Lecture 2 - Probability Theory & Inferential Statistics

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Review of Probability Theory & Inferential Statistics

Warning - the following content *will* seem abstract at first, but I promise it is essential to understanding statistical inference.

Probability theory has to do with assigning expectations for events in the 'long run' that you can generally only observe in a more limited way (e.g., limited by time or the number of 'trials').

A prominent example is a coin toss - we expect a 50/50 chance to obtain heads/tails on any given toss. How about if we tossed a coin 1000 times - how many times do you think we will see heads appear?

First Things First - Some Definitions

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- **Outcome**: Each distinct result of a trial
- **Sample Space**: The set of all possible outcomes, denoted S

Some Classic Examples

Coins, Cards, and Dice

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- **Trial**: flip of a coin, roll a die, draw a card
- **Outcome**: heads, '2', ace of spades
- **Sample space**: [H,T], [1,2,3,4,5,6], [all 52 cards]

Some Additional Definitions

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'Success' is defined as some event occurring:

$$p(A) = \frac{\text{\# of observations favoring Event 'A'}}{\text{total \# of observations in sample space}}$$

Some Additional Definitions (Continued)

Complement of an event: The set of all possibilities of an event not occurring

'Failure' is defined as the event not occurring

$$p(\text{not A}) = p(\sim A) = p(A') = 1 - p(A)$$

By definition, $p(A) + p(\sim A) = 1$

Here's Some Examples to Reinforce The Concept of 'Events'

Flipping a coin: S=[H,T]

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$$p(H) = \frac{p(H)}{p(H) + p(T)}$$

$$p(H) = \frac{.50}{.50 + .50}$$

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Rolling a die: S=[1,2,3,4,5,6]

$$p(1) = \frac{p(1)}{p(1)+p(2)+p(3)+p(4)+p(5)+p(6)}$$

$$p(1) = \frac{(1/6)}{(1/6)+(1/6)+(1/6)+(1/6)+(1/6)+(1/6)}$$

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$$p(even) = \frac{p(2) + p(4) + p(6)}{p(1) + p(2) + p(3) + p(4) + p(5) + p(6)}$$

Here Are Some Examples to Reinforce the Concept of 'Events'

Rolling a die: S=[1,2,3,4,5,6]

$$p(even) = \frac{p(2)+p(4)+p(6)}{p(1)+p(2)+p(3)+p(4)+p(5)+p(6)}$$

$$p(even) = \frac{(1/6)+(1/6)+(1/6)}{(1/6)+(1/6)+(1/6)+(1/6)+(1/6)+(1/6)}$$

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Drawing a card: S=[all 52 cards]

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$$p(\text{jack of diamonds}) = \frac{p(\text{jack of diamonds})}{p(\text{all 52 cards})}$$

$$p(\text{jack of diamonds}) = \frac{1/52}{52/52}$$

Here Are Some Examples to Reinforce the Concept of ‘Events’

Drawing a card: S=[all 52 cards]

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Drawing a card: S=[all 52 cards]

$$p(\text{ace}) = \frac{p(\text{ace})}{p(\text{all 52 cards})}$$

$$p(\text{ace}) = \frac{(1/52) + (1/52) + (1/52) + (1/52)}{52/52}$$

More on Events

When we talk about the **probability** of an event occurring we are referring to them over the *long run*, or over an *infinite* number of trials.

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Probability also called a 'limiting relative frequency' - contrast to a *proportion* (or relative frequency).

Building on the Basics - Counting Rules

From this foundation, we can conceive of a range of possible outcomes that *may* occur over a series of trials with a set sample space S.

To count these outcomes, there are a few basic rules.

Counting Rule #1 - Basic Counting Rule

Basic counting rule: the number of total possible outcomes from 'n' *independent* trials

$$k_1 \text{ x } k_2 \text{ x } k_3 \text{ x } \cdots \text{ x } k_n$$

$k_i = \#$ total outcomes from trial 'i'

Counting Rule #2 - Permutation Rule

Permutation rule: the # of possible *ordered* arrangements of ‘r’ objects from a group of ‘n’ objects.

