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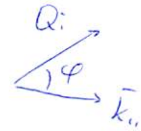
Doping combinations for the gr-hBN-gr tunnelling device

$$J(\omega) = \frac{e^2}{4\pi^4} \int d^2\bar{Q}_i \int d^2\bar{k}_{\parallel} \left[1 + \cos(\phi_i - \phi_j)\right] \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega)\, I_+(k_{\parallel}, \omega)$$

$$\bar{Q}_j = \bar{Q}_i - \bar{k}_{\parallel}$$

$$\cos(\phi_i - \phi_j) = \frac{\bar{Q}_i \cdot \bar{Q}_j}{Q_i Q_j} = \frac{Q_i Q_j \cos\varphi}{Q_i Q_j}$$



$$Q_j = |\bar{Q}_i - \bar{k}_{\parallel}| = \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi}$$

$$\bar{Q}_i \cdot \bar{Q}_j = \bar{Q}_i \cdot (\bar{Q}_i - \bar{k}_{\parallel}) = Q_i^2 - \bar{Q}_i \cdot \bar{k}_{\parallel} = Q_i^2 - Q_i k_{\parallel} \cos\varphi$$

$$\cos(\phi_i - \phi_j) = \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j} = \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}}$$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\bar{k}_{\parallel} \int_0^{\pi} d\varphi \int_0^{\infty} dQ_i\, Q_i \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}}\right] \Delta(Q_i, k_{\parallel}, \omega)$$

$$1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = \frac{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi} + Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}}$$

Solving the J's:

$Q_i > 0$

$Q_j > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int_0^{\infty} dk_{\parallel} \int_{-\pi}^{\pi} d\eta \int_0^{\pi} d\varphi\, I_+(k_{\parallel}, \omega) Q_{i0} k_{\parallel} \left[1 + \frac{Q_{i0} - k_{\parallel}\cos\varphi}{\sqrt{Q_{i0}^2 + k_{\parallel}^2 - 2Q_{i0} k_{\parallel}\cos\varphi}}\right] \Delta(Q_{i0}, k_{\parallel}, \omega)$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dk_{\parallel} \int_0^{\pi} d\varphi\, I_+(k_{\parallel}, \omega)\, \overbrace{Q_{i0} k_{\parallel} \left[1 \underset{-}{+} \frac{Q_{i0} - k_{\parallel}\cos\varphi}{Q_{j0}}\right] \frac{1}{|F'(Q_i)|}}^{= A(Q_{i0})}\, \Theta$$

"−" for e-h & h-e

$$Q_{j0} = \sqrt{Q_{i0}^2 + k_{\parallel}^2 - 2Q_{i0} k_{\parallel}\cos\varphi}$$

$$\Delta \propto \delta(\hbar v_F \tilde{Q} + g) \quad \begin{cases} \tilde{Q} = \tilde{Q}(Q_f, Q_i) = \tilde{Q}(Q_i, k_{\parallel}, \varphi) \\ g = g(\epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) \end{cases}$$

Possibilites:

$$\tilde{Q}_1 = Q_f - Q_i$$

$$\tilde{Q}_2 = Q_f + Q_i$$

$$\tilde{Q}_3 = -Q_f - Q_i$$

$$\tilde{Q}_4 = -Q_f + Q_i$$

Dirac-delta properties:

$$\delta(F(x)) = \sum_j \frac{\delta(x - x_j)}{|F'(x_j)|}$$

$$\left[\begin{array}{l} F(x_j) = 0 \\ F'(x) = \frac{dF(x)}{dx} \end{array} \right]$$

$$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$$

$$\delta(\hbar v_F \tilde{Q} + g) = \frac{1}{\hbar v_F} \delta\left(\tilde{Q} + \frac{g}{\hbar v_F}\right)$$

$$F(Q_i) = \tilde{Q} + \frac{g}{\hbar v_F} = \tilde{Q} + g_0$$

ZEROS

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 + Q_f - Q_i = g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi} = 0$$

$$Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi = Q_i^2 + g_0^2 - 2Q_i g_0 \quad ; \quad 2Q_i k_{\parallel} \cos\varphi = -g_0^2 + 2Q_i g_0 + k_{\parallel}^2$$

$$Q_i(2k_{\parallel}\cos\varphi - 2g_0) = k_{\parallel}^2 - g_0^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2k_{\parallel}\cos\varphi - 2g_0} = Q_{i01}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_f + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi = Q_i^2 + g_0^2 + 2Q_i g_0$$

$$k_{\parallel}^2 - g_0^2 = Q_i \left(2k_{\parallel} \cos\varphi + 2g_0 \right)$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2(k_{\parallel}\cos\varphi + g_0)} = Q_{i02}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0 - Q_f - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$-2Q_i g_0 + g_0^2 + Q_i^2 = Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi$$

$$Q_i^2 - Q_i k_{\parallel} \cos\varphi + \frac{k_{\parallel}^2 - g_0^2}{2} = 0$$

