{16}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{21}+2c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{12}+c^{11}+c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1^2 x_3^2 + \left( \frac{c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{19}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{22}+c^{20}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_1 x_3^3 + \left( \frac{2c^{27}+c^{26}+c^{24}+c^{23}+2c^{22}+c^{21}+c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{23}+2c^{22}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c} \right) x_0 x_3^3 + \left( \frac{2c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{20}+2c^{19}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c}{c^{22}+2c^{21}+c^{19}+c^{18}+2c^{16$$

[illegible]



$$\begin{aligned}
& \left( \frac{c^{25}+2c^{23}+2c^{22}+c^{21}+c^{20}+2c^{17}+2c^{16}+c^{14}+c^{13}+2c^{12}+c^{11}+c^{10}+2c^7+2c^6+2c^5+c^4+c^3+2c^2+1}{c^{22}+2c^{21}+c^{19}+c^{17}+2c^{16}+2c^{14}+c^{13}+2c^{11}+c^{10}+c^9+c^8+2c^7+c^6+c^5+c^3+c^2} \right) x_2 x_3^4 + \left( \frac{c^{26}+c^{25}+c^{24}+c^{23}+c^{21}+2c^{20}+2c^{18}+2c^{17}+c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+c^7+c^6+c^5+2c^2+2c+1}{c^{23}+c^{22}+2c^{21}+2c^{20}+c^{19}+2c^{17}+c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+c^7+c^6+c^5+2c^2+2c+1} \right) x_0^2 x_4 + \\
& \left( \frac{2c^{34}+2c^{32}+c^{31}+2c^{29}+2c^{26}+c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{14}+c^{13}+2c^{12}+2c^{10}+2c^9+2c^8+c^7+c^6+c^5+2c^2+2c+1}{2c^{29}+2c^{28}+2c^{26}+c^{25}+c^{24}+2c^{22}+c^{21}+c^{20}+c^{19}+c^{18}+c^{16}+c^{15}+2c^{14}+2c^{13}+c^{12}+2c^8+2c^7+2c^6} \right) x_0^3 x_4 + \\
& \left( \frac{2c^{34}+c^{32}+2c^{30}+2c^{28}+c^{27}+2c^{24}+2c^{19}+c^{17}+c^{16}+2c^{15}+c^{14}+2c^{13}+c^{12}+2c^{11}+2c^9+c^8+c^6+2c^5+2c^4+c^3+2c^2+c+1}{c^{31}+c^{30}+c^{29}+c^{27}+c^{25}+2c^{24}+c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{19}+2c^{18}+c^{17}+2c^{13}+c^{12}+2c^{10}+2c^9+c^8+2c^7+2c^6} \right) x_0^2 x_1 x_4 + \left( \frac{c^{34}+c^{33}+2c^{32}+2c^{30}}{c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+2c^{19}+2c^{18}+2c^{17}+2c^{12}+2c^9+c^8+2c^7+c^6} \right) x_1^3 x_4 + \\
& \left( \frac{2c^{35}+c^{33}+c^{32}+2c^{31}+2c^{30}+c^{29}+2c^{28}+c^{27}+c^{26}+c^{25}+c^{23}+2c^{21}+c^{19}+2c^{18}+2c^{11}+c^{10}+c^8+c^7+c^6+2c^5+2c^4+c^3+2c^2+c+2}{c^{30}+2c^{29}+2c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+2c^{19}+2c^{17}+2c^{12}+2c^9+c^8+2c^7+c^6} \right) x_1^3 x_4 + \left( \frac{c^{36}+2c^{35}+2c^{34}+2c^{32}}{c^{27}+2c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+2c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+2c^8+c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_0 x_1 x_2 x_4 + \\
& \left( \frac{2c^{33}+2c^{31}+c^{30}+2c^{29}+c^{25}+2c^{24}+c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+2c^8+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{28}+2c^{27}+2c^{25}+2c^{24}+c^{23}+2c^{22}+c^{20}+2c^{19}+2c^{18}+2c^{16}+2c^{15}+c^{14}+c^{12}+2c^{10}+c^6+2c^5+2c^4+2c^3+c^2+2c} \right) x_0 x_1^2 x_4 + \\
& \left( \frac{2c^{38}+c^{37}+c^{36}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{27}+c^{26}+c^{22}+c^{21}+2c^{19}+c^{18}+2c^{17}+c^{16}+c^{12}+c^{11}+c^{10}+c^9+2c^6+2c^4+c^3+c^2+2c+1}{c^{33}+2c^{32}+2c^{30}+2c^{29}+c^{28}+2c^{27}+c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{19}+c^{17}+2c^{15}+c^{11}+2c^{10}+2c^9+2c^8+c^7+2c^6} \right) x_0^2 x_2^2 x_4 + \\
& \left( \frac{2c^{38}+c^{37}+2c^{36}+c^{35}+2c^{34}+2c^{32}+c^{31}+2c^{30}+2c^{29}+2c^{28}+c^{27}+c^{26}+2c^{25}+2c^{24}+c^{23}+2c^{22}+c^{20}+c^{18}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{33}+2c^{32}+2c^{30}+2c^{29}+c^{28}+2c^{27}+c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+c^{19}+c^{17}+2c^{15}+c^{11}+2c^{10}+2c^9+2c^8+c^7+2c^6} \right) x_0^2 x_2^2 x_4 + \\