Order matters!

$$\text{with replacement: } P_r^n = n^r$$

$$\text{without replacement: } P_r^n = \frac{n!}{(n-r)!}$$

Factorial notation: 3!=(3*2*1) = 6 and 0! = 1

Counting Rule #3 - Combination Rule

Combination rule: the number of possible *unordered* arrangements of r objects from a collection of ‘n’ objects.

Order does **not** matter!

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Applying the Permutation and Combination Rules - An Example

Let's apply these rules using the same criteria - we have n=10 trials and r=5 successes.

Permutations $= P_5^{10}$

$$
\begin{aligned}
P_5^{10} &= \frac{10!}{(10-5)!} \\
&= \frac{10 \text{ x } 9 \text{ x } 8 \text{ x } 7 \text{ x } 6 \text{ x } 5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1}{5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1} \\
&= 10 \text{ x } 9 \text{ x } 8 \text{ x } 7 \text{ x } 6 \\
&= 30240
\end{aligned} \tag{1}
$$

Applying the Permutation and Combination Rules - An Example

Combinations $= C_5^{10}$

$$\begin{aligned} C_5^{10} &= \frac{10!}{5!(10-5)!} \\ &= \frac{10 \text{ x } 9 \text{ x } 8 \text{ x } 7 \text{ x } 6 \text{ x } 5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1}{(5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1)(5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1)} \\ &= \frac{10 \text{ x } 9 \text{ x } 8 \text{ x } 7 \text{ x } 6}{5 \text{ x } 4 \text{ x } 3 \text{ x } 2 \text{ x } 1} \\ &= \frac{30240}{120} \\ &= 252 \end{aligned} \tag{2}$$

Some Applied Examples

License plates in NC: 3 letters, 4 numbers - How many different sequences?

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Which counting rule do we use?

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$$\# \text{ of sequences} = 26 * 26 * 26 * 10 * 10 * 10 * 10 = 175760000$$

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Combination lock: 36 numbers, 3 in the combo - How many different sequences?

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$$\# \text{ of sequences} = 36 * 36 * 36 = 46656$$

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Poker: 5 card hands - How many different sequences?

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Same counting rule?

$$\# \text{ of sequences} = \frac{52!}{(52-5)!} = 311875200$$

Another Applied Example - Counting Marbles

Let's suppose we have a bag of four colored marbles (Red, Orange, Green, & Blue).

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How many *ordered* arrangement of all 4 exist?

$$P_4^4 = \frac{4!}{(4-4)!} = 24$$

Another Applied Example - Counting Marbles

Here are the 'receipts' for the above calculation. Below is a table for *each* of the 24 possible outcomes:

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ROGB	ORGB	GROB	BGRO
ROBG	ORBG	GRBO	BGOR
RBOG	OBRG	GBOR	BRGO
RBGO	OBGR	GBRO	BROG
RGBO	OGBR	GORB	BORG
RGOB	OGRB	GOBR	BOGR

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What about the # of *ordered* arrangments of just two marbles at a time?

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What about the # of *ordered* arrangments of just two marbles at a time?

$$P_2^4 = \frac{4!}{(4-2)!} = 12$$

Another Applied Example - Counting Marbles

And again, the 'receipts':

RO	OR	BO	GO
RB	OB	BR	GR
RG	OG	BG	GB

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$$C_4^4 = \frac{4!}{4!(4-4)!} = \frac{4!}{4!} = 1$$

Another Applied Example - Counting Marbles

What about the # of *unordered* arrangements of all four?

What rule do we use here?

$$C_4^4 = \frac{4!}{4!(4-4)!} = \frac{4!}{4!} = 1$$

That's right - if order doesn't matter, there's only one combination of all 4 marbles being selected.

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Last one, I promise. What about the # of *unordered* arrangements of two out of the four marbles?

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Last one, I promise. What about the # of *unordered* arrangements of two out of the four marbles?

$$C_2^4 = \frac{4!}{2!(4-2)!} = 6$$

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Although the above result might seem strange at first, consider the following table of results:

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Although the above result might seem strange at first, consider the following table of results:

RO/OR	BG/GB
RG/GR	BO/OB
RB/BR	GO/OG

Make more sense now?

Rules of Probability

Before we begin to talk about *distributions* of probability, it is helpful to review some basic rules.