$$Q_i = \frac{1}{2} k_{\parallel} \cos\varphi \pm \frac{1}{2} \sqrt{k_{\parallel}^2 \cos^2\varphi - k_{\parallel}^2 + g_0^2} =$$

$$= \frac{1}{2} \left[k_{\parallel} \cos\varphi \pm \sqrt{k_{\parallel}^2 (\cos^2\varphi - 1) + g_0^2} \right] =$$

$$= \frac{1}{2} \left[k_{\parallel} \cos\varphi \pm \sqrt{g_0^2 - k_{\parallel}^2 \sin^2\varphi} \right] = Q_{i03}^{\pm}$$

$$Q_i (2k_{\parallel} \cos\varphi - 2g_0) = k_{\parallel}^2 - g_0^2$$

$$Q_{i03} = \frac{k_{\parallel}^2 - g_0^2}{2(k_{\parallel}\cos\varphi - g_0)} = Q_{i01}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$g_0 - Q_j + Q_i = g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$g_0^2 + Q_i^2 + 2g_0 Q_i = Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi$$

$$2Q_i (g_0 + k_{\parallel} \cos\varphi) = k_{\parallel}^2 - g_0^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2(g_0 + k_{\parallel}\cos\varphi)} = Q_{i02} \longrightarrow \text{same as case } 2!$$

$$Q_{i04} = Q_{i02}$$

DERIVATIVES

1) $\tilde{Q}_1 + g_0 = 0$

$$F(Q_i) = g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi}$$

$$F'(Q_i) = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} \qquad Q_{j01} = Q_{i01} - g_0$$

$$A(Q_i) = \left[1 \pm \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}\right] \frac{1}{|F'(Q_i)|} = \frac{Q_j \pm Q_i \mp k_{\parallel}\cos\varphi}{Q_j} \frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} =$$

$$= \frac{Q_j \pm Q_i \mp k_{\parallel}\cos\varphi}{|-Q_j + Q_i - k_{\parallel}\cos\varphi|} = \frac{Q_j \underline{+} Q_i \underline{-} k_{\parallel}\cos\varphi}{|Q_j - Q_i + k_{\parallel}\cos\varphi|}$$

$$A_1(Q_{i01}) = \frac{2Q_{i01} - g_0 - k_{\parallel}\cos\varphi}{|-g_0 + k_{\parallel}\cos\varphi|}$$

e-h $\quad A_1(Q_{i01}) = \operatorname{sgn}(Q_j - Q_i + k_{\parallel}\cos\varphi) = \operatorname{sgn}(-g_0 + k_{\parallel}\cos\varphi)$

2) $\tilde{Q}_2 + g_0 = 0$

$$F(Q_i) = g_0 + Q_j + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = 1 + \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2+k_{11}^2-2Q_i k_{11}\cos\varphi}} = 1 + \frac{Q_i - k_{11}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j + Q_i - k_{11}\cos\varphi|} \qquad Q_{j_{02}} = -g_0 - Q_{i_{02}}$$

$$A(Q_i) = [\quad]\frac{1}{|F'(Q_i)|} = \frac{Q_j \pm Q_i \mp k_{11}\cos\varphi}{|Q_j + Q_i - k_{11}\cos\varphi|} = \operatorname{sgn}(Q_j + Q_i - k_{11}\cos\varphi)$$

$\hookrightarrow A_2(Q_{i_{02}}) = \operatorname{sgn}(-g_0 - k_{11}\cos\varphi) = A_2$

$$A_2(Q_{i_{02}}) = \frac{-g_0 - 2Q_{i_{02}} + k_{11}\cos\varphi}{|g_0 + k_{11}\cos\varphi|}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = -1 - \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2+k_{11}^2-2Q_i k_{11}\cos\varphi}} = -1 - \frac{Q_i - k_{11}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{-Q_j - Q_i + k_{11}\cos\varphi} \qquad Q_{j_{03}} = g_0 - Q_{i_{03}}$$

$$A(Q_i) = \frac{Q_j \pm Q_i \mp k_{11}\cos\varphi}{|-Q_j - Q_i + k_{11}\cos\varphi|} = \operatorname{sgn}(Q_j + Q_i - k_{11}\cos\varphi) \longrightarrow \text{same as case } 2!$$

$\hookrightarrow A_3(Q_{i_{03}}) = \operatorname{sgn}(g_0 - k_{11}\cos\varphi) = A_3$

$$A_3(Q_{i_{03}}) = \frac{g_0 - 2Q_{i_{03}} + k_{11}\cos\varphi}{|g_0 - k_{11}\cos\varphi|}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j + Q_i = g_0 + Q_i - \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = 1 - \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2+k_{11}^2-2Q_i k_{11}\cos\varphi}} \quad \Big\| \quad \frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j - Q_i + k_{11}\cos\varphi|}$$

$$Q_{j_{04}} = g_0 + Q_{i_{02}} = -Q_{j_{02}}$$

$$A(Q_i) = \frac{Q_j \pm Q_i \mp k_{11}\cos\varphi}{|Q_j - Q_i + k_{11}\cos\varphi|} \longrightarrow \text{same as case } 1!$$