& \left( \frac{2c^{36}+c^{35}+2c^{31}+c^{30}+2c^{29}+c^{28}+c^{27}+c^{26}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{12}+2c^{10}+c^8+c^6+2c^4+c^3+c^2+c+2}{c^{32}+2c^{29}+c^{28}+2c^{27}+c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{19}+c^{18}+2c^{17}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^6} \right) x_2^3 x_4 + \\
& \left( \frac{c^{35}+c^{34}+2c^{33}+c^{32}+2c^{30}+c^{28}+c^{27}+2c^{25}+c^{23}+2c^{20}+c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+2c^{10}+c^9+c^8+c^7+c^6+c^5+2c^4+c^3+2c^2+2c+2}{c^{31}+2c^{28}+c^{27}+2c^{26}+c^{25}+c^{24}+2c^{23}+c^{22}+2c^{19}+c^{18}+2c^{17}+2c^{16}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^8+c^7+c^5} \right) x_0^2 x_3 x_4 + \\
& \left( \frac{2c^{32}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+c^{24}+2c^{23}+2c^{19}+c^{18}+2c^{17}+c^{15}+2c^{14}+2c^{12}+c^{11}+2c^8+2c^7+c^6+c^4+2c^3+c^2+c+1}{c^{28}+c^{23}+2c^{20}+2c^{19}+c^{18}+2c^{15}+c^{14}+c^{13}+c^{12}+c^{11}+2c^{10}+c^9+c^6+c^5} \right) x_0 x_1 x_3 x_4 + \left( \frac{2c^{32}+2c^{31}+c^{30}+2c^{29}}{c^{31}+c^{30}+2c^{29}+2c^{27}+2c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+2c^{20}+c^{19}+c^{18}+2c^{17}+2c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_0 x_2 x_3 x_4 + \\
& \left( \frac{2c^{34}+c^{32}+c^{31}+c^{30}+2c^{29}+c^{27}+2c^{26}+2c^{25}+c^{24}+2c^{22}+2c^{21}+2c^{20}+2c^{18}+2c^{17}+2c^{16}+2c^{14}+2c^{12}+2c^{11}+2c^8+2c^5+c^4+c^3+2c^2+2}{c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+c^{23}+2c^{21}+2c^{20}+c^{18}+2c^{17}+c^{15}+2c^{14}+2c^{12}+c^{11}+c^{10}+c^9+c^7+c^5} \right) x_2^2 x_3 x_4 + \\
& \left( \frac{2c^{40}+c^{39}+c^{36}+2c^{35}+2c^{32}+c^{31}+c^{30}+2c^{29}+c^{28}+c^{27}+2c^{26}+c^{25}+c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+c^{19}+c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{35}+c^{34}+2c^{33}+c^{32}+c^{30}+2c^{28}+c^{26}+c^{25}+c^{23}+c^{21}+c^{20}+2c^{19}+2c^{18}+c^{16}+2c^{15}+c^{13}+c^{12}+c^{11}+2c^{10}+c^9+2c^7+2c^6} \right) x_2^2 x_3 x_4 + \\
& \left( \frac{2c^{40}+c^{39}+c^{38}+c^{36}+c^{34}+2c^{32}+c^{31}+c^{30}+c^{28}+c^{27}+2c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+c^{20}+c^{18}+c^{17}+c^{16}+2c^{14}+c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^7+c^6+c^5+2c^4+c^3+2c^2+2c+1}{c^{35}+c^{34}+2c^{33}+c^{32}+c^{30}+2c^{28}+c^{26}+c^{25}+c^{23}+c^{21}+c^{20}+2c^{19}+2c^{18}+c^{16}+2c^{15}+c^{13}+c^{12}+c^{11}+2c^{10}+c^9+2c^7+2c^6} \right) x_2^2 x_3 x_4 + \\
& \left( \frac{c^{36}+c^{35}+2c^{32}+2c^{31}+2c^{29}+c^{28}+2c^{27}+c^{26}+c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+c^{15}+c^{12}+c^{11}+2c^{10}+c^9+c^8+2c^6+2c^5+c^4+c+1}{c^{35}+c^{34}+2c^{33}+c^{32}+c^{30}+2c^{28}+c^{26}+c^{25}+c^{23}+c^{21}+c^{20}+2c^{19}+2c^{18}+c^{16}+2c^{15}+c^{13}+c^{12}+c^{11}+2c^{10}+c^9+2c^7+2c^6} \right) x_2 x_2^3 x_4 + \\
& \left( \frac{c^{39}+2c^{38}+c^{34}+c^{32}+2c^{31}+2c^{30}+c^{29}+2c^{28}+2c^{24}+c^{23}+2c^{21}+2c^{20}+2c^{19}+c^{16}+2c^{14}+c^{13}+2c^9+c^5+2c^2+1}{c^{34}+2c^{33}+c^{32}+2c^{31}+2c^{30}+2c^{27}+2c^{26}+c^{24}+c^{23}+2c^{22}+2c^{21}+c^{19}+2c^{17}+2c^{16}+2c^{14}+2c^{13}+c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6} \right) x_3^3 x_4 + \left( \frac{c^{45}+c^{43}+2c^{42}+2c^{41}+c^{39}}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{35}+2c^{34}+2c^{33}+c^{32}+2c^{31}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+c^{25}+c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+c^{19}+c^{18}+2c^{17}+2c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_1^4 x_4 + \\
& \left( \frac{2c^{45}+2c^{44}+2c^{43}+c^{41}+c^{40}+2c^{39}+c^{38}+2c^{32}+2c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{24}+2c^{23}+2c^{21}+2c^{20}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{20}+2c^{17}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^7} \right) x_1^4 x_4 + \\
