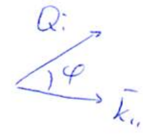
11 feb 2019

Doping combinations for the gr-hBN-gr tunnelling device

$$J(\omega) = \frac{e^2}{4\pi^4} \int d^2\bar{Q}_i \int d^2\bar{k}_{\parallel} \left[1 + \cos(\phi_i - \phi_j) \right] \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega)\, I_1(k_{\parallel}, \omega)$$

$$\bar{Q}_j = \bar{Q}_i - \bar{k}_{\parallel}$$

$$\cos(\phi_i - \phi_j) = \frac{\bar{Q}_i \cdot \bar{Q}_j}{Q_i Q_j} = \frac{Q_i Q_j \cos\varphi}{Q_i Q_j}$$



$$Q_j = |\bar{Q}_i - \bar{k}_{\parallel}| = \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi}$$

$$\bar{Q}_i \cdot \bar{Q}_j = \bar{Q}_i (\bar{Q}_i - \bar{k}_{\parallel}) = Q_i^2 - \bar{Q}_i \cdot \bar{k}_{\parallel} = Q_i^2 - Q_i k_{\parallel} \cos\varphi$$

$$\cos(\phi_i - \phi_j) = \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j} = \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}}$$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\bar{k}_{\parallel} \int_0^{\pi} d\varphi \int_0^{\infty} dQ_i\, Q_i \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \right] \Delta(Q_i, k_{\parallel}, \omega)$$

$$1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = \frac{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi} + Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}}$$

Solving the J's:

$Q_i > 0$

$Q_j > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int_0^{\infty} dk_{\parallel} \int_{-\pi}^{\pi} d\eta \int_0^{\pi} d\varphi\, I_1(k_{\parallel}, \omega) Q_{i0} k_{\parallel} \left[1 + \frac{Q_{i0} - k_{\parallel}\cos\varphi}{\sqrt{Q_{i0}^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \right] \Delta(Q_{i0}, k_{\parallel}, \omega) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dk_{\parallel} \int_0^{\pi} d\varphi\, I_1(k_{\parallel}, \omega)\, Q_{i0} k_{\parallel} \overbrace{\left[1 + \frac{Q_{i0} - k_{\parallel}\cos\varphi}{Q_{j0}} \right] \frac{1}{|F'(Q_i)|}}^{= A(Q_{i0})} \Theta_0$$

$$Q_{j0} = \sqrt{Q_{i0}^2 + k_{\parallel}^2 - 2Q_{i0} k_{\parallel}\cos\varphi}$$

$$\Delta \propto \delta(\hbar v_F \tilde{Q} + g) \quad \begin{cases} \tilde{Q} = \tilde{Q}(Q_j, Q_i) = \tilde{Q}(Q_i, k_{\parallel}, \varphi) \\ g = g(\epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) \end{cases}$$

Possibilites:

$$\tilde{Q}_1 = Q_j - Q_i$$

$$\tilde{Q}_2 = Q_j + Q_i$$

$$\tilde{Q}_3 = -Q_j - Q_i$$

$$\tilde{Q}_4 = -Q_j + Q_i$$

Dirac-delta properties:

$$\delta(F(x)) = \sum_j \frac{\delta(x - x_j)}{|F'(x_j)|}$$

$$\left[\begin{array}{l} F(x_j) = 0 \\ F'(x) = \dfrac{dF(x)}{dx} \end{array} \right]$$

$$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$$

$$\delta(\hbar v_F \tilde{Q} + g) = \frac{1}{\hbar v_F} \delta\left(\tilde{Q} + \frac{g}{\hbar v_F}\right)$$

$$F(Q_i) = \tilde{Q} + \frac{g}{\hbar v_F} = \tilde{Q} + g_0$$

ZEROS

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 + Q_j - Q_i = g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi} = 0$$

$$Q_i^2 + k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi = Q_i^2 + g_0^2 - 2Q_i g_0 \quad ; \quad 2Q_i k_{\parallel} \cos\varphi = -g_0^2 + 2Q_i g_0 + k_{\parallel}^2$$

$$Q_i (2k_{\parallel} \cos\varphi - 2g_0) = k_{\parallel}^2 - g_0^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2k_{\parallel}\cos\varphi - 2g_0} = Q_{i01}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_j + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{\shortparallel}^2 - 2Q_i k_{\shortparallel} \cos\varphi} = 0$$

