

# Computations

$$\bar{h} = 1$$

## 1 $S_2$ , char 2, $\tau$ trivial

### 1.1 Hilbert polynomial

$$t^4 + 2t^3 + 2t^2 + 2t + 1 = (t + 1)^2(t^2 + 1).$$

### 1.2 Generators

$$x_0^2 + x_1^2$$

$$x_0^4.$$

## 2 $S_3$ , char 3, $\tau$ trivial

### 2.1 Hilbert polynomial

$$t^{12} + 3t^{11} + 6t^{10} + 8t^9 + 9t^8 + 9t^7 + 9t^6 + 9t^5 + 9t^4 + 8t^3 + 6t^2 + 3t + 1 = (t^2 + t + 1)^3(t^6 + t^3 + 1).$$

### 2.2 Generators

$$x_0^3 + x_1^3 + x_2^3$$

$$((2c + 2)/c)x_0^3 - x_0^2x_1 + x_0x_1^2 + ((c + 1)/c)x_1^3 + x_0^2x_2 - x_1^2x_2 - x_0x_2^2 + x_1x_2^2$$

$$x_0^9.$$

## 3 $S_4$ , char 2, $\tau$ trivial

### 3.1 Hilbert polynomial

$$t^6 + 4t^5 + 7t^4 + 8t^3 + 7t^2 + 4t + 1 = (t + 1)^4(t^2 + 1).$$

### 3.2 Generators

$$x_0^2 + x_1^2 + x_2^2 + x_3^2$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3$$

$$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3$$

$$x_0^4.$$

## 4 $S_5$ , char 5, $\tau$ trivial

### 4.1 Hilbert polynomial

PARTIAL

### 4.2 Generators

$$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5$$

$$\begin{aligned} & ((4c+4)/c)x_0^5 + x_0^4x_1 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 - x_0x_1^4 + x_0^4x_2 + (3c/(c+2))x_0^3x_1x_2 + \\ & ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_2 + (c/(c+2))x_0x_1^3x_2 + ((4c+1)/(c+2))x_0^3x_2^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_2^2 + \\ & ((4c^2+c)/(c^2+1))x_0x_1^2x_2^2 + ((c+4)/(c+2))x_0^2x_3^2 + (c/(c+2))x_0x_1x_2^3 - x_0x_2^4 + 2x_0^4x_3 + (2c/(c+2))x_0^3x_1x_3 + \\ & ((4c^2+c)/(c^2+1))x_0^2x_1^2x_3 + x_1^4x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2+1))x_0^2x_1x_2x_3 + (4c/(c+2))x_1^3x_2x_3 + ((4c^2+ \\ & c)/(c^2+1))x_0^2x_2^2x_3 + ((c^2+4c)/(c^2+1))x_1^2x_2^2x_3 + (4c/(c+2))x_1x_2^3x_3 + x_2^4x_3 + ((3c+2)/(c+2))x_0^3x_3^2 + ((c^2+ \\ & 4c)/(c^2+1))x_0x_1^2x_3^2 + ((4c+1)/(c+2))x_1^3x_3^2 + (2c^2/(c^2+1))x_0x_1x_2x_3^2 + ((2c^2+3c)/(c^2+1))x_1^2x_2x_3^2 + ((c^2+ \\ & 4c)/(c^2+1))x_0x_2^2x_3^2 + ((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2 + ((4c+1)/(c+2))x_2^3x_3^2 + ((2c+3)/(c+2))x_0^2x_3^3 + (3c/(c+ \\ & 2))x_0x_1x_3^3 + ((c+4)/(c+2))x_1^2x_3^3 + (3c/(c+2))x_0x_2x_3^3 + (2c/(c+2))x_1x_2x_3^3 + ((c+4)/(c+2))x_2^2x_3^3 + 3x_0x_3^4 - \\ & x_1x_3^4 - x_2x_3^4 + ((c+1)/c)x_3^5 + x_0^4x_4 + (3c/(c+2))x_0^3x_1x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_4 + (c/(c+2))x_0x_1^3x_4 + \\ & (3c/(c+2))x_0^3x_2x_4 + (c^2/(c^2+1))x_0^2x_1x_2x_4 + (3c^2/(c^2+1))x_0x_1^2x_2x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_2^2x_4 + (3c^2/(c^2+ \\ & 1))x_0x_1x_2^2x_4 + (c/(c+2))x_0x_2^3x_4 + (2c/(c+2))x_0^3x_3x_4 + (3c^2/(c^2+1))x_0^2x_1x_3x_4 + (4c/(c+2))x_1^3x_3x_4 + (3c^2/(c^2+ \\ & 1))x_0^2x_2x_3x_4 + (2c^2/(c^2+1))x_1^2x_2x_3x_4 + (2c^2/(c^2+1))x_1x_2^2x_3x_4 + (4c/(c+2))x_2^3x_3x_4 + (2c^2/(c^2+1))x_0x_1x_3^2x_4 + \\ & ((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4 + (2c^2/(c^2+1))x_0x_2^2x_3^2x_4 + (4c^2/(c^2+1))x_1x_2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_2^2x_3^2x_4 + \\ & (3c/(c+2))x_0^3x_4^2 + (2c/(c+2))x_1x_3^3x_4 + (2c/(c+2))x_2x_3^3x_4 - x_3^4x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_4^2 + \\ & ((4c^2+c)/(c^2+1))x_0x_1^2x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_2x_4^2 + (3c^2/(c^2+1))x_0x_1x_2x_4^2 + ((4c^2+c)/(c^2+1))x_0^2x_3x_4^2 + \\ & ((c^2+4c)/(c^2+1))x_1^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_0x_2^2x_4^2 + ((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_2^2x_3x_4^2 + \\ & ((c^2+4c)/(c^2+1))x_2x_3^2x_4^2 + ((c+4)/(c+2))x_3^3x_4^2 + ((c+4)/(c+2))x_0^3x_4^3 + (c/(c+2))x_0x_1x_4^3 + (c/(c+2))x_0x_2x_4^3 + (4c/(c+ \\ & 2))x_1x_3x_4^3 + (4c/(c+2))x_2x_3x_4^3 + ((4c+1)/(c+2))x_3^3x_4^3 - x_0x_4^4 + x_3x_4^4 \end{aligned}$$