$$\hookrightarrow A_4(Q_{i_{0}}) = \frac{2Q_{i_{02}} + g_0 - k_{11}\cos\varphi}{|g_0 + k_{11}\cos\varphi|}$$

$$A_4 = \operatorname{sgn}(g_0 + k_{11}\cos\varphi)$$

Spectrally resolved probabilities

$$S(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i \; Q_i \, I_1(k_\parallel, \omega) \left[1 + \frac{Q_i - k_\parallel \cos\varphi}{Q_j}\right] \delta(\tilde{Q} + g_0) \; \Theta(Q_j, Q_i) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i \; Q_i \, I_1(k_\parallel, \omega) \left[1 + \frac{Q_i - k_\parallel \cos\varphi}{Q_j}\right] \sum_0 \frac{\delta(Q_i - Q_{i0})}{|F'(Q_{i0})|} \; \Theta(Q_j, Q_i) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \sum_0 k_\parallel \, Q_{i0} \, I_1(k_\parallel, \omega) \; A(Q_{i0}) \; \Theta(Q_{j0}, Q_{i0}) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; k_\parallel \, I_1(k_\parallel, \omega) \; B(Q_{i0}) \qquad \begin{array}{l} Q_{j0} > 0 \\ Q_{i0} > 0 \end{array}$$

$$B(Q_{i0}) = B(k_\parallel, \epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) = \sum_0 Q_{i0} \; A(Q_{i0}) \; \Theta(Q_{j0}, Q_{i0})$$

Combinations

a) electron - hole : $g_0^{eh} = \frac{1}{\hbar v_F} \left(\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta(\tilde{Q}_3 + g_0^{eh}) \; \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}) \; \Theta(\epsilon_{F2} - \hbar v_F Q_{j03}) +$$
$$+ \delta(\tilde{Q}_1 + g_0^{eh}) \; \Theta(\epsilon_{F1} - \hbar v_F Q_{i0}) +$$
$$+ \delta(\tilde{Q}_4 + g_0^{eh}) \; \Theta(\epsilon_{F2} - \hbar v_F Q_{j0}) +$$
$$+ \delta(\tilde{Q}_2 + g_0^{eh})$$

red A's !

$$B^{eh} = Q_{i03} \; A_3 \; \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}) \; \Theta(\epsilon_{F2} - \hbar v_F Q_{j03})$$
$$+ Q_{i01} \; A_1(Q_{i01}) \; \Theta(\epsilon_{F1} - \hbar v_F Q_{i01}) +$$
$$+ Q_{i02} \; A_4(Q_{i02}) \; \Theta(\epsilon_{F2} + \hbar v_F Q_{j02}) +$$
$$+ Q_{i02} \; A_2$$

b) hole - electron: $g_0^{he} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_2 + g_0^{he}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$

red A_2!

$$B^{he} = Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

c) hole - hole: $g_0^{hh} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_4 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} - \hbar v_F Q_f\right) +$$
$$+ \delta\left(\tilde{Q}_2 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)$$

$$B^{hh} = Q_{io2}\, A_4(Q_{io2})\,\Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} + \hbar v_F Q_{fo2}\right) +$$
$$+ Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)$$

d) electron - electron: $g_0^{ee} = \frac{1}{\hbar v_F}\left(\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\hat{Q}_1 + g_0^{ee}\right)\Theta\left(\epsilon_{F1} - \hbar v_F Q_i\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$
$$+ \delta\left(\tilde{Q}_2 + g_0^{ee}\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$

$$B^{ee} = Q_{io1}\, A_1(Q_{io1})\,\Theta\left(\epsilon_{F1} - \hbar v_F Q_{io1}\right)\Theta\left(\hbar v_F Q_{fo1} - \epsilon_{F2}\right) +$$
$$+ Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

SUMMARY

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; k_\parallel \, I_1(k_\parallel, \omega) \, B^{ij}$$

ij : electron and hole doping combinations

$$Q_{io1} = \frac{k_\parallel^2 - g_0^2}{2k_\parallel \cos\varphi - 2g_0} \qquad Q_{io2} = \frac{k_\parallel^2 - g_0^2}{2(k_\parallel \cos\varphi + g_0)} \qquad Q_{io4} = Q_{io2}$$

$$\parallel$$

$$Q_{io3} = \frac{k_\parallel^2 - g_0^2}{2(k_\parallel \cos\varphi - g_0)} = Q_{io1}$$

$$g_0 = g_0^{ij}$$

$$Q_{io} = Q_{io}(g_0^{ij})$$

$$A_1(Q_{io1}) = \frac{2Q_{io1} - g_0 - k_\parallel \cos\varphi}{|-g_0 + k_\parallel \cos\varphi|}$$

$$A_2 = \mathrm{sgn}(-g_0 - k_\parallel \cos\varphi)$$

$$A_1 = \mathrm{sgn}(-g_0 + k_\parallel \cos\varphi)$$

$$A_2 = \frac{-g_0 - 2Q_{io2} + k_\parallel \cos\varphi}{|g_0 + k_\parallel \cos\varphi|}$$