$$Q_i^2 + k_{\shortparallel}^2 - 2Q_i k_{\shortparallel} \cos\varphi = Q_i^2 + g_0^2 + 2Q_i g_0$$

$$k_{\shortparallel}^2 - g_0^2 = Q_i\,(2k_{\shortparallel}\cos\varphi + 2g_0)$$

$$Q_i = \frac{k_{\shortparallel}^2 - g_0^2}{2(k_{\shortparallel}\cos\varphi + g_0)} = Q_{i02}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0 - Q_j - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{\shortparallel}^2 - 2Q_i k_{\shortparallel} \cos\varphi} = 0$$

$$-2Q_i g_0 + g_0^2 + Q_i^2 = Q_i^2 + k_{\shortparallel}^2 - 2Q_i k_{\shortparallel}\cos\varphi$$

$$Q_i^2 - Q_i k_{\shortparallel}\cos\varphi + \frac{k_{\shortparallel}^2 - g_0^2}{2} = 0$$

$$Q_i = \frac{1}{2}k_{\shortparallel}\cos\varphi \pm \frac{1}{2}\sqrt{k_{\shortparallel}^2\cos^2\varphi - k_{\shortparallel}^2 + g_0^2} =$$

$$= \frac{1}{2}\left[k_{\shortparallel}\cos\varphi \pm \sqrt{k_{\shortparallel}^2(\cos^2\varphi - 1) + g_0^2}\right] =$$

$$= \frac{1}{2}\left[k_{\shortparallel}\cos\varphi \pm \sqrt{g_0^2 - k_{\shortparallel}^2\sin^2\varphi}\right] = Q_{i03}^{\pm}$$

$$Q_i(2k_{\shortparallel}\cos\varphi - 2g_0) = k_{\shortparallel}^2 - g_0^2$$

$$Q_{i03} = \frac{k_{\shortparallel}^2 - g_0^2}{2(k_{\shortparallel}\cos\varphi - g_0)} = Q_{i0 1}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$g_0 - Q_j + Q_i = g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$g_0^2 + Q_i^2 + 2g_0 Q_i = Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi$$

$$2Q_i\left(g_0 + k_{\parallel}\cos\varphi\right) = k_{\parallel}^2 - g_0^2$$

$$Q_i = \frac{k_{\parallel}^2 - g_0^2}{2(g_0 + k_{\parallel}\cos\varphi)} = Q_{i02} \quad \rightarrow \quad \text{same as case } 2!$$

$$Q_{i04} = Q_{i02}$$

DERIVATIVES

1) $\tilde{Q}_1 + g_0 = 0$

$$F(Q_i) = g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$F'(Q_i) = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} = -1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} \qquad Q_{j01} = Q_{i01} - g_0$$

$$A(Q_i) = \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{Q_j}\right]\frac{1}{|F'(Q_i)|} = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{Q_j}\,\frac{Q_j}{|Q_i - Q_j - k_{\parallel}\cos\varphi|} =$$

$$= \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|-Q_j + Q_i - k_{\parallel}\cos\varphi|} = \frac{Q_j + Q_i - k_{\parallel}\cos\varphi}{|Q_j - Q_i + k_{\parallel}\cos\varphi|}$$

$$A_1(Q_{i01}) = \frac{2Q_{i01} - g_0 - k_{\parallel}\cos\varphi}{|-g_0 + k_{\parallel}\cos\varphi|}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$F(Q_i) = g_0 + Q_j + Q_i = g_0 + Q_i + \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = 1 + \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}} = 1 + \frac{Q_i - k_{11}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j + Q_i - k_{11}\cos\varphi|} \qquad Q_{j_{02}} = -g_0 - Q_{i_{02}}$$

$$\Delta(Q_i) = [\quad]\frac{1}{|F'(Q_i)|} = \frac{Q_j + Q_i - k_{11}\cos\varphi}{|Q_j + Q_i - k_{11}\cos\varphi|} = \mathrm{sgn}(Q_j + Q_i - k_{11}\cos\varphi)$$