& \left( \frac{c^{43}+2c^{41}+2c^{40}+2c^{39}+2c^{37}+2c^{34}+c^{32}+c^{31}+c^{30}+c^{28}+c^{26}+2c^{25}+c^{24}+c^{23}+2c^{21}+2c^{20}+c^{17}+c^{15}+2c^{12}+c^{11}+c^{10}+c^9+c^7+c^6+2c^5+c^4+2c^2+2}{c^{38}+2c^{37}+2c^{36}+2c^{35}+c^{33}+2c^{32}+c^{31}+c^{30}+2c^{27}+2c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{20}+2c^{17}+2c^{16}+c^{15}+2c^{13}+c^{12}+2c^9+2c^8} \right) x_1^2 x_4^2 + \\
& \left( \frac{c^{45}+c^{44}+c^{42}+c^{41}+c^{39}+2c^{38}+c^{36}+2c^{34}+c^{33}+c^{31}+2c^{30}+2c^{29}+c^{28}+c^{27}+c^{26}+2c^{25}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{17}+2c^{16}+c^{14}+2c^{13}+c^{10}+c^9+2c^7+2c^6+2c^3+c^2+c+1}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{20}+2c^{17}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^7} \right) x_1^2 x_4^2 + \\
& \left( \frac{c^{45}+2c^{43}+2c^{42}+2c^{41}+c^{38}+2c^{35}+c^{34}+2c^{33}+2c^{31}+c^{30}+2c^{29}+2c^{27}+2c^{25}+c^{24}+c^{23}+2c^{21}+2c^{20}+2c^{16}+2c^{15}+2c^{13}+c^{12}+2c^9+2c^8}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{20}+2c^{17}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^7} \right) x_2^2 x_3^3 x_4 + \\
& \left( \frac{c^{44}+c^{43}+c^{41}+2c^{39}+2c^{38}+c^{37}+2c^{36}+2c^{35}+c^{34}+c^{33}+2c^{32}+2c^{31}+2c^{29}+2c^{28}+2c^{26}+2c^{24}+2c^{23}+2c^{21}+2c^{20}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+c^{10}+c^9+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_2^2 x_3^3 x_4 + \\
& \left( \frac{c^{44}+2c^{43}+c^{42}+c^{41}+c^{40}+2c^{39}+c^{38}+2c^{36}+c^{34}+2c^{33}+c^{32}+c^{31}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{20}+2c^{17}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^7} \right) x_2^2 x_3^3 x_4 + \\
& \left( \frac{c^{43}+c^{42}+c^{41}+2c^{40}+c^{39}+2c^{37}+c^{36}+2c^{35}+2c^{34}+2c^{33}+2c^{31}+c^{30}+2c^{28}+2c^{26}+c^{24}+2c^{23}+c^{21}+2c^{19}+2c^{18}+2c^{17}+c^{16}+c^{13}+c^{11}+c^{10}+c^8+c^6+c^5+c^4+c^2+c+1}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+c^{32}+c^{31}+c^{28}+2c^{26}+2c^{25}+2c^{23}+c^{22}+2c^{20}+2c^{17}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^7} \right) x_2^2 x_3^3 x_4 + \\
& \left( \frac{2c^{41}+2c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{34}+2c^{33}+c^{32}+2c^{29}+2c^{28}+c^{27}+c^{26}+2c^{25}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{17}+2c^{16}+c^{14}+2c^{13}+c^{10}+c^9+2c^7+2c^6+2c^3+c^2+c+1}{c^{36}+2c^{34}+c^{32}+2c^{31}+c^{28}+2c^{27}+2c^{26}+c^{25}+2c^{24}+c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{17}+c^{16}+2c^{13}+c^{11}+c^{10}+c^8+2c^7} \right) x_2 x_2^3 x_3^3 + \\
& \left( \frac{c^{45}+c^{44}+c^{43}+2c^{42}+c^{40}+2c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{33}+c^{31}+2c^{29}+2c^{27}+2c^{25}+c^{24}+c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+c^{15}+2c^{14}+c^{13}+2c^{12}+c^{11}+c^8+c^6+2c^4+c^3+2c^2+c+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_2^2 x_3^3 x_3^3 + \\
& \left( \frac{2c^{45}+c^{43}+2c^{42}+c^{41}+2c^{40}+2c^{39}+c^{37}+c^{36}+2c^{35}+c^{34}+c^{31}+c^{30}+2c^{28}+c^{26}+c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{14}+c^{13}+2c^{11}+c^{10}+c^9+c^8+c^7+2c^6+2c^4+c^3+c^2+c+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_2^2 x_3^3 x_3^3 + \\
& \left( \frac{2c^{41}+c^{39}+c^{38}+2c^{37}+2c^{36}+2c^{35}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{36}+c^{35}+c^{32}+2c^{28}+2c^{26}+2c^{24}+2c^{23}+c^{21}+2c^{19}+2c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+2c^{12}+2c^{11}+c^8} \right) x_2^2 x_3^3 x_3^3 + \\
& \left( \frac{2c^{44}+c^{43}+2c^{42}+c^{41}+c^{40}+c^{38}+c^{37}+c^{36}+c^{35}+c^{34}+c^{33}+2c^{32}+2c^{31}+2c^{30}+c^{29}+2c^{27}+2c^{25}+2c^{24}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_2^2 x_3^3 x_3^3 + \\
