Computations
$\bar{h} = 1$

1 S_2, char 2, τ trivial

1.1 Hilbert polynomial

$t^4 + 2t^3 + 2t^2 + 2t + 1 = (t+1)^2(t^2+1)$.

1.2 Generators

$x_0^2 + x_1^2$

x_0^4.

2 S_3, char 3, τ trivial

2.1 Hilbert polynomial

$t^{12} + 3t^{11} + 6t^{10} + 8t^9 + 9t^8 + 9t^7 + 9t^6 + 9t^5 + 9t^4 + 8t^3 + 6t^2 + 3t + 1 = (t^2+t+1)^3(t^6+t^3+1)$.

2.2 Generators

$x_0^3 + x_1^3 + x_2^3$

$((2c+2)/c)x_0^3$
$+ ((c+1)/c)x_1^3$
$- x_0^2x_1 - x_1^2x_2 - x_0x_2^2$
$+ x_0x_1^2 + x_0^2x_2 + x_1x_2^2$

x_0^9.

3 S_4, char 2, τ trivial

3.1 Hilbert polynomial

$t^6 + 4t^5 + 7t^4 + 8t^3 + 7t^2 + 4t + 1 = (t+1)^4(t^2+1)$.

3.2 Generators

$x_0^2 + x_1^2 + x_2^2 + x_3^2$

$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3$

$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3$

x_0^4.

4 S_5, char 5, τ trivial

4.1 Hilbert polynomial

PARTIAL

$1 + 5t + 15t^2 + 35t^3 + 70t^4 + 122t^5 + 190t^6 + 270t^7 + \ldots$

4.2 Generators

PARTIAL

$x_0^5 + x_1^5 + x_2^5 + x_3^5 + x_4^5$

$((4c+4)/c)x_0^5 + x_0^4x_1 + x_0^4x_2 - x_0x_1^4 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 + (3c/(c+2))x_0^3x_1x_2 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_2 + (c/(c+2))x_0x_1^3x_2 + ((4c+1)/(c+2))x_0^3x_2^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_2^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_2^2 + ((c+4)/(c+2))x_0^2x_2^3 + (c/(c+2))x_0x_1x_2^3 - x_0x_2^4 + 2x_0^4x_3 + (2c/(c+2))x_0^3x_1x_3 + ((4c^2+c)/(c^2+1))x_0^2x_1^2x_3 + x_1^4x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2+1))x_0^2x_1x_2x_3 + (4c/(c+2))x_1^3x_2x_3 + ((4c^2+c)/(c^2+1))x_0^2x_2^2x_3 + ((c^2+4c)/(c^2+1))x_1^2x_2^2x_3 + (4c/(c+2))x_1x_2^3x_3 + x_2^4x_3 + ((3c+2)/(c+2))x_0^3x_3^2 + ((c^2+4c)/(c^2+1))x_0x_1^2x_3^2 + ((4c+1)/(c+2))x_1^3x_3^2 + (2c^2/(c^2+1))x_0x_1x_2x_3^2 + ((2c^2+3c)/(c^2+1))x_1^2x_2x_3^2 + ((c^2+4c)/(c^2+1))x_0x_2^2x_3^2 + ((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2 + ((4c+1)/(c+2))x_2^3x_3^2 + ((2c+3)/(c+2))x_0^2x_3^3 + (3c/(c+2))x_0x_1x_3^3 + ((c+4)/(c+2))x_1^2x_3^3 + (3c/(c+2))x_0x_2x_3^3 + (2c/(c+2))x_1x_2x_3^3 + ((c+4)/(c+2))x_2^2x_3^3 + 3x_0x_3^4 - x_1x_3^4 - x_2x_3^4 + ((c+1)/c)x_3^5 + x_0^4x_4 + (3c/(c+2))x_0^3x_1x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_4 + (c/(c+2))x_0x_1^3x_4 + (3c/(c+2))x_0^3x_2x_4 + (c^2/(c^2+1))x_0^2x_1x_2x_4 + (3c^2/(c^2+1))x_0x_1^2x_2x_4 + ((3c^2+2c)/(c^2+1))x_0^2x_2^2x_4 + (3c^2/(c^2+1))x_0x_1x_2^2x_4 + (c/(c+2))x_0x_2^3x_4 + (2c/(c+2))x_0^3x_3x_4 + (3c^2/(c^2+1))x_0^2x_1x_3x_4 + (4c/(c+2))x_1^3x_3x_4 + (3c^2/(c^2+1))x_0^2x_2x_3x_4 + (2c^2/(c^2+1))x_1^2x_2x_3x_4 + (2c^2/(c^2+1))x_1x_2^2x_3x_4 + (4c/(c+2))x_2^3x_3x_4 + (2c^2/(c^2+1))x_0x_1x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4 + (2c^2/(c^2+1))x_0x_2x_3^2x_4 + (4c^2/(c^2+1))x_1x_2x_3^2x_4 + ((2c^2+3c)/(c^2+1))x_2^2x_3^2x_4 + (3c/(c+2))x_0x_3^3x_4 + (2c/(c+2))x_1x_3^3x_4 + (2c/(c+2))x_2x_3^3x_4 - x_3^4x_4 + ((4c+1)/(c+2))x_0^3x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_4^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_4^2 + ((3c^2+2c)/(c^2+1))x_0^2x_2x_4^2 + (3c^2/(c^2+1))x_0x_1x_2x_4^2 + ((4c^2+c)/(c^2+1))x_0x_2^2x_4^2 + ((4c^2+c)/(c^2+1))x_0^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_1^2x_3x_4^2 + (2c^2/(c^2+1))x_1x_2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_2^2x_3x_4^2 + ((c^2+4c)/(c^2+1))x_0x_3^2x_4^2 + ((2c^2+3c)/(c^2+1))x_1x_3^2x_4^2 + ((2c^2+3c)/(c^2+1))x_2x_3^2x_4^2 + ((c+4)/(c+2))x_3^3x_4^2 + ((c+4)/(c+2))x_0^2x_4^3 + (c/(c+2))x_0x_1x_4^3 + (c/(c+2))x_0x_2x_4^3 + (4c/(c+2))x_1x_3x_4^3 + (4c/(c+2))x_2x_3x_4^3 + ((4c+1)/(c+2))x_3^2x_4^3 - x_0x_4^4 + x_3x_4^4$