$\hookrightarrow A_2(Q_{i_{02}}) = \mathrm{sgn}(-g_0 - k_{11}\cos\varphi) = A_2$

3) $\tilde{Q}_3 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j - Q_i = g_0 - Q_i - \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = -1 - \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}} = -1 - \frac{Q_i - k_{11}\cos\varphi}{Q_j}$$

$$\frac{1}{|F'(Q_i)|} = \frac{Q_j}{-Q_j - Q_i + k_{11}\cos\varphi} \qquad Q_{j_{03}}^{\pm} = g_0 - Q_{i_{03}}^{\pm}$$

$$A(Q_i) = \frac{Q_j + Q_i - k_{11}\cos\varphi}{|-Q_j - Q_i + k_{11}\cos\varphi|} = \mathrm{sgn}(Q_j + Q_i - k_{11}\cos\varphi) \longrightarrow \text{same as case 2!}$$

$\hookrightarrow A_3(Q_{i_{03}}) = \mathrm{sgn}(g_0 - k_{11}\cos\varphi) = A_3$

4) $\tilde{Q}_4 + g_0 = 0$

$$F(Q_i) = g_0 - Q_j + Q_i = g_0 + Q_i - \sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}$$

$$F'(Q_i) = 1 - \frac{Q_i - k_{11}\cos\varphi}{\sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11}\cos\varphi}} \quad \Big\| \quad \frac{1}{|F'(Q_i)|} = \frac{Q_j}{|Q_j - Q_i + k_{11}\cos\varphi|}$$

$Q_{j_{04}} = g_0 + Q_{i_{02}} = -Q_{j_{02}}$

$$\Delta(Q_i) = \frac{Q_j + Q_i - k_{11}\cos\varphi}{|Q_j - Q_i + k_{11}\cos\varphi|} \longrightarrow \text{same as case 1!}$$

$$\hookrightarrow A_4(Q_{i_{02}}) = \frac{2Q_{i_{02}} + g_0 - k_{11}\cos\varphi}{|g_0 + k_{11}\cos\varphi|}$$

Spectrally resolved probabilities

$$S(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i \; Q_i \, I_1(k_\parallel, \omega) \left[1 + \frac{Q_i - k_\parallel \cos\varphi}{Q_j} \right] \delta(\tilde{Q} + g_0) \, \Theta(Q_j, Q_i) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \, k_\parallel \int_0^\pi d\varphi \int_0^\infty dQ_i \; Q_i \, I_1(k_\parallel, \omega) \left[1 + \frac{Q_i - k_\parallel \cos\varphi}{Q_j} \right] \sum_0 \frac{\delta(Q_i - Q_{i0})}{|F'(Q_{i0})|} \, \Theta(Q_j, Q_i) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \sum_0 k_\parallel \, Q_{i0} \, I_1(k_\parallel, \omega) \, A(Q_{i0}) \, \Theta(Q_{j0}, Q_{i0}) =$$

$$= \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; k_\parallel \, I_1(k_\parallel, \omega) \; B(Q_{i0})$$

$g_0 > 0$

$Q_{i0} > 0$

$$B(Q_{i0}) = B(k_\parallel, \epsilon_{F1}, \epsilon_{F2}, \omega, eV_b) = \sum_0 Q_{i0} \, A(Q_{i0}) \, \Theta(Q_{j0}, Q_{i0})$$

Combinations

a) electron - hole : $g_0^{eh} = \frac{1}{\hbar v_F} \left(\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega \right)$

$$\delta(\tilde{Q}_3 + g_0^{eh}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io_3}) \, \Theta(\epsilon_{F_2} - \hbar v_F Q_{jo_3}) +$$

$$+ \delta(\tilde{Q}_1 + g_0^{eh}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{i0}) +$$

$$+ \delta(\tilde{Q}_4 + g_0^{eh}) \, \Theta(\epsilon_{F_2} - \hbar v_F Q_{j0}) +$$

$$+ \delta(\tilde{Q}_2 + g_0^{eh})$$

$$B^{eh} = Q_{io_3}^{+} A_3 \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io_3}^{+}) \, \Theta(\epsilon_{F_2} - \hbar v_F Q_{jo_3}^{+}) + Q_{io_3}^{-} A_3 \, \Theta(\epsilon_{F1} - \hbar v_F Q_{io_3}^{-}) \Theta(\epsilon_{F_2} - \hbar v_F Q_{jo_3}^{-})$$

$$+ Q_{io_1} A_1(Q_{io_1}) \, \Theta(\epsilon_{F_1} - \hbar v_F Q_{io_1}) +$$

$$+ Q_{io_2} A_4(Q_{io_2}) \, \Theta(\epsilon_{F_2} + \hbar v_F Q_{jo_2}) +$$

$$+ Q_{io_2} A_2$$

b) hole - electron: $g_0^{he} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_2 + g_0^{he}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$