Probability Rule #1 - The Bounding Rule

All probabilities are bounded by 0 and 1

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This means that a probability value may not be negative or exceed a value of 1

$$0 \leq p(A) \leq 1$$

Probability Rule #2 - The Addition Rule

Addition rule: the probability of observing either of two events, or the union of two or more events.

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$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

Addition rule for mutually exclusive events: if two events cannot simultaneously occur, there's no joint probability.

$$p(A \cup B) = p(A) + p(B)$$

Probability Rule #3 - The Multiplication Rule

Multiplication rule: the probability of observing two or more events simultaneously, or the *intersection* of two or more events (aka, joint probability).

$$p(A \cap B) = p(A) \text{ X } p(B \mid A)$$

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Multiplication rule for independent events: Where the probability of event B is unaffected by the occurrence of event A (e.g., sampling with replacement).

$$p(A \cap B) = p(A) \text{ X } p(B)$$

A Word on Conditional Probabilities

A conditional probability is defined as the probability of one event occurring *given* that another event has occurred.

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These are generally important to the social sciences, as they are central to causal reasoning in the absence of experimental data.

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Some examples of conditional probability questions include:

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- What is the probability that a person will receive an offer for an interview if they have a felony conviction?
- What is the probability a person will be convicted as an adult if they were convicted as a juvenile?
- What is the probability that someone will relapse into drug use after they had participated in a drug abuse program?

Practice - Coins, Dice, and Cards

Let's work through some examples!

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- $p(H \cup T) = p(H \cup H') = \frac{1}{2} + \frac{1}{2} = ???$

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- $p(H \cup T) = p(H \cup H') = \frac{1}{2} + \frac{1}{2} = 1.0$

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- $p(H \cup T) = p(H \cup H') = \frac{1}{2} + \frac{1}{2} = 1.0$
- $p(HH \cup TT) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} =???$

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- $p(H \cup T) = p(H \cup H') = \frac{1}{2} + \frac{1}{2} = 1.0$
- $p(HH \cup TT) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = 0.50$

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- $p(H \cap T) = \frac{1}{2}$ x $\frac{1}{2} = \frac{1}{4}$ =???

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- $p(H \cap T) = \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{4} = 0.25$
- $p(HH \cap TT) = \frac{1}{4} \text{ x } \frac{1}{4} = \frac{1}{16} = ???$

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- $p(HH \cap TT) = \frac{1}{4} \text{ x } \frac{1}{4} = \frac{1}{16} = 0.06$

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- $p(H \cap H \cap H \cap H \cap H) = \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{32}$ =???

Practice - Coins, Dice, and Cards

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- $p(H \cap H \cap H \cap H \cap H) = \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{32} = 0.031$

Practice - Coins, Dice, and Cards

Let's work through some (more) examples!

- $p(1 \cup 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = ???$

Practice - Coins, Dice, and Cards

Let's work through some (more) examples!

- $p(1 \cup 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 0.33$

Practice - Coins, Dice, and Cards

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- $p(1 \cup 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 0.33$
- $p(1 \cup 2 \cup 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = ???$

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- $p(1 \cap 6) = \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{36} =???$

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- $p(1 \cap 6) = \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{36} = 0.028$

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- $p(1 \cap 6) = \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{36} = 0.028$
- $p(1 \cap 2 \cap 3) = \frac{1}{6} \text{ x } \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{216} =???$

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- $p(1 \cap 6) = \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{36} = 0.028$
- $p(1 \cap 2 \cap 3) = \frac{1}{6} \text{ x } \frac{1}{6} \text{ x } \frac{1}{6} = \frac{1}{216} = 0.004$

Practice - Coins, Dice, and Cards

- $p(\text{ace} \ \cup \ \text{king}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$ =???

