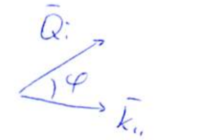
11 feb 2019

Doping combinations for the gr-hBN-gr tunnelling device

$$J(\omega) = \frac{e^2}{4\pi^4} \int d^2\bar{Q}_i \int d^2\bar{k}_\parallel \left[1 + \cos(\phi_i - \phi_j)\right] \Delta(\bar{Q}_i, \bar{k}_\parallel, \omega)\, I_1(k_\parallel, \omega)$$

$$\bar{Q}_j = \bar{Q}_i - \bar{k}_\parallel$$

$$\cos(\phi_i - \phi_j) = \frac{\bar{Q}_i \cdot \bar{Q}_j}{Q_i Q_j} = \frac{Q_i Q_j \cos\varphi}{Q_i Q_j}$$



$$Q_j = |\bar{Q}_i - \bar{k}_\parallel| = \sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}$$

$$\bar{Q}_i \cdot \bar{Q}_j = \bar{Q}_i \, (\bar{Q}_i - \bar{k}_\parallel) = Q_i^2 - \bar{Q}_i \cdot \bar{k}_\parallel = Q_i^2 - Q_i k_\parallel \cos\varphi$$

$$\cos(\phi_i - \phi_j) = \frac{Q_i - k_\parallel \cos\varphi}{Q_j} = \frac{Q_i - k_\parallel \cos\varphi}{\sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}}$$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\bar{k}_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i \; Q_i \left[1 + \frac{Q_i - k_\parallel \cos\varphi}{\sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}}\right] \Delta(Q_i, k_\parallel, \omega)$$

$$1 + \frac{Q_i - k_\parallel \cos\varphi}{\sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}} = \frac{\sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi} + Q_i - k_\parallel \cos\varphi}{\sqrt{Q_i^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}}$$

Solving the δ's:

$Q_i > 0$
$Q_j > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int_0^\infty dk_\parallel \int_{-\pi}^{\pi} d\eta \int_0^\pi d\varphi \; I_1(k_\parallel, \omega) Q_{i0} k_\parallel \left[1 + \frac{Q_{i0} - k_\parallel \cos\varphi}{\sqrt{Q_{i0}^2 + k_\parallel^2 - 2Q_i k_\parallel \cos\varphi}}\right] \Delta(Q_{i0}, k_\parallel, \omega)$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; I_1(k_\parallel, \omega)\, Q_{i0} k_\parallel \overbrace{\left[1 + \frac{Q_{i0} - k_\parallel \cos\varphi}{Q_{j0}}\right] \frac{1}{|F'(Q_i)|}}^{= A(Q_{i0})} \; \Theta$$

$$Q_{j0} = \sqrt{Q_{i0}^2 + k_\parallel^2 - 2Q_{i0} k_\parallel \cos\varphi}$$

$$\Delta \propto \delta(\hbar v_F \tilde{Q} + g) \quad \begin{cases} \tilde{Q} = \tilde{Q}(Q_j, Q_i) = \tilde{Q}(Q_i, k_{\parallel}, \varphi) \\ g = g(\epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) \end{cases}$$

Possibilites:

$$\tilde{Q}_1 = Q_j - Q_i$$

$$\tilde{Q}_2 = Q_j + Q_i$$

$$\tilde{Q}_3 = -Q_j - Q_i$$

$$\tilde{Q}_4 = -Q_j + Q_i$$

Dirac-delta properties:

$$\delta(F(x)) = \sum_j \frac{\delta(x - x_j)}{|F'(x_j)|}$$

$$\left[\begin{array}{l} F(x_j) = 0 \\ F'(x) = \dfrac{dF(x)}{dx} \end{array} \right]$$

$$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$$

$$\delta(\hbar v_F \tilde{Q} + g) = \frac{1}{\hbar v_F} \delta\left(\tilde{Q} + \frac{g}{\hbar v_F}\right)$$