$$A_4(Q_{io2}) = \frac{2Q_{io2} + g_0 - k_\parallel \cos\varphi}{|g_0 + k_\parallel \cos\varphi|}$$

$$A_3 = \mathrm{sgn}(g_0 - k_\parallel \cos\varphi)$$

$$A_3 = \frac{g_0 - 2Q_{io3} + k_\parallel \cos\varphi}{|g_0 - k_\parallel \cos\varphi|}$$

$$A_4 = \mathrm{sgn}(g_0 + k_\parallel \cos\varphi)$$

$$Q_{fo1} = Q_{io1} - g_0$$

$$Q_{fo2} = -g_0 - Q_{io2}$$

$$Q_{fo3}^{\pm} = g_0 - Q_{io3}^{\pm}$$

$$Q_{fo4} = -Q_{fo2}$$

electron - electron : $g_0^{ee} = (\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{ee} = Q_{io1} A_1(Q_{io1}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io1}) \, \Theta(\hbar v_F Q_{fo1} - \epsilon_{F2}) + \quad \text{c-c} \quad \text{reg 1}$$
$$+ \, Q_{io2} A_2 \, \Theta(\hbar v_F Q_{fo2} - \epsilon_{F2}) \quad \text{c-v} \quad \text{reg 2}$$

electron - hole : $g_0^{eh} = (\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{eh} = Q_{io3} A_3 \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io3}) \Theta(\epsilon_{F2} - \hbar v_F Q_{fo3}) + \quad \text{c-v} \quad \text{reg 3}$$
$$+ \, Q_{io1} A_1(Q_{io1}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io1}) + \quad \text{c-c} \quad \text{reg 1}$$
$$+ \, Q_{io2} A_4(Q_{io2}) \, \Theta(\epsilon_{F2} + \hbar v_F Q_{fo2}) + \quad \text{v-v} \quad \text{reg 4} \qquad Q_{io2} A_2 \quad \text{v-c} \quad \text{reg 2}$$

hole - electron. $g_0^{he} = (-\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{he} = Q_{io2} A_2 \, \Theta(\hbar v_F Q_{io2} - \epsilon_{F1}) \, \Theta(\hbar v_F Q_{fo2} - \epsilon_{F2}) \quad \text{v-c} \quad \text{reg 2}$$

c: conduction
v: valence

hole - hole : $g_0^{hh} = (-\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{hh} = Q_{io2} A_4(Q_{io2}) \Theta(\hbar v_F Q_{io2} - \epsilon_{F1}) \, \Theta(\epsilon_{F2} + \hbar v_F Q_{fo2}) + \quad \text{v-v} \quad \text{reg 4}$$
$$+ \, Q_{io2} A_2 \, \Theta(\hbar v_F Q_{io2} - \epsilon_{F1}) \quad \text{v-c} \quad \text{reg 2}$$

Allowed energies

$$g_0 = \eta_0 + \hbar\omega/\hbar v_F$$

$$Q_i - k_{11} < Q_j < Q_i + k_{11}$$

1) $\tilde{Q}_1 + g_0$

$$g_0 - k_{11} < g_0 + Q_j - Q_i < g_0 + k_{11}$$

$$\eta_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j - Q_i = 0$$

reg 1

$$\hbar\omega = Q_i - Q_j - \eta_0 \longrightarrow \boxed{-k_{11} - \eta_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \eta_0} \qquad \hbar\omega > 0$$

$$Q_i - Q_i + k_{11} - \eta_0 = k_{11} - \eta_0$$

$$Q_i - Q_i - k_{11} - \eta_0 = -k_{11} - \eta_0$$

2) $\tilde{Q}_2 + g_0$

$$\eta_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j + Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = -\eta_0 - Q_j - Q_i$$

reg 2

$$\boxed{0 < -\eta_0 - 2Q_i - k_{11} < \frac{\hbar\omega}{\hbar v_F} < -\eta_0 - 2Q_i + k_{11}}$$

$$-\eta_0 - Q_i - k_{11} - Q_i = -\eta_0 - 2Q_i - k_{11}$$

$$-\eta_0 - Q_i + k_{11} - Q_i = -\eta_0 - 2Q_i + k_{11}$$

3) $\tilde{Q}_3 + g_0$

$$\eta_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j - Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j + Q_i - \eta_0$$

reg 3

$$Q_i - k_{11} + Q_i - \eta_0 = -k_{11} - \eta_0 + 2Q_i \qquad \boxed{2Q_i - k_{11} - \eta_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \eta_0 + 2Q_i} \qquad \frac{\hbar\omega}{\hbar v_F} > 0$$

$$Q_i + k_{11} + Q_i - \eta_0 = k_{11} - \eta_0 + 2Q_i$$

4) $\tilde{Q}_4 + g_0$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j - Q_i - \eta_0$$

$$\eta_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j + Q_i = 0$$

reg 4 = reg 1

$$Q_i - k_{11} - Q_i - \eta_0 = -k_{11} - \eta_0$$

$$\boxed{-k_{11} - \eta_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \eta_0}$$

$$Q_i + k_{11} - Q_i - \eta_0 = k_{11} - \eta_0$$

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CALCULATIONS ELIMINATING $k_{\parallel}$