$((4c+4)/c)x_0^5 + x_0^4x_1 + ((4c+1)/(c+2))x_0^3x_1^2 + ((c+4)/(c+2))x_0^2x_1^3 - x_0x_1^4 + 2x_0^4x_2 + (2c/(c+2))x_0^3x_1x_2 + ((4c^2+c)/(c^2+1))x_0^2x_1^2x_2 + x_1^4x_2 + ((3c+2)/(c+2))x_0^3x_2^2 + ((c^2+4c)/(c^2+1))x_0x_1^2x_2^2 + ((4c+1)/(c+2))x_1^3x_2^2 + ((2c+3)/(c+2))x_0^2x_2^3 + (3c/(c+2))x_0x_1x_2^3 + ((c+4)/(c+2))x_1^2x_2^3 + 3x_0x_2^4 - x_1x_2^4 + ((c+1)/c)x_2^5 + x_0^4x_3 + (3c/(c+2))x_0^3x_1x_3 + ((3c^2+2c)/(c^2+1))x_0^2x_1^2x_3 + (c/(c+2))x_0x_1^3x_3 + (2c/(c+2))x_0^3x_2x_3 + (3c^2/(c^2+1))x_0^2x_1x_2x_3 + (4c/(c+2))x_1^3x_2x_3 + (2c^2/(c^2+1))x_0x_1x_2^2x_3 + ((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3 + (3c/(c+2))x_0x_2^3x_3 + (2c/(c+2))x_1x_2^3x_3 - x_2^4x_3 + ((4c+1)/(c+2))x_0^3x_3^2 + ((3c^2+2c)/(c^2+1))x_0^2x_1x_3^2 + ((4c^2+c)/(c^2+1))x_0x_1^2x_3^2 + ((4c^2+$

$c)/(c^2+1))x_0^2x_2x_3^2+((c^2+4c)/(c^2+1))x_1^2x_2x_3^2+((c^2+4c)/(c^2+1))x_0x_2^2x_3^2+((2c^2+3c)/(c^2+1))x_1x_2^2x_3^2+((c+4)/(c+2))x_2^3x_3^2+((c+4)/(c+2))x_0^2x_3^3+(c/(c+2))x_0x_1x_3^3+(4c/(c+2))x_1x_2x_3^3+((4c+1)/(c+2))x_2^2x_3^3-x_0x_3^4+x_2x_3^4+x_0^4x_4+(3c/(c+2))x_0^3x_1x_4+((3c^2+2c)/(c^2+1))x_0^2x_1^2x_4+(c/(c+2))x_0x_1^3x_4+(2c/(c+2))x_0^3x_2x_4+(3c^2/(c^2+1))x_0^2x_1x_2x_4+(4c/(c+2))x_1^3x_2x_4+(2c^2/(c^2+1))x_0x_1x_2^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_4+(3c/(c+2))x_0x_2^3x_4+(2c/(c+2))x_1x_2^3x_4-x_2^4x_4+(3c/(c+2))x_0^3x_3x_4+(c^2/(c^2+1))x_0^2x_1x_3x_4+(3c^2/(c^2+1))x_0x_1^2x_3x_4+(3c^2/(c^2+1))x_0^2x_2x_3x_4+(2c^2/(c^2+1))x_1^2x_2x_3x_4+(2c^2/(c^2+1))x_0x_2^2x_3x_4+(4c^2/(c^2+1))x_1x_2^2x_3x_4+(2c/(c+2))x_2^3x_3x_4+((3c^2+2c)/(c^2+1))x_0^2x_3^2x_4+(3c^2/(c^2+1))x_0x_1x_3^2x_4+(2c^2/(c^2+1))x_1x_2x_3^2x_4+((2c^2+3c)/(c^2+1))x_2^2x_3^2x_4+(c/(c+2))x_0x_3^3x_4+(4c/(c+2))x_2x_3^3x_4+((4c+1)/(c+2))x_0^3x_4^2+((3c^2+2c)/(c^2+1))x_0^2x_1x_4^2+((4c^2+c)/(c^2+1))x_0x_1^2x_4^2+((4c^2+c)/(c^2+1))x_0^2x_2x_4^2+((c^2+4c)/(c^2+1))x_1^2x_2x_4^2+((c^2+4c)/(c^2+1))x_0x_2^2x_4^2+((2c^2+3c)/(c^2+1))x_1x_2^2x_4^2+((c+4)/(c+2))x_2^3x_4^2+((3c^2+2c)/(c^2+1))x_0^2x_3x_4^2+(3c^2/(c^2+1))x_0x_1x_3x_4^2+(2c^2/(c^2+1))x_1x_2x_3x_4^2+((2c^2+3c)/(c^2+1))x_2^2x_3x_4^2+((4c^2+c)/(c^2+1))x_0x_3^2x_4^2+((c^2+4c)/(c^2+1))x_2x_3^2x_4^2+((c+4)/(c+2))x_0^2x_4^3+(c/(c+2))x_0x_1x_4^3+(4c/(c+2))x_1x_2x_4^3+((4c+1)/(c+2))x_2^2x_4^3+(c/(c+2))x_0x_3x_4^3+(4c/(c+2))x_2x_3x_4^3-x_0x_4^4+x_2x_4^4$