$$F(Q_i) = \tilde{Q} + \frac{g}{\hbar v_F} = \tilde{Q} + g_0$$

ZEROS

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 + Q_j - Q_i = g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi} = 0$$

$$Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi = Q_i^2 + g_0^2 - 2Q_i g_0 \quad ; \quad 2Q_i k_{\parallel} \cos\varphi = -g_0^2 + 2Q_i g_0 + k_{\parallel}^2$$

$$Q_i (2k_{\parallel} \cos\varphi - 2g_0) = k_{\parallel}^2 - g_0^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2k_{\parallel}\cos\varphi - 2g_0} = Q_{i01}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_g + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi = Q_i^2 + g_0^2 + 2Q_i g_0$$

$$k_{\parallel}^2 - g_0^2 = Q_i \left(2k_{\parallel} \cos\varphi + 2g_0 \right)$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2(k_{\parallel}\cos\varphi + g_0)} = Q_{i02}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0 - Q_g - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$-2Q_i g_0 + g_0^2 + Q_i^2 = Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi$$

$$Q_i^2 - Q_i k_{\parallel} \cos\varphi + \frac{k_{\parallel}^2 - g_0^2}{2} = 0$$

$$Q_i = \frac{1}{2} k_{\parallel} \cos\varphi \pm \frac{1}{2} \sqrt{k_{\parallel}^2 \cos^2\varphi - k_{\parallel}^2 + g_0^2} =$$

$$= \frac{1}{2} \left[k_{\parallel} \cos\varphi \pm \sqrt{k_{\parallel}^2 (\cos^2\varphi - 1) + g_0^2} \right] =$$

$$= \frac{1}{2} \left[k_{\parallel} \cos\varphi \pm \sqrt{g_0^2 - k_{\parallel}^2 \sin^2\varphi} \right] = Q_{i03}^{\pm}$$

$$Q_i (2k_{\parallel} \cos\varphi - 2g_0) = k_{\parallel}^2 - g_0^2$$

$$Q_{i03} = \frac{k_{\parallel}^2 - g_0^2}{2(k_{\parallel}\cos\varphi - g_0)} = Q_{i01}$$

4) $\tilde{Q}_4 + g_o = 0$

$$g_o - Q_j + Q_i = g_o + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$g_o^2 + Q_i^2 + 2g_oQ_i = Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi$$

$$2Q_i\,(g_o + k_{\parallel}\cos\varphi) = k_{\parallel}^2 - g_o^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_o^2}{2(g_o + k_{\parallel}\cos\varphi)} = Q_{io2} \longrightarrow \text{same as case } 2!$$

$$Q_{io4} = Q_{io2}$$

DERIVATIVES

1) $\tilde{Q}_1 + g_o = 0$

$$F(Q_i) = g_o - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$F'(Q_i) = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} \qquad Q_{jo1} = Q_{io1} - g_o$$

$$A(Q_i) = \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}\right]\frac{1}{|F'(Q_i)|} = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{Q_j}\;\frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} =$$

$$= \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|-Q_j + Q_i - k_{\parallel}\cos\varphi|} = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|Q_j - Q_i + k_{\parallel}\cos\varphi|}$$

$$A_1(Q_{io1}) = \frac{2Q_{io1} - g_o - k_{\parallel}\cos\varphi}{|-g_o + k_{\parallel}\cos\varphi|}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$F(Q_i) = g_0 + Q_j + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi}$$

$$F'(Q_i) = 1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = 1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j + Q_i - k_{\parallel}\cos\varphi|} \qquad Q_{j_{o2}} = -g_0 - Q_{i_{o2}}$$

$$A(Q_i) = [\quad] \frac{1}{|F'(Q_i)|} = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|Q_j + Q_i - k_{\parallel}\cos\varphi|} = \text{sgn}(Q_j + Q_i - k_{\parallel}\cos\varphi)$$

$\hookrightarrow A_2(Q_{i_{o2}}) = \text{sgn}(-g_0 - k_{\parallel}\cos\varphi) = A_2$