$Q_i\,,\,Q_f\,,\,k_{\parallel} > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\vec{Q}_i \int_0^{\pi} d\varphi \int_0^{\infty} dk_{\parallel} \quad k_{\parallel} \left[1 \pm \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \right] \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega)$$

$$\delta(F(k_{\parallel})) = \sum_0 \frac{\delta(k_{\parallel} - k_{\parallel 0})}{|F'(k_{\parallel 0})|} \qquad Q_f = \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$J(\omega) = \frac{e^2}{2\pi^4 \hbar v_F} \int_0^{\infty} dQ_i \cdot 2\pi \int_0^{\pi} d\varphi \sum_0 Q_i\, k_{\parallel 0} \left[1 \pm \frac{Q_i - k_{\parallel 0}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0}\cos\varphi}} \right] \frac{1}{|F'(k_{\parallel 0})|} \Theta_0$$

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dQ_i \int_0^{\pi} d\varphi \sum_0 Q_i\, k_{\parallel 0}\; \Delta(Q_i, k_{\parallel 0})\, \Theta_0$$

As before: $F(k_{\parallel}) = \tilde{Q} + g_0 \qquad g_0 = g/\hbar v_F$

$\hookrightarrow \tilde{Q} = \tilde{Q}(k_{\parallel})$ also!

= zeros:

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 01}^{\pm} = Q_i\cos\varphi \pm \sqrt{Q_i^2\cos^2\varphi + g_0^2 - 2Q_i g_0}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 - 2Q_i g_0 = 0$$

$$k_{\parallel 02}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0^2 + Q_i^2 - 2Q_i g_0$$

$$g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi \cancel{+2Q_i^2} - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 03}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 - 2Q_i g_0}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi - g_0^2 - 2g_0 Q_i = 0$$

$$k_{\parallel 04}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0} = k_{\parallel 02}^{\pm}$$

= derivatives

$$F'(k_{11}) = \frac{k_{11} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11} \cos\varphi}}$$

same for all of them!

$$F'(k_{110}) = \frac{k_{110} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{110}^2 - 2Q_i k_{110} \cos\varphi}}$$

$$A = \left[1 \pm \frac{Q_i - k_{110}\cos\varphi}{\sqrt{Q_i^2 + k_{110}^2 - 2Q_i k_{110}\cos\varphi}} \right] \frac{1}{|F'(k_{110})|} = \left[\quad \right] \frac{\sqrt{Q_i^2 + k_{110}^2 - 2Q_i k_{110}\cos\varphi}}{|k_{110} - Q_i \cos\varphi|} =$$

$$= \frac{\sqrt{Q_i^2 + k_{110}^2 - 2Q_i k_{110}\cos\varphi}}{|k_{110} - Q_i\cos\varphi|} \pm \frac{Q_i - k_{110}\cos\varphi}{|k_{110} - Q_i \cos\varphi|}$$

1) $\tilde{Q}_1 + g_0$

$$Q_{g_0} = \sqrt{Q_i^2 + k_{110}^2 - 2Q_i k_{110} \cos\varphi}$$

$$-g_0 + k_{01}^{\pm}\cos\varphi$$

$$A_{01}^{\pm} = \frac{2Q_i - g_0 - k_{01}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_0^2 - 2Q_i g_0}}$$

$$Q_{g_{01}} = Q_{i01} - g_0$$

2) $\tilde{Q}_2 + g_0$

$$-g_0 - 2Q_{i02} + k_{1102}^{\pm}\cos\varphi$$

$$A_{02}^{\pm} = \frac{-g_0 - k_{1102}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_0^2 + 2Q_i g_0}}$$

$$Q_{g_{02}} = -g_0 - Q_{i02}$$

3) $\tilde{Q}_3 + g_0$

$$g_0 - 2Q_{i03} + k_{1103}^{\pm}\cos\varphi$$

$$A_{03}^{\pm} = \frac{g_0 - k_{1103}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_0^2 - 2Q_i g_0}}$$

$$Q_{g_{03}} = g_0 - Q_{02}$$

4) $\tilde{Q}_4 + g_0$

$$g_0 + k_{1102}^{\pm}\cos\varphi$$

$$A_{04}^{\pm} = \frac{g_0 + 2Q_i - k_{1102}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_0^2 + 2Q_i g_0}}$$

$$Q_{g_{04}} = g_0 + Q_{i04}$$

Change of variable:

$$\int_0^{\pi} d\varphi \; F(\cos\varphi) = \int_1^{-1} -\frac{du}{\sqrt{1-u^2}} \; F(u) = \int_{-1}^{1} \frac{F(u)}{\sqrt{1-u^2}} \; du$$

$$u = \cos\varphi$$

$$du = -\sin\varphi \; d\varphi = -\sqrt{1-\cos^2\varphi} \; d\varphi$$

$$= -\sqrt{1-u^2} \; d\varphi$$

$$\frac{-du}{\sqrt{1-u^2}} = d\varphi$$

$$u_L = \cos 0 = 1$$ → not much better

$$u_U = \cos\pi = -1$$

Integral: $J(\omega) = \frac{e^2}{\pi^3 \hbar v_F} \int_0^{\infty} dQ_i \int_0^{\pi} d\varphi \sum_0 Q_i \, k_{\parallel 0}(\cos\varphi) \, A(Q_i, \cos\varphi) \, \Theta_0 =$