$$\begin{aligned} & ((4c+4)/c)x_0^5 + x_0^4x_1 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 - x_0x_1^4 + 2x_0^4x_2 + (2c/(c+2))x_0^3x_1x_2 + ((4c^2+ \\ & c)/(c^2+1))x_0^2x_1^2x_2 + x_1^4x_2 + ((3c+2)/(c+2))x_0^3x_2^2 + ((c^2+4c)/(c^2+1))x_0^2x_1x_2^2 + ((4c+1)/(c+2))x_1^3x_2^2 + ((2c+ \\ & 3)/(c+2))x_0^2x_3^2 + (3c/(c+2))x_0x_1x_2^3 + ((c+4)/(c+2))x_1^2x_3^2 + 3x_0x_2^4 - x_1x_2^4 + ((c+1)/c)x_2^5 + x_0^4x_3 + (3c/(c+ \\ & 2))x_0^3x_1x_3 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_3 + (c/(c+2))x_0x_1^3x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2+1))x_0^2x_1x_2x_3 + \\ & (4c/(c+2))x_1^3x_2x_3 + (2c^2/(c^2+1))x_0x_1x_2^2x_3 + ((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3 + (3c/(c+2))x_0x_2^2x_3 + (2c/(c+ \\ & 2))x_1x_2^3x_3 - x_2^4x_3 + ((4c+1)/(c+2))x_0^3x_3^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_3^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_3^2 + ((4c^2+ \\ & c)/(c^2+1))x_0^2x_2x_3^2 + ((c^2+4c)/(c^2+1))x_1^2x_2x_3^2 + ((c^2+4c)/(c^2+1))x_0x_2^2x_3^2 + ((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2 + \\ & ((c+4)/(c+2))x_2^3x_3^2 + ((c+4)/(c+2))x_0^2x_3^3 + (c/(c+2))x_0x_1x_3^3 + (4c/(c+2))x_1x_2x_3^3 + ((4c+1)/(c+2))x_2^2x_3^3 - \\ & x_0x_3^4 + x_2x_3^4 + x_4^4x_4 + (3c/(c+2))x_0^3x_1x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_4 + (c/(c+2))x_0x_1^3x_4 + (2c/(c+2))x_0^3x_2x_4 + \\ & (3c^2/(c^2+1))x_0^2x_1x_2x_4 + (4c/(c+2))x_1^3x_2x_4 + (2c^2/(c^2+1))x_0x_1x_2^2x_4 + ((2c^2+3c)/(c^2+1))x_1^2x_2^2x_4 + (3c/(c+ \\ & 2))x_0x_2^3x_4 + (2c/(c+2))x_1x_2^3x_4 - x_2^4x_4 + (3c/(c+2))x_0^3x_3x_4 + (c^2/(c^2+1))x_0^2x_1x_3x_4 + (3c^2/(c^2+1))x_0x_1^2x_3x_4 + \\ & (3c^2/(c^2+1))x_0^2x_2x_3x_4 + (2c^2/(c^2+1))x_1^2x_2x_3x_4 + (2c^2/(c^2+1))x_0x_2^2x_3x_4 + (4c^2/(c^2+1))x_1x_2^2x_3x_4 + (2c/(c+ \\ & 2))x_2^3x_3x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_3^2x_4 + (3c^2/(c^2+1))x_0x_1x_2^3x_4 + (2c^2/(c^2+1))x_1x_2x_2^3x_4 + ((2c^2+3c)/(c^2+ \\ & 1))x_2^2x_3^2x_4 + (c/(c+2))x_0^3x_4^2 + (4c/(c+2))x_2x_3^3x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_4^2 + \end{aligned}$$