$((4c+4)/c)x_0^5+2x_0^4x_1+((3c+2)/(c+2))x_0^3x_1^2+((2c+3)/(c+2))x_0^2x_1^3+3x_0x_1^4+((c+1)/c)x_1^5+x_0^4x_2+(2c/(c+2))x_0^3x_1x_2+(3c/(c+2))x_0x_1^3x_2-x_1^4x_2+((4c+1)/(c+2))x_0^3x_2^2+((4c^2+c)/(c^2+1))x_0^2x_1x_2^2+((c^2+4c)/(c^2+1))x_0x_1^2x_2^2+((c+4)/(c+2))x_1^3x_2^2+((c+4)/(c+2))x_0^2x_2^3+((4c+1)/(c+2))x_1^2x_2^3-x_0x_2^4+x_1x_2^4+x_0^4x_3+(2c/(c+2))x_0^3x_1x_3+(3c/(c+2))x_0x_1^3x_3-x_1^4x_3+(3c/(c+2))x_0^3x_2x_3+(3c^2/(c^2+1))x_0^2x_1x_2x_3+(2c^2/(c^2+1))x_0x_1^2x_2x_3+(2c/(c+2))x_1^3x_2x_3+((3c^2+2c)/(c^2+1))x_0^2x_2^2x_3+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_3+(c/(c+2))x_0x_2^3x_3+(4c/(c+2))x_1x_2^3x_3+((4c+1)/(c+2))x_0^3x_3^2+((4c^2+c)/(c^2+1))x_0^2x_1x_3^2+((c^2+4c)/(c^2+1))x_0x_1^2x_3^2+((c+4)/(c+2))x_1^3x_3^2+((3c^2+2c)/(c^2+1))x_0^2x_2x_3^2+((2c^2+3c)/(c^2+1))x_1^2x_2x_3^2+((4c^2+c)/(c^2+1))x_0x_2^2x_3^2+((c^2+4c)/(c^2+1))x_1x_2^2x_3^2+((c+4)/(c+2))x_0^2x_3^3+((4c+1)/(c+2))x_1^2x_3^3+(c/(c+2))x_0x_2x_3^3+(4c/(c+2))x_1x_2x_3^3-x_0x_3^4+x_1x_3^4+x_0^4x_4+(2c/(c+2))x_0^3x_1x_4+(3c/(c+2))x_0x_1^3x_4-x_1^4x_4+(3c/(c+2))x_0^3x_2x_4+(3c^2/(c^2+1))x_0^2x_1x_2x_4+(2c^2/(c^2+1))x_0x_1^2x_2x_4+(2c/(c+2))x_1^3x_2x_4+((3c^2+2c)/(c^2+1))x_0^2x_2^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_2^2x_4+(c/(c+2))x_0x_2^3x_4+(4c/(c+2))x_1x_2^3x_4+(3c/(c+2))x_0^3x_3x_4+(3c^2/(c^2+1))x_0^2x_1x_3x_4+(2c^2/(c^2+1))x_0x_1^2x_3x_4+(2c/(c+2))x_1^3x_3x_4+(c^2/(c^2+1))x_0^2x_2x_3x_4+(4c^2/(c^2+1))x_1^2x_2x_3x_4+(3c^2/(c^2+1))x_0x_2^2x_3x_4+(2c^2/(c^2+1))x_1x_2^2x_3x_4+((3c^2+2c)/(c^2+1))x_0^2x_3^2x_4+((2c^2+3c)/(c^2+1))x_1^2x_3^2x_4+(3c^2/(c^2+1))x_0x_2x_3^2x_4+(2c^2/(c^2+1))x_1x_2x_3^2x_4+(c/(c+2))x_0x_3^3x_4+(4c/(c+2))x_1x_3^3x_4+((4c+1)/(c+2))x_0^3x_4^2+((4c^2+c)/(c^2+1))x_0^2x_1x_4^2+((c^2+4c)/(c^2+1))x_0x_1^2x_4^2+((c+4)/(c+2))x_1^3x_4^2+((3c^2+2c)/(c^2+1))x_0^2x_2x_4^2+((c^2+4c)/(c^2+1))x_1x_2^2x_4^2+((4c^2+c)/(c^2+1))x_0x_2^2x_4^2+((c^2+4c)/(c^2+1))x_1x_2x_3x_4^2+((4c^2+c)/(c^2+1))x_0x_3^2x_4^2+((c^2+4c)/(c^2+1))x_1x_3^2x_4^2+((c+4)/(c+2))x_0^2x_4^3+((4c+1)/(c+2))x_1^2x_4^3+(c/(c+2))x_0x_2x_4^3+(4c/(c+2))x_1x_2x_4^3+(c/(c+2))x_0x_3x_4^3+(4c/(c+2))x_1x_3x_4^3-x_0x_4^4+x_1x_4^4$