Practice - Coins, Dice, and Cards

- $p(\text{ace} \ \cup \ \text{king}) = \dfrac{4}{52} + \dfrac{4}{52} = \dfrac{8}{52} = 0.16$

Practice - Coins, Dice, and Cards

- $p(\text{ace } \cup \text{ king}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.16$
- $p(\text{ace } \cup \text{ diamond}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = ???$

Practice - Coins, Dice, and Cards

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- $p(\text{ace} \ \cup \ \text{diamond}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = 0.31$

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- with replacement: $p(\text{ace } \cap \text{ ace}) = \frac{4}{52} \text{ x } \frac{4}{52} = \frac{16}{2704} = ???$

Practice - Coins, Dice, and Cards

- $p(\text{ace} \ \cup \ \text{king}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.16$
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- with replacement: $p(\text{ace} \ \cap \ \text{ace}) = \frac{4}{52} \text{ x } \frac{4}{52} = \frac{16}{2704} = 0.006$

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- with replacement: $p(\text{ace} \ \cap \ \text{ace}) = \frac{4}{52} \text{ x } \frac{4}{52} = \frac{16}{2704} = 0.006$
- without replacement: $p(\text{ace} \ \cap \ \text{ace}) = \frac{4}{52} \text{ x } \frac{3}{51} = \frac{12}{2652} = ???$

Practice - Coins, Dice, and Cards

- $p(\text{ace} \ \cup \ \text{king}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.16$
- $p(\text{ace} \ \cup \ \text{diamond}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = 0.31$
- with replacement: $p(\text{ace} \ \cap \ \text{ace}) = \frac{4}{52} \text{ x } \frac{4}{52} = \frac{16}{2704} = 0.006$
- without replacement: $p(\text{ace} \ \cap \ \text{ace}) = \frac{4}{52} \text{ x } \frac{3}{51} = \frac{12}{2652} = 0.005$

This Leads Us to…Contingency Tables

A contingency table displays the joint distribution of two variables and is also referred to as a *cross-tab*.

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Here's an example using data from the NLSY97 - a data set we will become very familiar with this semester.

	Employed?		
Delinquent?	No	Yes	Total
No	3642	3046	6688
Yes	955	1291	2246
Total	4597	4337	8934

Computing Probabilities from the Contingency Table

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- $p(\text{D} \mid \text{E}) = \text{ ??? } = 0.2976712$

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And here's another Contingency Table!

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	School Performance				
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Yes	1290	1679	1127	252	4348
Total	3168	3216	1835	330	8549

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- $p(\mathrm{A} \cup \mathrm{B}) = ??? + ??? = \frac{6384}{8549} = 0.747$

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- $p(\text{D} \mid \text{B}) = \ ??? \ = 0.522$

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- $p(\text{D} \mid \text{C}) = \;\; ??? \;\; = 0.614$

Computing Probabilities from the Contingency Table

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- $p(\mathrm{D} \mid \mathrm{C}) = \frac{1127}{1835} = 0.614$
- $p(\mathrm{D} \mid \mathrm{F}) = \;???\; = 0.764$

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This Brings Us to... Probability Distributions!

We need to distinguish a frequency distribution (with proportions) from a probability distribution (with probabilities).

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We need to distinguish a frequency distribution (with proportions) from a probability distribution (with probabilities).

The former is observable - the latter is not. Understanding the latter, though, is **very** important for understanding statistical inference generally.

So, What is A Probability Distribution?

Again, helpful to contrast to a frequency distribution.

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Frequency distribution: Empirical, contains proportions which are *relative frequencies* we observe with real data.

Probability distribution: Theoretical, contains probabilities which are *limiting relative frequencies* we do not observe with real data, but expect over the long run.

What do you think it means to expect something over the *long run*?

A Practical Example - Flipping a Coin

Flipping a single coin: S=[H,T]; $p(H) = .50$; $p(T) = .50$

Outcome	f1	f2	f3	f4	f5
Heads	0	4	45	490	4990
Tails	1	6	55	510	5010
Number of Flips	1	10	100	1000	10000

Probability Distributions - How Are They Helpful?

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This is what forms the link between empirical observations and statistical inference (and, by proxy, causal inference).

An Example - Building a Probability Distribution

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Sex is independent across children: $p(\text{boy}) = 0.52$; $p(\text{girl}) = 0.48$

Building the Probability Distribution

Sequence	Probability
BBB	

Building the Probability Distribution

Sequence	Probability
BBB	$(.52)(.52)(.52) = .141$
BBG	

Building the Probability Distribution

Sequence	Probability
BBB	$(.52)(.52)(.52) = .141$
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BGB	

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BGG	$(.52)(.48)(.48) = .120$
GBG	$(.48)(.52)(.48) = .120$
GGB	$(.48)(.48)(.52) = .120$
GGG	$(.48)(.48)(.48) = .111$

Probabilities for Sex Composition

What is the probability of observing…

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Probabilities for Sex Composition

What is the probability of observing...