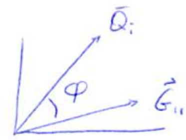
$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dQ_i \int_{-1}^{1} du \sum_0 Q_i \, k_{\parallel 0}(u) \, A(Q_i, u) \frac{\Theta_0}{\sqrt{1-u^2}}$$

$$Q_i^2 \cos^2\varphi + g_0^2 - 2Q_i g_0$$

Rotated graphene layers in gr-hBN-gr

$$J(\omega) = \frac{e^2}{4\pi^4} \int d^2\bar{Q}_i \int d^2\bar{k}_{\parallel} \; [1 + \cos[\phi_i - \phi_j' + \theta)] \; \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega) \; I_1(k_{\parallel}, \omega)$$

$$\Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega) = \delta(\hbar v_F Q_j - \hbar v_F Q_i + \epsilon_{F_1} - \epsilon_{F_1} - eV_b + \hbar\omega) \; \Theta(\epsilon_{F_1} - \hbar v_F Q_i)\,\Theta(\hbar v_F Q_j - \epsilon_{F_2})$$

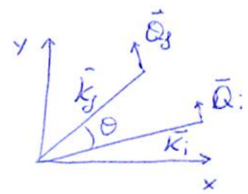


$$\hbar v_F Q_j = \hbar v_F \, |\bar{Q}_i - \bar{G}_{\parallel}|$$

$$\vec{k}_j \cdot \vec{k}_i = k^2 \cos\theta$$

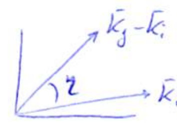
$$\bar{G}_{\parallel} = \bar{k}_j - \bar{k}_i + \bar{k}_{\parallel}$$

$$K = \frac{4\pi}{3\sqrt{3}a_{cc}} \simeq 17\,\mathrm{nm}^{-1}$$



$$\phi_i = \mathrm{arctg}\left(\frac{Q_{y_i}}{Q_{x_i}}\right) \qquad \phi_j' = \mathrm{arctg}\left(\frac{Q_{y_i} - G_{\parallel y}}{Q_{x_i} - G_{\parallel x}}\right)$$

$$\bar{k}_j - \bar{k}_i = \frac{2\pi}{3a_{cc}}\left(\cos\theta - \frac{\sin\theta}{\sqrt{3}} - 1 \,,\; \sin\theta + \frac{\cos\theta}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$$



Comparison:

$$\phi_j \longrightarrow \phi_j' - \theta$$

$$k_{\parallel} \longrightarrow \vec{G}_{\parallel}$$

$$Q_j \longrightarrow |\bar{Q}_i - \bar{G}_{\parallel}|$$

$$g_0 = \frac{1}{\hbar v_F}\left(\epsilon_{F_1} - \epsilon_{F_2} - eV_b + \hbar\omega\right)$$

$$G_{\parallel}^2 = |\bar{k}_j - \bar{k}_i|^2 + k_{\parallel}^2 + 2(\bar{k}_j - \bar{k}_i)\cdot\bar{k}_{\parallel} =$$

$$= k_j^2 + k_i^2 - 2\bar{k}_j\cdot\bar{k}_i + k_{\parallel}^2 + 2|\bar{k}_j - \bar{k}_i|\,k_{\parallel}\cos\eta =$$

$$= 2K^2 - 2K^2\cos\theta + k_{\parallel}^2 + 2\sqrt{2}\,k_{\parallel}\cos\eta \cdot K\sqrt{1-\cos\theta}$$

$$= 2K^2(1-\cos\theta) + k_{\parallel}^2 + 2\sqrt{2}\,K k_{\parallel}\cos\eta\sqrt{1-\cos\theta}$$