conjecture: x_0^{25}

5 S_6, char 3, τ trivial

5.1 Hilbert polynomial

PARTIAL

$1+6t+21t^2+51t^3+96t^4+147t^5+192t^6+222t^7+\dots$

5.2 Generators

PARTIAL

$x_0^3+x_1^3+x_2^3+x_3^3+x_4^3+x_5^3$

$((2c+2)/c)x_0^3$

$+((c+1)/c)x_4^3$
$+x_0^2x_1+x_0^2x_2+x_0^2x_3+x_1^2x_4+x_2^2x_4+x_3^2x_4+x_0x_4^2+x_0^2x_5+x_4x_5^2$
$-x_0x_1^2-x_0x_2^2-x_0x_3^2-x_0^2x_4-x_1x_4^2-x_2x_4^2-x_3x_4^2-x_4^2x_5-x_0x_5^2$
$+(c/(c+2))x_0x_1x_2+(c/(c+2))x_0x_1x_3+(c/(c+2))x_0x_2x_3+(c/(c+2))x_0x_1x_5+(c/(c+2))x_0x_2x_5+(c/(c+2))x_0x_3x_5$
$+(2c/(c+2))x_1x_2x_4+(2c/(c+2))x_1x_3x_4+(2c/(c+2))x_2x_3x_4+(2c/(c+2))x_1x_4x_5+(2c/(c+2))x_2x_4x_5+(2c/(c+2))x_3x_4x_5$

$((2c+2)/c)x_0^3$
$+((c+1)/c)x_3^3$
$+x_0^2x_1+x_0^2x_2+x_1^2x_3+x_2^2x_3+x_0x_3^2+x_0^2x_4+x_3x_4^2+x_0^2x_5+x_3x_5^2$
$-x_0x_1^2-x_0x_2^2-x_0^2x_3-x_1x_3^2-x_2x_3^2-x_3^2x_4-x_0x_4^2-x_3^2x_5-x_0x_5^2$
$+(c/(c+2))x_0x_1x_2+(c/(c+2))x_0x_1x_4+(c/(c+2))x_0x_2x_4+(c/(c+2))x_0x_1x_5+(c/(c+2))x_0x_2x_5+(c/(c+2))x_0x_4x_5$
$+(2c/(c+2))x_1x_2x_3+(2c/(c+2))x_1x_3x_4+(2c/(c+2))x_2x_3x_4+(2c/(c+2))x_1x_3x_5+(2c/(c+2))x_2x_3x_5+(2c/(c+2))x_3x_4x_5$

$((2c+2)/c)x_0^3$
$+((c+1)/c)x_2^3$
$+x_0^2x_1+x_1^2x_2+x_0x_2^2+x_0^2x_3+x_2x_3^2+x_0^2x_4+x_2x_4^2+x_0^2x_5+x_2x_5^2$
$-x_0x_1^2-x_0^2x_2-x_1x_2^2-x_2^2x_3-x_0x_3^2-x_2^2x_4-x_0x_4^2-x_2^2x_5-x_0x_5^2$
$+(c/(c+2))x_0x_1x_3+(c/(c+2))x_0x_1x_4+(c/(c+2))x_0x_3x_4+(c/(c+2))x_0x_1x_5+(c/(c+2))x_0x_3x_5+(c/(c+2))x_0x_4x_5$
$+(2c/(c+2))x_1x_2x_3+(2c/(c+2))x_1x_2x_4+(2c/(c+2))x_2x_3x_4+(2c/(c+2))x_1x_2x_5+(2c/(c+2))x_2x_3x_5+(2c/(c+2))x_2x_4x_5$

$((2c+2)/c)x_0^3$
$+((c+1)/c)x_1^3$
$-x_0^2x_1-x_1^2x_2-x_0x_2^2-x_1^2x_3-x_0x_3^2-x_1^2x_4-x_0x_4^2-x_1^2x_5-x_0x_5^2$
$+x_0x_1^2+x_0^2x_2+x_1x_2^2+x_0^2x_3+x_1x_3^2+x_0^2x_4+x_1x_4^2+x_0^2x_5+x_1x_5^2$
$+(c/(c+2))x_0x_2x_3+(c/(c+2))x_0x_2x_4+(c/(c+2))x_0x_3x_4+(c/(c+2))x_0x_2x_5+(c/(c+2))x_0x_3x_5+(c/(c+2))x_0x_4x_5$
$+(2c/(c+2))x_1x_2x_3+(2c/(c+2))x_1x_2x_4+(2c/(c+2))x_1x_3x_4+(2c/(c+2))x_1x_2x_5+(2c/(c+2))x_1x_3x_5+(2c/(c+2))x_1x_4x_5$

conjecture: x_0^9.