- Just 1 girl among three children ($p(1 \text{ girl})$)?
  - $p(1 \text{ girl}) = p(BBG) + p(BGB) + p(GBB) = 0.130 + 0.130 + 0.130 = 0.390$

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- 2 or more boys among three children ($p(2+\text{ boys})$)?
  - $p(2+\text{ boys}) = p(BBB) + p(BBG) + p(BGB) + p(GBB) = 0.141 + 0.130 + 0.130 + 0.130 = 0.531$

The Binomial Distribution

This brings us to the **Binomial Distribution** - a probability distribution for dichotomous outcomes.

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Events are labeled as successes or failures: $p(\text{success}) = p$; $p(\text{failure}) = q = 1 - p$; n = # of independent trials

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We are interested in the probability of observing r successes in n trials - order does not matter.

The Binomial Distribution (cont)

Combines the multiplication rule with the combination rule.

Mutliplication rule tells us the probability a *one* specific sequence. The Combination Rule tells us how many times we would observe that sequence.

For example, having 1 girl out of three kids:

1) $p(GBB) = (0.48)(0.52)(0.52) = 0.48^1 \text{ x } 0.52^2 = 0.130$

2) $C^3_1 = \dfrac{3!}{1!(3-1)!} = \dfrac{3!}{2!} = 3$

3) $p(1 \text{ girl}) = 3 \text{ x } 0.130 = 0.390$

The Binomial Distribution (cont)

Formally, the combined mathematics:

$$p(r) = C_r^n p^r q^{n-r} = \binom{n}{r} p^r q^{n-r}$$

We can then construct the full probability distribution using the above equation, where:

1) Success = the birth of a girl
2) N = the # of trials = 3
3) p = p(success) = 0.48
4) q = p(failure) = 1-0.48 = 0.52

Sex Composition Probability Distribution

#of Girls (r)	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1		
Total			

Sex Composition Probability Distribution

#of Girls (r)	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.48^0$ x $.52^3 = .141$	
Total			

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#of Girls (r)	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.48^0$ x $.52^3 = .141$	1 x $.141 = .141$
1	3		
Total			

Sex Composition Probability Distribution

#of Girls (r)	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.48^0$ x $.52^3 = .141$	1 x $.141 = .141$
1	3	$.48^1$ x $.52^2 = .130$	
Total			

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0	1	$.48^0$ x $.52^3 = .141$	1 x $.141 = .141$
1	3	$.48^1$ x $.52^2 = .130$	3 x $.130 = .390$
2	3		
Total			

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1	3	$.48^1$ x $.52^2 = .130$	3 x $.130 = .390$
2	3	$.48^2$ x $.52^1 = .120$	3 x $.120 = .360$
3	1		
Total	8		

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#of Girls (r)	C^n_r	$p^r q^{n-r}$	$p(r)$
0	1	$.48^0$ x $.52^3 = .141$	1 x $.141 = .141$
1	3	$.48^1$ x $.52^2 = .130$	3 x $.130 = .390$
2	3	$.48^2$ x $.52^1 = .120$	3 x $.120 = .360$
3	1	$.48^3$ x $.52^0 = .111$	
Total	8		