& \left( \frac{c^{44}+2c^{43}+2c^{40}+2c^{39}+2c^{38}+2c^{36}+2c^{35}+c^{34}+2c^{32}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{20}+2c^{19}+2c^{18}+c^{14}+c^{10}+2c^8+2c^6+2c^3+2c^2+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7+c^6+c^5+c^4+2c^3+2c+1} \right) x_3 x_3^3 x_3^3 + \\
& \left( \frac{2c^{40}+2c^{39}+c^{36}+c^{35}+c^{34}+2c^{32}+2c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{25}+c^{24}+2c^{23}+2c^{21}+2c^{20}+c^{19}+c^{16}+c^{12}+2c^{10}+2c^9+c^8+2c^7+c^6+2c^4+2}{c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+2c^{33}+c^{31}+c^{27}+2c^{26}+2c^{24}+c^{22}+2c^{21}+2c^{17}+2c^{15}+c^{14}+c^{13}+2c^9} \right) x_4^4
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{c^2+2} \right) x_0^4 + \left( \frac{c^{21}+c^{19}+2c^{18}+c^{17}+c^{16}+c^{13}+c^{10}+c^9+c^8+2c^7+c^6+2c^5+c^4+2c^2+2c}{c^{17}+2c^{15}+c^{14}+2c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^6+c^5+2c^3+2c^2+c+2} \right) x_0^3 x_1 + \left( \frac{2c^{24}+c^{21}+2c^{20}+c^{19}+2c^{17}+2c^{16}+c^{10}+c^8+c^7+2c^6+c^5+c^4+2c^3}{c^{20}+2c^{19}+c^{17}+c^{15}+2c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+2c^5+c^4} \right) x_0^2 x_1^2 + \left( \frac{c^{27}+2c^{26}+c^{25}+c^{24}+2c^{19}+2c^{15}+c^{13}+2c^{10}+c^9+2c^8+c^7+2c^6+2c^4+c^3+c^2+c+1}{c^{22}+c^{20}+c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+2c^{14}+2c^{11}+c^9+c^8+c^7+c^6+2c^5+c^4} \right) x_0 x_0^3 + \left( \frac{c^{22}+c^{20}+c^{19}+c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+c^{10}+2c^8+c^7+2c^5+c^4+2c^3+2c}{c^{19}+c^{16}+c^{15}+2c^{14}+c^{13}+c^{12}+c^{10}+c^9+c^7+2c^6+2c^4+c^3+c^2+c+2} \right) x_0^3 x_2 + \left( \frac{2c^{27}+2c^{26}+2c^{25}+c^{24}+2c^{23}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{12}+2c^{11}+2c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+2c^3+2c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+2c^3} \right) x_0^2 x_2^2 + \left( \frac{2c^{20}+2c^{19}+c^{17}+c^{15}+c^{11}+c^{10}+c^8+c^7+2c^4+2c^3+c^2+c+1}{c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+c^{13}+c^{12}+2c^{10}+c^9+c^6+c^4+2c^3} \right) x_0^2 x_2^2 + \left( \frac{2c^{16}+2c^{15}+c^{14}+2c^{12}+c^{10}+c^9+c^8+2c^7+2c^6+c^4+c^3+c^2}{c^{21}+2c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+c^{13}+c^{12}+2c^{10}+c^9+c^6+c^4+2c^3} \right) x_0^2 x_2^2 + \left( \frac{2c^{27}+c^{26}+c^{25}+2c^{24}+2c^{23}+c^{22}+c^{19}+c^{17}+2c^{16}+c^{15}+2c^{12}+c^{11}+c^8+c^7+c^6+2c^4+c^3+c^2+c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+2c^3} \right) x_1^2 x_2^2 + \left( \frac{2c^{26}+c^{25}+c^{23}+2c^{22}+2c^{21}+2c^{19}+2c^{18}+2c^{1$$

$$\begin{aligned} & \frac{c^{27}+2c^{24}+2c^{19}+c^{15}+5c^{12}+c^{11}+5c^9}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+2c^9+2c^8+c^7+c^6+c^4+c^3}x_0x_2^2x_3 + \left( \frac{2c^{26}+c^{25}+c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{18}+c^{17}+2c^{16}+c^{11}+2c^{10}+c^9}{c^{22}+c^{21}+2c^{20}+2c^{19}+c^{18}+2c^{16}+c^{15}+c^{14}+2c^{11}+2c^{10}+2c^9+c^{19}+2c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+2c^9+2c^8+2c^7+c^6+c^5+c^4} \right)x_0x_2^2x_3 + \left( \frac{c^{25}+2c^{24}+2c^{23}+2c^{20}+c^{19}+2c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+2c^9+2c^8+c^7+2c^6+c^5+c^4}{c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{13}+2c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+2c^3+2c^2+c+1} \right)x_0x_2x_2^3 + \left( \frac{2c^{26}+2c^{25}+2c^{23}+c^{22}+c^{21}+2c^{19}+2c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+2c^3+2c^2+c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+c^3} \right)x_2^2x_3^2 + \left( \frac{2c^{19}+c^{17}+c^{16}+2c^{15}+c^{14}+c^{11}+2c^9+c^8+c^7+2c^6+2c^4}{c^{16}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+c^5+c^4+2c^2+1} \right)x_0x_3^3 + \left( \frac{2c^{23}+2c^{22}+2c^{21}+c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+c^{15}+2c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+2c^5+c^4+1}{c^{20}+2c^{19}+c^{17}+c^{15}+2c^{14}+2c^{12}+c^{11}+2c^9+c^8+c^7+c^6+2c^5+c^4+c^3+c+1} \right)x_1x_3^3 + \left( \frac{2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{21}+c^{20}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+2c^{12}+2c^9+c^8+c^7+c^6+2c^5+c^4+c^3+c+1}{c^{23}+2c^{22}+c^{20}+c^{18}+2c^{17}+2c^{15}+c^{14}+2c^{12}+c^{11}+c^{10}+c^9+2c^8+c^7+c^6+c^5+c^4+c^3} \right)x_2x_3^3 + \left( \frac{2c^{30}+c^{29}+c^{28}+2c^{27}+2c^{25}+2c^{24}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+2c^{12}+c^{11}+c^{10}+2c^9+2c^8+c^7+c^6+c^5+c^4+c^3+c+1}{c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+c^{15}+c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+2c^9+2c^8+c^7+c^6+c^5+c^4+2c^3+2c^2+c+1} \right)x_0^2x_1x_4 + \left( \frac{c^{31}+2c^{29}+2c^{27}+2c^{26}+c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+2c^{11}+c^{10}+c^9+c^8+c^7+c^6+2c^5+c^4+c^3+c+1}{c^{32}+c^{31}+2c^{29}+2c^{25}+c^{22}+2c^{20}+c^{17}+2c^{16}+2c^{14}+c^{13}+c^{12}+c^{11}+2c^{10}+2c^9+c^8+c^7+c^6+c^5} \right)x_0x_1^2x_4 + \left( \frac{2c^{34}+c^{32}+2c^{29}+2c^{28}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+2c^{13}+c^{12}+c^{11}+c^{10}+c^8+c^7+c^6+2c^5+2c^4+2c^3+2c^2+c+2}{c^{29}+2c^{27}+2c^{25}+c^{24}+2c^{23}+2c^{22}+c^{21}+c^{20}+c^{19}+2c^{17}+2c^{16}+2c^{14}+2c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^7+2c^6} \right)x_1^3x_4 + \left( \frac{c^{35}+2c^{34}+2c^{33}+c^{32}+c^{31}+c^{27}+2c^{24}+2c^{21}+c^{19}+2c^{18}+2c^{16}+c^{15}+c^{13}+2c^{11}+2c^{10}+c^9+c^7+c^6+c^5+c^4+2c^3+2c^2+c+2}{c^{30}+2c^{29}+2c^{28}+c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{20}+c^{19}+c^{15}+2c^{12}+c^{10}+2c^9+c^8+c^5+2c^4} \right)x_0^2x_2x_4 + \left( \frac{c^{38}+2c^{37}+c^{36}+2c^{35}+2c^{34}+2c^{32}+c^{31}+c^{28}+2c^{27}+c^{23}+c^{20}+2c^{19}+2c^{18}+2c^{16}+c^{14}+2c^{10}+2c^8+2c^6+c^5+c^4+2c^3+2c^2+2}{c^{33}+2c^{31}+2c^{30}+c^{29}+2c^{26}+c^{25}+c^{23}+2c^{22}+2c^{21}+c^{20}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{11}+c^9+c^8+2c^5} \right)x_1^2x_2x_4 + \left( \frac{c^{37}+c^{34}+c^{33}+2c^{32}+2c^{31}+2c^{30}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{22}+2c^{21}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+2c^{13}+c^{12}+c^{10}+2c^8+c^6+2c^5+2c^3+2c^2+c+2}{c^{32}+c^{30}+2c^{29}+c^{27}+2c^{25}+c^{24}+2c^{23}+2c^{20}+c^{19}+c^{18}+c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^6} \right)x_1 + \left( \frac{2c^{34}+c^{33}+c^{31}+2c^{30}+2c^{29}+c^{27}+c^{25}+c^{23}+c^{22}+2c^{21}+2c^{20}+2c^{15}+2c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^5+2c^3+2c+1}{c^{29}+2c^{27}+2c^{25}+c^{24}+2c^{23}+2c^{22}+c^{21}+c^{20}+c^{19}+2c^{17}+2c^{16}+2c^{14}+2c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^7+2c^6} \right)x_3^2x_4 + \left( \frac{c^{37}+2c^{35}+c^{34}+c^{33}+2c^{31}+c^{28}+c^{27}+2c^{26}+2c^{25}+2c^{23}+2c^{22}+2c^{21}+c^{18}+c^{15}+c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+2c^7+2c^6+2c^4+c^2+2c+1}{c^{32}+2c^{31}+2c^{30}+c^{29}+2c^{28}+c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{20}+c^{19}+c^{18}+c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^6} \right)x_0x_1x_3x_4 + \left( \frac{c^{34}+c^{33}+2c^{31}+2c^{30}+c^{27}+c^{26}+c^{25}+2c^{24}+2c^{23}+c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{17}+c^{16}+c^{14}+2c^{13}+c^{12}+c^{10}+c^8+c^7+c^6+c^5+c^4+c^3+c^2+c+2}{c^{30}+2c^{29}+2c^{27}+2c^{24}+2c^{23}+c^{22}+2c^{21}+c^{20}+2c^{19}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+c^{12}+2c^{11}+c^{10}+c^9+2c^8+c^7} \right)x_1^2x_3x_4 + \left( \frac{c^{39}+2c^{38}+2c^{37}+2c^{36}+2c^{35}+2c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{28}+2c^{27}+2c^{23}+2c^{17}+c^{16}+2c^{13}+c^{12}+2c^{10}+2c^9+2c^8+2c^4+c^2+1}{c^{34}+2c^{32}+2c^{$$

$$\begin{aligned}