6 S_6, char 2, τ trivial

6.1 Hilbert polynomial

$t^8+6t^7+16t^6+26t^5+30t^4+26t^3+16t^2+6t+1=(t+1)^6(t^2+1)$

6.2 Generators

$x_0^2+x_1^2+x_2^2+x_3^2+x_4^2+x_5^2$

$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_0x_3 + x_1x_4 + x_2x_4 + x_3x_4 + ((c+1)/c)x_4^2 + x_0x_5 + x_4x_5$

$((c+1)/c)x_0^2 + x_0x_1 + x_0x_2 + x_1x_3 + x_2x_3 + ((c+1)/c)x_3^2 + x_0x_4 + x_3x_4 + x_0x_5 + x_3x_5$

$((c+1)/c)x_0^2 + x_0x_1 + x_1x_2 + ((c+1)/c)x_2^2 + x_0x_3 + x_2x_3 + x_0x_4 + x_2x_4 + x_0x_5 + x_2x_5$

$((c+1)/c)x_0^2 + ((c+1)/c)x_1^2 + x_0x_2 + x_1x_2 + x_0x_3 + x_1x_3 + x_0x_4 + x_1x_4 + x_0x_5 + x_1x_5$

x_0^4.

7 S_9, char 3, τ trivial

7.1 Hilbert polynomial

PARTIAL

$\cdots + 157t^3 + 45t^2 + 9t + 1$

7.2 Generators

PARTIAL

$x_0^3 + x_1^3 + x_2^3 + x_3^3 + x_4^3 + x_5^3 + x_6^3 + x_7^3 + x_8^3$

$((2c+2)/c)x_0^3$
$+ ((c+1)/c)x_7^3$
$+ x_0^2x_1 + x_0^2x_2 + x_0^2x_3 + x_0^2x_4 + x_0^2x_5 + x_0^2x_6 + x_1^2x_7 + x_2^2x_7 + x_3^2x_7 + x_4^2x_7 + x_5^2x_7 + x_6^2x_7 + x_0x_7^2 + x_0^2x_8 + x_7x_8^2$
$- x_0x_1^2 - x_0x_2^2 - x_0x_3^2 - x_0x_4^2 - x_0x_5^2 - x_0x_6^2 - x_0^2x_7 - x_1x_7^2 - x_2x_7^2 - x_3x_7^2 - x_4x_7^2 - x_5x_7^2 - x_6x_7^2 - x_7^2x_8 - x_0x_8^2$
$+ (c/(c+2))x_0x_1x_2 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 + (c/(c+2))x_0x_1x_6 + (c/(c+2))x_0x_2x_6 + (c/(c+2))x_0x_3x_6 + (c/(c+2))x_0x_4x_6 + (c/(c+2))x_0x_5x_6 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (c/(c+2))x_0x_6x_8$
$+ (2c/(c+2))x_1x_2x_7 + (2c/(c+2))x_1x_3x_7 + (2c/(c+2))x_2x_3x_7 + (2c/(c+2))x_1x_4x_7 + (2c/(c+2))x_2x_4x_7 + (2c/(c+2))x_3x_4x_7 + (2c/(c+2))x_1x_5x_7 + (2c/(c+2))x_2x_5x_7 + (2c/(c+2))x_3x_5x_7 + (2c/(c+2))x_4x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 + (2c/(c+2))x_1x_7x_8 + (2c/(c+2))x_2x_7x_8 + (2c/(c+2))x_3x_7x_8 + (2c/(c+2))x_4x_7x_8 + (2c/(c+2))x_5x_7x_8 + (2c/(c+2))x_6x_7x_8$

$((2c+2)/c)x_0^3 + x_0^2x_1 - x_0x_1^2 + x_0^2x_2 + (c/(c+2))x_0x_1x_2 - x_0x_2^2 + x_0^2x_3 + (c/(c+2))x_0x_1x_3 + (c/(c+2))x_0x_2x_3 - x_0x_3^2 + x_0^2x_4 + (c/(c+2))x_0x_1x_4 + (c/(c+2))x_0x_2x_4 + (c/(c+2))x_0x_3x_4 - x_0x_4^2 + x_0^2x_5 + (c/(c+2))x_0x_1x_5 + (c/(c+2))x_0x_2x_5 + (c/(c+2))x_0x_3x_5 + (c/(c+2))x_0x_4x_5 - x_0x_5^2 - x_0^2x_6 + x_1^2x_6 + (2c/(c+2))x_1x_2x_6 + x_2^2x_6 + (2c/(c+2))x_1x_3x_6 + (2c/(c+2))x_2x_3x_6 + x_3^2x_6 + (2c/(c+2))x_1x_4x_6 + (2c/(c+2))x_2x_4x_6 + (2c/(c+2))x_3x_4x_6 + x_4^2x_6 + (2c/(c+2))x_1x_5x_6 + (2c/(c+2))x_2x_5x_6 + (2c/(c+2))x_3x_5x_6 + (2c/(c+2))x_4x_5x_6 + x_5^2x_6 + x_0x_6^2 - x_1x_6^2 - x_2x_6^2 - x_3x_6^2 - x_4x_6^2 - x_5x_6^2 + ((c+1)/c)x_6^3 + x_0^2x_7 + (c/(c+2))x_0x_1x_7 + (c/(c+2))x_0x_2x_7 + (c/(c+2))x_0x_3x_7 + (c/(c+2))x_0x_4x_7 + (c/(c+2))x_0x_5x_7 + (2c/(c+2))x_1x_6x_7 + (2c/(c+2))x_2x_6x_7 + (2c/(c+2))x_3x_6x_7 + (2c/(c+2))x_4x_6x_7 + (2c/(c+2))x_5x_6x_7 - x_6^2x_7 - x_0x_7^2 + x_6x_7^2 + x_0^2x_8 + (c/(c+2))x_0x_1x_8 + (c/(c+2))x_0x_2x_8 + (c/(c+2))x_0x_3x_8 + (c/(c+2))x_0x_4x_8 + (c/(c+2))x_0x_5x_8 + (2c/(c+2))x_1x_6x_8 + (2c/(c+2))x_2x_6x_8 + (2c/(c+2))x_3x_6x_8 + (2c/(c+2))x_4x_6x_8 + (2c/(c+2))x_5x_6x_8 - x_6^2x_8 + (c/(c+2))x_0x_7x_8 + (2c/(c+2))x_6x_7x_8 - x_0x_8^2 + x_6x_8^2$