Zeros:

$$Q_{i01} = \frac{G_{\parallel}^2 - g_0^2}{2G_{\parallel}\cos\varphi - 2g_0}$$

$$G_{\parallel} = G_{\parallel}(k_{\parallel}, \eta, \theta)$$

$$G_{\parallel x} = G_{\parallel}\cos\varphi$$

$$\longrightarrow \; Q_{i01} = Q_{i01}(\varphi, \eta, g_0, k_{\parallel})$$

$$G_{\parallel y} = G_{\parallel}\sin\varphi$$

$$J(\omega) = \frac{e^2}{\hbar 4\pi^4 v_F}\, 2\int_0^{\infty} dk_{\parallel}\; k_{\parallel}\, I_1(k_{\parallel},\omega)\int_0^{\pi} d\eta \int_{-\pi}^{\pi} d\varphi \;\; Q_{i01}\left[\,1 \underbrace{\pm}_{\text{in case of e-h or h-e}} \cos(\phi_{i0} - \phi_{j0}' + \theta)\,\right]\,\Theta(Q_{i01})\,\frac{1}{|F'(Q_{i01})|}$$

$$F(Q_i) = Q_j - Q_i + g_0 = |\bar{Q}_i - \bar{G}_{\parallel}| - Q_i + g_0$$

$G_{\parallel} \neq G_{\parallel}(Q_i)$

$$Q_j = |\bar{Q}_i - \bar{G}_{\parallel}| = \sqrt{Q_i^2 + G_{\parallel}^2 - 2\bar{Q}_i \bar{G}_{\parallel}} = \sqrt{Q_i^2 + G_{\parallel}^2 - 2 Q_i G_{\parallel} \cos\varphi}$$

$$F'(Q_i) = -1 + \frac{Q_i - G_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + G_{\parallel}^2 - 2Q_i G_{\parallel}\cos\varphi}}$$

$$Q_{j0} = -g_0 + Q_{i0} > 0$$

$$F'(Q_{i0}) = -1 + \frac{Q_{i0} - G_{\parallel}\cos\varphi}{-g_0 + Q_{i0}} = \frac{+g_0 - G_{\parallel}\cos\varphi}{-g_0 + Q_{i0}}$$

$$\frac{1}{|F'(Q_{i0})|} = \frac{|g_0 - Q_{i0}|}{|g_0 - G_{\parallel}\cos\varphi|}$$

$$J(\omega) = \frac{e^2}{2\pi^4 \hbar v_F} \int_0^{\infty} dk_{\parallel}\, k_{\parallel}\, I_1(k_{\parallel}, \omega) \int_0^{\pi} dy \int_{-\pi}^{\pi} d\varphi\, \frac{Q_{i0}}{|F'(Q_{i0})|} \left[1 \pm \cos(\phi_{i0} - \phi'_{j0} + \theta)\right] \Theta(Q_{j0})$$

14 marzo 2019

PARALLEL ELECTRON WAVE - FUNCTIONS IN GLBNG

$$\varphi_i^+(\bar{r}) \cdot \varphi_j(\bar{r}) \quad \varphi_j^+(\bar{r}') \cdot \varphi_i(\bar{r}') \;:$$

electrons: $\varphi_e(\bar{r}) = \frac{1}{\sqrt{2A}} e^{i\bar{Q}\cdot\bar{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} e^{i\bar{k}\bar{r}}$

Perfect stacking: $\bar{k}_j = \bar{k}_i$

holes: $\varphi_h(\bar{r}) = \frac{1}{\sqrt{2A}} e^{i\bar{Q}\bar{r}} \begin{pmatrix} -e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} e^{i\bar{k}\bar{r}}$

$\boxed{e-e}$

$$\frac{1}{4A^2} e^{i(\bar{r}-\bar{r}')(\bar{Q}_j-\bar{Q}_i+\bar{k}_j-\bar{k}_i)} \left| \begin{bmatrix} e^{-i\phi_i/2} & e^{i\phi_i/2} \end{bmatrix} \begin{bmatrix} e^{i\phi_j/2} \\ e^{-i\phi_j/2} \end{bmatrix} \right|^2 =$$

$$= \frac{1}{4A^2} e^{i\tilde{r}\tilde{Q}} \left[e^{i(\phi_j-\phi_i)/2} + e^{-i(\phi_j-\phi_i)/2} \right]^2 = \frac{1}{4A^2} e^{i\tilde{r}\tilde{Q}} \left[2\cos\left(\frac{\phi_j-\phi_i}{2}\right) \right]^2 = \circledast$$

NOTE:

$$\begin{pmatrix} e^{-i\phi_j/2} & e^{i\phi_j/2} \end{pmatrix} \begin{pmatrix} e^{i\phi_i/2} \\ e^{-i\phi_i/2} \end{pmatrix} = e^{-i(\phi_j-\phi_i)/2} + e^{i(\phi_j-\phi_i)/2} \quad \checkmark$$

$$\cos\alpha = \frac{1}{2}\left(e^{i\alpha} + e^{-i\alpha}\right) \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\circledast = \frac{1}{4A^2} e^{i\tilde{r}\tilde{Q}} \, 4\left(1+\cos(\phi_j-\phi_i)\right)\frac{1}{2} = \frac{1}{2A^2} e^{i\tilde{r}\tilde{Q}} \left[1+\cos(\phi_j-\phi_i)\right] =$$

$$= \frac{1}{2A^2} e^{i\tilde{r}\tilde{Q}} \left[1 + \frac{\bar{Q}_j \cdot \bar{Q}_i}{Q_j \cdot Q_i}\right]$$