& \left( \frac{2c^{31}+2c^{30}+c^{29}+c^{28}+2c^{27}+c^{26}+c^{24}+c^{23}+2c^{21}+c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{15}+2c^{14}+c^{13}+2c^{12}+2c^{11}+2c^{10}+2c^9+c^8+2c^6+2c^5+2}{c^{26}+2c^{25}+2c^{24}+2c^{23}+c^{22}+c^{21}+c^{19}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{11}+c^{10}+2c^9+2c^8+2c^7+2c^5} \right) x_0^3 x_3 + \\
& \left( \frac{2c^{37}+2c^{36}+2c^{34}+2c^{31}+c^{30}+c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+c^{24}+c^{23}+2c^{21}+c^{20}+c^{19}+c^{18}+2c^{17}+c^{14}+c^{13}+2c^{12}+c^{10}+c^6+2c^4+2c^2+2}{c^{33}+c^{32}+2c^{30}+2c^{26}+c^{23}+2c^{21}+c^{18}+2c^{17}+2c^{15}+c^{14}+c^{13}+c^{12}+2c^{11}+2c^{10}+c^8+c^7+c^6} \right) x_0^2 x_1 x_3 + \\
& \left( \frac{2c^{33}+c^{32}+2c^{30}+2c^{27}+c^{26}+c^{25}+c^{23}+2c^{22}+2c^{21}+c^{20}+c^{19}+2c^{18}+2c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{11}+2c^{10}+c^9+c^6+c^4}{c^{29}+2c^{28}+2c^{22}+2c^{21}+c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+2c^{14}+c^{13}+c^{11}+2c^{10}+c^9+c^6+c^4} \right) x_0 x_1^2 x_3 + \left( \frac{2c^{37}+c^{35}+c^{34}+c^{32}+2c^{30}+c^{29}}{c^{32}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{22}+2c^{20}+c^{17}+2c^{16}+2c^{14}+c^{13}+c^{12}+c^{11}+2c^{10}+2c^9+c^7+c^6+c^5} \right) x_0^2 x_2 x_3 + \\
& \left( \frac{2c^{32}+2c^{31}+c^{30}+2c^{29}+2c^{27}+c^{26}+2c^{25}+c^{24}+c^{23}+2c^{22}+c^{20}+c^{19}+2c^{18}+c^{16}+2c^{15}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+c^7+2c^5+2c+1}{c^{28}+c^{27}+c^{26}+2c^{24}+c^{22}+2c^{21}+2c^{17}+c^{14}+c^{13}+c^{12}+c^{10}+c^8+2c^7+2c^6+c^5} \right) x_0 x_1 x_2 x_3 + \\
& \left( \frac{c^{36}+c^{34}+2c^{33}+2c^{32}+c^{31}+c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{23}+c^{22}+c^{20}+2c^{19}+c^{18}+2c^{14}+c^{13}+2c^{12}+c^{11}+c^4+2c^3+2c^2+2c+1}{c^{32}+2c^{30}+2c^{29}+c^{28}+2c^{25}+c^{24}+c^{22}+2c^{21}+2c^{20}+c^{19}+c^{17}+c^{16}+c^{15}+2c^{14}+2c^{13}+c^{10}+c^8+c^7+c^5+c^4+2c^2+1} \right) x_1^2 x_2 x_3 + \\
& \left( \frac{c^{38}+c^{36}+2c^{33}+2c^{31}+2c^{29}+2c^{28}+c^{27}+2c^{25}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{17}+c^{16}+c^{14}+c^{13}+c^{12}+2c^{11}+c^{10}+c^7+2c^5+2c^4+2c+1}{c^{34}+2c^{32}+2c^{31}+c^{30}+2c^{27}+c^{26}+2c^{25}+2c^{23}+c^{22}+c^{21}+c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+c^{12}+c^{10}+c^9+2c^6} \right) x_0 x_2^2 x_3 + \\
& \left( \frac{2c^{36}+c^{35}+c^{33}+c^{32}+2c^{31}+c^{28}+c^{27}+2c^{26}+2c^{25}+c^{24}+2c^{22}+c^{21}+2c^{20}+2c^{19}+c^{17}+c^{16}+c^{14}+2c^{12}+2c^{11}+2c^8+2c^7+2c^6+c^4+2c^3+2}{c^{32}+c^{30}+2c^{29}+c^{27}+c^{25}+2c^{24}+2c^{23}+2c^{20}+c^{19}+c^{18}+c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+c^9+c^8+2c^6} \right) x_1 x_2^2 x_3 + \\
& \left( \frac{2c^{32}+2c^{31}+2c^{30}+2c^{28}+2c^{25}+c^{24}+2c^{23}+c^{21}+c^{19}+2c^{17}+c^{15}+2c^{14}+c^{13}+2c^{12}+2c^{11}+c^{10}+c^8+2c^7+c^5+2c^4+c^3+c^2+2c+2}{c^{27}+2c^{26}+2c^{25}+c^{24}+2c^{23}+c^{20}+2c^{19}+c^{17}+2c^{16}+2c^{15}+c^{13}+2c^{11}+2c^6+c^5} \right) x_2^3 x_3 + \left( \frac{c^{36}+c^{33}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{22}+2c^{20}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5}{c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5} \right) x_0 x_3^2 + \\
& \left( \frac{2c^{37}+c^{36}+2c^{35}+2c^{34}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5}{c^{32}+2c^{29}+c^{28}+2c^{27}+c^{26}+c^{25}+2c^{24}+c^{23}+2c^{22}+c^{21}+c^{20}+c^{19}+2c^{18}+2c^{17}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^9+c^8+c^6} \right) x_1^2 x_3^2 + \left( \frac{c^{37}+c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{22}+2c^{20}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5}{c^{34}+2c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5} \right) x_0 x_3^3 + \\