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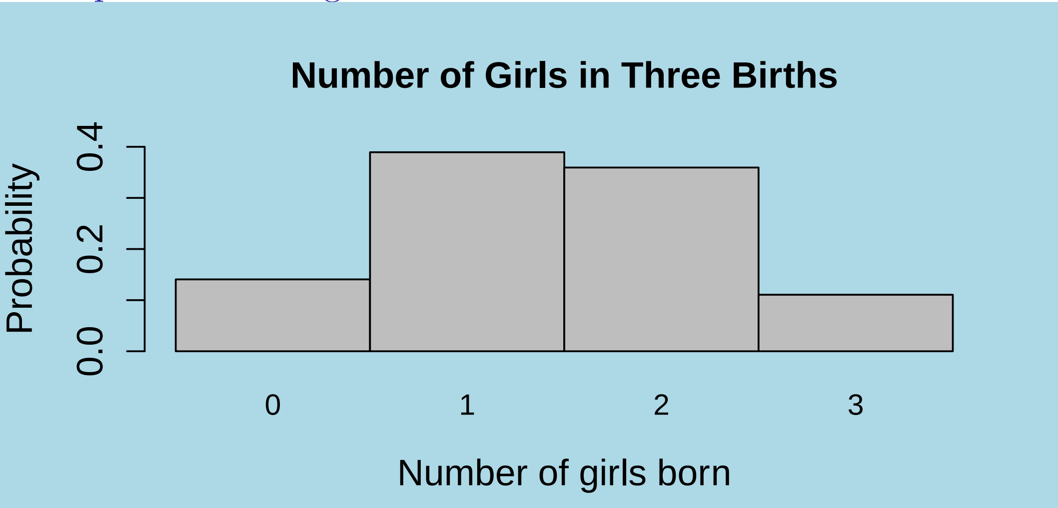
#of Girls (r)	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.48^0$ x $.52^3 = .141$	1 x $.141 = .141$
1	3	$.48^1$ x $.52^2 = .130$	3 x $.130 = .390$
2	3	$.48^2$ x $.52^1 = .120$	3 x $.120 = .360$
3	1	$.48^3$ x $.52^0 = .111$	1 x $.111 = .111$
Total	8		1.002

Sex Composition Histogram



Further Examples - Running Red Lights

Let's explore another example - running a red light.

Observations at a dangereous intersection in Charlotte indicate that cars passing through the intersection run the red light with a 0.60 probability.

That is, 6 out of 10 cars passing through the intersection will run a red light.

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1		
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5		
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10		
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10		
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10	$.6^3$ x $.4^2 = .035$	
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10	$.6^3$ x $.4^2 = .035$	10 x $.035 = .350$
4	5		
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10	$.6^3$ x $.4^2 = .035$	10 x $.035 = .350$
4	5	$.6^4$ x $.4^1 = .052$	
Total			

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10	$.6^3$ x $.4^2 = .035$	10 x $.035 = .350$
4	5	$.6^4$ x $.4^1 = .052$	5 x $.052 = .260$
5	1		
Total	32		

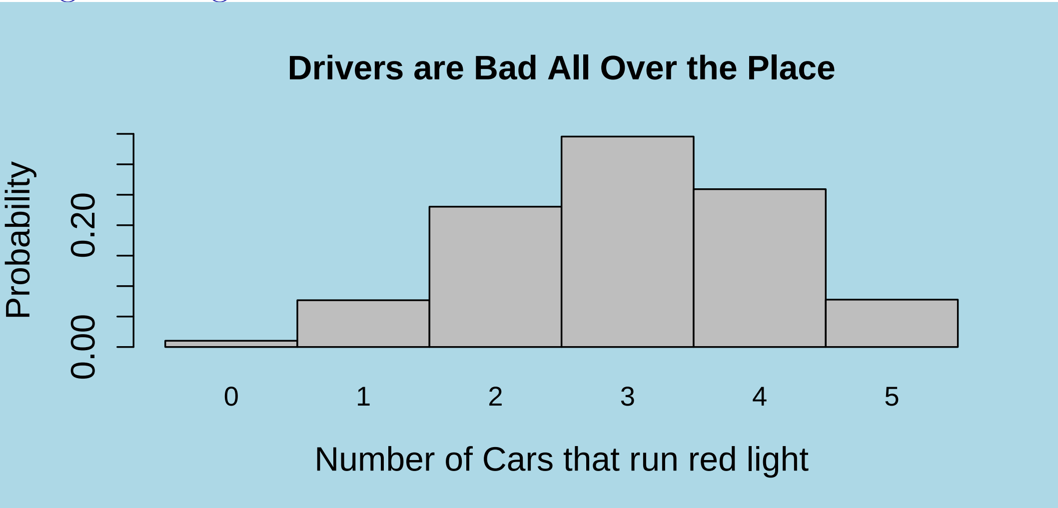
Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x $.010 = .010$
1	5	$.6^1$ x $.4^4 = .015$	5 x $.015 = .075$
2	10	$.6^2$ x $.4^3 = .023$	10 x $.023 = .230$
3	10	$.6^3$ x $.4^2 = .035$	10 x $.035 = .350$
4	5	$.6^4$ x $.4^1 = .052$	5 x $.052 = .260$
5	1	$.6^5$ x $.4^0 = .078$	
Total	32		