$$((4c^2 + c)/(c^2 + 1))x_0x_1^2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1^2x_2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_2^2x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1x_2^2x_4^2 + ((c + 4)/(c + 2))x_2^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3x_4^2 + (3c^2/(c^2 + 1))x_0x_1x_3x_4^2 + (2c^2/(c^2 + 1))x_1x_2x_3x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_2^2x_3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^3x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_2x_2^3x_4^2 + ((c + 4)/(c + 2))x_0^2x_4^3 + (c/(c + 2))x_0x_1x_4^3 + (4c/(c + 2))x_1x_2x_4^3 + ((4c + 1)/(c + 2))x_2^2x_4^3 + (c/(c + 2))x_0x_3x_4^3 + (4c/(c + 2))x_2x_3x_4^3 - x_0x_4^4 + x_2x_4^4$$

$$((4c + 4)/c)x_0^5 + 2x_0^4x_1 + ((3c + 2)/(c + 2))x_0^3x_1^2 + ((2c + 3)/(c + 2))x_0^2x_1^3 + 3x_0x_1^4 + ((c + 1)/c)x_1^5 + x_0^4x_2 + (2c/(c + 2))x_0^3x_1x_2 + (3c/(c + 2))x_0x_1^3x_2 - x_1^4x_2 + ((4c + 1)/(c + 2))x_0^3x_2^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_2^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_2^2 + ((c + 4)/(c + 2))x_1^3x_2^2 + ((c + 4)/(c + 2))x_0^2x_3^2 + ((4c + 1)/(c + 2))x_1^2x_3^2 - x_0x_2^4 + x_1x_2^4 + x_0^4x_3 + (2c/(c + 2))x_0^3x_1x_3 + (3c/(c + 2))x_0x_1^3x_3 - x_1^4x_3 + (3c/(c + 2))x_0^3x_2x_3 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_3 + (2c^2/(c^2 + 1))x_0x_1^2x_2x_3 + (2c/(c + 2))x_1^3x_2x_3 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2^2x_3 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_3 + (c/(c + 2))x_0x_2^3x_3 + (4c/(c + 2))x_1x_2^3x_3 + ((4c + 1)/(c + 2))x_0^3x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_3^2 + ((c + 4)/(c + 2))x_1^3x_3^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_3^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2x_3^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_3^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_2^2x_3^2 + ((c + 4)/(c + 2))x_0^2x_3^3 + ((4c + 1)/(c + 2))x_1^2x_3^3 + (c/(c + 2))x_0x_2x_3^3 + (4c/(c + 2))x_1x_2x_3^3 - x_0x_3^4 + x_1x_3^4 + x_0^4x_4 + (2c/(c + 2))x_0^3x_1x_4 + (3c/(c + 2))x_0x_1^3x_4 - x_1^4x_4 + (3c/(c + 2))x_0^3x_2x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_2x_4 + (2c^2/(c^2 + 1))x_0x_1^2x_2x_4 + (2c/(c + 2))x_1^3x_2x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2^2x_4 + (c/(c + 2))x_0x_2^3x_4 + (4c/(c + 2))x_1x_2^3x_4 + (3c/(c + 2))x_0^3x_3x_4 + (3c^2/(c^2 + 1))x_0^2x_1x_3x_4 + (2c^2/(c^2 + 1))x_0x_1^2x_3x_4 + (2c/(c + 2))x_1^3x_3x_4 + (c^2/(c^2 + 1))x_0^2x_2x_3x_4 + (4c^2/(c^2 + 1))x_1^2x_2x_3x_4 + (3c^2/(c^2 + 1))x_0x_2^2x_3x_4 + (2c^2/(c^2 + 1))x_1x_2^2x_3x_4 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3^2x_4 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_3^2x_4 + (3c^2/(c^2 + 1))x_0x_2x_3^2x_4 + (2c^2/(c^2 + 1))x_1x_2x_3^2x_4 + (c/(c + 2))x_0x_3^3x_4 + (4c/(c + 2))x_1x_3^3x_4 + ((4c + 1)/(c + 2))x_0^3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0^2x_1x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_0x_1^2x_4^2 + ((c + 4)/(c + 2))x_1^3x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_2x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_2x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_2^2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_2^2x_4^2 + ((3c^2 + 2c)/(c^2 + 1))x_0^2x_3x_4^2 + ((2c^2 + 3c)/(c^2 + 1))x_1^2x_3x_4^2 + (3c^2/(c^2 + 1))x_0x_2x_3x_4^2 + (2c^2/(c^2 + 1))x_1x_2x_3x_4^2 + ((4c^2 + c)/(c^2 + 1))x_0x_3^2x_4^2 + ((c^2 + 4c)/(c^2 + 1))x_1x_3^2x_4^2 + ((c + 4)/(c + 2))x_0^2x_4^3 + ((4c + 1)/(c + 2))x_1^2x_4^3 + (c/(c + 2))x_0x_2x_4^3 + (4c/(c + 2))x_1x_2x_4^3 + (c/(c + 2))x_0x_3x_4^3 + (4c/(c + 2))x_1x_3x_4^3 - x_0x_4^4 + x_1x_4^4$$

## 5 $S_6$ , char 3, $\tau$ trivial

### 5.1 Hilbert polynomial

PARTIAL

$$1 + 6t + 21t^2 + 51t^3 + 96t^4 + 147t^5 + 192t^6 + 222t^7 + \dots$$

(agrees with  $(t^2 + t + 1)^6(t^6 + t^3 + 1)$  conjecture)

### 5.2 Generators

PARTIAL

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3$$

$$((2c + 2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c + 2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c + 2))x_0x_1x_3 + (c/(c + 2))x_0x_2x_3 - x_0x_3^2 - x_0^2x_4 + x_1^2x_4 + (2c/(c + 2))x_1x_2x_4 + x_2^2x_4 + (2c/(c + 2))x_1x_3x_4 + (2c/(c + 2))x_2x_3x_4 + x_3^2x_4 + x_0x_4^2 - x_1x_4^2 - x_2x_4^2 - x_3x_4^2 + ((c + 1)/c)x_4^3 + x_0^2x_5 + (c/(c + 2))x_0x_1x_5 + (c/(c + 2))x_0x_2x_5 + (c/(c + 2))x_0x_3x_5 + (2c/(c + 2))x_1x_4x_5 + (2c/(c + 2))x_2x_4x_5 + (2c/(c + 2))x_3x_4x_5 - x_4^2x_5 - x_0x_5^2 + x_4x_5^2$$

$$((2c + 2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c + 2))x_0x_1x_2 - x_0x_2^2 - x_0^2x_3 + x_1^2x_3 + (2c/(c + 2))x_1x_2x_3 + x_2^2x_3 + x_0x_3^2 - x_1x_3^2 - x_2x_3^2 + ((c + 1)/c)x_3^3 + x_0^2x_4 + (c/(c + 2))x_0x_1x_4 + (c/(c + 2))x_0x_2x_4 + (2c/(c + 2))x_1x_3x_4 + (2c/(c + 2))x_2x_3x_4 - x_3^2x_4 - x_0x_4^2 + x_3x_4^2 + x_0^2x_5 + (c/(c + 2))x_0x_1x_5 + (c/(c + 2))x_0x_2x_5 + (2c/(c + 2))x_1x_3x_5 + (2c/(c + 2))x_2x_3x_5 - x_3^2x_5 + (c/(c + 2))x_0x_4x_5 + (2c/(c + 2))x_3x_4x_5 - x_0x_5^2 + x_3x_5^2$$