$$B^{he} = Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

c) hole - hole: $g_0^{hh} = \frac{1}{\hbar v_F}\left(-\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_4 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} - \hbar v_F Q_f\right) +$$
$$+\,\delta\left(\tilde{Q}_2 + g_0^{hh}\right)\Theta\left(\hbar v_F Q_i - \epsilon_{F1}\right)$$

$$B^{hh} = Q_{io2}\, A_4(Q_{io2})\,\Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)\Theta\left(\epsilon_{F2} + \hbar v_F Q_{fo2}\right) +$$
$$+\, Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{io2} - \epsilon_{F1}\right)$$

d) electron - electon: $g_0^{ee} = \frac{1}{\hbar v_F}\left(\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega\right)$

$$\delta\left(\tilde{Q}_1 + g_0^{ee}\right)\Theta\left(\epsilon_{F1} - \hbar v_F Q_i\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$
$$+\,\delta\left(\tilde{Q}_2 + g_0^{ee}\right)\Theta\left(\hbar v_F Q_f - \epsilon_{F2}\right)$$

$$B^{ee} = Q_{io1}\, A_1(Q_{io1})\,\Theta\left(\epsilon_{F1} - \hbar v_F Q_{io1}\right)\Theta\left(\hbar v_F Q_{fo1} - \epsilon_{F2}\right) +$$
$$+\, Q_{io2}\, A_2\, \Theta\left(\hbar v_F Q_{fo2} - \epsilon_{F2}\right)$$

SUMMARY

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^\infty dk_\parallel \int_0^\pi d\varphi \; k_\parallel \, I_1(k_\parallel, \omega) \, B^{ij}$$

ij : electron and hole doping combinations

$$Q_{i01} = \frac{k_\parallel^2 - g_0^2}{2k_\parallel \cos\varphi - 2g_0} \qquad Q_{i02} = \frac{k_\parallel^2 - g_0^2}{2(k_\parallel \cos\varphi + g_0)} \qquad Q_{i0_4} = Q_{i02}$$

$$\parallel$$

$$Q_{i0\bar{2}} = \frac{k_\parallel^2 - g_0^2}{2(k_\parallel \cos\varphi - g_0)} = Q_{i01}$$

$g_0 = g_0^{ij}$

$Q_{i0} = Q_{i0}(g_0^{ij})$

$$A_1(Q_{i01}) = \frac{2Q_{i01} - g_0 - k_\parallel \cos\varphi}{|-g_0 + k_\parallel \cos\varphi|} \qquad A_2 = \mathrm{sgn}(-g_0 - k_\parallel \cos\varphi)$$

$$A_4(Q_{i02}) = \frac{2Q_{i02} + g_0 - k_\parallel \cos\varphi}{|g_0 + k_\parallel \cos\varphi|} \qquad A_3 = \mathrm{sgn}(g_0 - k_\parallel \cos\varphi)$$

electron - electron: $g_0^{ee} = (\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{ee} = Q_{i01} A_1(Q_{i01}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{i01}) \, \Theta(\hbar v_F Q_{f01} - \epsilon_{F2}) + \quad ^{c-c} \text{ reg 1}$$
$$+ Q_{i02} A_2 \, \Theta(\hbar v_F Q_{f02} - \epsilon_{F2}) \quad ^{c-v} \text{ reg 2}$$

$Q_{f01} = Q_{i01} - g_0$

$Q_{f02} = -g_0 - Q_{i02}$

$Q_{f03}^{\pm} = g_0 - Q_{i03}^{\pm}$

$Q_{f04} = -Q_{f02}$

electron - hole: $g_0^{eh} = (\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{eh} = Q_{i03}^{+} A_3 \, \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}^{+}) \Theta(\epsilon_{F2} - \hbar v_F Q_{f03}^{+})^{c-v} + Q_{i03}^{-} A_3 \Theta(\epsilon_{F1} - \hbar v_F Q_{i03}^{-}) \Theta(\epsilon_{F2} - \hbar v_F Q_{f03}^{-})^{c-v} \text{ reg 3}$$
$$+ Q_{i01} A_1(Q_{i01}) \, \Theta(\epsilon_{F1} - \hbar v_F Q_{i01})^{c-c} + \quad \text{reg 1}$$
$$+ Q_{i02} A_4(Q_{i02}) \, \Theta(\epsilon_{F2} + \hbar v_F Q_{f02})^{v-v} + \quad Q_{i02} A_2^{\,v-c}$$