3) $\tilde{Q}_3 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$F'(Q_i) = -1 - \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = -1 - \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{-Q_j - Q_i + k_{\parallel}\cos\varphi} \qquad Q_{j_{o3}}^{\pm} = g_0 - Q_{i_{o3}}^{\pm}$$

$$A(Q_i) = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|-Q_j - Q_i + k_{\parallel}\cos\varphi|} = \text{sgn}(Q_j + Q_i - k_{\parallel}\cos\varphi) \longrightarrow \text{same as case } 2!$$

$\hookrightarrow A_3(Q_{i_{o3}}) = \text{sgn}(g_0 - k_{\parallel}\cos\varphi) = A_3$

4) $\tilde{Q}_4 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j + Q_i = g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$Q_{j_{o4}} = g_0 + Q_{i_{o2}} =$

$= -Q_{j_{o2}}$

$$F'(Q_i) = 1 - \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \quad // \quad \frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j - Q_i + k_{\parallel}\cos\varphi|}$$

$$A(Q_i) = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|Q_j - Q_i + k_{\parallel}\cos\varphi|} \longrightarrow \text{same as case } 1!$$

$$\hookrightarrow A_4(Q_{i_{o4}}) = \frac{2Q_{i_{o2}} + g_0 - k_{\parallel}\cos\varphi}{|g_0 + k_{\parallel}\cos\varphi|}$$

Spectrally resolved probabilities

$$S(\omega) = \frac{e^2}{\hbar\pi^3 v_F} \int_0^\infty dk_\parallel\, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i\; Q_i\, I_1(k_\parallel,\omega) \left[1 + \frac{Q_i - k_\parallel\cos\varphi}{Q_f}\right] \delta(\tilde{Q} + g_0)\; \Theta(Q_f, Q_i) =$$

$$= \frac{e^2}{\hbar\pi^3 v_F} \int_0^\infty dk_\parallel\, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i\; Q_i\, I_1(k_\parallel,\omega) \left[1 + \frac{Q_i - k_\parallel\cos\varphi}{Q_f}\right] \sum_0 \frac{\delta(Q_i - Q_{i0})}{|F'(Q_{i0})|}\; \Theta(Q_f, Q_i) =$$

$$= \frac{e^2}{\hbar\pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \sum_0 k_\parallel\, Q_{i0}\, I_1(k_\parallel,\omega)\, A(Q_{i0})\, \Theta(Q_{f0}, Q_{i0}) =$$

$$= \frac{e^2}{\hbar\pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi\; k_\parallel\, I_1(k_\parallel,\omega)\, B(Q_{i0}) \qquad \begin{matrix} g_0 > 0 \\ Q_{i0} > 0 \end{matrix}$$

$$B(Q_{i0}) = B(k_\parallel, \epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) = \sum_0 Q_{i0}\, A(Q_{i0})\, \Theta(Q_{f0}, Q_{i0})$$

Combinations

a) electron - hole : $g_0^{eh} = \frac{1}{\hbar v_F}\left(\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta(\tilde{Q}_3 + g_0^{eh})\; \Theta(\epsilon_{F1} - \hbar v_F Q_{i03})\; \Theta(\epsilon_{F2} - \hbar v_F Q_{f03}) +$$

$$+ \delta(\tilde{Q}_1 + g_0^{eh})\; \Theta(\epsilon_{F1} - \hbar v_F Q_{i0}) +$$

$$+ \delta(\tilde{Q}_4 + g_0^{eh})\; \Theta(\epsilon_{F2} - \hbar v_F Q_{f0}) +$$

$$+ \delta(\tilde{Q}_2 + g_0^{eh})$$

$$B^{eh} = Q_{i03}^{+}\, A_3\, \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}^{+})\, \Theta(\epsilon_{F2} - \hbar v_F Q_{f03}^{+}) + Q_{i03}^{-}\, A_3\, \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}^{-})\, \Theta(\epsilon_{F2} - \hbar v_F Q_{f03}^{-})$$