$$((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 - x_0^2x_2 + x_1^2x_2 + x_0x_2^2 - x_1x_2^2 + ((c+1)/c)x_2^3 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (2c/(c+2))x_1x_2x_3 - x_2^2x_3 - x_0x_3^2 + x_2x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (2c/(c+2))x_1x_2x_4 - x_2^2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_2x_3x_4 - x_0x_4^2 + x_2x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (2c/(c+2))x_1x_2x_5 - x_2^2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_2x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_2x_4x_5 - x_0x_5^2 + x_2x_5^2$$

$$((2c+2)/c)x_0^3 - x_0^2x_1 + x_0x_1^2 + ((c+1)/c)x_1^3 + x_0^2x_2 - x_1^2x_2 - x_0x_2^2 + x_1x_2^2 + x_0^2x_3 - x_1^2x_3 + (c/(c+2))x_0x_2x_3 + (2c/(c+2))x_1x_2x_3 - x_0x_3^2 + x_1x_3^2 + x_0^2x_4 - x_1^2x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_1x_3x_4 - x_0x_4^2 + x_1x_4^2 + x_0^2x_5 - x_1^2x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_1x_4x_5 - x_0x_5^2 + x_1x_5^2$$

conjecture:  $x_0^9$ .

## 6 $S_6$ , char 2, $\tau$ trivial

### 6.1 Hilbert polynomial

$$t^8 + 6t^7 + 16t^6 + 26t^5 + 30t^4 + 26t^3 + 16t^2 + 6t + 1 = (t+1)^6(t^2+1)$$

### 6.2 Generators

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_0x_3 + x_1x_4 + x_2x_4 + x_3x_4 + ((c+1)/c)x_4^2 + x_0x_5 + x_4x_5$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_1x_3 + x_2x_3 + ((c+1)/c)x_3^2 + x_0x_4 + x_3x_4 + x_0x_5 + x_3x_5$$

$$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3 + x_0x_4 + x_2x_4 + x_0x_5 + x_2x_5$$

$$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3 + x_0x_4 + x_1x_4 + x_0x_5 + x_1x_5$$

$$x_0^4.$$

## 7 $S_9$ , char 3, $\tau$ trivial

### 7.1 Hilbert polynomial

PARTIAL

$$\dots + 157t^3 + 45t^2 + 9t + 1$$

(fits with  $(t^2 + t + 1)^9(t^6 + t^3 + 1)$  conjecture)