Probability Distribution for Running a Red Light

r	C_r^n	$p^r q^{n-r}$	$p(r)$
0	1	$.6^0$ x $.4^5 = .010$	1 x .010 = .010
1	5	$.6^1$ x $.4^4 = .015$	5 x .015 = .075
2	10	$.6^2$ x $.4^3 = .023$	10 x .023 = .230
3	10	$.6^3$ x $.4^2 = .035$	10 x .035 = .350
4	5	$.6^4$ x $.4^1 = .052$	5 x .052 = .260
5	1	$.6^5$ x $.4^0 = .078$	1 x .078 = .078
Total	32		1.003

Red Light Histogram



Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	$.000$
1	$(10)(.6^1)(.4^9) = .002$	

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2		

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6		

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7		

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8		

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	.954
9		

Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	.954
9	$(10)(.6^9)(.4^1) = .040$	

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	.954
9	$(10)(.6^9)(.4^1) = .040$	.994
10		

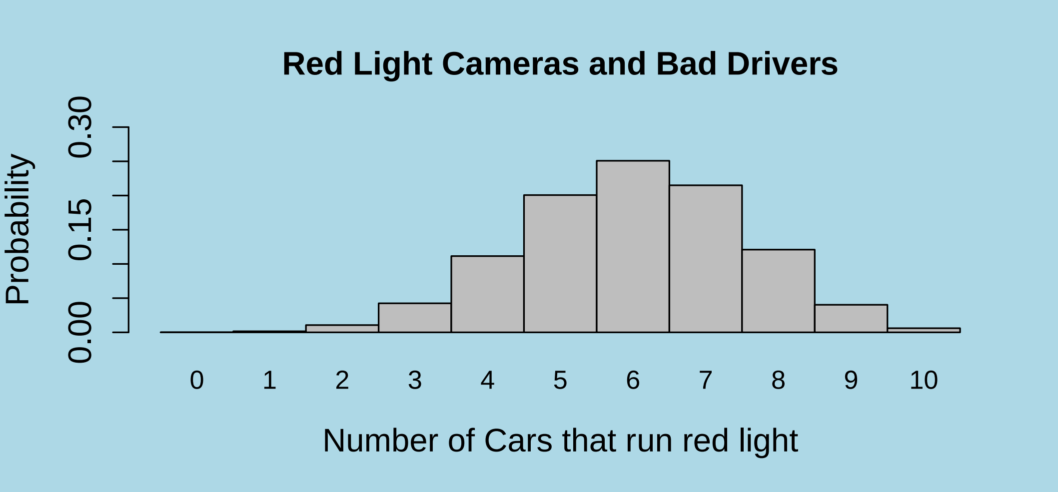
Running Red Lights - Extending the Example

r	$p(r) = C_r^n p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	.954
9	$(10)(.6^9)(.4^1) = .040$	.994
10	$(1)(.6^{10})(.4^0) = .006$	

Running Red Lights - Extending the Example

r	$p(r) = C^n_r p^r q^{n-r}$	cp
0	$(1)(.6^0)(.4^{10}) = .000$	.000
1	$(10)(.6^1)(.4^9) = .002$	.002
2	$(45)(.6^2)(.4^8) = .011$	.013
3	$(120)(.6^3)(.4^7) = .042$	.055
4	$(210)(.6^4)(.4^6) = .111$	.166
5	$(252)(.6^5)(.4^5) = .202$	.368
6	$(210)(.6^6)(.4^4) = .250$	.618
7	$(120)(.6^7)(.4^3) = .215$	.833
8	$(45)(.6^8)(.4^2) = .121$	.954
9	$(10)(.6^9)(.4^1) = .040$	.994
10	$(1)(.6^{10})(.4^0) = .006$	1.000

Running Red Lights - Extended Histogram



Cal Ripken

Here's one for the baseball fans.