$$((2c+2)/c)x_0^3+x_0^2x_1-x_0x_1^2+x_0^2x_2+(c/(c+2))x_0x_1x_2-x_0x_2^2+x_0^2x_3+(c/(c+2))x_0x_1x_3+(c/(c+2))x_0x_2x_3-x_0x_3^2+x_0^2x_4+(c/(c+2))x_0x_1x_4+(c/(c+2))x_0x_2x_4+(c/(c+2))x_0x_3x_4-x_0x_4^2-x_0^2x_5+x_1^2x_5+(2c/(c+2))x_1x_2x_5+x_2^2x_5+(2c/(c+2))x_1x_3x_5+(2c/(c+2))x_2x_3x_5+x_3^2x_5+(2c/(c+2))x_1x_4x_5+(2c/(c+2))x_2x_4x_5+(2c/(c+2))x_3x_4x_5+x_4^2x_5+x_0x_5^2-x_1x_5^2-x_2x_5^2-x_3x_5^2-x_4x_5^2+((c+1)/c)x_5^3+x_0^2x_6+(c/(c+2))x_0x_1x_6+(c/(c+2))x_0x_2x_6+(c/(c+2))x_0x_3x_6+(c/(c+2))x_0x_4x_6+(2c/(c+2))x_1x_5x_6+(2c/(c+2))x_2x_5x_6+(2c/(c+2))x_3x_5x_6+(2c/(c+2))x_4x_5x_6-x_5^2x_6-x_0x_6^2+x_5x_6^2+x_0^2x_7+(c/(c+2))x_0x_1x_7+(c/(c+2))x_0x_2x_7+(c/(c+2))x_0x_3x_7+(c/(c+2))x_0x_4x_7+(2c/(c+2))x_1x_5x_7+(2c/(c+2))x_2x_5x_7+(2c/(c+2))x_3x_5x_7+(2c/(c+2))x_4x_5x_7-x_5^2x_7+(c/(c+2))x_0x_6x_7+(2c/(c+2))x_5x_6x_7-x_0x_7^2+x_5x_7^2+x_0^2x_8+(c/(c+2))x_0x_1x_8+(c/(c+2))x_0x_2x_8+(c/(c+2))x_0x_3x_8+(c/(c+2))x_0x_4x_8+(2c/(c+2))x_1x_5x_8+(2c/(c+2))x_2x_5x_8+(2c/(c+2))x_3x_5x_8+(2c/(c+2))x_4x_5x_8-x_5^2x_8+(c/(c+2))x_0x_6x_8+(2c/(c+2))x_5x_6x_8+(c/(c+2))x_0x_7x_8+(2c/(c+2))x_5x_7x_8-x_0x_8^2+x_5x_8^2$$