### 7.2 Generators

PARTIAL

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + x_8^3$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\ & x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + \\ & (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 - x_0x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + \\ & (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (c/(c+2))x_0x_5x_6 - x_0x_6^2 - x_0^2x_7 + x_1^2x_7 + (2c/(c+2))x_1x_2x_7 + x_2^2x_7 + \\ & (2c/(c+2))x_1x_3x_7 + (2c/(c+2))x_2x_3x_7 + x_3^2x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 + \\ & x_4^2x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 + x_5^2x_7 + (2c/(c+2))x_1x_6x_7 + \\ & (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 + x_6^2x_7 + x_0x_7^2 - \\ & x_1x_7^2 - x_2x_7^2 - x_3x_7^2 - x_4x_7^2 - x_5x_7^2 - x_6x_7^2 + ((c+1)/c)x_7^3 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + \\ & (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_1x_7x_8 + (2c/(c+2))x_2x_7x_8 + \\ & (2c/(c+2))x_3x_7x_8 + (2c/(c+2))x_4x_7x_8 + (2c/(c+2))x_5x_7x_8 + (2c/(c+2))x_6x_7x_8 - x_7^2x_8 - x_0x_8^2 + x_7x_8^2 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\ & x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + \\ & (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 - x_0x_5^2 - x_0^2x_6 + x_1^2x_6 + (2c/(c+2))x_1x_2x_6 + x_2^2x_6 + \\ & (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 + x_3^2x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 + \\ & x_4^2x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + (2c/(c+2))x_4x_5x_6 + x_5^2x_6 + x_0x_6^2 - x_1x_6^2 - \\ & x_2x_6^2 - x_3x_6^2 - x_4x_6^2 - x_5x_6^2 + ((c+1)/c)x_6^3 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + \\ & (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + \\ & (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 - x_6^2x_7 - x_0x_7^2 + x_6x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + \\ & (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_6x_8 + (2c/(c+2))x_2x_6x_8 + (2c/(c+2))x_3x_6x_8 + \\ & (2c/(c+2))x_4x_6x_8 + (2c/(c+2))x_5x_6x_8 - x_6^2x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_6x_7x_8 - x_0x_8^2 + x_6x_8^2 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\ & x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 - x_0^2x_5 + x_1^2x_5 + (2c/(c+2))x_1x_2x_5 + \\ & x_2^2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 + x_3^2x_5 + (2c/(c+2))x_1x_4x_5 + (2c/(c+2))x_2x_4x_5 + \\ & (2c/(c+2))x_3x_4x_5 + x_4^2x_5 + x_0x_5^2 - x_1x_5^2 - x_2x_5^2 - x_3x_5^2 - x_4x_5^2 + ((c+1)/c)x_5^3 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + \\ & (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + \\ & (2c/(c+2))x_4x_5x_6 - x_5^2x_6 - x_0x_6^2 + x_5x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + \\ & (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 - x_5^2x_7 + \\ & (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_5x_6x_7 - x_0x_7^2 + x_5x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + \\ & (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_5x_8 + (2c/(c+2))x_2x_5x_8 + (2c/(c+2))x_3x_5x_8 + (2c/(c+2))x_4x_5x_8 - x_5^2x_8 + \\ & (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_5x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_5x_7x_8 - x_0x_8^2 + x_5x_8^2 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - \\ & x_0x_3^2 - x_0^2x_4 + x_1^2x_4 + (2c/(c+2))x_1x_2x_4 + x_2^2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 + x_3^2x_4 + x_0x_4^2 - \\ & x_1x_4^2 - x_2x_4^2 - x_3x_4^2 + ((c+1)/c)x_4^3 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_4x_5 + \\ & (2c/(c+2))x_2x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_4^2x_5 - x_0x_5^2 + x_4x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + \\ & (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 - x_4^2x_6 + (c/(c+2))x_0x_5x_6 + \\ & (2c/(c+2))x_4x_5x_6 - x_0x_6^2 + x_4x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + \\ & (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 - x_4^2x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_4x_5x_7 + \\ & (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_4x_6x_7 - x_0x_7^2 + x_4x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + \\ & (2c/(c+2))x_1x_4x_8 + (2c/(c+2))x_2x_4x_8 + (2c/(c+2))x_3x_4x_8 - x_4^2x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_4x_5x_8 + \\ & (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_4x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_4x_7x_8 - x_0x_8^2 + x_4x_8^2 \end{aligned}$$

$$\begin{aligned} & ((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 - x_0^2x_3 + x_1^2x_3 + (2c/(c+2))x_1x_2x_3 + x_2^2x_3 + \\ & x_0x_3^2 - x_1x_3^2 - x_2x_3^2 + ((c+1)/c)x_3^3 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_3x_4 + (2c/(c+2))x_2x_3x_4 - \\ & x_3^2x_4 - x_0x_4^2 + x_3x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_3x_5 + (2c/(c+2))x_2x_3x_5 - \\ & x_3^2x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_3x_4x_5 - x_0x_5^2 + x_3x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + \\ & (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 - x_3^2x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_3x_4x_6 + (c/(c+2))x_0x_5x_6 + \\ & (2c/(c+2))x_3x_5x_6 - x_0x_6^2 + x_3x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (2c/(c+2))x_1x_3x_7 + \\ & (2c/(c+2))x_2x_3x_7 - x_3^2x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_3x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_3x_5x_7 + \\ & (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_3x_6x_7 - x_0x_7^2 + x_3x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (2c/(c+2))x_1x_3x_8 + \\ & (2c/(c+2))x_2x_3x_8 - x_3^2x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_3x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_3x_5x_8 + \\ & (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_3x_6x_8 - x_0x_8^2 + x_3x_8^2 \end{aligned}$$