& \left( \frac{2c^{38}+2c^{37}+2c^{36}+c^{35}+2c^{33}+c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+c^7+c^4+2c^3+2c^2+2}{c^{34}+2c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+c^{10}+c^9+2c^8} \right) x_0^2 x_3^3 + \\
& \left( \frac{c^{44}+c^{43}+2c^{41}+2c^{37}+c^{36}+c^{35}+c^{34}+c^{33}+c^{30}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{21}+c^{20}+c^{19}+2c^{18}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{11}+c^{10}+c^9+c^7+c^5+2c^3+2c^2+c}{c^{39}+2c^{38}+c^{36}+2c^{35}+2c^{33}+2c^{32}+c^{30}+c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_1 x_2 x_3^3 + \left( \frac{c^{37}+c^{35}+2c^{33}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{22}+2c^{20}+c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5}{c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5} \right) x_0 x_3^4 + \left( \frac{c^{36}+c^{35}+c^{34}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5}{c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{13}+c^{10}+c^9+2c^8+2c^7+2c^6+2c^5} \right) x_2 x_3^4 + \\
& \left( \frac{2c^{38}+2c^{37}+2c^{36}+c^{35}+2c^{33}+c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+2c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+c^7+c^4+2c^3+2c^2+2}{c^{34}+2c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+c^{10}+c^9+2c^8} \right) x_0^2 x_3^4 + \\
& \left( \frac{c^{44}+c^{43}+2c^{41}+2c^{37}+c^{36}+c^{35}+c^{34}+c^{33}+c^{30}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{21}+c^{20}+c^{19}+2c^{18}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{11}+c^{10}+c^9+c^7+c^5+2c^3+2c^2+c}{c^{39}+2c^{38}+c^{36}+2c^{35}+2c^{33}+2c^{32}+c^{30}+c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_1 x_2 x_3^4 + \\
& \left( \frac{c^{44}+c^{42}+c^{41}+2c^{40}+2c^{39}+c^{38}+c^{37}+c^{36}+2c^{34}+2c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{29}+c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7}{c^{39}+2c^{38}+c^{36}+2c^{35}+2c^{33}+2c^{32}+c^{30}+2c^{29}+c^{28}+2c^{27}+c^{26}+c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_0 x_1 x_2 x_4 + \\
& \left( \frac{c^{45}+c^{44}+c^{41}+2c^{40}+2c^{39}+2c^{38}+c^{37}+2c^{36}+c^{35}+c^{34}+2c^{33}+c^{32}+c^{28}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+c^{16}+c^{15}+c^{13}+c^{12}+2c^{11}+2c^9+2c^8+c}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_0 x_1 x_2 x_4 + \\
& \left( \frac{c^{45}+2c^{44}+2c^{42}+2c^{41}+2c^{40}+c^{39}+c^{38}+2c^{37}+2c^{36}+c^{35}+c^{34}+2c^{33}+c^{32}+c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_0 x_2^2 x_4 + \\
& \left( \frac{c^{45}+2c^{43}+2c^{42}+c^{41}+c^{40}+c^{38}+2c^{37}+c^{36}+2c^{35}+2c^{33}+2c^{32}+c^{31}+c^{30}+2c^{27}+c^{25}+c^{20}+2c^{19}+2c^{18}+c^{17}+2c^{16}+2c^{15}+2c^{14}+c^{12}+c^{10}+2c^9+2c^7+2c^4+c^2+c+2}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_0 x_2^2 x_4 + \\
& \left( \frac{2c^{44}+c^{40}+c^{39}+c^{38}+2c^{35}+2c^{34}+2c^{32}+2c^{31}+c^{29}+c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8}{c^{40}+c^{39}+2c^{38}+2c^{37}+2c^{34}+c^{31}+c^{29}+c^{28}+2c^{27}+2c^{23}+c^{22}+2c^{21}+2c^{19}+c^{17}+c^{16}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9} \right) x_2^3 x_4 + \\
& \left( \frac{2c^{34}+2c^{33}+2c^{31}+2c^{30}+c^{29}+2c^{28}+c^{27}+c^{26}+c^{24}+c^{23}+2c^{20}+2c^{19}+2c^{17}+2c^{16}+2c^{15}+2c^{13}+c^{10}+2c^9+c^6+2c^5+1}{c^{29}+2c^{27}+2c^{26}+c^{25}+2c^{23}+2c^{22}+2c^{20}+2c^{19}+2c^{18}+2c^{11}+2c^{10}+c^7} \right) x_0^2 x_3 x_4 + \left( \frac{c^{43}+c^{42}+2c^{41}+c^{40}+2c^{39}+c^{38}+c^{37}+2c^{36}+2c^{35}+2c^{34}+2c^{32}+c^{31}+c^{30}+2c^{27}+c^{26}+2c^{25}+2c^{24}+2c^{23}+c^{21}+2c^{20}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+c^{11}+2c^8+c^7+c^6+2c^5+c^4+c^3+c^2+c+1}{c^{37}+2c^{36}+2c^{35}+2c^{34}+2c^{33}+2c^{32}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_1 x_2 x_3 x_4 + \\