$$+ Q_{i01}\, A_1(Q_{i01})\, \Theta(\epsilon_{F1} - \hbar v_F Q_{i01}) +$$

$$+ Q_{i02}\, A_4(Q_{i02})\, \Theta(\epsilon_{F2} + \hbar v_F Q_{f02}) +$$

$$+ Q_{i02}\, A_2$$

b) hole - electron: $g_0^{he} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_2 + g_0^{he}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_j - \epsilon_{F2}\right)$$

$$B^{he} = Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

c) hole - hole: $g_0^{hh} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_4 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} - \hbar v_F Q_j\right) +$$
$$+ \delta\left(\tilde{Q}_2 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)$$

$$B^{hh} = Q_{io2}\, A_4(Q_{io2})\,\Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} + \hbar v_F Q_{fo2}\right) +$$
$$+ Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)$$

d) electron - electon: $g_0^{ee} = \frac{1}{\hbar v_F}\left(\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_1 + g_0^{ee}\right)\Theta\left(\epsilon_{F1} - \hbar v_F Q_i\right)\Theta\left(\hbar v_F Q_j - \epsilon_{F2}\right)$$
$$+ \delta\left(\tilde{Q}_2 + g_0^{ee}\right)\Theta\left(\hbar v_F Q_j - \epsilon_{F2}\right)$$

$$B^{ee} = Q_{io1}\, A_1(Q_{io1})\,\Theta\left(\epsilon_{F1} - \hbar v_F Q_{io1}\right)\Theta\left(\hbar v_F Q_{fo1} - \epsilon_{F2}\right) +$$
$$+ Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

SUMMARY

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; k_\parallel \, I_1(k_\parallel, \omega) \, B^{ij}$$

ij : electron and hole doping combinations

$$Q_{io1} = \frac{k_\parallel^2 + g_0^2}{2k_\parallel \cos\varphi} \qquad Q_{io2} = \frac{k_\parallel^2 - g_0^2}{2(k_\parallel \cos\varphi + g_0)} \qquad Q_{io4} = Q_{io2}$$

$$Q_{io3}^{\pm} = \frac{1}{2}\left[k_\parallel \cos\varphi \pm \sqrt{g_0^2 - k_\parallel^2 \sin^2\varphi} \right]$$

$g_0 = g_0^{ij}$

$Q_{io} = Q_{io}(g_0^{ij})$

$$A_1(Q_{io1}) = \frac{2Q_{io1} - g_0 - k_\parallel \cos\varphi}{|-g_0 + k_\parallel \cos\varphi|} \qquad A_2 = \operatorname{sgn}(-g_0 - k_\parallel \cos\varphi)$$

$$A_4(Q_{io2}) = \frac{2Q_{io2} + g_0 - k_\parallel \cos\varphi}{|g_0 + k_\parallel \cos\varphi|} \qquad A_3 = \operatorname{sgn}(g_0 - k_\parallel \cos\varphi)$$

$Q_{jo1} = Q_{io1} - g_0$

$Q_{jo2} = -g_0 - Q_{io2}$

$Q_{jo3}^{\pm} = g_0 - Q_{io3}^{\pm}$

$Q_{jo4} = -Q_{jo2}$

electron - electron: $g_0^{ee} = (E_{F1} - E_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{ee} = Q_{io1} A_1(Q_{io1}) \Theta(E_{F1} - \hbar v_F Q_{io1}) \Theta(\hbar v_F Q_{jo1} - E_{F2}) + \quad {}^{c-c} \quad \text{reg } 1$$
$$+ \; Q_{io2} A_2 \Theta(\hbar v_F Q_{jo2} - E_{F2}) \quad {}^{c-v} \quad \text{reg } 2$$