$$((2c+2)/c)x_0^3+x_0^2x_1-x_0x_1^2+x_0^2x_2+(c/(c+2))x_0x_1x_2-x_0x_2^2+x_0^2x_3+(c/(c+2))x_0x_1x_3+(c/(c+2))x_0x_2x_3-x_0x_3^2-x_0^2x_4+x_1^2x_4+(2c/(c+2))x_1x_2x_4+x_2^2x_4+(2c/(c+2))x_1x_3x_4+(2c/(c+2))x_2x_3x_4+x_3^2x_4+x_0x_4^2-x_1x_4^2-x_2x_4^2-x_3x_4^2+((c+1)/c)x_4^3+x_0^2x_5+(c/(c+2))x_0x_1x_5+(c/(c+2))x_0x_2x_5+(c/(c+2))x_0x_3x_5+(2c/(c+2))x_1x_4x_5+(2c/(c+2))x_2x_4x_5+(2c/(c+2))x_3x_4x_5-x_4^2x_5-x_0x_5^2+x_4x_5^2+x_0^2x_6+(c/(c+2))x_0x_1x_6+(c/(c+2))x_0x_2x_6+(c/(c+2))x_0x_3x_6+(2c/(c+2))x_1x_4x_6+(2c/(c+2))x_2x_4x_6+(2c/(c+2))x_3x_4x_6-x_4^2x_6+(c/(c+2))x_0x_5x_6+(2c/(c+2))x_4x_5x_6-x_0x_6^2+x_4x_6^2+x_0^2x_7+(c/(c+2))x_0x_1x_7+(c/(c+2))x_0x_2x_7+(c/(c+2))x_0x_3x_7+(2c/(c+2))x_1x_4x_7+(2c/(c+2))x_2x_4x_7+(2c/(c+2))x_3x_4x_7-x_4^2x_7+(c/(c+2))x_0x_5x_7+(2c/(c+2))x_4x_5x_7+(c/(c+2))x_0x_6x_7+(2c/(c+2))x_4x_6x_7-x_0x_7^2+x_4x_7^2+x_0^2x_8+(c/(c+2))x_0x_1x_8+(c/(c+2))x_0x_2x_8+(c/(c+2))x_0x_3x_8+(2c/(c+2))x_1x_4x_8+(2c/(c+2))x_2x_4x_8+(2c/(c+2))x_3x_4x_8-x_4^2x_8+(c/(c+2))x_0x_5x_8+(2c/(c+2))x_4x_5x_8+(c/(c+2))x_0x_6x_8+(2c/(c+2))x_4x_6x_8+(c/(c+2))x_0x_7x_8+(2c/(c+2))x_4x_7x_8-x_0x_8^2+x_4x_8^2$$

$$((2c+2)/c)x_0^3+x_0^2x_1-x_0x_1^2+x_0^2x_2+(c/(c+2))x_0x_1x_2-x_0x_2^2-x_0^2x_3+x_1^2x_3+(2c/(c+2))x_1x_2x_3+x_2^2x_3+x_0x_3^2-x_1x_3^2-x_2x_3^2+((c+1)/c)x_3^3+x_0^2x_4+(c/(c+2))x_0x_1x_4+(c/(c+2))x_0x_2x_4+(2c/(c+2))x_1x_3x_4+(2c/(c+2))x_2x_3x_4-x_3^2x_4-x_0x_4^2+x_3x_4^2+x_0^2x_5+(c/(c+2))x_0x_1x_5+(c/(c+2))x_0x_2x_5+(2c/(c+2))x_1x_3x_5+(2c/(c+2))x_2x_3x_5-x_3^2x_5+(c/(c+2))x_0x_4x_5+(2c/(c+2))x_3x_4x_5-x_0x_5^2+x_3x_5^2+x_0^2x_6+(c/(c+2))x_0x_1x_6+(c/(c+2))x_0x_2x_6+(2c/(c+2))x_1x_3x_6+(2c/(c+2))x_2x_3x_6-x_3^2x_6+(c/(c+2))x_0x_4x_6+(2c/(c+2))x_3x_4x_6+(c/(c+2))x_0x_5x_6+(2c/(c+2))x_3x_5x_6-x_0x_6^2+x_3x_6^2+x_0^2x_7+(c/(c+2))x_0x_1x_7+(c/(c+2))x_0x_2x_7+(2c/(c+2))x_1x_3x_7+(2c/(c+2))x_2x_3x_7-x_3^2x_7+(c/(c+2))x_0x_4x_7+(2c/(c+2))x_3x_4x_7+(c/(c+2))x_0x_5x_7+(2c/(c+2))x_3x_5x_7+(c/(c+2))x_0x_6x_7+(2c/(c+2))x_3x_6x_7-x_0x_7^2+x_3x_7^2+x_0^2x_8+(c/(c+2))x_0x_1x_8+(c/(c+2))x_0x_2x_8+(2c/(c+2))x_1x_3x_8+(2c/(c+2))x_2x_3x_8-x_3^2x_8+(c/(c+2))x_0x_4x_8+(2c/(c+2))x_3x_4x_8+(c/(c+2))x_0x_5x_8+(2c/(c+2))x_3x_5x_8+(c/(c+2))x_0x_6x_8+(2c/(c+2))x_3x_6x_8+(c/(c+2))x_0x_7x_8+(2c/(c+2))x_3x_7x_8-x_0x_8^2+x_3x_8^2$$