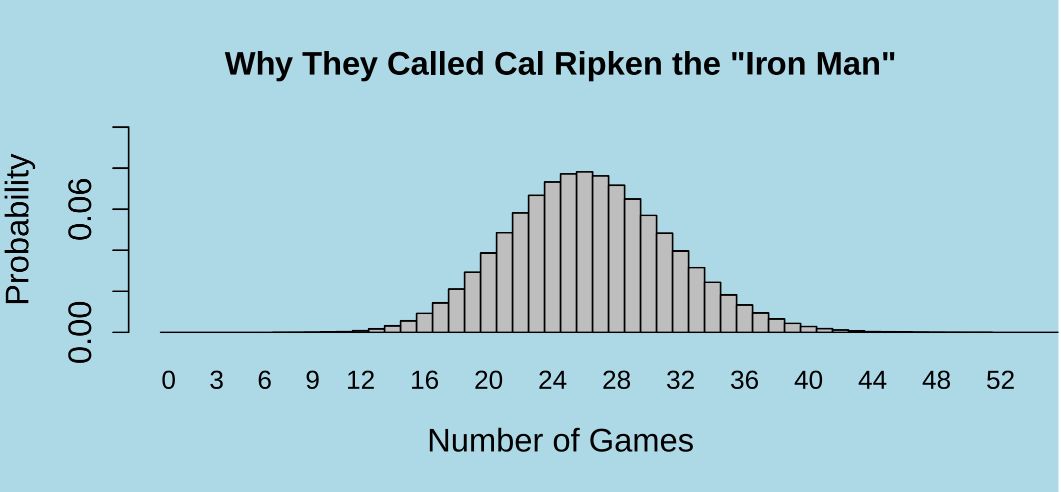
Let's suppose a baseball player has only 1/100 a chance to miss any given game due to injury.

Cal Ripken played an astonishing 2632 *straight* games without missing due to injury.

Just how **unlikely** are we to observe such a feat?

Stated differently, what is the probability a player will miss at least one game ($p(r \geq 1)$) due to illness/injury?

Cal Ripken Histogram



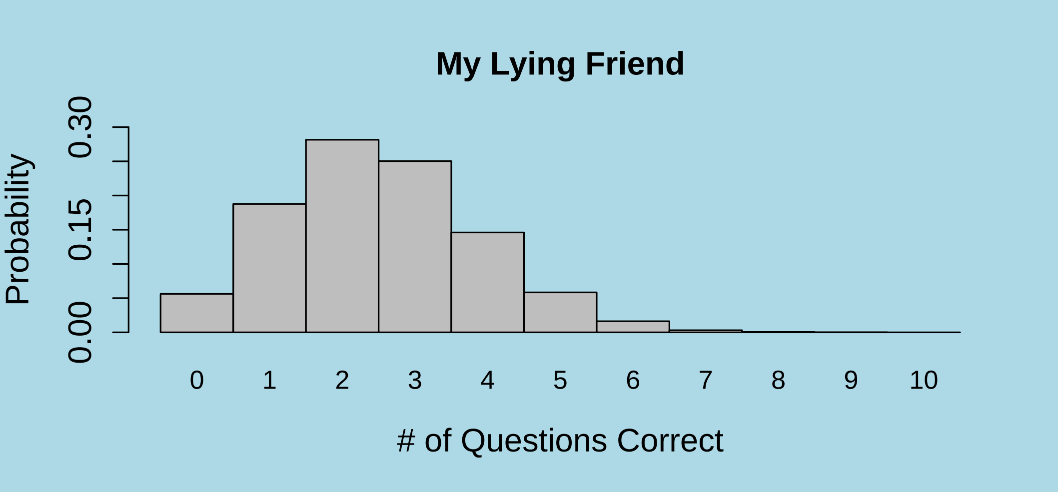
A Final Example - Your Roommate is A Liar

Set the stage - a 10-question multiple choice test with 4 answers per question.

Your roommate scored an 8 and said that she randomly guessed for each question.

What is the probability that she would actually guess correctly on 8 or more ($p(r \geq 8)$) questions?

Your Lying Roommate - A Histogram



Two Questions

Time for your two questions before the end of class!