& \left( \frac{2c^{42}+c^{41}+2c^{40}+2c^{37}+2c^{36}+2c^{35}+c^{31}+2c^{28}+2c^{25}+2c^{24}+c^{23}+2c^{22}+2c^{20}+c^{17}+c^{16}+c^{15}+2c^{14}+c^{13}+c^{11}+c^{10}+2c^9+2c^8+2c^7+c^6+2c^5+c+1}{c^{37}+c^{36}+2c^{35}+2c^{32}+2c^{30}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{16}+c^{15}+c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_2^2 x_3 x_4 + \\
& \left( \frac{c^{39}+c^{38}+2c^{36}+c^{35}+2c^{32}+2c^{31}+c^{30}+2c^{28}+2c^{27}+c^{25}+2c^{22}+2c^{21}+c^{20}+2c^{18}+c^{15}+c^{14}+c^{13}+2c^{12}+c^9+c^8+c^7+c^6+2c^5+c^4+2c^3+c^2+2c+1}{c^{34}+c^{32}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+c^{24}+2c^{23}+c^{22}+c^{19}+c^{18}+2c^{16}+2c^{15}+c^{14}+c^{11}+c^9+2c^7} \right) x_1 x_3^2 x_4 + \\
& \left( \frac{c^{45}+2c^{44}+c^{43}+c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+2c^{35}+c^{33}+c^{32}+2c^{31}+2c^{29}+c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+c^{21}+c^{20}+c^{18}+2c^{13}+c^{10}+2c^9+2c^8+2c^7+c^6+c^5+c^4+c^3+c^2+c+1}{c^{40}+2c^{39}+c^{37}+2c^{36}+2c^{34}+2c^{33}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_0 x_1 x_2 x_4^2 + \\
& \left( \frac{c^{38}+c^{36}+2c^{35}+c^{34}+2c^{32}+2c^{31}+c^{29}+2c^{28}+2c^{26}+c^{25}+2c^{23}+c^{21}+c^{19}+2c^{18}+2c^{13}+c^{12}+c^{11}+c^8+c^7+c^6+2c^5+2c^4+c+1}{c^{34}+c^{33}+2c^{32}+2c^{31}+c^{30}+2c^{28}+2c^{24}+c^{22}+c^{21}+2c^{20}+2c^{19}+2c^{16}+c^{15}+2c^{14}+2c^{13}+2c^{12}+c^9+c^8} \right) x_3^3 x_4 + \left( \frac{c}{c+2} \right) x_0^2 x_4^2 + \\
& \left( \frac{2c^{44}+c^{41}+c^{40}+2c^{39}+c^{37}+c^{35}+2c^{34}+c^{33}+c^{30}+2c^{27}+c^{26}+2c^{24}+c^{17}+c^{15}+2c^{14}+c^{12}+2c^9+2c^8+2c^7+2c^6+2c^5+2c^4+2c^3+c^2+2c+2}{c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+c^{34}+2c^{33}+c^{31}+c^{30}+2c^{27}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+c^{10}+c^6} \right) x_0 x_1 x_4^2 + \\
& \left( \frac{2c^{43}+c^{42}+2c^{41}+2c^{40}+c^{39}+2c^{38}+c^{36}+c^{35}+2c^{34}+c^{33}+c^{32}+c^{31}+c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+c^{16}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7}{c^{39}+2c^{38}+2c^{35}+2c^{33}+2c^{32}+c^{30}+c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_1^2 x_4^2 + \\
& \left( \frac{2c^{45}+c^{42}+c^{41}+c^{40}+2c^{34}+c^{33}+c^{32}+2c^{31}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+c^9+c^6+c^5+c+1}{c^{40}+c^{39}+c^{38}+c^{37}+c^{36}+c^{35}+2c^{34}+2c^{33}+2c^{32}+2c^{31}+2c^{30}+2c^{29}+2c^{28}+2c^{27}+2c^{26}+2c^{25}+2c^{24}+2c^{23}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+2c^{18}+2c^{17}+2c^{16}+2c^{15}+2c^{14}+2c^{13}+c^{12}+2c^{11}+2c^{10}+2c^9+2c^8+2c^7} \right) x_2 x_3 x_4^2 + \\
& \left( \frac{2c^{31}+2c^{29}+2c^{28}+c^{26}+2c^{25}+2c^{24}+2c^{21}+2c^{20}+2c^{19}+c^{17}+c^{16}+2c^{14}+2c^{13}+c^{12}+c^{10}+c^9+c^8+c^7+2c^6+2c^5+2}{c^{27}+c^{26}+2c^{23}+2c^{21}+c^{20}+2c^{19}+c^{17}+c^{16}+2c^{14}+2c^{13}+c^{12}+c^{10}+c^7+2c^6+c^5} \right) x_3^2 x_4^2 + \left( \frac{c}{c^2+2} \right) x_0 x_4^3 +
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{2c^{41}+c^{40}+2c^{39}+c^{37}+c^{35}+2c^{34}+c^{33}+2c^{32}+2c^{31}+2c^{30}+2c^{29}+2c^{28}+2c^{26}+2c^{25}+2c^{24}+2c^{22}+2c^{21}+2c^{20}+2c^{19}+c^{18}+c^{17}+2c^{15}+2c^{14}+c^{13}+c^{12}+2c^{11}+2c^7+2c^6+2c^5+2c^4+2c^3+2c^2+2c+1}{c^{39}+2c^{38}+c^{36}+2c^{35}+2c^{33}+2c^{32}+c^{30}+c^{29}+c^{28}+2c^{27}+2c^{26}+c^{25}+2c^{22}+2c^{19}+2c^{18}+2c^{17}+c^{16}+2c^{15}+c^{14}+c^{13}+2c^{12}+c^{11}+2c^{10}+2c^9+2c^8+2c^7+2c^6+2c^5+2c^4+2c^3+2c^2+2c+1} \right) x_2 x_4^3 + \\
& \left( \frac{c^{40}+c^{39}+2c^{36}+2c^{35}+2c^{34}+c^{32}+c^{31}+2c^{30}+c^{29}+c^{28}+c^{25}+2c^{24}+c^{23}+c^{21}+c^{20}+2c^{19}+2c^{16}+2c^{12}+c^{10}+c^9+2c^8+c^7+2c^6+c^4+1}{c^{38}+2c^{37}+2c^{36}+2c^{35}+c^{33}+2c^{32}+c^{31}+c^{30}+c^{27}+2c^{26}+c^{25}+2c^{24}+2c^{23}+c^{22}+2c^{20}+2c^{17}+2c^{16}+2c^{15}+2c^{13}+c^{12}+2c^9+2c^8+2c^7+2c^6+2c^5+2c^4+2c^3+2c^2+2c+1} \right) x_3 x_4^3
\end{aligned}$$