reg 4 reg 2

c: conduction
v: valence

hole - electron: $g_0^{he} = (-\epsilon_{F1} - \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{he} = Q_{i02} A_2 \, \Theta(\hbar v_F Q_{i02} - \epsilon_{F1}) \, \Theta(\hbar v_F Q_{f02} - \epsilon_{F2}) \quad ^{v-c} \text{ reg 2}$$

hole - hole: $g_0^{hh} = (-\epsilon_{F1} + \epsilon_{F2} - eV_b + \hbar\omega)/\hbar v_F$

$$B^{hh} = Q_{i02} A_4(Q_{i02}) \Theta(\hbar v_F Q_{i02} - \epsilon_{F1}) \, \Theta(\epsilon_{F2} + \hbar v_F Q_{f02}) + \quad ^{v-v} \text{ reg 4}$$
$$+ Q_{i02} A_2 \, \Theta(\hbar v_F Q_{i02} - \epsilon_{F1}) \quad ^{v-c} \text{ reg 2}$$

Allowed energies

$$g_0 = \gamma_0 + \hbar\omega/\hbar v_F$$

$$Q_i - k_{11} < Q_j < Q_i + k_{11}$$

1) $\tilde{Q}_1 + g_0$

$$g_0 - k_{11} < g_0 + Q_j - Q_i < g_0 + k_{11}$$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j - Q_i = 0$$

$\hbar\omega = Q_i - Q_j - \gamma_0 \longrightarrow$ reg 1 $\boxed{-k_{11} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \gamma_0}$ $\quad \hbar\omega > 0$

$$Q_i - Q_i + k_{11} - \gamma_0 = k_{11} - \gamma_0$$
$$Q_i - Q_i - k_{11} - \gamma_0 = -k_{11} - \gamma_0$$

2) $\tilde{Q}_2 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} + Q_j + Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = -\gamma_0 - Q_j - Q_i$$

reg 2 $\boxed{0 < -\gamma_0 - 2Q_i - k_{11} < \frac{\hbar\omega}{\hbar v_F} < -\gamma_0 - 2Q_i + k_{11}}$

$$-\gamma_0 - Q_i - k_{11} - Q_i = -\gamma_0 - 2Q_i - k_{11}$$
$$-\gamma_0 - Q_i + k_{11} - Q_i = -\gamma_0 - 2Q_i + k_{11}$$

3) $\tilde{Q}_3 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j - Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j + Q_i - \gamma_0$$

reg 3 $\boxed{2Q_i - k_{11} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \gamma_0 + 2Q_i}$ $\quad \frac{\hbar\omega}{\hbar v_F} > 0$

$$Q_i - k_{11} + Q_i - \gamma_0 = -k_{11} - \gamma_0 + 2Q_i$$
$$Q_i + k_{11} + Q_i - \gamma_0 = k_{11} - \gamma_0 + 2Q_i$$

4) $\tilde{Q}_4 + g_0$

$$\gamma_0 + \frac{\hbar\omega}{\hbar v_F} - Q_j + Q_i = 0$$

$$\frac{\hbar\omega}{\hbar v_F} = Q_j - Q_i - \gamma_0$$

reg 4 = reg 1

$$Q_i - k_{11} - Q_i - \gamma_0 = -k_{11} - \gamma_0$$
$$Q_i + k_{11} - Q_i - \gamma_0 = k_{11} - \gamma_0$$

$\boxed{-k_{11} - \gamma_0 < \frac{\hbar\omega}{\hbar v_F} < k_{11} - \gamma_0}$

15 Feb 2019

CALCULATIONS ELIMINATING $k_{\parallel}$ $\qquad$ $Q_i, Q_f, k_{\parallel} > 0$

$$J(\omega) = \frac{e^2}{2\pi^4} \int d^2\tilde{Q}_i \int_0^{\pi} d\varphi \int_0^{\infty} dk_{\parallel} \quad k_{\parallel} \left[1 + \frac{Q_i - k_{\parallel}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}} \right] \Delta(\bar{Q}_i, \bar{k}_{\parallel}, \omega)$$

$$\delta(F(k_{\parallel})) = \sum_0 \frac{\delta(k_{\parallel} - k_{\parallel 0})}{|F'(k_{\parallel 0})|} \qquad Q_f = \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi}$$