$$((2c+2)/c)x_0^3+x_0^2x_1-x_0x_1^2-x_0^2x_2+x_1^2x_2+x_0x_2^2-x_1x_2^2+((c+1)/c)x_2^3+x_0^2x_3+(c/(c+2))x_0x_1x_3+(2c/(c+2))x_1x_2x_3-x_2^2x_3-x_0x_3^2+x_2x_3^2+x_0^2x_4+(c/(c+2))x_0x_1x_4+(2c/(c+2))x_1x_2x_4-x_2^2x_4+(c/(c+2))x_0x_3x_4+(2c/(c+2))x_2x_3x_4-x_0x_4^2+x_2x_4^2+x_0^2x_5+(c/(c+2))x_0x_1x_5+(2c/(c+2))x_1x_2x_5-x_2^2x_5+(c/(c+2))x_0x_3x_5+(2c/(c+2))x_2x_3x_5+(c/(c+2))x_0x_4x_5+(2c/(c+2))x_2x_4x_5-x_0x_5^2+x_2x_5^2+x_0^2x_6+(c/(c+2))x_0x_1x_6+(2c/(c+2))x_1x_2x_6-x_2^2x_6+(c/(c+2))x_0x_3x_6+(2c/(c+2))x_2x_3x_6+(c/(c+2))x_0x_4x_6+(2c/(c+2))x_2x_4x_6+(c/(c+2))x_0x_5x_6+(2c/(c+2))x_2x_5x_6-x_0x_6^2+x_2x_6^2+x_0^2x_7+(c/(c+2))x_0x_1x_7+(2c/(c+2))x_1x_2x_7-x_2^2x_7+(c/(c+2))x_0x_3x_7+(2c/(c+2))x_2x_3x_7+(c/(c+2))x_0x_4x_7+(2c/(c+2))x_2x_4x_7+(c/(c+2))x_0x_5x_7+(2c/(c+2))x_2x_5x_7+(c/(c+2))x_0x_6x_7+(2c/(c+2))x_2x_6x_7-x_0x_7^2+x_2x_7^2+x_0^2x_8+(c/(c+2))x_0x_1x_8+(2c/(c+2))x_1x_2x_8-x_2^2x_8+(c/(c+2))x_0x_3x_8+(2c/(c+2))x_2x_3x_8+(c/(c+2))x_0x_4x_8+(2c/(c+2))x_2x_4x_8+(c/(c+2))x_0x_5x_8+(2c/(c+2))x_2x_5x_8+(c/(c+2))x_0x_6x_8+(2c/(c+2))x_2x_6x_8+(c/(c+2))x_0x_7x_8+(2c/(c+2))x_2x_7x_8-x_0x_8^2+x_2x_8^2$$

$$((2c+2)/c)x_0^3-x_0^2x_1+x_0x_1^2+((c+1)/c)x_1^3+x_0^2x_2-x_1^2x_2-x_0x_2^2+x_1x_2^2+x_0^2x_3-x_1^2x_3+(c/(c+2))x_0x_2x_3+(2c/(c+2))x_1x_2x_3-x_0x_3^2+x_1x_3^2+x_0^2x_4-x_1^2x_4+(c/(c+2))x_0x_2x_4+(2c/(c+2))x_1x_2x_4+(c/(c+2))x_0x_3x_4+(2c/(c+2))x_1x_3x_4-x_0x_4^2+x_1x_4^2+x_0^2x_5-x_1^2x_5+(c/(c+2))x_0x_2x_5+(2c/(c+2))x_1x_2x_5+(c/(c+2))x_0x_3x_5+(2c/(c+2))x_1x_3x_5+(c/(c+2))x_0x_4x_5+(2c/(c+2))x_1x_4x_5-x_0x_5^2+x_1x_5^2+x_0^2x_6-x_1^2x_6+(c/(c+2))x_0x_2x_6+(2c/(c+2))x_1x_2x_6+(c/(c+2))x_0x_3x_6+(2c/(c+2))x_1x_3x_6+(c/(c+2))x_0x_4x_6+(2c/(c+2))x_1x_4x_6+(c/(c+2))x_0x_5x_6+(2c/(c+2))x_1x_5x_6-x_0x_6^2+x_1x_6^2+x_0^2x_7-x_1^2x_7+(c/(c+2))x_0x_2x_7+(2c/(c+2))x_1x_2x_7+(c/(c+2))x_0x_3x_7+(2c/(c+2))x_1x_3x_7+(c/(c+2))x_0x_4x_7+(2c/(c+2))x_1x_4x_7+(c/(c+2))x_0x_5x_7+(2c/(c+2))x_1x_5x_7+(c/(c+2))x_0x_6x_7+(2c/(c+2))x_1x_6x_7-x_0x_7^2+x_1x_7^2+x_0^2x_8-x_1^2x_8+(c/(c+2))x_0x_2x_8+(2c/(c+2))x_1x_2x_8+(c/(c+2))x_0x_3x_8+(2c/(c+2))x_1x_3x_8+(c/(c+2))x_0x_4x_8+(2c/(c+2))x_1x_4x_8+(c/(c+2))x_0x_5x_8+(2c/(c+2))x_1x_5x_8+(c/(c+2))x_0x_6x_8+(2c/(c+2))x_1x_6x_8+(c/(c+2))x_0x_7x_8+(2c/(c+2))x_1x_7x_8-x_0x_8^2+x_1x_8^2$$

conjecture: x_0^9

8 Conjecture for $p = 3, 3 \mid n$

Variables are $x_0, \ldots, x_{n-1}$.

Generators are:

x_0^9 in degree 9, and $\sum x_i^3$ in degree 3. There are $n-2$ remaining generators in degree 3, each with the following form:

$$\tfrac{2c+2}{c}x_0^3 + \tfrac{c+1}{c}x_1^3 - x_0^2x_1 + x_0x_1^2 + \left(\textstyle\sum_{i\geq 2} x_0^2x_i - x_1^2x_i - x_0x_i^2 + x_1x_i^2\right) + \tfrac{c}{c+2}\left(\textstyle\sum_{i,j\geq 2; i\neq j} x_0x_ix_j - x_1x_ix_j\right).$$

(The other generators are created from this one by switching x_1 with x_k for some $k \geq 2$.)