electron - hole: $g_0^{eh} = (E_{F1} + E_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{eh} = Q_{io3}^{+} A_3 \Theta(E_{F1} - \hbar v_F Q_{io3}^{+}) \Theta(E_{F2} - \hbar v_F Q_{jo3}^{+})^{c-v} + Q_{io3}^{-} A_3 \Theta(E_{F1} - \hbar v_F Q_{io3}^{-}) \Theta(E_{F2} - \hbar v_F Q_{jo3}^{-})^{c-v} \quad \text{reg } 3$$
$$+ \; Q_{io1} A_1(Q_{io1}) \Theta(E_{F1} - \hbar v_F Q_{io1})^{c-c} + \quad \text{reg } 1$$
$$+ \; Q_{io2} A_4(Q_{io2}) \Theta(E_{F2} + \hbar v_F Q_{jo2})^{v-v} + Q_{io2} A_2^{\;v-c}$$
$$\text{reg } 4 \qquad \text{reg } 2$$

c: conduction
v: valence

hole - electron: $g_0^{he} = (-E_{F1} - E_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{he} = Q_{io2} A_2 \Theta(\hbar v_F Q_{io2} - E_{F1}) \Theta(\hbar v_F Q_{jo2} - E_{F2}) \quad {}^{v-c} \quad \text{reg } 2$$

hole - hole: $g_0^{hh} = (-E_{F1} + E_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{hh} = Q_{io2} A_4(Q_{io2}) \Theta(\hbar v_F Q_{io2} - E_{F1}) \Theta(E_{F2} + \hbar v_F Q_{jo2}) + \quad {}^{v-v} \quad \text{reg } 4$$
$$+ \; Q_{io2} A_2 \Theta(\hbar v_F Q_{io2} - E_{F1}) \quad {}^{v-c} \quad \text{reg } 2$$

Allowed energies

$$g_0 = \gamma_0 + \frac{\hbar\omega}{\hbar v_F}$$

$$Q_i - k_{\parallel} < Q_j < Q_i + k_{\parallel}$$

1) $\tilde{Q}_1 + g_0$

$$g_0 - k_{\parallel} < g_0 + Q_j - Q_i < g_0 + k_{\parallel}$$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j - Q_i = 0$$

$$\hbar\omega = Q_i - Q_j - \gamma_0 \longrightarrow \boxed{-k_{\parallel} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{\parallel} - \gamma_0}$$

reg 1

$\hbar\omega > 0$

$$Q_i - Q_i + k_{\parallel} - \gamma_0 = k_{\parallel} - \gamma_0$$

$$Q_i - Q_i - k_{\parallel} - \gamma_0 = -k_{\parallel} - \gamma_0$$

2) $\tilde{Q}_2 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j + Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = -\gamma_0 - Q_j - Q_i$$

reg 2

$$\boxed{0 < -\gamma_0 - 2Q_i - k_{\parallel} < \frac{\hbar\omega}{\hbar v_F} < -\gamma_0 - 2Q_i + k_{\parallel}}$$

$$-\gamma_0 - Q_i - k_{\parallel} - Q_i = -\gamma_0 - 2Q_i - k_{\parallel}$$

$$-\gamma_0 - Q_i + k_{\parallel} - Q_i = -\gamma_0 - 2Q_i + k_{\parallel}$$

3) $\tilde{Q}_3 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j - Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j + Q_i - \gamma_0$$

reg 3

$$Q_i - k_{\parallel} + Q_i - \gamma_0 = -k_{\parallel} - \gamma_0 + 2Q_i \qquad \boxed{2Q_i - k_{\parallel} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{\parallel} - \gamma_0 + 2Q_i}$$