$$J(\omega) = \frac{e^2}{2\pi^4 \hbar v_F} \int_0^{\infty} dQ_i \cdot 2\pi \int_0^{\pi} d\varphi \sum_0 Q_i k_{\parallel 0} \left[1 + \frac{Q_i - k_{\parallel 0}\cos\varphi}{\sqrt{Q_i^2 + k_{\parallel 0}^2 - 2Q_i k_{\parallel 0}\cos\varphi}} \right] \frac{1}{|F'(k_{\parallel 0})|} \Theta_0$$

$$J(\omega) = \frac{e^2}{\hbar \pi^3 v_F} \int_0^{\infty} dQ_i \int_0^{\pi} d\varphi \sum_0 Q_i k_{\parallel 0} \, \Delta(Q_i, k_{\parallel 0}) \, \Theta_0$$

As before: $\quad F(k_{\parallel}) = \tilde{Q} + g_0 \qquad\qquad g_0 = g/\hbar v_F$

$\hookrightarrow \tilde{Q} = \tilde{Q}(k_{\parallel})$ also!

$\Rightarrow$ zeros:

1) $\tilde{Q}_1 + g_0 = 0$

$$g_0 - Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel}\cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 01}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2\cos^2\varphi + g_0^2 - 2Q_i g_0}$$

2) $\tilde{Q}_2 + g_0 = 0$

$$g_0 + Q_i + \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2k_{\parallel} Q_i \cos\varphi - g_0^2 - 2Q_i g_0 = 0$$

$$k_{\parallel 02}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0}$$

3) $\tilde{Q}_3 + g_0 = 0$

$$g_0^2 + Q_i^2 - 2Q_i g_0$$

$$g_0 - Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi \cancel{+ 2Q_i^2} - g_0^2 + 2Q_i g_0 = 0$$

$$k_{\parallel 03}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 - 2Q_i g_0}$$

4) $\tilde{Q}_4 + g_0 = 0$

$$g_0 + Q_i - \sqrt{Q_i^2 + k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi} = 0$$

$$k_{\parallel}^2 - 2Q_i k_{\parallel} \cos\varphi - g_0^2 - 2 g_0 Q_i = 0$$

$$k_{\parallel 04}^{\pm} = Q_i \cos\varphi \pm \sqrt{Q_i^2 \cos^2\varphi + g_0^2 + 2Q_i g_0} = k_{\parallel 02}^{\pm}$$

= derivatives

$$F'(k_{11}) = \frac{k_{11} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{11}^2 - 2Q_i k_{11} \cos\varphi}}$$ same for all of them!

$$F'(k_{11o}) = \frac{k_{11o} - Q_i \cos\varphi}{\sqrt{Q_i^2 + k_{11o}^2 - 2Q_i k_{11o} \cos\varphi}}$$

$$A = \left[1 + \frac{Q_i - k_{11o}\cos\varphi}{\sqrt{Q_i^2 + k_{11o}^2 - 2Q_i k_{11o}\cos\varphi}} \right] \frac{1}{|F'(k_{11o})|} = \left[\quad \right] \frac{\sqrt{Q_i^2 + k_{11o}^2 - 2Q_i k_{11o}\cos\varphi}}{|k_{11o} - Q_i \cos\varphi|} =$$

$$= \frac{\sqrt{Q_i^2 + k_{11o}^2 - 2Q_i k_{11o}\cos\varphi}}{|k_{11o} - Q_i\cos\varphi|} + \frac{Q_i - k_{11o}\cos\varphi}{|k_{11o} - Q_i\cos\varphi|}$$

1) $\tilde{Q}_1 + g_o$

$$Q_{g_o} = \sqrt{Q_i^2 + k_{11o}^2 - 2Q_i k_{11o}\cos\varphi}$$

$$A_{o1}^{\pm} = \frac{2Q_i - g_o - k_{o1}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_o^2 - 2Q_i g_o}}$$

2) $\tilde{Q}_2 + g_o$

$$A_{o2}^{\pm} = \frac{-g_o - k_{11o2}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_o^2 + 2Q_i g_o}}$$

3) $\tilde{Q}_3 + g_o$

$$A_{o3}^{\pm} = \frac{g_o - k_{11o3}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_o^2 - 2Q_i g_o}}$$

4) $\tilde{Q}_4 + g_o$

$$A_{o4}^{\pm} = \frac{g_o + 2Q_i - k_{11o2}^{\pm}\cos\varphi}{\sqrt{Q_i^2\cos^2\varphi + g_o^2 + 2Q_i g_o}}$$