$$2))x_1x_3x_8 + (2c/(c+2))x_2x_3x_8 - x_3^2x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_3x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_3x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_3x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_3x_7x_8 - x_0x_8^2 + x_3x_8^2$$

$$((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 - x_0^2x_2 + x_1^2x_2 + x_0x_2^2 - x_1x_2^2 + ((c+1)/c)x_2^3 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (2c/(c+2))x_1x_2x_3 - x_2^2x_3 - x_0x_3^2 + x_2x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (2c/(c+2))x_1x_2x_4 - x_2^2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_2x_3x_4 - x_0x_4^2 + x_2x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (2c/(c+2))x_1x_2x_5 - x_2^2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_2x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_2x_4x_5 - x_0x_5^2 + x_2x_5^2 + x_0^2x_6 + (c/(c+2))x_0x_1x_6 + (2c/(c+2))x_1x_2x_6 - x_2^2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_2x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_2x_4x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_2x_5x_6 - x_0x_6^2 + x_2x_6^2 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (2c/(c+2))x_1x_2x_7 - x_2^2x_7 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_2x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_2x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_2x_5x_7 + (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_2x_6x_7 - x_0x_7^2 + x_2x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (2c/(c+2))x_1x_2x_8 - x_2^2x_8 + (c/(c+2))x_0x_3x_8 + (2c/(c+2))x_2x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_2x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_2x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_2x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_2x_7x_8 - x_0x_8^2 + x_2x_8^2$$

$$((2c+2)/c)x_0^3 - x_0^2x_1 + x_0x_1^2 + ((c+1)/c)x_1^3 + x_0^2x_2 - x_1^2x_2 - x_0x_2^2 + x_1x_2^2 + x_0^2x_3 - x_1^2x_3 + (c/(c+2))x_0x_2x_3 + (2c/(c+2))x_1x_2x_3 - x_0x_3^2 + x_1x_3^2 + x_0^2x_4 - x_1^2x_4 + (c/(c+2))x_0x_2x_4 + (2c/(c+2))x_1x_2x_4 + (c/(c+2))x_0x_3x_4 + (2c/(c+2))x_1x_3x_4 - x_0x_4^2 + x_1x_4^2 + x_0^2x_5 - x_1^2x_5 + (c/(c+2))x_0x_2x_5 + (2c/(c+2))x_1x_2x_5 + (c/(c+2))x_0x_3x_5 + (2c/(c+2))x_1x_3x_5 + (c/(c+2))x_0x_4x_5 + (2c/(c+2))x_1x_4x_5 - x_0x_5^2 + x_1x_5^2 + x_0^2x_6 - x_1^2x_6 + (c/(c+2))x_0x_2x_6 + (2c/(c+2))x_1x_2x_6 + (c/(c+2))x_0x_3x_6 + (2c/(c+2))x_1x_3x_6 + (c/(c+2))x_0x_4x_6 + (2c/(c+2))x_1x_4x_6 + (c/(c+2))x_0x_5x_6 + (2c/(c+2))x_1x_5x_6 - x_0x_6^2 + x_1x_6^2 + x_0^2x_7 - x_1^2x_7 + (c/(c+2))x_0x_2x_7 + (2c/(c+2))x_1x_2x_7 + (c/(c+2))x_0x_3x_7 + (2c/(c+2))x_1x_3x_7 + (c/(c+2))x_0x_4x_7 + (2c/(c+2))x_1x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_5x_7 + (c/(c+2))x_0x_6x_7 + (2c/(c+2))x_1x_6x_7 - x_0x_7^2 + x_1x_7^2 + x_0^2x_8 - x_1^2x_8 + (c/(c+2))x_0x_2x_8 + (2c/(c+2))x_1x_2x_8 + (c/(c+2))x_0x_3x_8 + (2c/(c+2))x_1x_3x_8 + (c/(c+2))x_0x_4x_8 + (2c/(c+2))x_1x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_5x_8 + (c/(c+2))x_0x_6x_8 + (2c/(c+2))x_1x_6x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_1x_7x_8 - x_0x_8^2 + x_1x_8^2$$

conjecture:  $x_0^9$