$\frac{\hbar\omega}{\hbar v_F} > 0$

$$Q_i + k_{\parallel} + Q_i - \gamma_0 = k_{\parallel} - \gamma_0 + 2Q_i$$

4) $\tilde{Q}_4 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j + Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j - Q_i - \gamma_0$$

reg 4 = reg 1

$$Q_i - k_{\parallel} - Q_i - \gamma_0 = -k_{\parallel} - \gamma_0 \qquad \boxed{-k_{\parallel} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{\parallel} - \gamma_0}$$

$$Q_i + k_{\parallel} - Q_i - \gamma_0 = k_{\parallel} - \gamma_0$$

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CALCULATIONS ELIMINATING $k_{\parallel}$ $\qquad$ $Q_i, Q_y, k_{\parallel} > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\vec{Q}_i \int_0^{\pi} d\varphi \int_0^{\infty} dk_{\parallel} \quad k_{\parallel} \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \right] \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega)$$

$$\delta(F(k_{\parallel})) = \sum_0 \frac{\delta(k_{\parallel} - k_{\parallel 0})}{|F'(k_{\parallel 0})|} \qquad Q_y = \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$J(\omega) = \frac{e^2}{2\pi^4 \hbar v_F} \int_0^{\infty} dQ_i \cdot 2\pi \int_0^{\pi} d\varphi \sum_0 Q_i k_{\parallel 0} \left[1 + \frac{Q_i - k_{\parallel 0}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0}\cos\varphi}} \right] \frac{1}{|F'(k_{\parallel 0})|} \Theta_0$$

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dQ_i \int_0^{\pi} d\varphi \sum_0 Q_i k_{\parallel 0} \, \Lambda(Q_i, k_{\parallel 0}) \, \Theta_0$$

As before: $F(k_{\parallel}) = \tilde{Q} + g_0$ $\qquad g_0 = \omega/\hbar v_F$

$\hookrightarrow \tilde{Q} = \tilde{Q}(k_{\parallel})$ also!

= zeros:

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 01}^{\pm} = Q_i\cos\varphi \pm \sqrt{Q_i^2\cos^2\varphi + g_0^2 - 2Q_i g_0}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 - 2Q_i g_0 = 0$$

$$k_{\parallel 02}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi + 2Q_i^2 - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 03}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 - 2Q_i^2 - 2Q_i g_0}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi - g_0^2 - 2 g_0 Q_i = 0$$

$$k_{\parallel 04}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0} = k_{\parallel 02}^{\pm}$$

$=$ derivatives

$$F'(k_{\parallel}) = \frac{k_{\parallel} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi}}$$

same for all of them!

$$F'(k_{\parallel 0}) = \frac{k_{\parallel 0} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0} \cos\varphi}}$$

$$A = \left[1 + \frac{Q_i - k_{\parallel 0} \cos\varphi}{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0} \cos\varphi}} \right] \frac{1}{|F'(k_{\parallel 0})|} = \left[\quad \right] \frac{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0} \cos\varphi}}{|k_{\parallel 0} - Q_i \cos\varphi|} =$$

$$= \frac{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0} \cos\varphi}}{|k_{\parallel 0} - Q_i \cos\varphi|} + \frac{Q_i - k_{\parallel 0} \cos\varphi}{|k_{\parallel 0} - Q_i \cos\varphi|}$$

1) $\tilde{Q}_1 + g_0$

$$Q_{g_0} = \sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0} \cos\varphi}$$

$$A_{01}^{\pm} = \frac{2Q_i - g_0 - k_{0i}^{\pm} \cos\varphi}{\sqrt{Q_i^2 \cos^2\varphi - g_0^2 - 2Q_i g_0}}$$

2) $\tilde{Q}_2 + g_0$

$$A_{02}^{\pm} = \frac{-g_0 - k_{\parallel 02}^{\pm} \cos\varphi}{\sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0}}$$

3) $\tilde{Q}_3 + g_0$

$$A_{03}^{\pm} = \frac{g_0 - k_{\parallel 03}^{\pm} \cos\varphi}{\sqrt{Q_i^2 \cos^2\varphi + g_0^2 - 2Q_i^2 - 2Q_i g_0}}$$

4) $\tilde{Q}_4 + g_0$

$$A_{04}^{\pm} = \frac{g_0 + 2Q_i - k_{\parallel 02}^{\pm} \cos\varphi}{\sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0}}$$