Problem set-6: Simulation and Markov Chain Monte Carlo

Course: R Progamming Lab

Course Number: MA588

L-T-P-C: 0-0-3-3

Semester 2, 2019-20

Do all the following exercises in R. You need to submit the pdf file using knitr where R-codes are visible along with output. Do proper documentation.

Deadline to submit this assignment: 11.59PM, 16 February 2020

1. Suppose U_1, U_2 and U_3 are independent uniform random variables on the interval (0, 1). Use simulation to estimate the following quantities.
   (a) $E[U_1 + U_2 + U_3]$.
   (b) $\mathrm{Var}(U_1 + U_2 + U_3)$ and $\mathrm{Var}(U_1) + \mathrm{Var}(U_2) + \mathrm{Var}(U_3)$.
   (c) $E[\sqrt{U_1 + U_2 + U_3}]$.
   (d) $P(\sqrt{U_1} + \sqrt{U_2} + \sqrt{U_3} \geq 0.8)$.

2. Write an R function which simulates the outcomes of a student guessing at a True-False test consisting of n questions.

   (a) Use the function to simulate one student guessing the answers to a test with 10 questions; calculate the number of correct answers for this student.

(b) Simulate the number of correct answers for a student who guesses at a test with 1000 questions.

3. Simulate 10000 binomial pseudorandom numbers with parameters 20 and 0.3, assigning them to a vector called `binsim`. Let X be a Binomial(20, 0.3) random variable. Use the simulated numbers to estimate the following.
   (a) $P(X \leq 5)$.
   (b) $P(X = 5)$.
   (c) $E[X]$.
   (d) $\mathrm{Var}(X)$.
   (e) The 95th percentile of X. (You may use the `quantile()` function.)
   (f) The 99th percentile of X.
   (g) The 99.9999th quantile of X.
   In each case, compare your estimates with the true values. What is required to estimate extreme quantities accurately?

4. Simulate 100 realizations of a normal random variable having mean 51 and standard deviation 5.2. Estimate the mean and standard deviation of your simulated sample and compare with the theoretical values.

5. Simulate 1000 realizations of a standard normal random variable Z, and use your simulated sample to estimate
   (a) $P(Z > 2.5)$,
   (b) $P(0 < Z < 1.645)$,
   (c) $P(1.2 < Z < 1.45)$,
   (d) $P(-1.2 < Z < 1.3)$.
   Compare with the theoretical values.

6. Simulate from the conditional distribution of a normal random variable X with mean 3 and variance 16, given that $|X| > 2$.

7. Using the fact that a χ^2 random variable on one degree of freedom has the same distribution as the square of a standard normal random variable, simulate 100 independent realizations of such a χ^2 random variable, and estimate its mean and variance. (Compare with the theoretical values: 1, 2.)

8. A χ^2 random variable on n degrees of freedom has the same distribution as the sum of n independent standard normal random variables. Simulate a χ^2 random variable

on eight degrees of freedom, and estimate its mean and variance. (Compare with the theoretical values: 8, 16.)

9. Use Monte Carlo integration to estimate the following integrals. Compare with the exact answer, if known.

$$\int_0^1 x\,dx, \qquad \int_1^3 x^2\,dx, \qquad \int_0^{\pi} \sin(x)dx, \qquad \int_1^{\pi} e^x\,dx, \qquad \int_0^{\infty} e^{-x}\,dx,$$

10. Write a function to generate standard normal pseudorandom numbers on the interval [-4, 4], using runif() and the rejection method. Can you modify the function so that it can generate standard normal pseudorandom numbers on the entire real line?