$\boxed{h-h}$

$$\frac{1}{4A^2} e^{i\tilde{r}\tilde{Q}} \begin{bmatrix} -e^{-i\phi_i/2} & e^{i\phi_i/2} \end{bmatrix} \cdot \begin{bmatrix} -e^{i\phi_j/2} \\ e^{-i\phi_j/2} \end{bmatrix} \begin{bmatrix} -e^{-i\phi_j/2} & e^{i\phi_j/2} \end{bmatrix} \cdot \begin{bmatrix} -e^{i\phi_i/2} \\ e^{-i\phi_i/2} \end{bmatrix} =$$

$$= \frac{1}{4A^2} e^{i\tilde{r}\tilde{Q}} \left[e^{i(\phi_j-\phi_i)/2} + e^{-i(\phi_j-\phi_i)/2} \right]^2 = \frac{1}{2A^2} e^{i\tilde{r}\tilde{Q}} \left[1 + \frac{\bar{Q}_j \cdot \bar{Q}_i}{Q_j \cdot Q_i}\right] \quad \checkmark$$

$\boxed{e-h}$

$$\frac{1}{4A^2} e^{i\tilde{Q}\tilde{r}} \begin{bmatrix} e^{-i\phi_i/2} & e^{i\phi_i/2} \end{bmatrix} \cdot \begin{bmatrix} -e^{i\phi_j/2} \\ e^{-i\phi_j/2} \end{bmatrix} \begin{bmatrix} -e^{-i\phi_j/2} & e^{i\phi_j/2} \end{bmatrix} \cdot \begin{bmatrix} e^{i\phi_i/2} \\ e^{-i\phi_i/2} \end{bmatrix} =$$

$$= \frac{1}{4A^2} e^{i\tilde{Q}\tilde{r}} \left[-e^{i(\phi_j-\phi_i)/2} + e^{-i(\phi_j-\phi_i)/2} \right] \cdot \left[-e^{-i(\phi_j-\phi_i)/2} + e^{i(\phi_j-\phi_i)/2} \right] =$$

$$= \frac{-1}{4A^2} e^{i\tilde{Q}\tilde{r}} \left[e^{i(\phi_j-\phi_i)/2} - e^{-i(\phi_j-\phi_i)/2} \right]^2 = \frac{-1}{4A^2} e^{i\tilde{Q}\tilde{r}} (2i)^2 \sin^2 \frac{(\phi_j-\phi_i)}{2} =$$

$$= \frac{1}{2A^2} e^{i\tilde{Q}\tilde{r}} \left[1 - \cos(\phi_j - \phi_i) \right] = \frac{1}{2A^2} e^{i\tilde{Q}\tilde{r}} \left[1 - \frac{\vec{Q}_j \vec{Q}_i}{Q_j Q_i} \right]$$

$\boxed{h-e}$

$$\frac{1}{4A^2} e^{i\tilde{Q}\tilde{r}} \begin{bmatrix} -e^{-i\phi_i/2} & e^{i\phi_i/2} \end{bmatrix} \cdot \begin{bmatrix} e^{i\phi_j/2} \\ e^{-i\phi_j/2} \end{bmatrix} \begin{bmatrix} e^{-i\phi_j/2} & e^{i\phi_j/2} \end{bmatrix} \begin{bmatrix} -e^{i\phi_i/2} \\ e^{-i\phi_i/2} \end{bmatrix} =$$

$$= \frac{1}{2A^2} e^{i\tilde{Q}\tilde{r}} \left[1 - \frac{\vec{Q}_j \vec{Q}_i}{Q_j Q_i} \right]$$

Modify A's for e-h and h-e:

$$A = \left[1 - \frac{Q_{i0} - k_{11} \cos\varphi}{Q_{j0}} \right] \frac{1}{|F'(Q_{i0})|}$$

$\boxed{1}$

$$A_1 = \frac{Q_j - Q_i + k_{11} \cos\varphi}{Q_j} \quad \frac{Q_j}{|Q_i - Q_j - k_{11} \cos\varphi|} = \frac{Q_j - Q_i + k_{11} \cos\varphi}{|Q_j - Q_i + k_{11} \cos\varphi|} =$$

$$= \operatorname{sgn}(Q_j - Q_i + k_{11} \cos\varphi)$$

$$A_1(Q_{i01}) = \operatorname{sgn}\left(- g_0 + k_{11} \cos\varphi\right)$$

$\boxed{2}$

$$A_2(Q_{i02}) = \frac{-g_0 - 2Q_{i02} + k_{11} \cos\varphi}{|-g_0 - k_{11} \cos\varphi|} = \frac{-g_0 - 2Q_{i02} + k_{11} \cos\varphi}{|g_0 + k_{11} \cos\varphi|}$$

$\boxed{3}$

$$A_3(Q_{i03}) = \frac{g_0 - 2Q_{i03} + k_{11} \cos\varphi}{|g_0 - k_{11} \cos\varphi|}$$

$\boxed{4}$ $\quad A_4 = \operatorname{sgn}(g_0 + k_{11} \cos\varphi)$